COO-2245-48TR

DEPARTMENT OF NUCLEAR ENGINEERING
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge, Massachusetts 02139

June 1977

HYDRODYNAMIC BEHAVIOR OF A BARE ROD BUNDLE

by

J.G. Bartzis
N.E. Todreas

ERDA Research and Development
Contract E(11-1)-2245
U.S. Energy Research and Development Administration

DISCLAIMER

"This report was prepared as an account of Government-sponsored work. Neither the United States, or the Energy Research and Development Administration nor any person acting on behalf of the Commission

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness or usefulness of the information contained in this report, or that the use of any information, apparatus method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of, any information, apparatus, method, or process disclosed in this report.

As used in the above, 'person acting on behalf of the Commission' includes any employee or contractor of the Administration or employee of such contractor, to the extent that such employee or contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Administration or his employment with such contractor."

PATENT STATUS

This document, transmitted in advance of patent clearance, is made available in confidence solely for use in performance of work under contracts with the U. S. Energy Research and Development Administration (ERDA). It is not to be published nor its contents otherwise disseminated or used for purposes other than specified above before patent approval for such release or use has been secured from the cognizant assigned ERDA patent group.

Reports and Papers Published under
MIT Coolant Mixing in LMFBR Rod Bundles Project

A. Quarterly Progress Reports (Available from National Technical Information Service, U.S. Department of Commerce, Springfield, VA 22151)

COO-2245-1	Period June 1, 1972 - November 30, 1972
COO-2245-2	Period December 1, 1972 - February 28, 1973
COO-2245-3	Period March 1, 1973 - May 31, 1973
COO-2245-6	Period June 1, 1973 - August 31, 1973
COO-2245-7	Period September 1, 1973 - November 30, 1973
COO-2245-8	Period December 1, 1973 - February 28, 1974
COO-2245-10	Period March 1, 1974 - May 31, 1974
COO-2245-13	Period June 1, 1974 - August 31, 1974
COO-2245-14	Period September 1, 1974 - November 31, 1974
COO-2245-15	Period December 1, 1974 - February 28, 1975
COO-2245-23	Period March 1, 1975 - May 31, 1975
COO-2245-25	Period June 1, 1975 - August 31, 1975
COO-2245-26	Period September 1, 1975 - November 30, 1975
COO-2245-28	Period December 1, 1975 - February 29, 1976
COO-2245-30	Period March 1, 1976 - May 31, 1976
COO-2245-31	Period June 1, 1976 - August 31, 1976
COO-2245-34	Period September 1, 1976 - November 30, 1976
COO-2245-38	Period December 1, 1976 - February 28, 1977

Reports Issued Under This Contract

B. Topical Reports (Available from National Technical Information Service, U.S. Department of Commerce, Springfield, VA 22151)

E. Khan and N. Todreas, "A Review of Recent Analytical and Experimental Studies Applicable to LMFBR Fuel and Blanket Assembly Design," COO-2245-4TR, MIT, Sept. 1973.

E. Khan, W. Rohsenow, A. Sonin and N. Todreas, "A Simplified Approach for Predicting Temperature Distribution in Wire Wrapped Assemblies," COO-2245-5TR, MIT, Sept. 1973.

T. Eaton and N. Todreas, "Instrumentation Methods for Interchannel Coolant Mixing Studies in Wire-Wrap Spaced Nuclear Fuel Assemblies," COO-2245-9TR, MIT, June 1974.

Y.B. Chen, K. Ip, N.E. Todreas, "Velocity Measurements in Edge Subchannels of Wire Wrapped LMFBR Fuel Assemblies," COO-2245-11TR, MIT, September 1974.

E. Khan, N. Todreas, W. Rohsenow, A.A. Sonin, "Analysis of Mixing Data Relevant to Wire-Wrapped Fuel Assembly Thermal-Hydraulic Design," COO-2245-12TR, MIT, September 1974.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "A Porous Body Model for Predicting Temperature Distributions in Wire Wrapped Fuel and Blanket Assemblies of a LMFBR," COO-2245-16TR, MIT, March 1975.

E. Khan, W.M. Rohsenow, A. Sonin, N. Todreas, "Input Parameters to the ENERGY Code (To be used with the ENERGY Code Manual) COO-2245-17TR, MIT, May 1975.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "Manual for ENERGY Codes I, II, III," COO-2245-18TR, MIT, May 1975.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "Manual for ENERGY Codes I, II, III Computer Programs," COO-2245-18TR Revision 1, MIT, July 1976.

P. Carajilescov and N. Todreas, "Experimental and Analytical Study of Axial Turbulent Flows in an Interior Subchannel of a Bare Rod Bundle," COO-2245-19TR, MIT,

B. Chen and N. Todreas, "Prediction of Coolant Temperature Field in a Breeder Reactor Including Interassembly Heat Transfer," COO-2245-20TR, MIT, May 1975.

F. Carre and N. Todreas, "Development of Input Data to ENERGY Code for Analysis of Reactor Fuel Bundles," COO-2245-21TR, MIT, May 1975.

Reports Issued Under This Contract

B. Topical Reports, Continued

H. Ninokata and N.E. Todreas, "Turbulent Momentum Exchange Coefficients for Reactor Fuel Bundle Analysis," COO-2245-22TR, MIT, June 1975.

R. Anoba and N. Todreas, "Coolant Mixing in LMFBR Rod Bundles and Outlet Plenum Mixing Transients," COO-2245-24TR, August 1975.

B. Bosy, "Fabrication Details for Wire Wrapped Fuel Assembly Components," COO-2245-27TR, MIT, November 1975.

Ralph G. Bennett and Michael W. Golay, "Interferometric Investigation of Turbulently Fluctuating Temperature in an LMFBR Outlet Plenum Geometry," COO-2245-29TR, MIT, June 1976.

N.E. Todreas, "Analysis Methods for LMFBR Wire Wrapped Bundles," COO-2245-32TR, MIT, November 1976.

K.L. Basehore and N.E. Todreas, "Development of Stability Criteria and an Interassembly Conduction Model for the Thermal-Hydraulics Code SUPERENERGY," COO-2245-33TR, MIT December 1976.

Robert Masterson and Neil E. Todreas, "Analysis of the Feasibility of Implementing an Implicit Temporal Differencing Scheme in the SUPERENERGY Code," COO-2245-35TR, MIT, February 1977.

S. Glazer, C. Chiu and N. Todreas, "Collection and Evaluation of Salt Mixing Data with the Real Time Data Acquisition System," COO-2245-36TR, MIT, April 1977.

B. Mikic, E.U. Khan and N.E. Todreas "An Approximate Method for Predicting Temperature Distribution in Wire Wrapped Fuel Assemblies of a LMFBR," COO-2245-37TR, MIT, April 1977.

C. Chiu and N. Todreas, "Development of a Technique for Subchannel Flow Rate Measurements in LMFBR Wire Wrapped Assemblies," COO-2245-39TR, July 1977

C. Chiu and N. Todreas, "WARD Blanket Assembly Pre-Test Predictions by SUPERENERGY," COO-2245-40TR, July 1977

C. Chiu and N. Todreas, "Flow Split for a LMFBR 4" Wire Wrapped Blanket Assembly," COO-2245-41TR, July 1977

C. Chiu and N. Todreas, "Static Pressure and Pressure Drop for a LMFBR 4" Wire Wrapped Blanket Assembly," COO-2245-42TR, July 1977

Reports Issued Under This Contract

B. Topical Reports, Continued

C. Chiu and N. Todreas, "Mixing Experiments for a LMFBR 4" Wire Wrapped Blanket Assembly," COO-2245-43TR, July 1977

Yi Bin Chen and Michael W. Golay, "Coolant Mixing in the LMFBR Outlet Plenum," COO-2245-44TR, June 1977.

J. Kelly and N. Todreas, "Turbulent Interchange in Triangular Array Bare Rod Bundles," COO-2245-45TR, July 1977

K.L. Basehore and N.E. Todreas, "Assessment of the Need to Incorporate a Variable Swirl Flow Model into the ENERGY Code," COO-2245-46TR, July 1977.

K.L. Basehore and N. Todreas, "Analysis of the Thermal-Hydraulic Behavior in the CRBR Secondary Control Assembly, Including Interassembly Heat Transfer Effects," COO-2245-47TR, July 1977.

J.G. Bartzis and N.E. Todreas, "Hydrodynamic Behavior of a Bare Rod Bundle," COO-2245-48TR, June 1977.

Reports Issued under this Contract

C. Papers and Summaries

Yi Bin Chen, Ka-Lam Ip, Neil E. Todreas, "Velocity Measurements in Edge Channels of Wire-Wrapped LMFBR Fuel Assemblies," American Nuclear Society Transactions Vol. 19, 1974, pp. 323-324.

P. Carajilescov, N. Todreas, "Experimental and Analytical Study of Axial Turbulent Flows in an Interior Subchannel of a Bare Rod Bundle," J. of Heat Transfer, Vol. 98, No. 2, May 1976, pp. 262-268 (Included as Appendix to Quarterly Progress Report, COO-2245-15).

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "A Porous Body Model for Predicting Temperature Distribution in Wire-Wrapped Fuel Rod Assemblies," Nuclear Engineering and Design, 35 (1975) 1-12.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "A Porous Body Model for Predicting Temperature Distribution in Wire-Wrapped Rod Assemblies Operating in Combined Forced and Free Convection," Nuclear Engineering and Design, 35 (1975) 199-211.

Ralph G. Bennett and Michael W. Golay, "Development of an Optical Method for Measurement of Temperature Fluctuation in Turbulent Flows," American Nuclear Society Transactions, Vol. 22, 1975, p. 581.

B. Chen and N. Todreas, "Prediction of the Coolant Temperature Field in a Breeder Reactor Including Interassembly Heat Transfer," Nuclear Engineering and Design 35, (1975) 423-440 (Included as Appendix to Quarterly Progress Report, COO-2245-23).

R. Bennett and M.W. Golay, "Interferometric Investigation of Turbulently Fluctuating Temperature in an LMFBR Outlet Plenum Geometry," Accepted for the ASME Winter Annual Meeting, Dec., 1976, (Included as Appendix in Quarterly Progress Report, COO-2245-30).

B.B. Mikic, E.U. Khan, N.E. Todreas, "An Approximate Method for Predicting Temperature Distribution in Wire Wrapped Fuel Assemblies of a Liquid Metal Fast Breeder Reactor," Mech. Res. Comm., Vol. 3, 353-360 (1976).

Reports Issued Under this Contract

C. Papers and Summaries (Continued)

L. Wolf, R. Karimi, I.Y. Kim, C.N. Wong, M.K, Yeung "2-D Thermoelastic Analysis of LMFBR Fuel Rod Claddings," Paper C4/d, 4th International Conf. Structural Mechanics in Reactor Technology, San Francisco, August 1977.

M. Yeung, L. Wolf, "Effective Conduction Mixing Lengths for Subchannel Analysis of Finite Hexagonal LMFBR Bundles," ANS Meeting, New York, June 1977.

C. Chiu and N. Todreas, "Flow Split Measurements In An LMFBR Radial Blanket Assembly," ANS Meeting, New York, June 1977.

ABSTRACT

Temperature distributions within the rod bundle of a nuclear reactor is of major importance in nuclear reactor design. However temperature information presupposes knowledge of the hydrodynamic behavior of the coolant which is the most difficult part of the problem due to complexity of the turbulence phenomena. In the present work a 2-equation turbulence model - a strong candidate for analyzing actual three dimensional turbulent flows - has been used to predict fully developed flow of infinite bare rod bundle of various aspect ratios (P/D). The model has been modified to take into account anisotropic effects of eddy viscosity.

Secondary flow calculations have been also performed although the model seems to be too rough to predict the secondary flow correctly. Heat transfer calculations have been performed to confirm the importance of anisotropic viscosity in temperature predictions.

All numerical calculations for flow and heat have been performed by two computer codes developed in the present work which were based on the TEACH code [71]. Also expeirmental measurements of the distribution of axial velocity, turbulent axial velocity, turbulent kinetic energy and radial Reynolds stresses were performed in the developing and fully developed regions. A 2-channel Laser Doppler Anemometer working on the Reference mode with forward scattering was used to perform the measurements in a simulated interior subchannel of a triangular rod array with P/D=1.124.

Comparisons between the analytical results and the results of this experiment as well as other experimental data in rod bundle array available in literature are presented. The predictions are in good agreement with the results for the high Reynolds numbers.

ACKNOWLEDGEMENTS

The author is pleased to acknowledge the guidance, encouragement and inspiration provided by his supervisor, Professor Neil Todreas.

The author wishes to thank his thesis reader, Professor Michael Golay, for his valuable comments in this work.

The author is grateful to Mr. Paul Menadier, Mr. David Gwinn and Mr. Albert T. Supple, Jr., members of the MIT Reactor staff, for the useful help in the laboratory.

Thanks are also due to Betty Wytias, Virginia O'Keefe, Cindi Mitaras and Eva Hakala who performed most of the typing of this thesis.

The author wishes to express his sincere appreciation to his wife Georgia, for her patience and encouragement during this research effort.

This research was conducted as part of the "Coolant Mixing in LMFBR Rod Bundles Project," contract E(11-1)-2245 sponsored by ERDA. The author gratefully acknowledges this support.

TABLE OF CONTENTS

PAGE NO.

PAGE NO.

PAGE NO.

PAGE NO.

LIST OF TABLES

LIST OF FIGURES

PAGE NO.

PAGE NO.

NOMENCLATURE

AE, AW, AN, AS, AP	FDE coefficients (Eq. 4.44)
c_L	constant (Eq. 3.58)
c_P	coolant specific heat
c_μ	constant (Eq. 3.42)
c_2	constant (Eq. 3.39)
c_1, c_2	constants (Eq. 3.47)
$c_{\varepsilon 1}$, $c_{\varepsilon 2}$	constants (Eq. 3.72)
d	scattering particle size
D	rod diameter
D_H	hydraulic diameter of the infinite array
D_H^T	hydraulic diameter of the test section
D_i	clad inner diameter
E	constant (Eq. 3.80)
f_D	Doppler frequency
g	gap length
k	turbulent kinetic energy
l_r, l_θ	length scales
L	entrance length
P	pitch
q'	linear heat generation rate
q_o''	outside clad heat flux
q_i''	inside clad heat flux
r	radius
$2r_o$	diameter of the pipe or the distance between parallel plates
T_c	coolant temperature

T_{cl}	clad temperature
T_{co}	outside clad temperature
$\bar{T}_{co}$	average outside clad temperature
u_b	bulk velocity of the infinite array
u_b^T	test section bulk velocity
u_o	maximum channel velocity at L/DH = 113
u_{oL}	maximum channel velocity in position L
u^*	local friction velocity
$\bar{u}^*$	average friction velocity
v_r, v_r'	mean and rms radial velocity
v_θ, v_θ'	mean and rms tangential velocity
v_z, v_z'	mean and rms axial velocity
y	radial distance from the wall
$y+$	$y+ = \frac{yu^*}{\nu}$
$\hat{y}$	radial distance between the wall and zero shear stress line

GREEK LETTERS

α	constant (Eq. 3.63)
α_H	coolant heat diffusivity
$\delta\phi$	error in quantity ϕ
ε	energy dissipation
ζ	general turbulent quantity
θ	angle
κ	Von Karman constant
λ_c	coolant heat conductivity
λ_{cl}	clad heat conductivity

ν	viscous kinematic viscosity
$\nu^T_{\alpha\beta}$	turbulent viscosity ($\alpha,\beta=r,\theta,z$)
ρ	coolant density
$\sigma_1,\sigma_2,\sigma_3$	dimensions of light intersection volume (Fig. 2.6)
$\sigma_\kappa,\sigma_\varepsilon,\sigma_T$	diffusion constants for k, ε and T_c
$(\tau_w)_{avg}$	average wall shear stress
ϕ	quantity $v_z,k,\varepsilon,v_\theta,v_r,P/\rho$
ψ_m	eddy viscosity ratio

CHAPTER 1

INTRODUCTION

1.1 General

Heat transfer within the rod bundle of a nuclear reactor is of major importance in nuclear reactor design. The temperature field affects the mechanical and physical behavior of the reactor materials such as fuel and clad, as well as the neutron physics parameters. Thus, the temperature directly or indirectly imposes a limit on the thermal power produced by the fuel element. Especially in the LMFBR, the clad temperature is one of the significant design limits. Thus a detailed knowledge of the temperature within the fuel element and especially in the clad region is very important. However, temperature information presupposes knowledge of the hydrodynamic behavior of the coolant which is the most difficult part of the problem due to the complexity of the turbulence phenomena.

1.2 Assessment of Existing Methods for Rod Bundle Analysis

The methods which yield the detailed knowledge of the hydrodynamic (and/or thermal) behavior of the rod bundle are called Distributed Parameter (DP) Techniques. The DP techniques with respect to flow, as presently formulated, are restricted to fully developed flow and bare rod bundles which correspond

to special cases in a reactor such as nearly incompressible flow in an interior subchannel far away from disturbances.

The advantages of such a simplified case which is 2-dimensional are evident, i.e.,

1. Low computer time, since the problem is 2-dimensional and the solution is limited to a characteristic triangle (Fig. 1.1).
2. Modeling of the turbulent quantities is easier here than for 3-dimensional problems.

In addition such a simplified case is a powerful tool for understanding phenomena such as shear stress distribution and heat transfer coefficient variation along the periphery of the rod, secondary flow effects and thermal exchange coefficients between subchannels. Such information can be used for improving the solution from Lumped Parameter Techniques (LP) in which the properties are lumped over a characteristic region of the core (subchannel, bundle, etc.)

It is evident that any attempt to use a DP technique for analysis of a rod bundle of a reactor will have to cope with the following problems:

1. Modeling of the turbulent flow in 3-dimensions.
2. Very long computer time (which is the most severe difficulty).

With respect to modeling by existing methods which treat turbulent flows in 3 dimensions, the turbulence model methods

are the most attractive due to

a) relative simplicity and flexibility to handle complicated problems since the prediction is based on adjustment of constants;

b) use of some turbulence "information" for predicting flow characteristics in a turbulent domain.

With respect to the long computer time the high computational costs will be a persistent difficulty. Because of the computer cost a 3-D DP technique can be most beneficial in design when it is applied "locally" at regions where one is interested in detailed information (e.g., hot spot, blockage) with boundary condtions supplied by an LP technique.

So it is evident from the above discussion that there is a need for developing 3-D DP techniques which can be applied in a local region of interest. A strong candidate technique is the turbulence model methods.

The turbulence models can be distinguished with respect to Reynolds shear stress closure as follows:

a. Models using the concept of eddy viscosity

b. Models using transport equations for Reynolds stresses.

In this approach the models using the concept of eddy viscosity are considered because they are simple, widely used for engineering problems, and require less computer cost. The most advanced eddy viscosity approach is the two equation turbulence model and therefore, it is used in the present approach.

However, before starting on the analysis of three-dimensional flows, it is better to fully understand the two-D fully developed flows as predicted by the turbulence equation models.

Therefore, in the present work the following goals were established:

1. Obtain, through experimental and analytical work a good prediction of the hydrodynamic behavior of the channel through a two-equation turbulence model in fully developed flow. Utilize this prediction method to make some preliminary calculations of heat transfer performance.
2. Extend experiments into the developing region to provide a data base for future model development.

1.3 Summary of Present Work

1.3.1 Experimental work

The experimental task was a continuation of the experimental work initiated by P. Carajilescov [1] in a test section designed to simulate a typical interior subchannel of a triangular array of a bare rod bundle with P/D ratio equal to 1.124.

A two-channel Laser Doppler Velocimeter used by Y.B. Chen [84] with 120 mm lens has been used for the measurements instead of the one-channel approach used in [1].

Measurements of the hydrodynamic parameter (axial velocity, turbulent kinetic energy, Reynolds stresses) were made in the

fully developed region (L/D_H= 113) at Reynold numbers of 65,000, 26,500 and 9,000.

With respect to the developing region, the above measurements were repeated for L/D_H = 15 and 46 for Re = 65,000 and 26,000.

Attempts were made to measure the secondary flow at Re = 65,000, but were unsuccessful.

1.3.2 Analytical work

For prediction of the flow in the fully developed region, the two-equation turbulent model using transport equation for turbulent kinetic energy and energy dissipation have been used. A non-isotropic model of eddy viscosity has been introduced to improve the model.

Secondary flow effects have been studied using the Lauder and Ying [37] model and the Bobkov et al. [2] model.

All calculations have been performed by a computer program developed in the present work based on the TEACH code [71].

For the convection terms in the finite difference equation the Up-Wind Derivative Difference Scheme Developed in the present work by the author has been used.

CHAPTER 2

EXPERIMENTAL WORK

2.1 Introduction

Experiments on test section simulating the infinite bare rod bundle have been performed by several authors. These experiments refer mainly to axial velocity measurements and to a lesser extent wall shear stress distribution. Very limited measurements exist for parameters such as turbulent kinetic energy, Reynolds stresses and secondary flow.

Table 2.1, which is based on Table II.1 of Ref. 1, summarizes the experimental work done in bare rod bundles.

The purpose of the present experimental work is the extension of P. Carajilescov's work as follows.

I. Fully Developed Region

Comparisons of the mean velocity profiles between positions L/D_H = 46 and 77 done in Ref. 1 did not confirm that the flow in position L/D_H = 77 was fully developed. Measurements are performed at L/D_H = 113 by extending the test section length.

To study the effect of Reynolds Number, measurements at Re = 9,000 and 65,000 are performed besides the Re = 27,000 used in Ref. 1. The Reynolds stresses are performed by simultaneous measurements of both velocity components utilizing a two-channel Laser Doppler Anemometer (see §2.2.3.)

II. Developing Region

The above measurements are also performed for L/D_H = 15 and L/D_H = 46 to examine entrance effects.

2.2 Apparatus

2.2.1 Test section

As mentioned earlier, the test section used by P. Carajilescov [1] has been used in the present work. For corrosion protection, however, the test section has been anodized. The cross section is shown in Fig. 2.1 and the dimensions are given in Table 2.2.

The test section used in Ref. 1 is composed of three 2-foot long sections, one of which is provided with windows for the laser light to pass through the moving fluid. The interchange of the three sections permits measurements at positions L/D_H = 15, 46, and 77.

In order to examine whether the data obtained at position L/D_H = 77 represents data in the fully developed region, the following modifications have been made in the test section.

I. A new section 28 inches long has been added which permits measurements at the position L/D_H = 113.

II. In order to decrease the entrance length for the flow, an entrance region has been inserted between the test section and the hydraulic loop pipe as shown in Figures 2.2 and 2.3. The entrance region has been added to suppress the undesired large-scale turbulence coming from the two-inch loop pipe. Such turbulence, the decay of which is roughly proportional to the inverse of the square of the eddy size, increases the entrance length. Thus by killing the pipe turbulence, the entrance region reduces the entrance length.

