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ANTENNA-PLASMA COUPLING THEORY FOR ICRF HEATING OF LARGE TOKAMAKS*

A. Ram and A. Bers

March 1982

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ANTENNA-PLASMA COUPLING THEORY FOR ICRF HEATING OF LARGE TOKAMAKS

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A number of experiments (TFR, PLT) have reported significant heating of ions in a plasma by waves in the ion-cyclotron range of frequencies (ICRF). These waves are excited in the plasma by an antenna structure consisting of a current carrying conductor near the plasma wall in the shadow of the limiter. An effective coupling of the external radiofrequency (rf) power to the plasma is achieved when these rf current carrying conductors are shielded from the plasma by a metal screen which locally shorts out the toroidal component of the electric field leaving the poloidal component unaffected. The screen also prevents the plasma from penetrating to the rf conductor. In this paper we study the coupling characteristics of such an antenna structure by analysing a model where a thin current sheet is placed between a fully conducting wall and a sheet of anisotropic conductivity representing the screen. The inhomogeneous plasma in the shadow of the limiter is assumed to extend from the screen onwards away from the antenna. The excitation of the fields inside the plasma are found by analysing the radiation properties of this current sheet antenna. We assume that the current distribution of the antenna is given and that the fields excited inside the plasma are absorbed in a single pass. In all experiments to-date the cross-sectional plasmas are relatively small so that the rf conductor is a half-loop around the plasma in the poloidal direction. However, for reactor size plasmas this cannot be done and the antenna dimensions will be small compared to the plasma cross-sections. We, thus, assume an antenna of finite poloidal and toroidal extent with dimensions small compared to the plasma minor radius. We further approximate the coupling geometry by a slab model (Figure 1). The x-axis is taken to be along the plasma inhomogeneity, the y-axis along the poloidal direction and the z-axis along the toroidal magnetic field.

The fields in the vacuum and the plasma are Fourier transformed in y and z and are taken to have a dependence of the form $\exp(ik_y y + ik_z z - i\omega t)$. The plasma in the coupling region near the screen is assumed to be described by its cold dielectric tensor:

$$\begin{pmatrix} K_\perp & -iK_X & 0 \\ iK_X & K_\perp & 0 \\ 0 & 0 & K_\parallel \end{pmatrix} \tag{1}$$

where

$$K_\perp = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} - \sum_i \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} \tag{2}$$

$$K_X = \frac{\omega_{pe}^2}{\omega_{ce}^2 - \omega^2}\frac{\omega_{ce}}{\omega} + \sum_i \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}\frac{\omega_{ci}}{\omega} \tag{3}$$

$$K_\perp = 1 - \frac{\omega_{pe}^2}{\omega^2} - \sum_i \frac{\omega_{pi}^2}{\omega^2} \tag{4}$$

the summation being over the ion-species, ω_c is the cyclotron frequency, ω_p is the plasma frequency and, ω is the external frequency. The inhomogeneity in the toroidal magnetic field is introduced through ω_c and that in the density through ω_p. The cold plasma dispersion relation leads to the existence of two components of the ICRF wave - the slow and the fast modes. These waves decouple as $|K_\parallel| \gg |K_\perp|,\ K_X$ inside the plasma. The slow wave has a resonance at the point where $|K_\perp| = 0$ which occurs at low densities near the edge of the plasma. In order to avoid the coupling to the slow wave we choose the density at the screen to be such that the slow wave is not excited inside the plasma. This density also ensures that $|K_\parallel| \gg |K_\perp|,\ K_X$ from the screen onwards into the plasma. So we consider only the coupling to the fast wave. The fast wave z-component of the electric field in the plasma E_z^p is zero. The other two components of the electric field inside the plasma are given by[1]:

$$E_x^p = \frac{i}{K_\perp - n_y^2 - n_z^2}\left(n_y \frac{c}{\omega}\frac{dE_y^p}{dx} + K_X E_y^p\right) \tag{5}$$

