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5TH HIGH-ENERGY HEAVY-ION STUDY

MAY 18-22, 1981

**LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
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The **FRONT COVER** shows graphic results of heavy-ion collisions with increasing energies. **Lower left:** Nuclear breakup at ~ 15 MeV/nucleon as seen in a streamer chamber at the 88 Inch Cyclotron. **Center:** A high multiplicity Ar + Pb collision at 1.8 GeV/nucleon recorded at the Bevalac streamer chamber. **Upper left:** Computer reconstruction of tracks obtained in α - α collisions at ~ 15 GeV/nucleon, from studies at the Split-Field Magnet facility at the CERN ISR.

The **BACK COVER** shows the Pb bricks introduced at the beginning of the Study to remind participants that very heavy mass collisions in the GeV/nucleon range will become available in 1982 at the Bevalac facility. Several speakers used these bricks during their presentations.

FOREWORD

This was the fifth of a continuing series of Summer Studies held at LBL to discuss high energy heavy ion collisions. Recently, a similar meeting has been held on alternate years at GSI (Darmstadt); and, in 1979, we held a meeting at LBL exclusively devoted to ultra-relativistic nuclear collisions.

Two new features distinguish this Study from earlier meetings in the series. First, we significantly broadened the energy range for discussion by including collisions from about 20 MeV/nucleon to the highest available in the cosmic radiation. The lower range, particularly below 100 MeV/nucleon, will be under intense study in the near future with machines such as the upgraded Bevalac, Michigan State University Superconducting Cyclotron, GANIL in France, and the SC at CERN. Recently, the high energy collision regime has been expanded by the successful operation of the CERN ISR with alpha particles. Second, in addition to an extensive program of invited talks, we decided for the first time to actively solicit contributions. Over 60 were received, and we were able to schedule oral presentations by over half of the contributors. The Study was held over the four-and-a-half day period from May 18 to May 22, with a remarkable level of interest being maintained by participants throughout the entire meeting.

The Organizing Committee, which planned this meeting, included Hans Gutbrod, Dave Hendrie, Howel Pugh, and Jorgen Randrup. The Study would not have been possible without the efforts of Eileen Eiland (Nuclear Science Division), Peggy Little (Technical Information Department), and Kathy Williams (Accelerator Research Coordination Office). Our many thanks to them. Finally, George S. Pappas has done an excellent job in editing these Proceedings and getting them out ahead of schedule.

LEE S. SCHROEDER

Chairman, Organizing Committee

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HEAVY ION INTERACTIONS FROM 20 TO 150 MeV/NUCLEON*

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Features of the reaction mechanisms that are expected to be prominent at intermediate bombarding energies are outlined in terms of the hydrodynamic model, TDHF calculations, and Pauli-blocking considerations. Existing experimental information is reviewed, including data from the LBL 88"-cyclotron for 20 MeV/u ^{16}O -induced reactions, results from the CERN-SC accelerator up to 86 MeV/u and information from the Bevalac Streamer Chamber for 30-150 MeV/u ^{12}C - and ^{40}Ar -induced reactions.

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I. INTRODUCTION

The past year has seen increased activity in intermediate energy heavy ion research, principally due to the availability of suitable accelerators at CERN for lighter ions up to 96 MeV/u and at the LBL Bevalac which currently extracts beams as heavy as ^{40}Ar with energies as low as 30 MeV/u. The VICKSI machine in Berlin and recent success at Oak Ridge at 20-25 MeV/u, and the projects at MSU, ANL, Caen, and the upgraded Bevalac hold a great deal of promise for this field in the near future. Also, the problems of instrumentation appear to be solvable, with detectors developed for low energy and relativistic heavy ion research, including large-area gas ionization and avalanche counters, multi-detector arrays of telescopes, phoswiches [1], etc., and streamer chambers, which seem to be sufficient to handle the particle multiplicities with much more ease than at high energies. Figures 1(a) and (b) show pictures of collisions taken in a 1.2 m streamer chamber for 60 MeV/u ^{40}Ar ions on targets of KCl and BaI_2 , respectively. For contrast, Figure 2 shows the reaction of 1.8 GeV/u $^{40}\text{Ar} + ^{208}\text{Pb}$, taken in the same 1.2 m streamer chamber at the LBL Bevalac, which demonstrates the difference in complexity due to high multiplicity encountered at high energy. One should not minimize the problems at intermediate energy, however, since 4π detection is necessary to understand most reaction mechanisms, and unlike the situation encountered in high energy p+p collisions, instrumentation must be sensitive to a wide variety of ionization densities and ranges, e.g., for pions, slow heavy fragments, etc.

Conceptually, the intermediate energy range for heavy ion reactions is regarded as a transition region, where explanations of phenomena

require an extension of the mean field description, one-body viscosities etc., to a nucleon-nucleon description as collision velocities are increased to values comparable to the Fermi velocity and beyond. Several interesting mechanisms are expected to have thresholds in the range of 20-150 MeV/u, as well as some exclusion principal effects concentrated only in this energy range, which will be discussed later. Also, one hopes to learn about the properties of nuclear matter under extreme conditions, i.e., the nuclear equation of state, some aspects of which may be more accessible than at relativistic energies. One may put the intermediate energy regime in perspective with a plot of temperature as a function of collision energy, i.e., available energy per participant nucleon in the center of mass shown in Figure 3. It can be seen that the temperature, or at least the momentum perpendicular to the beam axis, $P_{\perp}$, per emitted light particle, increases rapidly above a laboratory energy of 20 MeV/nucleon, becoming greater than the binding energy, in agreement with expectations for a Fermi gas. In a temperature plot such as this, in fact, one might think of the entire Bevalac energy range of 30-2100 MeV/u as a transition region, compared to center-of-mass energies attainable at the ISR at CERN.

II. HYDRODYNAMICS AND TDHF

Results of hydrodynamic model calculations for ≈ 100 MeV/u heavy ions are encouraging for studies of the compressibility of nuclear matter, with thermal properties being more accessible at relativistic energies. This point has been made by Stöcker and collaborators in the past [9,10], and recently an effort was made to quantify this statement

through calculations of the sensitivity of observables to the parameters and functional form of the nuclear equation of state, $W(\rho, T)$, and the affect on the maximum pressure $P(E)$, and the entropy $S(E)$ produced in a reaction [11]. The pressure and entropy are somewhat easier to relate to observables than $W(\rho, T)$, being related to P_L , and deuteron/proton production ratios, respectively. In Figure 4(a) is shown the comparison of $P(E)$ and $S(E)$ relative to values obtained for a Fermi gas (i.e., with no compression energy) as a function of bombarding energy, for four different values of the compressibility constant in the nuclear equation of state, ranging from 100 to 800 MeV/c. The best sensitivity for the pressure is obtained at the lowest bombarding energy. Similarly, in Figure 4(b) the maximum differentiation among several favorite functional forms of $W(\rho)$ is obtained at low energies, with very little dependence on thermal properties at low energy, shown in Figure 4(c). In the lower parts of Figure 4, it can be seen that the entropy shows a similar, but somewhat reduced dependence on bombarding energy in the calculations. Also, it has been apparent for some time that values of $S(E)$ derived from deuteron/proton ratios do not follow predictions of a simple model by Siemens *et al.* [12] with regard to the bombarding energy dependence. One observes a nearly constant d/p ratio of ≈ 0.3 from 20 to 2100 MeV/ μ , which is explained by Stöcker *et al.* [13] using a breakup mechanism, after chemical equilibrium ceases in the expanding system. Thus one measures protons which originally were tied up in heavier resonances, and the deuteron formation is similarly depleted. Also, one must temper arguments about sensitivity to the compressibility by noting that the results of the hydrodynamic model depend critically on the

assumption of a very short nucleon mean free path, λ_n . Thus, peripheral heavy ion reactions in low density regions, which are somewhat easier to study than central collisions, would not be valid cases. Exclusion principal blocking can easily make λ_n comparable to the nuclear size, as shown by Randrup [14], in Figure 5. Thus, at an entrance channel energy of ≈ 30 MeV/u, λ_n is approximately three times the non-Pauli blocked value or $\lambda_n \approx 4-6$ F. Thus, instead of getting the "bounce-off effect" illustrated in Figure 6, with positive-angle scattering, Fermi-jet nucleons can penetrate a heavy target nucleus, a shock will not be set up, and, as demonstrated by TDHF calculations [16], the projectile residue can orbit to negative angles, as shown in Figure 7 for 30 MeV/u $^{12}\text{C} + ^{197}\text{Au}$. A near-central collision is shown on the left-hand side of Figure 7, with snapshots of the matter density contours. Time increases from top to bottom, and the ^{12}C ion is incident from the top. First, a small contribution of high energy promptly emitted particles (PEPS) escape from coupling of the incident ion velocity with the Fermi velocity of nucleons in the target. In the second snapshot at $t = 119.8$ fm/c, approximately eight fast particles escape due to preequilibrium emission. For more grazing collisions, shown in the middle set of density contours, the same sequence is seen, but at $t = 147$ fm/c only two nucleons are produced by the preequilibrium emission, and in the latest snapshot the heavy projectile-like residue is slowed down and orbits to negative angles at later times. The far right-hand series of contour maps demonstrate the orbiting feature for 85 MeV/u at a late stage of the reaction over a large range of impact parameters, $b = 4-9$ fm. The results of these TDHF calculations indicate that the angle of emission of the

heavy slow (1-5 MeV/u) projectile residue is a good indicator of the impact parameter. This feature may be important since an initial scan of streamer chamber data in the range of 30-100 MeV/u, seems to indicate that charged-particle-multiplicity may not be useful for b-selection, as discussed later. Thus, we see the contrast between hydrodynamic and TDHF calculations, with the truth between these two extremes. There may be a narrow, but interesting energy range in which an amplification effect operates during the course of a reaction, with the collapse of the Pauli blocking from the first set of nucleon collisions which, in turn, allows a short mean free path for following collisions, as discussed by Randrup [14]. Signatures of this effect may show up in multiplicity distributions, or in nonthermal $P_{\perp}$ distributions, i.e., with structure, especially in particle-exclusive data.

III. SINGLE PARTICLE INCLUSIVE DATA

In the intermediate energy regime many mechanisms are expected to be operating, such as breakup and fragmentation for the more peripheral collisions, and as shown previously for TDHF calculations, perhaps a remnant of the deep inelastic process may exist. These large-impact-parameter mechanisms can easily dominate the more central, "explosive" processes which we are attempting to study especially in single particle inclusive data. The grazing collisions are troublesome due to large cross sections from the $2\pi b_{db}$ factor, and contribute fast light particles, much like the central, entrance channel energy-damping processes where hydrodynamics may apply. Unless a sophisticated detector system is used (e.g., streamer chamber or plastic ball), it is difficult to

separate the various processes. In Figure 8 some representative events are shown from the LBL 1.2 m streamer chamber for the reaction of 60 MeV/u $^{40}\text{Ar} + \text{BaI}_2$. Figure 8(a) is an example of a "deep" fragmentation process. A beam momentum projectile-like fragment, $Z \approx 14$, creates the heavy track. But notice the five narrow tracks, caused by proton and deuteron emission at fairly large angles, $\theta_{\text{LAB}} \approx 25\text{-}45^\circ$, or nearly 90° in the nucleon-nucleon center of mass. Also, in Figure 8(b) another prominent mechanism is shown, which might be called "projectile shattering". Many of the eighteen protons of the projectile appear in a tight cluster at the beam momentum, but five fast particles are emitted to large angles. This break-up process may stem from the "volcano effect", pointed out by Wong [17]. This is an idea of Archibald Wheeler *et al.* [18], dating back about twenty years, and was used to explain the origin of fission neutrons. In a heavy ion collision a sudden density increase can result in a more repulsive mean field potential, and can lead to a disintegration of one of the interacting heavy ions. With single particle inclusive data, one could not distinguish the processes shown in Figure 8(a) and (b) from the more central hydro-collisions, an example of which is shown in Figure 9. The class of events shown in Figure 8(a) and (b) are representative of about half of the total events recorded, of which the trigger conditions ($Z \leq 14$ in a narrow cone around the beam direction) have already eliminated extreme grazing collisions, or another $\approx 20\%$ of the reaction cross section. (Figure 8(a) and (b) are not rare events.) It would appear to be difficult to obtain meaningful information with single particle inclusive measurements, or with a low-geometry apparatus, due to the mixture of all of the processes shown in

Figures 8 and 9. One should keep this point in mind as existing measurements are reviewed.

Recently, the MSU group [2] has taken an extensive set of single particle inclusive measurements for ^{16}O bombardments at energies of 20 MeV/u and lower at the LBL 88"-cyclotron. The angle and energy information were fit with a two-parameter procedure, to extract a "moving source" velocity and a "temperature", T , mainly from the tails of the energy spectra. The "temperatures" obtained are shown in Figure 10 for targets of ^{27}Al , ^{90}Zr , and ^{197}Au . The "moving source velocity" consistently comes out to be $\approx 1/2 v_{\text{beam}}$, in agreement with Symons *et al.* [19]. It can be seen that the T values correlate nicely with the Coulomb-corrected entrance channel energy and are in reasonable agreement with a Fermi gas prediction. For 58 and 86 MeV/u ^{12}C -induced reactions, the same conclusion is reached about $v_{\text{source}} \approx 1/2 v_{\text{beam}}$ [3]. The temperature-parameter values extracted from $\theta_{\text{LAB}} = 90^\circ$ (Figure 11) are shown in Figure 12, and it can be seen that an appreciable target-dependence exists, much like that seen in the data of Nagamya *et al.* [20,21] at 800 MeV/u. A finite cluster size-effect seemingly accounts for this T variation [22,23]. For 150 MeV/u $^{12}\text{C} + \text{BaI}_2$, an effective temperature was extracted from streamer chamber data [5] from $\langle P_1 \rangle$ values as outlined by Hagedorn *et al.* [24] in collisions that had zero or one leading particle, as discussed later. A value of $T \approx 35$ MeV is only slightly higher than the Fermi gas prediction. It is remarkable that the T parameters extracted from the exponential tails of energy spectra give values similar to those extracted with a procedure using $\langle P_1 \rangle$ values for central collisions.

Another prominent feature shown in Figure 10 is the difference in the T parameter for composite particles, with increasing T for deuterons and tritons, compared to protons. Several explanations come to mind. The same situation exists for slope differences in π , p (and d) energy spectra as shown in Figure 13 at higher energies [25]. Riding on the velocity front of a "blast wave" in a hydrodynamic model explanation [26], the heavier the emitted particle, the greater the fractional energy boost due to the collective motion at an assumed freeze-out density. But this T difference also comes about in a natural way from the Hagedorn fireball model, essentially from resonance decay. The heavier the resonance (Δ , d , t etc.), the earlier it will freeze out, and the higher the temperature. Although differences in low energy data shown here are probably caused by other mechanisms, such as breakup, the conflict between hydrodynamics and fireball must be resolved in the higher energy work for an important reason. The hydro-calculations coupled with the coalescence model predict that the heavier the composite particle, the *colder* the region of origin [27], but Hagedorn theory predicts that the heavy resonances originate from the *hot* state of the fireball. Of course, the magnitude of the temperature difference in the Hagedorn model depends on the equation of state used, with results of calculations in Figure 13 produced by a system with essentially all of the energy tied up in resonances, with little in random kinetic motion.

Returning to the 20 MeV/u reactions, p, d, t spectra from the $^{15}\text{O} + ^{238}\text{U}$ reaction were fit with some success with a Coulomb-corrected coalescence model [28], which is remarkable at this low energy. One might ask where the reservoir of free neutrons and protons might be, from

which the coalescence proceeds. For a further test, we measured the neutrons emitted in 20 MeV/u $^{16}\text{O} + ^{238}\text{U}$ by time-of-flight techniques. Spectra taken at several angles are shown in Figure 14. A comparison with proton spectra in Figure 15 demonstrates, even with Coulomb corrections, that the simple coalescence model doesn't work, since there is a difference in slope of the high energy tails. The ratio $n/p < 1$ seen on the tails of the spectra is similar to the situation found at higher energies [29], explained by Gyulassy [30] as a momentum shift due to Coulomb repulsion. However, a difference in slope for the tails of the energy spectra is not explained. The results may be a further demonstration that breakup of emitted clusters (i.e., after the chemical equilibrium stage) is important, as calculated by Stöcker [13] for entropy-related considerations in this intermediate energy range. Thus we have a situation involving fast light-particle emission, which is similar to that of pion emission at ISR energies, when it is said that one is dealing with second- or third-hand information due to the decay of resonances.

IV. STREAMER CHAMBER METHODS

The need for complete particle-exclusive data in the intermediate energy range is apparent, if, for no other reason, to separate different types of processes. One type of detector that is proving useful in understanding heavy ion reaction mechanisms is the streamer chamber, originally designed for high energy physics studies. Operating with an internal target, these devices are capable of nearly 4π geometry for charged particles, with good momentum resolution. Figure 16 shows the

basics of this gas counter. A short duration (≈ 10 ns) high voltage (≈ 750 KV) pulse is fed in, which causes the gas (e.g., He-Ne mixture) to start avalanching along ionization trails, i.e., where charged particles have passed. An important point, which distinguishes this device from emulsions, etc., is the triggerability, allowing one to select a desired class of events as determined by electronic detectors. The memory time in the gas, that is, the time duration over which the chamber is sensitive, is typically microseconds, but can extend to milliseconds, and is determined principally by the amount of SF_6 introduced for quenching the avalanche. The whole device is placed in a high magnetic field (≈ 3 -20 KG) and the rigidity is determined from the track curvature. Information is recorded on photographic film or, at lower resolution, with videotape or CCD cameras. For heavy ion research, a small chamber is operating at the LBL 88"-cyclotron [33]. This chamber is well suited to beam energies of 20-40 MeV/u when augmented with specialized electronic detectors. Essentially it is a charged-particle multiplicity detector operating with a single camera view. At the LBL Bevalac, a large chamber (1.2 m in length) has been adopted from high energy physics and upgraded somewhat for heavy ion studies. In the past year we have applied this device to the problem of intermediate energy studies, with bombardments of KCl and BaI_2 targets (chosen to be non-conductors in the 750 KV field) with beams of ^{12}C and ^{40}Ar at energies of 30-150 MeV/u. A diagram of the experimental set-up is shown in Figure 17. Three camera views allow complete event reconstruction, that is, the momenta of all charged particles that escape from the target are determined with high angular and curvature resolution over most of the

the chamber volume. Electronic detectors have been used only for determining some very general trigger conditions. A pair of high resolution 100 μ -thick silicon detectors act as an active collimator to define the incoming beam trajectory and Z before entering the chamber. Downstream of the target assembly, inside the active gas volume, a thin plastic scintillator coupled to a photomultiplier is used to select events in which a target interaction occurred. A spectrum from this downstream scintillator is shown in Figure 18, which demonstrates one of the trigger conditions used for ^{40}Ar interactions. Essentially, ionization loss in the plastic requires a beam velocity particle with $Z \approx 1-14$ to be detected. Thus the grazing collisions with $Z \approx 15-18$ are rejected. Except for the multiple target arrangement, Figure 17 represents the normal mode of operating this streamer chamber for relativistic heavy ion research. It can be seen that this is a powerful device, providing charged-particle-exclusive momentum, or more correctly, rigidity determinations. Two principal weaknesses exist, however, in the use of the chamber: (a) the slow, expensive processing of photographic information due to manual scanning, and (b) the determination of rigidity information only, with no mass and energy data. The latter consideration is important for intermediate energy reactions, since many composite particles, including heavy fragments, are emitted. Both of these problems have been alleviated somewhat with a common solution, the use of computer digitization of photographic information. With observed multiplicities of $\approx 5-10$, which are relatively low compared to relativistic energy results, the events are simple enough to allow automatic scanning and measuring, which saves approximately a factor of ten in time and

expense compared to manual scanning techniques. Using the ALICE facility at Argonne [34], the image processing group has digitized tracks with photoreader techniques and we have studied the problem of Z, A determination by using ionization density information. Thus track widths, peak intensities, peak half-widths, tenth-widths, and integrated intensities are extracted from background subtracted data while computer-vectoring along the tracks.

In Figure 19 we show an example of the particle identification capability of this procedure with test-run data for the 150 MeV/u ^{12}C beam from the Bevalac. Using the integral of the FWHM of track profiles and correcting for camera angles relative to the tracks, we obtain a particle ID spectrum with fairly clean, p,d,t, α separation, even for a rather broad momentum slice. While not proportional to dE/dX , there is still enough sensitivity left in the streamer process to allow separation. Although we have restricted this determination to a 20° cone around the beam axis, and the sensitivity does decrease when larger dip angles are used, the results are most encouraging. To enhance the dE/dX proportionality for larger angles and higher Z, recent experiments were performed with image intensifiers in the cameras. These light multipliers provide a gain of ≈ 10 -100 which allows operation of the chamber at lower light levels; therefore at lower electric fields. A side effect of the image intensifiers is the image distortion which can be seen in most of the sample photographs shown here. With the use of a calibration grid, this distortion is easily corrected with the computer processing. Results are expected in the near future. The expectations of extending the Z identification up to rather high values comparable to

that of the beam, led to the design of the target arrangement shown in Figure 17. A series of relatively thin ($\approx 25 \text{ mg/cm}^2$) foils allow heavy fragments to escape, but the overall target thickness remains comparable to the amount of gas and window material in the chamber. The question of the maximum Z which we can identify is still open, but Figure 20 shows an interesting fragmentation and secondary reaction for the 100 MeV/u ^{40}Ar beam, interacting first with a BaI_2 target. Assuming this is projectile breakup, and counting tracks, ten protons are emitted, leaving an ^{16}O ion at beam velocity which interacts in the trigger scintillator, emitting eight more protons. Thus the difference in ionization densities and track widths for the incident ^{40}Ar on the far left and the ^{16}O fragmentation product is evident. The frequency of occurrence of such secondary processes is consistent with normal fragmentation cross section estimates. (One does not need a Judek-Friedlander effect here.) Also, the delta electron count aids in the Z identification. Slow heavy fragments which stop in the counter gas (Figure 21) have a characteristic signature of tiny islands of ionization removed from the track. This "spattering" of charge is present over a velocity window from $\approx 5\text{--}30 \text{ MeV/u}$. Although not fully understood, this process can be used in the identification of slow but incompletely stopped fragments.

In Figure 22 an example of a ^4He fragment which stops in the counter gas can be seen as the uppermost track that extends forward, along with a number of faster light particles. Multiple heavy fragment emission is an important process, even for light element targets as shown in Figure 23 for 100 MeV/u $^{40}\text{Ar} + \text{KCl}$. The streamer chamber pictures are

informative, to a point, giving us ideas on the general topology of the reactions, but one needs more quantitative information from reconstructed events to draw conclusions about the reaction mechanisms that operate at intermediate energies.

V. CHARGED-PARTICLE-EXCLUSIVE DATA

A small set of data has been reconstructed and analyzed for 150 MeV/u ^{12}C on a thick (3 mm) BaI_2 target. The statistical accuracy of the quantities that have been extracted is not good, except for gross properties such as $\langle P_{\perp} \rangle$ or "temperature". But it is of interest to start thinking about some of the quantities available from streamer chamber data. In order to avoid assumptions about "moving source velocities", or the correct coordinate system for the data, we can adopt the usual procedure from high energy physics for fixed target work by using the projection of the particle momentum on the plane perpendicular to the beam axis, $P_{\perp}$. From the $\langle P_{\perp} \rangle$ of the class of events selected, a value of the temperature was extracted following Hagedorn *et al.* [24]. A value of $T = 35 \text{ MeV}$ ($\pm 5\%$) was found for protons with no increase in temperature, within statistics, for deuterons. This value of temperature was shown previously in Figures 3 and 12. The uppermost graph in Figure 24 plots the distribution of total $P_{\perp}$ determined event-by-event, that is, the scalar sum of $P_{\perp}$ -values. The distribution is broad and extends up to an island of counts, or at least a long tail of high values. Here we have the total momentum converted from the entrance channel to $P_{\perp}$. The middle part of Figure 24 becomes more interesting by dividing the total $P_{\perp}$ for each event by the multiplicity M . The few

counts that were isolated at a total $P_{\perp} \approx 3000$ MeV/c remain isolated at $P_{\perp}/M \approx 350$ MeV/c. The high $P_{\perp}$ region is not made up of unusually large numbers of particles; instead, each particle has a high momentum. Perhaps a unique high- $P_{\perp}$ process is responsible. Also, for the bulk of the events there is a focusing around $P_{\perp}/M \approx 200$ MeV/c, the meaning of which is not fully understood. One might suggest that with this procedure the thermal fluctuations become less important and $P_{\perp}/M$ may reflect a "blast wave" or a "shock". The data shown here are from events selected with a "leading particle" criterion in an attempt to sample central collisions. That is, events were rejected after reconstruction if there were two or more beam velocity particles ($v_{\text{beam}} \pm 10\%$) in a 5° cone around the beam rigidity axis. The lower left-hand part of Figure 24 shows the dependence of $\langle P_{\perp}/M \rangle$ on the number of leading charged particles, N . The insensitivity for $N = 0$ compared to $N = 1$ may be related to the lack of sensitivity to neutrons so that events with $N = 0$ often have a fast leading neutron. The drop-off of $P_{\perp}/M$ (and $P_{\perp}$) for $N > 2$ is appreciable, and suggests that the leading particle criterion is a good procedure for selecting events with a high degree of thermalization of the entrance channel energy. We must keep in mind that this procedure can bias the data heavily, but it is believed to be a much better, more decoupled method than, for instance, the use of particle multiplicity.

The lower right-hand part of Figure 24 demonstrates one type of procedure that we have been using for several years, which characterizes the multi-particle data with a "moment analysis". Event shapes are associated with the number of major axes, $\ell = 1, 2, 3, \dots$, and the degree of the symmetry can be seen from the value of the coefficient as it

varies between zero and one (perfect symmetry). Many other schemes have been devised for performing multi-particle correlations, including thrust, sphericity, and minimum spanning tree, which are discussed by R. Stock in these Proceedings. The moment analysis scheme as outlined by Fox and Wolfram [35] has some useful properties, including the least amount of "input bias". In most correlation schemes, it must be decided, at the outset, what type of a correlation effect is being studied. In the moment analysis scheme, a more general approach can be taken, using spherical harmonics to define a set of observables which for a non-space fixed axis reduces to

$$H_{\ell} = \sum_{i,j} \frac{|\vec{p}_i| |\vec{p}_j|}{s} P_{\ell}(\cos \phi_{ij})$$

for particles i,j . For our purposes in which $P_{\perp}$ is used, the two-dimensional analog of the set of H_{ℓ} is

$$C_{\ell} = \frac{\sum_i |\vec{p}_i| e^{i\ell\phi_i}}{\sqrt{s}}$$

where the ϕ_i are particle angles in the plane perpendicular to the beam axis, related to some arbitrary zero in that plane. One may further simplify this expression by assuming unit projections of all tracks, N . In the data analysis, particles with $P_{\perp} \leq 50$ MeV/c have been ignored. Then

$$\begin{aligned}
C'_2 &= \frac{1}{N^2} \left| \sum_i e^{i\ell\phi_i} \right|^2 \\
&= \frac{1}{N^2} \sum_i \sum_j e^{i\ell(\phi_i - \phi_j)} \\
&= \frac{1}{N^2} (N + \sum_{i,j} \cos \ell\phi_{ij})
\end{aligned}$$

over all pairs of relative angles between all particles. The result is plotted as the solid line in the lower right-hand portion of Figure 24, for the C'_2 (two-jet) coefficient since it shows a much larger symmetry than C_1 or C_3 . The dashed line represents the random situation and was generated by choosing tracks from different events which have the same multiplicity as the event of interest. For this type of procedure in which the magnitude of the momentum is ignored (except for the 50 MeV/c lower cut-off), this is a large correlation effect and essentially tells us that the particle emission patterns in central collisions tend to have two major "jets". From scanning a large number of events like that shown in Figure 9, it is apparent that there tend to be two major sprays of particles. And here we have another feature of such correlation analyses; that is, if one cannot see at least a hint of the symmetry effect, especially from something as unabstract as streamer chamber photographs, then a correlation analysis has little chance of bringing out anything new. Such procedures are of principal use in quantifying effects. Another technical matter is one of statistics in

each event, which limits many correlation schemes. Unless there are enough tracks $\geq 20-30$, thrust and minimum spanning tree methods are not useful in most instances. The moment analysis method also suffers from statistics problems, but to a lesser degree. Of course, nothing is superior to the existence of a large symmetry effect, which seems to be the case in Figure 24, and results in the large affect on C_2' for multiplicities of only 6-10.

More information is needed to understand the "jetiness" observed here, to distinguish among several possible explanations involving Fermi jets, hot spot decay, hydrodynamic bounce-off, or perhaps a remnant of the initial quasielastic scattering with associated cascades. We have barely begun studies in the intermediate energy range but interesting effects are already emerging from correlation analyses shown here and in results presented in these Proceedings, among others, by Lynen for heavy fragment emission, and by Jakobsson, especially on pion production.

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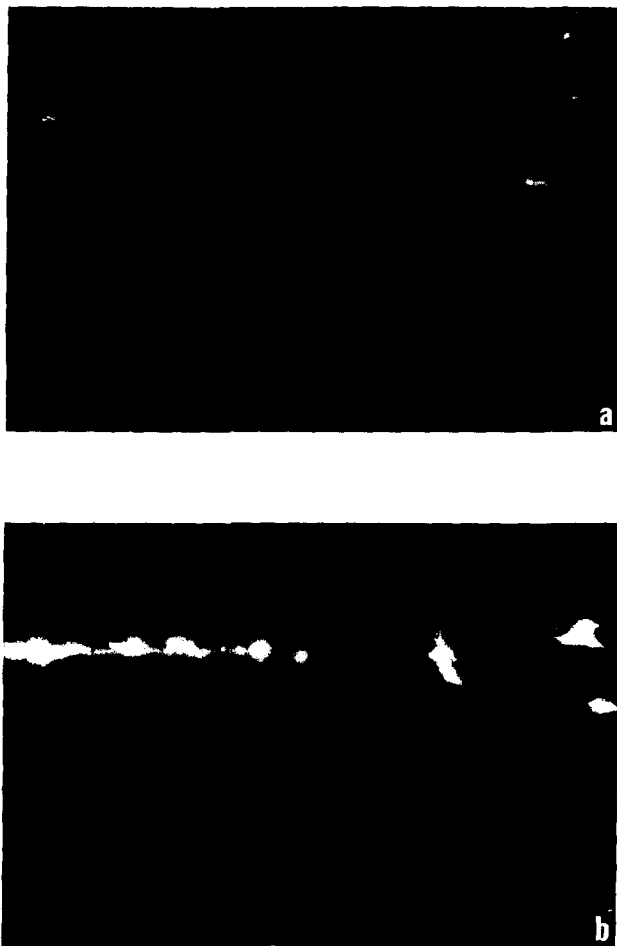


Figure 1. Pictures of events taken with a hand-held camera in the 1.2 m streamer chamber at the LBL Bevalac for 60 MeV/u ^{40}Ar on thin targets of KCl and BaI_2 for (a) and (b), respectively. The beam entered from the left and the magnetic field was approximately 3 KG. (Original in color).

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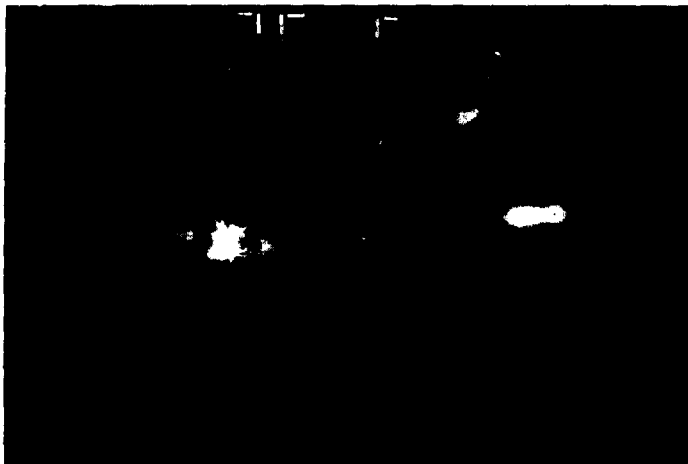


Figure 2. Same as Figure 1, except the reaction is $1.8 \text{ GeV/u } ^{40}\text{Ar} + \text{PbO}_2$. The magnetic field was 13 KG. (Original in color).

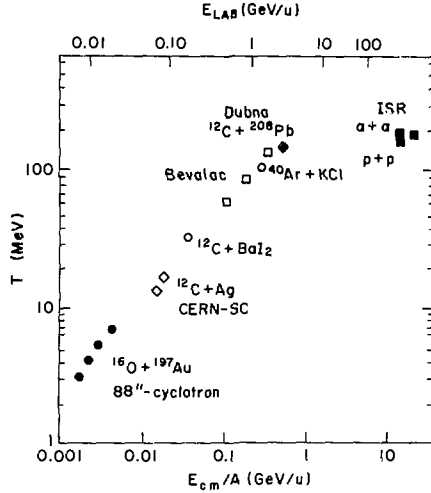


Figure 3. The effective temperature T as a function of energy in the center of mass per participant nucleon, assuming a contribution from the target equal to the projectile mass. The upper scale of incident laboratory energy of the projectile is nonlinear. Most values of T were extracted from the tails of proton energy spectra using single particle inclusive data. Points denoted as "O" were extracted from particle-exclusive streamer chamber data from $\langle P_{\perp} \rangle$. Legend of points: ● LBL 88"-cyclotron [1,2], ◆ CERN,SC [3], □ LBL Bevalac [4], ○ LBL Bevalac [5], ◆ Dubna [6], ■ CERN,ISR [7,8].

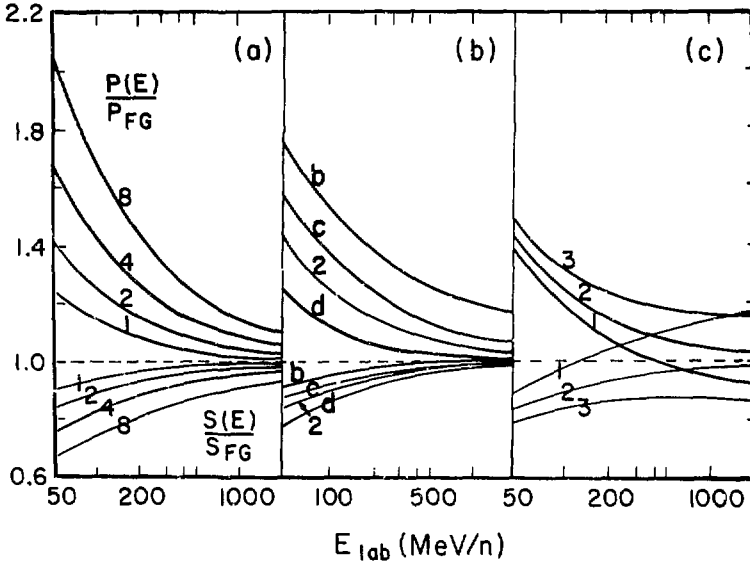


Figure 4. The bombarding energy dependence of the maximum pressure $P(E)$ and energy $S(E)$ attained in a heavy ion collision relative to Fermi gas values. (a) Results of using compressibility (or incompressibility) constants of $K = 100, 200, 400$, and 800 MeV/c are shown for the equation of state $W(\rho) = K(\rho - \rho_0)^2 / 18\rho\rho_0$ where ρ is the nucleon density, ρ_0 is the stable ground state value. (b) A fixed value of $K = 200$ MeV/c was used for various forms of the equation of state, with curve 2 reproduced from from part (a). Curves b, c, and d represent ρ^2 , $\rho^{1/2}$, and $\ln \rho$ forms, respectively. (c) Thermal properties, see Stöcker *et al.* [11].

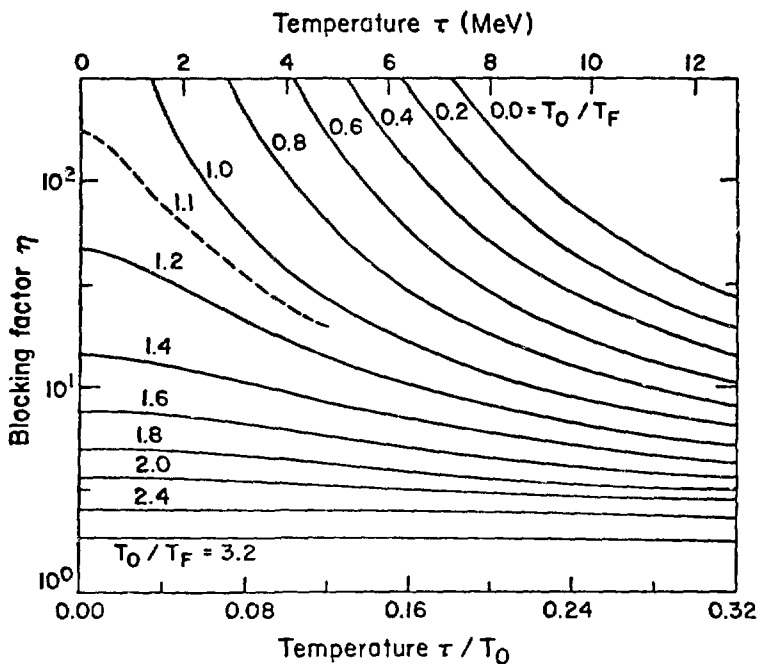


Figure 5. The blocking factor η from the relationship $\lambda_n = \eta \lambda_0$ where λ_0 is the high temperature limit (no Pauli blocking) of the nucleon mean free path. The abscissa is the nuclear temperature. Each curve is labeled with the incident kinetic energy T_0 of an incident nucleon relative to the Fermi energy. $T_0/T_F = 2.0$ represents ~ 30 MeV/u. From Randrup [14].

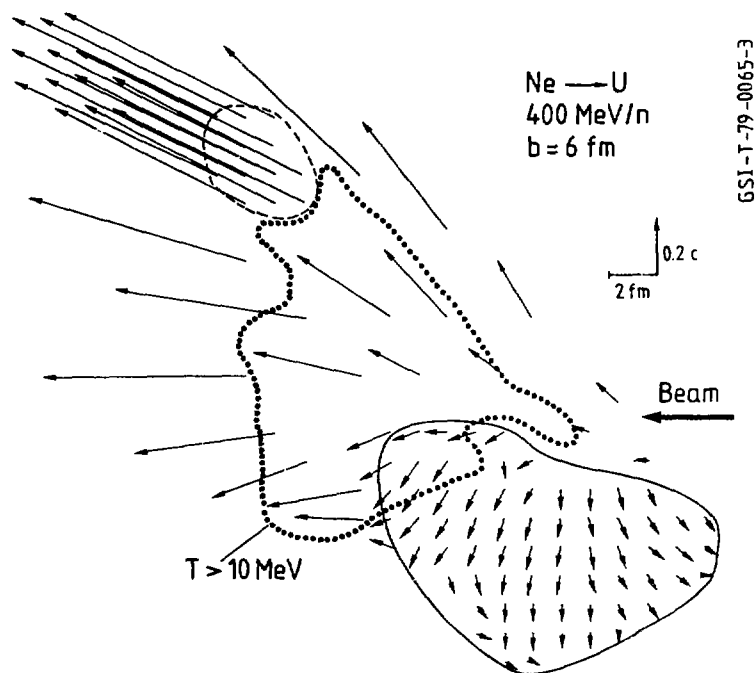


Figure 6. Density and temperature envelopes, and velocity vectors of the matter flow for an intermediate impact parameter collision of $393 \text{ MeV/u } ^{20}\text{Ne} + ^{238}\text{U}$, in a late stage of the reaction. From a hydrodynamic calculation of Stöcker *et al.* [15] demonstrating the "bounce-off" effect.

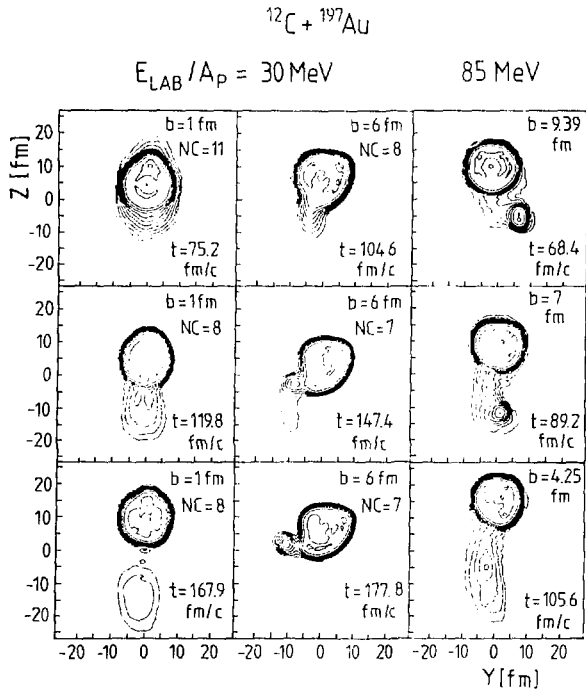


Figure 7. Matter density contours as calculated in a TDHF model for the $^{12}\text{C} + ^{197}\text{Au}$ reaction. The left-hand and middle columns represent results for a bombarding energy of 30 MeV/u. Reaction time increases from top to bottom. The right-hand column shows results for 85 MeV/u at a late stage of the reaction for grazing or near-grazing impact parameters of 9, 7, and 4 fm from top to bottom. The projectile approaches from top to bottom, is offset to the left, and the projectile residue "orbits" to the right-hand side of the figure (left-bank of the beam). Ref. [16].

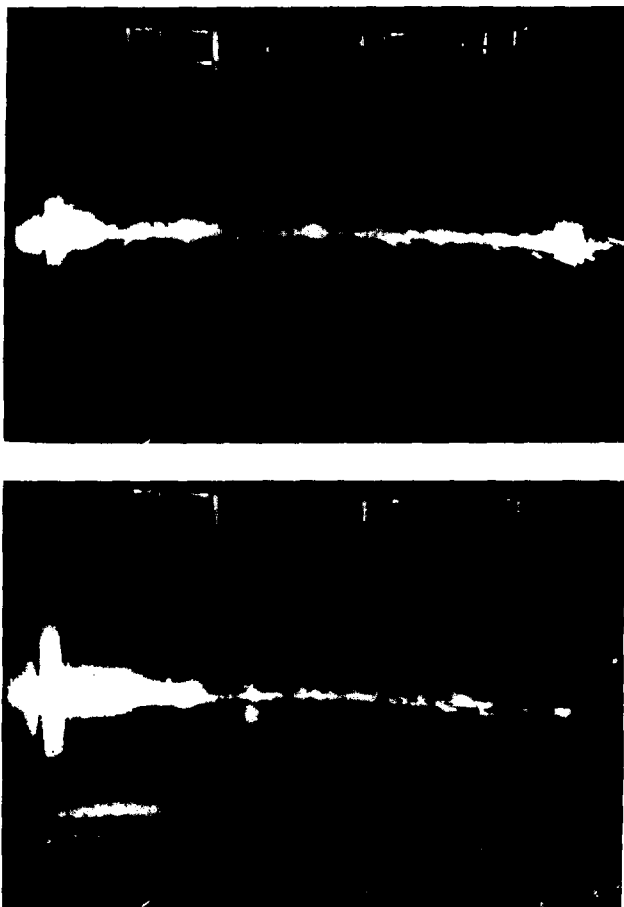


Figure 8. Photographs of events taken at the LBL 1.2 m streamer chamber for 60 MeV/u ^{40}Ar that entered from the left and reacted with a 25 mg/cm² BaI₂ target. Track curvature is produced by a 3.5 KG magnetic field. (a) An example of a fragmentation process. (b) Projectile shattering. From the GSI-LBL-ANL streamer chamber group.

[(a) XBB 817-6962; (b) XBB 817-6963]

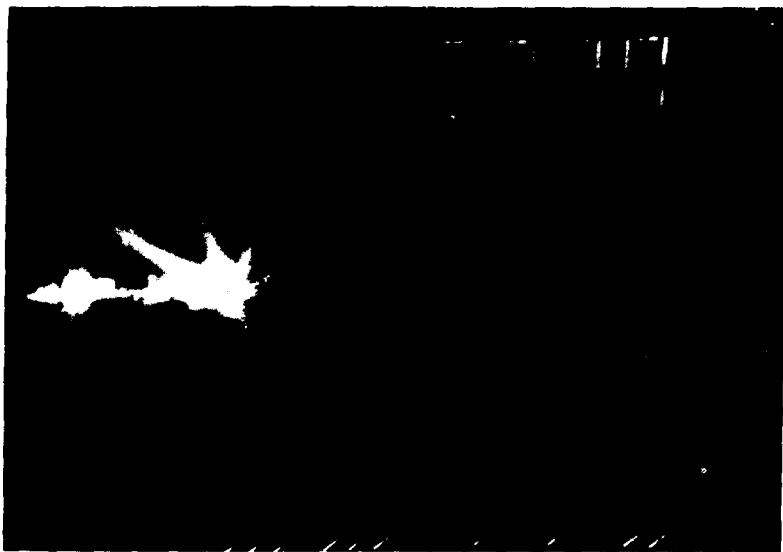


Figure 9. Same as Figure 8 except an example of a more central collision, with no leading particles.

(XBB 817-6964)

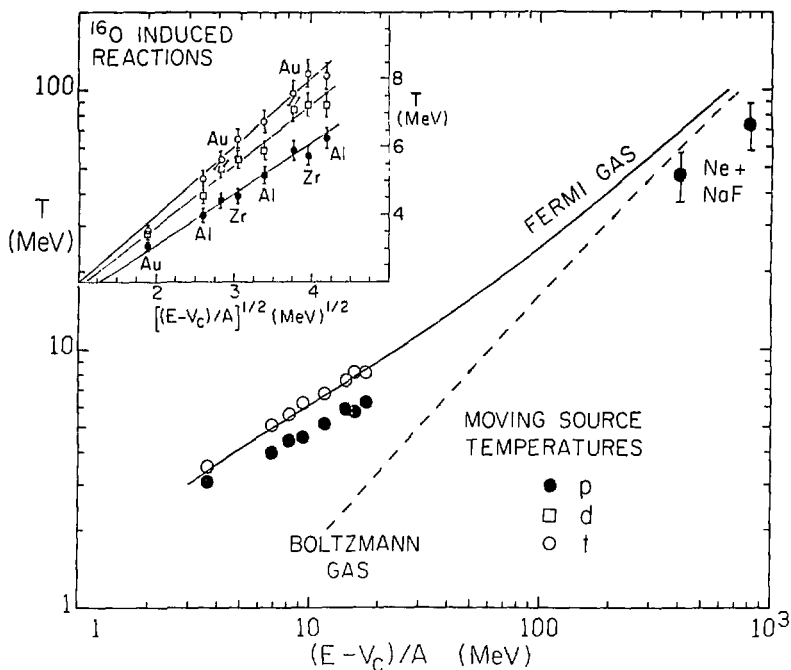


Figure 10. Temperature parameter from the tails of single particle inclusive energy spectra as a function of the entrance channel energy, corrected for the Coulomb barrier, for ^{16}O bombardment of ^{27}Al , ^{90}Zr , and ^{197}Au . The inset shows the apparent difference in "temperature" for emitted p, d, t. From Awes *et al.* [2].

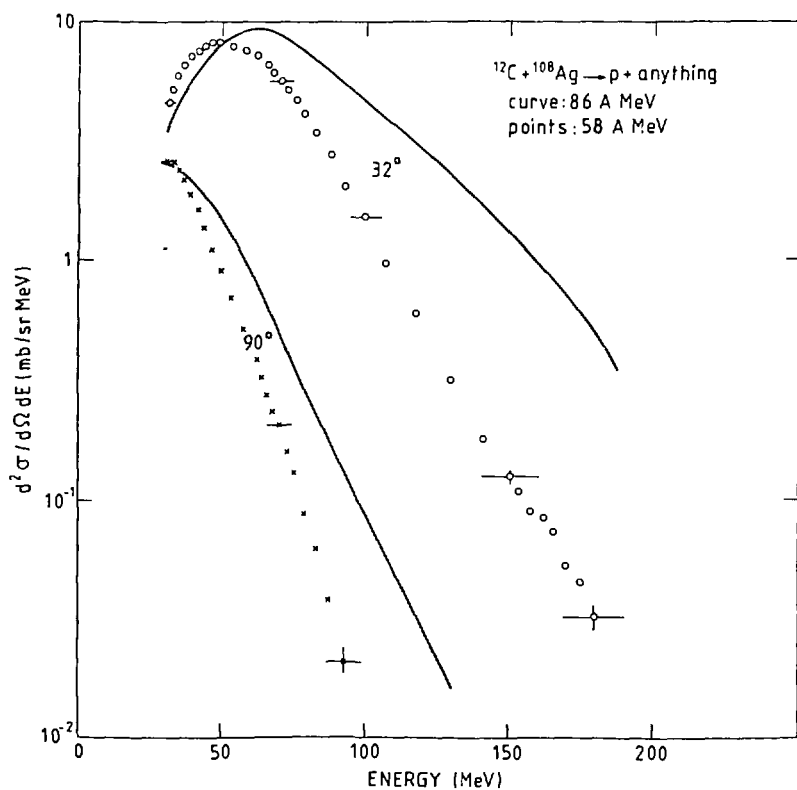


Figure 11. Representative spectra taken at the CERN-SC for the $^{12}\text{C} + \text{Ag}$ reaction at 58 and 86 MeV/u. From Jakobssen *et al.* [3].

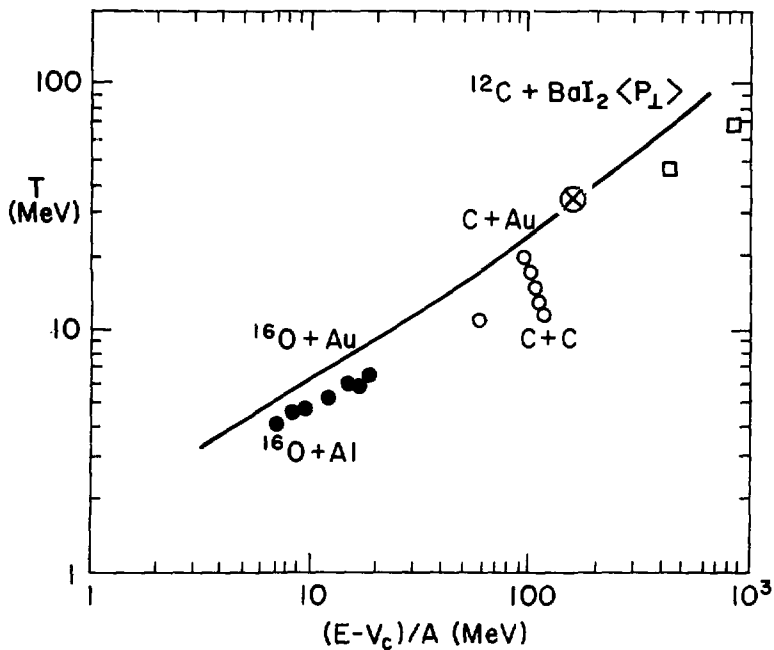


Figure 12. Same as Figure 10 except additional data are presented.

○ Jakobssen *et al.* [3], □ Nagamiya data [20], all from fitting tails of energy spectra, ⊗ Streamer Chamber Group [5], from $\langle P_\perp \rangle$ using charged-particle-exclusive data in central collisions.

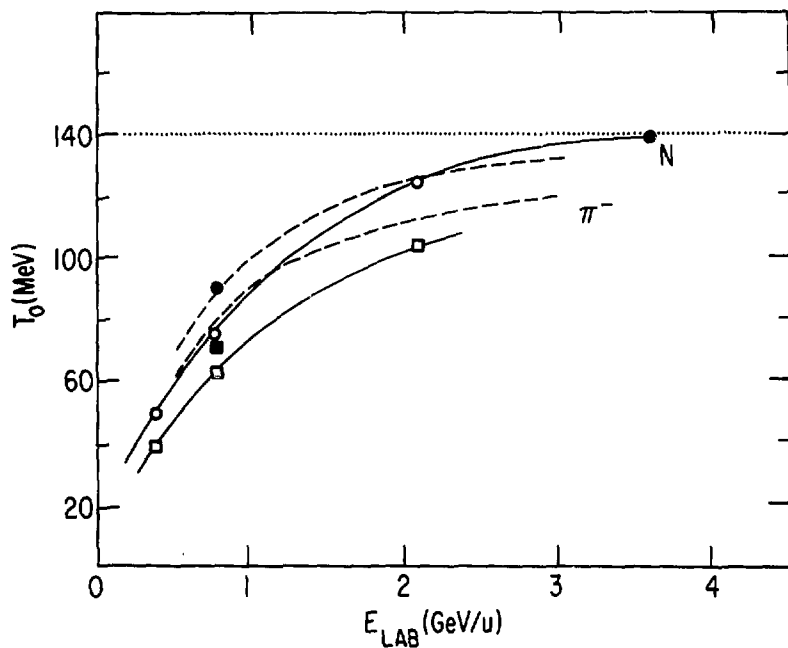


Figure 13. Slope factors T_0 extracted from the exponential tails of energy spectra for proton and pion spectra [20,21]. Solid lines are drawn through the data for the $^{20}\text{Ne} + \text{NaF}$ reaction (open points), while solid points represent heavy target cases. The dashed lines represent temperature values that were calculated with the statistical bootstrap model [24,25].

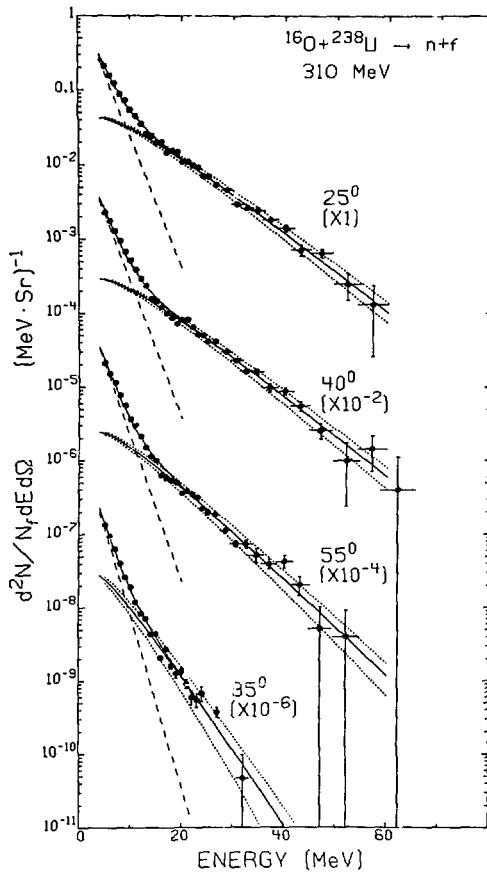


Figure 14. Energy spectra of neutrons emitted at laboratory angles of 25, 40, 55, and 95 degrees for the reaction of 310 MeV ^{16}O on ^{238}U . Time-of-flight techniques were used with fission fragments providing the time reference. Dashed lines represent fission neutron contributions, while the solid lines show the preequilibrium components [31].

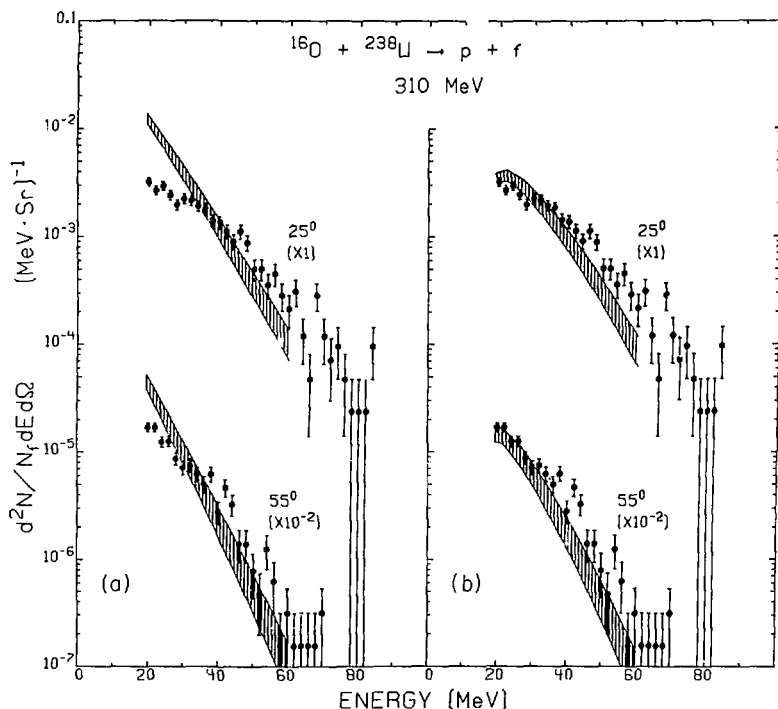


Figure 15. The data points in (a) and (b) are energy spectra of protons emitted at laboratory angles of 25 and 55 degrees in the reaction 310 MeV $^{16}\text{O} + ^{238}\text{U}$. (a) The shaded area represents preequilibrium neutron data and uncertainties. (b) Neutron data which have been modified with a penetrability calculation assuming $Z = 1$, for a more direct comparison with the proton data [31].

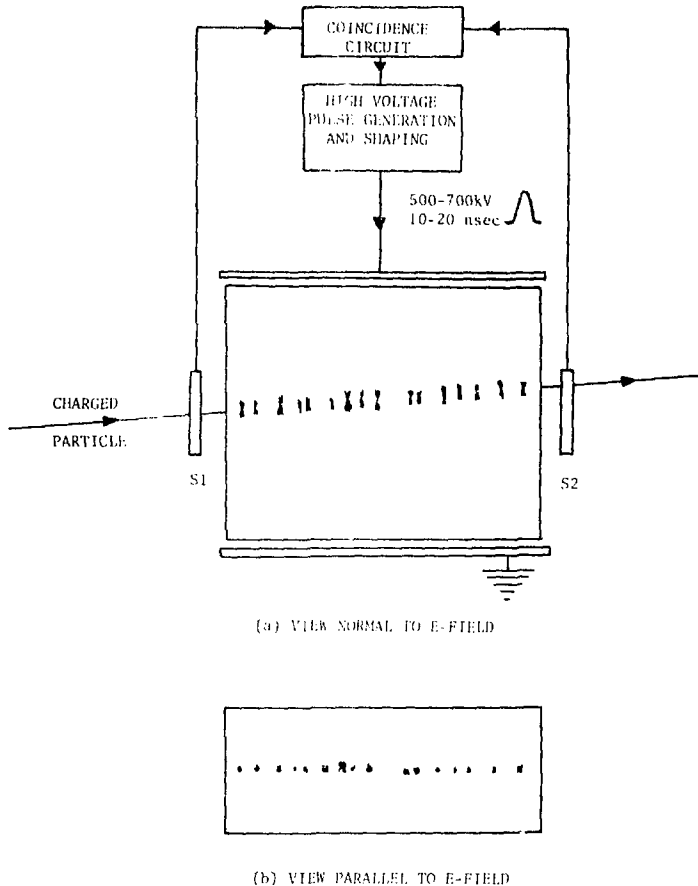


Figure 16. A diagram of a streamer chamber with external trigger counters S1 and S2. (a) Along the applied electric field, streamers can develop to a length of ≈ 1 mm to 1 cm which (b) appear as a series of dots or a narrow trail of ionization when viewed end-on. From Schroeder [32].

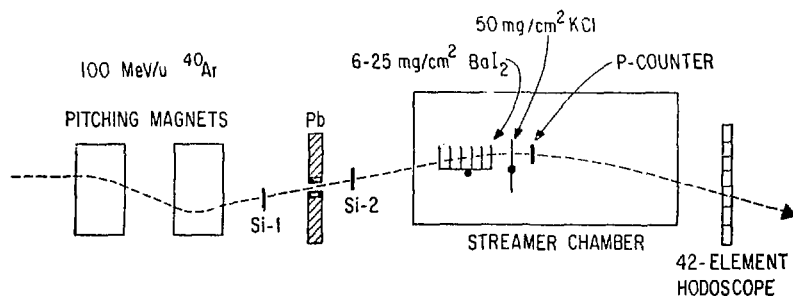


Figure 17. A schematic of the 1.2 m streamer chamber and associated set-up at the LBL Bevalac for intermediate energy heavy ion experiments. Six 25 mg/cm^2 BaI_2 targets and one 50 mg/cm^2 KCl target were mounted in series which allowed observation of heavy composite particles between the targets. A $1/16$ " plastic scintillator coupled via a 6' light guide to a photomultiplier (outside of the B- and E-fields) was used to interrogate beam-like particles for triggering the chamber (P-counter). An extensive array of magnetic bends, thin silicon surface barrier detectors and lead collimators was used upstream of the chamber to clean up the beam since no vacuum line was available in the beam transport system.

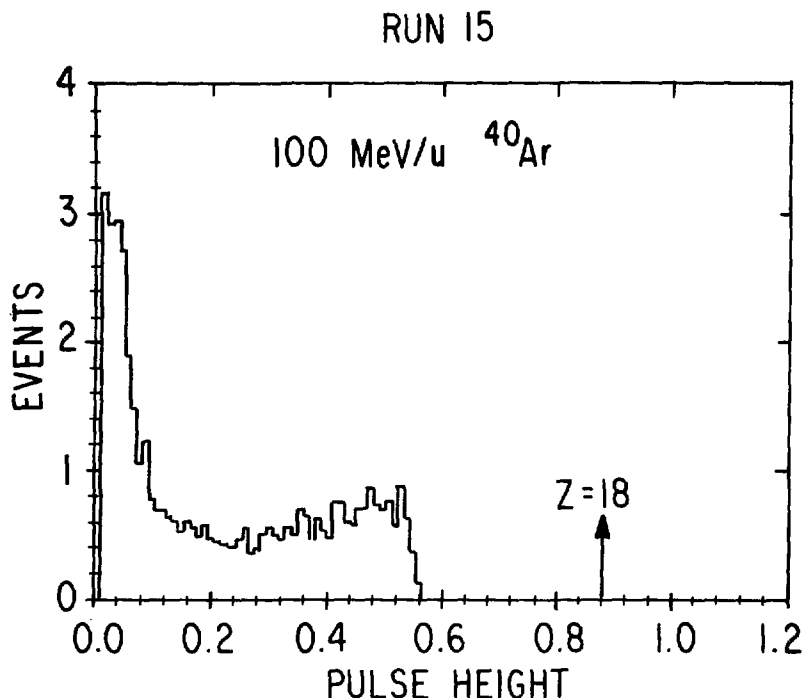


Figure 18. Pulse-height spectrum from the P-counter in Figure 17, used in the trigger conditions for the photographs shown throughout. Upper and lower level discriminators were set in order to require a pulse height equivalent to a Z range of 1-14 for beam velocity fragments. The arrow indicates the position of ^{40}Ar ions in the ungated spectrum.

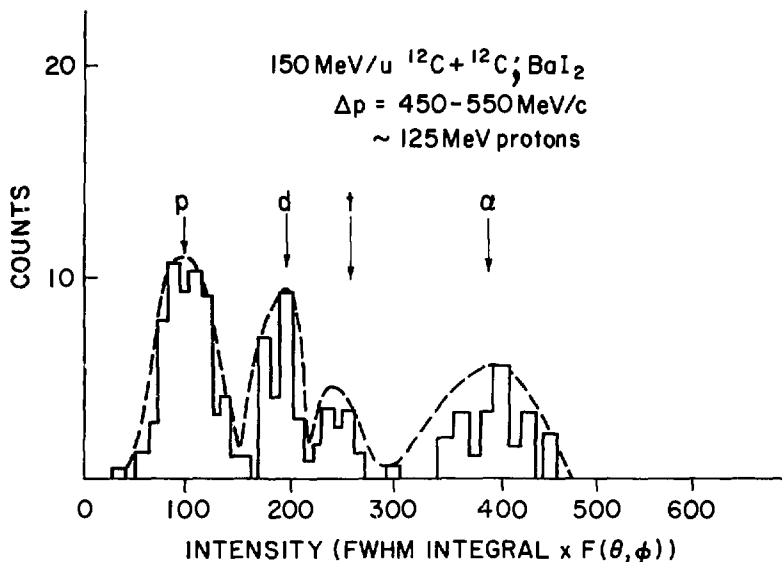


Figure 19. Particle identification in the streamer chamber. Results from the image processing facility ALICE at ANL [34] obtained by integration of track profiles with 6-bit intensity storage. The integral of a track profile between limits corresponding to the FWHM is corrected for film background and for broadening due to differences in the angle between the respective position on the track and the film plane of the camera. Intensities fall into groups which correspond to p, d, t, α when a window is placed on the rigidity. Approximately forty sampling areas are taken from each track. No image intensifiers were used for these data. [5]



Figure 20. An example of a primary and secondary interaction which demonstrates the difference in brightness between $Z = 18$ and $Z \approx 8$. The reaction of 100 MeV/u $^{40}\text{Ar} + \text{BaI}_2$ (left-hand vertex) produces a fragmentation product with $Z \approx 8$ which, in turn, interacted in the plastic scintillator of the P-counter (Figure 17). (XBB 817-6965)



Figure 21. A 50 MeV/u chlorine ion stopped in the gas of the streamer chamber, which allows one to "see" a Bragg peak. (XBB 817-6966)



Figure 22. 100 MeV/u $^{40}\text{Ar} + \text{BaI}_2$ interaction. A slow ^4He ion (along the middle fiducials) stops in the gas showing a Bragg peak. (XBB 817-6967)

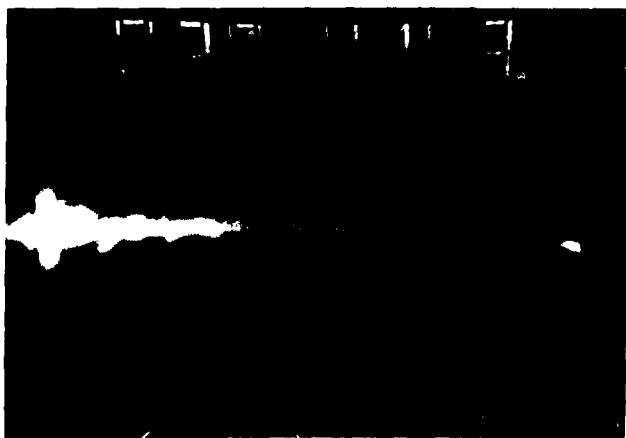


Figure 23. 100 MeV/u $^{40}\text{Ar} + \text{KCl}$, showing the emission of two slow heavy fragments and one fast heavy fragment. (XBB 817-6968)

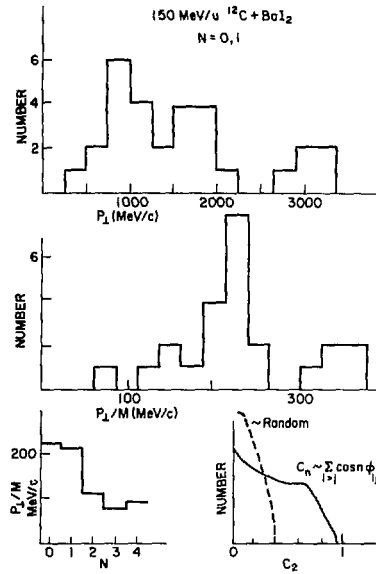


Figure 24. Charged-particle-exclusive data from the Bevalac streamer chamber [5], for p,d,t, α emitted in the 150 MeV/u $^{12}\text{C} + \text{BaI}_2$ reaction. The uppermost graph shows the distribution of the scalar sum of $P_{\perp}$ values in each event. In the middle graph, the $P_{\perp}$ for each event has been divided by the multiplicity. Both of the upper two graphs show data that have been selected with the condition that no more than one leading particle (with $Z \leq 2$), was present in the event. The lower left-hand graph shows the quantity $P_{\perp}/M$ as a function of the number of leading particles, N . The lower right-hand graph shows an example of a moment analysis of event shapes.

FIRST RESULTS FROM THE LBL 88-INCH CYCLOTRON
HYBRID STREAMER CHAMBER

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ABSTRACT

We report preliminary measurements of associated charged particle multiplicities in the reaction $^{16}\text{O} + \text{CsI}$ at 16.5 MeV/nucleon using a hybrid streamer chamber.

The detection system consists of a small volume streamer chamber in conjunction with an array of solid state particle telescopes located at forward angles. The telescopes serve as event trigger and also provide the energy and charge identification of the trigger particle. In the initial run the charge of the trigger particle was recorded directly in the event photograph via an electronic particle identifier and a light-emitting diode display. In the most recent run the telescope data and the event number were event-mode recorded by a computer.

The observed charged-particle multiplicity distributions as a function of ejectile charge and angle are consistent with a calculation which suggests that the broadening of heavy ion spectra with bombarding energy near 20 MeV/nucleon is due to a transition from transfer reactions populating quasibound states of the target to three (or more) - body final states, and that the transition is governed largely by angular momentum.

INTRODUCTION

One of the basic concerns in nuclear physics is charting the energy ranges over which various types of reactions occur, and then finding the causes for their onset and later cessation. Now, for the general class of peripheral reactions, transfer is the characteristic low-energy phenomenon, and projectile fragmentation the characteristic high energy phenomenon. A question of contemporary interest is - when and why do transfer reactions cease, and higher multiplicity events such as fragmentation begin to occur?

There are theories and some evidence that the transition from transfer to fragmentation begins near 15 MeV/nucleon. Our group at the LBL 88-Inch Cyclotron has begun studying the transition by observing 15 MeV/nucleon reactions with a hybrid streamer chamber. Such a detection system provides a complete visual observation of both transfer and higher multiplicity events, and may be triggered to select events of a preferred topology.

Before discussing our preliminary results, we wish to summarize some current evidence for the transition, and ideas for its cause.

Evidence for a transition from transfer to fragmentation near 15 MeV/nucleon.

Contemporary data show that below 15 MeV/nucleon transfer coexists with projectile breakup mechanisms. Above 100 MeV/nucleon transfer has virtually ceased; projectile fragmentation is observed instead. At what energy does transfer cease and fragmentation begin?

Gelbke, et. al.¹ have observed that for 140 MeV $^{16}\text{O} + \text{Pb}$ the

reaction product cross-sections agree with the ground state Q-value systematics of 2-body final states, while for 315 MeV $^{16}\text{O} + \text{Pb}$ the cross-sections follow Q-value systematics for 3- or more body final states. Richardson, et. al.² have found that the mean gamma multiplicity observed following quasi-elastic heavy ion reactions saturates at 15 MeV/nucleon and then decreases at higher energies. Now, high ℓ -states are known to be produced in massive transfer and incomplete fusion, and the ℓ -transfer is proportional to the mass transfer.³ Therefore the Richardson observation implies that the transfer of successively less massive clusters ceases as one passes 15 MeV/nucleon. Gelbke's results imply a simultaneous increase in the mean multiplicity of peripheral reactions at 15 MeV/nucleon.

A possible explanation of the transition.

It has been suggested⁴ that transfer ceases when the transferred angular momentum is sufficiently high to put the cluster in an unbound state. Wilczynski calculates⁵ a critical angular momentum ℓ_{crit} which is the limiting value for transfer to a bound state. If one considers a Gaussian ℓ -window for transfer, of width σ and centroid

$$\ell_{\text{opt}} = M_{\text{CLUSTER}} v_{\text{PROJ}} r_{\text{TGT}} / \hbar$$

then the fraction of this window lying below ℓ_{crit} (Figure 1) shows the effect which Wilczynski's ℓ -cutoff would have on transfer probability as a function of mass and energy.

Wilczynski combines this ℓ -cutoff with ground state Q-value systematics to calculate transfer cross-sections as a function of cluster charge and mass.⁵ For the case of $^{16}\text{O} + \text{CsI}$ at 16.4 MeV/nucleon, this

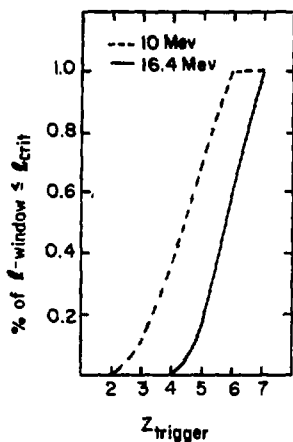


Fig. 1

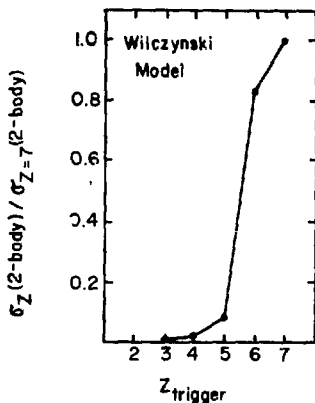


Fig. 2

model predicts the transfer cross-sections shown in Figure 2. The sharp drop at ^{12}C is due both to l_{crit} (cf. Figure 1) and to a sudden change in ground state Q -values.

Preliminary experimental results.

In order to begin a study of transfer and fragmentation in the transition region, we have measured charged particle multiplicities associated with one projectile remnant of known charge for the reaction $^{16}\text{O} + \text{CsI}$ at 16.4 MeV/nucleon. These measurements are being made with a small streamer chamber triggered by forward angle particle telescopes. Figure 3 is a schematic for the operating mode of the system.

In our initial run the telescope was coupled to a particle identifier to obtain the charge of the trigger particle, which was then digitally encoded and recorded on the film. A later, not-yet analyzed run

used a computer-assisted system to add the energy of the trigger particle. The associated charged particle multiplicity recorded in the event photograph was corrected for target evaporation. The resulting multiplicities for projectile residues are shown in Figure 4 in two simple formats-

- a) for each trigger particle Z ,
the ratio of the number of transfer events to the number of $Z = 1$ transfers;
b) for each trigger Z , the ratio of transfer events to higher multiplicity events.

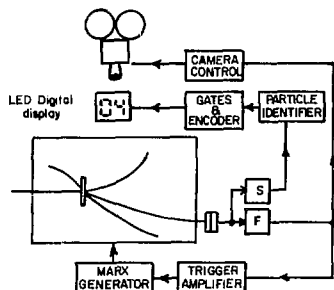


Fig. 3

These data show features which are important to the transition from

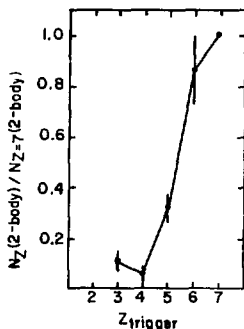


Fig. 4a

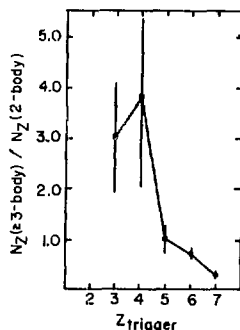


Fig. 4b

transfer to fragmentation. First, there is a decrease in relative probability of transfer as the transferred cluster mass increases. This decrease closely follows the Wilczynski calculation (cf. Figure 2); in particular, its position and slope are consistent with the effects of an

l -cutoff for transfer (cf. Figure 1). However, according to Wilczynski a sudden change in Q -values also causes a sharp decrease in transfer probability at $Z = 6$. With the present data it is not possible to separate the effects of the Q -values and the l -cutoff. Figure 4b shows that as the trigger mass decreases, the relative probability of higher-multiplicity events increases; i.e., there is a trade-off between transfer and fragmentation. This is consistent with fragmentation having a contribution from transfers to unbound states.

In conclusion, we have observed at 16.4 MeV/nucleon a transition from transfer to fragmentation as a function of reaction product masses. The notion of a critical angular momentum predicts the observed features of the transfer probabilities and provides a plausible explanation for the cessation of transfer reactions near 15 MeV/nucleon. However, in Wilczynski's model of transfer the qualitative behaviour of the observed transfer probabilities might also be due mainly to the ground state Q -values for the $^{16}\text{O} + \text{Cs}$ system.

There are obvious additional measurements to be made. A different reaction energy will change the l -cutoff but will not affect the reaction Q -values. A different projectile will provide different Q -values. At present, though, we are encouraged by our first direct observation of both transfer and fragmentation events in the critical transition region.

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New experimental data on particle production at 86 AMeV.

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Abstract:

We present inclusive spectra of p,d,t from 86 AMeV ^{12}C induced reactions at CERN-SC. The results are discussed in terms of a moving thermal Boltzmann source. Typical temperatures, deduced from the high energy part of 90° spectra, are between 13 and 17 MeV, with the temperature increasing with the target mass. Corresponding deuteron spectra may be incorporated in the same simple picture. Measurements of π^+ production in the reactions $^{12}\text{C}+\text{C}$ and $^{12}\text{C}+\text{Au}$ at 86 AMeV are announced. Preliminary results are presented.

¹ The data presented in this report has been obtained within the CERN-Copenhagen-Grenoble-Lund collaboration.
 Carlén², Jakobsson², Kristiansson², Krumlinde², Norén¹, Oskarsson², Otterlund², Schröder², Gustafsson¹, Johansson², Ryde¹, Tibell¹, Bondorf⁵, Fai⁵, Karvinen⁵, Nielsen⁵, Buenerd⁶, Cole⁶, Lebrun⁶, Loiseaux⁶, Martin⁶, Ost⁶, de Saintignon⁶, Guet⁷, Monnard⁷, Mougey⁷, Nifenecker⁷, Perrin⁷, Pinston⁷, Ristori⁷, Schussler⁷, Julien⁸

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Much theoretical interest has been involved in intermediate energy, heavy ion physics and a variety of reaction mechanisms has been proposed. Typical high energy as well as low energy models approach here their limits of validity. The poor experimental knowledge has however, been a disturbing drawback in the development of the field. The experiment reported in this paper offers the first exploratory mapping of light particle emission in the region around 100 AMeV, which has recently become accessible at some accelerator laboratories.

The initial stage of the experiment has been devoted to inclusive measurements of p,d,t spectra, to study the particle emission from the violent collision process in the overlapping volumes of the colliding nuclei [1]. A special effort to investigate subthreshold pion production has also been made. Properties of projectile fragmentation and elastic scattering have been presented elsewhere [2,3]. The ambition of this presentation is to give a general survey over the experimental data, by choosing some interesting examples from the huge collection of data and to describe the most obvious features in very simplified terms, rather than going into the very deep details.

The CERN K800 synchrocyclotron has been equipped with a heavy ion source. Its high intensity 86 AMeV ^{12}C beam has been used as projectile on the targets C, Al, Cu, Ag, Au. Inclusive energy spectra of p,d,t at angles from $30-140^\circ$ have been obtained with a combined Si-plastic scintillator-time of flight telescope, giving excellent particle identification, and a maximum uncertainty in energy of $\pm 5\%$ at the highest energies. Absolute cross-sections are determined to within 30%.

Fig. 1 shows double-differential cross-sections for protons at different angles. A broad peak and an exponential falloff to-

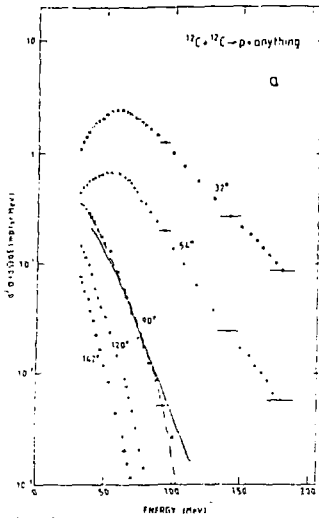


Fig.1

and T , the source temperature. The best fit to 90° data is achieved with $T=13.0$ MeV. When only energies above 60 MeV are considered, we get lower temperatures, possibly due to contributions from quasielastic first order nucleon-nucleon scattering, which results in a steeper slope (dashed curve). The latter distribution has been calculated in the framework of two colliding Fer-

Table 1

Target	T_{40} (MeV)
86A MeV	
C	13.0
Al	13.8
Cu	13.8
Ag	16.6
Au	17.0
58A MeV	
Ag	11.4

mi spheres ($E_f=25$ MeV) with inclusion of Pauli blocking. Table 1 shows a summary of the temperatures found in a two parameter fit to 90° data for all targets. The best fit is obtained for a source velocity close to half the beam velocity, for all targets. We observe an increasing temperature with increasing target mass.

Comparing with 32° spectra, fig 2, it is

wards higher energies is characteristic for the emission in the forward direction, while large angle spectra are typically exponential. An attempt to describe the hard part of the 90° spectra, by a thermal fireball-like source is represented by the solid curve in fig. 1. In the case of $^{12}\text{C}+\text{C}$ any such source is expected to move with half the beam velocity. We calculate the classical thermal Boltzmann spectrum:

$$\frac{d^2\sigma}{dE d\Omega} = C \cdot \bar{E} \exp \left[- (E + E_s - 2 \cdot \bar{E} \bar{E}_s \cos \theta) / T \right]$$

E is the proton kinetic energy,

E_s the source energy per nucleon

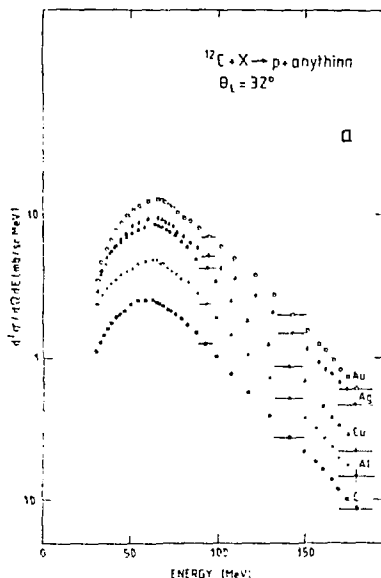


Fig. 2
beam degraded to 58 AMeV
for the reaction $^{17}\text{C} + \text{Ag}$.
The slopes are here significantly steeper than in the 86 AMeV case (lower T , see table 1). Similar observations are done in composite particle spectra. Since the ratio between the temperatures at 86 and 58 AMeV is the same as the ratio between the beam energies, the same projectile to target ratio in the participant is found provided

impossible to account for the whole distribution, only by this simple model. In the peak region it is necessary to incorporate other processes. The peak positions are independent of the target nucleus, which strongly suggests that they are built up mainly from quasielastic nucleon-nucleon scattering. The n - n scattering calculation, with a somewhat changed value of the Fermi energy, gives in fact a better description of the data.

Fig. 3 shows energy distributions obtained with the

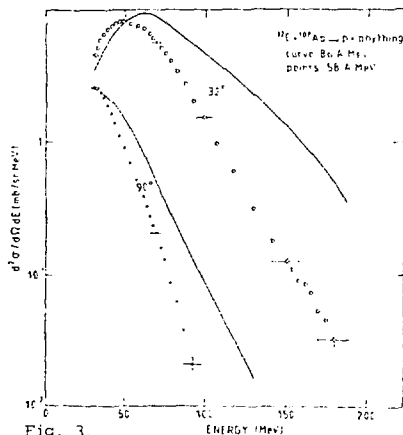
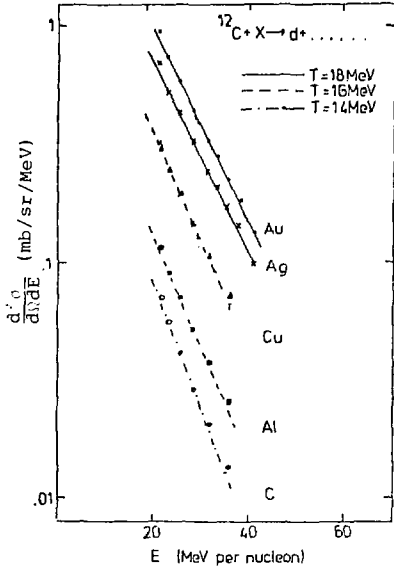


Fig. 3.

that all energy available is used to heat up the participant.

The formation of light composite particles in heavy ion collisions is a topic of special interest. Deuteron spectra are supposed, via the coalescence relation, to reveal information about the spatial size and breakup properties of the emitting source, as well as the entropy production in the collision.



In fig. 4 we show 90° deuteron spectra for five different targets. The exponential slopes have been compared with thermal Boltzmann distributions with the same procedure as described for protons. With a hot source, moving with half the beam velocity, we find temperatures, quite comparable to the corresponding temperatures for protons. The cross-sections are increasing proportionally to A_T (the target mass) why these high energy deuterons are assumed to come mainly from central collisions.

Fig. 4

Pion production.

Pions produced below the kinematic threshold for free nucleon-nucleon collisions have been a topic of hot theoretical and experimental debate during the last years. Observations of pions at ~ 100 A MeV could point towards exotic phenomena like pion condensation, collective behaviour of the nuclear matter or at least extreme tails of the Fermi momentum distribution of nucleons inside the colliding nuclei. An emulsion experiment repor-

ted a huge production cross-section $\sim 3b$ [4]. This result was however rejected by two others [5,6] setting an upper limit of $\sim 10 \mu b$. A Bevalac experiment [7] which measured pions in the extreme forward direction, verified a low but measurable level.

The CERN SC machine provides with its intensity of $> 10^{10}$ particles per sec an excellent tool for investigations of low cross-section phenomena. We have measured π^+ cross-sections at 55° ,

90° and 130° from the reactions $^{12}C+C$ and $^{12}C+Au$ at 86 AMeV. The detector was a multielement plastic scintillator telescope (fig. 5). For a particle stopping in one scintillator, we demand a delayed signal within 200 ns that could be associated with the muon in the $\pi^+ \rightarrow \mu^+ \nu$ decay. If the muon is detected we record $\Delta E, E$, the time between prompt and delayed signal and the amplitude of the delayed signal. An illustration of the particle resolution and the unquestionable observation of π^+ is given in fig. 5. Particles in the lower band decay with a measured halflife of 26ns to a charged particle

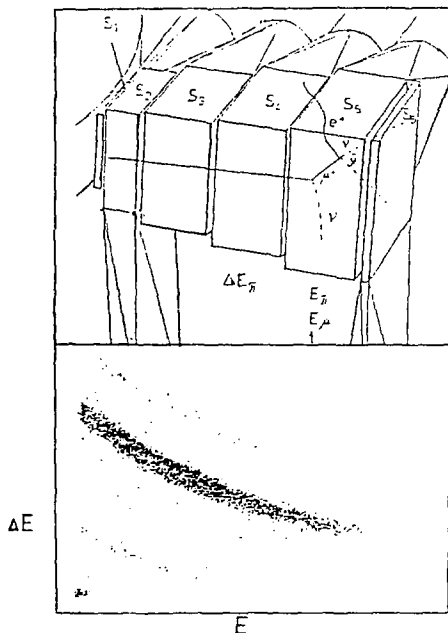


Fig. 5

which if it is assumed to be a muon has a total energy loss of ~ 4 MeV. The data analysis is in progress and only the very first

preliminary energy distributions are discussed here. Fig. 6 shows double differential cross-sections for π^+ at 55° , 90° and 130° in the reactions $^{12}\text{C}+\text{C}$ and $^{12}\text{C}+\text{Au}$.

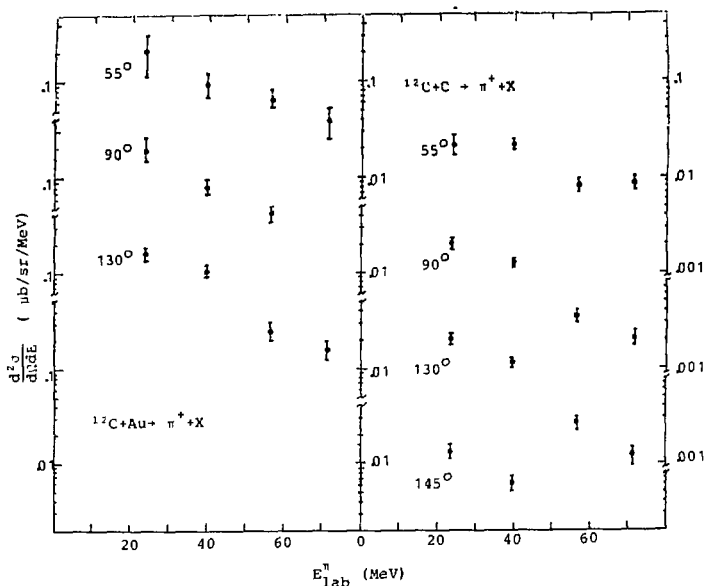


Fig. 6

The emission is almost isotropic in the lab for the lowest energies and gets more forward directed going to higher energies. Also at 130° we observe π^+ with as high energy as 70-80 MeV. The A dependence of the cross-section is weaker than A_T , only a factor of 9 changing from C to Au target. The emission is symmetric in the nucleon-nucleon CM-frame for the case $^{12}\text{C}+\text{C}$. For Au target the relevant system of symmetry seems to be slower, presumably close to the nucleus-nucleus CM.

Some concluding remarks: We have done some interesting obser-

vations concerning the emission of light particles in intermediate energy heavy ion collisions. Inclusive proton spectra cannot be understood in a thermal source approach alone. Necessary measurements of two particle correlations in order to sort out the contribution from quasi-elastic nucleon-nucleon scattering are planned. Coincidences between light particles and projectile fragments should establish any effects of a sideways push of the projectile during the collision.

All data discussed in this contribution come from the CERN-Copenhagen-Grenoble-Lund collaboration. The service-minded personell at the CERN SC is gratefully acknowledged for the support during the experiment. This experiment has been sponsored by the research councils of Denmark and Sweden as well as the Nordic committee for accelerator based research.

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PERIPHERAL AND CENTRAL COLLISIONS AT
AROUND 100 MeV per NUCLEON*

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Abstract

We propose a basic distribution model for heavy-ion collisions at energies around 100 MeV per nucleon and beyond, applicable to both the peripheral fragmentation and the central collision processes. The model assumes a basic single-particle distribution out of which the distribution of the other particles can be deduced. It recovers the fragmentation model of Goldhaber and Feshbach in the case of peripheral collisions. It can be applied successfully to analyze the central collisions data of Ar + KCl at 800 MeV per nucleon with an ellipsoidal Gaussian distribution. It leads to the well-known coalescence relation for composite particle production. While systematizing old results, the model is useful in paving a way for new investigations in intermediate energy heavy-ion reactions.

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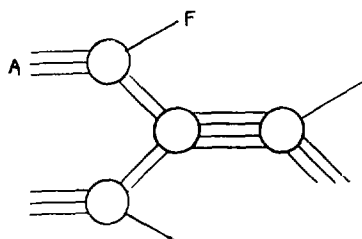
I. INTRODUCTION

In intermediate energy heavy-ion reactions, the reaction product can be usually classified as coming from a peripheral process or a central collision process. This classification pertains not so much to the totality of a collisional event, but rather to the relationship between the detected products and the other parts of the colliding system. For some impact parameters, a peripheral process occurs at the same time as a central collision process, although their reaction products usually fall into different kinematic regions.

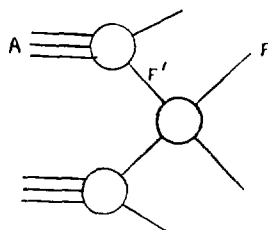
The peripheral process can be further separated into the (direct) peripheral fragmentation and the hard-scattering processes¹ (Fig. 1a). In the peripheral fragmentation process, the detected fragment F comes out of the parent nucleus A as a spectator without scattering with nucleons in the other nucleus, while in the hard-scattering process, it suffers a single collision with a constituent of the other nucleus. In a central collision process (Fig. 1b), the detected fragment comes out from the participant region where many collisions between the nucleons of the two nuclei occur.

Recent experimental data for the peripheral process in heavy-ion reactions around 100 MeV per nucleon have been obtained at Berkeley²⁻⁴ and CERN.^{5,6} The fragmentation model developed by Feshbach, Huang^{7,8} and Goldhaber⁹ for high-energy, heavy-ion reactions, appears to provide a good description for the peripheral fragmentation process around 100 MeV per nucleon when it is modified to include the transverse motion of the projectile.²

(a) PERIPHERAL REACTIONS



DIRECT FRAGMENTATION



HARD SCATTERING

(b) CENTRAL COLLISIONS

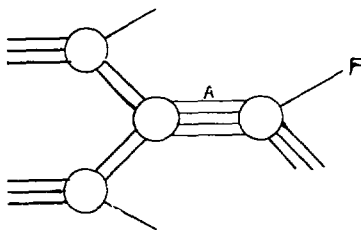


Figure 1. Types of processes involved in intermediate-energy, heavy-ion collisions. (a) In the peripheral reactions, there is the direct fragmentation process in which the detected fragment F is a spectator and encounters no collision with the other nucleus. There is also the hard-scattering process in which the detected fragment F emerges when a constituent F' in the nucleus A suffers a single collision with a constituent of the other nucleus. (b) In the central collision process, the fragment F comes out of the participant region A where many collisions between nucleons of the two nuclei occur.

Recently, experimental data for the central collision process in the energy region around 100 MeV per nucleon also became available.^{10,11} One approach to study central collisions theoretically is to examine the dynamics as an initial value problem.¹²⁻¹⁹ Another useful approach which we shall take up here is to go backward in time. We start with the detected fragments and try to construct the configuration of the nuclear matter before the fragments fly apart. When we view central collisions in this manner, we see similarities between the direct fragmentation process and the central collision process (Fig. 1). Furthermore, the coalescence relation²⁰⁻²² for central collisions is a natural result in the peripheral fragmentation model of Goldhaber⁹ when the underlying single-particle distribution is a Gaussian. It is of interest to develop a model for both the peripheral fragmentation and the central collision processes in a parallel manner. We call such a model the basic distribution model to emphasize the basic distribution as the important element. It includes the fragmentation model of Goldhaber and Feshbach as a special case.

II. THE BASIC DISTRIBUTION MODEL

The fundamental assumption of the basic distribution model is that there is a basic single-particle distribution $f(\vec{r}, \vec{p})$ from which other distributions can be constructed. To make the problem simple, we assume further that there are no additional correlations between the nucleons other than those contained in $f(\vec{r}, \vec{p})$. The many-body distribution for the system is

$$W(\phi_1 \phi_2 \dots \phi_A) = c f(\phi_1) f(\phi_2) \dots f(\phi_A) \delta\left(\vec{k} - \sum_{i=1}^A \vec{p}_i\right) \quad (2.1)$$

where $\phi = (\vec{k}, \vec{p})$, $\vec{k}$ is the momentum of the parent nucleus A and c is a normalization constant. Additional antisymmetrization may have effects²³ which need to be considered in a future refinement of the model. The momentum distribution $w_{F/A}(\vec{k})$ of finding a fragment F with a momentum $\vec{k}$ is then given by

$$w_{F/A}(\vec{k}) = c_F^A \int d\phi_1 \dots d\phi_A W(\phi_1 \phi_2 \dots \phi_A) g_F(\phi_1 \phi_2 \dots \phi_F) \delta\left(\vec{k} - \sum_{j=1}^F \vec{p}_j\right) \quad (2.2)$$

where g_F specifies the condition for the coalescence of F nucleons into a cluster. To make the problem simple, we make the assumption that the coalescence function g_F is a function only of the momenta; spatial dependence will be considered at a later date. Then, the probability function $w_{F/A}(\vec{k})$ becomes

$$w_{F/A}(\vec{k}) = a_F \int \prod_{i=1}^A d\vec{p}_i w(\vec{p}_i) g_F(\vec{p}_1 \vec{p}_2 \dots \vec{p}_F) \delta\left(\vec{k} - \sum_{j=1}^F \vec{p}_j\right) \delta\left(\vec{k} - \sum_{j=1}^A \vec{p}_j\right) \quad (2.3)$$

where

$$w(\vec{p}) = \int d\vec{k} f(\vec{k}, \vec{p}) \quad (2.4)$$

and

$$a_F = c_F^A c. \quad (2.5)$$

III. MOMENTUM DISTRIBUTION FOR PERIPHERAL FRAGMENTATION

In the peripheral fragmentation process the distribution $w(\vec{p})$ is approximately a Fermi sphere. As the coalescence function g_F involves relative momentum separation of the order greater than the Fermi momentum p_F , we can approximate g_F by unity and obtain

$$w_{F/A}(\vec{F}) = a_F B_F(\vec{F}) B_{A-F}(\vec{A}-\vec{F}), \quad (3.1)$$

where

$$B_F(\vec{F}) = \int \prod_{i=1}^F d\vec{p}_i w(\vec{p}_i) \delta\left(\vec{F} - \sum_{j=1}^F \vec{p}_j\right). \quad (3.2)$$

We can also construct the two-fragment momentum distribution:

$$w_{(F,G)/A}(\vec{F}, \vec{G}) = a_{FG} B_F(\vec{F}) B_G(\vec{G}) B_{A-F-G}(\vec{A}-\vec{F}-\vec{G}) \quad (3.3)$$

where

$$a_{FG} = \frac{A!}{F!G!(A-F-G)!} \cdot c. \quad (3.4)$$

In the case when there is no dispersion in the center-of-mass momentum $\vec{A}$, as in high-energy, heavy-ion reactions, we can work in the frame in which $\vec{A} = 0$ and consider an isotropic basic distribution $w(\vec{p})$. We have

$$B_1(\vec{F}) = w(\vec{F}) \quad (3.5)$$

and by the central-limit theorem, we have for $F \gg 1$

$$B_F(\vec{F}) \approx \frac{1}{(\sqrt{2\pi F}\sigma_0)^3} \exp\left[-\frac{\vec{F}^2}{2F\sigma_0^2}\right] \quad (3.6)$$

where
$$\sigma_o^2 = \frac{1}{3} \int d\vec{p} \vec{p}^2 w(\vec{p}). \quad (3.7)$$

Therefore, the momentum distribution $w_{F/A}(\vec{F})$ is

$$w_{F/A}(\vec{F}) = \frac{a_F}{(2\pi)^3 [F(A-F)]^{3/2} \sigma_o^6} \exp \left[-\frac{\vec{F}^2}{2\sigma_F^2} \right] \quad (3.8)$$

where
$$\sigma_F^2 = \frac{F(A-F)}{A} \sigma_o^2 \quad (3.9)$$

and the two-fragment distribution function $w_{(FG)/A}(\vec{F}, \vec{G})$ is

$$w_{(FG)/A}(\vec{F}, \vec{G}) = \frac{a_{FG}}{(\sqrt{2\pi})^9 (\sqrt{FG(A-F-G)})^3 \sigma_o^9} \exp \left[-\frac{\vec{F}^2}{2F\sigma_o^2} - \frac{\vec{G}^2}{2G\sigma_o^2} - \frac{(\vec{F}+\vec{G})^2}{2(A-F-G)\sigma_o^2} \right] \quad (3.10)$$

Equations (3.8-3.10) are valid for all F and G if $w(\vec{p})$ is a Gaussian and valid for F, G, A-F, A-F-G $\gg 1$ if $w(\vec{p})$ is not a Gaussian. Equation (3.9) is essentially the result of Goldhaber.⁹ Equation (3.10) for a Gaussian distribution $w(\vec{p})$ was first given by Feshbach.⁷

At high enough bombarding energies when the plane-wave-impulse approximation applies, the invariant cross section for the production of a fragment F is proportional to $w_{F/A}(\vec{F})$. The experimental cross section can therefore be compared directly with $w_{F/A}(\vec{F})$.

As is well known, the longitudinal momentum dispersion in peripheral fragmentation processes around 100 MeV per nucleon is properly described by the Goldhaber results of Eq. (3.9) and need not be discussed again.²⁻⁴

For the case when $w(\vec{p})$ is given by a Gaussian distribution, which is appropriate for light nuclei, we have from Eq. (3.8)

$$\frac{d\sigma_{AB+FX}}{d\vec{F}}(\vec{F}) \propto \left[\frac{d\sigma_{AB+PX}}{d\vec{p}}(\vec{p} = \vec{F}/F) \right]^{F \cdot \frac{A}{A-F}} \quad (3.11)$$

We obtain the power law relating the cross sections for the production of the composite particles ($F \ll A$) in a peripheral reaction. The similarity between Eq. (3.11) and the coalescence relation for central collisions provides an important motivation to study central collisions within the same mathematical structure, as we have explained previously.

Equation (3.11) does not hold when $w(\vec{p})$ is not a Gaussian. We have instead

$$\left[\frac{d\sigma_{AB+FX}}{d\vec{F}}(\vec{F}) \right]^F \propto \left[\frac{d\sigma_{AB+GX}}{d\vec{G}}(\vec{G}) \right]^G. \quad (3.12)$$

for F , G , $A-F$, and $A-G \gg 1$. In the special case when $w(\vec{p})$ is given by a step function, which is a reasonable approximation for the momentum distribution in heavy nuclei, we have

$$\frac{d\sigma_{AB+(A-1)X}}{d\vec{F}}(\vec{F}), \frac{d\sigma_{AB+PX}}{d\vec{F}}(\vec{F}) \sim \theta(p_f - |\vec{F}|) \quad (3.13)$$

and

$$\frac{d\sigma_{AB \rightarrow (A-2)X}}{d\vec{F}}(\vec{F}), \frac{d\sigma_{AB \rightarrow dX}}{d\vec{F}}(\vec{F}) \sim \left[1 - \frac{3}{4} \frac{|\vec{F}|}{p_f} + \frac{1}{16} \frac{|\vec{F}|^3}{p_f^3} \right]. \quad (3.14)$$

Thus, the two-nucleon removal cross section and the deuteron production cross section are approximately in the form of a triangle. The cross sections for three-nucleon removal and triton production cross sections assume a Gaussian shape as the B-function converges to a Gaussian distribution when $F \gtrsim 3$. Clearly, the power law relating the production of protons, deuterons and tritons does not hold.

The result of Eq. (3.10) indicates that for the detection of two fragments, the probability is the greatest when the two fragments F and G have equal and opposite momenta in the frame in which the parent nucleus is at rest. This correlation is the stronger, the smaller is the value of $A-F-G$. Such a correlation is essentially a momentum-conservation correlation. It is important to verify Eq. (3.10) experimentally in order to assess the validity of the fragmentation model.

IV. CENTRAL COLLISIONS

The analysis we have given for the peripheral reactions needs some modifications and amendments before it can be applied to the case of central collisions. The mass number A, the center-of-mass momentum $\vec{A}$ of the source, and the basic distribution depend on the impact parameter b. Furthermore, unlike the peripheral case in which g_F can be taken to be unity, the coalescence function needs to be accounted for.

From Eq. (2.3), the invariant cross section $S_F(\vec{F})$ for producing a

fragment F with a momentum $\vec{F}$ is given by

$$S_F(\vec{F}) = \int \pi b db a_F(b) B_F'(\vec{F}, b) B_{A-F}(\vec{A}(b) - \vec{F}, b) \quad (4.1)$$

where

$$a_F(b) = \frac{[A(b)]! c}{F! [A(b) - F]!} \quad (4.2)$$

$$B_F'(\vec{F}, b) = \int \prod_{i=1}^F d\vec{p}_i w_b(\vec{p}_i) \delta\left(\vec{F} - \sum_{j=1}^E \vec{p}_j\right) g_F(\vec{p}_1 \dots \vec{p}_F) \quad (4.3)$$

and

$$B_{A-F}(\vec{A}(b) - \vec{F}, b) = \int \prod_{i=1}^F d\vec{p}_i w_b(\vec{p}_i) \delta\left(\vec{A}(b) - \vec{F} - \sum_{i=1}^F \vec{p}_i\right). \quad (4.4)$$

The function B_F' differs from the function B_F in the coalescence function $g_F(\vec{p}_1 \dots \vec{p}_F)$. In the case of collision of two equal nuclei, the center-of-mass momentum is independent of the impact parameters. This is the case we shall take up in this paper. We choose to work in the frame in which the center-of-mass is at rest so that the center-of-mass momentum $\vec{A}$ does not enter into our consideration.

The basic momentum distribution $w(p)$ depends on the impact parameter. It is an interesting question to examine what this dependence is. In the absence of impact parameter selection, we shall use a single basic momentum distribution $w(\vec{p})$ as an average over the different impact parameters involved in the central collision process. The invariant cross section $S_F(\vec{F})$ for the production of F is simplified to be

$$S_F(\vec{F}) = \bar{a}_F B_F'(\vec{F}) B_{A-F}(-\vec{F}), \quad (4.5)$$

where
$$\bar{a}_F = \int \pi b db a_F(b), \quad (4.6)$$

and there is no impact parameter dependence on B_F' and B_{A-F} .

A central collision region is characterized by a large number of participants $A \gg 1$. Therefore, for small values of the fragment number F , the function B_{A-F} has a large dispersion and can be considered to be a constant in the momentum region of interest. We have approximately

$$S_1(\vec{k}) \propto B_1'(\vec{k}) = w(\vec{p}) \quad (4.7)$$

and
$$S_F(\vec{k}) \propto B_F'(\vec{k}) \quad (4.8)$$

for small values of F . Thus, from the experimental data, we can construct approximately what $w(\vec{p})$ is. Once $w(\vec{p})$ is obtained, we can use it to generate the cross sections for the production of composite particles.

It will be useful to parameterize $w(\vec{p})$ into a form flexible enough that it can be used to discuss central collisions at various energies. This momentum distribution $w(\vec{p})$ is the integral of the distribution $f(\vec{k}, \vec{p})$ over the spatial coordinates. We can examine some typical momentum distributions we expect in different models (Fig. 2).

In the TDHF approximation where only the mean field is important, the expected momentum distribution is given by two Fermi spheres separated by a distance. The separation $2p_0$ can be so large that the two Fermi spheres become disjointed (Fig. 2a). Some useful parameterization of $w(\vec{p})$ can be

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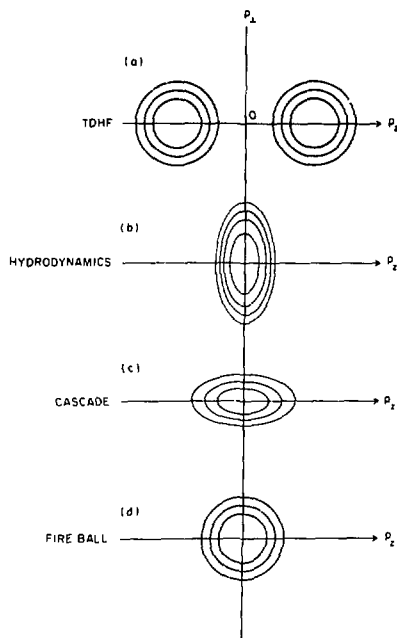


Figure 2. Different types of momentum distributions in different dynamical models. Here p_z is chosen to be along the beam direction.

$$w(\vec{p}) = \frac{A}{1 + \exp[(p_f - |\vec{p} \pm \vec{p}_0|)/a]} \quad (4.9)$$

or

$$w(\vec{p}) = A \exp \left[- \frac{|\vec{p} \pm \vec{p}_0|^2}{2\sigma^2} \right] \quad (4.10)$$

where the positive and the negative signs are for $p_z < 0$ and $p_z > 0$, respectively. The quantities a and σ are diffuseness and width parameters.

In hydrodynamics, while the local momentum distribution is taken to be that of a spherically symmetric shape, these distributions are shifted by the average velocity field of the fluid element. In intermediate-energy central collisions, the high-density region forms a barrier in the longitudinal direction, and thus the flow is deflected towards the transverse direction. The momentum distribution, when integrated over the spatial coordinates, will tend to have a wider width in the transverse direction (Fig. 2b). In a cascade calculation, the momentum distribution is expected to be ellipsoidal for high-energy collisions,^{18,19} while in the fireball model, it is assumed to be a spherical shape²⁴ (Fig. 2c,2d). Some simple parameterizations of $w(\vec{p})$ for these types of dynamics can be given as follows:

$$w(\vec{p}) = A \left\{ 1 + \exp \left[- \left(\frac{p_z^2}{b_z^2} + \frac{\vec{p}_\perp^2}{b_\perp^2} - p_o^2 \right) / a \right] \right\}^{-1} \quad (4.11)$$

or

$$w(\vec{p}) = A \exp \left[- \frac{p_z^2}{2\sigma_z^2} - \frac{\vec{p}_\perp^2}{2\sigma_\perp^2} \right] \quad (4.12)$$

In all the above equations (4.9)-(4.12), the momentum $\vec{p} = (p_z, \vec{p}_\perp)$ refers to the momentum of the nucleon as measured in the frame in which the center-of-mass is at rest. We can relate this to the momentum in the laboratory frame using relativistic kinematics. The transverse momentum $\vec{p}_\perp$ is frame-independent. The momentum p_z is explicitly given in terms of the relativistically invariant quantities x and $\vec{p}_\perp$

$$p_z = \frac{m^2 (x^2 - 1) - \vec{p}_\perp^2}{2mx} \quad (4.13)$$

where

$$x = \frac{p_z^L + p_o^L}{s_z + s_o}, \quad (4.14)$$

$$s_z = \sqrt{2\epsilon m}/2, \quad (4.15)$$

$$p_o^L = \sqrt{m^2 + (p_z^L)^2 + \vec{p}_\perp^2}, \quad (4.16)$$

and

$$s_o = \sqrt{m^2 + s_z^2}. \quad (3.17)$$

Here, m is the rest mass of a nucleon, p_z^L is the longitudinal momentum of the nucleon in the laboratory system, s_z is the momentum of a nucleon traveling with the velocity of the center-of-mass of the participant nucleons and ϵ is the kinetic energy per nucleon in the laboratory system.

We analyze the data of $^{40}\text{Ar} + \text{KCl} \rightarrow p + X$ at 800 MeV per nucleon in terms of a momentum distribution of the above types. It is found that the ellipsoidal Gaussian distribution Eq. (4.12) fits the experimental data quite well (Fig. 3). The unnormalized momentum distribution $w(\vec{p})$, which is taken to be the invariant cross section for proton production,

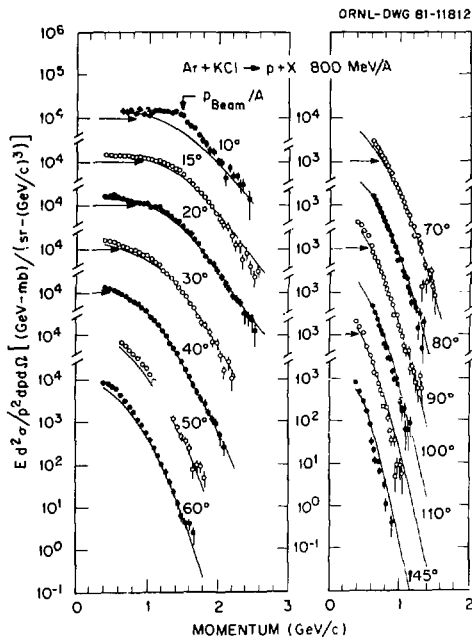


Figure 3. The proton production invariant cross section in the collision of $^{40}\text{Ar} + \text{KCl}$ at an energy of 800 MeV per nucleon. The data points are from Nagamiya, *et al.*²⁰ The solid curves are theoretical results with an ellipsoidal Gaussian distribution given by Eqs. (4.18) and (4.19). The beam momentum per nucleon is 1464 MeV/c and is indicated by an arrow.

is found to be

$$w(\vec{p}) = A \exp \left[-\frac{p_z^2}{2\sigma_z^2} - \frac{\vec{p}_\perp^2}{2\sigma_\perp^2} \right] \quad (4.18)$$

where $A = 1.5 \times 10^4 \text{ GeV-mb/sr-(GeV/c)}^3$

$$\sigma_z = 427 \text{ MeV/c} \quad (4.19)$$

and $\sigma_\perp = 332 \text{ MeV/c}.$

For the 10° data, there are additional contributions due to the peripheral processes, and thus the experimental cross sections are larger than those given by Eq. (4.18) near the beam rapidity region (Fig. 3).

We observe that the longitudinal momentum distribution has a larger width as compared to that of the transverse distribution, as one expects from the cascade model.^{18,19} Clearly, there are substantial nucleon-nucleon collisions which transform the two-center momentum distribution into an ellipsoidal Gaussian distribution by filling in the space in between. The collisions are not numerous enough to erase the initial preference of the longitudinal momentum direction. The anisotropy in the momentum distribution indicates that complete thermal equilibrium of the whole system is not yet achieved.

The invariant cross section for the production of a cluster of F nucleons $S_F(\vec{p})$ is approximately proportional to $B_F'(\vec{p})$ [Eq. (4.8)]. It is necessary to evaluate $B_F'(\vec{p})$, taking into account the coalescence function $g_F(\vec{p}_1 \dots \vec{p}_F)$. In order to have an understanding of the general features of this function, we shall evaluate $B_F'(\vec{p})$ with nonrelativistic kinematics. Relativistic generalization will be taken up at a later

date.

For a momentum distribution $w(\vec{p})$ of the type given by Eq. (4.18), we have

$$B_F^1(\vec{p}) = A^F \int d\vec{p}_1 \dots d\vec{p}_F \exp \left[- \sum_{i=1}^F \left(\frac{p_{iZ}^2}{2\sigma_z^2} + \frac{\vec{p}_{i\perp}^2}{2\sigma_\perp^2} \right) \right] \delta(\vec{p} - \vec{p}_1 \dots - \vec{p}_F) g_F(\vec{p}_1 \dots \vec{p}_F). \quad (4.20)$$

We can make the well-known transformation to Jacobian coordinates $\vec{y}_i$

$$\left\{ \begin{aligned} \vec{y}_i &= \sqrt{\frac{i}{i+1}} \left(\frac{1}{i} \sum_{j=1}^i \vec{p}_j - \vec{p}_{i+1} \right) \quad i = 1, \dots, F-1 \end{aligned} \right. \quad (4.21)$$

$$\left\{ \begin{aligned} \vec{y}_F &= \frac{1}{\sqrt{F}} \sum_{j=1}^F \vec{p}_j. \end{aligned} \right. \quad (4.22)$$

Then, we have

$$\sum_{i=1}^F p_{iZ}^2 = \sum_{i=1}^F y_{iZ}^2 \quad (4.23)$$

$$\text{and} \quad d\vec{p}_1 \dots d\vec{p}_F = d\vec{y}_1 \dots d\vec{y}_F, \quad (4.24)$$

with similar transformations for the transverse components.

The Jacobian coordinate $\vec{y}_i$ with $i = 1, \dots, F-1$ is essentially the relative momentum coordinate between the $(i+1)$ th nucleon and a cluster of i nucleons. Any coalescence condition for the formation of F will involve these coordinates of $\vec{y}_1, \dots, \vec{y}_{F-1}$ and not the coordinate $\vec{y}_F$ which is the total momentum coordinate of the cluster with F nucleons. Thus, Eq. (4.20) is separable in terms of the Jacobian coordinates

$$B_F^1(\vec{F}) = A^F \int d\vec{y}_F \exp \left[-\frac{y_{FZ}^2}{2\sigma_Z^2} - \frac{\vec{y}_{F\perp}^2}{2\sigma_{\perp}^2} \right] \delta(\vec{F} - \sqrt{F} \vec{y}_F) \quad (4.25)$$

$$\times \left(\int d\vec{y}_1 \dots d\vec{y}_{F-1} \exp \left[-\sum_{i=1}^{F-1} \left(\frac{y_{iZ}^2}{2\sigma_Z^2} + \frac{y_{i\perp}^2}{2\sigma_{\perp}^2} \right) \right] g_F(\vec{y}_1, \dots, \vec{y}_{F-1}) \right).$$

The second factor is clearly independent of $\vec{F}$ and thus

$$B_F^1(\vec{F}) = A' \exp \left[-\frac{F_Z^2}{2F\sigma_Z^2} - \frac{\vec{F}_{\perp}^2}{2F\sigma_{\perp}^2} \right] = [W(\vec{F}/F)]^F. \quad (4.26)$$

Hence, we obtain the relation²⁰⁻²²

$$S_F(\vec{F}) \propto [S_1(\vec{F}/F)]^F. \quad (4.27)$$

This relation, obtained in the frame in which the center-of-mass is held fixed, can be shown to hold also for other coordinate frames.

When we apply the results of Eq. (4.26) to analyze the data for deuteron and triton production in the collision of $^{40}\text{Ar} + \text{KCl}$ at 800 MeV per nucleon, we find good general agreement (Figs. 4 and 5). However, the peripheral processes contribute substantially near the beam rapidity and target rapidity and show up as a discrepancy for the 10° and 15° data at the beam rapidity and at small momenta and large angles. It is of interest to investigate whether these discrepancies can be quantitatively accounted for by peripheral processes.

V. CENTER-OF-MASS MOMENTUM DISTRIBUTION

For intermediate energy, heavy-ion peripheral reactions around 10^3

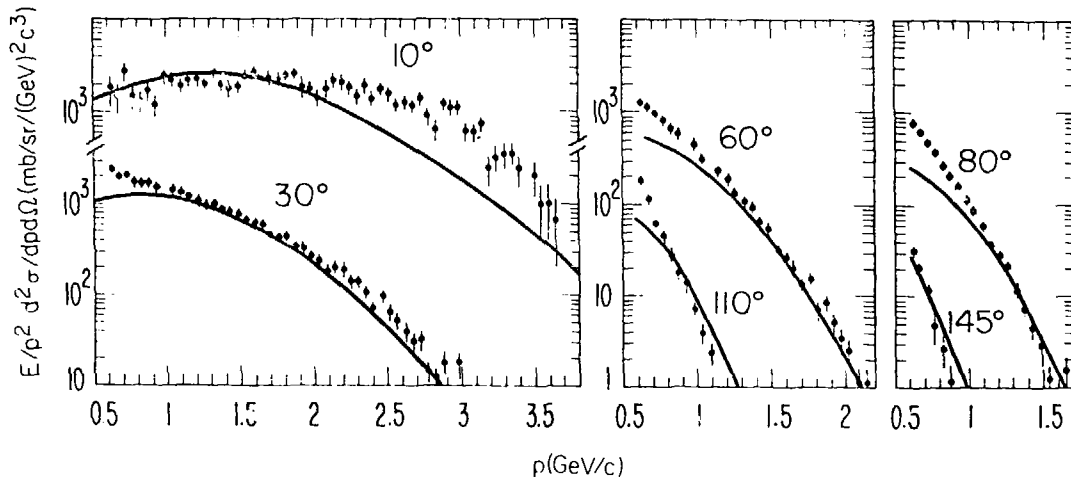
$\text{Ar} + \text{KCl} \rightarrow \text{d} + \text{X} \quad 800 \text{ MeV/A}$


Figure 4. The deuteron production invariant cross section in the collision of $^{40}\text{Ar} + \text{KCl}$ at an energy of 800 MeV per nucleon. The data points are from Nagamiya, *et al.*²⁰ The solid curves are theoretical results calculated with the ellipsoidal Gaussian distribution given by Eq. (4.26) with $A' = 2500 (\text{GeV}\cdot\text{mb})/\text{sr}-(\text{GeV}/c)^3$ and values of σ_z , $\sigma_\perp$ determined from the proton production data.

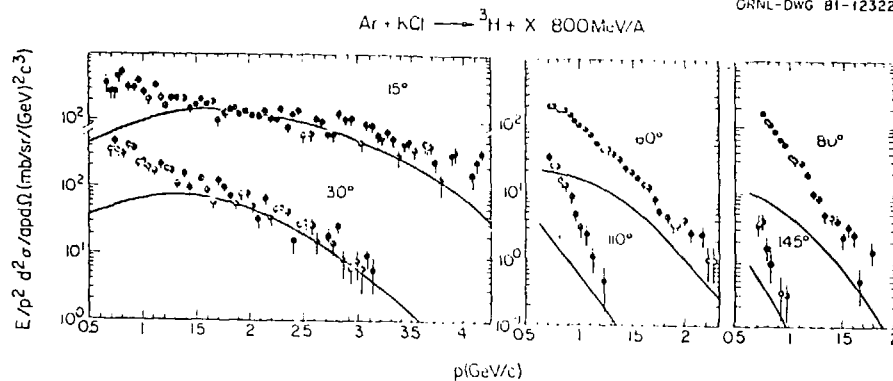


Figure 5. The triton production invariant cross section in the collision of ${}^{40}\text{Ar} + \text{KCl}$ at an energy of 800 MeV per nucleon. The data points are from Nagamiya, *et al.*²⁰ The solid curves are theoretical results calculated with the ellipsoidal Gaussian distribution given by Eq. (4.26) with $A^1 = 240$ ($\text{GeV}\cdot\text{mb}/\text{sr}\cdot(\text{GeV}/c)^3$) and values of σ_z and $\sigma_\perp$ determined from the proton production data.

MeV per nucleon, the trajectories of all the different orbits need not be in the form of a straight line. There is a distribution of the transverse momentum of the different orbits. This additional dispersion of the orbits gives rise to an additional width to the transverse momentum distribution of the fragments. The center-of-mass momentum distribution is also important for central collisions of unequal nuclei where the center-of-mass of the source moves with different velocity depending on the impact parameters.

The center-of-mass motion can be taken into account by including a distribution for the center-of-mass momentum $P(\vec{A})$ so that the probability differential is

$$dW(\phi_1 \phi_2 \dots \phi_A, \vec{A}) = C f(\phi_1) f(\phi_2) \dots f(\phi_A) P(\vec{A}) \delta\left(\vec{A} - \sum_{i=1}^A \vec{p}_i\right) d\vec{A} \prod_{i=1}^A d\vec{p}_i. \quad (5.1)$$

For the direct fragmentation and when $P(\vec{A})$ is a Gaussian distribution of the form

$$p(\vec{A}) \propto \exp \left[-\frac{(A_Z - \langle A_Z \rangle)^2}{2\sigma_{AZ}^2} - \frac{(\vec{A}_\perp - \langle \vec{A}_\perp \rangle)^2}{2\sigma_{A\perp}^2} \right], \quad (5.2)$$

we obtain

$$W_{F/A}(\vec{F}) \propto \exp \left[-\frac{(F_Z - (F/A)\langle A_Z \rangle)^2}{2\sigma_{FZ}^2} - \frac{(\vec{F}_\perp - (F/A)\langle \vec{A}_\perp \rangle)^2}{2\sigma_{F\perp}^2} \right] \quad (5.3)$$

where the width σ_{F1} is

$$\sigma_{F1}^2 = \frac{F(A-F)}{A} \sigma_o^2 + \frac{F^2}{A^2} \sigma_{A1}^2. \quad (5.4)$$

This is essentially the result derived in Ref. 2 with a small difference of $F(F-1)$ and $A(A-1)$ instead of F^2 and A^2 . Equation (5.4) has been found to agree well with experiment² when a transverse momentum dispersion is allowed for in the collision of ^{16}O at 90 MeV per nucleon.

The center-of-mass momentum distribution for central collisions must be taken into account for the collision of unequal nuclei. Such studies in the framework of the basic distribution model have not been made but deserve special attention in the future.

VI. SUMMARY AND DISCUSSIONS

The basic distribution model we have introduced allows us to discuss two processes involved in intermediate-energy, heavy-ion reactions. These are the direct peripheral fragmentation process in which the fragment suffers no collision with the other nucleus and the central collision process in which the fragments come out of the participant region where many collisions occur. These are two different and opposite extremes. Clearly, they are not the only processes present in intermediate-energy, heavy-ion collisions. For example, there is also the hard-scattering process in which the fragment suffers a single collision with nucleons in the other nucleus before it is detected. From our results in the relativistic case, we expect the hard-scattering process to become important in the region of transverse momentum slightly beyond the direct fragmentation peak.¹ The inclusion of such a

process will supplement the two different processes discussed here and will probably cover the dominant processes of interest.

The data for the proton production in the collision of $^{40}\text{Ar} + \text{KCl}$ at 800 MeV per nucleon indicate that the distribution of the nucleons before flying apart in a central collision is approximately that of an ellipsoidal Gaussian distribution having unequal transverse and longitudinal momentum dispersions. In fact, the ratio of the longitudinal momentum dispersion to that of the transverse is approximately 4 to 3. This is consistent with the type of momentum distributions one obtains in a cascade calculation and may perhaps rule out other types of dynamics such as hydrodynamics. How the distribution changes as a function of collision energy is clearly an important piece of information. Systematic investigations of such a variation are of great interest.

It is interesting to note that the coalescence power law relation can be obtained from the basic distribution model. However, this is true for a Gaussian form of the basic momentum distribution where the Jacobian coordinate separation can be carried out. For low-energy collisions around 50 MeV per nucleon, the Pauli exclusion principle is still important. One expects the basic distribution to assume a shape given by Eq. (4.9) or (4.10) with $2|\vec{p}_0| < p_F$. In that case, the coalescence relation need not be valid, as the momentum coordinates may not be separable. Furthermore, as one can judge from Figs. 4 and 5, the coalescence power law relation holds well for high momenta but may not hold for low momenta at large angles; there may be other processes which contribute to the spectra at low momenta. An experimental investigation of particle production around the energy of 50 MeV per nucleon will be

of interest in clarifying these points.

So far, correlation between nucleons is included only from the construction of the momentum distribution $w(\vec{p})$. It is necessary to examine, in the future, correlations due to the symmetry or antisymmetry of the particle coordinates which were found to be important in narrowing the widths of the fragment momentum distribution.²³

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THE EMISSION OF LIGHT NUCLEAR FRAGMENTS IN REACTIONS
INDUCED BY 86 MeV/u ^{12}C PROJECTILES

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If a heavy nucleus, e.g., Au, is bombarded with relativistic protons¹⁾ light nuclear fragments (LF) with element numbers $\sim 5 \leq Z \leq 20$ are emitted which have a nearly isotropic angular distribution in the Lab system. Their energy spectrum shows an exponential falloff, with a maximum being determined by the Coulomb energy of the total system. These fragments are believed to result from nuclear spallation or some sort of evaporation process.

Using the 86 MeV/u ^{12}C beam of the CERN-SC, several targets in the range between ^{58}Ni and ^{238}U have been irradiated. The experimental setup consisted of the following detectors:

- a large area position sensitive ionisation chamber.
- a heavy ion hodoscope (HIL) which consisted of 10 position sensitive parallel plate detectors covering a solid angle of 2.5π . These detectors were sensitive to nuclear fragments heavier than carbon.

¹⁾ A.M. Poskanzer, G.W. Butler, and F.K. Hyde, Phys.Rev. C3, 882 (1971).

- a 0° hodoscope consisting of 27 plastic scintillators which covered most of the angular range between 1.5° and 10° around the beam axis. These detectors were used to measure spectator fragments of the projectile.
- two solid state dE/dx -E telescopes measuring the spectra of protons and alpha particles.

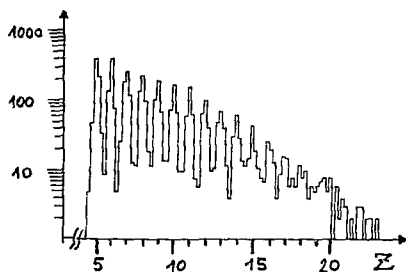


Fig. 1. Element distribution of light fragments for an Au target at a Lab angle of 18° . The intensities have been integrated over energy between 1.5 and 15 MeV/u.

In this experiment light fragments were also observed, and their distribution is shown in Fig. 1. Both the relative intensities of different elements as well as their absolute intensities are very similar to the distributions observed with relativistic protons of 5.5 GeV. Because of the large differences in size and speed of the incident projectile as well as the available energy this result is rather striking. From this similarity one can conclude that the LF are not the remnants of some sort of fireball, but originate from one of the colliding nuclei - in our case the target nucleus. Going to lighter nuclei, similar intensities and energy spectra of LF are observed; the main differences are caused by the reduced Coulomb energies. There is a rather low coincidence probability between a LF and the 0° hodoscope, which

confirms that the LF predominantly originate from nonperipheral collisions.

Interesting information could be obtained from coincidences measured with the heavy-ion hodoscope (HIL) which allowed determination of correlations between a LF and other nuclear fragments (NF) with masses $\sim 10 \leq A \leq 140$. Triggering on a LF the probabilities for associate NF observed for an Au target are given below:

- No other nuclear fragments;
the residue survives ~50%
- Two heavy fission-like
fragments ~50%
- More than two fragments $\leq 5\%$

These numbers have been corrected for the efficiency of the heavy-ion hodoscope, assuming a $1/\sin \theta$ distribution of the fission fragments in the Lab system and the LF have been integrated over energy between 1.5 and 15 MeV/u. The errors in the numbers are ~15% and result mainly from the uncertainties of the speed of the emitting system.

The most interesting observation is the very high probability with which the residual nucleus survives after the emission of a fragment as large as Ne or even Ca. This is difficult to explain if spallation is assumed to be the production mechanism.

In order to obtain some information about the time at which the LF is emitted as compared to the fission process, we have looked for correlations between the energy spectrum of LF and the angle between the light and the forward-going fission fragment. No correlations have been observed. Furthermore, the energy spectrum of the LF is the same

if the residual nucleus is fissioning or not. From this we conclude that the LF is emitted prior to fission.

One further information is the variation of the energy spectrum of LF with emission angle, which under the assumption of a two-step process (isotropic emission from a moving source) can be used to determine the speed of the source. This is a rather general assumption and should be valid for both very fast (explosion) and very slow processes (evaporation). With relativistic protons¹⁾ this speed has been determined to 0.0005 c, which is much smaller than the speed of the compound system (0.016 c). In this experiment a speed of 0.03 c has been determined for oxygen fragments, which is slightly higher than the compound nuclear limit (0.025 c). Triple coincidences between a light and two fission fragments, however, have shown that the average speed is considerably smaller, around 0.01 c. We therefore conclude that the basic assumption of a two-step process is in disagreement with the experimental observations.

Summarizing all points, the emission of light fragments can be explained neither by spallation nor by evaporation. The exact mechanism is not yet known.

A RELATIVISTIC QUANTUM FIELD THEORY OF NUCLEAR STRUCTURE*

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ABSTRACT

We review a model relativistic quantum field theory of the nuclear many-body problem containing baryons and vector and scalar mesons. A simple solution to the field equations exists which becomes increasingly valid as the density of the system increases. We review the implied properties of nuclear and neutron matter at all baryon densities. The properties of finite nuclei are discussed within the framework of the mean-field Thomas-Fermi approximation. Serot's work extending the theory to include π and ρ mesons in a renormalizable non-abelian gauge theory is reviewed. Recent calculations of Horowitz and Serot of the properties of finite nuclei within relativistic Hartree theory based on this model are presented. Calculations of isoscalar and isovector collective excitations of nuclei within this coupled meson-baryon field theory are discussed.

Since the model retains Lorentz covariance and includes the meson degrees of freedom explicitly, it may be used to obtain insight into the behavior of nuclear matter under extreme conditions. The equation of state of neutron matter at all temperatures is examined. Kleppinger's study of the behavior of the moving nuclear fluid and of sources in the moving fluid, and Bielajew's analysis of the behavior of the nuclear medium undergoing extremely rapid rotation are discussed.

Corrections to the mean-field theory arising from quantum fluctuations, correlations (both short-range and long-range), and exchange are examined. Finally the motivation and validity of this approach to nuclear structure theory is summarized.

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I. Introduction

Nuclei are unique forms of matter, consisting of many baryons in close proximity. They are the principal sources of mass and energy in the universe. All of the fundamental forces of nature, strong, electromagnetic and weak are active in nuclei, as is gravity if one includes neutron stars. These are just enormous nuclei, with radii a factor of $\sim 10^{19}$ larger than those of atomic nuclei and densities up to an order of magnitude higher, held together by the gravitational attraction.

The goal of nuclear theory is to first develop a description of nuclear structure and behavior starting from a minimal number of assumptions. For example, one seeks a concise description of nuclear charge and mass densities, and of the simple modes of motion of nuclei, both single-particle and collective. One would then like to extrapolate away from the properties of observed atomic nuclei into new regimes where the nuclear system is subject to extreme conditions of density, temperature, size, flow, and rotation. It is just such extrapolations which are required when nuclei are subject to astrophysical conditions present in massive condensed stellar objects, or laboratory conditions obtained through energetic heavy-ion interactions. It is essential to have a consistent underlying theoretical framework in carrying out these extrapolations.

The traditional approach to the theory of nuclear structure has involved proton and neutron coordinates and static two-body potentials. The Schrödinger equation is then solved within the framework of non-relativistic many-body theory. This approach has, of course, led to a

very successful understanding of many features of nuclear structure and behavior. Here, however, I would prefer to dwell on the inadequacies of this approach:

- 1) It is an approximation to describe nuclei in terms of only nucleon coordinates. The theory should be formulated in terms of a minimal and complete set of underlying degrees of freedom; for all relevant nuclear energies and densities, these appear to be mesons and baryons.
- 2) At nuclear densities the velocity is already $v/c \sim 1/4$ for the fastest nucleons at the Fermi surface and as the density, temperature, and flow velocities are raised, the relativistic propagation of the baryons, and relativistic propagation and retardation of the mesons generating the interactions, must be taken into account.
- 3) Correct, causal restrictions on the propagation of the modes of excitation of the interacting system must be contained in the theory.

The only consistent theoretical framework we have for describing such an interacting many-body system is relativistic quantum field theory. Given a lagrangian density, we in principle have a complete calculational framework with which to answer any question which can be asked about the system. It is essential to work with a renormalizable theory if one is to have a consistent theoretical framework in which to make finite, well-defined predictions for nuclear phenomena to all orders in the interaction strength. Imposing renormalizability also substantially limits the types of theories and interactions available. Whether the theory is really "correct" at sufficiently small distances is a question which can be asked of all existing theories of nature; it is always a matter of over what range of variables a theory is

meaningful. In the final analysis a theory is demonstrated to be correct or incorrect only by comparing precise theoretical predictions with experiment.

I want to discuss a model relativistic quantum field theory which we have been studying at Stanford for the past several years (1-16) and some of the results that have been obtained. Many of the ideas here are similar in spirit to those of Johnson and Teller (17) as developed by Duerr (18), and applications of the present model and similar ones (19,20) have been made by many others. Here I would just like to single out in particular Kerman and Miller (21), Boguta and Bodmer (22), and Boguta and Rafelski (23) for their important contributions.

II. The Model

As a first simple model (Model I) consider the fields in Table 1a. We assume that the neutral

	Field	Description	(particles)	Mass
1a	ψ	baryon	(p,n,...)	M
	ϕ	neutral scalar meson	(σ)	m_s
	V_λ	neutral vector meson	(ω)	m_v
1b	π	charged pseudoscalar meson	(π)	m_π
	ρ_λ	charged vector meson	(ρ)	m_ρ

Table 1. Fields in the model

scalar meson couples to the scalar baryon density through a coupling $g_s \bar{\psi}\psi\phi$ and that the neutral vector meson couples to the conserved baryon current through $ig_v \bar{\psi}\gamma_\lambda \psi V_\lambda$. As motivation for this model, we observe that in the limit of heavy, static baryons the one-meson ex-

change graphs give rise to an effective nucleon-nucleon potential of the form (we take $\hbar = c = 1$)

$$V_{\text{eff}} = \frac{g_v^2}{4\pi} \frac{e^{-m_v r}}{r} - \frac{g_s^2}{4\pi} \frac{e^{-m_s r}}{r} \quad (1)$$

and for appropriate choices of the coupling constants and masses, this potential can be made attractive at large separations and repulsive at short distances in accord with the nucleon-nucleon force. The reader will ask, "Where are the pions?". In fact, the effects of the pion interaction largely average out in the description of the bulk properties of nuclei due to the strong spin dependence of the coupling. We must ultimately include additional fields for the pi, rho, etc. if we hope to achieve a truly quantitative description of the nuclear system. We shall return to this shortly.

The field equations for this Model I are

$$(\square - m_s^2)\phi = -g_s \bar{\psi}\psi \quad (2)$$

$$\frac{\partial}{\partial x_\nu} F_{\mu\nu} + m_v^2 V_\mu = ig_v \bar{\psi}\gamma_\mu \psi \quad (3)$$

$$[\gamma_\mu \left(\frac{\partial}{\partial x_\mu} - ig_v V_\mu \right) + (M - g_s \phi)] \psi = 0 \quad (4)$$

Here $F_{\mu\nu} = \partial V_\nu / \partial x_\mu - \partial V_\mu / \partial x_\nu$ is the antisymmetric field tensor. Equation (2) is simply the Klein-Gordon equation with a scalar source; Eq.

(3) looks like massive Q.E.D. with the conserved baryon current

$B_\mu = i\bar{\psi}\gamma_\mu\psi$ rather than the (conserved) electromagnetic current as

source; and Eq. (4) is the Dirac equation with the scalar and vector

fields introduced in a minimal fashion. The free-field lagrangians for this system are contained in any text, and the interaction lagrangian is just $\mathcal{L}' = ig_V \bar{\psi} \gamma_\mu \psi V_\mu + g_S \bar{\psi} \psi \phi$. The theory is renormalizable for it is similar to massive QED with a conserved current and additional scalar interaction. Since the dimensionless coupling constants will turn out to be large, we have really not made much progress unless a sensible starting solution to the theory can be found. Note that this strong-coupling aspect is always present in nuclear theory and just because the strong coupling is incorporated into internucleon potentials does not mean that this difficulty has been avoided. The main point of the present work is that we can find a simple solution to the field equations which becomes increasingly valid as the baryon density of the system increases; this solution then provides a meaningful starting description of the system as well as a consistent basis for computing corrections through the use of relativistic quantum field theory and standard many-body techniques (24).