Additionally, the entrance region has been added to provide approximately the boundary conditions of uniform laminar mean flow at the entrance of the test section useful for the developing region measurements.

This is obtained by including within the entrance region a honeycomb consisting of 2mm coffee straws three inches long. The flow within each straw has a Reynolds Number below 2,000 in all cases, so that the flow at the exit of the honeycomb is approximately laminar.

The overall view of the modified test section is shown in Figure 2.2.

2.2.2 The loop

Since our original purpose was to obtain a Reynolds Number of 100,000 or even higher, the following modifications have been made in the loop used in Ref. 1.

I. Insertion of a 5 HP centrifugal pump in parallel with the already existing 1 HP pump

II. Replacement of the one-inch piping system by one and a half-inch pipe to decrease the pipe resistance and keep the cost relatively low

III. Addition of a heat exchanger connected with cold and/or hot water. The reasons for adding the heat exchanger were:

a. to cool the water due to pump warm-up

b. to heat the water to obtain higher Reynolds Number by decreasing the viscosity

IV. Increase the flow meter maximum capacity from 30 GPM to 74 GPM by changing the float

V. Addition of a purification loop using a fixed bed demineralizer to protect the whole loop from corrosion

The whole hydraulic loop is shown shematically in Fig. 2.4.

2.2.3 Laser Doppler Velocimeter

For the flow measurements, the two-channel Doppler Velocimeter operated in a Reference Beam Mode [58] used by Y. B. Chen [84] has been used. The details of the system as used in the present work are as follows:

2.2.3.1 Optical arrangement

The Laser light is provided by a SPECTRA-PHYSICS Laser Model 164. The beam consists of green light of wavelength 0.5145μ and a diameter of 1.5mm as shown in Table 2.2, and passes through a DISA 55L88 LDA Transducer [56].

Within the transducer, the laser beam is 50% split. One of the splitted beams is 40 MHZ shifted through a Bragg cell acousto-optic modulator which is part of LDA transducer and then is again 50% splitted to form what we call reference beams. The remaining beam is the so-called scattering beam. In other words, what has been obtained through the LDA transducer is a set of three parallel beams, one relatively strong unshifted (the scattering beam) and two weak 40 MHZ shifted (the reference beams.)

The relative position of the three beams is shown in Fig. 2.5, whereas the geometric distances are given in Table 2.2. In the present work the three beams pass through a 120mm lens. The use of 120mm focal length instead of 300mm used in [84] is due to the desire to obtain relatively small spatial resolution. The three beams intersect at the point under consideration. A drawing of the intersection region is shown in Fig. 2.6. The dimensions of the intersection regions are given in Table 2.2.

The scattering light generated within the intersection region is collected along the directions of the reference beams by two DISA 55L12 Photomultipliers, as shown in Fig. 2.5 and Fig. 2.7.

The beat frequence f_B of the reference and scattering light is directly related to the flow velocity component U_1 perpendicular to the symmetry line as shown in Fig. 2.6 as follows:

$$f_{PM} = 40 \text{ MHZ} - f_D \qquad (2.1)$$

where f_D is the Doppler frequency given by the relation [58]

$$f_D = \frac{1}{C} U_1 \qquad (2.1a)$$

where C the calibration factor given by the relation

$$C = \frac{\lambda}{2n \sin \frac{\beta}{2}} \qquad (2.2)$$

where λ is the light wavelength;

n is the fluid refractive index;

β is the intersection angle shown in Fig. 2.6.

2.2.3.2. Electronics

For processing the photomultiplier signal output, a number of techniques exist with associated advantages and disadvantages, given by F. Durst, A. Melling and T. H. Whitelaw in Ref. 58 Chapter 6.

In the present work the frequency tracking procedure has been used since it was available and it was used by previous investigators [1], [84.] The PM tube signal passes through a frequency mixer of a preselected frequency f_{LO} so that the frequency mixer output becomes f_{LO} + (40 MHZ - f_D.)

The final signal which passes through a frequency tracker is translated to a continuous voltage signal from which mean values, rms values and cross correlations are obtained by analog electronics.

The main advantages of the frequency tracker are its [57]:

I. Capability to provide real time demodulation of the Doppler signal

II. Relatively rapid data processing rate

III. Relative insensitivity to variations in particle concentration.

The main disadvantages are:

I. Performance deteriorates as the signal-to-noise ratio decreases.

II. Since its output is based on scattering light for more than one particle simultaneously, a relatively large broadening of the Doppler signal occurs due to random phase fluctuation [65] from the many particle system.

2.3 The Measurements

2.3.1 General

The parameters of interest in the present work are

$$\overline{v_z},\ \overline{v_r},\ \overline{v_\theta},\ v_z',\ v_r',\ v_\theta',\ \overline{v_r'v_z'},\ \overline{v_\theta'v_z'}$$

The parameters that are convenient to measure are (see Fig. 2.8)

$$v_z,\ v_1,\ v_2,\ v_z',\ v_1',\ v_2',\ \overline{v_1'v_z'},\ \overline{v_2'v_z'}$$

The relations between these two groups of parameters are simple and are given in Table 2.4.

2.3.2 Experimental Grid

With respect to the experimental grid, points have been selected which lie on lines parallel to the two-axis (see Fig. 2.1) and corresponding to angle differences of

3° for axial velocity

6° for stresses

The distance between two adjacent 2-lines is taken as:

$$(\delta s)_2 = \begin{cases} 0.3\text{mm for axial velocity} \\ 0.6\text{mm for stresses} \end{cases}$$

Some stress measurements close to the wall have been performed at $(\delta s)_2 = 0.3$mm.

The detailed grid is given in Appendix C.

2.3.3 Point positioning

Due to different refraction indices between water and air, the intersection point displacement measured in the air has to be corrected to correspond to real displacement within the water. According to Figure 2.9

$$(\delta s)_{air} = \frac{tg7.6°}{tg9.46°} (\delta s)_{water} = 0.7474(\delta s)_{water}$$

where

$(\delta s)_{water}$ stands for the actual displacement

$(\delta s)_{air}$ is the displacement as measured in the air

2.3.4 Scattering particles

One of the primary concerns was the scattering particle size since it affects the signal-to-noise ratio [85]. With respect to the particle size Yu. N. Dubnitshev, et al [12]

using Fourier optics has proved that for small sizes of particles and photomultiplier aperture, the optimum signal is obtained by

$$d = (2m + 1)\frac{\lambda f}{2a} \quad \text{for differential mode} \qquad (2.3)$$

$$d = (2m + 1)\frac{\lambda f}{4a} \quad \text{for reference mode}$$

where m is an integer

f is the focal distance

2a is the two-beam distance at the lens.

Present geometry gives for m = 1, an optimum particle size of .77μ.

In present work, polystyrene particles of sizes 0.5, 0.79, 1.0, and 2.0 were investigated. The best signals were obtained utilizing 0.79μ and 1.00μ particles. This result supports the correctness of Eq.(2.3.)

2.3.5 Measurements

Measurements were performed at the axial position L/D_H = 15, 46, and 113 and Reynolds Numbers close to 9,000, 26,500, and 65,000. Results are given in Appendix C and in Figures 5.1-5.18.

The measurements are characterized by good results for axial velocity and axial velocity fluctuations.

The measurements in the lateral and radial directions are less precise due to relatively low signal-to-nosie ratio.

The measurements and their errors will be discussed below.

Table 2.5 gives the frequency ranges and the Tracker used to obtain the results.

As we can see from Table 2.5, DISA 55L20 tracker has been used more in the low frequencies due to greater flexibility of frequency ranges and frequency bandwidths. For the care of Re = 65,000 due to low signal-to-noise ratio, the only way to get lateral velocity measurements is a TSI Tracker combined with a TSI (0.6 - 2.0 MHZ) bandbass filter.

2.4 Experimental Errors

2.4.1 Axial Velocity

As usual, we can distinguish between random errors and systematic errors.

2.4.1.1 Random errors

This error represented by $\delta v_z'$ is due to the various signal frequency broadening mechanisms. While these broadening mechanisms cannot be eliminated, their effect on the error can be compensated by adopting a large integration time since the error is inversely proportional to $(T)^{1/2}$ [38], i.e.,

$$\delta v_z' \approx (2v_z'^2 \frac{F}{T})^{1/2} \qquad (2.4)$$

where v_z' is the rms value of axial velocity

F is the time scale defined as [38]

$$F = \int_0^{\infty} \frac{(v_z(t) - \bar{v}_z)(v_z(t+t) - \bar{v}_z)}{v_z'^2} \qquad (2.4a)$$

T is the integration time

The integration time has been taken equal to 10s so that the error has been limited for all cases below 0.5%. Achievement of this range of error has been confirmed by the spread of the data output of the digital voltmeter.

2.4.1.2 Systematic errors

The systematic errors can be mainly categorized as:

I. Nonuniform velocity errors

II. Position errors

III. Frequency Tracker errors

2.4.1.2.1 Nonuniform velocity error--The velocity gradient is associated with the fact that the velocity is not constant within the control volume. Therefore, what we take as an output is not the velocity at the center position of the control volume but some average over the control volume.

An exact analytical expression for velocity gradient broadening is very difficult to obtain. Approximate expressions have been obtained by many authors [57], [59], [60], [61.] Melling [57] considers Gaussian intensity distribution within the elliptic intersection region and velocity gradient only along one of the principal axes of the elliptic control volume whose length is $2\sigma_1$ (see Fig. 2.6.)

In this case,

$$(v_z)_{meas} - v_z \simeq \frac{\sigma_1^2}{2}\left(\frac{d^2 v_z}{dy^2}\right)_0 \qquad (2.5)$$

where 0 stands for the probe volume center point.

Preliminary calculations based on Formula (2.5) show that the above error is negligible.

2.4.1.2.2 Position error--The position error is of order of the probe volume size, i.e.

$$\Delta y \simeq 2b_o \simeq 0.053\text{mm}$$

Since the important velocity change is along the radial line, we can approximate

$$(v_z)_{meas} - v_z = \pm \Delta y \frac{dv_z}{dy} \qquad (2.6)$$

Taking

$$\frac{dv_z}{dy} = \frac{u^*}{\kappa y} \qquad (2.7)$$

we find finally

$$\frac{(v_z)_{meas} - v_z}{u_o} \simeq \pm \frac{u^*/u_o}{\kappa \frac{y}{\Delta y}} \simeq \pm \frac{u^*/u_o}{\kappa \frac{y}{2b_o}} \qquad (2.8)$$

It is evident from above expressions that above error is significant only for the point close to the wall, whereas for the points far from the wall the error is insignificant. For example, for the point ($\theta = 0$ $\frac{y}{2b_o} = 8.96$) which is closer to the wall and Re = 65,000 which is the worst case, the error is calculated to be 1.2% of the maximum velocity.

2.4.1.2.3 Frequency tracker errors-- Such errors can be mainly due to dynamic response of the instrument, slew rates limitation, and finite bandwidth of the tracker.

The error is taken as 1% full-scale deflection of the range in use [1], [75], i.e. 0.1v for all cases, since the range in use is always 10 volts.

Thus for Re = 9,000 (f_{max} = 500 KHZ, 1v = 50 HZ,) the error is of order 2% of the maximum velocity. For Re = 26,500 (i.e. f_{max} = 1500 KHZ, 1v = 150 HZ) the error is of order 2% of the maximum velocity, whereas for Re = 65,000 (i.e. f_{max} = 5000 HZ, 1v = 500 HZ,) the error is of the order of 3% of the maximum velocity.

2.4.2 Turbulent axial velocity

The major errors in obtaining rms values of velocities in general are due to various frequency signal broadenings.

If we could estimate the broadening effect, the real rms value, assuming Gaussian distributions, is given by the relation

$$v_z'^2 = \left[\left(v_z'^2\right)_{MEAS} - \left(v_z'^2\right)_{BROAD} \right]^{1/2} \qquad (2.9)$$

The broadening effects can be distinguished mainly as

1. Finite transit time broadening $\left(\delta v_z'\right)_L$
2. The non-uniform mean velocity broadening $\left(\delta v_z'\right)_G$
3. The turbulent velocity gradient error $\left(\delta v_z'\right)_T$
4. Signal processing error $\left(v_z'\right)_{LAM}$

Broadenings due to particle Gaussian motion, particle size and population fluctuation, Laser linewidth and flow rate fluctuation have been neglected as small compared to above-mentioned broadening effects. The quantity $v_z'^2$ will be given by the relation

$$\left(v_z'^2\right)_{BROAD} = \left(\delta v_z'\right)_L^2 + \left(\delta v_z'\right)_G^2 + \left(\delta v_z'\right)_T^2 + \left(v_z'^2\right)_{LAM} \qquad (2.10)$$

2.4.2.1 Finite transit time broadening $(\delta v_z')_L$

The finite transit broadening is due to the fact that a particular particle scatters light only for a finite time—the time required to cross the intersection volume. Assuming that the aperture of photomultiplier is such that all the scattered light in the intersection region is heterodyned, the frequency broadening due to transit time effect is expected to be given by a relation [57]

$$(\delta v_z')_L = \text{CONST}\ \frac{v_z}{\ell} \qquad (2.11)$$

where L is the mean particle velocity and ℓ a length scale representing the finite size of the probe volume.

It is evident that ℓ will depend mainly on

a. Beam waist at the intersection region (probe volume)

b. Velocity vector orientation

c. Angle of intersection of the two beams

Thus a precise knowledge of $(\delta v_z')_L$ is rather difficult to obtain.

George and Lumley [62], assuming Gaussian intensity distribution within the intersection volume and a many particle system arriving at random times, obtained

$$(\delta v_z')_L = \frac{\lambda}{2\sqrt{2}\pi n_w \sin\frac{\beta}{2}} \cdot \frac{v_z}{2b_o} \qquad (2.12)$$

From the data of Table 2.3 we find

$$\frac{(\delta v_z')_L}{v_z} = 0.01 \qquad (2.13)$$

The typical error due to finite transit time broadening based on Eq.(2.13) and a typical value $v_z'/v_z = 0.05$ and calculated from Eq.(2.9) is of order 4%.

2.4.2.2 Non-uniform mean velocity broadening $(\delta v_z')_G$

The nature of the error is described in §2.4.1.2.1.

Melling [57] gives the following approximate relation for turbulent velocity error due to non-uniform velocity

$$(\delta v_z')_G = \sigma_1 \frac{\partial v_z}{\partial y} \qquad (2.14)$$

where $\sigma_1 \equiv b_o$

As in the case of the mean axial velocity, the error is more important close to the wall. As that region

$$\frac{\partial v_z}{\partial y} = \frac{u^*}{ky} \qquad (2.15)$$

Thus $$(\delta v_z')_G = \frac{1}{2k} \frac{a^*}{y/2\sigma_1} \qquad (2.16)$$

The error due to non-uniform mean velocity is higher close to the wall as seen in Eq. (2.16.) Calculations based on (2.66) show an error of no more than 1%.

2.4.2.3 Turbulent velocity broadening $(\delta v_z')_T$

Due to finite scattering volume there is a fluctuation of the volumetric averaged instantaneous velocity which introduces additional ambiguity $(\delta v_z')_T$.

W. E. George and J. L. Lumley [62], considering Gaussian statistics, end up with the following approximate relation for the velocity broadening

$$(\delta v_z')_T^2 = \int_{-\infty}^{+\infty} \phi_{11}(k_1,k_2,k_3) \exp\left[-(\sigma_1 k_1)^2 - (\sigma_2 k_2)^2 - (\sigma_3 k_3)^2\right] dk_1 dk_2 dk_3 \qquad (2.17)$$

where $\phi_{11}(k_1,k_2,k_3)$ is the 3-dimensional turbulence spectra.

Further, the authors [62] assuming isotropic turbulence, Pao's [66] form of energy spectra and $\sigma_3 >> \sigma_2, \sigma_1$ (which is actually our case) obtain Fig. 7 in Ref. 62 which gives

$$\frac{(\delta v_z')_T}{(\nu\varepsilon)} \quad \text{vs} \quad \frac{\sigma_3\sqrt{2}}{\eta}$$

where ε is the energy dissipation

and $\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$ is Kolmogorov's microscale

For the case $\frac{\sigma_3\sqrt{2}}{\eta} < 0.3$

the authors [62] suggest the simplified relation

$$(\delta v_z')_T = \frac{2\sigma_3}{\sqrt{15}} \left(\frac{\varepsilon}{\nu}\right)^{1/2} \qquad (2.18)$$

In our case we can use Fig. 7 directly since the range of

$\frac{\sigma_3\sqrt{2}}{\eta}$ of interest is in the region where $\frac{(\delta v_z')_T}{(\nu\varepsilon)^{1/4}}$ can be roughly taken as a constant equal to 1.6. Specifically for our case, using the definition of Komogorov's microscale

$$\frac{\sigma_3\sqrt{2}}{\eta} = \frac{\sigma_3\sqrt{2}}{\left(\frac{\nu}{\varepsilon}\right)^{1/4}} = \frac{\sigma_3\sqrt{2}}{\nu}\cdot\left(\frac{0.4k_o^{3/2}}{\ell}\right)^{1/4} \simeq 11$$

and $(\delta v_z')_T / (\nu\varepsilon)^{1/4} \simeq 1.6$ (2.19)

where $2\sigma_3 = 0.6\text{mm}$

$\nu = 1.0 \times 10^{-6}\ \text{m/s}^2$

k_o = a typical turbulent kinetic energy level

$= 0.003v_b$

$\ell = 0.2D_H = 4\text{mm}$

and $v_b = 2\text{m/s}$ (Re = 65,000)

Eq. (2.19) gives a typical error of 3% for Re = 65,000, 5% for Re = 26,500, and 7% for Re = 9,000.

2.4.2.4 Signal processing broadening

The electronic equipment introduces some broadening due to instrument noise. The above broadening is measured in our case as follows. The rms value of the laminar flow (Re = 1500) has been measured by using frequency shift for each DISA frequency range and each IF bandwidth of the Tracker used in present measurements. The rms values in mV are presented in Table 2.6.

Fig. 2.10 shows the $(v_z'^2)_{LAM}$ vs % BW in the range of 1.5 MHZ. $(v_z')_{LAM}$ varies linearly with % BW, which means that $(v_z')_{LAM}$ is a white noise effect in which the Tracker acts as a low pass filter.

The signal processing broadening cannot be attributed to transit time broadening since Formula (2.13) gives very low values compared with what have been measured.

More specifically, for the mean velocity measured (100mv) Eq.(2.13) gives a finite transit time broadening of 1mv which is much smaller than any value given in Table 2.6.

Since the above broadening is measured, it is not treated as an error but it has been subtracted from the measured rms values, i.e.

$$v_z'^2 = (v_z'^2)_{rms} - (v_z'^2)_{LAM}$$

2.4.2.5 Other errors

A reading error is introduced also from the DISA 55D35 RMS Voltmeter. It is taken 2% of the output [76].

2.4.3 Turbulent velocities v_r', v_θ'

The main characteristic of these measurements is the low signal-to-noise ratio; therefore the main error comes from the relatively low performance of the tracker at low signal-to-noise ratios. The reason is that the tracker can be locked into frequencies other than the Doppler signals (e.g. the shifted frequency.)

With respect to the broadening mechanisms mentioned in §2.4.2 the only significant broadening mechanisms seem to be

a. Turbulent velocity broadening

b. Signal processing broadening

which are taken of the same order of magnitude as in the case of v_z'.

The overall error based on best judgment is taken of order $\pm$ 15%.

The results are tabulated in Appendix C.

2.4.4 The Reynolds stresses $\overline{v_r v_z}$ and $\overline{v_\theta v_z}$

Above stresses are obtained by measuring the quantities $\overline{v_1 v_z}$ and $\overline{v_2 v_z}$ and applying the relations shown in Table 2.4. In these measurements again the main difficulty was the low signal to noise ratio in the signals v_1' and v_2', especially very close to the wall. We could estimate based on best

judgement an error 15% for $\overline{v_l' v_z'}$ and 100% for $\overline{v_\theta' v_z'}$

The results for $\overline{v_r' v_z'}$ are shown in APPENDIX C.

CHAPTER 3

THE ANALYTIC MODEL DEVELOPMENT

3.1 The Equations

Our ultimate purpose is to be able to solve the Reynolds equations within the characteristic triangle shown in Fig. 1.1. The Reynolds equations are the time-smoothed momentum equations obtained from the Navier-Stokes equations. In cylindrical coordinates the equations are as follows:

I. Continuity Equation

$$\nabla \cdot \underline{V} = 0 \tag{3.1}$$

II. Momentum Equations

r - direction: $$\nabla v_r \underline{V} + \bar{\nabla} \underline{\tau}_r = -\frac{\partial (p/\rho)}{\partial r} + \frac{v_\theta^2}{r} + \tau_{\theta\theta} \tag{3.2}$$

θ - direction: $$\nabla r v_\theta \underline{V} + \bar{\nabla} r \underline{\tau}_\theta = -\frac{\partial (p/\rho)}{\partial \theta} \tag{3.3}$$

z - direction: $$\nabla v_z \underline{V} + \bar{\nabla} \underline{\tau}_z = -\frac{\partial (p/\rho)}{\partial z} \tag{3.4}$$

where $\underline{V}$ is the velocity vector; i.e.

$$\underline{V} = (v_r, v_\theta, v_z) \tag{3.5}$$

$\bar{\nabla}$ is the divergence operator; i.e.

$$\bar{\nabla} \cdot = \left(\frac{1}{r}\frac{\partial}{\partial r} r \cdot , \frac{1}{r}\frac{\partial}{\partial \theta} \cdot , \frac{\partial}{\partial z} \cdot\right) . \tag{3.6}$$

p is the pressure

ρ is the density

$\underline{\tau}_r$, $\underline{\tau}_\theta$, $\underline{\tau}_z$ are stress vectors; i.e.

$$\begin{aligned} \underline{\tau}_r &= (\tau_{rr} , \tau_{r\theta} , \tau_{rz}) \\ \underline{\tau}_\theta &= (\tau_{r\theta} , \tau_{\theta\theta} , \tau_{\theta z}) \\ \underline{\tau} &= (\tau_{rz} , \tau_{\theta z} , \tau_{zz}) \end{aligned} \qquad (3.7)$$

consisting of a laminar part, the laminar stresses and a turbulent part, the Reynolds stresses.

$$\tau_{\alpha\beta} = \tau_{\alpha\beta}^{(\ell)} + \tau_{\alpha\beta}^{(t)} \qquad (\alpha,\beta = r,\theta,z)$$

where

$$\left.\begin{aligned} \tau_{rr}^{(\ell)} &= -2\nu \frac{\partial v_r}{\partial r} , \quad \tau_{rr}^{(t)} = \overline{v_r'^2} \\ \tau_{\theta\theta}^{(\ell)} &= -2\nu\left(\frac{\partial v_\theta}{v\partial\theta} + \frac{v_r}{r}\right) , \quad \tau_{\theta\theta}^{(t)} = \overline{v_\theta'^2} \\ \tau_{zz}^{(\ell)} &= -2\nu \frac{\partial v_z}{\partial z} , \quad \tau_{zz}^{(t)} = \overline{v_z'^2} \\ \tau_{r\theta}^{(\ell)} &= -\nu\left[r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial\theta}\right] , \quad \tau_{r\theta}^{(t)} = \overline{v_r'v_\theta'} \\ \tau_{\theta z}^{(\ell)} &= -\nu\left[\frac{\partial v_\theta}{\partial z} + \frac{\partial v_z}{r\partial\theta}\right] , \quad \tau_{\theta z}^{(t)} = \overline{v_\theta'v_z'} \\ \tau_{rz}^{(\ell)} &= -\nu\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right] , \quad \tau_{rz}^{(t)} = \overline{v_r'v_z'} \end{aligned}\right\} \qquad (3.8)$$

Obviously, in order to be able to solve Equations (3.1 - 3.4) some knowledge of the Reynolds stress is required. Accurate knowledge of Reynolds stresses does not exist due to

the complexity of the turbulence, especially in the shear flows. Thus, approximate relations, rather crude from the scientific point of view, are used, which however are very useful in solving engineering problems. The additional relations used in order to be able to solve Equations (3.1 - 3.4) consist of the Reynolds stress closure.

