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Title: **OPTIMIZATION OF STABILITY INDEX VERSUS FIRST STRIKE COST**

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Maximizing the stability index rather than minimizing first strike cost in allocating offensive missiles increases stability by 10-15% at the cost of a comparable and seemingly illogical increase in cost to the first striker.

This note studies the impact of maximizing the stability index rather than minimizing the first strike cost in choosing offensive missile allocations. It does so in the context of a model in which exchanges between vulnerable missile forces are modeled probabalistically, converted into first and second strike costs through approximations to the value target sets at risk, and the stability index is taken to be their ratio. The value of the allocation that minimizes the first strike cost for both attack preferences are derived analytically. The former recovers results derived earlier. The latter leads to an optimum at unity allocation for which the stability index is determined analytically.

For values of the attack preference greater than about unity, maximizing the stability index increases the cost of striking first 10-15%. For smaller values of the attack preference, maximizing the index increases the second strike cost a similar amount. Both are stabilizing, so if both sides could be trusted to target on missiles in order to minimize damage to value and maximize stability, the stability index for vulnerable missiles could be increased by about 15%. However, that would increase the cost to the first striker by about 15%. It is unclear why--having decided to strike--he would do so in a way that would increase damage to himself.

Review of earlier results. Exchanges between symmetric missiles forces can be described in terms of the first, F, and second, S, strikes that they could deliver. For a force of M vulnerable missiles with m weapons each, of which a fraction f is directed at the opponent's missiles, the first strike on value targets is

$$F = (1 - f)mM. \quad (1)$$

The average number of weapons delivered on each opponent vulnerable missile is

$$r = fmM/M = fm. \quad (2)$$

For r large, the average probability of survival is approximately[1]

$$Q \approx q^r, \quad (3)$$

where q = 1 - p, and p is the attacking missile's single shot probability of kill, which is taken to be the same for all missiles. The second strike is

$$S = mMQ \approx mMq^r. \quad (4)$$

S is delivered on value targets, as missiles remaining at the end of the exchange have no value.

Costs and stability index. These first and second strike magnitudes can be converted into the costs of striking first and second through exponential approximations to the fraction of the value targets destroyed. The cost of damage to self and of incomplete damage to the other fall on different parties; thus, they are incommensurate, but a conventional approximation is to take their weighted sum[2]

$$C_1 = (1 - e^{-kS} + Le^{-kF})/(1 + L), \qquad (5)$$

where $k \approx 0.001$ is a constant roughly equal to the inverse of the size of the value target sets held at risk[3] and L is a constant that represents the attacker's relative preference for inflicting damage on the other and preventing damage to self. This construction of C_1 as a weighted averages of the cost to self and other is plausible but not unique.[4] The normalized second strike costs cost to the second striker, who must ride out the first strike F, is

$$C_2 = (1 - e^{-kF} + Le^{-kS})/(1 + L), \qquad (6)$$

using the same constant L used above.[5] There is some arbitrariness in converting C_1 and C_2 into stability indices.[6] The ratio of costs C_1/C_2 is used below based on the basis of the argument that if the cost of striking first, C_1, is large, the first striker should be deterred from initiating an exchange, but if the cost of striking second, C_2, is small, both sides should see little penalty in riding out a crisis. The ratio C_1/C_2 captures both of these effects in a single stability index

$$I = C_1/C_2 = (1 - e^{-kS} + Le^{-kF})/(1 - e^{-kF} + Le^{-kS}) \qquad (7)$$

To minimize first strike costs, the attacker minimizes C_1, which for moderate forces, i.e., $F, S << 1/mk$, reduces to

$$C_1 = [L + k(S - LF)]/(1 + L) = \{L + k[mMq^{fm} - L(1 - f)mM\}/(1 + L), \qquad (8)$$

whose minimum can be found by differentiation with respect to f

$$f_1 = \ln(-L/m \ln q) / (m \ln q), \qquad (9)$$

which depends inversely on m and only logarithmically on L and q. Figure 1 compares the optimal allocation f_1 of vulnerable $m = 3$ warhead missiles with the result of the iterative solution of Eq. (8) for $M = 200$ missiles. The agreement is good throughout. Both curves fall gradually from ≈ 1 to 0.15 as L increases from $L = 0.2$ to $L = 1$.