III. Mean Field Theory

Consider a uniform system of baryons in a box of volume V . As the baryon density increases, so do the source terms on the right-hand-sides of Eqs. (2,3). When the source terms are large, the meson fields can be replaced by their expectation values which are then classical fields

$$\phi \rightarrow \langle \phi \rangle \equiv \phi_0 \quad (5)$$

$$V_\lambda \rightarrow \langle V_\lambda \rangle \equiv i\delta_{\lambda 4} V_0 \quad (6)$$

and Eqs. (2,3) can be immediately solved for a static, uniform system to give

$$\phi_0 = \frac{g_s}{m_s} \langle \bar{\psi} \psi \rangle \equiv \frac{g_s}{m_s} \rho_s \quad (7)$$

$$V_0 = \frac{g_v}{m_v} \langle \psi^\dagger \psi \rangle = \frac{g_v}{m_v} \rho_B \quad (8)$$

When these classical fields are substituted into the Dirac Eq. (4), this equation is linearized and may be solved exactly. The stationary state solutions for a uniform system $\psi = \exp\{-i\epsilon t + i\mathbf{k} \cdot \mathbf{x}\} U(\mathbf{k}, \lambda)$ satisfy a modified Dirac equation with effective mass

$$M^* = M - g_s \phi_0 \quad (9)$$

and energy spectrum.

$$\epsilon_k^{(\pm)} = g_v V_0 \pm (k^2 + M^{*2})^{1/2} \quad (10)$$

The solutions satisfy the relation

$$\bar{U}U = \frac{M^*}{(M^{*2} + k^2)^{1/2}} U^\dagger U \quad (11)$$

The scalar density is reduced relative to the baryon density by Lorentz contraction. The Dirac fields may be expanded in terms of the solutions to the Dirac equation with coefficients $(A_{\mathbf{k}\lambda}, B_{\mathbf{k}\lambda})$ in standard fashion, and the hamiltonian density becomes

$$\begin{aligned} \hat{H}_{\text{MFT}} = & g_V V_0 \rho_B + \frac{1}{V} \sum_{\underline{k}\lambda} (\underline{k}^2 + M^*{}^2)^{\frac{1}{2}} (A_{\underline{k}\lambda}^\dagger A_{\underline{k}\lambda} + B_{\underline{k}\lambda}^\dagger B_{\underline{k}\lambda}) \\ & + \frac{1}{2} m_s^2 \phi_0^2 - \frac{1}{2} m_V^2 V_0^2 \end{aligned} \quad (12)$$

The new normal-mode amplitudes can be interpreted as creation and destruction operators and the system quantized in canonical fashion. Here we have chosen to define the mean field theory (MFT) by normal ordering in these creation and destruction operators. (Corrections to this procedure will be discussed with other modifications at the conclusion of this talk). Since the hamiltonian density in Eq. (12) is diagonal, the exact solution for the MFT is known and all the properties of the system can be computed within this exactly solvable model.

IV. Nuclear Matter

Consider uniform nuclear matter. The ground state of Eq. (12) is obtained by filling the states with wave number $\underline{k}$ and spin-isospin degeneracy γ up to the fermi level ℓ_F . The equation of state in the MFT is then easily found to be

$$e = \frac{1}{2} C_V^2 n_B^2 + \frac{1}{2C_s^2} (1-\chi)^2 + \frac{\gamma}{(2\pi)^3} \int_0^{\ell_F} d\underline{t} (t^2 + \chi^2)^{\frac{1}{2}} \quad (13)$$

$$p = \frac{1}{2} C_V^2 n_B^2 - \frac{1}{2C_s^2} (1-\chi)^2 + \frac{1}{3} \frac{\gamma}{(2\pi)^3} \int_0^{\ell_F} d\underline{t} \frac{t^2}{(t^2 + \chi^2)^{\frac{1}{2}}} \quad (14)$$

$$n_B = \frac{\gamma}{(2\pi)^3} \int_0^{\ell_F} d\underline{t} \quad (15)$$

where e , p , and n_B are the dimensionless energy density, pressure, and baryon density and all quantities are made dimensionless through the

nucleon mass M . In these equations

$$\chi \equiv M^*/M \quad (16)$$

is the effective mass. The first two terms in Eqs. (13, 14) arise from the mass terms for the vector and scalar fields, the final two terms are those of a relativistic Fermi gas of baryons of mass χ . The equation of state is given in parametric form $e(n_B)$, $p(n_B)$. It remains to determine the effective mass. This can be done by minimizing $e(\chi)$ with respect to the parameter χ (equivalently with respect to ϕ_0) which leads to the self-consistency relation

$$\chi = \left[1 + C_s^2 \frac{\gamma}{(2\pi)^3} \int_0^{\lambda_F} \frac{dt}{(t^2 + \chi^2)^{3/2}} \right]^{-1} \quad (17)$$

This is identical to the scalar meson field equation where Eq. (11) has been used in the computation of the scalar density. It is evident that at high density $\lambda_F \rightarrow \infty$ the energy density is dominated by the vector repulsion, as is the pressure. In this limit $p \rightarrow e$, which implies that the thermodynamic speed of sound approaches the velocity of light from below. At lower densities, the attractive scalar interaction can be made to dominate if the coupling constants are chosen properly. The nuclear medium then saturates. The coupling constants can be chosen to reproduce the equilibrium properties of nuclear matter ($\gamma=4$, $(E - B m_\pi c^2)/B = -15.75$ MeV, $k_F = 1.42 F^{-1}$) according to

$$C_v^2 = g_v^2 (M^2/m_v^2) = 195.7 \quad (18)$$

$$C_s^2 = g_s^2 (M^2/m_s^2) = 266.9 \quad (19)$$

Note that only the ratio of coupling constants to masses enters here. The resulting saturation curve is shown in Fig. 1 (1,2). Note that it is the relativistic nature of the scalar and vector couplings that is responsible for saturation in this model; a Hartree-Fock variational estimate built on the potential limit in Eq. (1) demonstrates that in this limit, the system is unstable against collapse (24).

The corresponding curve for neutron matter obtained by simply replacing $\gamma = 2$ is also shown in Fig. 1, and the equation of state (pressure vs. mass density) for neutron matter at all densities in Fig. 2. In Model I there is a phase transition similar to the liquid-gas transition in the Van der Waal's equation of state, and the properties of the two phases are deduced through a Maxwell construction. The approach to the causality limit $p \rightarrow e$ is also noted. There are only two parameters in the MFT of nuclear matter, and all other properties may now be calculated from the constants determined in Eqs. (18, 19); these are compared with experiment in Table 2. The quantity a_4 is the coefficient in the symmetry energy of the semi-empirical mass formula (24,11) and K_V is the bulk modulus (24). If the scalar field is identified as the σ meson (a broad two-pion resonance) at $m_s/M \cong .59$ and the vector field with the ω meson ($m_v/M = .835$), then we obtain from Eqs. (18,19) the coupling constants shown in Table 2. These are compared with values obtained from a phase-shift analysis of nucleon-nucleon scattering (2).

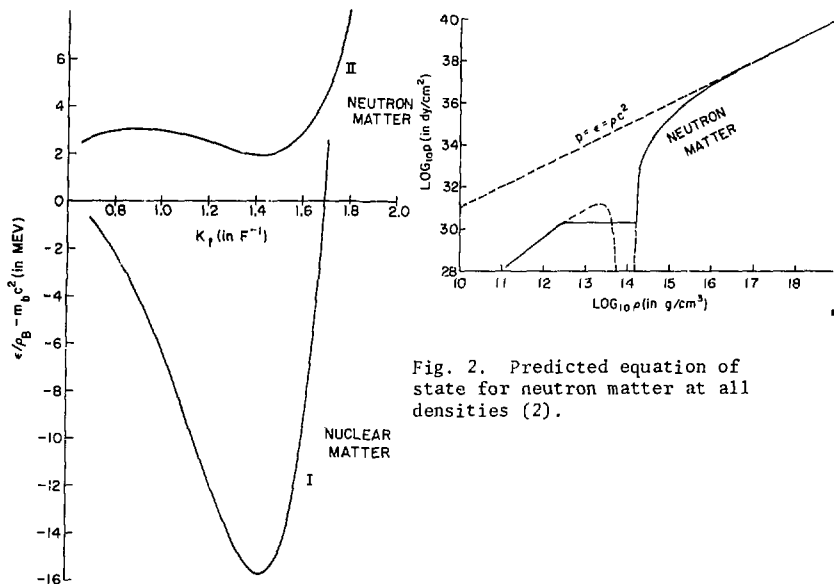


Fig. 2. Predicted equation of state for neutron matter at all densities (2).

Fig. 1. Saturation curve for nuclear matter. Coupling constants chosen to fit value and position of minimum. Prediction for neutron matter ($\gamma=2$) also shown (2).

Table 2. Calculated properties of nuclear matter and basic couplings (2)

	a_4	M^*/M	K_V^{-1}	$g_S^2/4\pi$	$g_V^2/4\pi$
Theory	22.0 MeV	0.56	550 MeV	7.39	10.8
Experiment	33.2 MeV	(0.6)	(200-300 MeV)	8.2(σ)	17.3(ω)

V. Finite Nuclei

Although the MFT has so far only been justified at high density, it is of interest to investigate the phenomenology of the MFT as ap-

plied to existing nuclei. Does it give sensible results? Of course, any real theoretical justification for these applications has to lie in systematically evaluating the corrections to the theory. Since we have a completely consistent relativistic theoretical framework, we are in a position to do this. I will return to these points at the end of my talk. First, however, let us discuss applications to finite nuclei.

To extend the previous analysis to finite systems with fixed number of baryons we allow spatial variations in the mean fields $\phi_0(|\underline{x}|)$, $V_0(|\underline{x}|)$. This adds the terms $(m_s^2/2)(\nabla\phi_0)^2 - (m_v^2/2)(\nabla V_0)^2$ to the hamiltonian density. It will initially be assumed that these fields vary slowly enough so that at each point the baryons can be treated as moving in locally constant fields and the previous analysis from nuclear matter applied. The ground state of the system is then obtained by minimizing the total energy E as a function of the local Fermi wave number $k_F(|\underline{x}|)$. The constraint of a fixed number of baryons B is incorporated through a lagrange multiplier μ . Thus we set

$$\delta E - \mu \delta B = 0 \quad (20)$$

Since V_0 and ϕ_0 satisfy the field equations

$$(\nabla^2 - m_s^2)\phi_0(r) = -g_s \rho_s(r) \quad (21)$$

$$(\nabla^2 - m_v^2)V_0(r) = -g_v \rho_B(r) \quad (22)$$

the variations with respect to these quantities vanish and they may effectively be held constant during the variation. The result of the

variational principle is that $k_F(r)$ must satisfy the equation

$$g_V V_0 + (k_F^2 + M^{*2})^{1/2} = \mu = \text{const} \quad (23)$$

which can be used to find $k_F(\phi_0, V_0, \mu; r)$. The local baryon and scalar densities are expressed in terms of $k_F(r)$ and $M^*(r)$ as

$$\rho_B = \frac{Y}{(2\pi)^3} \int_0^{k_F(r)} dk \quad (24)$$

$$\rho_S = \frac{Y}{(2\pi)^3} \int_0^{k_F(r)} \frac{M^*}{(M^{*2} + k^2)^{1/2}} dk \quad (25)$$

Since in this Thomas-Fermi MFT the baryon and scalar densities vanish identically past a certain radius r_0 , the asymptotic form of the solution is known and hence $(V_0'/V_0)_{r_0}$ and $(\phi_0'/\phi_0)_{r_0}$ are determined. The coupled, non-linear differential Eqs. (21, 22) can be integrated in from r_0 , and the pair $(\mu, V_0(r_0))$ [or $(\mu, \phi_0(r_0))$] varied until a solution is found which satisfies the interior boundary conditions $V_0'(0) = \phi_0'(0) = 0$. The total number of baryons is determined at the end of the calculation by integrating Eq. (24). This has been carried out in Refs. (8, 11) [see also (23)]. There are now two additional parameters in the theory since (m_s, m_v) must now be specified independently. We choose m_s/M to fit the observed surface thickness of nuclei $t \cong 2.4 F$. The dependence on m_v/m_s is not strong (8) and if we arbitrarily choose $m_v/m_s = 1.5$ in rough accord with the observed meson masses ($m_v/m_s \cong 1.42$) then the resulting fields and densities for $B = 40$ are those shown in Fig. 3. (Here it is assumed that $e = 0$ so that

$N = Z$). It is often numerically useful to simplify Eq. (22) by assuming that the baryon density does not vary significantly over the compression wavelength of the vector mesons $1/m_V$; in that case Eq. (22) reduces to

$$V_0(r) = (g_V/m_V^2)\rho_B(r) \quad (26)$$

and one is left with the single non-linear differential Eq. (21) to integrate. Comparison with the solutions to the full set of coupled differential equations for different values of m_S/m_V shows that this is an unessential approximation (8). The dimensionless quantities (n_B, χ) calculated with Eq. (26) are plotted vs. $r \equiv m_S|x|$ for a nucleus with $r_0 = 20$ (and resulting $B = 67.19$) in Fig. 4.

A Foldy-Wouthuysen reduction of the single-particle Dirac equation for non-relativistic nucleons

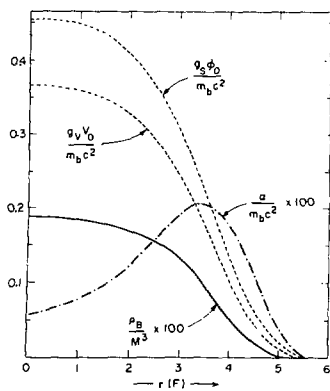


Fig. 3. Fields and baryon density for a finite system with $B=40$ ($N=Z$). $\alpha \equiv \alpha_{s.o.}(r)$ is the spin-orbit density (8).

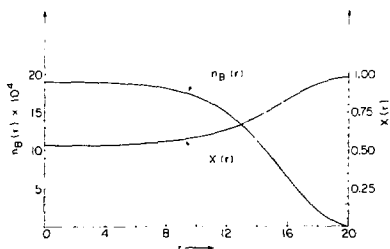


Fig. 4. Calculated static baryon density $n_B(r) = \rho_B/M^3$ and effective mass $\chi = M^*/M$ vs. $r = m_S|x|$ for nucleus with $r_0 = 20$ and $B = 67.19$ ($N=Z$) (13). Uses couplings of Eqs. (18,19) and $m_S/M = 0.736$.

moving in the potential generated by the scalar and vector fields $\phi_0(r)$ and $V_0(r)$ allows us to identify the effective single-particle spin-orbit interaction $-\alpha_{so}(r)\underline{s}\cdot\underline{\ell}$. The quantity $\alpha_{so}(r) = -(g_V V_0'(r) + g_S \phi_0'(r))/2M^2 r$ is also shown in Fig. 3. Note that whereas the effects of V_0 and ϕ_0 cancel in the binding energy, they add in the spin-orbit interaction, which of course, forms the basis of the nuclear shell model. Brockmann and Weise (25) have also investigated the spin-orbit interaction in this theory.

In order to realistically discuss nuclei with $N \neq Z$ it is necessary to extend Model I to at least include rho mesons which couple to the isovector current, as well as the Coulomb interaction.

VI. Serot's Model

Serot has extended Model I to include fields π and b_μ for the pi and rho mesons (Table 1b - we reserve the symbol ρ for nuclear densities). He develops a renormalizable quantum field theory using a gauge theory for the rho field b_μ , a complex Higgs doublet χ , and spontaneously broken gauge symmetry (10). The general approach is that reviewed by Abers and Lee (26). Serot includes four additional terms in the previous lagrangian, and we shall refer to this as Model II. The new contributions are a rho-nucleon coupling $\frac{1}{2} g_\rho \bar{\psi} \gamma_\mu \underline{\tau} \cdot b_\mu \psi$ and

$$\begin{aligned} \mathcal{L}_\pi = & -\frac{1}{2} \left\{ \left(\frac{\partial}{\partial x_\mu} + g_\rho b_\mu \times \right) \underline{\pi} \right\}^2 + m_\pi^2 \underline{\pi}^2 \} \\ & + i g_\pi \bar{\psi} \gamma_5 \underline{\tau} \cdot \underline{\pi} \psi + \frac{1}{2} g_\phi \pi^2 \phi \end{aligned} \quad (27)$$

$$\mathcal{L}_H = - \left[i \left(\frac{\partial}{\partial x_\mu} - \frac{i}{2} g_\rho \tau \cdot \underline{b}_\mu \right) \chi \right]^2 + m_0^2 \chi^\dagger \chi + \lambda_0 (\chi^\dagger \chi)^2 \quad (28)$$

$$\mathcal{L}_{\text{Gauge}} = - \frac{1}{4} \left[\frac{\partial}{\partial x_\mu} \underline{b}_\nu - \frac{\partial}{\partial x_\nu} \underline{b}_\mu + g_\rho \underline{b}_\mu \times \underline{b}_\nu \right]^2 \quad (29)$$

This theory now possesses a local $SU(2)$ symmetry. The symmetry is broken by taking $m_0^2 < 0$. The χ field is redefined in terms of a new ("Higgs") field η which has zero vacuum expectation value, and the massless $\underline{b}_\mu$ field now acquires a mass m_ρ . The lagrangian can be rewritten in the unitary gauge which makes the particle content and the Feynman rules manifest. It is given in Serot's paper. Here we simply summarize some of his principal results:

1) The coupling constant $g_{\phi\pi}$ can be chosen so that a calculation of pion-nucleon scattering at the "tree-graph" level gives scattering lengths in reasonable agreement with experiment. The implied width of the σ due to the free decay $\sigma \rightarrow 2\pi$ is then comparable to that observed experimentally.

2) If the coupling constant g_ρ is determined from the decay $\rho \rightarrow 2\pi$ calculated in the same approximation, the result is $C_\rho^2 = g_\rho^2 (M^2/m_\rho^2) = 54.71$.

3) At high densities, the MFT will again hold, and for $N \neq Z$ the third component of the rho field develops an expectation value. We thus replace $\underline{b}_\mu$ by

$$\langle \underline{b}_\mu^{(j)} \rangle = i \delta_{\mu 4} \delta^{j3} b_0 \quad (30)$$

In the limit of a large Higgs mass $m_H \rightarrow \infty$, Serot shows that the Higgs sector has negligible effect on the low-energy behavior of the theory, and from the field equations for uniform nuclear matter

$$2b_0 = \frac{g_\rho}{m_\rho} \langle \psi^\dagger \tau_3 \psi \rangle \equiv \frac{g_\rho}{m_\rho} \rho_3 \quad (31)$$

Equations (30, 31) imply an additional contribution to the symmetry energy of nuclear matter and yield the new value $a_4 = 33.6$ MeV, in substantially better agreement with the experimental value quoted in Table 2.

4) In Serot's Model II the field Eqs. (21-25) for finite nuclei must be extended to include terms for the condensed rho field $b_0(r)$ as well as for the Coulomb potential $A_0(r)$.

$$(\nabla^2 - m_\rho^2) b_0 = -\frac{1}{2} g_\rho \rho_3(r) \quad (32)$$

$$\nabla^2 A_0 = -e \rho_p(r) = -e \frac{1}{2} [\rho_B(r) + \rho_3(r)] \quad (33)$$

and the chemical potential conditions for protons and neutrons become respectively

$$\mu_p \equiv g_v v_0 + \frac{1}{2} g_\rho b_0 + e A_0 + (k_{Fp}^2 + M^{*2})^{\frac{1}{2}} = \mu \quad (34)$$

$$\mu_n \equiv g_v v_0 - \frac{1}{2} g_\rho b_0 + (k_{Fn}^2 + M^{*2})^{\frac{1}{2}} = \mu \quad (35)$$

The charge density of ^{40}Ca as recomputed by Serot is shown in Fig. 5. It is compared with the charge density obtained from electron scatter-

ing (11). In this calculation $e=0$, the scalar mass is fit to the surface thickness, and Eq. (26) is used for the massive vector mesons.

Figure 6 shows the results of Serot's calculation for ^{208}Pb . Here a value

$$C_\rho^2 = g_\rho^2 (M^2/m_\rho^2) = 100.2 \quad (36)$$

is used to get the precise value of Z/A . (With the value of C_ρ^2 determined from $\rho \rightarrow 2\pi$, the equilibrium lead nucleus is ^{219}Pb .) A slightly different value of $k_F = 1.325 \text{ F}^{-1}$ is used for nuclear matter

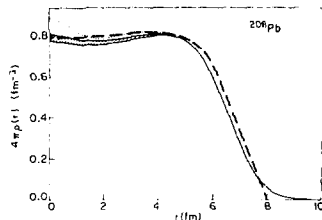
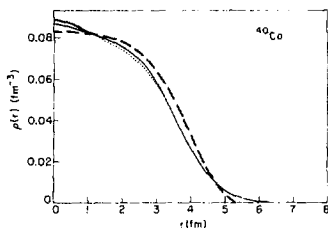


Fig. 5. Charge density of ^{40}Ca calculated in relativistic MFT (dashed line) and experimental value from electron scattering (solid curve). Surface thickness of Ca determines m_s (11).

Fig. 6. Same as Fig. 5 for ^{208}Pb (11).

to reproduce the value of the central density in ^{208}Pb . The resulting shapes of all densities are then predicted. The semi-empirical mass parameters obtained from the computed properties of ^{40}Ca and ^{208}Pb expressed in the form

$$\frac{E}{B} = -a_1 + \frac{a_2}{B^{1/3}} + \frac{a_3 Z^2}{B^{4/3}} + \frac{a_4 (B-2Z)^2}{B^2 (1+\zeta/B^{1/3})}$$

are given in Table 3, where they are compared with the best experimental values of these quantities.

	a_2	a_3	a_4	ζ	m_s/M	a_2/t
theory	27.2	0.740	37.0	3.18	0.745	11.5
experiment	17.8	0.710	33.2	3.28	0.59	7.4

Table 3. Comparison of semi-empirical mass formula parameters determined from the calculated properties of ^{40}Ca and ^{208}Pb in Model II with the experimental data (11). All a_i in MeV and a_2/t in MeV/F.

ζ represents the "surface symmetry energy."

Serot has found the single-nucleon spectra by solving the Dirac equation in the fields $\phi_0(r)$, $V_0(r)$, $b_0(r)$, and $A_0(r)$ obtained for ^{208}Pb . This is shown in Fig. 7. This nuclear model predicts the spectra with reasonable accuracy using static meson-field potentials determined solely from the bulk properties of massive nuclei.

VII. Collective Modes

The collective modes of the coupled meson-baryon system may be investigated by allowing the condensed meson fields and baryon density to acquire a slow time dependence. If a velocity field

$$v = -\vec{\nabla} \psi \quad (37)$$

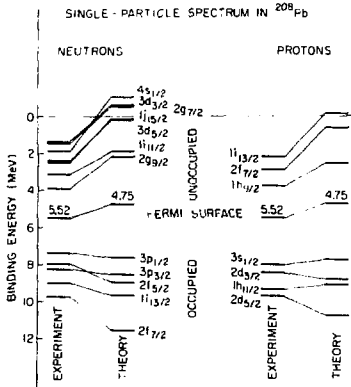


Fig. 7. Single-nucleon spectra for nucleons moving in the Thomas-Fermi MFT meson-fields of ^{208}Pb . Experimental values are taken from neighboring nuclei (11).

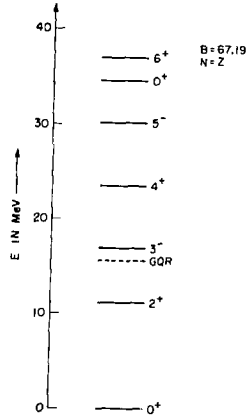


Fig. 8. Calculated excitation energies of isoscalar collective modes of nucleus in Fig. 2. Also shown is the observed energy of the 2^+ giant isoscalar quadrupole resonance (GQR) (13).

(assumed irrotational for simplicity) is assigned to each element of the nuclear medium then the equations of motion for isoscalar modes of $N=Z$ nuclei where the neutron and proton fluids move in phase are

$$-\frac{\partial \psi}{\partial t} + \frac{1}{2} (\nabla \psi)^2 + \frac{\mu}{m_b} = 0 \quad (38)$$

$$\frac{\partial \rho_B}{\partial t} - \nabla \cdot (\rho_B \nabla \psi) = 0 \quad (39)$$

$$(\square - m_v^2) V_0 = -g_v \rho_B \quad (40)$$

$$(\square - m_s^2) \phi_0 = -g_s \rho_s \quad (41)$$

The first equation is just Bernoulli's equation with μ the local chemical potential given by Eq. (23); the second is the baryon continuity equation, and the last two are meson field equations. Static solutions to these equations reproduce the previous Eqs. (21-25). The fields can now be allowed to have additional, small, time-dependent increments and the equation of motion linearized about the equilibrium static solution. If normal-mode solutions are sought, the result is a set of coupled eigenvalue equations. These may be derived from a variational principle which allows variational estimates of the eigenvalues. This is carried out in Ref. (13). The resulting spectrum for the nucleus in Fig. 2 is given in Fig. 8. Here Eq. (26) has been used. Also shown is the giant isoscalar quadrupole resonance (GQR) which is experimentally observed to occur in medium-to-heavy mass nuclei at an energy $E_{\text{GQR}} = 63 \text{ MeV/B}^{1/3}$.

Isovector collective modes for this same system are investigated in Ref. (14). Equations (38, 39) are first written separately for protons and neutrons, using Eqs. (34, 35) (with $e=0$). A similar variational calculation of the isovector spectrum is shown in Fig. 9 (for the value of C_ρ^2 in Eq. (36).) The observed energies of the giant dipole resonance $E_{\text{GDR}} = 76 \text{ MeV/B}^{1/3}$ and giant isovector quadrupole resonance $E_{\text{GQR}} = 130 \text{ MeV/B}^{1/3}$ are also shown. In Fig. 10, the Coulomb form factors for (ee') excitation of the 1^- , 2^+ , and 0^+ $T = 1$ states of Fig. 9 are shown.

Horowitz has obtained the exact eigenvalues in the isovector case by numerical integration; they lie within approximately 1 MeV of the

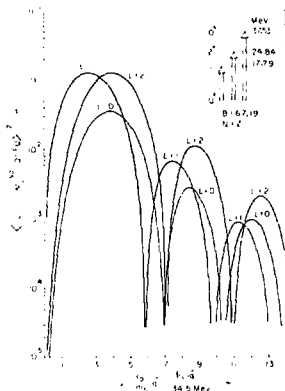
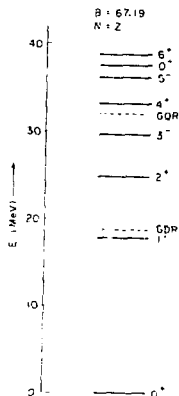


Fig. 9. Same as Fig. 8 for isovector modes (14).

Fig. 10. Coulomb form factors for (ee') excitation of the states in Fig. 9. Uses variational wave functions (13,14).

variational estimates (14).

VIII. Relativistic Hartree

Horowitz and Serot have recently carried out a relativistic Hartree calculation of the properties of spherical nuclei within the framework of Model II (15) (see also Ref. [27]). The parameters (g_s, g_v, g_ρ, m_s) are again determined from the properties of nuclear matter (binding energy, density, symmetry energy) and the rms radius of ^{40}Ca . All other constants are fixed at their experimental values. (This procedure yields $m_s/M = .554$) In this approach, the source terms in Eqs. (21, 22, 32, 33) are determined self-consistently by summing the individual baryon contributions; these are obtained by solving the

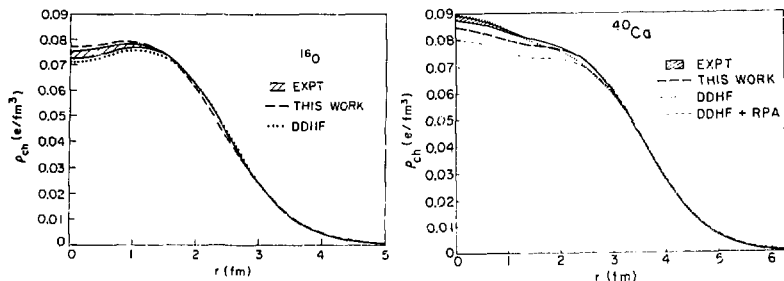


Fig. 11. Charge distribution for ^{16}O . Relativistic Hartree results are indicated by the dashed line. Solid line is experimental curve obtained from (e,e) (15).

Fig. 12. Same as Fig. 11 for ^{40}Ca (15).

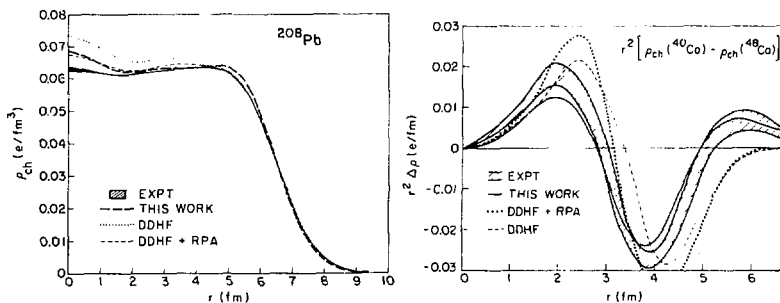


Fig. 13. Same as Fig. 11 for ^{208}Pb (15).

Fig. 14. Same as Fig. 11 for charge density difference between ^{40}Ca and ^{48}Ca (15).

Dirac equation for the baryons in the meson fields. The charge densities obtained for ^{16}O , ^{40}Ca , and ^{208}Pb are shown in Figs. 11, 12, 13, and the charge density difference between ^{48}Ca and ^{40}Ca in Fig. 14.

All are compared with the best experimental results obtained from electron scattering. Figure 15 shows the difference in neutron distributions for ^{48}Ca and ^{40}Ca , together with experimental results from high energy (800 MeV) proton scattering at LAMPF. Figure 16 shows the complete relativistic Hartree spectrum for ^{208}Pb .

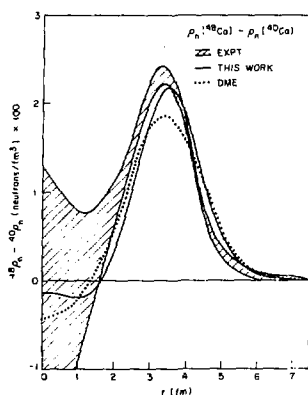


Fig. 15. Calculated neutron density difference between ^{48}Ca and ^{40}Ca compared with experimental result from (p,p) (15).

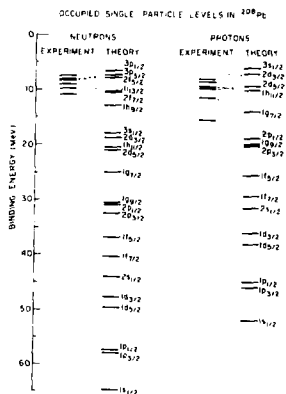


Fig. 16. Predicted spectrum for occupied single-nucleon levels in ^{208}Pb . Experimental levels are from neighboring nuclei (15).

IX. Nuclear Matter Under Extreme Conditions

We have seen that this relativistic quantum field approach to the theory of nuclear structure is able to provide a reasonable description of observed nuclei with a minimal number of underlying assumptions and parameters. Since the theory is Lorentz covariant and deals explicitly with the underlying baryon and meson degrees of freedom, it is of interest to extrapolate to see what the theory now predicts for nuclear matter under various extreme conditions:

- 1) Size Let us first extrapolate to larger nuclei. A Thomas-Fermi MFT calculations just as in Fig. 6 but with a starting radius of $r_0 = 30$ yields a nucleus with $B = 238.3$ and $Z = 93.5$, not far from the actual nucleus ${}^{238}_{94}\text{Pu}$. The resulting baryon and proton densities are shown in Fig. 19. (This is not yet a great extrapolation in nuclear size, but it is all the author has had the strength to compute!) This calculation uses the couplings of Eqs. (18, 19, 36) and $m_s/M = .724$.
- 2) Density The neutron matter equation of state shown in Fig. 2 gives the stress tensor through the relation

$$T_{\mu\nu} = P \delta_{\mu\nu} + (\rho + P) u_\mu u_\nu \quad (42)$$

where $u_\mu = (Q, i)$ is the four-velocity of the neutron fluid. The Tolman-Oppenheimer-Volkoff equations for the metric in general relativity can now be integrated to give the mass of a neutron star as a function of central density. The result is shown in Fig. 20. The MFT gives a maximum mass for a neutron star of $2.57 M_\odot$; at this mass the central baryon density is one order-of-magnitude larger than the cen-

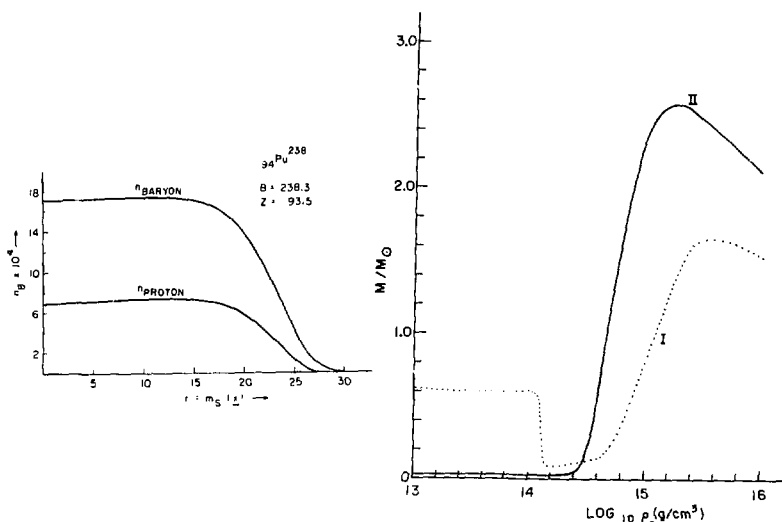


Fig. 19. MFT calculation of baryon and proton densities for $r_0 = 30$ (compare Fig. 4). Uses couplings of Eqs. (18, 19, 36) and $m_s/M = 0.724$.

Fig. 20. Calculated mass of neutron star as function of central density in Model I-curve II (2).

tral density of ^{208}Pb .

3) Temperature To investigate the equation of state at finite temperature in Model I, we calculate the thermodynamic potential

$$\Omega(\mu, V, T; \phi_0, \psi_0) = -k_B T \ln [\text{Tr} e^{-(\hat{H} - \mu \hat{B})/k_B T}] \quad (43)$$

where μ is the chemical potential, k_B is Boltzmann's constant, and $\hat{H}$ is the hamiltonian calculated from Eq. (12). In terms of the dimensionless temperature $\theta \equiv k_B T/m_p c^2$ the required modifications of the equa-

tails of the model, for example it disappears when the ρ is included in Model II (11); however, it does illustrate the richness of the physics which can be described in this simple MFT.

- ii) As $\theta \rightarrow 0$ at any $n_B > 0$ the baryon density goes to a step function and the previous results are reproduced.
- iii) As $n_B \rightarrow \infty$ at any θ , the same is true.
- iv) For $\theta \ll 1$ and $n_B \rightarrow 0$ the equation of state is that of a classical non-relativistic gas.
- v) As $\theta \rightarrow \infty$ at any n_B , baryon pairs will be produced and the equation of state takes a form very similar to that of a black body with $e = 7\pi^2 \theta^4 / 60$ and $p = e/3$.

The solution to the self-consistency equation is such as to decrease the mass of a nucleon as the temperature is raised due to pair formation; it is shown at vanishing baryon density in Fig. 22. The most striking feature of this result is the sudden decrease in the mass of the baryon with increasing θ well below $\theta = 1$. At high temperatures (as at high density) the baryons are effectively massless. As the temperature is lowered, the baryons suddenly acquire mass due to the self-consistent freezing out of the vacuum pairs and corresponding decrease in ϕ_0 .

4) Flow Kleppinger has investigated the behavior of uniform nuclear matter under the same hydrodynamic assumptions discussed in Sec. VII.(28). Since the theory is covariant one can discuss all possible fluid velocities (28, 12). He derives a relativistic equation for the excitation of the medium (sound waves) for any fluid flow. A point source at the

tion of state are the replacements $\theta(k_F - t) \rightarrow n(\theta) \pm \bar{n}(\theta)$ in the baryon density, Eq. (15) (minus sign) and in the energy density, pressure, and self-consistency Eqs. (13, 14, 17) (plus sign) respectively (3). Note the crucial difference in signs in these expressions. In these relations the Fermi distribution functions are given by $[\exp[(t^2 + \chi^2)^{1/2} v] / \theta + 1]^{-1}$ for $n(\theta)$, $\bar{n}(\theta)$ where $v = n_B c_V^2 = \mu$; they depend self-consistently on the mass χ . The calculated isotherms for this MFT are shown in Fig. 21. Several feature of these results are of interest:

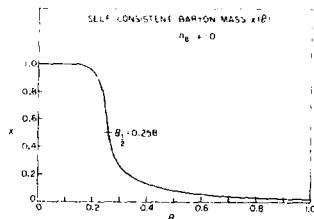
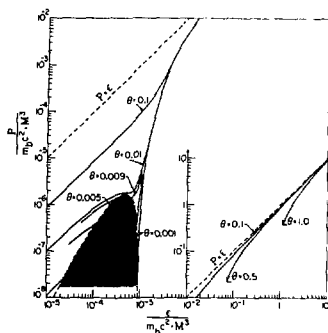


Fig. 21. Calculated isotherms for neutron matter in Model I (3). Here $\theta \equiv k_B T / m_B c^2$.

Fig. 22. Baryon mass vs. temperature at vanishing baryon density in the MFT (3).

i) The phase transition is again seen, and there is a critical point and critical isotherm. This phase transition is sensitive to the de-

origin $\sim \delta^{(4)}(x)$ may be added to the equations and the resulting disturbance investigated. Kleppinger is able to evaluate the required Green's function analytically. The contours of constant amplitude disturbance at later times for the nuclear fluid flowing with velocity $\underline{u}$ in the z direction are illustrated in Figs. 23 and 24 for the two cases of subsonic and supersonic flow. We note the Mach cone in the second case with opening angle $\sin \theta_m = (s/u)[(c^2 - u^2)/(c^2 - s^2)]^{1/2}$. In Model I, $s/c = 0.256$ for equilibrium nuclear matter (16,28).

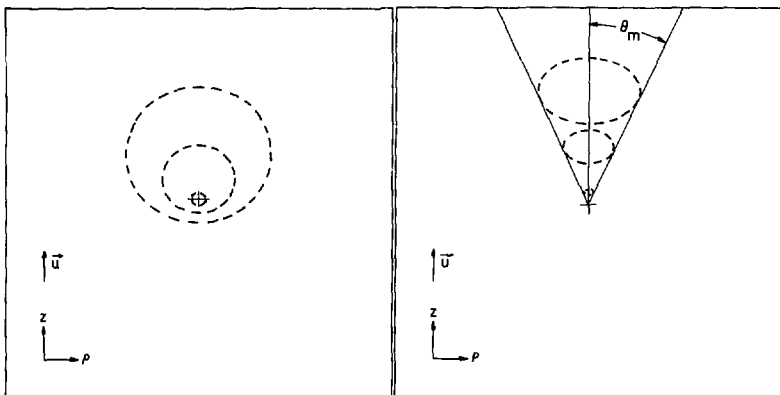


Fig. 23. Contours of constant amplitude at later times for a point source $\sim \delta^{(4)}(x)$ in nuclear fluid moving with velocity $\underline{u}$ in z direction-subsonic flow (28). Sound velocity $s/c = 0.256$

Fig. 24. Same as Fig. 23 for supersonic flow (28).

5) Rotation Bielaiew has investigated the behavior of nuclear matter undergoing extremely rapid rotation (29). The resulting density profile for a long self-bound cylinder of nuclear matter rotating about the symmetry axis is shown in Fig. 25 for various edge velocities. Note how the rotation changes the shape of the density profile. Since the system is self-bound, too rapid rotation will cause it to fly apart. Figure 26 illustrates the behavior of the nuclear medium confined to a cylinder and put into rapid rotation. The nuclear matter is pushed against the wall and at rapid-enough angular velocities, the density develops a central hole. These particular calculations are somewhat academic, but the results are interesting.

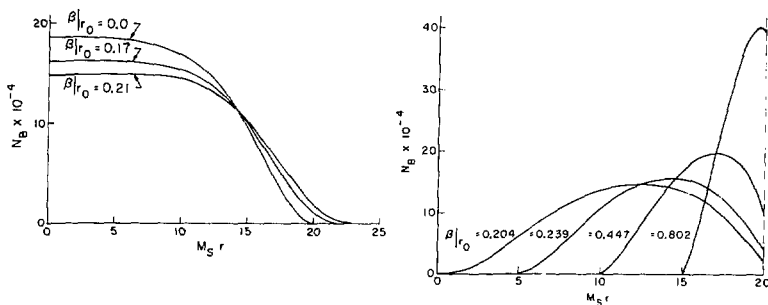


Fig. 25. Behavior of density profile (cf. Fig. 4) for long self-bound cylinder of nuclear matter under rotation about symmetry axis. Here $\beta = v/c$ is the edge velocity at $r = r_0$ (29).

Fig. 26. Density profile for contained cylinder of nuclear fluid rotating with different edge velocities. Note the development of the central hole (29).

X. Extensions

A fully consistent relativistic Hartree calculation must include the effect of the mean meson fields on virtual baryon-antibaryon pairs. A prescription for computing this correctly involves adding counter terms.

$$\delta \mathcal{L} = \sum_{n=1}^4 (C_n / n!) \phi^n \quad (43)$$

to the lagrangian density (Model I). If the one, two, three, and four scalar meson amplitudes are required to vanish in the real vacuum for vanishing meson four-momenta, thus minimizing the role of possible many-body forces in the theory, the result is to add an additional "vacuum fluctuation" term to the energy density. This term provides a

$$\delta e_{VF} = - \frac{\gamma}{16\pi^2} \left[\chi^4 \ln \chi + (1-\chi) - \frac{7}{2} (1-\chi)^2 + \frac{13}{3} (1-\chi)^3 - \frac{25}{12} (1-\chi)^4 \right] \quad (44)$$

proper treatment of the vacuum energy remaining after extracting the normal-ordered part of the hamiltonian in Eq. (12). The effect of this term on the zero-temperature equation of state has been investigated by Chin (4, 5), and on the finite-T equation of state by Freedman (6). The coupling constants required to fit the saturation properties of nuclear matter are exhibited in Table 4. The effect on the charge density for ^{40}Ca , although essential to the mathematical consistency of the theory is not large, as shown in (8).

The inclusion of exchange corrections to the MFT has been investigated at $T=0$ for nuclear matter by Chin (4,5) and Brittan (7), for

	$C_V^2 / (C_V^2)_{\text{MFT}}$	$C_S^2 / (C_S^2)_{\text{MFT}}$	K_V^{-1}	M^*/M
with vacuum fluctuations	.586	.687	470 MeV	.72
with exchange and correlations	.721	.802	413 MeV	.65

Table 4. Results for nuclear matter when separately including modifications to the MFT discussed in text (4,5,7)

$T \neq 0$ by Freedman (6), and for finite nuclei in (8). Again the effects are not large at nuclear matter densities if the coupling constants are readjusted to give nuclear matter, except for $T \rightarrow \infty$ where the ring diagrams must be summed to achieve physically sensible results.

The effects of correlations in nuclear matter have been investigated to second-order in the meson exchanges for the ladder diagrams contributing to the baryon self-energy by Brittan (7). He shows that the correlation corrections become negligible with respect to the MFT results as $k_F \rightarrow \infty$. Inclusion of the exchange and correlation corrections at nuclear matter densities gives rise to the results shown in Table 4 (7). Brittan has also solved the Bethe-Goldstone equation (24) in the potential limit of Eq. (1). This sums the ladder diagrams to all orders. With the coupling constants of Table 4, the two-nucleon correlations in the Bethe-Goldstone wave function are weak and the second-order correlation energy is the dominant correlation correction. Note that the correlations have the desirable effect of somewhat reducing the compressibility.

Collective modes in this model have also been investigated by Chin (4,5) and Matsui (16), and the thermal conductivity by Freedman (30).

Horowitz and Serot are currently carrying out a self-consistent relativistic Hartree-Fock calculation for nuclear matter at all densities and for finite nuclei. In Model I, peculiar things appear to happen to the self-consistent single-nucleon spectrum at high densities. In Model II, because of the very large g_π in Eq. (24), any pion loop integral causes these peculiarities to appear at lower densities. We do not yet understand how to treat the pions correctly in loop integrals.

XI. Conclusions

In summary, despite its over simplification of the actual meson degrees of freedom involved in nuclear physics, the present approach has four great advantages:

- 1) The concept of a static two-body potential is never introduced
- 2) Lorentz covariance is retained throughout and the meson degrees of freedom are dealt with explicitly
- 3) The theory can be solved simply in the mean-field approximation to give results which appear to be correct at high baryon density and which provide a remarkably consistent description of observed nuclei
- 4) There is a well-defined procedure for calculating corrections within the framework of a renormalizable quantum field theory.

In a larger perspective, any model of ordinary nuclear matter based on baryons and mesons must eventually break down when the

interior quark structure of the baryons manifests itself. The transition from ordinary nuclear matter to quark matter is one of the most interesting and challenging problems in nuclear physics.

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A RELATIVISTIC QUANTUM FIELD THEORETICAL MODEL
FOR NUCLEAR SLAB COLLISION DYNAMICS*

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The relativistic mean field model proposed by J.D. Walecka was recently used with great success to describe the properties of nuclear matter as well as the structure of the finite static nuclei. Starting from Walecka's Lagrangian the nucleons are represented by single particle spinors, which are determined by a Dirac equation containing a repulsive mean neutral vector field V_μ and an attractive mean isoscalar field σ . Both fields satisfy Klein-Gordon equations. Their source terms are again determined by the spinors. We applied this relativistic mean field model to the dynamics of two colliding slabs. They are translational invariant in two transverse dimensions and consist of spin and isospin symmetric nuclear matter. By specification of appropriate initial conditions for the collision of two equal slabs, we solved the system of coupled Klein-Gordon-Dirac equations for lab bombarding energies per nucleon up to 400 MeV. At low energies the results are identical with TDHF results. The damping of the collision, the phenomenon of high nuclear density, and the effect of retardation during the reaction are studied.

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To describe a heavy ion collision from the basis of a relativistic quantum field theory is very desirable because in principle one has a complete calculational framework with which to answer any question that can be asked about the system. J.D. Walecka¹⁾ has proposed a relativistic quantum field theory for nuclear matter that recently was used successfully to determine the structure of finite nuclei.²⁻⁴⁾ This gives the motivation to extend the Walecka model to describe the dynamics of colliding nuclei.⁵⁾

To limit the computational scope we consider two colliding equal nuclear slabs of spin and isospin symmetric nuclear matter that are finite in z-direction and infinite in the transverse directions. In the Walecka model, the interaction between the nucleons is due to the exchange of a neutral scalar meson (σ) of mass m_σ and a neutral vector meson (V_μ) of mass m_V . The scalar meson couples to the scalar nucleon density through a coupling $-g_\sigma \bar{\psi} \psi$ and the vector meson to the conserved nucleon current through $ig_V \bar{\psi} \gamma_\lambda \psi V_\lambda$. The field equations become manageable by using the mean field approximation¹⁾ where one substitutes the meson fields σ and V_μ by their expectation values σ and V_μ and introduces the normal ordering to calculate the expectation values of the source terms. To do so, we assume the relativistic many-body state, which describes the colliding nuclear slabs to be of the form

$$|\phi\rangle = a_{\alpha_1}^+ a_{\alpha_2}^+ \dots a_{\alpha_A}^+ |0\rangle \quad (1)$$

$a_{\alpha_i}^+$ creates a nucleon characterized by quantum numbers $\alpha \equiv (j, K_\perp)$. j is the z-quantum number and $K_\perp$ the transverse wave number. With this the Euler-Lagrange equations yield Klein-Gordon equations for the

mean meson fields and a Dirac equation for the functions $f_{A_{jK_L}}$ and $f_{B_{jK_L}}$, which correspond to the large and small component of the relativistic single nucleon wave functions.

$$\frac{\partial^2 \sigma}{c^2 \partial t^2} - \frac{\partial^2 \sigma}{\partial z^2} + m_S^2 \sigma = -g_S \rho_S \quad (2a)$$

$$\frac{\partial^2 V_0}{c^2 \partial t^2} - \frac{\partial^2 V_0}{\partial z^2} + m_V^2 V_0 = g_V \rho_V \quad (2b)$$

$$\frac{\partial^2 V_z}{c^2 \partial t^2} - \frac{\partial^2 V_z}{\partial z^2} + m_V^2 V_z = g_V j_z \quad (2c)$$

and

$$\frac{\partial}{\partial t} f_{A_{jK_L}} + \left(\frac{\partial}{\partial z} - i g_V V_z \right) f_{B_{jK_L}} = i(-g_V V_0 - \sqrt{K_L^2 + m^{*2}}) f_{A_{jK_L}} \quad (3a)$$

$$\frac{\partial}{\partial t} f_{B_{jK_L}} + \left(\frac{\partial}{\partial z} - i g_V V_z \right) f_{A_{jK_L}} = i(-g_V V_0 + \sqrt{K_L^2 + m^{*2}}) f_{B_{jK_L}} \quad (3b)$$

Here the effective mass is $m^* = m + g_S \sigma$. The scalar density ρ_S , the nucleon density ρ_V , and the nucleon current j_z are determined by the functions f_A and f_B .

$$\rho_S = \frac{2}{\pi} \sum_j \int dK_L K_L \frac{m^*}{\sqrt{K_L^2 + m^{*2}}} \left\{ |f_A|^2 - |f_B|^2 \right\} \quad (4a)$$

$$\rho_V = \frac{2}{\pi} \sum_j \int dK_L K_L \left\{ |f_A|^2 + |f_B|^2 \right\} \quad (4a)$$

$$j_z = \frac{2}{\pi} \sum_j \int dK_L K_L \left\{ f_A^* f_B + f_B^* f_A \right\} \quad (4b)$$

The appropriate initial condition for the approaching slabs is constructed by an inhomogeneous Lorentz transformation applied to the

static relativistic nucleon wave functions, which correspond to the ground state of a single slab at rest.

The half-side momentum per area unit

$$\frac{P_z}{F} = \frac{1}{c} \int dz \left\{ \langle \phi | \psi^\dagger \frac{\partial}{\partial z} \psi | \phi \rangle - \frac{\partial \sigma}{c \partial t} \frac{\partial \sigma}{\partial z} - \frac{\partial V_z}{\partial z} \left(\frac{\partial V_0}{\partial z} + \frac{\partial V_z}{c \partial t} \right) \right\} \quad (5)$$

determines the amount of energy that is transferred during the reaction from initial kinetic energy to intrinsic excitation.

We solved the coupled system of differential equations (2),(3) numerically for different bombarding energies. Initially, each slab is in its ground state. With the constraint of 1.4 nucleons per unit area for each slab, self-consistency leads to a slab built up by four relativistic single nucleon wave functions. The slab thickness corresponds to a nucleus of $A \approx 35$. Since the functions $f_{A/B}$ depend weakly on $K_\perp$, we consider in our computation only wave functions with $K_\perp = 0$. At energies lower than $E_{c.m.}/A = 100$ MeV, we neglect the second time-derivations in the Klein-Gordon equations (2); that means, we neglect retardation effects.

Figure 1 shows the resulting density ρ_v during the slab collision at the low energy of $E_{c.m.}/A = 3.5$ MeV. The time t is in units of 10^{-21} sec. After the collision about 70% of the initial energy is transferred into intrinsic excitation. These results are almost identical with the corresponding TDHF calculations⁶⁾. At the intermediate energy of $E_{c.m.}/A = 25$ MeV, the profiles for ρ_v are shown in Fig. 2. While at this energy the TDHF model predicts several lumps of nuclear matter to appear at the surface which separate off, here a neck of matter between two big separating fragments is formed. Finally, the neck contracts, forming two clusters.

Figure 3 displays the density profiles at the high bombarding energy of $E_{c.m.}/A = 100$ MeV. The maximum density reached is less than $2\rho_0$. The dashed curves in Fig. 3 show the results taking the second time-derivations in Eqs. (2) into account. Obviously, retardation smooths the density profiles. Finally, Fig. 4 displays the meson fields at three different times during the slab collision. During the stage of initial penetration the vector field becomes very strong, slowing down the colliding slabs.⁷⁾

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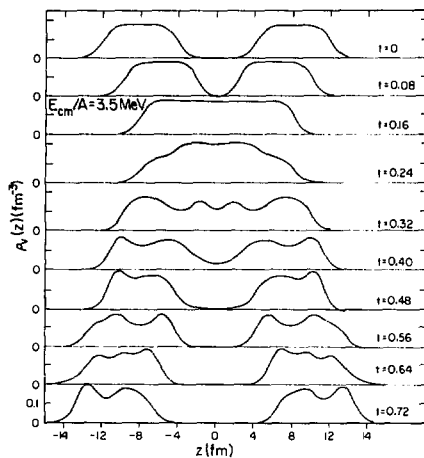


Fig. 1

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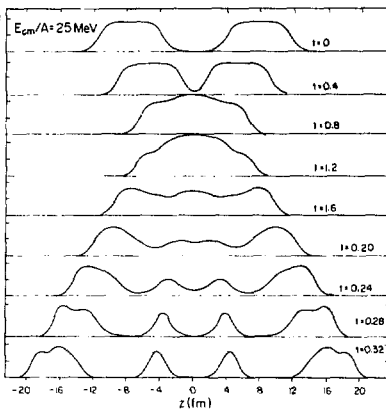


Fig. 2

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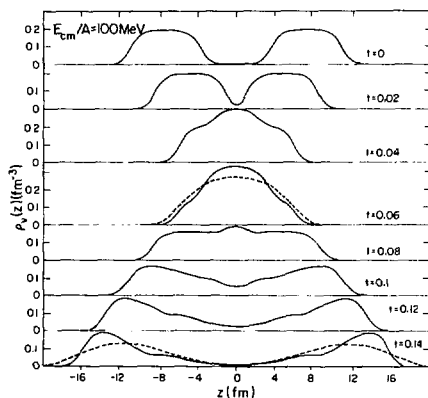


Fig. 3

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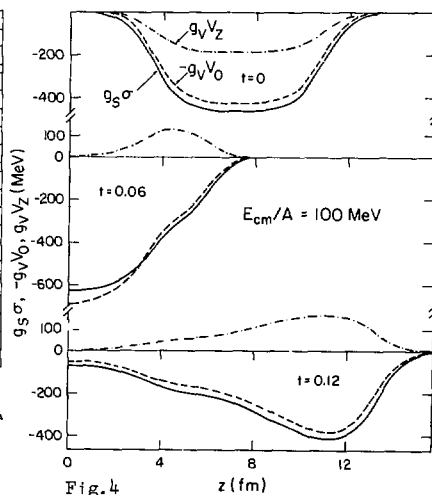


Fig. 4

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Pion Condensation and the Evolution of Neutron Stars*

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A new property of pion condensation was noticed recently, that the pressure is anisotropic.¹ This follows from the broken spatial symmetry. The anisotropy can be established by studying the stress-energy tensor. The chiral σ -model including the σ , π and ω mesons is used to prove that the pressure anisotropy is finite at the equilibrium value of the condensate wave number. Consequences of an anisotropic pressure on the evolution of neutron stars are investigated. The anisotropic pressure and gravity will establish an equilibrium deformation of 0.006 which is prolate because $P_3 > P_1 = P_2$, for a star of average density $\rho = 10\rho_0$. Such a star, rotating about any axis, not its symmetry axis, will radiate gravitational waves, which will damp the rotation. In addition pulsars are observed because of their electromagnetic radiation. The dispersion of period derivatives as a function of period, is compared with the data² for a typical neutron star, having a solar mass, a radius 10 km and an initial period $T=0.1$ sec. It will evolve along the lines from left to right depending on the relative strength of gravitational and electromagnetic dipole power radiated, parameterized by x in the equation for period derivative,

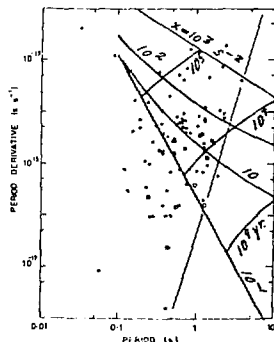
$$\dot{T} = \left(\frac{1}{T^3} + \frac{x}{T} \right) 2 \times 10^{-16} \quad , \quad (T \text{ in sec})$$

computed for the above star. It appears that such a deformation is consistent with the data and can account for the wide dispersion observed for $\dot{T}$ vs T .

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AN EXPERIMENTAL PROGRAM TO STUDY THE PHYSICAL VACUUM:
HIGH-ENERGY NUCLEUS-NUCLEUS COLLISIONS

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1. THE PHYSICAL VACUUM IN CONFINING QCD

Quarks and gluons exist; they are nearly massless, but it is very hard or even impossible to knock them out of the proton. It is now widely believed that this strange state of affairs is due to the properties of the physical vacuum state as it now exists in our part of the Universe. On this view, the ground state of the vacuum is not that familiar in quantum electrodynamics (QED). That state is basically empty space, perturbed by fluctuations which occasionally give rise to a virtual electron-positron pair. In the quantum chromodynamic (QCD) theory of quarks and gluons, the stronger and more complicated forces give rise to a state which cannot be described as a perturbation on empty space. Instead, the physical vacuum has properties which resemble those of a physical medium. For example, the colour field is completely excluded, or at least strongly repelled, from a macroscopic volume of physical vacuum. This effect confines the quarks and gluons, which carry colour, inside the hadrons. On the scale of hadrons, quantum fluctuations make the phenomena more complex, but a simple picture postulates that the strong colour fields inside the hadron create a local volume of space more like the perturbative vacuum state, reverting to the physical vacuum state outside. This concept has been quantitatively expressed by the bag model, with some success.

This physical vacuum is also supposed to explain the origin of broken symmetries. An analogy is a perfectly symmetrical sphere of iron. Above the Curie temperature the state has spherical symmetry. At low temperature, the ground state will be magnetized, with the magnetic field pointing in an arbitrary direction determined by quantum fluctuations. The symmetry of the state has been broken, without any arbitrary direction entering in the laws of nature. By a quite similar mechanism, the parameters of the physical vacuum could determine the seemingly arbitrary breaking of symmetries in particle physics, though the fundamental laws remain symmetrical.

It seems that the physical vacuum has acquired properties reminiscent of Maxwell's ether. At least, so we are asked to believe. Maxwell introduced his ether for plausible reasons, but crucial experimental tests were found, and the theory was found wanting. In this talk I discuss experiments for testing the idea that the physical vacuum is not identical to the perturbative one!).

Our vacuum state has no consequences for the testing of special relativity, and probably none for (macroscopic) general relativity. Fortunately, another classical experiment on the vacuum is predicted to show striking results. The effect is due to the predicted instability of the physical vacuum state in the presence of high-energy density or matter density. Under these conditions, the lower-energy state is that based on the perturbative vacuum: empty space with real and virtual quarks and gluons traversing it, without colour confinement. This change to a qualitative different state is in fact expected to occur, under suitable conditions, as a sharp phase transition. The origin in this transition is that the physical vacuum state is supposed to arise from ordered virtual constituents which are disrupted by thermal agitations, or the colour fields of dense matter. The analogy of the iron sphere is again valid: the spontaneous symmetry breaking of the physical vacuum is a

PLANCK EXPERIMENT ON VACUUM MELTING

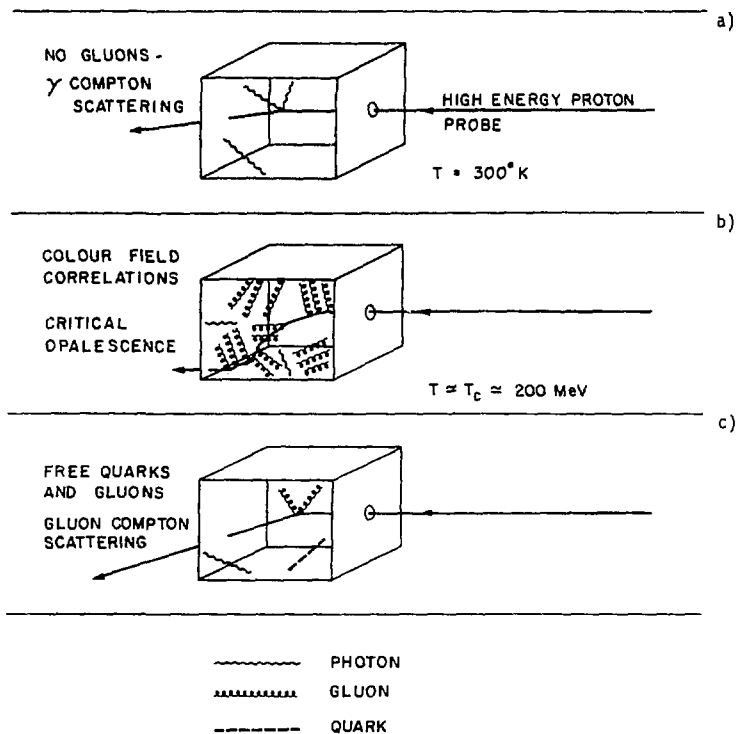


Fig. 1 Idealized experiment on "melting of the vacuum": a) Box at ambient temperature, showing thermal photons detected by Compton scattering of high-energy protons; b) at critical temperature, with large-scale fluctuations of the colour dielectric constant, and critical opalescence for protons; c) above transition, free gluons and quarks are detected in the middle of the box.

low-temperature phenomenon. The "Curie temperature" of the vacuum is of the order of the QCD scale parameter Λ .

2. AN IDEALIZED EXPERIMENT TO OBSERVE THE MELTING OF THE PHYSICAL VACUUM

Planck showed how far-reaching conclusions can be arrived at by analysing a volume of vacuum surrounded by walls in thermal equilibrium with the radiation in the interior. Let us follow him, adding equipment which will measure gluons as well as photons. In Fig. 1a we see a large box with thick walls at temperature T . The radiation emitted through a small aperture is measured. Alternatively, if we want to be sure of what happens in the middle of the box, a high-energy proton beam is sent through the aperture, and Compton scattering of photons and gluons is measured.

At low temperature, $T < \Lambda$, we will detect photons filling the box with the Planck distribution, but no gluons. Why not, since massless thermal gluons should be emitted by the walls? (If a mass is attributed to the gluons it is surely $\ll \Lambda$.) The answer is supposed to be that the physical vacuum filling the box forces a thermal gluon back into the wall.

As the temperature of the wall is raised, there are more -- and more energetic -- thermal gluons emitted. They penetrate slightly further into the vacuum. Finally, the temperature approaches where the ordered structure of the virtual particles in the physical vacuum is so much disrupted by these assaults that the perturbative vacuum state is energetically preferred. Very near this temperature, large-scale fluctuation appears in the vacuum, with a mixture of colour-confining and unconfining regions. The phenomenon of critical opalescence will render the box opaque to the high-energy protons at that point (Fig. 1b).

Above the transition temperature, we will find freely propagating gluons and quarks filling the box (Fig. 1c). The situation at the small aperture is more complex, since it is a boundary with the physical vacuum in the world outside. Only constituent combinations which are colourless can make it to the outside world.

Suppose the walls are heated further. We note that the thermal energies of the constituents are $> \Lambda$, so that they are entering the regime of asymptotic freedom and their interactions are decreasing as they are heated. It seems there is no limit to the temperature. The "limiting temperature" observed in hadronic interactions must be a confinement effect, and indeed the Hagedorn temperature of 160 MeV is close to that estimated for the critical temperature.

The elements of this analysis which must be transferred to a real experiment are the following:

- i) The size of the box. The scale is given by Λ , $\sim \frac{1}{2}$ fermi. The size must be larger than that. Evidently, the proton is not large enough.
- ii) The temperature. One should be able to sweep through the region 100-400 MeV, or thereabouts.
- iii) A sufficient degree of thermal equilibrium must be established.
- iv) The probes must be able to examine the interior of the "box" -- affording measurements of sufficient subtlety to distinguish the conditions above and below the transition, and the critical phenomena.

3. REAL EXPERIMENTS

First, I will mention some possible approaches along conventional experimental lines. Consider, first, proton-proton collisions. We know that the distributions of the particles in the "beam jets" as well as in high transverse momentum jets closely resemble those in the jets from high-energy e^+e^- annihilations²⁾. The latter we may take to reflect the characteristics of the fragmentation of single quarks. It follows that ordinary pp collisions show no sign of the presence of many constituents spread over a volume and in some sort of equilibrium -- the conditions we wish to produce. It is possible that some rare events in pp collisions are somewhat more suitable for our purpose, but it does not seem likely that they will go far enough towards satisfying the first three conditions above.

We can think of using protons incident on a nuclear target. Here again we can profit by a considerable body of knowledge from recent experiments³⁾. For example, if we consider the system in which the proton is at rest, and consider the proton fragmentation products after it has been struck by the incident nucleus, we know that they are not very different from those after the proton has been struck by another proton. Consider, instead, the nucleus to be at rest. The proton passes through, making several collisions. The fast forward products do not fragment until they have left the nucleus (see the previous remark). The slower particles are emitted at larger angles, and do fragment inside the nucleus. Their fate is a hard one, however. These fragmenting particles have energies of a few GeV or less, and they enter a volume of cold nuclear matter where they are outnumbered by "stationary" nucleons at the odds of typically ten to one. They create feeble cascades, where the creation of a few pions is partially counterbalanced by pion absorption. No wonder that the observed increase in pion multiplicity, in comparison with pp collisions, is only between two and three in the heaviest nuclei. There is no possibility of heating a large volume to an interesting temperature. Instead, the energy provided is dissipated in a large mass of cold nuclear matter.

It comes rather naturally to consider nucleus-nucleus collisions at high energy. First we note that accelerators, linear or circular, act upon the charge. A fully stripped heavy ion has charge Z times that of a proton, and A times the mass, with $A \approx 22$. The total energy of a nucleus produced by the accelerator is thus about $Z/2$ times that of that of a proton from the same accelerator. Even for a medium-size nucleus, say argon, this is a huge factor. Given that we needed to heat a large volume, the fact that the energy is distributed over a number of particles is not a disadvantage. Quite the contrary, since this energy can be deposited in the target with reasonable efficiency, which is of course not the case when trying to heat a nuclear volume with one very high energy proton.

To give an idea of what should happen in such a collision, I shall estimate the number of pions produced, always assuming that there is no new physics at the level of the individual nucleon-nucleon collisions. I should like to suppose that the energy is high enough so that there is a well-defined central region in rapidity, though the pion multiplicity may start to saturate at somewhat lower energies. In pp collisions, this occurs for lab. energies of about 100 GeV. In nuclear collisions, the leading quarks are further degraded by multiple collisions, and the energy required is greater by $\approx v$, where v is the average number of collisions of the primary, $v = 2-4$, depending on the A of the nucleus, or 0.5-1.0 TeV lab. energy per nucleon, or (for comparison with cosmic ray events) a total energy of $\approx 10^{14-15}$ ev. I shall also consider only central, i.e. head-on, collisions, since we can surely select them experimentally. In the next section, I will consider the applications of non-central collisions.

Suppose the target nucleus is struck by one proton of such an energy. We know what happens: the number of pions (including π^0) is, from the CERN Intersecting Storage Rings (ISR), about 20 on the average. The effect of the nuclear target is to multiply this to about 50. Consider now that the nucleus is struck at the same time but at different points. Surely the number of pions produced is twice that produced by one proton. As the number of nucleon projectiles increases, the possibility of coherence between nearby nucleons arises. It is hard to see a motivation for such a coherence, and I believe it is not suggested by the cosmic-ray data⁴⁾. A multiplicity linear in A cannot be far wrong, and thus for A of 200, the multiplicity could reach 10,000 pions.

Naïvely, we could suppose that these pions are created in the volume of the two nuclei before the system has had time to disassemble. Note, however, that if each pion is supposed to occupy the volume attributed to it in the bag model, there is not room for that many pions. We may suppose that the matter is rather in the form of quarks and gluons, forming pions as the density falls to the appropriate value. Here, however, we make contact with the considerations on the role of the physical vacuum.

We know that the nucleus is made of nucleons, not a big bag of quarks. In fact, most of the volume inside a nucleus is occupied by the vacuum -- not by the nucleon bags. In the collisions just described, it seems very likely that the conditions are created where that physical vacuum is unstable, and at each point there is a transition to a perturbative vacuum filled with quarks and gluons. We then indeed have a big bag. The surface presumably emits pions as long as the temperature is high enough. In suggestive language, "the surface boils pions at the Hagedorn temperature". Arguments have been given that this state lasts "long enough"⁵⁾.

From another point of view, a novel aspect of the de-confining phase transition is that confinement is of necessity a long-range effect, and the transition necessarily produces long-range order. In the past, it has not been clear why there should be collective effects among many hadrons at particle physics energies. Now the confining properties of the physical vacuum guarantee such effects, within the orthodox theory. Having found circumstances where they are likely to occur, we must see if they can be observed. The problem of observables is considered in Section 5.

4. GLANCING COLLISIONS

For the study of the high-energy density states described in the previous sections, central collisions are clearly the best. We may ask if there is any interest in glancing collisions of very high energy nuclei. I believe I can suggest at least one topic for which these collisions offer a unique opportunity to study a physical configuration otherwise inaccessible: the irradiation of cold nuclear matter by an intense wave of soft pions.

To see how this comes about, consider the sequence of events in a glancing collision, as shown in Fig. 2. In Fig. 2a, a collision is shown which involves somewhat less than half the matter in the nucleus. The frame in which one of the nuclei is at rest is used. Figure 2b shows how the incident nucleus appears as a Lorentz-contracted disk which sweeps through the target nucleus. The energy deposited in this process can be estimated by an appeal to limiting fragmentation, as shown in Ref. 5. At least, this should give a lower limit to the true value. Most of this energy will be radiated as pions. Figure 2c shows the situation after the incident nucleus has passed on. The nucleons in the interacting portion of the target nucleus pick up an average velocity in the projectile direction, and the corresponding group of nucleons in the projectile nucleus is degraded in velocity. Most of the pions which reach the non-interacting piece of the nucleus are radiated from the target group of interacting nucleons. The kinematics are such that the pions reaching the non-interacting part of the nucleus are mostly of very low energy, in this frame.

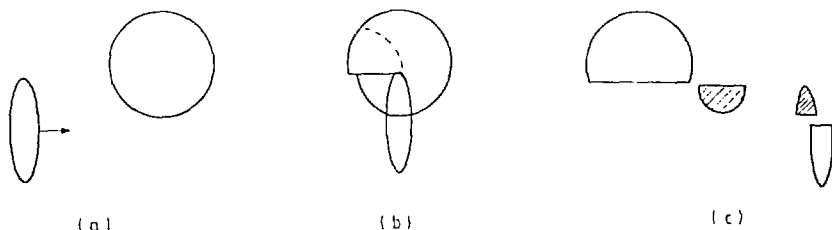


Fig. 2 A picture of the evolution of a glancing collision of a very high energy heavy nucleus, in the rest frame of the target nucleus, at three different stages.

Though only a small fraction of all pions traverse the non-interacting portion of the target, the number can easily reach ten or more. The wavelength of these pions will be of the same order as the nuclear size. The effects of such an intense wave of strongly interacting pions may produce an interesting ordering in the nuclear matter, which may be detectable by observing the nucleons, which are unlikely to have the characteristics expected for "spectators", or by observations of the pions with velocities near that of the target.

5. THE PROBLEM OF OBSERVABLES

The literature on this subject does not provide many good discussions of the quantities to be observed. One of the weaknesses, as well as strength, of the thermodynamical method is that one can proceed happily in a discussion using the thermodynamic variables without the necessity of explaining how they are to be measured. The problem becomes acute when there are strong temporal and spatial variations. A correct procedure would be to perform a Monte Carlo simulation at the constituent and vacuum level, but that is out of reach for the moment. We cannot yet renounce thermodynamical considerations.

We can begin the discussion by noting that most of the common observables are not very useful. Most hadrons will have at last scattered near the surface of the interaction volume, largely erasing the information about their previous history. It is not sensible to go to such trouble to provide a good surface-to-volume ratio, and then selectively to observe the surface. Weakly interacting probes are called for. Most of our considerations must then deal with photons, or virtual photons observed as lepton pairs⁶).

The photons in question are of course direct photons, not those from meson decays. This suggests a rare particle, of order $1/137$, compared to pions, but that can be misleading. For example, the point-like nature of the photon causes it to be much more common at high transverse momenta, where $\gamma/n^0 > 10\%$ beyond 5 GeV/c. More complex phenomena are probably present at low p_T , where observations, so far all depending on lepton pairs, show a relatively copious production of virtual photons. The virtual photons have the advantage that the mass distribution carries some information, so that the temperature of an equilibrium source could in principle be read by either the mass distribution or the transverse momentum distribution. Experimentally, they have the advantage of avoiding contamination from pion decay, though the conversion to lepton pairs costs a factor of 10^3 in rate. As A increases, the ratio of volume (producing photons) increases more quickly than surface (producing pions). This further enhances the γ/π^0 ratio, probably to values ($> 10\%$) which can be measured directly⁷).

The photons and leptons could be used in an attempt to observe the phase transition. The c.m.s. energy of the nuclei is varied, and the temperature indicated by the transverse momentum and mass distribution is determined. The rate of photon emission is then determined as a function of temperature. As the transition temperature is passed, the character of the particles producing the radiation changes, and one would expect a change in the number of the photons produced, or in the slope of the photon production versus temperature.

A variation of the baryon, or quark, density at fixed temperature will also allow a sweep across the transition. It is known from ISR data that the ratio of baryons to mesons varies strongly with rapidity. Though the variation will be somewhat smoothed out in nuclear collisions, this will give another convenient parameter to vary in the region of the phase transition.

Since we have only rough estimates of the transition temperature⁸⁾, rather crude notions of "temperature" in pp collisions, and as yet no direct data relevant to the temperature inside nuclear collisions, we cannot say anything precise about the energies necessary to produce temperatures above the critical temperature. It seems clear that the energies investigated at Berkeley and Dubna, a few GeV per nucleon, are not sufficient. Conservative estimates in the region of limiting fragmentation, roughly corresponding to the energy range defined earlier for central region formation, seem to show energy densities of the required magnitude. In practical terms, keeping to accelerators at present existing or under construction, it seems that we must speak in terms of FNAL, SPS, ISR, or ISABELLE. Of course, experiments at 10-15 GeV/nucleon would at least allow a better estimate to be made.

Another technique of observing the new phase is based on calculations which show that heavy quarks will be much more common⁹⁾. One then has to believe that they will survive the hadronization process, which seems plausible, but not certain.

Of course, it is easy to imagine many exotic and exciting forms of matter which can be created in these collisions, but I prefer to stick with an analysis of the fundamental phenomenon as we now understand it.

There are many interesting experimental questions associated with the large energy releases and large multiplicities present in these events, which I have discussed elsewhere¹⁰⁾.

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WHAT CAN WE LEARN FROM INCLUSIVE SPECTRA?

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1. INTRODUCTION

I was asked to give a review at this Conference on the existing data of single particle inclusive spectra. Before I discuss this subject I would like to show one picture in Fig. 1 which was taken by a streamer chamber.¹ Here more than 50 charged particles are observed. In such a case it seems hopeless to learn anything meaningful from the detection of only one particle out of more than 50. In the first place, therefore, I would like to point out that there is a certain limitation in physics which we can extract from single particle inclusive data.



Fig. 1 An example of the streamer chamber picture of nuclear collision.

However, in Japan and China, there is an adage which says

一を聞いて十を知る . (Hearing 1, you must learn 10; Japan)

拳一反三 (Hearing 1, you must learn 3; China)

I don't know why 10 is decreased down to 3 in China.² Anyway, I would like to try to extract as much information as possible from the available data.

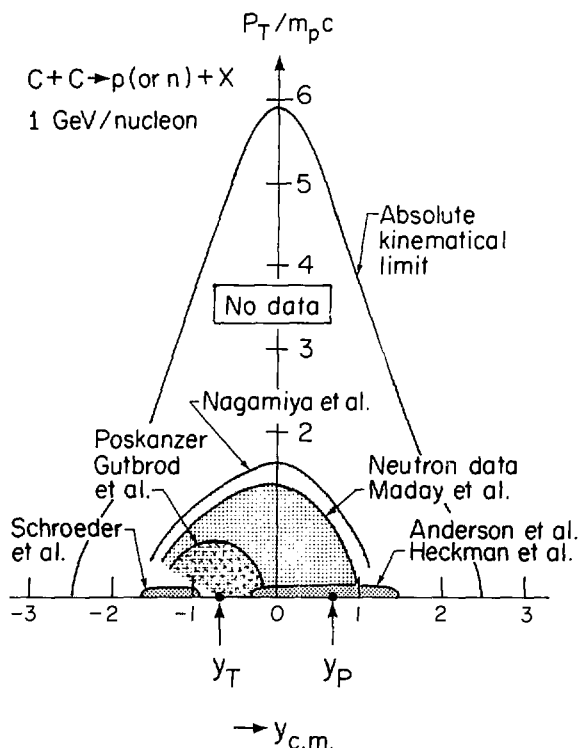
The organization of my talk is as follows.*¹) First I will describe very briefly the present experimental status on single particle inclusive measurements (Sec. 2). Then, I discuss the geometrical aspect of the collision from the data of total integrated cross sections of nuclear charge or mass (Sec. 3). The dynamical aspect of the collision, especially that for the participant region, forms the major part of my talk, and it will be discussed in connection with proton spectra (Sec. 4), composite fragment spectra (Sec. 5), pion production (Sec. 6), ratios of π^-/π^+ , n/p and $t/^3\text{He}$ (Sec. 7), and production of strange particles (Sec. 8). Then, I will describe the spectator physics from the data on projectile fragments (Sec. 9), and finally give a summary (Sec. 10).

2. CURRENT EXPERIMENTAL STATUS

So far a large number of experimental groups at the Bevalac have measured protons, neutrons, nuclear fragments and mesons at various kinematical regions. For example, the available proton and neutron data already cover a fairly wide kinematical region in the plane of rapidity (y) and transverse momentum (p_T/mc), as shown in Fig. 2. If the cross section is normalized to 1 in the projectile or target fragmentation region, the measured

*¹) In the actual presentation in the Conference I skipped Secs. 3, 4, and 7.

cross sections extend down to $10^{-(5-6)}$ which is about $10-100 \mu\text{b}/\text{sr}/(\text{GeV}/c)^3$. However, I would like to point out also that the area allowed by the kinematics in the y - p_T/mc plane is still much wider than the area covered by the available data. With a reasonable experimental device we can measure the cross section down to 10^{-10} at the Bevalac. Such measurements of small cross sections are definitely important in the near future.

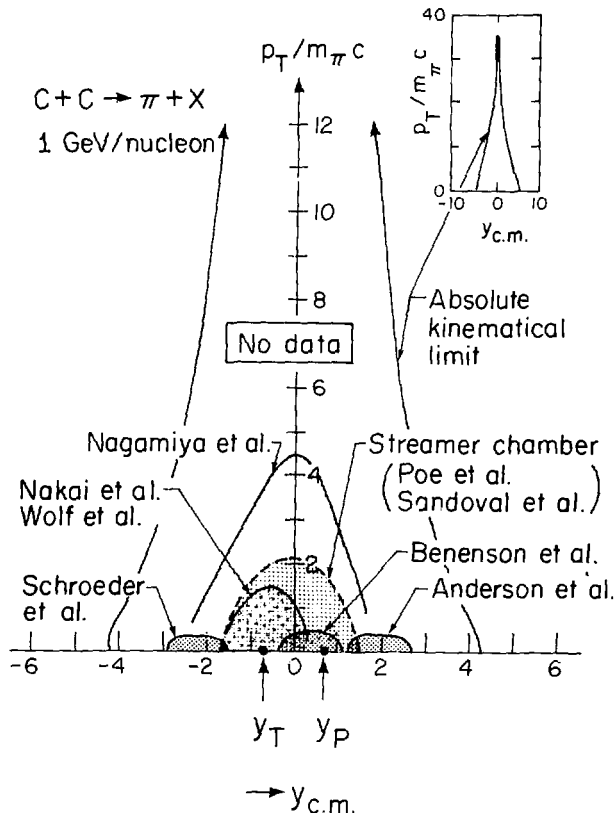


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Fig. 2 Current experimental status for measurements of protons and neutrons.

A similar situation is observed for pion measurements, as shown in Fig.

3. Data are available over a fairly wide kinematical region, but still there is a wide region over which the cross sections have not yet been measured. In the near future we should try to measure cross sections down to $1 \text{ nb/sr}/(\text{GeV}/c)^3$.



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Fig. 3 Current experimental status for measurements of pions.

3. TOTAL INTEGRATED CROSS SECTIONS OF NUCLEAR CHARGE OR MASS

In high energy nuclear collisions it has been rather well known that the participant-spectator model³ explains gross features of the emission of nuclear fragments including protons and neutrons. For example, a large fragment yield at $(y, p_T) = (y_P, 0)$ or $(y_T, 0)$ is mainly due to the spectator while the smooth proton or pion spectra at large angles are primarily due to the participant. By using this model we first calculate how many protons (nucleons) are classified as participant protons (nucleons) and how many are as spectator protons (nucleons).

Let us assume that the projectile nucleus consists of Z_P protons and N_P neutrons (and therefore, $A_P = Z_P + N_P$), and similarly assume that the target nucleus consists of A_T nucleons. Then, the average number of participant protons from the projectile nucleus, which represents the participant charge from the projectile, is given by Z_P times the target cross section divided by the total geometrical cross section, because projectile protons which hit the target are regarded as participant protons. We thus have

$$\begin{aligned} \langle Z_{\text{Proj}}^{\text{Parti}} \rangle &= Z_P \times (\text{Target cross section}) / \sigma_0 \\ &= Z_P A_T^{2/3} / (A_P^{1/3} + A_T^{1/3})^2, \end{aligned} \quad (1)$$

where σ_0 is the total cross section which is approximately given by

$$\sigma_0 = \pi r_0^2 (A_P^{1/3} + A_T^{1/3})^2. \quad (2)$$

Similarly for the average number of participant protons from the target nucleus we have

$$\langle Z_{\text{Targ}}^{\text{Parti}} \rangle = Z_T A_P^{2/3} / (A_P^{1/3} + A_T^{1/3})^2. \quad (3)$$

The total integrated yield of participant protons, namely the total

integrated yield of nuclear charge from the participant region, is the sum of the above participant proton numbers multiplied by the total cross section σ_0 , and is given by

$$\begin{aligned}\sigma_{\text{tot}}^{\text{charge}}(\text{Participant}) &= (\langle Z_{\text{Proj}}^{\text{Parti}} \rangle + \langle Z_{\text{Targ}}^{\text{Parti}} \rangle) \times \sigma_0 \\ &= \pi r_0^2 (Z_P A_T^{2/3} + Z_T A_P^{2/3}).\end{aligned}\quad (4)$$

Similarly for the total integrated cross sections of nuclear charge for projectile and target spectators we have

$$\begin{aligned}\sigma_{\text{tot}}^{\text{charge}}(\text{Proj Frag}) &= (Z_P - \langle Z_{\text{Proj}}^{\text{Parti}} \rangle) \times \sigma_0 \\ &= \pi r_0^2 Z_P (A_P^{2/3} + 2A_P^{1/3} A_T^{1/3}),\end{aligned}\quad (5)$$

$$\sigma_{\text{tot}}^{\text{charge}}(\text{Targ Frag}) = \pi r_0^2 Z_T (A_T^{2/3} + 2A_P^{1/3} A_T^{1/3}).\quad (6)$$

These formulas are the natural consequence of the Glauber theory⁴ and have been derived also in Refs. 5 and 6. For A + A collisions with $Z = N$ we should note that the above values of σ in Eqs. (4)-(6) are proportional to $A^{5/3}$. This value of 5/3 for the power dependence is referred to as the geometrical limit.

In Figs. 4-6 the above three formulas are compared with data. Shown in Fig. 4 are the values of $\sigma_{\text{tot}}^{\text{charge}}$ measured by Lindstrom et al.⁷ for C and O projectiles. First we notice that the yields are almost independent of the beam energy. Secondly, the observed yields are approximately proportional to $A_T^{1/4}$. Thirdly, the absolute values of $\sigma_{\text{tot}}^{\text{charge}}$ are very well explained by Eq. (5) with $r_0 = 0.95$ fm. When these data were published, it was thought that the observed $A_T^{1/4}$ dependence showed evidence that projectile fragments are produced mostly from "peripheral" collisions. However, this conclusion is not obvious, because the participant-spectator model does not specify any impact

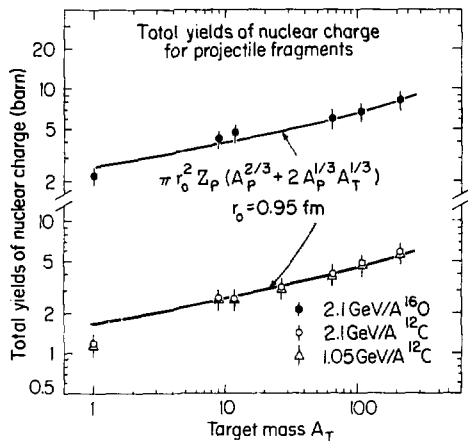


Fig. 4 Total integrated cross sections of nuclear charge for projectile fragments as compared with Eq. (5).

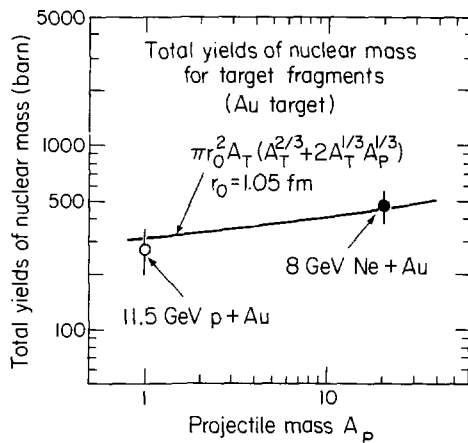


Fig. 5 Total integrated cross sections of nuclear mass for target fragments as compared with Eq. (6).

parameter, and some projectile fragments can be produced even from small impact parameters. Still, this model predicts approximately the $A_T^{1/4}$ dependence due to the existence of the $A_p^{2/3}$ term which is added to the term of $2A_p^{1/3}A_T^{1/3}$ in Eq. (5).

A similar plot for target fragments is shown in Fig. 5. Here, the total integrated cross sections of nuclear mass in the two collisions, 11.5 GeV p + Au (Ref. 8) and 400 MeV/nucleon Ne + Au (Ref. 9), are plotted. The data are again explained by Eq. (6) with $r_0 = 1.05$ fm [Note that in this case we should replace Z_T by A_T in Eq. (6), since we plot the total mass yields instead of the total nuclear charge yields].

Fig. 6 shows the sum of charges for p, d, t, and ^3He evaluated from the relatively high-energy fragment data observed at large angles.¹⁰ These data

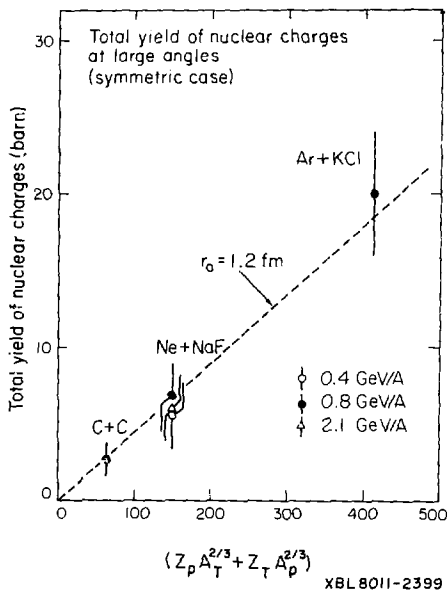


Fig. 6 Total integrated cross sections of nuclear charge for the participant region as compared with Eq. (4).

should now be compared with Eq. (4), since most of such fragments are expected to originate from the participant. We again observe that the total integrated cross sections are almost independent of the beam energy, and the agreement of the data with Eq. (4) is good if we take $r_0 = 1.2$ fm.

SUMMARY:

1. The total integrated cross sections of nuclear charge or mass are almost independent of the beam energy. The observed absolute values of cross sections are well explained by the participant-spectator model. Thus, the total integrated cross sections are determined largely by the collision geometry only.
2. The observed $A_T^{1/4}$ dependence of the cross section for the projectile fragments can be explained by the participant-spectator model, and thus, this power dependence does not immediately imply that projectile fragments are produced from peripheral collisions.
3. The value of r_0 for the participant region ($r_0 \approx 1.2$ fm) is larger than that for the region of projectile or target fragmentation ($r_0 \approx 1.0$ fm), implying that the clean-cut participant-spectator model might not be appropriate to describe the details of the collision geometry. Some nucleons which are located in the boundary region would carry an intermediate nature between the participant and spectator. In the present analysis these intermediate nucleons are probably counted as participant.

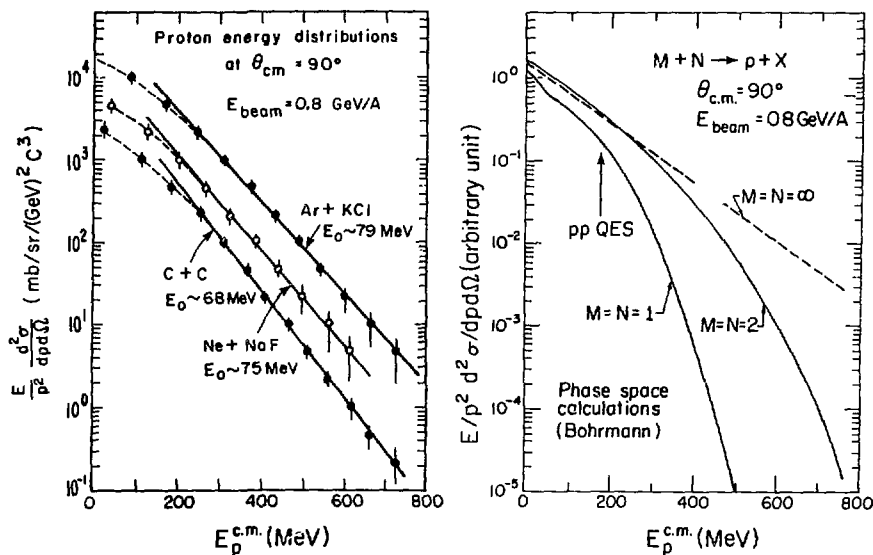
4. PROTON SPECTRA

Once we understand the geometry, the next immediate interest is the collision dynamics. Since the spectator part does not experience any violent nucleon-nucleon interaction, the most interesting region from the viewpoint of the reaction mechanism is the participant part. I will therefore discuss the

the participant dynamics from this section through to Sec. 8. Physics related to the spectator will be described in Sec. 9.

So far a large number of proton data have been reported.¹⁰⁻¹⁵ Since these data have been frequently mentioned in several conferences, I will skip most of them in the present talk and show only one piece of the data. Shown in Fig. 7 are the proton energy spectra at c.m. 90° , which corresponds to $y = (y_p + y_T)/2$, from three sets of nearly equal-mass nuclear collisions, C + C, Ne + NaF, and Ar + KCl.^{13,10} Here the invariant cross sections, σ_{inv} ,

$$\sigma_{inv} \equiv E(d^3\sigma/d^3p) \quad (7)$$



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Fig. 7 Proton energy spectra at $\theta(\text{c.m.}) = 90^\circ$ measured in collisions of 800 MeV/nucleon C + C, Ne + NaF, and Ar + KCl (left), and the phase space calculation of the proton emission at 800 MeV/nucleon done by Bohrman (right). Fermi motion as well as elastic and inelastic processes are included in the calculation.

are plotted as a function of the kinetic energy, $E_p^{C.m.}$. The nominal beam energy was 800 MeV/nucleon for all cases. These data tend to reflect more of the features of the participant region, because at $\theta_{C.m.} = 90^\circ$ the influence from the spectator is the smallest.

First we notice that the spectrum shape is almost identical for the three collisions. This fact implies that the beam energy per nucleon rather than the total beam energy plays a key role in determining the collision dynamics. Secondly, the spectrum shape approaches an exponential at high energies but it deviates substantially from the exponential shape at low energies. Such a "shoulder-arm" type distribution is a typical feature seen in the proton spectra at 800 MeV/nucleon. The turning point between the shoulder and arm sits at around $E_p^{C.m.} \approx 200$ MeV, which is very close to the projectile (or target) energy per nucleon in the C.M. frame (182 MeV in this case). This fact implies that protons from quasi-elastic pp or pn scatterings might be responsible for creating such a turning point.

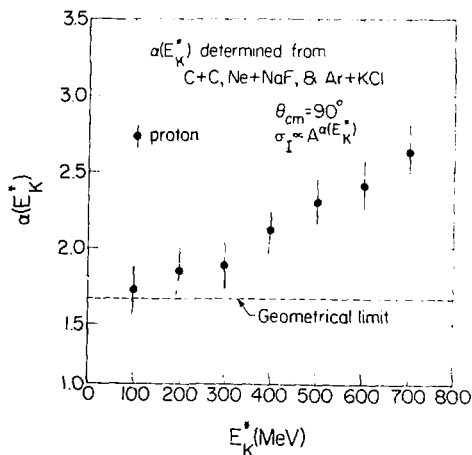
In order to study these observed spectra in more detail I show in Fig. 7 a phase-space calculation by Borhmann.¹⁶ If we consider the case in which a nucleus-nucleus collision is a simple incoherent sum of single NN collisions without any succeeding multiple collisions, then two nucleons, one from the projectile and the other from the target, determines the whole dynamics. As shown in the $M = N = 1$ system in Fig. 7, the spectrum in this case has a rather clear turning point at around $E_p^{C.m.} \approx 182$ MeV (indicated by pp QES in the figure). This spectrum, however, is quite different from the observed one, especially in the high energy region. If multiple NN collisions take place, then the available energy tends to be shared among larger number of nucleons than two, and the yields of high energy protons could increase. In fact, if four nucleons, two from the projectile and two from the target, share the available energy, then the shape of the calculated spectrum drastically

changes at high energies, and the calculated spectrum becomes much closer to the observed one. The extreme limit of multiple NN collisions is the thermal case in which we have an almost pure exponential shape which disagrees with the data. Therefore, the observed spectrum indicates that both the single and multiple NN collision processes are important, and especially the latter in the region of the high energy tails.

The importance of multiple NN collisions in producing the high energy tails is observed also in data on the A dependence. For nearly equal-mass collisions the observed cross sections shown in Fig. 7 are parameterized, to a good approximation, by

$$\sigma_{\text{inv}}(E_p^{\text{c.m.}}) \propto A^\alpha, \quad (8)$$

where A is the projectile (and target) mass. The observed values of α are displayed in Fig. 8 for various c.m. kinetic energies, $E_K^{\text{c.m.}}$. For low energy



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Fig. 8 A dependence of proton emission in A on A collisions.

protons we observe $\alpha \approx 5/3$ which is the geometrical limit mentioned in the previous section. For high energy protons, however, α is substantially larger than this geometrical limit. A larger power than $5/3$ implies that more nucleons are involved in a collision than geometrically expected, which is possible through multiple NN collisions. Therefore, these data strongly suggest the importance of the multiple NN collisions for the production of high energy protons at c.m. 90° , or in other words, for the production of high p_T protons.

The reason why I mention such classical data is because I would like to remind you again the importance of both "direct" and "multiple-collision" components in particle production in high energy nuclear collisions. Although such a mixture of the two components has been clearly observed in the two proton correlation data,^{17,13} some evidences of it are already observed in inclusive proton spectra. When the fireball model was proposed,¹¹ the fit to the data was done only for low energy protons below the per nucleon beam energy, since thermal protons tend to be of low energy. However, we now observe that low energy protons substantially deviate from the exponential shape and cannot be explained by any thermal model. The exponential shape is only valid at high energies where non-thermal particles are emitted. Here we find a dilemma of the thermal model.

SUMMARY:

1. The spectrum shape of protons at c.m. 90° at the beam energy of around 1 GeV/nucleon is characterized by the "shoulder-arm" shape.
2. The beam energy per nucleon (not the total beam energy) determines the major feature of proton emission.
3. The comparison of the observed spectra with a phase-space calculation as well as the observed A dependence suggest the importance of the multiple NN collision process for the production of large p_T protons.

4. Low energy protons originate both from direct and multiple collisions.

This was also confirmed by the two proton correlation experiment. In addition, the known experimental fact that $\lambda \approx R$ at the beam energy of 1 GeV/nucleon supports this conjecture, where λ is the nucleon mean free path inside the nucleus (which is about 2.4 fm^{18}) and R is the size of the interaction region (which is 2-3 fm according to the discussion in Sec. 5.2.).

5. FORMATION OF COMPOSITE FRAGMENTS

5.1. Power Law

From the participant region protons and neutrons are emitted over a wide range of energy and angle. In this case there is a certain chance that these nucleons stick together to form a composite fragment. Naively we therefore expect that the probability of forming a deuteron at a velocity $\vec{v}_d$ is proportional to the product of the probabilities of finding a proton and a neutron at the same velocity:

$$P_d(\vec{v} = \vec{v}_d) \approx P_p(\vec{v} = \vec{v}_d) \cdot P_n(\vec{v} = \vec{v}_d). \quad (9)$$

If neutron spectra can be replaced by proton spectra, then we expect the cross section of the composite fragment with mass number A to be given by,

$$E_A(d^3\sigma_A/d^3p_A) = C_A [E_p(d^3\sigma_p/d^3p_p)]^A, \quad (10)$$

where $p_A = A \cdot p_p$, and C_A is a constant.

In Fig. 9 the spectra of deuterons¹⁰ from 800 MeV/nucleon $C + C$ collisions are shown and they are compared with the squares of the observed proton spectra. With one normalization constant C_A the above power law of Eq. (10) holds extremely well.

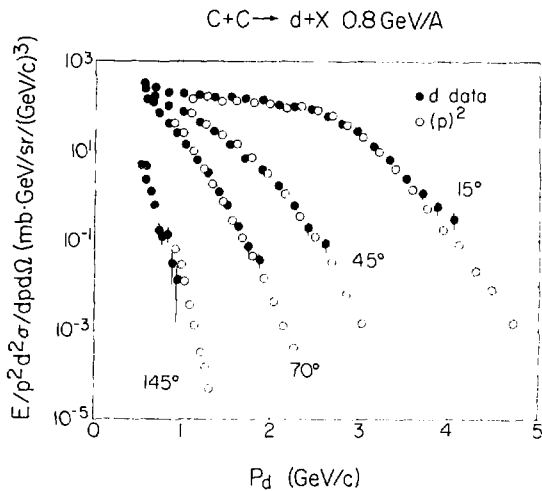


Fig. 9 Deuteron spectra as compared with the squares of proton spectra.

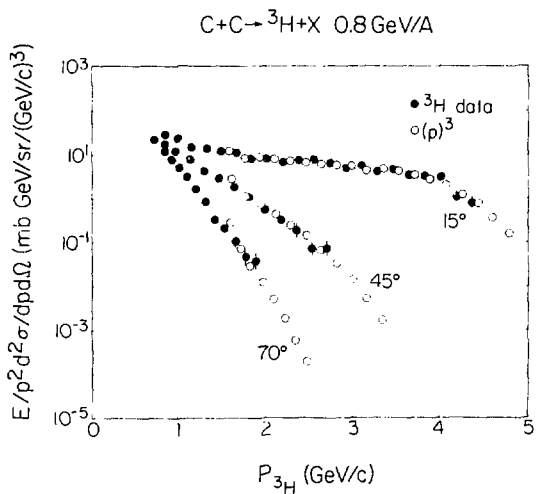


Fig. 10 Triton spectra as compared with the cubes of proton spectra.

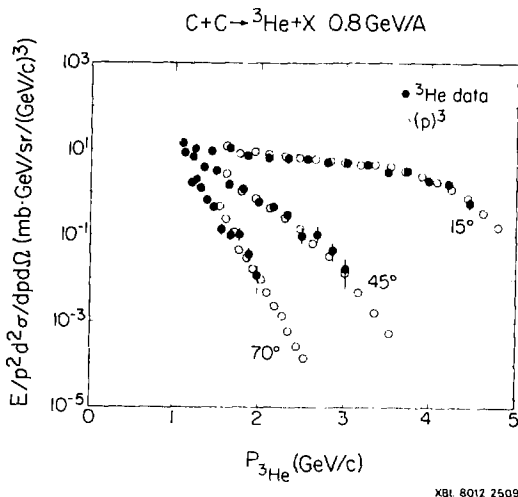


Fig. 11 ${}^3\text{He}$ spectra as compared with the cubes of proton spectra.

The triton spectra from the same reaction are shown in Fig. 10. In this case the spectra agree very well with cubes of the observed proton spectra. Similarly the power law holds very well for ${}^3\text{He}$, as shown in Fig. 11, and of course, we expect nearly the same yields for tritons and ${}^3\text{He}$ from $C + C$ collisions, as seen in Figs. 10 and 11.

Let us study in more detail the implication of this power law. In Fig. 13 I show the observed deuteron cross sections divided by the squares of the observed proton cross sections for $\text{Ne} + \text{NaF}$ collisions at three bombarding energies, 400, 800, and 2100 MeV/nucleon.¹⁰ The value C_A is almost independent of the deuteron momentum as well as the deuteron emission angle. In addition, the value C_A is almost independent of the projectile energy. What do we learn from these data?

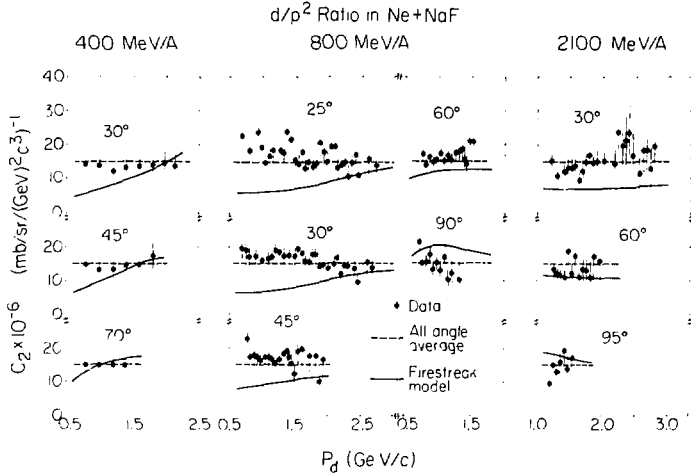


Fig. 12 Observed d/p^2 ratios for Ne + NaF collisions.

If the particle density in unit phase space volume is given by $f(\vec{p})$ such that

$$(1/V) \cdot (d^3n/d^3p) = f(\vec{p}), \quad (11)$$

then the ratio C_A is given by

$$C_A = (1/\gamma V)^{A-1} \cdot f(\vec{p}_A) / [f(\vec{p}_p)]^A \quad \text{with } \vec{p}_A = A \cdot \vec{p}_p. \quad (12)$$

Now, what do we expect for C_A in terms of various models? In a simple fireball model¹¹ $f(\vec{p})$ is simply given by

$$f(\vec{p}) = e^{-E/T}, \quad (13)$$

Consequently, C_A is given by

$$C_A = \text{const.}/(\gamma V)^{A-1}, \quad (14)$$

where γ is the Lorentz factor of the emitted particle measured relative to the c.m. frame of the fireball.¹⁹ In the data shown in Fig. 12 the range of γ is 1.0 - 1.3 for 400 MeV/nucleon, 1.0 - 1.6 for 800 MeV/nucleon, and 1.1 - 2.0 for 2.1 GeV/nucleon. Therefore, within a factor two the simple thermal model agrees with the observed fact the C_A is almost constant.

However, it has been well known that the simple fireball model does not explain any data. Especially the large anisotropy of the observed angular distribution¹⁰ in the c.m. frame cannot be explained by this model. In order to reproduce the angular anisotropy the firestreak model^{20,21} was proposed and it has been often used to fit the data. In this model the nucleus is divided into several tubes in order to take into account the geometry more accurately than the simple fireball model. As a result, the temperature is different from tube to tube. Here, $f(\vec{p})$ is given by

$$f(\vec{p}) = \sum a_i e^{-E/T_i}, \quad (15)$$

and C_A is no longer constant but has a strong fragment-energy dependence. For example, the predicted value²¹ of C_A in the case of 400 MeV/nucleon at 30° (left upper corner in Fig. 12) changes from 5 to 20 which clearly disagrees with the data. Therefore, the firestreak model has trouble in reproducing the observed power law. This fact further indicates that the composite fragments may not be produced from a macroscopic chemical equilibrium inside the fireball.

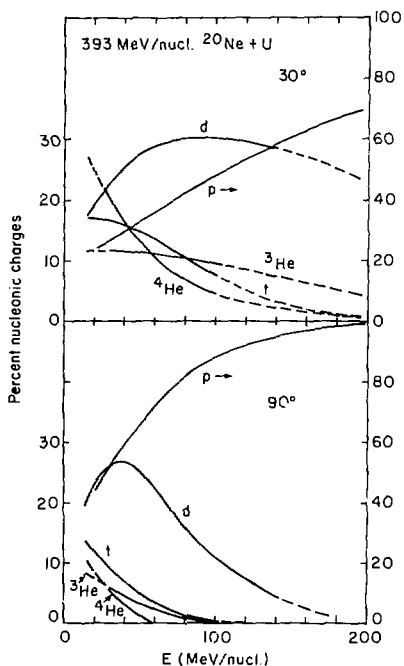
In the coalescence model²²⁻²⁵ nucleons which are located within a radius of p_0 in the momentum space stick together to form a composite fragment. In this case C_A is given by

$$C_A \propto [(4\pi/3) \cdot p_0^3]^{A-1}. \quad (16)$$

We should note, however, that in this model, if there are two or more nucleons inside the radius p_0 , then the assembly of these nucleons is immediately regarded as a real composite particle. Therefore, the cross section of composite fragment A is expressed as

$$\sigma_A = C_A [\sigma(\text{original nucleon})]^A, \quad (17)$$

namely, the cross section of a composite fragment is proportional to the A^{th} power of the cross section of the original nucleons before the formation of



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Fig. 12 Percentage fractions of nuclear fragments in 393 MeV/nucleon Ne + U.

composite fragments. However, the experimental fact is that the power law holds between observed cross sections:

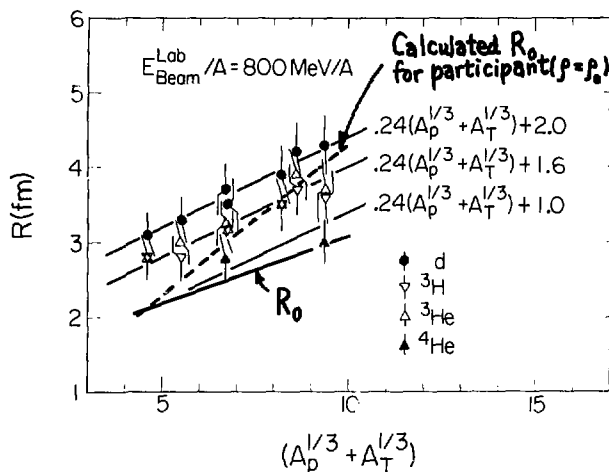
$$\sigma_A(\text{observed}) = C_A [\sigma(\text{observed proton})]^A. \quad (18)$$

Of course, if the observed proton cross sections are much larger than the observed cross sections of composite fragments, then there is no significant difference between Eqs. (17) and (18). But, as shown in Fig. 13, the yield ratios between composite particles and protons are sometimes comparable to each other in certain kinematical regions.¹⁴ Still, the power law holds well between the observed composites and the observed protons. Therefore, when we apply the coalescence model, we have to assume that the local chemical equilibrium exists between the formation and break-up of composite fragment, such as $d \rightleftharpoons p + n$. If the local chemical equilibrium holds, then why not the macroscopic chemical equilibrium? This is a very interesting subject which has to be studied in the future.

5.2. Size of the Interaction Region

Next, we discuss another aspect of the ratio C_A . As seen from Eq. (12), C_A is related to the phase space volume V or the radius R ($= (3V/4\pi)^{1/3}$). In Fig. 14 the radii obtained in various collisions at 800 MeV/nucleon¹⁰ are shown. Here, the theoretical model of Mekjian²⁶ has been used. The observed radius increases as the projectile and target masses increase. Also, the radius increases as the mass of the detected composite fragments decreases. Why do we observe such features?

Recently Sato and Yazaki²⁷ proposed a theory to relate the observed radius R with the real source radius R_0 and the radius of the composite fragment R_A by the following formula:



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Fig. 14 The radius of the phase space volume determined from the observed power law. The formula of Mekjian is used. Also, the empirical source radius R_0 as well as the calculated R_0 are shown.

$$R = 1/p_0 = \kappa \times \sqrt{R_0^2 + R_A^2}. \quad (19)$$

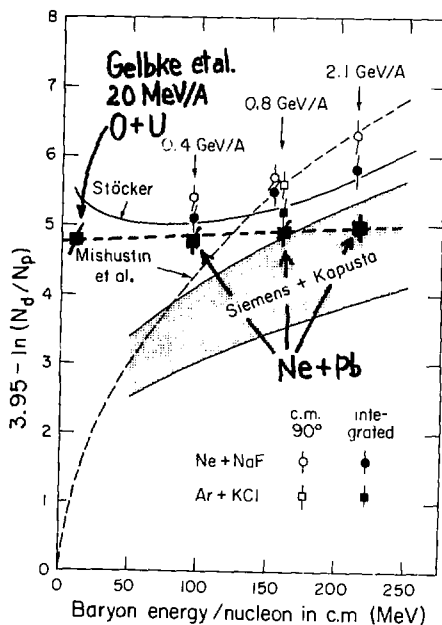
At the present moment the predicted value of κ cannot be used to explain the whole set of available data, but qualitatively this formula explains the observed larger tendency that R is larger for lighter mass composite fragments, since the radius R_A is 2.2 fm for d , 1.7 fm for t and ${}^3\text{He}$, and 1.3 fm for α , and thus monotonically decreases as the mass of the emitted fragment increases.

If we admit that the observed radius R is determined only by R_0 and R_A , then we can empirically evaluate R_0 by plotting the observed values of R^2 as a function of R_A^2 and by extrapolating R^2 to the limit of $R_A^2 = 0$. Such empirical values of R_0 are also shown in Fig. 14, which ranges from 2 to 3 fm. Also, in Fig. 14 the calculated values of R_0 based on the participant-spectator model are plotted for normal nuclear density ($\rho = \rho_0$). We observe good agreement

between the calculated and observed values for light mass systems such as C + C, whereas the calculated value is much larger than the observed one for heavy mass systems such as Ar + Pb. Of course, it is too early to say that this may be an evidence on the formation of high density, but it is certainly interesting to study in more detail the source radius from the composite spectra.

5.3. Entropy

So far I have discussed mainly the power law. A simple yield ratio between composite fragment and proton tells us another type of information.



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Fig. 15 The observed d/p ratio and the entropy.

One of the topics related to this yield ratio is the entropy. This discussion was initiated by the work of Siemens and Kapusta.²⁸ They pointed out two aspects. One is that the d/p ratio is related to the quantity of entropy, S , by the relation of

$$S = 3.95 - \ln(N_d/N_p), \quad (20)$$

and the other is that the observed entropy is much larger than the theoretical entropy, as shown in Fig. 15. Therefore, they thought that perhaps a new degree of freedom is created in high energy nuclear collisions.

In this Conference this subject of the entropy will be discussed at length by G. Bertsch²⁹ and H. Stöcker³⁰, and therefore I will not go into details in the present talk. Instead, I would like to point out three aspects related to this entropy work. The first point is that a new calculation by Mishustin et al.³¹ who took into account the pion degree of freedom explains reasonably well the data except for that with 400 MeV/nucleon beams. The second point is that, if we plot the data for non-symmetric system such as Ne + Pb (Ref. 10) or O + U (Ref. 32), then the d/p ratio stays almost constant at all bombarding energies from 20 to 2000 MeV/nucleon. This fact implies that the d/p ratio may not be directly related to the quantity of entropy. In fact, Stöcker has recently shown that the observed d/p ratio has no relationship to the entropy, and has predicted the almost beam-energy independent d/p ratio, as shown in Fig. 15. The third point which is a more philosophical question, is the production mechanism of composite fragments. As mentioned previously, the macroscopic chemical equilibrium may not be the real production mechanism of composite fragments. The quantity of entropy is directly related to the chemical potential μ by the formula $S = 5/2 - \mu/T$, where T is the temperature. Therefore, unless it is proved that the macroscopic chemical equilibrium is the real production mechanism of composite

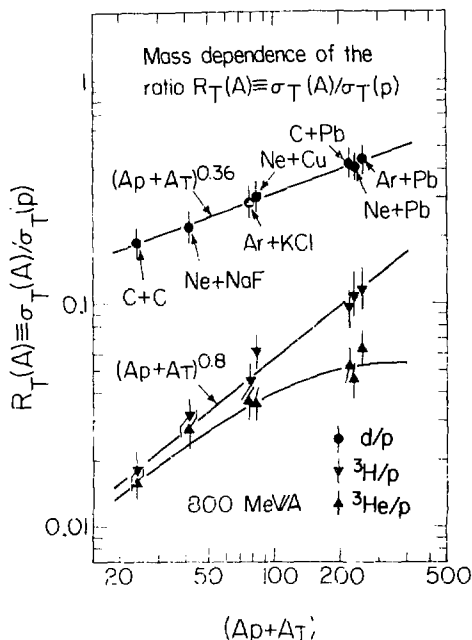


Fig. 16 Ratios of d/p , t/p and ${}^3\text{He}/p$ plotted as a function of the total mass number of the system.

formation, it is rather difficult at least for me to immediately accept the relationship between the entropy and the observed d/p ratio.

Finally I would like to point out the finite size effect of the system on the composite formation. In Fig. 16 the d/p ratios or t/p ratios are plotted as a function of the projectile mass plus the target mass.¹⁰ Larger ratios are observed for heavier mass systems. This is understandable, because more combinations between nucleons are available for heavier mass systems. However, most of the theoretical models have not correctly treated this finite size effect of the system, and predict a constant yield ratio for any combination between the projectile and target.³³ This point has to be

correctly solved especially before discussing the entropy.

SUMMARY:

1. A power law given by Eq. (10) holds very well between the observed proton and composite-fragment spectra. If we use the coalescence model, we have to assume local chemical equilibrium between the formation and break-up.
2. The observed C_A is independent of beam energy as well as the fragment energy and angle, and depends only on the projectile and target masses. This fact is inconsistent with the prediction of the firestreak model and suggests that macroscopic chemical equilibrium may not be the mechanism responsible for producing composite fragments.
3. Source radii were derived from the observed values of C_A . The results are 2-3 fm, depending on the mass of the system.
4. With regard to the yield ratio between the composite particles and protons, all theoretical models should first consider the finite size effect of the system.
5. The discussion of the entropy is certainly interesting, but there is a question on the relationship between the d/p ratio and the entropy.

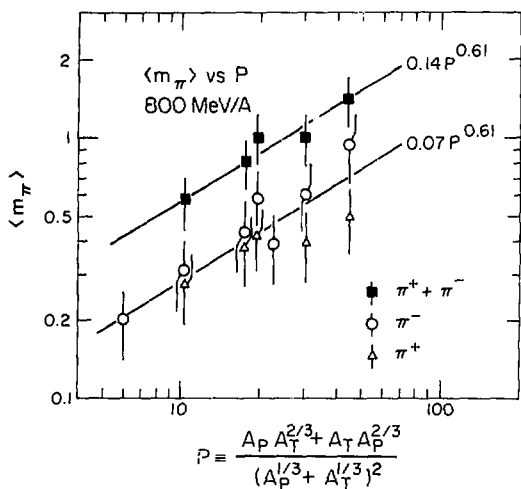
6. PION PRODUCTION

6.1. Multiplicity

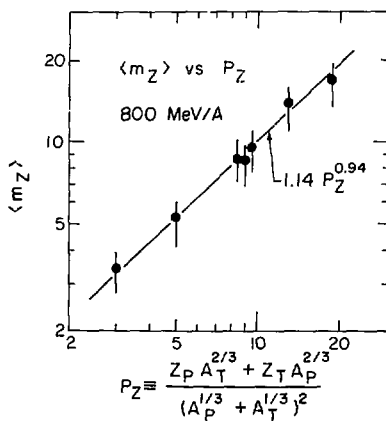
For beam energies around 1 GeV/nucleon the dominant secondary particles created in the collisions are pions. In this section we review recent progress on pion production.

As an introduction I discuss the pion multiplicity. Since the total integrated inclusive cross section, σ_{incl}^{tot} is given by

$$\sigma_{incl}^{tot} = \langle m \rangle \cdot \sigma_0, \quad (21)$$



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Fig. 17 Observed pion multiplicities $\langle m_\pi \rangle$.

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Fig. 18 Observed multiplicities of total nuclear charge, $\langle m_Z \rangle$.

where σ_0 is the total cross section, we can evaluate the average multiplicity $\langle m \rangle$ from the inclusive data. In Fig. 17 the observed values of $\langle m_\pi \rangle$ are plotted as a function of the average participant nucleon number, P , for various projectile and target combinations at 800 MeV/nucleon. The multiplicity is roughly parameterized as³⁴

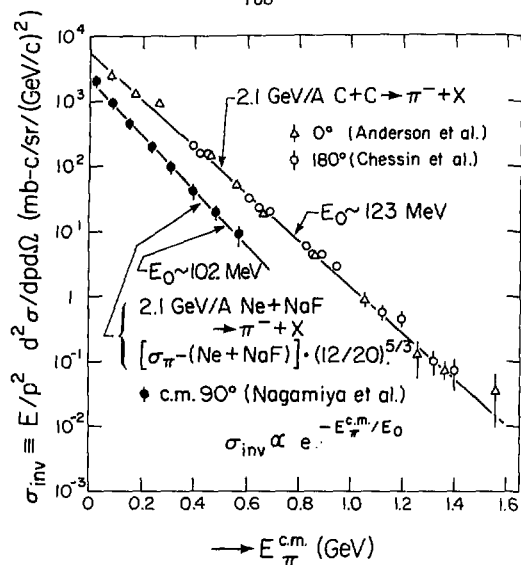
$$\langle m_\pi \rangle = a \cdot P^x, \quad (22)$$

with $x \approx 2/3$. This implies that the pions are emitted from the surface of the participant and not from deep inside the participant region. If we compare these data with the average multiplicity of total nuclear charge, $\langle m_Z \rangle$, the difference is much clearer. As seen in Fig. 18, the multiplicity $\langle m_Z \rangle$ increases almost linearly with the average participant proton number, P_Z . Therefore, the $P^{2/3}$ dependence of the pion multiplicity strongly suggests the importance of the pion absorption process. So far a number of theoretical models^{21,35} have overestimated the pion yields, since they do not take into consideration the pion absorption effect. I would like to emphasize here that all the theoretical models should first include the pion absorption before they are compared with the data.

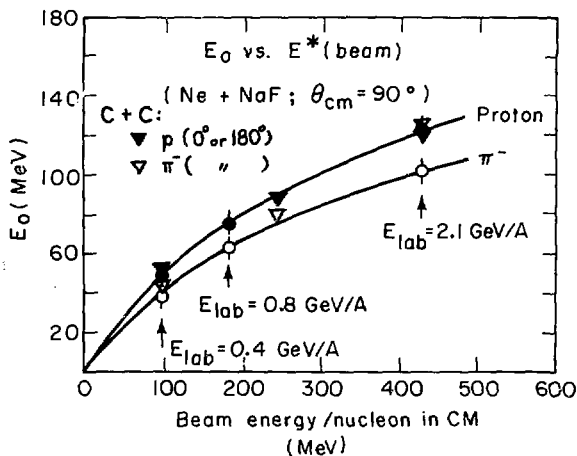
6.2. Energy and Angular Distributions

Then, how about the energy spectra? In Fig. 19 the energy distributions of negative pions at 0° , 180° , and at 90° are plotted in the c.m. frame. Data were taken with 2.1 GeV/nucleon beams, but unfortunately no direct comparison can be done for the same projectile and target combinations. An important message obtained from this figure is that the energy distribution is almost exponential at any emission angle. Also, the 0° and 180° data currently cover much wider kinematical regions than the 90° data.

In Fig. 20 the inverse exponential slopes E_0 are plotted as a function of



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Fig. 19 Pion spectra at $E_{\text{Lab}}^{\text{Beam}} = 2.1 \text{ GeV/nucleon}$.

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Fig. 20 Observed inverse slopes E_0 at 90° as compared with those at 0° and 180° .

the beam energy per nucleon in the c.m. frame. The value of E_0 monotonically increases with the beam energy. Also, the data at 0° and 180° show almost the same behavior with the data at 90° except at $E_{\text{Lab}} = 2.1 \text{ GeV/nucleon}$.³⁶

Of course, there is no a priori reason that the spectrum shape should be exponential. In fact, Anderson et al.³⁷ showed that the energy distributions in the nucleon-nucleon c.m. frame are not exponential for proton and deuteron beams. For projectiles heavier than α the spectrum shape approaches exponential, as shown in Fig. 21. In this figure we also observe that yields of high energy pions are significantly higher for heavier mass projectiles. This is an interesting feature and suggests cumulative effects.

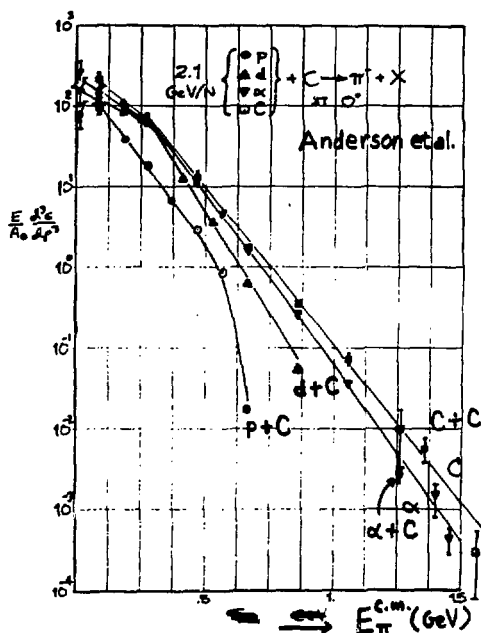
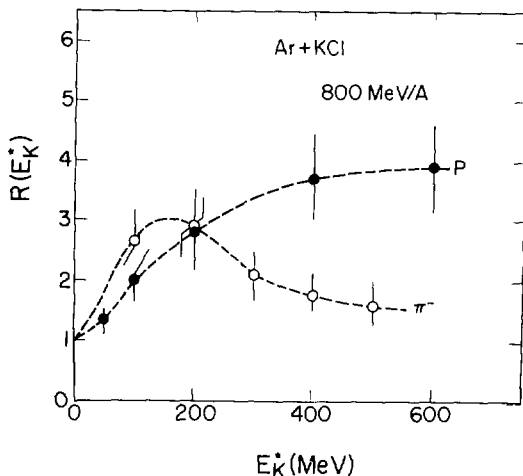


Fig. 21 Pion spectra by various projectiles at 2.1 GeV/nucleon.

Next I discuss the angular distribution. I discuss it for a beam energy of around 1 GeV/nucleon in Ar + KCl or Ar + Ca collisions. Shown in Fig. 23 are the ratios of the π^- cross sections between 30° and 90° in the c.m. frame for 800 MeV/nucleon Ar + KCl collisions, plotted as a function of the pion energy in that frame. The ratio has a peak at around $E_\pi^{c.m.} \approx 150$ MeV. This observed feature has not yet been explained by any conventional theoretical model, but it seems that such a feature of pion emission is consistent with the expectation that most of the pions are from Δ_{33} resonances, since the decay from Δ increases the pion yield at around 150 MeV and might have a large anisotropy there.

Another interesting subject of the angular distribution is its detailed structure for low energy pions at $E_\pi^{c.m.} \approx 10$ -20 MeV. Wolf et al.³⁸ and Nakai et al.³⁹ reported a broad 90° peaking for π^+ emission at $E_\pi^{c.m.} \approx 10$ -20 MeV in



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Fig. 22 The 30° to 90° yield ratio in the c.m. frame, plotted as a function of the kinetic energy in that frame.

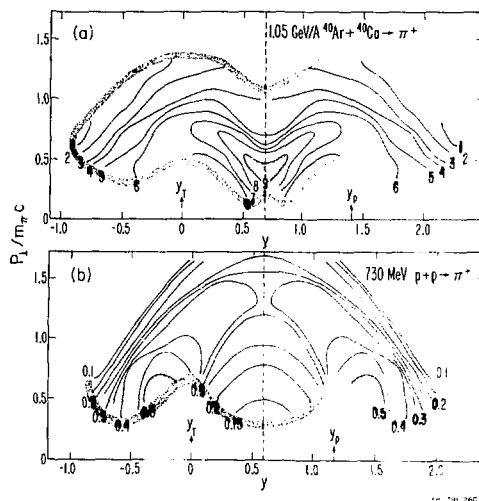


Fig. 23 Data of π^+ by Wolf et al. to show 90° peaking.

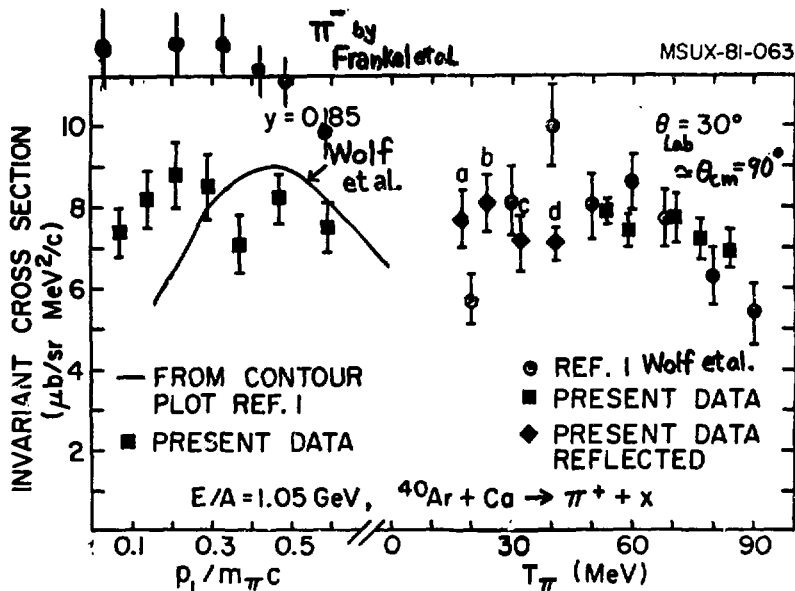


Fig. 24 Preliminary data of π^+ and π^- by Frankel et al. to show no evidence on the 90° peaking.

Ar + Ca and Ne + NaF collisions. The well-known plot for this broad peaking is shown in Fig. 23. Libbrecht and Koonin⁴⁰ studied Coulomb effects on pion emission and showed that this broad peaking might be due to the Coulomb focusing effect. If the Coulomb effect is responsible, then it is rather interesting to measure both π^+ and π^- .

Recently Frankel et al.⁴¹ measured both π^+ and π^- by a magnetic spectrometer in 1.05 GeV/nucleon Ar + Ca collisions, the same projectile and target combinations used by Wolf et al. In Fig. 24 their preliminary data are displayed. In the left-hand side of the figure the p_T distributions of both π^- and π^+ at c.m. 90° are shown. The yield of π^- is consistently larger than that of π^+ , probably because of the Coulomb effect. However, these data do not show any peaking, which strongly contradict the data by Wolf et al. According to Frankel et al., the difference may have originated from the trick of the contour plot, because if they plot the raw data at $\theta_{\text{Lab}} = 30^\circ$ (which roughly corresponds to c.m. 90°), then their data agree reasonably well with the data by Wolf et al. except for one or two points, as seen in the right-hand side of Fig. 24. However, if the data points are connected by a smooth curve, one set of the data produces a peak while the other set does not. The peaking is anyway about a 20 % effect in the cross section, and the measurements are not easy. We should perhaps wait for another independent experiment to pin down this 90° puzzle.

6.3. Subthreshold Pion Production

In nucleon-nucleon collisions the threshold energy of pion production is 290 MeV. Therefore, any pion production in nuclear collisions with beams below 290 MeV/nucleon is due to the nuclear effect.

Benenson et al.⁴² have measured both π^- and π^+ at 0° in Ne + NaF collisions and showed that pions are produced even at beam energy of 80

MeV/nucleon, as shown in Fig. 25. For the production of 70-80 MeV pions in the laboratory frame with 80 MeV/nucleon beams, if we ignore Fermi motion, then about 9 nucleons have to sum their kinetic energies to create one pion, because in the c.m. frame the kinetic energy plus rest mass of these pions is about 185 MeV while the beam energy per nucleon in this frame is only 20 MeV/nucleon. If these pions are created in single nucleon-nucleon collisions, then we have to assume a Fermi motion of about 400 MeV/c.

Subthreshold pion production has also been studied at c.m. 90° in 200 MeV/nucleon Ne + NaF collisions.⁴³ Very preliminary data are shown in Fig. 26 (The absolute value is reliable only within a factor of 2-3). In the c.m. frame the kinetic energies of the observed pions extend up to 260 MeV. For the production of such high energy pions about 8 nucleons have to sum their kinetic energies. This situation is very similar to the case studied just above for 0° pion production by 80 MeV/nucleon beams. However, if we

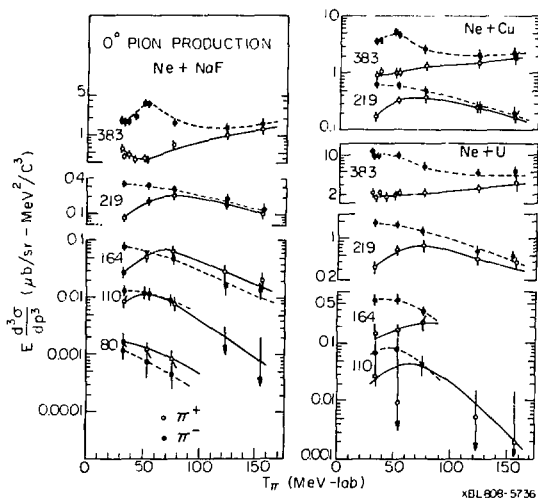


Fig. 25 Pion production at 0° with low energy beams.

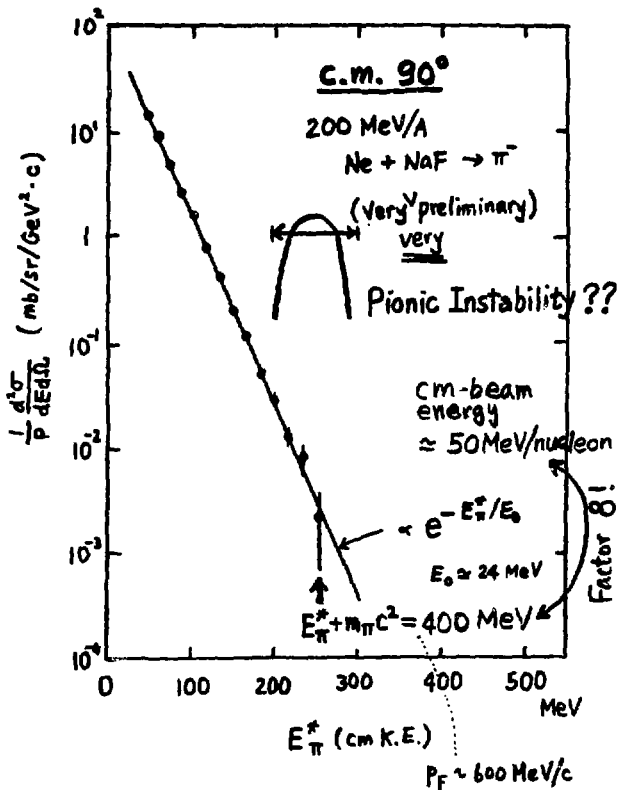


Fig. 26 Pion production at 90° with 200 MeV/nucleon beams.
Data are very preliminary.

look at the data more carefully we notice that the cross section to produce 260 MeV pions (in the c.m.) as seen in Fig. 26 is about 1000 times smaller than the cross section to produce pions at 0° with 60 MeV/nucleon beams. This is an interesting aspect. In terms of single nucleon-nucleon collisions we need 600 MeV/c Fermi motion in the former case while 400 MeV/c in the latter. This fact may perhaps be responsible for creating such a large difference in cross sections. Another interesting aspect seen in Fig. 26 is that the energy spectrum is almost of a pure exponential type down to such low cross sections.

This is rather surprising.

SUMMARY:

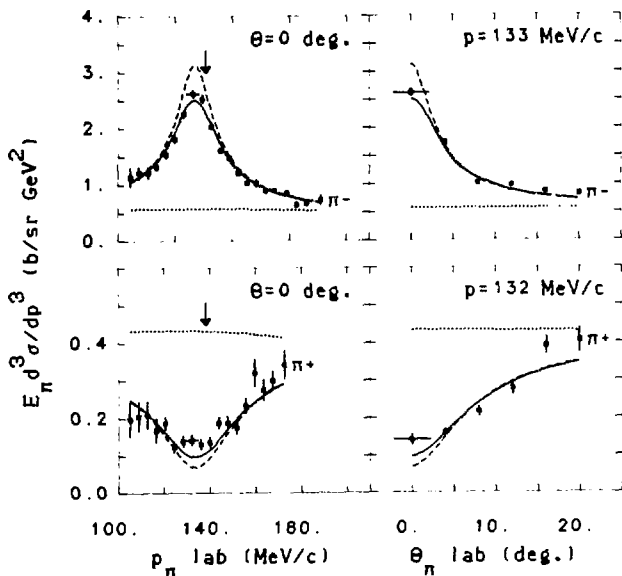
1. The average pion multiplicity $\langle m_\pi \rangle$ is proportional to $P^{2/3}$ where P is the average nucleon number in the participant region. This fact suggests the importance of pion absorption process.
2. Energy spectra in the c.m. frame are almost exponential at any angle and at any beam energy.
3. At 1 GeV/nucleon beam energies the angular distribution in the c.m. frame shows forward peaking at $E_\pi^{c.m.} \approx 150$ MeV, presumably due to the formation of Δ . The 90° peaking at $E_\pi^{c.m.}$ is now uncertain.
4. Subthreshold pion production is a useful tool to study energy concentration in high energy nuclear collisions and has been studied at both 0° and c.m. 90° . Pions which require an energy concentration of about 8-9 times the average beam energy per nucleon are observed.
5. Although I have not discussed it, no messages have yet been obtained on the evidence of pion condensation.

7. RATIOS OF π^-/π^+ , n/p AND $t/^3\text{He}$

7.1. π^- to π^+ Ratio

In Fig. 27 the data of π^- and π^+ measured at 0° in collisions of 380 MeV/nucleon Ne + NaF are displayed. To a first-order approximation we expect that both π^- and π^+ yields should be the same, since both Ne and NaF contain almost equal proton and neutron numbers. However, as seen from the figure, the yields of π^- are much higher than those of π^+ . In addition, the yields of π^- have a sharp peak at a certain momentum, whereas those of π^+ have a valley there. The pion velocity at this peak (or valley) is very close to the beam velocity which is indicated by an arrow in Fig. 27. The ratio of π^- to π^+

Ne+NaF $\rightarrow$ X + π
 E/A = 380 MeV



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Fig. 27 Spectra of π^- and π^+ emitted at forward angles.

there reaches about 20-30.

It is expected that projectile fragments are clustered dominantly at 0° at the beam velocity. Since these clusters carry positive charges, negative pions produced in the collisions will be attracted by these clusters due to Coulomb interactions, whereas positive pions will be repelled from them. Therefore, we expect large yields of π^- and small yields of π^+ . In fact, theoretical calculations of Coulomb interactions^{40,44} explain reasonably well these observed features.

Although it is expected that Coulomb effects are the strongest at 0° , it is also interesting to study how these effects appear in the data at large

gles. In Fig. 28 the π^- to π^+ ratios measured at 30° , 60° , and 90° are ted as a function of the measured pion momentum for various collisions at a energies of 800 MeV/nucleon. In the Ne + NaF case the ratio is larger than one only in the small momentum region at 30° , while it is close to one at 60° and 90° for all momenta. A similar tendency is observed for Ar + KCl. These facts imply that the Coulomb effects are anisotropic in the laboratory frame and are stronger at smaller angles. This is consistent with the expectation that most of the nuclear charges are clustered along the beam axis. For Ar + Pb stronger Coulomb effects are observed mainly because of the larger nuclear charge of the system.