3.2 Reynolds Stress Closure - Previous Work

3.2.1 General

As was pointed out in the previous section, the difficulty of solving the continuity and momentum equations is strongly related to the difficulty of obtaining a Reynolds stress closure. In this section an attempt has been made of reviewing the main methods used by other investigators to obtain a Reynolds closure. The methods that will be of interest to the present work will be described in more detail.

We can distinguish three main approaches toward Reynolds stress closure.

1. Analytical expressions (i.e. semi-empirical expressions not included in the concept of eddy viscosity),
2. The eddy viscosity approach,
3. Transport equations for the Reynolds stresses.

3.2.1.1 Analytical Expressions (Algebraic relations)

The simplest way that one could think of studying the behavior of Reynolds stresses is to try to express them as a

function of the local and global characteristics of the flow that fits the experimental data. However, such an approach has disadvantages, such as:

a. Different expressions are generally needed for different geometries,

b. The larger the number of independent parameters, the more difficult it is to obtain such analytical expressions.

Therefore, such an approach seems more attractive for simple geometries (e.g. pipe flow, flow between parallel plates, boundary layer in a flat plate) or the wall regions of more complex geometries where parameters such as friction velocity u^* and local Reynolds number $y^+ = \frac{yu^*}{\nu}$ are enough to describe the Reynolds stresses. However, in the case of the simple geometries it is more logical to find analytical expressions directly for the velocity field rather than to go first into the Reynolds stresses.

Parenthetically it is noticed here that in the case of pipe flow and flow between parallel plates the shear stress is given by the relation (Ref. 1, p. 50):

$$\overline{v'_r v'_z} = -\nu \frac{dv_z}{dr} + \frac{r}{r_o} u_*^2 \qquad (3.9)$$

where

ν is the kinematic viscosity

r is the distance from the center line

r_o is the diameter of the pipe or the distance between the parallel plates.

According to (3.9) an analytical expression for $\frac{dv_z}{dr}$ gives an analytical expression for $\overline{v_r' v_z'}$.

With respect to the normal stresses the only analytical expressions as far as the author knows that are valid for closed ducts of different form are those obtained by V. P. Bobkov, M. Ch. Ibragimov and G. I. Savelev [2]. We will mention below their expressions for the normal stress of a circular pipe since we will assume in Section 3.3.3 that these are also approximately valid for rod bundles.

$$\left.\begin{aligned} \overline{v_r'^2} &= A_r v_z \left(1 - \frac{u_b}{u_o}\right) e^{-A_r \xi} \\ \overline{v_\theta'^2} &= A_\theta v_z \left(1 - \frac{u_b}{u_o}\right) e^{-A_\theta \xi} \\ \overline{v_z'^2} &= A_z v_z \left(1 - \frac{u_b}{u_o}\right) e^{-A_z \xi} \end{aligned}\right\} \quad (3.10)$$

where A's and B's are constants with values:

$$\left.\begin{aligned} A_r &= 0.4 \qquad B_r = 1.48 \\ A_\theta &= 0.6 \qquad B_\theta = 1.17 \\ A_z &= 0.95 \qquad B_z = 1.48 \end{aligned}\right\} \quad (3.10a)$$

$$\xi = 1 - \frac{r}{r_o} \quad (3.10b)$$

u_b is the bulk velocity

u_o is the maximum velocity in the channel

Equations (3.10) are valid for the center region of the flow up to $y^+ \approx 11$. The constants (3.10) fit well the experimental data in the range of $Re = 7.5 \times 10^4$ up to 2.4×10^5.

With respect now to using algebraic expressions into the wall region, limited work has been done towards this direction. One of the reasons for this is the difficulty of obtaining reliable experimental data due to the presence of the wall. For $\overline{v_r'v_z'}$ we can use again Eq. (3.9). We will mention here the expression given by A. P. Ivanov [3] who claims that it is consistent with the experimental data of Laufer [6,7] and it is valid in the outer wall region as well. Ivanov's relation is as follows:

$$\frac{\overline{v_r'v_z'}}{u^{*2}} = \frac{r}{r_o} \exp\left(-\frac{n_*^2}{1-n^*} \frac{r}{r_o-r}\right) \tag{3.11}$$

where

$$n_* = 6.6\ Re^{-0.44} \tag{3.11a}$$

With respect to the normal stress — very close to the wall — as far as the author knows, analytical expressions exist only for $\overline{v_z'^2}$ very close to the wall ($y^+ < 5$).

H. Eckelman [4] gives

$$\frac{(\overline{v'^2})^{1/2}}{u^*} = 0.24y^+ e^{1/2\alpha y^+} \qquad (y^+ < 5) \tag{3.12}$$

where

$$\alpha \approx 0.3 \tag{3.12a}$$

3.2.1.2 Eddy Viscosity Approach

Instead of inventing analytical expressions for each case, it seems much more logical and practical to try to understand the general behavior of the Reynolds stresses — in other words to try to build a model for the Reynolds stresses.

Boussinesq [5] was the first to attack the problem by introducing the concept of eddy viscosity assuming that the behavior of the Reynolds shear stresses are similar to the viscous stresses. The eddy viscosity concept is the most widely used in engineering problems. The eddy viscosity concept in its most general form [8] can be expressed as:

$$\tau_{\alpha\beta}^{(t)} = \frac{\nu_{\alpha\beta}^{T}}{\nu} \tau_{\alpha\beta}^{(\ell)} + \frac{2}{3} k \delta_{\alpha\beta} \qquad (3.13)$$

where

$\tau_{\alpha\beta}^{(t)}$: the turbulent Reynold stress

$\tau_{\alpha\beta}^{(\ell)}$: the viscous stress

k : the turbulent kinetic energy

ν : the kinematic viscosity

$\nu_{\alpha\beta}^{T}$: the eddy viscosity

and

$$\delta_{\alpha\beta} = \begin{cases} 0 & \text{for } \alpha \neq \beta \\ 1 & \text{for } \alpha = \beta \end{cases} \qquad (3.14)$$

The first term of the right hand side of Eq. (3.13) expresses the analogy between the Reynolds and the viscous stresses whereas the second term was added to satisfy two additional requirements:

a. The sum of the normal stresses is equal to turbulent kinetic energy.

b. In the case of absence of viscous stresses, i.e. in isotropic turbulence, the normal Reynolds stresses are equal.

Such an approach could be completely successful if we could assume that the introduced eddy viscosity $\nu_{\alpha\beta}^{T}$ was constant like the viscosity ν. Indeed, in some cases, like turbulent jets and wakes propagated in an infinite space or turbulence in a free atmosphere, such an assumption is satisfactory [Ref. 9, p. 365]. However, in the case of closed ducts with their boundary layers, such an assumption completely fails.

So in general $\nu_{\alpha\beta}^{T}$ is position dependent and, even worse, is a tensorial quantity rather than a scalar. It is thus evident that in order for this approach to become practical some knowledge of $\nu_{\alpha\beta}^{T}$ is required. Since we are going to use the same approach in the present work, this topic will be examined in more detail in §3.2.2.

3.2.1.3 Transport Equations

The idea here is that the Reynolds stresses, as the other hydrodynamic quantities, obey a transport equation of the following form, which can be obtained from the Navier-Stokes equations [10]

$$\text{CONVECTION} + \text{DIFFUSION} + \text{SOURCE} + \text{SINK} = 0$$

However, such equations cannot be solved since they are func-

tions of triple velocity correlations and double pressure velocity correlations. So modeling of the various terms in such equations seems necessary. Such an effort has been made by several authors such as Rotta [11], Davydov [12], Donalson [13], Daly and Harlow [14], Hanjalic and Launder [15], Naot et al. [16], Launder et al. [17].

In the present work we will not follow this approach. Our reason is that application of an eddy viscosity approach to our geometry should give reasonable results with far less computation time than a transport equation approach would require.

3.2.2 Eddy Viscosity Approach

As we have pointed out, the eddy viscosity is generally not constant, especially in the case of a duct flow which the rod bundle flow resembles. Therefore, detailed knowledge of eddy viscosity within the flow region is necessary. There are several approaches towards this direction. The main approaches can be classified as follows:

1. Analytical Expressions (Algebraic Expressions)
2. Prandtl Mixing Length Hypothesis
3. Van Karman Similarity Hypothesis
4. Kolmogorov-Prandtl Hypothesis
5. Transport Equations

3.2.2.1 Analytical Expressions

Efforts to express the eddy viscosity as an analytical function of the local and global characteristics of the flow

in simple terms have appeared in literature. Reichardt [18] has developed an eddy viscosity expression for the parallel plate geometry.

$$\nu_{rz}^{T} = \frac{k}{3}\left(1 - \frac{r}{r_o}\right)\left[0.5 + \left(\frac{r}{r_o}\right)^2\right]\left(1 + \frac{r}{r_o}\right) u^* \qquad (3.15)$$

where

$$k \simeq 0.4 \qquad (3.15a)$$

Eifler [19,20] has generalized the expression for ν_{rz}^{T} for round tubes, parallel plates and concentric annuli and the generalized expression has been applied satisfactorily for the rod bundle geometry [21]:

$$\nu_{rz}^{T} = y_o \frac{K'}{2(1+b)} [b+x^2][1-x^2] \frac{2K - x(k-1)}{(K+1)[K-2(K-1)]} u^* \qquad (3.16)$$

$$x = 1 = \frac{y}{y_o} \qquad (3.16a)$$

$$K = \begin{cases} 1 & \text{for parallel plates} \\ 0 & \text{for circular tubes} \\ 1 + \dfrac{y_o}{R} & \text{for flows outside a rod of radius R} \end{cases} \qquad (3.16b)$$

$$K' = \begin{cases} 0.407 & \text{for parallel plates or pipe flow} \\ 0.407 + 0.02\ y_o/R & \text{for flows outside a circular rod} \end{cases} \qquad (3.16c)$$

$$b = \begin{cases} 0.55 & \text{for parallel plates and pipe flows} \\ (1.8) - 1.3(y_o/R)[1. + 0.14(\log Re_\ell - 5)]^{-1} & \text{for flows outside a circular rod} \end{cases} \qquad (3.16d)$$

y_o is the radial distance between the wall and the zero shear stress line

and

Re_ℓ is the local Reynolds number at the peripheral position considered.

With respect to the lateral eddy viscosity, Nijsing and Eifler [21,22] use an average of $\nu^T_{\theta z}$ over the radial direction of the form

$$\nu^T_{\theta z} = c y_o u^* \tag{3.17}$$

with

$$c = 0.185 . \tag{3.18}$$

Rapier and Redman [24] take

$$c = 0.1 . \tag{3.19}$$

3.2.2.2 Prandtl Mixing Length Hypothesis

Efforts to understand the general behavior of eddy viscosity has been carried out by several authors. The main idea behind these efforts is that the eddy viscosity can be expressed as a product of a velocity scale U times a length scale ℓ; i.e.

$$\nu^T = U \cdot \ell \tag{3.20}$$

in analogy with the approach used in modeling laminar viscosity. The velocity scale U can be associated with representative velocity of the fluid eddies in a particular position. The

length scale ℓ can be considered as the representative length an eddy travels before losing its identity. This is the basis of Prandtl's mixing length theory [25]. In developing his hypothesis, we will refer to the simplest case of parallel flow. According to Prandtl

$$v'_z \sim v'_r \sim \ell' \frac{dv_z}{dz} \tag{3.21}$$

where v'_r, v'_z represent the fluctuation velocities of fluid lumps arriving at r from the position $r + \ell'$ and $r - \ell'$. A reasonable assumption is to take

$$U \sim (\overline{v'^2_z})^{1/2} \simeq (\overline{v'^2_r})^{1/2}$$

$$\ell \sim (\overline{\ell'^2})^{1/2}$$

where ℓ is Prandtl's mixing length. Hence, from Eq. (3.21) we obtain

$$U \sim \ell \frac{dv_z}{dz} \tag{3.22}$$

Finally, Eq. (3.20) becomes

$$\nu^T_{rz} = \text{const} \times \ell^2 \left|\frac{dv_z}{dz}\right| \tag{3.23}$$

ℓ is not a constant but depends generally on the position under consideration and is usually given by analytical expressions based on experimental data. A more general form for the mixing length has been given by Buleev [26]:

$$\frac{1}{\ell} = \text{const} \times \int_\Omega \frac{d\Omega}{\ell(\Omega)} \tag{3.24}$$

where Ω denotes the direction

and $\ell(\Omega)$ is the distance of the point under consideration from the wall in the direction Ω.

Prandtl's similarity hypothesis has been successfully applied to the study of turbulent motion along walls (pipe, channel, plate) and in free turbulent flows [see, for example, Ref. 44, Chapters XIX, XX, XXI, XXIV].

In rod bundles it has been applied by Bender et al. [27, 28]. For ℓ Buleev's formula (3.24) is used.

However, Prandtl's similarity hypothesis proved to be unsatisfactory in the region where $dv_z/dy = 0$ since it gives zero eddy diffusivity, which is in contrast to the experimental evidence that gives considerably higher values of eddy viscosity.

3.2.2.3 Von Karman Similarity Hypothesis

Prandtl's theory does not solve completely the problem of eddy viscosity behavior since it does not say anything about the behavior of ℓ. A general hypothesis that allows us to establish a general connection between the length ℓ and the mean velocity field was proposed by Von Karman [29]. His approach is called "the local kinematic similarity hypothesis." According to this approach the turbulent fluctuations are similar at all points of the field. They differ from point to point only by time and length scales or by equivalent length and velocity scales. Quantitatively, such an assumption leads to the following relation for the velocity scale and length scale:

$$U \sim \frac{dv_z}{dz} \tag{3.25}$$

$$\ell \sim \frac{dv_z/dz}{d^2v_z/dz^2} \tag{3.25a}$$

Consequently the Karman similarity hypothesis gives the following relations for the shear stress, which is essentially the same as Eq. (3.23)

$$\nu_{rz}^T = \ell^2 \left| \frac{dv_z}{dz} \right| \frac{dv_z}{dz} \tag{3.26}$$

where

$$\ell = k \frac{dv_z/dy}{d^2v_z/dy^2} \tag{3.26a}$$

and k is called the Von Karman constant and is equal to 0.4. Von Karman's hypothesis imposes excessively rigid constraints in the velocity fluctuations that do not generally agree with experimental evidence [30, p. 107].

3.2.2.4 Kolmogorov - Prandtl Hypothesis

Prandtl's mixing length hypothesis and Von Karman's similarity hypothesis reflect the effort to find a relation between eddy viscosity and the characteristics of the mean flow. Other efforts on the same line of approach have been made, such as Taylor's vorticity transfer theory [31], improved theories of Prandtl's mixing length hypothesis [44, p. 549], Buleev's theoretical model for turbulent transfer [26] and its modification by Ramm [32] for the case of rod bundle geometry.

The above models do not directly take into consideration statistical characteristics of turbulence upon which the eddy

viscosity depends. The first attempt to express eddy viscosity as a function of the turbulent characteristics of the turbulence is due to Kolmogorov [33] who postulated a characteristic turbulent frequency f.

If we assume that the length scale ℓ and velocity scale U (of Eq. 3.20) are functions of the turbulent kinetic energy k and the frequency f we can find immediately from dimensional analysis that

$$U \sim k^{1/2} \tag{3.27a}$$

$$\ell \sim \frac{k^{1/2}}{f} \tag{3.27b}$$

and consequently Eq. 3.20 becomes

$$\nu^T = \text{const} \times \frac{k}{f} \tag{3.28}$$

Prandtl [34] has substituted in Eq. (3.20) only the scale U given by the relation (3.27a). In such a case, equation (3.20) becomes

$$\nu^T = \text{const} \times k^{1/2} \times \ell \tag{3.29}$$

where ℓ is the length scale equivalent to the mixing length used in (3.23).

It is evident from the above discussion that any quantity ζ which is assumed to be a function of k and ℓ only (like f in 3.27b) can be expressed by dimensional analysis in the form of:

$$\zeta = k^{\alpha} \ell^{\beta} \tag{3.30}$$

Then, one can write ν^T from Eq. 3.20 as a function of k and

ζ as follows:

$$\nu^T = \text{const} \times k^{1/2 - \alpha/\beta} \, \zeta^{1/\beta} \tag{3.31}$$

Eqn. (3.31) is a general relation of eddy viscosity as a function of the statistical characteristics k and z. However, the utilization of (3.31) requires the knowledge of k and ζ. This can be obtained either by semi-empirical equations or by transport equations for k and ζ. The way of evaluating k and ζ defines the type of the turbulence model as 0-equation model, 1-equation model and 2-equation model where the term "equation" refers to transport equations for k, ζ.

0-Equation Model — In this case k and ℓ are prescribed by semi-empirical equations. This model will be applied in this work for comparison with 1- and 2-equation models.

1-Equation Model — In this case a transport equation for k is used, whereas ℓ is described by a semi-empirical relation. This model has been widely used (see references in 35 and 36). With respect to the closed channel flows and rod bundles the 1-equation model is applied by Launder and Ying [37] and P. Carajilescov [38] in square ducts and rod bundles respectively.

The transport equation for k can be obtained from the Navier-Stokes equation. The way of modeling it is similar to that discussed in §3.2.1.3 concerning the Reynolds stresses themselves although the task is much easier, as we will see later.

2-Equation Model — In this model, transport equations are required for both k and ζ. In the literature one can find a variety of ζ's one used by different authors, as we can see from Table 3.1 taken from [39].

The transport equation for ζ usually is found by one of two ways:

1) Derivation from Navier-Stokes equation. For example, a transport equation for the turbulent energy dissipation ε can be obtained [40, 41]. Such a procedure, however, is not easy for all ζ. Therefore, the second procedure is sometimes used.

2) Derivation from turbulent kinetic energy and dimensional analysis. To be more specific, the turbulent kinetic energy according to what we have said in §3.2.3 can be written:

$$\text{convection } (k) + \text{diffusion } (k) + \text{source } (k) + \text{sink } (k) = 0 \tag{3.32}$$

The transport equation for ζ can be assumed to be

$$\text{convection } (\zeta) + \text{diffusion } (\zeta) + (\text{const}) \times \frac{\zeta}{k} \text{ source } (k) + (\text{const})' \times \frac{\zeta}{k} \text{ sink } (k) = 0 \tag{3.33}$$

where the constants have to be determined from experimental data.

From the relations (3.31) and (3.33) it is evident that such an approach presupposes the adequacy of k and ζ for describing the "integral" behavior of turbulence at particular points. However, this cannot be true for nonisotropic turbulence where more than one velocity scale and/or length scale

exists. Monin [9] has proposed the following alternative relation for the Reynolds stresses based on the fact that the Reynolds stress is a second order symmetric tensor and the assumption that the Reynolds stress can be expressed as a linear combination of the mean strains:

$$\overline{u'_\alpha u'_\beta} = \frac{2}{3} k\delta_{ij} - \frac{1}{2} k^{1/2} \sum_i \left(\ell_{\alpha i} \frac{\tau^{(\ell)}_{i\beta}}{\nu} + \ell_{\beta i} \frac{\tau_{i\alpha}}{\nu} \right) \quad (3.34)$$

where $\frac{\tau^{(\ell)}_{i\beta}}{\nu}$, $\frac{\tau^{(\ell)}_{\alpha}}{\nu}$ is the mean strain and $\ell_{\alpha\beta}$ are the length scales to be determined. However, such a relation which includes the unknowns $\ell_{\alpha\beta}$ is rather difficult to utilize in solving engineering problems.

3.2.2.5 Transport Equation

A logical thought originating from Eq. (3.31) is to use for ζ the eddy viscosity itself, assuming that it is isotropic. This idea has been put forward by Nee and Kovasznay [42] and is used in the code VARR-II [43]; i.e., a transport equation for ν^T is used, which deals with the solution of two-dimensional transient flows.

This approach is another alternative based on Kolmogorov-Prandtl approach, which makes more difficult the introduction of a non-isotropic eddy viscosity.

3.3 Reynolds Stress Closure — Present Approach

3.3.1 General

As we have pointed out in the previous section, the eddy

viscosity approach will be used in this work for obtaining a Reynolds stress closure. From all the eddy viscosity approaches, the Kolmogorov-Prandtl hypothesis is adopted since it relates eddy viscosity directly to the turbulence characteristics of the flow that are the "natural" parameters. The most advanced model based on Kolmogorov-Prandtl hypothesis is the two-equation model in which transport equations for turbulent kinetic energy k and another characteristic quantity of the turbulence, ζ, are used. The selection of ζ varies from author to author. We select for ζ the energy dissipation ε which is the most widely used. Calculations with 0-equation model and one-equation model with be performed for comparison with two-equation model.

