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Cabibbo Current and CP Violation in a Six Quark Gauge Model*

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Abstract

The extended Cabibbo current in a six quark gauge model is obtained in terms of Eulerian angles θ_c, φ and ψ which are functions of quark mass ratios and phases in the quark mass matrices. Particular attention is payed to arbitrary phases that lead to CP violation. In the limit $m_u/m_c \to 0$ there is no CP violation for $|\Delta S| = 1$ processes. Some phenomenological results of the model are discussed.

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DISCLAIMER

Recently several authors have computed the Cabibbo angle in terms of quark mass ratios for gauge models supplemented by various discrete symmetries.[1-4] It now appears relevant to extend such models to incorporate the effects of heavier quarks (t or b) suggested by the discovery of the Υ(9.5 GeV).[5] In this note we present an extended Cabibbo current in a six quark $SU(2)_L \times SU(2)_R \times U(1)$ gauge model which depends on Eulerian angles θ_c, φ, and ψ. Our purpose is to express the CP violation and Eulerian angles in terms of quark mass ratios and phases which appear in the quark mass matrix. CP invariance is violated for $|\Delta S| = 1$ processes due to a phase difference between the left- and right-handed currents. Some phenomenological consequences of the present model are discussed.

A general quark mass matrix can be generated by a quark-Higgs Yukawa interaction of the form

$$H = \sum_{i,j=1,2,3} g_{ij} \bar{Q}_{iR} \phi_{ij} Q_{jL} + \text{h.c.} , \tag{1}$$

where the Higgs scalars ϕ_{ij} transform as $(\frac{1}{2},\frac{1}{2},0)$. The $Q_{iL,R}$ represent the quark doublets $Q_{1L,R} = (u_o,d_o)_{L,R}$, $Q_{2L,R} = (c_o,s_o)_{L,R}$ and $Q_{3L,R} = (t_o,b_o)_{L,R}$. The coupling constants g_{ij} are taken to be real so that CP invariance is violated spontaneously. Parity violation is due to the inequality of the left- and right-handed intermediate boson masses $M_{WL} < M_{WR}$. We require that the interaction be invariant under left-right symmetry $Q_{iL,R} \to Q_{iR,L}$, $\phi_{ij} \to \phi_{ij}^\dagger$ and further require invariance under the following discrete symmetry:

$$\begin{array}{lll} Q_{1L} \to i Q_{1L} & Q_{1R} \to -i Q_{1R} , & Q_{2L,R} \leftrightarrow Q_{3L,R} , \\ \phi_{11} \to \phi_{11} , & \phi_{22} \leftrightarrow \phi_{33} , & \phi_{ij} \to -i \phi_{ij} \ (i \neq j) \end{array} \tag{2}$$

This discrete symmetry is one of the simplest for left-right symmetric models if one requires the GIM-mechanism[6] for the case $m_{b,t} \gg m_{u,d,s,c}$ and further requires that mixing angles be expressible in terms of quark mass ratios and phases.

After spontaneous symmetry breaking, the quark mass terms can be written in the form $\sum_{i=1,2} \bar{\psi}^{o}_{iR}\, m^{o}_{i}\, \bar{\psi}^{o}_{iL}$ + h.c., where $\psi^{o}_{1L,R} = (u_o, c_o, t_o)_{L,R}$, $\psi^{o}_{2L,R} = (d_o, s_o, b_o)_{L,R}$ and

$$m_i^o = \begin{pmatrix} 0 & \epsilon_i e^{i\delta_i} & \epsilon_i e^{i\delta_i} \\ \epsilon_i e^{i\delta_i} & \beta_i & 0 \\ \epsilon_i e^{i\delta_i} & 0 & b_i e^{i\gamma_i} \end{pmatrix}. \tag{3}$$