$$\frac{d^2E_y^p}{dx^2} + f(x)\frac{dE_y^p}{dx} + g(x)E_y^p = 0 \tag{6}$$

where $n_{y,z} = ck_{y,z}/\omega$

$$f(x) = \frac{-n_y^2}{(n_z^2 - K_\perp)(n_y^2 + n_z^2 - K_\perp)}\frac{dK_\perp}{dx} \tag{7}$$

$$g(x) = \frac{\omega^2}{c^2}\left\{K_\perp - n_z^2 - n_z^2 - \frac{n_y(c/\omega)(dK_X/dx) - K_X^2}{n_z^2 - K_\perp} - n_y \frac{K_X}{(n_z^2 - K_\perp)(n_y^2 + n_z^2 - K_\perp)}\frac{c}{\omega}\frac{dK_\perp}{dx}\right\} \tag{8}$$

Previously, the field solutions for finite n_y were examined analytically for the Eqs. (5-8) with all the derivatives of the dielectric tensor elements set to zero [2]. In this paper we give the results of solving (numerically) Eqs. (5-8) without neglecting these derivatives. Only those solutions which give power flow into the plasma are considered.

The fields in the free space region between the wall and screen, and containing the current sheet, are described by a superposition of the full set of transverse-electric (TE) and transverse-magnetic (TM) modes. (The TEM fields related to the feed of the antenna sheet are ignored). These fields are given by:

$$E_x^v = \frac{\beta^2}{\omega\epsilon_0}(H_+^s e^{-\gamma x} + H_-^s e^{\gamma x}) \tag{9}$$

$$E_y^v = -\frac{i\gamma}{\omega\epsilon_0}k_y(H_+^s e^{-\gamma x} - H_-^s e^{\gamma x}) + k_z(E_+^s e^{-\gamma x} + E_-^s e^{\gamma x}) \tag{10}$$

$$E_z^v = -\frac{i\gamma}{\omega\epsilon_0}k_z(H_+^s e^{-\gamma x} - H_-^s e^{\gamma x}) - k_y(E_+^s e^{-\gamma x} + E_-^s e^{\gamma x}) \tag{11}$$

$$s = I, II\,; \quad \beta^2 = k_y^2 + k_z^2\,, \quad \gamma^2 = \beta^2 - \frac{\omega^2}{c^2}$$

where $s = I$ refers to the vacuum region between the wall and the antenna, and $s = II$ to the region between the antenna and the screen. Setting $H_+^s = H_-^s = 0$ gives the TE mode and setting $E_+^s = E_-^s = 0$ gives the TM mode. We assume that the antenna is carrying a current in the y direction at frequency ω with the current density given by:

$$\bar{J} = K_0\,\delta(x)\,F(y)\,G(z)\;\hat{y} \tag{12}$$

where K_0 is the surface current density amplitude and $F(y)$ and $G(z)$ are the dimensionless profile factors of the current. Letting E_{y0} be the complex amplitude of the electric field component E_y^p inside the plasma, the constants H_+^s, H_-^s, E_+^s, E_-^s, and E_{y0} can be determined by satisfying the boundary conditions at the wall, antenna and the screen.

The radiation impedance of the antenna is obtained by applying the complex Poynting theorem at the surface of the antenna [2].

$$Z_A = \frac{1}{(2\pi)^2}\frac{2}{|I|^2}\int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} dk_z \left(\frac{1}{2}E_y H_z^*\right)_{x=0} = R_A + iX_A \tag{13}$$

$$\text{where} \quad I = K_0 \int G(z)\,dz \quad \text{is the current}$$

On matching the boundary conditions at the conducting wall, antenna, and the screen the fields at the antenna can be expressed in terms of the fields inside the plasma. Then, the complex impedance can be expressed as:

$$Z_A = \frac{|K_0|^2}{(2\pi)^2|I|^2}\int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} dk_z\,|\tilde{F}(k_y)\,\tilde{G}(k_z)|^2\,Z_k \tag{14}$$

where the intrinsic impedance, Z_k, is given by

$$Z_k = \sqrt{\frac{\mu_0}{\epsilon_0}}\frac{e^{2\gamma a} - 1}{2(e^{2\gamma(a+b)} - 1)}\left[(e^{2\gamma b} - 1)\frac{i\omega}{c\gamma}(n_y^2 - 1) - 2\sqrt{\frac{\epsilon_0}{\mu_0}}\left(\frac{\tilde{E}_y^p(x=b)}{K_0}\right)e^{\gamma b}\right] \tag{15}$$