Detailed study of Coulomb effects may tell us useful information on the nuclear transparency. If the nucleus is completely transparent, then both projectile and target pass through each other, and consequently, most of nuclear charges are clustered at $(y, p_T) = (y_p, 0)$ and $(y_T, 0)$. On the other

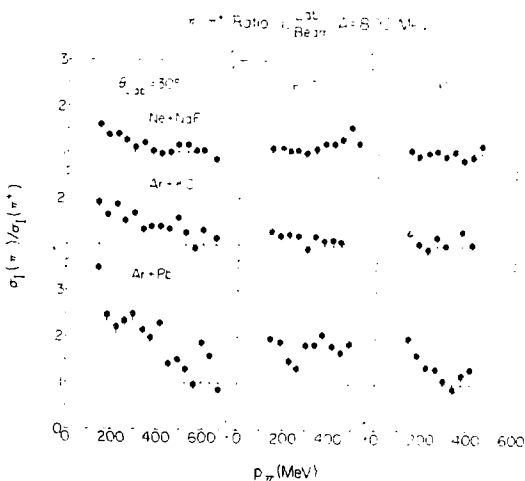
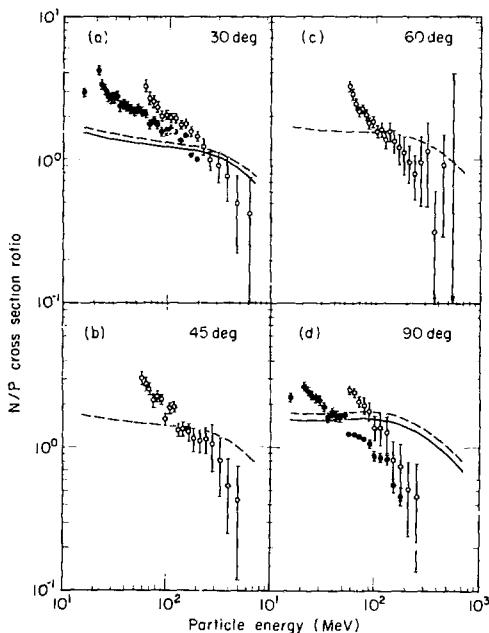


Fig. 28 Ratios of π^- to π^+ at large angles.

hand, if the nucleus is completely black, then the projectile and target stop each other completely in the c.m. frame. A theoretical value of the π^- to π^+ ratio at $P_T = 0$ at c.m. 90° for Ar + KCl collisions (at around 1 GeV/nucleon) is 2.8 for the complete black case and 1.6 for the complete transparent case.⁴⁵ The observed value is about 2 which is in between the two extreme cases.

7.2. Neutron to Proton and Triton to ^3He Ratios

If π^- to π^+ ratios show Coulomb effects, we expect also that the neutron to proton as well as the triton to ^3He ratios should be affected by Coulomb

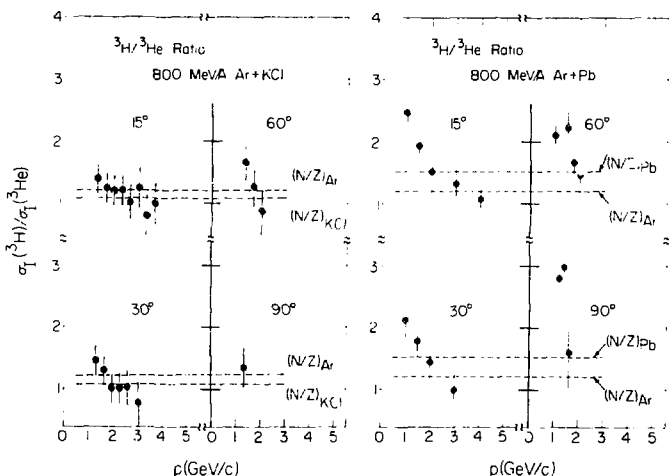


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Fig. 29 Neutron to proton ratios measured in Ne + (U,Pb) collisions.

interactions. In Fig. 29 the observed neutron to proton ratios⁴⁶ in 300-400 MeV/nucleon Ne + (Pb,U) collisions are displayed. Unfortunately these neutron and proton measurements were done at slightly different beam energies. Nevertheless, we clearly observe that the n/p ratio is larger at smaller fragment energy. A similar tendency is observed in the data of triton to ^3He ,¹⁰ as shown in Fig. 30.

These facts immediately suggest that Coulomb effects may also be important in explaining the observed n/p and t/ ^3He ratios. However, if we compare the observed momentum dependence of the n/p or t/ ^3He ratio with the π^-/π^+ ratio, we notice that the mechanism which causes an enhancement in the ratio in the small momentum region may not be necessarily the same. For example, in the case of π^- to π^+ , the ratio is larger than the N/Z of the system (N: neutron number, Z: proton number) in the small momentum region at forward angles but not at large angles (Fig. 28), whereas such a large ratio



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Fig. 30 Triton to ^3He ratios in Ar + KCl and Pb collisions.

is observed at all angles in the case of n/p and $t/^3\text{He}$ ratios (Figs. 29 and 30).

For a system with $N/Z > 1$, such as $\text{Ne} + \text{Pb}$, one would naively expect that a different mechanism might be involved in the case of the n/p or $t/^3\text{He}$ ratio as compared to the π^-/π^+ ratio. The neutron to proton ratio for the originally produced nucleons (in the sense of cascade calculations) would be almost equal to the N/Z ratio of the system. However, some of these nucleons will be combined to form composite fragments before they are actually emitted. Among them the deuteron emission would be the largest. Since deuterons carry equal numbers of protons and neutrons, the remaining neutrons and protons might form a ratio which is much larger than the N/Z ratio of the system. This idea has theoretically been formulated by Randrup and Koonin⁴⁷ and also by Stevenson⁴⁸ in order to explain the observed neutron to proton ratios. According to them, a large ratio is expected in the small momentum region at all angles because deuterons are more easily produced there. In fact, Stevenson showed that this effect without any Coulomb effect already explains the major feature of the data of the n/p ratio shown in Fig. 29. Therefore, a careful treatment is required when one applies the theory of Coulomb effects on the ratios of n/p and $t/^3\text{He}$.

SUMMARY:

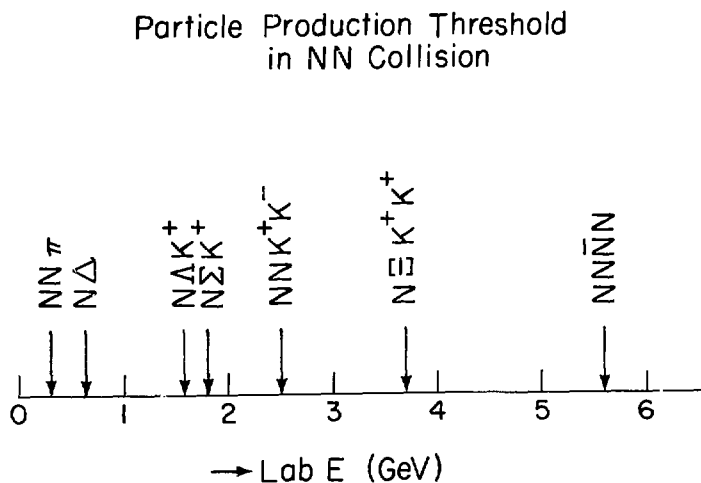
1. The cross section of π^- has a sharp peak at 0° at the beam velocity and that of π^+ has a valley there. This fact strongly suggests that Coulomb interactions distort pion spectra especially at the region of the beam velocity.
2. At large laboratory angles the π^- to π^+ ratio is larger than the N/Z ratio of the system especially in the small momentum region at forward angles.
3. The n/p and $t/^3\text{He}$ ratios are larger than the N/Z ratio of the system in the small momentum region at all angles. Coulomb effects as well as other

mechanisms, such as isospin equilibration, must be considered in order to explain these ratios.

8. PRODUCTION OF STRANGE PARTICLES

In Fig. 31 various threshold energies for particle production in nucleon nucleon collisions are listed. At around 2 GeV the threshold energies for the production of strange particles are clustered. In this talk I will discuss K^+ and Λ .

The motivation of measuring K^+ or Λ is as follows. Since the cross section of $K^+ + N$ [$\sigma(K^+N) \approx 10$ mb] is much smaller than that of $N + N$ [$\sigma(NN) \approx 40$ mb] or $\pi + N$ [$\sigma(\pi N) \approx 200$ mb], K^+ is less likely to be rescattered by

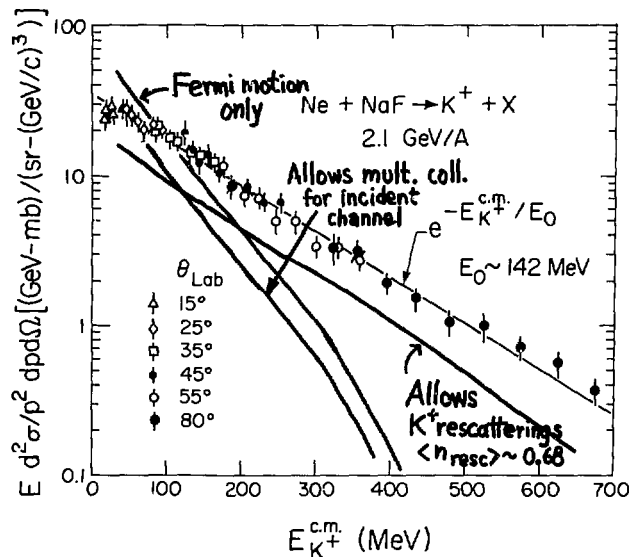


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Fig. 31 Particle production threshold energies in nucleon nucleon collisions.

surrounding nucleons (compared to π or N), and thus reflects more of the features of the violent stage of the collision at which particles are created. In addition, because of the strangeness conservation, K^+ is less likely absorbed inside the nucleus. These situations for K^+ production are quite different from those for pions.

Schnetzer et al.⁴⁹ have measured K^+ spectra and its associate multiplicity with a magnetic spectrometer and 16 multiplicity counters. In Fig. 32 an example of the measured energy spectrum in the c.m. frame is plotted for 2.1 GeV/nucleon Ne + NaF collisions. The spectrum shape is almost exponential with the inverse exponential slope, E_0 , around 142 MeV. If this



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Fig. 32 The K^+ spectrum in 2.1 GeV/nucleon Ne + NaF collisions.

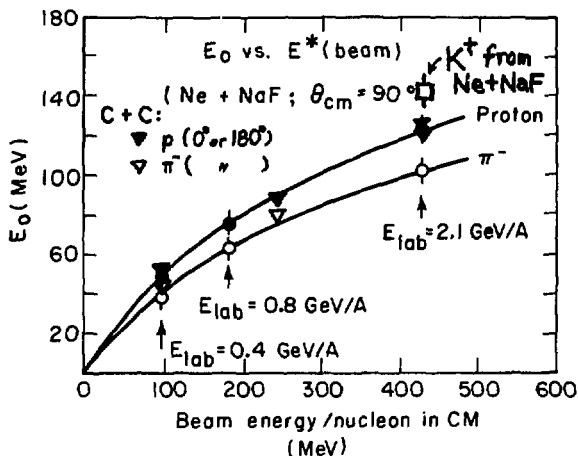


Fig. 33 Observed E_0 for K^+ as compared with E_0 for protons and pions.

E_0 is plotted on the same graph shown before, then the value of E_0 for K^+ is much larger than E_0 for protons or pions (see Fig. 33). This fact implies that K^+ reflects more violent stage than protons and pions do.

Then how much useful information can we obtain from the kaon spectra? In order to create 0.7 GeV (kinetic energy) K^+ in the c.m. frame by a single nucleon-nucleon collision, one needs a Fermi motion of 7-800 MeV/c which is unrealistically too large for one nucleon to carry. Therefore, one can have a strong hope that the cumulative effect can most sensitively be studied from the K^+ spectrum.

However, the actual situation is rather complicated. According to Randrup and Ko⁵⁰, the data are, in fact, not explained by Fermi motion only. Also, it is not possible to reproduce the data even if the multiple collisions for the incident channel are taken into account. However, if we allow slight

rescatterings of K^+ after its creation, for example, at the level of an average number of rescattering of 0.68, then such rescatterings broadens the spectrum up to the observed slope, as seen in Fig. 32. This is somewhat discouraging. Nature is not always kind when we strive for a new physics.

The projectile and target mass dependences of the K^+ production has also been studied by Schnetzer et al. The data shows that

$$\sigma \propto A_P \times A_T, \quad (23)$$

where A_P and A_T are the projectile and target mass numbers respectively. This relation is somewhat different from that expected by a simple participant-spectator model in which we have $A_P^{2/3} A_T^{2/3} + A_T^{2/3} A_P^{2/3}$. The above relation, Eq. (23), is consistent with the prediction by Randrup and Ko.

We should also mention the thermal model. If we assume chemical equilibrium, then a calculation showed a factor of 20 larger yield than the data, although the shape of the momentum spectra as well as the angular distributions are reasonably well reproduced.⁵¹

The Λ production has been studied recently by Harris et al.⁵² with a streamer chamber in 1.8 GeV/nucleon Ar + KCl collisions. In this measurement the decay of Λ ,

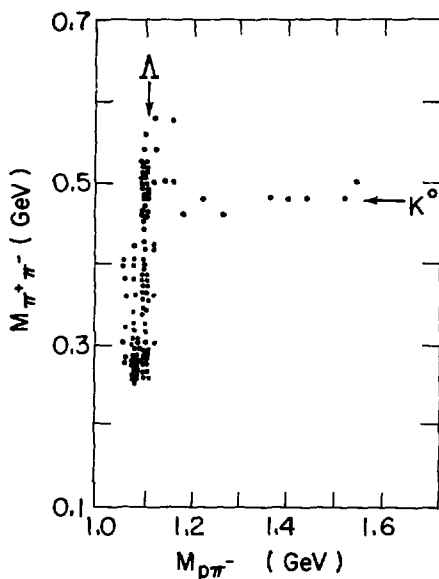
$$\Lambda \rightarrow p + \pi^- \quad (64 \% \text{ branching}), \quad (24)$$

was used for the identification. A plot of the measured invariant mass for the $p\pi^-$ system is shown in Fig. 34. We clearly observe the signal of Λ .

In Fig. 35 the observed momentum distribution of Λ is plotted in the plane of the parallel and transverse momenta in the c.m. frame. The circle in the figure indicates the expected kinematical region for Λ production in 1.8

GeV nucleon + nucleon collisions. Most of the observed points are scattered outside this nucleon-nucleon kinematical limit. Very few points inside this circle are somewhat inconsistent with the previous K^+ data, but it is consistent with the K^+ data that the observed points are scattered over a wide kinematical region.

I would like to mention here one interesting aspect of Λ production. Since the decay of Λ shown in Eq. (24) is via weak interactions, the angular distribution of p or π^- is not isotropic if Λ has a polarization. In general, the angular distribution of p or π^- with respect to the polarization axis of Λ is given by



XBL 804-753

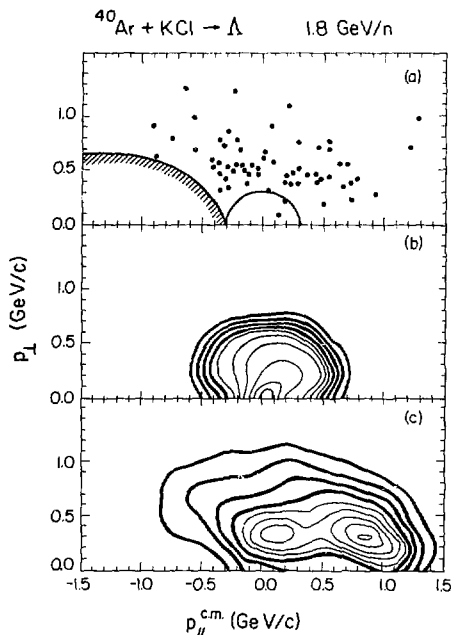
Fig. 34 Invariant mass plot for positive + negative charged particles.

$$W(\theta) = 1 + \alpha P \cos \theta, \quad (25)$$

with α in this case is -0.64 . By defining the reaction plane such that the beam and the emitted Λ form this plane, Harris et al. determined the polarization, P , to be

$$P = -0.10 \pm 0.05. \quad (26)$$

So far, inclusive measurements have neglected the quantity of angular momentum or spin, but such a variable is extremely interesting measurement for the future.



XBL 813-437

Fig. 35 Distribution of the observed Λ particles in the c.m. momentum space (top figure). Circle indicate the kinematical limit in nucleon-nucleon collisions.

SUMMARY:

1. Production of K^+ or Λ has recently been observed. Especially for K^+ the observed spectrum is almost exponential with the inverse slope E_0 being larger than the corresponding values for protons and pions.
2. The cross section of the K^+ production is approximately proportional to $A_p \times A_T$, where A_p and A_T are, respectively, the projectile and target mass numbers.
3. In terms of the quark model K^+ and Λ are described as $(u\bar{s})$ and (uds) , respectively. Strange quarks have to be created to produce these particles. Especially for Λ , spins of u and d are coupled to zero,⁵³ and therefore the polarization of Λ readily measures the polarization of the s-quark. Measurements of Λ polarization is thus interesting and perhaps useful for a study of the roles of quarks in high energy nuclear collisions.

9. PHYSICS RELATED TO PROJECTILE FRAGMENTS

From the viewpoint of the study of the collision mechanism the spectator region seems to be less interesting, because it is the region where no strong nucleon-nucleon collisions take place. However, several interesting physics phenomena are hiding in this region, and perhaps, this region will supply one of the most promising futures of high-energy nuclear collisions. I select two topics related to the projectile fragments; the high momentum component of nucleons inside the nucleus and the production of neutron-rich isotopes.

9.1. High Momentum Component inside the Nucleus

Since the projectile fragment is formed from a cluster which has not experienced any strong interaction, it tends to keep the various static properties that the projectile nucleus had before the collision. Recently

Fujita, Hüfner, and Nemes^{54,55} pointed out that the parallel momentum distribution in the fragmentation process of the one-nucleon removal reaction, such as $\alpha \rightarrow {}^3\text{He}$ or ${}^{16}\text{O} \rightarrow {}^{15}\text{O}$, will directly reflect the internal motion of nucleons inside the nucleus. In high energy nuclear collisions the momentum transfer, $\vec{q}$, in the process of projectile fragmentation is much smaller than the original projectile momentum, $\vec{p}_{\text{Beam}}$, since $|\vec{q}| \approx \hbar/R \approx$ (a few 100) MeV/c while $|\vec{p}_{\text{Beam}}| > 10$ GeV/c. We therefore expect that $\vec{q} \perp \vec{p}_{\text{Beam}}$. In other words, the transverse momentum distribution of fragments may contain both the effects of the nuclear reaction and the internal motion, whereas the longitudinal momentum distribution tends to reflect only the internal motion.

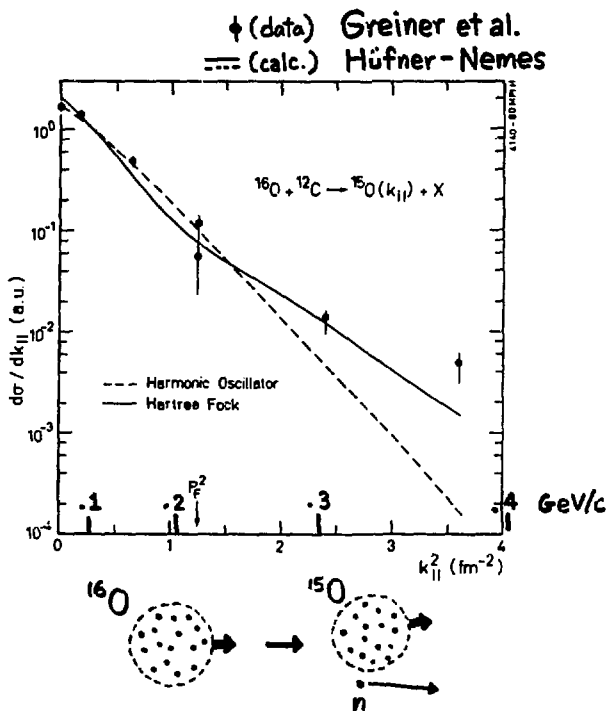


Fig. 36 Momentum distribution of ${}^{15}\text{O}$ at 0° in the projectile rest frame.

The one-nucleon removal reaction has a further merit for the study of the internal motion of nucleons. This reaction is an almost exclusive reaction, because if one takes out (A-1) cluster from A, then only one nucleon is left over which was scraped out by the target. From momentum conservation, therefore, the longitudinal momentum distribution of the observed (A-1) cluster in the projectile rest frame would immediately reflect the momentum distribution of the single nucleon (which was scraped out by the target) inside the projectile nucleus. In Fig. 36 the data by Greiner et al.⁵⁶ for the process of $^{16}\text{O} \rightarrow ^{15}\text{O}$ are shown. Hüfner et al. concluded that a simple Gaussian momentum distribution of a single nucleon inside the nucleus does not explain the data and suggested that an appropriate modification of the nuclear wavefunction is required in the high momentum region. This analysis neglects multi-step processes in creating the observed ^{15}O , and perhaps needs a more careful consideration. But, I think that it is quite interesting and important to extend such measurements into a much higher momentum region.

9.2. Production of Neutron Rich Isotopes

The second interesting subject for projectile fragments is the production of neutron rich isotopes. To a first order approximation, the neutron to proton ratio (N/Z ratio) of the projectile spectator is nearly equal to the N/Z ratio of the projectile nucleus, since the former is a part of the latter. For example, if we use ^{238}U as a projectile, then $N/Z \approx 1.6$. It is well known that the stability line of the nucleus extends along $N = Z$ for light nuclei, and thus the projectile spectators from U beams tend to fill the unstable neutron-rich-isotope region. Of course, the N/Z ratio of the detected projectile fragment is not really the same as that of the projectile spectator because of the the existence of the evaporation process (ablation) which occurs after the scraping process of the projectile nucleus by the target

(abrasion).^{57,58} However, the important point here is that the neutron rich isotopes are rather easily produced by using high-energy nucleus beams.

So far, three sets of experimental data⁵⁹⁻⁶¹ have been reported on the discovery of new neutron rich isotopes. Symons et al.⁵⁹ and Westfall et al.⁶⁰ have accelerated ^{40}Ar and ^{48}Ca beams and discovered 16 new isotopes, as shown in Fig. 37. This type of study will open up a variety of new applications of high energy nucleus beams, especially if we have U beams. Is there a new region of stability? How about a new region of deformation? Is the proton radius significantly different from the neutron radius? How about static properties of new isotopes, such as lifetimes or magnetic moments? Are there any polarization or alignment associated with the isotopes production? These are some of a number of interesting questions.

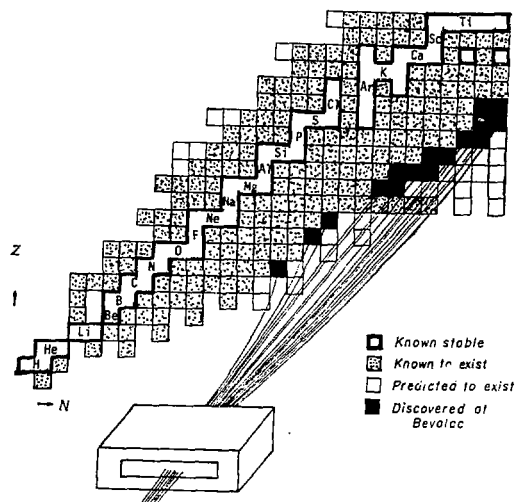


Fig. 37 New isotopes produced by ^{40}Ar and ^{48}Ca beams.

Another interesting application of neutron rich isotopes is their use as secondary beams. Because projectile fragments have their velocity which is almost equal to the projectile velocity, we may have a chance to use them as secondary beams. So far, only stable nuclei were used as projectiles, but in the future, we may be able to use unstable nuclei as projectiles. For example, if U beams become available, it may be possible to use ^{52}Ca as a projectile. I think we should pursue this rather seriously.

In connection with neutron rich isotopes I would like to mention another interesting result. Moeller et al.⁶² have recently measured the cross

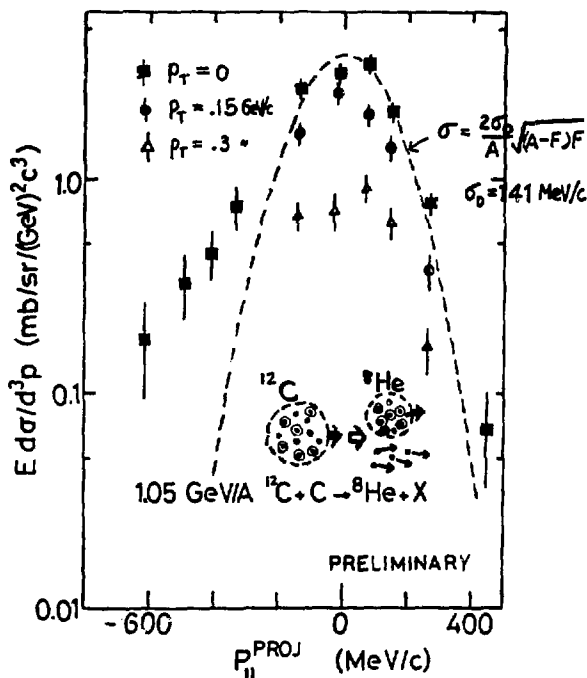


Fig. 38 Production of ^8He at 0° in C + C collisions.

sections of ^8He production in 2.1 GeV/nucleon C + C collisions, as shown in Fig. 38. In this case, four protons are scraped out from the projectile carbon nucleus, and it is a quite interesting process from the viewpoint of the reaction mechanism. The production cross section is about 35 μb which is about 30 time larger than the production cross section of ^8He as a target fragment in collisions of 2.2 GeV p + C (1.2 μb).⁶³ Using current theories it is very hard to explain such a large discrepancy, and it may be an interesting subject to be studied in the future.

SUMMARY:

1. It is useful to study the parallel momentum distribution in the one nucleon removal reaction for the purpose of studying the nucleon momentum distribution inside the static nucleus. Using an electron machine it is rather hard to extract the internal momentum which is larger than 0.3 GeV/c. High energy nucleus beams may offer a unique opportunity for the study of internal momentum distribution.
2. Production of neutron rich isotopes may open up a variety of future applications of high energy nucleus beams. Many topics can be studied in relation to these neutron rich isotopes.
3. I would like to emphasize again the future usage of neutron rich isotopes as secondary beams.

10. SUMMARY

Let me summarize my talk. Because I already listed a summary for each individual topic at the end of each section, I would like to summarize the whole talk from a somewhat different viewpoint.

At a beam energy of around 1 GeV/nucleon the de Broglie wave length of the incident nucleons inside the projectile ($\lambda_{\text{de Broglie}} \approx 0.3\text{--}0.5 \text{ fm}$) is much

shorter than the typical internucleon separation ($d \approx 1.8$ fm) inside the nucleus. This fact implies that nucleons inside the projectile nucleus can recognize the individuality of nucleons inside the target nucleus. Therefore, to a first order approximation, the high energy nucleus beam is a simple assembly of nucleon beams.

Then, what is the uniqueness of the nucleus-nucleus collision as compared with the nucleon-nucleon collision. The first obvious feature of the nucleus beam is the clustering of nucleons inside the projectile nucleus (see the upper half of Fig. 39). For a projectile with a mass number of 40-50, there are about $10 \text{ nucleons}/(\text{a few fm})^2 \approx 10^{26} \text{ nucleons}/\text{cm}^2$, and this nucleon

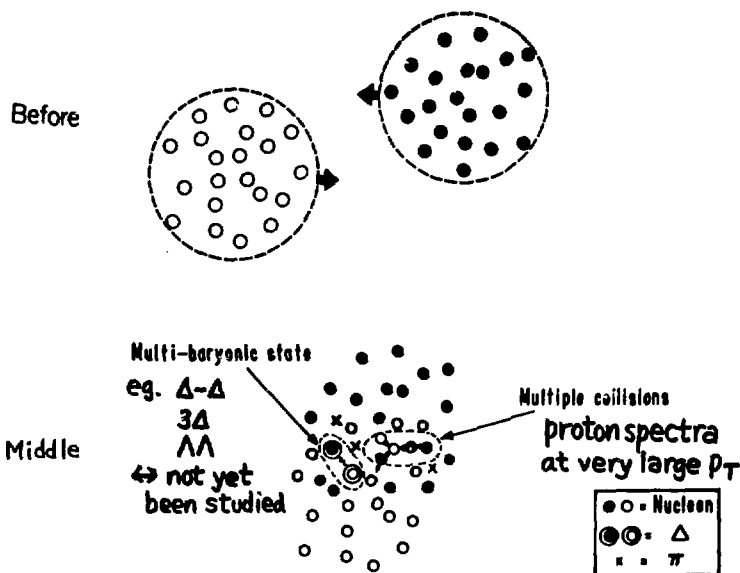


Fig. 39 Features of nuclear collisions (I).

beam is also sharply bunched in time; a few nucleons per $t = (a \text{ few fm})/c \approx 10^{-23}$ s. As a result, the product of these two numbers leads to an instantaneous flux of about $10^{49}/\text{cm}^2/\text{s}$, which cannot be obtained by any conventional proton accelerator. Thus, the nucleus beam may be regarded as a locally high flux nucleon beam.

The clusterization of nucleons induces several effects that are unique to high-energy nuclear collisions. During the course of a collision we have multiple nucleon-nucleon collisions, as shown in the lower half of Fig. 39. Macroscopically, these multiple collisions are a very important process for creating high density, and thus it is essential to study where we can more sensitively observe such a process. The importance of this process has been studied, in this talk, especially for large p_T proton emission in Sec. 4. At a beam energy of around 1 GeV/nucleon we also expect a copious production of Δ particles. These Δ particles may interact with each other to create multi-baryonic excited states, such as Δ - Δ or 3Δ state. Specifically, a possibility to form a Δ - Δ bound state has theoretically been studied in the past.⁵⁴⁻⁶⁶ This multi-baryonic excited state is a very interesting subject to be studied in the future.

After the collision we expect the following situation as illustrated in Fig. 40. Two groups of nucleons which have not experienced any hard nucleon-nucleon collision will be created. These two groups are called the projectile and target spectators, respectively, and they produce projectile and target fragments. Nucleons which have experienced hard nucleon-nucleon collisions are scattered out at all angles, and are called the participant. In this talk the usefulness of this participant-spectator model has been studied especially from the total integrated cross sections of nuclear charge or mass in Sec. 3.

Some of the participant nucleons may stick together to form a composite particle. We studied the formation mechanism of composite fragments in Sec.

5. Also, from the composite spectra the radius of the participant region has been estimated to be 2-3 fm.

Pions are also created. We have studied in Sec. 6 that some pions are absorbed, as shown in the Fig. 40, by surrounding nucleons. Some pions experience large Coulomb forces which induce a large π^-/π^+ ratio, as studied in Sec. 7.

Although the production cross section of strange particles are small, we have seen K^+ or Λ in Sec. 8. These particles have relatively long mean free paths and thereby less likely be rescattered as also shown in Fig. 40.

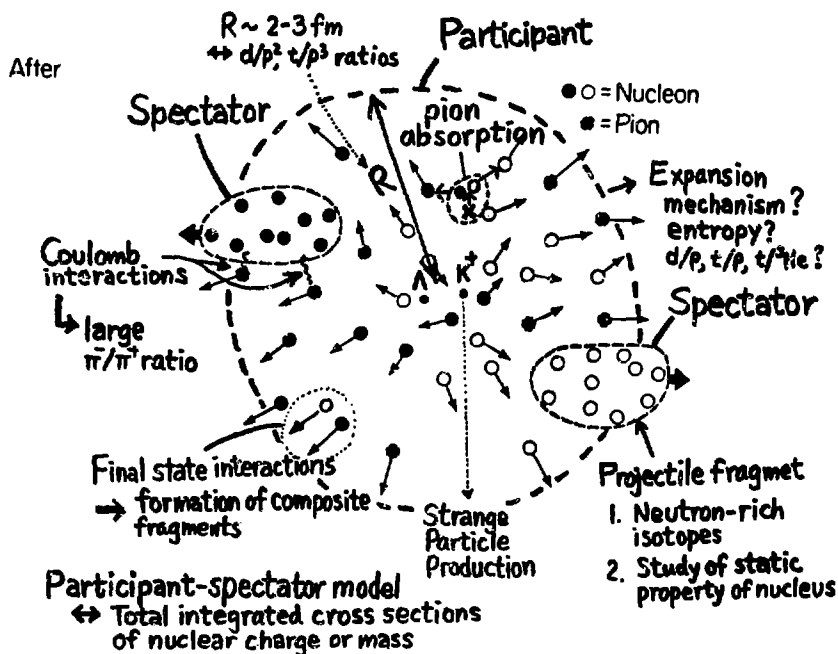


Fig. 40 Features of nuclear collisions (II).

One of the interesting questions related to the participant region is how the whole system expands after the collision. The quantity of entropy is certainly an interesting one which is related to the macroscopic expansion mechanism, but I feel that we need a more careful work here. Also, it is certainly an interesting question to study if d/p , t/p , or $t/{}^3\text{He}$ ratios are related to the macroscopic expansion mechanism.

With regard to the spectator, we have learned in Sec. 9 that the projectile fragment may be a useful probe to study the internal motion of clusters inside the static nucleus. In addition, we learned that the neutron rich isotopes will open up a variety of future applications.

I think that the physics learned from inclusive spectra are more or less displayed in these two figures; in Figs. 39 and 40, and particularly in the latter.

So far, nothing exotic have been observed in the inclusive data. However, the inclusive data have been (and still are) very useful in studying the actual reaction mechanism of the complicated nucleus-nucleus collision. As I mentioned in Sec. 2, the available data cover only a small portion of the kinematical area allowed by the collision kinematics. In the future it is definitely important to measure inclusive spectrum in such an unmeasured region. Theoretically, no satisfactory explanations have yet been available for most of the data shown in this talk. Therefore, my final conclusion is that both experimental and theoretical studies of inclusive spectra must still to be done in the future.

ACKNOWLEDGMENTS

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TARGET FRAGMENTATION FROM 10 MeV/u TO 2100 MeV/u*

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ABSTRACT

We have measured the average target fragment energies and momenta in the reaction of ^{12}C , ^{16}O , and ^{20}Ne with ^{181}Ta and ^{197}Au at 12 different projectile energies (7,13,17,19,50,75,80,86,250,400,1000, and 2100 MeV/u). The data support the conclusion there is a slow continuous evolution of reaction mechanism(s) with increasing projectile energy.

The momentum imparted to the target fragments in the interaction of heavy ions with ^{197}Au increases with increasing projectile energy from 7 MeV/u to 19-50 MeV/u and then decreases with further increases in projectile energy. There is considerable momentum transfer to the target in the interaction of 50-86 MeV/u ^{12}C with heavy targets. These results are best described by the generalized firestreak model. They are inconsistent with the generalized critical angular momentum model, and with the notion of time dilation being responsible for decreasing momentum transfer at higher projectile energies and do not agree with a number of models of peripheral reactions.

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I. Introduction

In recent years, there has been increasing interest in studying how the mechanism(s) of nuclear heavy ion reactions change as the projectile energy increases from low (~ 10 MeV/u) energies to relativistic (2100 MeV/u) energies. We thought it would be valuable to use a single set of experimental techniques (radiochemical measurements of target fragment yields, average momenta and energies, and angular distributions) to characterize the changes in target fragmentation over this projectile energy regime. Accordingly, we report in this paper the results of measurements of target fragmentation in the reactions of ^{12}C , ^{16}O , and ^{20}Ne with ^{181}Ta and ^{197}Au at twelve different projectile energies from 7 - 2100 MeV/u. At the lower energies, our work complements that of Gelbke et al.¹ on projectile fragmentation and Dyer et al.² on target fragmentation (in which the target residue fissions). Some of the data for the higher projectile energies (>250 MeV/u) measured by our group^{3,4} and by Kaufman et al.⁵ has been published. Much of the new data reported herein is from studies of reactions induced by projectiles whose energy ranged from 12-86 MeV/u.

II. Results and Discussion

The F/B ratios for typical high mass (^{167}Tm), intermediate mass (^{146}Gd), medium mass (^{96}Tc) and light mass (^{46}Sc) fragments increase with increasing total ^{12}C projectile kinetic energy until ~ 1 GeV and then decrease with further increases in the total projectile kinetic energy for the interaction of heavy ions with ^{197}Au (Figure 1). It is interesting to note that this behavior is qualitatively similar to that observed⁶ in the interaction of energetic protons with ^{197}Au . This de-

crease in F/B with increasing proton energy in proton-induced reactions has been qualitatively understood⁷ as a consequence of time dilation. If the same mechanism(s) are occurring in heavy ion reactions as in proton-induced reactions, then this explanation must be incorrect because at the heavy ion velocities encountered in this work, time dilation effects are negligible.

The two-step model¹¹ was used to deduce values of p_{11} , the longitudinal component of the momentum transferred to the target fragment in the initial projectile-target interaction. As an aid to understanding the variation of p_{11} with fragment mass and projectile energy, let us define a parameter called the "inelasticity" as the ratio of the measured p_{11} value for a given fragment to the maximum momentum that could be imparted to that fragment, i.e., the initial momentum it would receive when the projectile and target nuclei fused, followed by a breakup into that fragment. The variation of the inelasticity with fragment mass and projectile energy is shown in Figure 2. One immediately notices the large inelasticities occurring in the interaction of 0.6 and 1.0 GeV ^{12}C (50 and 86 MeV/u) with ^{197}Au . The magnitude of these momentum transfers indicates that the generalized critical angular momentum model⁸ does not correctly describe incomplete fusion at these energies, since it predicts no significant momentum transfer to the target fragments can occur at projectile energies greater than ~20 MeV/u. The variation of inelasticity with product mass number appears to be different below 50 MeV/u as compared to higher energies. The fragment inelasticities appear to vary smoothly and decrease continuously with increasing projectile energy from 50 MeV/u to 2100 MeV/u indicating con-

tinuous evolution of reaction mechanism(s) in this energy regime. If one examines the variation of the absolute value of p_{11} with projectile energy one observes an increase in p_{11} from 7 to 19-50 MeV/u with a decrease at higher energies. The complex variation of the properties of target fragmentation with projectile energy is to be contrasted with the observations of projectile fragmentation¹ where the phenomena observed at a projectile energy of 20 MeV/u were similar to those observed at projectile energies of 2.1 GeV/u.

Many models of peripheral collisions predict⁹ that $\beta p_{11}/\Delta E_T$ should linearly correlate with $\sqrt{1-\beta^2}$ where β is the projectile velocity (in units of c) and ΔE_T is approximately the excitation energy of the target fragment precursor. The values of p_{11} for the interaction of 50-2100 MeV/u ^{12}C with ^{197}Au do not show such a linear correlation. The measured variation of p_{11} for fragments with $A \leq 167$ with projectile energy (from 86 MeV/u to 400 MeV/u) is in approximate (20%) agreement with the predictions of the generalized firestreak model.¹⁰ (This model overestimates the fragment momentum by a factor of 2 at 2.1 GeV/u and does not account for the production of fragments with $A < 150$ at 50 MeV/u). It is interesting to note that this model predicts the capture of large pieces of the projectile at projectile energies of 86 MeV/u.

V. Acknowledgements

We gratefully acknowledge the assistance of R.J. Otto in making the measurements at projectile energies of 7-19 MeV/u. One of us (PLM) gratefully acknowledges the support of CERN for the period of the experiments there while KA and WDL thank their home institutions for over-all support.

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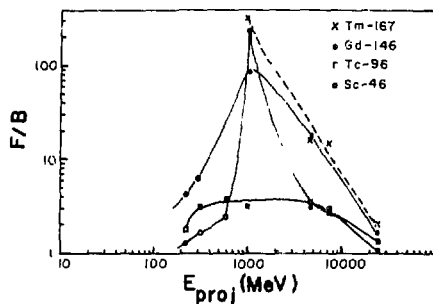


Figure 1. Variation of F/B for selected products from heavy ion reactions with Au. The data for $E_{\text{proj}} > 4800$ MeV are from Ref. 5.

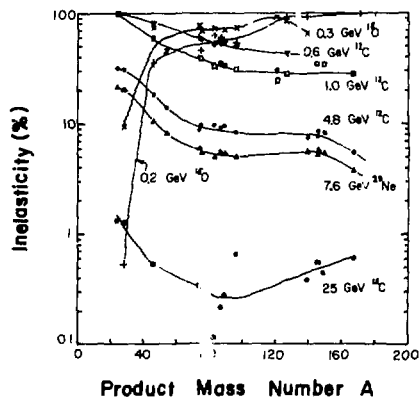
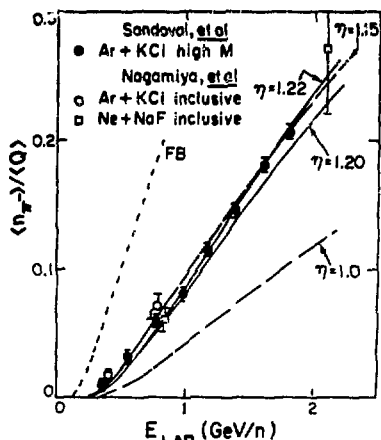


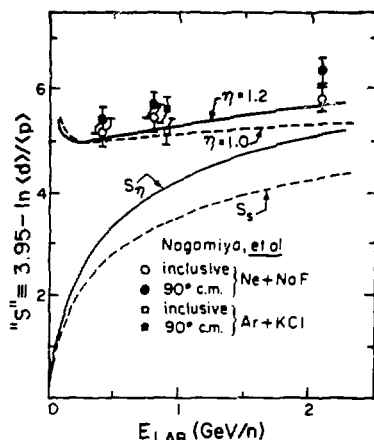
Figure 2. Variation of inelasticity with product mass number for reactions of various heavy ions with Au. The data for the three highest energies are from Ref. 5.

Pion, Light Fragment, and Entropy Production in Nuclear Collisions*
 Horst Stöcker, LBL

The production of pions and light fragments in fast nuclear collisions is calculated in a relativistic fluid dynamical model. We include viscous effects and a chemical equilibrium model of particle production. The linear bombarding energy dependence and, in particular, the absolute values of the calculated pion, proton and deuteron yields are found to be in agreement with the experimental data. The previous discrepancy between the calculated entropy and the measured d/p ratio is resolved.



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The figure shows the bombarding energy dependence of the mean number of pions per nuclear fragment (left) and the connection between the d/p ratio and the entropy (right)

*Based on LBL-preprint 12302

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STATISTICS VERSUS DYNAMICS
IN
INTERMEDIATE ENERGY HEAVY ION REACTIONS

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Abstract

Some selected aspects of nuclear reactions in the intermediate energy regime covered by the BEVALAC are reviewed. Concentrating on the violent part of the reaction we discuss the principal differences between and the limitations of the hydrodynamical and the dilute gaskinetic pictures from the perspective of isolating collective flow effects in a near model-independent way. The space-time development of the reaction is discussed within the cascade picture. While in the near future the sophisticated models will not be able to provide a detailed discussion of the spectra, phenomenological approaches already allow a transparent survey of the systematics of inclusive data up to the far end of phase-space. As particular examples the extreme momentum components of protons and pions are discussed. The question, to which extent do we understand the pion production, still remains open.

I. Introduction

Since the early days, when bare multiplicities and inclusive cross-sections were studied, the field of high energy heavy ion reactions recorded a considerable progress. Previous speakers /1/ reported on explorations at the borders of phase-space at high momenta, the observations of pions far down into the 'sub-threshold' regime and the first production rates on strange particles. Two particle correlations were as equally well studied as spectra of single particles associated with certain emission patterns of other ejectiles. Soon projectiles as heavy as Uranium will be available and with the plastic ball (GSI/LBL) and Diogen (Saclay) the path is paved to observe complex emission patterns of charged particles in near 4π -geometry.

The theoretical description of these reactions likewise showed progress though they could not quite keep pace with the evolution on the data front. A broad variety of models has been developed and studied. Yet, most of them are limited to deal with one-body observables, that is to say inclusive spectra, mainly protons and some attempts to describe the pion yields. The theoretical description of composite-particle production was left in an unsatisfactory state for quite a while. Only over the last months new approaches came into the discussion.

The role of theory thereby is manyfold. Besides the suggestion of all the possible novel effects that could occur in hot and dense nuclear matter that ultimately expands to form stable nuclei and hadrons, it is important to gain insight into the various patterns the reaction can evolve, to suggest specific observations that isolate specific dynamical effects and as a last point to provide a brought and systematic description of various data in simple physical terms (phenomenology).

The rich variety of present approaches served a significant insight into the dynamical evolution of these reactions. Recall the role of geometry, the notions of spectators and participants; with the two step fragmentation picture of abrasion and ablation at peripheral impact covering the low momentum parts relative to the projectile or target restframes, and with the violent or 'hot' participant component prevailing in central collisions. The latter component covers a relatively broad spectrum of final momenta centered between projectile and target rapidities with tails reaching the borders of phase-space at high momenta.

Rather than going into a detailed comparative discussion of the various approaches as already done by others on earlier occasions /2/, rather than exposing the respective nice (or pretended) agreement with the data, I like to discuss some selected aspects in my lecture. After a survey over the various existing approaches from the perspective of the Bogoliubov systematic, I like to discuss the question of how to isolate collective flow effects. This brings us to a new type of data analysis that came into the discussion over the last year: the exclusive analysis of emission patterns event by event. Further, at this time, I feel that some comments on the importance of quantum effects and their role in particular for the formation of composite particles are in order. The study of the intra nuclear cascade has made a step forward so as to disentangle its complicated evolution in those simple terms that also are used in simple phenomenological approaches. As a last point from the perspective of a simple phenomenological model I like to draw your attention to the systematics of single particle spectra away from the bulk part at high momenta.

II. The systematic of models

The most fundamental theoretical frame to begin with would be a relativistic quantum field theory of hadronic interactions. However, for the kinetic regime of reactions that we are interested in such an approach will not be practicable even in the far future. The first results of a time dependent mean field approach /3.1/ within the phenomenological Walecka model /3.2/ are promising, nevertheless all the limitations discussed already in connection with TDHF will also apply there and limit its application to an energy regime far below 100 MeV/u in my opinion. To quote M. Gyulassy: we have to settle for less.

Ignoring particle production for a while, the quantum many-body density matrix in its time development is the central quantity to study. Its Wigner or phase space representation brings us close to classical concepts. In fact the less demanded above is classical mechanics: a quasi classical theory. It is an attempt to simulate the evolution of the quantum Wigner distributions by means of classical concepts, as classical mechanics or classical stochastic processes as for example suggested by Remler /4 /. Thereby 'simulation' means that the respective classical forces or processes are effective ones (possibly momentum and phase space density dependent) as to accommodate most of the desired quantum features as 'off-shell' effects or exclusion principle⁺. It also implies that the approximate A-body density matrix obtained has only limited validity, e.g. being reliable on the one particle level or for a few lower moments of many-body observables, but not for a full exclusive (coherent) observation.

Although a full classical mechanic treatment involving 80 nucleons has been proven to be practicable (the respective model being called the classical

⁺c.f. the discussion of quantum effects given later

equation of motion (CEOM) approach / 5 /) further simplification is desirable from the theoretical point of view. The final goal is to retain only those degrees of freedom that essentially determine the observables in concern. Thus, there is an interest in macro-physical types of theories. Inclusive measurements for example bring the one-body density into the centrum of interest.

The various possible macro approaches can be nicely brought into a systematic perspective, Fig. 1. According to the duration of the process τ relative to the relevant micro to macro time scales $\tau_{int}, \tau_{loc}, \tau_{eq}$, of the duration of individual NN collisions, local equilibration times and global equilibration time, respectively one enters different macro regimes. As soon as many-body correlations have decayed (i.e. their influence on the one-body density effectively ceased) ($\tau \leq \tau_{loc}$) we enter the kinetic regime. The respective kinetic equation of the one-body density $f(r,p)$ is governed by a collision term besides the usual streaming (Vlasov) part. Usually the collision term is of Boltzmann type, i.e. non retarded (Markovian) accounting for two body collisions. Thus, its application is limited to the dilute gas regime. Later on when near local equilibrium is established ($\tau > \tau_{loc}$) one enters the hydrodynamical regime, the respective dynamical equations emerging from a moment expansion in momentum space of the kinetic equations.

The intra nuclear cascade is commonly quoted as a tool to simulate the solution of the Boltzmann equation. In fact, it is more as it retains memory effects on the many body level. Besides its main assumption of successive binary 'on-shell' collisions it is a genuine preequilibrium model on the micro scale. Quite a variety of cascade versions exist by now* / 6 /.

* The main difference between them is the way they treat the incident nuclei prior to the individual collisions: as a medium with certain depletion recipes (due to the collisions) or as a collection of nucleons frozen in positions and momenta relative to the respective nuclear frames.

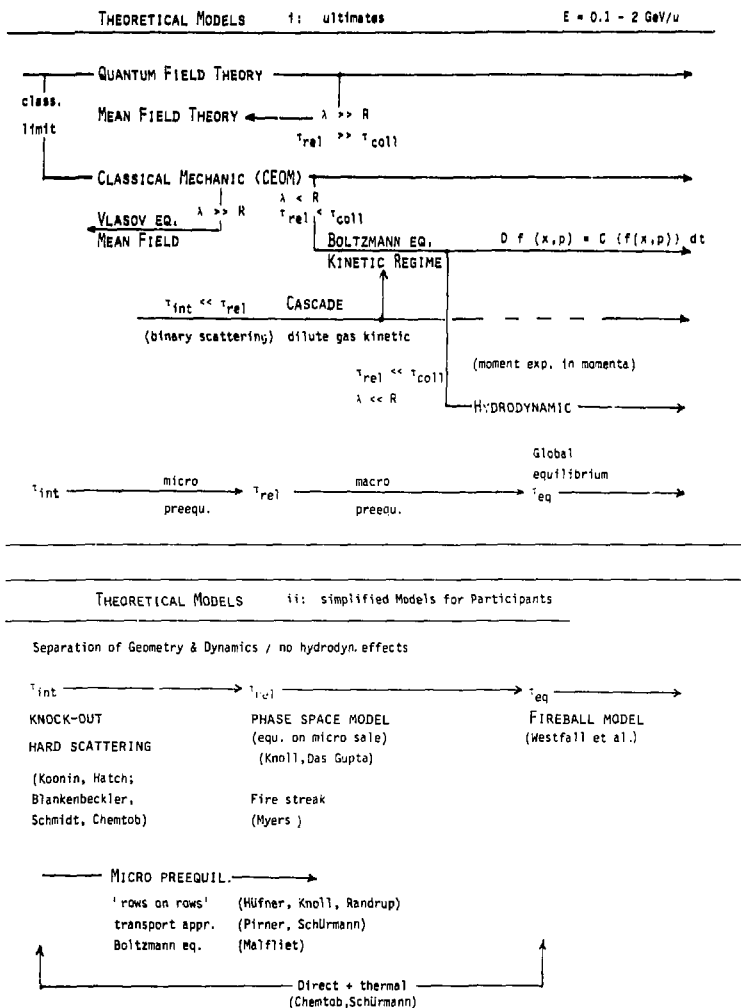


Fig. 1. Systematic of the different theoretical models / 5-10/ in terms of the duration of the collision process τ_{coll} relative to the micro to macro scales τ_{int} , τ_{loc} , τ_{eq} .

On the other hand in order to elucidate the micro preequilibrium phase, quite a variety of simplified models have been suggested and studied / 7 /. Guided by high energy pictures most of them employ straight line communication assumption in order to separate geometrical from dynamical effects. The transition from the quasi-free scattering of two nucleons out of their respective Fermi-seas (knock-out) / 9 / over many collisions towards the formation of a local equilibrium / 8 / has been studied. Within the frame of classical transport, as Markovian processes in time or in the number of collisions of individual constituents, they studied the amount of equilibration attained during the reaction as far as the kinetic and/or the chemical degrees of freedom (pion production, formation of composite nuclei) are concerned.

As a result of these studies a nucleon loses its memory on its incident oriented motion after about 3 collisions; at low energies (0.1 - 0.5 GeV/u) due to the near isotropy of the NN cross section at higher energies due to the onset of inelastic channels in the basis NN process. Hence, $\tau_{loc} \approx 6\text{fm}/v$ where v is the relative velocity of the two nuclei. To be more precise, at $\tau = \tau_{loc}$ those nucleons that were in interaction contact (c.f. the linked cluster picture discussed later) attain a single particle spectrum that is close to a statistical equilibrium, if averaged over many cascade events. This may not yet justify a hydrodynamical regime where in each event local equilibrium is required. The question under which circumstances the cascade shows hydrodynamical features is still not settled. As for the present experience with realistic nuclear dimensions the hydro regime seems not yet reached.

On the other hand, facing the limit of local and instantaneous equilibration, the hydrodynamical approaches / 10 / set off to test the properties of the nuclear equation of state. The compression to dense and hot matter leads far away from normal nuclear stability and different properties of the equation of state

should lead to quite different patterns of the resulting collective nuclear flow. The dynamical pictures discussed are those of shock-waves, 'bounce-off' and 'side-splash' effects due to the central pressure built up, the physical phenomena expected are a sensitivity of the observables on the equation of state in general and the possible formation of density-isomers, pion condensates or ultimately quark matter, in particular.

III. Search for collective flow effects

Compared to the observational capability in the macro world, where in each micro second millions of photons are lifting the secret of the space-time evolution of the object observed like the beauty of a milk drop's splash, Fig. 2, the experimental devices at our disposal for nuclear reactions are rather poor: a collection of ejectiles observed at certain asymptotic momenta. Clearly we rely on our imagination and a lot of theoretical experience with various kinds of models to come closer to the secrets of the nuclear world.

Although out of the three sophisticated models, the intra nuclear cascade INC, the nuclear hydrodynamical models NHD and the classical equation of motion approach CEOM, the last one is certainly the most appropriate to describe the nuclear collision dynamics as it uniformly covers all dynamical regimes, it is still interesting to compare the space time evolution of all three models. Rather different physical regimes are thereby accommodated from the dilute gas kinetic regime to the hydro-regime.

For simplicity and brevity let me concentrate on the evolution of central collisions in equal mass systems. It is amazing to see how due to the initial inertia of the system up to the moment of highest compression all three approaches lead to similar spatial configurations of oblate shapes. However,



SPLASH OF A MILK DROP

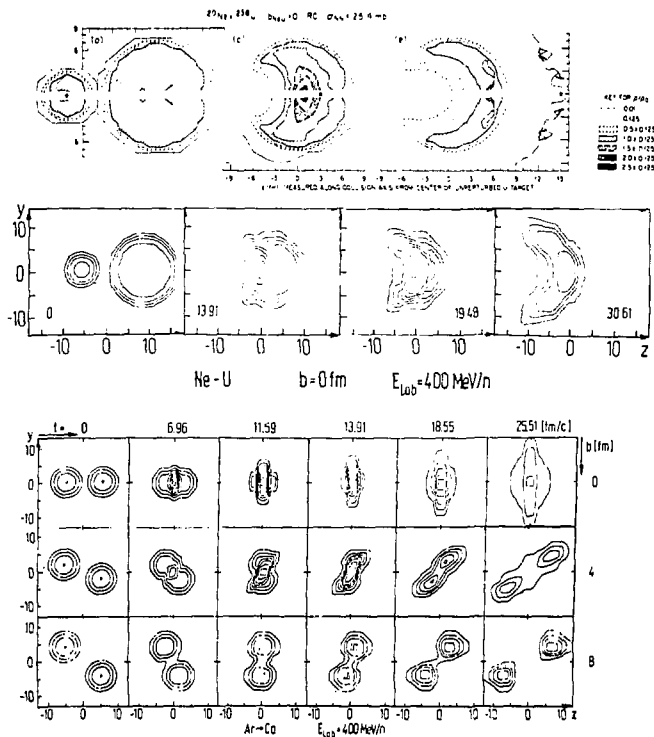
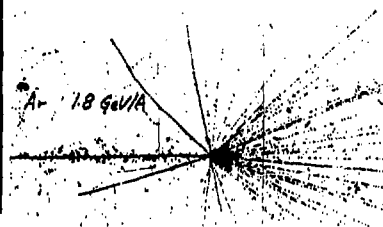


Fig. 2. Collective dynamics: a) macro observation of a milk drop splashing from a height of 18 cm onto a plate covered by a thin layer of milk, photo: H.E. Edgerton, M.I.T. Cambridge, Mass.; b) central collision event of 1.8 GeV/u Ar on emulsion; c) spatial evolution of a central Ne (400 MeV/u) collision on Uranium in the SIMON cascade (E. Halbert /6/) and the NHC model (H. Stöcker /10/); d) NHC calculation for an equal-mass collision system.

the expansion phases, that is to say the ultimate emission patterns look quite different. While hydrodynamic due to the isotropic pressure bends the incoming stream sideways, and therefore leads to a strong sideward emission, the INC emission grows preferentially in the forward-backward direction with increasing beam energy. The CEOM result falls right in the middle with near isotropic emissions at 400 MeV/u that tend to show a slight sideward preference in the larger systems calculated as $\text{Ca} + \text{Ca} / 5$ /. Interesting in this respect are also the relatively moderate compressions attained in the CEOM model they stay below 2^* , compared to the strong compressions of 3 to 5 above normal nuclear matter density in both other pictures for beam energies from 0.4 to 2 GeV/u.

As there are such significant differences in the emission patterns depending on the dynamics accounted for, we may ask the question how could we possibly observe these effects from the asymptotically available momenta. Our present experience tells us that inclusive measurements of a single ejectile do not suffice. Most of the structural differences are washed out as only one particle is observed from each event and due to the average over all impact parameters. In particular if one is interested in collective flow effects that result from the response on the high compression we have to discard peripheral collisions where we believe that non equilibrium cascading processes are prevailing. In fact, recently observed proton spectra associated with high multiplicities in 400 MeV/u Ne on U /19.1/ show a slight forward suppression in the emission, Fig. 3, an effect that has not yet been explained by the INC while the NHD model shows similar features (c.f. the contribution of H. Stöcker / 11/). Although there are still many effects to be considered such as Coulomb, formation of composites ... that may perturb the spectra at these low ejectile energies, we have at least a signal!

* This was also seen in the hard sphere dynamics (billiard balls) of E. Halbert / 6 /.

As we have not yet really a decisive clue one may ask: is there a near model-independent way to isolate collective flow effects from the asymptotically observed momenta? My tentative answer is: yes! As the collective flow is a genuine many-body effect we have to exploit many-body observables. The idea is similar to those used in particle physics for the identification of jets in terms of thrust or sphericity /12/. Our situation however, being more subtle because of the near random appearance of our high multiplicity events. It is clear that if we want to isolate collective flow effects we have to look for bulk properties of the emission pattern. Any high energy ejectile results from fluctuations on the micro scale and not from a certain preferred direction of the macro flow. Thus any confirmation of a collective flow-effect and its possible sensitivity on certain physical effects can be found at best in the study of a few lowest moments of the asymptotic observables.

Suppose for each event we know all momenta $\vec{p}^\mu$ of a certain number of emitted particles (the more of them the better). Relative to their c.m. frame one can construct the respective second rank momentum tensor

$$Q_{ij} = \sum_{\mu} p_i^\mu p_j^\mu \gamma(p^\mu) \quad i, j = \{x, y, z\}$$

where $\gamma(p)$ is a suitably chosen scalar measure. While thrust⁺ /12.1/ is only capable of isolating one preferred emission axis the above second rank tensor has the distinct advantage that it represents already the global shape structure for each event. In fact Q_{ij} represents an ellipsoid and through diagonalisation one can express its contents through its three

⁺J. Kapusta and D. Strottman /12/ pointed out the advantage of thrust of being unaffected by coalescence as it is linear in p^μ . The very same advantage can be recovered in Q_{ij} if $\gamma(p^\mu) = 1/|p^\mu|$. With $\gamma(p^\mu) = 1/p_\mu^2$ one recovers the angular emission analysis suggested by G. Bertsch and A.A. Amsden /12.4/.

major axis $\{\vec{q}_1, \vec{q}_2, \vec{q}_3\}$ with $q_1 < q_2 < q_3$. Inserting the relative length of the major axis

$$\lambda_1 = q_1 / \sum_{i=1}^3 q_i$$

into a Dalitz kind of triangular plot /12.3/ opens the possibility for a shape analysis (c.f. the talk of R. Stock). The sphericity

$$S = \frac{3}{2} (\lambda_1 + \lambda_2)$$

is a scalar measure with isotropic events if $S=1$ and pencil like events if $S=0$.

There are certainly many ways to represent the shape structure in terms of the tensor Q_{ij} . For a first survey to be presented at this conference we chose a representation that directly shows the direction of the preferred emission and the amount of elongation $\epsilon = 1-S$ in that direction. Thus we represented it by one vector

$$\vec{\epsilon} = (1-S) \vec{q}_3$$

It explores the inside of a unit sphere, with isotropic events for $\vec{\epsilon}=0$ and pencil like events at its periphery ($|\vec{\epsilon}|=1$). This vector is precisely of the desired kind (a low moment) and has the ability to isolate the occurrence of collective flow effects as I shall demonstrate with some simple examples below. Thus, 'elongation' cross sections $d^3\sigma/d^3\epsilon$ taken for a certain but high multiplicity range may contain the clue.

Guided by the bounce-off pattern of hydrodynamics /9.3/ a simple two source model is employed for the testscenarios; the two thermal sources with half the particles placed at collective velocities $\pm \vec{v}_{coll}$ respectively.

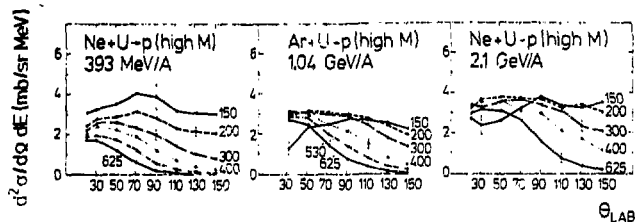


Fig. 3. Angular distributions of protons emitted from high multiplicity selected events observed at lab momenta ranging from 150 to 625 MeV/c /19.1/.

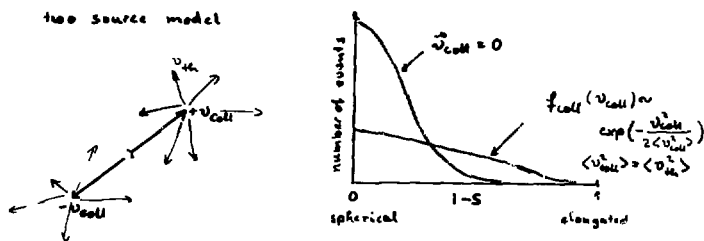


Fig. 4. Sphericity distribution of the two source test scenarios described in the text: a) no-collectivity $\langle v_{coll}^2 \rangle = 0$; b) $\langle v_{coll}^2 \rangle = \langle v_{th}^2 \rangle$.

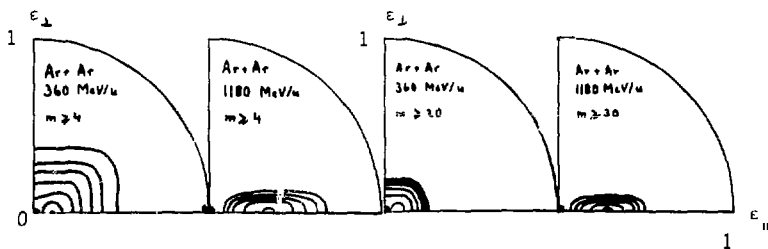


Fig. 5. Distribution of cascade events (counting only participants) over the elongation plane $\vec{e} = \{e_{\parallel}, e_{\perp}\}$; parallel denoting the beam directions. The contour lines separate a factor of two, respectively. Isotropic events at $\vec{e} = 0$; pencil like events at $|\vec{e}| = 1$. Calculations by J. Knoll et al. /12/.

$$P(\vec{v}) \sim \exp \left\{ - \frac{(\vec{v} \pm \vec{v}_{\text{coll}})^2}{2 \langle v_{\text{thermal}}^2 \rangle} \right\} .$$

For no collectivity ($\vec{v}_{\text{coll}} = 0$) we expect distributions near to isotropy except for finite number effects. The corresponding elongation distribution peaks at $\epsilon=0$ with a variance of about $1/\sqrt{m-1}$ where m (≈ 20) is the multiplicity, the subtraction of one resulting from the transformation into the respective c.m. frame. In a realistic scenario one has, of course, to allow that each event has a different collective velocity. The worst possible, since it is invisible in the respective inclusive spectrum, is a Gaussian spectrum for $\vec{v}_{\text{coll}}$. Still the respective elongation distribution shows a significant deviation from the no-collectivity case, Fig. 4. Note, however, the limited resolution at our disposal, which restricts

$$\langle v_{\text{coll}}^2 \rangle > \langle v_{\text{thermal}}^2 \rangle / (m-1).$$

This implies for example that two particle correlations still do not serve as conclusive evidence for the genuine many-body effect of a collective flow.

With the above encouragement let us now proceed and discuss the first 'elongation' results as obtained from a cascade calculation. Fig. 5 shows as a contour diagram over the elongation plane $\vec{\epsilon} = (\epsilon_{\parallel}, \epsilon_{\perp})$ a cascade analysis at two energies and for two multiplicity cuts. While at 360 MeV/u the resulting events show near isotropic emission patterns ($\epsilon=0$) with a slight preferential elongation in beam direction there is a strong elongation in beam direction of $\sqrt{\langle P_{\parallel}^2 \rangle / 2 \langle P^2 \rangle} \approx 1.6$ at 1.2 GeV/u. Besides fluctuations due to finite particle number effects (low multiplicities have larger fluctuations) this finding appears to be independent of the multiplicity. Thus, cascade makes

a strong prediction. Irrespective of the type of event (peripheral or central) the global shape of the resulting participant pattern is about the same and carries the memory of the incident beam direction. This is in strong contradiction to the predictions from hydrodynamics, c.f. /11/.

In conclusion with the heavier beams and the new 4π multiparticle detecting devices I see the possibility to observe the existence of collective flow effects in confined pieces of dense nuclear matter.

IV. Quantum effects and the formation of composite particles

Let me come to another subject that deserves some critical considerations at this time. It concerns the importance of quantum effects. With the opening of the new energy decade below 100 MeV/u and the growing interest in the production of composite particles relative to the nucleon yields this becomes a vital problem to my opinion.

The usual criterion cited for the validity of classical concepts is the wavelength of the relative motion of the two nuclei λ in comparison with the nuclear dimensions R . And for sure for that degree of freedom we are in a classical regime and the notion of an impact parameter makes perfect sense. Further, except for coherent or partly coherent processes⁺ phase informations are not important due to the inclusiveness of the observations. Rather, it concerns the relative motion of the nucleons where quantum effects are important.

⁺e.g. the 'deeply inelastic nuclear Mössbauer effect' seen in the reaction
 $^{12}\text{C} (86 \text{ MeV/u}) + ^{12}\text{C} \rightarrow ^{12}\text{C} + X$ at high momentum transfers /19.4/.

Thus, we consider the following effects to be important: a) Pauli exclusion, if the phase space density of each fermion component $f(x,p)$ comes close to one; b) binding effects, if the two- or many-particle distribution in phase space e.g. $f(x_1, p_1, x_2, p_2)$ approaches unity and c) off-shell effects if the energy uncertainty ΔE_{coll} due to collision broadening $\Delta E_{coll} = 1/\tau_{coll}$ is comparable to the mean kinetic energies.

Now, we can consult any of the microscopic models the INC or the CEOM to settle these questions. Lets consult the cascade version of J. Cugnon /6/ at 400 MeV/u. We infer collision rates of 5 fm/c, hence $\Delta E_{coll} \approx 40$ MeV, temperatures of about 70 MeV and a maximal compression of about 3. Since the momentum spectra at the moment of highest compression are close to the asymptotic ones we can estimate the one fermion occupation in phase space. We find its maximum to be larger than 0.5. Hence a) and b) are true: quantum effects are important and we definitely have to set off for a description that accommodates these effects in a quasi-classical way. The inclusion of off-shell effects opens the phase-space available in each microscopic collision, this will fasten the local equilibration time; the Pauli blocking⁺ will certainly give rise to pressure effects.

While the above considerations concentrated on the hot phase of the reaction, the compression and expansion phase, it is the ultimate transition from that phase to stable nuclei and hadrons (i.e. the break-up) where quantum effects definitely play a role. Thus, the formation of composite particles like deuterons and alpha particles are the respective study objects. Their production becomes of growing importance with lower bombarding energies. Yet,

⁺not with respect to the initial Fermi seas as accounted for in some cascade versions but relative to the actual occupation in the one fermion phase space.

the approaches to study the production of composites used so far employ quite classical concepts like a) the coalescence model /13/ and b) the chemical equilibrium model /8/. Both pictures certainly provide a survey of the possible production rates. However they are not at the level to allow quantitative predictions⁺. Let me explain my concerns with respect to the chemical equilibrium picture. This is a concept that only applies in the dilute gas limit where the compounds move relatively freely except for occasional reactions. At densities close to or above nuclear matter densities we see facts that let us doubt the validity of the simple detailed balance equations employed /8,14/.

Starting from low densities, the onset of a Mott-transition /15/ is a well known phenomenon in plasma physics. It occurs at densities ρ_M at which the proximity of the surrounding matter alters the effective binding forces to such an extent that the bound states emerge into the continuum. Thus, no compounds exist beyond ρ_M . Recent nuclear matter estimates (G. Röpke et al. /15/) showed $\rho_M \approx 0.1 \rho_0$. This implies at least association rates of compounds lower than estimated by detailed balance. Moreover, at densities where the internuclear distance is comparable or smaller than the range of the bonds, already at the classical level the distinction between free and bound objects loses its sense. This is even more true for quantum fermion systems, if also the thermal wavelength $\lambda_{th} = \sqrt{2\pi\hbar^2/MT}$ is still comparable to the range of the bonds, c.f. the respective arguments given by G. Bertsch /14/ for the counting rule of compounds.

Thus, employing rate equations between various compounds throughout the whole reaction (in particular the hot phase) seems to make little sense. I rather

⁺as for example the entropy puzzle constructed on the measured d/p ratio /14/; c.f. also G. Bertsch, these proceedings

like to take the view point given by Randrup and Koonin /8 / who consider the chemical equilibrium as a tool to estimate the phase space available for the various compounds at the moment of break-up.

With respect to the importance of quantum effects there is an interesting new approach to calculate the formation of composite particles given by Remler and Gyulassy /16/. Starting from a T-matrix description of the process, e.g. deuteron production, and casting it into a Wigner-phase space representation they obtain an expression where the exact two particle Wigner function of the system is examined with respect to its overlap with the final state deuteron Wigner function. The essential approximation employed is to replace the exact two particle Wigner function by the classical two particle phase-space distribution obtained from a cascade calculation. As to their result, it is essentially in the last collision a nucleon makes that the decision is made, whether it, together with a near neighbour, can form a deuteron. First results with this technic seem to give promising results for the deuteron production even for cases where the classical concepts above failed.

V. The micro regime

The CEOM and the INC models in connection with simplified multiple collisions approaches are the tools to study the evolution on the micro scale. Whereas the major goal was to calculate inclusive spectra, fig.6 , in the meanwhile some effort has been put forward to bring more transparency into these complicated codes. Interesting in this respect is an attempt to disentangle the cascade in terms of those subsets of nucleons (they are called clusters) that are connected by 'violent' collisions, fig.7 , / 17 /. These clusters comprise the micro dynamics of violent collisions that ultimately lead to micro equilibrium while the gentle collisions omitted in the analysis will govern the macro dynamics. A similar idea

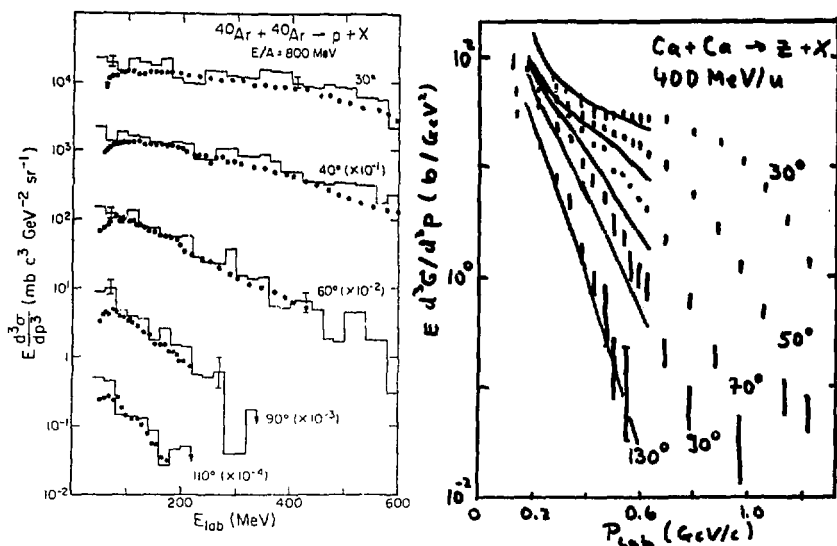


Fig. 6. Some typical inclusive spectra from cascade calculations; left: calculation from J. Cugnon et al. /6/, data ref. /19.2/; right: calculation including statistical error bars from C. Noack /6/, data (full lines) from/19.3/.

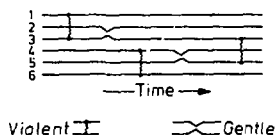


Fig. 7. Illustration of the decomposition into violently interacting clusters (1,3,5), (4,6) and (2) for a six particle collision.

has been suggested by Bodmer et al. / 5 / for the analysis of the CEOM approach in dividing the NN force into a long and short range part; the long range part governing the macro (collective) dynamics in terms of an averaged force (Vlasov part of the kinetic equation) while the short range force enters as a stochastic collision term. Given such a picture also the CEOM calculations could be subjected to a corresponding cluster analysis. The main advantage of the cluster representation is that for one-body observables (i.e. inclusive cross sections) it brings us to a formulation of the cross section as a sum over the single particle spectra pertaining to all possible cluster. This form is of the type that is most commonly employed in simple phenomenological approaches /7-8/.

Let me summarize the results of the cluster analysis /17 /. Characterising each type of cluster by the numbers of incident projectile and target participants respectively one finds:

- i) given a specific cluster, the resulting spectrum does not depend on the colliding nuclei (universality)
- ii) The spectral shapes are found to be in close agreement with those calculated with the simple rows-on-rows prescription. Beyond a cluster size of about 8 nucleons the shape of these spectra does not change any more, indicating that a kind of bulk limit is attained.
- iii) The gentle collisions lead only to minor additional momentum transfers due to the fact that they are also treated stochastically. Here the CEOM may actually show more collective behaviour due to the long range part of the NN forces.

The greatest discrepancy with simplified models appears in the size of the clusters formed. While the straight-line concept in the rows-on-rows model /7/ leads to fairly small clusters, the intranuclear cascade links many nucleons

into the same clusters. The main reason for these large sizes produced in the INC was traced to the finite value in the NN interaction radius which leads to an increase in transverse communication as the size of the cluster grows. Still, a structural analysis of these large clusters would probably reveal them as built up of several subclusters linked by only one or two interactions. Hence the subclusters may reflect more directly the micro dynamics of the process.

I think of such an analysis as an aid to reducing the inherent complexity of the intranuclear cascade description. In particular, the cluster decomposition may improve the practical utility of the INC for the study of relatively rare events, such as high-momentum components or multi-particle coincidences.

This brings me to my last subject the high momenta from the perspective of a simple model-systematic. As already mentioned, this is a regime where collective flow effects play no role. Rather, the high momenta result from the pre-macro regime, from a couple of violent NN collisions that generate them.

The first proton backward production results /19.5/ with momentum transfers of 2 GeV/c stimulated quite a variety of explanations /13/. Nucleons with a high momentum at the instant of impact get released and observed through a single scattering: the picture of high Fermi momentum components, and data were discussed in terms of a so called two body scaling representation. On the other side the simultaneous reaction of the beam particle with several target nucleons also open the backward hemisphere accompanied

by the emission of the other participating nucleons in the forward direction. Terms like 'fluctuons' or 'correlated clusters' / 18 / came up in the sense of an accidental local density enhancement, on which the projectile hits. The latter models have in common that the incident energy will be shared among a few nucleons, actually irrespective of whether this happens in a local vicinity or not. This altogether motivated a systematic study in terms of a simple phasespace model, that I like to report on in brevity.

Recalling the cluster picture explained above the nuclear phase space model / 8 / is easily explained. Taking the rows on rows geometry (i.e. a transverse communication range of $\sigma_{NN}^{\text{tot}} = 40$ mb) to specify the formation cross section for each possible cluster, it assumes micro equilibrium distribution for the cluster spectra. In general we expect the one particle spectra of the form

$$F_C(p_1) = \int \frac{d^3 p_2}{\epsilon_2} \cdots \int \frac{d^3 p_{M+N}}{\epsilon_C} |S(p_1 \dots p_C)|^2 \delta^4(\sum_i p_i - p_C)$$

where p_C is the incident four momentum of the cluster $\{C\}$ and $|S|^2$ describes the exclusive probability for the final state $\{\vec{p}_1 \dots \vec{p}_C\}$. As the S-matrix cannot be obtained from first principles at this time, one follows Fermi's statistical theory and assumes the least, which is the most chaotic possible: the equal opportunity employment of all accessible final states. Thus $S = \text{const}$ and F_C only governed by the conservation laws acquires a so called phase-space distribution. The intrinsic nuclear Fermi motion is included by a folding procedure over the incidentally available four momentum p_C of each cluster. Note, that opposed to thermal models which employ Maxwell-Boltzmanns law, these distributions explicitly take care of finite particle-number effects and thereby the finiteness of the available phase space. This leads to a significant change of the momentum distributions at high momenta and

is the main reason for the relative success of the model. Thus, the model accounts for equilibrium on the micro scale, while it is of preequilibrium nature on the macro scale (no communication between different clusters).

It is almost of no surprise that the above model served a quantitative description⁺ of inclusive high energy heavy ion data. Noteworthy is that as well as /the dependence of the transverse spectrum on the mass number of the collision system, the shouldering feature discussed by Nagamiya also found a quantitative description. Even the difference in the slopes between pions and protons can be accommodated by the model to a large extent, if a proper description of the pion production rates is used⁺.

While for the high transverse momenta it seems more or less obvious that they are built up by multiple collision contributions (that is to say participants) the high longitudinal momenta were always brought in connection with fragmentation processes (i.e. the fate of spectators). The subsequent analysis suggests the opposite conclusion: they are nothing but the extension of the violent reaction part (i.e. participants) beyond the 'spectator' peak. Fig. 8 gives an impressive confirmation of the last statement. Over one and a half orders of magnitude in energy for proton or heavy-ion induced reactions the phase-space model gives an absolute reproduction of the forward backward yields over four orders of magnitudes. The main contribution to the high momenta arises from processes where about 4 nucleons share their energy. Note further, that even an extension of the present calculational effort by a factor 100 will keep these data out of reach of a detailed discussion by cascade models. Thus, we rely on phenomenological studies.

⁺S. Böhrmann, J. Knoll /18/

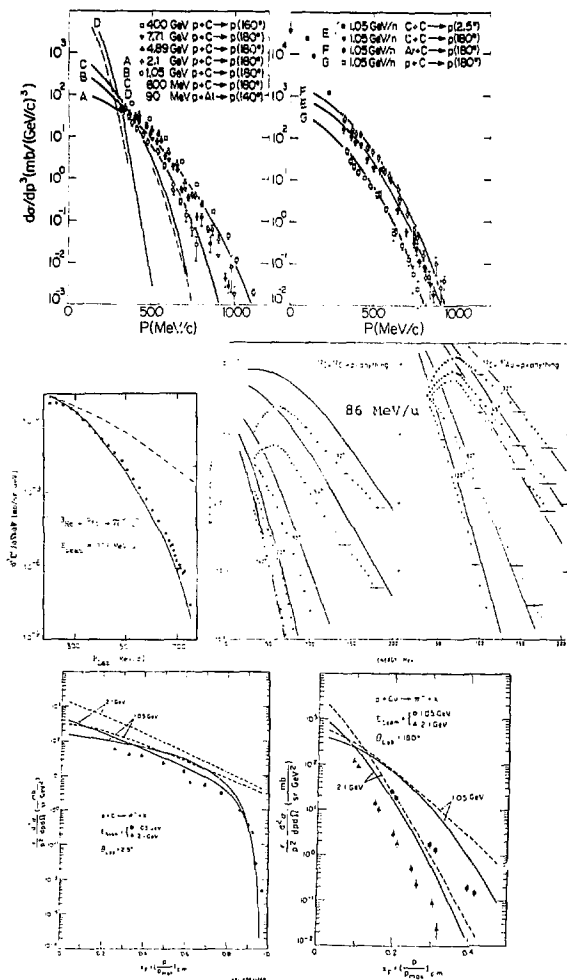


Fig. 8. Inclusive protons and pions at high momenta from various nuclear reactions /19/ compared to the nuclear phase-space model /8 micro eq./ (full lines); above: the dashed lines represent the data at 90 and 600 MeV respectively; else: the dashed-lines give the fire streak calculation.

Beyond 2 GeV/u the phase-space model is no longer applicable, since it does not account for the limiting character of the transverse momenta. Further up in energy the coherent tube model provides that type of phenomenology that the nuclear phase-space model does in the Bevalac energy regime.

Even at energies as low as 36 MeV/u where recently data have been taken⁺ the high energy components of the spectrum are seen to fall into the very same systematics, while strong deviations show up at moderate momenta. There the model drastically overestimates the proton yield since it does not include the formation of composite particles ranging from deuterons even up to the fused system in heavy mass collision systems. The onset of the latter processes with degrading energy which may tell something about the tensile strength of nuclear matter is one of the interesting aspects of the new research that intends to cover the 10 - 100 MeV/u energy decade, as already discussed.

The production of pions as an endothermic reaction becomes important above 600 MeV/u. Their production leads to a considerable cooling of the system, so their inclusion is already important for the proton spectra shown. As a 'chemical' process we require one parameter in terms of a density (similar to the freeze-out density in the chemical equilibrium model). This parameter, as a function of the available c.m. energy per particle, is adjusted so as to cover the observed pion multiplicities. Thus, the production rates are not predicted by the model. In particular, since ρ_c varies from 0.1 - 2 GeV/u, the chemical equilibrium picture seems not to be the right one. Thus, the production rates are still a subject of future investigations. Yet, once the rates are fixed, the high momentum parts of the spectra are seen to be given by the assumption of kinetic equilibrium.

⁺B. Jakobsson /19/

The observed scaling behaviour for the production of pions at 0° , with respect to Feynman's variable $x = P/P_{\text{max}}/19.8/$ motivated an explanation in terms of a hard scattering model on the quark constituents (Schmidt /18/). Yet, it failed to explain non-scaling at backward angles, note the asymmetry of projectile and target. As the phase-space model can cope with both features, we doubt more exotic explanations and in particular the role that quarks should play in that context. See also the respective discussion of Gyulassy and Landau / 18 /. Even the near sub-threshold data /19.13/, which trace the pion production over 8 orders of magnitude down to the kinematical limit are nicely followed by the model.

Conclusion

In summary, I discussed various aspects of the collision dynamics of nuclear reactions in the BEVALAC energy range: from the micro to the macro dynamics. While a signature of the micro dynamics seems to be best seen in the high momentum components of the emitted particles, macro-collective effects essentially coin the moderate to low momentum components.

We saw signs that both the high longitudinal and transverse momenta preferentially result from a cooperative action of a couple (≈ 4) of nucleons. Whether this is due to initial state correlation or multiple scattering, whether it happens coherently or incoherently is not yet settled. Further measurements, in particular coincidences of these rare components with other ejectiles, may help clarify the reaction dynamics. As for the pion spectra, they also shown strong signs of kinetic equilibration. There is not yet a clear signature on the role the delta resonance plays and better π^+, π^- data in the mid-rapidity region /19.15/ are necessary to settle this

problem, besides the role of Coulomb effects. Further, the relatively strong pion production at 'subthreshold' energies deserves our future attention as they clearly may point towards cooperative effects.

Clearly with the entropy puzzle boosted, a growing interest to understand the production of composite particles could be recorded over the last year. Here I see a promising perspective for those types of approaches that evaluate quantum formulations of the respective cross sections with quasi-classical approximants.

As for the search for collective effects, hydrodynamical pictures suggested quite a variety of phenomena. On the other hand cascade calculations made quite different predictions for the emission patterns. Therefore with the new beams up to heavy nuclei and the new 4π multi-particle detecting devices I see a challenging perspective to settle some of the questions concerning the macro dynamics. From there, however, to some detailed statements about the behaviour of nuclear matter under extreme conditions there is still a long path to go.

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A SIMPLIFIED MULTIPLE SCATTERING APPROACH TO INCLUSIVE SPECTRA
OF NUCLEONS PRODUCED IN HIGH-ENERGY HEAVY-ION COLLISIONS *)

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Abstract:

We provide a microscopic basis for a previously used phenomenological model of transport theory by starting from a simplified Boltzmann equation. This enables us to calculate the friction and diffusion coefficients of a Fokker-Planck equation. These are compared with the transport coefficients extracted from experimental data. Finally, we point out limitations of the current background models.

1. Introduction

Over the past few years, near-analytical multiple collision models based on independent nucleon-nucleon collisions have helped to understand particle inclusive production in high-energy heavy-ion reactions. These models permit an easy study of single and multiple scattering components. In addition, they make possible an analysis of data in extreme regions of phase space where numerical approaches fail because of poor statistics. Despite of their simplicity, these models yield at least semiquantitative results in comparison with the data.

The characteristic features of the measured nucleon inclusive spectra are the shoulder seen at forward angles /1/, and the near-exponential fall-off at high

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energies /2/. The phenomenological model of transport theory /3/ incorporates these two features in a simple and appealing way: The single scattering contribution is simplified by making use of the momentum distribution of the "test" nucleon resulting after its first collision, with a partner nucleon out of the Fermi sea. The higher-order scattering contributions are calculated from a transport equation of Fokker-Planck type, with the single scattering momentum distribution as initial condition.

2. Transport Theory /3/

Restricting to identical nuclei, we assume the nucleon inclusive cross section to be given by /3/

$$E \frac{d\sigma}{d\vec{p}} = A \sum_{n=1}^{\infty} \sigma_n \left[P_n^B(\vec{p}) + P_n^T(\vec{p}) \right] \quad (1)$$

with the nuclear mass number A , geometrical factors σ_n for n collisions of the test nucleon /4/, and momentum distributions P_n^B and P_n^T of beam-like (fast) and target-like (slow) nucleons normalized to unity. Since P_n^T can be obtained from P_n^B by symmetrizing the NN cross section, we only consider beam-like nucleons and abbreviate P_n^B by P_n . The single scattering contribution to (1) is replaced by a strong knock-out ("direct") component where "strong" is defined by requiring the momentum-integrated single scattering - and direct spectra to be the same. This approximation has been tested /5/ for the example of 800 MeV/N Ne + NaF-N + X within the framework of the rows-on-rows model /6/. The result, plotted in the mid-rapidity frame, is shown in Fig. 1. The agreement between the single scattering and direct spectra is quite good, except for the highest energies, but in that regime the higher-order scattering contributions dominate.

The higher-order collisions ($n \geq 2$) can be considered as a Brownian motion process. This means, the momentum distribution P_n of (1) are obtained from the recurrence relation

$$P_{n+1}(\vec{p}) = \int d^3p' w(\vec{p}' \leftarrow \vec{p}) P_n(\vec{p}') \quad (2)$$

with the transition probability w for a momentum transfer $\vec{\xi} = \vec{p} - \vec{p}'$ as given in the theory of conventional Brownian motion [7].

The Fokker-Planck equation

$$\frac{\partial P_n(\vec{p})}{\partial n} = (\beta \vec{\nabla}_p \cdot \vec{p} + D \Delta_p) P_n(\vec{p}) \quad (3)$$

is obtained by an expansion of the r.h.s. of (2) in $\vec{\xi}$, keeping terms to second order, and by treating the number of collisions n as a continuous variable. The transport coefficients β and D are connected by the Einstein relation

$$m\tau\beta = D \quad (4)$$

with the nucleon mass m and the temperature τ of the host medium, i.e. the fire-ball. This is a consequence of requiring the Maxwell distribution

$$P_{TH} = (2\pi m\tau)^{-3/2} \exp(-\vec{p}^2/2m\tau) \quad (5)$$

to be the stationary solution of (2) and (3) *). Because of the phenomenological nature of this approach the transport coefficients cannot really be calculated. They have been obtained in [3a] by connecting the diffusion coefficient D to the average momentum transfer squared obtained from the NN-differential cross section.

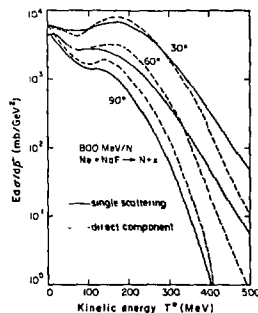


Fig. 1

*For quantitative results it is essential to use the relativistic Maxwell distribution and a corresponding generalization of the Fokker-Planck equation [3b].

3. Boltzmann-Equation Approach /8/

In this section, we put the phenomenological approach of /3/ on a more microscopic basis. We start from the linearized Boltzmann equation

$$m \frac{\partial}{\partial t} P(\vec{r}, \vec{q}, t) + \vec{q} \cdot \vec{\nabla}_x P(\vec{r}, \vec{q}, t) = -\sigma g(\vec{r}) \bar{q} P(\vec{r}, \vec{q}, t) + \sigma g(\vec{r}) \int d\vec{q}' \omega(\vec{q}' \rightarrow \vec{q}) P(\vec{r}, \vec{q}', t). \quad (6)$$

Here, P is the distribution function of the test nucleon at time t , at a position $\vec{r}$ and a momentum $\vec{q}$ in the lab system, σ is the total NN cross section, and $g(\vec{r})$ is the nuclear density. The average momentum $\bar{q}$ in the loss term on the r.h.s. of (6) is explicitly given by

$$\bar{q} = \int d\vec{q}_2 |\vec{q} - \vec{q}_2| (2\pi m\tau)^{-3/2} \exp[-(\vec{q}_2 - \vec{p}_{TH})^2 / 2m\tau] \quad (7)$$

and the transition kernel ω of the gain term by

$$\omega(\vec{q}' \rightarrow \vec{q}) = \int d\vec{q}_2 d\vec{q}_2' \delta\left[\frac{1}{2}(\vec{q}' + \vec{q}_2') - \frac{1}{2}(\vec{q} + \vec{q}_2)\right] \delta(|\vec{q} - \vec{q}_2| - |\vec{q}' - \vec{q}_2'|) \quad (8)$$

$$\cdot \frac{1}{|\vec{q} - \vec{q}_2|} \frac{1}{\sigma} \frac{d\sigma}{d\Omega} \left(\frac{1}{2}|\vec{q} - \vec{q}_2|, \sqrt{s}\right) (2\pi m\tau)^{-3/2} \exp[-(\vec{q}_2' - \vec{p}_{TH})^2 / 2m\tau].$$

By transforming (6) into an integral equation, by making the eikonal approximation and solving the integral equation by iteration, (1) and (2) can be derived. (For a more general and very extensive study of the Boltzmann-equation see ref. /9/).

However, we don't have to rely on a phenomenological expression for the transition probability anymore, but obtain a parameter-free expression instead. In the region of small c.m. momenta, this expression is

$$w(\vec{p}' \rightarrow \vec{p}) = \frac{1}{2\pi r^2} \frac{1}{s} e^{-s^2/r^2} e^{-p^2(\vec{p}' \cdot \vec{q})^2 / 2m\tau} \quad (9)$$

with the slope Γ^{-2} of the NN-differential cross section, approximated by a Gaussian. A Fokker-Planck equation can be obtained, with constant coefficients β and D , obeying the Einstein relation (4).

Experimental nucleon inclusive angular distributions at high energy, obtained from central (i.e. high-multiplicity-selected) collisions permit an extraction of the friction coefficient β /3b/. This is shown in Fig. 2 for the example of 800 MeV/N Ar+KCl+p+X, high-M, as measured in ref./2/. The large c.m. kinetic energy of the outgoing protons guarantees the direct contribution to the cross section to be negligibly small /10/ so that there is no interference with the statistical contributions. A value of $\beta=1/2$ fits the data best. This agrees well with the calculated value $\beta=r^2/6m = 0.46$ of /8/. (Corresponding diffusion coefficients are obtained from the Einstein relation (4)). From this value of β one finds, using the approximate relation

$$\beta^{-1} \approx n_{\text{relax}}^{-1} \quad (10)$$

where n_{relax} is the number of collisions it takes the test nucleon to relax into equilibrium, $n_{\text{relax}} \approx 3$ which is a reasonable number.

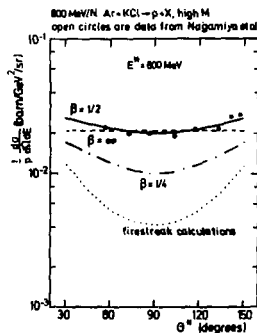
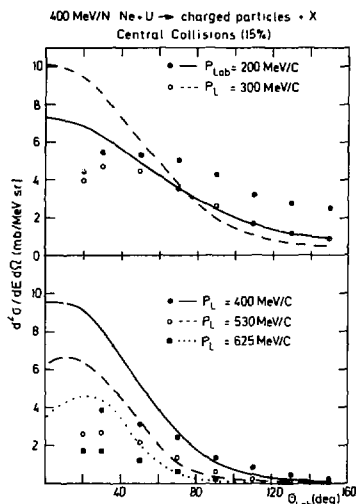


Fig. 2

4. Forward Angle Suppression in 400 MeV/N Ne + U → p + X, Central Collisions

Finally, we present an example where the current models based on independent NN collisions face some problems in comparison with the data. This concerns the slight sideways peak of the high-M proton inclusive angular distributions observed in /11/. In Fig. 3 we compare these data with results obtained from the two-component model "direct plus thermal" of /10/ which is a simplified version of the transport approach /3/. We see that the forward angle suppression cannot be reproduced.

Similar results are obtained with sophisticated intranuclear cascade codes. Hydrodynamical models, on the other hand, can qualitatively explain these data, the peak arising from a collective sideward flow of nuclear matter /12/. If these results are supported by more detailed measurements as well as calculations, this would be an indication that strong compression of nuclear matter can indeed occur under suitable conditions. To reach such conclusions, a comparison of the data with the results obtained from conventional models as discussed in this contribution is clearly needed.



The symbols denote the data by Stock et al., the lines are the results of the two-component model 'direct plus thermal'.

Fig. 3

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Entropy production in high energy collisions

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Abstract

The entropy production in high-energy collisions is computed in a Monte Carlo cascade model. For collisions of ^{40}Ca on ^{40}Ca at 800 MeV/n beam energy, the computed entropy is 4.4 per particle, about a unit higher than estimated from bulk dynamics. The particle correlation function of the final state is also computed, and is found to be in reasonable accord with a thermal distribution of the same entropy. With such low entropy values, most of the particles emerge in clusters, contrary to experiment. Thus the cascade calculation supports the conclusion of Siemens and Kapusta, that additional degrees of freedom become accessible in heavy ion collisions, beyond those in a conventional nuclear description.

1. Introduction

There is a close connection between deuteron production in high energy heavy ion collisions and the entropy generated in the collisions, pointed out by Siemens and Kapusta.¹ These authors also made the provocative claim that the generated entropy is larger than can be understood with conventional models of the dynamics of nuclear matter. However, this result relies on the validity of a macroscopic treatment of the dynamics, ignoring size effects due to the finite mean free path or the nucleons. In this paper we study the entropy generation and deuteron formation in a finite system using a Monte Carlo cascade model. We do find that additional entropy is produced in a finite system, compared to the bulk value. Nevertheless the predicted entropy is much lower than the empirical data on deuteron emission suggests.

We begin with the definition of entropy in a system of noninteracting fermions,

$$S = - \int d\gamma [f \ln f + (1-f) \ln(1-f)] \quad . \quad (1)$$

Here f is the occupation probability of phase space, and $d\gamma$ is the phase space volume element,

$$\int d\gamma = 4 \int \frac{d^3x \, d^3p}{(2\pi\hbar)^3} \quad . \quad (2)$$

The spin-isospin degrees of freedom give rise to the factor of 4 in Eq. (2). We shall consider only the dilute limit of Eq. (1), $f \ll 1$, which will be justified later. Then $\ln(1-f)$ may be expanded and Eq. (1) reduces to the classical equation for the entropy,

$$S = A - \int d\gamma f \ln f = A[1 - \langle \ln f \rangle] \quad (3)$$

where A is the number of particles. As a practical matter, f

will be computed in the cascade model by counting particles in cells of phase space. The optimum choice of cell configuration requires some numerical analysis, which we discuss in Section 2. The heavy ion collision we consider is ^{40}Ca on ^{40}Ca at 800 MeV per nucleon bombarding energy. The actual details of the entropy computation are presented in Section 3. For the cascade calculations, we have used the model described in Ref. [2]. We used the computer results of Ref. [2] together with an independently written computer program based on the same cross sections. We will not describe the cascade aspect of the calculations, except to note that pions are included via Δ excitations of nucleons. The Δ 's are assumed to have decayed to pions and nucleons when the nucleon distribution function is calculated. We will only examine the entropy associated with the nucleus; there is of course additional entropy associated with the pions in the final state. In Section 4 we discuss the relation between the entropy, the average phase space density, and the pair correlation function. The cascade results are found to be in reasonable accord with approximations based on thermal equilibrium. Finally, we confirm the finding of Ref. [1] that too many deuterons are predicted, compared to what is observed.

2. Numerical Analysis

To calculate the distribution function f in the cascade model, we first divide the phase space into cells, labelled by index i . The number of particles n_i in each cell is found at some fixed time in the cascade. Statistics can be improved by summing n_i over many runs of the cascade program. The distribution function f is then estimated as

$$\bar{f}_i = \frac{n_i}{R \int_i d\gamma} \quad , \quad (4)$$

where R is the number of runs of the cascade. The division of phase space into cells must be done with some care.

Ideally the cells should be small enough so that f is uniform within the six-dimensional volume of the cell. Thus the cell division shown in Fig. 1a would underestimate f and give too high an entropy. Conflicting with the demand for small cell size is the requirement that the cells include enough particles so that statistical errors in the numbers n_i do not become significant. An extreme case of this is shown in Fig. 1b. Fortunately, the entropy is such a smooth function of f that there is a wide tolerance on these requirements.

The effect of nonuniformity of f within the cells can be examined with a simple model problem. Suppose there is some phase space variable $z \geq 0$, with the distribution given by

$$f = f_0 e^{-z/z_0} \quad . \quad (5)$$

Then Eq. (3) yields an entropy

$$S/A = [2 - \ln f_0] \quad . \quad (6)$$

We compare this result with the estimate obtained by dividing z into intervals of length $\Delta z = \xi z_0$. This result is

$$\begin{aligned} \left. \frac{S}{A} \right|_{\text{est}} &= 1 - \frac{\sum_i \Delta z \bar{f}_i \ln \bar{f}_i}{\sum_i \Delta z \bar{f}_i} \\ &= 1 - \ln(1 - e^{-\xi}) f_0 / \xi + \xi / (e^{\xi} - 1) \quad . \end{aligned} \quad (7)$$

For $\xi = 1$, corresponding to a ratio of particles e in adjacent intervals, the difference between (6) and (7) is 0.04, i.e., the error in the entropy is only .04 per particle. Thus if we aim to compute the entropy to an accuracy of ~ 0.1 , we can permit cell configurations that have f varying by a factor of 3 or so between adjacent cells. It is necessary that the cell boundary allow all the variations in f to be displayed, but

only at this crude level.

The next question is the error due to counting statistics with a finite number of particles. There is a systematic bias in estimates based on Eq. (4) with n_i small. This is illustrated by the cell configuration of Fig. 1b, having most cells empty. The density in cells containing particles is then estimated too high. Since only occupied cells contribute in the calculation of $\sum_i \bar{x}_i \ln \bar{x}_i$, the computed entropy would be low. It is straightforward to determine this bias, treating the fluctuations in n_i about the mean $\langle n_i \rangle$ with Poisson statistics. The average deviation of the estimate from the true cellular entropy is

$$\begin{aligned} \left. \frac{\Delta S}{A} \right|_{\text{statistical}} &= \frac{\langle \sum_i (\langle n_i \rangle \ln \langle n_i \rangle - n_i \ln n_i) \rangle}{\sum_i n_i} \\ &= - \frac{\langle \sum_i n_i \ln(1 + \frac{\delta n_i}{\langle n_i \rangle}) \rangle}{\sum_i n_i} \end{aligned} \quad (8)$$

where $\delta n_i = n_i - \langle n_i \rangle$. We expand the logarithm to second order and use $\langle \delta n_i^2 \rangle = \langle n_i \rangle$, to evaluate (8) as

$$\left. \frac{\Delta S}{A} \right|_{\text{statistical}} \approx - \frac{\sum_i \frac{1}{2}}{\sum_i n_i} = - \frac{1}{2} \frac{C}{RA} \quad (9)$$

Here C is the number of occupied cells and RA is the total number of particles in R runs. Naively we might have expected to require a high accuracy in each cell but Eq. (9) is much less demanding. For example, to achieve an accuracy of 0.1, we need only an average of 5 particles in each occupied cell.

Besides this statistical bias, there will be a statistical uncertainty due to the fluctuations in the computed S about the mean. We will not attempt to evaluate this analytically, but will be content to determine this error "experimentally" by generating an ensemble of computed entropies.

3. Practical Details

The distribution function f depends on six variables and it would not be practical to make a brute force division into cells in all six dimensions. We have to select the variables to which f is most sensitive and ignore the others. We may start from the variables in polar coordinates: the momentum $\vec{p}$ specified by its magnitude p , its polar angle θ_p and azimuthal angle ϕ_p ; the magnitude r and its direction relative to p specified by polar angle θ_{pr} and azimuthal angle ϕ_{pr} . Obviously, the dependence of f on r and p needs to be treated explicitly. We will limit our calculation to head-on collisions, so that ϕ_p will be unnecessary. The remaining variables θ_{pr} , θ_p and ϕ_{pr} will be considered in turn.

The cascade model describes ^{40}Ca on ^{40}Ca at 800 MeV per nucleon bombarding energy. The particle numbers are accumulated over 10 runs of the program, giving 800 particles all together. We examine the distribution at a time 15 fm/c after the nuclei touch. By this time the N-N collisions have essentially ceased. We first look at the distribution function in r and p alone, dividing phase space up into spherical shells with $\Delta r = 2$ fm and $\Delta p = 0.2$ GeV/c. Table I shows the particle numbers n_i for this cell configuration. We only count particles that have been struck at least once, so the number of particles in the table (776) is less than 800. Notice that the particles with large r tend to have large p . At $t = 15$ fm/c the system is a nearly spherical expanding gas, and there are obviously going to be correlations between the directions of p and r as well. Accordingly, we subdivide the

cells depending on θ_{pr} . Table II shows the n_i with the subdivision into three, using $\theta_{pr} = 45^\circ$ and 18° as boundaries. Notice that the particles with largest r tend to be in the cell with smallest angle θ_{pr} . The entropy computed without the θ_{pr} division is 5.5 per particle, and with the division it is 4.5. Clearly it is essential to treat this correlation. A convenient way to treat the θ_{pr} correlation without using addition cell subdivisions is to shift the origin in the momentum space shells. The momentum vector is redefined

$$\vec{p}' = \vec{p} - c\vec{r} \quad . \quad (10)$$

The parameter c is chosen to reduce the residual θ_{pr} correlation. In Table III we display the number distribution with $c = 0.07$ GeV/c-fm, divided according to θ_{pr} greater than or less than 90° . The entropy without this θ_{pr} division is computed to be 4.40 per particle; with the division it is only reduced by 0.03 units. Thus we can drop the θ_{pr} dependence, once the transformation of coordinates has been made.

We next consider the dependence on θ_p . A subdivision of the cells using $|\cos \theta_p| = 0.5$ as a boundary lowers the computed entropy per particle by 0.04 units, i.e., there is no perceptible dependence on θ_p . This is to be expected with the cascade model of Ref. [2]. The authors found that the central collisions resemble a fireball, with the momentum distributed more or less equally in all directions. For completeness we also checked the dependence on θ_r , and found a negligible change in S , $\Delta S/A = 0.02$. The statistical fluctuations in S/A appear to be very small, with three independent sets of runs giving $S = 4.41, 4.41$ and 4.40 .

Having found an appropriate cell configuration to compute the entropy, we now examine S as a function of time. Initially, the particles have zero entropy in the ground states of the nuclei, but we cannot handle S in the quantum regime. We

therefore only consider particles that have been struck in NN collisions in computing $\bar{f}_1$. Figure 2 shows how the entropy of the struck particles evolves with time. At the very beginning phase space is sparsely populated and the specific entropy is high. As time goes on the development becomes a steady process and the entropy approaches 4.3 per particle. Once the collisions have ceased, the flow of f in phase space is incompressible and S should not change. In fact the computed S increases by less than 0.1 from $t = 15$ to $t = 22$, showing the inherent accuracy of the computation.

4. From Bulk Dynamics to Deuteron Production

There is a large difference between the entropy we have calculated and the predicted entropy from bulk dynamics, which we would like to understand better. The maximum density reached in the collision is about four times nuclear matter density [2]. In thermal equilibrium at this density, the entropy would be 3.2 per nucleon, compared to the 4.4 we calculated. The first thing we note is that the average density is much less than the maximum. At $t = 8$ fm/c, the time maximum density is achieved, the average density is only twice nuclear matter density. Here a thermal distribution would give $S/A = 3.8$. The remaining half unit of entropy can possibly be understood as arising from the finite mean free path of the nucleons. If we study $f(p,r)$ for fixed p different from the initial momentum, there must be a smearing in the r -dependence by a distance of the order of the mean free path. Thus the effective volume of the system is larger by a dimension of the order of the mean free path.

We now turn to the questions of correlation functions and deuteron formation, and their relation to the entropy and the single-particle distribution function. If the system is in chemical equilibrium at low density, then the relation between entropy and deuteron abundance can be derived from

the Sackur-Tetrode equation. With the temperature large compared to the deuteron binding energy, this is

$$\frac{S}{A} = \frac{5}{2} + \frac{3}{2} \ln 2 - \ln 2N_d / 3N_p^{\text{free}} \quad (11)$$

Here N_d is the number of deuterons produced and N_p^{free} is the number of free protons. However, the conditions in a heavy ion collision do not obviously satisfy the low density assumption. The N-N collisions cease at $\sim \frac{1}{2}$ nuclear matter density, too high to neglect the interaction between chemical species. A formula with identical appearance to Eq. (11) can be derived with assumptions other than low density. The first step in the argument is to note that the deuteron abundance is related to the two-particle correlation function by¹

$$N_d = 3 \int \frac{d^3r d^3p}{(2\pi\hbar)^3} g_d(r,p) g_{np}(r,p) \quad (12)$$

Here g_d is the deuteron density matrix, and g_{np} is the correlation function between neutrons and protons of specific spin orientations,

$$g_{np}(r_{12}, p_{12}) = \int \frac{d^3R d^3P}{(2\pi\hbar)^3} f^{(2)}(r_n, p_n; r_p, p_p) \quad (13)$$

where $r_{12} = r_n - r_p$, $p_{12} = (p_n - p_p)/2$, $R = (r_n + r_p)/2$ and $P = p_n + p_p$. If we assume that g_{np} is uniform over a sufficiently large region of phase space, we may use the normalization condition of g_d to replace Eq. (12) by

$$N_d \approx 3g_{np}(0,0) \quad (14)$$

We also assume that the two particle distribution function is given by the product of single-particle functions, $f_{np}^{(2)} = f_n f_p$. Then $g_{np}(0,0)$ can be evaluated from Eq. (13) as

$$g_{np}(0,0) = \int d\gamma f_p(r,p/2) f_n(r,p/2) = 8N_{p\uparrow} \langle f_n \rangle, \quad (15)$$

where $N_{p\uparrow} = N_p/2$ is the total number of protons with given spin orientation. We also need to invoke the relation between S and $\langle f \rangle$ for a system in thermal equilibrium,

$$\frac{S}{A} = 1 + \frac{3}{2} (1 - \ln 2) - \ln \langle f \rangle. \quad (16)$$

Equations (14)-(16) can be combined to give an equation with identical appearance to Eq. (12). However, the interpretation of the particle numbers are different. In Eq. (14) N_d is the number of deuteron-like pairs, including pairs contained in larger clusters. Similarly N_p in the high density derivation is the total number of protons in the system, including bound protons.

The assumptions in Eq. (14) and (16) can be tested with the cascade results. Let us first consider the relation between $\langle f \rangle$ and S . Computing $\langle f \rangle$ as $\langle \bar{f}_i \rangle$, the $^{40}\text{Ca} + ^{40}\text{Ca}$ cascade yields

$$\langle f \rangle \approx 0.07. \quad (17)$$

Alternatively, $\langle f \rangle$ can be calculated from the two-particle correlation function via Eq. (15). We estimate the correlation function by counting pairs in cells of r_{12} and p_{12} , summing over R and P ,

$$\bar{g}_{np} = \frac{n_i}{\int_i \frac{d^3 r_{12} d^3 p_{12}}{(2\pi\hbar)^3}}. \quad (18)$$

The computed $\bar{g}_{np}$ is shown in Table IV for cells in the form of spherical shells with $\Delta p = 0.1$ GeV/c and $\Delta r = 1$ fm.

At the moment we are just interested in $g_{np}(0,0)$, which we estimate by the entry in the first cell, $0 \leq r \leq 1$ fm, $0 \leq p \leq 0.1$ GeV/c. Then $\langle f \rangle$ can be found from Eq. (15), and we find the value shown in the second entry of Table V. If the distribution were thermal, Eq. (16) could be applied yielding the value for $\langle f \rangle$ shown in the third entry to Table V. The thermal value is less than the other estimates. We conclude that the actual distribution function is less homogeneous than a thermal distribution, and there will be more clusters than predicted from the entropy of a thermal distribution.

We next examine the validity of the approximation leading to Eq. (14), that g_{np} is uniform. Comparing the first two cells in r in Table IV, we see that $\bar{g}$ is reasonably uniform on the spatial scale of the deuteron. This is not the case for the p -dependence. At small r_{12} , the p_{12} dependence can be characterized by a temperature, $T \approx 25$ MeV. Parenthetically, this is a good illustration of the cooling of a system in isentropic expansion.³ In any case, the fall-off in p is significant over the scale of the momentum in the deuteron density matrix $p \sim 0.2$ GeV/c. We thus cannot affirm the assumption underlying Eq. (14). The various possible assumptions lead to different predictions for the clustering in the final state, and we compare these in Table VI. Remember that N_d/N_p is the ratio of deuteron-like pairs in composite particles to the total number of protons. We see that a major fraction of the protons are predicted to be found in clusters.

Experimentally, far less clustering is found. Nagamiya et al.⁴ have measured the particle abundances emerging from Ar + KCl collisions at 800 MeV/u. These are quoted in Table VII. Let us ask for the number of protons clustered in deuterons or in deuteron-like pairs in heavier nuclei, relative to the total number of protons observed. The number of

deuteron-like pairs in the heavier composite nuclei are shown in the second line of the table. The partial cross sections for these deuteron-like pairs is shown in the third line, and the total cross section for "deuterons" is 6 bn, to be compared with 20 bn for all protons, including bound ones. The ratio of these cross sections is shown at the bottom of Table VI. We see that clusters are a much smaller fraction of the total than predicted with the cascade model. Assuming that the cascade results do not change substantially at non-zero impact parameter, the conclusion follows that there is more disorder produced in the final state than is compatible with a description based on independent nucleon collisions.

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Table I Nucleon distribution in p and r for $^{40}\text{Ca} + ^{40}\text{Ca}$ collisions at 800 MeV/n, summed over ten collisions. The shell thickness of the cells is $p = 0.2$ GeV/c and $r = 2$ fm.

		p (GeV/c)					
		0,0	0,2	0,4	0,6	0,8	1,0+
r (fm)	0	10	35	32	16	4	
	2	25	82	107	50	12	
	4	5	47	88	93	22	
	6	0	6	31	67	22	
	8	0	0	1	10	11	
	10						

Table II Nucleon distribution in p and r , as in Table I, divided according to θ_{pr} .

$\theta_{pr} < 18^\circ$					$18^\circ < \theta_{pr} < 45^\circ$					$45^\circ < \theta_{pr}$				
0	1	1	0	1	1	4	8	2	1	9	30	23	14	2
3	3	19	3	3	6	24	46	24	2	16	55	42	23	7
1	7	14	15	6	2	26	56	63	16	2	14	18	15	0
0	3	15	24	11	0	3	16	43	11	0	0	0	0	0
0	0	1	7	9	0	0	0	3	2	0	0	0	0	0

Table III Nucleon distribution, as in Table I, with shifted momentum origin according to Eq. (10).

$\theta_{pr} < 90^\circ$					$\theta_{pr} > 90^\circ$				
4	18	19	6	1	6	19	14	8	2
18	62	51	13	4	22	65	35	4	2
12	57	29	12	1	25	81	38	0	0
13	32	10	1	0	15	42	3	0	0
3	4	0	0	0	7	8	0	0	0

Table IV The pair correlation function $g_{np}(r,p)$ near $(0,0)$.

		$p_{12} = (p_n - p_p)/2 \text{ GeV/c}$				
		0.0	0.1	0.2	0.3	0.4
r (fm)	0	15.1	7.8	4.9	0.6	
	1	12.1	6.7	3.3		
	2	8.8	5.8			
	3	6.4				
	4					

Table V Average occupation probability $\langle f \rangle$ computed various ways.

	$\langle f \rangle$
Direct cell average, Eq. (17)	0.07
Eq. (15)	0.09
Eq. (16)	0.05

Table VI Pair clustering in final state, N_d/N_p .

<u>Method</u>	<u>N_d/N_p</u>
Eq. (14), with $g_{np}(0,0)$ from Eq. (15) and (17).	0.8
Eq. (14), with $g_{np}(0,0) = 15.1$ from Table IV	1.0
Eq. (12), with $g_{np}(r,p) = \theta(0.2-p)\theta(2.4-r)$	0.5
Experiment	0.3

Table VII Cross sections for particle production in Ar + KCl collisions at 800 MeV/n, from Ref. 4.

	p	d	^3H	^3He	α
σ (barns)	14.1	4.0	0.62	0.52	0.16
deuteron clusters	0	1	3/2	3/2	3
$\sigma_{\text{"d"}}$	0	4.0	0.9	0.8	0.5

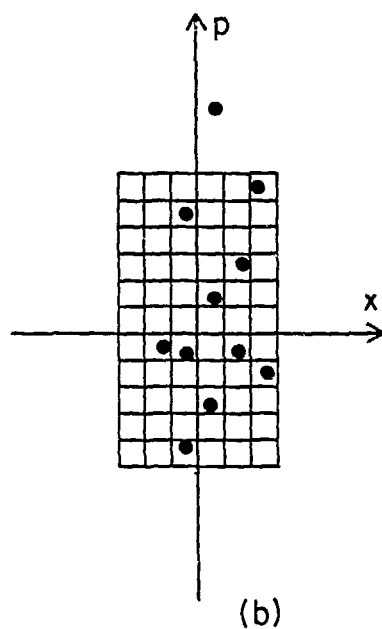
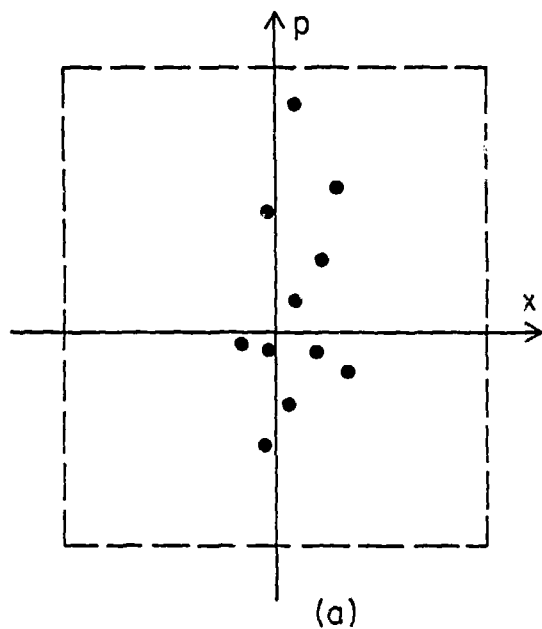


Figure 1 Phase space density estimated by counting particles in cells.
 In (a) the cell is too large, and the density will be underestimated.
 In (b) the cells are too small with the opposite effect.

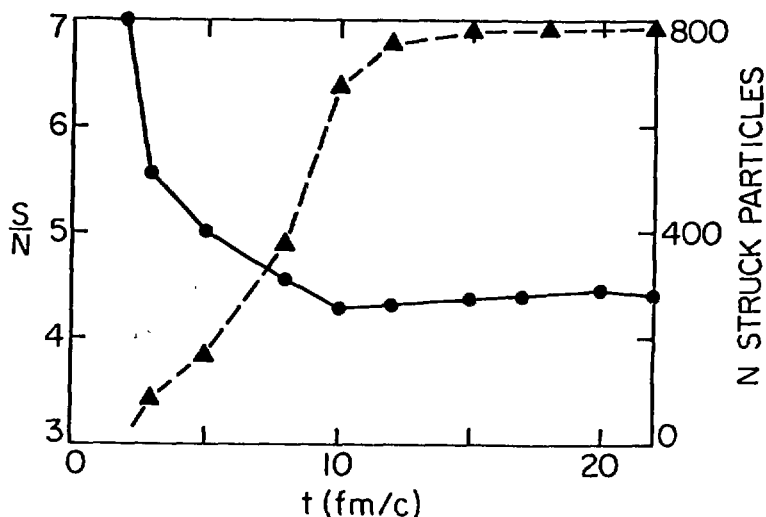


Figure 2 Time evolution of entropy for collisions of ^{40}Ca on ^{40}Ca at 800 MeV/n. Also shown (triangles) is the number of particles that have undergone collisions as a function of time. Nearly all the particles have collided by 12 fm/c, and the collisions essentially cease after 15 fm/c.

Entropy Generation in Nonviscous and Viscous Fluid Dynamics

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Los Alamos National Laboratory

The spectra of pions, protons, and deuterons from collisions of Ar on KCl at a beam energy of 800 MeV/A have been computed in a conventional relativistic hydrodynamic approach. Extreme softening of the finite temperature form of the nuclear equation of state leads to significant differences in the number of emitted particles of each species. Nevertheless there is still too little entropy and too few pions as compared with experiment. This discrepancy may be due to interpenetration, viscosity, and heat conduction, since the nucleon mean free path at this energy is probably not negligible compared to the size of the system. To estimate the influence of dissipation and heat conduction due to a finite mean free path ℓ , calculations were performed for a spherical expansion of a hot fireball of radius R in a nonrelativistic viscous fluid dynamic model. The entropy generated during the expansion is calculated as a function of ℓ/R . The hydrodynamic (isentropic) limit is approached as $\ell/R \rightarrow 0$.

Central Collisions of Heavy Ions--

Indication for Fluid Dynamical Behavior?*

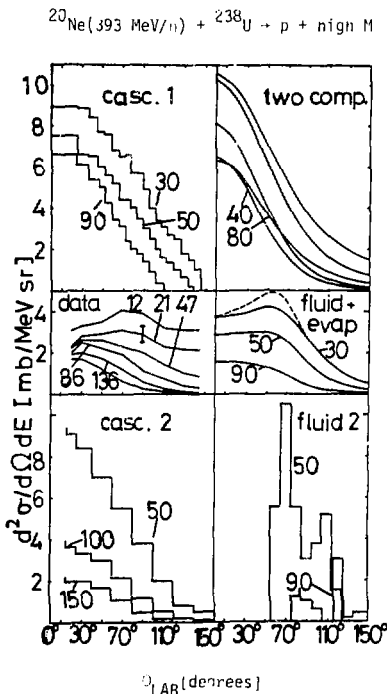
H. Stöcker^{1,4}, C. Riedel¹, G. Buchwald², L.P. Csernai²,
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The first detailed comparison of a variety of models with high multiplicity selected proton cross sections is presented. The reaction studied is $\text{Ne } (393 \text{ MeV/n}) + \text{U} \rightarrow \text{p} + \text{X}$ (high associated multiplicities). The observed preferential sideways emission of the matter is reproduced by a hydrodynamical calculation with final evaporation of protons, but the intranuclear cascade models and the two component (phase space) model fail in reproducing the data. The results seem to indicate that collective nuclear flow, as described by nuclear fluid dynamics, does occur in nuclear collision. The

forward peaking of the protons predicted by the cascade calculations, on the other hand, provides an example of a qualitative failure of such models, which are based on subsequent independent nucleon-nucleon collisions. While previous data in AgCl detectors have also indicated sideways peaking of α -particles, even more precise and detailed data on central collisions will be necessary to extract information on the nuclear equation of state.

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.

*Based on LBL preprint 12634



EXPLOSION-EVAPORATION MODEL FOR FRAGMENT PRODUCTION IN
INTERMEDIATE-ENERGY NUCLEAR COLLISIONS*

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ABSTRACT:

Nuclear collisions at intermediate energies may create transient systems of hot nuclear matter that decay into many nuclear fragments. The disassembly of such a nuclear "fireball" is described as a two-stage process. In the primary explosion stage the system quickly fragments into nucleons and composite nuclei according to the available phase space. The explosion produces excited nuclei with half-lives longer than the time associated with the breakup. In the secondary evaporation stage, these nuclei decay, first by sequential emission of light particles (neutrons, protons, alphas), later by electromagnetic radiation.

The secondary stage in general changes the relative abundancies of the various fragment species. This general feature makes it essential to take account of the composite fragments before using d/p as a measure of the entropy of the initial source.

The formation of unbound nuclei at the explosion stage also has the desirable effect of enhancing the final abundancies of particularly stable nuclei, e.g., ^4He . For neutron-excessive sources the presence of composite nuclei amplifies the ratio of observed neutrons and protons; this effect persists for heavier mirror systems. Predictions of the model are qualitatively compared to available experimental data.

The model offers a convenient way to augment existing dynamical models, such as intra-nuclear cascade and nuclear fluid dynamics, to yield actual nuclear fragments rather than merely matter distributions.

*Work supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.

Introduction

Heavy ion collisions at intermediate energies may create nuclear systems with excitation comparable to their total binding energy. A large number of multi-fragment channels are then accessible and many fragments are indeed formed in actual collisions [1]. The dynamics of the process can be extremely complicated and difficult to describe in detail. It is, however, precisely in these situations that a statistical model can provide a useful tool in evaluating experimental results. It may also serve as a theoretical background against which it is possible to judge detailed dynamical models.

In the present contribution we address the question of the disassembly of a piece of hot nuclear matter in the framework of a statistical model. We apply statistical ideas to describe the multifragmentation process in nuclear reactions with center-of-mass energies 20-200 MeV/nucleon. The model can be considered a direct extension of the one developed by Randrup and Koonin [2]. Similar ideas have been used by Mekjian and others [3] at higher energies.

The Model

The disassembly of a piece of hot nuclear matter is described as a two-stage process. In the primary, fast stage (which is referred to as the explosion) the system quickly fragments into free nucleons and composite nuclei according to available phase space. (At higher energies pion creation may also be significant.) The ejectiles produced in the explosion have to be stable on a time scale characteristic of this violent process. In the present work we include all states having a width $\Gamma < 1$ MeV.

The metastable products of the explosion will subsequently decay further on a slower time scale in a relaxation process, first by sequential emission of light particles, later by gamma decay. This secondary stage is referred to as the evaporation.

Following ref. 2, we assume that the explosion populates phase space statistically. Employing the grand canonical approximation, we calculate the partition function

$$Z = \sum_f e^{-\beta(E_f - \mu A_f - \nu T_f)} \quad (1)$$

The sum extends over all possible final states f . The Lagrange multipliers, β, μ, ν are adjusted to ensure that the ensemble average of the total energy E_f , baryon number A_f , and isospin T_f are equal to the prescribed values E_0, A_0 , and T_0 .

The quantity $\omega = (\ln Z)/A_0$ is an intensive quantity, which (neglecting fragment-fragment interactions) may be written as a sum over contributions from different fragment species AT . Each such contribution is of the form

$$\omega_{AT} = \chi \cdot \frac{4\pi}{3} r_0^3 \frac{2\pi m A^{3/2}}{Bh^2} \cdot \zeta_{AT} \cdot e^{-B(V_{AT} - U_A - V_T)} \quad (2)$$

with the different factors arising from integration over different parts of phase space. The volume factor $\frac{4\pi}{3} r_0^3$ is a result of the integration over fragment position and expresses the effective size of the system at disassembly. The dimensionless parameter χ is the ratio of the available volume at disassembly to the reference volume $\frac{4\pi}{3} r_0^3 A_0$. Note that for extended fragments χ also contains the effect of the eigenvolume of the fragments; therefore, we regard χ as a somewhat adjustable parameter.

The second factor in (2) arises from the integration over fragment momentum (ensemble temperature $\tau = 1/\beta$), while ζ_{AT} is the intrinsic partition function of the fragment species AT . In the exponent in (2) V_{AT} is the ground state mass excess of the nucleus AT .

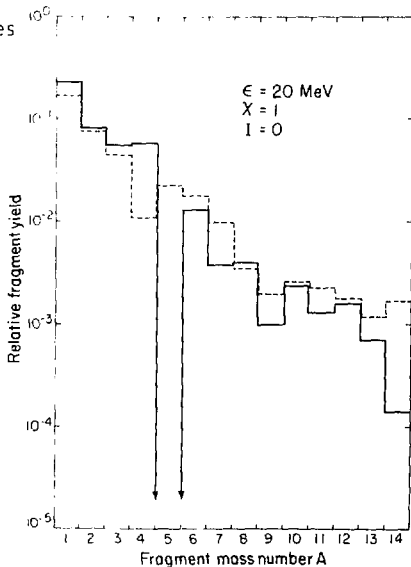
For given values of the available excitation energy per nucleon ϵ , the asymmetry $I = (N-Z)/A$ and the Lagrange multipliers β, U, V are determined by solving the three constraint equations. Subsequently, the partition function can be constructed, yielding the information on the system immediately after the primary explosion.

The treatment of the evaporation stage is in principle a standard procedure. In practice, however, to our knowledge there is no reliable evaporation theory for highly excited, very light ($A \lesssim 16$) nuclei. Therefore, we use the maximum possible experimental information on the positions and lifetimes of the nuclear levels [4]. Where the experimental information is scarce, we use a level density $\rho(\epsilon) = k \cdot \exp(2\sqrt{a(A)\epsilon})/A^{5/3}$ with a surface correction to the level density parameter $a(A) = (1 - c/A^{1/3})A/8$ MeV designed to fit low-lying levels of nuclei $5 \leq A \leq 16$. The fitted values of the parameters are: $k = 0.2$, $c = 0.8$. The results do not depend strongly on moderate variations of these parameters.

In describing the evaporation stage, we include the most frequently emitted light particles: neutrons, protons (with a branching possibility), and alpha particles.

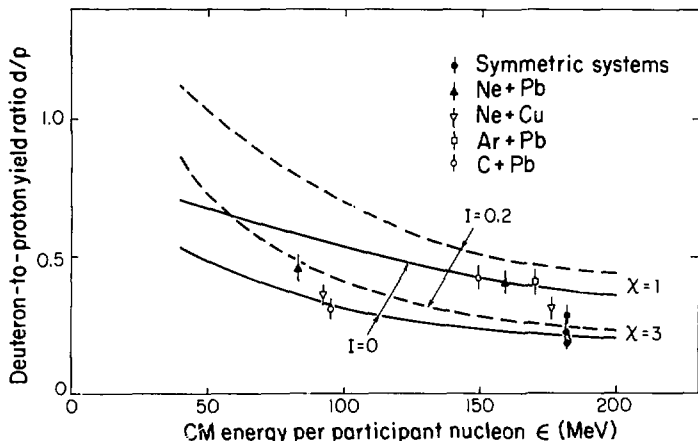
Results

In order to demonstrate the importance of the evaporation stage in shaping the final distributions, in Fig. 1 we compare the relative yields of final fragments (full histogram) to those prior to the evaporation stage (dashed histogram) in one particular example with the parameter values $\epsilon = 20$ MeV, $\chi = 1$, $I = 0$. The most spectacular



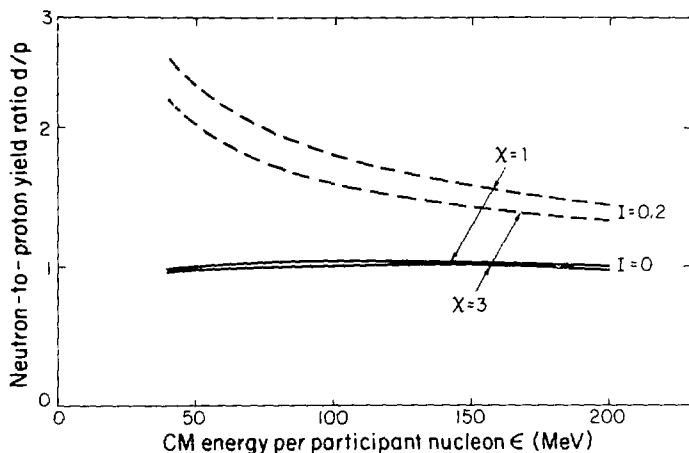
difference is, of course, that all the unstable $A = 5$ nuclei disappear from the final distribution. Furthermore, the yield of alpha particles is increased by about a factor of six and actually becomes a local maximum in the distribution. A similar effect is found for other especially stable nuclei. (Without unbound nuclei in the explosion stage the picture would be inverted [2].) The number of free nucleons is also increased significantly (more than 30% in the present example), while the deuterons receive only a relatively small contribution from the evaporation. All the heavy fragments, on the other hand, tend to suffer substantial losses, in particular, the less stable odd-mass nuclei.

This change of the relative abundancies of the various fragment species significantly affects the final deuteron-to-proton yield ratio, d/p . This general feature makes it essential to take account of the decay of composite fragments before using d/p as a measure of the entropy of the initial source, as has been attempted in [5]. The inadequacy of the expression $\sigma = 3.95 - \ln(d/p)$ to calculate the specific entropy from the observed d/p ratio has also been emphasized in the contribution by Stöcker [6].



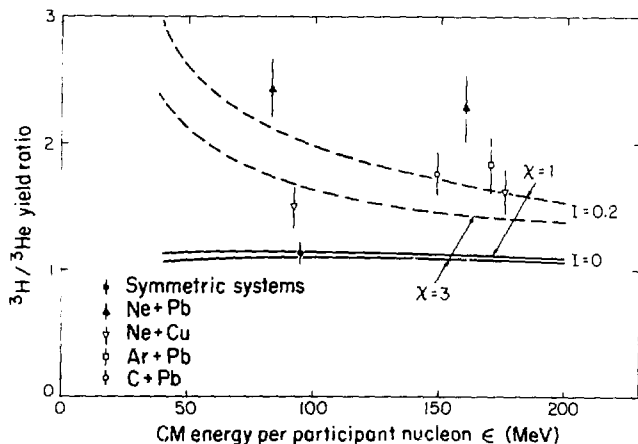
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Figure 2 displays the d/p ratio obtained for several values of the parameters χ and I in our model as a function of the available excitation energy per participant nucleon ϵ . The comparison of our results to the experimental data of Nagamiya et al. [10] can only be considered a qualitative one at this point. Our model in its present form does not treat the geometrical aspects of the collision. In order to compare with data, we have therefore proceeded as follows: The source is identified with the participant nucleons and is assumed to consist of projectile and target matter according to the standard sharp sphere geometry; the total center-of-mass energy is assumed to be available for excitation. The data points are included in Fig. 2. The theoretical curves represent expected, extreme limits of the parameter range. In this preliminary stage of comparison we can only state that the data are compatible with the model.



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Figure 3 displays the neutron-to-proton ratio n/p as a function of the available energy per participant nucleon, ϵ . The curves do not depend strongly on X , but for neutron-rich sources the presence of composite nuclei amplifies the resulting n/p ratio [1,2].



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Figure 4 shows that this effect persists in the triton-to- ${}^3\text{He}$ ratio, which is easier to measure. It is seen that the symmetric systems have ${}^3\text{H}/{}^3\text{He}$ slightly larger than unity both in the data and in the calculation. Experimental points representing the more asymmetric systems fall in the region around the theoretical curves labeled by $I = 0.2$.

Conclusion

We have pointed out that it is essential to include metastable states in the description of the disassembly of hot nuclear matter. Doing so substantially increases the available phase space for the primary explosion and requires treating the secondary, evaporation stage of the process. The final relative abundancies are sensitive to the extension of the primary breakup in space and time, and the measurement of composite fragment yields may therefore elucidate the details of the disassembly process.

The preliminary comparison with data appears encouraging but we believe that much careful work is still needed (both experimentally and theoretically).

Finally we emphasize that the present model addresses a question usually ignored in present theories for the time evolution of the collision. Existing dynamical theories, such as the intra-nuclear cascade and nuclear fluid dynamics, only yield a kind of matter distribution emerging from the nuclear collision. Our aim is to incorporate actual physical fragments in the theory. So far we have focused on the relative fragment yields but the spectra can also be obtained in a straightforward manner. The model may thus offer a convenient augmentation of existing dynamical theories for intermediate-energy nuclear collisions.

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FRAGMENT CORRELATIONS IN NUCLEAR BREAKUP INDUCED BY RELATIVISTIC PROJECTILES*

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The interactions of 5 GeV protons, 5 GeV ^4He ions, and 5-42 GeV ^{20}Ne ions with ^{197}Au have been investigated in experiments at the LBL Bevalac. Masses, kinetic energies, and angular correlations between coincident heavy ($A > 12$) fragments were measured along with the associated multiplicity of fast charged particles.

Two arrays of silicon surface-barrier detectors (one of 12 detectors, the other of 16 detectors) were mounted on either side of the beam to measure the kinetic energies and masses of coincident fragments by time of flight. The start signal was provided by two gas avalanche counters. Four sets of ionization chambers, each with three silicon surface barrier stopping detectors, determined the Z of accompanying light fragments by ΔE - E measurement. The number and angular distribution of energetic, light charged particles (e.g., π^+ , H^+ , He^{++}) emitted at angles greater than 9° to the beam were measured in coincidence by an array of 80 plastic scintillators outside a thin-walled, 1-meter diameter, spherical chamber. Particles emitted at smaller angles were measured by a 2m x 2m plastic wall of 60 pairs of scintillators located about 6 meters downstream of the chamber. A schematic diagram of the apparatus is shown in Figure 1.

The present discussion will be limited to observations of coincident pairs of heavy fragments and the associated multiplicity of fast, charged particles. Other aspects of the interactions will be covered

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in the next paper to be presented by A. I. Warwick. Although the analysis of the extensive data obtained in these experiments is not yet complete, the results to date provide further evidence and details of an unusual nuclear breakup mechanism first observed with protons at the Argonne ZGS⁽¹⁾ and the importance of the total projectile energy, rather than the energy per nucleon, in determining the reaction mechanisms.⁽²⁾

The geometrical arrangement of the two arrays of fragment detectors used for mass measurement covered the region around a correlation angle of 180° , thus emphasizing events with low momentum transfer from the beam. The sum of the two fragment masses, $M_t = M_1 + M_2$, is a measure of the extent of disruption of the target. The distributions in M_t for projectiles of protons, ^4He , and ^{20}Ne , all at 5 GeV kinetic energy, show a peak at $M_t = 160$ -180, with a broad distribution down to low masses (Figure 2). The peak is due to binary fission of the near-target residues which result primarily from peripheral collisions, in which little energy and momentum are transferred. The events of much lower total mass probably arise from more nearly central collisions, in which extensive disruption of the target occurs.

The angular correlation between these fragments provides another way to characterize different kinds of nuclear breakup. As shown in Figure 3, for events with $M_t > 160$ the correlation is relatively narrow, with a most probable angle a few degrees forward of 180° , as expected for the binary fission mechanism. The distribution broadens for lower values of M_t , and is essentially flat for $M_t < 100$. It is not isotropic at these low M_t values, however, as shown by the lower coincidence rate between the array detectors and the ΔE -E telescopes at angles far from 180° .

The total kinetic energy of the two fragments is shown as a function of the sum of the fragment masses in Figure 4 for 5 GeV protons and ^{20}Ne . The intensity contour lines differ by factors of two. The dashed lines represent the expectation of the total kinetic energy for the fission of a species of given total mass from fission systematics.⁽³⁾

Although some events do follow the fission systematics, a large fraction of the events occur with a total fragment kinetic energy considerably

greater than that expected for a fission mechanism. This suggests that the nascent fragments were closer together at the time of breakup than in a normal, deformed fission configuration.

Correlated light charged particle multiplicity distributions are shown in Figure 5 for 5 GeV ^{20}Ne . For total fragment masses >160 the distribution peaks at low multiplicity, consistent with a fission mechanism. For lower mass totals, the distributions show an increasing probability of high multiplicity associated with the binary events. The multiplicity values represent the observed number of events in the 80 plastic scintillators and have not yet been corrected for coincidence loss, shadowing, or solid angle. The mean values are plotted as a function of the total fragment mass in Figure 6 and indicate an increasing light particle emission associated with lower mass totals. The distributions scale with the total projectile energy rather than the energy per nucleon - a feature common to a number of aspects of these reactions.⁽²⁾

These studies have demonstrated the complexity of the breakup of a heavy nucleus by relativistic projectiles. The features of binary events suggest a new mechanism distinct from fission, but further analysis of the data and further studies are required to establish a detailed interpretation.