Figure 2 shows the first and second strike costs and index, C_1, C_2, and I for the f_1 of Fig. 1. C_1 increases with L, but C_2 increases more rapidly, so I falls with L. I is slightly above unity for $L < 0.2$, but for larger L, I drops to about 0.8, which denotes about a 20% drop in stability for more aggressive attacker behavior.

To maximize the stability index, the attacker would chose the f that maximizes I. For moderate forces, $F, S << 1/k$, I reduces to

$$I \approx C_2 - C_1 = [k(S - F) \approx k[mMq^r - (1 - f)mM]. \qquad (10)$$

whose extremum can be found by differentiation to be $f_1 = \ln(-1/m \ln q)/(m \ln q) \approx 0.4$. However, Fig. 3 shows that this corresponds to the minimum of I. The maximum is produced by f = 1 for a variety of values of L; thus,

$$I_{max} \approx kmMq^m, \qquad (11)$$

for all L >> kF, S. Figure 4 shows C_1 as a function of L for the values of f that maximize I and that minimize C_1. The costs are similar for small L, but diverge for L > 0.5. By L = 2, the cost for maximizing I is about 20% higher. Figure 5 shows C_2 as a function of L for f that maximize I and minimize C_1. Costs are similar for L = 0.2, but diverge for larger values, converging again at about 1.5. For L = 0.5 to 1, the cost for maximizing I is 10-20% higher. Figure 6 shows the indices from maximizing I and minimizing C_1. The former is roughly constant at about unity in accord with Eq. (10). The latter falls for reasons discussed in conjunction with Fig. 2. Thus, the difference between them--and the penalty for maximizing I--grows to about 15% for intermediate values of L.

Summary and conclusions. This note studies the impact of maximizing the stability index rather than minimizing the first strike cost in choosing offensive missile allocations. It does so in the context of a model in which exchanges between vulnerable missile forces are modeled probabalistically. Results are converted into first and second strike costs through approximations to the value target sets at risk, and the stability index is taken to be their ratio. It is possible to analytically derive the value of the allocation that minimizes the first strike cost for both attack preferences studied. The former recovers results derived earlier. The latter leads to an optimum at unity allocation for which the stability index is determined analytically.

For values of the attack preference greater than about unity, maximizing the stability index increases the cost of striking first about 15%. For smaller values of the attack preference, maximizing the index increases the second strike cost about 15%. Each is stabilizing; thus, if both sides would target missiles in order to minimize damage to value and to maximize stability, stability indices for vulnerable missiles could be in creased by about 15%. However, that would increase the cost to the first striker by about 15%. It is unclear why--having decided to strike--the first striker would do so in a way that would increase damage to himself.

References

1. G. Canavan, "Probability of Survival from Multiple Weapon Attacks," Los Alamos report LA-UR-97-, February 1997.

2. G. Kent and R. DeValk, "Strategic Defenses and the Transition to Assured Survival," RAND Report R-3369-AF, October, 1986.

3. G. Canavan, "Costs of Strikes Between Vulnerable Missile Forces," Los Alamos report LA-UR-97-, February 1997.

4. G. Canavan, "Stability at Symmetric Low Force Levels," op. cit.

5. G. Canavan, "Destabilizing Effects of Perceptions," Los Alamos LA-UR-96-1742, May 1996

6. G. Canavan, "Impact of Differing Metrics on Crisis Stability Analyses," A. Zichichi ed., International Seminar on Nuclear War and Planetary Emergencies, 18th Session: Global Stability Through Disarmament (London, World Scientific, 1993).