It is repeated here that authors who apply the Kolmogorov-Prandtl hypothesis for channel flow analysis, including the rod bundle and square duct flows, and consider isotropic viscosity are not consistent with the experimental evidence [23,88]. In fact, present analysis will show (see Chapter 5) that non-isotropic eddy viscosity considerably affects the prediction of flow and heat in rod bundles. The importance of anisotropic viscosity is fluid flow and heat transfer calculations in a bare rod bundle has also been confirmed by other investigators [94, 32]. In this approach, turbulent anisotropy will be introduced by using more than one length scale in modeling turbulent viscosity.

3.3.2 Shear stress relations

In the present approach, the shear stresses will be expressed as follows:

$$\overline{v_r'v_z'} = -\nu_{rz}^T \frac{\partial v_z}{\partial r} \tag{3.35}$$

$$\overline{v_\theta'v_z'} = -\nu_{\theta z}^T \frac{\partial v_z}{r\partial\theta} \tag{3.36}$$

$$\overline{v_r'v_\theta'} = -\nu_{r\theta}^T \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r}\right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] \tag{3.37}$$

We will further assume

$$\nu_{r\theta}^T \approx \nu_{rz}^T \tag{3.38}$$

for the following reasons:

1) $\overline{v_r'v_\theta'}$ does not play an important role in the creation of the secondary flow [52] so a very accurate knowledge of $\overline{v_r'v_\theta'}$ is not necessary. In fact, M. Kh. Ibragimov et al. [53] and P. Carajilescov [1] neglect $\overline{v_r'v_\theta'}$ in their calculations so a more accurate knowledge of $\nu_{r\theta}^T$ seems worthless.

2) $\overline{v_r'v_\theta'}$ includes v_θ' fluctuations and v_θ' is expected to have a length scale close to v_z'.

3.3.2.1 Eddy viscosity for shear stresses

In this approach the eddy viscosities ν_{rz}^T and $\nu_{r\theta}^T$ will be expressed as follows:

<u>0- or 1-Equation Model</u>

$$\nu_{rz}^T = c_\nu k^{1/2} \ell_r \tag{3.39}$$

$$\nu_{\theta z}^T = c_\nu k^{1/2} \ell_\theta \tag{3.40}$$

where c_ν will be a constant consistent with experimental evidence.

2-Equation Model — For the energy dissipation ε Eq. (3.30) gives for $\zeta = \varepsilon$

$$\varepsilon = k^{3/2}/\ell \tag{3.41}$$

Based on the relations (3.41), (3.39) and (3.40) the eddy viscosity relations can be put in the following form

$$\nu_{rz}^{T} = c_{\mu} k^2/\varepsilon \tag{3.42}$$

$$\nu_{\theta z}^{T} = c_{\mu} \frac{\ell_{\theta}}{\ell_{r}} k^2/\varepsilon \tag{3.43}$$

It is evident from the above formulations that the eddy viscosity ratio is given by the relation

$$\psi_m = \frac{\nu_{rz}^{T}}{\nu_{r\theta}^{T}} = \frac{\ell_r}{\ell_\theta} \tag{3.44}$$

Thus, additional knowledge of ψ_m is needed.

3.3.3 Normal stress relations

For the normal stresses, i.e. $\overline{v_r'^2}$ and $\overline{v_\theta'^2}$, which are mainly responsible for the secondary flow [52], two options have been introduced in the present work. The reason for selecting two options is primarily for comparison, and secondly because both options have been used for prediction of the secondary flow in square ducts.

First option — In this case the normal stress modeling used by Launder and Ying and P. Carajilescov [1] has been used. The idea behind this approach is that the $\overline{v_r'^2}$ and $\overline{v_\theta'^2}$ transport equation that can be obtained from the Navier-Stokes Equation can be simplified considerably if one realizes that:

1) they are important with respect to secondary flow only

close to the wall region, and

2) the convection and diffusion mechanisms are insignificant compared with generation and dissipation mechanisms.

Under these circumstances the $\overline{v_r'^2}$ and $\overline{v_\theta'^2}$ transport equations are reduced to [Ref. 1, p. 227]:

$$\overline{\frac{p'}{\rho}\frac{\partial v_r'}{\partial r}} \simeq \frac{\varepsilon}{3} \tag{3.45}$$

$$\overline{\frac{p'}{\rho}\frac{\partial v_\theta'}{r\partial\theta}} \simeq \frac{\varepsilon}{3} \tag{3.46}$$

What remains to be done is the modeling of pressure-velocity correlations.

This has drawn the attention of several investigators. Here we will adopt the approach discussed in Ref. [17]. The velocity-pressure correlations are expressed as functions of shear stresses and mean velocity gradients, which for our case, neglecting the secondary flow effects, are expressed as follows:

$$\overline{\frac{p'}{\rho}\frac{\partial v_r'}{\partial r'}} = -\frac{c_1}{2}\frac{\varepsilon}{k}\left(\overline{v_r'^2} - \frac{2}{3}k\right) - \frac{8c_2 - 2}{11}\nu_{rz}^T\left(\frac{\partial v_z}{\partial r}\right)^2 + \frac{9c_2 + 6}{33}\left\{\nu_{rz}^T\left(\frac{\partial v_z}{\partial r}\right)^2 + \nu_{\theta z}^T\left(\frac{\partial v_z}{r\partial\theta}\right)^2\right\} \tag{3.47a}$$

$$\overline{\frac{p'}{\rho}\frac{\partial v'}{r\partial\theta}} = -\frac{c_1}{2}\frac{\varepsilon}{k}\left(\overline{v_\theta'^2} - \frac{2}{3}k\right) - \frac{8c_2 - 2}{11}\nu_{\theta z}^T\left(\frac{\partial v_z}{r\partial\theta}\right)^2 + \frac{9c_2 + 6}{33}\left\{\nu_{rz}^T\left(\frac{\partial v_z}{\partial r}\right)^2 + \nu_{\theta z}^T\left(\frac{\partial v_z}{r\partial\theta}\right)^2\right\} \tag{3.47b}$$

where c_1 and c_2 are constants. Combining Eqs. (3.46) and (3.47) we find finally

$$\overline{v'^2_r} = \frac{2}{3}(1. - 1./c_1)k + \frac{2(9c_2 + 6)}{33c_1}\frac{k}{\varepsilon}\left[\nu^T_{rz}\left(\frac{\partial v_z}{\partial r}\right)^2 + \nu^T_{\theta z}\left(\frac{\partial v_z}{r\partial\theta}\right)^2\right] - \frac{2(8c_2 - 2)}{11c_1}\frac{k}{\varepsilon}\nu^T_{rz}\left(\frac{\partial v_z}{\partial r}\right)^2 \qquad (3.47a)$$

$$\overline{v'^2_\theta} = \frac{2}{3}(1 - 1./c_1)k + \frac{2(9c_2 + 6)}{33c_1}\frac{k}{}\left[\nu^T_{rz}\left(\frac{\partial v_z}{\partial r}\right)^2 + \nu^T_{\theta z}\left(\frac{\partial v_z}{r\partial\theta}\right)^2\right] - \frac{2(8c_2 - 2)}{11c_1}\frac{k}{\varepsilon}\nu^T_{\theta z}\left(\frac{\partial v_z}{r\partial\theta}\right)^2 \qquad (3.47b)$$

The constants c_1, c_2 have to be found by experiment. Launder et al. [17] suggest

$$\left.\begin{aligned} c_1 &\simeq 1.5 \\ c_2 &\simeq 0.4 \end{aligned}\right\} \qquad (3.48)$$

Second option — In this case the analytical expressions (3.10) will be used.

3.3.4 The length scales

Viscosity ratios are given in the literature, especially for the fully developed flow condition in rod bundles, from authors who do not use Kolmogorov-Prandtl's approach, e.g. Nijsing and Eifler [22]. However, such a ratio is strongly related to the models for $\nu^T_{\theta z}$ and ν^T_{rz} adopted which are different from our approach. Here we will try to estimate ℓ_r and ℓ_θ from empirical relations based on experimental observations.

3.3.4.1 The length ℓ_r

We will restrict our discussion to the region $y^+ \geq 30$ since it represents the region where the calculations are to be performed. It is called the outer region of the flow. The region $y^+ < 30$ which is very close to the wall is characterized by sharp velocity gradients and strongly anisotropic turbulent kinetic energy distribution. In this region a model based on a Kolmogorov-Prandtl hypothesis would not seem useful although efforts towards this direction have been made by some authors [50, 52]. Even worse, the calculation of hydrodynamic characteristics within the wall region directly from the set of equations needs relatively high computational effort due to the large number of points needed for the numerical calculations.

However, things can be simplified considerably in the case of moderate acceleration or deceleration due to the fact that the hydrodynamic characteristics are satisfactorily described by u^* and y^+ only. Returning to the outer region (i.e. $y^+ \geq 30$) let us review the length scales taken by some authors.

Launder and Ying [37] for the case of rectangular duct flow use Buleev's formulation, Eq. (3.24),

$$\frac{1}{\ell} = \int_0^{2\pi} \frac{d\phi}{s} \tag{3.49}$$

Such a relation gives ℓ proportional to the wall distance for points very close to the wall.

D. Naot et al. [54] for the case of rectangular duct flow used

$$\left.\begin{aligned} \ell &= y \qquad y \leq 0.54\, D_H \\ \ell &= 0.135 D_H \qquad 0.135 D_H < y \leq \frac{D_H}{2} \end{aligned}\right\} \quad (3.50)$$

where y is the distance from the wall and D_H is the hydraulic diameter of the flow.

P. Carajilescov [1] for the case of rod bundles developed the following relation

$$\left.\begin{aligned} \ell &= y \quad \text{for} \quad y \leq 0.44\hat{y} \\ \ell &= 0.44\hat{y} + 0.066\hat{y} \sin\left|\frac{\pi}{0.38}\left(\frac{y}{\hat{y}} - 0.44\right)\right| \\ &\qquad \text{for} \quad y > 0.44\hat{y} \end{aligned}\right\} \quad (3.51)$$

where $\hat{y}$ is the radial distance of the centerline from the wall at a particular angle θ.

Relations (3.46), (3.47) and (3.48) agree that close to the wall

$$\ell \simeq y$$

In this approach we will distinguish three subregions within the outer region:

A. The dynamic equilibrium subregion

B. The central subregion

C. The subregion in between.

3.3.4.1.1 <u>The dynamic equilibrium subregion</u> — The dynamic equilibrium subregion is the region closer to the wall and has the following characteristics:

a) The diffusion and convective mechanisms of turbulent kinetic energy transport are small compared with source and dissipation mechanisms. In this region turbulent kinetic energy has its highest values whereas the turbulent kinetic energy gradients are relatively small. In other words, if δk is the change of kinetic energy within this region and k_o is a representative value of the turbulent kinetic energy level, the following relation holds:

$$\frac{\delta k}{k_o} << 1$$

Thus, in such a case we can approximate

$$k \simeq k_o \tag{3.52}$$

b) The shear stress $\overline{v_r' v_z'}$ is approximately constant. A good approximation consistent with the experimental evidence is that

$$\overline{v_r' v_z'} \text{ is } \simeq u_*^2 \neq f(r) \tag{3.53}$$

If we want to put $\overline{v_r' v_z'}$ in the form of (3.37) using the relation (3.41), Eq. (3.53) leads to the following relation for ℓ_r:

$$\ell_r \simeq \frac{I}{c_\nu} \frac{u^*}{k_o^{1/2}} \frac{u^*}{dv_z/dr} \tag{3.54}$$

Since k_o is mainly a function of u^*

$$\frac{1}{c_\nu} \frac{u^*}{k_o^{1/2}} \simeq \text{const}$$

In order to have the same order of magnitude for the constants with other investigators who take $\ell_r = y$ we will define the above constant as follows.

Within dynamic equilibrium region the logarithmic law is expected to be valid, i.e.

$$\frac{dv_z}{dr} = \frac{u^*}{\kappa y} \tag{3.55}$$

when κ is the Von Karman constant, i.e.

$$\kappa \simeq 0.4$$

So we take

$$\frac{1}{c_\nu} \frac{u^*}{k_o^{1/2}} \simeq \frac{1}{\kappa} \tag{3.56}$$

Thus, from Eq. (3.54)

$$\ell_r = \frac{1}{\kappa} \frac{u^*}{dv_z/dr} \tag{3.57}$$

3.3.4.1.2 The central region — The central region is characterized by small or zero velocity energy gradients. The selection of ℓ constant and proportional to the distance between the wall and the zero shear stress line gives satisfactory results; i.e.

$$\ell_r = c_L \hat{y} \tag{3.58}$$

The value of c_L has to be found from the experimental data. A quick estimation can be found, however, from the simple geometries as pipe flow

$$\text{for } c_\nu \simeq 0.2 \tag{3.59a}$$

$$\left.\frac{k^{1/2}}{u^*}\right|_{y=\hat{y}} \simeq 0.9 \quad \text{(See Ref. 2)} \tag{3.59b}$$

$$\left.\nu_{rz}^T\right|_{y=\hat{y}} \simeq 0.07\, u^* \hat{y} \quad \text{(See Ref. 94)} \tag{3.60}$$

Substituting Equations (3.58), (3.59), and (3.60) into Eq. (3.39) gives

$$0.07\, u^* \hat{y} = 0.2(0.9\, u^*)\, c_L \hat{y} \qquad (3.61)$$

or

$$c_L = 0.4$$

which is in good agreement with P. Carajilescov [1] who takes at zero shear stress line $c_L = 0.374$ and higher values for points outside the zero shear stress line.

3.3.4.1.3 The intermediate subregion -- The bigger subregion in the outer flow region is the intermediate region which has neither the properties of the dynamic equilibrium region nor the properties of the central region. Taking into consideration equations (3.57) and (3.58), it is reasonable to assume for this region that the

$$\ell_r = \text{function}\left(\frac{dv_z/dr}{u^*}, \hat{y}\right) \qquad (3.62)$$

A logical approximation for the function can be the following:

$$\left(\frac{1}{\ell_r}\right)^{\alpha} = \left(\kappa \frac{dv_z/dr}{u^*}\right)^{\alpha} + \left(\frac{1}{c_L \hat{y}}\right)^{\alpha} \qquad (3.63)$$

where α is a real number to be determined from the experiments. The characteristic behavior of Eq. (3.63) is that in the vicinity of the central region the first term of the right hand side is negligible compared to the second term. Thus Eq. (3.63) reduces to Eq. (3.58).

On the other hand, in the vicinity of the wall the first term dominates in Eq. (3.63), which then reduces to Eq. (3.57).

With respect to the constant α, one expects α to be a function of the Reynolds number, since the higher the Reynolds number the larger the central region becomes, i.e. the larger the α becomes. However, from the nature of the relation (3.63) for large α's (i.e. $\alpha > 4$), large changes in α have little effect. For example, as Fig. 3.2 shows, the prediction of velocity with the 1-equation model is not substantially affected by taking $\alpha = 20$ versus $\alpha = 10$. The significance of α is further diminished in the 2-equation model, as Fig. 3.3 shows. In our case, α is taken as

$$\alpha \simeq 10$$

The advantages of introducing the length scale of Eq. (3.63) are the following:

1. A continuous function is given for the mixing length which is much more accurate than Buleev's length scale (3.49) which is also continuous.

2. As we can see from Fig. 3.2, a large α (i.e. $\alpha \simeq 10$) gives results close to those obtained by using P. Carajilescov's mixing length (3.51).

3. Formula (3.57), which represents the first term of Eq. (3.63), identifies the length scale as not being a pure geometric characteristic of the flow. In a sense, it reflects the distortion of the turbulent kinetic energy spectrum due to the presence of the nonuniform velocity field, which is quite reasonable. Therefore, it is expected to be a better formula for length scales where the logarithmic law of velocity is not valid (e.g. for non-fully developed flows or in

the regions very close to the wall. For example, in the laminar sublayer, formula (3.57) predicts

$$\ell_r \simeq \nu/u^*$$

which is the proper length scale; whereas Eq. (3.52) predicts

$$\ell_r = 0$$

4. Extrapolation of Eq. (3.57) in the tangential direction, as we will see below, leads to an estimate of the length scale ℓ_θ.

The main disadvantage seems to be that Eq. (3.63) does not predict a maximum value within the intermediate region for the length scale, as the relation of P. Canajilescov predicts. However, the absence of this peaked portion in the mixing length prediction has no substantial significance in flow predictions with the turbulence model, especially in the 2-Equation model.

3.3.4.2 The length scale ℓ_θ

In order to estimate ℓ_θ the following assumptions will be made.

1. $\nu^T_{rz} \simeq \nu^T_{\theta z}$ in the central region. This is a reasonable assumption, taking into consideration the fact that $\overline{v_r'^2}$, $\overline{v'^2}$, $\overline{v_z'^2}$ are of the same order of magnitude; i.e., $\overline{v_z'^2}:\overline{v_r'^2}:\overline{v'^2} = 1.16: 1: 1$ [2].

2. Neglect the effect of dv_z/dr on ℓ_θ as a second order effect.

3. An expression similar to (3.63) holds for ℓ_θ; i.e.

$$\left(\frac{1}{\ell_\theta}\right)^\alpha = \left(\kappa \frac{dv_z/rd\theta}{u^*}\right)^\alpha + \left(\frac{1}{c_L L_\theta^*}\right)^\alpha \qquad (3.64)$$

where L^* is the "free" tangential length scale. Since $dv_z/rd\theta$ is expected to be small, we can approximate

$$\ell_\theta \simeq c_L L_\theta^* \qquad (3.65)$$

where

$$c_L \simeq 0.4$$

as given by (3.61)

The length scale ratio (or the eddy viscocity ratio) is given by the relation

$$\psi_m = \frac{\nu_{rz}^T}{\nu_{r\theta}^T} = \frac{\ell_r}{\ell_\theta} = \left\{\left(\frac{L_r^*}{L_\theta^*}\right)^\alpha + \left(c_L \frac{dv_z/dr}{u^*/\kappa L_\theta^*}\right)^\alpha\right\}^{\frac{1}{\alpha}} \qquad (3.66a)$$

In Fig. 3.1 ψ_m is compared with experimental data taken by A. Quarmby and R. Quirk [68]. These experimental data are obtained by measuring the mass and heat diffusivities (Sc ≃ 0.77, Pr ≃ 0.71) in the fully developed region of a plain tube.

For the utilization of (3.66) in the case of plain tubes, take $L_r^* = L_\theta^* = r_o$, $dv_z/dr = u^*/\kappa y$, and $\alpha = 1, 2, 10$ respectively, yielding

$$\psi_m = \left\{1 + \left(\frac{0.4}{y/r_o}\right)^\alpha\right\}^{1/\alpha} \qquad (3.66b)$$

Equation (3.66b) suggests that ψ_m should not vary significantly for $\alpha \geq 2$. This behavior is confirmed by the data of Quarmby [68] as illustrated in Fig. 3.1. Also of even more importance is the consistency of the data with the prediction

of Eq. (3.66b). Note that while the measurements are heat and mass transfer results, since the Schmidt Number and Prandtl numbers were close to unity (0.77 and 0.71, respectively), ψ_m is effectively equal to ψ_D and ψ_H.

With respect to the proper selection of ψ_m for the case of rod bundles, we have tested the following possible cases:

a) Isotropy of the eddy viscosity along the zero stress line, i.e., $L_\theta^* = \hat{y}$

b) Isotropy of the eddy viscosity at $y = \hat{y}_{max}$, i.e., $L_\theta^* = \hat{y}_{max} \equiv L_r^*$

In Figure 3.3, the quantity v_z/v_o is plotted vs. $y/\hat{y}$ at $\theta=0$. The fitting of data by taking $L_\theta^* = y_{max}$ considerably improves the data prediction.

It must be pointed out here that experimental data obtained by K. Rehme in a side subchannel of a rod bundle [88] indicate isotropyic eddy viscosity in the central channel region together with nonisotropy in the gap region, which is consistent with case (b).

H. Ramm and K Johannsen [32], working with an approach based on Buleev's theoretical model [26] have also tested isotropy along the zero shear stress line versus isotropy at the point $y=y_{max}$ but with respect to length scales, not the eddy viscosity. Although the length scales used in [32] have similar physical meaning compared with the present length scales, they are defined differently due to the different

overall approach. Therefore, the length scale ratio in [32] is not equal to the eddy viscosity ratio in the present approach. The above authors have chosen a method of calculation similar to case (a) on the ground that it gives more conservative heat transfer results with their model.

From Figure 3.3 the importance of introducing the eddy viscosity anisotropy is very clear. The improvement of the match between data and turbulence model predictions using the above introduced length scales has been considerable.

3.3.5 The final form of the equations

3.3.5.1 The differential equations

3.3.5.1.1 General form — In the above discussion, it has been made clear that the equations to be solved are those of

continuity and momenta conservation and transport equations for turbulent kinetic energy and energy dissipation.

The momentum and transport equations can be put in the general form

$$\left.\begin{array}{l} \text{|<---Convection--->|<---Diffusion--->|<---Source--->|} \\ \nabla\phi\underline{V} \quad + \quad \nabla\underline{T}\phi \quad = \quad \nabla S\phi + S_\phi^* \\ \underline{T}\phi = (T\phi r,\ T\phi\theta) \\ \underline{S}\phi = (S\phi r,\ S\phi\theta) \end{array}\right\} \quad (3.67)$$

where ϕ stands for v_z, v_r, v_θ, k and ε as shown in Table 3.2a. The separation of the diffusion term into a T_ϕ term and an S_ϕ term which is treated as a source term is made on the ground that T_ϕ includes only the terms that depend explicitly on ϕ.

3.3.5.1.2 Momenta equations — The momenta equations are obtained from the momenta Reynolds Equations given in §3.1.1 and the Reynolds stress selections given in §3.3.1. Momentum equations can be put in the general form (3.67). Each individual term is given in Table 3.2a.