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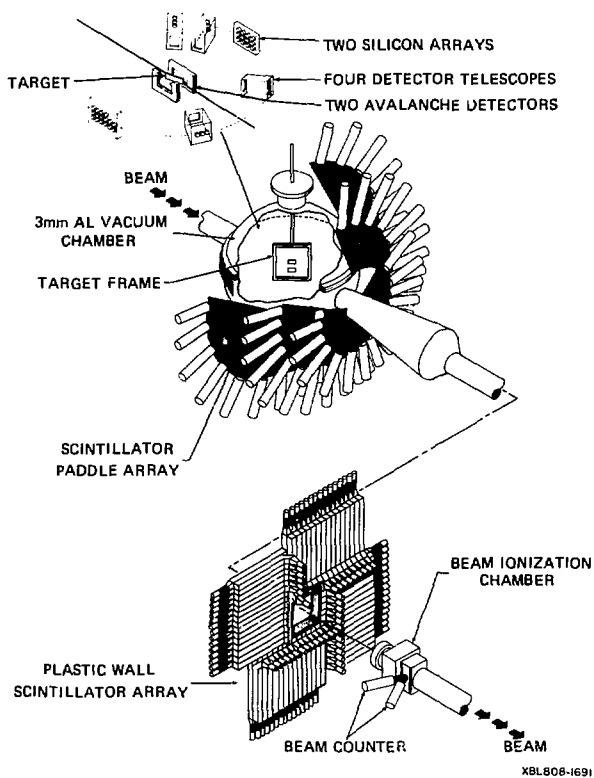


Figure 1. Schematic diagram of apparatus.

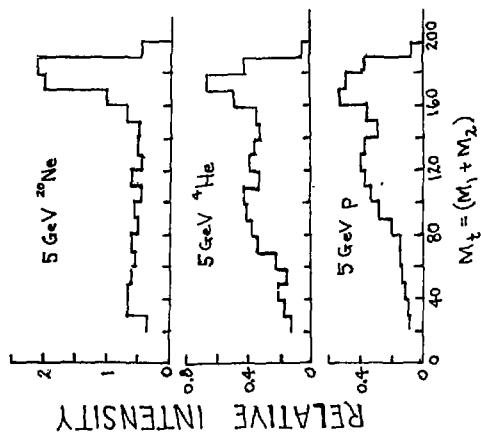


Figure 2. Coincident fragment mass distribution.

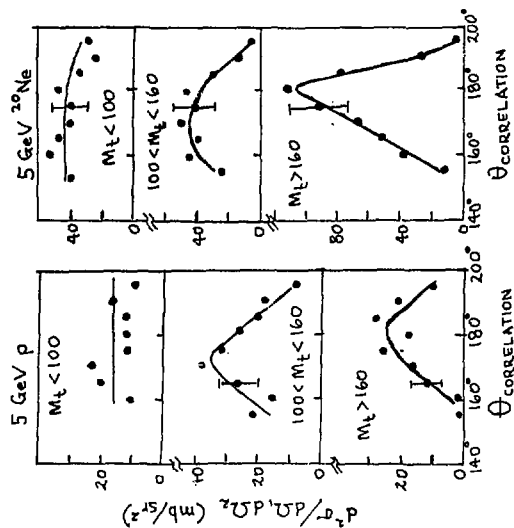


Figure 3. Angular correlation of coincident fragments.

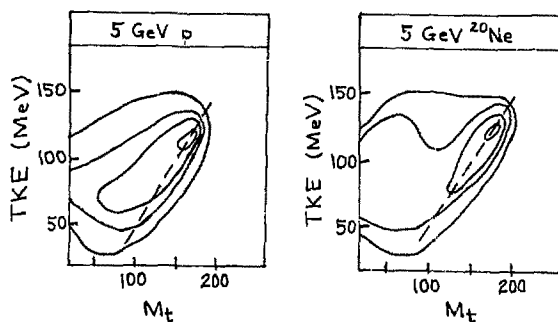


Figure 4. Total kinetic energy contours.

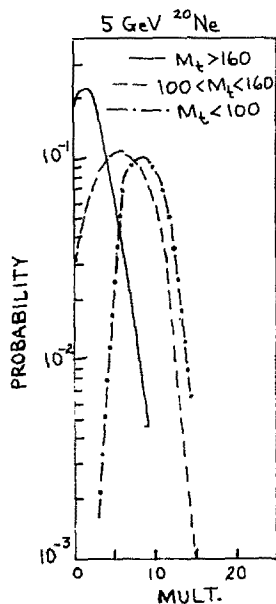
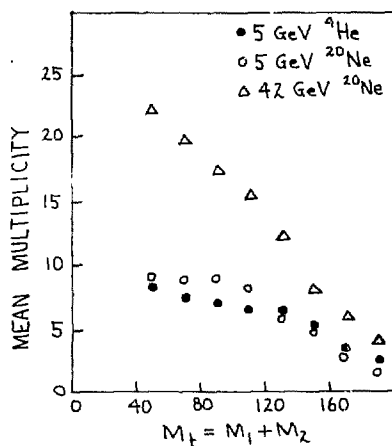


Figure 5. Observed multiplicity distributions.

Figure 6. Mean observed multiplicities as a function of M_t .

THE MEASUREMENT OF β -DECAY LIFETIMES
VIA PROJECTILE FRAGMENTATION

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ABSTRACT

It has recently been shown^{1,2} that heavy ion projectile fragmentation produces a broad spectrum of isotopes extending far from the valley of β stability. This discovery has enabled the development of a significant new technique to measure the β decay lifetimes of large numbers of very neutron-rich nuclei in a single experiment. This technique utilizes a multi-element position-sensitive Si(Li) telescope to stop and identify each projectile fragment; the same detector array is then used with higher electronic gain to observe the delayed electron from the embedded fragment's decay. In the first application of this method, the fragmentation of ⁴⁰Ar at 285 MeV/u was used to measure several known and new β decay lifetimes in the range from 40 msec to 600 msec. The success of the initial run indicates that, with higher intensities and heavier beams which will become available from the upgraded Bevalac, this technique can make a major contribution to the study of β decay systematics, with applications to nuclear structure and to astrophysics.

INTRODUCTION

We wish to report the development of a significant new technique to measure β^- -decay lifetimes of very-neutron-rich isotopes, using high energy heavy ion fragmentation reactions. The method has just been used for the first time, and the results presented here are quite preliminary.

Systematic β -decay measurements are quite important to such varied subjects as the gross theory of weak interactions, nuclear structure systematics, and astrophysical processes involving large networks of exotic nuclei. But there are a variety of programs now under way to measure β -lifetimes. What are the advantages of our new method?

There are several inherent difficulties in the observation of β -decay far from stability. The exotic parent isotopes have very small production cross-sections, and very short lifetimes ($\tau_\beta \ll 1s$). There are high backgrounds from target area activation and from strong production of the longer-lived isotopes nearer to stability. Very far from stability the identification of the parent isotope is nontrivial. The contemporary techniques deal with these problems in different ways, but each method limits what can be measured. For instance, if the reaction products are transported, one is usually limited to lifetimes longer than several hundred milliseconds. Chemical separation and certain ion source techniques severely restrict the Z of the parent isotope. And in general, the measurements involve long, low-yield experiments, and provide very few new lifetimes. Our new method suffers from none of these hindrances.

A new isotope production mechanism inspires a new method of β -decay measurement.

It was discovered a few years ago that heavy ion projectile fragmentation at several hundred MeV/nucleon produces a broad spectrum of beam-velocity isotopes ranging far from stability.^{1,2} If a light target is used, the fragments are very strongly peaked around 0° , which allows very high detection efficiency. For example, in the fragmentation of 9.6 GeV ^{48}Ca on ^9Be , twenty seven neutron-rich isotopes of unknown lifetime were observed.²

In order to measure the β^- -decay lifetimes of these projectile fragments, it is necessary to

- a) stop the fragments,
- b) identify the parent isotopes by charged particle spectroscopy,
- c) observe their β -decay without transporting the products,
- d) interrupt the beam while observing β 's,
- e) correlate the delayed β 's with the parent nuclei, and
- f) minimize the background from more abundantly produced fragments.

All of these conditions are satisfied by the following technique (see Fig. 1 for the elements of the measurement system). The forward-peaked projectile fragments are deflected through a magnetic spectrometer to select isotopes of a particular rigidity (this clearly separates the rare isotopes from the more abundantly produced isotopes nearer to stability). The fragments are then stopped in a multi-element solid state telescope; their range and energy loss provide charge and mass identification. Upon seeing a particular isotope stop,

the beam is interrupted, the detector electronics are switched to the higher gain necessary to detect low energy electrons, and the delayed β from the embedded fragment is observed. The stopping detectors are position-sensitive, so that we may determine that the observed β originated at the position of the fragment.

In our first experimental run, the solid state telescope used to stop the fragments consisted of two 5 cm $\times$

800 μ m Si(Li) detectors (for a particle trigger) backed by six 7 cm $\times$ 5 mm position-sensitive Si(Li) detectors. The telescope was surrounded by scintillators and an active collimator to suppress background. Aluminum degraders fixed the stopping point of the chosen fragments in the center of the telescope.

The experiment was controlled by a Microprogrammable Branch Driver (MBD) linked to the data acquisition computer. The MBD examined a parallel hardware particle identification for each fragment event in the telescope; if specified identification gates were satisfied the MBD signalled the beam interruption and the change to the high gain β^- detection mode.

Preliminary Results

In the first run isotopes were observed from the fragmentation of

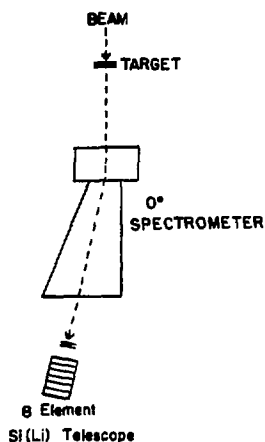


Fig. 1

285 MeV/nucleon ^{40}Ar on ^9Be . Figure 2 shows a rough particle identification spectrum for $A = 22$ fragments stopping in a particular one of the six detectors. Mass resolution for $Z = 12$ is already apparent. By carefully calibrating the gains of the detectors, and averaging the multiple ΔE -E measurements which a six-element telescope provides, we can obtain clear isotope resolution for all charges. Figure 3 shows the resulting particle spectrum for Aluminum fragments of a particular rigidity. When gates are set around each of these two Aluminum peaks, and histograms made of the delay times for β^- events originating in the stopping

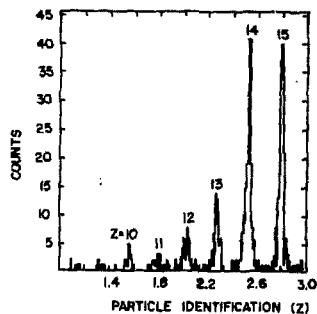


Fig. 2

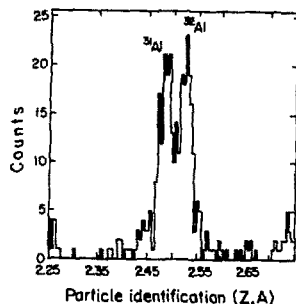


Fig. 3

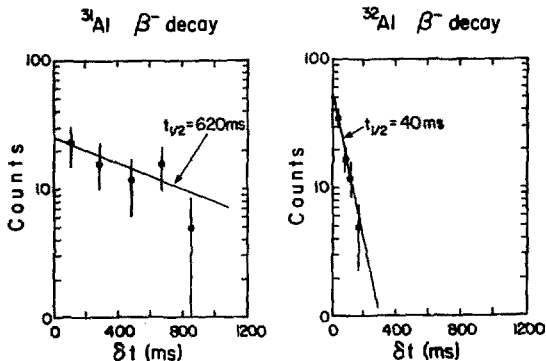


Fig. 4

detector, we obtain the β -decay time spectra shown in Figure 4. Our measurement of 620 ms for the ^{31}Al half-life is in good agreement with the known half-life of 640 ms. An expanded plot of the fast, ^{32}Al curve is shown in Fig. 5. The ^{32}Al life-time was not previously known; we obtain a value of 40 ms.

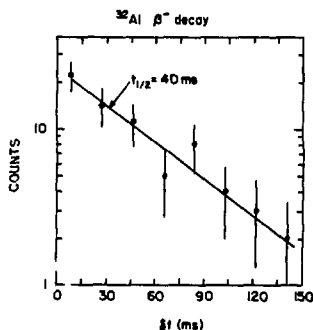


Fig. 5

It must be emphasized that these data are quite preliminary - they represent only a fraction of the total Aluminum data, and have not benefited from all of the background reduction possible in the analysis. More importantly, the Aluminum isotopes represent only a fraction of the total number of neutron-rich isotopes observed in this first run.

The power of this new method should now be clear - with the heavier beams and higher intensities which will be available from the up-graded Bevalac, it will be possible to measure dozens of new β^- lifetimes in a single run. These measurements will include lifetimes well beyond the current limits of other techniques.

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Acknowledgment

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Duf

Multi-Particle Correlation Observables in High Energy

Nucleus-Nucleus Collisions

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I. Introduction

Studies of high energy nucleus-nucleus collisions have mostly focused on single particle inclusive measurements up to now¹⁾. The wide range of particle species in the final state, from nucleons, π , K, Λ to clusters and heavy nuclei, with their different specific energy ranges, required a set of devoted experiments exploiting the various specific messages such particles provide about the complicated overall process of a nucleus-nucleus collision. Multiplicity selection²⁾ and triggering on the extent of projectile degradation³⁾ have helped to study specifically peripheral and central collisions, thus removing in part the loss of information detail due to impact parameter averaging. If we describe the final state of N particles in the continuum by

$$O(\vec{p}_1 \dots \vec{p}_N) = |S(A_1, A_2, E, b; \vec{p}_1 \dots \vec{p}_N)|^2 \phi(\vec{p}_1 \dots \vec{p}_N) ,$$

where S is the matrix element for the flux into this configuration from a collision of nuclei A_1, A_2 at energy E/A and impact parameter b , and ϕ is the appropriate phase space function, then we obtain single particle observables by taking the integral

$$\int O(\vec{p}_1 \dots \vec{p}_N) d\vec{p}_1 \dots d\vec{p}_{N-1} db = O(\vec{p}_N) \quad .$$

It is clear that features of the reaction mechanism, as contained in S , will only in exceptional cases, as in remote regions of phase space, leave a distinct signature in this average. With N ranging up to $A_1 + A_2$ in central collisions, each event constitutes an ensemble average in this mode of observation, dominated by the phase space functional, which takes care of overall energy, momentum, and particle number conservation.

Statistical models are naturally linked to such observables.

In order to obtain qualitatively different information, we want to study multiparticle exclusive observables, up to the extreme of a fully exclusive experiment identifying all the final state particles, including neutrals, leptons, and photons. The instrumentation effort is about linear with the number of particles per event, whereas the data analysis computer time grows nearly quadratically with N . This analysis, however, is now carried out event by event. From the N_i momenta obtained for each particle species i of the event, we shall construct certain collective variables, like the mean transverse and longitudinal momentum, overall momentum/energy flux directions, measures of symmetry and isotropy of the event, invariant mass spectra, etc. The high number of identical particles in each event provides some elementary statistics per event in this analysis, such that the total number of events required in order to satisfy statistical needs is not too high, thus keeping the overall

analysis effort within manageable limits. Consider the example of event-by-event analysis of $\langle p_{\perp}^2 \rangle / \langle p_{\parallel}^2 \rangle$. This ratio may be linked to the question of transparency in a nucleus-nucleus collision, measuring the overall degradation of the initial relative longitudinal motion of target and projectile nuclear matter. For complete thermalization we would expect $\frac{1}{2} \sum_i^N p_{i\perp}^2 \approx \sum_i^N p_{i\parallel}^2$ for each event in its center-of-mass frame. The ratio of the two sums, taken event by event and then averaged over a set of events, is a more sensitive measure of equilibration than picking at random one particle out of each event (in a single particle inclusive measurement) and then constructing the same ratio where the sum is now taken over the number of events considered. The reason is that, in each event, equilibration takes momentum flux out of the longitudinal c.m. motion and puts it into transverse flux. This cannot be seen if only one particle is observed per event. Another simple example for the benefits of exclusive measurement is the analysis of two particle correlations. A double arm experiment gets one pair per event, whereas an exclusive measurement yields $N(N-1)/2$ pairs if N is the multiplicity per event of the particle species considered. With $N \approx 30$ protons in a central Ca + Ca collision³⁾, one event is equivalent in information to 435 double arm spectrometer events. Moreover, other types of observables may be constructed from the N exclusively measured protons.

The main purpose of this talk is, therefore, to consider the potential of exclusive measurements, a field that we are just beginning to get acquainted with in experiments like the streamer chamber^{3,4)}, the plastic ball⁵⁾, or emulsions⁶⁾. There are concepts of analysis available from particle physics, but we will have to find out which types of observables are of specific interest towards our understanding of nucleus-nucleus collisions.

After these introductory remarks, the structure of the talk will be as follows:

II. Global features of exclusively measured events:

- A. Number correlations, where we shall exploit the multiplicities of pions, protons, clusters, strange particles measured per event, as well as certain simple quantities as $\langle p_{\perp} \rangle$, $\langle p_{\parallel} \rangle$.
- B. Vector correlations, where we shall consider appropriate measures for isotropy, symmetries, clustering, or substructure in the final state like jets or preferred flux directions.

III. Hybrid analysis:

Here we consider the measurement of one or two specific fragments, like heavy spectator nuclei, high $p_{\perp}$ particles, polarization of one particle, etc., along with a characterization of certain global features of the event such as a localization of the reaction plane or another anisotropy of the final state.

IV. Conclusions:

About future experiments employing the concepts developed, with a side glance at two-particle correlation studies.

II. Global features of exclusively measured events

How do we approach an exclusive measurement? Up to now, only charged particle exclusive observation has been employed in our field, using emulsions, the streamer chamber, and the Plastic Ball spectrometer. Simultaneous measurement of neutrals and protons will be a next significant step in the future⁷⁾. A streamer chamber picture³⁾ shown in fig. 1 for a central collision of $^{40}\text{Ar} + \text{KCl}$ at 1.8 GeV/A may serve

in order to illustrate the situation we encounter. We see about 40 charged particles, with negative and positive charge distinguished by curvature in the magnetic field and fast/slow particles identified by their ionization density in the chamber gas. Measuring of the tracks results in the picture of fig. 2 where we see a blind spot caused by a flare around the target. From three stereoscopic views measured for each event, track reconstruction in the magnetic field leads to a measurement of rigidity for each track. Combining rigidity, sign of curvature, and ionization density information, we proceed to an exclusive determination of charged particle identity and momentum/energy. Before we consider observables that may be associated with all these momentum vectors of each event, let us first look at certain simple types of analysis regarding multiplicities and mean values of momenta, event by event.

A. Number correlations

What we mean with such correlations is indicated in fig. 3. It shows event frequency contour lines in the plane of π^- multiplicity vs total charged particle multiplicity, for Ar + KCl at 1.8 GeV/A in the streamer chamber³⁾. The data are taken in a "minimum bias" trigger mode, registering each event in which there is no beam particle, $Z = 18$, detected within a 5° cone about the 0° direction, thus making sure that the projectile has undergone an interaction. The bulk of the cross section is found in peripheral interactions with small n_{π^-}, n_{ch} . Towards the more central collisions, nearly complete explosions into all the original single constituent charges, plus emission of more pions, is observed; the events approach the complete disintegration line along which $n_{ch} = Z_1^{Ar} + Z_2^{K,Cl} + n_{\pi^-} + n_{\pi^+}$ (where we assume $\langle n_{\pi^-} \rangle \approx \langle n_{\pi^+} \rangle$). The dashed line indicates the ridge of the cross section. The smooth

shift to higher n_{π^-} with increasing degree of disintegration indicates that there is no unusual pion production mechanism taking over in head-on collisions, at least not visibly at the multimillibarn level of cross sections to which this plot is sensitive. This disappoints the hopes of a phase transition setting in at small impact parameters when the reaction volume and the degree of momentum equilibration are large--at least as far as the idea of "copious pion production", originally associated with such a phase transition⁸⁾, is concerned. In fact, a closer analysis³⁾ shows that the mean number of π^- increases linearly with the number of participant nucleons.

However, we may not have chosen the interaction such that there is a chance to observe drastic signatures of a phase transition. First, with $A_1 = 40$, $A_2 = 37/39$ the reaction volume may still be too small. Furthermore, at 1.8 GeV/A the internal free energy per nucleon in central collisions may be too high, leading to an effective temperature that is far above the critical temperature associated, for example, with pion condensation. The high temperature, of about 100 MeV, is reflected in the close approach of the cross section ridge towards the complete disintegration line. The mean multiplicity of clusters with $Z \geq 2$ does not exceed one or two in the most violent events. It has been shown that the ratio of cluster to nucleon multiplicity can serve as a reaction thermometer⁹⁾.

Figure 3 also includes Λ production in this reaction¹⁰⁾, with the corresponding events given as dots in the contour diagram. The Λ particles were measured in a central trigger mode of the streamer chamber, which cuts out events with $n_{ch} \lesssim 25$, leaving the multiplicity contour lines unchanged at higher n_{ch} . We note that the Λ -producing events fall

mostly on the low n_{π^-} side of the cross section ridge. Λ particles emerge from associated production, $N + N \rightarrow \Lambda + K + N$ having the lowest threshold energy; rest mass and kinetic energy require about 1 GeV from the total available c.m. energy. Such events are, therefore, somewhat depleted in energy, resulting in less pion production. On the other hand, the nucleon "temperature" appears not to be decreased; the Λ events closely approach the complete disintegration limit. This aspect deserves further study when better Λ statistics become available.

It is obvious how rich in information such plots of rather simple results of an event-by-event analysis can be. Special "trigger" particle events, beyond Λ particles, may be studied this way, such as events with a high $p_{\perp}$ particle, with several transverse clusters or with $\bar{p}$, $\bar{\Lambda}$ towards higher than Bevalac energies. Also, other variables may be used, like the total energy observed in π^- and π^+ vs the transverse energy of protons in each event, etc.

As a final example for number correlations, fig. 4 shows preliminary data for the sum of $|p_{\perp}|$ and $|p_{\parallel}|$ for the first 20 events of central Ar + KCl collisions¹¹⁾ at 1.8 GeV/A that have been completely analyzed. This analysis is quite time consuming at the present instrumentation level of measuring without computer automatization. The semiautomatic PEPR system at the University of Heidelberg is getting ready to take over this task; it will process about 5-10 full events per hour. In fig. 4, each event is represented in the c.m. system by a dot in the plane $\frac{2}{\pi N} \sum_i^N |p_{\perp}^i|$ vs $\frac{1}{N} \sum_i^N |p_{\parallel}^i|$. At these preliminary statistics, thermalization appears to be somewhat incomplete, but much less so than in cascade model predictions¹²⁾ that indicate a very high degree of transparency for this reaction at 1.8 GeV/A. The lower part of fig. 4 gives similar data for the reaction Ar + Pb at

800 MeV/A. We observe an almost isotropic distribution of momenta in the fireball center of mass.

B. Vector correlations

1. Visualizing complete events

Before we turn to a quantitative analysis of $\vec{r}$ -space or $\vec{p}$ -space global features of events, in terms of collective variables constructed from all the particle vectors, we may stop to consider which features we see, or expect to see, in nucleus-nucleus collision final states. Our expectations are mostly based on the recent discussion of participants, spectators, thermalization (isotropy in the c.m. system) versus transparency (elongation along the beam direction), cascades versus hydrodynamic flux (phase space statistics vs. sideways flow into certain preferred directions), etc. Can we identify such features in the confusing richness of information provided by the 50 to 100 vectors of each event? Can we guess about the appropriate observables? And finally, are there features obvious in the events that point towards other, new observables that might take us beyond the present horizon of models?

Let us proceed from the Ar + KCl "central" event shown in figs. 1 and 2. Figure 5 gives a side view of the momentum vectors in the lab frame, with x being the beam direction. Always remembering that we do not see the neutrals, carrying about half the momentum flux, we may still say the picture looks as expected; in a near-central collision of equal size nuclei, there is no target fragmentation region left, with particles rather slow in the lab frame. A thermalized system decaying (with high internal energy) in the overall c.m. frame would produce exactly the focusing into a 45° cone about 0° that we see. Figure 6 gives a "top" view but now in the c.m. frame. We think about momentum balancing, and

there appears to be some flux anisotropy. No leading particles are apparent. Figure 7, a side view, reveals an entirely different aspect of our event. It offers four jets, forward/backward and up/down, in a distinctly nonspherical pattern. Hydrodynamical prejudices: do we see target/projectile spectator flux because we did not really hit head-on, plus sideways splashing? Final view, fig. 8, in beam direction: the downward jet is still visible, but an obvious lack of $p_{\perp}$ balancing reminds us of the missing neutrals and of all the various ways to be fooled by statistics in one single event.

What are the collective features? First, this event is flat; it stands upright in a plane made up by the x-z direction. Furthermore, it exhibits four jets, i.e. it is not even isotropic in that plane. Thus, we need a measure of flatness and of multi-jettiness. It is also clear from this sequence of views how rewarding it is to have a display capable of rotating the events into any desired orientation, in order to help our limited stereoscopic perception. An experimentalist's approach towards the collective observables would be to look at such displays of hundreds of events, but nobody has done this cheap, visual experiment yet.

Striking differences are obvious in comparing central events in Ca + Ca with Ca + Pb. Figures 9 and 10 show examples of these reactions at 1.8 GeV/A. The first exhibits a similar forward focus although the main part of the momentum flux seems to be diverted to the upper half plane, in which we also see a pair of π^- very close in momentum. Such two π^- correlations at $\Delta\vec{p} \rightarrow 0$, imbedded in the final state, exhibit the Hanbury-Brown, Twiss effect¹³). Figure 10, Ca + Pb, looks qualitatively different, with about 30 rather slow particles covering all polar angles around the target, plus the forward cone of mostly fast particles.

Apparently, no common moving source is governing this pattern; do we see target "spectator" breakup plus a fireball? The incoming projectile energy is considerably degraded into transverse and even backward motion in the lab frame. In any moving frame, we expect this pattern to break up into two (or more) separate particle manifolds, calling for a collective analysis treating several superimposed flux patterns. Again, energy degradation is obvious in the Ca + Pb event shown in fig. 11, with no charged particle at all within about 15° to the beam axis. Whether or not this event is cylinder-symmetric and thus a candidate for near-zero impact parameter selection can only be seen in the 3-dimensional analysis.

From these pictures, we expect to need measures for substructure like clustering or jetting, for cylinder symmetry, for isotropy or flatness and, particularly in light projectile and heavy target reactions, for two separate components far separated in phase space. Furthermore, in semiperipheral reactions, we wish to localize the effective reaction plane between projectile fragments and recoiling target remnants.

2. Axes and scalar measures of event structure

In this section, we shall introduce the concepts of sphericity¹⁴⁾ and thrust¹⁵⁾ analysis, developed in particle physics. After looking at some applications to relativistic heavy ion reactions we shall be in the position to see their benefits and limitations in our case and to look out for other, more flexible pattern analysis methods, such as the minimum spanning tree¹⁶⁾.

Sphericity analysis

Here we are fitting a triaxial ellipsoidal shape to the square of the momentum components of the $i = 1 \dots N$ final state particles, in their c.m. frame; the method is formally identical to the inertia axis determination

of a rigid body, with the local mass density replaced by the local square momentum (scalar) density. The usefulness for identification of back-to-back jets in hadron or e^+e^- physics is obvious; the high momentum flux along the jet axes is accentuated by the p^2 weight, which suppresses the other reaction products that emerge from soft scattering.

Formally, one proceeds as follows:

Construct the quantity

$$S(\hat{n}) = 3 \frac{\sum_i^N p_{i\perp\hat{n}}^2}{\sum_i^N p_i^2},$$

where $\hat{n}$ is an arbitrary direction unit vector, and $p_{i\perp\hat{n}}^2$ is the component squared of the i th particle momentum perpendicular to $\hat{n}$. The axis of maximum elongation is the one of minimum inertia $\approx$ flux perpendicular to this axis; therefore, the sphericity is obtained as

$$S = \min_{\hat{n}} S(\hat{n})$$

by variation of the direction $\hat{n}$. This is equivalent to finding the highest eigenvalue Q_3 and, in addition (for further information) the other two, Q_2 and Q_1 , of the symmetrical matrix

$$M_{\alpha\beta} = \sum_i^N p_{i\alpha} p_{i\beta}, \quad i = 1 \dots N, \alpha, \beta = 1 \dots 3.$$

Normalizing $Q_1 + Q_2 + Q_3 = 1$, and ordering $0 \leq Q_1 \leq Q_2 \leq Q_3$, we can express the sphericity by

$$S = \frac{3}{2}(Q_1 + Q_2) \begin{cases} \rightarrow 0 \text{ for } Q_3 \gg Q_1, Q_2, \text{ i.e. a "two jet-pencil"} \\ \rightarrow 1 \text{ for } Q_1 \approx Q_2 \approx Q_3, \text{ i.e. a sphere.} \end{cases}$$

If $Q_1 \approx Q_2$ we have a rotation ellipsoid, but if $Q_1 \ll Q_2$ the event is flat. We thus define

$$\text{Flatness } F = \frac{\sqrt{3}}{2}(Q_2 - Q_1) \begin{cases} \rightarrow 0 \text{ for cylinder symmetry} \\ \text{about } Q_3\text{-direction} \\ \rightarrow \frac{S}{\sqrt{3}} \text{ for coplanar events.} \end{cases}$$

The coplanarity is a characteristic feature of e^+e^- , 3-jet events where the third jet results from gluon bremsstrahlung.

What we do in sphericity analysis is to fit a four-parameter collective flux ellipsoid (Q_1, Q_2, Q_3 , direction of Q_3) to the square of the N momenta of each event. It is clear that N has to be large in order to attach significance to four variables; we shall illustrate this below. Furthermore, in the application to nucleus-nucleus collisions we have to avoid being misled by the square-momentum analysis; one heavy projectile or target fragment with its high p in the c.m. is sufficient to dominate all the other particles, pulling the Q_3 direction along with it. Therefore, sphericity is very sensitive to spectator motion. The positive aspect of this: if we search for the reaction plane of a semiperipheral reaction, $S(\hat{n})$ is the appropriate measure to select it from an isotropic participant background.

Thrust analysis

Here we simply look at momentum flux density. The quantity

$$T(\hat{n}) = \frac{\sum_i^N |\vec{p}_i \cdot \hat{n}|}{\sum_i^N |p_i|}, \quad \hat{n} \text{ arbitrary direction unit vector,}$$

may be visualized as the flux density in direction θ, ψ as given by a scalar, which we may take as the radius in θ, ψ direction of a deformed flux spheroid. The direction of maximum elongation is called thrust direction, and

$$T = \text{Max}_{\hat{n}} T(\hat{n}) \begin{cases} \rightarrow 1 \text{ for 2 jets back to back along } \hat{n} \\ \rightarrow \frac{1}{2} \text{ for isotropic emission} \end{cases}$$

is defined as the (maximum) thrust of an event.

Thrust analysis determines just one scalar plus a direction, as the collective features of the event. Clusters that are "coalesced" out of nucleons are given the same weight in T as their constituents would have. On the other hand, one axis may not be sufficient in all cases of interest. We might then want to construct the whole $T(\hat{n})$ flux spheroid and fit certain simple shapes to it (pancakes, cigars, etc.).

3. Applications of sphericity and thrust analysis

a. Negative pion emission in Ar + KCl at 1.8 GeV/A

We shall illustrate sphericity analysis in an application to π^- emission in central Ar + KCl events studied in the streamer chamber¹¹⁾. At 1.8 GeV/A, the mean π^- multiplicity is about 6 and the mean participant proton number is 30, indicating a selection of impact parameters $b \lesssim 0.3 b_{\text{max}}$ in this trigger mode³⁾.

Figure 12 gives a contour plot in $p_{\perp}$ and rapidity y_{CM} of the invariant pion cross section, for all π^- from an analyzed sample of about 3000 events. The contours are approximately symmetric about midrapidity, with a slight shift towards positive rapidity caused probably by the fact that $A = 40$ collides with $A = 38$ and by the neutron excess of the projectile (π^- are produced by $n + N \rightarrow \pi^- + p + N$). So the pion source is moving with about the velocity of the $N + N$ center of mass. Emission is not quite isotropic, however, as can be seen from the dashed lines representing an isotropic source at midrapidity; there is a forward-backward enhancement of the yield. The same feature is obvious

from fig. 13, showing the distribution in $\cos\theta$ of observed emission angles in the c.m. system. We might conclude that there is in the c.m. system a memory of the beam direction preserved in the nucleon motion, i.e. that participant nucleons are not completely thermalized, because then the known forward-backward enhancement of π emission in nucleon-nucleon collisions would explain our observation. However, this conclusion is not stringent from the data of figs. 12 and 13 for the following reasons:

1. Due to the finite number of nucleons in the colliding nuclei, there may occur a shift away from midrapidity in the effective center of mass of each individual event, caused by fluctuation in the projectile and target participant numbers. The individual c.m. rapidities would be symmetrically distributed about midrapidity, and a half-width $\Delta y \approx 0.3$ would be sufficient to explain the elongation of the contour plot along $\pm y$. We would, therefore, like to see whether this elongation is a feature of the individual events or one of ensemble averaging.
2. As to fig. 13, it is not so much the emission angle but the momentum flux angular distribution of the pions that we should analyze.

This appears to be a good case for sphericity analysis. There are no leading, high $\vec{p}$ pions that might obscure the picture. Figure 14 gives the angular distribution for the direction of the main axis, Q_3 , resulting from a fit of all events with $n_\pi \geq 4$. We note here, without proof, that the direction of Q_3 is the most statistically significant of the four sphericity analysis parameters, i.e. the most reliable in a situation of relatively small event multiplicity. The corresponding Monte Carlo simulation gives a flat distribution. We can now safely conclude that the square momentum flux in π^- shows a forward-backward enhancement on the event-by-event basis.

Proceeding to the other collective parameters, fig. 15 shows event frequency contours in the plane of coplanarity or flatness vs sphericity. The events cluster at $F \approx 0.2$ and $S \approx 0.5$, i.e. at drastically nonspherical, rather flat shapes. Figure 16 shows the projection onto the S axis; the length of the main axis appears to be about twice that of the other axes. The events thus look like flattened cigars, oriented preferentially along the beam axis. But comparing with a Monte Carlo simulation with respect to S and F leads to a big surprise! Figure 17 shows this for S, giving the ratio of true event to Monte Carlo event frequency along the S axis; the ratio is compatible with 1 for all S, within statistics. The same is found for the flatness variable. At the level of statistics provided by several thousand events and with a mean particle multiplicity of about 8 (resulting from the cut $m_{\pi^-} \geq 4$), the parameters S and F are governed by finite particle number effects; i.e., a multiplicity 8 event, under the square weighting of momenta, is on the average nonspherical by its very nature!. The full set of collective parameters, introduced by sphericity analysis, thus overstretches the information that 8 momentum vectors can provide. We need more particles in order to make the square momentum flux density functional continuous enough for the fit not to get hooked on statistical fluctuations.

We conclude that the elongation direction is the least critical parameter in sphericity analysis, which makes a maximum thrust analysis the more promising candidate in such low M cases because it focuses specifically on this feature of the event. Much better event statistics may help for the essay about S and F, with some small signal standing out above the Monte Carlo simulation.

At the present level of analysis, we may conclude that π^- emission is enhanced forward-backward in central Ar + KCl collisions at 1.8 GeV/A; but we cannot learn more about the collective features of the square momentum flux pattern.

b. Collective observables of proton emission, predicted by cascade and hydrodynamical models

After having absorbed the warnings about sufficiently high particle multiplicities from the pion case, let us turn to proton emission analysis from central collisions where the mean multiplicities are about 5 to 10 times larger. The analysis of experiments, like the streamer chamber or the plastic ball, will be progressing within the next year. Meanwhile, we may look at the predictions of the two most contradictory models for the reaction mechanism, cascades^{12,17)}, and hydrodynamics^{18,19)}.

The main difference in the predictions of these two models for central collisions of $A \approx 40$ nuclei is appropriately expressed in momentum flux variables; cascades predict drastic transparency effects towards the top Bevalac energies, i.e. $\langle p_{\parallel} \rangle > \langle p_{\perp} \rangle$ in the c.m. frame, whereas hydrodynamics predicts a 90° deflection of the incident momentum flux. Thus the cascade final state is a cigar along the beam direction, but hydrodynamical models predict a pancake momentum pattern, perpendicular to the beam axis at small impact parameters.

In terms of sphericity analysis, fig. 18 illustrates the cascade model predictions¹⁷⁾ for proton emission from $^{38}\text{Ar} + ^{38}\text{Ar}$ at 360 MeV/A and 1180 MeV/A. The variables used here are (1-sphericity), which we might call elongation, and the polar angle of the main axis Q_3 with respect to the beam direction. The plots refer to the c.m. frame, giving contour lines of event frequency in the $(1 - s)$ vs θ plane. Only

participant protons are included in the analysis of each event, i.e. those protons that did engage in the cascade. We are thus avoiding spectator effects; but note that these figures are not directly comparable to data except for the limiting ideal case of an extremely strict $b = 0$ selection in the data. Nevertheless, they may serve to enhance the specific features of cascade model predictions. For each energy, contours equivalent to an inelastic and central trigger mode are obtained by multiplicity cuts. At 360 MeV/A, the plot for $M \geq 4$ is again governed by small particle number effects; for $M \geq 20$ we observe shapes that are very close to perfect spheres: no transparency in central collisions at this energy but also no signal of a sideways enhancement ($\theta \rightarrow 90^\circ$). At 1180 MeV/A the picture is different. In the $M \geq 30$ selection we observe a statistically significant elongation, corresponding to a ratio of about 1.6:1 of the principal axis to the mean of the two smaller axes. The orientation is along the beam axis.

The cascade model thus predicts the onset of transparency in central Ar + Ar collisions already at intermediate Bevalac energies. Comparing with the $|p_\perp|$ vs $|p_\parallel|$ plot of the Ar + KCl data in fig. 4 we conclude, with the necessary reservations in view of very low statistics, that even at 1.8 GeV/A no such drastic transparency is observed. Note, moreover, that the data should be even more forward-backward enhanced because they still include some spectator motion (in the sense of the cascade model) due to the contributions from $b \neq 0$.

Turning to the hydrodynamical model, fig. 19 shows¹⁹⁾ the familiar plot of density time development in configuration space for a symmetric collision at 400 MeV/A. The three typical impact parameters range from peripheral to head-on. There are no spectators in hydrodynamics because

the collective pressure gradient, built up in the zone of geometrical overlap, acts on all the nucleons. The final state exhibits a collective sideways deflection ("bounce-off") of the momentum flow, increasing with smaller impact parameter. At $b = 0$, the final state is a pancake both in configuration and momentum space because $\theta = 90^\circ$ is the preferred flow direction, and we have cylinder symmetry about the beam axis. Figure 20 shows the result of a thrust analysis of these calculations, as a function of impact parameter $b/b_{\max}$. The lower horizontal axis gives the "bounce-off" angle θ , identical to the thrust direction, corresponding to the impact parameter shown on the upper horizontal axis. The maximum value of T decreases from complete elongation of the flux pattern along the beam direction for grazing collisions, to "isotropy" for head-on collisions. Here we observe an inadequacy of thrust analysis; it cannot, in the value of T , distinguish between a sphere and a cylinder symmetric but flat distribution. Generalized sphericity analysis, with its higher number of collective variables, could appropriately describe this final state: $Q_3 < Q_2 = Q_1$, $\theta(Q_3) = 0^\circ$.

We have to wait for more data analysis to decide how nature falls in between these two model extremes. There is one point important about these models: hydrodynamical predictions are A -invariant, whereas cascade predictions are not; at fixed incident energy, the cascade final state gets more spherical with increasing A . For heavy nuclei, the hydrodynamical approach appears to be more appropriate than for $A \lesssim 40$; we shall at Bevalac energies be trying to decide whether a slightly cigar-shaped cascade or a pancake hydrodynamical final state fits the data. The transient state of maximum compression, to which we have not yet found a direct experimental access, looks alike in both models (pancake shape in $\vec{r}$ -space). Figure 21 summarizes these features in a schematic way.

4. More general pattern analysis

We have seen that event-by-event analysis of momentum thrust and sphericity appears appropriate, most of all, in collisions of equal mass nuclei, in order to select more sharply for $b \approx 0$ events [cylinder symmetry: $Q_1 = Q_2$, $\theta_{(Q_3)} = 0$] and then check transparency/equilibration, and to decide between various models of the reaction dynamics by means of their characteristic flux predictions. However, the Ca + Pb events shown in figs. 10 and 11 indicated a problem not easily handled by thrust and sphericity; two or more separate components in the final state, perhaps to be identified with the spectator and participant regions of phase space, call for a more flexible treatment capable of identifying the particle "families" and then analyzing each one by itself. Furthermore, there may be other observable multiparticle correlation features, beyond our present concepts of equilibrium, cascades vs hydrodynamical flux, etc. Such features may be discovered by looking at other representations of the final state.

As an example for an $\vec{r}$ -space representation that may be useful, fig. 22 shows a polar plot of the emission angles θ, ψ in the c.m. system, for all charged particles of a central Ar + KCl event at 1.8 GeV/A. A similar plot could be made separately for each kind of particle, found with sufficiently high multiplicity in the event, like all pions vs protons vs d, t, He. Clustering in configuration space would show up as regions of high point density. A pattern analysis method appropriate to such a "family" search is the minimum spanning tree¹⁶⁾, illustrated in fig. 23. Three families of particles are shown in a plane that could be that of the fig. 22 graph or any other suitable $\vec{r}$ or $\vec{p}$ space, 2- or 3-dimensional representation of the event. The quantity minimized is the

total length of lines connecting all the particles; the families thus emerge as branches with internal short distances, connected to a "tree" by a minimum number of long lines. The method is applicable to any kind of multiaxial, multicluster features, where the meaning of the axes depends on the choice of metric and representation. In 3-dimensional $\vec{p}$ space, for example, this picture would be interpreted as a 3-jet event, and we might then look at the total momentum of each jet and obtain the mean $p_{\perp}$ within each family with respect to its jet axis. In 3-dimensional $\vec{r}$ space, we might check with the same method whether or not there is a preferred orientation of pion emission with respect to proton emission or whether there are participant and target/projectile spectator families. The resulting main branches would then fix the orientation of the impact plane. These examples may be sufficient to illustrate how the desired observable is connected to an appropriate choice of the metric in this method of analysis.

5. Experimental considerations

We have seen that collective analysis, event by event, may become meaningful provided that certain conditions are met:

- a. Multiplicities should be high for each family of particles, to be separately studied in the final state.
- b. The fraction of unobserved or misidentified particles should be kept as low as possible.
- c. The experiment should have a highly uniform detection efficiency over all 4π .

As to the multiplicities required, the sphericity analysis example of π^- from $\text{Ar} + \text{KCl}$ has served as a warning about finite particle number effects. In order for a collective degree of freedom (such as flatness)

to be a meaningful concept, there has to be a sufficient number of microscopic degrees of freedom (like single particle momenta) folding into its definition. Each event has thus to provide elementary statistics. The number of events considered can only in part make up for a lack of elementary statistics; it is obvious that even a million events, with two pions in each of them, cannot support a sphericity analysis. Requiring at least one order of magnitude in the ratio of microscopic to collective degrees of freedom, the mean number of about 40 charged particles in a central Ar + KCl collision may be just about sufficient to perform a 4-parameter sphericity analysis. We thus need all their individual momenta, and the experiment should have sufficient spatial resolution in order to register them. Track detectors and the streamer chamber are adequate in this respect. A multisegmented hodoscope array or a plastic ball type system must have a resolution of 2-4 degrees in both θ and ψ , which leads to the about 900 modules in the plastic ball/wall⁵⁾, as sketched in fig. 24.

At the extremes of attainable Bevalac reaction multiplicities, such as $\langle m_{ch} \rangle \approx 160$ in central Pb + Pb at 1 GeV/A, we may reach a critical point because neither of the familiar experiments will be fully adequate. How shall we go beyond? A general idea might be to give up individual particle identification, from which then to construct the collective observables, measuring local averages of certain particle, momentum or charge fluxes instead. The appropriate ways of calorimetry have not yet been worked out; the energy range of particles, from about 20 to 1000 MeV, mostly not yet minimum ionizing, is inconvenient. Particle physics calorimetry is not applicable in a straightforward manner. Furthermore, the desired flux observables would have to be built into the mode of

observation, thus limiting the range of accessible information unlike in the case of a fully exclusive experiment. At the present time when we are just starting to guess about the collective degrees of freedom, relevant to nucleus-nucleus collisions, a decision about the appropriate experiments seems premature. Let us wait to see what the next year of plastic ball and streamer chamber data analysis will tell us.

The fraction of unobserved particles in present 4π experiments is about $1/2$, i.e. the neutrals. The plastic ball offers some promise as to neutron detection; however, with a different efficiency from that for charged particles. On the event-by-event basis, a 4π detection with about 15% efficiency for neutrons is of no real use unless the neutron multiplicity gets so high that even sampling them becomes significant, i.e. in Pb + Pb collisions. In such collisions, another indirect solution comes to mind; the fraction of clusters, d, t, ^3He , ^4He , will have such a high multiplicity that we might guess the neutron flux from the p:d and ^3He :t ratio in phase space, on the event-by-event basis. This might help to understand semiperipheral collisions, in the colder "spectator" regions of phase space. We shall come back to this aspect in the next chapter.

Finally, any nonuniformity of the detection efficiency can be a major obstacle in collective analysis. There is always a difficulty in the target plane, due to particle absorption, increased γ conversion, and multiple scattering. In addition, the streamer chamber cannot momentum analyze the 90° particles emitted along the magnetic field direction, and to some extent all particles moving along the camera viewing angles. The plastic ball appears, at first sight, to be free of such additional "blind spots". However, its finite energy range of particle identification, from

about 30 to 200 MeV/A, leads to trouble after transforming into the center of mass or any other moving frame; only about half the relevant phase space is covered. In the streamer chamber data analysis, we thus have to watch out for an instrument-induced flatness of the events, with a part of $p_{\perp}$ missing. The plastic ball has a built in forward-backward anisotropy in the center of mass; its main strength thus lies with target fragmentation pattern analysis.

In general, all the three topics mentioned here, low multiplicities, lack of exclusivity, and detection efficiency anisotropy, result in a heavy leaning on Monte Carlo simulation of random but detector-typical background before significant information may be extracted.

III. Hybrid Analysis

In this chapter we shall sketch certain types of analysis in which there is a trigger particle selected, either by hardware or off line, and collective features of the event associated with it are considered at the same time. The simplest example is the familiar selection for small or large impact parameters²⁾, by requiring a high/low multiplicity of charged particles associated with a single reaction product. This is not an event-by-event analysis but just a selection of a restricted sample of events. Let us now look into examples that use a higher degree of exclusivity.

A. Localization of the impact plane

The concept for this experiment (yet to be carried out) originates from hydrodynamic model studies^{18,19)} of light projectile-heavy target reactions, like $\text{Ne} + \text{U}$. One approach towards testing the predictions of sideways collective flow, made by this model, is to identify $b = 0$ as closely as possible, by requiring cylinder symmetry about the beam axis and then look for directions of preferred emission of protons, clusters, etc. This selection is difficult because of the small cross section associated with $b \rightarrow 0$. To test hydrodynamics at finite impact parameters we have to select an effective b window and to localize the orientation of the impact plane, i.e. to find the plane that contains the beam axis and the two centers of the colliding nuclei.

This idea is illustrated in fig. 25 by the results of a hydrodynamical model calculation¹⁸⁾ of matter flux in $\text{Ne} + \text{U}$ at 393 MeV/A, for various impact parameters. Invariant nucleon cross section contour levels are given, projected onto the impact plane defined by $\pm p_{\perp}$ and rapidity. The impact vector is always pointing to the positive $p_{\perp}$

direction, from the target to the projectile center. At $b = 0$ we observe total degradation of the incident projectile matter and momentum flux; the bulk of it is emitted at small rapidities and $50^\circ \lesssim \theta \lesssim 90^\circ$, with azimuthal (= cylinder) symmetry. At $b = 8$ fm, on the other hand, we find the target- and projectile-like regions of phase space preferentially populated. More accurately, the projectile remnant flux centers at a finite $\theta \approx 15^\circ$; it has acquired a finite total $p_{\perp}$, due to the collective "bounce-off" from the target surface. For $b = 6, 4, 2$ fm, the two flux families merge more closely but all the time exhibiting two peaks falling onto the impact plane. The "bounce-off" angle increases with b . Figure 26 shows a Ca + Pb event at 2.1 GeV/A that might correspond to $b \approx 6$ fm.

How would a collective analysis of exclusively measured events of Ne + U or Ca + Pb proceed in order to check this prediction? The nucleon and cluster multiplicity will be high enough, for all but the largest impact parameters, to perform a flatness analysis in terms of sphericity variables, obtaining $Q_1 < Q_2$ and the direction of Q_3 . Conveniently, the limiting case of cylinder symmetry, $Q_1 = Q_2$, $\theta(Q_3) \rightarrow 0$, is contained in this. The directions of Q_2 and Q_3 define the impact plane. Of course, its orientation will only be approximately localized due to the finite particle number statistics of each event, perhaps to about $\pm 20^\circ$ in the azimuth. Alternatively, we might use the minimum spanning tree to identify the target/projectile particle families. The impact plane is then defined by the connecting main branch vector and the beam axis. After this, we shall rotate all the events into a common (arbitrary) impact plane orientation. Furthermore, we may sort the events into bins of Q_3 direction $\Delta\theta_i$ in this plane, corresponding in the hydrodynamical model to bins of impact parameters (bounce-off angle).

Finally, we shall take an ensemble of about 500 to 1000 events per $\Delta\theta_i$ bin, which means about 25 000 to 50 000 particle vectors per bin, at a mean multiplicity of about 50 in a minimum bias Ne + U experiment. Now we may construct invariant cross section contours and compare to the prediction of fig. 25 for the nucleon and cluster flux density distribution.

However, the check of hydrodynamics does not stop here. We may get one step further in looking at protons and clusters separately. This is significant because the regions of collective flux in phase space are relatively low in internal thermal energy, in the hydrodynamical model. In these regions, therefore, we expect an enhanced cluster abundance. Figures 27 and 28 illustrate this prediction for the proton and α -particle flux at $b = 6$ fm. Chemical equilibrium conditions have been assumed for the final state of the hydrodynamical calculation¹⁸⁾, at densities below that of normal nuclear matter, resulting in a prediction of the flux carried by each nucleon or cluster species. It is obvious that the α -particle flux, fig. 28, exhibits the asymmetry and flatness of fluid dynamical flux predictions in a much more drastic way than the proton flux, fig. 27. The protons originate more specifically from the hot fireball zones (called head shock in the model terminology) and are emitted more spherically.

Such a comparison of data with theory will certainly be much more conclusive than anything we have done thus far. The corresponding phase space calculation, by means of the cascade model, will require a chemical equilibrium treatment of the final state of that model. Such calculations are also becoming available now²⁹⁾. The difference in the predictions of these two models should be that hydrodynamical collective flux leads to

a more narrow confinement of the spectator and participant particle families in phase space. The experimental effort, going into this test of reaction dynamics, is fairly high but still manageable.

A final remark is in order now about the term "hybrid analysis", that I use for such studies. They proceed from an event-by-event analysis (impact plane and bounce-off direction) to a single particle density distribution, as obtained from an ensemble of events binned according to their collective features.

B. Heavy spectator nuclear fragments and the bounce-off effect

A special case of the bounce-off mechanism in hydrodynamics arises if, in a reaction like $\text{Ca} + \text{Pb}$, a large fraction of the target spectator remains intact, with the emission of a heavy fragment that carries most of the target part of the collective momentum flux. This type of event will be associated with rather large impact parameters. In hydrodynamics terminology, the mechanism would be understood in terms of a collective pressure field associated with the high density in the target-projectile overlap region that is localized on the surface of the heavy target nucleus. The target spectator region is pushed to the side, as is the projectile remnant. The energy transfer is relatively small (unlike in central collisions) and shared equally between the target and projectile remnants. It will be sufficient to fragment the small projectile remnant into nucleons and light clusters, but in the target remnant there may be regions left cold enough in order to survive as heavy fragments, still carrying the collective bounce-off momentum transfer. From elastic α -particle scattering at 180° it is known that the target mean field can absorb several GeV/c of recoil momentum without fragmentation.

The resulting final state: light projectile fragments, near beam velocity, are emitted at small but finite $\theta \lesssim 10^\circ$. Also on the projectile side in the impact plane, we shall observe the expansion products from the hot overlap zone, mostly nucleons at about midrapidity. The heavy target remnant recoils to the opposite direction in the impact plane. The experimental program is then to

1. Identify the projectile fragment family and determine the corresponding thrust direction that, together with the beam direction, fixes the impact plane and the projectile bounce-off angle.
2. Identify the "fireball" particles near midrapidity. Their number is a measure of the size of the overlap zone and, thus, of the impact parameter. Their total energy approximates the energy transfer between target and projectile (the degree of inelasticity).
3. Identify the target fragments, triggering the experiment by the occurrence of a heavy ($A \gtrsim 50$) nucleus that is slow in the laboratory frame. At 10 MeV/A, such a fragment carries $\gtrsim 7$ GeV/c of collective momentum! At 2 GeV/A incident ^{40}Ar energy, about 30 projectile fragment nucleons balancing this momentum would be distributed around $\theta \approx 5^\circ$.

In connection with this type of expected event, the remark has often been made that all its features follow directly from energy/momentum conservation. This is not true, however. Of course, any final state is consistent with the conservation laws, but the hydrodynamical model predicts a much sharper focusing of the momentum flux into the two bounce-off directions than any phase space model (cascade), due to the collective pressure gradient established in the interaction zone. The shape of the colliding nuclear matter distribution, at given impact

parameter, plays a decisive role in the orientation of the resulting pressure field, unlike in cascade calculations in which each binary nucleon scattering is assumed to proceed as in free space, and the shapes are of secondary importance, affecting the outcome only through subsequent rescattering. A different way of stating this: the overlap zone in such a semiperipheral event has a thickness of only about two mean free paths, in the cascade view of things. These two binary scatterings per nucleon are governed by free N-N collision phase space, with little chance of a cooperative effect setting in that could simulate the collective deflection of flow in hydrodynamics. Thus, an event with sharp focusing of the flux is rare in the cascade model, whereas it is predominant in the hydrodynamical model. Of course, the applicability of hydrodynamics appears to come to its very limit in this situation. Zero mean free path is assumed locally, in the rather small subvolume of target and projectile overlap, which accommodates only about two binary collisions, in the cascade approach.

This rather risky theoretical situation has attracted several experiments, reported at this conference^{21,22}). Figure 29 shows a sketch of one²¹⁾, using three major components: a set of detectors for heavy, slow target fragments inside a vacuum chamber; a multiplicity array for fast particles surrounding the forward hemisphere of the chamber (fireball products); and the segmented plastic wall array, identifying the projectile fragment flux distribution at small forward angles by means of time of flight and specific ionization measurement. The data of this experiment, which is only exclusive to the extent required by the sought observables, are not yet fully analyzed. We are also waiting for a cascade model treatment of the bounce-off effect.

C. Pion condensation search

Let us now reach out, with some speculative remarks, beyond the familiar territory of models. Of course, hydrodynamics will only be conclusively tested in exclusive analysis, and in this way we may indirectly approach the still evasive conditions at high density in the transient, short-lived state before expansion. One qualitative step further into the unknown do we go in thinking about phase transitions and their possible characteristic observables, because such theories require a detailed microscopic state of nuclear matter, for the short transient time scale of some 10^{-23} s, beyond the development towards local equilibrium or the onset of collective flux. At Bevalac energies, pion condensation is the only likely candidate for a phase transition because the total energy density of at most 0.8 GeV per cubic Fermi, reached at density $\rho = 4\rho_0$ in a 2 GeV/A Ca + Ca collision, falls well below the present estimates for a quark matter transition²³⁾.

Let us disregard, for the moment, the critical questions about short lifetime, finite size, and high temperature of the fireball, assuming that a small but finite fraction of near-central collision events result in pion condensation. This results in symmetry breaking²⁴⁾; a spin-isospin ordered lattice is being established in the interaction zone, leading to long-range orientation along a plane that is fluctuating from one event to the other. This state may be reflected in an emission anisotropy because the nucleons in the remote regions of target and projectile, streaming into the overlap zone where a condensate is already established, will scatter anisotropically with respect to the lattice direction (critical opalescence). Thus, at least at $b \approx 0$, there are no leading fragments but the event still shows no cylinder symmetry. We are referring to a collision of equal size nuclei.

The experiment should thus select nonspherical events of maximum multiplicity, no leading particles, and a thrust or sphericity orientation towards large θ . Unfortunately, such an event pattern might also be caused by hydrodynamical flow, at small but finite impact parameters (fig. 19). We would then localize nothing but the impact plane. We need another additional observable to be selective to pion condensation. Can we make use of the spin orientation along the lattice? One idea might be to identify a signal of magnetic moment orientation in observing γ -emission anisotropy. However, let us stay within the capabilities of existing instrumentation, like the streamer chamber. We may think, then, about a polarization measurement of the produced particles. Proton polarization can be measured by secondary scattering from spin zero nuclei. This would require something like a carbon foil surrounding the target, measuring the up/down asymmetry of secondary interactions in it. Each event provides a sufficient number of emitted protons, such that at least one such reaction takes place, per event, even in a thin foil. However, the analyzing power of double scattering is low, and we may be unable to satisfy statistical requirements, within the limits imposed by the effort of exclusive event analysis.

An alternative approach may be to look at Λ particle polarization¹⁰⁾. The Λ particles produced in the interaction decay in flight, away from the primary vertex, $\Lambda \rightarrow \pi^- + p$ with 67% branching ratio. In this weak decay, the Λ polarization direction is preserved (by parity nonconservation) in the spatial orientation of the Λ -p production plane. The analyzing power of this process is high (0.64). The rationale would be that Λ particles are produced in the high energy density zone, i.e. in the pion condensate. They rescatter from the lattice, orienting

their spin direction relative to it. We would thus have to obtain an ensemble of high multiplicity, no leading particle, nonspherical events with a Λ in it. This set of requirements implies a lot of running time because the mean Λ multiplicity in Ca + Ca central collisions at 2.1 GeV/A is only about 0.08. We should gather several thousand Λ -producing events, to look for a change of polarization with the orientation of Λ emission relative to the event asymmetry plane, as resulting from sphericity analysis of the event protons. Unless a 4π - Λ trigger is used--a detector system by far exceeding familiar experiment dimensions at the Bevalac--this amounts to gathering about 200 000 central trigger events in the streamer chamber (≈ 200 Bevalac hours), scanning all of them for Λ events, and exclusively analyzing the resulting sample of several thousand. Figure 30 shows how Λ particles are identified from an invariant mass analysis of the measured negative and positive track rigidity in a "Vee" event¹⁰⁾. Competing Vee-producing events are $K^0 \rightarrow \pi^+ + \pi^-$ decay and $\gamma \rightarrow e^+ + e^-$ conversion; they are discriminated by invariant mass analysis.

We see that the effort is very high; we are certainly approaching the level of particle physics experiments here. Furthermore, this experiment proposal is based on theoretical dreams. In particular, one might argue that the Bevalac maximum energies, at which we reach the Λ production threshold, come along with reaction temperatures that are beyond the critical temperature for pion condensation. With considerations such as these, we have obviously exhausted our present experimental and theoretical capacities. They may still serve for an orientation as to what lies ahead.

IV. Conclusions

In this talk, I have focused entirely on multiparticle correlations because this field is new and relatively unfamiliar to all of us, but our experimental developments, like the plastic ball or the streamer chamber, do already allow for this type of analysis. Two particle correlation analysis is certainly as promising a tool, to proceed beyond single particle inclusive observables. Such correlations as

- a. "Anomalous" decay studies by $\alpha \rightarrow d + d$ invariant mass reconstruction with the HISS spectrometer²⁵⁾,
- b. Di-lepton measurements²⁶⁾,
- c. Back-to-back proton or pion emission correlation²⁷⁾,
- d. Proton-proton quasifree scattering kinematical correlation²⁸⁾,
- e. Identical pion and nucleon correlation²⁹⁾ at $\Delta \vec{p} \rightarrow 0$ (Hanbury-Brown, Twiss effect), and
- f. "Vee" events from Λ , K^0 decay, and γ conversion, imbedded in the final state¹⁰⁾

are being actively pursued at the present time, and contributions to this conference illustrate their richness in information.

Let me end with some conclusions about exclusive, event-by-event analysis:

- A. As examples of collective analysis of the final state, we have described the concepts of thrust, sphericity, and the minimum spanning tree. The first two lead to the construction of multi-axial 3-dimensional surfaces, representing the density distribution of momentum and square momentum flux, respectively. Two jet structure, flatness, leading particle motion, and the orientation of the impact plane, as well as zero impact parameter selection by cylinder symmetry, are the observables most

easily accessible in this framework. The minimum spanning tree is the most flexible approach, applicable to any 2- or 3-dimensional, $\vec{r}$ or $\vec{p}$ or E space representation of the event. It identifies any clustering or particle family structure, depending on the choice of metric, and thus appears to be suitable for exposing unexpected features for which we do not yet have theoretical prejudices.

Furthermore, we should exploit scalar quantity correlations in the events, such as proton, pion, cluster multiplicities, or $|p_{\perp}|$ and $|p_{\parallel}|$ sums and the total transverse energies carried by various particle species.

B. To apply collective analysis, we need a sufficient number of microscopic degrees of freedom per collective degree of freedom. Monte Carlo simulation of events becomes extremely significant, to assess the essaying power of collective analysis. Thus, we should be delighted, rather than frightened, about the high multiplicities of particles in Bevalac energy collisions! Proton (and neutron) multiplicities are always adequate, but pion and cluster statistics will be marginal. At CERN PS energies of up to 15 GeV/A, pion analysis will come to the foreground, and strange particle statistics will be marginal.

Looking ahead to Bevalac Pb + Pb reactions at 1 GeV/A, with proton multiplicities exceeding 100, or to VENUS or CERN ISR reactions producing even more pions, we shall reach a point at which individual particle momentum reconstruction becomes both overwhelmingly tedious and perhaps unnecessary. Calorimetric experiments that measure local averages of particle, energy, or charge flux appear more appropriate. However, we shall then lose some of the nice aspects of fully exclusive identification, for example the possibility to construct any desirable two particle correlation observable. Furthermore, at Bevalac energies, a

large fraction of the final state particles will be too low in energy to apply familiar calorimetry. New experimental designs, not provided by present particle physics, have to be developed.

C. The presently available 4π instrumentation, i.e. the plastic ball and wall, the streamer chamber, and emulsions, are perfectly suitable for us to test conclusively the dynamical models, discussed now. Kinematical equilibrium vs transparency, local chemical equilibrium, etc. are approached in observables like event-by-event $p_{||}$ to $p_{\perp}$ ratios, cluster and nucleon flux comparison, and the distribution of the charge to neutral ratio in phase space. For example, in collisions of zero isospin nuclei like ^{40}Ca with high isospin targets, the distribution of the $t:^3\text{He}$ ratio in phase space, along with a determination of the impact plane, will serve to pin down the flux pattern of projectile and target matter and the extent of equilibration (mixing).

Beyond this, the selection of events corresponding to finite impact parameter domains, by cylinder symmetry for $b \rightarrow 0$ and impact plane plus "bounce-off" pattern identification for finite b , will enable us to discriminate phase space/cascade mechanisms from collective fluid dynamics patterns. The overall result of this might be that one of the models will be substantiated, in all its predicted observable features, by the features of a certain class of reaction, like $\text{Fe} + \text{Fe}$ or $\text{Ca} + \text{Pb}$, to such a degree that its implications about density and temperature in the transient compressional phase gain credibility. Such that, although we are still not finding direct probes for the ratio of potential (compressional) to thermal energy, as a function of reaction time, the features of the nuclear matter equation of state become known.

D. Beyond our present model discussion of hydrodynamical collectivity versus phase space/cascade statistics, the guidance provided by theory is scarce, as far as suitable observables, corresponding to phase transitions, are concerned. If pion condensation occurs, and if it leads to a macroscopic plane of lattice order in the collision (and not to a multitude of randomly ordered domains as in a ferromagnet), we may identify it by decay anisotropy, detected event by event, and by additional probes of the spin/magnetic moment orientation.

Finally, beyond the guidelines and prejudices established by theory, there is always the empirical approach. Let us look, by means of exclusive analysis, for unexpected and surprising features of the event.

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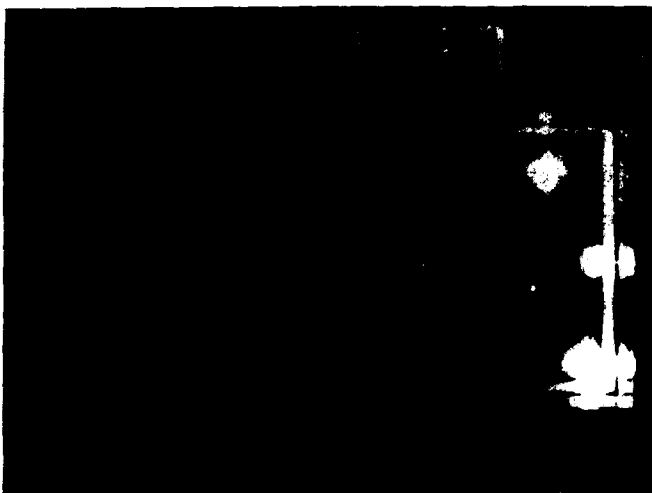
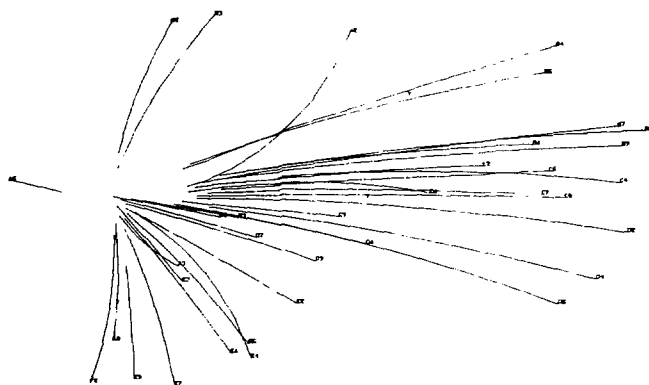


Fig. 1. Streamer chamber view of a central $^{40}\text{Ar} + \text{KCl}$ interaction at 1.8 GeV/A.



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Fig. 2. The same event as fig. 1, after measurement.

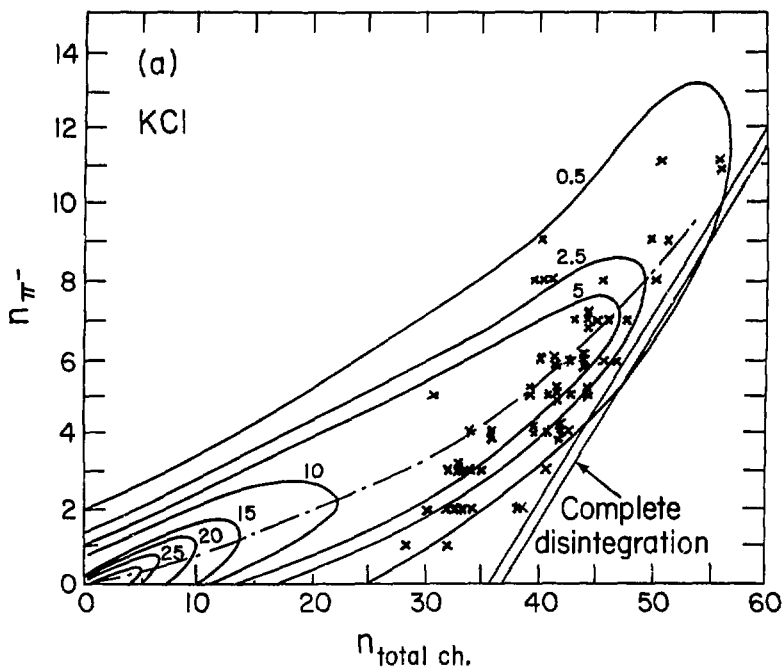


Fig. 3. Correlation between negative pion and total charged particle multiplicity in Ar + KCl at 1.8 GeV/A. The smooth lines give contours of event frequency. Events that produce a Λ particle are given as crosses.

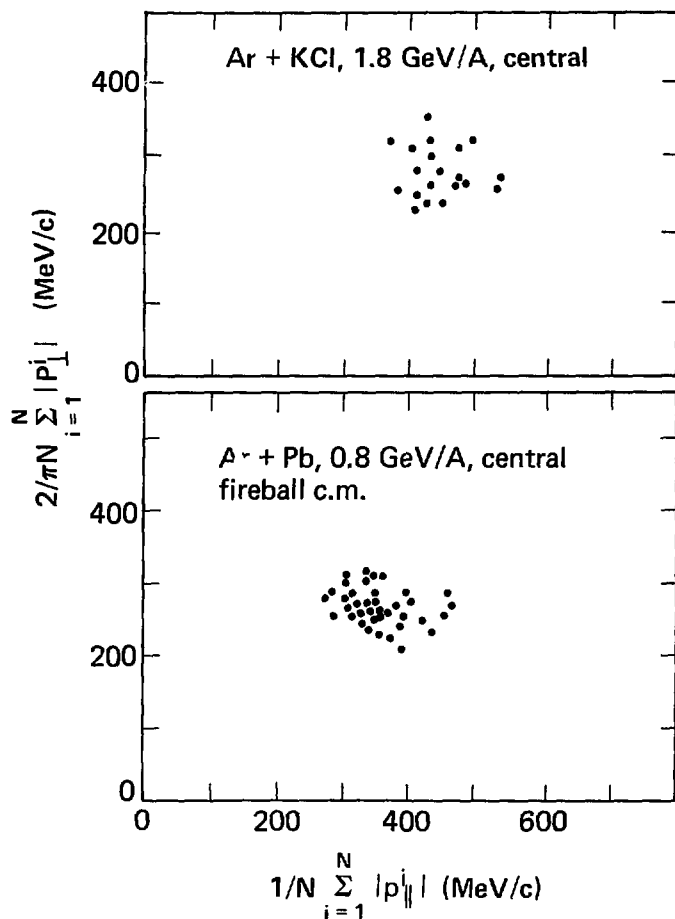


Fig. 4. Preliminary data from an exclusive analysis of Ar + KCl central collisions at 1.8 GeV/A for the event-by-event values of total $\sum_{i=1}^N |p_{\perp}^i|$ and $|p_{\parallel}|$ in the center of mass, as normalized by the multiplicity M of charged particles. The lower part shows Ar + Pb at 0.8 GeV/A with $p_{\parallel}$ referring to the fireball center of mass.

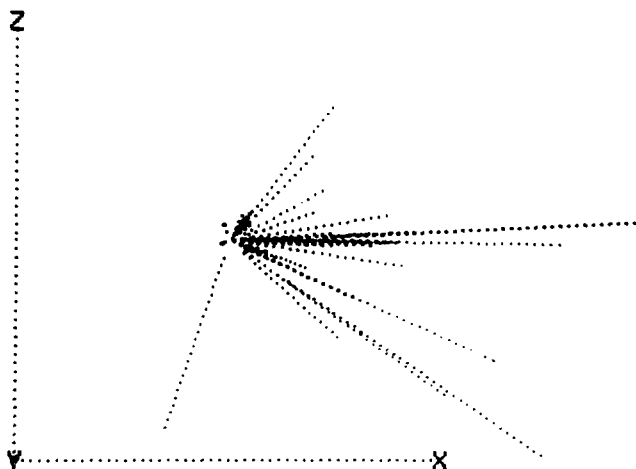


Fig. 5. The laboratory momentum vectors of the event shown in figs. 1 and 2 in a side view. The beam is in x direction.

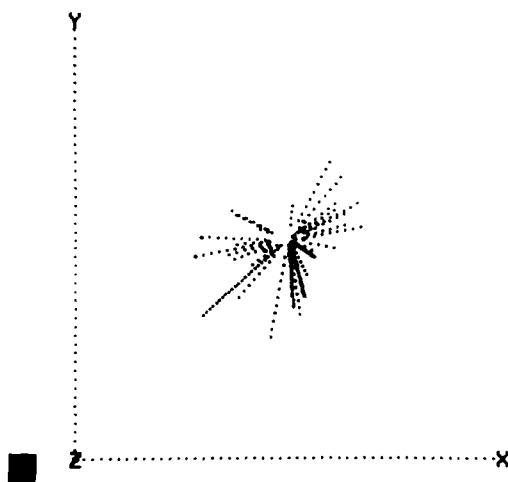


Fig. 6. Top view of the same event, as transformed into the center of mass system.

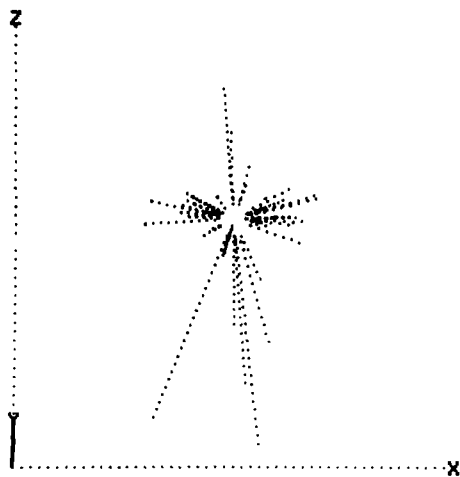


Fig. 7. Side view of the same event in the c.m. system. Four jets are clearly distinguished in this projection.

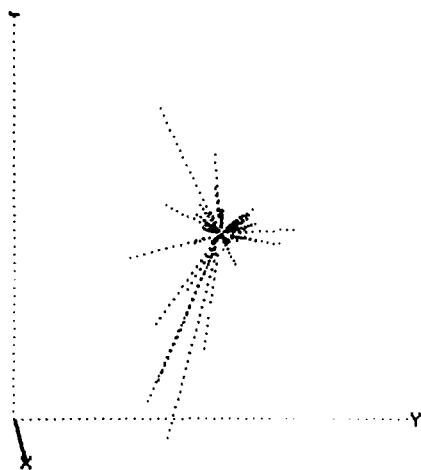


Fig. 8. The same event, as viewed in beam direction.

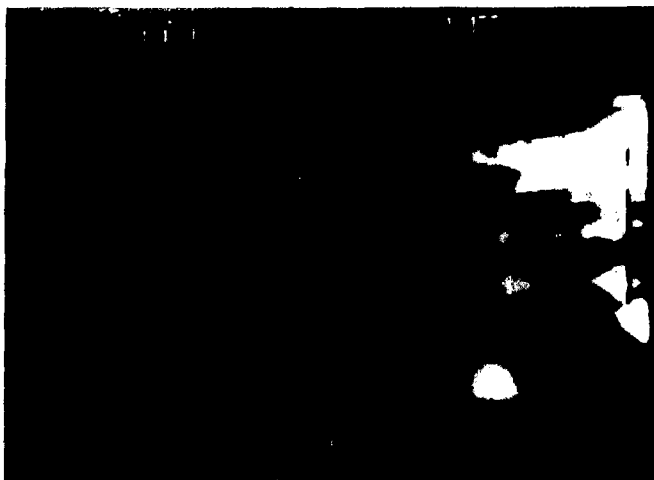


Fig. 9. Central collision of $^{40}\text{Ca} + ^{40}\text{Ca}$ at 2.1 GeV/A, exhibiting the typical forward focus in the lab frame of equal mass central collisions.



Fig. 10. Central collision of $^{40}\text{Ca} + \text{Pb}$ at 2.1 GeV/A. Note the different emission pattern in the target fragmentation region.



Fig. 11. Another example of $^{40}\text{Ca} + \text{Pb}$ at 2.1 GeV/A, showing complete projectile energy degradation.

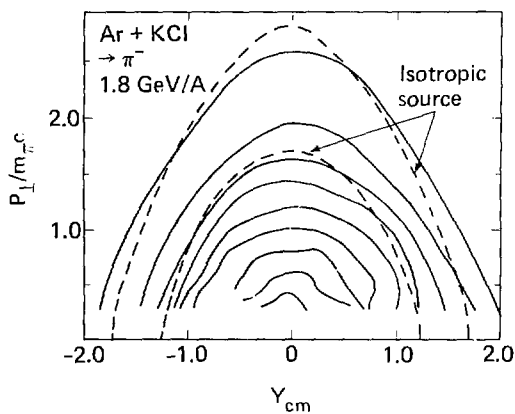


Fig. 12. Contour plot of invariant $\text{Ar} + \text{KCl} \rightarrow \pi^-$ cross section, for central collisions at 1.8 GeV/A. The dashed lines correspond to an isotropical pion source at midrapidity.

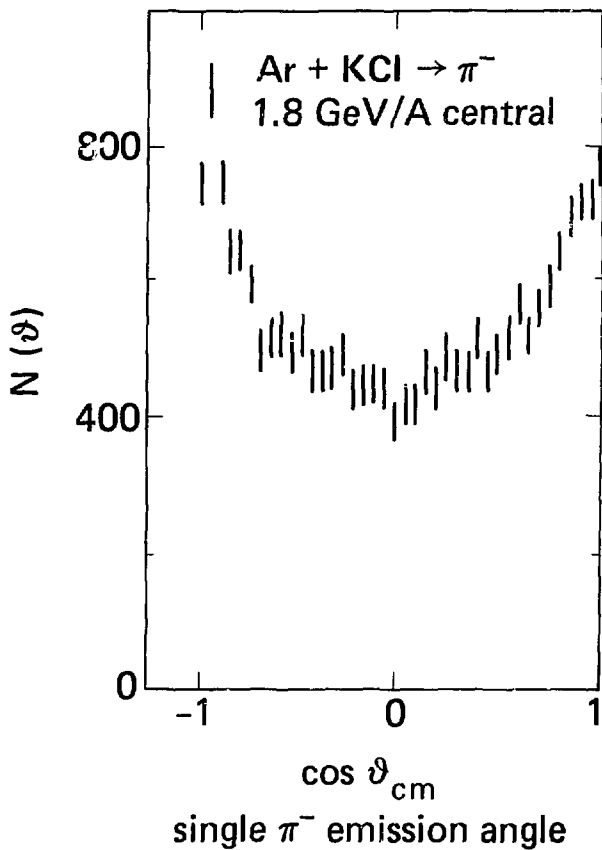


Fig. 13. Distribution of polar angle in the c.m. system of π^- produced in Ar + KCl at 1.8 GeV/A.

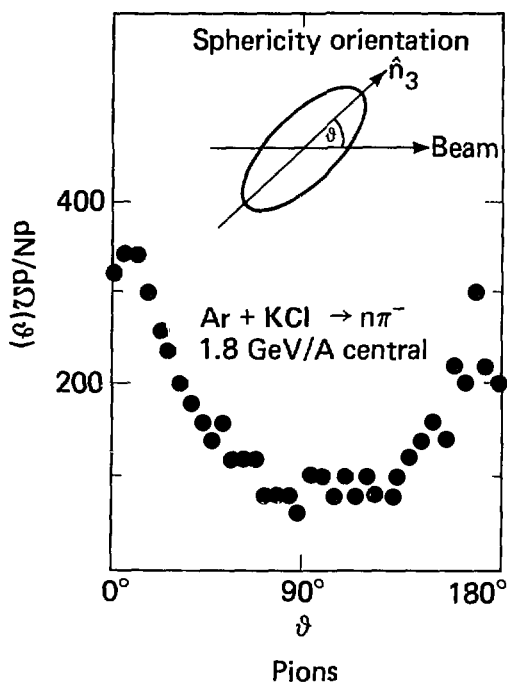


Fig. 14. Distribution of polar angle in the c.m. system of the Q_3 sphericity direction, from an event-by-event analysis of Ar + KCl + $n\pi^-$, $n \geq 4$.

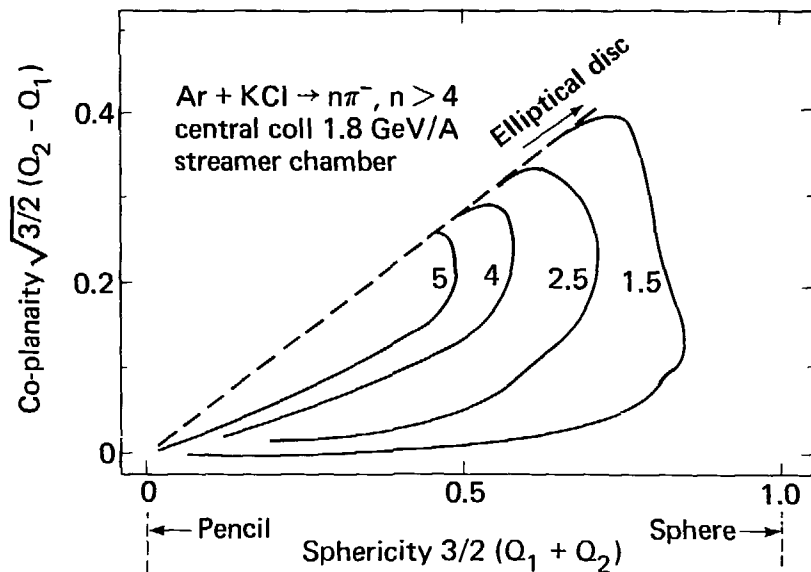


Fig. 15. Event frequency contour lines, in the plane of sphericity vs flatness/coplanarity, from collective analysis of the negative pions in the c.m. system of Ar + KCl at 1.8 GeV/A.

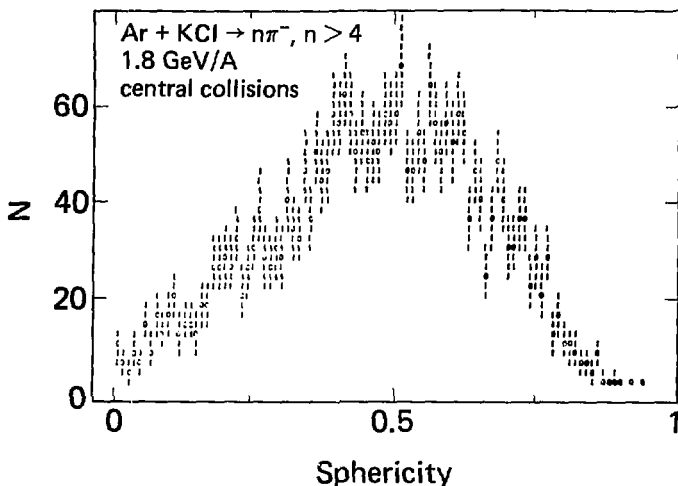
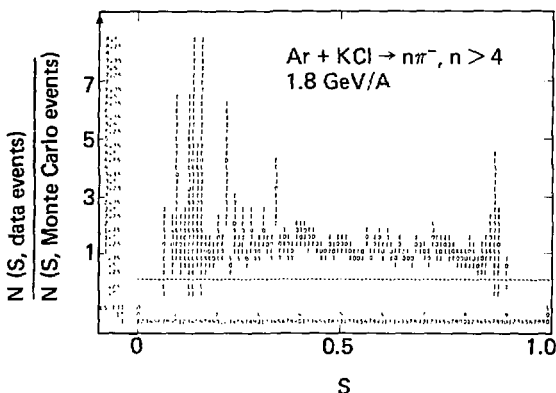


Fig. 16. Projection of fig. 15 onto the sphericity axis.



→ At $\langle n\pi^- \rangle \approx 8$, the sphericity signal does not stand out above Monte Carlo random sampling event sphericities!

Fig. 17. Same plot as fig. 16, now showing the ratio of data events to Monte Carlo events, for each bin in S . The ratio is compatible with one, for all S .

Ar + Ar $\rightarrow$ protons, Cascade model
Knoll, Riedel, Yariv

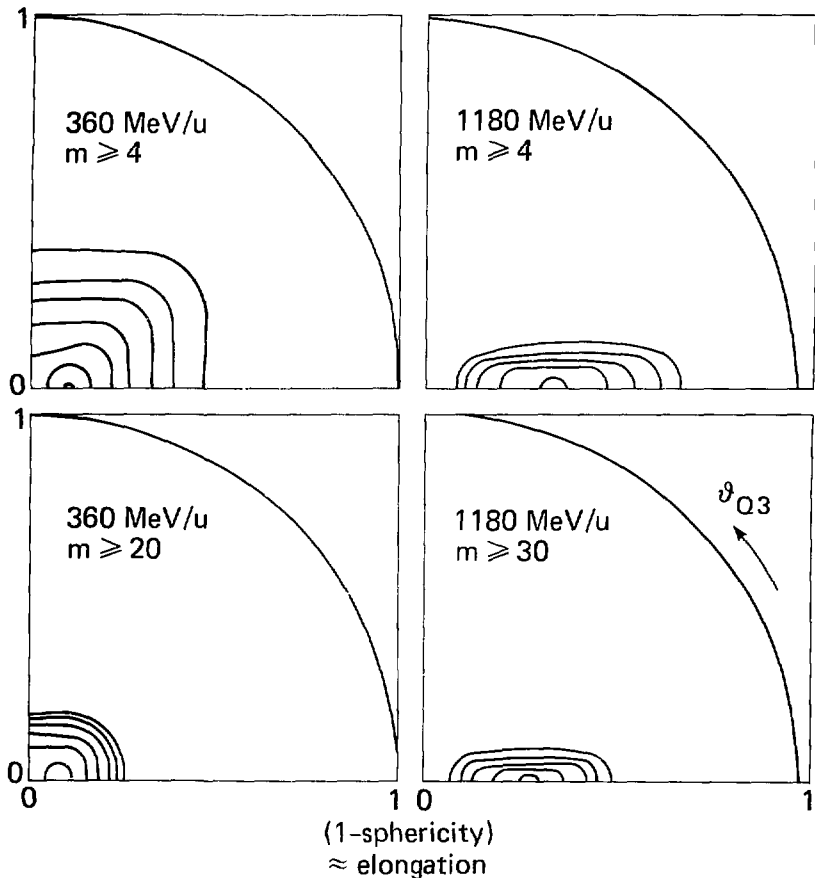


Fig. 18. Plot of event frequency contours, in the plane of maximum elongation = $(1 - S)$ vs polar orientation of $(1 - S)$, resulting from sphericity analysis of the participant protons in a cascade model calculation of $^{38}\text{Ar} + ^{38}\text{Ar}$ at 360 and 1180 MeV/A. Multiplicity cuts corresponding to a minimum bias trigger ($m \geq 4$) and to a central collision selection are shown.

$A + A$

182 MeV/A

CM

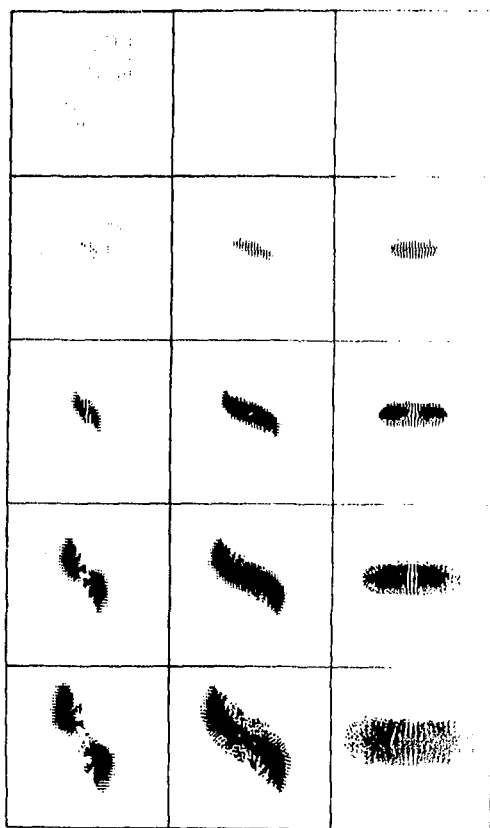
 $b = 0.8 b_{max}$ $b = 0.4 b_{max}$ $b = 0$ 

Fig. 19. Configuration space plot of the density development with time, from a hydrodynamic calculation of equal mass nuclei colliding at three different impact parameters.

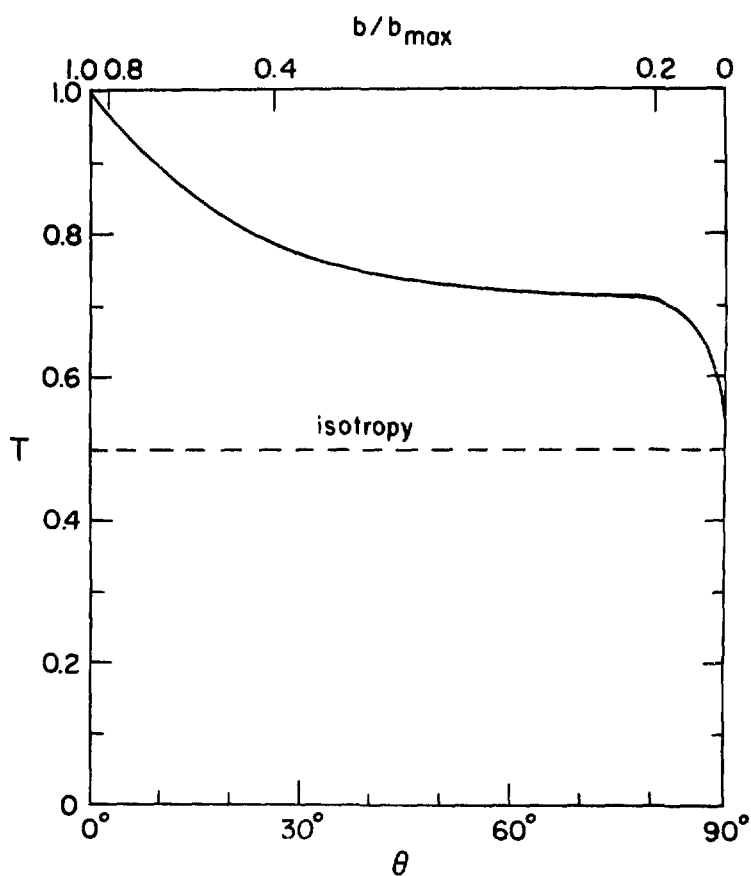
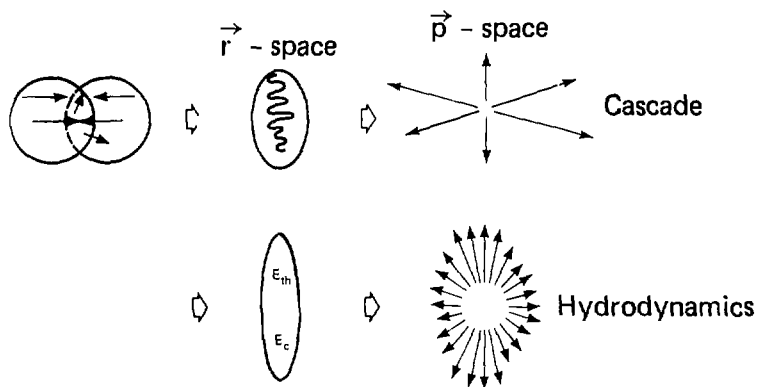


Fig. 20. Results of nucleon thrust analysis of the hydrodynamical calculation, shown in fig. 19, exhibiting T as a function of $b/b_{\max}$ and bounce-off angle relative to the beam axis.

Symmetric collisions in CM



XBL 81B-1260

Cascade: Not scale and E invariant

→ sphericity increases with A
decreases with E
thrust in beam direction

Hydrodynamics: A invariant

sphericity increases with E?
thrust $\perp$ to beam

Nature ?

Fig. 21. Sketch of the global features of a central A + A collision, as predicted by hydrodynamical and cascade models.

Ar + KCl $\rightarrow$ charged particles, 1.8 GeV/A

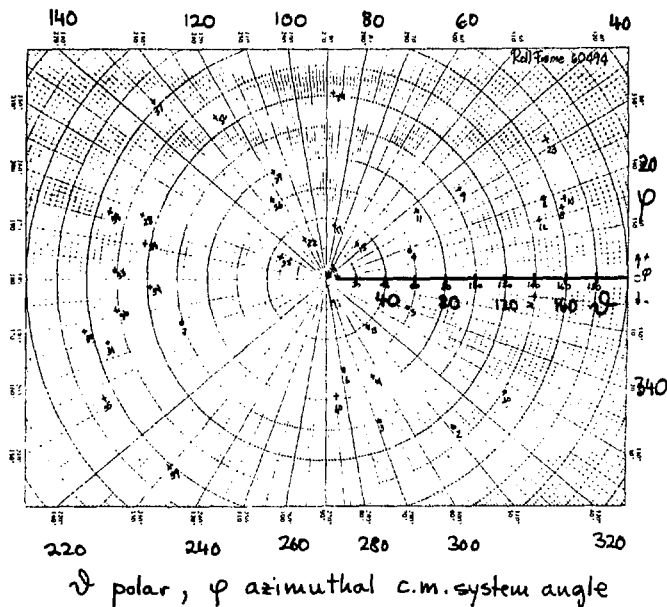


Fig. 22. Plot of polar vs azimuthal angle in the c.m. system of charged particles in a central Ar + KCl event at 1.8 GeV/A.

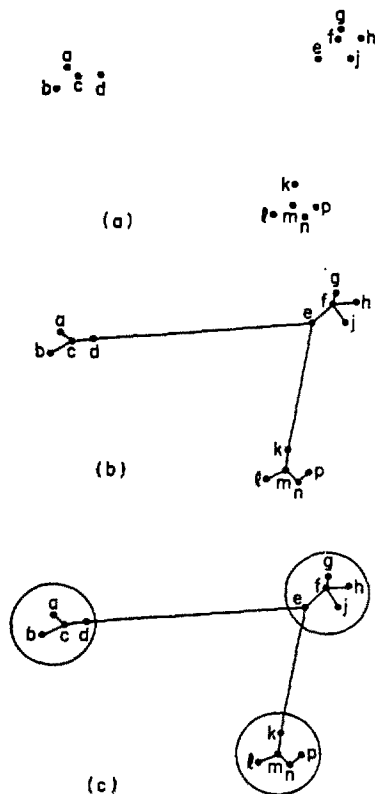


Fig. 23. Sketch of an application of the minimum spanning tree analysis to three groups of particles, clustering in a 2-dimensional representation.

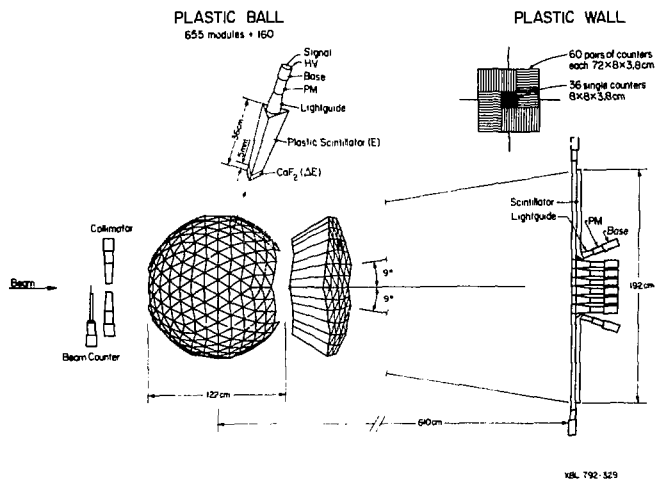


Fig. 24. Sketch of the plastic ball, plastic wall 4π -detector system.

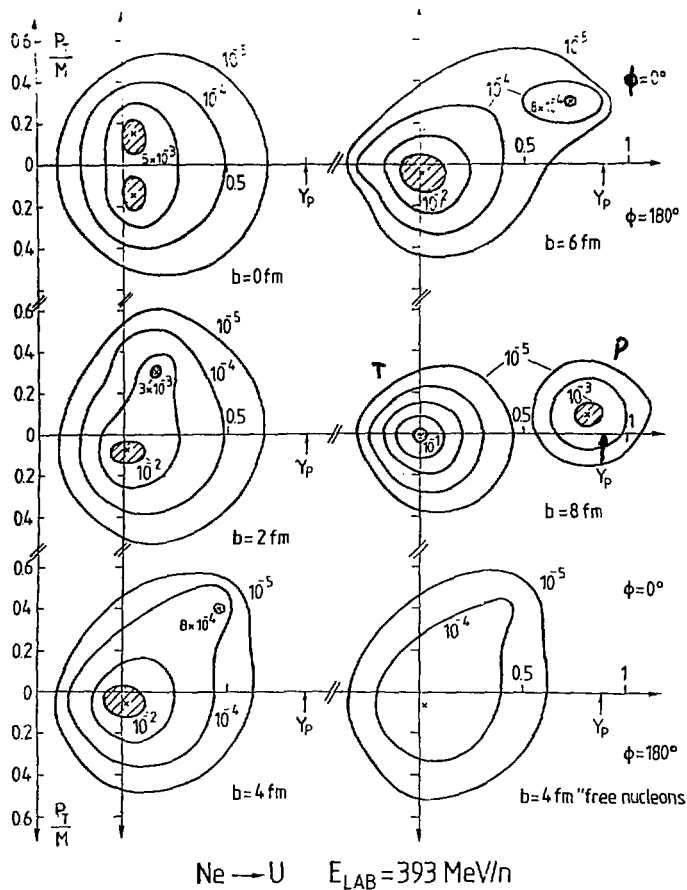


Fig. 25. Hydrodynamical model: contours of invariant nucleon + cluster cross section, projected onto the impact parameter plane, for $b = 1, 2, 4, 6, 8 \text{ fm}$, in Ne + U collisions at 393 MeV/A .



Fig. 26. Streamer chamber view of an event $^{40}\text{Ca} + \text{Pb}$ at 2.1 GeV/A, which exhibits the qualitative features of a bounce-off reaction.

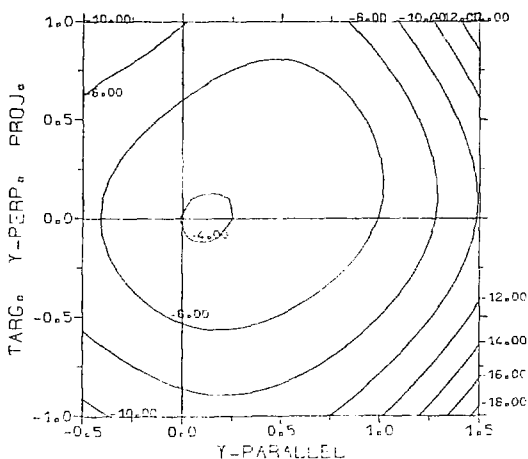


Fig. 27. Hydrodynamical model: contour plot for impact parameter $b = 5$ fm in $\text{Ne} + \text{U}$, showing the part of matter flux emerging in free protons.

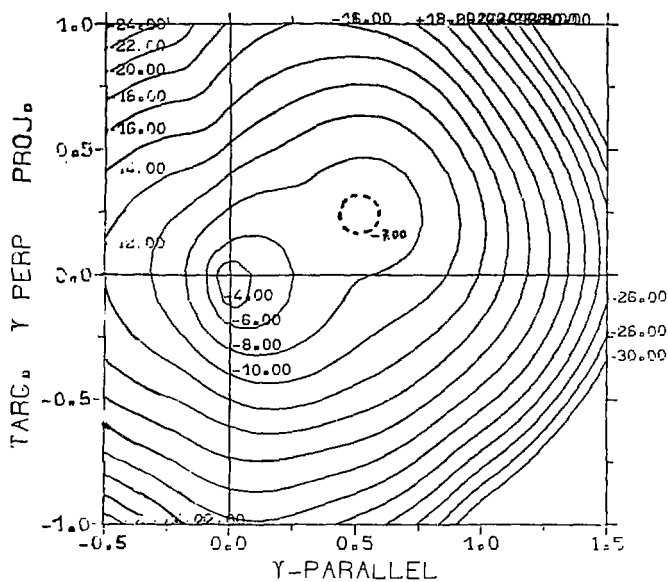


Fig. 28. The same plot, for α -particle emission, in chemical equilibrium.

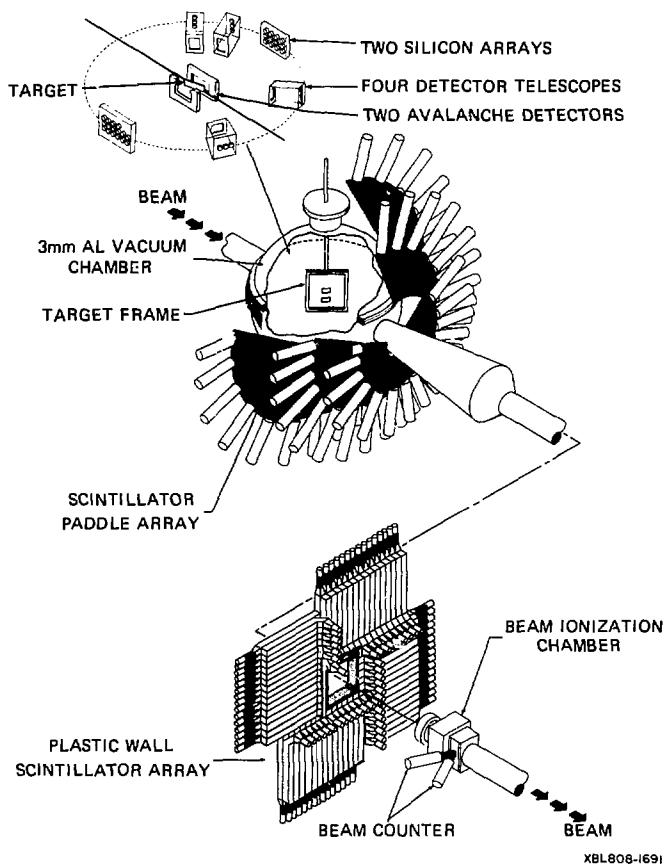


Fig. 29. Sketch of an experiment designed to identify the bounce-off effect predicted by the hydrodynamical model. The three main components are shown: a set of detectors for slow, heavy target fragments inside a vacuum chamber; surrounding it is a multiplicity array for fast fireball fragments; farther downstream, the segmented plastic wall registers projectile remnants at $\theta \lesssim 9^\circ$.

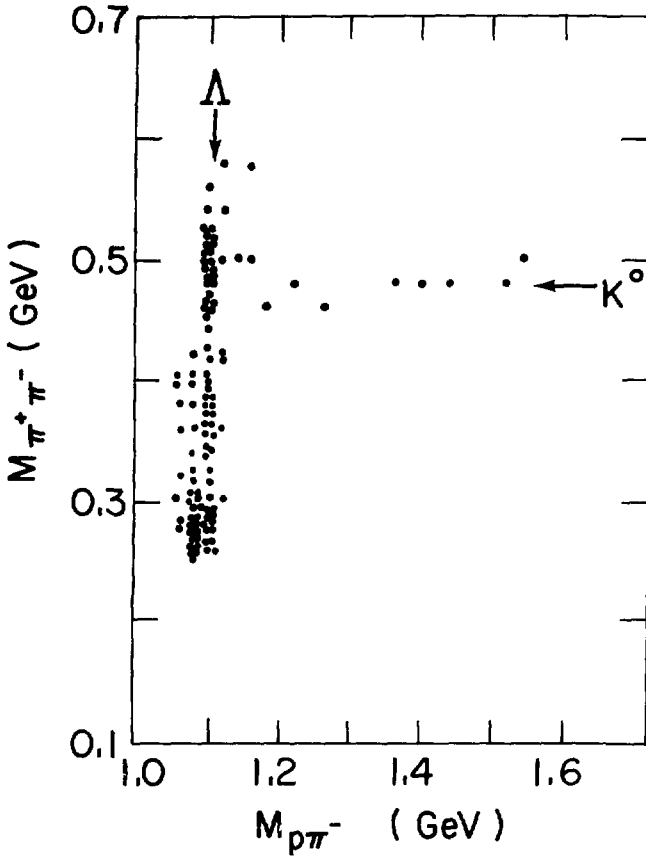


Fig. 30. Invariant mass plot constructed from "Vee" events in Ar + KCl central collisions at 1.8 GeV/A. The branches due to $\Lambda \rightarrow p\pi^-$ and $K^0 \rightarrow \pi^+\pi^-$ are exhibited.

TOTAL EVENT STRUCTURE: 425 MeV/N Ne on NaF

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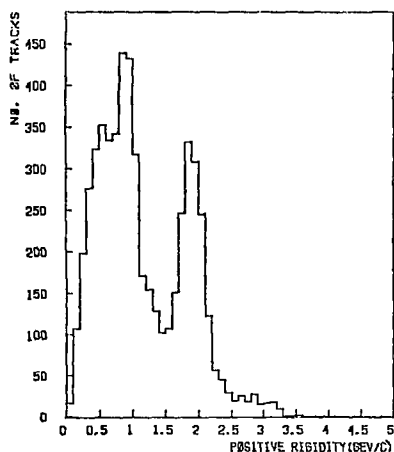
ABSTRACT

A preliminary study of the first exclusive data of total charged secondaries from Ne incident on a NaF target at 425 MeV/N is reported. A multi-particle final state analysis is discussed.

In relativistic heavy ion collisions, inclusive data on the production of pions, protons, and light fragments have been extensively pursued.¹ Single particle spectra and two particle correlations² have been systematically analysed. However, more stringent tests of the reaction mechanism are necessary and require exclusive event analysis. This report presents the first analysis based on the measurement of all charged secondaries emerging from high energy heavy ion collisions.

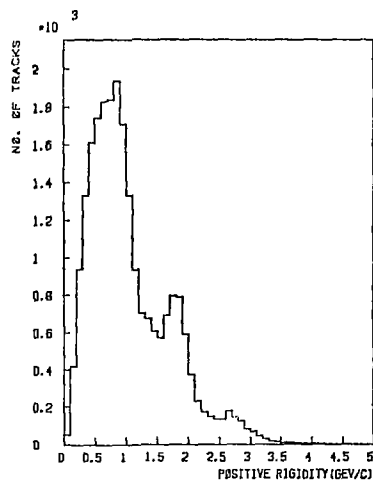
The present experiment utilized a 425 MeV/N Ne beam at the Bevalac incident on a NaF target. The 1.25 meter LBL streamer chamber was employed in two basic trigger modes, inelastic mode and central collision mode.³ The former trigger mode accepts all interactions but the most peripheral collisions, and provides virtually an unbiased sample of inelastic data. The latter selects only the most central collisions and favors high multiplicity events. Two subsamples of the events were chosen such that all the tracks satisfied the reconstruction criteria. They contain 718 events from the inelastic trigger mode and 1893 events from the central collision trigger mode. The inelastic events have an average charged multiplicity per event of 7.7 ± 0.3 and the average pion multiplicity of 0.06 ± 0.01 . The central collision sample has a average charged multiplicity per event of 12.3 ± 0.3 and an average pion multiplicity of 0.13 ± 0.01 . The maximum charge multiplicity for this data sample is 21.

The results of measurements are expressed in terms of rigidity, defined as the momentum per charge. Rigidity is a convenient variable when explicit particle identification is not available, as in this experiment. The rigidity distributions of the positive charged secondaries for the inelastic and central collision trigger modes are shown in Fig. 1(a) and 1(b) respectively. The most distinctive features are the enhancements near 900, 1800, and 2700 MeV/c per charge, corresponding to particles having a speed equal to the



INELASTIC TRIGGER

Figure 1(a)



CENTRAL TRIGGER

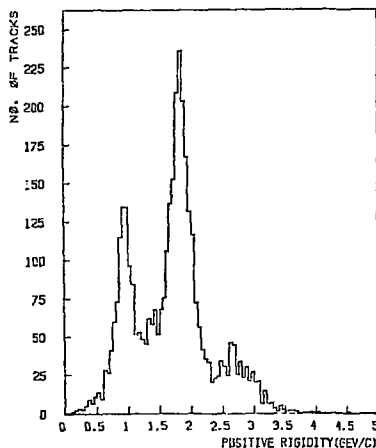
Figure 1(b)

projectile speed, with a mass to charge ratio of one, two and three respectively. The broad distribution below 900 MeV/c per charge is due mainly to the interaction participants. The identification of these enhancements as projectile fragments can be further demonstrated by selecting secondaries within the forward cone. In Fig. 2 the rigidity distribution for the central collision data is presented with only the positive charged secondaries that satisfy $\theta_{\text{LAB}} \leq 50^\circ$. The dominance of light projectile fragments over interaction participants is clearly evident. This dominance persists in both the inelastic and central collision data until θ_{LAB} is greater than 100° .

The relative production of light projectile fragments to protons can be studied. If we identify the particles in the rigidity peaks at 1800 and 2700 MeV/N per charge as deuterons and tritons, respectively, and all other positive charged secondaries as protons, we can obtain an estimate of N_d/N_p and N_t/N_p . These ratios are presented in Table I for various forward angular regions. The identification of all particles with low rigidity (≤ 750 MeV/N per charge) as protons overestimates the number of protons and therefore these ratios should be considered as lower limits. In Table II these ratios are presented for several multiplicity ranges. Both Tables I and II demonstrate the expected dependence of the production of light fragments on angle and multiplicity. This production rises rapidly in the extreme

forward angular region, between 0° and 5° ; and becomes insignificant in events with large multiplicity. Precise determination of these ratios will require more detailed analysis and possible identification of light fragments in the lower rigidity region.

There is no definitive approach for the study of total event structure when a large number of secondaries is involved. One approach is to parameterize the events using the jet variables employed in high energy physics. We have already demonstrated that the thrust variable⁴, T , can be useful in studying the negative pion momentum flow in heavy ion collisions⁵. We have applied the thrust analyses to both the inelastic and central collisions Ne-NaF events. Thrust is a useful tool to compare the model predictions by J. Cugnon and S. E. Koonin⁶, and by H. Stöcker.⁷ In our data sample most of the target fragments are absorbed in the target. Therefore, the azimuthal angle of the thrust axis, ϕ_T , should coincide with the reaction plane. Using ϕ_T to define the reaction plane we have found no evidence of enhanced participant production at 90° to the reaction plane. In addition, the $\cos \theta_T$ distribution of the thrust axis can be used to study the possible "bounce off" of projectile fragments. The $\cos \theta_T$ distribution indicates that the "bounce off" effect, if any, can be no larger than the predictions of Cugnon and Koonin.⁶



CENTRAL TRIGGER

Figure 2

In summary, a large number of charge exclusive events has been measured for 425 MeV/N Ne on NaF. Preliminary analysis of particle production, projectile fragmentation, and exclusive final state analysis has been presented. We emphasize that explicit and detailed comparison of present models and this data should provide important information and insight into the reaction process. More definitive results will be forthcoming.

It is a pleasure to acknowledge the continuing efforts of Jim Brannigan and the Bevalac staff. This work was supported by the Nuclear Science Division of the U.S. Department of Energy.

TABLE I

Lab Angle Ratio	$0 \leq \theta < 5^\circ$	$5^\circ \leq \theta < 10^\circ$	$10^\circ \leq \theta < 15^\circ$
d/p inelastic	2.18 ± 0.20	0.42 ± 0.06	0.37 ± 0.06
d/p central	1.20 ± 0.08	0.52 ± 0.04	
t/p central	0.13 ± 0.01	0.08 ± 0.01	

TABLE II

Multiplicity range Ratio	≤ 8	9-12	13-16	$17 \leq$
d/p	0.25 ± 0.02	0.10 ± 0.003	0.04 ± 0.002	0.
t/p	0.02 ± 0.005	0.01 ± 0.001	0.005 ± 0.001	0.

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TWO-PION CORRELATIONS IN HEAVY ION COLLISIONS

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ABSTRACT

The technique of pion interferometry has been applied to study the reaction $1.8 \text{ GeV/A } ^{40}\text{Ar} + \text{KCl} \rightarrow 2\pi^\pm + \text{X}$. Preliminary analysis of 4500 $2\pi^-$ pairs and 5500 $2\pi^+$ pairs give similar radii for the two charges ($3.18 \pm .33 \text{ fm}$ for $2\pi^-$ vs. $3.92 \pm .43 \text{ fm}$ for $2\pi^+$), lifetimes consistent with 1.5 fm/c , and no convincing evidence for coherence of the pion source.

This work was supported by the U.S. Department of Energy under Contract number W-7405-ENG-48.

The nature of the pion source in relativistic heavy ion collisions has been the object of considerable theoretical speculation. Suggested models range from a simple superposition of nucleon-nucleon results to production through coherent states of the pion field¹. Due to averaging over unobserved features of the collision process, many of these models lead to similar predictions for the single-particle inclusive spectra. Thus, one is naturally led to the measurement of two-pion observables for further information on the pion production mechanism.

In particular, pion interferometry has been proposed as a tool for measuring the radius and lifetime of the pion source². This sensitivity to the source size has recently been verified in streamer chamber measurements³. In addition, Gyulassy et al.⁴ have emphasized that the pion-pair correlation function also contains information related to the degree of coherence of the pion field. This report describes preliminary results for the radius, lifetime and second-order coherence of pion pairs from the reaction $1.8 \text{ GeV/A } ^{40}\text{Ar} + \text{KCl} \rightarrow 2\pi^{\pm} + X$.

Approximately 5500 $2\pi^{+}$ pairs and 4500 $2\pi^{-}$ pairs with laboratory momentum between 200 and 1000 MeV/c were analyzed with a magnetic spectrometer system situated at 45° to the beam axis. This momentum regime corresponds to pions emitted slightly backward of 90° in the center-of-mass. The pion momenta were determined by an effective-edge algorithm, which was combined with first-order ray optics to describe the effect of vertical focusing by the field. The measured momenta spectra are in good agreement with the predictions of a Monte-Carlo program, which has also been used to calculate the momentum resolution. The Monte-Carlo results indicate that the momentum resolution is $\leq 4\%$.

for $p \leq 250$ Mev/c and falls rapidly to $\sim 2\%$ for $p \geq 300$ Mev/c.

The measured pion momenta are transformed to the center-of-mass, where the relative momentum and energy of each pair are calculated and then used to determine the correlation function

$$C_2(q, q_0) = N(q, q_0)/B(q, q_0)$$

Here $q = |\vec{p}_2 - \vec{p}_1|$, $q_0 = |E_2 - E_1|$; $N(q, q_0)$ is the number of pions observed with q and q_0 . $B(q, q_0)$, the expected background in the absence of Bose-Einstein correlations, is generated by mixing pion momenta from different events. (See reference 5 for further details.) The resulting correlation function is then fit to the expression

$$C_2(q, q_0) = 1 + \lambda \exp(-\frac{1}{2}q^2 R^2 - \frac{1}{2}q_0^2 T^2)$$

In this expression R and T are the radius and lifetime of the corresponding spatial distribution function $\rho(r, t) \sim \exp(-r^2/R^2 - t^2/T^2)$. The parameter λ is expected to be 1 for a maximally chaotic pion source. If, however, the pion's wavefunctions are not mutually incoherent, substantial deviations from 1 may occur.

The results of performing such fits may be found in Table 1. In most cases, T has been fixed to 1.5 fm/c (the same value assumed in the analysis of reference 3). This is necessary in most cases since the small number of events, combined with the strong correlation between q and q_0 , often does not permit separate extraction of R and T . In the first fit, λ is also fixed to the value of one, and no Coulomb correc-

tions have been applied to the data.

This data and the corresponding fit are (summed over q_0) shown in Figure 1a. The summation over q_0 defines a spectrometer-averaged correlation function

$$\langle C_2(q) \rangle = \frac{\sum_{q_0} N(q, q_0)}{\sum_{q_0} B(q, q_0)} .$$

While no longer a true correlation function, this form is chosen to provide a convenient one-dimensional presentation of the data and the fitted function.

The small value of R implied by the raw data ($R = 1.84 \pm .27$ fm) results from the suppression of the like-charge correlation by the Coulomb interaction between the two pions. An event-by-event correction for this effect (Fig. 1b) leads to $R = 2.98 \pm .30$ fm, again with $\lambda = 1$. R is largely unchanged when λ is left as a free parameter (Fig. 1c): $R = 3.18 \pm .33$ fm, with $\lambda = .86 \pm .07$. The R value obtained from the corresponding $2\pi^+$ fit (Fig. 1d), $3.92 \pm .43$ fm, is somewhat larger, while $\lambda = .78 \pm .07$ is smaller.

It would be premature to conclude that these values of λ indicate appreciable source coherence. Aside from the question of statistical significance, λ is also affected by factors such as the event geometry, the construction of the background, spatial correlations within the source, etc.

Finally, allowing T to vary in the case of the $2\pi^+$, we obtain

$T = 1.34^{+1.31}_{-1.34}$ fm/c, consistent with our fixed value of 1.5 fm/c. At present, the $2\pi^-$ data set does not allow separate extraction of R and T. However, analysis of added data should produce similar results for these events.

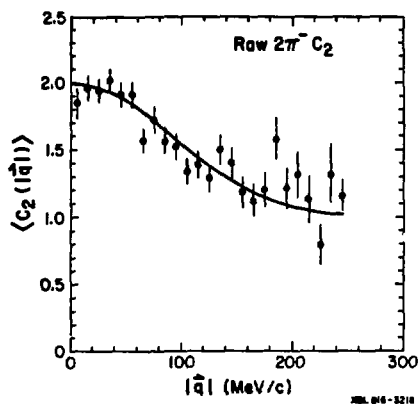
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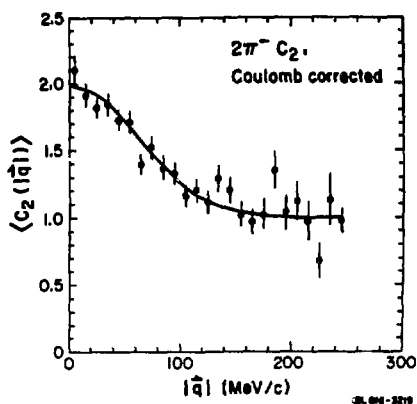
Table 1. Results from fits to the function
 $C_2(q, q_0) = 1 + \lambda \exp(-q^2 R^2/2 - q_0^2 T^2/2)$

Description	λ	R(fm)	χ^2/NDF
$2\pi^-$, raw	$\equiv 1$	$1.84 \pm .27$	114/113
$2\pi^-$, Coulomb corrected	$\equiv 1$	$2.98 \pm .30$	120/113
$2\pi^-$, "	$.86 \pm .07$	$3.12 \pm .33$	116/113
$2\pi^+$, "	$.78 \pm .07$	$3.92 \pm .43$	89/109

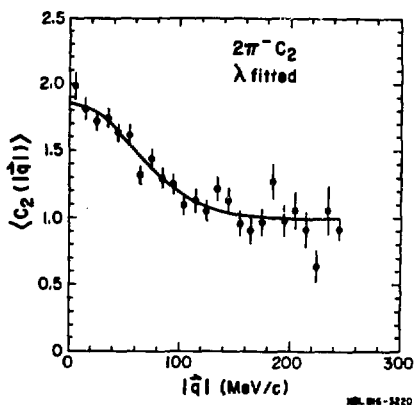
FIGURE 1.



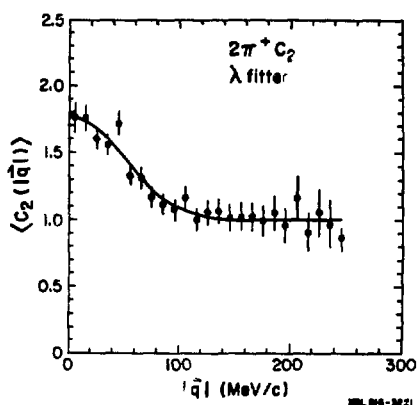
a.)



b.)



c.)



d.)

PROTON SOURCES IN CENTRAL COLLISIONS OF Ar + KCl AT 1.8 GeV/n

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Abstract

We have measured the single proton inclusive cross-sections, the two-proton correlation function and the associated multiplicity of high P fragments in collisions of Ar + KCl at 1.8 GeV/n. The average separation $\langle \Delta r \rangle$ between the protons is obtained from the correlation function of the proton pair at the beam rapidity and at $Y_B/2$. After selecting high multiplicity events, $\langle \Delta r \rangle$ increases for protons at Y_B and decreases for protons at $Y_B/2$. This indicates that the beam rapidity protons and the mid-rapidity protons come from two different sources.

Relativistic heavy ion collisions offer the exciting possibility of studying nuclear matter under conditions of high densities and temperatures. It has been suggested ⁽¹⁾ that the density will increase with bombarding energy because of the center of mass Lorentz contraction of the projectile and target along the beam dimension. This achievement of high density would be trivial if the projectile and target simply passed through each other without interacting. On the other hand, if there are a sufficient number of hadronic interactions during the interpretation

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phase to quasi-equilibrate at least some of the nuclear material, certain interesting phenomena such as phase transitions to quark matter² could occur. The signature for such transitions has not yet been unambiguously developed but it seems necessary to determine if a thermalized source is created in the projectile target center of mass.

The overestimate, by the fireball³ and diffuse firestreak⁴ models of the mid-rapidity single particle inclusive cross-sections^{5,6} suggests that not all if any of the overlapping projectile and target nucleons are thermalized for collisions of Ar + Cu at 1.8 GeV/n and Ne + U at 2.1 GeV/n.

We have attempted to study the space-time properties of the source which may be created in the center of mass by measuring the correlation between two protons emitted at small relative momenta from collisions of Ar + KCl $\rightarrow$ P + P + X at 1.8 GeV/n. Koonin⁷ has shown that the final state p-p interactions make the correlation function quite sensitive to the average separation between these two protons at the time they are emitted from a thermalized source. We have recently reported⁸ correlation function measurements for protons traveling at the beam rapidity Y_B and at $Y_B/2$. The source size we calculate from our data for protons at $Y_B/2$ with high associated multiplicities was significantly smaller than the size we obtain for protons at Y_B . This smaller size of ~ 2.2 fm seems too small to be consistent with the assumption that these protons come from a thermal source containing a large fraction of the projectile and target nucleons at normal or less than normal nuclear densities. We report here some additional results to further clarify the origin of these mid-rapidity protons.

In Figure 1, we show the acceptance of our apparatus in $P_{\perp}$ -rapidity space. The two protons were detected in a magnetic spectrometer that was adjusted to detect protons either at the beam rapidity Y_B (S1) or at $Y_B/2$ (S2). The rapidity $Y_B/2$ corresponds to that of the Ar-KCl center of mass. Seven multiplicity counters were at polar angles of 30° and nine were at 50° . The regions labeled M 30° and M 50° show their acceptances.

We show in Table 1 the invariant single particle inclusive cross-sections (in arbitrary units for settings S1 and S2. The column labeled $M \geq 0$ shows the invariant cross section with no multiplicity requirements. The column labeled $M > 4$ shows the cross-section when 5 or more particles are detected by the multiplicity counters. The quantity (C) defined as the ratio of (f) for $M > 4$ to f for $M \geq 0$ is larger at $Y_B/2$ than at Y_B . The larger value of (C) at $Y_B/2$ indicates that this multiplicity requirement is selecting more central collisions since this selection tends to decrease the emission of particles at Y_B more than at $Y_B/2$. On the other hand, (f) is larger for the data at Y_B than at $Y_B/2$ even when the high multiplicity requirement is imposed. This implies that even for smaller impact parameter central collisions, most of the nucleons are not thermalized in the Ar-KCl center of mass.

We now turn to the results for the correlation function R . Details about its definition and experimental determination are given in (8). We show in Figure 2 R as a function of the relative momentum $\Delta P = |P_1 - P_2|/2$ without any multiplicity requirement. Koonin has shown that the peak in R at $\Delta P \sim 20$ MeV/c varies inversely with the average separation $\langle \Delta r \rangle$ between the two protons at the time of their emission.

The peak is largest for data at $Y_B/2$ and smallest for data at Y_B . This indicates that protons with rapidities near that of the Ar-KCl center of mass have a smaller $\langle \Delta r \rangle$, while protons at the beam rapidity have a larger $\langle \Delta r \rangle$.

There may be difficulties in identifying $\langle \Delta r \rangle$ with the source size. The $\langle \Delta r \rangle$ we extract is a measure of the separation between the two protons at their time of emission, presumably at the time of their independent final scatterings from other hadrons. The two protons then emerge from the source both traveling with the velocity $\bar{V}$, the velocity of the p-p center of mass in the source rest frame ($\Delta P = 0$). If this emission consisted of small momentum transfer scatterings then the separation $\Delta \vec{r}$ we measure along $\bar{V}$ is limited to λ , the mean free path at the time of emission. On the other hand, the component of $\Delta \vec{r}$ along $\bar{V}$ will be increased if the two protons are emitted at separate times. We may also underestimate the source size if for some dynamical reason the requirement of small ΔP causes $\langle \Delta r \rangle$ to be small. Because of these complications, we feel it is necessary to study how $\langle \Delta r \rangle$ changes for different kinematical regions and under different trigger conditions.

To study how $\langle \Delta r \rangle$ changes for smaller impact parameter collisions, we obtain the correlation function for high multiplicity events. The results in Figure 3 show R for data at Y_B and $Y_B/2$ when $M \geq 4$. Imposing the multiplicity requirement of $M \geq 4$ increases the peak value of R for data at $Y_B/2$ while this same requirement decreases this peak for data at Y_B . The selection of smaller impact parameter collisions increases $\langle \Delta r \rangle$ for protons with velocities close to the beam rapidity and decreases $\langle \Delta r \rangle$ for protons with velocities near that of the Ar-KCl center of mass.

These results are not consistent with the fireball model in which all of the participants (over-lapping projectile and target nucleons) are slowed down to thermal energies in the Ar-KCl center of mass, while the projectile and target spectator nucleons emerge from the collision with velocities near those of the projectile and target, respectively. According to this model, imposing the $M \geq 4$ cut should increase $\langle \Delta r \rangle$ at $Y_B/2$ while decreasing it at Y_B . The increase in $\langle \Delta r \rangle$ for data at Y_B and the larger value of $\langle f \rangle$ for Y_B with $M \geq 4$ are good evidence that many of the participants are not equilibrated in the Ar-KCl center of mass.

The $Y_B/2$ data do, however, indicate that a significant number of interactions occur while the projectile and target are interpenetrating. The small $\langle \Delta r \rangle$ with $M > 4$ excludes the possibility that these mid-rapidity protons are emitted from projectile and target remnants after the colliding nuclei have passed through each other. If such a process were significant we would measure a large separation along the beam direction (a large $\langle \Delta r \rangle$) since, for equal mass colliding nuclei, any given $Y_B/2$ proton could come with equal probability from either the projectile remnant or the target remnant.

The small $\langle \Delta r \rangle$ is, however, consistent with two other possibilities. These $Y_B/2$ protons may come from an isolated low-density source that contains only a fraction of the participants. The smaller $\langle \Delta r \rangle$ for $M > 4$ could be consistent with this hypothesis because the present multiplicity counters do not require a large number of particles at $Y_B/2$ with low $P_{\perp}$. The other possibility is that these mid-rapidity protons are emitted at early stages of the collision while the projectile and

target are still interpenetrating. We measure a small $\langle \Delta r \rangle$ because of contraction along the beam direction and high density (small λ) effects along $\bar{V}$ which is perpendicular to the beam direction.

At the present time we cannot reliably estimate the density because of uncertainties of the source size perpendicular to the beam direction and of the multiplicity of nucleons near $Y_B/2$. We plan to measure R at 0^0 ($\bar{V}$ parallel to the beam direction) and the multiplicity of protons near $Y_B/2$ with the new large acceptance HISS facility. We plan to also measure R for different projectile-target combinations as the heavier ion beams become available at the Bevalac. With heavier projectiles it should be easier to thermalize a larger fraction of the nucleons in the nucleus-nucleus center of mass. It will be particularly interesting to study $\langle \Delta r \rangle$ and the density at the time of emission for such heavy systems since it will be easier to compare these measured quantities with theoretical predictions, including those of quark matter, if a higher degree of equilibration is achieved.

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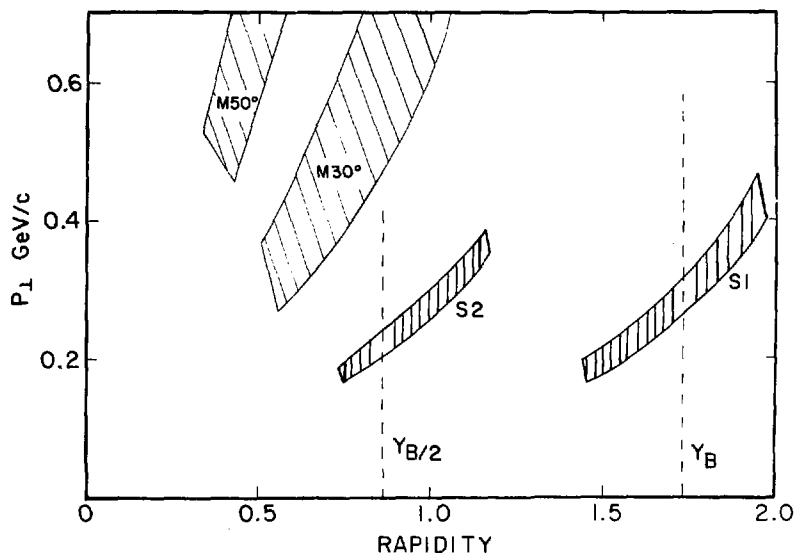


Figure 1. Acceptance of apparatus in $P_{\perp}$ - rapidity space.

Table 1. Invariant cross-sections $E/p^2 d\sigma/dPd\Omega$ (arbitrary units) for different multiplicity requirements M . Figure 1a

SETTING	$P_{\perp}$ (LAB)	(LAB)	$M \geq 0$	$M \geq 4$	$C = \frac{M \geq 4}{M \geq 0}$
S1, (Y_B)	2.4	6°	548	37.8	0.069
S2, ($Y_B/2$)	1.0	13.5°	100	9.4	0.096

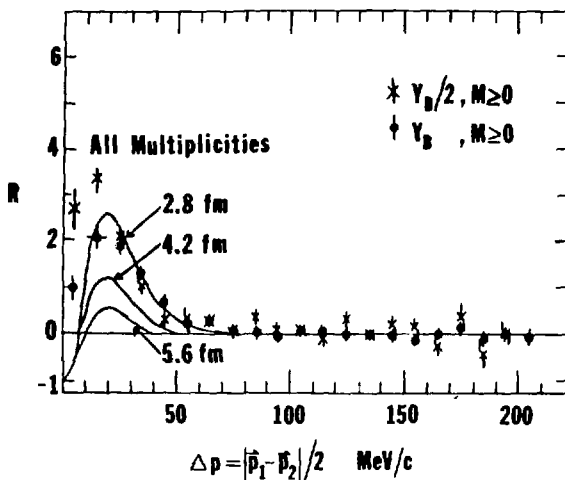


Figure 2. The correlation function, R , for $\text{Ar} + \text{KCl} \rightarrow p + p + X$ at 1.8 GeV/n (all multiplicities).

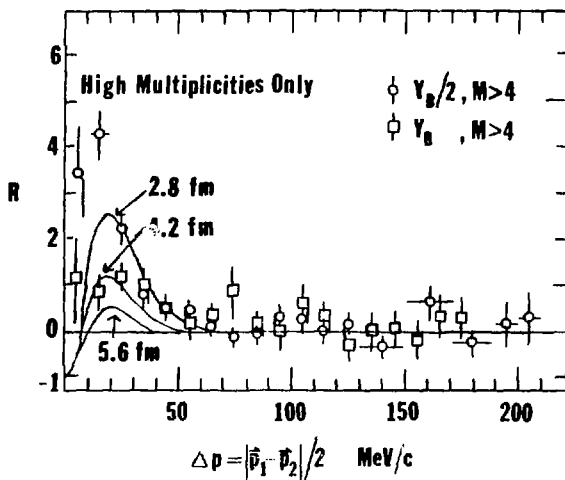


Figure 3. The correlation function, R , for $\text{Ar} + \text{KCl} \rightarrow p + p + X$ at 1.8 GeV/n (high multiplicities only).

Aug

Measurement of Two-particle Correlations in 800 MeV pA Collisions*

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ABSTRACT

Two-particle (p-p, p-d) coincidence has been measured in $p + A$ collisions. Production mechanism of backscattered proton has been studied by forward-backward coincidence events. Data show the importance of 2N-cluster scattering and/or multiple scattering processes involving a few nucleons. They also show that the nucleon-nucleon quasi-elastic-scattering is not the dominant part of the backscattered proton production.

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Two-particle (p-p, p-d) momentum and angular correlations for 800 MeV protons on nuclear targets (C, NaF, KCl, Ag, and Pb) have been measured to study particle production mechanisms, especially in the energy region beyond free nucleon-nucleon kinematics. A magnetic spectrometer ($400 \leq p \leq 2000$ MeV/c, $15^\circ \leq \theta \leq 100^\circ$, $\phi = 0^\circ$), four sets of time-of-flight counter telescopes ($0.2 \leq \beta \leq 1$, $15^\circ \leq \theta \leq 90^\circ$, $\phi = 90^\circ$ and 180°), and three sets of ΔE -E counter telescopes ($300 \leq p \leq 600$ MeV/c, $100^\circ \leq \theta \leq 160^\circ$, $\phi = 90^\circ$ and 180°) were used to detect two particles emitted in the same reaction plane ($\Delta\phi = 180^\circ$) as well as two particles emitted out-of-plane ($\Delta\phi = 90^\circ$).

We report here only the data with one particle (proton, p, or deuteron, d) forward, $\theta_1 = 15^\circ$ and the other (p) backward, $\theta_2 = 118^\circ$.

Since no proton is backscattered in free nucleon-nucleon collisions, backscattering of proton in p + A collision must be due to nuclear effects. Single-particle inclusive cross-sections of the backscattered proton were measured with incident proton of 600 and 800 MeV¹⁾ and 640 MeV²⁾. The inclusive cross-sections as a function of momentum have exponential tails extending out to hundreds MeV/c. Several models such as the quasi-elastic-scattering (QES)³⁾, cluster scattering⁴⁾, cascade and statistical models⁵⁾, have been applied to explain the inclusive cross-sections with reasonable success. However, the success of these models, which are based on different assumptions, is an indication that the production mechanism of backscattered proton is not uniquely determined by the analysis of the inclusive cross-sections. Two-particle coincidence measurements have the advantage of more direct information about QES, cluster scattering and some of the other processes, and thus we expect such measurement to provide useful constraints on these models.

A two-proton coincidence experiment was reported by Komarov, et al.⁶⁾ suggesting the existence of two-nucleon (2N) cluster scattering process for the backward proton production. However, the importance of these processes as compared to others was not clear, because their measurement covered a rather narrow kinematical region which is biased in favor of 2N-cluster scattering processes.

The present measurement covers a much wider kinematical region which includes QES and 2N-cluster reactions. Since the 2N-cluster

mechanism is closely related to deuteron production in the forward direction, a coincidence spectrum of a forward-going deuteron with a backscattered proton (d-p) was measured as well as the coincidence spectrum of two-protons (p-p). These coincidence spectra are measured in two modes, namely in-plane coincidence (IPC), ($\theta_1 = 15^\circ$, $\theta_2 = 118^\circ$, $\Delta\phi = 180^\circ$) and out-of-plane coincidence (OPC), ($\theta_1 = 15^\circ$, $\theta_2 = 118^\circ$, $\Delta\phi = 90^\circ$).

For all targets, more IPC events were observed than OPC events in both the p-p and d-p coincidences. This observation suggests that the backscattered protons are not emitted from an equilibrated system containing more than a few particles. Also incoherent multiple scattering which involves many particles is not likely to be the origin of the backscattered proton.

Fig. 1 a) and b) show the scatter plot of p-p and d-p IPC events in p + C reactions. Solid curves in the figures show the kinematics of p-p and d-p QES. The kinematic loci of the break-up of two and three nucleon clusters with zero excitation energy are represented by broken lines. The numbers along the curves indicate the initial momenta of the p, d or clusters inside the target nucleus. The IPC events show the clear existence of the p-p and p-d QES which are not seen in OPC. Besides the p-p QES, a broad but definite population is seen in the region of 2N-cluster break-up. In spite of the large excitation (300 ~ 400 MeV) of the remaining system, which includes all the particles other than detected ones, the number of IPC events are still larger than those of OPC. Fig. 2 shows the spectra of the backscattered proton associated with the different components, namely QES, non-QES IPC and OPC. All of the spectra, except d-p QES, are fitted well by a function Ae^{-P/P_0} as in the case of the inclusive spectra. The slope factors for these spectra are tabulated in Table 1. The slope factor, P_0 of the p-p QES and OPC are smaller than that of the inclusive spectra. Because of the fast fall-off of the spectra, these two components cannot contribute significantly to the backscattered high-energy protons. The slope factors P_0 of in-plane, p-p and d-p, non-QES are similar to the P_0 of the inclusive spectra and hence suggest that there is a close relationship between these spectra. The difference spectrum between in-plane p-p non-QES and

out-of-plane p-p is plotted in Fig. 3 as well as the proton spectrum associated with d-p QES. The two spectra are very similar in shape. If we consider OPc events as an isotropic background, a similarity of the spectra in Fig. 3 suggests that the excess in IPC is produced through a mechanism similar to that for deuteron production.