1 f vs L; num & anal

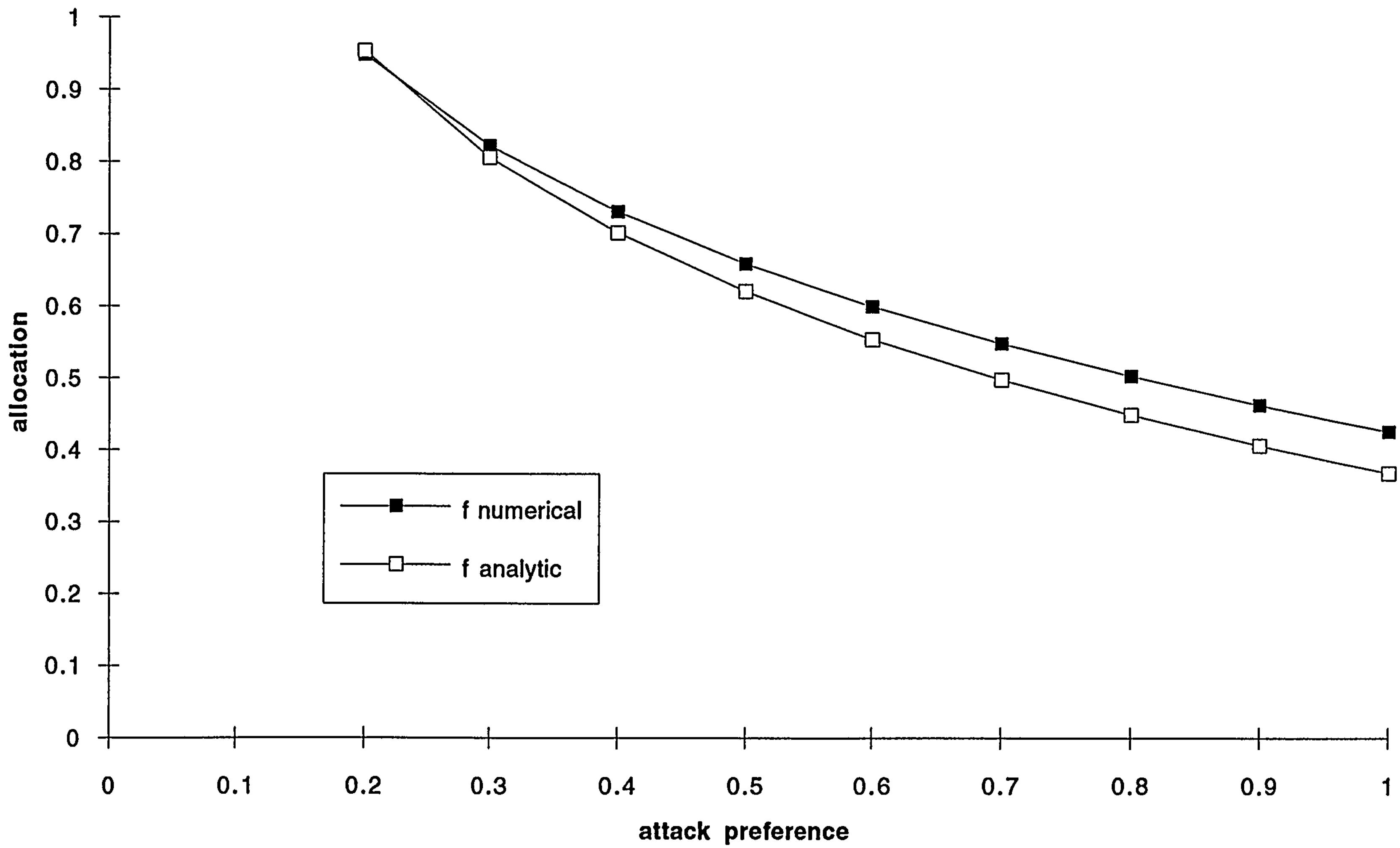

Fig. 1. Optimal missile allocation vs attack preference.