$\tilde{F}(k_y)$, $\tilde{G}(k_z)$ are the Fourier transforms of $F(y)$, $G(z)$ respectively, and $\tilde{E}_y^p(x) = E_y^p(x)/\{\tilde{F}(k_y)\tilde{G}(k_z)\}$

To determine $\tilde{E}_y^p$ we solve the Eqs. (5-8) by assuming an outgoing wave at a short distance away from the screen inside the plasma. Then the differential equation for E_y^p (Eq. (6)) is integrated backwards in x to determine $\tilde{E}_y^p$ at the screen ($x = b$).

The first term in the expression of Z_k is purely imaginary and contributes only to the reactive part of the impedance. This would be the radiation impedance of the antenna if the screen were to be replaced by a perfectly conducting wall. If we choose the antenna spectrum to be uniform in y and z, this reactive part has a logarithmic singularity as $k_y \to \infty$. This singularity is a consequence of the infinite charge required at the antenna ends in y to maintain the assumed uniform current. The experimental situation is closer to that shown in Figure (2) where the feed lines to the antenna are included. Here the current flows smoothly through the antenna and there is no singularity. We approximate this situation by having the antenna in a finite sized rectangular box with fully conducting walls whose y dimension is the same as the length of the antenna in y and whose z dimension is larger than the antenna length in z (Figure 3). Then the singularity at the y-ends of the antenna is removed and this should give a good approximation to the reactive loading of the antenna in an actual situation. This particular problem can again be solved in terms of the free-space modes of the box. The reactive part of the antenna impedance is found to be:

$$X_A = \frac{\omega\mu_0|K_0|^2}{|I|^2 d\, L_y} \sum_{m,n=0}^{\infty} \left(\frac{4m^2\pi^2}{L_y^2 k_0^2} - 1\right) \frac{1}{\bar{\alpha}} \frac{(e^{2\bar{\alpha} b} - 1)(e^{2\bar{\alpha} a} - 1)}{e^{2\bar{\alpha}(a+b)} - 1} |F_m|^2 |G_n|^2 \tag{16}$$

$$\text{where} \quad \bar{\alpha}^2 = \frac{4m^2\pi^2}{L_y^2} + \frac{(2n+1)^2}{d^2}\frac{\pi^2}{4} - k_0^2, \qquad k_0 = \frac{\omega}{c}$$

$$F_m = \int_{-\frac{L_y}{2}}^{\frac{L_y}{2}} dy\, F(y) \cos\left(\frac{2m\pi y}{L_y}\right), \qquad G_n = \int_{-d}^{d} dz\, G(z) \cos\left\{\frac{(2n+1)\pi}{2d} z\right\}$$

and, $2d(> L_z)$ is the length of the box in the z-direction. If we now assume a uniform current profile, then F_m is nonzero for $m = 0$ only. Thus, the double sum in Eq. (16) is reduced to just a single sum over n which converges rapidly to give a finite contribution to the reactive impedance. The contribution to the total reactive impedance from the second term of Eq. (15) is found to be very small compared to the reactive impedance evaluated from Eq. (16). The resistive part of the antenna impedance is just the real part of the second term of Eq. (15).