In conclusion, the fast fall-off of the p-p QES component indicates that quasi-elastic-scattering is not the dominant mechanism for producing backscattered high-energy protons. The dominance of the in-plane coincidence together with the steep slope of the spectra associated with the out-of-plane coincidence suggest that thermal or statistical equilibrium is not achieved. The similarity of the backscattered proton spectrum of d-p events and the difference spectrum between in-plane and out-of-plane coincidence events suggests the importance of scattering of the incident proton on correlated 2N-clusters. However, incoherent multiple scattering processes involving a few nucleons cannot be rejected at this stage of analysis. We need further analysis including angular distributions and also better theoretical calculations to distinguish these components.

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- 7) The QES component is defined rather arbitrarily here as the events in which the excitation of residual nuclear system is less than 100 MeV.

TABLE 1

Sl. factors P_0 of backscattered proton spectra
in 800 MeV p + A collisions

		$(P_0 \text{ in MeV/c})$		
Target		C	KCl	Pb
inclusive		88 ± 1	87 ± 2	85 ± 2
in-plane coincidence	p-p QES	56 ± 4	55 ± 3	56 ± 4
	p-p non-QES	85 ± 5	98 ± 5	92 ± 4
	p-d non-QES	-	94 ± 8	84 ± 9
out-of- plane coincidence	p-p all	72 ± 5	71 ± 3	80 ± 4
	p-d all	63 ± 20	73 ± 7	$77 \pm (8)$

The backscattered proton spectra are fitted by a function:

$$\frac{d\sigma}{dpd\Omega} = Ae^{-P/P_0}$$

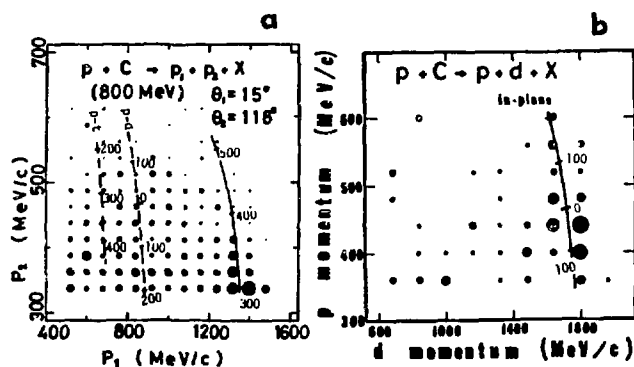


Fig. 1 Momentum-momentum scatter plot of the p-p (a) and p-d (b) events of in-plane-coincidence mode. The area of circle in the figure is roughly proportional to the number of events.

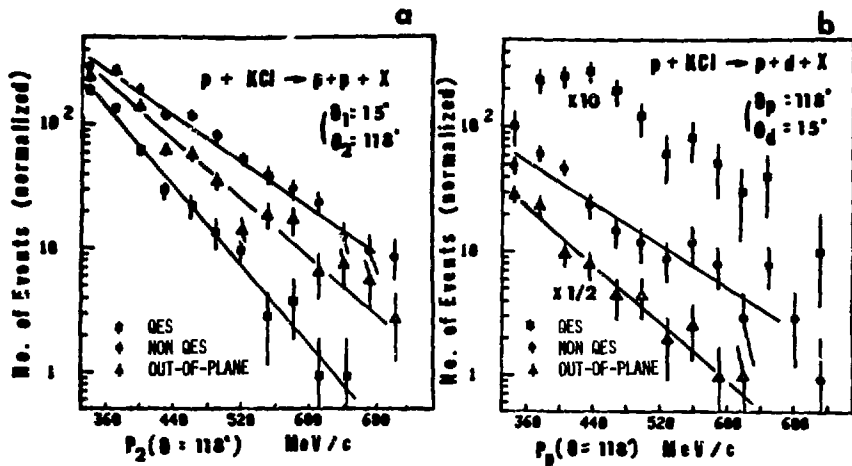


Fig. 2 Spectra of the backscattered protons for different components. Solid lines are the exponential fit to the data.

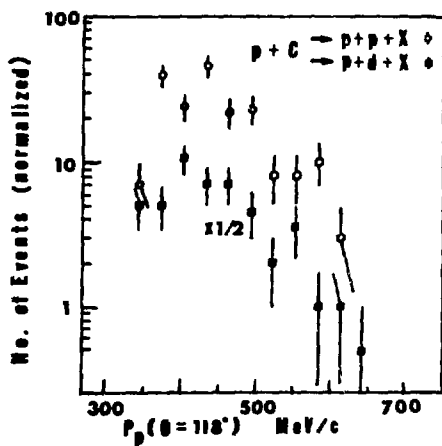


Fig. 3 The backscattered proton spectra associated with d-p quasi-elastic-scattering and the difference between in-plane p-p non-QES and out-of-plane p-p coincidences.

PARTICLE PRODUCTION IN NUCLEUS-NUCLEUS COLLISIONS AT 4.5 GeV/c/A
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Abstract: Recent results, from the Dubna streamer chamber experiment, on relativistic nucleus-nucleus collisions at incident momentum 4.5 GeV/c per nucleon will be reviewed. The multiplicity distributions for collisions of ⁴He, ¹²C, and ¹⁶O with various nuclear targets ranging from ⁶Li to ²⁰⁷Pb were studied for a sample of unbiased inelastic events and for central collisions. Inclusive rapidity (y) and transverse momentum (p_t) distributions and y - p_t -multiplicity correlations will be presented as well as their dependence on degree of "centrality" of nucleus-nucleus collisions. The data will be compared with recent theoretical models concerning soft collisions of nuclei.

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1. INTRODUCTION.

The purpose of this talk is to review recent experimental results on particle production in nucleus-nucleus collisions, coming from the Dubna streamer chamber experiment (SKM-200 Collaboration, Alma-Ata-Dubna-Moscow-Tbilisi-Warsaw)

During last five years the Dubna streamer chamber group has been taking data with different beams (^4He , ^{12}C , ^{16}O) incident on different targets ranging from ^6Li to ^{207}Pb and with different trigger conditions: minimum-bias, semi-central and central triggers. All data concern incident beam momentum of 4.5 GeV/c per incident nucleon. Part of the obtained results is already published [1-5]. Having a rather large amount of experimental data and only a small amount of time I have to perform some drastic cuts. The selection will serve the attempt to answer the question:

What are the main features of the production mechanism in nucleus-nucleus collisions (background production) at 4.5 GeV/c/A.

The basic information on experimental set-up is presented in Fig.1. The 2-meter streamer chamber ($60 \times 100 \times 200 \text{ cm}^3$) filled with pure neon gas at atmospheric pressure was placed in a magnetic field of about 0.8 T. Pure nuclear targets in the form of thin solid discs mounted within fiducial volume of the chamber, were used. The neon gas filling the streamer chamber served also as a target. Although mounting the conducting targets in the streamer chamber was pioneering, the solution was very simple [6]. Thin solid discs were mounted inside a 30 μm mylar transparent cylindrical container. Continuous flowing of nitrogen through the container was maintained,

giving sufficient insulation. The basic triggering set-up (Fig.1) consisted of a telescope of scintillation counters in front of the chamber (S_1, S_2, S_3, S_4) defining the beam particles and veto counters (S_5, S_6) placed behind the chamber.

I am not going to discuss experimental details here; all information on that point as well as other data which I will not mention in present brief survey one can find in original papers [1-5].

2. MULTIPLICITY DISTRIBUTIONS.

Although all charged particles are registered in the streamer chamber in a 4π geometry, the present review will concern only multiplicity and kinematical characteristics of negative secondaries, since they can be rather safely attributed to negative pions.

Three different triggering modes were used to study multiplicity distributions:

minimum-bias trigger (unbiased sample of all inelastic events);

semi-central collision trigger (suppressing events with charged fragments of the projectile, emitted at angles less than the veto-angle, θ_{ch});

central collision trigger (charged and neutral fragments suppressed in forward direction, with veto angles, θ_{ch} for charged and θ_n for neutrons as fragments).

I would like to point out that up to my knowledge in all experiments using so called "central collision" trigger only charged projectile fragments were suppressed. In not triggerable track detectors, such as nuclear emulsion and bubble chambers, off-line selection of central collision events, with respect to the registered

charged projectile fragments corresponds to our semi-central trigger. One should be careful about possible experimental bias of semi-central collision sample by peripheral events with neutrons as only projectile fragments.

The following notation will be used:

σ^{tot} , σ^{inel} , σ^{el} , σ^{Prod} - total, inelastic, elastic and production cross-sections, respectively.

$\langle n \rangle^{\text{inel}}$, $\langle n \rangle^{\text{Prod}}$ - average number of negative particles normalized to σ^{inel} and to σ^{Prod} , respectively.

It should be stressed that the careful normalization of data appears to be very important in the case of nucleus-nucleus collisions. Normalization of the same data to σ^{inel} or to σ^{Prod} for instance leads to more than 10% difference.

Fig.2 presents, as an example, minimum-bias multiplicity distributions for ^{12}C collisions with five nuclear targets. One can see a structure in multiplicity distributions for ^{12}C interactions with heaviest target nuclei. The multiplicity distributions in semi-central collisions of ^{12}C with ^{64}Cu and central collisions of ^{12}C with ^{91}Zr are presented in Fig.3. One can see smooth, Poisson-like distributions. Therefore, most likely trivial effects like eg. mixing-up multiplicity distributions of peripheral and central collisions are responsible for the bump in Fig.2.

Fig.4 presents data collected up to now, on minimum-bias and semi-central multiplicity distributions in the form of the Wroblewski plot (D vs $\langle n \rangle$). The lines corresponding to p-p data and Poisson multiplicity distribution are also given. It should be noted that

whereas the experimental data on minimum-bias sample obey approximately linear D vs $\langle n \rangle$ dependence (different slope from that of p-p data), those for semi-central (veto angle for charged projectile fragments $\theta_{ch} = 2^\circ$ or $\theta_{ch} = 4^\circ$) collisions do not follow the same dependence. In fact the data are rather close to a linear dependence of D^2 on $\langle n \rangle$. For heavier targets, changing θ_{ch} from 2° to 4° for semi-central sample leads to an increase of the average multiplicities $\langle n \rangle$, while the dispersions D are decreasing. For heaviest colliding nuclei the multiplicity distributions for semi-central collisions are even narrower than Poisson distribution!. More detailed comparison of semi-central (open triangles) and central (black triangles) multiplicity distributions for $^{12}\text{C}-\text{A}_t$ collisions with Poisson distributions is presented in Fig.5, where the ratio η ($\eta = D^2 / \langle n \rangle$) is plotted. For Poisson distribution $\eta = 1$. One can see systematic deviations from Poisson distributions for $\theta_{ch} > 4^\circ$. The multiplicity distributions for real central collisions are narrower than for semi-central. A more detailed discussion on that point is given in ref.[5].

It appears interesting to compare experimental average multiplicities with predictions of theoretical models. Fig.6 presents such a comparison¹ for $^4\text{He}-\text{A}_t$ and $^{12}\text{C}-\text{A}_t$ interactions. It is seen that predictions derived from only two models [7,8] (denoted by triangles) are consistent with the data. These two models are independent nucleon-nucleon collision models based on important assumption that

¹ The derivation of model predictions is described in ref [2,3]

particles are produced mostly in the first nucleon-nucleon collision, occurring when nucleons are traversing nuclear matter. Thus, the nuclear matter seems to be transparent for "wounded" nucleons. One can assume that the particle production is proportional to the average number of nucleon-nucleon collisions [9] or to the average number of wounded nucleons [10]. The predictions resulting from these two assumptions are denoted as black and open squares, respectively. The predictions of a full three dimensional cascade model [11] are denoted on Fig.6 by crosses. They are not too badly inconsistent with data.

One can try to understand the above presented experimental facts within the following consideration. Probably due to the energy dissipation after the first collision (and certainly after the second collision) a nucleon is not able to produce particles any more because its energy is close or even below the energy threshold for particle production. Models [9] and [10] do not take this into account² Models [7] and [8] consider this effect (energy dissipation), using it as an assumption, whereas the cascade model [11] takes energy and momentum conservation into account. From the above presented comparison of data and model predictions rather important question emerges:

To what extent observed nuclear transparency is due to "energy conservation" and to what extent to eg. formation zone effects?

² One should stress that the authors of the model proposed in ref.[10] never claimed that their model should work in the energy region of a few GeV per nucleon.

3. $y - p_t - n$ - "centrality" CORRELATIONS.

Let me now present and discuss some more advanced type of analysis, namely that concerning y (rapidity) - p_t (transverse momentum) - n (multiplicity) - centrality correlations. Momentum and angular measurements are completed by now for minimum-bias samples of ^4He interactions with Li, C, Ne, Al, Cu and Pb target nuclei. The number of nucleons registered among the projectile charged fragments can be used as a measure of the impact parameter of nucleus-nucleus collisions (in other words as a degree of "centrality"). A detailed description of experimental method used for the determination of projectile fragments as well as results on fragmentation process study is given in ref.[4].

Hereafter the following notation will be used:

"inelastic" events - minimum-bias sample (average over impact parameter);

"peripheral" events - events with at least two nucleons found among the projectile fragments (large impact parameters);

"semi-central" events - events with no charged projectile fragments (small impact parameters).

All quantities were calculated in the laboratory system and normalized to σ_{inel} . Only negative particle production (π^-) will be discussed.

Fig.7 presents rapidity ($y=0.5\ln((E+p_1)/(E-p_1))$) distributions of negative particles produced in ^4He interactions with several target nuclei. No target mass dependence in projectile fragmentation region is seen, while a marked dependence in the target fragmentation and

central rapidity regions is seen.

It is difficult to show in limited number of diagrams 4-dimensional (y - p_t - n -centrality) correlations. An attempt to present some of these results is given in Figs.8 and 9. In Fig.8 average multiplicity is correlated with centrality (Fig.8a), rapidity (Fig.8b) and transverse momentum (Fig.8c) for ^4He interactions with 5 nuclear targets. The data presented in these figures suggest the following conclusions. a) There is no dependence of $\langle n \rangle$ on target mass for peripheral events. An increase in $\langle n \rangle$ is due to central collision events. b) The matter is transparent for particles produced with large rapidity (projectile fragmentation region), whereas it is not for central and target fragmentation rapidity region. c) The increase of the multiplicity with the target mass is due to pions with relatively low transverse momenta.

In Fig.9 average p_t of negative pions is plotted against A_t . It is seen that: a) The average p_t value doesn't depend on centrality, at least within the present experimental accuracy. $\langle p_t \rangle$ seems to be constant or even decreases with increasing target mass. b) For events with higher multiplicity ($n > 2$) events, $\langle p_t \rangle$ is smaller than for low multiplicity events ($n \leq 2$). It means that if selection of central events is done by a multiplicity cut, than for such "central" collisions the $\langle p_t \rangle$ of produced π^- is smaller than in "peripheral" events!

Showing these three selected pictures I finished now presentation of quasi-exclusive data on pion production in ^4He - A_t interactions. Let me now dare to speculate a bit on implications resulting from the

data to present theoretical ideas on particle production in nucleus-nucleus collisions. It seems that some experimental observations are crucial for testing theoretical ideas. Note, that the $\langle p_t \rangle$ is in fact a measure of the temperature.

Recently Hagedorn and Rafelski applied a bootstrap model to nucleus-nucleus collisions [14] and derived the $\langle p_t \rangle$ value for the produced pions to be ≈ 320 MeV/c for nucleus-nucleus collisions at 4.5 GeV/c/A whereas in experiment $\langle p_t \rangle \approx 240$ MeV/c was observed. However may be ^4He is a too small nucleus to apply proposed model. In a collective-type model proposed by Meng Ta-chung [13,15] (Violent Collision model) the author claims that in violent (central) collisions average multiplicity ($\langle n \rangle$) and average transverse momentum ($\langle p_t \rangle$) of produced pions are higher than in normal collisions. Indeed, $\langle n \rangle$ is higher, but one can see from the presented data (Fig.9a) that there is no dependence of $\langle p_t \rangle$ on centrality, and $\langle p_t \rangle$ for large multiplicity events is lower (Fig.9b). So even such a qualitative comparison shows that the model predictions are not valid. Such thermodynamic models as fireball [16] and firestreak models [17] are also not able to reproduce simultaneously the observed $\langle n \rangle$ and $\langle p_t \rangle$ dependence on the target mass [18]. Fig.10 shows a comparison of rapidity distributions for $^4\text{He} - ^{64}\text{Cu}$ interactions derived from the Intranuclear Cascade model [11] with those obtained in the experiment. Marked disagreement in low rapidity region is seen.

Finally let me discuss implications of presented data to "universality assumption" used by collective type models such as Coherent Tube model (CTM) [12]. A tube-tube collision in this type of models

is considered as being similar to a nucleon-nucleon interaction (the universality assumption) with the same effective energy $\langle Q \rangle$ in the center of mass system of colliding objects. In Fig. 11 $\langle p_t \rangle$ for produced negative particles is plotted for nucleon-nucleon data [19,20] and for ${}^4\text{He}$ inelastic and semi-central interactions with Li, C, Al, Cu and Pb. Universality assumption means that ${}^4\text{He}$ -A_t data points should follow the nucleon-nucleon $\langle p_t \rangle$ vs $\langle Q \rangle$ dependence, which is not the case. Recently the necessity of including the "tube fragmentation" mechanism was proposed [21]. It is not clear, however, whether one can consider effects of the tube fragmentation in addition to "universality" assumption in the few GeV energy region. Nevertheless, the possibility of "saving" CTM by applying the tube fragmentation mechanism was carefully studied by Jedrzejczak and coworkers [22]. The authors showed that the discrepancies between the model predictions and experimental data could not be removed by taking into account fragmentation processes.

So, the conclusion of this brief comparison between theoretical approaches and experimental data is as follows:

The data are in a disagreement with the predictions of the theoretical models considered above. New concepts and modifications of theoretical approaches are needed.

Let me mention an attempt carried out by Danielewicz and coworkers [18]. They tried to get an agreement with the data by including a partial transparency of nuclear matter [23] into fireball, firestreak and firetube models. A reasonable agreement with data could be achieved when an additional assumption, namely that projectile

nucleons can collide at most once with target nucleons, has been taken into account.

It is amusing that whether one start from multiple scattering type models or from thermodynamic models the requirement of the reproduction of existing experimental data suggests dominant role of single nucleon interaction!

4. CONCLUSIONS.

In conclusion I have to stress that what I tried to present you was a study of "background" production processes. I believe that first we should understand this to be able to identify signatures of unusual states of hadronic matter.

It seems that basic production process occurring in nucleus-nucleus collisions in the present energy region is independent nucleon-nucleon scattering + small cascading (no model on this basis, predicting more than multiplicity distributions, is available).

From a comparison of presented data with collective type models one can say that there is no evidence that we deal with collective interactions in each event, what is claimed within the framework of these models. Obviously, it does not mean that there are no collective effects at all in nucleus-nucleus collisions!

The single nucleon interaction picture says that nuclear matter is rather transparent. May be, however it has not a chance at present energy range to be not transparent.

More experimental data of quasi-exclusive type needed, especially for heavier projectile nuclei, to clarify situation at present energies. I would like to mention two experiments in which similar stu-

dies are being carried out: Berkeley streamer chamber [24] and Dubna propane bubble chamber [25]. We may expect more data soon.

The situation will be much more clear at higher energies where we will be far from the production threshold. The entire volumes of colliding nuclei would thus be used more efficiently for particle production and particle production mechanism testing.

5. ACKNOWLEDGEMENT.

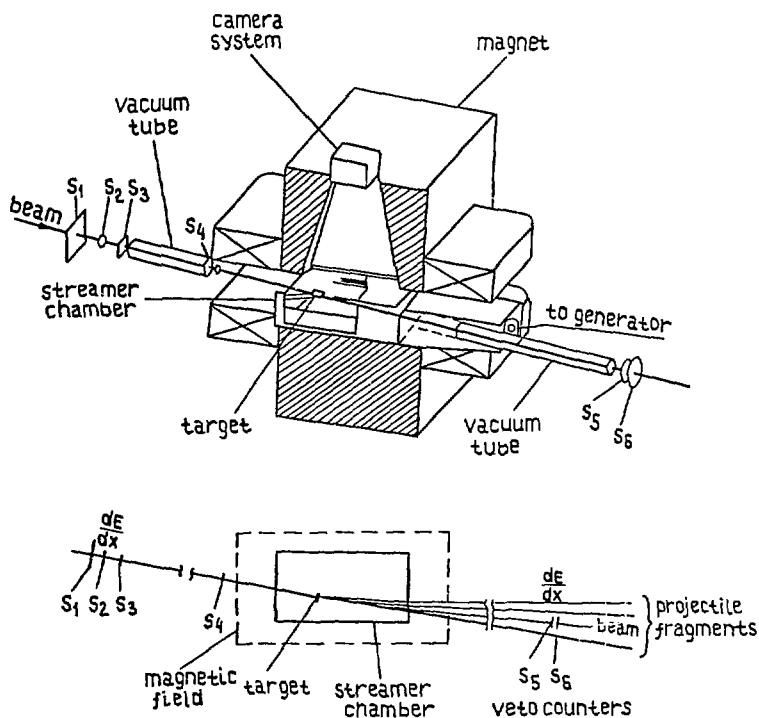
I would like to thank my colleagues in the SKM-200 Collaboration for an enjoyable experiment in Dubna, especially Dr E.O.Okonov, the leader of the collaboration, for illuminating discussions and encouragement. Most of ideas presented here were worked out together with them.

I would also like to thank prof. E.Skrzypczak for useful discussions and helpful comments on this manuscript.

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$$\text{TRIGGER} = S_1 \times S_2 \times S_3 \times S_4 \times \bar{S}_5 \times \bar{S}_6$$

Fig.1 The experimental set up.

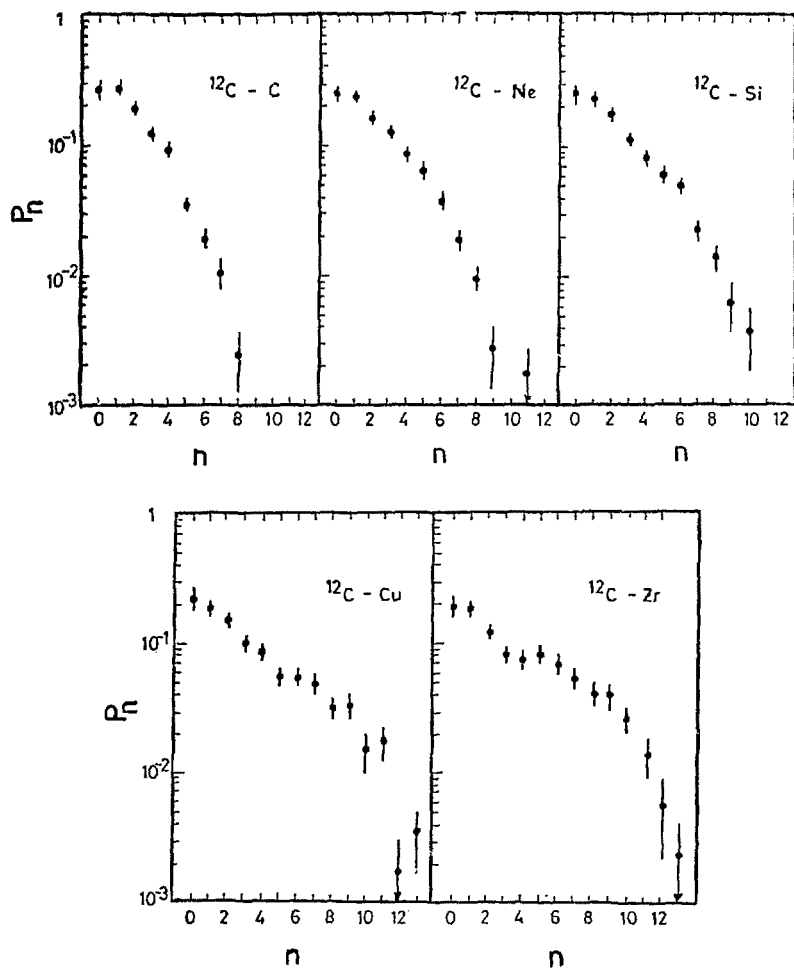


Fig.2 Minimum-bias multiplicity distributions. Normalization to σ_{inel} .

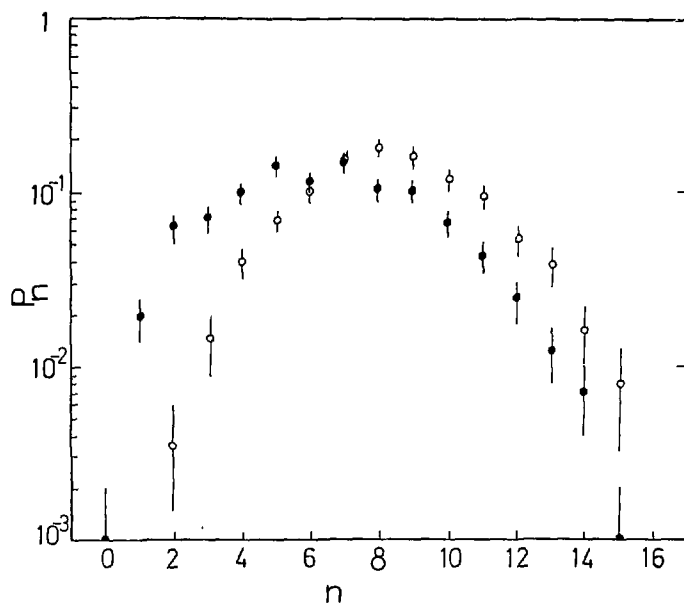


Fig.3 The multiplicity distributions for semi-central $^{12}\text{C}-^{64}\text{Cu}$ interactions ($\theta_{ch}=2^\circ$) - black points, and for central $^{12}\text{C}-^{91}\text{Zr}$ interactions ($\theta_{ch}=2^\circ, \theta_n=2^\circ$) - open points.

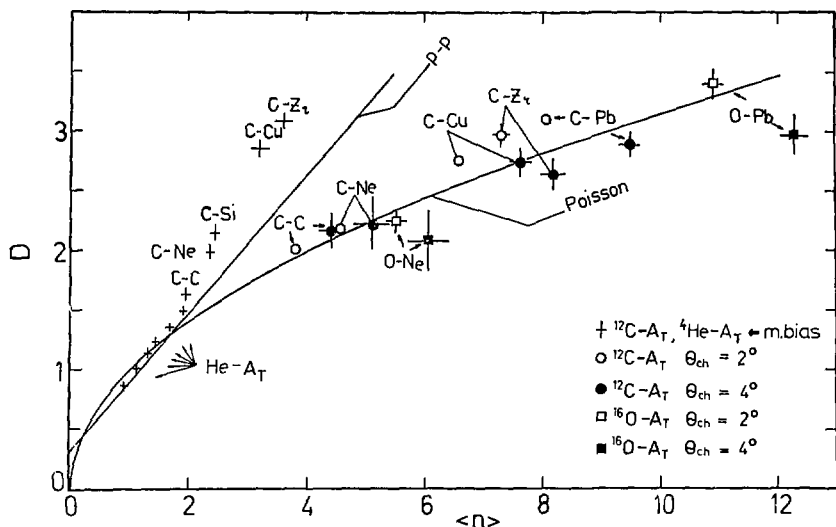


Fig.4 The dispersion $D = \sqrt{\langle n^2 \rangle - \langle n \rangle^2}$ versus the average value $\langle n \rangle$ of the multiplicity distributions of negative secondaries:

crosses - ${}^4\text{He}$ and ${}^{12}\text{C}$ minimum-bias data;

open circles - ${}^{12}\text{C}$ semi-central collisions, $\theta_{\text{ch}} = 2^\circ$;

open squares - ${}^{16}\text{O}$ semi-central collisions, $\theta_{\text{ch}} = 2^\circ$;

full circles - ${}^{12}\text{C}$ semi-central collisions, $\theta_{\text{ch}} = 4^\circ$;

full squares - ${}^{16}\text{O}$ semi-central collisions, $\theta_{\text{ch}} = 4^\circ$.

The curve $D = \sqrt{\langle n \rangle}$ corresponding to Poisson-type distributions and the straight line corresponding to multiplicity distributions for p-p interactions are also shown.

Normalization to σ^{prod} .

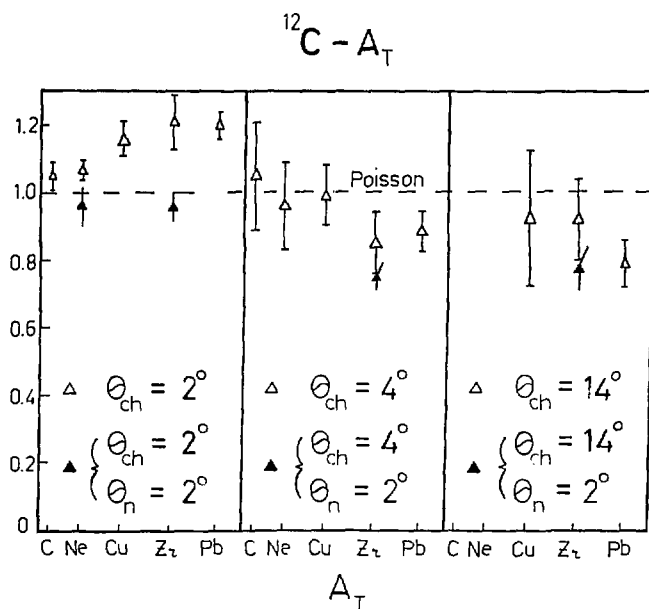


Fig.5 The values of the parameter $\eta = D^2 / \langle n \rangle$ versus A_T for semi-central (open triangles) and central collisions (black triangles) of ^{12}C and ^{16}O nuclei is plotted for different θ_{ch} .

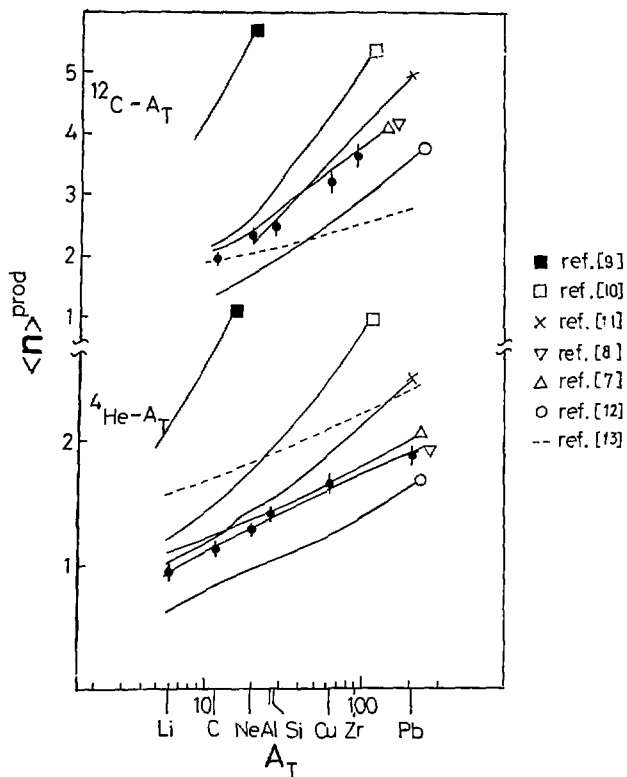


Fig. 6 A_T dependence of the average multiplicity values for ^4He and ^{12}C nucleus interactions. Experimental data and predictions of several theoretical models are plotted.

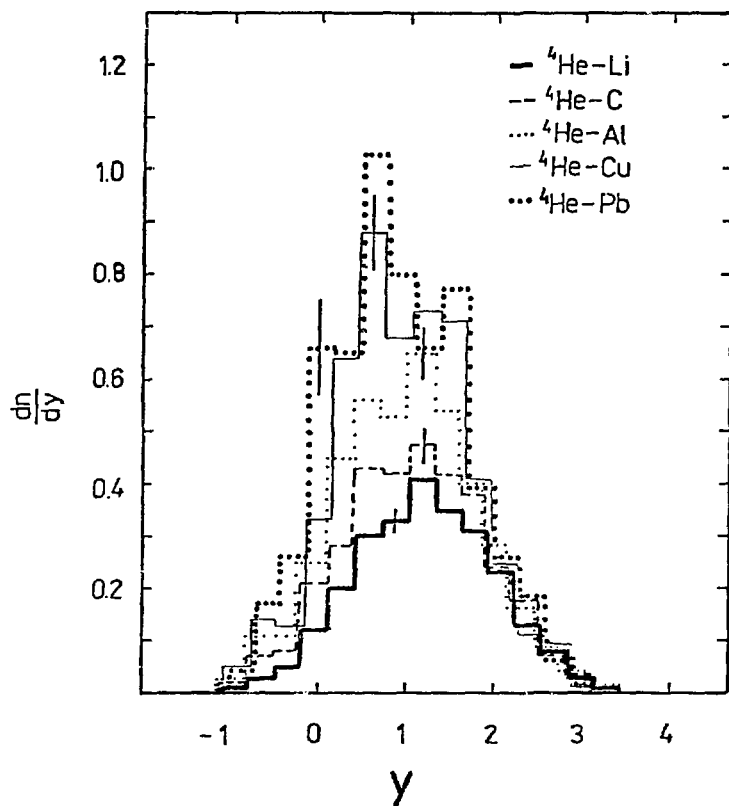


Fig.7 Rapidity distributions for negative pions from ${}^4\text{He}$ interactions with Li, C, Al, Cu and Pb target nuclei.

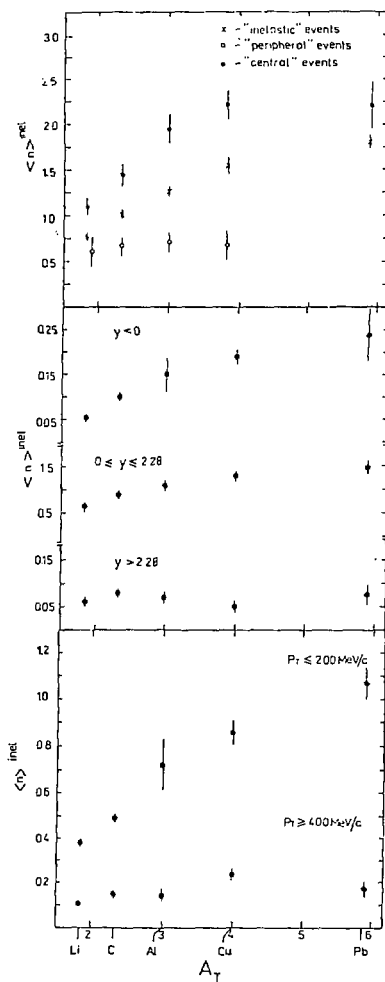


Fig.8 The A_T -dependence of the average multiplicity of negative pions for a) "inelastic", "peripheral" and "semi-central" events, b) pions from three rapidity regions, c) pions with high ($p_T \geq 0.4 \text{ GeV/c}$) and low ($p_T < 0.2 \text{ GeV/c}$) transverse momenta.

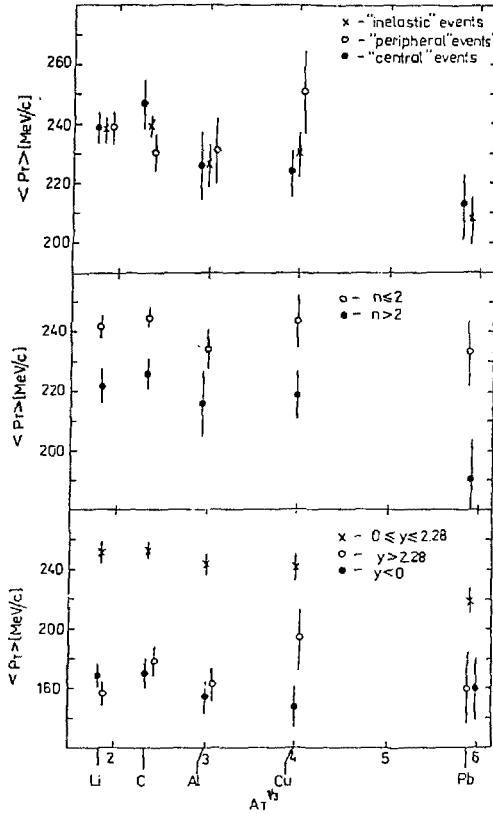


Fig.9 The A_t -dependence of the transverse momentum of negative pions for a) "inelastic", "peripheral" and "semi-central" events, b) low ($n \leq 2$) and high ($n > 2$) multiplicity events and c) pions from three different rapidity regions.

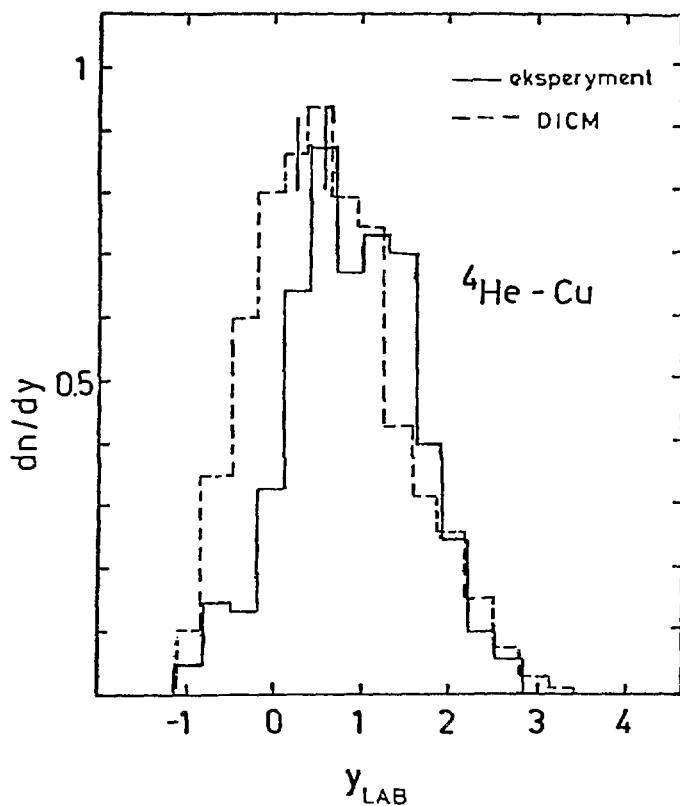


Fig.10 Comparizon of rapidity distribution for ${}^4\text{He} - {}^{63}\text{Cu}$ interactions obtained experimentally and calculated by Intranuclear Cascade model [11].

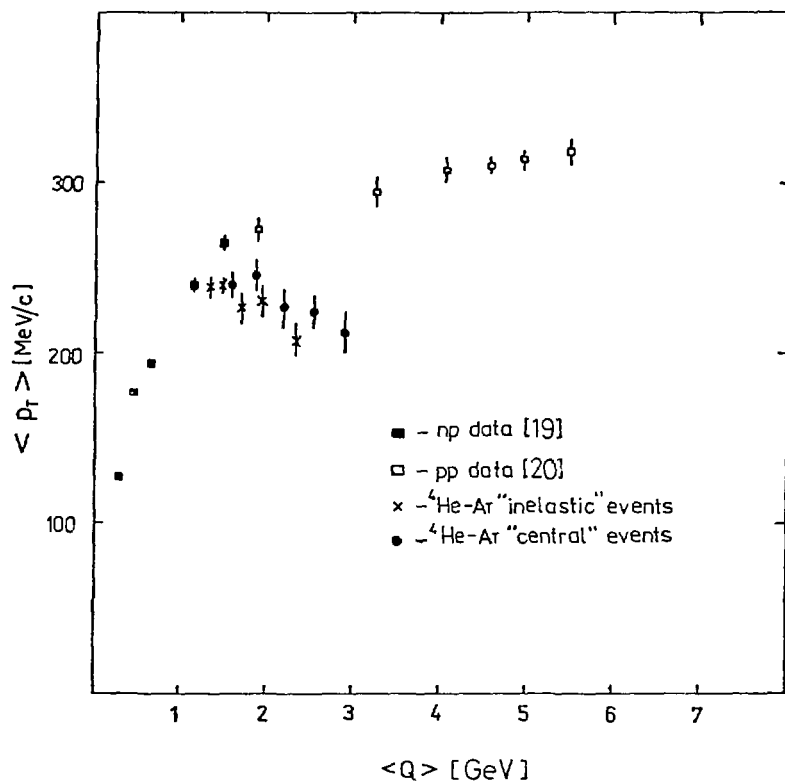


Fig.11 The dependence of average transverse momentum $\langle p_T \rangle$ values on average available energy $\langle Q \rangle$, in the CMS of colliding objects.

Asymptotic Diffraction Theory¹R. J. Glauber*, M. Bleszynski[†] and P. Osland*

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We describe an approximate approach to the theory of diffractive scattering which holds for strong interactions and large momentum transfers. The approximation is based on the assumption of complex straight-line trajectories for the colliding particles and an application of the method of stationary phase. The scattering amplitude is expressed as a sum of contributions from the neighborhoods of a small number of trajectories of stationary phase. The angular distributions which result are analogous to optical diffraction patterns for a corresponding number of slits (with complex parameters.) We have evaluated the asymptotic amplitudes for a wide variety of interactions both analytically and numerically. Quantitative comparisons of the asymptotic scattering amplitude with the amplitudes derived from diffraction theory and from the partial wave solution to the Schrödinger equation show its accuracy to be quite good at all save the smallest scattering angles. The asymptotic approach is of particular value where, as in heavy-ion scattering, both attractive and repulsive interactions are present. Simple qualitative observations about the behavior of the stationary paths are then sufficient to explain many complexities of the observed angular distributions.

¹ Submitted to Phys. Lett.

THEORY OF COULOMB DISTORTION OF PION SPECTRA BY HEAVY-ION SPECTATOR FRAGMENTS*

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1. Introduction

It has been known for over two years now that the π^- spectrum from heavy ion collisions peaks near beam velocity and that the π^+ spectrum has a corresponding depression. We have guessed that these features are Coulomb effects of projectile fragments. In papers just submitted for publication^{1,2} the experimental data and the theoretical interpretation are presented. We aim in the present paper to give a complementary report with somewhat more mathematical detail than in ref. 2. We also highlight and discuss implications of the theoretical parameters deduced from the pion spectra.

2. Outline of the theory

2.1. Construction of Coulomb Green's Function

We start by considering a fragment of charge Z_F and mass A_F and a pion of mass m_π at a distance r from the center of this fragment under consideration. For production of slow pions near the beam velocity, we seek a solution of a Schrodinger equation describing the relative motion (nonrelativistic) of the two particles of the form

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^{††}On leave from Institute for Nuclear Study, University of Tokyo

$$\left\{ -\frac{\hbar^2}{2\mu} \nabla^2 + V(r) + W(r) - E \right\} \psi(\vec{r}) = 0. \quad (1)$$

It is possible to attempt solving this equation via Green's Function if one writes as

$$\{E - H(\vec{r})\} \psi(\vec{r}) = \rho(\vec{r}), \quad (2)$$

where

$$H(\vec{r}) = -\frac{\hbar^2}{2\mu} \nabla^2 + V(r) \quad \text{with } V(r) = \frac{Z_\pi Z_F e^2}{r} \quad (3)$$

In Eq. (2) E stands for the kinetic energy of the pion when it is far from the fragment under consideration. We may write the solution of Eq. (2) as

$$\psi(\vec{r}) = \int d^3 r' G(\vec{r}, \vec{r}') \rho(\vec{r}'), \quad (4)$$

where $G(\vec{r}, \vec{r}')$ is the three-dimensional Green's Function and satisfies the differential equation

$$\{E - H(\vec{r})\} G(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}'). \quad (5)$$

G satisfies the homogeneous differential equation for $\vec{r} \neq \vec{r}'$ but is singular at $\vec{r} = \vec{r}'$. It can be shown that the three-dimensional Green's Function has a Legendre expansion of the form^{3,4}

$$G(\vec{r}, \vec{r}') = \frac{1}{4\pi} \frac{2\mu}{\hbar^2} \frac{1}{r r'} \sum_{\ell=0}^{\infty} (2\ell+1) P_{\ell}(\hat{r} \cdot \hat{r}') g_{\ell}(r, r'), \quad (6)$$

where the radial Green's Function $g_{\ell}(r, r')$ satisfies the differential equation

$$\left\{ \frac{d^2}{dr^2} + k_{\pi}^2 - \frac{\ell(\ell+1)}{r^2} - \frac{Z_{\pi} Z_F e^2}{r} \right\} g_{\ell}(r, r') = \delta(r - r') \quad \text{with } k_{\pi} = \frac{p_{\pi}}{\hbar}. \quad (7)$$

2.2. The delta source at a distance

We may solve Eq. (2) with the aid of the delta-function source at $\vec{r}' = \vec{r}_S$. In this case $\rho(\vec{r}) = \lambda \delta(\vec{r} - \vec{r}_S)$ and the solution to Eq. (1),

which we seek, becomes related to the three-dimensional Green's Function by

$$\psi(\vec{r}) = \lambda G(\vec{r}, \vec{r}_s) \quad (8)$$

One has to solve Eq. (7) for $g_\ell(r, r_s)$ for $r > r_s$ by imposing the boundary condition of an outgoing wave on the radial wave function. This could be done by changing the variable r to the dimensionless variable $\rho = kr$ and introducing the Sommerfeld parameter $\eta = Z_\pi Z_F e^2 / (\hbar c |\vec{\beta}_\pi - \vec{\beta}_F|)$, where $|\vec{\beta}_\pi - \vec{\beta}_F|$ is the velocity (in units of c) of the produced pion relative to the assumed spectator fragment. We see immediately that

$$g_\ell(r, r_s) = -k^{-1} F_\ell(\eta, \rho_s) [G_\ell(\eta, \rho) + i F_\ell(\eta, \rho)] \quad (9)$$

F and G stand for the regular and irregular Coulomb wave functions, respectively.

For an observation point far from the region of interaction, one can find for G the following:

$$G(\vec{r}, \vec{r}_s) \underset{r \rightarrow \infty}{\sim} F_\delta(\vec{r}, \vec{r}_s) \frac{e^{i(kr - \eta \ln 2\rho)}}{r} \quad (10)$$

The logarithmic term is a direct consequence of the long-range nature of the Coulomb force. This term produces no net current at infinity, so we know that F_δ can be interpreted as a wave amplitude for the production of pions from a δ -source at r_s and with

$$\sigma = |f_\delta(\vec{r}, \vec{r}_s)|^2, \quad (11)$$

where

$$F_\ell(\vec{r}, \vec{r}_s) = -\lambda \frac{Z_\pi}{\hbar^2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} (-1)^{\ell} e^{i\sigma_\ell} y_{\ell m}^*(\theta, \phi) y_{\ell m}(\theta_s, \phi_s) \left\{ \frac{F_\ell(\eta, \rho_s)}{\rho_s} \right\}, \quad (12)$$

and σ_ℓ is the well-known Coulomb phase shift.

For comparison with zero degree pions, we let $\vec{r} = (\infty, 0, 0)$ and take the

beam along the z-axis. This will eliminate the m-dependence of Eq. (12).

For $\vec{r}_s = (r_s, 0, 0)$ or $\vec{r}_s = (r_s, \frac{\pi}{2}, 0)$ f_δ is proportional to a sum over ℓ of $a_\ell F_\ell(n, \rho_s)$ plus $F_0(n, \rho_s)$. We found that F_0 is the predominant term in the expansion and it is good approximation to write

$$|F_\delta(\vec{r}, \vec{r}_s)|^2 \approx \lambda \left(\frac{2u}{h^2} \right)^2 \left| \frac{F_0(n, \rho_s)}{\rho_s} \right|^2 \quad (13)$$

Hence

$$\sigma_{\pi^\pm} \propto \left| \frac{F_0(n_\pm, \rho_{s_\pm})}{\rho_{s_\pm}} \right|^2 = G_\pm (1 + \rho_{s_\pm} n_\pm + \dots)^2 \quad (14)$$

where

$$G_\pm = \frac{2\pi n_\pm}{[\exp(2\pi n_\pm) - 1]}, \text{ and the } +, - \text{ signs stand for } \pi^+ \text{ and } \pi^-, \text{ respectively.}$$

2.3. Averaging over products and velocity dispersion

For comparison with inclusive pion data, we should calculate an average value of Eq. (14) over all products and relative velocity of pion and fragment. The velocity dispersion formula obtained by Greiner et al.⁵ is used and has the form

$$P \propto \exp[-\beta_F^2 / (2\alpha_F^2)] \quad (15)$$

$$\text{with } \alpha_F = \sigma_0 / (m_N c) \sqrt{(A_0 - A_F) / [A_F(A_0 - 1)]},$$

and where A_0 is the projectile mass number, σ_0 is a constant. The grand average of the pion cross section is given by

$$\langle \sigma_{\pi^\pm} \rangle = N_\pi \frac{\sum_{A_F, Z_F} N_F \alpha(A_F, Z_F) F(A_0 - A_F) G(\langle n \rangle_{\beta_F}) \left\{ 1 + \frac{m r_{0c}}{h} \beta_{\pi^\pm} \langle n \rangle_{\beta_F} A_F^{1/3} \right\}^2}{\sum_{A_F, Z_F} N_F \alpha(A_F, Z_F) F(A_0 - A_F)} \quad (16)$$

where

$$N_F = \sqrt{2\pi} \alpha_F^3$$

$\sigma(A_F, Z_F)$ is the cross section for forming a fragment of charge

Z_F and mass A_F

$F(A_0 - A_F)$ is the factor giving pion production dependence on mass

loss $A_0 - A_F$

$r_s = r_0 A_F^{1/3}$ is assumed to be the most probable distance where the pion produced

$$\langle n_{\pm} \rangle_{\beta_F} = \pm \left(\frac{Z_F e^2}{\hbar c} \right) \frac{\text{erf} \left[\frac{\beta_{\pm} / (\sqrt{2} \alpha_F)}{\beta_{\pm}} \right]}{\beta_{\pm}} .$$

3. Comparison with Data and Discussion

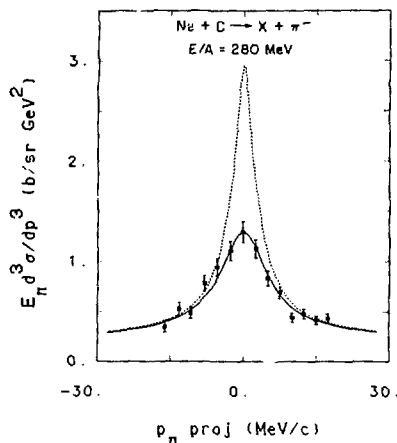
Figures 1 and 2 show comparisons with zero degree π^- of Sullivan et al.¹ The dashed curve is the direct result of eq. (16), and the solid curve is the direct result after folding in the experimental resolution. For the calculations, the fragment cross sections $\sigma(A_F, Z_F)$ were taken from the fast stage, before particle evaporation, of an abrasion-ablation firestreak code of Patrick McGaughey.⁶ It was necessary for best agreement to use a fragment velocity dispersion parameter σ_0 smaller than the experimental value from inclusive fragmentation experiments.^{5,7} That is, we use $\sigma_0 = 60$ MeV/c rather than the 86 MeV/c of refs. 5 and 7. Of course, the pion experimental data are sensing a different fragmentation velocity dispersion, that of the fast cascade (abrasion) product compound nuclei rather than the final products measured in the inclusive experiments. It may well be that the heavy ion abrasion stage knocks out correlated chunks of nucleons, leaving the primary fragments with narrow velocity dispersions. These narrow distributions are then broadened in the particle evaporation stage.

We found also that the pion source function $F(A_0 - A_F)$ needs to weight the heavier fragments (for the Ne data only) more than the first guess of simple linear dependence on fragment mass loss. That is, we used for the Ne data of Fig. 1 the function $F(A_0 - A_F) = (A_0 - A_F) - 0.06(A_0 - A_F)^2$. This quadratic term may be due to shadowing effects in pion production. That is, the observed pions must be formed near the surfaces of the nuclei, whereas fragment mass loss may be more nearly proportional to the number of nucleon-nucleon collisions throughout the nuclear volume.

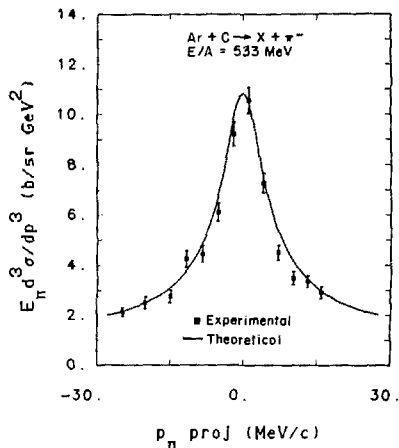
We look forward to the next generation experiments in HISS, where the correlations of pions with specific heavy fragments will be measured.

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6. We are grateful to Patrick McGaughey of Lawrence Berkeley Laboratory, Berkeley, for performing the fragmentation cross section theoretical calculations.
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FBL 816-10177



FBL 816-10170

Fig. 1. Lorentz invariant π^- cross sections at zero degrees vs. pion momentum in the projectile frame for $^{20}\text{Ne} + ^{12}\text{C}$. Points are data of ref. 1. The dashed (solid) curves are theoretical calculations with eq. (16) before (after) folding in experimental resolution.

Fig. 2. Same as Fig. 1 except for $^{40}\text{Ar} + ^{12}\text{C}$.

ANOMALOUS BEHAVIOR OF SECONDARY PARTICLES PRODUCED FROM
RELATIVISTIC HEAVY ION INTERACTIONS*

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ABSTRACT

The interaction mean free paths of relativistic projectile fragments, $3 \leq Z \leq 18$ emitted from two heavy-ion beams of ^{40}Ar and ^{56}Fe at an energy of ~ 2 GeV/nucleon in nuclear emulsion are measured and they are found to be shorter for a few centimeters after their emission than at larger distances or than predicted from experiments on accelerator beams. Some characteristics of the primary and the secondary interactions connected with the anomaly are presented.

*Work supported by NCI DHEW under contract number R01-CA24878 and the SUNY Research Foundation.

In the studies of heavy cosmic ray particles the reaction mean free path (mfp) of the projectile fragments (PF) of relativistic heavy-ion collisions in their first two centimeters was observed to be small.¹ In cosmic ray studies, the statistics have always been limited, with the result that the above stated fact was not taken seriously. Now heavy ion beams are available at the Bevalac to observe this effect.²

To confirm the short reaction mean free path in heavy secondary particles, we used two small stacks of Ilford G-5 emulsions exposed to ^{56}Fe and ^{40}Ar beams at ~ 2 GeV/nucleon parallel to the pellicle surfaces. Each pellicle was scanned by along-the-track method.³ We thus found about 2000 primary interactions and those interactions which have relativistic tracks of $Z \geq 3$ emitted from the primary interactions within 0.1 radian forward cone were followed until they either interacted or left the stack. For each PF we measured its charge Z to a prediction of one charge unit⁴ and distance T available for interaction in the emulsion (the potential length) or if it interacted, the distance L to the interaction point. The mfp of interactions in nuclear emulsion of a homogeneous beam of nuclei of charge Z is given by $\lambda_z^* = L/N$, where L is the total length of both interacting and noninteracting tracks followed with N observed interactions. This estimate is therefore independent of stack size or of the track segments in which λ is measured. As we are dealing with secondary particles of different charges we can parametrize the mfp of a secondary beam of particular Z -value by the relation $\lambda_z = \lambda_1 Z^{-b}$ where $\lambda_1 = \lambda_{\text{beam}} = 32.8 \pm 1.7$ cm and $b = 0.42 \pm 0.04$. This relation is consistent with

the trend of mfp's calculated from cross sections based on several versions⁵ of geometrical models. In following the PFs of charge $Z \geq 3$, they are divided into two charge groups as shown in Table I for $L = 1.6$ cm and 2.6 cm respectively. The mfp's of the anomalous components is shorter by at least 2.5 standard deviations than the normal mean free path.

For N_z number of secondary particles of charge Z and mfp λ_z , we get at fixed b , the expression for mfp $\lambda_1^* = \sum_z N_z \lambda_z^* Z^b / \sum_z N_z$. For all the PF's with charges $Z \geq 3$, we show in Fig. (1) the observed dependence of λ_1^* on distance L , for a number of independent segments. The dashed line is the prediction from λ_{beam} . We may note the low value of λ_1^* in the first few centimeters and beyond $L = 3$ cm, the estimate of λ_1^* is comparable with λ_{beam} . In Fig. (2), we present two distributions of interaction distances L for events with potential paths $T \geq T_1 = 3.9$ cm and 6.5 cm respectively. For small distance $L \leq 2.6$ cm, the experimental observation (solid line) shows an excess of events over the number predicted (dotted line) and for $T_1 = 3.9$ cm, it amounts to about 2.5 standard deviations. The occurrence of anomalous particles is of the order of a few percent of all the secondary particles observed.

As yet we do not know the reasons for the abnormal mfp of events within $L \leq 2.6$ cm. In order to understand the result of this experiment, we looked at the characteristics of the primary (parent) interactions to observe the following facts: (a) The mfps of PF's originating from primary events with $N_h \geq 7$ (N_h denotes the number of heavy

tracks with $\theta \leq 0.7$ and refers to the primary events for the following conclusions) are in general shorter than the mfp of PF's from $N_h < 7$ type primary events. This is shown in Table II for a group of charges (3-11). One observes that a large portion of anomalous interactions appear in events in $N_h \geq 7$. (b) The multiplicity of the anomalous events in $N_h \geq 7$ appears to be slightly less than the multiplicity for normal events, while for events with $N_h < 7$ there appears to be no change within the statistical errors and this is shown in Table III for a group of charges (3-16) for $L \leq$ and > 3 cm. (c) Multiplicity (n_s) distributions of all the secondary interactions with $L \leq 3$ cm peaks at low n_s value than for events with $L > 3$ cm. (d) We observe more secondary white stars (14%) for $L \leq 3$ cm than for $L > 3$ cm (10%). (e) We further observe that the percentage of white stars produced by PF's which are coming from the interactions with heavier targets ($N_h > 7$) is larger within $L \leq 3$ cm than in $L > 3$ cm. (f) We have also observed some multichain events and comparison of mfps emitted from later generations of PF in the extranuclear cascades show shorter mfps.

One of the possible explanations of the present findings is perhaps the quark-parton effect in soft (small-momentum transfer) processes. Our current understanding of strong interactions at high energies is based on the notion of weakly interacting quarks at short-distances. An extrapolation of this idea implies that as matter is compressed to high density or heated to extreme temperature, a phase of quasi-free quark matter or the formation of exotic objects like

Table I. Mean free paths (λ^*) at different distances L from the origins of PF's for grouped charges.

<u>Z</u>	<u>$\lambda^* (L \leq 1.6 \text{ cm})$</u>	<u>$\lambda^* (L > 1.6 \text{ cm})$</u>	<u>$\lambda^* (L \leq 2.6 \text{ cm})$</u>	<u>$\lambda^* (L > 2.6 \text{ cm})$</u>
3-8	8.9 ± 1.30	11.7 ± 1.40	9.4 ± 1.18	13.1 ± 1.82
9-16	7.6 ± 0.72	9.4 ± 0.72	7.6 ± 0.60	10.0 ± 0.83

Table II. Mean free paths (λ^*) at different distances L from the origins of PF's for grouped charges emitted from different types of primary interactions (with different N_h values). Last row indicates overall λ^* for $N_h < \text{or} \geq 7$.

<u>Z</u>	<u>$N_h < 7$</u>		<u>$N_h \geq 7$</u>	
	<u>$\lambda^* (L \leq 2 \text{ cm})$</u>	<u>$\lambda^* (L > 2 \text{ cm})$</u>	<u>$\lambda^* (L \leq 2 \text{ cm})$</u>	<u>$\lambda^* (L > 2 \text{ cm})$</u>
3-11	9.6 ± 1.33	13.1 ± 1.95	8.62 ± 1.20	10.6 ± 1.20
3-11	11.21 ± 1.14		9.7 ± 0.88	

Table III. Average multiplicity $\langle n_s \rangle$ of the secondary interactions for grouped charges emitted from different types of primary interactions (with different N_h values). Last row indicates overall $\langle n_s \rangle$ for $N_h < \text{or} \geq 7$.

<u>Z</u>	<u>$N_h < 7$</u>		<u>$N_h \geq 7$</u>	
	<u>$\langle n_s \rangle (L \leq 3 \text{ cm})$</u>	<u>$\langle n_s \rangle (L > 3 \text{ cm})$</u>	<u>$\langle n_s \rangle (L \leq 3 \text{ cm})$</u>	<u>$\langle n_s \rangle (L > 3 \text{ cm})$</u>
3-16	9.59 ± 0.92	9.74 ± 1.19	9.66 ± 1.22	11.6 ± 1.64
3-16	9.64 ± 0.71		10.4 ± 0.91	

a gluon-plasma should appear. If such a state of matter does exist, i.e. large bags filled with quarks and gluons then one likely way of finding it would be through high-energy collisions of heavy-ions on heavy targets. The present experiment meets the above requirements to produce the short lived (i.e. $\tau \leq 10^{-11}$ sec) particles at high energy.

We are very grateful to the staff of Bevalac at LBL Laboratory for their help with the exposure of emulsion stacks.

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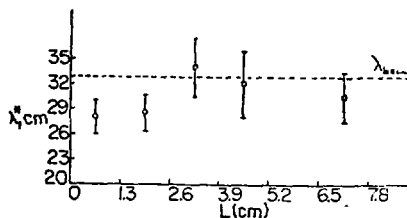


Fig. 1 Experimental estimation of λ_n^* at different distances L from the origins of projectile fragments. Dashed line is the prediction from λ_{beam}^* .

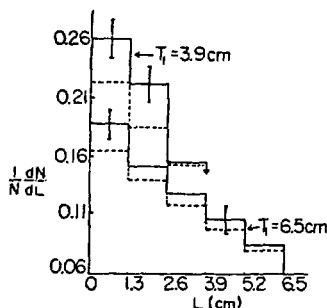


Fig. 2 Distributions of interaction distances L for events with potential paths $T > T_1$. Solid line is the experimental observation. Dashed line is the expected value.

BEHAVIOR OF SECONDARY PARTICLES EMITTED FROM
HEAVY-ION INTERACTIONS AT AN ENERGY <1 GeV/N

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ABSTRACT

The interaction mean free paths of relativistic projectile fragments, $3 \leq Z \leq 26$ emitted from heavy-ion beam of ^{56}Fe at an energy of <1 GeV/nucleon in nuclear emulsion are measured and they remain almost constant at different distances from their production point within their statistical errors. Some characteristics of the primary and the secondary interactions are presented.

*Work supported by NCI DHEW under contract number R01-CA24878 and by the SUNY Research Foundation.

Before the time of the operation of high energy accelerators at CERN, Chicago, Dubna and heavy-ion accelerator at Berkeley, California, most of the basic knowledge about the interaction of fundamental particles came from the studies of cosmic ray physics.¹

In these studies the reaction mean free path (mfp) of the projectile fragments (PF) of relativistic heavy-ion collisions in their first two centimeters was observed to be small.² With the availability of Heavy Ion beams at Bevalac (Berkeley) efforts are being made to study systematically^(3,4) anomaly of short mean free path (in first two centimeters) observed in cosmic ray studies.^(1,2) In a previous paper,⁴ using ^{56}Fe and ^{40}Ar emulsion interactions at ~ 2 GeV/nucleon (beam B_1), we observed that the projectile fragments in their first few centimeters, after emerging from a nuclear interaction (~ 10 gm/cm² of matter traversed or $\sim 10^{-11}$ seconds), have significantly shorter mfp's than those observed from 'normal' beams of the same charge z ; but at larger distances from the interaction point the mfp's return to 'normality'.

In order to know whether this effect is present at lower energy, we have studied ^{56}Fe -emulsion interactions at ~ 1 GeV/nucleon in a small stack of Illford G-5 emulsions exposed (beam parallel to pellicle surfaces) to Bevalac beam. Scanning was done by area scanning method in the first 1.5 cm. track length of beam particle in emulsion so that primary interactions in this case were produced at energy greater than ~ 0.85 GeV/nucleon. We thus found about 1500 primary interactions with Fe beam. There are interactions which

have relativistic tracks of $Z \geq 3$ emitted from the primary interactions within 0.1 radian forward cone were followed until they either interacted or left the stack. For each PF we measured its charge Z to a precision of one charge unit⁵ and distance T available for interaction in the emulsion (the potential length) or if it interacted, the distance L to the interaction point. The mfp of interactions in nuclear emulsion of a homogeneous beam of nuclei of charge Z is given by $\lambda_Z^* = L/N$, where L is the total length of both interacting and noninteracting tracks followed with N observed interactions. This estimate is therefore independent of stack size or of the location of the track segments in which λ is measured. The mfp's thus calculated are shown in Table I for two charge groups and cut at $L = 2.6$ cm. As we are dealing with a number of secondary particles of different charges coming from different interactions, we can combine the information of a secondary beam of particular Z -value by the relation $\lambda_Z = \lambda_1 Z^{-b}$ where $\lambda_1 = \lambda_{\text{beam}} = 35 \pm 1.8$ cm and $b = 0.42 \pm 0.024$. This relation is consistent with the trend of mfp's calculated from cross sections based on several versions⁶ geometrical models.

For N_Z number of secondary particles of charge Z and mfp λ_Z , we get at fixed b , the expression for mfp $\lambda_1^* = \sum_Z N_Z \lambda_Z^* Z^b / \sum_Z N_Z$.

For all the PF's with charges $Z \geq 3$, we show in Fig. (1) the observed dependence of λ_1^* on distance L , as calculated for a number of independent segments. The value of λ_1^* does not change with L , within the statistical errors.

In Fig. (2) we show a distribution of interaction distance L for

events with $T \geq T_1 = 6.5$ cm. The experimental values (solid line) with statistical errors are within the predicted value (dotted line) and this shows no real anomalous effect at low energy.

In order to investigate further the result of this experiment, we looked at the characteristics of the primary (parent) interactions from where the PFs originate and also the secondary events. In general their characteristics are similar. In particular we observe the following facts: (i) Evaporation fragments which are presumably emitted within all angles to the primary direction seem to behave normally and so do the shower particles. (ii) We divide the distribution of mfps of PF's further into different types of primary events (with different N_h values). In Table II, is shown the mfps of PF's from a group of charges (3-11) in 2 GeV/N (beam B_1) as well as in 1 GeV/N (beam B_2). One observes that a large portion of anomalous interactions appear in events with $N_h \geq 7$ in beam B_1 . For beam B_2 , mfps distributions are the same within the statistical errors. (iii) When we divide the primary events according to the target size (i.e. N_h value) the ratio of number of interactions per PF for different charge groups with $N_h < \text{or} \geq 7$ are shown in Table III for both the beams. For each group for beam B_1 interaction density is larger for events with $N_h \geq 7$ than for events with $N_h < 7$, while for beam B_2 this is not the case. At low energy (<1 GeV/nucleon) PFs from the heavy ion beam do not seem to produce the abnormal nuclei. We are checking this further by different projectiles at different energies.

Table I Mean free paths (λ^*) at distance $L \leq$ and > 2.6 cm from the origins of PF's for grouped charges.

<u>Z</u>	Beam energy (< 1.0 GeV/N)	
	λ^* ($L \leq 2.6$ cm)	λ^* ($L > 2.6$ cm)
3 - 14	13.31 ± 1.37	16.20 ± 1.83
15 - 26	9.95 ± 1.07	12.48 ± 1.32
3 - 26	12.05 ± 0.89	14.20 ± 1.10

Table II Mean free paths (λ^*) at distance $L = 2$ cm from the origins of PF's for grouped charges emitted from different types of primary interactions (with different N_h values) produced by beams B_1 and B_2 .

Beam B_1 (~ 2 GeV/N)			Beam B_2 (< 1.0 GeV/N)		
<u>Z</u>	<u>$N_h < 7$</u>	<u>$N_h \geq 7$</u>	<u>Z</u>	<u>$N_h < 7$</u>	<u>$N_h \geq 7$</u>
3 - 11	11.21 ± 1.14	9.7 ± 0.88	3 - 26	13.15 ± 1.02	12.90 ± 0.95

Table III Number of secondary interactions per PF for different charge groups emitted from different types of primary interactions (with $N_h <$ and ≥ 7) for both the beams.

<u>Z</u>	Beam B_1 (~ 2 GeV/N)		Beam B_2 (< 1.0 GeV/N)	
	<u>$N_h < 7$</u>	<u>$N_h \geq 7$</u>	<u>$N_h < 7$</u>	<u>$N_h \geq 7$</u>
3 - 14	0.35 ± 0.016	0.41 ± 0.02	0.29 ± 0.02	0.31 ± 0.017
15 - 26	0.40 ± 0.021	0.45 ± 0.005	0.49 ± 0.03	0.41 ± 0.031

We are very grateful to the staff of Bevalac at LBL Laboratory for their help with the exposure of emulsion stacks.

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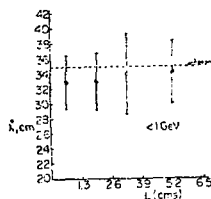


Fig. 1 Experimental estimation of λ^* at different distances L from the origins of projectile fragments. Dashed line is the prediction from λ_{beam} for beam B_2 ($< 1 \text{ GeV/N}$).

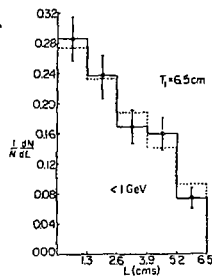


Fig. 2 Distributions of interaction distance L for events with potential paths $T \geq T_1$. Solid line is the experimental observation. Dashed line is the expected value for beam B_2 ($< 1 \text{ GeV/N}$).

EVIDENCE FOR ANOMALOUS NUCLEI AMONG RELATIVISTIC
PROJECTILE FRAGMENTS AT 2 A GeV*

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Abstract

Two independent emulsion experiments using Bevalac beams of ^{16}O and ^{56}Fe at ≈ 2 A GeV find with $\geq 99.7\%$ confidence that the reaction mean free paths of projectile fragments, $3 \leq Z \leq 26$, are shorter for a few centimeters after their emission than at larger distances, or than predicted from experiments on beam nuclei. This effect, which is enhanced in later generations of fragments, can be interpreted by the relatively rare occurrence of fragments that interact with an unexpectedly large cross section.

Two independent experiments using nuclear emulsion were conducted to examine the interaction characteristics of relativistic projectile fragments (PF), $3 \leq Z \leq 26$.¹ One experiment, scanned and measured at NRC, used 2.1 A GeV ^{16}O as the incident beam; the other, at LBL, used 1.88 A GeV ^{56}Fe .

The principal interaction characteristic examined in these data was the inclusive reaction mean free path (MFP), symbolized by " λ ". We have measured the MFPs of primary beams, and fit to these primary results a power law, $\lambda = \Lambda Z^{-b}$, where " Λ " is a parameter which is independent of the charge Z . For primary beams, $\Lambda = 30.4$ cm and $b = -0.44$. We have also measured the MFP parameter on PFs as a function of distance after emission (Fig. 1). As is readily observable, the MFP parameter Λ does vary as a function of distance after emission. The solid line in Fig. 1 is the result of a fit with 0.06 of the PFs having a constant MFP of 2.5 cm, independent of Z (which is a MFP for a nucleus on the order of Pb), and the rest of the PFs have the MFP values obtained from the primary beam fit. On the face of things, this hypothesis of a 6% admixture gives a reasonable fit to our observations. However, before taking it too seriously, one must question if the apparent shortening of Λ within the first few cm after emission might not be an artifact of the assumed relation between λ and Z which we have used to fit the MFP data.

To investigate this possibility, we construct a method which is independent of any fit or assumption about the functional dependence of λ on Z . The ratio of any two measured values of the MFP (for particles of the same intrinsic MFP) obeys the Fisher F-variance ratio distribution. Hence, we measure the MFP separately for each PF charge from each laboratory as a function of some fixed distance after emission. We form the ratio of the MFPs before and after this fixed distance; the probability of obtaining this ratio or a larger one from a pair of measurements should obey the uniform distribution $U(0,1)$. We calculate this probability from the F distribution, and choose the distance to be the 2.5 cm MFP of the "short MFP component" (dubbed "anomalous"). The result is shown in Fig. 2a. This histogram for the 24 PF charges at LBL and the 6 PF charges at NRC is clearly not the uniform distribution; indeed, the mean P value is 3.4 standard deviations (SD) away from its expected value $\langle P \rangle = 1/2$. Because of the way in which we have calculated the probability values, this corresponds to a significantly shorter PF MFP within the first 2.5 cm after emission, independent of any assumption concerning the expected value of any MFP.

In addition to examining the MFP as a function of distance after emission, we also consider the MFP as a function of the generation of the PF; we regard the incident beam nuclei as "primaries", their immediate PFs as "secondaries", the immediate PFs from secondaries as "tertiaries", etc. Proceeding via the F-test as we did above for distance but now by generation (secondaries versus tertiaryaries and later generations), we obtain the result in Fig. 2b. Here the mean P value is only about 2.1

SD away from its expectation; however, the largest P value recorded was only 0.778, and the binomial probability to observe 0 events in the uniform distribution out of 29 attempts with $P \geq 0.778$ is $7(10^{-4})$. We would expect about 6 events in this P region.

In the above discussion, we have considered total track length S_N observed for N interactions, whether or not all particles interacted. We may also limit consideration to those particles which did interact, neglecting those which did not. To investigate further the apparent shortening of the MFP by generation, we consider the topology in which a secondary interacted and emitted a tertiary which also interacted. Taking into account the expected MFP from the primary beam fit and the distance over which a particle could in fact have been observed to interact within our detectors (the potential path), one may calculate probability values $P_2(X)$ and $P_2(Y)$ for the secondary and tertiary particles respectively to have interacted within the observed distances X, Y, or shorter. These probabilities are independent because of the way in which we have calculated them; thus, a scatter plot of $P_2(X)$ vs. $P_2(Y)$ should populate uniformly the unit plane. Dividing each axis into two a priori equiprobable bins, we obtain the scatter plot, Fig. 3. We define a quantity C equal to the the number of counts in quadrant III minus the number in quadrant I all divided by the number in quadrant II plus quadrant IV. The expected value is $\langle C \rangle = 0 \pm 0.098$ from the multinomial distribution, whereas we observe $(74-46)/(47+47) = 0.298$; this value deviates by more than 3 SD from its expectation. Hence, we tentatively conclude that a secondary anomalon is more likely to give rise to a tertiary anomalon; we term this retention of the anomalon condition "memory".

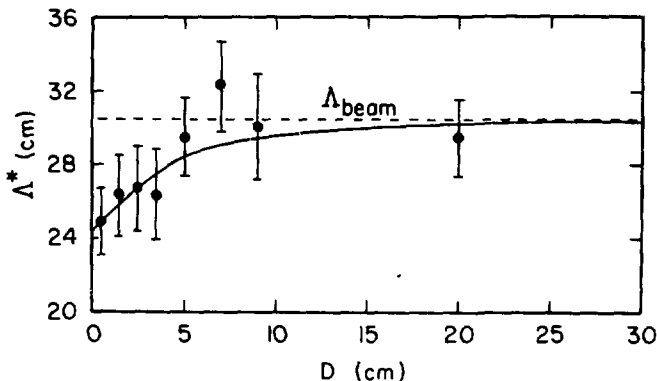
Standard concerns are those of coincident tracks, hyperfragments, and pionic atoms simulating PF interactions. To consider this, we again examine the topology: interacting secondary $\rightarrow$ interacting tertiary. We now consider the expected number of interactions as a function of the charge change ΔZ at the secondary-tertiary vertex. We construct R, the ratio of the observed number of interactions to the number expected from our primary beam fit. We find that $R(0 \leq \Delta Z \leq 1) = 1.25 \pm 0.09$, while $R(2 \leq \Delta Z \leq 5) = 1.29 \pm 0.10$. It is obvious that these potential background channels would enhance $R(0 \leq \Delta Z \leq 1)$, but the fact that $R(2 \leq \Delta Z \leq 5)$ has essentially the same value argues against such background contamination in the data.

As part of a new experiment, we are examining the above questions with more data, as well as exploring the relationship of anomalon production to the fragmentation patterns of both the projectile and the target. We also hope to be able to determine the lifetime and decay modes of anomalons. In addition, we are extending our results to PFs of $Z=2$; preliminary results on $Z=2$ PFs seem to have the same trend as the PF data reported above.

In conclusion, we find: a) over the first several

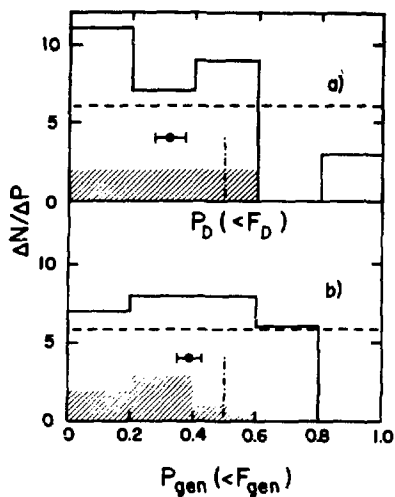
centimeters after emerging from an interaction, PFs are observed to have shorter MFPs than those obtained from primary beam nuclei; b) at distances greater than ≈ 5 cm, PF MFPs are compatible with our primary beam measurements; c) under the assumption that there are two populations of PFs, normal and anomalous, a best fit to the data is obtained with $\approx 6\%$ of the PFs having the anomalously short MFP ≈ 2.5 cm; and d) the anomalous property of PFs persists (shows "memory") in subsequent fragmentation reactions.

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NBL 808-11485

Fig. 1. Estimates Λ^* for the parameter Λ at different distances D from the origins of PFs: full circles, experiment; dashed line, prediction from Λ as fit to primary beams; solid line, fit assuming a 6% admixture of PFs with a constant 2.5 cm MFP.



JPL 808-10280

Fig. 2. Experimental frequency distribution of (a) $P_D(F_D)$ and (b) $P_{\text{gen}}(F_{\text{gen}})$; see text; the dashed line is the expected uniform distribution; the points with error bars are the experimental means $\bar{P}$, to be compared to their expectation $\langle P \rangle = 1/2$; the shaded area refers to the results from NRC.

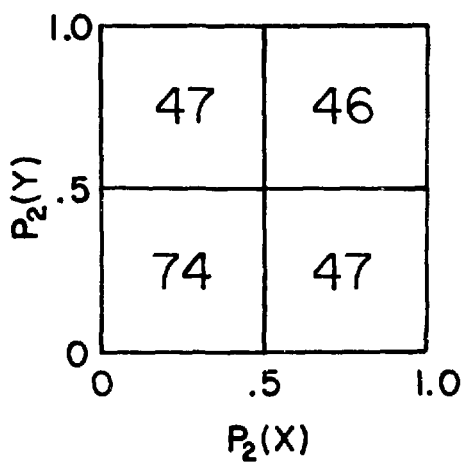


Fig. 3. The scatter plot of the $P_2(X)$ distribution versus the $P_2(Y)$ distribution.

ON THE POSSIBILITY OF TOROIDAL NUCLEAR ISOMERS

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Simple models suggest that toroidal configurations of light quarks are more stable than spherical configurations for $A > 4$. The baryon number density of such configurations could be comparable to that in ordinary nuclei, which suggests that toroidal "quark nuclei" may be fairly easily produced during relativistic central collisions of nuclei. Such tori would enjoy significantly larger cross-sections for nuclear interactions than ordinary nuclei and may possibly be identified with the nuclear fragments with anomalously small mean free paths found by Friedländer.

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DEMON DEUTERONS

A Heavy Ion-Study

by

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Abstract: It is suggested that the deuteron can be excited by strong interaction to an almost stable state consisting of three diquarks. We speculate that such an exotic colour configuration might have a high cross section and be related to the abnormal nuclear fragments found both in cosmic rays and here at the LBL.

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This talk will contain the results of some loose, wild, fragmentary and preliminary thinking inspired partly by the Judek-LBL "anomalons" ¹⁾, partly by a piece of highly unofficial data rumoured to us from the Dubna-Strasbourg group working with α p collisions at 15.5 GeV/ α in a Dubna bubble chamber. There it has been found that the deuterons produced as projectile fragments have a mean free path in the bubble chamber which is considerably shorter than that of normal beam deuterons.

We do not have any further comments on experimental data. Rather we will take it for granted in this talk that there exist anomalons among nuclear fragments, and in particular that there is a deuteron-like object with very strange properties.

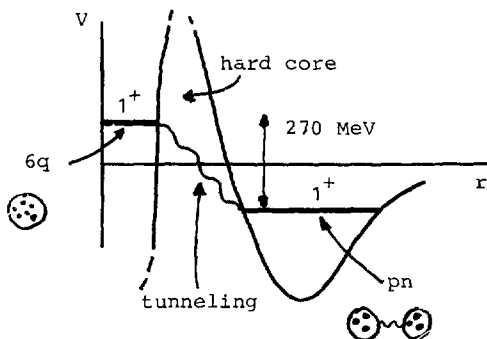
Quite a few questions about those demon deuterons must now be answered:

- Q1) What are they?
- Q2) How are they formed?
- Q3) Why are they long-lived enough to be seen ($\sim 10^{-10}$ s)?
- Q4) Why are they so large (perhaps a few hundred mb)?
- Q5) how do they relate to the "normal" Judek anomalous (with charge $Z \geq 3$)?

For all these questions we have so far only speculative answers, none of them well underbuilt with numerical calculations or fits to data.

A1) We believe they are deuterons excited to some of the six-quark states of a bound two-nucleon system.

To illustrate this we will recapitulate the bag model picture of a normal deuteron²⁾. There the deuteron is to 93.8% a bound pn state, to 0.6% a bound $\Delta\Delta$ state and to 5.6% a collective 6q state. If one prefers a potential picture, the 6q state is what one gets when the nucleon tunnels through the hard core, or when the system is excited by more than around 270 MeV. This corresponds to pushing the two bags together to form one single bag:



To explain why fragment deuterons are on average different from normal ones one clearly needs a production mechanism which gives them a strongly enhanced percentage of those $6q$ states.

A2) One can think of two completely different formation mechanisms. The first one has been suggested by Y. Karant³⁾: At the time of knock-out, the deuterons (or any fragment inside a nucleus) already have abnormal rates of exotic quark states. This comes about because the deuteron potential is deformed due to the presence of the other nuclear matter. Either the core becomes softer, the well deeper, or both.

We do not believe in that explanation, because the presence of exotic systems inside a nucleus would probably show up in the behaviour of the nucleus itself and not only in its fragments. Instead we think that the knock-out process helps the deuteron to fall into its $6q$ state:



These two ideas lead to different predictions for the angle distribution of anomalons. In our picture a momentum transfer is needed, so we predict that anomalons do not appear among evaporation products and hence have a broader angle distribution than all fragments taken together. We also predict anomalon production in processes like $dp \rightarrow dX$ and $pp \rightarrow dX$. Especially the first one should be more favourable to study than $\alpha p \rightarrow dX$, due to better statistics.

A3) The demon deuterons are long-lived because the $6q$ configuration ends up in a state with a mass which is too low to allow a strong decay to (pn) .

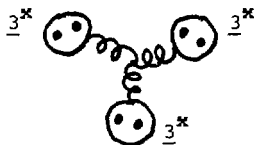
There are two suggestions for what that state could be. Karant's idea³⁾ is that it is a dibaryon state consisting of

two coloured $3q$ objects, each being an $\underline{8}$ representation of $SU(3)_{\text{colour}}$, forming a total singlet:



The idea is that once the $6q$ (one-bag) state has been formed it might snap back into the normal two-body deuteron via this dibaryon state. The problem is that its mass in the bag model is around 2.3 GeV, i.e. above the strong decay threshold. Again, the fact that the deuteron potential is deformed inside a nucleus might save the situation; the effective fragment $\underline{8}$ - $\underline{8}$ mass could be smaller than 2.3 GeV.

In our model there is a wider choice of states, because they are formed by strong knock-out processes, and hence not restricted to having $J^P = 1^+$. It turns out that there are states with $J^P = 0^-, 2^-, \dots$ consisting of three diquarks, each being a coloured $\underline{3}^*$ object, and with a mass around 2.0 GeV:



The low mass is due to a balancing of the mass term from the number of point-like objects in the bag with the term from the contact colourmagnetic interaction. The sum minimizes for the $\underline{3}^*-\underline{3}^*-\underline{3}^*$ configuration. The system will require a third-order electromagnetic interaction to decay to (pn) , which explains its long life-time.

A4) To get the high cross section the demon deuterons presumably need to be both geometrically large and have violently reacting constituents. The size of the $\underline{8}$ - $\underline{8}$ could be due to the fact that the confinement force is weaker than in

the $\underline{3-3}^*$ and $\underline{3-3-3}$ cases, while the diquark deuteron could be large because it has an even J and consists of three bo-
sons. Once these objects are large the anomalon-proton cross section is much more sensitive to the colour composition of the constituents. The octet-singlet and triplet-singlet interactions are stronger than that between two singlets.

A5) There are three ways to understand also the heavier anomalons once the demon deuterons are blamed on exotic colour excitations. The deuteron-like object might simply be so large that it dominates the total cross section even if it belongs to a heavier fragment. Another option is that a diquark or octet system might use its strong interaction to polarize the nuclear matter around it. A diquark, for instance, could try to compensate its colour with neighbouring nucleons, so that the whole fragment will end up as a three-centre system, each centre bearing the colour of the original diquark. Finally, one can also imagine that the first collision turns the whole fragment into octet or diquark matter. That would correspond to a sort of phase transition in nuclear matter.

We are grateful to G. Chapline, H. Heckman, Y. Karant and R. Stenbacka for inspiring discussions. One of us (S.F.) wants to thank L. Schroeder, LBL, and G. Eilam, R. Hwa and M. Moravcsik, University of Oregon, for hospitality during his stay in Berkeley and Eugene, where part of this work was done. Financial support from the LBL, the University of Oregon, the Wallenberg Foundation and the Swedish Natural Science Research Council is gratefully acknowledged.

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Evidence for two different reaction mechanisms in
relativistic heavy ion collisions

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(presented by E. Schopper)

An exclusive-type study of the transverse momentum distribution of α -particles emerging as relativistic projectile fragments from collisions of a 1.9 GeV/nucleon Fe^{56} -beam in nuclear emulsions reveals that the events fall in either one or the other of two distinct classes with sharply differing characteristics. Each of these classes is characterized by a Maxwell distribution in the projectile rest-frame, but the associated temperatures differ by a factor of ~ 4 (viz ≈ 10 MeV and ≈ 40 MeV). The same 40 MeV temperature is observed among the target related α -particles emitted from high-multiplicity selected C^{12} collisions in nuclear emulsion at 0.87 GeV/nucleon and 2.1 GeV/nucleon. We show that the "hot" ($T \approx 40$ MeV) events are not simulated by "projectile bounce" effects.

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Relativistic α -particles emitted in Fe-nucleus interactions at 1.7 A GeV.

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Relativistic α -particles have been studied in Fe-emulsion interactions at 1.7 A GeV. Comparisons of the observed angular distribution with that from ^{16}O -emulsion reactions at 2.1 A GeV reveal that comparatively more α -particles are observed at large angles in the Fe-emulsion reactions. The α -particles with large emission angles cannot be explained by fragmentation from a clean cut spectator. Comparison of the experimental data with moving relativistic Boltzmann distributions shows that a single Boltzmann distribution cannot fit the fragmentation peak and the tail simultaneously. A thermal source (fireball) explaining the tail part of the distribution need to be formed by a mechanism different from a simple clean cut participant-spectator picture. A transverse momentum transfer to the spectator before fragmentation may explain the tail.

THE EXPERIMENT¹⁾

423 ^{56}Fe -emulsion interactions at 1.7 A GeV have been measured for the study of produced relativistic α -particles. These interactions form a random subsample chosen from a collection of 1680 ^{56}Fe -interactions obtained by track scanning, the details of which are discussed elsewhere²⁾. Relativistic α -particles making angles upto 10 degrees with the beam have been identified and their angles, θ_α , are measured. For identification ionization measurements are made near the interaction and also when the tracks have traversed through a distance of 2 cm. If the ionization is four times minimum ionization and it does not change in the second measurement the track is identified as due to a relativistic α -particle.

THE RESULTS.

Fig. 1 shows the measured angular distributions for relati-

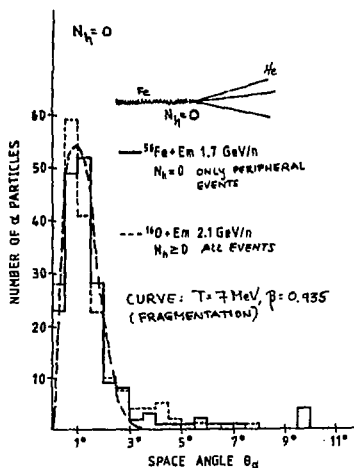


Fig.1. Angular distributions of relativistic α -particles.

vistic α -particles. We compare here peripheral $^{56}\text{Fe}+\text{Em}$ reactions ($N_h=0$) with unbiased $^{16}\text{O}+\text{Em}$ reactions ($N_h>0$ i.e. all collisions)³⁾. The angular distributions are similar.

For comparison we performed calculations based upon a gaussian distribution of momentum in the projectile rest frame (compare Greiner et al.⁴⁾). The predictions differ negligibly from a Boltzmann distribution if the width σ_0 of the gaussian distribution is taken to be $\sigma_0^2 = m_\alpha T$. We can draw the following conclusions:

- 1) Unbiased $^{16}\text{O}+\text{Em}$ reactions ($N_h>0$) exhibit distributions expected from projectile fragmentation. $^{56}\text{Fe}+\text{Em}$ reactions show this behaviour only in peripheral reactions ($N_h=0$).
- 2) Moving relativistic Boltzmann distributions and that given by Greiner et al.⁴⁾ are quite similar at this temperature.
- 3) Very few α particles emitted by pure fragmentation of the projectile have angles greater than 3° (at an incident energy of 1.7 A GeV). This has prompted us to define a tail to peak ratio R_{tp} .

$$R_{tp} = \frac{\text{Number of } \alpha \text{ particles with } 10^\circ \geq \theta_\alpha > 3^\circ}{\text{Number of } \alpha \text{ particles with } \theta_\alpha \leq 3^\circ}$$

R_{tp} is a measure of the contribution of relativistic α -particles which are not produced by fragmentation of a clean cut spectator.

We find that

$$R_{tp} = 0.101 \pm 0.024 \text{ for Fe reactions with } N_h = 0 \text{ (peripheral collisions)}$$

$$\text{and } R_{tp} = 0.09 \pm 0.021 \text{ for } ^{16}\text{O} \text{ reactions with } N_h \geq 0 \text{ (all collisions).}$$

Fig. 2 shows the measured angular distribution of relativistic α -particles in all the 423 interactions. This distribution exhibits a broader peak and a much larger tail than the corresponding distribution observed in $^{16}\text{O}+\text{Em}$ reactions i.e. unbiased $^{56}\text{Fe}+\text{Em}$ reactions ($N_h \geq 0$) have a substantial number of α -particles with emission angles much larger than expected from fragmentation of a clean cut spectator.

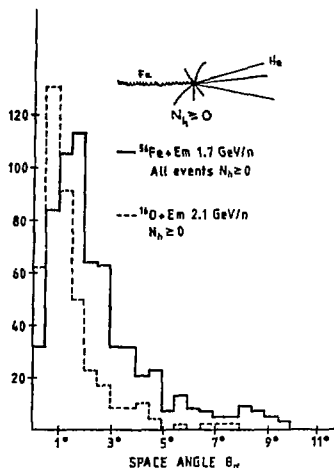


Fig. 2. Angle distributions for $N_h \geq 0$ events. The dotted histogram is the corresponding angular distribution for ^{16}O -emulsion reactions at 2.1 GeV/n. The distributions are normalized to the same number of α particles in the 0° - 2° range.

many sets of T and β values which give a good fit. The results are given in the β , T plot of Fig. 4a (Exp.fit.). Here we can see that if we insist on a velocity β equal to β_{beam} then a large value of the temperature T is required for the moving relativistic Boltzmann distribution. On the other hand the tail can also be fitted to a smaller value of

The angular distributions for two N_h groups i.e. $1 < N_h \leq 6$ and $N_h > 6$ are given in Fig. 3. One observes a slightly broader fragmentation peak for the $1 < N_h \leq 6$ group (Fig. 3b), whereas, the tail seems to be larger for the $N_h > 6$ group (Fig. 3a). $R_{\text{TP}} = 0.29$ for $N_h \leq 6$ events and $R_{\text{TP}} = 0.54$ for $N_h > 6$ sample.

PROCEDURE OF ANALYSIS

As an attempt to understand the production of relativistic α -particles the experimental distribution is fitted to that originating from a fireball moving with velocity β (in units of c) and heated to a temperature T (MeV). No assumption is made regarding the process of the formation of the fireball or the thermal source.

The calculations of the distribution were done using moving relativistic Boltzmann distributions.

The experimental distribution between 3° and 10° , termed as the tail of the angular distribution, is first fitted to moving relativistic Boltzmann distributions with β and T as free parameters. There are

the temperature but then the β value needed will be appreciably smaller than that of β_{beam} . In the same graph another curve (Kin.fit) is displayed which gives the 4-momentum conservation constrain if it is assumed that N_p and N_t nuc-

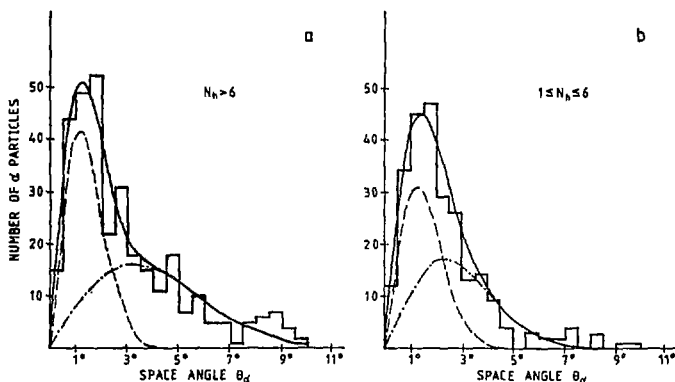


Fig.3. Angle distributions for $N_h > 6$ and $1 < N_h < 6$ samples respectively (dot-dashed curves are fit to the peak part, dashed to the tail part, and the full curve is the sum of these two).

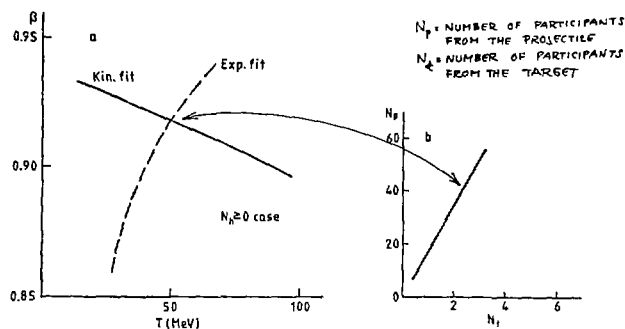


Fig.4. (a) The full line curve is due to kinematically allowed values of β and T whereas the dashed curve is from fitting of experimentally observed angular distribution with moving Boltzmann distributions.

(b) shows various N_p , N_t values corresponding to the point of intersection in Fig. 4(a).

leons participate from the projectile and target respectively and that all available kinetic energy in the nucleon common cms is completely randomized through elastic collisions. The intersection of the two curves gives us the kinematically allowed values of β_t and T_t of the thermal source which give fit to the experimental tail of the angular distribution.

The Boltzmann distribution corresponds to β_t and T_t is extrapolated for angles between 0° and 3° . The extrapolated distribution is subtracted from the experimental one. The resulting distribution is thereafter fitted to a moving Boltzmann distribution, assuming the velocity, β_p , to be equal to the velocity of the projectile. The best values of T_p is 10 - 14 MeV. It would be emphasized here that the peak can also be fitted to a lower temp (7-8 MeV), but in that case the peak velocity β_p will have to be lowered.

DISCUSSIONS.

A. We observe that the peak temperature is greater than 7-8 MeV in all the cases except in the $N_h = 0$ sample. This shows that the projectile is heated slightly more than the one which can be understood from a clean cut spectator fragmentation.

The calculated values of T_t and β_t corresponds to a rather peculiar fireball which is kinematically allowed. As shown in Fig. 4 (b) the allowed values of N_p and N_t suggest that in such a fireball only a few nucleons from the target participate which shows that a clean cut geometrical picture for participants and spectators cannot explain the tail of the angular distribution.

B. α clusters of the Fe projectile could scatter from individual nucleons and be observed at large angles i.e. in the tail area of the angular distribution. In order to explore this possibility Monte Carlo simulations were performed.

Based upon these calculations one can say that qualitatively α clusters scattered from nucleons could contribute to the observed relativistic α particles at large angles. However, the shape of the spectra is not in accordance to the experimental data.

C. Another possibility could be that these α -particles are produced in a secondary process. In the first step short lived fragments of the projectile are produced with large transverse momentum, the relativistic α -particles are subsequently produced by fragmentation of these fragments.

D. Recently the large angle α -production has been interpreted as a direct evidence for a highly collective mechanism⁵⁾.

CONCLUSIONS

The α particles with large angles cannot be explained by fragmentation from a clean cut spectator. Comparison with moving relativistic Boltzmann distributions shows that a simple Boltzmann distribution cannot fit the fragmentation peak and the tail simultaneously. A large transverse momentum transfer may explain the tail.

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DECONFINEMENT IN DENSE MATTER

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GermanyAbstract:

We survey recent developments in the study of strongly interacting matter by means of quantum chromodynamics on the lattice. The Monte Carlo evaluation of $SU(2)$ Yang-Mills systems is shown to provide a unified thermodynamic description from the hadronic regime through the deconfinement transition to a non-interacting gluon gas. We apply our results to determine physical deconfinement parameters and discuss possible signals for the formation of a quark-gluon plasma in relativistic heavy ion collisions.

...replied Alice; "and I wish you wouldn't keep appearing and vanishing so suddenly: you make one quite giddy."

"All right," said the Cat; and this time it vanished quite slowly, beginning with the end of the tail, and ending with the grin, which remained some time after the rest of it had gone.

Lewis Carroll, ALICE'S ADVENTURES IN
WONDERLAND

I. INTRODUCTION

With increasing density, hadronic matter is expected to undergo a transition to quark matter. With sufficient overlap or interpenetration of hadrons, the constituent quarks will no longer remember to whom they belong or where they come from. The resulting plasma of quarks and gluons presents us with a state of matter physically different from anything we have so far explored in the laboratory. Our universe in the first 10^{-5} seconds after the big bang probably consisted of such a plasma, in equilibrium with photons and leptons; only after this brief pregenesis interlude was the void created and hadrons appeared.

It is the prospect of simulating such a process in the laboratory, of creating quark matter and watching it "freeze" into hadrons, that makes relativistic heavy ion collisions a subject of such fundamental interest.

The idea of a phase transition from hadronic to quark matter is as old¹⁾ as the quark structure of hadrons. Since then, a great variety of more or less phenomenological approaches to the two-phase nature of strongly interacting matter have been pursued²⁾. They all have in common two phases as input. The advent of quantum chromodynamics (QCD) gave rise to the hope that both the two-phase character and the transition might be obtained from one basic theory. The exciting developments of the past

year seem to indicate that this hope is justified³⁻⁵⁾, and so the major part of my talk will be devoted to QCD results on the "deconfinement" transition in strongly interacting matter.

Once we know that QCD does predict the formation of quark matter: what are the transition parameters, and what are the observable signals we may expect as consequence of quark matter formation in heavy ion collisions? Since the detectors in our laboratories register only hadrons, photons and leptons, the little bang in a heavy ion collision, just like its cosmological big brother, will leave us with no more than "the smile of the absent cat". In the second part of my talk I shall give some parameter values for the transition and elaborate a little on ways to recognize and interpret the smile.

II. THE TRANSITION TO QUARK MATTER IN QCD

A Overview

Quantum chromodynamics specifies the basic interaction of quarks and gluons; from this we are to obtain the description of strongly interacting matter in its different states. Not surprisingly, the first attempts concentrated on limiting behaviour.

Asymptotic freedom makes interactions at very short distances (or high momenta) arbitrarily weak, so that a perturbation expansion in powers of the effective coupling is expected to converge in the limit of high temperature or density⁵⁾. Sufficiently hot and/or dense matter should therefore become a gas of non-interacting quarks and gluons.

In the confinement region, at lower temperatures and densities, strongly interacting matter should exhibit hadronic behaviour. One has here considered on the lattice an expansion in terms of the inverse coupling (strong coupling expansion⁷⁾) or used in the continuum semi-classical solutions to the field equations (instanton gas⁸⁾). The resulting descriptions indeed proved many aspects of hadron phenomenology, as given e.g. by dual string or bag models.

Strong and weak coupling approaches have evidently specified regions of applicability and thus basically give one-phase descriptions. Nevertheless, they already provide hints for a phase transition near the boundary of their regions of validity. In the perturbative treatment of the quark-gluon gas, the pressure (in first order) becomes negative at some temperature value, and this has been interpreted as the onset of confinement⁹⁾. In the strong coupling expansion, there are indications for a phase transition due to Debye screening of colour charges¹⁰⁾. The suppression of large scale instantons leads to similar conclusions¹¹⁾. It is clear, however, that these limiting approaches cannot give us the unified "whole-range" description we would like to obtain from a basic theory.

The Monte Carlo evaluation of Yang-Mills systems on the lattice now for the first time provides us with such a unified picture³⁻⁵⁾. The method itself was devised for and first applied to the study of the confinement problem¹²⁾. Its use in finite temperature statistical mechanics has followed two slightly different lines. We can either consider the thermodynamics of a pure Yang-Mills system⁵⁾, as non-Abelian extension of black body radiation, or we can study the effect of a finite temperature Yang-Mills heat-bath on the interaction of a static quark-antiquark pair as "external" colour sources^{3,4)}. In this presentation, the emphasis will be on the former: the interacting gluon system gives us within a conventional statistical mechanics framework most of the phase-structure information and in addition furnishes a very physical and clean-cut way of determining the transition temperature T_c .

The Monte Carlo evaluation of lattice gauge theories, both for confinement and for statistical mechanics studies, has up to now essentially been restricted to Yang-Mills fields - i.e., to gluon systems without quarks. The reason for this is of technical nature: in the path integral formulation employed for the Monte Carlo evaluation, the integration over fermion fields because of the antisymmetrization becomes much more complex than that over boson fields. Hopefully, however, the restriction to Yang-Mills systems is not so serious, both qualitatively and quantitatively: since gluons mediate the interaction between themselves in

much the same way as they do between quarks, we have reason to expect that they provide us with a realistic replica of the complete world. Moreover, ways to include fermions in the Monte Carlo approach have been suggested^{13,14)} and first results tend to follow the pure Yang-Mills pattern. Further work on this is presently in progress and further results are expected soon.

The Monte Carlo evaluation of finite temperature Yang-Mills thermodynamics on the lattice has so far produced three main results: there is a deconfinement phase transition at a critical temperature T_c ³⁻⁵⁾; sufficiently far above T_c , the system behaves like an ideal gluon gas^{5a)}, and below T_c it follows the thermodynamics of a hadron gas as given by bag or string models^{5b)}. In the next section we shall therefore first recall briefly the thermodynamic behaviour of an ideal gluon gas and of a hadronic resonance gas, before we present in section C the results of the Monte Carlo evaluation.

B Ideal Gluon Gas and Hadronic Resonance Gas

The statistical mechanics of an ideal gas of massless bosons, such as photons, starts from the partition function

$$\ln Z(\beta, V) = \frac{g V \beta}{(2\pi)^3} \int d^3k \ln \left[\frac{1}{1 - e^{-|k|\beta}} \right] . \quad (2.1)$$

Here $\beta^{-1} = T$ is the temperature, with the Boltzmann constant set equal to unity, V the spatial volume, and g the number of internal degrees of freedom (for photons, $g = 2$ from the two spin directions). In general, the partition function would also depend on the chemical potential μ ; for free massless particles, however, thermal equilibrium requires $\mu = 0$.

From the above $Z(\beta, V)$ we obtain for the energy density ϵ the well-known Stefan-Boltzmann form

$$\epsilon(\beta) \equiv \lim_{V \rightarrow \infty} \frac{-1}{V} \left(\frac{\partial \ln Z}{\partial \beta} \right)_V = \frac{\pi^2}{30} g \beta^{-4} . \quad (2.2)$$

The pressure

$$p(\beta) = \lim_{V \rightarrow \infty} \frac{1}{V} \left(\frac{\partial \ln Z}{\partial V} \right)_{\beta} = \frac{\pi^2}{90} g \beta^{-3}. \quad (2.3)$$

fulfills

$$p = \frac{1}{3} \varepsilon, \quad (2.4)$$

corresponding to a "limiting" speed of sound $c_0^2 = 1/3$. Finally, we note

$$c_V(\beta) = (\partial \varepsilon / \partial \beta^{-1})_V = \frac{2}{15} g \beta^{-3} \quad (2.5)$$

for the specific heat at constant volume.

An $SU(2)$ gluon system has three colour and two spin degrees of freedom, giving $g = 6$ and hence

$$\varepsilon(\beta) = \frac{\pi^2}{5} \beta^{-4} \quad (2.6)$$

as the corresponding Stefan-Boltzmann form.

For the statistical mechanics of hadronic matter, we consider an ideal gas of ground state hadrons and all their resonance excitations. In full QCD, hadrons and resonances can be constructed both from quarks (mesons, baryons, and their resonances) and from gluons (glueballs and excited gluonium states). In a pure Yang-Mills world, we are left with only glueballs and their excitations; we assume that these behave in the conventional hadronic way. The partition function of a resonance gas is

$$\ln Z(\beta, V) = \frac{V}{(2\pi)^3} \int_0^\infty dm \tau(m) \int d^3k e^{-\beta \sqrt{k^2 + m^2}} \quad (2.7)$$

for a system with zero chemical potential and, for simplicity, with Boltzmann statistics. From hadron dynamics (dual string¹⁵⁾, bag¹⁶⁾ models, the resonance spectrum $\tau(m)$ is known to have the form

$$\tau(m) = d \delta(m - m_0) + c \theta(m - 2m_0) m^{-a} e^{bm}, \quad (2.8)$$

$a, b, c, d = \text{const.}$

as first proposed by Hagedorn¹⁷⁾. The first term in eq. (2.8) corresponds

to a d-fold degenerate ground state; for $c = 0$, we would thus simply obtain an ideal gas of ground state hadrons. While a depends on the details of the model used¹⁸⁾, b is related quite generally to the string tension σ ($b^{-2} = 4\pi/3\sigma$ in four dimensions) or equivalently to the Regge slope.

From eq. (2.7) and (2.8) we obtain for the energy density of the resonance gas

$$\epsilon(\beta) \simeq \epsilon_0(\beta) + \frac{c}{(2\pi\beta)^{3/2}} \int_{m_0}^{\infty} dm m^{-a+5/2} e^{-m(\beta-b)} \quad , \quad (2.9)$$

with $\epsilon_0(\beta)$ denoting the energy density of an ideal gas of ground state hadrons m_0 . The corresponding specific heat becomes

$$c_V(\beta) \simeq c_V^{(0)}(\beta) + \beta \left\{ \frac{3}{2}(\epsilon - \epsilon_0) + \frac{c}{(2\pi\beta)^{3/2}} \int_{m_0}^{\infty} dm m^{-a+7/2} e^{-m(\beta-b)} \right\} \quad ; \quad (2.10)$$

again the first term describes the ideal ground state gas.

It is well-known¹⁹⁾ and from eq. (2.9/2.10) also immediately evident that the spectral form (2.8) leads to critical behaviour. Depending on the exact value of a , from some derivative of the partition function on we will have divergent expressions at and/or above the critical temperature $T_c = b^{-1}$.

To illustrate what happens, we choose $a = 4$, $c = d = 1$. The specific heat of the resonance gas then diverges at $T_c = b^{-1}$, while

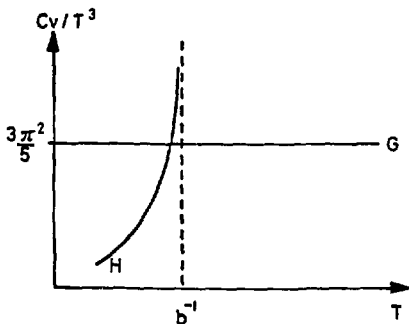


Figure 1 :

The specific heat of the hadronic resonance gas (H) and of the ideal gluon gas (G).

the energy density remains finite there. Both are not defined for $T > T_c$.

In fig. 1, we compare the specific heat of the ideal gluon gas with that of the hadronic resonance gas. Our hope is that QCD will give us a unified description which at low T follows the hadron curve, at high T converges to the Stefan-Boltzmann form for the ideal gluon gas, and in between gives a phase transition. In the next section we shall find these expectations indeed fulfilled.

C Gluon Thermodynamics on the Lattice

The partition function for a quantum system described in terms of fields $A(x)$ by a Hamiltonian $H(A)$ is defined as

$$Z = \text{Tr } e^{-\beta H} \quad , \quad (2.11)$$

with $T = \beta^{-1}$ is again the physical temperature. The conventional lattice formalism is obtained from this in three steps: (1) reformulation of Z as path integral; (2) introduction of the lattice; (3) change of "variables" from gauge field to gauge group. Let us look at this procedure in a little more detail.

The Lagrange density of gluon QCD is given by

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} \quad , \quad (2.12)$$

with

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g \lambda_{bc}^a A_\mu^b A_\nu^c \quad . \quad (2.13)$$

Here the λ^{abc} are the generators of the relevant underlying gauge group $\{[A_\mu^a, A_\mu^b] = \lambda_c^{ab} A_\mu^c\}$; for SU(2), the colour indices a, b, c each run from one to two, for SU(3) from one to three. If we set the generators equal to zero, we recover the photon gas structure: it is the non-Abelian nature which gives us the gluon-gluon interaction. The partition function Z can now be written²⁰⁾ in the form of a path integral

$$Z = \int \mathcal{D} A \exp - \left\{ \int_0^\beta dt \int_V d^3x \mathcal{L} \right\} \quad , \quad (2.14)$$

using the Euclidean Lagrange density, $\mathcal{L}$ with $it = \tau$, and with periodicity in τ , $A(\vec{x}, \tau) = A(\vec{x}, \tau + \beta)$. The three-dimensional integral of the Hamiltonian form ($H \sim \int d^3x \mathcal{H}(x)$) thus becomes an asymmetric four-dimensional one, with the "special" dimension measuring the temperature.

In the next step, we replace the Euclidean $x - \tau$ continuum by a finite lattice²¹⁾, with N_σ sites and spacing a_σ in the spatial part, N_β sites and spacing a_β in the temperature direction. To assure the required periodicity in τ , we choose a lattice closed on itself: $i = i + N_\beta$ for all sites i on the τ axis. For economy in the later calculations, we work with lattices which are symmetric and also periodic in the space part, although neither property would be necessary. - The integrals in the exponent of eq. (2.14) now become sums, and we have $V = (N_\sigma a_\sigma)^3$, $\beta = N_\beta a_\beta$. The thermodynamic limit requires $N_\sigma \rightarrow \infty$ at fixed a_σ ; the continuum limit is then obtained by $a_\sigma, a_\beta \rightarrow 0$ with fixed $N_\beta a_\beta$, which forces also $N_\beta \rightarrow \infty$. The success of the approach rests on the (lucky) fact that already rather small lattices ($N_\sigma \sim 5-10$, $N_\beta \sim 3-5$) seem to be asymptotic, and that scale changes (changes in lattice spacings) can by the renormalization group method be achieved through changes in the coupling strength g .

In the last step, we replace the gauge field "variable" $A_\mu((x_i + x_j)/2)$ associated to the link between two adjacent sites i and j by the gauge group element

$$U_{ij} = \exp \left\{ \frac{i}{2} g(x_i - x_j)^\mu A_\mu^a \left(\frac{x_i + x_j}{2} \right) \sigma_a \right\} \quad ; \quad (2.15)$$

here the σ_a are (for SU(2)) the usual Pauli matrices. With this transformation, the partition function becomes^{5a)}

$$Z = \int \prod_{\{\text{links}\}} dU_{ij} \exp - \left\{ \frac{4}{g^2} S(U) \right\} \quad , \quad (2.16)$$

$$S(U) = \frac{a_\beta}{a_\sigma} \sum_{\{P_\sigma\}} \left[1 - \frac{1}{2} \text{Tr}(U_{ij} U_{jk} U_{kl} U_{li}) \right] + \frac{a_\sigma}{a_\beta} \sum_{\{P_\beta\}} \left[1 - \frac{1}{2} \text{Tr}(U_{ij} U_{jk} U_{kl} U_{li}) \right] \quad (2.17)$$

Here the sum $\{P_\sigma\}$ runs over all purely spacelike lattice plaquettes $\{ijkl\}$, while $\{P_\beta\}$ runs over all those with two spacelike and two "temperature-like" links. - If we insert eq. (2.15) in eq. (2.16/2.17) and expand for small lattice spacings ($|x_i - x_j| \rightarrow 0$), then we recover in leading order the starting form (2.12/2.13).

From eq. (2.16/2.17), the energy density

$$\epsilon \approx \frac{-1}{V} \left(\frac{\partial \ln Z}{\partial \beta} \right)_V = \frac{-1}{N_\sigma^3 N_\beta a_\sigma^3} \left(\frac{\partial \ln Z}{\partial a_\beta} \right) a_\sigma \quad (2.18)$$

is found to be^{5a)}

$$\epsilon \approx 4(N_\sigma^3 N_\beta a_\sigma^3 a_\beta g^2)^{-1} \left\{ \left\langle \frac{a_\beta}{a_\sigma} \sum_{\{P_\sigma\}} \left[1 - \frac{1}{2} \text{Tr} UUUU \right] \right\rangle - \left\langle \frac{a_\sigma}{a_\beta} \sum_{\{P_\beta\}} \left[1 - \frac{1}{2} \text{Tr} UUUU \right] \right\rangle \right\} \quad (2.19)$$

with $\langle \rangle$ denoting the usual thermodynamic average

$$\langle X \rangle \approx \left\{ \int \Pi dU X e^{-4S/g^2} \right\} / \left\{ \int \Pi dU e^{-4S/g^2} \right\} \quad (2.20)$$

Eq. (2.19) is our starting point for the Monte Carlo evaluation of gluon thermodynamics.

The evaluation is now carried out as follows²²⁾. The computer simulates an $N_\sigma^3 \times N_\beta$ lattice; for convenience we choose $a_\sigma = a_\beta = a$. Starting from a given ordered (all $U = 1$, "cold start") or disordered (all U random, "hot start") initial configuration, successively each link is assigned a new element U' , chosen randomly with the weight $\exp \{-4S(U')/g^2\}$. One traverse of this procedure through the entire lattice is called one iteration. In general, we have found that five

hundred or so iterations provide reasonable first indications about the behaviour of the energy density (2.19), but for some precision one should have more. Differences in the first calculations of the critical temperature for deconfinement³⁻⁵⁾ appear to be partially due to this factor. The results given here are obtained with typically around three thousand iterations, after which we observe quite stable behaviour; we have moreover as a test checked and confirmed our results with finite subgroup approximations to $SU(2)$ ²³⁾. Our work was done with $N_\sigma = 7, 9, 10$ for $N_\beta = 2, 3, 4, 5$; apart from expected finite lattice size effects²⁴⁾ there was no striking N_σ dependence of ϵ , suggesting that in general the thermodynamic limit is reached. To give at least some intuitive grounds for this, note that a $10^3 \times 3$ lattice has about 12,000 link degrees of freedom.

As result of the Monte Carlo evaluation, we obtain for a lattice of given size (N_σ, N_β) the energy density ϵ as function of g . In the continuum limit, g and the lattice spacing a are related through

$$a\Lambda_L = (11g^2/24\pi^2)^{-51/121} \exp - (12\pi^2/11g^2) ; \quad (2.21)$$

this relation is found by requiring a dimensional parameter Λ_L to remain constant under scale changes accompanied by corresponding changes in coupling strength. Hence once we are in the region of validity of the continuum limit, eq. (2.21) gives us the connection between g and a . Since $(N_\beta a)^{-1}$ is the temperature in units of Λ_L , we then have the desired continuum form of $\epsilon(\beta)$. To check that at a given temperature we are indeed in the region of validity of the renormalization group relation (2.21), we calculate the energy density at fixed $T = (N_\beta a)^{-1}$ for different N_β and correspondingly different $g(a)$. In fig. 2, we show the results for $T = 500 \Lambda_L$ from $N_\beta = 2, 3, 4$ and 5 ; they agree very well.

Fig. 2 moreover provides us with our first conclusion on gluon thermodynamics: as is evident, the results from the Monte Carlo evaluation of Yang-Mills QCD at $T = 500 \Lambda_L$ agree quite well with the anticipated Stefan-Boltzmann form (2.6). Thus at this temperature (which

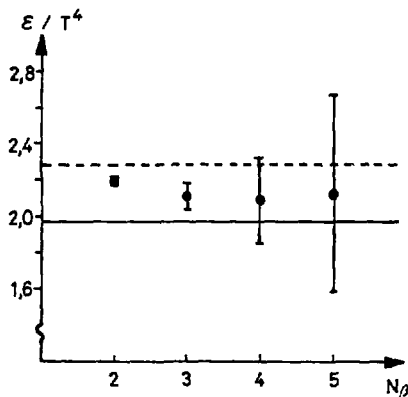


Figure 2 :
Energy density of the SU(2) Yang-Mills system at $T = 500 \Lambda_L$, for different temporal lattice sizes; the solid line is the Stefan-Boltzmann limit, the dashed line includes in addition perturbative corrections.

with physical values for Λ_L ²⁵⁾ becomes about 2.5 GeV), the interacting gluon system has already attained the high temperature limit corresponding to an ideal gas.

Let us now lower the temperature. The energy density then does not change much until about $T = 50 \Lambda_L$, when it drops sharply (fig. 3). The derivative of this curve gives us the specific heat, shown in fig. 4. It has a sharp singularity-like peak at $T_C \approx 43 \Lambda_L$, as signal of the transition from bound to free gluons. With the Λ_L of the physical world, this gives us $T_C = 215$ MeV. How do we know that it is the deconfinement transition which occurs here? There are two separate pieces of evidence. We shall see shortly that both energy density and specific heat of the SU(2) Yang-Mills system below T_C follow the behaviour of hadronic matter^{5b)}, as given in eq. (2.9) and (2.10). - In the alternative approach to our problem^{3,4)}, studying the behaviour of a static $q\bar{q}$ pair immersed in a gluon system of temperature T , the free energy F of an isolated quark serves to define $L = \exp -\beta F$ as order parameter. It is shown in ref 3) and 4) that L is essentially zero below and non-zero above T_C . Since $L = 0$ corresponds to an infinite free energy of an isolated colour source, we have confinement below T_C .

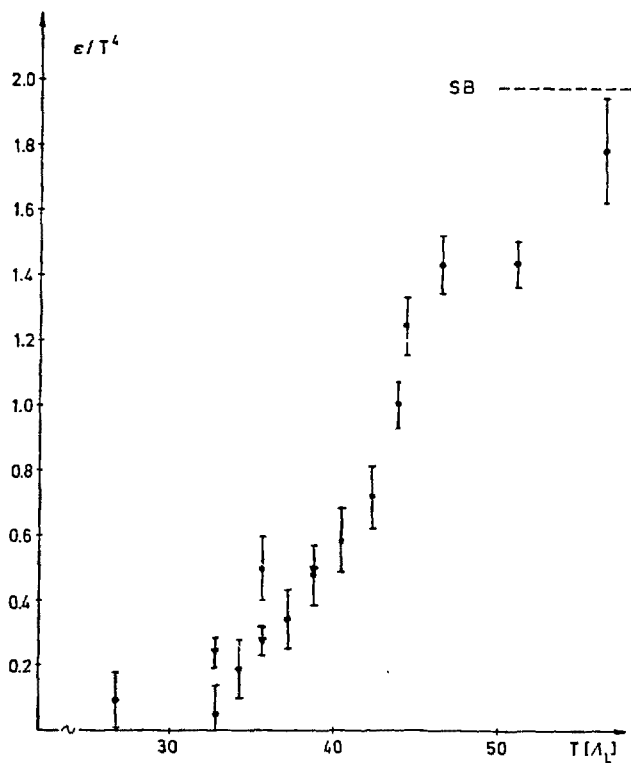


Figure 3 :

Energy density of the $SU(2)$ Yang-Mills system in the region of the de-confinement transition. The dashed line (SB) gives the Stefan-Boltzmann limit. Circles are from calculations using a $7^3 \times 3$ lattice, triangles from $9^3 \times 3$. From ref. 5b).

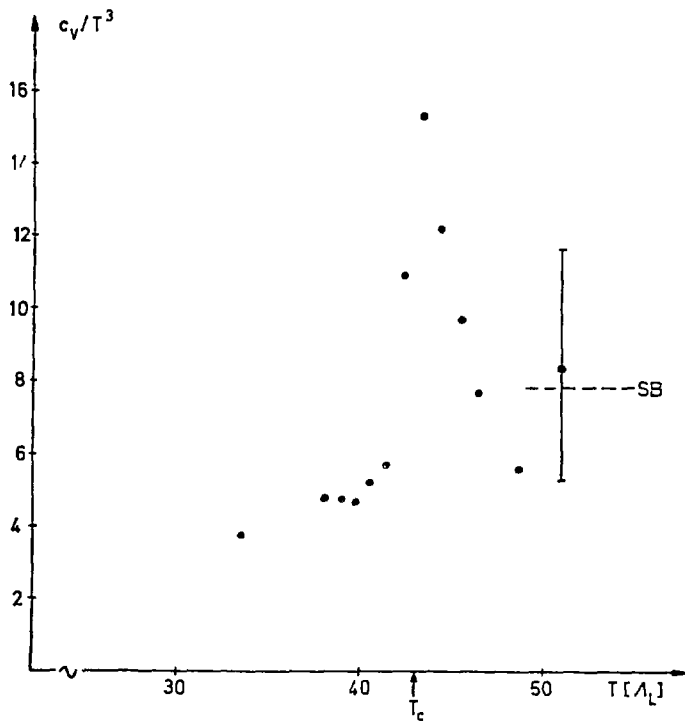


Figure 4 :

Specific heat of the SU(2) Yang-Mills system in the region of the de-confinement transition. The dashed line (SB) gives the Stefan-Boltzmann limit. The right-most point shows a typical error bar. From ref. 5b).

Coming now, as promised, to the temperature region just below T_c , we show in fig. 5 the energy density from the Monte Carlo evaluation compared to the hadronic resonance gas form (2.9), both as functions of $x = (T_c/T) - 1$. A word to the parameters entering eq. (2.9): $d = 6$, in accord with glueball models (spin zero and two for the ground state); c is determined by normalizing $\varepsilon(T)$ at T_c ; finally, it is found

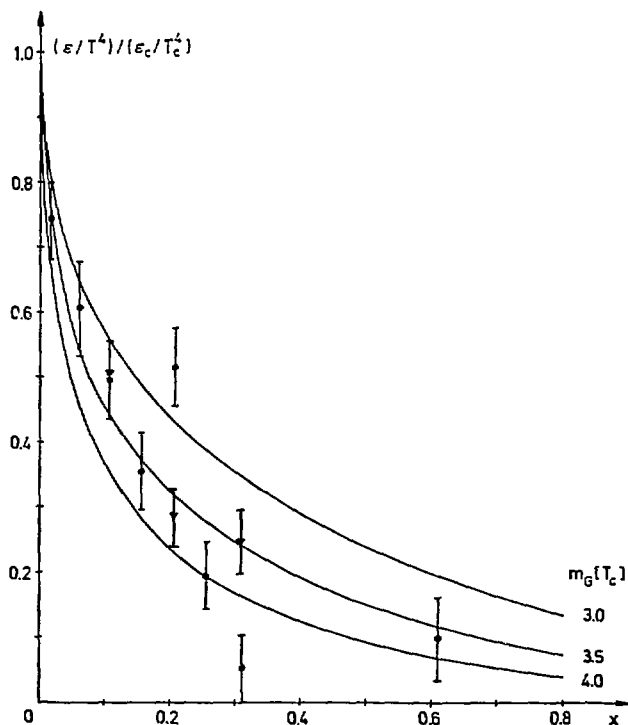


Figure 5 :

Energy density of the SU(2) Yang-Mills system below the deconfinement transition, as function of $x = (T_c/T) - 1$, for different glueball masses m_G ; circles are from calculations using a $7^3 \times 3$ lattice, triangles from $9^3 \times 3$. From ref. 5b).

that changes of a in a reasonable range ($7/2 < a \leq 9/2$ only gives $\epsilon(T_C)$ finite, $c_V(T_C)$ diverging) change the behaviour of ϵ only very little. Hence the comparison of eq. (2.9) with the Monte Carlo results, if it leads to agreement on functional behaviour, also allows us to determine the mass m_G of the glueball, as lowest gluonium state. We see from fig. 5 that eq. (2.9) and $m_G \approx 3.5 T_C = 150 \Lambda_L$ provides quite good agreement with the Monte Carlo data. Moreover, this value of m_G (with physical parameters about 750 MeV) is in reasonable accord with other lattice QCD determinations²⁶⁾.

In conclusion: we have seen that Monte Carlo techniques applied to lattice QCD allow us to evaluate gluon thermodynamics over the whole temperature range, and the resulting behaviour indeed shows the expected two-phase nature. At low temperatures, we have a hadronic resonance gas of gluonium states; heating brings us to a deconfinement transition and beyond that to an ideal gluon gas.

It is quite evident what the next steps should be in the approach just sketched for SU(2) gluon thermodynamics: the extension to SU(3) systems, and the inclusion of quarks. While the former seems feasible with present techniques²⁷⁾, the full quark-gluon thermodynamics may yet require some further computational progress²⁸⁾.

III. THE TRANSITION TO QUARK MATTER IN HEAVY ION COLLISIONS

A Transition Parameters

Before looking for possible quark matter signals in heavy ion collisions, we must determine at what temperature and density the deconfinement transition should occur and then convince ourselves that these values can be attained in the projected experiments.

Phenomenologically, there are various ways to estimate the transition point. Deconfinement may start

- when the density of strongly interacting matter becomes that of an individual nucleon;
- when the free energy of interacting nuclear matter becomes equal to

that of quark matter, taken as Fermi gas with bag pressure or some other type of interaction;

- when the quark cores of the hadrons form arbitrarily large clusters, i.e., "percolate";
- when the specific heat or the compressibility of hadronic matter in a given model becomes negative;

to name just some. The resulting transition densities cover quite a range. Consider the first picture in our list above: the charge radius of a nucleon is about 0.85 fm, which makes the hadron density 0.39 fm^{-3} and thus gives 2.3 times the standard nuclear density ($n_0 = 0.17 \text{ fm}^{-3}$) as transition point. On the other end, the MIT quark bag model compared to specific nuclear matter models gives 10-50 times the nuclear density for the transition. - The transition temperature for strongly interacting matter of zero chemical potential is generally thought to lie around 200 MeV, corresponding to the critical temperature in hadron thermodynamics.

The parameter values for the deconfinement transition in QCD are completely determined by the Monte Carlo evaluation. As long as this evaluation is carried out only for SU(2) Yang-Mills systems, any conclusion about transition points is, however, based on the assumption that these results will persist in an SU(3) world with quarks. More specifically: the lattice scale parameter Λ_L for the SU(2) theory of section II can be related to string tension¹²⁾ and this in turn to the Regge slope α' . If we here use the experimental value $\alpha' = 1 \text{ GeV}^{-2}$, we get $\Lambda_L \simeq 5 \text{ MeV}$. With this, we obtain for the transition temperature $T_C \simeq 43 \Lambda_L$ (see fig. 4)

$$T_C = 215 \pm 15 \text{ MeV} \quad , \quad (3.1)$$

where the error includes only uncertainties in the determination of T_C in terms of Λ_L . The corresponding mass for the ground state gluonium (glueball) is then with $m_G \simeq 3.5 T_C$ (see fig. 5) given by

$$m_G \simeq 750 \text{ MeV} \quad . \quad (3.2)$$

From fig. 3 we note that $\epsilon_C/T_C^4 \simeq 1$, so that

$$\epsilon_C = (280 \pm 80) \text{ MeV/fm}^3 \quad (3.3)$$

is the critical energy density. To obtain some feeling for this value, recall that the energy density of nuclear matter is 160 MeV/fm^3 , while that of a nucleon is 360 MeV/fm^3 . Alternatively, we can use the glueball as basic hadron and thus obtain

$$n_c = (0.37 \pm 0.10) \text{ fm}^{-3} = (2.2 \pm 0.6) n_0 \quad (3.4)$$

for the critical hadron density. Both eq. (3.3) and (3.4) again contain only the error in T_c , as given by eq. (2.1). We thus note that our SU(2) replica of the full world gives us transition parameters rather at the low side of the "phenomenological range" - with eq. (3.1) and (3.4), the deconfinement transition takes place at about twice nuclear density and for a temperature only slightly higher than the canonical Hagedorn temperature.

B Conditions in Heavy Ion Collisions

What are the energy density and temperature values which we may expect in heavy ion collisions? Before addressing this question, we recall a few empirical features from high energy hadron-hadron and hadron-nucleus collisions. The secondaries in these reactions are emitted over a comparatively long time scale ($\sim P_{\text{LAB}}$); in the overall c.m.s., the slow fragments appear first, the fast ones last. As a consequence, nuclei are rather transparent to incoming hadrons²⁹⁾: in the collision, the projectile is excited, but all except the target fragmentation secondaries are emitted outside the nucleus. The space-time development of such interactions does not produce a high density situation, because the secondaries do not appear simultaneously. Nevertheless, the interaction of a hadron with a heavy nucleus is more intense than that between two hadrons, as attested by a roughly three times higher multiplicity³⁰⁾. In the collision with a massive nucleus, a hadron thus experiences about three times more interactions than in the corresponding hadron-hadron collision, and thereby leads to about three times more secondaries per rapidity interval, at least in the central and in the target fragmentation region.

In a head-on collision between two nuclei with A nucleons each, we then have as lower bound A superimposed pp collisions, leaving aside for the moment the mentioned nuclear enhancement factor. The Lorentz

contraction of the two nuclear volumes in the overall c.m.s. assures us that these A collisions occur indeed simultaneously. They take place over the nuclear cross section $\pi(1.2 A^{1/3})^2 \text{ fm}^2$, so that per nucleon cross section $\pi(0.85)^2 \text{ fm}^2$ we have (see fig. 6)

$$A(0.85)^2 / (1.2 A^{1/3})^2 = 0.5 A^{1/3} \quad (3.5)$$

superimposed interactions, instead of one as in a pp collision. If we now bring in the nuclear enhancement factor of about three, then we have in the collision of two heavy nuclei

$$v = 1.5 A^{1/3} \quad (3.6)$$

interactions per nucleonic cross section. In a given spatial region, in which a proton-proton collision deposits the energy for the formation of one meson pair, a uranium-uranium collision leaves ten times that value.

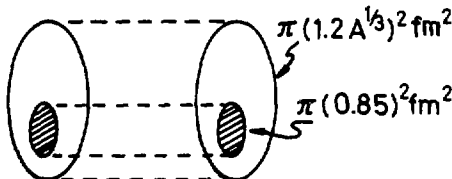


Figure 6 :
Comparison of nuclear
and nucleonic collision
geometries

To obtain still more specific estimates, we assume that in a pp collision the energy density at the hadronisation stage is roughly that of nuclear matter, 160 MeV/fm^3 . This leads us to expect for uranium-uranium collisions energy densities of

$$E_{AA} \simeq 1.6 \text{ GeV/fm}^3 \quad (3.7)$$

and more. We have in our picture assumed scaling pp behaviour; eq. (3.7) is therefore an underestimate if the number of secondaries in a hadron-hadron interaction grows as $(\ln P_{\text{LAB}})^2$.

A much more thorough and quantitative analysis of the conditions to be expected in the fragmentation region of heavy ion collisions has recently been carried out³¹⁾. Since L. McLerran is presenting the re-

sults at this meeting, we note here only that they agree well with what we obtained by our geometric considerations.

We can thus conclude with some assurance that high energy heavy ion collisions will indeed provide us with sufficient energy density for the formation of a quark-gluon plasma. This does not assure us, however, that thermal equilibrium will be established in such a collision. As a counter-example, we recall that in high energy e^+e^- interactions, an even larger amount of energy is deposited in an even smaller spatial volume because of the underlying dynamics, this only leads to a single $q\bar{q}$ pair, however, and not to anything "statistical".

It is therefore important to note that mean free path arguments³¹⁾ as well as considerations based on perturbative QCD cross sections³²⁾ suggest that there are indeed a sufficient number of interactions in relativistic heavy ion collisions to produce thermal equilibrium. The apparent increase of hard collisions in α - α processes³³⁾ might also support this experimentally.

C Signals for Quark Matter Formation

The search for observable consequences of quark matter formation in heavy ion collisions has recently led to a number of proposals³⁴⁻³⁸⁾. Up to now, even the evidence for nuclear matter in thermal equilibrium remains rather circumstantial in such collisions - so this search is not an easy one. The hadrons observed after the interaction presumably will not retain any kinetic information from a phase which existed before their creation. We therefore seem to be left with three types of tests:

- a study of photons and/or lepton pairs emitted in the early "hot" quark phase and thus unaffected by the subsequent cooling and hadronisation of the system^{35,37)};
- a study of photon and/or lepton pair spectra as function of the primordial temperature, passing from $T > T_c$ to $T < T_c$, in the hope of observing a discontinuity at T_c ;
- a study of the quantum number structure of the hadronic final state, in the hope of observing restrictions caused by the earlier quark phase³⁸⁾.

Unfortunately, both nuclear matter and non-equilibrium nucleon-nucleon interactions may lead to a behaviour which is at least qualitatively similar; hence any proposal for a decisive check should ideally include quantitative signal-to-noise information. Even though such quantitative studies are not yet always completed, let us look at some of the possible signals in a little more detail.

The production of massive lepton pairs by nuclear matter appears much suppressed because of lower temperatures³²⁾; on the other hand, non-equilibrium Drell-Yan production of such pairs will only suffer a power law mass damping (M^{-4}) instead of the thermal Boltzmann suppression $\exp - (M/T)$ ³⁹⁾. For a spectrum of the latter form, the production rate

$$\partial N / \partial M \sim \int_{T_0}^0 dT e^{-M/T} \sim e^{-M/T_0} \quad (3.8)$$

should be dominated by the highest temperature contribution. If we observe, at the same incident energy, different mass lepton pairs, then by plotting

$$-\frac{1}{M} \ln(\partial N / \partial M) \simeq \frac{1}{T_0} \quad (3.9)$$

as function of M , we can determine the primordial temperature T_0 of the quark matter formed. We expect T_0 to be the same for all (large) M in a given reaction, but to increase with increasing collision energy.

If low energy ion collisions lead to nuclear matter formation and high energy collisions to quark matter, then the above method should allow a determination of $T_0(P_{LAB})$ over a range including T_c . At $T_0(P_{LAB}) = T_c$, we expect a discontinuity in the rate, as the specific heat of the radiating system there becomes discontinuous. To study this in more detail, one could perhaps simulate the problem by considering photon emission from a system with discontinuous c_v .

In the fragmentation regions of the colliding nuclei, possible quark matter will contain a considerable excess of u and d quarks deposited by the incoming nucleons. In dense quark matter, these are expected to form at least partially degenerate Fermi seas, thus creating a block

against $u\bar{u}$ and $d\bar{d}$ creation in comparison to $s\bar{s}$ production: a u or a d would have to go to the top of the Fermi sea, and s not. This blocking mechanism should lead to an enhanced $\bar{\Lambda}/\bar{p}$ production ratio³⁸⁾ as well as to more energetic K^+ than K^- ³⁹⁾.

IV. CONCLUSIONS

Quantum chromodynamics provides us with a theoretical basis for strong interaction thermodynamics, which clearly predicts a transition from nuclear matter to quark-gluon matter; moreover, it gives us, in the lattice approach, a unified description over the entire temperature/density range. Actual calculations are at present for computational reasons restricted to pure Yang-Mills systems, but work on the full problem is in progress.

Estimates of the transition parameters, when compared with the values expected in relativistic heavy ion collisions, indicate that such experiments should yield sufficient energy and particle density for the formation of quark-gluon matter. There are moreover indications that the collision will also lead to thermal, perhaps even "chemical" equilibrium.