3.3.5.1.3 Turbulent kinetic energy equation — The transport equation for k can be obtained by multiplying Navier-Stokes equations for v_r, v_θ, v_z with velocity fluctuations v_r', v_θ', v_z' and taking the time average. The exact form of the k-equation for our particular geometry has been given by P. Carajilescov [Ref. 1, Eq. B-6, p 221-222]. Using the present shear stress relations, it can be reduced to Eq. (3.67)

which, neglecting the time and z-dependent terms, can be rewritten as follows:

$$\underbrace{\frac{1}{r}\frac{\partial}{\partial r}\, r v_r k + \frac{\partial}{r\partial\theta}\, v_\theta k}_{\text{Convection}} = \underbrace{\left[\frac{1}{r}\frac{\partial}{\partial r}\, r\left(\nu\frac{\partial k}{\partial r} - \overline{v_r' k'} - \frac{\overline{v_r' p'}}{\rho}\right) + \frac{\partial}{r\partial\theta}\left(\nu\frac{\partial k}{r\partial\theta} - \frac{\overline{v' p'}}{\rho} - \overline{v_\theta' k'}\right)\right]}_{\text{Diffusion}} = \underbrace{S_K - \varepsilon}_{\text{Source}} \qquad (3.68)$$

where

$$\left.\begin{aligned} S_K &= -\left\{\left(\overline{v_r'^2}\frac{\partial v_r}{\partial r} + \overline{v_r' v_\theta'}\frac{\partial v_\theta}{\partial r} + \overline{v_r' v_z'}\frac{\partial v_z}{\partial r}\right)\right. \\ &\quad \left. + \left(\overline{v_\theta' v_r'}\frac{\partial v_r}{r\partial\theta} + \overline{v_\theta'^2}\frac{\partial v_\theta}{r\partial\theta} + \overline{v_\theta' v_z'}\frac{\partial v_z}{r\partial\theta}\right)\right\} \\ \varepsilon &= \nu\left\{\left[\left(\frac{\overline{\partial v_r'}}{\partial r}\right)^2 + \left(\frac{\overline{\partial v_\theta'}}{\partial r}\right)^2 + \left(\frac{\overline{\partial v_z'}}{\partial r}\right)^2\right]\right. \\ &\quad + \left[\left(\frac{\overline{\partial v_r'}}{r\partial\theta}\right)^2 + \left(\frac{\overline{\partial v_\theta'}}{r\partial\theta}\right)^2 + \left(\frac{\overline{\partial v_z'}}{r\partial\theta}\right)^2\right] \\ &\quad + \left[\left(\frac{\overline{\partial v_r'}}{\partial z}\right)^2 + \left(\frac{\overline{\partial v_\theta'}}{\partial z}\right)^2 + \left(\frac{\overline{\partial v_z'}}{\partial z}\right)^2\right] \\ &\quad \left. + \left[\frac{\overline{v_r'^2} + \overline{v_\theta'^2}}{r^2} + 2\left(\overline{v_r'\frac{\partial v_\theta'}{r\partial\theta}} - \overline{v_\theta'\frac{\partial v_r'}{r\partial\theta}}\right)\right]\right\} \end{aligned}\right\} \qquad (3.69)$$

Equation (3.68) is modelled as follows.

Diffusion term — The diffusion term is put in a typical diffusion form, in accordance with several authors:

$$\left.\begin{aligned} \overline{v_r'k'} + \frac{\overline{v_r'p'}}{\rho} &\simeq - \frac{\nu_{rz}^T}{\sigma_\kappa} \frac{\partial k}{\partial r} \\ \overline{v_\theta'k'} + \frac{\overline{v_\theta'p'}}{\rho} &\simeq - \frac{\nu_{\theta z}^T}{\sigma_\kappa} \frac{\partial k}{r\partial \theta} \end{aligned}\right\} \quad (3.70)$$

where σ_κ is a constant of order 1.0 [67].

Source term — Term S, representing turbulence generation, can be reduced by neglecting mean radial and tangential velocity terms as negligibly small and substituting the shear stress relations as follows:

$$S_\kappa \simeq + \nu_{rz}^T \left(\frac{\partial v_z}{\partial r}\right)^2 + \nu_{\theta z}^T \left(\frac{\partial v_z}{r\partial \theta}\right)^2 \quad (3.71)$$

Term ε is defined as the energy dissipation.

With these steps the turbulent kinetic energy equation can be put in the general form (3.67) where each term is shown in Table 3.2a.

3.3.5.1.4 Energy Dissipation — The equation for energy dissipation ε is obtained as suggested by Lauder [67] and based on the philosophy of Eq. (3.33), i.e.

$$\underbrace{\frac{1}{r}\frac{\partial}{\partial r} r v_r \varepsilon + \frac{\partial}{r\partial\theta} v_\theta \varepsilon}_{\text{Convection}} \underbrace{- \frac{1}{r}\frac{\partial}{\partial r} r\left(\nu + \frac{\nu_{rz}^T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial r} - \frac{\partial}{r\partial\theta}\left(\nu + \frac{\nu_{\theta z}^T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{r\partial\theta}}_{\text{Diffusion}} = \underbrace{c_{\varepsilon 1}\frac{\varepsilon}{k} S_k - c_{\varepsilon 2}\frac{\varepsilon}{k}\varepsilon}_{\text{Source}} \quad (3.72)$$

where S_k is given from Eq. (3.71) and σ_ε, $c_{\varepsilon 1}$, $c_{\varepsilon 2}$ are constants determined from experiment.

From the decay of turbulence behind a grid the value of c_2 is suggested to be 1.8 — 2.0 [67].

A relation between $c_{\varepsilon 1}$, $c_{\varepsilon 2}$, σ_ε is obtained from near to the wall turbulence [67] using Eq. (3.69) and the near wall relations

$$\left.\begin{aligned} \frac{\partial v_z}{\partial y} &= \frac{u_*}{ky} \\ \varepsilon &= \frac{u_*^3}{ky} \\ k &= \frac{u_*^2}{\sqrt{c_\mu}} \end{aligned}\right\} \qquad (3.73)$$

Neglecting diffusion and tangential contributions and taking into consideration that

$$\frac{y}{D/2} \ll 1 \quad \text{and} \quad \frac{\nu}{\nu^T_{rz}} \ll 1$$

Eq. (3.73) reduces to

$$\frac{\partial}{\partial y}\left(\frac{\nu^T_{rz}}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial r} = c_{\varepsilon 1}\frac{\varepsilon}{k}S_k - c_{\varepsilon 2}\frac{\varepsilon}{k}\varepsilon \qquad (3.74)$$

If further the tangential component of S_k in Eq. (3.71) is neglected, the near wall relations of Eq. (3.73) are used and ν^T_{rz} is represented by Eq. (3.42), we further obtain

$$\frac{d}{dy}\left[y\frac{d}{dy}\left(\frac{1}{y}\right)\right] = \frac{(c_{\varepsilon 1} - c_{\varepsilon 2})\,\sigma_\varepsilon\sqrt{c_\mu}}{\kappa^2 y^2}$$

By performing the derivatives we end up with

$$c_{\varepsilon 2} - c_{\varepsilon 1} = \frac{\kappa^2}{\sigma_\varepsilon \sqrt{c_\mu}} \qquad (3.75)$$

Launder [67] suggests that

$$c_{\varepsilon 1} \simeq 1.44 \qquad c_{\varepsilon 2} \simeq 1.92 \qquad \text{and} \qquad \sigma_\varepsilon \simeq 1.3$$

3.3.5.2 The Solution Domain

The equations are to be solved within the characteristic triangle of the infinite rod bundle as shown in Fig. 1.1.

The secondary flow quantities (i.e., v_r, v_θ, p/ρ) are going to be solved in the whole flow area of the characteristic triangle, whereas the v_z momentum equation and the k, ε transport equations are going to be solved only in the outer region of the flow (i.e., $y_B^+ \geq 25\text{-}30$). As pointed out in Ref. 1 (p. 56) the exclusion of the wall region is made due to the following reasons:

a) The sharp gradients of the hydrodynamic quantities within the wall region need a relatively high number of points to describe the hydrodynamic field sufficiently well.

b) The strongly anisotropic turbulent kinetic equation makes a turbulent model based on the Kolmogorov-Prandtl hypothesis questionable. However, on the other hand, the prescription of the quantities themselves within the wall region is relatively not difficult since the main parameters of dependence are the wall shear stress, the distance from the wall and

the fluid viscosity.

The approach of excluding the wall region from the domain of solution has been used by several authors (see, e.g., [37, 54]) and in the efforts preceding this work at MIT [1].

3.3.5.2.1 Wall Layer Thickness Prescription — The wall layer thickness is defined from the relation

$$\frac{y_B \bar{u}^*}{\nu} \simeq 30 \text{ to } 60 \qquad (3.76)$$

where $\bar{u}^*$ is the average friction velocity, i.e.

$$\bar{u}^* = \left(\frac{\tau_w}{\rho}\right)^{1/2}_{avg}$$

y_B is the wall layer thickness

and ν is the fluid kinematic viscosity.

The exact value of relation (3.76) is left as problem-dependent, and its selection should take into consideration that

1) The higher the value of $y_B\bar{u}^*/\nu$ is, the fewer points are necessary to obtain the numerical solution; and
2) The closer to 30 the value of $y_B\bar{u}^*/\nu$ is, the more valid Eq. (3.52) is.

In case the bulk velocity v_B is given instead of $\bar{u}^*$, we use the friction factor relation

$$\bar{u}^* = \sqrt{\frac{f}{2}}\, v_b \qquad (3.77)$$

For the friction factor estimation we will use Ibragimov's expression, which has also been used in Ref. [1]. This expression is as follows:

$$\left.\begin{aligned} \frac{f}{f_o} &= (0.58 + 0.42\ \bar{e}^{\gamma})[1 + 0.1(\frac{2\bar{y}}{D})^{4/3}] \\ \text{where} \\ \gamma &= 0.21\ \frac{P}{D}(\frac{A}{\bar{y}^2})^{1/4}\ (\frac{1}{\sqrt{3}} - \frac{1}{2})\ \frac{D}{\bar{y}} \\ \bar{y} &= \frac{1}{A}\iint_A yrdrd\theta = \frac{1}{2}\ (\frac{3}{\pi\sqrt{3}}\ \frac{P}{D} - 1)\ D \end{aligned}\right\} \quad (3.78)$$

A is the flow area within the characteristic triangle and

$$f_o = 0.046\ Re^{-0.2} \quad (3.79)$$

is the friction factor for circular pipe.

3.3.5.3 Boundary Conditions — The boundary conditions are given in Table 3.2b. The nature of the boundary conditions at the symmetry lines is profound. We will discuss below the boundary conditions at the near wall boundary (i.e., at the position y_B).

3.3.5.3.1 Near Wall Boundary Conditions —

Axial velocity — Along the y_B circumferential line we will make the widely used assumption of logarithmic profile, i.e.

$$v_z(y_B,\theta) = \frac{u^*(\theta)}{\kappa}\ \ell n\ E\ \frac{u^*(\theta)y_B}{\nu} \quad (3.80)$$

where κ and E are constants.

According to Ref. [1], $\kappa \simeq 0.419$, $E \simeq 9.8$.

The velocity profile in the region $0 \leq y \leq y_B$ needed for calculation of the bulk velocity is taken as follows:

$$\frac{v_z(y,\theta)}{u^*(\theta)} = \frac{u^*(\theta)y}{\nu} \qquad \text{for} \quad \frac{u^*(\theta)y}{\nu} \leq 5- \qquad (3.81a)$$

$$\frac{v_z(y,\theta)}{u^*(\theta)} = \alpha + \beta \ell n \frac{u^*(\theta)y}{\nu} \qquad \text{for} \quad 5 \leq \frac{u^*(\theta)y}{\nu} \leq \frac{u^*(\theta)y_B}{\nu} \qquad (3.81b)$$

where

$$\beta = \frac{v_z(y_B,\theta)/u^*(\theta) - 5}{\ell n \frac{y_B u^*(\theta)}{5\nu}} , \qquad (3.81c)$$

$$\alpha = \frac{1}{5}\left[v_z(y_B,\theta)/u^*(\theta) + 5 - \beta \ell n \frac{5y_B u^*(\theta)}{\nu} \right] \qquad (3.81d)$$

The expressions for α, β come from the velocity continuity condition at the points

$$\frac{yu^*(\theta)}{\nu} = 5 \quad \text{and} \quad \frac{yu^*(\theta)}{\nu} = 45 \qquad (3.82)$$

3.3.5.3.2 Turbulent Kinetic Energy and Energy Dissipation — As we have pointed out in §3.3.1.1 the near wall boundary lies within the dynamic equilibrium subregion where the importance of the convection and diffusion mechanism are minor.

Thus the transport equation for k can be reduced to

$$(-\overline{v_r' v_z'})_{y=y_B} \left(\frac{\partial v_z}{\partial r}\right)_{y=y_B} \simeq \varepsilon_B \qquad (3.83)$$

where ε_B is the energy dissipation at $y=y_B$.

On the other hand the axial momentum equation neglecting the lateral gradient as small compared with the radial gradient

is reduced to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\overline{v_r'v_z'}\right) - \nu\frac{\partial^2 v_z}{\partial r^2} = \frac{4}{D_H}(\bar{u}^*)^2 \tag{3.84}$$

where D_H is the hydraulic diameter and $\bar{u}^*$ is the average friction velocity.

Integrating Eq. (3.80) over y_B and neglecting the friction effect at y_B we obtain

$$\left(-\overline{v_r'v_z'}\right)_{y=y_B} = \frac{r_1}{r_2}[u^*(\theta)]^2 - 2\frac{r_2^2 - r_1^2}{r_2 D_H}(\bar{u}^*)^2 \tag{3.85}$$

where

$$r_1 = D/2$$

$$r_2 = D/2 + y_B$$

Since

$$\left(\frac{\partial v_z}{\partial r}\right)_{y=y_B} = \frac{u^*(\theta)}{\kappa y_B}$$

and

$$c_\mu \frac{k_B^2}{\varepsilon_B}\left(\frac{\partial v_z}{\partial r}\right)_{y=y_B} = \left(-\overline{v_r'v_z'}\right)_{y=y_B}$$

we find finally

$$\varepsilon_B = \left(-\overline{v_r'v_z'}\right)_{y=y_B}\frac{u^*(\theta)}{\kappa y_B} \tag{3.86}$$

$$k_B = \frac{1}{\sqrt{c_\mu}}\left(-\overline{v_r'v_z'}\right)_B \tag{3.87}$$

3.3.5.4 Reynolds Number vs. Mean Wall Shear Stress — Since we are going to solve the problems for the fully developed flow, some information on flow level is needed. This has been done by two methods:

a) Specify the average wall shear stress $(\tau_w/\rho)_{avg}$ which appears in the source term of mean velocity equation (See Table 3.2a).

b) Specify the Reynolds number or equivalently the bulk velocity.

Both approaches are mathematically well defined. However, there is a difference when the problem is solved numerically.

In the second case, since there is an integral type of restriction and the problem has to be solved iteratively, some normalization of velocities is needed per iteration or per number of iterations. Since each numerical technique has a different effect per iteration, due to finite number of points, the answer will depend not only on the number of points but from the numerical method used as well.

This is illustrated in Fig. 3.4 where the shear stress distribution is plotted for a GRID 30×16, P/D = 1.124, Re ≈ 27,000, using a 0-equation model for simplicity.

The solution has been obtained for a Reynolds number input by line iteration (See Chapter 4) excluding secondary flow and successive relaxation method with and without secondary flow. The effect of the numerical method is clear.

It is worth pointing out here that the solution of the no secondary flow problem with line iteration technique (curve 1) and the solution of the secondary flow problem with successive relation (curve 2) leads to the conclusion that the secondary flow increases the non-uniformity of wall shear stress distribution -- which is WRONG!

Therefore, in the present work the input of average wall shear stress is used.

CHAPTER 4

NUMERICAL WORK

4.1 Grid

We have tried to make the grid space as uniform as possible. The reason is that although grid nonuniformity can improve the accuracy of the problem in some cases, it usually has a negative effect on rate of convergence and stability [70]. On the other hand, a uniform grid makes the numerical equations easier to handle. So in the θ direction a uniform spacing $\Delta\theta$ is used, i.e.

$$\Delta\theta = \theta_j - \theta_{j-1} = \frac{\pi}{6*(JN - 1)} \tag{4.1}$$

where JN is the number of mesh points in the θ-direction. With respect to the r-direction, (i.e., $\theta = 0$)

$$r_i - r_{i-1} = \left(\frac{P}{2} - \left(\frac{D}{2} + y_B \right) \right) / (M - 2); \quad i \leq M$$

$$M = IN - (JN - 1) \tag{4.2a}$$

where

M is the number of points along $\theta = 0$ line

IN is the number of points along $\theta = 30°$ line, and

y_B is the wall boundary thickness.

For $i > M$ the spacing cannot be uniform since it is defined from the zero shear stress line as shown in Fig. 4.1.

In this case

$$r_{i=M+j-1} = \frac{P}{2\cos\theta} - \frac{P}{2} \; ; \quad M < i \leq IN \tag{4.2b}$$

The grid is shown in Fig. 4.1.

So by the above procedure the mesh points (i,j) have been defined. However at these points only the scalar parameters k, ε, and p the axial velocity v_z and auxiliary quantities (e.g., eddy viscosities, mixing length) will be calculated. The parameters $v_r(i,j)$ and $v_\theta(i,j)$ will be defined in a slightly different position as shown in Fig. 4.3. This procedure is similar to that adopted in the TEACH code [71].

4.2 The Finite Difference Equations

4.2.1 Momenta and Transport Equations

The finite difference equations are obtained by integration of Eq (3.67) over the control volume. The control volume surrounding the point under consideration is shown in its general form in Fig. 4.2.

After integration over the control volume and taking into consideration the Divergence Theorem Eq. (3.67) becomes

$$\oint_{cs} \phi \, \underline{v} \, \underline{n} \, d\ell + \oint_{cs} \underline{T}_{\phi} \, \underline{n} \, d\ell = \oint_{cs} \underline{S}_{\phi} \, \underline{n} \, d\ell + \iint_{cv} S^*_{\phi} \, dA \tag{4.3}$$

where by $\iint_{cv}$ we denote the integration over the boundary of the control volume, and $\underline{n}$ is the unit vector perpendicular to the surface boundary as shown in Fig. 4.2. Let

A_e = length (ne, se)

A_w = length (nw, sw)

A_n = length (ne, nw)

A_s = length (se, sw)

V_p = area (ne, se, sw, nw, ne) (4.4)*

The value of above quantities for the various angles ϕ are given in Table 4.2. Equation (4.3) can be written

$$A_e\phi_e v_{re} - A_w\phi_w v_{rw} + A_n\, n v\phi_n - A_s\phi_s v_{\theta s} + T_{\phi re}A_e -$$

$$- T_{\phi rw}A_w + T_{\phi\theta n}A_n - T_{\phi\theta s}A_s = S_{\phi re}A_e - S_{\phi rw}A_w +$$

$$+ S_{\phi\theta n}A_n - S_{\phi\theta s}A_s + S^*_{\phi P}V_P \qquad (4.5)$$

In Eq. (4.5) the following approximation has been made

$$\iint_{cv} S^*_\phi dA \approx S_{\phi\underline{P}}^* V_P \qquad (4.6)$$

where

$S_{\phi\underline{P}}^*$ is the value of S* at point P

V_P is the control volume surface

The continuity equation gives after integration over the control volume

$$A_e\, v_{re} - A_w v_{rw} + A_n v_{\theta n} - A_s v_{\theta s} = 0 \qquad (4.7)$$

In Eq. (4.5) we can distinguish between the Convection term, the Diffusion term and the Source term as follows:

$$DT = T_{\phi re}A_e - T_{\phi rw}A_w + T_{\phi\theta n}A_n - T_{\phi\theta s}A_s \qquad (4.8a)$$

(*) the nomenclature used in Fig. 4.2 is almost the same as that used in Ref. 36.

Convection term:

$$CT = A_e v_{re}\phi_e - A_w v_{rw}\phi_w + A_n v_{\theta n}\phi_n - A_s v_{\theta s}\phi_s \qquad (4.8b)$$

Source term:

$$ST = S_{\phi re}A_e - S_{\phi rw}A_w + S_{\phi \theta n}v_{\theta n} - S_{\phi \theta s}v_{\theta s} + S^*_{\phi P}V_P \qquad (4.8c)$$

Thus Eq. (4.5) can be written

$$DT + CT = ST \qquad (4.8d)$$

4.2.1.1 Diffussion Term

4.2.1.1.1 Scalar Quantities k, ε and the Axial Velocity V_z

From the Table 3.9a we have

$$\left.\begin{aligned} T_{\phi r} &= -\frac{\nu_{rz}^{eff}}{\sigma_\phi}\frac{\partial\phi}{\partial r} \\ T_{\phi} &= -\frac{\nu_{\theta z}^{eff}}{\sigma_\phi}\frac{\partial\phi}{r\partial\theta} \end{aligned}\right\} \qquad (4.9)$$

We can make the following approximations

$$\left.\begin{aligned} T_{\phi re} &= -\frac{\left[\nu_{rz}^{eff}\right]_E + \left[\nu_{rz}^{eff}\right]_P}{2\sigma_\phi}\cdot\frac{\phi_E - \phi_P}{E - V_P} \\ T_{\phi rw} &= \frac{\left[\nu_{rz}^{eff}\right]_W + \left[\nu_{rz}^{eff}\right]_P}{2\sigma_\phi}\cdot\frac{\phi_P - \phi_W}{V_P - V_W} \end{aligned}\right\} \qquad (4.10a)$$

$$\left.\begin{aligned} T_{\phi\theta n} &= -\frac{\left[\nu_{\theta z}^{eff}\right]_N + \left[\nu_{\theta z}^{eff}\right]_P}{2\sigma_\phi}\cdot\frac{\phi_N - \phi_P}{V_P\Delta\theta} \\ T_{\phi\theta s} &= -\frac{\left[\nu_{\theta z}^{eff}\right]_S + \left[\nu_{\theta z}^{eff}\right]_P}{2\sigma_\phi}\cdot\frac{\phi_P - \phi_S}{V_P\ \Delta\theta} \end{aligned}\right\} \qquad (4.10b)$$

Thus Eq. (4.8a) becomes

$$DT = (DE+DW+DN+DS)\phi_P - DN \cdot \phi_N - DS \cdot \phi_S - DE \cdot \phi_E - DW \cdot \phi_W$$

where ϕ equals k, ε or V_z

where (4.11)

$$DE = \frac{BE}{\sigma_\phi}\left[\left(\nu_{rz}^{eff}\right)_E + \left(\nu_{rz}^{eff}\right)_P\right] \quad (4.12a)$$

$$DW = \frac{BW}{\sigma_\phi}\left[\left(\nu_{rz}^{eff}\right)_W + \left(\nu_{rz}^{eff}\right)_P\right] \quad (4.12b)$$

$$DN = \frac{BN}{\sigma_\phi}\left[\left(\nu_{\theta z}^{eff}\right)_N + \left(\nu_{\theta z}^{eff}\right)_P\right] \quad (4.12c)$$

$$DS = \frac{BS}{\sigma_\phi}\left[\left(\nu_{\theta z}^{eff}\right)_S + \left(\nu_{\theta z}^{eff}\right)_P\right] \quad (4.12d)$$

$$BE = \frac{Ae}{2(r_E - r_P)} \quad (4.13a)$$

$$BW = \frac{Aw}{2(r_P - r_w)} \quad (4.13b)$$

$$BN = \frac{An}{2r_P \Delta\theta} \quad (4.13c)$$

$$BS = \frac{As}{2r_P \Delta\theta} \quad (4.13d)$$

The values of DE, DW, DN, DS, BE, BW, BS are given in Table 4.2.