The power flow into the plasma is given by

$$P_{PL} = \mathrm{Re}\left[\frac{1}{2}\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} dk_z \;\; E_y^p H_z^{p*}\right] \tag{17}$$

As an example, we apply the above results to a tokamak plasma consisting of just deuterium and electrons with the following parameters: major radius = 3 m., minor radius (to the screen) = 1.27 m., density at the screen = 2×10^{11} cm^{-3}, peak density (at center) = 5×10^{13} cm^{-3}, toroidal magnetic field = 35 kG, $\omega = 2\omega_{CD} = 3.35 \times 10^8$ sec^{-1}, $a = 5$ cm, $b = 3$ cm, $L_y = 60$ cm, $L_z = 40$ cm, $d = 25$ cm and a uniform current profile in y and z. With the above choice of edge density we make sure that the slow wave is not excited at the edge. For a parabolic density profile the real part of the integrand of Eq. (14) is plotted as a function of n_y and n_z in Figure (4). We find that the major contribution to the resistive impedance of the antenna comes from the region $n_y \neq 0$. The total resistive impedance is calculated to be $R_A = 9.58\ \Omega$. If we set $n_y = 0$ in Eqs. (5-8) then we would get a resistive impedance of 6.7 Ω/meter of antenna length in y. For an antenna with $L_y = 60$ cm, this would give a resistive impedance of 4 Ω. This is less than half the value obtained when we take into account the appropriate n_y spectrum. In our previous work[2] where we accounted for the n_y spectrum, but the derivatives of the dielectric tensor elements were set to zero, a total resistive impedance of 21.0 Ω was obtained for the same parameters and profiles; this is more than twice the value obtained from the exact theory which does not set the derivatives of the dielectric tensor elements to zero.

From our results we find that the major contribution to R_A comes from the $n_y < 0$ part of the spectrum. *We, thus, conclude that a set of poloidal antennae phased to excite a current spectrum biased towards $n_y < 0$ would couple more effectively to the plasma.* For the above parameters, we also find that $(2P_{PL}/I^2)$ of Eq. (17) is 9.4 Ω. Comparing this to $R_A = 9.58\ \Omega$ we find that almost all of the power emitted from the antenna is going into the plasma. The reactive impedance from Eq. (16) is calculated to be $X_A = 22.5\ \Omega$. This reactive impedance is completely inductive.

If we keep all parameters fixed as above but change the density profile to a gaussian, we get $R_A = 18.3\ \Omega$. Hence, for the same edge density a more gradual increase in the density from the edge leads to better coupling.

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[1] A. Bers, J. Jacquinot, G. Lister, 2nd Joint Grenoble- Varennes Int. Symp., Como, Italy, Sept. 1980 and Report EUR-CEA-FC-1066 Fontenay-aux-Roses, France, Sept. 1980.

[2] A. Bers, L. Harten, A. Ram, Proceedings of the Fourth Topical Conference on RF Plasma Heating, 1981, Austin, Texas, pp. A16-1–A16-4.

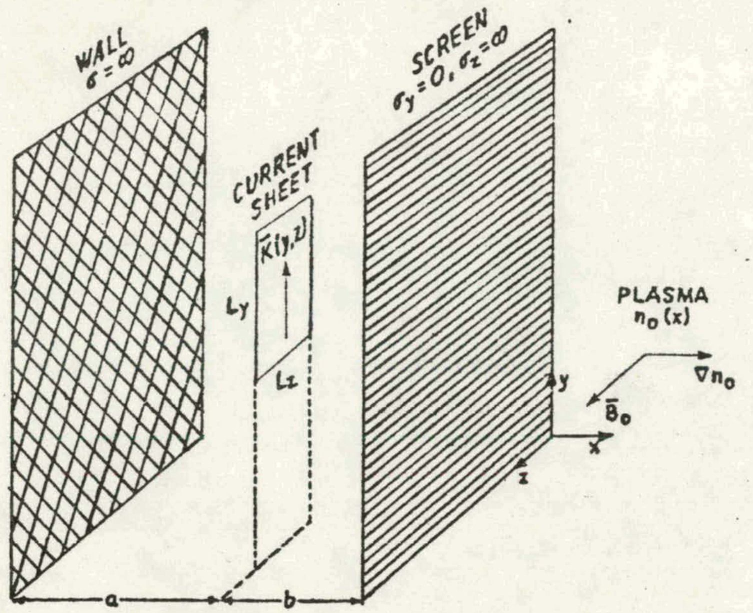

FIGURE I

Slab geometry model used for the analyses.