The quantities ϵ_i, β_i, b_i, δ_i and γ_i are real and are determined by the Higgs potential which we do not discuss here. Since the vacuum expectation value of one of the Higgs scalars can be chosen to be real, we take β_i to be real without loss of generality. The mass eigenstates are expressible in terms of the original states by a bi-unitary transformation[7] as $\psi_{iR} = P_i^\dagger V_i^\dagger \psi^o_{iR}$, $\psi_{iL} = P_i^\dagger U_i^\dagger \psi^o_{iL}$. The unitary matrix U is approximated to order ϵ_i/b_i and β_i/b_i because of the relation $m_{t,b} \gg m_{u,d,s,c}$ and is given by

$$U = \begin{pmatrix} \cos\theta\, e^{-i\delta} & \sin\theta\, e^{-i\delta} & (\epsilon/b)\, e^{-i(\delta-\frac{\gamma}{2})} \\ -\sin\theta & \cos\theta & 0 \\ -(\epsilon/b)\cos\theta\, e^{-i\gamma} & -(\epsilon/b)\sin\theta\, e^{-i\gamma} & e^{-i\gamma/2} \end{pmatrix}, \tag{4}$$

where the subscripts are suppressed. The unitary matrix V is given by $V = U^* K$ where the diagonal matrix K with elements $K_{11} = -K_{22} = -K_{33} = -1$ makes the masses of the u-quark and d-quark positive. The angle θ_i is given by

$$\tan 2\theta_i = 2\epsilon_i/\beta_i\,, \quad \tan^2\theta_1 = m_u/m_c\,, \quad \tan^2\theta_2 = m_d/m_s\,, \tag{5}$$

$$\epsilon_1 = (m_u m_c)^{1/2}\,, \quad \epsilon_2 = (m_d m_s)^{1/2}\,, \quad b_1 = m_t\,, \quad b_2 = m_b\,.$$

The diagonal phase matrices P_i will be chosen so that there are no relative phases in the u, d, s, c sector of the left-handed weak current.

When expressed in terms of the mass eigenstates, the extended left-handed Cabibbo current $J_{\mu L}$ takes the form

$$J_{\mu L} = \bar{\Psi}^{o}_{1L} \gamma_\mu \Psi^{o}_{2L} = \bar{\Psi}_{1L} \Gamma^{c}_{L} \gamma_\mu \Psi_{2L},$$

where[7]

$$\Gamma^{c}_{L} = P_1^\dagger U_1^\dagger U_2 P_2 = \begin{pmatrix} \cos\theta_c & \sin\theta_c & -\psi\sin\theta_c \\ -\sin\theta_c & \cos\theta_c & -\psi\cos\theta_c - \varphi e^{i\delta_c} \\ -\varphi\sin\theta_c & \varphi\cos\theta_c + \psi e^{i\delta_c} & e^{i\delta_c} \end{pmatrix}, \tag{6}$$

$$\begin{aligned} &\sin\theta_c\, e^{i\eta} = \sin\theta_2 \cos\theta_1 - \cos\theta_2 \sin\theta_1\, e^{-i\delta}, \\ &\cos\theta_c\, e^{i\xi} = \cos\theta_2 \cos\theta_1\, e^{-i\delta} + \sin\theta_2 \sin\theta_1, \\ &\varphi\sin\theta_c = k\cos\theta_2, \quad \psi\sin\theta_c = -k\cos\theta_1, \\ &k e^{i\omega} = (m_d m_s/m_b^2)^{1/2} e^{-i(\gamma-\delta)} - (m_u m_c/m_t^2)^{1/2}, \\ &\delta_c = \xi + \delta, \quad \gamma = \gamma_2 - \gamma_1, \quad \delta = \delta_2 - \delta_1 . \end{aligned} \tag{7}$$