4.2.1.1.2 The Velocity v_r

From Table 3.2a we see that

$$\left.\begin{aligned} T_{\phi r} &= -2\nu \frac{\partial v_r}{\partial r} \\ T_{\phi\theta} &= -\nu^{eff}_{r\theta} \frac{\partial v_r}{r\partial\theta} \end{aligned}\right\} \quad (4.14)$$

We make the following approximations

$$\left.\begin{aligned} T_{\phi re} &= -2\nu \left[v_{rE} - v_{rP}\right] / \left[v_E - v_P\right] \\ T_{\phi rw} &= -2\nu \left[v_{rP} - v_{rW}\right] / \left[v_P - r_w\right] \\ T_{\phi\theta n} &= -\nu^{eff}_{rzn} \left[v_{rn} - v_{rP}\right] / r_P / \Delta\theta \\ T_{\phi\theta s} &+ -\nu^{eff}_{r\theta s} \left[v_{rP} - v_{rS}\right] / r_P / \Delta\theta \end{aligned}\right\} \quad (4.15)$$

Thus the diffusion term (4.8a) for v_r becomes finally

$$\boxed{DT = (DE+DW+DN+DS)v_{rP} - DN\cdot v_{rN} - DS\cdot v_{rS} - DE\cdot v_{rE} - DW\cdot v_{rW}} \quad (4.16)$$

where

$$DE = BE \cdot \nu \quad (4.17a)$$

$$DW = BW \cdot \nu \quad (4.17b)$$

$$DN = BN \left[\left(\nu^{eff}_{r\theta}\right)Ne + \left(\nu^{eff}_{r\theta}\right)_e + \left(\nu^{eff}_{r\theta}\right)_w + \left(\nu^{eff}_{Nw}\right)_{Nw}\right] \quad (4.17c)$$

$$DS = BS \left[\left(\nu^{eff}_{r\theta}\right)Se + \left(\nu^{eff}_{r\theta}\right)_e + \left(\nu^{eff}_{r\theta}\right)_w + \left(\nu^{eff}_{r\theta}\right)_{Sw}\right] \quad (4.17d)$$

$$BE = \frac{2A_e}{r_E - r_P} \tag{4.18a}$$

$$BW = \frac{2A_W}{r_P - r_W} \tag{4.18b}$$

$$BN = BS = \frac{A_n}{4r_P \Delta\theta} \tag{4.18c}$$

The values of DE, DW, DN, DS, BE, DW, BS are given in Table 4.2.

4.2.1.1.3 The Velocity v_θ

From Table I we have

$$T_{\phi r} = - \nu_{r\theta}^{eff} \, r^2 \, \frac{\partial}{\partial r} \left(\frac{v_\theta}{r}\right) \tag{4.19a}$$

$$T_{\phi\theta} = - 2\nu \frac{\partial v_\theta}{\partial \theta} \tag{4.19b}$$

We make the following approximations

$$\left.\begin{aligned} T_{\phi re} &= \nu_{r\theta e}^{eff} \, r_w^2 \left[\frac{\frac{v_{\theta E}}{r_E} - \frac{v_{\theta P}}{r_P}}{r_E - r_P}\right] \\ T_{\phi rw} &= \nu_{r\,w}^{eff} \, r_w^2 \left[\frac{\frac{v_{\theta P}}{r_P} - \frac{v_{\theta W}}{r_W}}{r_P - r_W}\right] \end{aligned}\right\} \tag{4.20}$$

$$\left.\begin{aligned} T_{\phi\theta n} &= -2\nu \left(\frac{v_{\theta N} - v_{\theta P}}{\Delta\theta} \right) \\ T_{\phi\theta s} &= 2\nu \left(\frac{v_{\theta P} - v_{\theta S}}{\Delta\theta} \right) \end{aligned}\right\} \qquad (4.20)$$

Thus Eq. (4.8a) for v_θ becomes

$$\boxed{DT = \left(DN+DS+DE\cdot\frac{r_E}{r_P}+DW\frac{r_W}{r_P}\right) v_P - DE\cdot v_{\theta E} - DW\cdot v_{\theta W} - DN\cdot v_{\theta N} - DS\cdot v_{\theta S}} \qquad (4.21)$$

where

$$\left.\begin{aligned} DE &= BE \left[\left(\nu_{r\theta}^{eff}\right)_{nE} + \left(\nu_{r\theta}^{eff}\right)_{sE} + \left(\nu_{r\theta}^{eff}\right)_{s} + \left(\nu_{r\theta}^{eff}\right)_{n} \right] \\ DW &= BW \left[\left(\nu_{r\theta}^{eff}\right)_{nW} + \left(\nu_{r\theta}^{eff}\right)_{sW} + \left(\nu_{r\theta}^{eff}\right)_{s} + \left(\nu_{r\theta}^{eff}\right)_{n} \right] \end{aligned}\right\} \qquad (4.22)$$

$$\left.\begin{aligned} DN &= DW = BN \cdot \nu \\ BE &= \frac{r_e^2 \quad Ae}{4r_E(r_E - r_P)} \\ BW &= \frac{r_w^2 \quad Aw}{4r_W(r_P - r_W)} \\ BN &= BS = \frac{2An}{\Delta\theta} \end{aligned}\right\} \qquad (4.23)$$

The values of DE, DW, DN, DS, BE, BW, BS are given in Table 4.23.

4.2.4.2 The Convection Term

Recall that the convection term is given by the Eq. (4.8) i.e.,

$$CT = A_e v_{re} \phi_e - A_w v_{rw} \phi_w + A_n v_{\theta n} \phi_n - A_s v_{\theta s} \phi_s \qquad (4.8b)$$

where

ϕ is given in Table 3.2a

From Eq. (4.8b) it is evident that the common substitution

$$\phi_e = \frac{\phi_E + \phi_P}{2}$$

etc., can produce a finite difference equation with negative coefficients which can produce nonconvergence in the solution. Since our ultimate purpose is to obtain a finite difference equation with positive coefficients and on the other hand as much accuracy as possible we use the Upwind Derivative Difference Scheme (UDDS) proposed in Appendix A.

Let

$$\left.\begin{aligned} \phi_e &= \phi_P + \left(\frac{\partial \phi}{\partial r}\right)_P (r_e - r_P) \\ \phi_n &= \phi_P + \left(\frac{\partial \phi}{\partial \theta}\right)_P \frac{\Delta\theta}{2} \\ \phi_w &= \phi_P - \left(\frac{\partial \phi}{\partial r}\right)_P (r_P - r_w) \\ \phi_s &= \phi_P - \left(\frac{\partial \phi}{\partial \theta}\right)_P \frac{\Delta\theta}{2} \end{aligned}\right\} \qquad (4.24)$$

Substituting Eqs. (4.9) into (4.86) and taking into consideration continuity Eq. (4.7) we end up with

$$CT = \left[A_e v_{re}(r_e - r_P) = A_w v_{rw}(r_P - r_w)\right] \left(\frac{\partial \phi}{\partial r}\right)_P + \frac{A_n \Delta\theta}{2} (v_{\theta n} + v_{\theta s}) \left(\frac{\partial \phi}{\partial \theta}\right)_P \tag{4.25}$$

In Eq. (4.25) we have taken into consideration the fact that $A_n = A_s$.

We further approximate

$$\left(\frac{\partial \phi}{\partial r}\right)_P = \begin{cases} \dfrac{\phi_E - \phi_P}{r_E - r_P} & \text{if } A_e v_{re}(r_e - r_P) + A_w v_{rw}(r_P - r_w) \leq 0 \\ \dfrac{\phi_P - \phi_w}{r_P - r_w} & \text{otherwise} \end{cases} \tag{4.26}$$

$$\left(\frac{\partial \phi}{\partial \theta}\right)_P = \begin{cases} \dfrac{\phi_N - \phi_P}{\Delta\theta} & \text{if } v_{\theta P} \leq 0 \\ \dfrac{\phi_P - \phi_S}{\Delta\theta} & \text{otherwise} \end{cases} \tag{4.27}$$

Under these circumstances the convection term can be written

$$CT = (CE + CW + CN + CS)\phi_P - CE \cdot \phi_E - CW \cdot \phi_W - CN \cdot \phi_N - CS \cdot \phi_S \tag{4.28}$$

where

$$CE = 0.5\Big\{\big|A_e v_{re}(r_e - r_P) + A_w v_{rw}(r_P - r_w)\big| - A_e v_{re}(r_e - r_P) - A_w v_{rw}(r_P - r_w)\Big\}\Big/(r_E - r_P) \tag{4.29a}$$

$$CW = 0.5\Big\{\big|A_e v_{re}(r_e - r_P) + A_w v_{rw}(r_P - r_w)\big| + A_e v_{re}(r_e - r_P) + A_w v_{rw}(r_P - r_w)\Big\}\Big/(r_P - r_w) \tag{4.29b}$$

$$CN = A_n \frac{|v_{\theta n} + v_{\theta s}| - v_{\theta n} - v_{\theta s}}{4} \tag{4.29c}$$

$$CS = A_s \frac{|v_{\theta n} + v_{\theta s}| + v_{\theta n} + v_{\theta s}}{4} \tag{4.29d}$$

The value for CE, CW, CN, CS at a particular point (i,j) and for $\phi = k, \varepsilon, v_z, v_r$ are given in Table 4.3.

The convection term for the component v_θ itself is slightly different (*) than Eq. (4.28) as in the case of the diffusion term, i.e.,

$$CT = \left(CE\,\frac{r_P}{r_E} + CW\,\frac{r_P}{r_w} + CN + CS\right)v_{\theta P} - CE \cdot v_{\theta E} - CW \cdot v_{\theta W} - CN \cdot v_{\theta N} - CS \cdot v_{\theta S}$$

The values of CE, CW, CS for v_θ are given in Table 4.3.

(*) Equation (4.28 is valid for rv_θ instead of v_θ as Table 3.2a shows.

4.2.1.3 Source Term

Recall that the source term is given by Eq. (4.8c) i.e.,

$$ST = S_{\phi re} A_e - S_{\phi rw} A_w + S_{\phi \theta n} A_n - S_{\phi \theta s} A_s + S^*_{\phi} V_P \qquad (4.8c)$$

The form of the source term for each quantity is different in character so we examine ST for each quantity separately.

4.2.1.3.1 Axial Velocity v_z,

From Table we see that

$$ST = S^*_p V_p = \frac{4}{DH} \left(\frac{\tau_w}{\rho}\right)_{avg} V_P \qquad (4.30)$$

where

$\left(\frac{\tau_w}{\rho}\right)_{avg}$ is the average shear stress, and

DH is the hydraulic diameter.

4.2.1.3.2 Kinetic Energy k

From Table (3.2a)

$$ST = S^*_P V_P = \left[(\nu^T_{rz})_P \left(\frac{\partial v_z}{\partial r}\right)^2_P + (\nu^T_{\theta z})_P \left(\frac{\partial v_z}{r \partial \theta}\right)^2 - \varepsilon_P \right] V_P \qquad (4.31)$$

For reasons of numerical stability the energy dissipation term is calculated from Eq. (3.42) i.e.,

$$\varepsilon_P = \frac{c_\mu k_P^2}{(\nu^T_{rz})_P} \qquad (4.32)$$

Thus

$$ST = \left[(\nu^T_{rz})_P \left(\frac{\partial v_z}{\partial r}\right)_P + (\nu^T_{\theta z})_P \left(\frac{\partial v_z}{r \partial \theta}\right)^2_P \right] V_P - SP \cdot k_P \qquad (4.33)$$

where

$$SP = \frac{c_\mu k_P}{(\nu^T_{rz})_P} \tag{4.34}$$

4.2.3.1.3.3 Energy Dissipation ε

From Table (3.2a)

$$ST = \left\{ c_{\varepsilon 1} \frac{\varepsilon}{k} \left[\nu^T_{rz} \left(\frac{\partial v_z}{\partial r} \right)^2 + \nu^T_{\theta z} \left(\frac{\partial v_z}{r \partial \theta} \right)^2 \right] - c_{\varepsilon 2} \frac{\varepsilon^2}{k} \right\} \tag{4.35}$$

Since

$$\frac{\varepsilon}{k} = c_\mu \frac{k}{\nu^T_{\nu z}} \tag{4.36}$$

Then

$$ST = c_{\varepsilon 1} \frac{c_\mu k_P}{(\nu^T_{rz})_P} \left\{ (\nu^T_{rz})_P \left(\frac{\partial v_z}{\partial r} \right)^2 + \nu^T_{\theta z} \left(\frac{\partial v_z}{r \partial \theta} \right)^2_P \right\} - SP \; \varepsilon_P \tag{4.37}$$

where

$$SP = c_{\varepsilon 2} c_\mu \frac{k_P v_P}{(\nu^T_{rz})_P} \tag{4.38}$$

4.2.1.3.4 Radial Velocity v_r

From Table (3.2a) we have

$$S_{\phi r} = -\overline{(v'_{r\theta})} + \frac{p}{\rho}$$

$$S_{\phi\theta} = \nu^{eff}_r \; r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right)$$

$$S^*_\phi = \frac{v_\theta^2 + \overline{v'^2_\theta}}{r} - \frac{2\nu}{r} \left(\frac{\partial v_\theta}{r \partial \theta} + \frac{v_r}{r} \right) \approx \frac{\overline{v'^2_\theta}}{r}$$

Thus Eq. (4.86) becomes

$$ST = A_w\left[\overline{v_{rw}'^2} + (p/\rho)_w\right] - A_e\left[\overline{v_{re}'^2} + (p/\rho)_e\right]$$

$$+ (\nu_r^{eff})_n \, r_P\left[\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]_n A_n - \nu_r^{eff} \, r_P\left[\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]_s A_s$$

$$+ \frac{(\overline{v_\theta'^2})_P}{r_P} \, V_P \qquad (4.40)$$

The ST value for the point (i,j) is given in Table 4.4.

4.2.1.3.5 Velocity v_θ

From Table (3.2a) we have

$$S_{\phi r} = -\nu_{r\theta}^{eff} \frac{\partial v_r}{\partial \theta} \qquad (4.41)$$

$$S_{\phi\theta} = 2\nu v_r - r\overline{v_\theta'^2} - \frac{rp}{p} - r(\overline{v_\theta'^2} + \frac{p}{\rho}) \qquad (4.42)$$

Thus Eq. (4.8c) can be approximated

$$ST = \nu_{r\theta e}^{eff} \left(\frac{\partial v_r}{\partial \theta}\right)_e A_e - \nu_{r\theta w}^{eff} \left(\frac{\partial v_r}{\partial \theta}\right)_w A_w$$

$$+ r_p A_n\left[(\overline{v_\theta'^2})_s + (p/\rho)_s - (\overline{v_\theta'^2})_n - (p/\rho)_n\right] \qquad (4.43)$$

The ST value for the point (i,j) is given in Table 4.4.

4.2.1.4 General Form of FDE

The general form of the differential equations of all the quantities under consideration, has as follows

$$(AP + SP)\phi_P = AE \cdot \phi_E + AN \cdot \phi_N + AW \cdot \phi_W + AS \cdot \phi_S + SU \quad (4.44a)$$

where

$$\left.\begin{aligned} AE &= CE + DE \\ AW &= CW + DW \\ AN &= CN + DN \\ AS &= CS + DS \\ SU &= ST + SP \cdot \phi_P \end{aligned}\right\} \quad (4.44b)$$

$$AP = \begin{cases} \dfrac{r_P}{r_E} CE + \dfrac{r_E}{r_P} DE + \dfrac{r_P}{r_W} CW + \dfrac{r_W}{r_P} DW + AN + AS & \text{for } v_\theta \\ AE + AW + AN + AS & \text{otherwise} \end{cases} \quad (4.44c)$$

where CE, CW, CN, CS, DE, DW, DN, DS, SU, SP are given in Table 4.2, 4.3 and 4.4.

4.2.1.5 The Boundary Conditions

4.2.1.5.1 Quantities k, ε, v_z

4.2.1.5.1.1 Boundary I and II (Fig. 4.1)

According to Table 3.2b

$$\left(\frac{\partial \phi}{\partial \theta}\right)_{I,II} = 0 \quad (4.45)$$

where

$$\phi = k, \varepsilon \text{ or } v_z$$

Numerically Eq. (4.44) is translated to

$$\left.\begin{aligned} \phi_{i,j} &= \phi_{i,j+1} \quad \text{for Boundary I} \\ \phi_{i,j} &= \phi_{i,j-1} \quad \text{for Boundary II} \end{aligned}\right\} \quad (4.46)$$

4.2.1.5.1.2 Boundary III

From Table (3.2a) we have

$$\left(\frac{\partial \phi}{\partial n}\right)_{III} = 0 \tag{4.47}$$

Integrating over the triangular control volume shown in Fig. 4.4 and taking into consideration Eq. (4.47) we end up with

$$\left.\begin{aligned} (AP + SP')\phi_P &= AW \cdot \phi_W + AN \cdot \phi_N' + SU' \\ AW &= DW + CW' \\ AN &= DN + CN' \\ AP &= AW + AN \end{aligned}\right\} \tag{4.48}$$

Where DW, DN are given from Table 4.2 and 4.3 respectively

$$\left.\begin{aligned} CN' &= -AN(v_\theta)_{i,j+1}/2 \\ CW' &= A_W(v_r)_{i,j}/2 \\ SU' &= SU\,\frac{V_{P'}}{V_P} \\ SP' &= SP\,\frac{V_{P'}}{V_P} \end{aligned}\right\} \tag{4.49}$$

and A_e, A_n, V_P are given from Table 4.1. SU, SP are given from Table 4.4.

$V_{P'}$ is the area of the triangular control volume under consideration, i.e.,

$$(V_{P'}) = \left[(r_{i+1} + r_i)\sin\Delta\theta - (r_i + r_{i-1})\Delta\theta\right]\frac{r_i + r_{i+1}}{8} \tag{4.50}$$

It is evident from Eq. (4.49) that we have substituted for

$$\phi_e = \frac{\phi_E + \phi_P}{2}$$

and similarly for ϕ_w, ϕ_n, ϕ_s.

4.2.1.5.1.3 Wall Region

Axial Velocity

At the y_B line we will demand continuity of the velocity and the velocity gradient.

The velocity gradient can be calculated numerically with accuracy $0(r^2)$ from the relation

$$\left(\frac{dv_z}{dr}\right)_{2,j} = \frac{r_4 - r_2}{(r_3 - r_2)(r_4 - r_3)} (v_z)_{3,j} - \frac{r_3 - r_2}{(r_4 - r_2)(r_4 - r_3)} (v_z)_{4,j}$$

$$- \frac{r_4 + r_3 - 2r_2}{(r_3 - r_2)(r_4 - r_2)} (v_z)_{2,j} \qquad (4.51a)$$

since

$$(v_z)_{2,j} = \frac{u_j^*}{\kappa} \ln E \frac{u_j^* y_B}{\nu} \qquad (4.51b)$$

and

$$\left(\frac{dv_z}{dr}\right)_{2,j} = \frac{u_j^*}{\kappa y_B} \qquad (4.51c)$$

we can solve Eq. (4.51) for u_J^* and $(v_z)_{2,j}$ numerically.

Turbulent Kinetic Energy and Energy Dissipation

$k_{2,j}$ and $\varepsilon_{2,j}$ are calculated from the algebraic relations given in Table 3.2b.

4.2.1.5.2 v_r Velocity

Boundary I and II (Fig. 1.1)

According to Table 3.2b

$$\left(\frac{\partial v_r}{\partial \theta}\right)_{I,II} = 0 \tag{4.52a}$$

thus numerically Eq. (4.52a) is translated

$$\left.\begin{aligned} (v_r)_{i,j} &= (v_r)_{i,j+1} \quad \text{for Boundary I} \\ (v_r)_{i,j} &= (v_r)_{i,j-1} \quad \text{for Boundary II} \end{aligned}\right\} \tag{4.52b}$$

Boundary III (Fig. 1.1)

From Table 3.2b we have

$$\underline{V}_{sec}\ \underline{n} = 0 \tag{4.53}$$

Numerically Eq. (4.53) can be expressed as we can see from Fig. 4.4b.

$$(A_e)_{i-1}(v_r)_{i,j} = (A_n)_{i-1}(v_\theta)_{i-1,j} \tag{4.54}$$

where A_e and A_n are given in Table 4.3.

Wall

From Table 3.2b we have

$$(v_r)_{i,j} = 0 \tag{4.55}$$

from the continuity equation, since

$$\frac{\partial v_\theta}{\partial \theta} = 0 \quad \text{at the wall}$$

it is

$$\frac{\partial (r v_r)}{\partial r} = 0$$

which gives

$$(v_r)_{i,j} = 0 \tag{4.56}$$

as well

4.2.1.5.3 v_θ Velocity

Boundaries I and III

From Table 3.2b we have

$$(v_\theta)_{I,II} = 0 \tag{4.57}$$

Numerically we can approximate* (with accuracy of $0(\frac{\Delta\theta}{2})$)

$$\left.\begin{aligned}(v_\theta)_{i,2} &= 0\\ (v_\theta)_{i,N} &= 0\end{aligned}\right\} \tag{4.58}$$

where N is the maximum number of points for v_z along the tangential direction.

Boundary III

From Table 3.2b

$$\frac{\partial V_{sec}}{\partial_n} = 0 \tag{4.59}$$

This relation leads to (see Ref. 1)

$$\frac{\partial(rv_\theta)}{\partial r} = \frac{\partial v_r}{\partial\theta} \tag{4.60}$$

Numerically Eq. (4.59) as we can see from Fig. 4.46 can be expressed

$$\frac{r_i(v_\theta)_{i,j} - r_{i-1}(v_\theta)_{i-1,j}}{r_i - r_{i-1}} = \frac{(v_r)_{i,j} - (v_r)_{i,j-1}}{\Delta\theta} \tag{4.61}$$

---

*More accurate schemes for Eq. (4.57) have led to instabilities.

Wall

From Table 3.2b

$$(v_\theta)_{1,j} = 0 \tag{4.62}$$

4.2.2 Pressure Equation

Up to now we have obtained Finite Difference Equations for the quantities v_z, v_r, v_θ, k and ε. What remains is an equation for pressure which has to come from the continuity equation.

Here we will follow the procedure followed by A.D. Gosman [67,71] and Patankar and Spalding [72].

The idea is that the previously obtained equations for v_r and v_θ do not necessarily satisfy the continuity euquations. Thus pressure connections will lead to velocity connections so that the continuity equation is obeyed.

