A Few-Body Scattering Integral Equation Approach

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It is a common feature of N-body scattering integral equations like those of Faddeev-Yakubovski, Alt-Grassberger-Sandhas or Kouri-Levin-Tobocman that the kernels contain Green's functions or t-amplitudes of all subsystems. Thus, in order to calculate an N-body t-amplitude, one needs as input N-1, N-2, ..., two-body Green's functions or equivalent operators totally off shell.

For the investigation of few-body binding energy correlations, Lippmann-Schwinger-type equations with a simple structure of kernels turned out to be useful.[1] The aim of this note is to present N-body integral equations with simple kernel structures for the scattering of two composite particles in the incoming channel. We use only the bound state part of the subsystem Green's functions. This leads to coupled equations with kernels consisting of two-body interactions, the free Green's function, the bound state wave function of the N-1, N-2, ..., two-body subsystem and the corresponding poles. Thus, in order to calculate an N-body t-amplitude, one must first calculate only the N-1, N-2, ..., two-body bound states and binding energies. In the low energy region there is another advantage. If the scattering energy lies below some cluster breakup threshold, one can omit the pole corresponding to this cluster which reduces the number of coupled equations.

To derive our equations, we use the resolvent equation which correlates the full Green's function G, the free Green's function G_o, and the potential V. We decompose G in parts each of which contains only one pole, but the sum includes all poles. We find a coupled set of equations for the residues, which can be continued uniquely onto the real axis. Let us introduce the following notation: σ describes a subsystem, b is an overall quantum number, ϕ^b is a bound state wave function and E^b is the binding energy corresponding to a state b in the subsystem σ (ϕ^b_σ, E^b_σ includes also disjoint subsystems like $\phi^{dd}_{12,34} = \phi^d_{12} \times \phi^d_{34}$ and $E^{dd}_{12,34} = E^d_{12} + E^d_{34}$ in the N=4 case). $|\vec{p}\rangle$ is a plane wave state, where $\vec{p}$ stands for all Jacobi momenta. $|\vec{p},\phi^b\rangle_\sigma$ is a state with ϕ^b in the subsystem σ. V_σ is the interaction between particles contained in the cluster σ and $\bar{V}_\sigma = V - V_\sigma$. We define energy shells $ES^b_\sigma = \{\vec{p}_{os} | E - E^b_\sigma - p^2_{os} = 0\}$ and find that there is a positive distance between ES^b_σ and $ES^{b'}_{\sigma'}$ if $\sigma \subsetneq \sigma'$. We introduce an auxiliary pole split function $\tilde{\omega}^b_\sigma(p)$ which takes the value 1 if $p = p_{os} \in E^b_\sigma$, takes the value 0 if $|\vec{p}-\vec{p}_{os}| >$ 1/3 distance from ES^b_σ to the closest energy shell $ES^{b'}_{\sigma'}$, with $\sigma \subsetneq \sigma'$ or $\sigma' \subsetneq \sigma$, and is an arbitrary smooth interpolation in between. The operators I^b_σ, p^b_σ, ω^b_σ, O^b_σ, and K^b_σ are given by

$$I^b_\sigma = \int d\vec{P}\, |\vec{P},\phi^b\rangle_\sigma\, {}_\sigma\langle\vec{P},\phi^b|$$

$$\frac{1}{P^b_\sigma(z)}|\vec{P},\phi^b\rangle_\sigma = \frac{1}{z-E^b_\sigma-p^2}|\vec{P},\phi^b\rangle_\sigma$$

$$\omega^b_\sigma\, |\vec{P},\phi^b\rangle_\sigma = \tilde{\omega}^b_\sigma(p)\,|\vec{P},\phi^b\rangle_\sigma$$

$$O^b_\sigma = \omega^b_\sigma\, I^b_\sigma$$

$$K^b_\sigma = I^b_\sigma(1-\omega^b_\sigma G_0(z)V_\sigma)\frac{1}{P^b_\sigma(z)}I^b_\sigma$$

Finally, we define transition operators $X_{0\sigma}^{\ b}$, $X^{b'b}_{\sigma'\sigma}$ and residual states $|R^b\rangle_\sigma$, $|R\rangle_0$ via

$$G(z)O^b_\sigma = \sum_{\sigma'b'} O^{b'}_{\sigma'}G(z)O^b_\sigma + G_0(z)X_{0\sigma}^{\ b}(z)\frac{1}{P^b_\sigma(z)}I^b_\sigma$$

$$O^{b'}_{\sigma'}G(z)O^b_\sigma = \delta_{(\sigma'b',\sigma b)}\frac{1}{P^b_\sigma(z)}O^b_\sigma + I^{b'}_{\sigma'}\frac{1}{P^{b'}_{\sigma'}(z)}X^{b'b}_{\sigma'\sigma}(z)\frac{1}{P^b_\sigma(z)}I^b_\sigma$$

$$|R^{b'}\rangle_{\sigma'} = X^{b'b}_{\sigma'\sigma}(z)|\vec{P}_{os},\phi^b\rangle_\sigma - \bar{\delta}_{(\sigma'b',\sigma b)}\,O^{b'}_{\sigma'}P^{b'}_{\sigma'}(z)|\vec{P}_{os},\phi^b\rangle_\sigma$$

$$|R\rangle_0 = X_{0\sigma}^{\ b}(z)|\vec{P}_{os},\phi^b\rangle_\sigma + \sum_{\sigma'b'}\bar{\delta}_{(\sigma'b',\sigma b)}\,G_0^{-1}(z)O^{b'}_{\sigma'}|\vec{P}_{os},\phi^b\rangle_\sigma$$

Then the scattering equations read, where σ,b of the incoming channel are kept fixed,

$$K^{b'}_{\sigma'}(z)|R^{b'}\rangle_{\sigma'} = O^{b'}_{\sigma'}\Big(G_0(z)\bar{V}_\sigma|\vec{P}_{os},\phi^b\rangle_\sigma + G_0(z)VG_0(z)|R\rangle_0 + G_0(z)\bar{V}_{\sigma'}\frac{1}{P^{b'}_{\sigma'}(z)}|R^{b'}\rangle_{\sigma'} + G_0(z)V\sum_{\sigma''b''}\bar{\delta}_{(\sigma'b',\sigma''b'')}\frac{1}{P^{b''}_{\sigma''}(z)}|R^{b''}\rangle_{\sigma''}\Big)$$

$$|R\rangle_0 = G_0^{-1}(z)\Big(1-\sum_{\sigma'b'}O^{b'}_{\sigma'}\Big)\Big(G_0(z)\bar{V}_\sigma|\vec{P}_{os},\phi^b\rangle_\sigma + G_0(z)VG_0(z)|R\rangle_0 + G_0(z)V\sum_{\sigma''b''}\frac{1}{P^{b''}_{\sigma''}(z)}|R^{b''}\rangle_{\sigma''}\Big)$$

One can show the following properties.[2,3] The on-shell matrix elements of $X_{0\sigma}^{\ b}$ and $X^{b'b}_{\sigma'\sigma}$ and the on-shell scalar products of $|R^b\rangle_\sigma$ and $|R\rangle_0$ give the physical transition amplitudes. Unitarity relations among the operators $X_{0\sigma}^{\ b}$ and $X^{b'b}_{\sigma'\sigma}$ can be established, leading to the optical theorem. Finally, the scattering equations are unique for $Z = E+i\varepsilon$, $\varepsilon > 0$ and can be continued uniquely onto the real axis.

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[3] H. Kröger and R. Perne, Nuov. Cim. Lett., to be published.