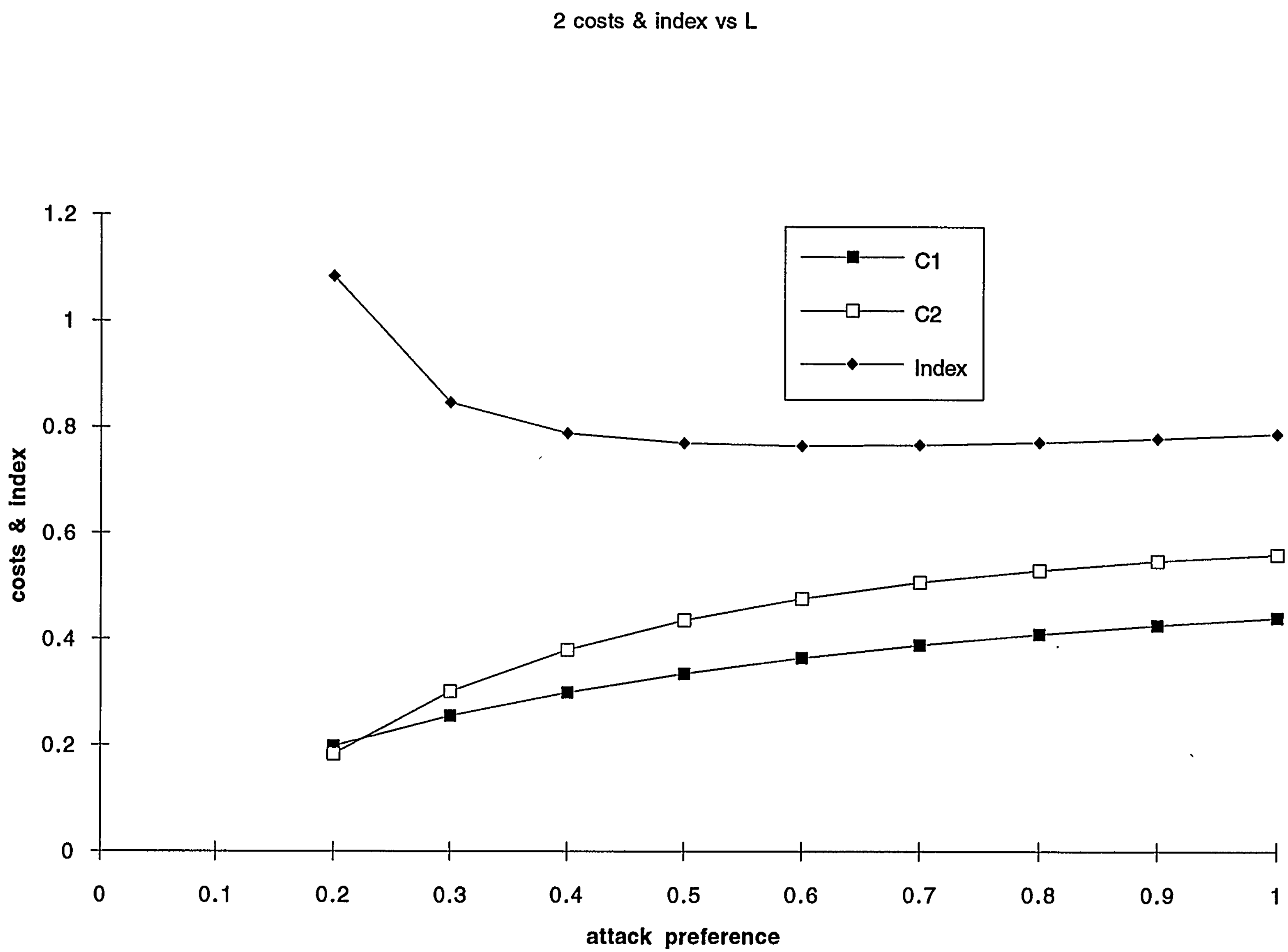

Fig. 2. Costs and index vs attack preference.