Thus in order to obtain a finite difference equation for p/ρ at a particular point P (see Fig. 4.2) the pressures p_P, p_E, p_W, p_N, p_S must change so that the continuity equation is obeyed. Let the new pressures and velocities be

$$\left.\begin{aligned} p_P^* &= p_P + p_P' \\ p_E^* &= p_E + p_E' \qquad v_{re}^* = v_{re} + v_{re}' \\ p_W^* &= p_W + p_W' \qquad v_{rw}^* = v_{rw} + v_{rw}' \\ p_N^* &= p_N + p_N' \qquad v_{rn}^* = v_{rn} + v_{rn}' \\ p_S^* &= p_S + p_S' \qquad v_{rs}^* = v_{rs} + v_{rs}' \end{aligned}\right\} \tag{4.63}$$

The continuity equation is

$$A_e v_{re} - A_w v_{rw} + A_n v_{\theta n} - A_s v_{\theta s} = SMP \tag{4.64a}$$

and

$$A_e v^*_{re} - A_w v^*_{rw} + A_n v^*_{\theta n} - A_s v_{\theta s} = 0 \qquad (4.64b)$$

SMP is not necessarily zero.

From the general Eq. (4.44) and Eq. (4.40) we can see that the equation for v_{re} is (see Fig. 4.2).

$$(AP)_e v_{re} = \text{length}(n,s) \cdot p_P/\rho - \text{length}(nE,sE) \cdot p_E/\rho + \text{other terms} \qquad (4.65)$$

By substituting v_{re} for v^*_{re} and p_E, p_P for p^*_E, p^*_W and neglecting any effect from the other terms we end up with

$$A_e(v^*_{re} - v_{re}) = DPE \frac{r_e}{r_E} p'_P/\rho - DPE \cdot p'_E/\rho \qquad (4.66)$$

and

$$DPE = \frac{\text{length}(nE, sE)}{(AP)_e} \cdot A_e \qquad (4.67)$$

Working similarly with the other velocities, substituting the resultant equations into Eq. (4.64a) and taking into consideration Continuity Eq. (4.64b), we end up with

$$DPE \frac{r_P}{r_E} + DPW \frac{r_P}{r_W} + DPN + DPS\ p'_P/\rho = DPE * p'_E/\rho + DPW\ p'_W/\rho + DPN\ p'_N/\rho + DPS \cdot p'_S/\rho - SMP \qquad (4.68)$$

The expression for DPE, DPW, DPN, DPS at the particular point (i,j) are given in Table 4.5. Equation (4.68) is the FDE for pressure correction and has exactly the same form as Eq. (4.44)

which is valid for the other quantities. If we write

$$(AP+SP)p_P/\rho = AE \cdot p_E/\rho + AW \cdot p_W/\rho + AN \cdot p_N/\rho + AS \cdot p_S/\rho + SU \quad (4.69)$$

where

$$\left.\begin{aligned} AE &= DPE \\ AW &= DPW \\ AN &= DPN \\ AS &= AE\,\frac{r_P}{r_E} + AW\,\frac{r_P}{r_W} + AN + AS \\ SU &= -SMP \\ SP &= 0 \end{aligned}\right\} \quad (4.70$$

4.3 Numerical Solution

4.3.1 General

Recall that the finite difference equtions to be solved are of the form

$$(AP)^*_{i,j}\phi_{i,j} = (AE)^*_{i,j}\phi_{i+1,j} + (AW)_{i,j}\phi_{i-1-j} + (AN)_{i,j}\phi_{i,j+1}$$

$$+ (AS)_{i,j}\phi_{i,j+1} + SU_{i,j}$$

$$AP \leq AE + AW + AN + AS$$

$$AP,\ AE,\ AN,\ AS \geq 0 \quad (4.71)$$

where ϕ stands for

v_z the axial velocity;

v_r the radial velocity;

v_θ the the tangential velocity

p'/ρ the pressure correction

k turbulent kinetic energy, and

ε energy dissipation

Equation (4.71) is a non-linear equation since its coefficients are function of the unkonwn quantities.

The behavior of a corresponding linear equation of form (4.71) is well known and a number of numerical techniques exist that guarantee convergence. However in the actual nonlinear problem, the applicability of such techniques is not known. In the present work our original purpose was to use the numerical technique used in TEACH [71], which here is called line iteration technique (LI). Actually LI gives a relatively good rate of convergence in the case where the secondary flow is suspended. However in the case where secondary flow is retained we have difficulties due to the very low rate of convergence. Therefore the secondary flows solutions have been worked by using the well known successive relaxation technique [74]. Both techniques as applied in our case are described below.

4.3.2 Line Iteration Technique [71]

In order to minimize the negative effect with respect to convergence that the non-linearity introduces, an under-relaxation factor is introduced as follows:

$$\phi_{i,j}^{(n)} = (1\text{-UREL})\ \phi_{i,j}^{(n-1)} + \text{UREL}\ \ \phi_{i,j}^{*(n)} \qquad (4.72)$$

where

n is the number of iteration

UREL the underrelaxation factor, and

$\phi^{*(n)}_{i,j}$ is given by a relation based on Eq. (4.71) as follows

$$(AP)^{(n-1)}_{i,j}\phi^{*(n)}_{i,j} = (AE)^{n-1}_{i,j}\phi^{(n)}_{n+1,j} + (AW)^{(n-1)}_{i,j}\phi^{(n)}_{i-1,j}$$

$$+ (AN)^{(n-1)}_{i,j}\phi^{(n-1)}_{i,j+1} + (AS)^{(n-1)}_{i,j}\phi^{(n)}_{i,j-1} + S^{(n-1)}_{i,j} \qquad (4.73)$$

Taking into consideration Eq. (4.73), Eq. (4.72) can be written in vector form as

$$[A]_j\Phi^{(n)}_j + [B]_j\Phi^{(n-1)}_{j+1} + [C]_j\Phi^{(n)}_{j-1} = 0 \qquad (4.74)$$

where

$$\Phi_j = \text{vector } \{\phi_{i,j}\} \qquad (4.74a)$$

$$[B]_j = \text{diagonal } \{(AN)_{i,j}\} \qquad (4.74b)$$

$$[C]_j = \text{diagonal } \{(AS)_{i,j}\} \qquad (4.74c)$$

$$[S] = \text{diagonal } \{S_{i,j} + \frac{1-UREL}{UREL} \cdot (AP)_{ij}\} \qquad (4.74d)$$

$$[A]_j = \begin{bmatrix} & & & & 0 \\ & \ddots & \ddots & \ddots & \\ & (AW)_{i,j} & -(AP)_{i,j}/UREL & (AE)_{i,j} & \\ & & \ddots & \ddots & \ddots \\ 0 & & & & \end{bmatrix} \qquad (4.74e)$$

From Eq. (4.74) we obtain

$$\underline{\Phi}_j = -[A]_j^{-1}\left\{[B]_j \Phi_{j+1}^{(n-1)} + [C]_j \Phi_{j-1}^{(n)} + [S]^{n-1}\right\} \tag{4.75}$$

The inversion of matrix A is made by using the matrix factorization technique [76].

4.3.3 Successive Relaxation and Successive Underrelaxation Technique

This technique is a point iteration technique. The underrelaxation factor is again used here.

$$\phi_{i,j}^{(n)} = (1-\text{UREL})\ \phi_{i,j}^{(n-1)} + \text{UREL} \cdot \phi_{i,j}^{*(n)} \tag{4.76}$$

where $\phi_{i,j}^{*(n)}$ is now given by the relation

$$(AP)_{i,j}^{(n-1)} \phi_{i,j}^{*(n)} = (AE)_{i,j}^{(n-1)} \phi_{i+1,j}^{(n-1)} + (AW)^{n-1} \phi_{i+1,j}^{(n)}$$

$$+ (AN)_{i,j}^{n-1}\ \phi_{i,j+1}^{(n-1)} + (AS)^{(n-1)} \phi_{i,j-1}^{(n)} + S^{(n-1)} \tag{4.77}$$

4.3.4 Line Iteration vs. Point Iteration

The application of Line Iteration Technique in cases without secondary flow creates no problem with respect to convergence. Trying to solve the secondary flow with Line Iteration Technique had led to instabilities. The instability can be removed by using underrelaxation factors below 0.5. From running the first 1000 iterations, it was clear that an extremely high number of iterations is needed to obtain the solution. It was also evident that the underrelaxation factor reduces the rate of convergence.

For illustration we have taken the simple linear problem of the laminar flow with P = 57.5mm P/D = 1.124 and $\tau_w/\rho = 2.7 \times 10^{-5}$ an underrelaxation factor of 1.0 to 0.5 has been studied. In Fig. 4.5 the CPU time required in a IBM 360 computer as a function of the underrelaxation factor is plotted. A grid of 29 × 16 points has been used. An accuracy of $v_z(I = 5, J = 5)$ up to 3 decimal points has been used as a convergence criterion. From Fig. 4.5 we can see that an UREL = 0.9 increases the number of iterations for solution 15 times! Figure 4.5 shows also that the Line Iteration Technique combined with an underrelaxation factor 0.7 requires more time for the solution than the point iteration technique with underrelaxation factor equal to unity.

The above example illustrates that a relatively fast technique (like Line Iteration Technique) combined with an underrelaxation factor probably is not the recommended technique to get fast solution. Some slower technique (like successive relaxation) with better underrelaxation may be preferrable.

That is exactly the case with secondary flow where solution of secondary flow has been obtained faster with successive relaxation technique (UREL = 1.0).

Thus in the present problem recommended technique for the problems with secondary flow suppressed is the Line iteration whereas for the secondary flow solution the Successive relaxation technique is recommended.

4.3.5 The Convergence Criteria

4.3.5.1 Convergence criteria in TEACH [71]

In TEACH the following quantity related to convergence criteria is defined:

$$X_\Phi = \left[\sum_{i,j} \left| (AE)^{(n)}_{i,j} \Phi^{(n-1)}_{i+1,j} + (AW)^{(n)}_{i,j} \Phi_{i-1,j} + (AN)^{(n)}_{k,j} \Phi^{(n-1)}_{i,j+1} + (AS)^{(n)}_{i,j} \Phi^{(n-1)}_{i,j-1} + (S_\Phi)^{(n)}_{i,j} - (AP)^{(n)}_{i,j} \Phi^{(n-1)}_{i,j} \right| \right] \Big/ \Phi \qquad (4\text{-}78)$$

where

n is the number of iterations,

Φ is a normalization constant

AE, AW, AN, AS, S are the coefficients of the finite difference equation for the quantity Φ.

The criterion for terminating the iteration is

$$X_\Phi \leq \varepsilon \qquad (4\text{-}79)$$

where ε is a specified sufficiently small number.

An additional criterion is related to the continuity equation and is expressed as follows

$$\sum_{i,j} \left| \frac{\int\int_{\text{control volume at (i,j)}} \nabla \underline{v} \, d(\text{vol})}{\Phi_M} \right| \leq \varepsilon \qquad (4.80)$$

where Φ_M is again a normalization constant

$\underline{v}$ is the velocity vector.

Although the above criteria seem to provide a correct picture of convergence, they make selection of ε relatively difficult, since it is determined by the coefficients of the difference equation of ϕ rather than the ϕ itself. In the present work, the convergence criteria have been selected to be easier and simpler than the TEACH approach and to take into consideration the peculiarities of the present problem.

4.3.5.2 Present convergence criteria

In the present work two criteria have been established. First,

$$\frac{\left|\phi_{i,j}^{(n)} - \phi_{i,j}^{(n-1)}\right|_{max}}{\Phi} \leq 1.0 \times 10^{-6} \qquad (4.81)$$

where Φ is also a normalization constant reflecting the size of ϕ.

ϕ stands for v_z, k, ε and v_θ

Φ has been selected as follows:

Axial velocity: $\Phi = u_b$

Turbulent kinetic energy: $\Phi = (\tau_w/\rho)_{avg}$

Energy dissipation: $\Phi = 4\ (\tau_w/p)_{avg}^{3/2}/DH$

v_θ velocity: $\Phi = 0.1\ u_b$

For relatively fast converging problems, which is usually the case for problems where the secondary flow is neglected, criterion (4.81) gives sufficiently accurate

results. For some slowly convergent cases, however, which includes the secondary flow cases, the above criterion cannot be met although the solution is accurate within .1%.

For such cases an alternative criterion based on more than one iteration, the following formula is used:

$$\frac{|v_{z,is,js}^{(n+ND)} - v_{z,is,js}^{(n)}|}{u_b} \leq 1.0\times10^{-3} \qquad (4.82)$$

where

ND = 60 for the line iteration scheme

ND = 500 for the successive relaxation scheme

(is,js) is a preselected point

Criteria (4.81) and (4.82) have been found to guarantee accurate enough solutions (i.e. within .1%) for all convergent cases.

The convergence and the accuracy is also tested by successive print outs of the quantities at a particular point. This is also done in TEACH code.

4.4 Computer Program

To obtain the solution of the Finite Difference Equations a computer program has been written. Its main structure is based on the TEACH code where subroutines calculating the grid, the wall thickness, and the average velocity are modified versions of the code used in Ref. 1. The code structure and listing are described in detail in Appendix E.

CHAPTER 5

RESULTS AND CONCLUSIONS

5.1 Experimental Results

5.1.1 Data evaluation

All the measurements performed for the estimation of the quantities v_z and v_z' were characterized by good signal to noise ratio and therefore the results were satisfactory.

The lateral measurements (i.e., measurements involving v_2 and v_1 components) are characterized by relatively low signal to noise ratio. Since the performance of the Frequency Tracker deteriorates with the decrease of signal to noise ratio, lateral measurements are characterized by relatively large errors as pointed out in Chapter 2. The reason for the low-to-signal ratio is attributed to the rather complicated geometry of the test section which for these particular directions can bring the signal below the satisfactory level. For measurement of the v_2 component in the gap region particularly, the measuring volume lies relatively close to the wall. This is because the largest dimension (0.66 mm) of the beam intersection volume lies across the gap region which is only half the thickness 3.175 mm. The close proximity of the measuring volume to the wall degrades the signal due to actual intersection and reflections of beams from the wall into the measuring volume.

Under these circumstances one can summarize the data evaluation performed in the lateral direction as follows:

Quantities v_r', v_θ': Although their size level seems to be reasonable, they give erroneous results for quantities like v_r'/v_θ' and $v_\theta'^2 - v_r'^2$. Specifically at some points v_r'/v_θ' is greater than one which is wrong. As a consequence of the relatively large errors, entrance effects cannot be distinguished.

Quantities $\overline{v_r'v_z'}$, $\overline{v_\theta'v_z'}$. For the quantity $\overline{v_r'v_z'}$, the results seem reasonably good, close to the gap up to a distance of 1.2 mm from the wall. The results of measurements closer to the wall give unacceptably low values for $\overline{v_r'v_z'}$. The reason is attributed to the low signal-to-noise ratio and to failure of the turbulent processor to respond to the "bursty" nature of turbulence close to the wall. Actually final measurements for points closer to the wall have not been performed.

One positive characteristic, however, of the cross-correlation measurements was that the noise carried by each signal is highly uncorrelated. To be more specific, the values for $\overline{v_r'v_z'}$ along the zero shear stress line were close to zero which means that the noise present in the signals v_1, v_2 and v_z is almost uncorrelated. This supports the suggestion [87] that results based on continuous Doppler signal are best obtained by collecting the signal in two different directions and correlating the outputs.

With respect to the quantity $\overline{v_\theta' v_z'}$ due to its relatively small size and the greater dependence on v_2 components, the results are characterized as rather inconclusive.

5.1.2 Fully Developed Region

One of the questions raised in [1] was whether the data taken at $L/D_H = 77$ corresponded to fully developed flow.

Figure 5.1 shows the velocity v_z normalized by the corresponding maximum channel velocity, at positions $\theta=0$, 12° and 30° as predicted by P. Carajilescov [1] at $L/D_H = 77$ and in the present work at positions $L/D_H = 77$ and 113.

Figure 5.1 shows that results taken in [1] were actually in the developing region. Also Figure 5.1 shows the positive effect of the entrance section fabricated and used only in this study versus [1] to reduce the entrance length.

With respect now to the question whether the position $L/D_H = 113$ lies in the developing region, one can distinguish in Figure 5.1 that there is a flow development from $L/D_H = 77$ to $L/D_H = 113$ which is, however, of the order of the experimental error. Therefore, we accept the data at $L/D_H = 113$ as data corresponding to fully developed flow within experimental error.

All the data taken are tabulated in APPENDIX C.

Experimental results for axial velocity v_z and turbulent axial velocity v_z' in the fully developed flow ($L/D_H = 113$)

are given in Figs. 5.2a-5.5c for Re=26,500 and 65,000 and in the Figs. 5.13 and 5.14 for Re=9,000 at θ=0°, 12° and 30°.

Experimental results for radial Reynolds stress and turbulent kinetic energy are given in Figs. 5.6a-5.9c for Re=26,500 and Re=65,000 and in the Figs. 5.15 and 5.16 at θ=0°, 12° and 24°.

Attempts also have been made to estimate wall shear stress from the logarithmic law ($v_z=\frac{u^*}{\kappa} \ln(Ey^+)$, κ=0.42, E=9.8) using the measured axial velocity at points closest to the wall. The logarithmic law as applied in §3.3.5.3.1 was used to calculate the bulk velocity. Average shear stress and bulk velocity data are given in Table 5.1. Wall shear stress distribution is given in Figs. 5.10, 5.11, 5.12 for Re=26,500, 65,000 and 9,000 respectively.

5.1.3. Developing Region

The entrance effects on the axial velocity and its rms value is clear as we can see in Figs. 5.2a-5.5c where data are presented at positions L/D_H=15, 46 and 113. To illustrate the combined effect of flow development and flow split, all quantities are normalized by the maximum velocity at L/D_H=113 for each Reynolds Number.

It is clear from the axial velocity results that

a) a flow split effect is present, i.e., higher flow rates for higher L/D_H because side subchannels experience lower flow rates than the interior subchannels. Fig. 2.1

shows that the present test section consists of twenty (20) characteristic triangles of the type of Figure 1.1, four (4) of which are side triangles.

The flow split effect also can be seen in Table 5.2 where the bulk velocities are given.

b) Flatter velocity profiles occur for smaller L/D_H as expected

c) The turbulent axial velocity is also developing as Figures 5.4a-5.5c show. However, the difference in turbulent axial velocity levels between $L/D_H = 46$ and 113 seems to be small.

For the other measured quantities it is rather difficult to make any definite conclusion based on the experimental results regarding development effects. One can see this from k and $\overline{v_r'v_z'}$ data plotted in Figures 5.6a-5.9c.

Comparison between all experimental data obtained in the present work at positions $L/D_H = 46$ and 113 show that the quantity mostly affected by the entrance length is the mean velocity. This suggests the mean velocity profile as the most reliable criterion of the onset of fully developed flow.

With respect to the wall shear stress distribution, we applied the logarithmic law as we did for the fully developed flow. The results are shown in Figures 5.10 and 5.11 for $3=26,500 and 65,000, respectively. Of course, use of the logarithmic law to describe the velocity distribution in the

developing region is not strictly correct. However it was done to explore the behavior of wall shear in the developing region since many experimenters may have inadvertently applied this approach to data in the developing region.

a) In the position $L/_{DH}=46$ the wall shear stress distribution is pretty much developed.

b) In position L/DH=15 a dip is observed towards the position $\theta=30^{o}$ for Re=26,500. For Re=65,000 there is a dip tendency between 18 and 24° but reversal of the dip occurred at $\theta=30^{o}$. These results raise the question whether the shear stress distribution dips that appear in the literature, representing fully developed conditions actually are associated with developing effects.

5.1.3.1 Flow Split

As is mentioned in the previous section and we also can see from Table 5.2 and Fig. 5.17 there is a flow difference in the flow between side and central subchannels in the test section.

The values for the ratio u_b/u_b^T for the position $L/D_H=113$ were 1.12 and 1.08 for Re=26,500 and 65,000 respectively.

The above values are close to the values suggested by the equal pressure split which is of order 1.36 to 1.40 for turbulent flow shown in APPENDIX D.

5.1.4 Secondary Flow

An attempt was made to measure the secondary flow in Re=65,000 but with no success.

The first negative result came from v_θ velocity measurements along the gap (θ=0) which yielded values of order 2% of the maximum velocity instead of the theoretical zero value.

Such results are attributed mainly to

a) Frequency Tracker Error which is of order 3% of the maximum velocity.

b) Low-signal to noise ratio.

c) Test section inclination errors. Actually such errors could not be higher than 0.5^o which introduces a maximum error of 0.9%.

All lateral velocity data used were below 3% i.e. within experimental error. It is evident from Figure 5.18 that data are inconclusive with respect to secondary flow.

The present data have been obtained with higher errors than those in [1]. (4% vs. 0.67%). The reason is that P. Carajilescov probably had a better signal to noise ratio which allowed the use of relatively small frequency range (500KHZ vs. 5.0 MHZ for the present case) The difference in signal to noise ratio can occur due to different Laser Beam arrangements. In this case the measurements in [1] were performed by a Dual Beam Mode instead of Reference Beam Mode used in the present approach.

5.2 Analytical Predictions

5.2.1 The Turbulence Models

Before entering into any discussion on turbulence model prediction the author wants to make clear that the turbulence models are not the tools for understanding the turbulence phenomena which are terribly complicated but are engineering tools for predicting useful hydrodynamic quantities. Therefore one has to examine their drawbacks and merits from this point of view.

The most attractive fe tures of turbulence models as pointed out in Chapter 1 are the following:

a) The ability to use turbulence parameters (e.g., turbulent kinetic energy, energy dissipation for this particular case as part of the model).

b) The relative simplicity and flexibility to handle complicated problems since the predictions are based on constant adjustments.

Such an approach of course has some disadvantages.

a) In reality for any given problem of non-homogeneous turbulence, the constants are not constants but parameters changing with position and flow characteristics. The selection of the constants is made by best fit to data. The author's experience from this work is that it is very difficult to reduce the error made in the predictions below 3% for mean velocities and 5-10% for other turbulent quantities. The author's opinion is that such error levels are due to the

change of constants with position and flow characteristics which can be affected little by more sophisticated modeling of the turbulent equations at least for eddy viscosity models (i.e.,up to 2-equation models).

b) The constants are not universal but depend in general on the problem under consideration. Such a drawback can be enhanced by errors introduced in modeling of the various terms in the transport equation of some turbulent quantity due to oversimplicity of the turbulence phenomena.

c) More than one set of constants fit the data of any particular problem. An illustrative example is shown in Figure 5.19 where predictions are obtained for the geometry of this thesis by using the constants of Launder et al.(95) and Hoffman (96) which are as follows:

Launder et al.	Hoffman
$c_\mu = 0.09$	$c_\mu = 0.09$
$\kappa = 0.42$	$\kappa = 0.41$
$c_1 = 1.44$	$c_1 = 1.81$
$c_2 = 1.92$	$c_2 = 2.0$
$\sigma_k = 1.0$	$\sigma_k = 2.0$
$\sigma_\varepsilon = 1.3$	$\sigma_\varepsilon = 3.0$

Both sets of constants look reasonable and yield identical results.