The Eulerian angles θ_c, φ and ψ in Γ^c_L are expressed in terms of the quark mass ratios and the phases δ and γ with the aid of Eq.(5). The phase δ_c parametrizes the CP violation. If $\tan\theta_1 = (m_u/m_c)^{\frac{1}{2}} \ll \tan\theta_2 = (m_d/m_s)^{\frac{1}{2}}$, then $\delta_c \approx 0$, so that CP violation is suppressed in the left-handed weak current. The Cabibbo angle θ_c ranges from $\theta_{c\,min} = \theta_2 - \theta_1$ to $\theta_{c\,max} = \theta_2 + \theta_1$ as δ varies from 0 to π. If we take the quark mass ratios $m_u/m_c = 1/320$ and $m_d/m_s = 1/20.5$ as given in Ref.1, $\theta_{c\,min} = 9.2°$, $\theta_{c\,max} = 15.7°$. It is interesting to observe that $\theta_c = 12.8°$ for $\delta = \pi/2$, which is close to the experimental value $\theta_c = 13°$. A rough estimate of the order of the magnitudes of the other angles is $|\varphi| \approx |\psi| \approx 10^{-1}$ rad, with the masses (MeV) $m_u \approx m_d \approx 10$, $m_s \approx 10^2$, $m_c \approx 10^3$ $m_b \approx 5 \times 10^3 \leqslant m_t$.

The right-handed Cabibbo current is given by

$$J_{\mu R} = \bar{\Psi}^o_{1R} \gamma_\mu \Psi^o_{2R} = \bar{\Psi}_{1R} \Gamma^c_R \gamma_\mu \Psi_{2R},$$

where[8]

$$\Gamma^c_R = P_1^\dagger V_1^\dagger V_2 P_2 = P_1^\dagger K U_1^T U_2^* K P_2 = (P_1^*)^2 K \Gamma_L^{c*} K (P_2)^2, \tag{8}$$

so that Γ^c_R is expressible in terms of Γ^{c*}_L. In view of the fact that the right-handed current interaction is suppressed by a factor $\left(M_{WL}/M_{WR}\right)^2$ relative to the left-handed current interaction, Γ^c_R is presented only to the zeroth-order in φ and ψ.

$$\Gamma^c_R = \begin{pmatrix} \cos\theta_c\, e^{-2i\xi} & -\sin\theta_c\, e^{2i(\eta+\delta)} & O(\psi) \\ \sin\theta_c\, e^{-2i\eta} & \cos\theta_c\, e^{2i(\xi+\delta)} & O(\varphi,\psi) \\ O(\varphi) & O(\varphi,\psi) & e^{i(\xi+\delta+\gamma)} \end{pmatrix}. \tag{9}$$

The Cabibbo angle for the right-handed current has the opposite sign to that of the left-handed current in the u, d, c, s sector. CP-invariance is violated for $|\Delta S| = 1$ nonleptonic decays: CP-violation $K \to 2\pi$ decay amplitudes satisfy[9,10] $\eta_{+-} = \eta_{oo}$ and are given by

$$|\eta_{+-}| \sim 2\left|\sin[2(\xi+\eta+\delta)]\right|(M_{WL}/M_{WR})^2 . \tag{10}$$

In the limit $\theta_1 = 0$, $\theta_c = \theta_2$ and there is no CP-violation by the right-handed current since $\delta_c = \xi + \delta = 0$ and $\eta = 0$ [see Eq.(7)]. As $\theta_1 = \tan^{-1}(m_u/m_c)^{\frac{1}{2}}$ is much smaller than $\theta_2 = \tan^{-1}(m_d/m_s)^{\frac{1}{2}}$, Eq.(10) can be written with the aid of (7)

$$|\eta_{+-}| \sim 4\left|\sin\delta/(\sin\theta_c \cos\theta_c)\right|(m_u/m_c)^{1/2}(M_{WL}/M_{WR})^2 . \tag{11}$$

Therefore, the data $|\eta_{+-}| \sim 2 \times 10^{-3}$ suggests that $M_{WR}/M_{WL} \leqslant 22$. In the same approximation, the CP-violating amplitude for the $|\Delta S| = 1$ process mediated by the left-handed current is given by $2k^2 \sin(\xi + \eta + \delta)$ and is negligible.