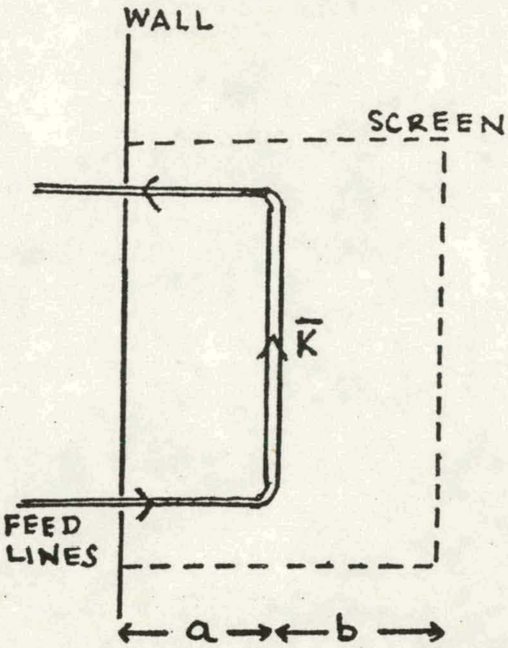

FIGURE 2

Schematic representation of the actual coupling structure.

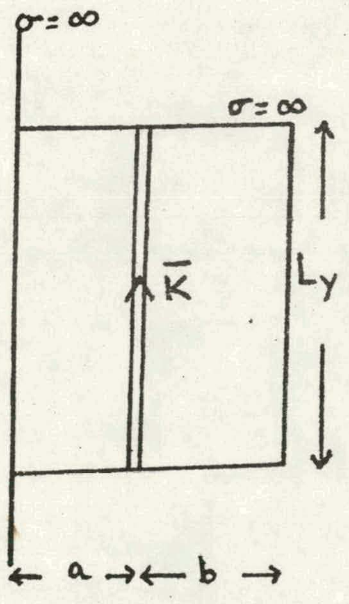

FIGURE 3

Geometry used for evaluating the reactive impedance of the antenna. The dimension in z is 2d.

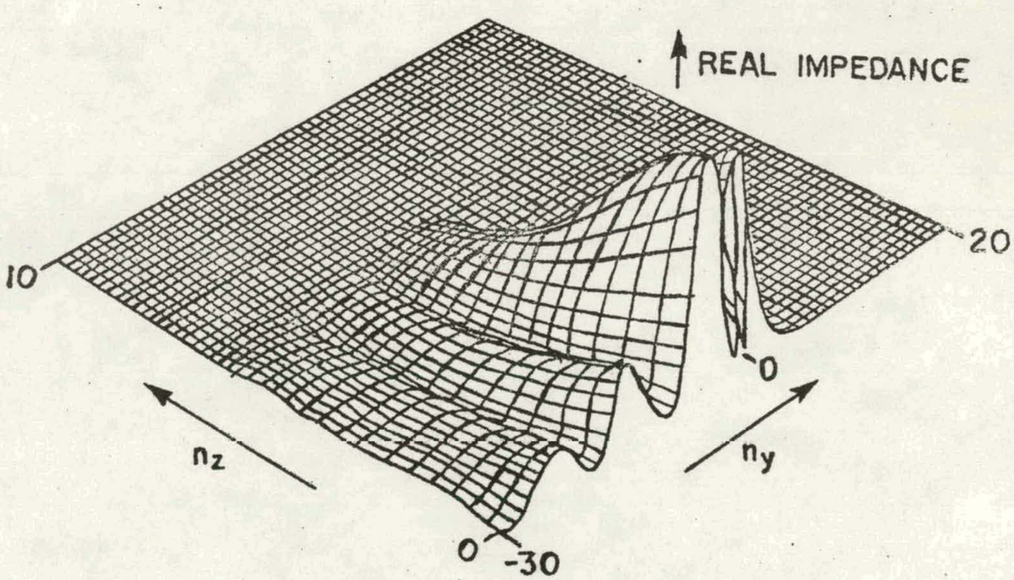

FIGURE 4

Real part of the antenna impedance plotted as a function of n_y and n_z.

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