3 I vs f; L

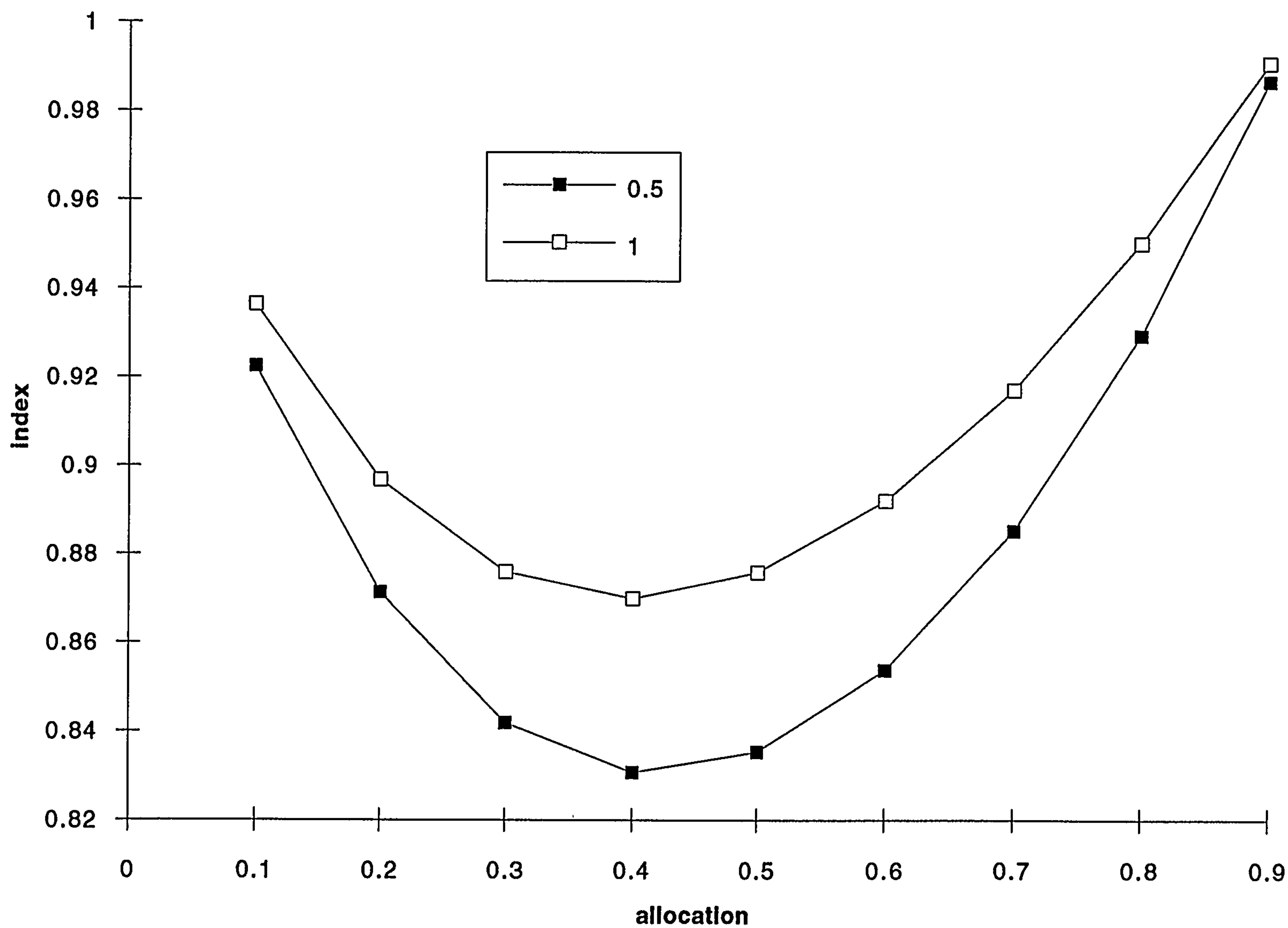

Fig. 3. Index versus allocation to missiles.