A number of possible signals for quark matter formation have been discussed, but more quantitative work seems necessary to obtain more conclusive experimental tests.

Acknowledgements

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HOT MATTER IN NUCLEAR COLLISIONS

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Abstract:

A theory of hot hadronic matter is being developed¹⁾ based on the present knowledge about strong interactions. Physically most important is the phase transition in which baryons and mesons are dissolved into the quark-gluon plasma^{2,3)}. Essential quantity controlling the occurrence of this transition is the energy density of hadronic matter. The heat and pions created in central high energy heavy ion collisions lower the necessary nuclear density to about 1-3 times normal nucleon density at temperatures of the order of the pion mass.

1) Introduction:

The theoretical descriptions of the nuclear phase consisting of individual hadrons and of the quark gluon plasma are entirely different: In the hadronic gas phase¹⁾ the hadronic cross section is being dominated by the formation of particle resonances, the essential physical input is the particle mass spectrum, here derived in the statistical bootstrap model. In the very dense domain of the matter it is essential to take into account the finite size and the clustering of individual quark bags. In the quark-gluon plasma, we have to deal with a many body gauge field theory at moderate interaction strength^{2,3)}. An insight into the behaviour of the plasma can be gathered studying the Fermi-Bose interacting quantum gases in a large quark bag^{3,4)}. In order to test our understanding of hot hadronic matter in kinetic and chemical equilibrium and to show the existence of the plasma phase different experiments are necessary. As an initial step I consider the temperatures of particles emitted in a hypothetical central fireball created in high energy nuclear collisions^{1,5)}. Suitable care must be taken in an experiment to eliminate the contributions of projectile and target nuclei - here we do not know the model dependent internal excitation. Among the results I discuss here a substantial entropy production in the explosion of the fireball due primarily to the production of new particles. Under certain conditions I also anticipate

that strangeness would equilibrate - the relative yields of (anti) strange particles can be used as a measure of the size of the reaction zone to be confronted with correlation experiments. If the phase transition occurs at moderate energy densities (500 MeV/fm^3), then the relative $\bar{\lambda}$ to $\bar{p}$ yields can be used as a measure of the relative antistrange quark abundance in the quark phase for heavy ion energies between 2 and 5 $\text{GeV/N}^{7)}$. Other phenomena will be studied at even higher energies: prompt photons, leptons and eventually heavy flavours (charm etc).

2) Hot Hadronic Gas Phase

The basic assumption will be the conceptual validity of the quark bag model⁴⁾ - what I propose below is practically thermodynamics of finite size bags⁸⁾ interacting through creation or destruction of new bag states. I will neglect quantum statistics - this approximation turns out to be permissible when the temperatures of individual hadrons are above 40-50 MeV.

From the partition function $Z(V, \mu, T)$ all physically interesting quantities, such as energy, pressure, entropy can be derived as they are simply first derivatives of Z . Here V is the total reaction volume, μ the baryonic chemical potential and $T = 1/\beta$ the temperature. We will also use the baryon fugacity $\lambda = \exp(\mu/T)$. λ or resp. μ are introduced to allow the conservation of the baryon number. The partition function can be expressed in terms of the hadronic mass - spectrum $\tau(p^2, b)$: $\tau(m^2, b) dm^2$ is the number of hadronic resonances of baryon number b in the mass intervall $(m^2, m^2 + dm^2)$.

We obtain^{1,6)}:

$$\ln Z = - \frac{2\Delta}{(2\pi)^3 H} \frac{\partial}{\partial \beta} \phi(\beta, \lambda) \quad (1a)$$

$$\phi(\beta, \lambda) = H \sum_{b=-\infty}^{+\infty} \lambda^b \int e^{-\beta p} \tau(p^2, b) d^4p \quad (1b)$$

The bootstrap constant H has been introduced here mainly for dimensional convenience: the dimensionless quantity ϕ will be derived from the statistical bootstrap model. Δ is the available volume in which hadrons are free to move after subtraction of their proper volumina. We know that in the bag model of massless relativistic quarks⁴⁾ the volume and mass of particle states is related:

$$V_{h,i} = M_i/4B \quad (2a)$$

and hence

$$\Delta = V - \langle E \rangle / 4B \quad (2b)$$

where $\langle E \rangle$ is the statistical average of the total energy. We emphasize that the quantity Δ must always remain positive; the total energy (mass) of the hadronic gas phase cannot exceed $4BV$. When this limiting value is approached, a tendency of quark bags to cluster together and to form one large bag is found. A necessary condition for this to occur is that the energy density of the (hypothetical) pointlike hadrons

$$\epsilon_p = -\frac{1}{\Delta} \frac{\partial}{\partial E} \ln Z = -\frac{2}{(2\pi)^3} \frac{\partial^2}{\partial E^2} \ln Z \quad (3)$$

diverges. The energy density $\epsilon = \langle E \rangle / V$ in terms of ϵ_p is just $\epsilon = \Delta / V \epsilon_p$ and with Eq. (2)

$$\epsilon = \frac{\epsilon_p}{1 + \epsilon_p / 4B} \quad \frac{\epsilon_p}{\epsilon_p} \rightarrow 4B \quad (4)$$

The factor $(1 + \epsilon_p / 4B)^{-1}$ represents the part of the van der Waals effect introduced by the finite size of individual hadrons.

In order to obtain a quantitative description of the hadronic phase we must derive an expression for the bootstrap function ϕ , Eq. (1b). By requiring that in the limit $\Delta \rightarrow 0$ the hadronic state as described by Eq. (1) is again just another particle contained in the mass spectrum τ a nonlinear equation for τ is obtained⁹⁾. In terms of the function ϕ it has the convenient form¹⁰⁾

$$\phi(\epsilon, \lambda) = 2\lambda e^{\epsilon} + 1 \quad (5)$$

The input function ϕ is defined in terms of the basic hadronic states $[q \bar{q}$ and $q q q]$ as

$$\phi = 2^{-HT} \left[3m_\pi K_1 \left(\frac{m_\pi}{T} \right) + 8 \cosh(u/T) m_N K_1 \left(\frac{m_N}{T} \right) \right] \quad (6)$$

We recall that Eq. (5) has a real solution $\tau = G(\epsilon)$ only for $\epsilon < \epsilon_0 = \ln(4/e)$. At this point there is a root singularity. Further study shows that Eq. (1b) then requires an exponential growth of $\tau e^{m/T_0}$. The highest hadronic gas temperature T_0 has the analogous physical meaning as the boiling point of water - for $T > T_0$ the phase containing individual hadrons cannot exist and the constituents - quarks - are liberated [in the sense that they can move within the hadronic volume V_h , Eq. (3)]. $T(u=0) = T_0$ has been

chosen to be 190 MeV ($H = .724 \text{ GeV}^{-2}$). This choice leads to agreement with the slope of inclusive cross sections: $6 \pm 1.15 \text{ GeV}$. The observed low pion temperatures $T \approx 120\text{--}150 \text{ MeV}$ are so small because pions are emitted mainly from the less dense and cooler domains of hadronic fireballs⁵⁾. As a consequence of the Van der Waals effect discussed the energy density along the phase boundary ϵ_c is equal to $4B$, while pressure vanishes. With $B \approx (170 \text{ MeV})^4 = 110 \text{ MeV/fm}^3$ we expect the phase transition at about 0.5 GeV/fm^3 .

To test these ideas about the hadronic gas phase the temperatures (slope parameters) of inclusive particle cross sections expected in relativistic heavy ion collisions have been computed⁵⁾. These calculations assume that:

- 1) small impact parameter collisions are identified,
- 2) only particles from the central fireball are counted.

Under these circumstances the initial kinetic energy of the projectile nucleus per nucleon, $E_{k,p}/A$, defines the available excitation energy per participating baryon in the fireball.

The equations of state that follow from Eq. (1) distribute this energy among kinetic and chemical degrees of freedom (collective motion and its energy has been neglected). We recall here that the resulting temperature of emitted nucleons is significantly higher from that of pions. The substantial rise of the temperature with the kinetic energy found is in agreement with experiment (BEVELAC-ISP)¹¹⁾.

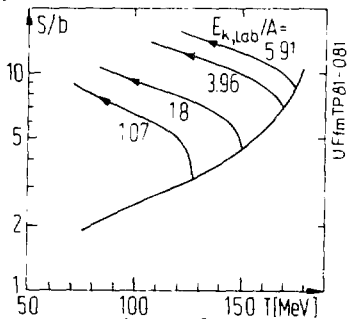


Fig. 1: Specific entropy per baryon for given projectile kinetic energy as function of temperature. The boundary is the specific entropy at the critical line.

In Fig. 1 I show another interesting result: Along the lines of constant energy per baryon, the entropy per baryon rises significantly from the (high) value computed at the critical line. This is due to the onset of strong pionization of the available energy. We thus keep in mind that hadronic fireballs do not expand adiabatically and that substantial cooperative entropy production is expected.

3) The Quark-Gluon Plasma

When hadrons have coalesced into a large quark bag at the phase boundary, we must change the theoretical model underlying the description of hot hadronic matter. The new central assumption, valid strictly only at very high energies, is the weakness of the quark-quark interaction: only in this case a description of interacting quantum Fermi-Bose gases^{2,3)}, may be successful. The quark masses in the relativistic quark-bag model are small: $m_q \approx 10-30$ MeV. Hence in the region of interest to us of μ and T , the quark chemical potential is $\mu_q = \frac{1}{3}\mu > m_q$. [The factor 1/3 arises in view of the quark baryon number 1/3]. As long as this condition is satisfied, the Fermi-Bose gas with interaction $O(\alpha_s)$ can be integrated analytically¹²⁾ and we find: $T \ln Z =$

$$\text{Quarks:} \\ = \frac{1}{3} \frac{gV}{2^{-2}} \left[\left(1 - \frac{2}{\pi} \alpha_s \right) \cdot \left(\frac{\mu^2}{4 \cdot 3^2} + \frac{\mu^2}{2 \cdot 3^2} (-T)^2 \right) + \left(1 - \frac{50}{21} \alpha_s \right) \frac{7}{60} (-T)^4 \right] \quad (7a)$$

$$\text{Gluons:} \\ = \frac{8V}{45 \cdot 2^{-2}} \left(1 - \frac{15}{4} \alpha_s \right) (-T)^4 \quad (7b)$$

$$\text{Vacuum:} \\ = -BV \quad (7c)$$

where $Z = Z_q Z_g Z_v$ is the total partition function.

Here $g = 2 \cdot 2 \cdot 3 = 12$ is the number of distinct quark modes with spin, isospin and colour. α_s is the QCD colour coupling constant $\alpha_s \approx 0.5$ for space-like q^2 and $\alpha_s \approx 2$ for time-like q^2 - but we ignored the q^2 dependence of α_s].

In Eq. (7) we show separately the contributions of quarks and gluons; the vacuum term is a phenomenological supplement at this stage of the discussion and has been chosen so as the bag energy density is B inside the region of the plasma and that an inside pointing pressure $P=B$ acts on the surface of the bag region. Contributions of heavy and strange quark flavours have been neglected.

The vacuum contribution, Eq. (7c) is as postulated in the quark bag model. However, at finite temperatures additional difficulty arises not shown by the perturbative expression. We recall that the vacuum structure term originates in the absence of the true vacuum gluonic structure from the region of space containing quarks. Therefore B should be calculated non-perturbatively together with the glue term, Eq. (7b) to yield some T-dependent quantity. In particular, this gauge pressure should vanish at some high temperature $T_{cr} \approx 1.5B^{1/4}$, when the structure of the vacuum is destroyed¹³⁾. Only above T_{cr} the expression (7b) is valid - while Eq. (7c) is only correct for $T=0$. For $T < T_{cr}$ there is only a partial restoration of symmetry and not all 8·2 gluonic degrees of freedom can be excited. Another aspect of this point is that gluons may not be able to exist as independent particles in the plasma region and are rapidly absorbed on the surface by the true vacuum. To estimate the lower boundary of the quark-gluon plasma phase in the μ -T plane we compute the zero pressure line: $PV = T \ln Z$, omitting the little important, but obscure glue term. The result is very similar to bootstrap nuclear matter with $T_c \sim B^{1/4}$ at $\mu=0$. This perhaps not accidental coincidence of the critical lines leads to the suggestion that both phases described here are directly adjacent to each other.

4) Strangeness in Nuclear Collisions

Unlike p-p collisions, strangeness may be close to kinetic equilibrium in nuclear collisions: it is the large reaction volume with typical lengths exceeding the Compton wavelength of the strange mass (280 MeV) that is of great importance here⁶⁾.

The crucial question is where the transition to 'infinitely' large systems occurs. The quantitative result is that in the hadronic volume $V_h = 4\pi/3 (1 \text{ fm})^3$ the strange mass 280 MeV matters, while already for $V \approx 6-8 \cdot V_h$ we are in the infinite limit - assuming a typical $T \sim 150$ MeV. I thus suppose that in nuclear collisions sufficiently large hadronic volumes are found to allow equilibration of strangeness. Furthermore, I have found from this study⁶⁾ that there is no information about the hadronic aggregate state that can be extracted from the total observed strangeness yields.

However, it is the antistrangeness that may be very useful in the search for quark matter: below the $\Lambda\bar{\Lambda}$ threshold [at 4-5 GeV taking account of

Fermi-motion in both nuclei] the anti-strangeness that must balance the strangeness produced in hadronic reactions will generally be found only in kaons, except if quark plasma states were formed. In that case the kinematic limit is of no importance and it is the relative abundance of antiquarks in the plasma that will control the antibaryon yields. Consider for example the $\bar{\Lambda}/\bar{N}$ ratio as a measure of the relative $\bar{s}$ and $\bar{u}$ or $\bar{d}$ abundances: on theoretical grounds I expect $\bar{s}$ to be as abundant as $\bar{u}$ and $\bar{d}$ taken together, supposing the typical T and μ values at 4-5 GeV/N. Hence if quark plasma is formed at these energies, $\bar{\Lambda}/\bar{N}$ ratio could become an important observable and in particular be equal or larger than one, as compared to $\sim 1/10$ in p-p collisions.

5) Conclusions

The study of the properties of very hot nuclear matter has just begun and it is too early to say if the features of strong interactions that have been included in these considerations are the most relevant ones. It is important to observe that the currently predicted pion and nucleon mean transverse momenta and temperatures show the required substantial rise as required by the experimental results available¹¹⁾ at $E_{k,lab}/A=2$ GeV [BEVALAC] and at 1000 GeV [ISR]. Further comparisons involving, in particular, particle multiplicities, entropy and strangeness production as discussed above are under consideration.

Perhaps the most interesting and far reaching aspect of this work is the realization that the transition to quark matter will occur at much lower baryon density for highly excited hadronic matter than for matter in the ground state ($T=0$). The precise baryon density of the phase transition depends somewhat on the phenomenological value of the bag constant; we estimate it to be at about $2-4 \nu_0$ at $T=150$ MeV. The detailed study of the different aspects of this phase transition, as well as of possible characteristic signatures of quark matter, must still be carried out. I have given here only a very preliminary report. However I believe that the occurrence of the quark plasma phase will certainly be an observable phenomenon. Here I have discussed a measurement of the $\bar{\Lambda}/\bar{p}$ relative yield at about 4-5 GeV/N kinetic energy nuclear collisions. In the quark plasma phase we expect a significant enhancement of $\bar{\Lambda}$ production which will be most likely visible in the $\bar{\Lambda}/\bar{p}$ relative rate. Also, characteristic high energy prompt gamma radiation will be used as an observable.

Many fruitful discussions with the GSI/LBL Relativistic Heavy Ion group stimulated many of the ideas presented here. I would like to thank R. Bock and R. Stock for their hospitality at GSI. Parts of this work were

performed in collaboration with R. Hagedorn, B. Müller, H.-Th. Elze, and M. Danos.

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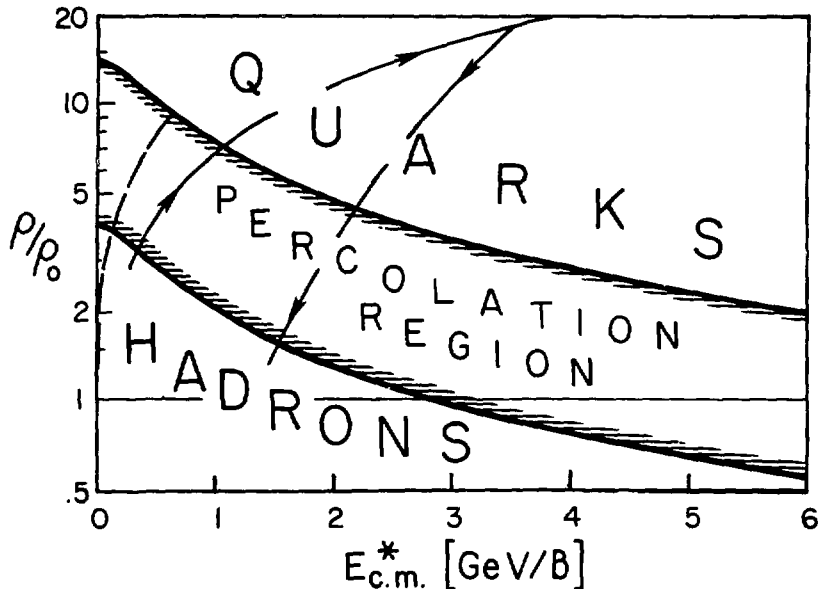
Percolation Versus Gibbs Equilibrium--
The Phase Transition from Hadron Matter to Quark Matter*

Horst Stöcker, LBL

The phase transition from hadron matter to quark matter is investigated in two distinct models: The percolation approach and the Gibbs phase equilibrium. We find a strong dependence of the onset of the percolation phase transition on the internal excitation energy E^* of the matter: For $E^* \approx 3$ GeV per baryon, the percolation sets in already at the ground state density of nuclear matter, $\rho \approx 0.17 \text{ fm}^{-3}$. The implications of our results for relativistic nuclear collisions are discussed. In the hydrodynamical model the onset of the hadron-quark matter phase transition is predicted at bombarding energies $E_{\text{LAB}} \geq 1 \text{ GeV/n}$ for central collisions of massive nuclei.

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*based on LBL preprint 12633



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OBSERVATION OF SUBTHRESHOLD K^-
PRODUCTION IN RELATIVISTIC NUCLEAR COLLISIONS

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The study of relativistic nuclear collisions may provide information pertaining to matter at high energies and densities ⁽¹⁾. A variety of nuclear phenomena ranging from the existence of abnormally dense nuclear states ⁽²⁾ to phase transition from hadron to quark matter ⁽³⁾ are under investigation. These effects necessitate some degree of thermalization or collective interactions among the colliding nucleons. Inclusive measurements of particles produced at forward angles in nuclear collisions at 2 GeV/nucleon suggest a fair degree of transparency and a lack of thermalization ⁽⁴⁾. However, energy spectra of particles produced at 90° with respect to the A + A center of mass exhibit an exponential falloff characteristic of a thermal system ⁽⁵⁾, and particles produced at backward angles imply some sort of collective interactions ⁽⁶⁾. Such data are suggestive of collective effects, but are not conclusive.

An unambiguous signature for collective or thermal effects is the production of particles which require more energy than is available in individual nucleon-nucleon collisions. At 2.1 GeV/n, the center of mass energy available for particle production in n-n collisions is 850 MeV. The lightest particle whose threshold is above this energy is the K^-

which requires a center of mass energy of 990 MeV, or 140 MeV above the available energy. Similarly, the anti-proton requires an energy of 1900 MeV in the center of mass. It is clear that individual nucleons must combine their energy to enable the subthreshold production of particles.

We have recently conducted a search for subthreshold K^- and $\bar{p}$ production in the reaction $^{28}\text{Si} + ^{28}\text{Si}$ at 2.1 GeV/nucleon. The negative secondary beam produced at 0° was guided along a magnetic beam line. Three bends were made for momentum selection and reduction of background. Detectors were placed at the three intermediate foci of the secondary beam. Scintillation counters were used to detect transmitted charged particles and to provide time of flight information. Cerenkov counters with an efficiency of better than one part in 10^4 for $\beta = .99$, were used to identify the dominant pion signal. A symmetric projectile-target system allowed us to define a unique center of mass frame. The central momentum was chosen for anti-protons with the velocity of the center of mass system. This corresponds to kaons with center of mass kinetic energy of 70 MeV.

We have observed a surprisingly large number of K^- . The time of flight spectrum between the first two sets of counters, with appropriate cuts on the respective Cerenkov counters, is shown in Figure #1. The event in the anti-proton channel has the $\frac{dE}{dx}$ of a pion and is consistent with background. A significantly higher level of sensitivity is needed for a meaningful anti-proton measurement. The kaon peak, containing 52 events, is distinct. This corresponds to the production of one K^- per 2,000 π^- (only 2.4% of the kaons survive the flight path). We note that

none of the four million events had both the pion time of flight and satisfied the Cerenkov counter cuts. Our measured differential cross section for K^- at .992 GeV/c and at 0° is

$$\frac{d^2\sigma}{dPd\Omega} = 1.3 \frac{\text{mb}}{\text{sr} \cdot \frac{\text{GeV}}{c}} \quad (\text{preliminary error about 50\%})$$

We would like to know if this result can be explained solely in terms of individual nucleon-nucleon collisions. Since there are no similar subthreshold kaon cross-sections available we have measured above threshold K^- production under identical conditions for protons on copper at 4.8 GeV projectile energy. We obtain a ratio of yields (per nucleon-nucleon collision) of

$$R_{\text{exp}} = \frac{\frac{d^2\sigma}{dPd\Omega} (\text{Si} + \text{Si} (2.1 \text{ GeV/n}) \rightarrow K^- \text{ at } 1 \text{ GeV/c } 0^\circ)}{\frac{d^2\sigma}{dPd\Omega} (\text{P} + \text{Cu} (4.8 \text{ GeV}) \rightarrow K^- \text{ at } 1 \text{ GeV/c, } 0^\circ)} \times \frac{63}{28^2} = .12$$

This ratio is compared to results of calculations which assume the nuclear interactions to consist of individual nucleon-nucleon collisions with the inclusion of Fermi momentum for the projectile and target nucleons. For the Fermi momentum distribution, we have used the double Gaussian reported by Geaga et al (7).

$$(F(p) \propto \exp^{(-p^2/2\sigma_1^2)} + .023 \exp^{(-p^2/2\sigma_2^2)}, \text{ with } \sigma_1 = 90 \text{ MeV/c}$$

and $\sigma_2 = 190 \text{ MeV/c}$).

This distribution, in addition to fitting the results for backward scattering of protons (7), also provide very good agreement with subthreshold anti-proton production data in p - nucleus collisions (see

Figure 2, data from Dorfman et al ⁽⁸⁾). Two independent calculations were made by members of our group and a third by John Harris of G.S.I. The result of these calculations, per nucleon-nucleon collision, is

$$R_{\text{calc.}} = .003$$

This is a factor of 40 smaller than the experimentally measured ratio. The assumptions inherent in the calculations are very conservative. If the fictitious kinetic energy introduced by the Fermi momentum is subtracted from the total available center of mass energy, the K^- rate in the N - N calculations at 2.1 GeV/n drops by a factor of four, while the rate for p - N at 4.8 GeV is hardly affected. Likewise, the assumptions of a constant transition matrix element, along with an $A_p A_t$ dependence (i.e., no nuclear shadowing or degradation of projectile energy) are more favorable for the subthreshold K^- production. The calculated ratio for the total integrated K^- yields is not significantly larger than the ratio calculated for K^- produced at the measured kinematic region, so that the discrepancy with the measured ratio cannot be attributed to rescattering of kaons produced in more favorable kinematic regions. Calculations which include multiple scattering of the colliding nucleons should be attempted for a comparison with our K^- yields.

We believe that the unexpectedly large number of K^- observed is an indication that some unusual phenomena may be taking place. The inclusion of nuclear Fermi momentum to boost up the energy available in individual nucleon-nucleon collisions does not account for the large K^-

rate, even though the Fermi distribution we use accounts for other experimental data quite well. It is difficult to imagine collective effects at such energies, since at 2 GeV the de-Broglie wavelength is significantly smaller than the inter-nucleon separation. However, people have suggested that pre-formed clusters or collective tubes (9) may provide more energy to create particles. Furthermore, it is not obvious that any degree of equilibration should occur in a system as small as $^{28}\text{Si} + ^{28}\text{Si}$ (10), although such a mechanism can probably account for the rates we observe.

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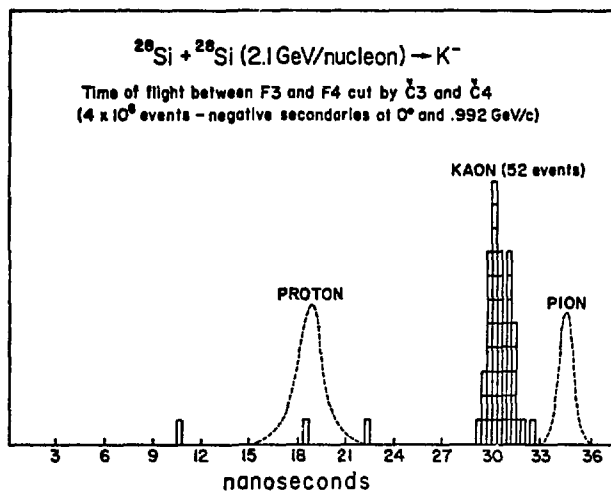


Figure 1

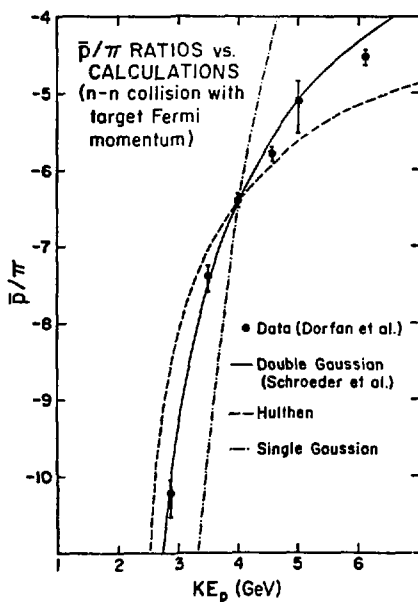


Figure 2

Dup

THE QUARK-GLUON PLASMA AND ULTRA-RELATIVISTIC
NUCLEUS-NUCLEUS COLLISIONS*

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A space-time picture of the fragmentation and central regions is presented for extremely high energy head-on heavy nucleus collisions. The energy densities of the matter produced in such collisions are estimated. Speculations concerning the possible formation of a quark-gluon plasma are discussed, as are possible experimental signals for analyzing such a plasma.

*Work supported in part by United States Department of Energy.

The potential experimental study of ultra-relativistic collisions between very heavy nuclei presents an exciting challenge to both theorists and experimentalists. The large multiplicities, $n \sim 10^3 - 10^4$, arising from head-on collisions at ISR energies will provide a fresh problem for detector technology. High energy densities and large multiplicities generated in such collisions offer a new panorama in which novel features of QCD may be probed.

The studies of Willis and colleagues have elucidated the possibilities for employing state-of-the-art detection systems to analyze heavy nucleus-nucleus collisions at ISR energies.⁽¹⁾ Such systems are capable of providing sophisticated calorimetry, tracking, momentum resolution, and particle identification. Equally sophisticated systems capable of carrying out similar analysis might be employed at the SPS, or the proposed Venus collider at LBL.

Recent theoretical studies provide a simple space-time description of ultra-relativistic nucleus-nucleus collisions.⁽²⁾ These studies concentrate on head-on collisions -- that is, collisions with impact parameters $b \lesssim 1$ fm. Geometrical considerations show that head-on collisions comprise $(1/2R)^2 \sim 1/2\%$ of all uranium-uranium collisions.

Head-on collisions are easily distinguished from peripheral collisions by the multiplicities of collision products. Assuming such multiplicities in nucleus-nucleus collision grow as the baryon number of the nuclei, A , the multiplicity in a head-on collision at ISR energies is $n \sim 10^3 - 10^4$. Fluctuations in peripheral collisions could rarely simulate such a large multiplicity.

We shall now proceed to present the results of an analysis of nucleus-nucleus collision, using the quark-parton model.⁽³⁻⁵⁾ Such a collision may be analyzed in the target rest frame. The projectile nucleus appears in this frame as a thin, Lorentz-contracted pancake. This projectile pancake passes through the target during the collision. The target is heated and compressed during this passage. The central region between the two nuclei is heated when the projectile emerges from the target. The projectile is heated and compressed in the final stage of the collision.

A hot, dense, baryon-rich configuration of hadronic matter is formed in the nucleus fragmentation region. The energy density of this hadronic matter may be $\epsilon \sim 2 \text{ GeV/fm}^3$ for heavy nuclei such as uranium. This energy density is larger than that of the energy density of matter inside a proton, $\epsilon_p \sim \frac{1}{4} \frac{1}{\pi r_p^3} \sim \frac{1}{2} \text{ GeV/fm}^3$ ($r_p \sim .8 \text{ fm}$), and may be large enough for formation of a quark-gluon plasma.⁽⁶⁾

The decay products of the nuclear fragmentation regions are identified by their rapidities,

$$y = \ln \frac{E + p''}{m}, \quad (1)$$

where E is the particle's energy, p'' its longitudinal momentum, and m its mass. For heavy nuclei such as uranium, the width of each of the nucleus fragmentation regions is $y \sim 3-4$ units. The total accessible rapidity interval at ISR energies is $y \sim 6-8$ units. A study of the central region at ISR energies is severely limited by this kinematic

restriction. The central region might, however, be studied at Isabelle energies.

If the multiplicity in the central region grows as the nuclear baryon number, A , and if the multiplicity continues to grow as it does at ISR energies, the energy density of hot, hadronic matter in the central region may be as high as $\epsilon \sim 2 \text{ GeV/fm}^3$ for heavy nuclei such as uranium. This energy density is as large as that of the fragmentation region, and a quark-gluon plasma may form. If the multiplicity grows as $A^{2/3}$, the energy density is probably too small ($\epsilon \sim 300 \text{ MeV/fm}^3$) to produce a plasma.

There is recent cosmic ray data for nucleus-nucleus collisions which favor a multiplicity growth proportional to A .⁽⁷⁾ In Fig. 1, a pseudo-rapidity plot for the interaction of a 10 TeV calcium nucleus with, presumably, a carbon nucleus is shown. About 600 charged particles are produced. The density of particles in the central region is consistent with a $\log^2 s$ multiplicity growth determined at ISR energies and an A dependence, but is inconsistent with an $A^{2/3}$ dependence and a $\log^2 s$ multiplicity growth.

A distinction between matter produced in the fragmentation regions and the central region is found in the baryon number density. This density is the difference between the number of baryons and antibaryons per unit volume. There should be only a small baryon number density in the central region. If a quark-gluon plasma forms in the central region, the study of the transition region between the fragmentation regions and central region would allow a study of the dependence of

characteristics of the quark-gluon plasma on baryon number density.

A hypothetical rapidity distribution for baryon number (baryons minus antibaryons) for a head-on collision between nuclei of baryon number A is shown in Fig. 2(a). The baryon number is concentrated in a nucleus fragmentation region of width $\Delta y \sim 2-3$. The heights of these fragmentation regions are proportional to A .

A hypothetical rapidity distribution for mesons for this collision is shown in Fig. 2(b). If the height of the central region were proportional to $A^{2/3}$, it would be cleanly isolated from the fragmentation regions at Isabelle energies.

A feature of nucleus-nucleus collisions which distinguishes them from hadron-hadron and hadron-nucleus collisions is the extremely large number of particles which initiate and are produced in the collision. If enough particles are produced in the primeval distribution of hot, hadronic matter, and if the matter stays hot and dense long enough, the constituents of the matter will come into thermal equilibrium. Estimates which employ the parton model indicate that if the height grows as A , kinetic equilibrium is established in the fragmentation region and in the central region. This conclusion is verified within perturbative QCD for the fragmentation region.⁽⁸⁾ These perturbative computations also tentatively conclude that chemical equilibrium may also be achieved.⁽⁸⁾

The results of the preceding paragraphs are obtained in the parton model picture of nucleus-nucleus scattering provided Bjorken and Gottfried.⁽³⁻⁴⁾ An implication of this picture is the inside-outside

development of cascades in hadronic collisions.

In the collision shown in Fig. 3, a massless, inelastically produced fragment separates from the projectile hadron. After a time t , the projectile and fragment have separated

$$\Delta r_{\perp} \sim \frac{p_{\perp}}{p''} t \quad (2)$$

and

$$\Delta r'' \sim \left(\frac{p_{\perp}}{p''}\right)^2 t. \quad (3)$$

In the local $p'' = 0$ frame of the fragment, this distance is

$$\Delta r_{\perp} \sim \Delta r'' \sim \frac{p_{\perp}}{p''} t. \quad (4)$$

The fragment may be properly included as part of the projectile hadron's wave function if $\Delta r_{\perp}, \Delta r'' \lesssim 1$ fm. The time needed for a fragment to materialize in a cascade is, therefore, $t \sim p''$, which is to say slow fragments materialize before the fast fragments.

Before a nucleus-nucleus collision takes place, the projectile nucleus appears as a Lorentz contracted pancake (Fig. 4). This pancake has a limiting thickness of $t \sim 1$ fm as a consequence of the low momentum wee parton component of the nuclear wave function.⁽³⁾

Not much intranuclear cascading takes place as the projectile nucleus traverses the target. Since the cascade is inside-outside, the

high momentum projectile fragments are produced after the projectile has traversed the target nucleus. The projectile nucleus does not slow significantly as it passes through the target nucleus. This phenomenon of nuclear transparency leads to the conclusion that approximate Feynman scaling should be appropriate for nucleus-nucleus interactions at energies high enough so that the projectile and target nuclei begin to pass through one another.

Low momentum pions, $p^\pi \lesssim R/R_0$ where $R_0 \sim 1$ fm and R is the nuclear radius, are produced within the nucleus and may become trapped in the nucleus fragmentation region. These pions are probably initially produced with a distribution typical of proton-proton interactions, and are generated by each of the A target nucleons.

Estimates of the number of pions trapped in the nucleus fragmentation region using pp inclusive data give about 3-4 pions per nucleon trapped for uranium-uranium collisions at ISR energies. At the highest ISR energies, this number should be a slowly varying function of energy, $N \sim \ln E$, and of nuclear baryon number $N \sim A^{1/3}$. Evaluated in the rest frame of the struck target, the energy per nucleon which is trapped is $E/N \sim 3-4$ GeV/nucleon. Uncertainties in the evaluation of this number are about a factor of two. A hypothetical distribution of the regions in rapidity space from which nucleons are trapped is shown in Fig. 5 for various reasonable assumptions on the cutoff in $p_\perp/p^\pi$.

After the target nucleons are struck, they attain longitudinal momentum. This longitudinal momentum is approximately that which would occur in a pp collision, corresponding to a $\gamma \sim 2$.⁽²⁾

The nucleons and trapped pions may form a baryon-rich fireball moving down the beam-pipe with a $\gamma \sim 2$.

As the projectile nucleus traverses the target, imparting longitudinal momentum to the target nucleons, the target is compressed. As shown in Fig. 6, this compression results from the sequential encounters of the projectile nucleus with target nucleons. After the left-most target nucleon is encountered, this nucleon acquires a velocity v . It travels a distance vR before the second nucleon is struck. In the lab frame, the apparent compression is $1/(1-v)$. In the rest frame of the struck nucleus, the compression is

$$C = \frac{1}{\gamma(1-v)} \sim 2\gamma, \quad (5)$$

which for $\gamma \sim 2$ is $C \sim 4$.

The energy density in the fragmentation region is estimated from the compression C , the trapped energy per baryon, E/N , and the energy density of nuclear matter, ρ_0 , as

$$\varepsilon \sim \rho_0 C E/N \sim 2\text{GeV}/\text{fm}^3 \quad (6)$$

A picture of the projectile nucleus sweeping through the target is shown in Fig. 7. The nucleus is shown being compressed, and pions are shown produced in the wake of the projectile.

After the projectile leaves the target, hot matter begins to form in the central region between the nuclei. This matter forms a hot fire

tube which eventually joins the two nuclei. The radiation from this fire tube forms the central rapidity region. The matter forming in this central region is shown in Fig. 8.

The matter produced in the central region initially has a large longitudinal momentum gradient. The mesons produced near the target have $p'' \sim 300$ MeV whereas the mesons produced near the projectile have $p'' \sim p''_{\text{projectile}}$. The separation between neighboring pions when they are initially produced in a pp collision is

$$\Delta \sim \left(\frac{p'_1}{p''_1} - \frac{p'_2}{p''_2} \right) d_0 \quad (7)$$

where $d_0 \sim 1$ fm.

The fire tube dynamics are most simply analyzed in a locally co-moving coordinate system. This system, the local $p'' = 0$ frame, is defined by choosing a reference point somewhere in the fire tube and Lorentz-boosting the local average longitudinal momentum of pions to zero. The longitudinal separation of nearest neighbor pions in this frame is

$$\Delta \sim \left(\frac{p_1}{p''} - 1 \right) d_0. \quad (8)$$

For $dN/dy \sim 2$, $\Delta \sim 1/2$ fm.

Many more mesons are produced in a nucleus-nucleus collision than in a pp collision, and the density of pions in the $p'' = 0$ frame

increases correspondingly. The cross sectional area of the region in which the pions are produced is $S \sim \pi R^2 \sim 4A^{2/3}$. Assuming the height of the central region in nucleus-nucleus collision is A , and assuming $\Delta \sim 1/2$ fm, the number density of pions in the central region is

$$n \sim 1/2 A^{1/3} \text{ pions/fm}^3. \quad (9)$$

This number is appropriate for ISR energies, and is roughly doubled at Isabelle energies if the observed $\ln^2 E$ growth in multiplicities may be extrapolated from ISR to Isabelle energies.

If each pion has an average energy $E \sim 600$ MeV in the local $p'' = 0$ frame, the energy density ϵ for uranium nuclei is

$$\epsilon \sim 2 \text{ GeV/fm}^3 \quad (10)$$

at ISR energies, and

$$\epsilon \sim 4 \text{ GeV/fm}^3 \quad (11)$$

at Isabelle energies. The energy densities obtained assuming $A^{2/3}$ heights of the central region are ~ 300 – 600 MeV/fm³.

The energy densities of Eqs. (10) and (11) are of the same order as those in the fragmentation region. These energy densities are larger than the energy density of a proton, and a quark-gluon plasma may form.

The characteristic expansion time of the matter produced in the central region and fragmentation region determines whether or not the matter comes into thermodynamic equilibrium. The characteristic expansion time for the fragmentation region is the time it takes a sound wave to travel across the compressed nucleus. For $v_s \sim 1/\sqrt{3}$, this time is $\tau \sim 4$ fm. Simple arguments employing the parton model⁽²⁾ and perturbative QCD⁽⁸⁾ suggest that this time is sufficient for the matter to achieve kinetic equilibrium. The perturbative QCD computations also tentatively suggest that chemical equilibrium may be attained.

The time scale in the central region differs from that of the fragmentation region. In the local $p'' = 0$ frame, the hadronic matter is produced with a momentum gradient

$$\frac{dp''}{dx} \sim 200 \text{ MeV/fm-pion.} \quad (12)$$

The matter is therefore initially produced expanding along the longitudinal direction. The characteristic expansion time which follows from Eqs. (9) and (12) is $\tau \sim 2$ fm. Arguments from the parton model suggest that this time may be sufficient for kinetic equilibrium to be established.

There are many potentially interesting avenues of research which may discover signals for the formation of a quark-gluon plasma in ultra-relativistic nucleus-nucleus collisions. Two potentially promising probes of nucleus-nucleus collisions are photons and lepton pairs, and strange particles.

Electromagnetic signals involving photons and lepton pairs may provide a clean probe of the hot, early stages of the quark-gluon plasma dynamics^(8,9). These particles are produced most copiously in the early stages of the plasma evolution, when the interactions between the plasma constituents occur most often and with the greatest average energy. Electromagnetic particles are also very penetrating and easily pass through the hot, hadronic matter produced in the nucleus-nucleus collision. Their contribution to nucleus-nucleus collisions are therefore enhanced by $A^{1/3}$ relative to surface production of low $p_{\perp}$ hadrons.

A large background for lepton pairs produced in the plasma is from Drell-Yan pairs produced in the initial nucleus-nucleus collision. The Drell-Yan background is reduced by inverse powers of the lepton pair mass for large pair masses. The plasma contribution dies off as an exponential of the pair mass scaled by the plasma temperature, and Drell-Yan pairs dominate at large masses. At large $p_{\perp}$, Drell-Yan pairs probably dominate also since gluon bremsstrahlung corrections to the lowest order Drell-Yan process generate high $p_{\perp}$ pairs.

The plasma contribution to lepton pairs may appear in the $X_F > 1$ region. This kinematic region is forbidden in pp collisions. A contribution will be found in nucleus-nucleus collisions since the nucleons have Fermi motion within the nuclei, and since these nucleons rescatter after the collision. The nucleon Fermi motion generates a Drell-Yan background for $X_F > 1$, but this background may be suppressed relative to the plasma contribution for $X_F \gg 1$.

A proper resolution of this potential signal deserves careful

theoretical and experimental study. Several parameters which may yield a resolution of this signal involve studying the $p_{\perp}$, M^2 , and X_f dependence of the pair production. Another parameter is the A dependence. High mass Drell-Yan pairs, and plasma produced lepton pairs are produced proportional to $A^{4/3}$ in a head-on collision. Pairs produced from hadron vector mesons on the surface of the hot fire tube vary as A .

Prompt photons also yield information about the plasma dynamics. A background to prompt photons produced in the plasma is provided by prompt photons produced in the initial nucleus-nucleus collision. The contribution of these prompt photons may be reduced by techniques discussed for reducing Drell-Yan pair backgrounds.

Another severe problem for measuring plasma photons is the background from π^0 decays. The total number of these γ 's produced in the fragmentation region is $N \sim 10^2 - 10^3$. The total number of energetic prompt γ 's from the plasma may be as small as 1-10.⁽⁸⁾ Some reduction in the π^0 background may be achieved at $X_f \gg 1$. The energetic γ 's produced in this region may arise from energetic quarks in the plasma rather than from the lower energy π^0 's produced at the fire tube surface.

As has been emphasized by Willis, low $p_{\perp}$ photons and low mass, $M < M_{\rho}$, low $p_{\perp}$ lepton pairs may provide a signal for the quark-gluon plasma dynamics. In this kinematic region, backgrounds from resonance decays and Drell-Yan pairs may be small. This kinematic region deserves careful theoretical study to determine if signals from the quark-gluon plasma may be disentangled from contributions arising from conventional processes in hadron-hadron scattering.⁽¹⁰⁻¹¹⁾

The electromagnetic component of the plasma yields information about the quark distributions inside the plasma. The high energy component might be calculable in perturbative QCD. The low momentum component may yield information on non-perturbative plasma dynamics. If variation of A and center of mass energy, E_{cm} , results in a significant temperature and density variation, a rapid variation in the production rates for lepton pairs as a function of temperature might signal a phase transition.

A hadronic signal for plasma formation might be found in strange particle production.⁽¹²⁾ The production of $s\bar{s}$ quark pairs takes place rapidly in the early stages of plasma formation. This production is not suppressed by the Zweig rule, since colored $s\bar{s}$ quark pairs may be produced by one gluon annihilation. If the plasma is hot and dense for a long enough time, the strange quark component may come into chemical equilibrium with up and down quarks and gluons.

Many of the strange quarks are radiated away as kaons. The $K^+(u\bar{s})$ and $K^0(d\bar{s})$ mesons are emitted preferentially to $K^-(u\bar{s})$ and $\bar{K}^0(\bar{d}s)$, since the u and d quarks have large Fermi energies. The strangeness of the plasma may tend to increase by kaon emission.

As the plasma begins expanding and evaporating, baryons are formed. These baryons might be formed in a sea of up and down quarks which is greatly enriched by strange quarks. In the $SU(3)$ symmetric limit where $\mathcal{N}_u \sim \mathcal{N}_d \sim \mathcal{N}_s \gg \mathcal{N}_u - \mathcal{N}_u \sim \mathcal{N}_d - \mathcal{N}_d \sim \mathcal{N}_s - \mathcal{N}_s$, there would be copious strange baryon production. Assuming an $SU(3)$ symmetric coalescence of quarks into baryons, the fraction of strange baryons is

$$f \sim 1 - \left(\frac{2}{3}\right)^3 = \frac{19}{27} \sim \frac{2}{3}. \quad (13)$$

There are, of course, many other potential signals of the dynamics of nucleus-nucleus collisions -- for example, high and low $p_{\perp}$ hadron production, and antibaryon production. Exotic QCD phenomena might also be studied, e.g., Prasad-Sommerfield chromo-magnetic monopoles, and instantons.

If free quarks were unconfined at zero temperature,⁽¹³⁻¹⁴⁾ truly exotic phenomena might occur in very high energy nucleus-nucleus collisions. Free quarks and gluons might be more easily produced in nucleus-nucleus collision than in a hadron-hadron collision.

In a finite temperature plasma, free quarks and gluons are present when $T > T_c$. This is true even for theories in which there are no light mass fermions. The octet color charge gluons may shield the triplet charge of heavy quark sources.

Suppose a quark or gluon surrounds itself with a color neutral ensemble of quarks and gluons as shown in Fig. 9. This ensemble might be regarded as a quark-gluon plasma with a finite color charge. The quark or gluon surrounded by hadronic matter is reminiscent of the picture of free quarks proposed by DeRujula, Jaffe, and Giles.⁽¹³⁾

In the plasma, the color field of the excess quark might generate a Coulombic strength field at the surface of the plasma-ball, $E \sim Q/4\pi R^2$. If the QCD vacuum does not confine color, this field strength might be insufficient to produce quark-antiquark or gluon-antigluon pairs as the

plasma-ball separates from another plasma-ball of opposing color charge. In hadron-hadron collisions, string-like configurations of color fields are favored in the hadron production processes. These string-like configurations generate quark-antiquark and gluon-antigluon pairs more easily than do Coulombic field configurations, and separated color is more easily neutralized.

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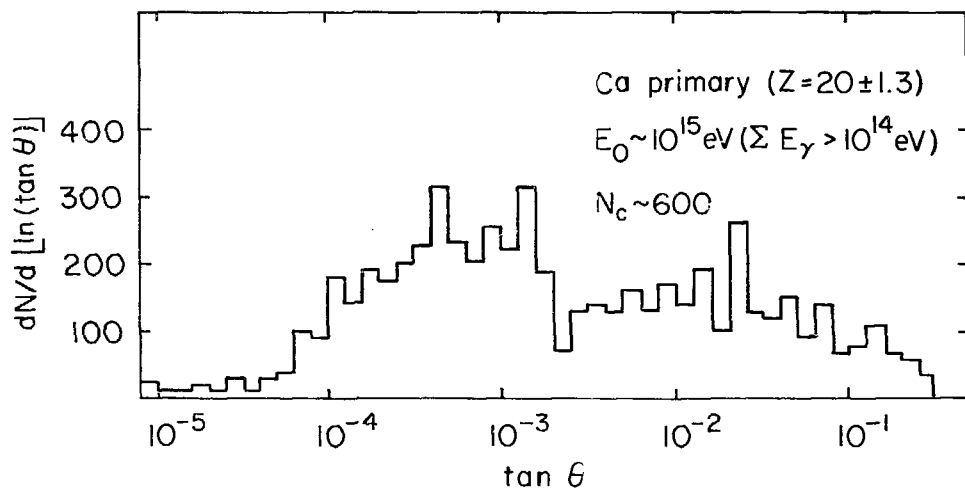


Fig. 1. A pseudo-rapidity distribution for a 10^3 TeV calcium nucleus-emulsion event.

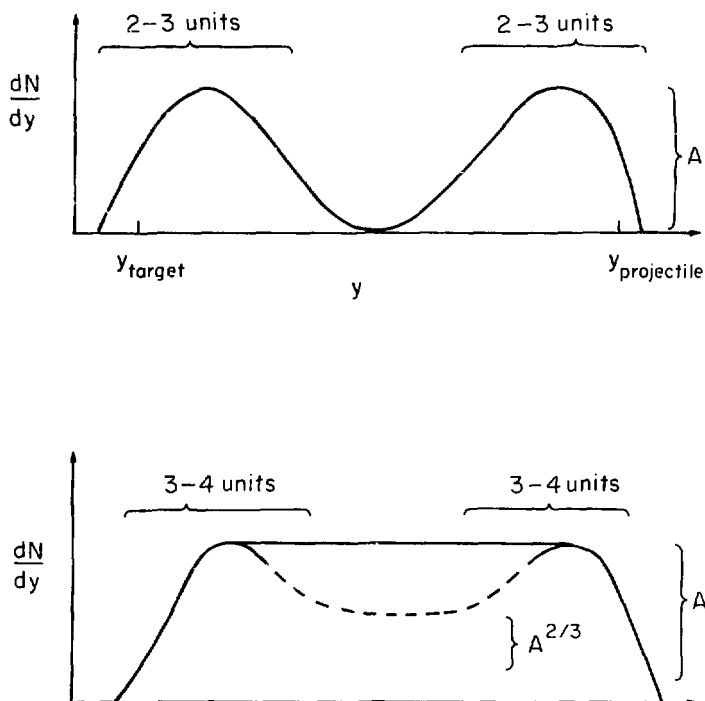


Fig. 2. Rapidity distributions for head-on nucleus-nucleus collisions:

- (a) The baryon number distribution (nucleon minus anti-nucleon),
- (b) The meson distribution assuming heights in the central region proportional to A (—) and $A^{2/3}$ (---).

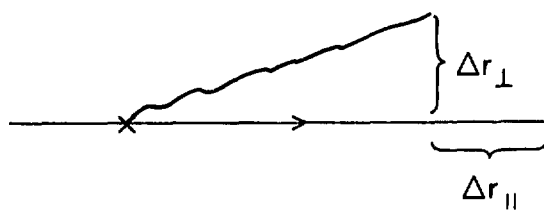


Fig. 3. Inelastic production of a particle.

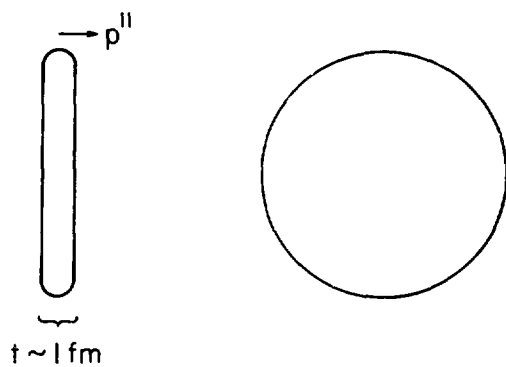


Fig. 4. Nucleus-nucleus scattering before a head-on collision.

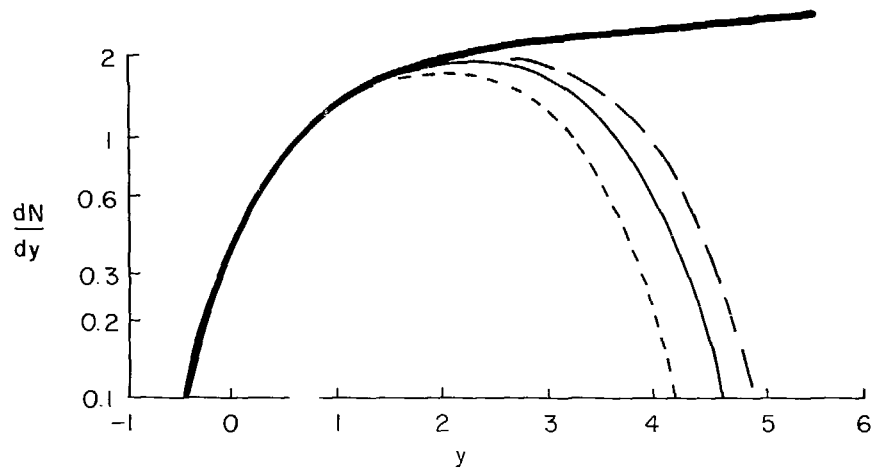


Fig. 5. The regions of phase-space from which nucleons are trapped under various seasonable trapping criteria. A plot of dN/dy for pp collisions at high energies is shown with the heavy line.

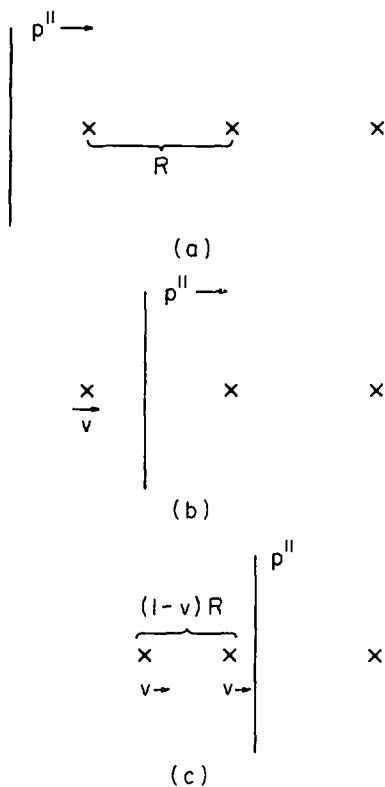


Fig. 6. Compression of the target nucleus:

- (a) Before encountering the first nucleon,
- (b) After encountering the first nucleon, and
- (c) After encountering the second nucleon.

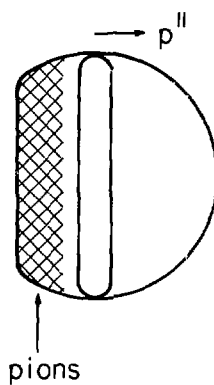


Fig. 7. The projectile nucleus sweeping through the target.

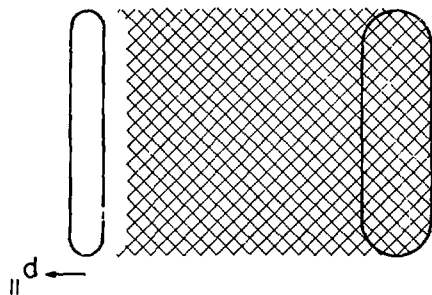


Fig. 8. Matter forming in the central region.

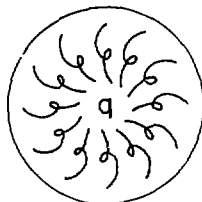


Fig. 9. An isolated quark surrounded by an ensemble of color neutral quarks and gluons.

STATISTICAL-HYDRODYNAMICAL MODELS OF
PARTICLE PRODUCTION IN HADRON-HADRON COLLISIONS*†

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A general discussion of the degrees of freedom suitable for the description of complex many particle production processes is given with emphasis on those averaged variables which can describe the gross features of various reactions. A review of the statistical-hydrodynamical model (initiated by Fermi and Landau and extended by many others) is given, with emphasis on simple hadronic processes. Despite some conceptual obscurities, the principal empirical consequences of this model are increasingly borne out in averaged quantities like single particle inclusive distributions and total multiplicities. Following speculations on the possible connection with QCD and quark-gluon matter, we briefly mention other collective and coherent effects to be studied in high energy multiparticle production reactions. Finally we relate such models to the underlying dynamics expressed in kinetic theory form by the use of phase space distributions introduced by Wigner. After showing how to set up scattering problems in this framework we analyze the N-body dynamics in terms of Boltzmann-Vlasov like equations.

*Work supported by the U. S. Department of Energy

I. INTRODUCTION

In 1973 I had the opportunity to summarize¹ the status of the hydrodynamical model of particle production at a meeting sponsored by the New York Academy of Sciences. At that time questions of scaling laws, the nature of rapidity distributions, the energy dependence of secondary multiplicities, etc. were in the center of attention and imagined to be supportive of the prevailing dogmas of parton thinking and modern j-plane theory. The point of my earlier talk was that these gross features of particle production data are better described by a suitably modernized Fermi-Landau statistical-hydrodynamical model, than by the then fashionable views. To quote from the introduction to that paper: "The fundamental averaged coordinates are not quarks, partons, Regge poles, or what have you, but are instead averaged thermodynamic field quantities such as energy density, entropy density, and so forth. There is, therefore, an emphasis on the collective motions, and the collective motions within the medium, of highly compressed and energetic hadronic matter. We propose to call this substance "prematter," calling matter the stuff of which S-matrix theory is composed (the asymptotic region). Since the behavior of prematter is presumably described by a dynamics more or less independent of the detailed laws of force as well as a knowledge of the fundamental constituents, it may very well happen that the study of such coarse quantities as inclusive distributions will not lead to any decisive information about fundamental issues."

These heretical views had a cold reception in the high energy community, although there remains a small and dedicated band of theorists who believe in the merit in this way of thinking.

Since 1973 the confluence of electro-weak gauge theories, QCD and unification models has diverted attention from the particle production

problem. Nevertheless a clear delineation of the space-time regions amenable to suitably averaged statistical descriptions remains a significant subject of research. Indeed the time seems ripe for a reassessment of the statistical-hydrodynamical models in the light of our increased understanding of the QCD description of strong interactions.

The basic coordinates used in the description of strong interactions have undergone a radical change in the past fifty years. It is probably premature to imagine that this evolution has terminated with the present dogma promulgated by QCD. In the 1930's we had protons and neutrons interacting by potentials. In the 1940's the addition of the pions gave substance to the force picture of Yukawa and the isospin concepts of Heisenberg and Kemmer. In the 50's and early 60's the discovery of many resonances gave rise to dominance of the S-matrix description of strong interactions and the central role of poles of the S-matrix. The 60's gave rise to Regge poles, primitive quark models, and currents generating the SU_3 (flavor) (and chiral SU_3 (flavor) suggested by the quark model. Further puzzling over awkward features of the quark model led Han and Nambu to introduce more SU_3 internal variables for the quarks (now called color variables) and finally to the idea of an unbroken local symmetry whereby an octet of massless gauge fields couples to the color degrees of freedom. This modern version of the theory was clearly expressed by Fritsch and Gell-Mann in 1972 and has provided lots of work for theorists ever since. Whether this preoccupation with quarks and glue proves to be yet another phase of an unknown evolution one cannot yet say.

There is another tradition, with still more ancient roots, in many ways independent of the details of the degrees of freedom. This lore deals

with statistically averaged variables, such as pressure, energy and entropy density, temperature, fluid four-velocities, etc. Naturally the idea of locally defined thermodynamical variables becomes suspect in transient states induced by high energy collisions. Nevertheless, the notion of collective variables is a venerable and useful idea, not necessarily dependent on the extreme conditions necessary for local thermodynamic equilibrium. Our task is then to identify and successfully use such variables, grafting the collective phase correctly onto the purely quantum phases of the evolution of the system. As usual the real difficulty is that the "basic" theory is too hard to solve, and the domain of applicability of the statistical-hydrodynamical models is too hard to assess.

At the end of this lecture I will outline briefly a theoretical framework, developed since 1973 in collaboration with Zachariasen, in which the complete quantum mechanical evolution is retained in a form where the transition to classical or kinetic descriptions is easily made when appropriate. This formalism is based on the Wigner phase space version of quantum mechanics.² In second quantization it is easy to extend this scheme to relativistic collisions and particle creation.

In broad outline the succeeding sections treat the following topics:

II. Review of the traditional statistical-hydrodynamical model initiated by Fermi and Landau.

III. Quark matter, QCD and the possible creation of new phases and liberation of diquarks in nucleus-nucleus collisions.

IV. Other collective and coherent effects, especially in hadron-nucleus collisions.

V. Connection with microscopic theory, via phase space distribution functions, and reduction of the N-body problem to the Vlasov-Boltzmann equation.

The traditional example of a simplified description of a many body system is the Boltzmann gas, where the elementary particles are the molecules. In this case the molecular collisions can establish local equilibrium if $\omega\tau \ll 1$ where $1/\tau$ is the collision frequency and ω is the frequency of the disturbance in question. In this case there is a local temperature $T(\vec{x}, t)$ independently of the details of the interatomic force, at least in the context of the Boltzmann equation. The traditional collective motions in this case are (a) hydrodynamic flow derived as moments of the Boltzmann equation and (b) sound vibrations derived secondarily from linearized versions of the hydrodynamic equations.

The preceding example is the prototype of the typical behavior of a many-body system, regardless of the existence of local thermodynamic equilibrium: namely flow behavior and the existence of internal excitations. At our meeting the most interesting situations involve three types of collisions. (a) proton-proton collisions, (b) proton-nucleus collisions and (c) nucleus-nucleus collisions.

In nuclear physics, the classical example of a statistical description of a collision process is the deposition of a low-energy neutron in a nucleus followed by thermalization of the neutron's energy resulting in a decay product spectrum described by an effective temperature T .

In heavy ion physics one can imagine the overlap of colliding nucleons in a geometrical sense if the velocities are sufficiently high to prevent effective communication of information to the non-overlapping regions. This simplest situation can be extended with sufficient care. For example at very low energies that best picture will be the usual sticky, viscous fluid with surface tension effects included. All these pictures, however compelling they may be intuitively, require further elucidation in a microscopic theory.

As for "elementary particle" collisions, nowadays we imagine a new situation both similar and different to that encountered in nuclear physics, where the colliding objects are imagined to be composed of the usual observable particles. For proton-proton collisions the colliding particles are supposed to be made of quarks and glue which never, or almost never, come out. The final particles are created by this conversion of the collision kinetic energy to field energy which finally materializes into asymptotic quanta. These various situations are described in Fig. 1.

Before going on to describe models we review a few important experimental facts from high energy proton-proton collisions. First of all, most of the produced particles are pions (~90%) even at the highest energies. These secondaries typically have their momentum directed along the collision axis, with the transverse momentum usually less than 300 MeV/c. A good fit to the p_T pion distribution is given by

$$\exp(-6p_T) \quad (1.1)$$

with p_T measured in GeV/c. Note that this is also $\exp(-p_T/m_\pi)$, a point we will return to later. In p_T - rapidity coordinates the secondaries

mostly live in a cylinder of radius 300 MeV/c and length $\ln s$, where s is the squared c.m. energy (see Fig. 1). It is not surprising then that the energy dependence of the multiplicity grows less rapidly than if production occurred all over momentum space. In fact $n \sim \ln s$ is not a bad fit and has been preferred by the wise men over the better power laws s^p , characteristic of statistical models. To see what the maximum multiplicity could be, imagine that we convert all the available energy in a p-p collision into n pions in the c.m. Energy conservation (plus conserving two baryons) gives

$$n m_\pi = \sqrt{s} - 2m_p \approx \sqrt{s} \quad (1.2)$$

which gives $n \sim s^{1/2}$, a much faster growth than the empirical $s^{1/4}$. In the next section we will discuss the multiplicity problem in much greater detail.

II. THE FERMI-LANDAU STATISTICAL-HYDRODYNAMICAL MODEL

These models of particle production in "elementary" particle production have roots going back over thirty years, and origins associated with such illustrious names as Heisenberg,³ Fermi,⁴⁻⁵ and Landau.⁶ (Heisenberg's idea about a turbulent particle production generated by nonlinearities in the field interactions might bear reexamination in view of recent developments in QCD.) Here we mainly pursue the Fermi-Landau tradition, whereby the kinetic energy of the projectiles is also converted to field energy via unspecified interactions and finally into asymptotic quanta. Since the mechanism of this conversion has in the past been unknown, this class of models has always relied on ad hoc initial conditions whose domain of validity eludes even the true believers.

Now it may become possible to illuminate this question in the context of QCD; indeed this would seem a most fruitful area of research at present. In the past it has been puzzling why the Fermi-Landau predictions worked so well even after some fudging involving leading particle effects, etc. At the end of our lecture we will outline a fully quantum mechanical framework within which kinetic-statistical behavior can be expected in favorable space-time regions.

The simplest version of Fermi's model imagines (Fig. 2a) the head-on collision of two Lorentz-contracted protons, which merge into a common blob of "stopped" matter (2b). At the time of the model's inception it was imagined that nuclear forces were so strong that the motion could be fully arrested and the projectile kinetic energy converted completely into internal thermal energy. Now we can imagine that nucleons are more transparent and moreover that peripheral collisions are significant even though they don't contribute so significantly to high multiplicity events as do the nearly central collisions. Hence a better picture of the situation immediately after a collision might be (2d) where the target and projectile fragments take off a substantial part ($\sim 1/2$) of the kinetic energy, leaving behind in the c.m. a blob of excited hadronic matter. In this situation one can imagine that the dynamics of the expanding blob should be described by hydrodynamics even though the extreme assumptions of the original Fermi picture (head-on collisions, complete stopping of the particles) are not believed. A proper quantitative analysis of this qualitative picture would constitute major progress.

In the simplest version of this model we have an energy $2E_{\text{cm}}$ dumped into a Lorentz contracted volume V_0/γ where γ is $E_{\text{cm}}/m = s^{1/2}/2m$ (m the proton mass) where $s = (2E_{\text{cm}})^2$ is the usual squared total c.m. energy.

V_0 is roughly the volume proton at rest, $4\pi/3\mu^3$. Then, when $T > \mu$, where μ is the rest mass energy of the produced quanta (pions), we can use standard black body radiation formulas. For example the equilibrium temperature is computed from

$$CT^4 = \epsilon = 2E_{cm}/V = \frac{s^{\frac{1}{2}}}{V_0/\gamma} = Ks \quad (2.1)$$

where C and K are known constants. Hence $T \propto s^{\frac{1}{4}}$ (for $T > \mu$) and the number density $n_\pi \propto T^3 \propto s^{3/4}$. The produced pions jumping into momentum space therefore have energy dependence going as

$$N_\pi = n_\pi V \propto s^{\frac{3}{4}}. \quad (2.2)$$

This is all fine, in that the coefficients as well as the s dependence, are in quite reasonable agreement with data. However the existence of other species of particles gives trouble with the observed dominance of pion secondaries at all energies. For example, when $T > m_K$ (already reached around 300 GeV in lab proton energy) we would by statistical weights expect

$$N_K = \frac{4}{3} N_\pi. \quad (2.3)$$

Hence Fermi's picture leads to too many heavy secondaries at very high energy, as well as a wrong isotropic angular distribution in momentum.

These defects were in part remedied by Pomeranchuk⁷ and Landau.⁶

Pomeranchuk noted that the matter created in the initial collision

interacts while it expands and cools. When T reduces to the mass of the lightest quantum (the pion) the number density drops sharply to zero so that interactions cease. Hence in this picture the heavy particles are suppressed by Boltzmann factors like $\exp(-m_K/m_\pi)$ if the reactions are sufficiently fast to establish any kind of local equilibrium. This problem of the relative population of heavy secondaries has been studied by Shuryak.³⁴

Landau proposed a more detailed dynamics which adopts the Fermi initial condition for the evolution of a perfect (i.e., no viscosity or heat conductivity) single component relativistic fluid of zero chemical potential, appropriate for non-conserved particles like pions. He took the ultra-relativistic equation of state

$$p = \frac{1}{3} \epsilon \quad (2.4)$$

appropriate for massless particles (or $T \gg m$) though in subsequent times many authors have parameterized data in terms of a more general formula

$$p = c_0^2 \epsilon \quad (2.5)$$

where c_0 is the adiabatic sound velocity.

The conservation of mass-energy and Newton's laws are subsumed in

$$\begin{aligned} \partial_\mu T_{\mu\nu} &= 0 \\ T_{\mu\nu} &= (\epsilon + p) u_\mu u_\nu - g_{\mu\nu} p \end{aligned} \quad (2.6)$$

where the stress-energy tensor involves the four velocity u , the pressure p , the energy density ϵ and the flat space metric $g_{\mu\nu}$.

At $t=0$ we assume Fermi's boundary condition, followed by hydrodynamic expansion until the breakup time t_B defined by the hypersurface

$$T(\underline{x}, t) = \mu. \quad (2.7)$$

For such $t_B(\underline{x})$ the appropriate pion mass is identified with the proper amount of c.m. fluid energy to obtain the differential cross section of the secondaries.

The different stages of this sequence are shown in Fig. 5. Unfortunately the results are not simple or transparent except in the case of Khalatnikov's one-dimensional solution.⁸ We may remark, however, that a weaker condition of the strong condition of local entropy current conservation

$$\partial_\mu s_\mu = 0 \quad (2.8)$$

namely overall entropy conservation, leads along with the identification $N_\pi \propto S_\pi$, to a multiplicity formula in accord with Fermi's original form (2.2). Indeed in Fig. (3) we see that this formula works for nearly all colliding objects whether hadronic or not, with nearly the same coefficient. This is all the more remarkable since the wise men of the j-plane and multiperipheral school have convinced experimentalists and theorists alike to think only logarithmically, and to fit the multiplicity with

$$A + B \ln s, \quad (2.9)$$

or when that fails, with

$$A + B \ln s + C (\ln s)^2 \quad (2.10)$$

etc. with no estimate of A, B, C, while the simple $Ks^{\frac{1}{2}}$ works perfectly (though rarely mentioned in current literature).

This universality was stressed⁹ at least as early as 1973. Now that $e^+ - e^- \rightarrow$ hadrons also exhibit¹⁰ the $s^{\frac{1}{2}}$ dependence, following earlier vicissitudes, it is really worth rethinking the whole issue.

To understand this point, however, we have to confess an error we made earlier (before the knowledge of jet structure in hadronic secondaries of $e^+ - e^- \rightarrow$ collisions) which were used¹¹⁻¹² in all subsequent hydrodynamic models of $e^+ - e^- \rightarrow$ hadrons. The hydrodynamic multiplicity formula depends on the initial hadron volume in a simple but sensitive way. Simplifying somewhat by concentrating on pions, we have

$$N = \alpha S_f \quad (2.11)$$

where $S_f = S_i$ by entropy conservation and

$$S_i = \sigma_i V_i \quad (2.12)$$

with σ_i the initial entropy density and V_i the initial hadronic volume.

Since $\sigma_i \propto \epsilon_i^{3/4}$ and $\epsilon_i = s^{\frac{1}{2}}/V_i$ we find for the universal multiplicity formula

$$N = C' s^{3/8} V_i^{\frac{1}{2}} \quad (2.13)$$

For pp collisions $N = Cs^{\frac{1}{2}}$, where $C = C'(2m_p V_0)^{\frac{1}{2}} 1.75$, all units in GeV. For πp , the same result holds for $\sqrt{s} \gg m_p$. In ep collisions a small Q

dependent correction is expected, $(1 + \frac{m_p^2 + Q^2}{s})^{\frac{1}{4}}$, at first seen and later not. In the case of $e^+ - e^-$ collisions, which proceed in the c.m. frame through a time like (spherically symmetric) virtual photon, we originally assumed that the initial hadrons would materialize in a spherically symmetric energy independent volume V_0 . This led to a multiplicity formula $N = C s^{3/8} (2m)^{\frac{1}{4}}$. Now it seems that, event by event, the photon materializes into back-to-back quark-anti-quark pairs which transform into jets more like those occurring in pp collisions. Indeed the greatly increased localization of matter in the final state jets improved the assumptions underlying the sort of equilibration underlying the hydrodynamical formulas. As before, we have yet to learn how to graft on the quantum structure of the initial condition (appropriate to asymptotic freedom) to the long range hadronic regime, presumably described by averaging processes at least similar to the hydrodynamical model. The clarification of this situation is the subject of current research.

In the foregoing we presented the question of the energy dependence of the multiplicity, the simplest of all problems, in an oversimplified way. We neglected viscosity (astonishing to low-energy nuclear physicists) and ignored the nuances of being not yet in a truly asymptotic regime. We also ignored the question of whether the contribution of 2 of the protons in pp collisions to the charged multiplicity can have an impact on data analysis, as pointed out by Friedlander,¹³ who believes that an $s^{3/8}$ law works well after subtracting out this effect. Hence a certain caution is advised even when approaching this much discussed topic.

We now pass to a brief description of the traditional Landau model, reviewed by this author and many others. Since the main results are well-known to some we shall be brief in our detailed description.

In the 1950's the model was developed in the relative absence of pertinent experimental data by Russian and Japanese workers among whom occur Pomeranchuk,⁷ Landau,⁶ Khalatnikov,⁸ Milekhin,¹⁴ Feinberg,¹⁵ Amai,¹⁶ Koba,¹⁷ Iso,¹⁸ Namiki,¹⁹ Ezawa,¹⁹ Ito, and Tanaka.²⁰ In the 1960's the rise of the S matrix and its multiperipheral perversions led to the suppression of research in this field. As good inclusive data became available it was noticed in the early 1970's by several workers (Carruthers and Duong-Van,^{21,22} Feinberg,¹⁵ Shuryak²³) that the original formulas of Landau gave a surprisingly good description of the available data. A small revival occurred through the work of Cooper,²⁴ Schonberg,²⁴ Weiner,²⁴⁻²⁶ Friedlander,²⁷ Hwa,²⁸ Chiu,²⁹ Masuda,³⁰ Anderson,³¹ Chaichan,³² Suhonen,³² and others, despite the negative attitude of many theorists toward the hydrodynamical model and especially its traditional assumptions. Nevertheless, the results of the Landau picture work and must be derived in the context of other approaches if the latter are to be taken seriously. In summary, these results are (written for pp collisions)

- (1) Confirmation of the Fermi multiplicity law

$$N = C s^{\frac{1}{2}} \quad (2.14)$$

- (2) Gaussian longitudinal rapidity distribution

$$\frac{1}{N} \frac{dN}{dy} \equiv \exp(-y^2/2L)/(2\pi L)^{\frac{1}{2}} \quad (2.15)$$

$$L \equiv \frac{1}{2} \ln s/4m^2 \quad (2.16)$$

$$y = \frac{1}{2} \ln \frac{E + p_{||}}{E - p_{||}} \quad (2.17)$$

- (3) Exponential transverse momentum distribution

$$\exp(-p_T/m_\pi) \approx \exp(-6p_T) \quad (2.18)$$

with p_T in GeV^{-1} .

(4) Rising so-called plateau

$$\left(\frac{d\sigma}{d\Omega}\right)_{90^\circ} \propto s^{1/4}/(\ln s)^{1/2} . \quad (2.19)$$

It should be noted that this distribution (2) is compatible with approximate Feynman scaling, except in the central region where deviations are correctly predicted,²² at least for the well studied case of pions. As for the transverse distribution, (2.18) corresponds to a uniform decay temperature. More detailed dynamical pictures lead to two separate slopes possibly coming from two separate regions characterized by differing sound velocities and possibly distinct hadronic phases. We return to this point in Sec. III.

In summary, the predictions of the Landau model are good enough to warrant further study to understand better which results are due to collective behavior and how they match up to more quantum-mechanical regimes. We believe that using QCD as a back drop will help in this analysis. As an example we mention a conjecture of Pikorski and van Hove³³ (described in Shuryak's review³⁴) which interprets Fig. (2d) in terms of a residual glue-ball excited in the collisions of the 3-quark bound state protons, which go away with $\frac{1}{2}$ the energy. This picture is buttressed by the quarks carrying $\frac{1}{2}$ the momentum of the proton in the parton model. The glue-glue-glue coupling is in many cases large enough to support such an interpretation. Right or wrong in detail, pictures like this will probably lead to great progress in elucidating the domain of validity of the hydrodynamical model.

III. QUARKIUM

The possible existence of quarkium, i.e., matter composed of quarks, anti-quarks and gluons, has become the subject of increased speculation in recent years. Today we are concerned mostly with the possibility of creation of this phase in hadronic collisions, and how one can know whether the elusive stuff has indeed been produced. In the case of pp collisions most workers now attempt to explain everything in terms of the underlying quarks and glue, although the collective or thermodynamic features are usually ignored. Dense astrophysical objects are a likely place to find quarkium, and many papers have been written on this subject. Whether very much quarkium can be produced in nucleus-nucleus collisions seems chancy to me on the basis of geometrical considerations. Nevertheless it is fun to consider what would happen if we could indeed make quarkium.

Although various authors have studied "the" phase transition from nuclear matter to quark matter, and the equations at state of each, perhaps not enough attention has been given to the various phases of quarkium. Even without the color degree of freedom (i.e., Abelian glue) we found many interesting possibilities in pre-QCD days for non-relativistic quarks treated as a Fermi liquid.³⁵ In analogy to the coulomb plasma one can have a lattice at low densities, a possible ferromagnetic phase when at lower densities the exchange energy is big enough that alignment of the spins lowers the energy faster than the concomitant increase in the kinetic energy of the Fermi sea. There are further possibilities of a self-consistent scalar flavor octet pair excitation and perhaps a superconducting phase. For QCD the number of degrees of freedom, and the number of physical possibilities

should be much greater, as exemplified by the existence of the instanton. A fairly up to date bibliography on this topic can be found in the reviews of Shuryak³⁴ and Hakim.³⁶

Since Professor Satz has already dealt with the properties of quark matter in this morning's lecture³⁷ I will only mention a few additional selected topics pertaining to this subject.

First consider a possible QCD underpinning suggested by Fig. 2d. If gluon self interactions dominate in production (even though color neutral $\bar{q}q$, and $3nq$ combinations will finally materialize) one could have graphs like those in Fig. 5 corresponding to Fig. 2d. Given enough time one could expect a kinetic theory description of thermodynamic equilibrium among the q , $\bar{q}$, g degrees of freedom. But the understanding of all this lies in the future.

Another interacting possibility relevant to detection of the phase transition between ordinary hadronic matter and quarkium has been pointed out by Friedlander and Weiner²⁷ in their analysis of large p_T particles produced in pp collisions. By taking very literally the predictions of the hydrodynamical model they interpret the change in slope as a change in the velocity of sound in the heated hadronic matter. The largest p_T particles are described by a velocity of sound $u^2 = 1/3$ characteristic of an ultra-relativistic fluid and identified with the hottest matter created in central collisions. For smaller values of p_T , $u^2 \approx 1/6$, which value corresponds to the equation of state of ordinary hadronic matter. As a consistency check, they note that the same approach gives a correct description of A dependent effects in proton-nucleus phenomenology.

Still more speculative is a recent idea of Goldman, Slansky and Shaw³⁸ concerning the possible production of diquarks in nucleus-

nucleus collisions. In their scheme SU_3^c (c for color) is not a perfect local gauge symmetry but is broken down to SO_3^c

$$SU_3^c \rightarrow SO_3^{c'}$$

where c' is the new "color" of the ordinary rotation group. We shall not describe the detailed mechanism of this breaking, which is quite natural, but only emphasize the consequences. $SO_3^{c'}$ is now the local gauge symmetry and the usual confinement lore is supposed to apply. Thus only c' singlets are allowed, but these can be ordinary diquarks as sketched below.

Under the breaking we have, labeling representation by their dimensions

$$q(3^c) \rightarrow q(3^{c'}) \quad (3.1)$$

$$g(8^c) \rightarrow g(3^{c'}, m=0) + g(5, m \neq 0) . \quad (3.2)$$

Thus the quark becomes a vector in $SO_3^{c'}$ while the gluon splits into 3 massless gluons and 5 massive vector particles. Now by the usual vector addition rule we see that we can have $SO_3^{c'}$ singlets in the combinations

$$(q^{c'} q^{c'}) \quad (3.3)$$

$$(q^{c'} g^{c'}) . \quad (3.4)$$

Where does the need for nucleus-nucleus collisions come in? In order to account for the scarcity of quarks the mass scale characteristic of the breaking has to be such that the breaking does not occur over

ordinary hadronic distances. Yet the scale must be such that there is some effect, at least in giant hadrons created in, say, uranium-uranium collisions. The usual scale in QCD is $\Lambda \approx 200\text{--}300$ MeV. So if we choose the massive gluon mass to be ≈ 20 MeV, the symmetry breaking in the propagator manifests itself over a distance of about $10f$. In other words we have effective SU_3^C for spatial separations less than $10f$. By creating a $q\bar{q}$ glue plasma (hopefully in U-U collisions) one could hope to create a situation where diquarks could leak out. We also mention that for energies in the TeV range the length of the Landau jet could satisfy this criterion. Numerical estimates need to be made of all these effects.

IV. OTHER POSSIBLE COLLECTIVE AND COHERENT EFFECTS

We mention briefly several other types of interesting collective and coherent effects whose careful study would lead to new insights. The subjects mentioned are:

1) Transparency of nuclei, as indicated by the weak A dependence of multiplicities in proton-nucleus collisions. Self-induced transparency might also occur.

2) Hot spots, diffusive transfer of energy.

3) Cumulative effect.

4) Bose correlations.

5) Pion condensates and other phases.

1) Early in the development of the hydrodynamical model it was noted that the multiplicity in proton-nucleus collisions was only weakly A dependent if the behavior of the nucleus was treated as that of a cylinder of fluid of cross section swept out by the incoming proton

(Bilenky and Milekhin³⁹). Since that time further studies have clarified the physical situation. As suggested by Landau's original model, materialization of the final hadrons probably does not occur inside the nucleus at all. This "pre-matter" of ref. 1 (now supposed to be quarks and glue) evidently does not participate in cascades that one might naively expect. For references to the many points of view expressed on this problem one may consult the summary of Ferbel⁴⁰ at the preceding conference of this series. We also must mention the energy-flux-cascade model of Gottfried,⁴¹ which shares many of the features of Landau's model. Here we want only to point out that once again the hydrodynamical model of proton-nucleus collisions works better than it should. The papers⁴² of Masuda and Weiner should be consulted for the detailed predictions.

More speculative is a nuclear analogue of the self-induced transparency effect in quantum optics.⁴³

2) The possible implications of a local deposition of energy have been especially pursued by Weiner and collaborators.⁴⁴ These "hot spots" are treated as a diffusive heat conduction process. Although this dynamical treatment is clearly only a first step, some interesting results have been obtained and provide the ground work for further studies. An especially interesting application is to $^{16}\text{O} + ^{58}\text{Ni}$ collisions, where the ^{16}O decay into $^{12}\text{C} + \alpha$ exhibits an angular dependence in accord with the expected excitation of one side of the ^{16}O nucleus.⁴⁵ The justification of local thermodynamic equilibrium appears difficult, as in the hydrodynamical model, the physical effect is there and disbelievers in the hot spot approach are invited to produce superior explanations of the asymmetries.

Truly impressive hot spots can be expected in a forthcoming Los Alamos/CERN experiment to be done at LEAR. The annihilation process

$$\bar{p} N \rightarrow n \pi \quad (4.1)$$

when $n \sim 6$, say at a "point" in a nucleus provides a fascinating probe of the nucleus to a highly localized excitation and should provoke intense theoretical activity.

3) The "cumulative effect", whereby pions and protons produced at 180° in the inclusive reactions

$$P + A \rightarrow p + \pi + X \quad (4.2)$$

far exceed the number expected from individual collisions, has been around for some time.⁴⁶ Proposed explanations include the "drifting hot spot",⁴⁷ and a parton interpretation.⁴⁸

4) Several authors^{49,50} have considered the possible existence of coherent Bose-Einstein correlations in production of identical particles, e.g. in reactions like

$$p + A \rightarrow \pi^- + \pi^- + X. \quad (4.3)$$

For electromagnetic radiation the analogue is the Hanbury-Brown, Twiss effect. By studying the correlations of the pions in (4.3) one can hope to learn about the nature of the emitting source. For example the correlation

$$C = \frac{d^2\sigma/d^3p_1 d^3p_2}{\frac{1}{\sigma} \cdot d\sigma/d^3p_1 \cdot d\sigma/d^3p_2} \quad (4.4)$$

one can compare with the extremes of a chaotic (black body) or coherent sources (Fig. 7).

Not unexpectedly, the strong and electromagnetic interactions cause complications in comparison with the photonic analogue, because of final state interactions, existence of isospin, etc. Nevertheless in the near future one can hope for interesting results on this topic.

5) Pion condensates, excited even virtually, could produce enhanced coherence reactions such as (4.4). Quantitative prediction of such effects remain frontier research problems, so far as I know.

V. UNIFIED APPROACH TO QUANTUM REACTION THEORY AND STATISTICAL-HYDRODYNAMICAL MODELS

By rewriting the equations of quantum mechanics in phase space language using the Wigner distribution function² it is possible to cast scattering theory in a form wherein kinetic and transport behavior are readily apparent. In the past decade this approach to scattering theory has been pursued independently in quantum chemistry,⁵¹⁻⁵² nuclear physics⁵³⁻⁵⁵ and particle physics.⁵⁶⁻⁵⁸ Workers in these specialities have unfortunately not been in close touch, a situation hopefully to be remedied by a review article currently in preparation.⁵⁹ The formalism can be cast in fully relativistic form and can deal with the case of particle creation and destruction.^{36, 56-58} For most of the present discussion we shall however restrict most of our attention to the non-relativistic N-body problem.

The basic objects of the theory are the reduced distribution functions

$$\begin{aligned}
 &f(p_1 R_1 t) \\
 &f(p_2 R_2, p_2 R_2 t) \\
 &f^{(n)}(p_1 R_1, p_2 R_2, \dots, p_n R_n t)
 \end{aligned}
 \tag{5.1}$$

$1 \leq n \leq N$ which obey a typical BBGKY-like set of hierarchy equations.

The advantages of this formulation of reaction theory are:

- (1) Explicit scattering boundary condition,
- (2) Manifest classical limit (but dangerous^{51,60} because of essential singularities in $\hbar$),
- (3) Kinetic equations (Vlasov, TDHF, RPA, Boltzmann) are easy to derive, though hard to justify,
- (4) Inclusive cross sections are directly related to the reduced distribution functions:

$$\begin{aligned}
 \frac{d\sigma}{d^3 p_1} &= \frac{\int d^3 R_1 f(p_1, R_1, \infty)}{\text{flux}} \\
 \frac{d^2 \sigma}{d^3 p_1 d^3 p_2} &= \frac{\int d^3 R_1 d^3 R_2 f(p_1 R_1, p_2 R_2, \infty)}{\text{flux}}
 \end{aligned}
 \tag{5.2}$$

and so forth.

- (5) Hydrodynamic equations, when valid, follow as moment equations of the foregoing.

Naturally, the N-body problem is typically very difficult, even in a homogeneous system. Our claim is that this formalism provides the best

technical apparatus to analyze kinetic behavior while retaining full quantum-mechanical accuracy. Naturally the calculation of the reduced distribution functions is an arduous procedure.

The functions (5.1) resemble the usual Boltzmann distribution in classical physics. For example

$$f(p,R,t)d^3Rd^3p \quad (5.3)$$

is the probability of finding at time t a particle between p and $p + dp$, R and $R + dR$. For a pure state of a single particle on an orbit we have

$$f(p,R,t) = \delta[p - p(t)]\delta[R - R(t)] \quad (5.4)$$

Here we do not pursue the classical description of ion-ion scattering, but refer to the work of Bodmer and collaborators.⁶¹

The Wigner functions of Eq. (5.1) do not always have the positive definite feature of their classical analogues. This property, connected with the uncertainty principle, prevents naive probabilistic approximations and has perhaps slowed the general acceptance of this formalism. With a little experience, one can easily learn to live with this situation.

We begin with the one-particle potential scattering problem. The position space number density is

$$n(\underline{R},t) = |\psi(\underline{R},t)|^2. \quad (5.5)$$

In terms of the Fourier coefficient $C(p, t)$

$$\psi(\underline{R}, t) = \int \frac{d^3 p}{(2\pi)^{3/2}} C(\underline{p}, t) e^{i \underline{p} \cdot \underline{R}} \quad (5.6)$$

the momentum space density is

$$n(\underline{p}, t) = |C(\underline{p}, t)|^2. \quad (5.7)$$

These features are united in the Wigner phase space distribution function, defined as the Fourier transform of the density matrix for a pure state on the relative coordinate:

$$\psi^*(\underline{x}_1, t) \psi(\underline{x}_2, t) \left\{ \begin{array}{l} \underline{x}_1 - \underline{x}_2 = \underline{r} \\ \frac{\underline{x}_1 + \underline{x}_2}{2} = \underline{R} \end{array} \right. \quad (5.8)$$

$$f(\underline{p}, \underline{R}, t) \equiv \int \frac{d^3 r}{(2\pi)^3} e^{-i \underline{p} \cdot \underline{r}} \psi^*(\underline{R} - \frac{1}{2} \underline{r}, t) \psi(\underline{R} + \frac{1}{2} \underline{r}, t); \quad (5.9)$$

$$f(\underline{p}, \underline{R}, t) = \int \frac{d^3 q}{(2\pi)^3} e^{-i \underline{q} \cdot \underline{R}} C^*(\underline{p} + \frac{1}{2} \underline{q}, t) C(\underline{p} - \frac{1}{2} \underline{q}, t). \quad (5.10)$$

Here and henceforth Planck's constant $\hbar$ is set equal to one. Here we have ignored spin and other internal degrees of freedom. We note the results

$$\int d^3 p f(\underline{p}, \underline{R}, t) = |\psi(\underline{R}, t)|^2 \quad (5.11)$$

$$\int d^3 R f(\underline{p}, \underline{R}, t) = |C(\underline{p}, t)|^2; \quad (5.12)$$

p is of course a c-number, not an operator. In the $(\underline{p}, \underline{R})$ phase space f is likely to be negative for small values of the action, especially for highly excited bound states. Though f is not necessarily positive it is clearly

real. In second quantization ψ is the field operator and the trace with the density matrix should be supplied.

The scattering problem is set up in Fig. 7 where the incident density f_0 will eventually become smooth and much bigger than the potential. By definition, for $p \neq p_0$ (the incident momentum)

$$\frac{dN_{\text{scatt}}}{d^3p} = \int d^3R f(\underline{p}, \underline{R}, \infty) . \quad (5.13)$$

This has to be divided by the incident integrated flux dN/dA which is given by

$$\text{flux} \cdot dA = \frac{dN}{dA} \cdot dA = \int_{-\infty}^{\infty} dt \underline{j}_0 \cdot d\underline{A} , \quad (5.14)$$

where the probability current is

$$\underline{j} = \frac{1}{2mi} (\psi^* \nabla \psi - \nabla \psi^* \psi) \quad (5.15)$$

$$= \int d^3p \, v_p f(\underline{p}, \underline{R}, t) ; \quad (5.16)$$

$d\underline{A}$ is the element of area perpendicular to the average momentum $\underline{p}_0$ present in the distribution f_0 .

In order to calculate f and hence the cross section we can use Schrödinger's equation to compute the equation of motion for f . We use the alternate forms

$$\begin{aligned} \frac{D}{Dt} f(\underline{p}, \underline{R}, t) &\equiv \left(\frac{\partial}{\partial t} + \underline{v}_p \cdot \nabla \right) f(\underline{p}, \underline{R}, t) \\ &= \int d^3p' K(\underline{p}' - \underline{p}, \underline{R}) f(\underline{p}', \underline{R}, t) \end{aligned} \quad (5.17)$$

where the kernel K is defined by

$$K(\underline{p}' - \underline{p}, \underline{R}) = \frac{1}{i} \int \frac{d^3 \underline{r}}{(2\pi)^3} e^{i(\underline{p}' - \underline{p}) \cdot \underline{r}} [V(\underline{R} + \frac{1}{2} \underline{r}) - V(\underline{R} - \frac{1}{2} \underline{r})] \quad (5.18)$$

or rewritten as the quantum Liouville equation

$$\frac{i\partial f}{\partial t} = Lf \quad (5.19)$$

where the (Hermitian) Liouville operator L is given by

$$Lf \equiv -i \underline{v}_{\underline{p}} \cdot \nabla f + i \int d^3 \underline{p}' K(\underline{p}' - \underline{p}, \underline{R}) f(\underline{p}', \underline{R}, t) . \quad (5.20)$$

To study the scattering problem we cast (5.19) in integral form

$$f(\underline{p}, \underline{R}, t) = f_o(\underline{p}, \underline{R}, t) + \int_{-\infty}^t dt' \int d^3 \underline{p}' K(\underline{p}' - \underline{p}, \underline{R}_{ret}) f(\underline{p}', \underline{R}_{ret}, t') \quad (5.21)$$

where the retarded point is

$$\underline{R}_{ret} = \underline{R} - \underline{v}_{\underline{p}}(t - t') . \quad (5.22)$$

The kernel K smears the classical orbits by momentum transfers $\Delta p \sim \hbar/a$, a the size of the potential. As $\hbar \rightarrow 0$ we easily obtain formally the classical Liouville equation

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \nabla f - \nabla V \cdot \nabla f = 0 . \quad (5.23)$$

In scattering problems, f typically has essential singularities and the classical limit can be quite tricky.⁵¹ For example, for a free Gaussian wave packet we have

$$f_o(\underline{p}, \underline{R}, t) \propto \hbar^{-3} \exp[-(\underline{p} - \underline{p}_o)^2 / 2\hbar^2 \sigma] \exp[-(\underline{R} - \underline{v}_p t)^2 / 2L] \quad (5.24)$$

where $L = \frac{1}{2}\sigma$ characterizes the underlying wave function.

In passing we note that the eikonal approximation follows directly on making the approximation $\underline{v}_p \cong \underline{v}_o$ in the left hand side of (5.17). For

this and other results for one-particle scattering refer to ref. 59.

Now we pass boldly to the $N_1 + N_2 = N$ -particle collision, whose sequences in time is imagined to evolve as shown in Fig. 8. We envision four stages

(a) incident nuclei

$$f \rightarrow f_{N_1}^{(0)} f_{N_2}^{(0)} \quad t \rightarrow -\infty$$

(b) Onset of kinetic regime,

(c) End of kinetic phase, beginning of separation, and

(d) Asymptotic regime

$$f \rightarrow \prod_i f_i^{(0)} \quad , \quad t \rightarrow +\infty$$

The details of this complex sequence have not yet been worked out even though intuitive ad hoc models have already been invented for various parts of the process, for example the intranuclear cascade models. The present scheme is so intricate mathematically that we believe the earliest application will be to establish the domain of validity of the intuitive models rather than to immediately give numerical results by explicit calculation.

Actual calculations in the N -particle problem including spin and isospin variables are best done in second quantization. Here we proceed with wave functions, ignoring particle identity, spin etc. Then the N -particle distribution function

$$f_N(p_1^R, p_2^R, \dots, p_N^R, t) \equiv$$

$$\int \prod_{j=1}^N \frac{d^3 r_j}{(2\pi)^3} e^{-i p_j \cdot r} j\psi^*(R_1 - \frac{1}{2} r_1, \dots, R_N - \frac{1}{2} r_N, t) \\ \times \psi(R_1 + \frac{1}{2} r, \dots, R_N + \frac{1}{2} r_N, t) \quad (5.25)$$

obeys the equation of motion

$$(\frac{\partial}{\partial t} + \sum_i v_i \cdot \nabla_i) f_N = \int \prod_i d^3 p_i' K(p_1' - p_1, R_1, \dots, p_N' - p_N, R_N, t) f_N(p_1' R_1 \dots p_N' R_N t) . \quad (5.26)$$

For the usual two-body potential K acts only on two momenta at a time.

The hierarchy equations are easily derived by integration over various coordinates. The analysis is easily done in Fourier transformed variables. The simplest equation is that for the one-particle distribution

$$(\omega - q_1 \cdot v_1) f^{(1)}_1(p_1, q_1, \omega) = \\ (N-1) \int \frac{d^3 q_1'}{(2\pi)^3} v(q_1') \int d^3 p_2 [f^{(2)}(p_1 - \frac{1}{2} q_1', q_1 - q_1', p_2 + \frac{1}{2} q_1', q_1' \omega) \\ - f^{(2)}(p_1 + \frac{1}{2} q_1', q_1 - q_1', p_1 - \frac{1}{2} q_1', q_1' \omega)] . \quad (5.27)$$

In time space (5.27) becomes the Vlasov equation when $f^{(2)}$ is factorized as $f^{(1)} f^{(1)}$, or TDHF when the equation is modified for Fermi statistics.

In general the equation of motion for the n th distribution function involves both the n th function and the $(n+1)$ st function. For example for $f^{(2)}$ we have a term $f^{(2)}$ corresponding to scattering of the selected particles, called 1 and 2, and a term involving $f^{(3)}$ due to the scattering

of particle 1 or 2 off one of the particles (called 3) of the medium. This situation is exhibited in Fig. 9. We shall not write these equations here because of their complexity and refer the ref. 59 for a more detailed discussion. We mention that a factorization of $f^{(3)}$ with the boundary condition $f_2 f_1 f_1$ at early times leads directly to the Vlasov-Boltzmann equation⁵⁹ in a uniform medium provided the collision time is substantially less than the relaxation time of the distribution function.

In the case of non-local spatial variations we can derive a Boltzmann-like equation with retardation effects included:

$$\begin{aligned}
 & \left(\frac{\partial}{\partial t} + \mathbf{v}_p \cdot \nabla \right) f(p, R, t) = \\
 & -(N-1) \int \frac{d^3 q_1 d^3 q_2}{(2\pi)^6} \int d^3 R' d^3 p' \int dt' e^{i(q_1 + q_2) \cdot (R - R')} v(q_1) v(q_2) \\
 & e^{-i q_2 \cdot \left(p - p' + \frac{1}{2}(q_1 - q_2)(t - t') \right)} \left\{ f_2 \left(p + \frac{q_1 + q_2}{2}, R - \frac{p + \frac{1}{2} q_1}{m} (t - t') \right); \right. \\
 & \quad \left. p', R' - p' + \frac{1}{2} q_2 (t - t'), t' \right\} \quad (5.28) \\
 & - f_2 \left(p_1 + \frac{q_1 + q_2}{2}, R - p + \frac{1}{2} q_1 (t - t') \right); \\
 & \quad \left. p' + q_2, R' - \frac{p' + \frac{1}{2} q_2}{m} (t - t'), t' \right\}
 \end{aligned}$$

+similar term

If we ignore correlations ($f^{(2)} = f^{(1)}f^{(1)}$) and can further neglect retardation, Eq. (5.28) reduces to a "local" Boltzmann equation with the Born approximation cross section. (The improvement on the Born approximation by setting $v \rightarrow t$ matrix is a solved problem.) All these matters are very complex and will not be treated today. The reader is referred to ref. 59 and to the results of work in progress. Applications to nuclei clearly involve many length or momentum parameters; nuclear size and surface thickness, incident momentum, Fermi momentum and potential range, all of which enter into a careful theory.

In ref. 59 we further discuss the connection of the formalism to the existence of hydrodynamic equations, and the relativistic field theory problem. Presently we are studying the possible justification of, and range of validity of the cascade, thermal, hydrodynamics and hot spot models. There is plenty of work to do, and your help is welcome.

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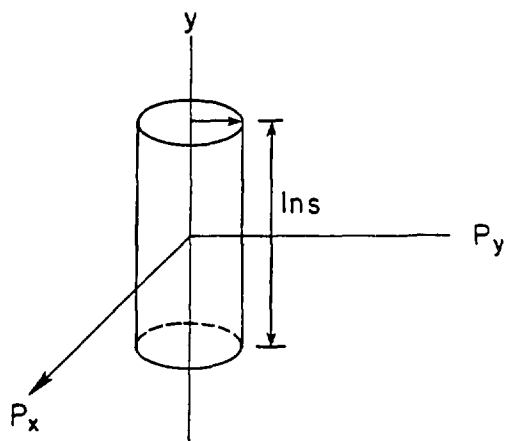


Fig. 1. The bulk of produced secondaries lie in a cylinder of radius $p_T \sim 350$ MeV/c and longitudinal rapidity $y \sim lns$ in (p_T, y) space.

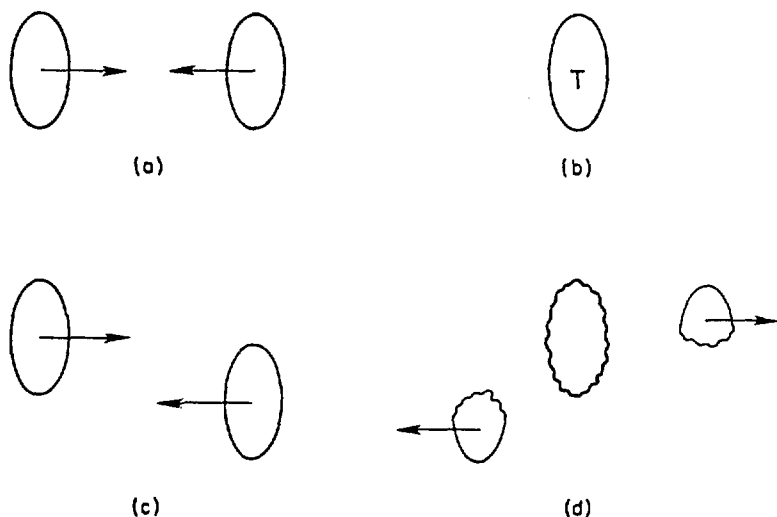


Fig. 2. The original Fermi model assumes the head on collision of two Lorentz-contracted protons (a), instantly equilibrating via very strong interactions (b). In (c) the non-zero impact parameter case is shown. Fig. (2d) shows a more modern version in which the projectile and target fragments move on, leaving behind a highly excited blob of matter at rest in the c.m. frame. The latter is then to be treated in the hydrodynamic model, and the extreme assumptions leading from (a) to (b) are avoided.

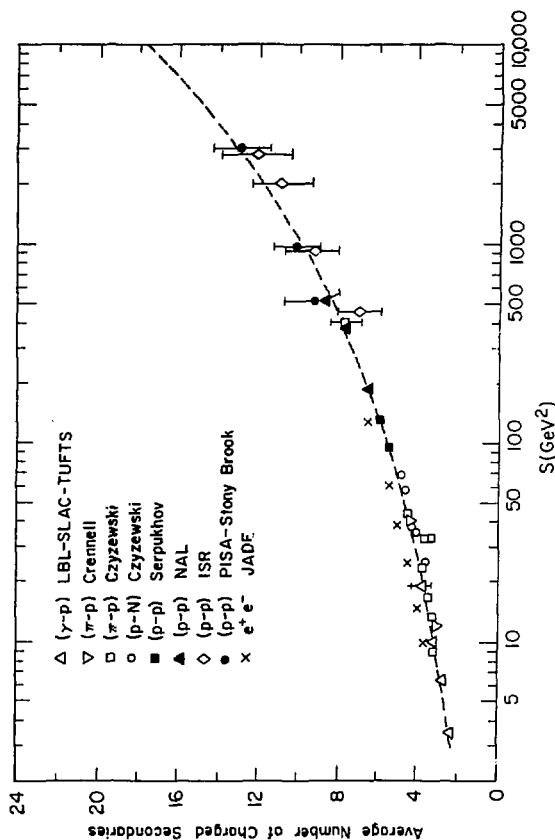


Fig. 3. The energy dependence of hadronic multiplicities is shown for an assortment of projectiles and targets.

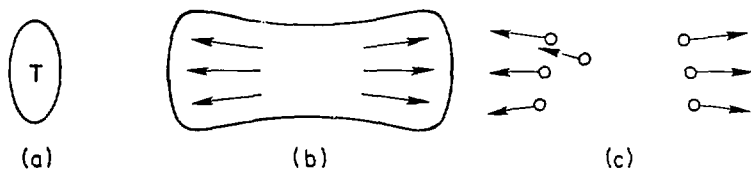


Fig. 4. The three stages of the Landau hydrodynamical model are indicated. Beginning with the thermalized Fermi disk in a, the fluid flow indicated in (b) is followed (c) by the breakup into asymptotic quanta.

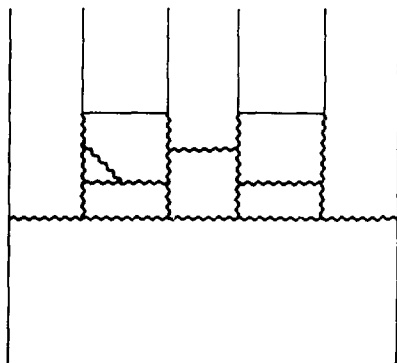


Fig. 5. A possibly significant contribution to the physics of Fig. 2d is shown. A glueball is formed, followed by the creation of $q \bar{q}$ pairs.

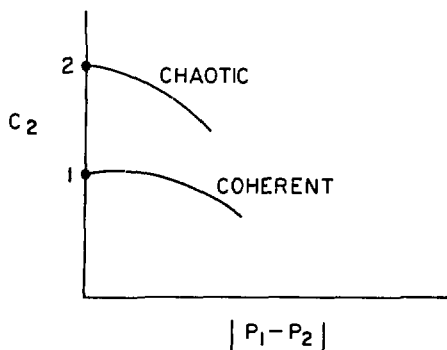


Fig. 6. The behavior the correlation C_2 defined in Eq. (4.4) is shown for the two extremes of a chaotic and an incoherent source.

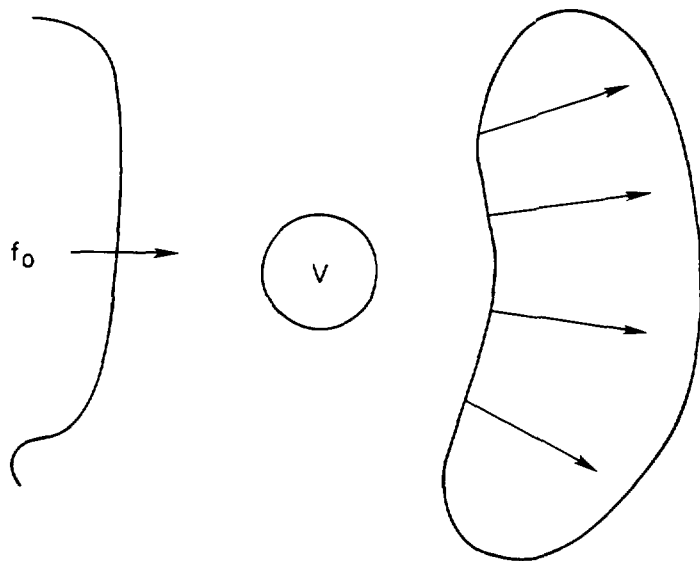


Fig. 7. The scattering process as described in the phase space formalism involves the incident density f_0 scattering off the potential V .

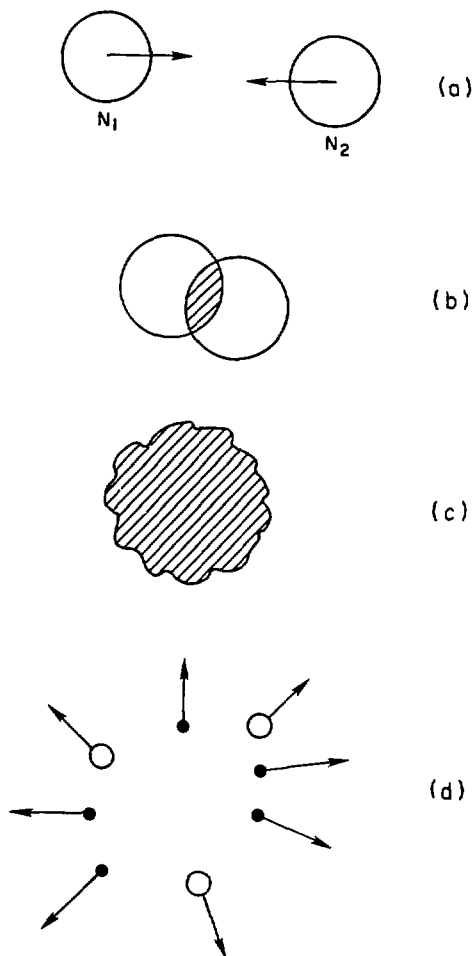


Fig. 8. Here we show a simplified version of the stages of a nucleus-nucleus collision. In (a) the incident density is $f_{N_1}^{(0)}, f_{N_2}^{(0)}$. In (b) the nuclei overlap, initiating a kinetic phase. (c) shows the excited system prior to separation (d).

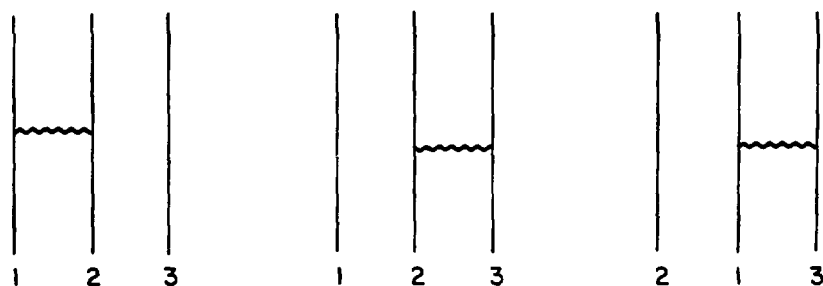


Fig. 9. The types of collisions entering into the equation of motion for the two particle distribution function $f^{(2)}(1,2)$ are shown. (a) shows particles 1 and 2 scattering while the particles of the medium watch; (b) and (c) show the selected particles (1 and 2) scattering off a particle (called 3) of the medium.

STUDY OF INELASTIC $\alpha\alpha$ AND αp COLLISIONS IN THE CERN ISR

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(Presented by M.A. Faessler)

ABSTRACT

Inelastic $\alpha\alpha$ and αp interactions have been measured at the CERN Intersecting Storage Rings at centre-of-mass energies of 125 and 88 GeV, respectively, with the Split Field Magnet Detector. Charged-particle densities and two-particle correlation functions in the central rapidity region are reported and compared with corresponding results for pp interactions and with theoretical predictions.

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1. INTRODUCTION

A general introduction to the physics in the ISR, and in particular to the physics with α particles, will be given by Jacob¹⁾. So I will make only two remarks as to what one hopes to learn from $\alpha\alpha$ and αp collisions in the ISR.

Firstly: with α particles in the CERN ISR we are entering a new energy regime for collisions involving nuclei. For $\alpha\alpha$ collisions, in particular, the step in energy which we have made is so large that we hope it may reveal some new phenomena, but in any case we think it should improve our understanding of old phenomena. This latter achievement has often been the neglected stepchild whenever the energy (or budget, etc.) was increased in searches for new phenomena.

Secondly: α particles are considered by heavy-ion physicists on the one hand as too-light nuclei. But particle physicists, on the other hand, consider them as too heavy, too complicated. Thus it only needs common sense to see that α particles must also have some advantages. I will skip the long list of advantages -- they are of both experimental and theoretical nature -- and only summarize them by saying that the α particle may establish a useful bridge between particle- and heavy-ion physics and between known phenomena and the dreamed-of new phenomena.

This contribution focuses on the production of particles in the central or pionization region in normal (soft) inelastic interactions. Knowing that central collisions are a favoured subject in these surroundings, it was tempting to select a topic which can be associated in one way or another with central collisions.

2. EXPERIMENT

The experiment was done using the Split Field Magnet Detector (SFMD). This is a large general-purpose device. It can detect charged particles and analyse their momenta over a solid angle close to 4π . Figure 1 shows an artist's view of the apparatus. The two beam tubes can be seen as well as the lower iron yoke (the upper one has been "removed" by the artist). The magnet gap (field volume of 30 m^3 with an average field of

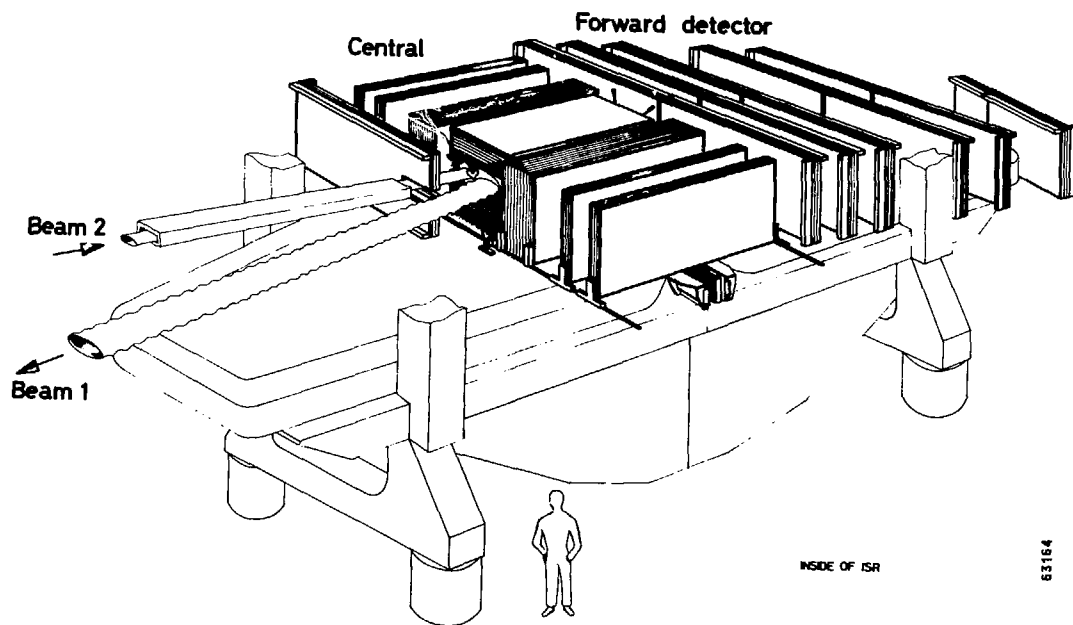


Fig. 1 Artist's view of the Split Field Magnet Detector

1 tesla) is filled with multiwire proportional chambers (MWPCs) with about 70,000 wires altogether. (Only some of the MWPCs are shown in the figure.)

In order to trigger this apparatus, combined wire signals are used -- signals of whole wire planes or of groups of neighbouring wires. To trigger on normal inelastic events ("minimum bias events"), only one track is required anywhere in the detector and one track is signalled by a coincidence of a reasonable sequence of at least five planes in any direction from the interaction vertex.

For the following results, events with at least one negative track were selected off line. Since fragmentation of nuclei without any particle production represents a good fraction [0(10%)] of inelastic $\alpha\alpha$ and $\alpha\alpha$ interactions, this cut was considered to yield a cleaner sample of events with particle production. On the other hand, the minimum bias requirement is met, because the efficiency of detecting a negative track is higher than 90% and does not suppress or prefer any special topologies in y-space.

3. RESULTS

3.1 Particle densities in the central region

Figure 2 shows the ratio of particle densities at equal nucleon-nucleon c.m. energies ($\sqrt{s_{NN}} = 31$ GeV) for $\alpha\alpha$ collisions to those for pp collisions, given separately for positive and negative tracks. These ratios have to be understood as qualitative, because the proton data were taken at a different magnetic field and at present there are uncertainties for the acceptance at this field value which can produce errors in the ratio of the order of 10-20%. But the following qualitative features can be seen.

The ratio is equal for positive and negative particles for c.m. rapidities (y_{cm}) around zero. This is consistent with the presence of a baryon-free central region and a rapidity gap of about 2 units between the two fragmentation regions. Towards the higher rapidities the negative ratio rises. Part of this rise can be explained by the different quark

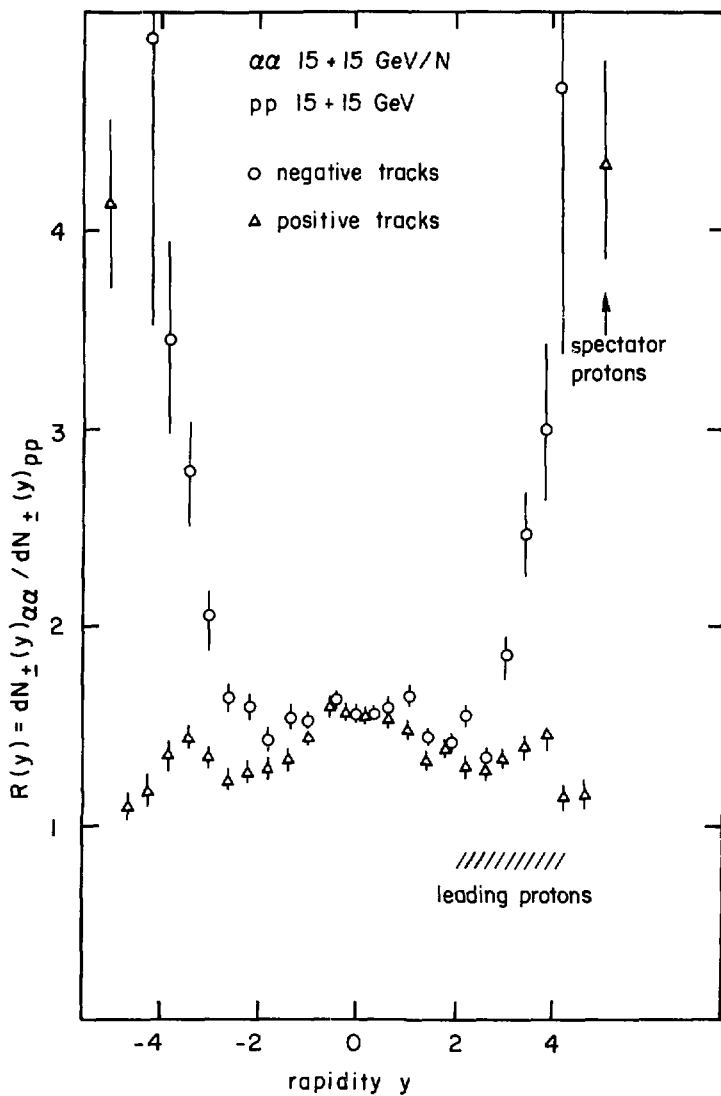


Fig. 2 Ratio of particle densities in $\alpha\alpha$ collisions to pp collisions versus rapidity y

composition of α 's and protons. The α 's have equal numbers of up and down quarks, so one expects equal yields for π^+ and π^- everywhere. Negative particles are dominantly pions. In pp interactions the π^-/π^+ ratio decreases to a value of $1/5$ towards the edges of the rapidity space²⁾. So the increase of the negative ratio in Fig. 2 can be explained in part by the decrease of fast negative pions at high y in pp interactions. But part of the rise could be due to an enhanced pion production in the fragmentation regions of the nuclei³⁾. A more careful study of the particle densities in the fragmentation regions is needed to see whether this is the case.

The positive ratio, however, decreases at high rapidities. Again, this can be due to the different quark compositions. Half of the leading baryons in $\alpha\alpha$ collisions are supposed to be neutrons -- which we do not see; in pp collisions they are mostly protons. Only at the highest y the positive ratio rises, owing to the spectator protons from the α particle. The two humps in the ratio of positive tracks are an artefact due to the assignment of a pion mass to all charged particles. Thus baryons get a higher rapidity (by $\ln [m_T(\text{baryon})/m_T(\text{pion})]$) than their true one.

Now let us focus on the central rapidities. If we compare the $\alpha\alpha$ and αp data with pp data taken at 62 GeV c.m. energy, we have the same magnetic field and the same acceptance. We find a central ratio R_c of 1.45 ± 0.05 for $\alpha\alpha$ to pp and 1.07 ± 0.05 for αp to pp. Errors include statistical and systematical uncertainties; the latter were estimated by a comparison of the ratio at different azimuthal angles. The rise of the central particle density with energy has been measured for pp collisions in the ISR by two experiments⁴⁾. The increase is $20 \pm 5\%$ going from 31 to 62 GeV and $11 \pm 5\%$ from 44 to 62 GeV. Using these correction factors to obtain R_c at equal c.m. energies (per nucleon-nucleon collision), we get

$$\begin{aligned} R_c \left(\frac{\alpha\alpha}{pp} \right) \Big|_{31 \text{ GeV}} &= 1.74 \pm 0.09 \\ R_c \left(\frac{\alpha p}{pp} \right) \Big|_{44 \text{ GeV}} &= 1.18 \pm 0.07 \end{aligned} \quad (1)$$

Why are the particle densities in the central (pionization) region interesting? Comparing particle densities in the central region for inelastic pp or πp interactions with the ones for e^+e^- annihilation or deep inelastic lepton scattering should tell us, at least qualitatively, which mechanism is at work in hadron-nucleon interactions, because we believe we understand fairly well the mechanism in e^+e^- annihilation and lepton-nucleon scattering within the framework of QCD⁵⁾.

Another approach to learn about this mechanism is to compare the central particle production in pp interactions with that in proton-nucleus and nucleus-nucleus interactions. Of course there is an additional challenge if one follows this approach. Once the mechanism in pp interactions is understood we can look for those phenomena which are bound to occur only in collisions of nuclei^{3,6)}.

We have some quantitative predictions for R_c . One was presented by Brodsky two years ago here at LBL⁷⁾. In his model sea partons of the projectile and target interact by the exchange of gluons. The central particles are produced by the subsequent colour neutralization which takes place between the sea partons and the valence partons. His prediction for the central ratio R_c can be entirely expressed in terms of production cross-sections:

$$R_c \left(\frac{\alpha\alpha}{pp} \right) = \frac{2A\sigma_{\alpha p}}{\sigma_{\alpha\alpha}} [1 - (\frac{1}{2})^{A\sigma_{pp}/\sigma_{\alpha p}}] = 1.80 \pm 0.13$$

$$R_c \left(\frac{\alpha p}{pp} \right) = \frac{A\sigma_{pp}}{2\sigma_{\alpha p}} + 1 - (\frac{1}{2})^{A\sigma_{pp}/\sigma_{\alpha p}} = 1.16 \pm 0.07.$$
(2)

The errors result from the uncertainties of the measured⁸⁾ production cross-sections.

In the model of Biaľas⁹⁾, the central particle ratio is equal to the number of colour strings between wounded quarks and can be expressed in terms of quark cross-sections on nucleons and nuclei¹⁰⁾

$$R_c \left(\frac{\alpha\alpha}{pp} \right) = \frac{\sigma_{\alpha q}^2}{\sigma_{qq}\sigma_{\alpha\alpha}} = 1.77, \quad R_c \left(\frac{\alpha p}{pp} \right) = \frac{3\sigma_{\alpha q}}{\sigma_{\alpha p}} = 1.16.$$
(3)

While for some other models¹¹⁾, the increase of the central particle density (1) is found to be surprisingly high, the agreement of the data with the predictions (2) and (3) is striking, but we find this agreement quite puzzling, because both models did not look so good when compared with data from another experiment, where we studied proton-nucleus collisions at energies between 20 and 150 GeV at the SPS¹²⁾. Extrapolating from these data one would expect no change of the central particle density in αp collisions as compared to pp collisions.

3.2 Two-particle correlations in the central region

One approach to learn more about the properties of the hadron matter created in the central region is to study particle correlations. We have started to study two-particle correlations and some examples for the normalized two-particle correlation function $R(y_1, y_2)$ will be shown. The latter is defined as

$$R(y_1, y_2) = \frac{\rho(y_1, y_2)}{\rho(y_1)\rho(y_2)} - 1, \quad (4)$$

with $\rho(y_1, y_2)$ being the inclusive two-particle density and $\rho(y_1)$ the inclusive single-particle density. Figure 3a shows $R(0, y_2)$ -- one particle was at $y_1 = 0 \pm 0.5$ -- for pp , $\alpha\alpha$, and αp interactions. The correlation function has a characteristic triangular shape, and for pp interactions both the width and height agree well with previous measurements in the ISR¹³⁾. For $\alpha\alpha$ interactions the width increases and a slight increase in the height can be observed. For αp interactions the proton-side (negative y_2) follows closely the 31 GeV pp data, while the α -side (positive y_2) follows the α data. This looks very systematic but what does it tell us? Indeed, not much yet because a large part of this correlation can be understood by the presence of at least two distinct components in the sample of inelastic events: the diffractive and the non-diffractive components, which have qualitatively different rapidity distributions. By selecting a particle with $y_1 \approx 0$ we very likely select a non-diffractive event, so the chance of finding another central particle is higher than for the average event.

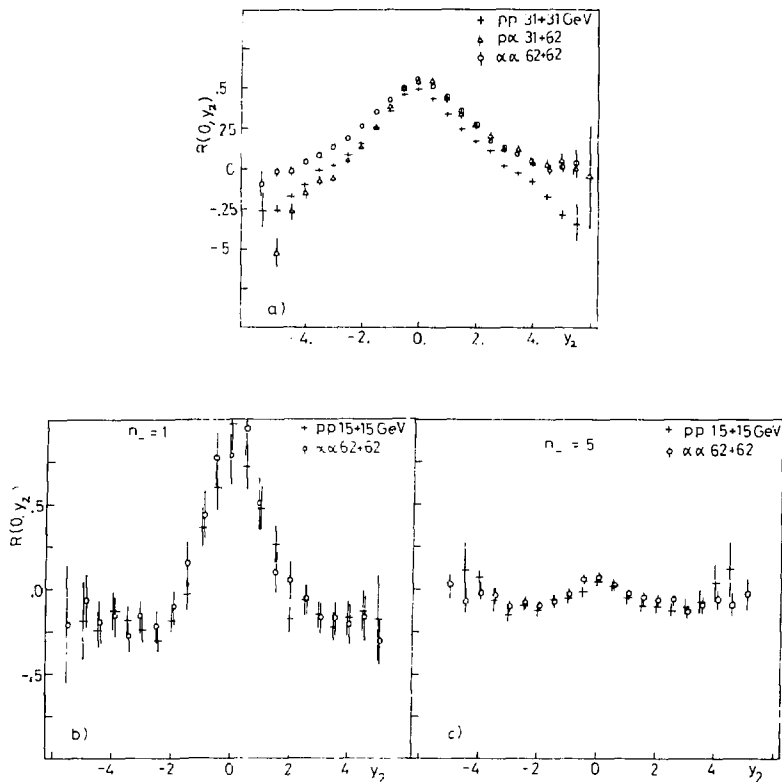


Fig. 3 Two-body correlation function [see Eq. (4)] for pp, αp , and $\alpha\alpha$ inelastic interactions:

- a) "minimum bias" events with at least one negative track;
- b) fixed multiplicity: 1 negative track;
- c) fixed multiplicity: 5 negative tracks.

A way to isolate true correlations from this sort of correlation is to study correlations at fixed multiplicity. Two examples are shown in Figs. 3b and c. In Fig. 3b events with a negative multiplicity of one (exactly 1 negative particle) were selected; for Fig. 3c the negative multiplicity was five. A much sharper peak can be seen now, indicating the presence of short-range correlations. The width of about 1 unit in y_2 is the correlation length. From an analysis of such correlations one may deduce the average multiplicity of a cluster decay in the framework of cluster production models¹³⁾. We have not done this yet, but it is apparent that the differences between $\alpha\alpha$ and pp interactions are small. The observed decrease of the correlation in both cases with increasing multiplicity can be readily interpreted in cluster production models. Larger multiplicity implies a larger number of clusters. If the clusters are independent from each other, the average correlation between particles should increase with increasing cluster number.

4. SUMMARY

A significant rise of particle densities in the central region has been observed in $\alpha\alpha$ and αp interactions compared with pp interactions at the same c.m. energy per nucleon-nucleon collision. The pattern of two-particle correlations in the central region seems to be very similar for pp , αp , and $\alpha\alpha$ interactions.

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Boltzmann-Equation Approach to High-Energy Nucleus-Nucleus Collisions

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In a collision between two nuclei we distinguish between participants and spectators. Spectators are those nucleons which have not undergone any interaction and are still embedded in the fermi sea of the projectile or target nuclear fragment. Participants are all other nucleons. These experience two types of interaction:

- their primordial creation as a participant through interaction of participants with spectators, and initially by the collision of projectile spectators with target spectators.
- collisions among participants which we call the process of rescattering.

In order to describe nucleus-nucleus collision along these lines we will use the Boltzmann equation. The validity of this equation rests on the assumption that we have a certain degree of diluteness in our system. Then all information is contained in the one-particle distribution function (no correlations) $N(\vec{r}, \vec{p}, \vec{t})$ and we can restrict ourselves to two-body collisions. The collision term, which describes the changes induced in N through interactions with other particles, is governed by the known nucleon-nucleon cross section and it takes the form:

$$C(N, N) \equiv \iiint [N_1' N_2' W(p_1' p_2' | p p_j) - N N_j W(p p_j | p_1' p_2')] d\omega_j d\omega_1' d\omega_2' \quad (1)$$

where W is the collision probability (related to cross sections) and $d\omega = d\vec{p}/E$.

Schematically the Boltzmann equation can then be written as:

$$D(N) = C(N, N) \text{ with } D(N) \equiv p^\mu \partial_\mu N = (E \frac{\partial}{\partial t} + \vec{p} \cdot \vec{\nabla}) N \quad (2)$$

Using the notation N_S and N_P respectively for the spectator and

participant distribution function we obtain:

$$D(N_S) = C(N_S, N_P) + C(N_S, N_S) \quad (3)$$

$$D(N_P) = C(N_P, N_S) + C(N_P, N_P)$$

Collisions between particles will finally lead to the attainment of equilibrium. Exact local equilibrium implies that $C(N, N) = 0$ for each $\vec{r}$, t separately. Since rescattering, denoted by the terms $C(N_P, N_P)$ and $C(N_S, N_S)$, is an active mechanism leading to local equilibrium we will impose local equilibrium separately for spectators and participants as a result of rescattering. Thus the equations (3) reduce to:

$$D(N_P) = C(N_P, N_S) \quad D(N_S) = C(N_S, N_P) \quad (4)$$

$$C(N_P, N_P) = 0 \quad C(N_S, N_S) = 0$$

where the last equations are valid, either by definition (the spectators take part in local equilibrium as fermi sea particles) or by assuming the rescattering to produce local equilibrium.

The set of coupled equations (4) is solved iteratively in order of scattering. The sequential depletion of N_S is taken into account on the average by renormalizing the overall density, total momentum and total excitation energy. Thus depletion is not treated in detail as a function of $\vec{r}$ and t through the second equation in (4) but in accordance with conservation laws in order of scattering. The multiple collision series for the participant distribution function takes the form:

$$D(N_P^{(n)}) = C_G(N_P^{(n-1)}, N_S^{(n)}) + C_L(N_P^{(n)}, N_S^{(n)}) \quad N_P = \sum_n N_P^{(n)} \quad (5)$$

with C_G and C_L denoting the gain and loss collision term (respectively with plus and minus sign in (1)) and $N_S^{(n)}$ is the n -th order renormalized spectator distribution function. Since we

furthermore assume that the primordial production of participants, described by (5), and the corresponding average depletion of the spectators takes place sequentially in increasing order of scattering one can eliminate the time dependence explicitly. The time evolution is then replaced in some sense by the order of scattering. The stationary distribution function $\bar{N}_p^{(n)}(\vec{r}, \vec{p})$ is defined as:

$$\bar{N}_p^{(n)}(\vec{r}, \vec{p}) = v \int_{-\infty}^{+\infty} dt N_p^{(n)}(\vec{r}, \vec{v}, t), \quad \vec{v} = \vec{p}/E \quad (6)$$

and is governed by an integral equation which can be obtained straightforwardly from (5)¹⁾.

The incorporation of rescattering is decoupled from the multiple collision transport equation (5) and will only be taken into account in an a posteriori manner. This assumption is based on the belief that the gross features of $\bar{N}_p^{(n)}$, in casu its moments in momentum space, are merely governed by (5) and only slightly altered through rescattering.

The real importance of rescattering comes from the fact that it is a mechanism for producing composites. This is in the spirit of the Butler-Pearson final-state interaction model where collisions between two participant nucleons in the mean potential field of the underlying target nucleus produce deuterons. Our treatment of rescattering will be thermodynamically as in the Hagedorn model²⁾. By imposing local (thermal and chemical) equilibrium we have taken into account the ultimate consequence of rescattering. At each $\vec{r}$ and n (order of scattering, equivalent with the time) the momentum distribution of particles of kind i (nucleons and all composites, resonances and newly created particles allowed by conservation laws) taking part in this local equilibrium is given, in its rest frame, by the familiar expression (with T and μ_i dependent on $\vec{r}$ and n):

$$f_i(\vec{p}, \vec{r}, n) = \frac{g_i}{2\pi} \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1} \quad (7)$$

where $E_1 = (m_1^2 + p^2)^{1/2} - B_1$, with B_1 the corresponding binding energy and $g_1 = (2S_1 + 1)(2I_1 + 1)$ denotes the multiplicity (spin S_1 , isospin I_1). Chemical equilibrium is expressed through the relation:

$$\mu(N, Z) = N\mu_n + Z\mu_p \quad (8)$$

with μ_n, μ_p the chemical potential for neutrons and protons respectively. The primordial distribution function $\bar{N}_p^{(n)}(\vec{r}, \vec{p})$ for the participants provides the necessary input for this thermodynamical approach¹⁾. At each $\vec{r}$ and n the baryon density, charge density and energy density are used to determine the chemical potentials μ_n, μ_p and the temperature T . The longitudinal momentum, which we denote as λ , is distributed according to a weight function $F(\lambda | \vec{r}, n)$ which is obtained from $\bar{N}_p^{(n)}(\vec{r}, \vec{p})$ by integrating over all other momentum components. Summing together all volume-elements and orders of scattering, and Lorentz transforming (indicated by the symbol $\mathcal{L}$) the thermodynamical distribution from its rest frame to the moving frame, one obtains the result:

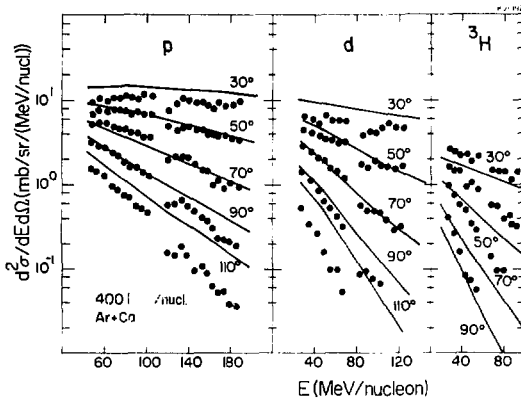
$$W_1(\vec{p}) = \frac{1}{N} \int d\vec{r} \mathcal{W}_1(\vec{p} | \vec{r}, n) \quad (9)$$

$$\mathcal{W}_1 = (\vec{p} | \vec{r}, n) = \int d\lambda F(\lambda | \vec{r}, n) \mathcal{L}_{\vec{p} \rightarrow \vec{p}'}(\lambda) f_1(\vec{p}' | \vec{r}, n)$$

where $W_1(\vec{p})$ is the complete momentum spectra (inclusive) for particles of kind i .

In ref. 3 we have presented theoretical results based on the method outlined before (and described in great detail in ref. 1) for proton, deuteron, triton, ^3H and ^4He inclusive particle spectra measured in 250 MeV/amu⁴⁾ and 400 MeV/amu⁵⁾ nucleus-nucleus collisionss. In fig. 1 we show a typical result for 400 MeV/nucleon $^{40}\text{Ar} + \text{Ca}$. In general the agreement with the data is reasonable to good. Remaining discrepancies are probably due to the method of depletion used (on the average instead of locally) and the influence

of finite particle effects which modify^{6,7)} the high energy tails of the thermal distribution (7). The latter aspect is probably also important in connection with the establishment of chemical equilibrium, especially in the beginning ($n = 1$) of the collision.



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ULTRA-HIGH ENERGY COLLISIONS IN
ASTROPHYSICS AND COSMIC RAYS*

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ABSTRACT

Knowledge of the chemical composition of primary cosmic ray nuclei is a prerequisite to understanding the origin, acceleration and propagation of the cosmic rays. Details of composition up to 100 GeV/nucleon have been studied extensively for many years, and the subject is a mature one in this low energy region. At much higher energies the flux becomes so low that the primaries cannot be studied directly, but can be observed only through their secondary cascades in the atmosphere. In the air shower energy region from 10^{14} to 10^{20} eV the subject of cosmic ray composition is therefore still in its infancy. In this talk I briefly review the subject of high energy cosmic rays above 10^{14} eV with emphasis on the role of meson production and fragmentation in collisions between ultrarelativistic nuclei.

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I. Introduction

Because the primary cosmic ray beam extends at least five orders of magnitude beyond 10^{15} eV, cosmic ray experiments can in principle provide information both about composition of the primary beam and about high energy physics well beyond accelerator energies. At energies above 10^{14} eV the primary flux is, however, so low that only indirect measurements of cascades in the atmosphere have been possible. Thus in practice ambiguities have so far made it difficult to draw definitive conclusions from experiment: there are too many parameters to be determined from data that necessarily has significant systematic uncertainties (see Table 1).

This situation should change soon. On the one hand, new colliding beam experiments will provide direct information about hadronic interactions around 10^{14} - 10^{15} eV (see Table 2). This should make it possible to determine at least the general features of primary composition in this energy range. Such a result will be of considerable interest for astrophysics because it is already known¹ that there is structure in the energy spectrum just above 10^{15} eV and there are hints² of changes in composition also. Table 3 illustrates the importance of knowledge of the energy dependence of the primary composition around 10^{15} eV for understanding cosmic ray origin and propagation. Keeping in mind that the thickness of the galactic disc is about 300 pc and that the scale of large magnetic irregularities is ~ 1 pc. it is clear, for example, that the interpretation of the change in the energy spectrum around 10^{15} eV¹ as well as the increasing anisotropy beyond

Table 1. Unknowns for high energy cosmic ray experiments

<u>Interaction properties</u>		<u>Primary beam</u>
σ	cross section-how does it depend on energy?	Chemical composition of primary nuclei
κ	inelasticity-what fraction of energy goes into particle production?	Energy spectra of components of primary beam.
$f(x)$	inclusive cross sections-how do shapes change with interaction energy	Possibility of new objects among high energy primaries.
$\langle n \rangle$	multiplicity	
P_T	Transverse momentum-jet structure?	
Effects of ultrarelativistic nucleus-nucleus collisions.		