A few remarks are in order in regard to the weak interaction of b- and t-quarks in this model. We assume $m_b < m_t$, then the weak decay of the b-quark is extremely slow due to the milliweak nature of its interaction, $\Gamma(b \to u\mu\nu)/\Gamma(\mu \to e\nu\bar{\nu}) \cong k^2\left(\frac{m_b}{m_\mu}\right)^5 \sim 10^4$ or $\tau(b \to u\mu\bar{\nu}) \sim 10^{-10}$ sec for $m_b \approx 4.5$ GeV. When the b-quark decays, it goes into the u-quark but not into the c-quark, and the dominant t-quark decay is into the b-quark, so $t \to b \to u$. The resulting dominant non-leptonic quark decay modes are $b \to u + \bar{u} + d$, $u + \bar{c} + s$ and $t \to b + u + \bar{d}$, $b + c + \bar{s}$. CP-violation is present in the $t \to b$ processes.

Production of a b- or a t-quark in the $\nu(\bar{\nu})$-induced reaction is severely suppressed by a factor k^2. Predictions resulting from our extended Cabibbo current can be tested by examining the decays of hadrons with b- and t-quark flavors.

The model discussed above is easily extended to leptons with the assignment $\psi_1 = (\nu_e, \nu_\mu, \nu_\tau)$ and $\psi_2 = (e, \mu, \tau)$. The assumption, $m_{\nu_e} = 0$ and $m_{\nu_\mu} \neq 0$, leads to the prediction that CP is invariant for left-handed leptonic current, and also in the eμ-sector for right-handed current. CP is violated in τ-decay via the right-handed interaction, which is suppressed by a factor $(M_{WL}/M_{WR})^2$ relative to the CP-conserving left-handed interactions.

A remark on the discrete symmetry (2) is in order here. The fact that a reasonable value of θ_c emerges in the u,d,s,c sector is a reflection of the discrete symmetry that requires $g_{11}=0$. Further requirements, left-right symmetry, the GIM-mechanism,[6] and that the Eulerian angles be expressible in terms of quark mass ratios and phases lead to the symmetry (2). Left-right symmetry requires

$$m^{\circ} = \begin{pmatrix} 0 & \epsilon & \epsilon' \\ \epsilon & \beta & \epsilon'' \\ \epsilon' & \epsilon'' & b \end{pmatrix}.$$

Since we require that all mixing angles are expressible in terms of mass ratios and phases there must be only three free parameters. Reduction of free parameters is achieved by imposing the $Q_{2L,R} \leftrightarrow Q_{3L,R}$ symmetry. Among the resulting three possible cases, (i) $\epsilon = \epsilon'$, $\epsilon'' = 0$, $\beta \neq b$, (ii) $\epsilon = \epsilon'$, $\epsilon'' \neq 0$, $\beta = b$, and (iii) $\epsilon \neq \epsilon'$, $\epsilon'' = 0$, $\beta = b$, only (i) is compatible with the GIM-mechanism for the u,d,s,c sector in the limit $m_{b,t} \gg m_{u,d,s,c}$. It would be of interest to check the observable consequences of the extended Cabibbo current resulting from the symmetry above.

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7. A non-hermitian matrix M can be diagonalized by a bi-unitary transformation $V^\dagger MU = M'$, where M' is diagonal, as follows: First diagonalize the positive definite matrix $M^\dagger M$ by the unitary transformation $U^\dagger M^\dagger MU = M'^2$. Then $V = MUM'^{-1}$ since $V^\dagger MU = M'^{-1}U^\dagger M^\dagger MU = M'$. This method is applicable only when $\det M \neq 0$. We would like to thank Professor R. L. Mills for a discussion.

8. The diagonal matrices P_1 and P_2 are chosen to be

$$P_1 = \begin{pmatrix} e^{i\xi} & & \\ & e^{i\eta} & \\ & & e^{i(\omega-\delta-\gamma_1/2)} \end{pmatrix} \quad \text{and} \quad P_2 = \begin{pmatrix} 1 & & \\ & e^{i(\xi+\eta+\delta)} & \\ & & e^{i(\xi+\omega-\gamma_1+\gamma_2/2)} \end{pmatrix}$$

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