4 C1 vs L; fix

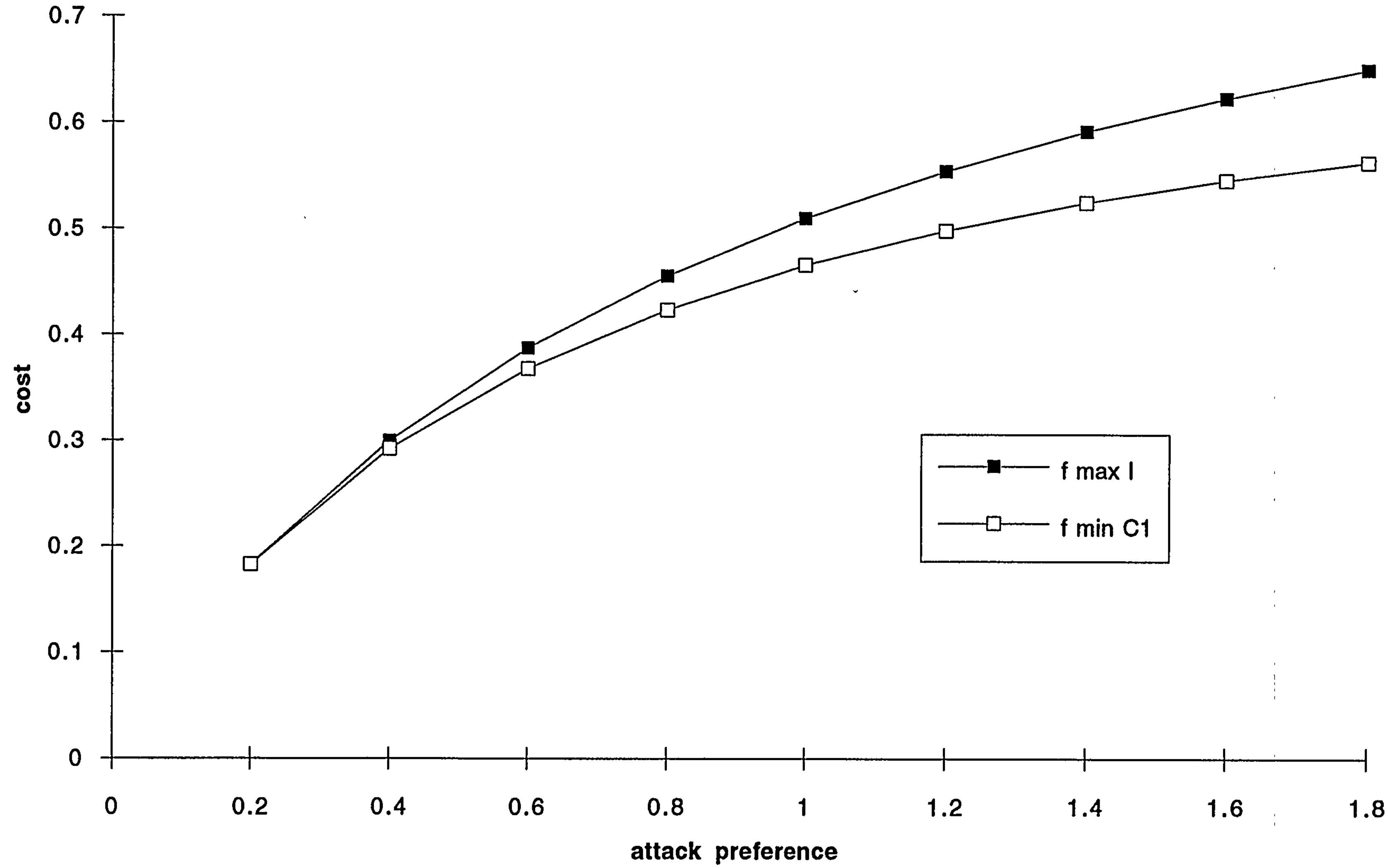

Fig. 4. First strike cost versus preference.