Table 2. Planned hadron colliding beam machines

Machine	Completion Date	C of m Energy	Equivalent Lab Energy
CERN $\bar{p}p$	1981	540 GeV	1.5×10^{14} eV
FNAL $\bar{p}p$	1984	2000 GeV	2.1×10^{15} eV
Isabelle	1986	800 GeV	3.4×10^{14} eV

Table 3. Larmor radius (pc) in $3\mu\text{G}$ field

$E_{\text{total}}(\text{eV})$	10^{15}	10^{17}	10^{19}
protons	0.3	30	3000
iron nuclei	0.01	1	100

10^{15} eV³ depend crucially on the composition. (Recall that cascade measurements determine total energy per nucleus rather than rigidity.)

At higher energies, several new cosmic ray air shower experiments now measure longitudinal development of individual air showers, including early portions of the shower before maximum. These include the atmospheric Cerenkov experiments of the Moscow State University⁴ and of the University of Durham⁵ groups, which operate around 10^{17} eV ($\sqrt{s} \sim 20$ TeV) and the Fly's Eye experiment at Utah⁶ which now operates in the range 10^{18} - 10^{19} eV ($\sqrt{s} \sim 100$ TeV). Because these techniques reflect early portions of shower development they are in a better position than conventional extensive air shower (EAS) experiments⁷ to determine simultaneously properties of particle interactions and of the primary beam. In particular it should be possible to measure both the relative fraction of heavy and light primary nuclei and the nucleon-air cross section in an energy range that is significantly higher than will be reached by machines in the foreseeable future.

In both cases it is necessary to understand fragmentation and meson production in collisions between ultrarelativistic nuclei in order to interpret the cascade measurements. In this talk I first review the situation around 10^{15} eV, including a digression to describe the status of the data from large emulsion chambers, including the "Centauro" events. Then I describe some work in progress⁸ on meson production in heavy ion collisions and its application to analysis of high energy Fly's Eye data.

II. Present Situation Around 10^{15} eV

Information about primary composition and properties of hadronic interactions in this energy range come at present from indirect observations of hadronic cascades in the atmosphere initiated by primary cosmic rays.⁹ Three types of experiments can be distinguished: a) energy and depth dependence of uncorrelated fluxes of hadrons, of muons and of γ -rays;¹⁰ b) fluxes and internal properties of γ -ray families^{11,12,13} (jets of particles associated with a single primary but involving a cascade of hadronic interactions); and c) properties of small EAS.^{14,15} There are two contrasting points of view about the implications of this data:

- 1) That hadronic scaling (limiting fragmentation in the form of radial scaling) is valid and that most observations can be understood by a suitable adjustment of the primary composition. In this case it is necessary to assume that the cross section rises with energy at the rate of about 15% per decade for $\sigma_{p\text{-air}}^{\text{inel}}$.^{16,9}
- 2) That scaling cannot under any circumstances fit all the data, and that drastic changes in hadronic interactions are required.^{10,14,15} Such changes should become significant already at beam energies of 100 TeV in the lab and would have the property that produced particles are relatively softer at high energies than at low; i.e. that interactions become more inelastic.

As I have suggested here, several different kinds of experimental

results lead to the same ambiguity. As an illustration I mention here the lateral spread of gamma-families as discussed by the Mt. Fuji emulsion chamber group.¹⁷

The emulsion chamber technique has been extensively reviewed.^{9,12} Briefly, a chamber consists of sandwich structures of Pb plates interspersed with photo-sensitive materials (γ -ray films and/or photographic emulsion). Sometimes there are two or more sandwiches separated by light material with a low ratio of interaction length to radiation length to allow for hadrons to interact. The large (~ 50 - 500 m^2) emulsion chambers are exposed at mountain altitudes (~ 4000 - 5000 m above sea level). The x-ray films are scanned for groups of dark spots due to electromagnetic cascades in the chamber which can be followed from layer to layer. Parallel cascades are atmospheric families due to photons (from π^0 's) and hadrons produced in the atmosphere by the interactions of a single primary cosmic ray. (Intensities and accuracies of measurement are such that unrelated cascades are seldom included.) To be visible a hadron must interact in the chamber, and only the portion of its energy that goes into neutral pions and hence photons is detected. A cascade which begins more than three cascade units in the detector is assumed to be a hadron. Cascade energies are estimated by fitting calibrated cascade curves to measured darkness of spots as a function of depth in the chamber. Minimum energy of a single detectable cascade is about 2 TeV, and families with total visible energy from tens to hundreds of TeV are studied. This corresponds to primary energies of 10^{14} - 10^{15} eV

and even higher.

The complexity of the events (in general several interactions are involved) requires computer simulations of the atmospheric cascades for comparison with the data. Akashi et al.¹⁷ find that the lateral distributions of families are broader than would be expected for mostly proton primaries and a conventional (scaling) picture of hadronic interactions. Agreement between calculation and experiment can be improved in at least two ways, either of which represents a significant change from known physics around 1 TeV:

- 1) by softening the hadronic interactions (i.e. producing more secondaries with a lower mean energy) or

- 2) by including a large contribution of heavy primaries.

Either change makes the showers develop higher in the atmosphere so that the families are more spread out at the observation level. Of course, increasing the mean transverse momentum of produced particles (by a factor of two) will also do the job, but this would not solve similar difficulties with other kinds of data. As I have shown elsewhere¹⁸ experiments at $\bar{p}p$ colliders will soon make it possible to distinguish between these two possibilities.

In addition to normal families the Brazil-Japan group have reported the existence of a class of exotic events which they call Centauros.¹² These have few photons incident on the chamber but many hadrons, indicating a deficiency of neutral pions in the atmospheric interaction. Multiplicities are ~ 50 and visible energies ~ 300 TeV. To investigate whether these events can be understood as a fluctuation

or normal events we¹⁹ made a simulation incorporating as well as we could the experimental selection criteria, which have an important influence on the interpretation of the data. We were particularly concerned to include the effects of heavy primaries, which tend to give a high ratio of hadrons to neutral pions because of the many nucleons in the incident nuclei. We found that two of the large Centauro events are truly anomalous both as compared to other data and as compared to the simulation. The other three large Centauros and a few of the lower multiplicity Centauros ("mini-Centauros") also appear to be very unusual, but I believe it is also possible that could be fluctuations of ordinary processes.

Figure 1 illustrates the result discussed above and also shows how the emulsion chamber data might be used to study primary composition. Following Lattes et al. we plot the number of hadrons (N_h) vs. the ratio q of visible hadronic energy to total visible energy. Two of the large Centauros are clearly anomalous. The other three are on the edge of the distribution of simulated events in a region populated mostly by iron primaries. Thus, if these Centauros are normal events, they are probably heavy nuclei. We note, however, that these three Centauros have a significantly smaller lateral spread of hadronic energy than the simulated events. Incidentally, the existence of the region $q > 0.3$, $N_h > 15$, which is populated almost entirely by heavy primaries in the simulation, illustrates the potential of emulsion chamber experiments for probing primary composition.

III. Role of Nucleus-Nucleus Collisions

In order to draw inferences about the primary composition, however, it is also necessary to assume some model of fragmentation and meson production in collisions between heavy primaries and atmospheric target nuclei near the beginning of the cascades. Direct measurements of collisions between ultra-relativistic heavy ions at hundreds of GeV/nucleon would be the best foundation for such a model. The use of this data would be somewhat analogous to the use of low energy fragmentation cross sections to interpret propagation of nuclei with several GeV/nucleon through the interstellar medium. In the absence of accelerator data at ultra-relativistic energies, the present author, with Stanev, Freier and Waddington, have used cosmic ray data on nuclear fragmentation and meson production around 10-50 GeV/nucleon as a basis of the required model.

The data²⁰ consist of 549 interactions produced by nuclei with Z from 8 to 28 in nuclear emulsion. The geomagnetic threshold energy at which these nuclei were collected was ~ 7 GeV/nucleon, giving a mean primary energy of ~ 20 GeV/n and a median of 11 GeV/n. For the present analysis we worked only with the 194 interactions with $8 \leq Z \leq 26$ and $2 \leq N_h \leq 7$. The latter criterion is intended to pick out those interactions in emulsion on CNO nuclei, which are comparable to the atmospheric targets relevant to very high energy cosmic ray experiments. The proportion of such events in the sample was 0.36, which should be compared with the fraction 0.32 expected from the composition of nuclear emulsion and the interaction cross sections. Thus

only about 11% of the sample with $2 \leq N_h \leq 7$ is due to peripheral collisions on Ag-Br. The quantities measured in the experiment are charges of incident and fragment nuclei, Z_{in} and $Z_{frag(s)}$ (to an accuracy of ± 1), number of produced alphas, N_α , number of minimum ionizing secondaries, N_s , and number of grey and black tracks, N_h . By inference the number of released protons from the projectile is

$$N_p = Z_{in} - Z_{frag(s)} - 2 N_\alpha,$$

and we assume the number of produced charged pions to be $N_\pi = N_s - N_p$. Average values for the data we use are shown in Table 4.

Table 4. Data summary

Z_{in}	# evnts	# with $\pi^\pm$	$\langle N_p \rangle$	$\langle N_\pi \rangle$
8-10	68	56	4.1	8.0
11-14	46	31	6.1	9.9
15-18	27	17	6.3	7.3
19-22	30	23	8.2	7.0
23-26	23	16	12.1	6.0

The fundamental problem we face is how to use this moderate energy data on collisions between nuclei to simulate ultra-high energy cascades initiated by heavy nuclei. In the absence of a theory of nucleus-nucleus collisions we make the following simple (but certainly not unique) assumptions:

- 1) Assume fragmentation probabilities are energy independent and

use the library of real events for Monte Carlo simulation to obtain the fragmentation history of the core nucleus in each shower.

- 2) Assume that the probability distribution of the number of released nucleons from a projectile nucleus is also independent of energy and is given by $N = 2 N_p$ to account for released neutrons.
- 3) Estimate the average number W_A of wounded nucleons in the projectile of mass A by comparing $\langle N_\pi \rangle$ with the average multiplicity $\langle m \rangle$ in pp collisions in the 10-50 GeV region where we have data on both nucleus-nucleus and pp collisions. The average fraction of the released nucleons that interact is then $C = W_A / \langle N \rangle_A$.
- 4) Choose the actual number of released nucleons that interact in a particular collision from a Poisson distribution with mean value $N * C = N * W_A / \langle N \rangle_A = N_p * W_A / \langle N_p \rangle_A$. This preserves the observed correlation between N_p and N_s .
- 5) Use whatever model is assumed for pp collisions at ultra-high energies to choose momenta of the secondary pions produced in the interactions of the wounded nucleons in the projectile nucleus. Released nucleons that do not interact simply propagate as free nucleons to the next interaction point.

To compute W_A from experiment we divide the nuclei into five mass groups as shown in Table 4 and compute

$$W_A = \langle N_\pi \rangle / \langle n_s \rangle_{p\text{-air}} \quad (1)$$

where $\langle n_s \rangle_{p\text{-air}} = \frac{1}{2} \langle m \rangle (1+w)$ and $\langle m \rangle \approx 2.15$ is the average multiplicity of charged mesons in pp collisions integrated over the cosmic ray spectrum above 7 GeV/n. We take the average number of wounded target nucleons in p-air collisions to be $w = 1.4$. This gives $R_A \equiv n_{p\text{-air}} / n_{pp} = \frac{1}{2} (1+w) = 1.2$. A better value²¹ is probably $R_A \approx 1.4$, but (for reasons that become apparent below) we want to minimize w and w_A .

For comparison we also estimate w_A and C from a simple geometrical picture:

$$\begin{aligned} \langle w_A \rangle_{\text{geometric}} &= A \sigma_{p\text{-air}} / \sigma_{A\text{-air}} \\ &\approx \frac{A \times 285}{68(A^{1/3} + 14^{1/3} - 1.12)^2} \end{aligned} \quad (2)$$

The characteristics of our data sample are summarized in Table 5.

Table 5.

Z_{in}	$w_A^{\text{"expt"}}$	w_A^{geom}	C_{exp}	$C_{\text{geom}}^{\sim 0.5}$
8-10	3.1	4.6	.38	.57
11-14	3.9	6.1	.31	.49
15-18	2.8	6.8	.22	.54
19-22	2.7	7.6	.16	.46
23-26	2.3	9.0	.09	.37

Notice that the number of interacting nucleons appears to be surprisingly low, especially in the heavier projectiles. Some of this effect could be due to contamination of peripheral collisions on

Ag-Br in the sample with $2 \leq N_h \leq 7$ (I. Otterlund, private communication). It seems difficult, however, to attribute the whole effect to this cause in view of the estimate mentioned above that this contamination amounts to only about 11%.

IV. Application to Fly's Eye

For purposes of cascade simulation, the effect of any model of nucleus-nucleus collisions (provided it can be expressed in terms of nucleon-nucleon collisions) is described by showing the distribution of points of first interaction of each nucleon in the nucleus. The histograms in Fig. 2 show the distributions of points of first interaction of nucleons per 5gm/cm^2 interval of the atmosphere for 500 incident Fe nuclei at each of three energies. The nucleon interaction lengths assumed are respectively 81, 64 and 48gm/cm^2 at 10^5 , 10^7 and 10^9 GeV energy per nucleus. The straight lines show the same distributions for the superposition model. (The superposition model assumes that the incident nucleus is equivalent to a beam of free nucleons that interact independently from the beginning with the interaction lengths stated above.) Note that with the realistic model, showers penetrate somewhat deeper than in the superposition model so that heavy primaries give showers that look somewhat more proton-like than might have been assumed.

The technique⁶ for simultaneously measuring $\sigma_{p\text{-air}}$ and primary composition above 10^{17} eV with Fly's Eye depends on unfolding the distribution of starting points of the showers. Since the actual starting point is invisible (too few particles to produce detectable atmosphere scintillation light), some practical measure, such as the depth at which the

shower reaches 1/4 of its maximum, will have to be used.

Fig. 3 shows the distribution of depth of 1/4 maximum for about 1000 simulated showers with an (arbitrary) assumed primary composition p: α : CNO + Mg: Fe = .50: .19: .19: .12. Showers which start late are predominantly protons, so σ_{p-air} can be determined from the tail of the distribution, as shown in the logarithmic inset to Fig. 3. The shoulder at $1/4 \text{ max} < 400 \text{ gm/cm}^2$ is almost entirely due to the iron component, suggesting that it should be possible to estimate the fraction of heavy nuclei with Fly's Eye provided it is not too small.

The major effect of the realistic model of nucleus-nucleus collisions we have used here is to broaden the distributions of depth of quarter maximum by about a factor of two compared to the superposition model. The average values of 1/4 maximum are also shifted deeper into the atmosphere by about 5%. The model parameters we have chosen (labelled "expt" in Table 5) tend to maximize these effects. I am therefore optimistic that the Fly's Eye experiment can achieve its goal of measuring the general features of cross sections and composition in the $10^{17} - 10^{19}$ eV region.

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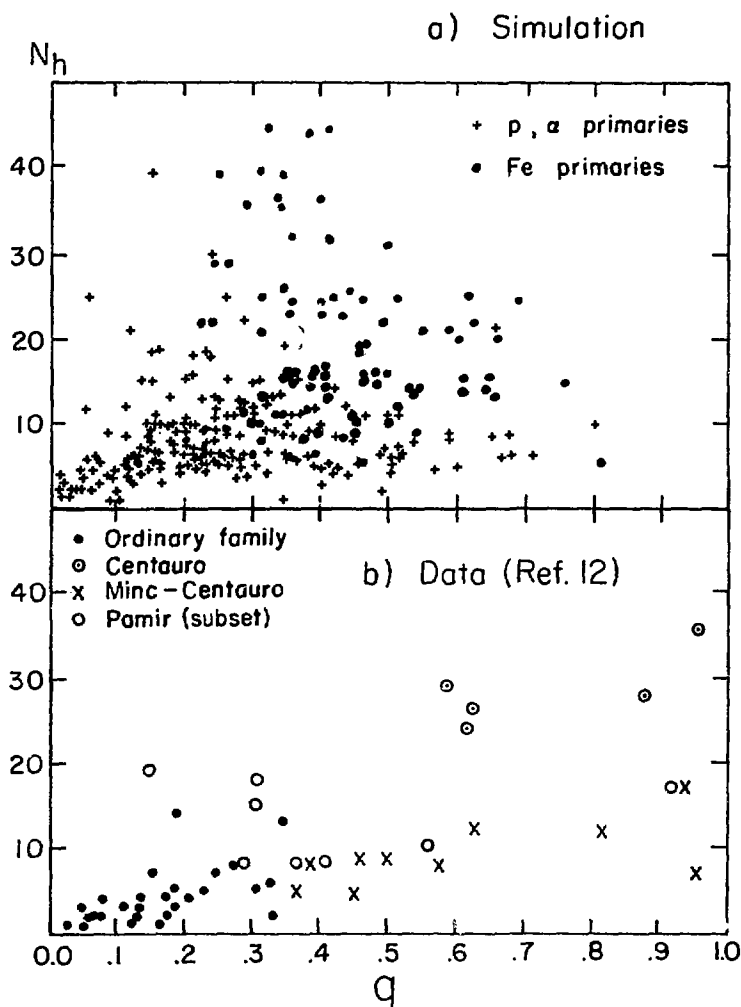


Figure 1

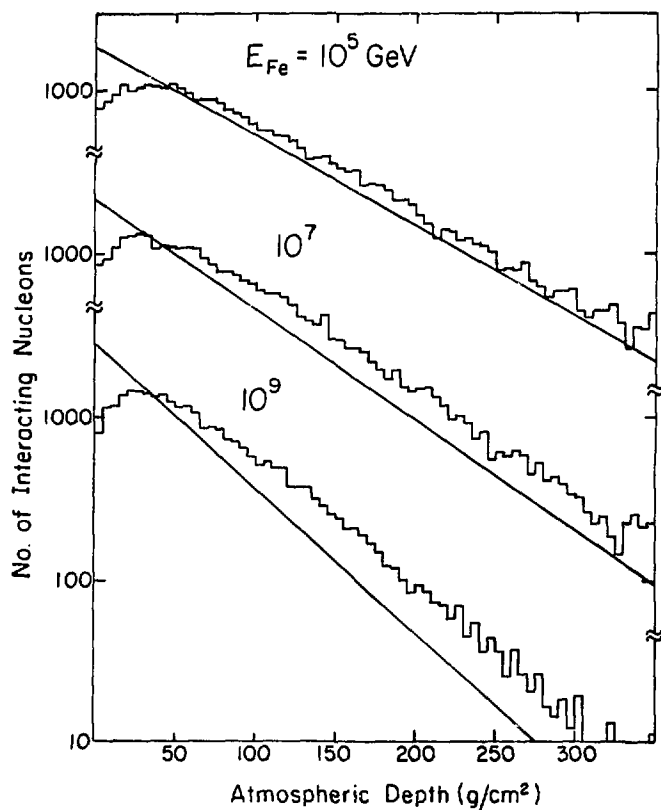


Fig. 2. Distributions of points of first interaction of nucleons in primary iron nuclei (simulation).

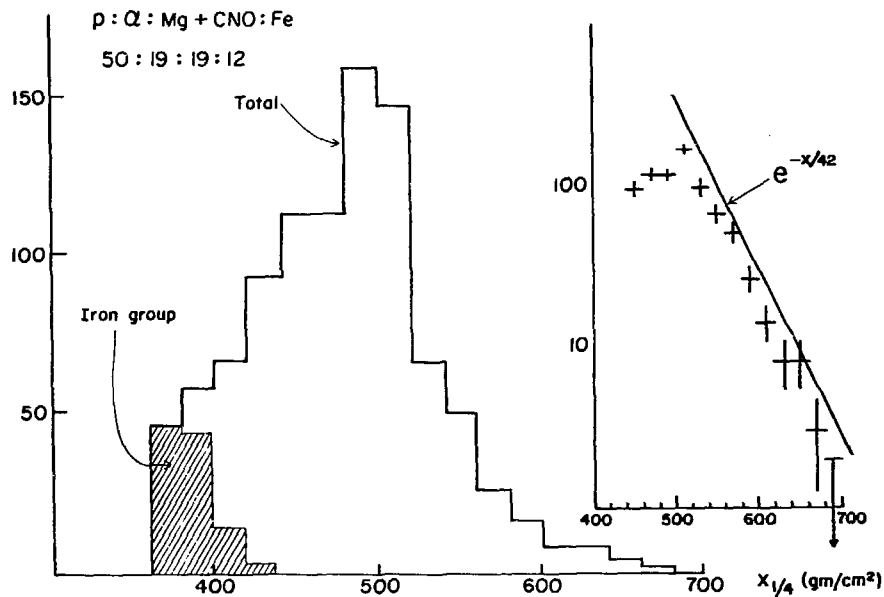


Fig. 3. Distribution of atmospheric depths at which showers reach one-fourth their maximum (simulation). Inset shows the same distribution on a logarithmic scale to illustrate how the interaction length can be extracted from the tail of the distribution. The input interaction length here was 42 gm/cm² for incident total energy of 3×10^{17} eV.

ANALYSIS OF NUCLEON AND HEAVY NUCLEUS COLLISIONS
AT ENERGIES GREATER THAN 10 TeV^{*}

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ABSTRACT

Two series of balloon flights were carried out in Australia in 1978 and 1979. Ninety high energy jet showers produced by cosmic ray protons, helium and heavier nuclei were observed. In proton-nucleus collisions at around 20 TeV, we observed a substantial increase of secondary charged particles in the central region. The average multiplicity of charged particles in helium-nucleus collisions is twice of that in proton-nucleus collisions at the same energy of 7 TeV. The average transverse momentum of π^0 meson is 0.46 ± 0.04 GeV/c for proton collisions, and 0.50 ± 0.04 GeV/c for helium or light nucleus collisions. These results are compared with the data of 400 GeV/c protons from accelerator.

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1. Introduction

Multiple production phenomena beyond 10 TeV energy range are very important for extending our insight deep into hadronic substance¹⁾⁻³⁾. It is also interesting to make clear the difference between Nucleon-Nucleus and Nucleus-Nucleus collisions at this superhigh energy region⁴⁾. This article describes 1) the primary energy spectra of proton and helium, 2) proton-lucite collisions at 20 TeV energy range, and 3) helium-lucite collisions at 7 TeV per nucleon.

2. Experimental Methods

The exposures were made for 32.5 hours and 12.0 hours at average height of 12 g/cm² and 20 g/cm², respectively. The total S × T amounts to 14 m² hours. Ninety high energy jet showers were observed; 50 of them are produced by protons, 23 by helium, 10 by light and 7 by heavier nuclei.

Energy of γ rays and momentum of charged particles are measured by the methods described in the reference⁵⁾. The invariant mass distribution of two γ rays is shown in Figure 1. The π^0 peak at $M_{\gamma\gamma} = 135 \text{ MeV}/c^2$ is clearly distinguished from the background. The energy resolution of γ rays is estimated to be 20 % by the width of this peak.

The incident axis of the jet shower is assumed to be the energy weighted center of γ rays and charged particles. The uncertainty of the axis is estimated to be 5×10^{-4} radians. This is confirmed by the analysis of jet showers produced by accelerated proton beams in which relative angle of secondary particles to non interacted beam tracks could be measured accurate enough.

The primary energy of observed jet showers were estimated by using the equation⁶⁾,

$$E(\text{per nucleon}) = \Sigma E_{\pi} / K_p / \langle \nu \rangle$$

$$\text{with } \langle \nu \rangle = A_B \times (\sigma_{PT} / \sigma_{BT}) \text{ and } \Sigma E_{\pi} = \Sigma E_{\gamma} + \Sigma E_{ch},$$

where we assumed that an incident nucleus with a mass number A_B and the energy E per nucleon collides with a target nucleus of a mass number A_T . K_p is the mean inelasticity coefficient for proton collision which is assumed to be 0.60. ΣE_{γ} and ΣE_{ch} are the sum of the energy carried away by neutral and charged mesons. ΣE_{γ} has been measured for 53 events listed in Table 1. In most cases, ΣE_{ch} is estimated by assuming constant P_T of 0.4 GeV/c. The above equation was checked by applying it to the jet showers produced by protons with energy of 303 GeV and 400 GeV. The primary energies thus estimated turned out to be 288 ± 34 GeV and 450 ± 77 GeV.

The average ratio of $(\Sigma E_{ch} / \Sigma E_{\gamma})$ is evaluated to be 1.8 for 32 proton events, and 2.0 for other 21 nucleus events in which both ΣE_{γ} and ΣE_{ch} were measured. These values are used in estimating primary energy for the other 37 events for which ΣE_{γ} is not measured and not listed in Table 1.

3. Experimental Results

The integral primary energy spectra for proton and helium nuclei are shown in Figure 2. These results are not inconsistent with the extrapolation from low energy regions obtained by Ryan et al.⁷⁾.

Average charged multiplicity for 20 proton-lucite collisions with energy more than 7 TeV is obtained as $\langle n_s \rangle_{PA} = 28 \pm 6$ after 9 % correction for the missing charged particles in the most backward region with

$\tan\theta_{ch} > 0.2$ rad. This number is converted to that of the proton-nucleon collisions taking into account of the nucleus target effect.⁶⁾ The result is $\langle n_s \rangle_{PP} = 20 \pm 4$ at $\langle E_p \rangle = 19 \pm 4$ TeV. These values are compared with average multiplicity at 400 GeV/c, $\langle n_s \rangle_{PA} = 13.5 \pm 1.1$ and $\langle n_s \rangle_{PP} = 9.6 \pm 0.8$. For 11 helium-lucite collisions with energy more than 3.5 TeV per nucleon, the average multiplicity is obtained to be $\langle n_s \rangle_{\alpha A} = 46 \pm 13$ at $\langle E_\alpha \rangle = 7 \pm 2$ TeV per nucleon. The ratio $\langle n_s \rangle_{\alpha A} / \langle n_s \rangle_{PA}$ is obtained to be 2.0 ± 0.2 at 7 TeV per nucleon.

The angular distribution of charged particles produced in proton-lucite collisions with energy over 7 TeV and helium-lucite collisions with energy over 3.5 TeV per nucleon are shown in Figure 3a). The result of 400 GeV/c proton collisions is also shown in the same figure for comparison. In our experiment, γ rays with energy higher than 30 GeV and emission angle $\tan\theta_\gamma < 5 \times 10^{-3}$ rad are detected with no bias. The angular distribution of γ rays produced in 32 proton-lucite collisions and that for 13 helium-lucite and 7 light nucleus-lucite collisions are shown in Figure 3b). The result of 400 GeV/c proton interactions is also shown in the same figure for comparison. The average multiplicity of charged particles in the central region of $|\eta_{cm}| < 1.5$, $\langle N_{ch} \rangle_{PA}$, is obtained as 8.1 ± 0.6 and 12.6 ± 1.0 for 400 GeV and 19 TeV. Assuming the correction factor of nucleus target effect⁹⁾, $R_A = 1.56$, $\langle N_{ch} \rangle_{PP}$ should be reduced to be 5.1 ± 0.4 and 8.3 ± 0.6 , respectively. The average multiplicity, $\langle N_{ch} \rangle_{\alpha A}$, is 23.3 ± 7.6 at the incident energy 7 ± 2 TeV per nucleon. The ratio $\langle N_{ch} \rangle_{\alpha A} / \langle N_{ch} \rangle_{PA}$ at 7 TeV per nucleon for lucite target is equal to 2.2 ± 0.2 .

The integral transverse momentum spectrum of γ rays is shown in

Figure 4 for 32 proton-lucite collisions. That for 13 helium-lucite and 7 light nucleus-lucite collisions is also shown in the same figure.

In the range of $0.08 < P_{TY} < 1.0$ GeV/c, it is well approximated by single exponential type distributions,

$$N(> P_{TY}) = N_0 \exp(-P_{TY}/\langle P_{TY} \rangle),$$

where $\langle P_{TY} \rangle$ and N_0 take the values 0.23 ± 0.02 GeV/c and 7.9 ± 0.5 for proton collisions, and 0.25 ± 0.02 GeV/c and 16.2 ± 1.0 for helium and light nucleus collisions, respectively. The average transverse momentum of π^0 meson is obtained as 0.46 ± 0.04 GeV/c and 0.50 ± 0.04 GeV/c by using the relation $\langle P_{T\pi^0} \rangle = 2 \times \langle P_{TY} \rangle$.

4. Conclusion

For proton-lucite collisions, a substantial increase of charged secondaries is observed in the central region as the primary energy goes up from 400 GeV to 20 TeV, giving evidence for an important violation of Feynman scaling. The average charged multiplicity in helium-lucite collisions is twice as large as that in proton-lucite collisions at the same energy 7 TeV per nucleon. The average transverse momentum of π^0 meson is obtained as 0.46 ± 0.04 GeV/c for proton collisions, and 0.50 ± 0.04 GeV/c for helium and light nucleus collisions.

5. References

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Fig.1. Distribution of invariant mass of γ rays, $M_{\gamma\gamma}$, of all events. The solid curve shows the background which was calculated by Monte Carlo program.

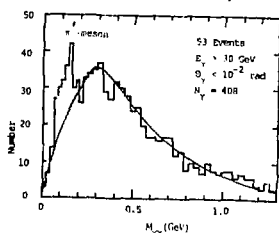


Fig.2. Integral primary energy spectra for proton and helium nuclei. The dotted line shows the extrapolation from low energy regions obtained by Ryan et al..

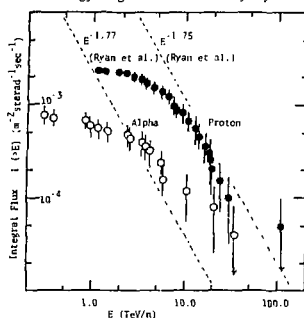


Fig.3. Angular distributions of secondary a) charged particles and b) γ rays in cosmic ray proton, helium and 400 GeV/c accelerator proton collisions.

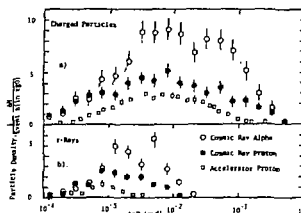
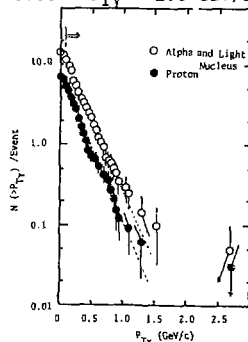


Table 1. Events selected for the present analysis.

Name	Pr1	4	Top	N _{ch}	E _T (GeV)	E ₀ (GeV)	E ₀ (GeV)	E ₀ (GeV)	E ₀ (GeV)	E ₀ (GeV)
REC10001	51	1	L2	5	10	1418	4171	947	0.088	
REC10011	51	1	L2	9	17	1287	1325	1520	0.090	
REC10021	51	1	L2	13	17	1757	1916	1418	0.109	
REC10031	51	1	L2	16	34	1285	12040	22108	0.208	
REC10041	51	1	L2	6	1	193	123	1244	0.184	
REC10051	51	1	L2	2	19	429	2892	1880	0.057	
REC10061	51	1	L2	7	11	1018	977	1391	0.152	
REC10071	51	1	L2	1	12	109	1371	1480	0.184	
REC10081	51	1	L2	1	21	1170	945	1109	0.124	
REC10091	51	1	EM	16	88	1543	2190	4932	0.215	
REC10101	51	1	L2	9	14	1945	1641	12644	0.228	
REC10111	51	1	L2	30	20	1464	1812	12475	0.279	
REC10121	51	1	L2	1	15	1430	3574	7839	0.196	
REC10131	51	1	L2	16	44	1910	5730	13061	0.250	
REC10141	51	1	L2	6	11	1171	2246	5655	0.217	
REC10151	51	1	EM	4	71	1704	8838	10283	0.409	
REC10161	51	1	L2	4	15	1210	4051	10607	0.191	
REC10171	51	1	EM	9	32	2160	1815	8291	0.455	
REC10181	51	1	L2	9	30	1600	1496	5164	0.235	
REC10191	51	1	L2	4	4	1290	429	3737	1.750	
REC10201	51	1	L2	1	12	1870	4464	12557	0.242	
REC10211	51	1	L2	2	7	3200	335	4225	0.047	
REC10221	51	1	L2	5	19	2800	1881	10807	0.419	
REC10231	51	1	L2	5	14	870	2178	4941	0.192	
REC10241	51	1	EM	1	47	870	1311	6431	0.145	
REC10251	51	1	L2	8	11	780	783	2204	0.093	
REC10261	51	1	L2	1	26	740	1352	6815	0.222	
REC10271	51	1	L2	5	15	370	1487	1476	0.284	
REC10281	51	1	L2	1	7	450	496	2747	0.172	
REC10291	51	1	L2	4	6	1510	217	2844	0.155	
REC10301	51	1	L2	1	17	1820	2327	9312	0.271	
REC10311	51	1	L2	11	43	510	1343	1555	0.175	
REC10321	AL	2	L2	18	47	2315	1846	9009	0.120	
REC10331	AL	2	EM	19	186	9711	26816	12861	0.192	
REC10341	AL	2	L2	10	57	2712	4044	5719	0.110	
REC10351	AL	2	L2	13	65	758	5533	3493	0.207	
REC10361	AL	2	L2	15	51	1385	2747	7741	0.149	
REC10371	AL	2	L2	44	3	1520	4914	1811	0.154	
REC10381	AL	2	L2	7	34	1520	1039	4118	0.139	
REC10391	AL	2	L2	4	18	1460	3373	4136	0.163	
REC10401	AL	2	L2	18	47	11430	10001	20677	0.147	
REC10411	AL	2	L2	4	84	1299	1448	5608	0.225	
REC10421	AL	2	L2	2	25	570	690	1164	0.192	
REC10431	AL	2	L2	4	44	1444	496	829	0.270	
REC10441	AL	2	L2	13	18	771	1728	2739	0.189	
REC10451	L2	4	L2	18	57	2064	4453	5860	0.202	
REC10461	L2	4	L2	14	67	1048	3511	5106	0.167	
REC10471	L2	4	L2	7	2840	1790	2304	1971	0.240	
REC10481	L2	4	L2	9	10	1244	642	1700	0.261	
REC10491	L2	4	EM	5	780	244	626	1084	0.184	
REC10501	L2	4	L2	9	20	1360	1440	1700	0.808	
REC10511	L2	4	L2	7	20	547	642	114	0.187	
REC10521	AL	2	L2	11	1444	369	1878	1355	0.183	

Fig.4. Integral transverse momentum spectrum of γ rays. The solid line shows the bias free regions. The dotted line shows the least square fit in the range of $0.08 < P_{T\gamma} < 1.0$ GeV/c.



$\alpha\alpha$ AND pp INTERACTIONS AT ISR ENERGIES ^{*})

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ABSTRACT

$\alpha\alpha$ and αp interactions could be studied during a short α run of the CERN Intersecting Storage Ring in the summer of 1980. The results thus obtained, which are still in a preliminary form, are here presented in parallel with matured pp results, in order to stress differences and similarities.

1. - INTRODUCTION

In the summer of 1980, α particles were stored in the ISR and a very successful 60 hour run took place. The beam momenta were the highest ones available on the machine, namely 63 GeV/c, with a resulting centre-of-mass energy of 125 GeV. The momentum per nucleon, which remains the key parameter, was, however, only 15.5 GeV/c. This is relatively low by present ISR standards, insofar as a large fraction of the running time in the standard pp mode has been at 31 GeV/c in recent years ¹⁾. The luminosity achieved in the $\alpha\alpha$ run was $2.8 \times 10^{28} \text{ cm}^{-2} \text{ s}^{-1}$, whereas a typical luminosity now achieved in pp runs is 2×10^{31} in a standard intersection. An αp run took place next to it in time. The beam momenta were 63 GeV/c (for α 's) and 31.5 GeV/c (for protons) with a resulting centre-of-mass energy of 88 GeV (22 GeV/c per nucleon in the centre-of-mass). The achieved luminosity was $8 \times 10^{29} \text{ cm}^{-2} \text{ s}^{-1}$.

Several experiments took data during these relatively limited runs. Whilst statistics are often limited and many results are to be considered as preliminary, with a fraction of the data analyzed and acceptance corrections not yet fully included, it is already possible to venture some general conclusions ²⁾.

These new results will be presented, considering in turn elastic scattering, particle production in a global sense, i.e., the dominant features of inelastic collision, and finally particle production at large transverse momentum, i.e., $p_T > 2 \text{ GeV/c}$. In each case we shall first briefly review the relevant key features of pp interactions ¹⁾.

Evidence for marked structures in the $\alpha\alpha$ and αp elastic differential cross-sections notwithstanding, the dominant new features appear

to be as follows :

- i) α collisions are far more efficient at producing large p_T secondary particles than one could expect from two sets of four nucleons acting incoherently ;
- ii) production of slow particles in the centre-of-mass (the so-called central region) is at least as important as if proportional to the mean number of nucleon collisions. This is where the rapidity distribution is maximum.

Both results are very interesting and somewhat challenging in the framework of present ideas about nucleus-nucleus interactions at very high energies ³⁾.

α particles are very light ions. Nevertheless this study of $\alpha\alpha$ collisions is a first glimpse at a new domain of physics. Indeed, the $\alpha\alpha$ ISR experiments correspond to an impressive leap in energy when considering ion collisions. The equivalent incident momentum per nucleon for an α beam impinging on a fixed Helium target is 515 GeV/c, when past experiments were limited to 2.9 GeV/c (Berkeley Bevalac) and 4.5 GeV/c (Dubna phasotron). At very high energy, the composite nature of the nucleon should become more important than the composite nature of the nucleus in terms of nucleons. The formation time of a hadron is determined by the level spacing of hadron excitations ($(0.3 \text{ GeV})^{-1}$ say). However, this applies to the hadron rest frame and, considering fast hadrons, the corresponding laboratory time exceeds by far the transit time during which the two nuclei pass each other (by two orders of magnitude, say, in the present case). The fast hadrons are therefore formed way outside the colliding nuclei and the cascade picture, familiar to low energy interactions, no longer applies. This has been already emphasized many

times ^{4),3)}. Nucleons in a nucleus do not oppose much screening to leading hadrons which pass through them as hadron constituents, as opposed to fully built hadrons ⁵⁾. Since these already well-documented features find a simple interpretation in terms of the composite nature of hadrons ³⁾, a detailed comparison between $\alpha\alpha$ and pp interactions may yield valuable clues about the nature of interactions at the constituent level. As discussed later, $\alpha\alpha$ and αp interactions at ISR energy illustrate very well the validity of these general ideas about leading particles. The leading fragments of a proton look very much the same whether the proton has been hit by a proton or an α particle.

Whilst this is already an interesting result, one may wish to grasp better the constituent-constituent interaction proper.

It is of course tempting to associate the relevant nucleon constituents with quarks and gluons. This is particularly the case for large p_T reactions where QCD used in a perturbative way may already provide a reasonable description of the observed processes ⁶⁾. In this case one may remark that the time scale for the reaction at the constituent level is now given by p_T^{-1} and, for $p_T \sim 5$ GeV/c, say, compares with the transit time (measured in the centre-of-mass of the collision) during which the two nuclei pass each other. It may then be possible to speak about rescattering processes, each collision being analyzed in the framework of perturbation theory with a relatively small effective coupling constant $\alpha_S(p_T^2)$ and resummation of the leading log series ^{6),7)}. While simple rescattering effects no longer apply at the nucleon level they may still apply at the quark-gluon level, hence the interest of large p_T production when relatively large numbers of quarks are brought together into collisions.

The importance of high p_T yields in $\alpha\alpha$ collisions, as compared to those measured in pp collisions, is certainly the most striking single effect discovered so far ²⁾. The observed enhancement (in the $2 < p_T < 6$ GeV/c range) far exceeds the factor 16 corresponding to incoherent interactions among nucleons. Whether or not the relatively large rates observed can be understood in terms of rescattering effects still remains an open question. This is certainly a challenging domain for further investigations. Insofar as QCD in its leading log approximation leads us to consider effects among quarks and gluons carrying finite fractions of the momentum of the hadrons they belong to $\alpha\alpha$ collisions bring into play 16 times more active constituents than a pp collision does. The complexity of nuclear matter may thus sometimes become an asset when trying to elucidate the nature of fundamental interactions.

For low p_T particle production it has also become usual to use QCD as a general framework but at present this can be done only with some non-perturbative algorithm, e.g., colour flux tubes extending between interacting quarks and penetrating the vacuum only as a magnetic field penetrates a type 2 superconductor. The energy thus stored in the flux tubes eventually materializes into jets of hadrons (mainly pions). In the picture advocated by Białas and Czyz ³⁾, for example, the number of "wounded" quarks and the number of extended flux tubes is larger in a nucleus-nucleus collision than it is in a proton-proton collision and this in a predictable way. This leads to specific features for the rapidity distribution of low p_T particles and in particular for the production of slow particles in the centre-of-mass which then result from the hadronization of the energy stored in the colour flux tubes.

The relation between the rapidity density of such particles (the so-called plateau density) and the number of nucleon collisions is, however, still a matter of conjecture and different models lead to different predictions ^{3),8)}.

In $\alpha\alpha$ collisions one finds a relatively high central density. It exceeds by a factor of the order of 1.8 the one observed in pp collisions at the same energy per nucleon ²⁾. This seems to be at least as high as the value expected from a full additive effect with several independent nucleon-nucleon collisions (with a mean value of the order of 1.6 nucleons hit in each α). The thus relatively large factor observed shows that a detailed study of particle production in $\alpha\alpha$ collisions should already be instructive, despite the relatively small size of the interacting nuclei.

Having very briefly surveyed the topics about which new information is available, we may say a few words about others, which are of great interest, but about which nothing can yet be said from the present results.

A very important question is that of the precise validity of the independent nucleon structure of the nucleus, as it could in principle be tested in a direct way through the observation of collective effects.

At present this is often phrased asking whether nucleons can merge into larger bags as the result of close interactions between nucleons. There is indeed evidence for particle production beyond the kinematical limit for nucleon-nucleon collisions which could perhaps be associated with such an effect ⁹⁾. If larger bags of quarks are actually involved, one should expect specific behaviours for the inclusive distribution of the very fast secondaries, as resulting from quark counting rules ¹⁰⁾. Whilst a $(1-x)^3$ behaviour is thus expected for pions produced near the

kinematical limit in pp collisions ($x=1$), a $(1-x)^9$ behaviour should result from a six-quark system, the momentum now being scaled according to the global momentum of the two nucleons. A $(1-x)^{21}$ component could even appear as corresponding to an α particle acting as a unique quark bag, albeit with probably a very small rate. The momentum would then be scaled according to the full α momentum. Such questions can, however, not yet be met with present results.

Collective interactions among nucleons are also related to the possibility of merging several nucleons into a "quark-gluon" plasma, a new phase of nuclear matter which could be reached at high energy and high density ^{11),12)}.

It has been argued that large p_T production can be looked at as the production of quarks and gluons outside of the realm of standard hadronic matter seen as a thermalized mixture of hadrons. During violent collisions among nuclei, some quarks and gluons could reach kinetic equilibrium through collisions rather than readily escaping as hadronic jets. Such an effect, but also the mere squeezing of several nucleons into a unique quark system, could lead to the transient existence of a quark-gluon plasma. Its high temperature equilibrium could manifest itself through its photon (lepton pairs) radiation, which would freely escape the high energy density zone, or also through other means ¹³⁾. Whilst α particles may be too small for such effects to occur at an observable rate, data should be watched with scrutiny for peculiar behaviours indicating something new. One cannot say anything at the present level of comparison between $\alpha\alpha$ and pp interactions.

The main body of this paper is organized as follows. We first discuss elastic scattering with results of two experiments, R210 (CERN-MIT

Naples-Pisa-Stony Brook) and R418 (CERN-Heidelberg-Lund). The latter experiment was actually the driving force in the $\alpha\alpha$ run at the ISR. This is done in Section 2. We next discuss particle production at low p_T with results from R418 again and from R807 (Brookhaven-CERN-Copenhagen-Lund-Rutherford-Tel Aviv-Pennsylvania-Johns Hopkins). This is done in Section 3. In Section 4, we turn to large p_T results with contributions from four different experiments, R418 and R807 again, for charged particles, and R806 (Athens-Brookhaven-CERN) and R110 (CERN-Oxford-Rockefeller) for neutral particles. In each case we compare $\alpha\alpha$ results to αp results (when also obtained during the same run) and also to pp results at the same energy (per nucleon)¹⁴.

Whilst present information is still fragmentary and results should be considered as preliminary, it already seems that $\alpha\alpha$ interactions show new features, specific of high energy collisions (e.g., large p_T production and central density, by which they distinguish themselves from a mere superposition of nucleon interactions.

2. - ELASTIC SCATTERING

Elastic scattering in pp collisions at ISR energy is characterized by a strong forward peak. An experimental parametrization of the type

$$\frac{d\sigma}{dt} = \frac{|F(s,t)|^2}{16\pi s^2} \frac{(\sigma_{tot})^2}{16\pi} e^{bt} \quad (1)$$

gives a value of b of the order of 13 (GeV/c)^{-2} for $|t| < 0.15 \text{ (GeV/c)}^2$. At larger $|t|$ values ($|t| > 0.2 \text{ (GeV/c)}^2$) the differential cross-section, which has then already fallen by an order of magnitude, can still be parametrized in terms of a single exponential but with $b \approx 11$ only.

Analyzed as purely diffractive scattering (a good first approximation), this corresponds to a scattering amplitude at impact parameter r

$$f(r,s) = \frac{1}{2\pi} \int \frac{|F(s,t)|}{8\pi s} e^{-iq \cdot r} d^2q \quad (2)$$

which is that of a relatively grey object, with a mean radius of about 0.9 f . The scattering amplitude (2) is 80% of the full absorption value (0.5) at $r = 0$ and has dropped to 10% of this value at $r = 1.5 f$. A striking feature is the presence of a dip at $|t| \approx 1.5 (\text{GeV}/c)^2$, which slowly moves in with increasing energy, as the forward peak shrinks. This is displayed in Fig. 1.

The over-all behaviour obeys geometrical scaling¹⁵⁾. With increasing energy the proton shape hardly changes. Its dimensions are, however, scaled up according to $\ln s$ and the cross-section as well as the slope increases.

Elastic scattering in pd and dd collisions has also been studied at the ISR and is then worth a mention¹⁶⁾. It shows patterns naturally associated with more extended objects. The forward peak in pd scattering has a slope of $b \approx 28 (\text{GeV}/c)^{-2}$ up to a "kink" around $|t| \approx 0.35 (\text{GeV}/c)^2$. The forward peak in dd scattering has a slope of $b \approx 57 (\text{GeV}/c)^2$ up to a dip at $|t| \approx 0.18 (\text{GeV}/c)^2$. This is shown in Fig. 2, which gives the pd and dd differential cross-section measured at $\sqrt{s} = 63$ and 53 GeV, together with predictions made using Glauber theory from the known pp differential cross-section. While the low $|t|$ behaviour corresponds to the size of the colliding objects, the larger $|t|$ behaviour is associated with multiple scattering effects. In connection

with that, the standard Glauber approach has, however, to be modified in order to include more intermediate states than a single nucleon¹⁶⁾. Such an introduction of excited diffractive state may be only a parametrization of a more complex process. It may fail with $\alpha\alpha$ scattering :

Figure 3 displays the αp results presently available. It combines the lower $|t|$ results of R210 ($\sqrt{s} = 88$ GeV) with the larger $|t|$ results of R418 ($\sqrt{s} = 88$ and 80 GeV). The forward peak slope is $b = 41 \pm 2$ (GeV/c)⁻². There is a marked dip at $|t| \approx 0.2$ (GeV/c)².

Since there may still be some normalization questions we prefer to present the R210 and the R418 results side by side. There is in any case over-all agreement in the region of overlap. In the latter case (R418) we also present previous Fermilab results¹⁷⁾.

Figure 4 presents the $\alpha\alpha$ results. It again puts together low $|t|$ data from R210 and larger $|t|$ data from R418. In this case one gets a very sharp forward peak with a slope $b \approx 100 \pm 10$ (GeV/c)² up to a marked dip at $|t| \approx 0.1$ (GeV/c)². There is a second dip at $|t| \approx 0.4$ (GeV/c)² with a three-order of magnitude difference between the two secondary maxima. Both sets of data correspond to $\sqrt{s} = 126$ GeV.

Experiment R210, which is sensitive to small angle scattering nicely complements the split field magnet experiment (R418), best suited for wider angle scattering.

In all four cases pd , $p\alpha$, dd and $\alpha\alpha$, the sharpening of the forward peak can be associated with the increasing region of space over which beam absorption occurs. One may simply see in the successive dips evidence for destructive interference among imaginary terms in the

amplitude obtained, according to (2), as successive Gaussian terms in r with decreasing width. This is a very rough picture which, as expected, signals an absorption zone with a much sharper edge than in pp scattering (Fig. 1c). There is, however, much more to it and the secondary maxima in $p\alpha$ and $\alpha\alpha$ scattering offer a challenge to the Glauber approach. The modified approach, including inelastic diffractive states, which is successful with pd and dd results¹⁶⁾ (Fig. 2) may fail in this case.

The forward peak observed in $p\alpha$ scattering is such that an extrapolation down to $t = 0$ can be attempted. This gives

$$\sigma_{\text{tot}}(p\alpha) = 135 \pm 20 \text{ mb}$$

$$\sigma_{\text{el}}(p\alpha) \approx 22 \text{ mb (using 1)}$$

Errors are large and such results cannot be deemed different from the more precise lower energy JINR results.

Comparing inelastic cross-sections in pp and $p\alpha$ scattering (when one should better compare non-diffractive inelastic collisions) one can estimate the number of nucleons involved in an inelastic $p\alpha$ collisions as 3),8)

$$v(p,\alpha) = \frac{4 \sigma_{\text{in}}(pp)}{\sigma_{\text{in}}(p\alpha)} \approx 1.20 \quad (3)$$

While this can be quoted only with a large error, one can say that the probability of double hit in $p\alpha$ collisions appears to be relatively small (20%, say). We shall come back to it when considering multiplicities.

3. - PARTICLE PRODUCTION AT LOW p_T

In pp collisions most secondary particles are produced with low p_T . The distribution is exponential with a slope most generally associated with the transverse dimension of the collision volume. Most particles are pions (~85% of slow secondaries in the centre-of-mass system).

The rapidity distribution shows a plateau, extending as $\ln s$ with increasing energy, with a mean density of about two charged particles per unit of rapidity. This has been known for a long time ¹⁾. It has more recently been found that the same plateau density applies to particle production in e^+e^- annihilation ¹⁸⁾. If one takes into account the fact that in pp collisions half of the incident energy is retained (on the average) by two leading baryons, an e^+e^- annihilation energy of 30 GeV (PETRA, PEP) does correspond to a typical ISR energy when considering pionization ¹⁹⁾. Figure 5 puts together inclusive particle distributions obtained at the ISR and at SLAC and PETRA, showing in both cases the extending plateau, with very similar heights.

The approximately constant (plateau) density is part of a more general property referred to as short-range order. The rapidity density does not change much with increasing energy. Neither does the mean value hardly change (Fig. 5) nor do correlations. This had been discovered long ago at the ISR. As shown in Fig. 6, the value of the two-body correlation function, while important, depends on the rapidity difference but hardly on the mean rapidity, provided that one remains within the plateau region.

Short range order has long been associated with general conclusions drawn from the multiperipheral model ²⁰⁾ whereby particle production results from a multi-exchange process. At each step down the multiperipheral chain (Fig. 7) each produced cluster (hence the short range correlation observed) takes on the average a fixed fraction of the remaining momentum. This readily leads to a rapidity plateau.

Such a property is also included in all successful models for jet fragmentation ²¹⁾. A more recent approach which uses an algorithm based on QCD in its non-perturbative form, sees such plateau distributions as resulting from the hadronization of an extended colour flux tube between a 3 and a $\bar{3}$ colour charge (the most economically meaningful colour separation, energetically speaking). The stored gluonic energy materializes into hadron clusters which can be considered as droplet of glue ²²⁾. The general scaling properties initially derived in the multiperipheral framework remain.

One should then use these general properties as a basis for discussion in nucleus-nucleus scattering. An expected difference is that a typical collision may now involve several (and independent ?) colour flux tubes ³⁾. According to such a picture, one should meet a higher plateau density than in proton-proton collisions, but short range correlations should remain the same, as an intrinsic property of the hadronization process, along a multiperipheral chain, or along an extended colour flux tube. Conversely anomalous correlations would signal something special to nucleus-nucleus interactions.

Rapidity correlations observed at ISR energies (Fig. 6) clearly display a short range component (dependent on Δy only) but also a long range component. One defines a correlation function :

$$R(y_1, y_2) = \frac{\rho^{(2)}(y_1, y_2)}{\rho(y_1) \rho(y_2)} - 1 \quad (4)$$

where ρ and $\rho^{(2)}$ are the one-particle and two-particle rapidity densities, respectively. In a very simple model whereby the production process is either standard (with an almost constant rapidity distribution) with probability β or clearly diffractive (with a large central rapidity gap) and probability $1-\beta$, one readily obtains ¹⁾

$$R(y_1, y_2) = \frac{r(y_1, y_2)}{\beta} + \frac{1-\beta}{\beta} \quad (5)$$

Here r is the correlation function proper to the standard mode, which should soon be comparable to charge-charge correlations measured in e^+e^- annihilation and defined according to (4). It is smaller (by a factor 2 to 2.5) than the global measured value (Fig. 6). Namely $r(0,0) \approx 0.25$.

In the case of nucleus-nucleus collisions there is an obvious extra source of long range correlations coming from the possibility of having 1,2,3,... independent nucleon collisions. One should then try to extract the value of $r(y_1, y_2)$ due to short-range effects and compare it to the value obtained from the analysis of correlation effects in pp collisions ²³⁾. There may be peculiar effects involving larger numbers of secondary particles. At present, analysis is under way (R418 and R807). One cannot yet conclude at any effect special to $\alpha\alpha$. We shall briefly comment on that later.

There is also a very interesting type of correlation which applies to identical particles. Pion production is high and one can consider a classical pion wave associating measured pion rates to its intensity

distribution. This is a generalization of the Hambury-Brown-Twiss effect well known in astronomy, to pion production²⁴⁾. Applied to pp collisions at the ISR (R407), this leads to a source size of the order of 1.3 ± 0.3 fermi¹⁾. A larger size should be found in $\alpha\alpha$ collisions. Analysis is under way (R807).

We now come back to present data, which refer to multiplicity distributions and rapidity distributions but first briefly comment about the rapidity "plateau" in pp collisions.

A constant plateau density is only a first approximation. As found at the ISR (R803), the plateau density slowly rises with incident energy and there are indications from cosmic ray experiments that it continues to do so up to incident energies of the order of 100 TeV²⁵⁾. In e^+e^- annihilation, an asymptotic rise can be interpreted as resulting from scaling violation effects understood in the framework of perturbative QCD²⁶⁾. In pp collisions similar arguments could apply but one is dealing with a far more complicated system, incoherent multiple collisions among low x partons becoming more probable with increasing energy²⁷⁾. Hydrodynamical arguments can also be used to interpret the observed increase in central density²⁸⁾. The rise of the plateau in $\alpha\alpha$ collisions would be an interesting parameter to have. Figure 8 shows the rise of the central rapidity yield as observed over the ISR energy range. It increases by 25% with respect with σ_{tot} .

While the present $\alpha\alpha$ run had only one energy and nothing can be said about the energy dependence of the central yield, there is already evidence for a central rapidity plateau.

Figure 9a shows the rapidity distribution of positive and negative secondaries obtained in $\alpha\alpha$ collisions at $\sqrt{s}_{NN} = 31$ GeV, the centre-of-mass energy at the nucleon-nucleon level, and Fig. 9b gives the ratio of rapidity densities between the $\alpha\alpha$ distributions and the distribution obtained in pp interactions, in the same experiment, but at $\sqrt{s} = 62$ GeV. The constancy of the ratio holds over three units of rapidity, with, there, identical yields for positives and negatives. The ratio is equal to 1.45 but one should acknowledge the rise of the plateau density with increasing energy (Fig. 8) and the ratio which one may then quote at the same energy per nucleon ($\sqrt{s}_{NN} = 31$ GeV) is of the order of $R \approx 1.8$. A direct comparison will eventually be possible at the same value of $\sqrt{s}_{NN}$. This involves, however, acceptance corrections which disappear for identical field values in the SFM detector (the same value is used for 62 GeV α 's and for 31 GeV protons) and which have not yet been fully analyzed. In any case one may already conclude at a very significant rise of the central density between pp and $\alpha\alpha$ collisions. R807 finds a similar enhancement.

Figure 9c shows the rapidity distributions observed in αp collisions (R418). There is no evidence for any central tilt in favour of the α side, where about 1.2 nucleons fragment, on the average. The "horns" seen on the positive distributions of Fig. 9a and Fig. 10 are far more pronounced than any feature associated with the leading protons in pp collisions (Fig. 9b). They correspond to the emergence of nuclear fragments. They can be sorted out and very preliminary percentages quoted by R418 are ²⁾ 64% for protons, 18% for deuterons, 5% for tritons and He^3 nuclei, the rest being quasi-elastically scattered α 's. The p_T width of the different distributions increases somewhat with mass from proton to triton. The associated multiplicity is larger when considering reactions with a proton fragment than those with a d or α fragment.

The mean multiplicity is the integral over the rapidity distribution as here normalized to the production cross-section. In view of the occurrence of positive nuclear fragments and of the presence of singly- or doubly-charged positive particles in the initial state, it is more instructive to compare negative multiplicities obtained in pp and $\alpha\alpha$ collisions. One then gets

$$\langle n_- \rangle = 6.80 \pm 0.07 \quad D_- = 4.96 \pm 0.05$$

for $\alpha\alpha$ at $\sqrt{s}_{NN} = 31$ GeV and for pp at the same momentum per nucleon.

$$\langle n_- \rangle = 3.92 \pm 0.05 \quad D_- = 3.11 \pm 0.04$$

D_- is the dispersion for negative particles, defined as $D_- = (\langle n_-^2 \rangle - \langle n_- \rangle^2)^{\frac{1}{2}}$.

The ratio between mean multiplicities at the same energy per nucleon ($\sqrt{s}_{NN} = 31$ GeV) is thus

$$R = \frac{\langle n_- \rangle_{\alpha\alpha}}{\langle n_- \rangle_{pp}} = 1.7 \pm 0.1 \quad (6)$$

It falls to 1.6 when normalizing according to counting rates with negative particles seen, in order to discriminate against fragmentation ($\alpha\alpha$) and diffractive (pp) reactions. This is what is actually done for the rapidity distributions of Figs. 9 and 10. One is led to conclude that double (or multiple) collisions are not very frequent. Comparing multiplicity distributions, as done in Fig. 11a (experiment R418) one indeed sees that the most frequent configurations in pp still appear to be

occurring about half as frequently in $\alpha\alpha$ collisions, as if two independent nucleon collisions would not occur more than half of the time. This is probably an overestimate since a collision may involve one fragmenting nucleon on one side and more than one on the other side, and hence an increased fragmentation multiplicity.

One sees, however, that most of the change is at wide angles when considering the multiplicity distributions observed by R807 in its central detector ($|\theta| < 30^\circ$ in the centre-of-mass system). They are displayed in Fig. 11b.

The charged particle multiplicity distributions observed when comparing pp and $\alpha\alpha$ and pp and αp , respectively, are here arbitrarily normalized at the same value at maximum. One notices then the widening of the distribution as one goes from pp to αp and to $\alpha\alpha$ which corresponds here to a widening of the multiplicity distribution in the central rapidity region. There is also a correlation between the very large multiplicities measured at wide angle and the one measured in the fragmentation region, the largest multiplicities being associated with the 4 nucleons in the α being hit during the collision²⁹⁾.

Since in pp collision it is well known that D^- increases linearly with $\langle n_- \rangle$, we show the corresponding points for $\alpha\alpha$ in Fig. 12 ("low" energy and ISR points). The ISR value is above the linearly extrapolated value. This is an unexpected feature if different types of reactions are present (1,2,... independent nucleon-nucleon collisions, say). The departure is, however, not very large, and indeed compatible with a single collision occurring about half of the time.

It seems that the ratio between the $\alpha\alpha$ and pp central rapidity distributions at the same energy per nucleon, which is of the order of 1.8, is somewhat large when compared to the enhancement expected from independent nucleon-nucleon collisions. We may discuss that in the framework of model calculations.

Experiment 807 had a calorimeter surrounding, the beam pipe, with which it could measure the energy deposited by fast fragments ²⁾. One could then compare the fragmentation of a 32 GeV/c proton whether it had been hit by a 15 GeV/c proton in an α particle or by a 32 GeV/c proton. In this case one knows that there is no appreciable dependence on the proton energy (scaling). As shown in Fig. 13a, one also sees that there is little difference, if any, to be associated with the fact that the collision occurred with an α particle. The similarity in shape corresponds to the proton fragmenting only much after traversing the α . There is also a similarity in rate.

In the quark model approach ³⁾, one would conclude that, at the level of accuracy reached, there is no appreciable difference between the number of "wounded quarks" in a proton whether it interacts with a proton or with an α particle. One would then roughly estimate (from the inelastic cross-section) that the number of wounded quarks $W_q(\alpha, \alpha)$ in each α in an $\alpha\alpha$ collision is of the order of 1.5-1.6.

One writes :

$$W_q(\alpha, \alpha) = W_N(\alpha, \alpha) W_q(p, \alpha) \quad (7)$$

with

$$W_N(\alpha, \alpha) = \frac{4\sigma_{in}(p\alpha)}{\sigma_{in}(\alpha\alpha)} \quad (8)$$

where W_N is the mean number of hit nucleons in an α particle colliding with another one. Similarly the number of wounded quarks in an α during an αp collision would be, as previously estimated, of the order of 1.2.

The pulse height distributions observed on the α side in αp and $\alpha\alpha$ collisions are shown in Fig. 13b. There is now some difference (albeit small) between the two distributions. It is compatible with a 25% excess of energy in the $\alpha\alpha$ case, in reasonable agreement with the estimates made.

If one considers the colour flux tube approach ³⁾, one would conclude that the central plateau density, which is proportional to the number of independent flux tubes, should be lower than the number of wounded quarks in each α , namely :

$$\lambda_{\alpha\alpha} = \frac{W_q(\alpha, \alpha)}{v(q, \alpha)} \quad (9)$$

where $v(q, \alpha)$, the mean number of quarks in an α involved in a quark α collision is of the order of 1.2. In view of this the observed value of 1.8 for the ratio of central density between $\alpha\alpha$ and pp collision may seem high.

Whilst α 's are probably ions too light to be fully relevant for considerations valid in the limit of large nucleon numbers ³⁾, the density is, however, clearly maximum at $y = 0$ (Fig. 9). This last property would agree with the conclusions of Brodsky et al. ⁸⁾ where the central multiplicity increases according to the number of nucleons hit in either particle. In this case the central density is of course higher

and could be of the order of 1.7. Nevertheless, the p result do not show the slant towards the α side, which one would then expect in this case (Fig. 9c). The large value obtained is then not very reliable. The two approaches considered correspond to two somewhat extreme views insofar as the central region is well separated from the fragmentation region in the former case ³⁾ and to some extent an extension of the fragmentation region in the latter one ⁸⁾.

One cannot conclude from α alone ! It seems, however, that the central rapidity density is somewhat large in view of estimates for the number of independent collisions, as if the density of slow partons, in the Feynman sense, would be enhanced in an α .

The p_T distribution does not show any appreciable change between the $\alpha\alpha$ and pp cases. This is shown in Fig. 14 (R418). Hadronization seems to proceed in a similar way in both cases.

Figure 10 shows the two-body charge-charge correlation, as defined by (4), measured in the R418 experiment. It combines $\alpha\alpha$ results with the first particle selected in the intervals $|y_1| < 0.5$ and $1 < |y_1| < 1.5$, showing them the corresponding variation of R as a function of y_2 . The peak shifts, the maximum in y_2 being at the value selected for y_1 as expected from short-range effects which are definitely present. Also shown is the correlation function measured for pp at the same value of $\sqrt{s}_{NN}$ for $|y_1| < 0.5$. The values of R measured in $\alpha\alpha$ and pp interactions are comparable.

One could expect $R(0,0)$ to be smaller in the $\alpha\alpha$ case since, as defined by (4), it should be inversely proportional to the rapidity density if clustering effects were identical. However, there is certainly

an extra long-range component in the $\alpha\alpha$ case which is due to the presence of different types of collisions. It may compensate for the increase of ρ . Only a refined analysis separating long- and short-range effects could tell. A crude estimate gives the needed compensation.

Whilst rapidity distributions cannot yet be compared at the same energy per nucleon (Fig. 9b) one may already say that at $|y| > 2$ there is a very significant increase in the number of negatives when comparing pp and $\alpha\alpha$ interactions (a factor 2 at $y = 3$, say²⁾). This is of course associated with the fragmentation of neutrons.

This is important to notice in view of its connection with a long-standing riddle associated with cosmic ray muons³⁰⁾. The μ^+/μ^- ratio is known to be 1.25 when an analysis based on scaling and pp production data at accelerator energy leads one to predict 1.56³⁰⁾. The α/p ratio at the same energy per nucleon (which is the relevant parameter when considering cosmic ray muon fluxes) has been measured to be 4%³¹⁾ and to be practically independent of energy. This corresponds to an over-all abundance of α (at the same energy) at the 25% level, which seems very reasonable.

While this may not be sufficient at removing the observed discrepancy, it goes a good fraction of the way in the right direction³¹⁾.

4. - PARTICLE PRODUCTION AT LARGE p_T

Whilst most produced particles have low transverse momentum, the probability of observing large p_T particles ($p_T > 3$ GeV/c, say) is anomalously large insofar as it exceeds by an important amount the exponential extrapolation of the low p_T yield. This important feature, discovered long ago at the ISR, is displayed in Fig. 15³²⁾. The

origin of large p_T secondaries is associated with hard scattering among hadron constituents, as schematically shown in Fig. 16. While one should not undermine standing problems, one may say that perturbative QCD offers a reasonable understanding of the observed effect in terms of quark and gluon scattering⁶⁾. In this framework the hard scattering proper takes a very short time. It is measured by the virtuality of the intermediate state in perturbation theory ($\tau \sim p_T^{-1}$). On the other hand, hadron formation, with jets associated with the scattered constituents (Fig. 16), takes a much larger time (which is proportional to p_T for the fastest hadrons, as a result of time dilation). Large p_T processes are therefore an a priori very interesting topic for high energy collisions among nuclei. The jet nature of the particles produced at large p_T has been ascertained through a long series of correlation experiments at the ISR¹⁾ and at Fermilab³³⁾. An important property is scaling in jet fragmentation, each large p_T particle observed on the away side from a large p_T particle used as a trigger, taking on the average a fixed fraction x_e of the trigger particle transverse momentum. This is shown in Fig. 17, which displays such a scaling property as observed in several experiments at the ISR¹⁾. There is also, as expected, a marked coplanarity structure among large p_T particles, the reaction plane being defined by the two-body reaction at the constituent level (Fig. 16). This is well illustrated by Fig. 18 which shows the azimuthal distribution of particles with $p_T > 0.6$ GeV/c associated with a trigger particle at $\phi = 0$ and with $p_T > 3$ GeV/c. The rapidity ranges covered are $|y| < 0.5$ (R413) and $|y| < 0.8$ (R807). It shows evidence for a strong backward jet and a much weaker trigger side forward jet (as a result of the trigger bias effect⁶⁾).

This rapid data sampling is meant at illustrating properties derived from the hard scattering approach (Fig. 16), with fragmentation of the scattered constituents into jets.

Considered as the result of a scattering process at the constituent level, large p_T production in proton-nucleus scattering should be naively proportional to the number of constituents, since the relevant cross-section is small and hard constituents suffer hardly any screening. It should rise linearly with the atomic number A , whereas low p_T production, which is a standard hadronic process, should rise only as $A^{2/3}$. Experiments at Fermilab came up, however, with a remarkable result³⁴⁾. The A dependence, parametrized as A^α is found to correspond to a value $\alpha < 1$ at low p_T (as expected) but to values becoming larger than 1 at large p_T ($p_T \gtrsim 2$ GeV/c, say). A sample of data, corresponding to large p_T pions produced by 400 GeV/c protons incident on different nuclear targets is shown in Fig. 19.

This effect is still a riddle. The most convincing explanation seems, however, to be in terms of multiple scattering³⁵⁾. Two successive scatterings at a very short time interval may contribute to building up the large transverse momentum found with the trigger particle.

The double scattering process then corresponds to an inclusive cross-section written as (neglecting triple scattering)

$$E \frac{d\sigma^{(2)}}{d^3p} = K A^2 \int d^2b \, dz \, dz' \, \rho(b,z) \, \rho(b,z') \quad (10)$$

where the integral runs over the impact parameter b and the longitudinal width of the nuclear target. The nuclear density is normalized to unity, hence the factor A^2 put in front. K corresponds to a folding integral

describing how the transverse momentum results from two successive scatterings. One may assume as a first approximation that scattering takes place on leading partons belonging to different nucleons. There may also be terms (potentially very interesting) whereby two partons in the same nucleon contribute ³⁶⁾.

The integral in (10) should decrease according to the nuclear surface, i.e., as $A^{\frac{2}{3}}$, since the density is normalized to 1. The double scattering term should therefore globally increase as $A^{\frac{1}{3}}$. One is then led to write the inclusive cross-section as

$$E \frac{d\sigma}{d^3p} \sim A(1 + \gamma A^{\frac{1}{3}} + \dots) \quad (11)$$

Numerical calculations ³⁵⁾ give a reasonable agreement with the data ³⁴⁾. While the integral in (10) behaves as R^2 , R being the nuclear radius, the dimension of K should be such as to provide a dimensionless correction factor $(1 + \gamma A^{\frac{1}{3}})$ to the inclusive yield. It should eventually decrease with p_T at least as fast as $(R^2 p_T^2)^{-1}$, this minimal regime corresponding to a scaling behaviour for the inclusive large p_T cross-section $E(d\sigma/d^3p) \sim p_T^{-4}$, which is not yet found in the p_T range presently probed ($p_T \sim 4$ GeV/c ^{1),6)}). The decrease with p_T should then be even steeper. One would then expect the parameter α to fall back eventually to 1 at larger p_T ^{*)}, ³⁷⁾. While the data may suggest such a trend (Fig. 19), it is not yet significantly supported !

*) Such considerations assume that the yield would be vanishingly small anyway when p_T values beyond the phase space limit for nucleon-nucleon scattering are considered.

We shall use these general remarks as a basis for a discussion for the $\alpha\alpha$ and αp results. For $\alpha\alpha$ single hard scattering is now expected to contribute a yield proportional to A^2 , and double scattering to $A^{\frac{7}{3}}$. We may then tentatively write the inclusive cross-section as

$$E \frac{d\sigma}{d^3p} \sim A^2(1 + \gamma' A^{\frac{1}{3}} + \dots) \quad (12)$$

and since double scattering can now occur on both sides, one is tempted to write, as a first approximation :

$$\gamma' \approx 2\gamma \quad (13)$$

As long as the correction term is numerically not very large one may then be tempted to square the correction factor when going from $p\alpha$ to $\alpha\alpha$ scattering. As it will be clear in the following, large p_T yield in $\alpha\alpha$ scattering shows surprisingly large values.

Figure 20a shows the inclusive π^0 cross-section measured by R806 in $\alpha\alpha$ collisions in the $2 < p_T < 4$ GeV/c range. Also shown is the inclusive cross-section measured by the same experiment for pp collisions, at the same value of $\sqrt{s}_{NN} = 31$ GeV. The yield in $\alpha\alpha$ scattering is larger and it increases with p_T relative to the value measured in pp collisions. The slope (of the approximately exponential distribution in this range) is definitely larger. $\alpha\alpha$ collisions are far more efficient at producing large $p_T \pi^0$, and the more so the larger p_T is. At 4 GeV/c the ratio far exceeds the value 16 expected from independent nucleon interactions. Errors are typically at the 15% level. Figure 20b shows a similar comparison, but for π^0 yields in αp and pp collisions at the same value of $\sqrt{s}_{NN}$, now equal to 44 GeV. In this latter

case, the relative enhancement slowly changes with p_T , being at the level of 4 only. Within errors this is compatible with the value expected from the Fermilab results (an enhancement by a factor 1.15 beyond the naïve value 4). Multiplying 16 by the square of this enhancement factor, one gets a value of 21 which is now somewhat lower than the ratio observed between $\alpha\alpha$ and pp induced reactions (Fig. 20a).

Experiment R110 also gives evidence for a similarly large factor. The data which now extend from 4 to 6 GeV/c are shown in Fig. 21. The enhancement factor, which is obtained using former pp data at the same value of $\sqrt{s}_{NN}$, is of the order of 28 at $p_T \approx 5$ GeV/c. It rises slowly with p_T over the 4 to 6 GeV/c range. The behaviour past 6 GeV/c is still statistically hardly significant in our quest for an eventual fall off, as previously advocated in the double-scattering approach.

One may then conclude at a rather high enhancement in $\alpha\alpha$ collisions. The relevant x_T values are comparable to that of the Fermilab experiment (Fig. 19) and kinematical conditions at the parton level should then be similar.

Two other experiments have data on charged particle production in $\alpha\alpha$ collisions. R807 finds a heavy/pion ratio similar to the one previously found for pp collisions at the same energy per nucleon ²⁾. It is at the level of 0.8 for positives and 0.25 for negatives, not changing significantly between 2 and 4 GeV/c. The relative abundance of pions is known to increase with energy at fixed p_T . Without proper pp data at $\sqrt{s} = 31$ GeV and large p_T , a precise comparison between yields in $\alpha\alpha$ and pp induced reactions is not yet possible here.

Experiment R418 triggers on a large p_T particle at 45° whereas all previous results correspond to 90° . The inclusive yields for positives and negatives thus measured are shown in Fig. 22. The ratios for positives and negatives are the same. This is probably a coincidence due to the fact (as discussed below) that positive and negative triggers are actually obtained at slightly different rapidity, although at the same angle (SFM field). This would otherwise imply (surprisingly) more π^- 's than π^+ 's. Particle ratios also measured in this experiment are indeed in agreement with the value previously quoted in connection with the R807 results.

A meaningful comparison with pp data should be made at the same rapidity since there is a non-negligible rapidity dependence at these energies. A trigger at 45° turns out to correspond to $y = 0.83$ for positives and $y = 0.62$ for negatives, the asymmetry being due to the SFM field. For that reason, comparison with existing data from R403¹⁾ can be made up to $p_T = 2.6$ GeV/c only, the two distributions also being shown on Fig. 22. One already finds a very significant enhancement. It is much larger for negatives than for positives (of the order of 40 for negatives and 25 for positives at $p_T = 2.6$ GeV/c). Such a difference is, however, expected on the ground that positives are detected at a slightly larger rapidity value than they are in the pp case, and lose in any case their privileged association with valence quarks when going from pp to $\alpha\alpha$ collisions.

After having reviewed present information about inclusive yields, we now turn to associated particles.

A large p_T particle is associated with a significant increase of the reaction multiplicity. Figure 21 shows the negative multiplicity distributions obtained by R418 in $\alpha\alpha$ and pp scattering without (minimum

bias) and with a large p_T trigger ($p_T > 3.6$ GeV/c). There is in both cases a similar shift of the mean associated multiplicity. The increase corresponds to the multiplicity associated with the recoiling jet (Fig. 16) and to the fact that observing a large p_T particle is a bias against low multiplicity (fragmentation and diffractive) configurations. At this level $\alpha\alpha$ and pp induced reactions look similar. The same also applies to the x_e distributions. This seems to rule out any important occurrence of two independent large p_T scatterings in an $\alpha\alpha$ collision with two recoiling jets of similar transverse momentum. Figure 24a shows the x_e distribution observed by R418 in $\alpha\alpha$ ($\sqrt{s}_{NN} = 31$ GeV) and pp ($\sqrt{s} = 63$ GeV) reactions. There are no appreciable differences. The distributions measured in αp are also similar. Figure 24b gives the x_e distributions for $\alpha\alpha$ and pp (same energies as R418) now observed by R807. Here again the two distributions are almost identical.

One does not expect the x_e distribution to change appreciably with energy and the comparison made should be meaningful. One may notice a slightly steeper dependence in the $\alpha\alpha$ case, as one should expect it from the partial occurrence of two jets on the away side. It is, however, not statistically significant. One may rather conclude that the recoiling systems to a large p_T trigger particle looks similar in $\alpha\alpha$ and pp collisions.

Significant differences occur when considering the azimuthal distribution of particles with a significant transverse momentum. Figure 25a gives the azimuthal distribution of particles with $p_T > 1$ GeV/c observed in association with a neutral trigger particle at $p_T > 2.5$ GeV/c by R110, normalized to that observed in pp collisions at the same energy per nucleon. While the rates are similar on the away side ($\phi \approx 180^\circ$) as

previously mentioned, one notices a large enhancement at wide angle ($\phi \approx 90^\circ$) in the $\alpha\alpha$ case. The distribution loses part of the coplanarity structure which it has in the pp case. The same effect is also seen on Fig. 25b, which presents results from R807. The p_T cut on the associated charged particles is the same and distributions are shown for two different trigger momenta ($p_T > 2.5$ GeV/c and $3.5 < p_T < 4.5$ GeV/c), the trigger particle now being a charged particle. The ratio corresponds this time to $\alpha\alpha$ at $\sqrt{s}_{NN} = 31$ GeV and to pp at $\sqrt{s} = 63$ GeV. This should be at the origin of the excess at $\phi \approx 0^\circ$, seen when comparing pp to $\alpha\alpha$. One again finds an excess at wide angles ($\phi \approx 90^\circ$) in the $\alpha\alpha$ case. If one wishes to see two recoiling systems part of the time, one would then conclude at a dominant configuration, whereby one jet recoils as in pp collisions while another one may take a smaller fraction of the transverse momentum, with fragments breaking the coplanarity structure while not contributing much to the transverse momentum balance.

5. - CONCLUSIONS

This attempt at a comparison between pp and $\alpha\alpha$ collisions at the ISR is still very limited in its scope. It is, however, impressive to see the many results which could be obtained during a very short run. This review focused on the data while venturing some general remarks, and in so doing, the reviewer feels somewhat like a "jaybird wearing the feathers of a peacock". No mention was made of the different apparatuses used. At present, one may consult Ref. 2) for a general survey of the experimental techniques as well as a more detailed presentation of the data. One sees that there is something special about reactions among α particles. It shows up most strongly in large p_T reactions, which are most closely linked to the hadronic structure.

ACKNOWLEDGEMENTS

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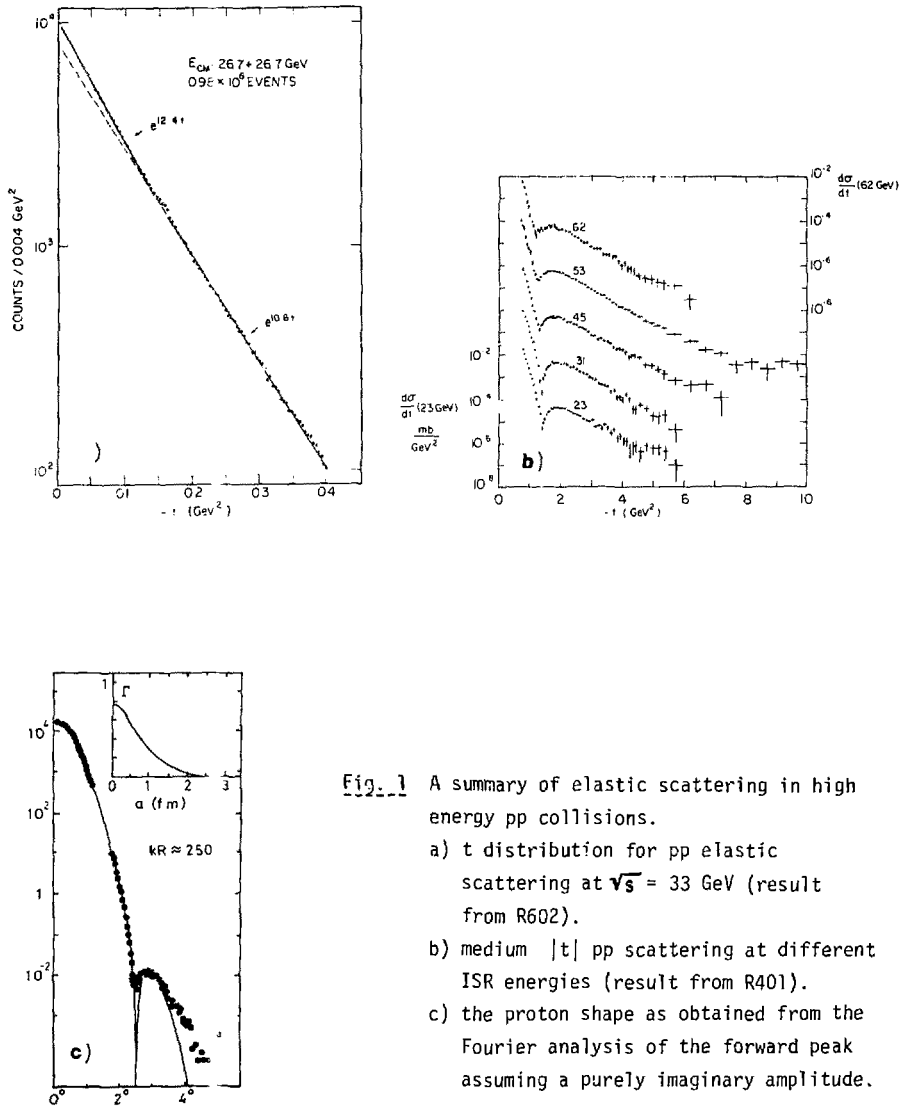


Fig. 1 A summary of elastic scattering in high energy pp collisions.

a) t distribution for pp elastic scattering at $\sqrt{s} = 33$ GeV (result from R602).

b) medium $|t|$ pp scattering at different ISR energies (result from R401).

c) the proton shape as obtained from the Fourier analysis of the forward peak assuming a purely imaginary amplitude.

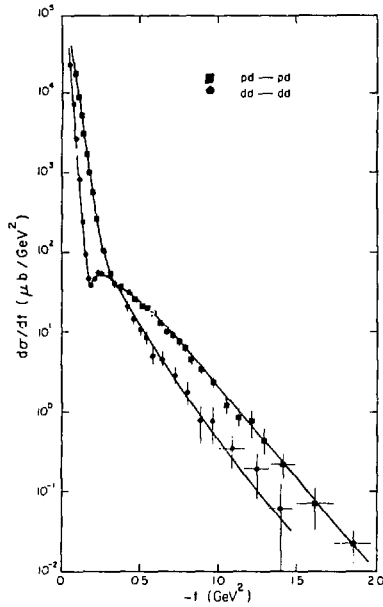


Fig. 2 pd elastic scattering (squares) at $\sqrt{s} = 63$ GeV and dd elastic scattering (dots) at $\sqrt{s} = 53$ GeV (result from R417). The solid lines are the absolute predictions of a modified Glauber model, which includes inelastic diffractive states, and is convoluted with the experimental resolution.

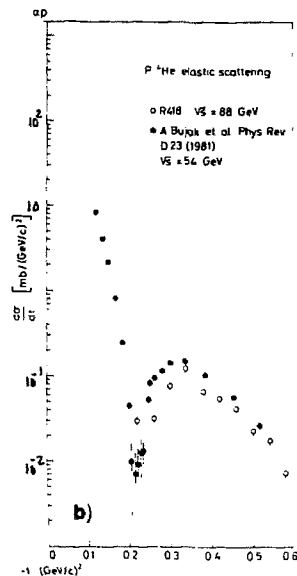
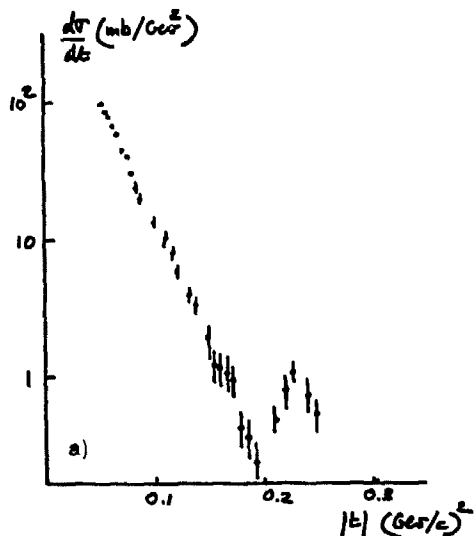


Fig. 3 $p\alpha$ elastic scattering at ISR energies. Results from R210 (a) and from R418 (b) are here combined with previous results from Fermilab ($\sqrt{s} = 48$).

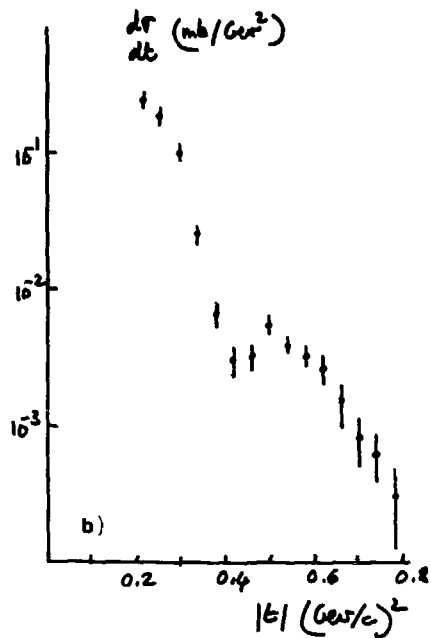
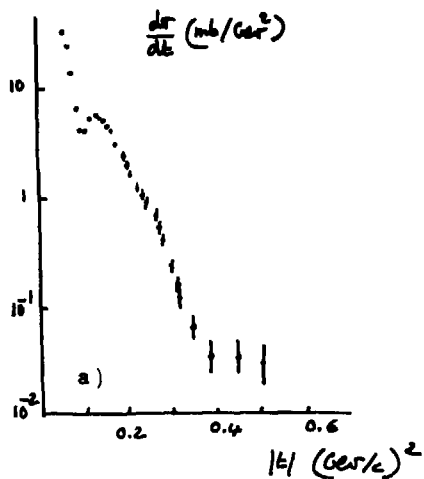


Fig. 4 $\alpha\alpha$ elastic scattering at ISR energies. Results from R210 (a) and from R418 (b) are presented side by side. The third maximum suggested in (a) is well displayed in (b).

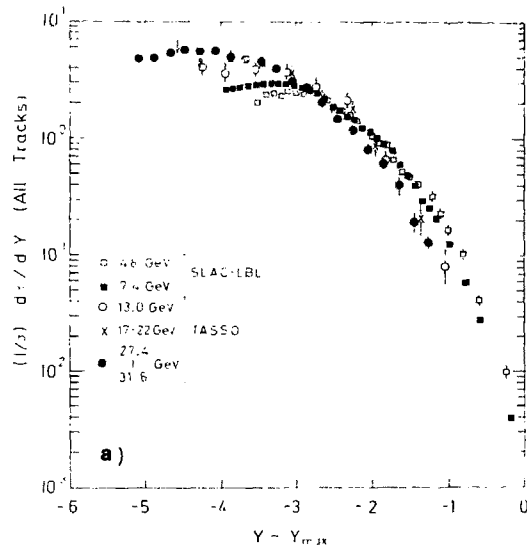
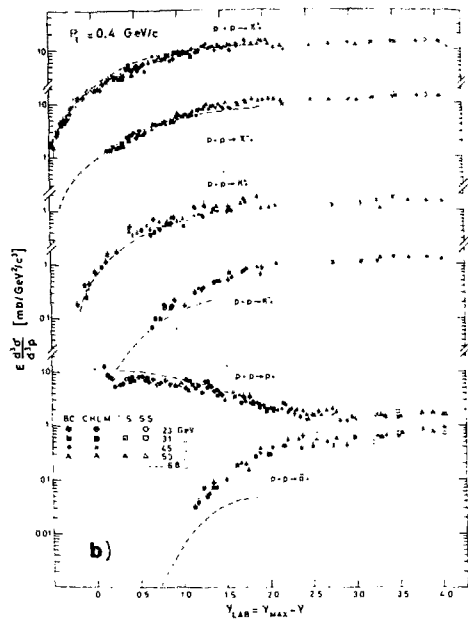


Fig. 5 The rapidity plateau. Rapidity distributions are shown as a function of $y - y_{\max}$, $y_{\max} \sim \frac{1}{2} \ln s$ being the highest value kinematically allowed for :

a) different types of secondary particles at ISR energies,

b) charged particles in e^+e^- annihilation (SPEAR and PETRA).

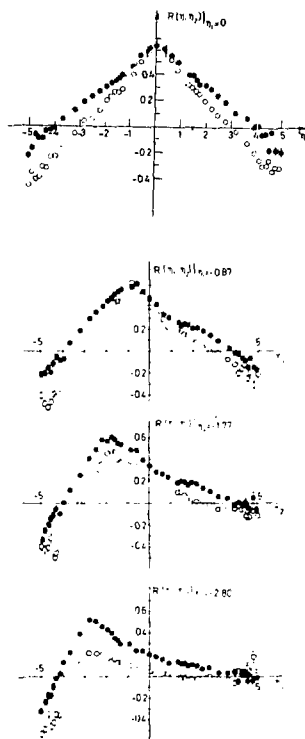


Fig. 6 Rapidity correlations among charged particles. $R(\eta_1, \eta_2)$ for different values of η_1 . Results from R801. One here uses the pseudorapidity $\eta = -\ln(p_L/p_T)$ as opposed to the rapidity $y = -\ln(E+p_L/E-p_L)$. Observing a particle enhances the probability of observing another one at the same rapidity. Features hardly change between $\sqrt{s} = 23$ GeV (open dots) and $\sqrt{s} = 63$ GeV (full dots).

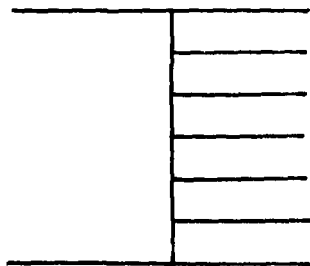


Fig. 7 Schematic view of a multiperipheral chain. In view of the observed short-range correlations (Fig. 6) it should refer to clusters of particles as opposed to individual particles.

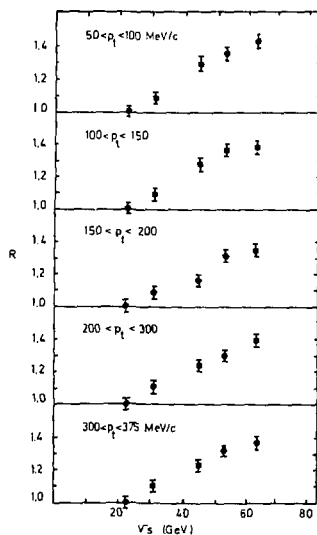


Fig. 8 The rise of the central rapidity density. The values are normalized to one at $\sqrt{s} = 23$ GeV. Results from R803.

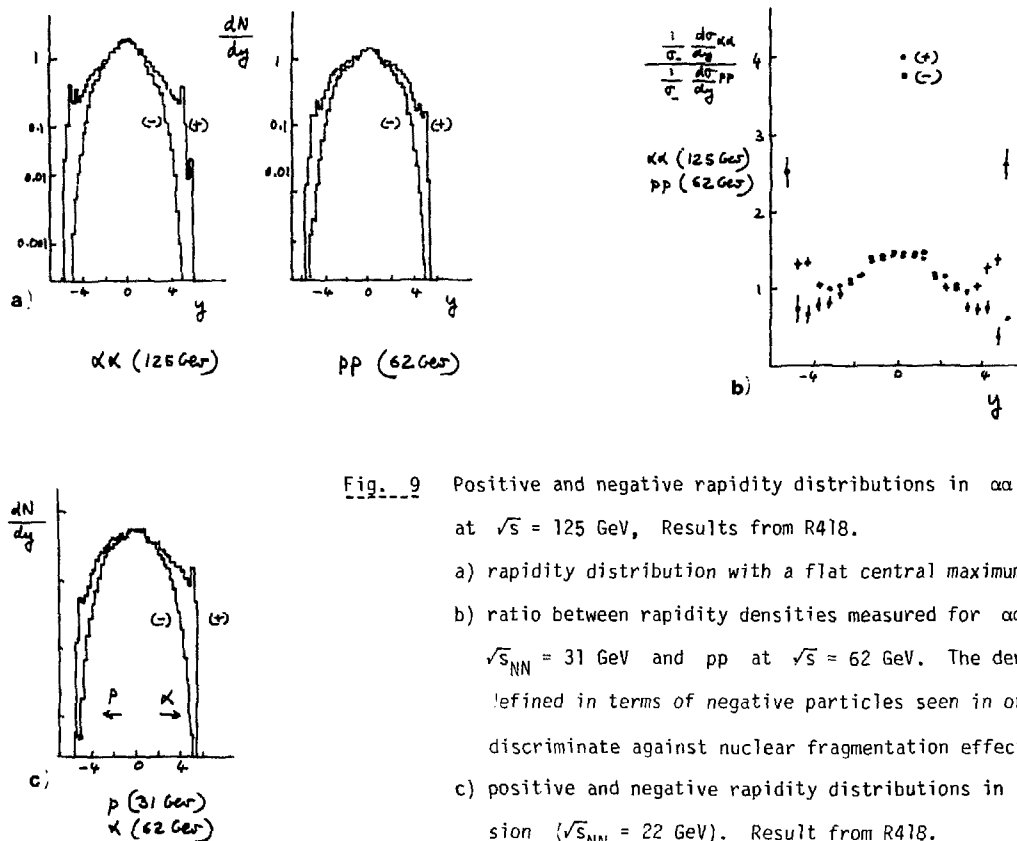


Fig. 9 Positive and negative rapidity distributions in α collisions at $\sqrt{s} = 125$ GeV, Results from R418.

a) rapidity distribution with a flat central maximum.

b) ratio between rapidity densities measured for αK at $\sqrt{s}_{NN} = 31$ GeV and pp at $\sqrt{s} = 62$ GeV. The densities are defined in terms of negative particles seen in order to discriminate against nuclear fragmentation effects.

c) positive and negative rapidity distributions in αp collision ($\sqrt{s}_{NN} = 22$ GeV). Result from R418.

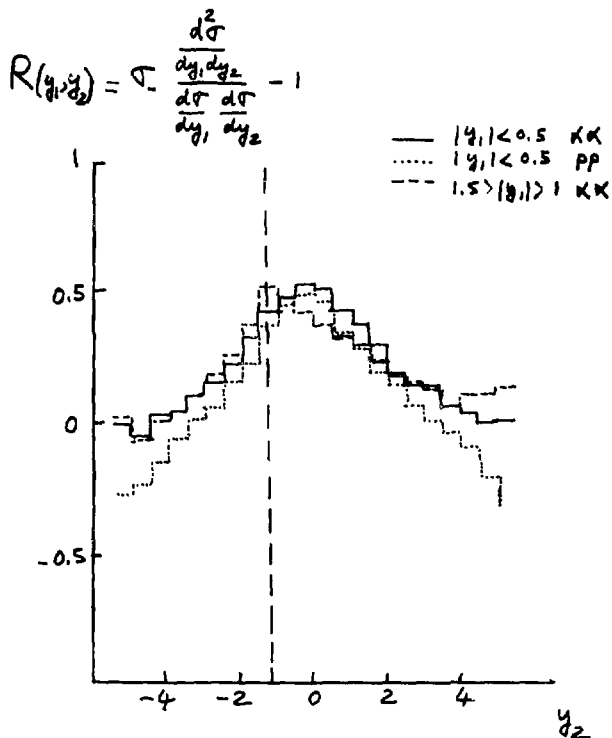


Fig. 10 Charge-charge correlation function in $\alpha\alpha$ collisions. $R(y_1, y_2)$ is shown as a function of y_2 for $|y_1| < 0.5$ and $1.5 < |y_1| < 1$. Also shown is the correlation function measured for pp collisions ($|y_1| < 0.5$) at the same energy per nucleon. Results from R418.

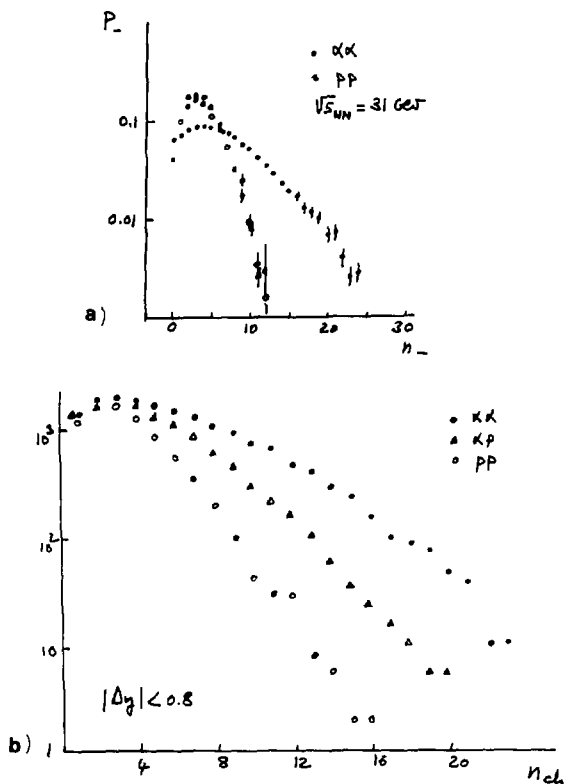


Fig. 11 Multiplicity distributions observed in $\alpha\alpha$ collisions and in pp collisions at the same momentum per nucleon.

a) negative global multiplicity distributions ; streamer chamber and SFM experiments, R418 for $\alpha\alpha$.

b) charge multiplicity in the central region ($|\theta| > 30^\circ$), R807 for pp , $p\alpha$ and $\alpha\alpha$. Each distribution is not separately normalized as done for (a). Notice the change in shape.

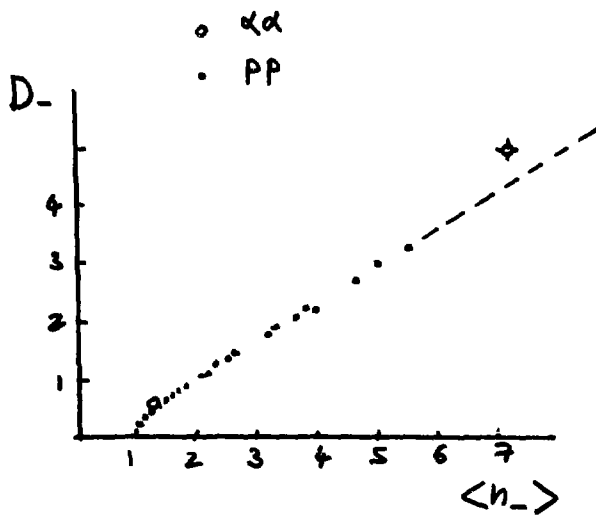


Fig. 12 The dispersion as a function of the mean multiplicity for negatives. pp data with two $\alpha\alpha$ points including the ISR point (R418).

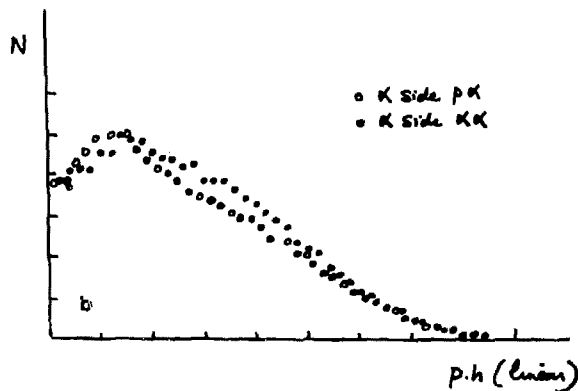
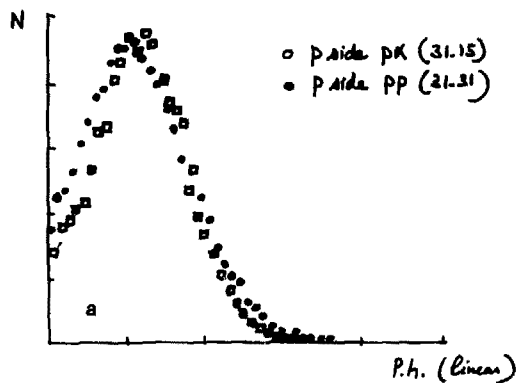


Fig. 13 Comparative value of the energy distribution (pulse height) collected in a forward calorimeter surrounding the beam pipes, as measured by R807.

- a) proton fragmentation when colliding with a proton (31-31) (square) and when colliding with an α (31-15) (dots). There is no appreciable difference.
- b) α fragmentation when colliding with a proton (open dots) and with an α (full dots). There is a slight excess in the latter case.

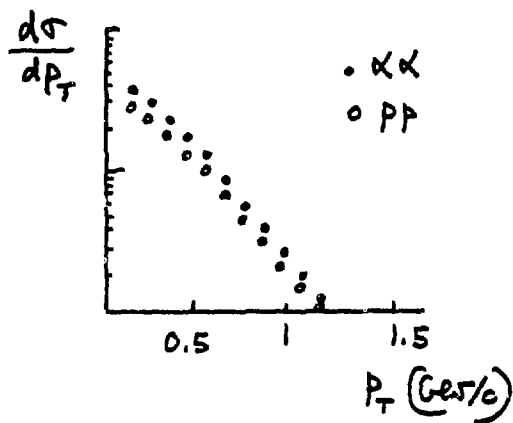


Fig. 14 p_T distributions in $\alpha\alpha$ and pp collisions normalized to the production cross-sections. There is no significant difference at low p_T , except for an over-all scaling. Results from R418.

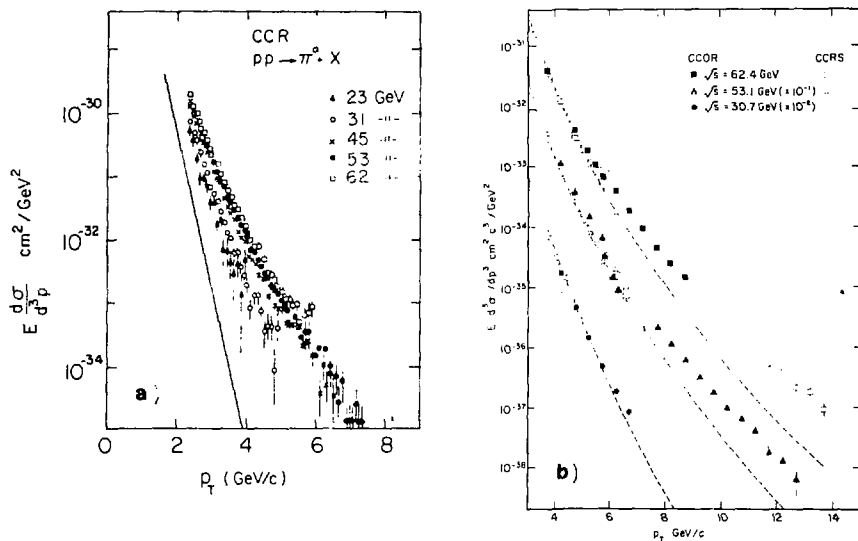


Fig. 15 Large p_T yields for π^0 at ISR energy.

- a) results from R103, showing how the production rates depart from the extrapolation of the exponentially decreasing low p_T yield (solid line). The production rates behave as $p_T^{-8} f(x_T)$ in the $2 < p_T < 8$ GeV/c range.
- b) results from R108, showing how the observed rates eventually depart from the $p_T^{-8} f(x_T)$ parametrization at medium p_T (dashed curves), in general agreement with expectation from perturbative QCD.

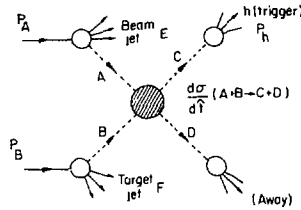


Fig. 16 Schematic view of a hard scattering process at the origin of a large p_T hadron.

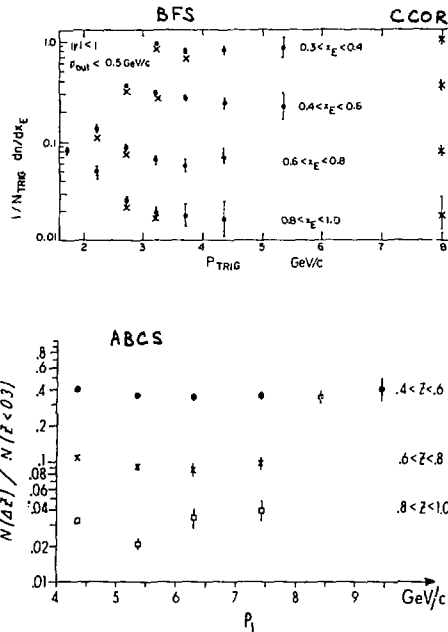


Fig. 17 Scaling in jet fragmentation. The particle yields for fixed values of x_e (fraction of the trigger momentum) are found to be independent of the momentum of the trigger particle. Results from R413, R108 and R806.

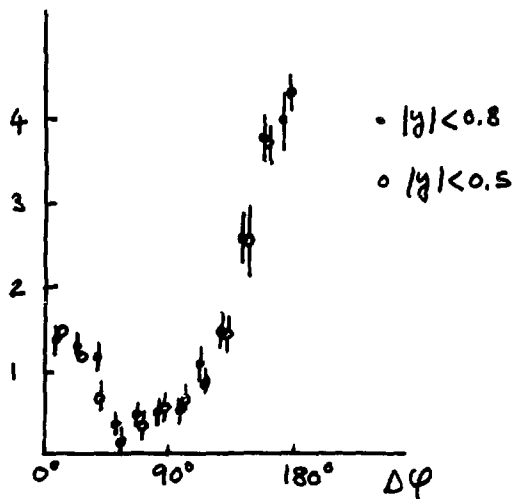


Fig. 18 Azimuthal distribution of particle with $p_T > 0.5$ GeV/c associated with a trigger particle at $\phi = 0$ and $p_T > 3$ GeV/c. This shows a clear planar configuration.

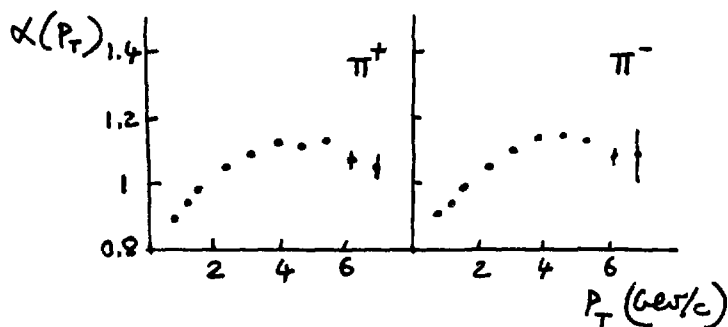


Fig. 19 Effective value of α in an A^α parametrization of pion production on nuclear targets as a function of p_T .

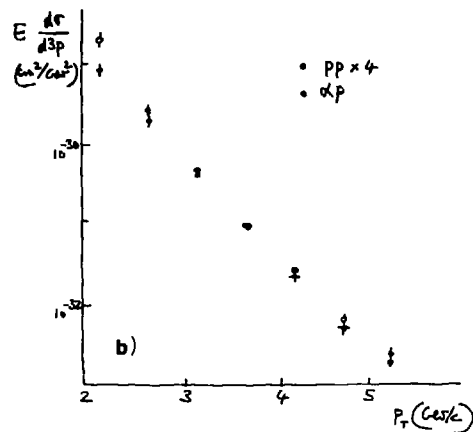
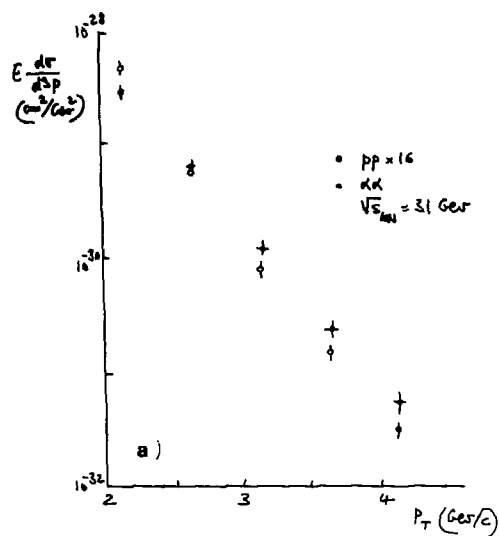


Fig. 20 Large p_T π^0 yields measured by R806.

- a) inclusive distribution in $\alpha\alpha$ collisions and 16 times the inclusive distribution in pp collisions at the same energy per nucleon.
- b) inclusive distribution in αp collisions and 4 times the inclusive distribution in pp collisions at the same energy per nucleon.

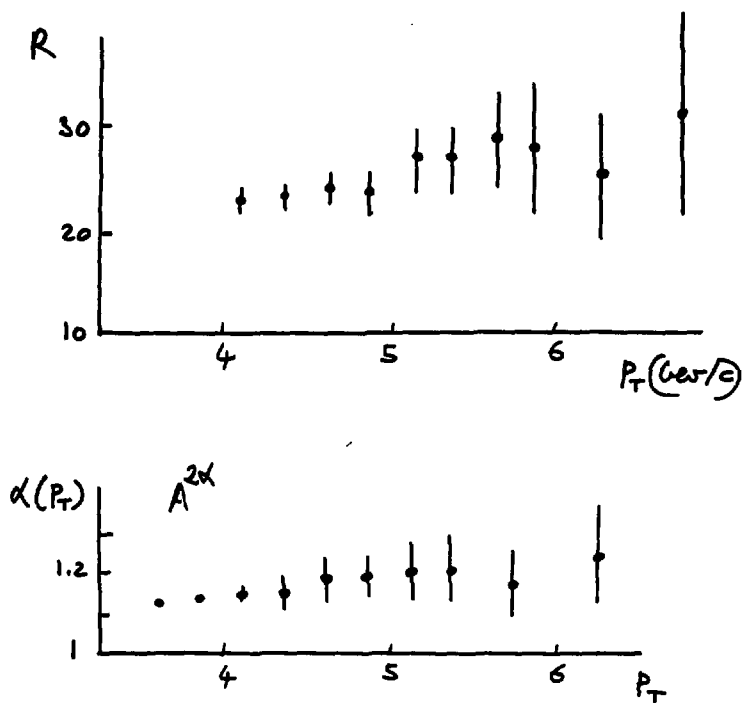


Fig. 21 Ratio between the π^0 inclusive yields in $\alpha\alpha$ and pp collisions at the same energy per nucleon (15 GeV). Results from R110.

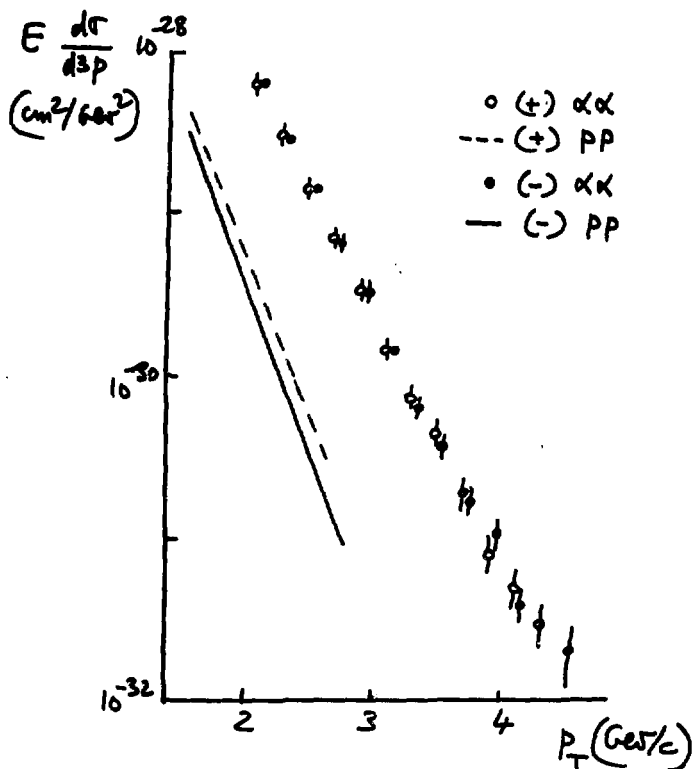


Fig. 22 Inclusive distribution for positive (a) and negative (b) particles as measured by R418. The mean rapidities are 0.83 and 0.62, respectively. Also shown are previous data from R203 at $y = 0.6$ at the same energy per nucleon. At $p_T \sim 2.5 \text{ GeV}/c$ the inclusive yield drops by a factor of 2 from $y = 0$ to $y = 0.6$.

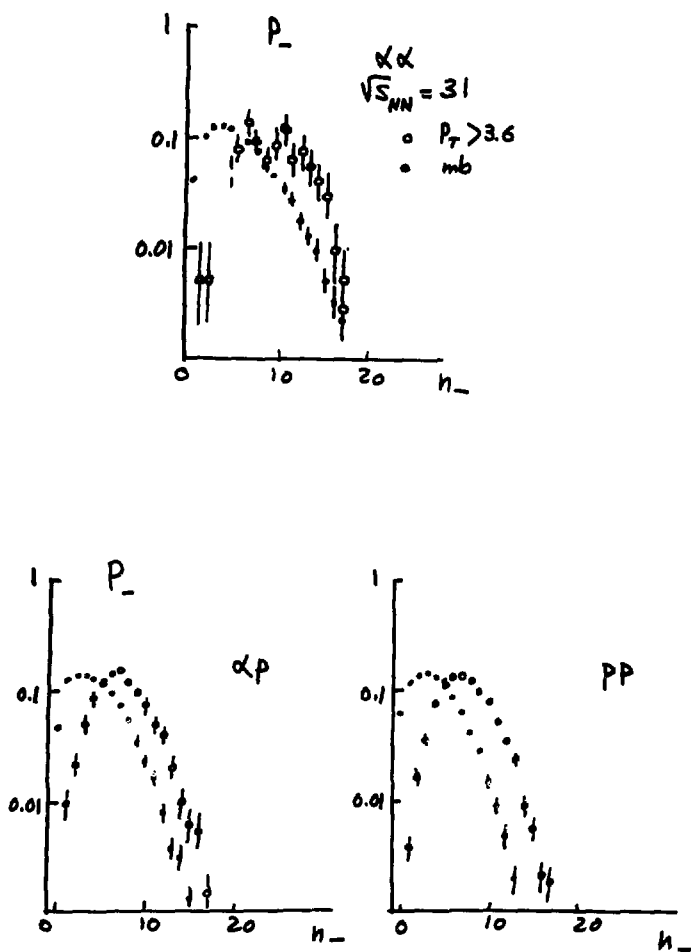


Fig. 23 Negative multiplicity distributions in $\alpha\alpha$, αp and pp collisions in the minimum bias case and in reactions triggered by a particle with $p_T > 3.6$ GeV/c. Results from R418.

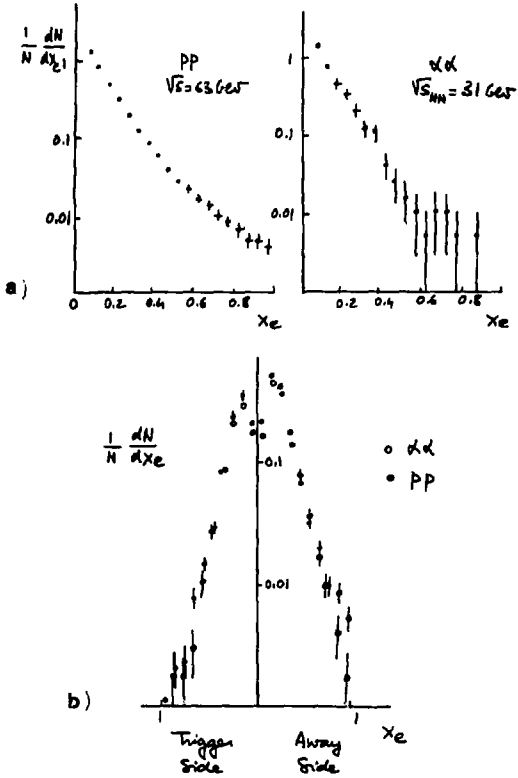


Fig. 24 The x_e distribution of particles associated with a large p_T trigger.

a) away side distributions in $\alpha\alpha$ and pp collisions at the same energy per nucleon. Results from R418.

b) same side and away side distribution in $\alpha\alpha$ and pp results at $\sqrt{s}_{NN} = 31$ and 63 GeV , respectively. Results from R807.

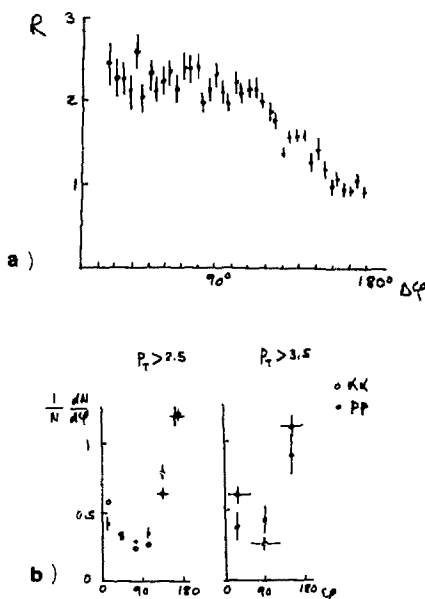


Fig. 25 Azimuthal distribution of associated secondaries to a large p_T trigger particle, normalized to the distribution observed in pp collisions (Fig. 18).

- a) azimuthal distribution of particles with $p_T > 1$ GeV/c (trigger momentum of 2.5 GeV/c). Ratio at the same energy per nucleon. Results from R110.
- b) azimuthal distribution of particles with $p_T > 1$ GeV/c (trigger momentum at 2.5 and 3.5 GeV/c). Comparison between $\alpha\alpha$ at $\sqrt{s}_{NN} = 31$ GeV and pp at $\sqrt{s} = 63$ GeV. Results from R807.

Comparable yields are observed at $\phi = 180^\circ$. Larger yields are observed at 90° in the $\alpha\alpha$ case.

INCLUSIVE SINGLE NEGATIVE PION PRODUCTION AT FORWARD ANGLES IN RELATIVISTIC NUCLEUS-NUCLEUS COLLISIONS

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We present spectra of negative pions produced at angles from 0° to 12° (lab) from collisions of 1.05 and 2.1 GeV/N protons, deuterons, alpha particles, and carbon nuclei with targets of C, Cu, Pb, and H (from a CH_2 -C subtraction). For alpha and carbon projectiles, pions are observed with nearly twice the kinetic energy per nucleon of the beam. The cross section per projectile nucleon as a function of kinetic energy gives evidence for a collective effect in the production of fast forward pions, or of a high momentum component in the internal momentum distribution in carbon which exceeds that in the alpha particle. Whereas the proton beam data exhibit scaling, the composite projectiles show a violation of scaling which increases with projectile mass. We present fits of the X_F and T_π dependence of the data, and the transverse momentum distributions, target and projectile mass dependence, and comparison with some proposed models.

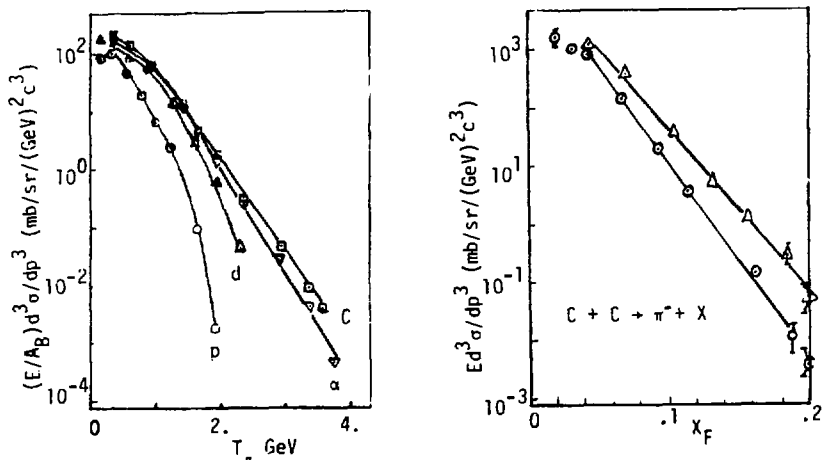


Fig. 1. (left): The Lorentz invariant cross section per beam nucleon versus the pion kinetic energy for 2.1 GeV/N p, d, α , C + C $\rightarrow \pi^- + X$ at 0° .

Fig. 2. (right): The Lorentz invariant cross section versus the Feynman scaling variable $X_F = p_{\text{cm}}/p_{\text{cm}}^{\text{max}}$ for 1.05 (triangles) and 2.1 (circles) GeV/N C + C $\rightarrow \pi^- + X$ at 0° .

This work was supported by the U.S. Department of Energy under contract number W-7405-ENG-48.

⁸HE PRODUCTION IN COLLISIONS OF 1.05 GEV/A CARBON WITH VARIOUS TARGETS IN THE BEAM FRAGMENTATION REGION

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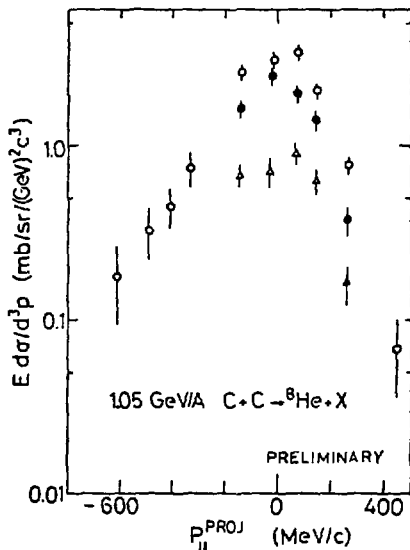
Lawrence Berkeley Laboratory, University of California, Berkeley.

We present data on single particle inclusive production of Helium isotopes, in particular ⁸He, in collisions of 1.05 GeV/A carbons with C, Cu and Pb targets (Bevalac experiment E205H). Positive fragments were measured in the region $0.5 \leq p_{\perp}^{\text{lab}}/Z \leq 8.0 \text{ GeV}/c$ and $0^\circ \leq \theta^{\text{lab}} \leq 12^\circ$.

As an example we show in the figure the invariant cross section of ⁸He vs. the longitudinal momentum in the projectile frame for three different transverse momenta. If the data at $p_T = 0 \text{ GeV}/c$ and $|p_{\perp}^{\text{proj}}| < 300 \text{ MeV}/c$ are fitted with a Gaussian the standard deviation is $\approx 170 \text{ MeV}/c$.

Comparison with ³He, ⁴He and ⁶He production under various kinematical conditions will be made. Target dependence will be discussed.

Fig. : The Lorentz invariant cross section vs. the longitudinal momentum in the projectile frame for 1.05 GeV/A C + C → ⁸He + X at $p_T = 0$ (○), 0.15 (●) and 0.3 (▲) GeV/c.



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Preequilibrium in relativistic heavy ion collisions

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During the past few years, several macroscopic (and microscopic) models have been proposed to interpret the experimental results obtained from relativistic heavy ion collisions. The macroscopic models¹, namely, abrasion-ablation, fireball-firestreak, hot spots, shock waves etc., expound, as a rule, the philosophy of collective behaviour although differing in details. It is conceivable that all these models contain some common elements of truth and at the present time, it is quite important to understand the interrelation between these models. This paper is an attempt in this direction. Essentially, we investigate theoretically the fate of the heavy spectator (defined in accordance to the original fireball model²) while it evolves toward equilibrium. In particular, we are interested in a configuration where the spectator and participant components, due to their large rapidity separation, may be distinguished and they evolve toward equilibrium on their own without contaminating each other. Such a situation is schematically depicted in fig. 1.

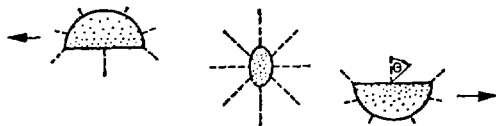


Fig. 1 Clean cut heavy ion collision leading to separated projectile and target fragments and a central fireball. The intensity of particle emission (dashed lines) depends on the value of the local temperature (dotted area).

Limiting Fragmentation in the Venus Regime[§]

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In 1972, Heckman et al.¹ introduced the concept of limiting fragmentation, known hitherto in elementary particle physics, to interpret their first heavy ion data from Bevatron. Soon afterwards, this led to the formulation of the abrasion-ablation model² followed by a multitude of models based on thermodynamic and/or hydrodynamic concepts.

In the present work, we investigate a plausible mechanism for limiting fragmentation in relativistic heavy ion collisions by looking at the fate of the target and projectile fragments after they have separated from the highly excited intermediate system of participants. For participants and spectators, we use the standard classification scheme (see, for example, Ref. 3).

Evidence from cosmic ray physics as well as heavy ion data^{4,5} at 2 GeV/A suggests that in the inclusive light particle spectra, one finds two characteristic inverse slopes of 10 MeV and 40 MeV. We associate the lower inverse slope with the temperature of the spectator fragments and the higher one with a temperature of the participants. This identification is quite natural in the hydrodynamical model which takes into account the dissipation of energy through the nuclear transport phenomena.⁶ Our approach⁷ differs from that of Feshbach and Huang⁶ and Goldhaber⁷ through the fact that it could eventually lead to a unified description of both participant and spectator nucleons. The essential point in our analysis is the formation of a local thermally equilibrated region in the spectator fragments which later evolves to overall equilibrium and this space-time evolution could leave a characteristic signature in the spectrum of the spectator fragmentation particles.

We shall, in the following, be concerned with ultra-relativistic heavy ion collisions so that the projectile and the target fragments as well as the participant fragments are well separated from one another in rapidity space. This may allow an observer to investigate the fragments from one of them without any contamination from the other two.

In the equal velocity frame, the participant nucleons

[§] Work supported in part by BMFT, DFG and GSI Darmstadt.

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in the colliding heavy ions may almost stop each other whereas the spectators continue with practically undiminished speeds along their original paths. This results in separating the participants from the spectators (see fig. 1). In the spirit of fluid dynamics, we can then postulate the following scenario. During the separation, the relative velocity between the participant part and the spectator part changes from its maximum value to zero over a finite thickness of the nuclear matter. If the nuclear matter is a viscous fluid, this would generate a friction force which tends to oppose this shear and the work done against this friction force manifests itself as internal thermal energy over the surface of separation upto a finite depth h . This internal energy later equilibrates the whole spectator fragment and causes the spectator fragmentation. Although h is not a calculable parameter of the theory, a reasonable guess for it would be the internucleonic distance.

For a viscous fluid, the relativistic dissipative stress energy tensor may be written as:

$$\tau_{ik} = -\eta \left(\frac{\partial u_i}{\partial x^k} + \frac{\partial u_k}{\partial x^i} + u_i u^l \frac{\partial u_l}{\partial x^k} + u_k u^l \frac{\partial u_l}{\partial x^i} \right) - \left(\int - \frac{\partial}{\partial x^l} \eta \right) \frac{\partial u_l}{\partial x^i} (g_{ik} + u_i u_k) \quad (1)$$

The u_i 's are the 4-velocities with indices ranging from 0 to 3, η is the coefficient of dynamic viscosity and $\int$ is the coefficient of bulk viscosity. By a suitable choice of coordinates, we orient the shearing plane parallel to the xz -plane so that a velocity gradient occurs only in the y -direction and the x -axis is the direction of fluid flow so that $u_y = u_z = 0$. Under the simplifying assumption that the velocity gradient is constant, we get

$$\tau_{xy} = -\eta \gamma \frac{v_x}{h} \quad (2)$$

where γ is the usual Lorentz factor corresponding to v_x ; τ_{xy} is the tangential friction force per unit area. Hence, we can calculate the total work done against this friction force by computing the area of contact between the participant part and the spectator part of a nucleus and the distance the participant has to travel across the spectator in the rest frame of the spectator during the shear. The area of contact has one length component along the direction of motion and thus suffers a Lorentz contraction. The spectator part being at rest, the crossing distance is not Lorentz contracted. Denoting the area of contact by S' ($=S/\gamma$), where S is the area of the spectator at the shearing surface and the crossing distance by D ($=2\sqrt{(S/\pi)}$), we get for the total work done against the friction force:

$$W_{\text{excited}} = \eta \gamma \frac{v_x S}{h} D = \eta \frac{v_x S D}{h} \quad (3)$$

This energy manifests itself as an internal thermal energy confined within a depth h from the surface in the spectator. Therefore the volume of the excited region is $\sim Sh$ and the number of excited nucleons is $n_e = n_s Sh$ where n_s ($\sim 0.18/\text{fm}^3$) is the number density of normal nuclear matter. Then the internal energy per nucleon is

$$\omega = \frac{W_{\text{excited}}}{n_e} = \eta \frac{v_x D}{h^2 n_0} \quad (4)$$

D is a function of the impact parameter and the sizes of the colliding nuclei. It has an upper limit of $2R$. v_x is also limited to the speed of light. So for a limiting value of η , ω tends to a limiting value which corresponds to, via an equation of state, a limiting temperature. We postulate a Fermi gas equation of state for nuclear matter which gives

$$\omega = a T^2 \quad (5)$$

with $a \approx 0.1 \text{ MeV}^{-1}$. Identifying T with the experimental number 10 MeV, we can calculate η which is found to be about $10^{-23} \text{ MeV} \cdot \text{sec}/\text{fm}^3$, for $h \sim 1 \text{ fm}$, D of the order of the diameter of a Pb^{207} nucleus and $v_x \sim 1$. Although this number is not to be taken seriously, we find it interesting that η from such an estimate should come out to be of the same order of magnitude as the value calculated for a Fermi gas model or the estimates from fission data¹⁰.

We can then look at the space-time development of this localized hot layer in the spectator through a heat diffusion mechanism. The problem involves a numerical calculation the details of which we omit here. A description of the equation solved and the numerical techniques may be found in Ref. 11. We would only present the results from a sample calculation for a collision of two equal Pb^{207} ions at an impact parameter of $b=R$ (where R is the radius of the nucleus) and the spectator parts are approximated by half spheres. The emission of nucleons is assumed to occur only from the nuclear surface and to be of the black body radiation type. In fig. 2, the nucleon differential cross-sections from one spectator are shown as a function of E of the emitted nucleon for several values of θ as defined in fig. 1. The effective temperatures range from 5 MeV to 10 MeV for the different angles for $E < 50 \text{ MeV}$. Above $E \approx 150 \text{ MeV}$, the effective temperature at all angles is equal to the initial temperature of 10 MeV. When the emission spectra of both the spectators are measured, the sign of the asymmetry between them should reverse. In this way, an "up-down" direction is defined.

If the coefficient of thermal conductivity for nuclear matter is not too large, the "up-down" asymmetry characterizes such a locally equilibrated excitation (hot spot). While many other models explain the phenomenon of nuclear limiting fragmentation, the hydrodynamic scenario discussed above has these specific features and future experiments may elucidate the situation by looking for such asymmetries.

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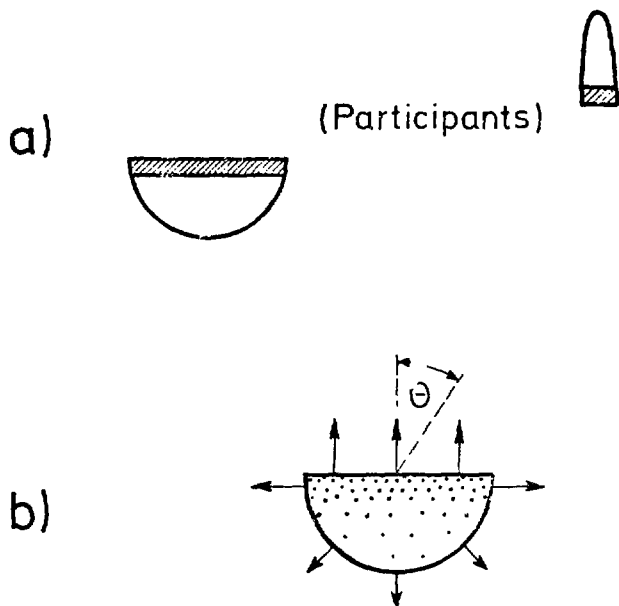


Fig.1

- a) Schematic representation of the fragmentation processes with clean cut geometry in the frame of one fragment.
- b) Decay of the local excitation into vacuum and body of spectator fragment seen in its rest frame.

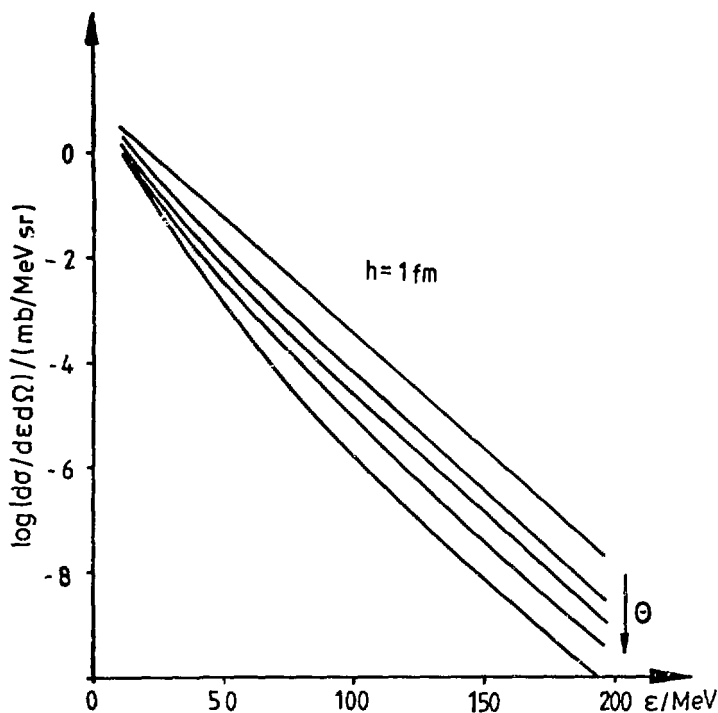


Fig. 2 Differential cross section $d^2\sigma/d\epsilon d\Omega$ as function of the kinetic energy ϵ of the emitted nucleons for detector angles $\theta = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ$ and $h = 1 fm$. θ is defined in Fig. 1.

PRODUCTION OF CHARGED HADRONS AT LARGE TRANSVERSE MOMENTUM
IN $\alpha\alpha$ AND $p\alpha$ COLLISIONS AT THE ISR

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ABSTRACT

The production of charged hadrons with high transverse momentum (p_T between 2 and 4.5 GeV/c) has been measured in $\alpha\alpha$ and $p\alpha$ collisions at nucleon-nucleon center-of-mass energies ($\sqrt{s_{NN}}$) of 31 and 44 GeV, respectively. The invariant cross sections $d^2\sigma/\pi dy dp_T^2$ are considerably larger in $\alpha\alpha$ collisions than A^2 ($=16$) times the cross sections in pp collisions. They exhibit a rapidity (y) dependence in $p\alpha$ collisions which contrasts to the rapidity independence observed in pp interactions, at equal $\sqrt{s_{NN}}$ and in the same p_T and y range. Various distributions of charged particles associated with the large p_T trigger track were studied and compared in $\alpha\alpha$, $p\alpha$ and pp collisions in order to understand the role of the nuclear target in large p_T particle production and, in particular, to pin down the mechanism responsible for the enhancement of the inclusive cross sections.

Interaction and Collective Effects in Classical-Equations-
of-Motion Calculations

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We discuss results obtained with the classical-equations-of-motion (CEOM) approach (ref. 1), with particular reference to interaction (potential energy) and collective effects in central collisions of equal mass nuclei. The essence of the CEOM approach is the classical calculation of all $A = A_P + A_T$ trajectories using a 2-body potential V between all pairs of nucleons; $V = V_{\text{short}} + V_{\text{long}}$ has a short range repulsion and a longer range attractive tail. In contrast to hydrodynamics, the CEOM approach is microscopic and includes transparency and nonequilibrium effects. In the hydrodynamic limit it corresponds to a dense fluid with a non ideal-gas equation of state and with transport coefficients which are not directly related to the 2-nucleon cross section as for a dilute (Boltzmann) gas appropriate to the hydrodynamic limit of cascade calculations. The latter which are also microscopic and which include

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nonequilibrium effects through collisions, correspond to an ideal gas equation of state and do not include potential energy effects.

In the CEOM calculations, many nucleons in general interact simultaneously, especially for dense conditions. Thus strictly one cannot speak about collisions as in cascade calculations. Nevertheless the evolution of the momentum distributions clearly shows evidence of single and multiple "collisions" (ref. 1). At first, single "collisions" dominate, followed during conditions of large overlap by the rapid onset of multiple "collisions." Our interpretation of these "collisions" is that they are mostly due to V_{short} . Furthermore, our calculations show evidence that the longer range attraction V_{long} is mostly effective, especially for dense conditions, via the corresponding average single-particle potential $\bar{V}(\vec{x}) = \int V_{\text{long}}(|\vec{x}-\vec{x}'|) \rho(\vec{x}') d^3x'$.