5 C2 vs L

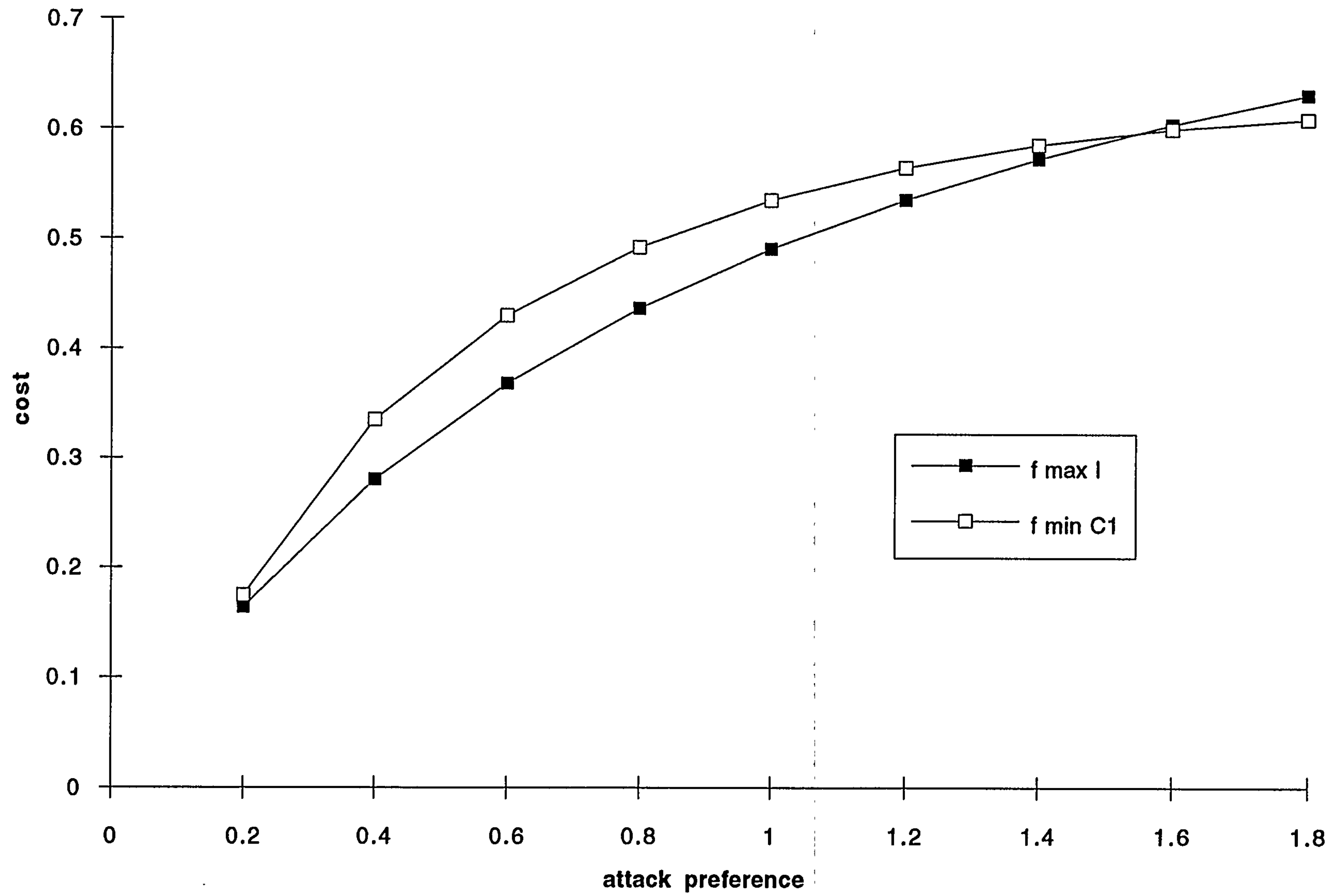

Fig. 5. Second strike costs versus attack preference.