We discuss interaction and collective effects in the c.m. system for central ($b=0$) collisions of $^{20}\text{Ne} + ^{20}\text{Ne}$ at $E_{\text{lab}} = 400$ and 800 MeV/n and for $^{40}\text{Ca} + ^{40}\text{Ca}$ at 400 MeV/n . The table shows results, at several collisions times t^* , for the momentum and position (beam to transverse beam direction) asymmetries $\omega_p = \langle p_{\parallel}^2 / p_{\perp}^2 \rangle^{1/2}$, $\omega_x = \langle z^2 / \frac{1}{2}(x^2 + y^2) \rangle^{1/2}$, the density ratio ρ/ρ_0 within a radius of $2f_m$ and the average density ratio $\bar{\rho}/\rho_0$, where $\bar{\rho} = A/(4\pi/3) R^3$ with $R = 5/3 \langle r^2 \rangle^{1/2}$. The time $t^* = t$ for 400 MeV but is reduced with respect to the ratio of beam velocities for 800 MeV ($t^* = 1.41 t$). If there was perfect scaling with the beam velocity then the results for different E_{lab} at the same t^* would be the same. $\omega_p, \omega_x > 1$ and < 1 for elongation and contraction in the beam direction, respectively.

Initially ($t^* \leq 6$ fm/c) there is dominance of single "collisions" up to quite large overlap. The large initial value of ω_p (determined by ratio of beam to Fermi momentum) is rapidly reduced by a factor ~ 2 but is still appreciably larger than unity at $t^* \sim 6-7$ fm/c, whereas the position distribution (characterized by ω_x) becomes much less elongated in the beam direction ($\omega_x \sim 1.2$). Subsequently ($7 \leq t^* \leq 10$ fm/c), corresponding to large overlap, there is rapid development of multiple scattering leading to approximate thermalization and momentum isotropy ($\omega_p \sim 1-1.3$) but to a position distribution contracted along the beam ($\omega_x \sim 0.7$). The density is maximum at $t^* \sim 9$ fm/c, near the end of this stage. Both these stages are not equilibrated, and especially the initial stage, cannot be described by hydrodynamics. For $9 \leq t^* \leq 13$ fm/c the approximately thermalized system expands with collisions effective, followed by a final expansion for which collisions are relatively ineffective and which is dominated by streaming in the average potential $\bar{V}$.

Collective effects and transparency. The progression Ne + Ne (800 MeV), Ne + Ne (400 MeV), Ca + Ca (400 MeV) shows increasingly rapid thermalization with final values of ω_p which decrease from ~ 1.08 , corresponding to forward peaking, to ~ 0.88 (for Ca + Ca), corresponding to sideways peaking. This trend is consistent with decreasing transparency as measured by Λ/L , where Λ is the mean free path and $L = O(R)$. (The NN cross section increases with decreasing energy.) The sideways peaking seen for Ca + Ca develops during $9 \leq t^* \leq 13$ fm/c. For Ne + Ne at the same energy (400 MeV) there is still some slight residual forward

peaking ($\omega_p \approx 1.02$). Euler hydrodynamics which shows sideways peaking (ref. 2) would show no difference between these two cases. Thus our calculations show clear indication of the onset of collective (hydrodynamic) effects with these apparent for Ca + Ca, but not yet clearly developed for Ne + Ne. Cascade calculations show no sideways peaking (ref. 2), although transparency effects are of course present. We then interpret the CEOM results as an interplay between transparency effects and interaction effects due to the repulsive core of the 2-body potential. First we note that well before approximate thermalization is attained ($t^* \leq 10$ fm/c) the position distribution is contracted in the beam direction whereas ω_p is still significantly larger than unity. This is because the nucleons at the "front" end of the projectile or target are the first to collide and to be slowed down, whereas the remaining "rear end" nucleons remain closer to the beam velocity. For Ne + Ne, especially at 800 MeV, the transparency is sufficiently large that there is some persistence of momentum in the beam direction even at $t^* \geq 10$ fm/c and conditions approach those necessary for hydrodynamic flow. There will then be a tendency for nucleons in the interior to be forced out sideways because of pressure from the nucleons in the rear which on average are still moving in the beam direction. In the CEOM calculations this is both because of the kinetic pressure and the potential energy contributions to the pressure due to the repulsive cores, whereas in the cascade calculations only the former contributes and is presumably inadequate by itself to give sideways flow. In this connection it would be of interest to make hydrodynamic calculations with an

ideal gas equation of state corresponding to the hydrodynamic limit of cascade calculations.

Expansion; collisions and average potential energy. At $t = 0$ the momentum distribution is peaked at the beam momentum with a spread due to the initial Fermi motion. Near the beginning of the expansion stage ($t^* \approx 10$ fm/c) the now approximately isotropic momentum distribution still has a peak (especially for Ne + Ne at 800 MeV) but at a considerably lower energy because of single and multiple scattering (ref. 1). As the expansion proceeds this peak moves inward to smaller momenta, approximately in step with the increase in potential energy, the kinetic energy of the peak nucleons decreasing as these move "up hill" in the average (attractive) potential $\bar{V}$. Concurrently, especially during the early expansion phase ($9 \leq t^* \leq 13$ fm/c), there is some increase in thermalization and presumably an increase in entropy. Only for Ne + Ne at 800 MeV, for which Λ/L is greatest, does the peak persist to $t = \infty$, and collisions have effectively ceased by 12 fm/c. For the other two cases, the final momentum distribution is approximately thermalized (apart from the sideways peaking for Ca + Ca) and the early expansion phase, during which there is approximate thermalization and during which collisions are still effective, thus lasts about 3-4 fm/c and terminates when $\bar{\rho} \approx 0.6\rho_0$. This is the phase which corresponds most closely to hydrodynamic conditions and during which sideways peaking develops for Ca + Ca.

The final expansion for $t^* \geq 13$ fm/c seems to involve mainly screaming in the average potential $\bar{V}$, with collisions playing a minor role. Most of the changes in the momentum distribution are then due to

the effect of $\bar{V}$. This situation is very clearly indicated for Ne + Ne at 800 MeV (ref. 1). An indication of this dominance of streaming is that ω_p has approximately attained its final values for $t^* \geq 13$ fm/c whereas ω_x is still increasing and approaching ω_p , corresponding to the build up of position and momentum correlations, with the (angular) position distribution eventually approaching that of the momentum distribution. During this final expansion phase the entropy is expected to be constant, appropriate to negligible collisions and to single particle Liouville streaming in an average single particle potential. At no stage of the expansion do we see any evidence for blast wave behavior (ref. 3) where a peak builds up and moves outward to larger momenta during the expansion.

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		Table						
τ^* (fm/c)		0	5	7	8	10	12	14
Ne+Ne 800 MeV/n	ω_p	5.54	4.4	2.7	2.05	1.25	1.12	1.09
	ω_x		1.81	1.21	0.95	0.68	0.70	0.76
	ρ/ρ_0		1.3	2.4	2.6	2.5	1.5	0.9
	$\bar{\rho}/\rho_0$		0.9	1.8	2.1	1.8	0.8	0.4
Ne+Ne 400 MeV/n	ω_p	3.98	3.27	2.03	1.61	1.14	1.05	1.04
	ω_x		1.88	1.23	0.96	0.71	0.71	0.75
	ρ/ρ_0		1.4	2.2	2.35	2.0	1.3	0.85
	$\bar{\rho}/\rho_0$		0.95	1.8	1.8	1.3	0.6	0.3
Ca+Ca 400 MeV/n	ω_p	4.02	2.57	1.65	1.34	1.02	0.91	0.88
	ω_x		1.49	1.10	0.88	0.70	0.67	0.68
	ρ/ρ_0		1.4	1.65	1.6	1.3	1.0	0.7
	$\bar{\rho}/\rho_0$		1.3	1.8	1.9	1.4	0.8	0.45

Huf

"CENTRAL" NEEDS NOT MEAN "COLLECTIVE" *

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ABSTRACT

Spectra of high momentum protons from high energy nuclear collisions are analyzed in terms of a simple statistical model: independently interacting groups of nucleons contribute incoherently. The nucleon momentum distributions within one group follow phase space. It can be shown that

- 1) the spectra can clearly be decomposed into spectator and participant contributions,
- 2) the average size of the nucleon groups is very insensitive to impact parameter restrictions.

The latter finding suggests that (at least for the high momentum tails of measured spectra) central collisions do not necessarily enhance the chance for seeing indications of effects due to coherence in the interaction of a larger number of particles ("collective effects").

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The statistical model of high energy nuclear collisions as introduced by J. Knoll [1] has proven to be a valuable tool for studying the high momentum parts of inclusive spectra [2]. Its main ingredients are:

- 1) geometrically defined groups of nucleons interact independently from each other
- 2) momentum distributions within one group are determined by phase space.

Formally, the one-nucleon inclusive cross section is written as an incoherent sum over contributions from ensembles containing M nucleons of the projectile A and N nucleons of the target B:

$$E \frac{d^3\sigma}{dp^3} = \sum_{M,N} (M+N) \sigma_{AB}(M,N) F_{MN}(\vec{p}) \quad (1)$$

Each contribution is factorized into a formation cross section $\sigma_{AB}(M,N)$ (determined by geometry) and a momentum distribution $F_{MN}(\vec{p})$ (determined by phase space). The factor (M+N) accounts for the relative weight of the different ensemble sizes. - Further details of the model are given in [1,2].

There is a number of implications which can be derived from the apparent success of this model explaining a large amount of experimental data. These will be discussed elsewhere [3]. Here we want to concentrate on some interesting consequences derived from the comparison between model calculations and measured proton spectra from high energy nuclear collisions in kinematical regions inaccessible to single nucleon-nucleon collisions. Such spectra (protons being scattered to high momenta at 180° laboratory angle) have been published recently [4]; fig.1 shows an example. Obviously, there are two different components contributing to the spectrum. One possible interpretation is that this structure is a reflection of the subdivision of the colliding nuclei into participant (geometrically overlapping) and spectator parts; a widespread concept among reaction dynamical models. The statistical model as it is designed describes participants only and in fact is only able to reproduce the high momentum part of the observed spectrum (curve "Σ" in fig.1). Thus

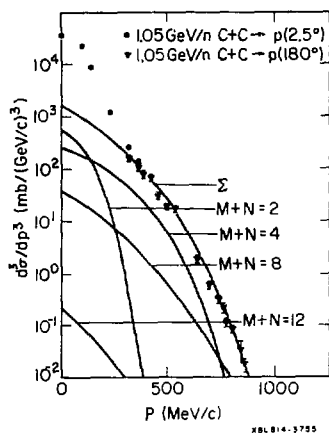


Fig.1. Experimental proton spectrum in comparison with a statistical model calculation. The 180° data are shown in the laboratory frame, the 2.5° data have been transformed to the projectile rest system [4]. The model prediction (curve " Σ ") has been decomposed into the contributions of nucleon groups of different size.

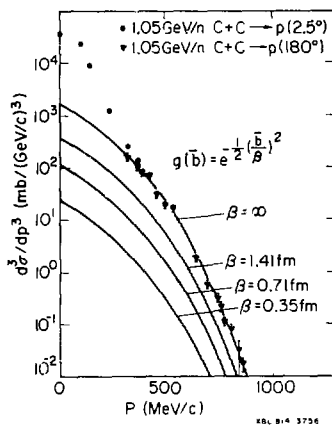


Fig.2. The influence of the restriction to central collisions on the outcome of the model calculation. The parameter β is the width of the Gaussian weight for the impact parameter $\vec{b}$.

we interpret the difference between our prediction and the measured data at low momenta as the contribution of the decay of the spectator parts: the two-component structure of the spectrum is simply due to the different decay schemes of participants and spectators.

Fig.1 shows that the shape of the (participant part of the) spectrum strongly depends on the size $M+N$ of the group the observed proton has been belonging to. From the similarity of shapes it can be deduced

that the main contribution comes from groups containing four nucleons. This is a rather small number; especially if one is looking for signatures of "collective effects" (effects to which several or many nucleons contribute coherently) one would rather like to look at data representing the mutual interaction of a larger number of nucleons. One hope has been that this can be accomplished by restricting the interactions to (nearly) central collisions, a restriction which is feasible experimentally (cf. [5] e.g.). The changes occurring when going from impact parameter averaged data to data from central collisions can be studied in the statistical model by introducing a Gaussian weight for the impact parameter $\vec{b}$ with a variable width β [6]. Fig.2 shows that except for a change in normalization the spectra of backward scattered protons are practically independent of impact parameter restrictions: the average nucleon group size does not seem to change significantly. This is surprising at first as one intuitively would expect major changes. Fig.3 shows that major changes do in fact occur if one looks into the details

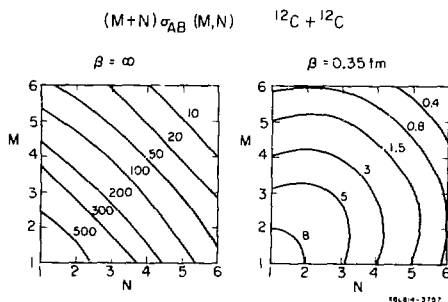


Fig.3. The weight of different combinations of projectile- and target-nucleon numbers in the inclusive cross section eq.1 in millibarns. Left: impact parameter averaged. Right: central collision. The parameter β has the same meaning as in fig.2.

of the group size distribution: the two dimensional plot shows the weight with which a certain group size contributes to the inclusive spectrum (cf. eq.1) as a function of projectile nucleon number M and target nucleon number N . Averaged over impact parameter the distribution is very flat perpendicular to the main diagonal, but it becomes lined up along the main diagonal for central collisions: the groups become more symmetric, i.e. are more likely to contain equal numbers of projectile and target nucleons. Despite these changes it turns out, however, that the mean of the distributions, the average group size, is practically the same in both cases: it increases by about 10% only! Table 1 shows that

Table 1. Average nucleon group size for various projectile-target-combinations and the influence of impact parameter restrictions.

	β [fm]	$\overline{M+N}$
$^{12}\text{C} + ^{12}\text{C}$	∞	3.8
	0.35	4.2
$^{40}\text{Ca} + ^{40}\text{Ca}$	∞	5.4
	0.70	6.1
$^{20}\text{Ne} + ^{238}\text{U}$	∞	7.0
	1.70	9.4
$^{238}\text{U} + ^{238}\text{U}$	∞	9.9
	1.26	11.0

this is true for heavier systems as well. Though the effect is more pronounced for an asymmetric system, it still is not very big: about 30% for $\text{Ne} + \text{U}$.

One can argue that this result is merely due to the linear geometry concept applied and thus completely model dependent. The above finding, however, has found support recently by a cluster analysis of three-dimensional cascade calculations [7]. Again the authors find a striking insensitivity of the mean cluster size on impact parameter restrictions (cf. their fig.4).

It should be mentioned that all this does not mean that there is no difference between peripheral and central collisions. On the contrary it is undoubtedly true that the reaction mechanism is largely different in these two cases. Rather, the claim is that the transition from impact parameter averaged data to data from central collisions is much less dramatic than expected.

In conclusion it can be said that -at least for the high momentum components of spectra- there exists a remarkable insensitivity to impact parameter restrictions. Going over to central collisions does hardly increase the number of mutually interacting nucleons and thus -if one believes in a kind of proportionality between nucleon number and buildup of "collective" effects- it does hardly increase the chance for finding signatures of some kind of "collectivity".

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Coherence and Self-Induced Transparency
in High-Energy Proton-Nucleus Collisions

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It is shown that even partial coherence of the mesonic radiation emitted in the first nucleon-nucleon encounter in a nucleus will lead to penetration of nuclear matter by the forward cone through the phenomenon of self-induced transparency and at the same time to Poisson fluctuations of the meson multiplicity in this con

Fluctuations of meson multiplicities in finite pseudo-rapidity intervals have been measured in proton-nucleus collisions recorded in nuclear emulsions exposed to 69,200 and 300 GeV proton beams. Good consistency with the Poisson law is observed at large pseudo-rapidities; at low rapidities important deviations occur indicating a predominant chaotic emission.

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Confinement and phase transitions*

by

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and

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University of Marburg, GermanyAbstract

It is argued that confinement plays an essential role in the determination of the critical density for the phase transition between nuclear and quark matter and that before the particular mechanism of confinement is known, no conclusions about the role of QCD corrections can be drawn. A new mechanism for confinement is proposed which follows from simple statistical arguments and in which the quark mass m_q is a smooth function of density $n_q \approx m_q = \frac{B}{n_q}$, where B is a constant of the order of the MIT bag constant.

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Mid-Rapidity Pion Cross Sections: New Data Confront Theories*

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Pion production by collisions of relativistic heavy ions has been studied recently by Wolf *et al.*¹ For the reaction $1.05 \text{ GeV/A } ^{40}\text{Ar} + \text{Ca} \rightarrow \pi^+$, they discovered a peak in the invariant cross section at mid-rapidity at $p_{\perp} \sim 0.5 m_{\pi} c$. This peak is not apparent in the 730 MeV $p+p$ data of Cochran *et al.*² and was attributed to possible 'hydrodynamic flow effects'. Several authors have suggested that the π^+ peak is a Coulomb effect^{3,4,5} - the π^+ at $p_{\perp} = 0$, $p_{\parallel \text{cm}} = 0$ would be depleted if there is a charged fireball in the center-of-mass, leaving a π^+ peak at larger $p_{\perp}$. These models predict the π^- would be drawn in towards the fireball, leaving an essentially flat distribution in the mid-rapidity region.

We have measured mid-rapidity π^+ and π^- cross sections using a 1.05 GeV/A argon beam with calcium, uranium, and carbon targets. The pions were measured with a magnetic spectrometer set at 15° to the beam and were stopped in a scintillator range telescope. A description of the spectrometer and detection techniques can be found in a recently submitted publication.⁶

In the region that we studied, our $^{40}\text{Ar} + 2 \text{ gm/cm}^2 \text{ Ca} \rightarrow \pi^+$ data are essentially flat. The π^-/π^+ ratio we measure of 1.5:1 is much less than those predicted by the models^{3,4,5}. Our data overlap the data of Wolf *et al.* only at a laboratory angle of 30° . As shown in Fig. 1, there is excellent agreement between the two sets data. There is no renormalization correction. When the data of Wolf *et al.* are reflected across the center-of-mass, some of their points lie within our acceptance, and some of these points differ from ours by $\sim 25\%$. The cause of this discrepancy could possibly be explained by noting that it is not strictly legitimate to reflect about the C.M. as we are not studying a symmetric system.

We further compare our data with that of Wolf et al. with the aid of computer drawn contour plots of the invariant cross section. Figure 2 shows these plots. The bottom half uses only the data of Wolf et al. and are quite similar to their published plot. Our data combined with that of Wolf et al. are shown in the top half of the figure. The dashed line is our spectrometer acceptance. With the addition of our data the peak appears to extend all the way to $p_{\perp} = 0$.

We next investigate the possibility that our spectrometer was capable of observing the mid-rapidity peak.

The first possible problem is that the $\pi\mu$ correction may be large due to our flight path. The decay correction is approximately 20% and was obtained by Monte Carlo calculation. This correction can be almost correctly estimated by using the survival probabilities for flight through a magnetic field plus a short flight path (~ 50 cm) outside the magnetic field, averaged over the spectrometer's acceptance. We note that the survival probability in a magnetic field is momentum independent. The $\pi\mu$ decay correction in our spectrometer has only a slight momentum dependence due to the path traveled outside of the spectrometer, and we believe we have made this correction properly.

A second possible problem is the resolution of our spectrometer. We used a 2 gm/cm^2 thick target while Wolf et al. used a target that was 200 mg/cm^2 , which is $1/10$ the thickness of ours. The dominant term in the formula for the scattering angle in multiple Coulomb scattering is proportional to the square root of the length traversed.⁷ Our angular resolution is dominated by multiple scattering in the target; however this is only $\sqrt{3}$ times the multiple scattering in the target of Wolf et al. As they measure points every 20° , the peak must be at least 20° wide, while we have data every 4° . Our resolution calculation includes corrections for $\pi\mu$ decay, multiple scattering in the target, and scattering in the scintillator range stack. Our resolution in the region of our acceptance is given approximately by $\langle \Delta p^2 \rangle^{1/2} \sim 0.5 \text{ MeV/c}$, $\langle \Delta \theta^2 \rangle^{1/2} \sim 3.5^\circ$, or alternatively by $\langle \Delta y^2 \rangle^{1/2} \sim 0.2 - 0.3$ and $\langle \Delta p_{\perp}^2 \rangle^{1/2} \sim 0.05-0.06 \text{ mpc}$. We estimate the effect of our resolution on the mid-rapidity peak. As a 'worst case' let us assume that the data of Wolf et al. represent the true spectrum and can be represented by a gaussian in $p_{\perp}$ and y with full widths at half maximum of 0.3 mpc in $p_{\perp}$ and 0.4 in y . These numbers are estimated from their contour plot in the region of our acceptance assuming a peak at 9 and a smooth background at 6 units of cross section. In this case the width in y would increase from 0.4 to 0.41 and the width in $p_{\perp}$ would increase from 0.3 to 0.33 . The height of the peak above background would be reduced from $9-6=3$ to 2.7 , or a height of 8.7 . If our resolution is twice as poor as we claim, the height of the peak above background would still be 2.1 , or a total height of 8.1 . Thus even in our worst case estimates, we should be able to see the peak.

The discrepancies between the two sets of data could be resolved by a high statistics experiment with good resolution,

with data on both sides of the center of mass. More interesting information could be obtained with the use of symmetric systems such as Ca + Ca, and also higher mass systems such as Pb + Pb.

The mid-rapidity π^-/π^+ ratios that we measured appeared to be consistent with Coulomb calculations that required the projectile and target to be transparent to one another. As this seems unlikely, we decided to perform Coulomb trajectory calculations with boundary conditions in which the pion originates on the nuclear surface and never travels within the nuclei. Preliminary calculations show that for pion velocities corresponding to the center of mass velocity, the Coulomb effects on the π^-/π^+ ratio are less than 5%. The π^-/π^+ ratio we measure may be nearly the primitive ratio at "freeze-out" or loss of chemical equilibrium with nuclear matter. The π^-/π^+ ratio in excess of unity may be primarily due to the neutron excess of ^{40}Ar spoiling isobaric symmetry.

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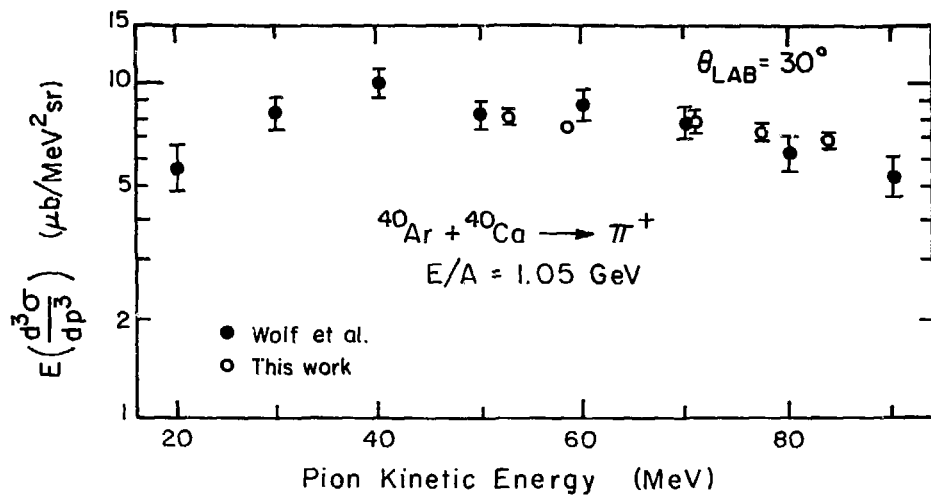


Fig. 1. Data at 30 degrees for this experiment (open circles) and for the experiment of Wolf *et al.*, (closed circles).

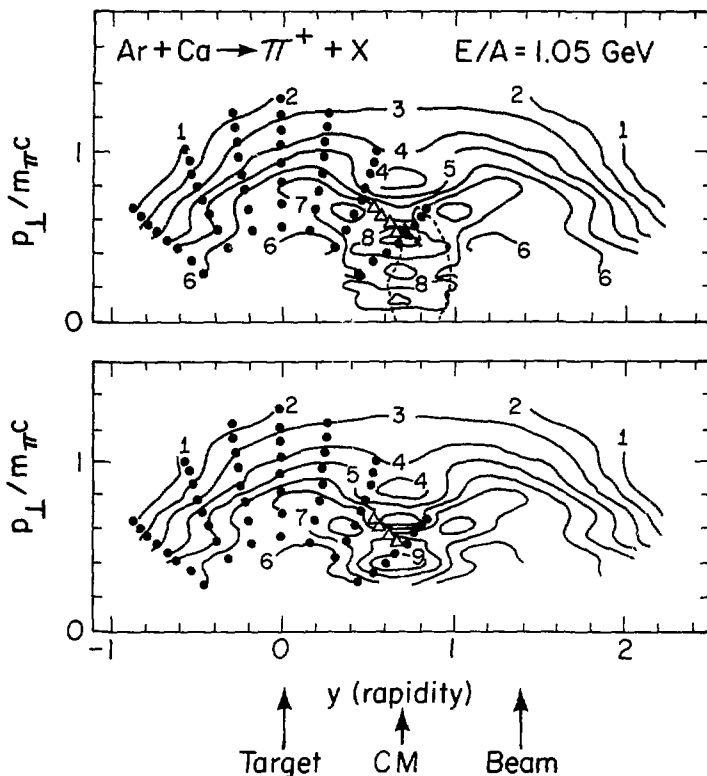


Fig. 2. Computer drawn contour plots of Lorentz-invariant cross sections of the $^{40}\text{Ar} + \text{Ca}$ system at $E/A = 1.05 \text{ GeV/n}$. Dots show the locations of data points of Wolf *et al*, and the dashed line encircles the region of our data. The open circles show reflected data points. The plot in the lower half was made with the data of ref. 1 alone, and the plot in the upper half with data from both experiments. The abscissa is the rapidity variable and ordinate is perpendicular momentum in units of $m_{\pi} c$. Contours are at intervals of $1 \mu\text{sr}^{-1} \text{ MeV}^{-2}$.

A Check on the Geometrical Assumptions in Models for Relativistic Heavy Ion Collisions

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Theoretical models for relativistic heavy ion collisions have two ingredients; (a) geometrical aspect which defines the participants and (b) dynamical aspect (one fireball, two fireballs, firestreak etc.). The purpose of this note is to point out that a recent experiment¹⁾ allows one to check on the geometry with virtually no contamination from the dynamics; a great deal can be learned by comparing this data with predictions from current models.

The cross-sections were obtained in a streamer-chamber experiment in which Ar hits KCl at 1.8 GeV/A in the lab. For our purpose $\text{Ar} \approx \text{K}$ or Cl . At a given impact parameter there will be projectile spectators (n_{proj}^S), target spectators (n_{tar}^S) as well as participants. If Q is the charge in the participating region, as a result of the violent collision, this will appear as $Q = n_p + n_d + n_{\pi^+} - n_{\pi^-}$. We have assumed that productions of composites higher than deuteron is negligible, a justifiable assumption at this energy. The total number of charged tracks is $z_{\text{tot}} = n_{\text{proj}}^S + n_{\text{tar}}^S + n_p + n_d + n_{\pi^+} + n_{\pi^-}$. Thus we have the equality $Q = z_{\text{tot}} - n_{\text{proj}}^S - n_{\text{tar}}^S - 2n_{\pi^-}$. Thus by identifying n_{π^-} , n_{proj}^S and n_{tar}^S one can obtain $\sigma(Q)$.

This cross-section $\sigma(Q)$ is just geometry. In Fig. 1 we show the experimental cross-section as a function of Q . Note that $\sigma(Q)$ is almost constant from $Q=10$ to $Q=26$ and then falls smoothly towards zero. Also shown in the figure is the cross-section calculated using sharp spheres of radius $1.2A^{1/3}$ fm. In this calculation we intergerized the

the charges; i.e. if for a range of impact parameter b , the charge in the overlapping region is between 24.5 and 25.5, then this range of b contributes to $\sigma(25)$. This model does not reproduce the flatness of the data and is in appreciable error around $Q=27$. Realistic density distributions must be used to explain this data.

For realistic density distribution, it will be useful to define ξ as the projection of density on the plane perpendicular to the beam direction: $\xi(s) = \int dz \rho(s,z)$. With a diffuse surface, the overlap between two ions is nowhere strictly zero and a definition for the participating region has to be introduced. One approach is to define participants as in the Firestreak model. In that model all particles that are in the region where $\eta = \xi_p / (\xi_p + \xi_t)$ lies between $\eta_{\min} = 0.05$ and $1 - \eta_{\min}$ are considered to be participants. As in the previous model the number of charges is integerised. The fit to the data is improved (Fig. 1) except that the cross-section for $Q=36$ is anomalously large, 233 mbs. The reason is simple; at $b=0$ for equal ions $\eta=0.5$ everywhere and b has to change significantly before $\eta=0.05$ or 0.95 can be reached. This entire region of b contributes to $\sigma(36)$. The implications are significant for past firestreak calculations.

The trouble with this model is that near $b=0$ all 36 charges always participate. In reality, when the diffuse edge of one nucleus hits the diffuse edge of another nucleus there is a finite probability that it will pass through without interacting. We therefore define two regions for each nucleus; a central region for $s < R_c$ and a diffuse region for $s > R_c$. When a central region hits a central region we assume that multiple collisions occur and all nucleons participate;

loosely speaking this is the fireball region. For other regions (diffuse on diffuse or diffuse on central) we assume that nucleons either pass through or at the most suffer one single knock out collision. We use a formalism due to W. Myers²⁾. Fluctuations in the clean knock out region are easily included. There is one parameter R_C ; for $R_C = 4.5$ fm, rather pleasing agreement is obtained (Fig. 2).

The partitioning of a nucleus into "opaque" and "transparent" regions is only an approximation. It is still a useful one as it allows one to calculate separately single knock out and multiple collision components in inclusive cross-sections. However a more general formula based on approximations of the Glauber theory can be obtained by extrapolating the work of Hufner et. al.³⁾. For equal ions we obtain the following relations for the cross section of having Q charges as participants out of a total of $2z$ charges.

$$\begin{aligned}\sigma_Q(\vec{b}) &= \int d^2b P_Q(\vec{b}) \\ P_Q(\vec{b}) &= \binom{2z}{Q} [1-P(\vec{b})]^Q P(\vec{b})^{2z-Q} \\ P(\vec{b}) &= \frac{1}{A} \int d^2s \xi(\vec{s}) e^{-\sigma_T \xi(\vec{s}+\vec{b})}\end{aligned}$$

The fit with $\sigma_T = 40$ mb is shown in Fig. 3 (calculation 1). Calculation 2 in the same figure refers to the Koonin and Hatch model⁴⁾ used in conjunction with realistic density distribution.

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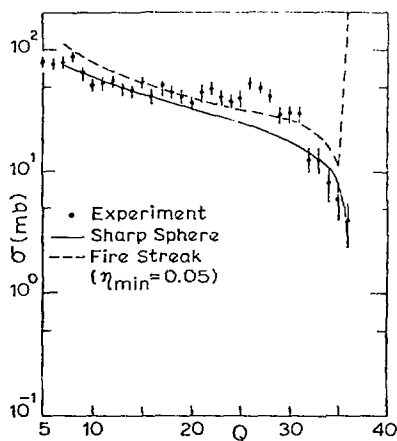


Fig. 1. The cross section for Q primordial charges compared with calculations (a) using sharp sphere nuclei (solid line) and (b) realistic density distribution (dashed line) where participating regions are defined through η between 0.05 and 0.95. An anomalously large cross-section for $\sigma(36) = 233$ mb is obtained.

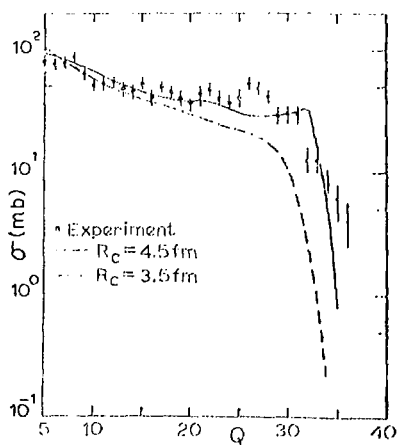


Fig. 2. The same data compared with a model in which each nucleus is broken up into an opaque and a transparent region. R_c is a parameter in the model.

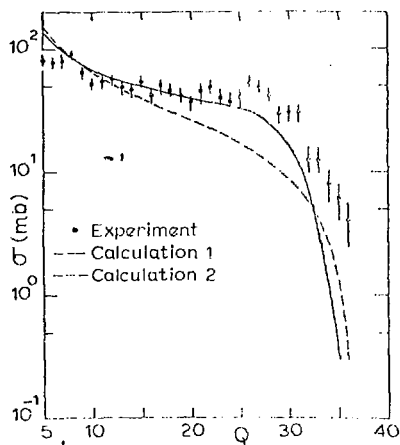


Fig. 3. The same data compared with a Glauber like model (calculation 1) and the Koonin-Hatch model (calculation 2) used in conjunction with a realistic density distribution.

MODELS FOR DEUTERON PRODUCTION IN RELATIVISTIC HEAVY ION COLLISIONS

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Both the thermodynamic model and the coalescence model have been used as models for deuteron production. Kapusta¹⁾ has also proposed a quantum mechanical sudden approximation model. We show that with a slight redefinition the coalescence model is equivalent to the sudden approximation model and is quite close to the simple thermodynamic prescription. All these prescriptions are different approximations to the second virial coefficients of a interacting hot gas of nucleons.

The simple thermodynamic prescription is as follows. Consider a fireball which contains nucleons. Because of interactions a deuteron can form and we assume that the only role of the forces is to allow the deuterons to form and dissociate. Otherwise the nucleons and deuterons are treated as free particles inside a box of volume V . The following relationship then follows from equilibrium laws

$$\frac{d^3 n_d}{d^3 p_d}(\underline{p}_d) = \frac{3}{4} \frac{h^3}{V} \frac{d^3 n_p}{d^3 p}(\underline{p}) \frac{d^3 n_n}{d^3 p}(\underline{p}) e^{\beta \bar{\epsilon}_B} \quad (1)$$

Here $\underline{p}_d = 2\underline{p}$; we note that $\frac{d^3 n_p}{d^3 p}(\underline{p})$ here refers to measured protons, not to primordial protons.

One often quoted criticism of the thermodynamic model for deuteron production is that it may not be possible for deuterons to exist inside the fireball. We now argue that because of high temperature in the fireball, it makes sense to talk of a deuteron in the fire-

ball. Consider a deuteron and another nucleon. If the third nucleon is outside the "volume" of the deuteron then clearly it is reasonable to talk of a deuteron and a nucleon. Now switch from configuration space to momentum space. If in the momentum space the third nucleon is outside the volume of the deuteron, then it is just as good to talk of a deuteron. At zero temperature and normal nuclear density, the probability of finding a third nucleon inside the momentum space volume occupied by a deuteron is large; but at high temperature and somewhat reduced density, the occupation is uncrowded and the probability of finding a third nucleon in the momentum space volume is small. In such a case we still have a deuteron and a third nucleon.

One might argue that in configuration space the nucleon-nucleon force is short range whereas in momentum space it is not. But that only means that because of the third particle, the deuteron can break up. But if it breaks up, it can recombine as well. According to previous calculations^{1,2)} there is enough time for equilibration.

It can be shown³⁾ that the condition that a deuteron can exist is that

$$\frac{\rho h^3}{2(2\pi m\tau)^3} \ll 1 \quad (2)$$

where ρ is the proton (or neutron) density and we have used non-relativistic kinematics. Eq. (2) is precisely the condition under which a Fermi-Dirac gas degenerates into a Maxwell-Boltzmann gas. This quantity in eq. (2) is $\approx .1$ for a typical fireball.

The coalescence model does not assume that deuterons pre exist in the fireball; the assumption here is that a neutron and a proton leaving the fireball with about the same momentum will coalesce into a

deuteron. Consider $n_p = n_n = n/2$ and the formation of a deuteron in the $S=1, M=1$ state. Consider a volume element $\Delta^3 p$ centred at p . The probability of finding a given proton in this volume element with spin up is

$$\tilde{P}(p) = \frac{1}{(n/2)} \frac{1}{2} \Delta^3 p \frac{d^3 n_p}{d^3 p}(p)$$

However, in order to form a deuteron the nucleons have to be correlated not only in momentum space but in configuration space as well. Thus we consider the probability that the proton is not only in the given momentum space volume but also in a given configuration space volume $\tilde{V}$ in the fireball. This volume $\tilde{V}$ is the characteristic volume of the deuteron. We are thus led to consider

$$P(p) = \frac{\tilde{V}}{V} \frac{1}{(n/2)} \frac{1}{2} \Delta^3 p \frac{d^3 n_p}{d^3 p}(p)$$

We have assumed that the characteristic volume of the deuteron is smaller than the volume of the fireball. The probability that both a neutron and a proton with spin up appear in the same part of the phase space with no other nucleon in the same phase space is

$$\frac{n}{2} P (1-P)^{\frac{n}{2}-1} \frac{n}{2} P (1-P)^{\frac{n}{2}-1} \approx \frac{n^2}{4} P^2 \quad (3)$$

We have assumed $\frac{n}{2} P \ll 1$ which is equivalent to eq. (2). We are however not interested in knowing in which of the volumes $\tilde{V}$ in the fireball the two nucleons appeared. The number of such elements is $(V/\tilde{V})$.

We thus obtain

$$\frac{V}{\tilde{V}} \frac{n^2}{4} P^2 = \frac{\tilde{V}}{V} \frac{1}{4} \Delta^3 p_1 \Delta^3 p_2 \left[\frac{d^3 n_p}{d^3 p}(p) \right]^2$$

To obtain $\frac{d^3 n_d}{d^3 p_d}$ note that $d^3 p_1 d^3 p_2 = d^3 p_d d^3 p_r$ where p_d and p_r are centre of mass and relative momenta. We now equate $d^3 p_r \tilde{V} = h^3$, the phase space volume of one quantum state. We only considered $S=1, M=1$ state. Including other M states, we obtain

$$\frac{d^3 n_d}{d^3 p_d}(\underline{p}_d) = \frac{3}{4} \frac{h^3}{V} \left(\frac{d^3 n_p}{d^3 p}(\underline{p}) \right)^2 \quad (4)$$

This equation is now close to eq. (1); the difference is the factor $\exp(\beta E_B)$ and also that $\frac{d^3 n_p}{d^3 p}$ in eq. (4) refers to primordial protons, not to the measured protons. Note that in contrast to the usual coalescence model we have a factor $1/V$.

Kapusta¹⁾ considered a model, in which deuterons arise because proton and neutron wavefunctions in the fireball have an overlap with a deuteron wavefunction; the absolute square of this overlap gives the number of deuterons. Let us denote the nucleon wavefunctions in the fireball by $\frac{1}{L^{3/2}} e^{\frac{i}{\hbar} \underline{p} \cdot \underline{r}}$; here L^3 is the volume of the fireball. We ignore the spin factors for the moment and put them in later. The number of protons in a momentum interval is $\frac{d^3 n_p}{d^3 p}(\underline{p}_1) d^3 p_1$ and the number of neutrons is $\frac{d^3 n_n}{d^3 p}(\underline{p}_2) d^3 p_2$. Let the deuteron interval wavefunction be $\chi(r)$. The overlap of a two particle wavefunction with the deuteron wavefunction can be calculated from

$$\begin{aligned} & \int \chi^*(\underline{r}) \frac{1}{L^3} e^{\frac{i}{\hbar} \underline{p}_1 \cdot \underline{r}} e^{\frac{i}{\hbar} \underline{p}_2 \cdot \underline{r}} d^3 r \\ &= \frac{1}{L^3} \int \chi^*(\underline{r}) e^{\frac{i}{\hbar} (\underline{p}_1 + \underline{p}_2) \cdot \underline{r}} e^{\frac{i}{\hbar} \frac{1}{2} (\underline{p}_1 - \underline{p}_2) \cdot \underline{r}} d^3 r \\ &= \frac{1}{L^3} \int e^{\frac{i}{\hbar} \underline{p}_d \cdot \underline{r}} \chi^*(\underline{r}) e^{\frac{i}{\hbar} \underline{p} \cdot \underline{r}} d^3 r \\ &= \left(\frac{2\pi\hbar}{L} \right)^{3/2} \frac{1}{L^{3/2}} e^{\frac{i}{\hbar} \underline{p}_d \cdot \underline{R}} \psi^*(\underline{p}) \end{aligned}$$

In the last step we have assumed that the fireball is bigger than the characteristic volume of the deuteron since we have equated

$$\int_{\text{fireball}} \chi^*(r) e^{\frac{i}{\hbar} \mathbf{p} \cdot \mathbf{r}} d^3r = (2\pi\hbar)^{3/2} \psi^*(\mathbf{p})$$

where $\psi(\mathbf{p})$ is the Fourier transform of the deuteron internal wavefunction. The number of deuterons is

$$\begin{aligned} d^3n_d &= \int \frac{d^3n_p(\mathbf{p}_1)}{d^3p} \frac{d^3n_n(\mathbf{p}_2)}{d^3p} d^3p_1 d^3p_2 \frac{\hbar^3}{L^3} \psi^*(\mathbf{p}) \psi(\mathbf{p}) \\ &= \frac{\hbar^3}{L^3} \int \frac{d^3n_p(\mathbf{p}_d/2 + \mathbf{p})}{d^3p} \frac{d^3n_n(\mathbf{p}_d/2 - \mathbf{p})}{d^3p} d^3p_d d^3p \psi^*(\mathbf{p}) \psi(\mathbf{p}) \end{aligned}$$

Assuming that the neutron or proton distributions do not change significantly within the range of $\mathbf{p}$ for which $\psi(\mathbf{p})$ is significant and putting in the spin factors we have

$$\frac{d^3n_d}{d^3p_d} = \frac{3}{4} \frac{\hbar^3}{v} \frac{d^3n_p(\mathbf{p}_d/2)}{d^3p} \frac{d^3n_n(\mathbf{p}_d/2)}{d^3p}$$

This is identical to eq. (4).

We now show that the simple thermodynamic prescription and the coalescence model are two limits of a more complete theory. The quantity of central importance in equilibrium statistical mechanics is the grand partition function. An expansion of the quantum mechanical grand partition function up to the second virial coefficient is well known. We consider a box of nucleons interacting by a two body force; for simplicity the force acts only in the spin triplet, $\ell=0$ state (See ref. 3 for more general case). The grand partition function can be written as

$$\begin{aligned} \ln Z_{gr} &= A + B \quad ; \quad A = \frac{V}{\hbar^3} \left[e^{\lambda_p} 2 (2\pi m \tau)^{3/2} + e^{\lambda_n} 2 (2\pi m \tau)^{3/2} \right] \\ B &= \frac{V}{\hbar^3} 3 e^{\lambda_p + \lambda_n} (4\pi m \tau)^{3/2} \left(e^{\beta \epsilon_B} - 1 + \frac{\beta}{\pi} \int \delta(\epsilon) e^{-\beta \epsilon} d\epsilon \right) \end{aligned}$$

For non-interacting nucleons B is zero. Some comments on B are useful. At zero temperature a well respected approximation for nuclear matter is the independent pair approximation. The terms in B are precisely due to an independent pair approximation in the simpler case of high temperature. In the simple thermodynamic prescription one has set

$$B = \frac{V}{h^3} 3 e^{\lambda_p + \lambda_n} (4\pi m \tau)^{3/2} e^{\beta \bar{E}_B}$$

In the coalescence model since $\beta \bar{E}_B \approx 0$ one has set

$$B = \frac{V}{h^3} 3 e^{\lambda_p + \lambda_n} (4\pi m \tau)^{3/2} (e^{\beta \bar{E}_B} - 1)$$

Numerical calculations with the full virial coefficients show that the simple thermodynamic prescription underestimates deuteron production by about 35% (at 800 MeV/n), the coalescence model overestimates by about 45%.

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Study of Highly Ionizing Particles at Mountain Altitude

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The motivation for this experiment was the widely acknowledged peculiar nature of the "Centauro" class of ultrahigh-energy interactions discovered at Mt. Chacaltaya about a decade ago by a Brazil-Japan collaboration. The rate of Centauro events is quite low, $\sim 0.02 \text{ m}^{-2}\text{yr}^{-1}$. Among the proposed explanations is the possibility that they might be initiated by a highly charged primary particle. To look for such particles we deployed a series of CR-39 plastic track detectors with large collecting power at the summit of White Mountain, CA (603 g/cm^2). Two experiments have been completed and analyzed: a single layer with a $10 \text{ m}^2 \text{ yr}$ exposure, and three layers in coincidence with a $0.025 \text{ m}^2 \text{ yr}$ exposure. The results demonstrate the superiority of CR-39 as a detector of highly ionizing particles ($Z/\beta \geq 30$) and its particular suitability as a collector of very rare particles. The low-energy spectra of nuclei with $Z \leq 3$ have been measured and are found to be consistent with spectra calculated from a model where the source of ions is atmospheric collisions of energetic hadrons. This is the first time that energetic Li has been identified at mountain altitude and that enough He has been seen to permit measurements of its energy spectrum. A density $\sim 10 \text{ m}^{-2}$ of fast particles with $Z \geq 4$ found in individual layers is consistent with the exposure received in a 10-hour commercial jet flight. In the interval $30 \leq Z/\beta \leq 100$ no events were found, from which we infer an upper limit of $0.4 \text{ m}^{-2}\text{yr}^{-1}$ (95% CL) on the flux of electrically charged particles with $Z/\beta \sim 30$ to ~ 100 and on the flux of superheavy magnetic monopoles with $\beta \geq 0.02$. Two objects in the single-layer array were located which, if they are indeed tracks, would correspond to particles with $Z/\beta \geq 100$, with abundance $\sim 0.2 \text{ m}^{-2} \text{ yr}$. New detectors with three layers interleaved with absorbers have been exposed at 603 g/cm^2 and at 200 g/cm^2 . We hope that data from these detectors can be included in the talk.

LANDAU SUM RULES AND SPIN DEPENDENT LANDAU
PARAMETERS WITH MESON EXCHANGES

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There are two Landau sum rules for the particle-hole interaction parameters in nuclear matter. One is the isovector triplet ($T=1, S=1$) sum rule, the other is the isoscalar singlet ($T=0, S=0$) sum rule. Both of the sum rules are applied to examine several sets of Landau parameters which have been used on various occasions. It is found that the set with spin-independent parameters based on our present knowledge of the compression modulus, the effective mass and the symmetry energy and with the spin dependent parameters derived from meson exchanges seems to satisfy the sum rules fairly well.

It is learned from the sum rule constraint that the expected variations of the compression modulus and the effective mass as a function of the nuclear matter density implies a density dependence in the spin and isospin dependent Landau parameter G'_0 , even though it might be weak. This differs from the belief that G' arises almost exclusively from the short-range ρ meson exchange and G'_0 will therefore remain as a constant. The origin of this density dependence is attributed to the density of states and the exchange of ρ , ω and σ mesons which produce a non-negligible G' . As the nuclear matter density increases, a non-decreasing G'_0 and the sum rule will dictate a decreasing effective mass m^* . Since the density of state depends on m^* , both the decrease of m^* and the increase of G'_0 will make pion condensation harder to realize.

In nuclear matter, the Pauli principle leads to two sum rules for the Landau-Migdal parameters. One is the isovector triplet ($T=1$, $S=1$) sum rule

$$S^{33} = \sum_{\ell} \left[\frac{F_{\ell}}{1+F_{\ell}/\Delta_{\ell+1}} + \frac{F'_{\ell}}{1+F'_{\ell}/\Delta_{\ell+1}} + \frac{G_{\ell}}{1+G_{\ell}/\Delta_{\ell+1}} + \frac{G'_{\ell}}{1+G'_{\ell}/\Delta_{\ell+1}} \right] = 0 \quad (1)$$

the other one is the isoscalar singlet ($T=0$, $S=0$) sum rule

$$S'' = \sum_{\ell} \left[\frac{F_{\ell}}{1+F_{\ell}/\Delta_{\ell+1}} - \frac{3F'_{\ell}}{1+F'_{\ell}/\Delta_{\ell+1}} - \frac{3G_{\ell}}{1+G_{\ell}/\Delta_{\ell+1}} + \frac{9G'_{\ell}}{1+G'_{\ell}/\Delta_{\ell+1}} \right] = 0 \quad (2)$$

Now let's apply these two sum rules to examine several sets of Landau-Migdal parameters. The sets of parameters are listed in Table 1 together with their respective compression moduli K , symmetry energies β , effective masses m^* and the Landau sum rules S^{33} and S^{11} . We see, from Table 1, that the empirical parameters of Migdal [1] violate the sum rules simply because all the parameters are positive. The large positive F_0 derives from the large compression modulus (~ 700 Mev) used. Similarly, the parameters used by Speth et al. [2], which are based on Migdal's theory, also violate the sum rules with all the parameters being positive. In the case of the modified Sky BWL1 interaction [3], the sum rules are violated by a large compression modulus and the negative spin-dependent parameter G_0 . This is a general feature of all the Skyrme interactions used. Bäckman [4] calculated the parameters from a G -matrix without taking into account the collective excitation. Therefore, it is not surprising to find [5] that the parameters he obtained do not satisfy the sum rules. On the other hand, Sjöberg [6] constructed

Table 1. Landau-Migdal parameters from various sources and their respective compression moduli, symmetry energies, effective masses and sum rules are listed. The number in the parenthesis is the result where the higher ℓ cases are included.

	Ref. ℓ	F_{ℓ}	F'_{ℓ}	G_{ℓ}	G'_{ℓ}	$K(\text{MeV})$	$\beta(\text{MeV})$	m^*/m	S^{33}	S^{11}
Migdal	[1] 0	~ 2	~ 0.6	~ 1	~ 1	~ 700	~ 20	1	~ 2.0	~ 2.5
Speth et al.	[2] 0	0.18	0.83	1.44	1.81	272	23.4	1	1.84	2.82
Modified	0	0.42	0.94	-0.27	0.40	180	30	0.8	1.33	3.15
Sky BWL 1	[3] 1	-0.61	0.55	0.55	0.55					
Bückman	[4] 0	-0.60	0.28	0.53	0.40	128	23	0.73	-1.04	-1.04
	1	-0.80	0.29	0.24	0.26					
	2	-0.30	0.06	0.14	0.10					
Sjöberg	[6] 0	-0.32	1.27	1.98	-0.10	208	38	0.74	0.37	-5.90
	1	-0.77	0.52	0.15	0.30				(0.03)	
	2	-0.47	0.13	0.01	0.12					
Phenomeno-logical and meson exchange theory	0	-0.39	0.97	1.5	1.5	200	36	0.7	0.41	0.51
	1	-0.90	0.3	0.2	0.2					

the quasi-particle interaction from G-matrix with the collective excitations included. Hence, antisymmetry between quasiparticles is expected and indeed he found a small value (0.03) for S^{33} . However, we find that S^{11} , calculated to $\ell=2$, is far from vanishing ($S^{11} = -5.9$). This probably reflects the fact that G' in this work is not correctly calculated [11]. The last row in Table 1 lists a set of Landau parameters with the spin-dependent parameters determined phenomenologically and the spin-

dependent parameters derived from meson exchanges. They seem to satisfy the sum rules fairly well.

The sum rules also provide information on the behavior of some of these parameters as a function of the nuclear matter density. To see this, we combine the sum rules S^{33} and S^{11} linearly to obtain

$$\sum_{\ell} \left[\frac{F_{\ell}}{1 + F_{\ell}/2\ell+1} + \frac{3G'_{\ell}}{1 + G'_{\ell}/2\ell+1} \right] = 0 \quad (3)$$

Assuming that F'_{ℓ} and G'_{ℓ} are negligible for $\ell \geq 2$, we can learn the behavior of G' in terms of those of F_0 and F_1 which are in turn related to the behavior of the compression modulus and the effective mass. It is clear that the compression modulus K decreases with the nuclear matter density since nucleons experience each other's repulsive core less. Whereas, the effective mass m^* will increase with decreasing nuclear matter density as nucleons behave more like free nucleons. These imply that both F_0 and F_1 are density dependent quantities. This is confirmed by Migdal's study of the isotope shift [1] where a strongly density dependent F_0 is required and by the fact that empirically m^*/m is found to be close to unity on the surface of the finite nuclei instead of the nuclear matter value of $0.7 \sim 0.8$. From Eq. (3) we see that if G'_{ℓ} with $\ell > 0$ are assumed to be negligibly small which is the case in several previous works [7,8], then G'_0 will be a density-dependent quantity in general. To see how this density dependence could come about in the meson exchange picture, we calculate G'_0 and G'_1 following the works in Ref. [7,8]. The results are summarized in Table 2. From Table 2, we see that σ and ω mesons contribute 0.56 and -0.26

Table 2. Meson exchange contributions to G_0^*/N_0 , G_1^*/N_0 , G_0^* and G_1^* are evaluated at different nuclear matter densities. N_0 is the density of state $2k\text{fm}^3/\pi^2$. The meson-nucleon coupling constants are taken as the following: $f_\rho^2/4\pi = 4.95$, $g_\omega^2/4\pi = 10$ and $g_\sigma^2/4\pi = 5$. The masses of the mesons are: $m_\rho = 773$ Mev., $m_\omega = 780$ Mev and $m_\sigma = 500$ Mev.

k_F	1.36 fm ⁻¹				1.71 fm ⁻¹				0.94 fm ⁻¹			
m^*/m	0.70				0.61				0.90			
N_0	0.918 fm ⁻²				1.01 fm ⁻²				0.816 fm ⁻²			
q_c	3.93 fm ⁻¹				4.32 fm ⁻¹				3.42 fm ⁻¹			
	ρ	ω	σ	$V_{2\pi}$	ρ	ω	σ	$V_{2\pi}$	ρ	ω	σ	$V_{2\pi}$
$G_{\sigma, \text{dir}}^*/N_0 (\text{fm}^2)$	1.35	-	-	0.27	1.48	-	-	0.30	1.17	-	-	0.23
$G_{\sigma, \text{ex}}^*/N_0 (\text{fm}^2)$	-0.23	-0.68	0.92	-	-0.21	-0.63	0.78	-	-0.24	-0.70	1.10	-
$G_1^*/N_0 (\text{fm}^2)$	-0.10	-0.29	0.61	-	-0.13	-0.38	0.67	-	-0.06	-0.17	0.42	-
TOTAL												
$G_0^*/N_0 (\text{fm}^2)$	1.63				1.72				1.56			
$G_1^*/N_0 (\text{fm}^2)$	0.22				0.16				0.19			
G_0^*	1.50				1.74				1.27			
G_1^*	0.20				0.16				0.16			

respectively to G_1^* which are not small. Including $G_{1\rho}^*$ ($= -0.09$) from the ρ -exchange, the final G_1^* has the value of 0.20 which is not negligible and presumably will yield a density dependence in G_0^* .

To study how the finite-rangeness of the interaction gives rise to a density dependence in G_0^* , we calculate the contributions from ρ , σ and ω exchanges at one third and twice the nuclear matter densities. From Table 2, we find that the interaction G_0^*/N_0 changes only about 4-6%

from that at the nuclear matter density. This is not due to the short-rangeness of the ρ meson whose contributions may vary by ~ 13 – 17% , but rather due to the cancellation between contributions from the σ meson and the ρ , ω mesons. Another density dependence in G'_0 derives from the density of state N_0 which depends on k_F and m^* . We shall take a typical value of $m^*/m = 0.61$ (0.9) at $k_F = 1.71 \text{ fm}^{-1}$ (0.94 fm^{-1}) as evaluated in Brueckner's theory [9]. As a result, N_0 changes by 10% (11%) at twice (one third) of the nuclear matter density. Together with the density variation from the finite-rangeness of the meson exchanges, it changes G'_0 from 1.50 at nuclear matter density to 1.74 (1.27) at twice (one-third) the nuclear matter density. Since the density of states depend on m^* , both the decrease of m^* and the increase of G'_0 will make phenomena like pion-condensation more difficult to realize.

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Pion Production in Central Collisions

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We present our recent pion production data in relativistic central collisions of 1.8 GeV/N ^{40}Ar beams on a Pb target, using the streamer chamber facility at Lawrence Berkeley Laboratory. Through the momentum distribution of negative pions, the temperature of the interaction fireball is inferred to be $86.6 \pm 3.9 \text{ MeV}$. The ratio of total pions produced to the total number of participating nucleons is obtained. Negative pion multiplicity cross-section suggests a suppression of high multiplicity events and a non-Poisson distribution. The size of pion-emitting source is studied as a function of pion multiplicity, which is closely correlated with impact parameter. The results are used to assess the nuclear thermodynamic fireball model. Comparison with other pion data will be presented.

Importance of Nucleon Knock-out in Inelastic α and ${}^6\text{Li}$ Scattering*

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To investigate the role of the nuclear continuum in inelastic hadron scattering we have studied¹ inelastic α and ${}^6\text{Li}$ scattering on ${}^{28}\text{Si}$ at 26 MeV/A under kinematical conditions favorable for giant resonance excitation. Spectra were taken in singles (Fig.1, top) and in coincidence with discrete γ transitions in residual nuclei (e.g., ${}^{27}\text{Al}$ and ${}^{27}\text{Si}$ in Fig.1). Surprisingly, substantial yield is seen in the $A=27$ channels even at high energy losses where statistical decay of highly excited states in $A < 27$ nuclei. We present this as evidence for a fast direct nucleon removal process from ${}^{28}\text{Si}$ by inelastic scattering. With that assumption one can correct for the unobserved ground state $d_{5/2}$ knock-out using known spectroscopic factors. As a result one obtains the histogram in Fig.1 (top) which shows that the inclusive spectrum is quantitatively described assuming knock-out processes alone. The flat shape of the inclusive α spectrum may be shown to result from an additional cross section component which corresponds to excitation of the target nucleus and subsequent evaporation. These results raise questions as to the potential of heavy ions to excite high-lying states of the target nucleus.

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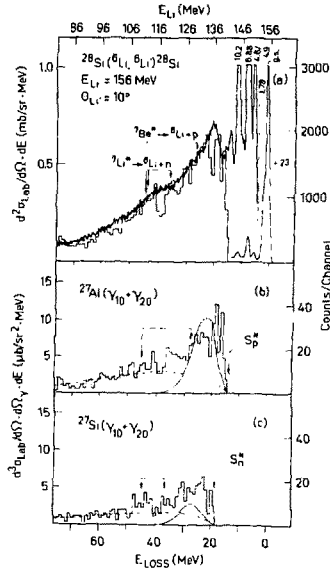


Fig. 1. Spectra of inelastically scattered ${}^6\text{Li}$ particles on ${}^{28}\text{Si}$ (a) singles; in coincidence with $1+0$ and $2+0$ transitions in (b) ${}^{27}\text{Al}$ and (c) ${}^{27}\text{Si}$. Solid and dashed curves represent statistical model and PWIA calculations. The histogram in (a) represents the cross section for direct one-nucleon removal.

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An S-Matrix Approach to Heavy Ion Reactions*

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We present an s-matrix formalism for the analysis of (α, α') scattering on light and heavy ions. We feel that utilization of this method can be extended to understand resonant and direct reaction phenomena in intermediate and high energy heavy ion reactions. Thus, internal nuclear structure may become better understood.

The detailed mechanism of alpha decay and its implications about the structure of nuclear matter has been a study of much interest over the last fifty years.¹⁻⁵ We have examined four major approaches to the theoretical calculation of alpha decay widths for light and heavy, even and odd-mass nuclei.⁶⁻⁹ Application of the microscopic shell model rate theory as well as macroscopic models utilizing the coupled channel formalism have been studied. Use of the R-matrix and S-matrix theories have been applied in order to overcome problems involving dependency on the connection radius and nuclear potential parameters of the relative and absolute alpha decay

widths.⁸⁻¹⁰

In the formalism of alpha decay, in which the alpha decay rate or decay width for alpha decay can be identified with a scattering event, one identifies a resonance width with a pole in the S-matrix.¹¹ When the real and imaginary parts of the energy-dependent S-matrix elements are plotted and these are identified with a loop which appears in the unit phase circle, the residue of the pole of this loop can be identified with the energy of the compound nuclear resonance state.

For a resonance S-matrix coupled channel model with a complex optical model nuclear potential,^{6,9,12} we calculate the alpha decay resonance width from the energy position of resonance loops in the complex S-matrix plane Argand diagrams. Because of the restrictions imposed by the memory of present computers, and the large number of partial waves necessary for calculations involving the alpha decay of intermediate heavy nuclei for the purposes of the present calculation, we consider alpha decay from parent nuclei in the first half of the s-d shell. Alpha particle resonance states occur, particularly in the deformed region between the Z (proton) and N (neutron) magic numbers of 8 and 20. We consider alpha - ²⁰Ne resonance (See Figure 1).

In analogy to the work in high-energy physics for the scattering of nucleons or nuclei off nuclei, the residue of the pole gives the decay as reaction width. The complex

S-matrix plots have a similar trend for both $W = -1.0$ MeV and $W = 0$ or imaginary nuclear potential parameter, which may indicate that the resonance loops and the curve shapes are due primarily to channel coupling or absorption into higher coupled wave channels. Absorption represented by the complex part of the potential also effects the resonance.¹³ Dependence on W is examined by K.W. McVoy.¹⁴ An example of Argand plots is given in Figure 2.

If the decay or reaction rate is very rapid, then the reaction is more likely to occur as a direct reaction and the compound nuclear state may not form. If the decay proceeds more slowly, the decay time decreases, then the decay width becomes larger. If W , the complex part of the potential becomes larger, then the decay width reflects the increase in absorption and the width becomes smaller and the decay time becomes longer. The existence of narrow resonances shows directly that long-lived compound systems, formed by the target nucleus and the incident nucleon or nuclei exist. The resonance structure and deduced pole residues, give some measure of the lifetime and nature of the resonance formation.

The lifetime will be effected by the incident alpha energy on the target nucleus. The qualitative structure of the S-matrix for the (α, α') scattering on the ^{24}Mg , 0^+ ground state nuclei was examined with a view towards developing techniques to help elucidate certain aspects of the alpha decay mechanism through multi-particle ion scattering and

and which can be expanded to heavier target nuclei. The qualitative features of the complex S-matrix elements in the energy and momentum planes may yield information about nuclear surface and interior clustering phenomena.¹¹ It is hoped that these high powered techniques from high energy nuclear scattering will give us a more comprehensive understanding of the alpha decay process and of nuclear structure, and allow us to calculate absolute decay width reasonably independent of the nuclear optical model parameters.¹⁵

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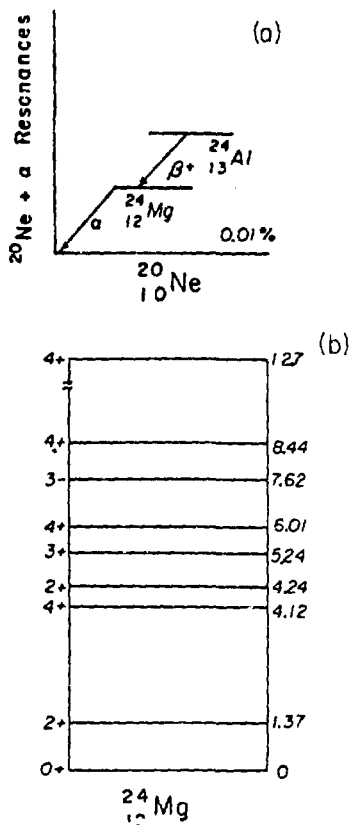


Fig. 1. Energy levels of even-even nuclei in the $2s1d$ shell for ^{24}Mg , including α - ^{20}Ne resonance excitation. (XBL 798-2433)

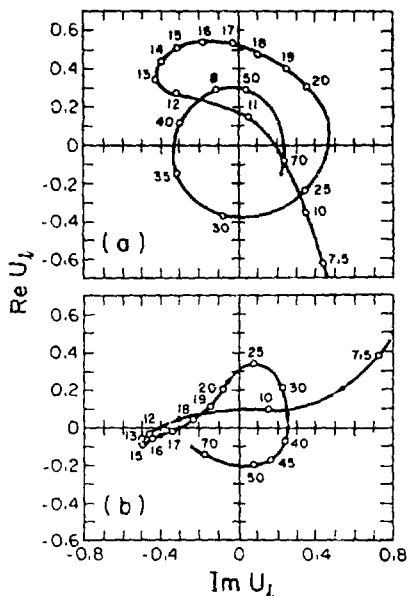


Fig. 2. The real and imaginary parts of the scattering matrix elements are plotted as $\text{Re } U_l$ vs. $\text{Im } U_l$ in the energy plane. Each labeled point denotes a laboratory energy in MeV from 7.5 to 70 MeV. Angular momentum values in Fig. 2a, b are $l=1$ and $l=4$, respectively. All cases have the imaginary parts of the Woods-Saxon potential, $W = -1.0$ MeV. The predominant "motion" from small to larger energy for small l is in the clockwise direction, which is the hard-sphere background. Counterclockwise motion becomes more predominant for larger l values, where more resonances occur. The onset of resonances occurs around $E = 13$ to 15 MeV in both Fig. 2a, b.

(XBL 798-2434)

P O L Y N O M I A L C H R O M O D Y N A M I C S *

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ABSTRACT

A model of classical field theory with internal symmetry $Z(3)$ is partially solved in 1+1 dimensions. The dynamical variables are two pairs of scalar and spinor fields Φ_1, Φ_2 and Ψ_1, Ψ_2 . The Φ and Ψ fields form composite finite-size particles. We study the mass spectrum and the interactions of these particles.

1. INTRODUCTION: The action of the model is [1]:

$$S(\Phi_1, \Phi_2, \Psi_1, \Psi_2) = \int dt dx \left[\frac{1}{2} (\partial_\mu \Phi_1)^2 + \frac{1}{2} (\partial_\mu \Phi_2)^2 \right. \\ \left. - \lambda (\Phi_1^2 + \Phi_2^2)^2 + \alpha (\Phi_1^3 - 3\Phi_1 \Phi_2^2) + \beta (\Phi_1^2 + \Phi_2^2) + \delta + i \bar{\Psi}_1 \gamma_\mu \partial_\mu \Psi_1 \right. \\ \left. + i \bar{\Psi}_2 \gamma_\mu \partial_\mu \Psi_2 + g (\bar{\Psi}_1 \Psi_1 - \bar{\Psi}_2 \Psi_2) \Phi_1 - g (\bar{\Psi}_1 \Psi_2 + \bar{\Psi}_2 \Psi_1) \Phi_2 \right]. \quad (1)$$

With $\alpha = 2\lambda\Phi_V/3$, $\beta = \lambda\Phi_V^2$, $\delta = -\beta\Phi_V^3$; $\partial_0 = \partial/\partial t$, $\partial_1 = \partial/\partial x$; $\gamma_0 = -\tau_2$, $\gamma_1 = i\sigma_1$ and $\bar{\Psi} = \Psi^\dagger \sigma_2$. $\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices. The coupling constants λ , g and the vacuum field Φ_V are free positive parameters.

The symmetry $Z(3)$ -- a rotation of the Φ and Ψ fields by 120° -- leaves the action invariant.

"Toy" models of nuclear physics in 1+1 dimensions have been studied at the level of mesons and nucleons [2], e.g. the σ -model. The aim of the present work is to start at a deeper level, and derive first the structure of the nucleons and from thereon their properties.

Using the vocabulary of QCD we consider the Φ -fields to play the role of scalar gluons [3], due to their cubic and quartic selfinteraction. The spinor fields Ψ and the $Z(3)$ group are assumed to play the role of quarks [4] and the color group respectively [5].

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2. STRUCTURE OF COMPOSITE PARTICLES: Particular solutions of the field equations are obtained in a closed form when the effect of the quarks upon the gluons is neglected-- an approximation also made in the MIT-bag model. We find soliton solutions for the Φ -fields [6], and localized solutions for the ψ -fields:

$$\Phi_1 = \Phi_V(1 + 3 \tanh(ax))/4, \quad \Phi_2 = \sqrt{3}\Phi_V(1 - \tanh(ax))/4; \quad (2)$$

$$\psi_1 = NU \exp(iEt - H(x)), \quad \psi_2 = i\sigma_3 \psi_1 \quad (3)$$

With $E = -\frac{1}{2}g\Phi_V$, $H(x) = (2\lambda)^{-\frac{1}{2}}g \ln \cosh(ax)$, $a = \sqrt{3\lambda/2}$. U is the the spinor part and N the normalization constant.

The total field energy generated by these solutions defines the mass of the ground-state of a finite-size composite particle:

$$M = a\Phi_V^3 - g\Phi_V. \quad (4)$$

A perturbational calculation of the time dependent oscillations of the Φ -fields gives two excited states with the masses $M^* = M + \sqrt{3}a$, and $M^{**} = M + 2a$.

We can think of removing one quark ($\psi_1=0$ or $\psi_2=0$). This process breaks the symmetry of the field equations and thus

costs energy. The mass of this "colored" state is larger: $M^c = M + 7g\Phi_V/8$. Removing finally both quarks ($\psi_1 = \psi_2 = 0$) we obtain a "glueball" with the mass $M^g = M + g\Phi_V$.

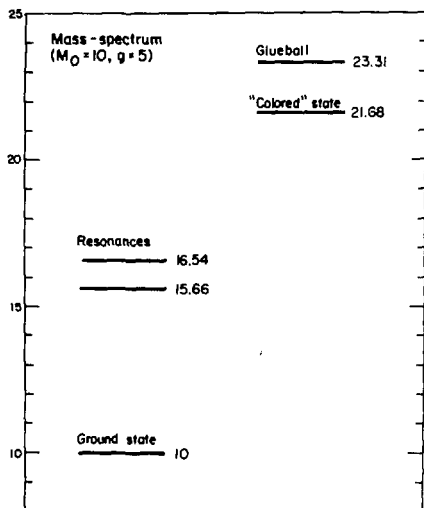


Fig. 1. Example of mass spectrum. The ground-state is chosen to have mass ten to give an easy reference point for the rest of the spectrum. The parameters are $\lambda = 1$, $g = 5$ and $\Phi_V = 2.6717$. The mass is measured in units of $\sqrt{\lambda}$.

3. TWO PARTICLE INTERACTIONS: Two-center Φ -fields are constructed by combining two soliton solutions with the centers localized at x_1 and x_2 ($x_2 \leq x_1$) as follows:

$$\Phi_1(x, x_1, x_2) = \Phi_v(1 + 3 \tanh(ax - ax_1))(-1 + 3 \tanh(ax - ax_2))/8, \quad (5)$$

$$\Phi_2(x, x_1, x_2) = \sqrt{3} \Phi_v(1 - \tanh(ax - ax_1))(1 + \tanh(ax - ax_2))/8. \quad (6)$$

The ψ -fields are superposed according to the Pauli exclusion principle [7] (applied to color):

$$\psi_1(x, x_1, x_2) = \psi_1(x - x_1) - \psi_1(x - x_2), \quad (7)$$

$$\psi_2(x, x_1, x_2) = \psi_2(x - x_1) - \psi_2(x - x_2). \quad (8)$$

The interaction potential between two particles at a distance $d = x_1 - x_2$ is defined as the difference between the field energy generated by the two-center fields and their total mass at large distance of separation.

The potential for two ground-state particles shows a shallow attraction where the particles begin to touch, and

a strong repulsion at complete overlap. The particles deficient in quarks have strong attractive potentials.

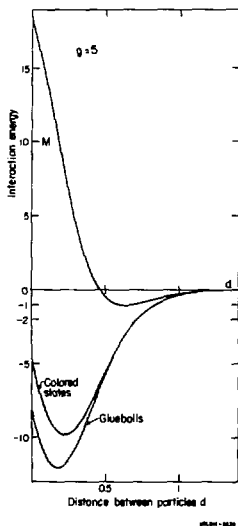


Fig. 2. Static potential of two composite particles in their ground-state with $M=10$ (upper curve). The attraction is due to the gluons, whereas the repulsion is due to the quarks. The extension of such particles calculated as an average value is 0.387 in units of $1/\sqrt{\lambda}$. The potential of colored-states and glueballs are purely attractive (lower curves). The parameters are the same as in Fig. 1.

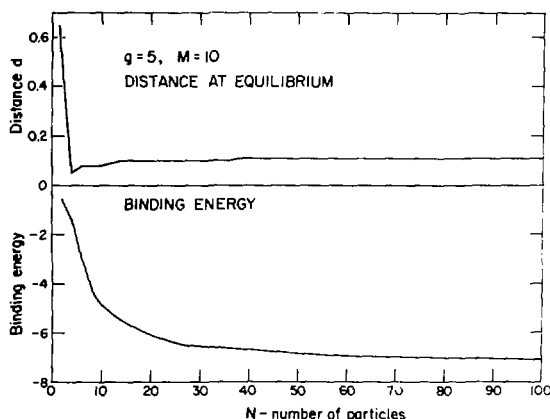
4. MANY PARTICLE INTERACTIONS: The ansatz in Eqs. (5-8) is generalized to systems with 2P particles, with the centers of the particles located at x_m , $m=1, 2P$. The centers have to be ordered: $x_{2P} \leq x_{2P-1} \leq \dots \leq x_2 \leq x_1$. The Φ -fields are:

$$\Phi_1 = 8^{-P} \Phi_V \prod_{m=1}^P (1 + 3 \tanh(ax - ax_{2m-1})) (-1 + 3 \tanh(ax - ax_{2m})), \quad (9)$$

$$\Phi_2 = 8^{-1} \sqrt{3} \Phi_V \sum_{m=1}^P (1 - \tanh(ax - ax_{2m-1})) (1 + \tanh(ax - ax_{2m})), \quad (10)$$

$$\psi_1 = \sum_{k=1}^{2P} (-1)^{k+1} \psi_1(x, x_k), \quad \psi_2 = \sum_{k=1}^{2P} (-1)^{k+1} \psi_2(x, x_k). \quad (11)$$

Calculations for a string of equidistant particles, $d = x_m - x_{m-1}$, show a marked dependence of the many-body



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Fig. 3. The interparticle distance and binding energy per particle for a string of N particles at equilibrium.

Saturation of many-body forces is observed [8]. The largest drop in distance and binding energy occurs when passing from two particle systems to four particle systems. The asymptotic value of the binding energy for large numbers of particles is about 70% of the rest mass $M = 10$ of the particles.

forces on the number of particles. When the number of particles in a system at equilibrium is increased the binding energy per particle increases and the distance between particles decreases, until they reach their saturation limit. The equilibrium point is defined by the minimum of the total energy of the system.

The particle (number) density in one dimensional systems is simply the inverse of the interparticle distance, provided all particles are equally spaced: $\rho=1/d$.

By computing the energy in a system of N particles at a distance d , we can get a representation of the equation of state -- energy as a function of density Fig.4. [9,10]. At the upper end of the curve, where the particles are squeezed into a very small volume, the value of the binding energy depends on the number of particles originally in the system.

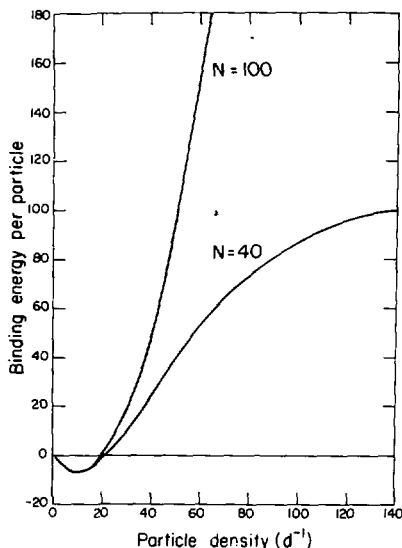


Fig.4. Equation of state for an ensemble of 40 and 100 particles of rest mass $M=10$.

CONCLUSIONS:

We present a mathematical model in 1+1 dimensions, that mimics i. the quark gluon picture of the structure of nucleons, ii. the two nucleon interaction and iii. some properties of nuclei and nuclear matter.

The calculations are done in a unified way using only the fields of the constituent particles of the "nucleons".

There are some new features as "colored" states, obtained when one fermion (quark) is removed (and transferred to a neighbor particle), and "glueballs" when both quarks are removed.

We calculate the interaction properties of these states (Fig.2), that might turn up in high energy experiments.

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NUCLEUS-NUCLEUS PROXIMITY IMAGINARY POTENTIAL

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Abstract : The imaginary part of the nucleus-nucleus interaction potential, for simultaneous excitations of the target and the projectile triggered by the two-body effective interaction and for one-body excitation induced by the single-particle field of the nucleus has been computed in the proximity approximation. A suitable two-body effective interaction has been devised to calculate the potential. For comparatively high relative energy, the two-body excitation mechanism seems to dominate, for low energy however, the one-body excitation is more important. The theoretical predictions agree reasonably well with appropriate experimental form-factors.

For two colliding nuclei, excitations of nuclear states can be initiated either by the single-particle field of the projectile exciting the target states (one-body excitation) or by the two-body nucleon-nucleon interaction potential inducing simultaneous excitations in both the colliding nuclei. Transfer of one or more particle across the neck of the nuclei, in proximate touch, can also lead to excitations - collective degrees of freedom, evidently, can be excited by any of the above mentioned mechanisms. To compute the imaginary potential for the one-body and the two-body excitation, we have used the traditional format of the second-order perturbation which, in general, is given by

$$U_2 = \sum_{\substack{m \neq 0 \\ n \neq 0}} \left[\langle \phi_0 \psi_0 | \sum_{i,j} v_{ij} | \phi_m \psi_n \rangle G_{mn} \langle \phi_m \psi_n | \sum_{i',j'} v_{i'j'} | \phi_0 \psi_0 \rangle \right] \quad (1)$$

where the propagator G_{mn} is given by

$$G_{mn} = \frac{1}{(2\pi)^3} \int d^3k \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{(\hbar^2/2\mu)(k_{mn}^2 - k^2) + i\epsilon} \quad (2)$$

a plane wave approximation has been used for relative motion between the nuclei and

$$\frac{\hbar^2}{2\mu} k_{mn}^2 = E - E_{mn} \quad (3)$$

E being the centre of mass energy for relative motion and E_{mn} is the sum of the excited state energies.

For two colliding nuclei, the centre of mass energy is usually much larger than the excitation energy - we shall, in this work assume that (i)

$E \gg E_{mn}$ and (ii) E_m or E_n can be approximated by an average constant value $\langle \Delta E \rangle$, the magnitude

of which would depend on the characteristic of the entrance phase^{1, 2}. With these two assumptions, one can now perform the integration over the kinetic energy of the two nuclei in the intermediate states, giving

$$\bar{G}(s) = G_{mn} \sim (\mu/2\pi\hbar^2) e^{iKs/s}$$

$$K^2 = (2\mu/\hbar^2)(E - \langle E \rangle)$$

where sensitivity of the average magnitude of $\langle E \rangle$ is investigated in detail.

$$U_2 = \bar{G}(s) \sum_{\substack{m \neq 0 \\ n \neq 0}} \left[\langle \phi_0 \psi_0 | \sum_{i,j} v_{ij} | \phi_m \psi_n \rangle \langle \phi_m \psi_n | \sum_{i',j'} v_{i'j'} | \phi_0 \psi_0 \rangle \right] \quad (4)$$

In this series, we now add and subtract the contribution for $\phi_m \equiv \phi_0$;

$$\psi_n \equiv \psi_0$$

Applying "closure" we get²

$$U_2 = \bar{G}(s) \left[\langle \phi_0 \psi_0 | \sum_{i,j} v_{ij} \sum_{i',j'} v_{i'j'} | \phi_0 \psi_0 \rangle - |\langle \phi_0 \psi_0 | \sum_{i,j} v_{ij} | \phi_0 \psi_0 \rangle|^2 \right] \quad (5)$$

There can be four types arising from the first matrix corresponding to

(a) $i = i', j = j'$ (b) $i \neq i', j \neq j'$ (c) $i \neq i', j = j'$ (d) $i = i', j \neq j'$ out of which

the terms corresponding to simultaneous excitations of particle-hole states arise from conditions (a) and (b) and the corresponding one-body excitation term arises from conditions (c) and (d); in configuration space the one-body excitation term takes the form

$$U_2'(R, s) = \bar{G}_1(s) \left[\int \rho_t^2(r) U_{12}(\vec{R} - \vec{r} + \frac{1}{2}\vec{z}_1) U_{12}(\vec{R} - \vec{r} - \frac{1}{2}\vec{z}_1) d\vec{r} - \right. \\ \left. \frac{1}{4} \int \rho_t^2(R_0 s_0) U_{12}(\vec{R} - \vec{r} + \frac{1}{2}\vec{z} - \frac{1}{2}\vec{z}_0) U_{12}(\vec{R} - \vec{r} - \frac{1}{2}\vec{z} + \frac{1}{2}\vec{z}_0) d\vec{r} d\vec{s} \right] \quad (6)$$

where U_{12} is the one-body potential responsible for exciting the particle-hole states in a nucleus, $\rho_t^2(R_0 s_0)$ is the density-matrix

On the other hand, starting from conditions (a) and (b) we have the potential for

simultaneous excitations yielding ($\vec{s}_0 = |\vec{s}_1 - \vec{s}_2|$)

$$U_2^2(R, s) = \bar{G}_2(s) \left[\int \rho_1(r_1) \rho_2(r_2) \psi(|\vec{r}_1 - \vec{r}_2 + \vec{R} + \frac{1}{2}\vec{s}|) \psi(|\vec{r}_1 - \vec{r}_2 + \vec{R} - \frac{1}{2}\vec{s}|) d^3r_1 d^3r_2 \right. \\ \left. - \frac{1}{16} \int \rho_1^2(R_1, s_1) \rho_2^2(R_2, s_2) \psi(|\vec{R}_1 - \vec{R}_2 + \frac{1}{2}\vec{s} - \frac{1}{2}\vec{s}_0 + \vec{R}|) \psi(|\vec{R}_1 - \vec{R}_2 - \frac{1}{2}\vec{s} + \frac{1}{2}\vec{s}_0 + \vec{R}|) d^3R_1 d^3R_2 d^3s_1 d^3s_2 \right] \quad (7)$$

It turns out that the second integral in eq. (7) is negligible but for eq. (6),

the second integral is significant. We calculate the one-body single-

particle potential U_{12} by folding in a suitable Gaussian representation of the $M3Y$ interaction of Bertsch et al³ yielding approximately a Woods-Saxon form factor with an average depth $V = 40$ MeV,

$r_0 = 1.05$ and $a = 0.65$ for - these results one should note depend on the combination of the isotopes chosen. For the two-body case, in eq. (7), not only the central component but the tensor component of the two-body interaction is also important⁴, the gaussian representation of $\psi(r)$ used in eq. (7) inclusive of the tensor component was chosen as

$$\psi(r) = -10.25 e^{-0.26 r^2} \quad (8)$$

The motivation of approximating eqns. (6) and (7) to a proximity form⁴ is primarily guided by the fact that for elastic scattering of two nuclei, the geometrical factors of the potential at or around the touching radius are the only quantities determined with any degree of confidence.

In eq. (7), the second term is neglected. Now both eqs. (6) and (7) can be expressed in terms of universal proximity functions, which depends nuclear density distributions and on the range of the two-body interaction

$$U_2^m(R) = (\mu/2\pi\hbar^2)(V_0^m)^2 \int_m^{\infty} (k) \frac{R_1 R_2}{R} \left(\epsilon_0^m + \frac{1}{2} y \frac{R_1 + R_2}{R_1 R_2} \epsilon_1^m \right) dy \quad (9)$$

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QUARK RECOMBINATION IN INCLUSIVE SPECTRA

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Abstract : The reaction $A_1 A_2 \rightarrow \left(\begin{smallmatrix} \pi^+ \\ K^+ \end{smallmatrix} \right) X$ has been analysed using the quark recombination model where a secondary meson is mediated by combining spectator constituent quarks of the baryons belonging to the incident nucleus with the anti-quarks from the sea, the even taking place at a specific value of the Bjorken variable $x_F \sim 1/3$. The inclusive cross-section $R(E \frac{d^2\sigma}{dp^3})$ is scaled in terms $R(PA_2 \rightarrow \pi^+ X)$ and/or $R(PP \rightarrow \pi^+ X)$, the scaling turns out to be a function of the probability functions that one or two constituent quarks will be wounded and also of the geometrical factors, depending on the location of the interaction. The model is moderately successful for low momenta secondary mesons. For high momentum component, the tripple Regge mechanism with the Δ and nucleon trajectory is invoked, where the elementary cross-sections are convoluted with the momentum distribution of the nucleons, bearing in mind the Pauli principle. The internal fermi motion is important. Finally, a generalised recombination model, depending on the probability distribution functions of up and down quarks is also developed to analyse the inclusive spectra.

As per contemporary wisdom, a hadron is made up of three (baryon) or two (a meson) quasi free, spatially discrete objects called quarks. For the collision of two nuclei at high energy there is a finite probability that a certain fraction of the constituent quarks go through the nucleus without interacting, determined by the density distributions of the interacting nuclei and the cross-section σ_{qN} . The secondary hadrons produced in this model are mediated by the non interacting spectator quarks recombining with either a quark or an antiquark from the sea and thus forming a baryon or a meson respectively, the process occurring at a specified value of the Bjorken scaling variable x_q of the constituents. For secondary pions, the region of x_q we shall concentrate is $x_q \sim 1/3$, as argued by Anisovich et al.¹. Evidently for outgoing pions this regime corresponds to low or medium momentum regions.

There remains the other important mechanism for large x^m of the secondary mesons, referred to in the literature as the Triple Regge Mechanism (TRM)², shown in fig. 1. The elementary $p\bar{p} \rightarrow \pi^+ X$ cross-section in TRM depends on the trajectory chosen, on x^m and t values. For $p\bar{p} \rightarrow \pi^- X$ the Δ trajectory is chosen and for $p\bar{p} \rightarrow K^+ X$ the relevant trajectories are of Λ and Σ . As will be pointed out in what follows that in TRM, the internal Fermi motion of the nucleons make the secondaries reach out for regions kinematically inaccessible for a nucleon-nucleon system not "dressed" by Fermi motion.

Apart from the recombination model for hadron nucleus collision at a fixed value of $x_q \sim 1/3$, for elementary $p\bar{p} \rightarrow \pi^+ X$ there has been

the striking experimental results³ at rather high energy, that the inclusive meson cross-section follow the spectator quark probability distribution function $u(x)$ for π^+ and $d(x)$ for π^- ; for large x^m , I have attempted to predict the inclusive data based on these functions.

We propose that the inclusive spectra for the species C for the reaction $A_1 A_2 \rightarrow CX, R_c = E_c \frac{d^3\sigma}{d^3p}$ consists of two components, recombination component and the TRM component denoted respectively by R_M and R_T , such that

$$R_c(p) = \frac{\sigma}{\sigma_{A_1 A_2}} \int d^2b \left[R_M(p, b) + R_T(p, b) \right] \quad (1)$$

where $\sigma_{A_1 A_2}$ is the total nucleus-nucleus cross-section and σ the total isospin averaged nucleon-nucleon cross-section; the quantities $R_M(p, b)$ and $R_T(p, b)$ are given by⁴ the product of the geometrical quantity and the momentum dependent quantity as given in ref. (4).

$$R_{M,T}(p, b) = \frac{dN_{M,T}(b)}{db} \tilde{R}_{M,T}^{A_1 A_2}(p) \quad (2)$$

In calculating $\tilde{R}_M$ and $\tilde{R}_T$ we need to know the probability that one or two of the constituent quarks get wounded while interacting with the target.

The probabilities depend on the density distributions of A_1 and A_2 and can be written in terms of the integrals over the impact parameters¹

$$V_n^{A_1}(A_2) = \left(3C_n / \sigma_{\mathcal{P}}^{A_1 A_2} \right) \int d^2b e^{-(3-n)\sigma_{\mathcal{P}} \tilde{T}_t(b)} \left[1 - e^{-\sigma_{\mathcal{P}} \tilde{T}_t(b)} \right]^n \quad (3)$$

where $n = 1, 2, 3$ correspond to the number of wounded quarks. From the normalisation condition $\sum_{n=1}^3 V_n = 1$ we get $\sigma_{\mathcal{P}}^{A_1 A_2}$; $\sigma_{\mathcal{P}} \sim 10 \text{ mb}$ and $\tilde{T}_t(b)$ is the standard profile function. Mesonic fragments in the recombination model can be produced when (1) two spectator quarks pick up two antiquarks

from the sea and one constituent quarks gets absorbed and/or (ii) one spectator quark picks up one antiquark from the sea, two other constituent quarks being absorbed in the nuclear medium. Now, applying the quark combinatorics¹

one can show

$$\tilde{R}_M^{A_1 A_2}(p) / \tilde{R}_M^{NN}(p) = (\sigma_P^{A_1 A_2} / \sigma_P^{NN}) (V_1^{A_1}(A_2) + \frac{4}{5} V_2^{A_1}(A_2)) \quad (4)$$

the factor 4/5 comes from the ratio of the mesonic fragments for the two processes mentioned above. For K^+ , the ratio is identical as expressed in eq. (4). One can also scale $A_1 A_2 \rightarrow \pi^- X$ in terms of $PA_2 \rightarrow \pi^- X$ in that case,

$$\tilde{R}_M^{A_1 A_2}(p) / \tilde{R}_M^{NA_2}(p) = (\sigma_P^{A_1 A_2} / \sigma_P^{NA_2}) (V_1^{A_1}(A_2) + 0.8 V_2^{A_1}(A_2)) / (V_1(A_2) + 0.8 V_2(A_2)) \quad (5)$$

Using eq. (5) and the data of ref.(5), we have computed $^{12}C + ^{12}C \rightarrow \pi^- X$ the inclusive data⁶.

Going on to TRM for two colliding hadrons Triple Regge limit is a special case of the single Regge limit namely, the region near the phase space boundary. For the case of $p\bar{p} \rightarrow \pi^+ X$ there will be Pomeron dominance in the $p\bar{p}$ vertex; the exchanged trajectory for $p\pi^+$ vertex there will be a nucleon or Δ whereas for the $p\pi^-$ vertex it will be only Δ trajectory, fig. 1; the elementary inclusive cross-section is given by²

$$\tilde{h}_t(p) = (s/\pi) \frac{d^2\sigma}{dt dM^2} = \beta(t) (1-x^m)^{1-2\alpha(t)}$$

where the missing mass square is $M^2 = s(1-x^m)$, x^m is measured in terms of maximum momentum: $\alpha(t) = 0.13 + 0.90t$ whereas $\beta(t) = 27 e^t \mu b \text{ GeV}^{-2} \text{ Sr}^{-1}$; $\tilde{h}_t(p)$ is then convoluted with the momentum distribution of the individual nucleons, bearing in mind the Pauli principle⁷; the internal Fermi motion effectively increasing the total

available S . Similarly, if one starts from the probability distribution function $u(x)$ for π^+ given by $(1-x^m)^{3.0}$ and $d(x)$ for π^- given by the form-factor $(1-x^m)^{4.5}$, the elementary cross-sections, exactly as in eq. (13), can be properly convoluted with the momentum distribution of the nucleons to obtain the $\tilde{R}_M(p)$. The results are shown in figs. 2 and 3.

In conclusion therefore, I would like to suggest that the formalism developed here is general enough and can be applied over a wide range of energy - the inclusive spectra for secondary hadrons for nucleus-nucleus collision at high momenta can be extremely useful for exploring the high momentum components of nucleonic motion and the dynamics of meson production in terms of the quark structure of hadrons. Microscopic models for hadron production in terms of quark degree of freedom and Regge mechanism appear to enjoy a modest success even for relatively low energy 1, 2 GeV/nucleon. Still higher energy data holds out tremendous promise for testing quark distribution functions and the behaviour of nucleons near the phase space boundary.

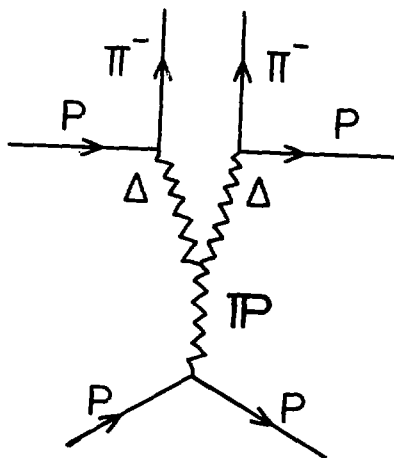


fig. 1.

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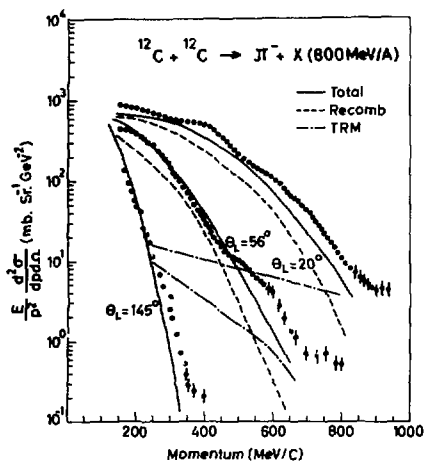


fig. 2.

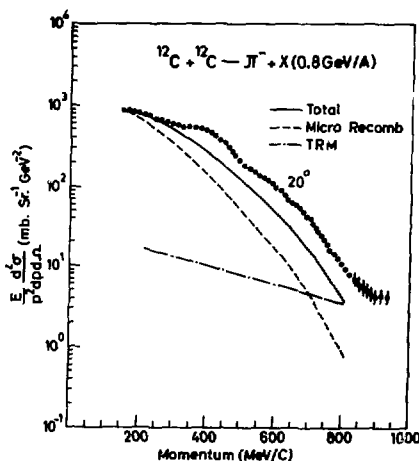


fig. 3.

Backward Emission in Hadron-Nucleus Collisions*

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Backward emission of hadrons in reactions of the type

$$P + T \rightarrow a + \text{anything} \quad (1)$$

where the projectile P is a hadron, T a nuclear target and a a hadron or a light nucleus has been the subject of experimental investigation in the last decade in an energy range E starting in the hundred MeV region and extending up to 400 GeV projectile energy. The main interest in these reactions lies in our view in the fact that they provide information about collective behavior of nucleons in nuclei (cumulative effect, i.e. the presence of secondary particles in a region of momentum space which cannot be populated by nucleon-nucleon interactions) although some authors² have recently "partonized" this effect. In particular the consequences of nuclear limiting fragmentation together with the cumulative effect can be used to obtain important information on transport properties and the equation of state of nuclear matter. Limiting fragmentation is a phenomenon discovered in the GeV region and applied to reaction (1) implies that in the high E limit two separate rapidity regions exist, one for the projectile and another for the target so that in each of the regions the inclusive cross section $d\sigma/d\epsilon d\Omega$ becomes independent of the incoming energy. Here ϵ and Ω refer to the kinetic energy and solid angle of the emitted particle. If we consider e.g. the target fragmentation region at a certain detector polar angle Θ all the data available so far in the energy range $0.6 \text{ GeV} \leq E \leq 400 \text{ GeV}$ obey roughly an exponential behavior in the variable ϵ

$$d\sigma/d\epsilon d\Omega = f(A_T, E) \exp(-\epsilon/\zeta(A_T, E)) \quad (2)$$

Limiting fragmentation which sets in at about 5 GeV projectile energy manifests itself by the fact that ζ and f become independent of E .

The analytic form of (2) is very suggestive for a statistical process and it will be shown that, indeed, the main features of the dependence of the inclusive cross section on the momentum of the emitted particle a can be understood in terms of "hot spots"^{3,4} created in the target. The concept of "hot spots" associated with thermodynamical preequilibrium has proven to be useful both in low energy nuclear physics⁵ as well as in high energy particle physics and it leads to an inclusive

* Work supported in part by BMFT and GSI Darmstadt

cross section of form (2) in which the inverse slope $\mathcal{T}$ is closely connected to an effective temperature.

As will be seen below the characteristic temperatures we get are of the order of tens of MeV and are therefore not consistent with an equilibrated nuclear system as treated in reference (6) but rather with a locally heated system.

We assume the existence of a class of events in which after the first collision of the incident projectile with nuclear matter an amount of energy-momentum $P^{h.s.}$ is deposited in a region which is small as compared with the dimensions of the target. This region will be called "hot spot" (h.s.). At small incident energies the projectile can be completely absorbed in the hot spot so that $P^{proj.} = P^{h.s.}$. At high energies however, because of limiting fragmentation in the target region we expect $P^{h.s.}$ ($P^{h.s.}$ we take parallel to the beam direction) to tend towards a constant limit when $E \rightarrow \infty$. As a consequence of momentum conservation the hot spot drifts into the nucleus. The drift distance d is measured from the point of first interaction to the center of the hot spot. It can be shown that only hot spots initiated at the backward surface are relevant for the cumulative effect. For this reason we measure d from the nuclear surface (cf. Fig. 1). At the same time the hot spot dissipates its $P^{h.s.}$ in the surrounding nuclear medium. This leads to an increase in time of the number of nucleons contained in the space region affected by the excitation. We assume that emission of particles occurs through excitation from and normal to the nuclear surface. The differential cross section $d^2\sigma/d\epsilon d\Omega$ (in the laboratory frame) then reads

$$\begin{aligned} d^2\sigma/d\epsilon d\Omega = & C W(P^{h.s.}) \sigma_{inv} \int_{\tau_0}^{\tau_R} dt \int_0^R db b h(b) \int_0^\infty d\epsilon' d^3z' \\ & \times \delta(\epsilon - \epsilon(\epsilon', z')) \delta(z - z(\epsilon', z')) \int (\vec{v}) (m + \epsilon') p'^2 \exp(-(\epsilon' + S)/\mathcal{T}) \end{aligned} \quad (3)$$

C is a normalization constant which can be determined from the equilibrium cross section. $W(P^{h.s.}) \times h(b)$ is the probability that a hot spot with four-momentum $P^{h.s.}$ is initiated by the incident projectile per surface element. $h(b)$ is the projection of the incident flux on the nuclear surface at impact parameter b . $W(P^{h.s.})$ is projectile dependent. σ_{inv} is the inverse cross section which depends on the emitted particle. Preequilibrium emission occurs at any time between a minimal time τ_0 which is needed to form a hot spot and the time τ_R after which equilibrium has been reached. The probability for surface emission of particles at temperature $\mathcal{T}$ is $(m + \epsilon') \cdot p'^2 \cdot \exp(-(\epsilon' + S)/\mathcal{T})$ where ϵ' is the kinetic energy of the emitted particle of mass m in the local rest frame (primed variables) of the excited nuclear matter. p' is the momentum of the emitted particle and $z = \cos \Theta$. S depends on the type of emitted particle:

$$S = \begin{cases} \text{separation energy for an emitted nucleon} \\ m_\pi \text{ for an emitted pion} \end{cases}$$

and $J(v)$ is the Jacobi determinant of the transformation from the local rest frame to the laboratory frame. The presence of the two d -functions and the two integrations over ξ' and z' are due to the z' dependence of $\vec{v}$.

Both $\vec{v}$ and $\vec{\tau}$ are functions of t , $\vec{r}_e$ and $\vec{r}_f$, where $\vec{r}'_e = (R, \Theta', \varphi')$ and $\vec{r}'_f = (R, \varphi'(b), \varphi')$ denote the points where particle emission and formation of the hot spot take place respectively. $\vec{\tau}$ and $\vec{v}$ can be obtained as solutions of the hydrodynamical equations of motion with appropriate boundary conditions which depend on $P^{h.s.}$. Here also nuclear matter properties enter the problem through ρ , c_p , χ and ν , where ρ is the nuclear matter density, c_p is the specific heat, χ is the thermal conductivity and ν stands for the viscosity coefficients. They are temperature dependent and carry therefore an additional $P^{h.s.}$ dependence with them. If we restrict ourselves to pre-equilibrium energies of $E \gtrsim 100$ MeV one can show that the detailed structure of the $\vec{v}$ -field has practically no influence on the spectrum. Therefore we take for $\vec{v}$ the average velocity which the hot spot has at the moment when the probability for particle emission is maximal. This is equivalent to a maximal surface temperature coming from a hot spot with a center which lies a distance $u = f(d, \Theta')$ away from the surface. d is the distance the hot spot has already drifted until this moment (cf. Fig. 1). It is a quantity which is directly measurable via the A_m -dependence of the cross section γ , so that it can replace a variable in the set ρ , c_p , χ and ν . We then have $\vec{v} = \vec{P}^{h.s.} / M^*$ where M^* is the mass of the hot spot, $M^* = [(\vec{P}^{h.s.})^2 + M^2]^{1/2}$ and $M = 4\pi/3 \rho u^3$. We are left then with $\vec{v}(P^{h.s.}, d, \Theta')$ and $\vec{\tau}(d, \rho, c_p, \chi, \nu, P^{h.s.}, \vec{r}_e, \vec{r}_f)$. We approximate $\vec{\tau}$ by the solution of the linear diffusion equation which can be written as $\vec{\tau}(d(P^{h.s.}), \chi, c_p, \rho, \nu, P^{h.s.}, \vec{r}_e, \vec{r}_f)$. In the range where it is applicable formula (3) reproduces the available data quantitatively. At backward angles formula (3) can be approximated by

$$\gamma = C W(P^{h.s.}) \sigma_{inv} (d(R-d)^2/\chi) (P^{h.s.}/\rho c_p)^3 (m + \epsilon') p' p \times \left(\frac{1}{(\epsilon' + S)^2 \tau_{max}^2} + \frac{2}{(\epsilon' + S)^2 \tau_{max}} + \frac{2}{(\epsilon' + S)^2} \right) \exp(-(\epsilon' + S)/\tau_{max}) \quad (4)$$

where $\tau_{max} = (2/3) (P^{h.s.}/\rho c_p) / (4\pi/3 \cdot d^3)$ and R is the target radius. This formula turns out to be applicable in the angular range $180^\circ \geq \Theta \geq 120^\circ$. It is remarkable that although there exists a continuum of temperatures, because of its exponential form the differential cross section depends essentially on one temperature τ_{max} . The transformation to the laboratory system does not change the exponential behaviour in the kinetic energy of the emitted particle; however, the inverse slope is no longer equal to the maximum temperature but becomes also a function of the hot spot velocity. The decrease of the inverse slope with angle is within the approximation of formula (4) merely a consequence of the relation $\epsilon' = \gamma(\epsilon + m - \vec{p} \cdot \vec{v}) - m$ i.e. of the hot spot velocity. γ and $\vec{v}$ depend on $P^{h.s.}$ and $\vec{P}^{h.s.}$.

respectively via the drift parameter. In addition $\bar{v}$ depends on the equation of state. In inclusive experiments, because of the dominance of one temperature one can also assume that only one single $P_0^{h.s.}$ is important. Then it is possible to determine $\bar{v}$ and $\bar{v}$ directly from the angular dependence of Ψ . As the drift parameter can be determined independently from the A_T -dependence $P_0^{h.s.}$ is fixed and if there exists a relation between $P_0^{h.s.}$ and P_0^c , ρ_c can also be fixed.

In the following we present some results of our computations and their comparison with data. In particular we consider reactions



where p and π denote protons and pions respectively. Fig.2 shows the A_T -dependence of the differential cross section for $27 \leq A_T \leq 207$ as given by equation (4) and the comparison with data. Excellent agreement is obtained for $d = 2$ fm independent of the projectile energy, as can be seen from Fig.3, where we present the dependence of Ψ on the kinetic energy of the emitted particles for different targets and projectile energies in the range $600 \text{ MeV} \leq E \leq 400 \text{ GeV}$ with $d = 2$ fm. Limiting fragmentation manifests itself through the existence of a limiting slope and cross section for high energies. From the polar angle θ dependence the hot spot velocity v is determined to be $v = 0.22 c$ and γ_{max} turns out to be 73 MeV (the inverse experimental slope is about 50 MeV). A comparison of the minimal $P_0^{h.s.}$ which must be transferred to the hot spot in order to explain the observed spectra with the $P_0^{h.s.}$ obtained from $\bar{v}$ shows that the formation of a hot spot is a highly inelastic process in which a local high temperature is achieved. We get a specific heat $c_p \gtrsim c_p^B$, where c_p^B corresponds to the Boltzmann equation of state.

A further confirmation of our picture is given by Fig.4, where the emission of pions and protons is compared. The same source parameters determine both the proton and pion spectra which means that they originate from one common source, i.e. the hot spot.

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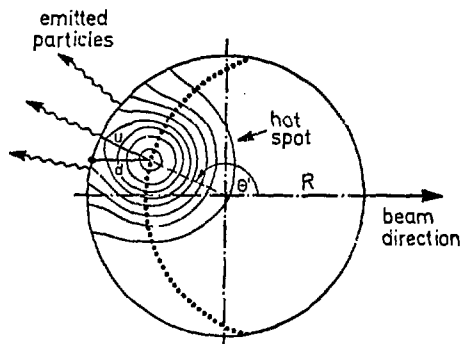


Fig. 1- Distribution of hot spots in a nucleus (their centers are represented by the dotted line) assumed to be created at the backward surface and drifted by the distance d in beam direction. The shortest distance of the hot spot center to the surface u depends on the polar angle. The hot spot is represented by its isothermals.

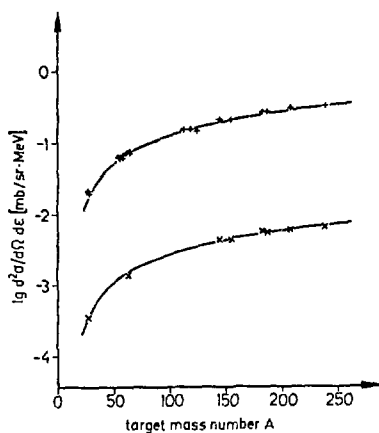


Fig. 2- Dependence of the cross section Ψ for reaction $p + T \rightarrow p + X$ on mass number A_T of the target for two values of the kinetic energy $\zeta = 133$ MeV (+) and $\zeta = 341$ MeV (x) at $\theta = 180^\circ$. The data points are from reference 12. The solid lines represent the theoretical results.

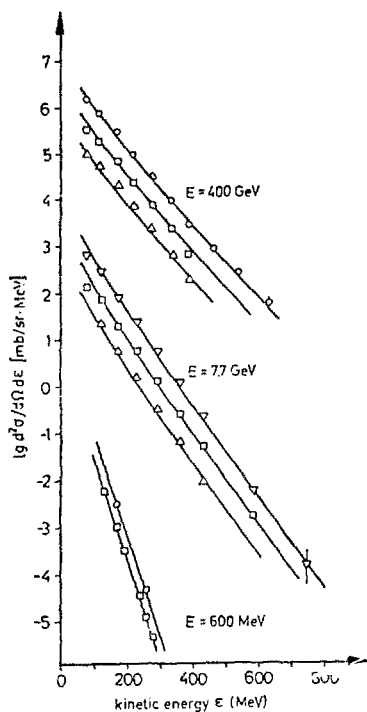


Fig. 3- Dependence of the cross section γ for reaction $p+T \rightarrow p+X$ on kinetic energy ϵ of the emitted protons for various targets ($^{27}\text{Al}(\Delta)$, $\text{Cu}(\square)$, $^{181}\text{Ta}(\circ)$, $^{207}\text{Pb}(\nabla)$) and incoming energy $E = 600$ MeV¹¹⁾, 7.7 GeV¹²⁾ (γ is multiplied by 10^3) and 400 GeV¹³⁾ (γ is multiplied by 10^6). $\Theta = 180^\circ$ for $E = 600$ MeV and 7.7 GeV. $\Theta = 160^\circ$ for $E = 400$ GeV.

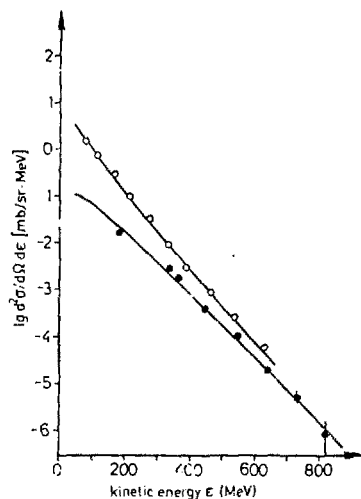


Fig. 4- Comparison of the dependence of the cross section γ for reaction $p+^{181}\text{Ta} \rightarrow p(\circ)+X$ and $p+^{181}\text{Ta} \rightarrow \pi(\bullet)+X$ on kinetic energy ϵ of the emitted particle for $\Theta = 160^\circ$ and $E = 400$ GeV¹³⁾.

Measurement of the Projectile Spectator Mass Distribution
with a Hadron Calorimeter

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University of California, Berkeley

We have recently measured the response of a hadron calorimeter to 2.1 GeV/nucleon ions ranging from ^2H to ^{20}Ne . The calorimeter was found to respond linearly to the total kinetic energy of the ion. The RMS resolution ranged from 50% for ^2H to 8.5% for ^{20}Ne .

In addition to the calibration we have performed an experiment to measure the total mass of projectile fragments by measuring their total kinetic energy. The calorimeter was placed such that it subtended an angle of $\pm 5^\circ$ vertically and horizontally about 0° . The energy deposited in the calorimeter was measured for each inelastic collision. The energy deposited in the calorimeter is proportional to the total mass of projectile fragments. In the framework of the abrasion-ablation model we measured the mass distribution of the projectile spectator. Single particle inclusive measurements of projectile (target) fragments are known to have mass distributions independent of the choice of target (projectile). By comparison our measurements of the total mass of projectile fragments depends strongly on the relative size of the projectile and target. For 2.1 GeV/nucleon $^{20}\text{Ne} + \text{Mo}$ the projectile mass distribution is relatively flat however for $^{20}\text{Ne} + \text{C}$ there are virtually no projectile residues smaller than mass ~ 5 . This result is in good quantitative agreement with a simple abrasion model. Modifications of the abrasion model to include transparency can be used to place limits on the nucleon-nucleon mean free path in nucleus-nucleus collisions.

CORRELATIONS CAUSED BY COLLECTIVE FLUID - DYNAMICAL FLOW*

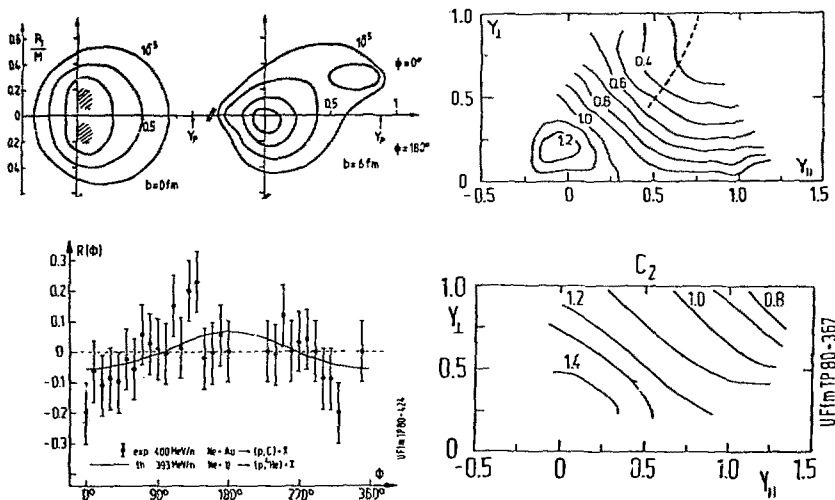
H.Stocker, LBL/GSI

L.P.Csernai, G.Buchwald, G.Graebner, J.A.Maruhn, and W.Greiner, University Frankfurt

Three dimensional fluid dynamical calculations with the description of final evaporation were performed for Ar + Pb and Ne + U at $E_{lab} = 400$ and 800 MeV/n. The triple differential cross sections show the signs of the Bounce - Off effect: An azimuthal anticorrelation between the projectile and target fragments. The correlation function calculated from this triple differential cross sections show a good agreement with experiments on 1) correlations between light and heavy fragments (see left side of the figure) and 2) two nucleon correlations (right). Thus our calculations confirm that correlations in collisions of heavy ions are mainly determined by the collective flow of the nuclear matter and microscopic correlations play only a minor role for collisions of very heavy systems.

The figure shows the azimuthal dependence of the triple differential cross section for the system Ne + U (upper left) at $E_{lab} = 393$ MeV/n and the corresponding correlation function for light to heavy fragments (lower left), together with the data.

The right hand side of the figure shows the calculated (upper right) and measured two nucleon correlation function (lower right) for the system Ar + Pb at $E_{lab} = 800$ MeV/n.



Data from H. Gutbrod et al. (left) and I. Tanihata et al. (right)

*Based on L.P.Csernai et al., to be published

Jets of Nuclear Matter from High Energy Heavy Ion Collisions

H. Stöcker

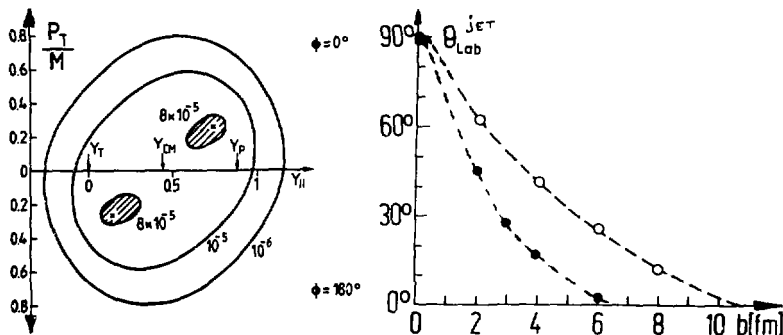
LBL/GSI

and

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Abstract: The nuclear fluid dynamical model with final thermal breakup is used to study the reaction $^{20}\text{Ne} + ^{238}\text{U}$ and $^{40}\text{Ar} + ^{40}\text{Ca}$ at $E_{\text{LAB}} = 390$ MeV/u. The calculated double differential cross sections $d^2\sigma/d\Omega dE$ are in agreement with recent experimental data. However, it is shown that the azimuthal dependence of the triple differential distributions $d^3\sigma/dE d\cos\theta d\psi$, to be obtained from 4π exclusive experiments with single event analysis, can yield considerably deeper insight into the collision process and allow for snapshots of the reactions. Strongly correlated jets of nuclear matter are predicted. The jet angle, θ_{jet} , relative to the beam axis increases from 0° for peripheral collisions to 90° for central collisions. A transition is predicted from a 180° azimuthally correlated two jet structure at intermediate impact parameters to a doughnut-shaped azimuthally symmetric distribution for central collisions. The figure shows the azimuthal dependence of the triple differential rapidity distribution (left) and the impact parameter dependence of the angle of the jet axis, θ_{jet} relative to the beam axis.



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$^{12}\text{C} + ^{12}\text{C}$ Reaction Cross Section between 10 and 24 MeV/A.*

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Measurements of the elastic scattering of $^{12}\text{C} + ^{12}\text{C}$ have been carried out at laboratory energies between 120 and 290 MeV. Our results, together with results at lower energies¹ have been analysed using a parameterized phase shift model placing special emphasis on the extraction of values of the reaction cross section σ_R . Examples of fits to our data are shown in Fig. 1 (plotted as a function of momentum transfer) and values of σ_R obtained in the energy range 70-290 MeV are displayed in Fig. 2. The reaction cross section apparently reaches a maximum near 8 MeV/nucleon (center of mass) in approximate agreement with a recent theoretical prediction of Di Giacomo et. al.²

Ref. 1. R. G. Stokstad et. al. ORNL/TM-5935.

Ref. 2. N. Di Giacomo et. al. - to be published.

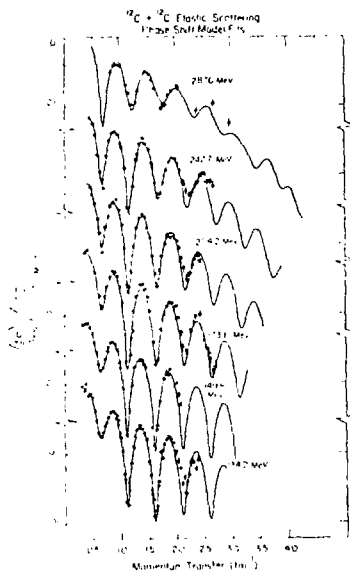


Fig. 1

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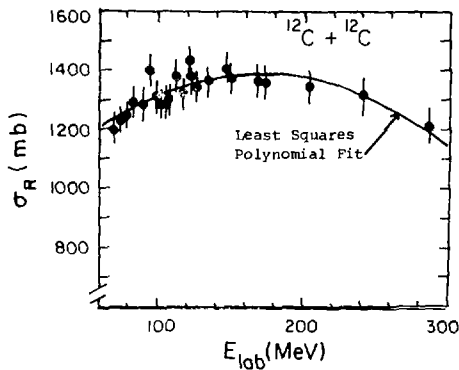


Fig. 2

Global Analysis of Relativistic Heavy-Ion Collisions

submitted by D. Strottman

A global analysis of relativistic heavy-ion collisions is proposed which entails a measurement of the momentum of all emitted particles. The resultant thrust is a measure of the final-state momentum flow and provides information on the global dynamics governing the collisions. This proposal is tested with a relativistic hydrodynamic model for collisions of equal-mass nuclei. The angle at which the thrust, or maximal directed momentum flow, occurs is 0° CM for grazing collisions and increases monotonically to 90° CM for central collisions. To facilitate interpretation and to develop some physical intuition we have made a color movie depicting collisions at four different impact parameters. Some other models of heavy-ion collisions predict qualitatively different results. Experimental observation of this dramatic behavior would imply an effective mean free path much less than the size of the colliding nuclei.

Some Systematics of Charged Pion Spectra Near Beam Velocity

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Doubly differential cross sections for the production of both π^+ and π^- near the incident beam velocity with pion lab angles from 0 to 20 degrees have been measured. Beams of ^{20}Ne with $E/A \approx 140, 280, 380, 480$ and 660 MeV and ^{40}Ar with $E/A \approx 535$ incident on targets from Be to U were used. A peak in the π^- spectrum and a depression in the π^+ spectrum was observed at 0° near the incident projectile velocity. The effect can be explained in terms of Coulomb interactions between pions and fragments of the incident beam. Least squares fits to the data using the Coulomb correction formulas of Gyulassy and Kauffmann¹ and an effective projectile fragment charge have been made. The systematics of the parameters extracted by this procedure and the relationship between these data and projectile fragmentation data^{2,3,4} will be discussed.

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AN OPTICAL MODEL APPROACH TO ABRASION-ABLATION

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Heavy-ion fragmentation has been described in terms of an abrasion-ablation model where the abrasion process is usually analyzed using high-energy Glauber theory or from classical geometrical overlap arguments. Recently, an alternative abrasion formulation utilizing an optical model potential approximation to the nucleus-nucleus multiple-scattering series has been developed, and theoretical estimates of abrasion cross sections for carbon and oxygen projectile nuclei reported. This optical model approximation to the multiple-scattering series converges more rapidly than Glauber's optical limit, and appears to be accurate for even very light nuclei. In the work reported herein, the optical model fragmentation theory is extended to include ablation effects. Theoretical cross sections are presented and compared with experimental results.

The high-energy heavy-ion component of galactic cosmic rays is likely to pose a significant hazard in future manned space operations for career astronauts/space workers in long-duration missions and/or numerous short-duration missions due to the cumulative effects of cell destruction and microlesion formation in non-regenerative tissues. Typical spacecraft material thicknesses would yield insignificant ionization energy losses. Hence attenuation through projectile fragmentation may be the only viable shielding mechanism in the near-term. Fragmentation of high-energy heavy-ion projectiles has typically been investigated within the framework of an abrasion-ablation collision model. The abrasion part of the process is usually analyzed from classical geometric overlap arguments^{1,2} or by utilizing high-energy Glauber theory.³ Recently an alternative abrasion formulation utilizing an optical model potential approximation⁴ to the nucleus-nucleus multiple-scattering series has been developed and reported.⁵ This particular approximation to the multiple-scattering series converges much more rapidly than the optical limit of Glauber's approximation and is accurate for even very light nuclei. In the work discussed herein, comparisons with recently reported⁶ experimental abrasion results will be made. Preliminary estimates for fragmentation cross sections, obtained by using the abrasion results of this theory in the abrasion-ablation model of Bowman et al.,¹ show remarkably good agreement with experiment.

In terms of the optical model potential approximation, the cross section for abrading m projectile nucleons is given by

$$\sigma_m = \binom{A}{m} \int d^2\vec{b} \{1 - \exp[-A_T \sigma(e) I(\vec{b})]\}^m \exp[-A_T A_F \sigma(e) I(\vec{b})] , \quad (1)$$

where

$$I(\vec{b}) = [2\pi B(e)]^{-3/2} \int d\vec{z} \int d^3\xi_T \rho_T(\vec{\xi}_T) \int d^3\vec{y} \rho_P(\vec{b} + \vec{z} + \vec{y} + \vec{\xi}_T) \\ \times \exp[-y^2/2B(e)] , \quad (2)$$

and the mass number of the residual projectile nucleus is A_F .

The total absorption (reaction) cross section is obtained by summing over all values of m according to

$$\sigma_{\text{abs}} = \sum_m \sigma_m . \quad (3)$$

The projectile (ρ_P) and target (ρ_T) densities used in equation (2) are ground-state single-particle number densities, which are extracted from nuclear charge distributions according to the procedure of reference 7. Average values for the energy-dependent nucleon-nucleon cross sections, $\sigma(e)$, and slope parameters $B(e)$, are taken from the compilations in reference 8. Nuclear charge distribution parameters are taken from the compilations in reference 9. Figure 1 displays results obtained from equation (1) for the collision of Ne projectiles at 2.1 GeV/nucleon incident kinetic energy with carbon target nuclei.

In order to compare with experiment, the theoretical relative probability is determined by folding the experimental detector efficiency with the relative probability (R.P.) given by

$$\text{R.P.} = \frac{\sigma_m}{\sigma_{\text{abs}}} \quad (4)$$

Since the experimental results include only residual fragments with $Z < 9$, the theoretical results from (4) are determined by excluding that half of $\sigma_1(m=1)$, which corresponds to single neutron removal, in both σ_m and σ_{abs} . Agreement between theory and experiment, as displayed in Figure 1, is excellent for residual masses, A_F , 8 or heavier, and indicates that this simple abrasion model is very accurate. Woods-Saxon charge distributions are used. Summing the abrasion cross sections according to (3) gives a theoretical absorption cross section of 1057 mb which agrees with the experimental value of 1040 ± 60 mb reported by the Dubna group.¹⁰

A preliminary investigation of ablation effects is made by using the results of Bowman et al.¹ Let the fragmentation (abrasion-ablation) cross section, σ_F , be

$$\sigma_F \equiv \alpha \sigma_{\text{abr}} \quad , \quad (5)$$

where σ_{abs} is the cross section for producing a particular projectile fragment by abrasion and α is an "ablation factor" which relates that particular abrasion cross section to the fragmentation

cross section for the same projectile fragment. In (5), the optical model abrasion cross sections of this work will be used with the ablation factors from reference 1 to estimate fragmentation cross sections for the reaction

$$O(2.1 \text{ GeV/n}) + Pb + \begin{bmatrix} N \\ C \\ B \end{bmatrix} + \text{anything} \quad . \quad (6)$$

The results are shown in Table 1 and compared to the experimental results of reference 11. Woods-Saxon charge densities were used for both nuclei. The agreement is again excellent.

In order to calculate σ_{abr} for use in (5), isotope production cross sections were determined from (1) by using a hypergeometric distribution according to

$$\sigma(Z, A) = \left(\frac{N}{n} \right) \left(\frac{Z}{z} \right) \frac{\binom{A}{m}}{\binom{A}{n}} \sigma_m(A) \quad , \quad (7)$$

with $n + z = m$ and $N + Z = A$. The σ_{abr} were then found by summing over all isotopes for a given fragment species

$$\sigma_{abr} = \sum_A \sigma(Z, A) \quad . \quad (8)$$

In summary, a simple optical model abrasion formalism has been shown to yield excellent agreement with recent experimental results. Preliminary fragmentation results, obtained by combining this abrasion

formalism with the simple ablation picture of Bowman et al., are in quantitative agreement with experimental results for relativistic oxygen projectiles colliding with lead targets. These results suggest that a simple quantum mechanical description of projectile fragmentation may yet be possible.

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Table 1. Optical model abrasion and fragmentation cross sections compared to experiment. Ablation factors are also shown.

Process	α	$\sigma_{abr}(\text{mb})$	$\sigma_F(\text{mb})$	$\sigma_{exp}(\text{mb})$
O + N	0.955	336	321	290.2 ± 34.5
O + C	0.956	215	206	227.8 ± 27.1
D + B	0.860	118	102	92.7 ± 12.8

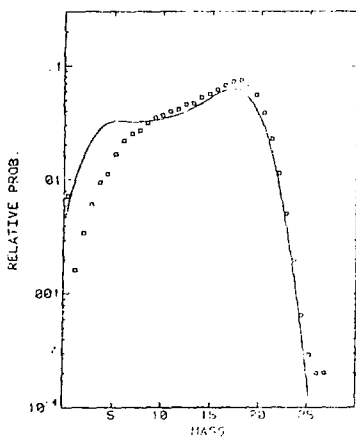


Figure 1. Abrasion results for 2.1 GeV/n Ne + C versus residual mass number. The solid curve gives the theoretical predictions. The squares are experimental results from reference 6.

FRAGMENT-SPRAY CORRELATIONS IN HIGH ENERGY HEAVY ION COLLISIONS*

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We report here the partially analysed results of an experiment performed at the Bevalac last year, focussing on heavy ion collisions leading to final states with heavy target fragments moving slowly in the laboratory frame. Typically, fragment energies are in the range 0.3 MeV/n to 3 MeV/n. Fragments across the entire mass range are measured.

We particularly wish to investigate the reaction mechanism. One of the models to be considered is a hydrodynamic description of such collisions. This model predicts distinctive 'bounce-off' features in

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which the transverse momentum transfer to the heavy fragment is accomplished through the expansion of a pressure zone.¹⁾ A more traditional approach has been to consider the Coulomb force as the origin of the kinetic energy of the heavy fragment, which, in this model, emerges as an evaporation product or fission fragment.²⁾ We do observe events for which such a Coulomb description is appropriate; however, we also observe, due to the exclusive nature of our experiment, events for which a Coulomb interaction between the fragment and a target remnant seems not to be an appropriate description of the final state. Such events could be candidates for a hydrodynamic description.

A further observation to be understood within the various models is the fact that the data are dominated by the total incident kinetic energy, independent of projectile mass.³⁾

Figure 1 shows, schematically, the experimental setup. The heavy fragments are detected inside the spherical scattering chamber by means of ΔE -E telescopes and TOF arrays. The spray of fast light charged particles (energies above about 20 MeV/n) is detected in the array of plastic scintillators, the 'paddles' covering a range of $9^\circ < \theta < 90^\circ$ and the 'wall' at $2^\circ < \theta < 9^\circ$. At this point we discuss the energy spectra of the heavy fragments at $\approx 90^\circ$ and the azimuthal correlation between these fragments and the spray in the paddles.

Figure 2 shows features which have invited the use of a two-step Coulomb description. Spectra of fragments in the mass range $24 \leq M < 28$ are shown for reactions induced by protons, alpha particles and ^{20}Ne projectiles at the same total bombarding energy (5 GeV). In each case

the spectrum has a peak and falls off exponentially, as in a Coulomb dominated evaporation process. Note the close similarity of the three spectral shapes, indicating that the fragment emission is a second stage of the reaction, more or less independent of the preceding energy dissipation stage, which is expected to differ widely from proton projectiles to ^{20}Ne projectiles. The mean free path of protons at this energy is such that they penetrate a considerable distance into the Au target nucleus before they undergo their first collision. At this stage there is some uncertainty in the normalization of these spectra.

Figure 3 shows the azimuthal correlation between the heavy fragments $M < 50$ and the spray of fast charged particles in the paddle array. The reaction is 400 MeV/n $\text{Ne} + \text{Au}$ and the fragments are detected at $\theta \approx 90^\circ$, $\phi \approx 0^\circ$. There is a strong azimuthal correlation between the spray and the fragment. Thus the fast spray plays a role in balancing the transverse momentum of the heavy target remnant. This coupling of the fragment emission to the fast stage of the reaction is in direct contradiction to the two-step interpretation and furthermore is an effect explicitly predicted by the hydrodynamic models. This experimental correlation has been called 'bounce-off' but has not been explicitly shown to originate with an expanding pressure zone.

In figure 4 we see some hope of resolving the contradictions between the two models discussed above. This contour plot is of yield against fragment energy and observed multiplicity of fast particles in the paddle array. The fragments are detected at $\theta \approx 90^\circ$ with $90 < M < 100$ from the reaction $\text{Ne} + \text{Au}$ at 250 MeV/n. There is a low multiplicity

component arising from peripheral collisions and exhibiting a Coulomb--type peak in the energy spectrum. The majority of the yield is at higher multiplicities, coming from more central, more violent collisions and apparently not dominated by a Coulomb interaction with a bound target remnant in the final state. At this stage it appears that this latter class of events is the most likely proving ground for a hydrodynamic description. In any case there is clearly room for several mechanisms operating simultaneously and we seem to have the data to separate them.

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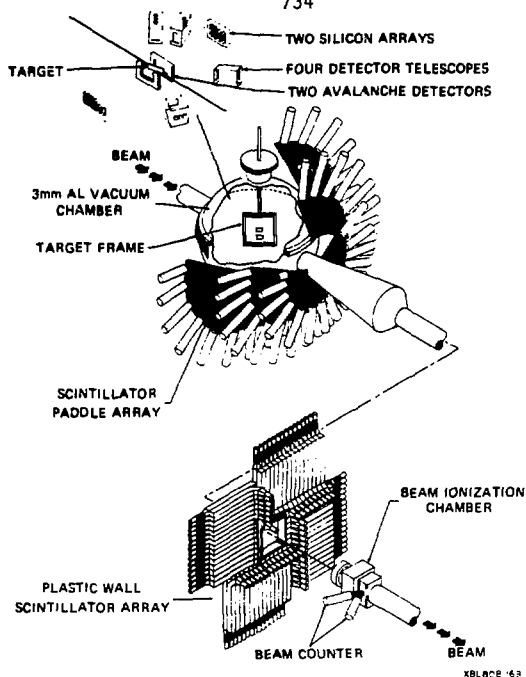


Figure 1: Schematic illustration of the experiment, the plastic wall is shown on a reduced scale.

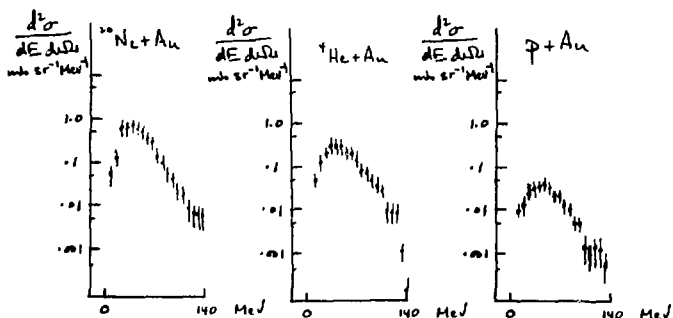


Figure 2: Comparison of energy spectra of fragments with masses in the range $24 \leq M < 28$, from reactions induced by projectiles with the same total energy (5 GeV)

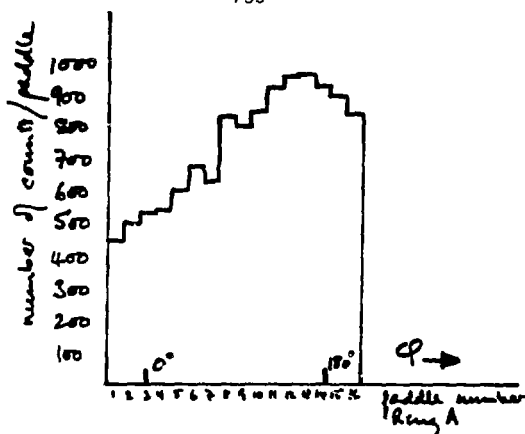


Figure 3: Yield of fast charged particles in paddle ring A as a function of azimuthal angle, observed in coincidence with a heavy fragment ($M > 50$) at $\theta = 90^\circ$, $\phi = 0$

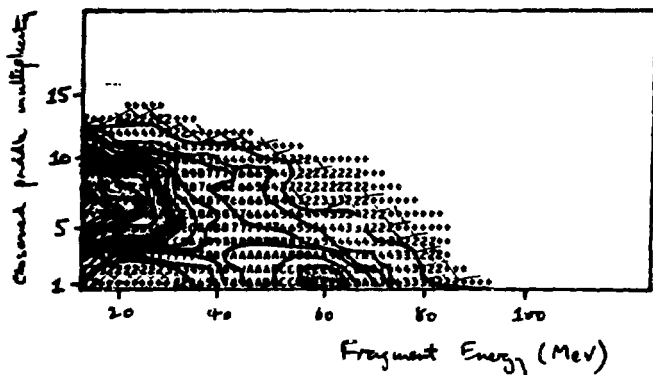


Figure 4: Contour plot of fragment yield against fragment energy and associated observed multiplicity of fast charged particles. The plot is for heavy fragments ($90 \leq M < 100$) observed at $\theta = 90^\circ$ from the reaction $\text{Ne} + \text{Au}$ at 250 MeV/n.

PION ANNIHILATION IN NUCLEI, SPIN, ISOSPIN AND ENERGY DEPENDENCE[†]

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To describe the propagation of different types of pions through nuclear matter with differing numbers of neutrons and protons, we have extended the optical potential to include terms describing the effect of pion annihilation on pn and pp/nn pairs with the accompanying possibility of spin flip. We modified and updated Mandelstam's isobar model of pion production in NN collisions, generalized the quasideuteron model of Eckstein and the Ericson, and fit the latest two nucleon pion production cross sections in all charge channels from threshold to $T_{\pi} \geq 400$ MeV. Then we extracted the spin, isospin and energy dependence of the annihilation strengths via projection of our phenomenological amplitudes.

Our results for the threshold values of the isoscalar B and C amplitudes are similar to those of other theoretical studies and somewhat smaller than the average phenomenological values. Naturally, our isobar model's predictions for the energy dependences of the microscopic $\text{Im}B$ and $\text{Im}C$ is of resonance shape. We find, however, that the s wave strength B is greatly reduced for $T_{\pi} > 30$ MeV by the "angle transformation" and loses much of its resonance shape - in agreement with some phenomenological studies.

The most important results of our work are the new determinations of the strengths, relative to the standard isoscalar (pn) term, of the potential terms describing annihilation on like (nn/pp) nucleon pairs and of pion spin flip scattering arising from the annihilation process (they can both be large). These should be particularly useful in studies of pion production in heavy ion collisions and of the nuclear structure of light nuclei deduced from pion scattering.

In the Figure below we show preliminary results for the relative strengths of "s-wave" absorption on like or unlike pairs, and p wave absorption on unlike pairs. λ is the probability for the NN pair to be in a P wave state.

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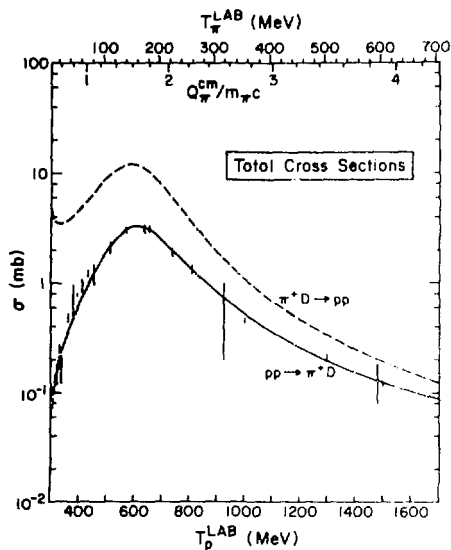


Fig. 1.

Theoretical cross section for single pion production $pp \rightarrow \pi^+ D$ (solid curve), and absorption $\pi^+ D \rightarrow pp$ (dashed curve).

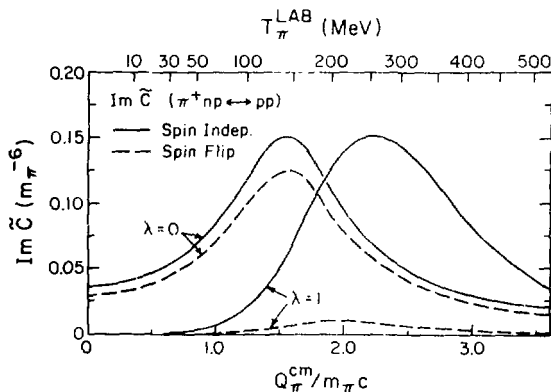


Fig. 2. The imaginary part of the π -NN p wave absorptive strength $\tilde{C}$ for absorption on a "like" pair of nucleons in a relative P state ($\lambda=1$). The dashed curve gives the strength of the spin flip (spin orbit) term.