6 I vs L; opt I & C1

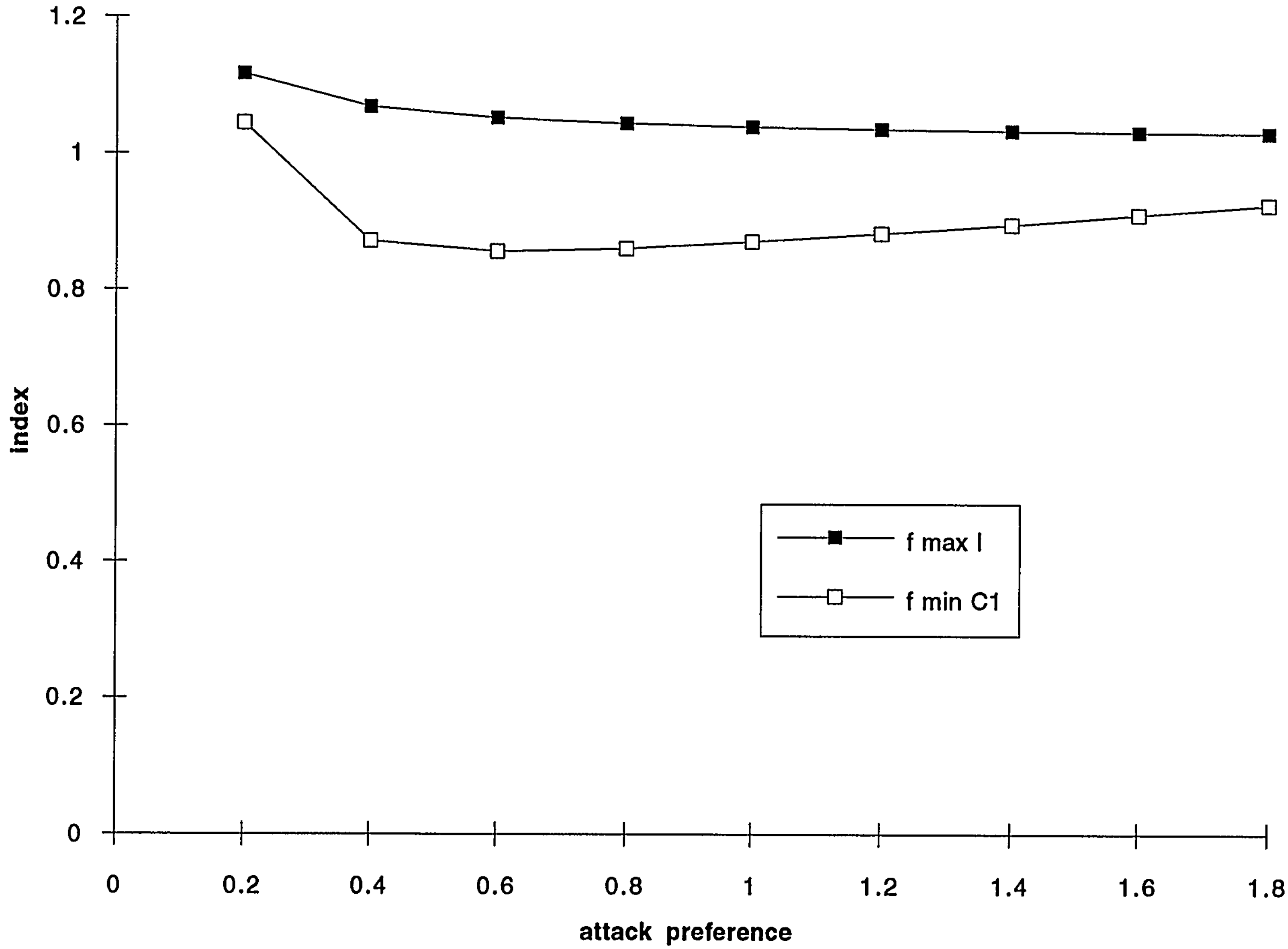

Fig. 6. Stability index versus preference.