However, on the positive side if for some particular experimental set you have a number of constant sets, you can single out a set that fits the experimental sets "acceptably" well. At this point two questions can be posed:

i) Is that set of constants "universal"?

ii) Is that set of constants physically correct for that particular family of experimental sets?

With respect to first question the answer is "not necessarily" for the reasons discussed in (a) and (b).

With respect to the second question the answer is again "not necessarily" for the simple reason that even if something is modeled wrongly or inadequately two or more wrong constants can have offsetting effects yielding the right answer.

However there is a third consequence of the relatively high number of constants. There is a danger that one can come out with a particular set of constants that fits only one particular experimental set adequately.

The immediate question which arises from the above discussion is:

How useful are the turbulence models with above advantages and disadvantages for prediction of flow in areas of Nuclear Engineering and especially in rod bundle flow prediction?

In developing the turbulent model of this thesis the following guidelines have been adopted.

a) Accepting that the constants cannot be universal there is a need to define the family of problems for which a particular set of constants can give satisfactory results.

We will consider at least the flow through a rod bundle, the duct flow and the flow along a cylinder as belonging to the same family on the grounds that they have one main direction of the flow. This decision leads us to look for experimental constants that fit simple one-dimensional flows belonging to above family (i.e., plain tube, circular pipe). At this stage we are obliged to pick something already existing in the literature. We will accept the Launder et al. constants on the ground that probably they are the most extensively tested.

Parenthetically it is noticed here that more systematic work is needed by analyzing a number of simple geometries at various Reynolds numbers and by identifying if possible all sets of constants applying to a particular case.

b) In Chapter 3 and in Figure 5.19 it has been made clear that utilization of an isotropic eddy viscosity model does not yield good predictions.

Therefore the anisotropic eddy viscosity model introduced in Chapter 3 will be used in the remainder of this chapter to predict the data.

c) The geometric peculiarities of the rod bundles will be reflected in the eddy viscosity model, e.g., the constant c_L in Eq. (3.63) can be expressed as a function of P/D ratio.

5.2.2 Secondary Flow

For the secondary flow calculations a 30 x 16 grid has been used. The geometrical characteristics of the experimental set has been used with an input average wall shear stress $(\tau_w/\rho)_{avg} = 4 \times 10^{-3}\ m^2/s^2$ (Re=26,000). For normal stresses the models mentioned in §3.3.3 are used, i.e.:

a) Launder and Ying model as expressed by the Equations (3.47a) and (3.47b).

b) Bobkov et al. model as expressed by the Equation (3.10).

In Figure 5.20 the velocity v_θ at $y=y_B$ is plotted for the above two cases which shows that case (a) gives a 3 loop (!) system and case (b) a 2 loop system. The latter is in agreement with P. Carajilescov's [1] predictions. Figure 5.21 shows the prediction of v_θ velocity along the $\theta=24°$ radial line for both cases.

Figures 5.20 and 5.21 show clearly the significance of the normal shear stress modeling in prediction of secondary flow independent of how well the rest of modeling has been done.

However both predictions have in common a negative loop close to the vicinity of $\theta=0$ which is smaller in strength compared with the positive one at larger θ.

However we cannot consider such predictions as conclusive for the following reasons:

a) The 2-eq. model (as well as the 1-eq. model) as pointed out in the previous section, due to over-simplification of the description of turbulence, is a relatively rough engineering tool for predicting flow characteristics (i.e., mean velocity, Reynolds stresses, turbulent kinetic energy). It is therefore very unrealistic to use such a model to predict some detailed features as secondary flow which is purely a turbulence phenomenon (there is no secondary flow in laminar flow) and is of order 1% (or below for some regions) of the mean velocity.

b) The 30 x 16 grid used for the present calculations is rather inadequate due to the existence of sharp v_θ gradients especially at $\theta=30^o$ (Fig. 5.20) and at the wall (Fig. 5.21). However increase of the grid requires more computer storage and exceedingly high computer cost. The solutions presented here already required 4,000 iterations and 90 min computer time on a IBM 360/65.

c) The small magnitude of the calculated secondary flow can be affected by computer round-off error although both cases are run with double precision.

In spite of above negative conclusions, one question still remains:

Is the secondary flow as predicted by the 2-equation turbulent model important in prediction of other hydrodynamic quantities and especially of the mean velocity which is the most important parameter? Or at least how important is secondary flow compared with anisotropic eddy viscosity as introduced in the present work? These factors are compared in Fig. 5.22 which shows v_z/u_o along the $\theta=0$ radial line.

The anisotropy is much more significant than the secondary flow effect. Under these circumstances further investigation of secondary flow has been deferred. Such decision has also been prompted by the high computer cost due to the very low convergence rate. All subsequent data predictions are made with secondary flow suppressed.

5.2.3 Data Predictions

As mentioned before all the predictions presented below are made with the 2-equation model and the anisotropic eddy viscosity as introduced in Chapter 3.

5.2.3.1 Numerical Grid

The adequacy of the numerical grid is tested by examination of the ratio of calculated average shear stress to input average wall shear stress. The correct value of course is unity which is expected to be met with an infinite grid. However, due to use of a finite grid the ratio is always less than one.

There are two ways to improve that ratio:

i) Grid point increase

ii) y_B^+ increase.

How y_B^+ affects the shear stress ratio is illustrated in Figure 5.40 where the above ratio is plotted vs y_B^+ for a 30 x 16 grid.

In present calculations a 56 x 21 grid is used. The y_B^+ parameter has been selected based on the following criteria:

a) To be as close to the wall as possible.

b) To give a wall shear stress ratio around 0.95 (i.e., a calculation error for shear stress of order 5%). The selection of above ratio has been on the following grounds:

i) Quantities like v_z/u_o are affected very little by the above ratio.

ii) Quantities like $\overline{v_r'v_z'}/u_o^2$, k/u_o^2 which are affected the most by the above ratio are not measured better than 5% in present measurements.

iii) A finer grid than 56 x 21 is needed to increase the above wall shear stress ratio which means higher computer cost.

The values of y_B^+ used in the present analysis are given in Table 5.3. All solutions are obtained by a Line Iteration scheme.

5.2.3.2 Present Data Prediction

The present model has been tested against the present

fully developed data (P/D=1.124, Re=65,000, 26,500 and 9,000) and with mean axial velocity predictions by Eifler [19] (P/D=1.08, Re=52,400), Trupp [80] (P/D=1.20, 1.35, with Re= 49,000 and 60,000 respectively) and Kjellstrom [82] (P/D= 1.217, Re=149,000).

The above data are representative of a variety of P/D ratios (1.08 to 1.35) and Reynolds numbers (9,000 to 150,000). Figures 5.20-5.34 show mean axial velocity, Radial Reynolds stress, Turbulent kinetic energy and wall shear stress distribution predictions compared to the present experimental results (i.e., P/D=1.124, Re=65,000, 26,500 and 9,000).

Figures 5.35-5.39 show mean velocity predictions of Eifler's data, Trupp's data and Kjellstrom's data.

The above results suggest that for P/D > 1.20 a value for c_L around 0.4 whereas for P/D < 1.20, c_L has to be increased. This actually is in agreement with the experimental data obtained by Rehme [88] in a side subchannel with P/D=1.07 which suggest an eddy viscosity along the zero shear stress line of

$$\nu^T_{rz} = (0.10 - 0.16)\, u^* \delta$$

Using this expression for ε in relations (3.39), (3.58) and (3.50), we find

$$c_L = 0.55 \text{ to } 0.9$$

A relation for c_L consistent with above evidence satisfactorily fitting the above data ($P/D \geq 1.08$) is the following

$$c_L = 1.62 \exp(-17\,(P/D-1)) + 0.4$$

All data predictions shown in Figures 5.23-5.38 were made with above relation. It is worth pointing out here that although c_L=0.4 gives less accurate results, it gives more conservative heat transfer results as shown in Figure 5.39.

The degree of success of the analytic predictions is a function of the Reynolds number range.

a) High Reynolds Number Predictions (Re≥26,500)

The mean axial velocity and the wall shear stress distribution predictions are satisfactory. The predictions for $\overline{v_r'v_z'}$ and k seem reasonably good although more accurate measurements are needed for more definite conclusions.

b) Low Reynolds Number Predictions (Re=9,000)

The predictions for v_z and k are not as satisfactory as for the High Reynolds Number. The predictions for $\overline{v_r'v_z'}$ and $\tau_w/(\tau_w)_{avg}$ are reasonably good. The present predictions for Re=9,000 support the approach of modifying the 2-equation turbulence model to take into account Low Reynolds Number effects already initiated by other investigators [e.g., Ref. 97]. For example Jones and Launder [97] introduce some position dependents of the constants close to the wall.

5.2.4 Heat Transfer Calculations

Two region (clad + coolant) heat transfer calculations have been performed to examine the effect of anisotropic viscosity as introduced in the present work versus isotropic viscosity working with the 2-equation turbulence model.

The calculations have been performed with the following parameters:

Coolant	Sodium
Clad	Stainless Steel
Pitch ratio	P/D=1.10
Rod Diameter	D=0.25"
Inner Clad Diameter	D_2=0.22"
Average Wall Shear Stress	$\frac{\tau_w}{(\tau_w)_{avg}}=0.325\ \frac{m^2}{s^2}$ (Re=86,000)
Linear Heat Generation Rate	$q'=15\ \frac{kw}{ft}$

All material properties were taken at 1000°F. For the numerical solution a grid of 30 x 16 points has been used. The Finite Difference Equations used are described in Appendix B. A ratio of heat diffusivity to eddy viscosity equal to one for both radial and tangential directions was used.

The numerical calculations have been performed by the Computer Code developed in the present work and described in Appendix E.

In Figure 5.40 the non-dimensional quality $\lambda_c \frac{\bar{T}_{co} - T_{co}}{q'}$ is plotted versus angle where T_{co} is the local outside clad temperature, $\bar{T}_{co}$ is the average outside clad temperature and λ_c is the coolant heat conductivity.

Figure 5.39 shows the strong effect of anisotropic

eddy viscosity where the value of $T_{co}^{max} - \overline{T}_{co}$ shows a difference almost 2 to 1. Figure 5.39 shows clearly the importance of anisotropic model of eddy viscosity in turbulence models for heat transfer calculation as demonstrated earlier by other investigators [94], [32].

5.3 Conclusions

From the present work the following conclusions can be drawn.

<u>On the Analytical Work</u>

1. The 2-equation turbulence model as presently formulated can be a satisfactory engineering tool for predicting useful hydrodynamic quantities (i.e., mean axial velocity, Reynolds stresses, shear stress) in a rod bundle at high Reynolds numbers (presently, it has been tested satisfactorily with Reynolds numbers above 26,500). The model does not seem to be a sufficient tool for predicting secondary flows in rod bundles.

2. The model has been used to successfully predict the data obtained in this study and published rod bundle data (principally axial velocity) utilizing one consistent set of constants for 1-dimensional flow from the literature.

3. The introduction of eddy viscosity anisotropy through length scale ratios, while keeping the model simple, also makes the model inherently extendable to 3 dimensions. (One can follow the same logic and introduce a length scale l_z similar to l_θ for axial diffusion.)

4. The strong effect of the anisotropy in eddy viscosity

on heat transfer calculations shown by other investigators is also confirmed in the present work by the present model.

On Numerical Work

1. For fully developed flows solved numerically using average velocity input instead of average shear stress input, the answer is a function of the numerical technique used due to the finite number of points. Therefore, use of an average velocity input is not recommended.

2. A relatively fast numerical technique (like the line Iteration Scheme used in TEACH code) combined with an under-relaxation factor is not necessarily recommended over a common point iteration technique without under-relaxation.

On Experimental Work

1. Experimental data on mean axial velocity v_z and the associated rms value v_z' in the test section used show clearly the presence of a developing flow-region.

2. The flow split model based on equal pressure drop and fully developed flow predicts higher flow rates for the interior subchannels close to those measured (within 10%).

3. The data suggest that the safest criterion of fully developed flew occurrence is the mean velocity profile.

4. Calculations of wall shear stress distribution using the experimental data of the axial velocities at the points closest to the wall and applying the logarithmic law indicate that dips observed in the vicinity of $\theta=30°$ may be associated with the developing region of the flow rather than with the fully developed region.

CHAPTER 6

RECOMMENDATIONS FOR FUTURE WORK

From the experience drawn from present work the following recommendations for future work can be made.

On Turbulence Models and Flow Predictions

1. In the present work the representative constants for 1-dimensional flow are taken from Launder et al. [67] which are based on "best fits" of cases representing one and multidimensional flows. Re-examination of the 2-equation turbulence model prediction in 1-dimensional flows (plane tube, circular pipe, annulus) and identification of all reasonable constant sets that fit the data seems necessary. Reynolds number effects on constants need to be studied especially at low Reynolds numbers.

2. A new radial scale has been introduced for the wall region which seems to be reasonable for the non-logarithmic region very close to the wall. One has to examine how this can improve predictions in the hydrodynamic quantities within that region since this thesis has been focused outside this region.

3. The eddy viscosity anisotropy introduced in the present work has significant effect on flow predictions and is consistent with experimental evidence. All predictions done in 2-dimensional or 3-dimensional flows using isotropic viscosity turbulence models (like recirculating flows) could profitably be re-examined with the present

anisotropic model.

4. The present model as formulated is easily extendable to three dimensions. It can be used to predict present experimental data in the developing region of the test section used.

On Numerical Work

1. The Up-Wind-Derivative Difference Scheme (UDDS) used in present work is superior to Upwind-Difference-Scheme (UDS) since it is one-order higher in accuracy. Existing codes using UDS can be modified to use the UDDS scheme.

2. Present calculations show that in order for the wall shear stress (or equivalently the axial velocity gradients) to be calculated to better than 5% of the true value, a large number of grid points (larger than 56 x 21) is recommended. This poses a drawback to the finite difference schemes. Other methods like Finite Element methods need to be examined for similar flow calculations.

3. The computer program developed in the present work works with pressure and velocity (instead of vorticity and stream function) and therefore can be relatively easily extended in three dimensions.

On Heat Transfer Calculations

In the present work some heat transfer calculations were performed to study the anisotropic eddy viscosity effect in heat transfer calculations. More extensive

studies are needed to actually predict heat transfer in the fully developed flow.

On Experiments

The measurements in this thesis were performed by a 2-channel LDA working on Reference Mode using frequency Tracking Techniques.

Characteristic of the above measurements were Doppler ambiguities, instrument noise, low performance of the turbulent processor near the wall, and low performance of the Tracker at low signal-to-noise ratio.

Using the same LDA equipment the following improvements can be made.

1. Substitute for the tracking technique at least in one channel other techniques which have a better performance at low signal to noise ratio (e.g., by spectrum analysis or individual particle techniques.

2. Look for better ways of turbulence signal processing.

On Secondary Flows

1. Direct Measurement

Measurement of the secondary flow is a rather difficult task. The obvious reasons are that the mean values of secondary velocities are much smaller than their rms values. Approximate calculations of the ratio of secondary flow rms value to mean value assuming

$$v'_\theta \approx u^* \approx 0.05\ v_b \qquad (6.1)$$

give
$$\frac{v'_\theta}{v_\theta} = \begin{cases} 5 \text{ for } v_\theta = 0.01\ v_b \\ 10 \text{ for } v_\theta = 0.005\ v_b \end{cases} \qquad (6.2)$$

Thus an experimentalist needs an instrument that will respond properly at instantaneous velocities at least $\pm 3v'_\theta$ (i.e., $\pm .15$ to 30 v_θ) and will measure at the same time correctly the mean velocity v_θ itself.

Usually the wider the range of the instrument, the less accurate is the measurement of small quantities. Besides the above drawback the relative large magnitude of v'_θ requires high integration time which poses additional demands on test section dimensions and mean axial velocity magnitude as follows.

Recall that the error made due to finite integration time assuming a Gaussian Distribution is given by the relation (Eq. 2.4).

$$\frac{(v_\theta)_{MEAS} - v_\theta}{v_\theta} = \frac{v'_\theta}{v_\theta} \sqrt{2} \left(\frac{F}{T}\right)^{1/2} \qquad (6.3)$$

where F is the time scale of the turbulence. Taking

$\frac{v'_\theta}{v_\theta} = 10$ and $\frac{(v_\theta)_{MEAS} - v_\theta}{v_\theta} = 0.2$ (i.e., 20%)

Eq. (6.3) gives

$$F = T/5000$$

Taking further T = 100 sec,

$$F = 0.02 \text{ sec} \qquad (6.4)$$

Recall that

$$F = \frac{\text{Test section dimension}}{v'_\theta}$$

For $F = 0.02$ sec

and $v'_\theta = 0.05\ v_b$

we find that

$$v_b \geq 1000 \text{ (Test section dimension)/sec}$$

This result represents the required v_b necessary for a given test section dimension to keep the error within 20% and the measuring time equal to or less than 100 sec.

For Test section dimension 5 mm: $v_b \geq 5$m/s.

For Test section dimension 10 mm: $v_b \geq 10$m/s.

The above preliminary calculations suggest use of a test section as small as possible and use of air as a working fluid to more easily achieve this minimum required velocity.

Future attempts to measure secondary flow in rod bundles have to take into consideration above restrictions.

2. Indirect Measurement and/or Predictions

In case direct measurements of secondary flow is not possible, a mixed scheme of experimental results and analytical calculations can be used as described below.

i. Measure mean velocities and Reynolds stresses.

ii. Fit the above data with analytic expressions sensitive enough to give confidence that first and second derivations can be extracted. Note that both the data and the analytic expressions must be sensitive enough.

iii. Utilize the above analytic expressions in the corresponding Reynolds equations by performing the neces-

sary differentiations to obtain the analytical expressions for the secondary flow.

The author's deep belief is that no existing turbulence model can confidently predict secondary flows which are of the bulk velocity.

On Methods for Reactor Design

1. The present model is easily extendable in three dimensions. It can be used in reactor design for flow and heat predictions in local regions of special interest (e.g., hot spot, blockage) by isolating them and applying boundary conditions from a Lumped Parameter Technique.

REFERENCES

1. P. Carajilescov, "Experimental and Analytical Study of Axial Turbulent Flows in an Interior Subchannel of a Bare Rod Bundle", MIT Ph.D. Thesis, 1975.

2. V.P. Bobkov et al., "Correction of Experimental Data on the Pulsation Velocity Intensity for Turbulent Fluid Flow in Channels of Different Form: Izv AN SSR Mekhanica Zhidkosti i Gaza Vol. 3, No. 3, (English Translation pps 111-113) (1968).

3. A. P. Ivanov, "Turbulent Friction in the Flow of Liquid in Tubes and Channels" Izv. AN SSSR Mekhanika pps 311-316) 1972.

4. H. Eckelmann, "The Structure of the Viscous Sublayer and the Adjacent Wall Region in a Turbulent Channel Flow", J.Fluid Mechanics, Vol. 65, Part 3, pp. 439-450, 1974.

5. J. Boussinesq, "Essai sur la Theorie des Eaux Counautes", Mem pres par div. savants a l'Acad Sci Paris, 23, No. 1, 1-680, 1877.

6. J. Laufer, "The Structure of Turbulence in Fully Developed Pipe Flow," NACA Report No. 1174, 1954.

7. J. Laufer, "Investigation of Turbulent Flow in a Two Dimensional Channel", NACA Report No. 1053, 1951

8. F.H. Harlow and P. I. Nakayama, "Turbulence Transport Equations", The Physics of Fluids, Vol. 10, No. 1, pps 2323 - 2332, 1967.

9. A. S. Monin and A.M. Yaglom, "Statistical Fluid Mechanics" Vol. 1, The MIT Press, 1961.

10. P. Y. Choi, "On Velocity Correlation and the Solutions of the Equations of Turbulent Fluctuation" Quart. Appl. Math 3, 31, 1945.

11. J. Rotta, "Statishische Theorie Nichthomogener Turbulentz" Zeitschrift für physik Bd 129, S. 547-572, 1951.

12. B. I. Davydov, "On the Statistical Dynamics of an Incompressible Turbulent Fluid" Dokl, AN SSSR Vol. 4, (English Translation pps 769-772), 1968.

13. C. P. Donalson, "A Computer Study of an Analytical Model of Boundary Layer Transition, AIAA, 1968.

14. Daly, B.J., and Harlow, F.H., "Transport Equations in Turbulence" Physics of Fluids 13 2634, 1976.

15. Hanjalic, K. and Lauder, B.E., "A Reynolds Stress Model of Turbulence and its Application to Thin Shear Flows", J. Fluid Mech. 52, 609, 1972.

16. Dan Naot, "The Influence of the Pressure Fluctuation on the Redistribution of the Velocity Correlation in Turbulent Flow, DScThesis Technion-Israel Institute of Technology, Haifa, May, 1972.

17. Launder, B.E., Reece, G.J. and Rodi, W., "Progress in the Development of a Reynolds Stress Turbulence Closure" J. Fluid Mechanics, 1975.

18. Reichardt, H. "Vollständige Darstellung der Turbulenten Gesh Windigkeitsverteilung in Glatten Leitungen. Z. Angew Math. Mech 31, 208-219, 1951

19. Eifler, W. Über die Turbulente Gesch -windigkeitsverteilung und Wandreibung in Shrömungskanälen Verschiede ner Querschnitte" Doctoral Thesis T.H., Darmstadt, 1968.

20. Eifler, W. "Calculations of Turbulent Velocity Distribution and Skin Friction in Concentric Annuli" Warme und Stoffuber tragung 2, 36-46, 1969.

21. R. Nijsing and W. Eifler, "Axial Development of Temperature Fields in Hexagonal Fast Reactor Fuel Rod Assemblies" Progress in Heat and Mass Transfer, Vol. 7, pp. 115-149, 1973.

23. Nijsing, R., Eifler, W., Delfau B. "Lateral Turbulent Diffusion for Longitudinal Flow in a Rectangular Channel" Nuclear Engineering and Design 32 (221-238) 1915

24. Rapier, A.C. and Redman, J.D.R., "The Calculations of the Velocity Distributions in Rod Clusters" TRG - Report [W] Reactor Development Laboratory, Windscale, UK, 1964.

25. Prandtl, L., "Über die Ausgelbildete Turbulentz ZAMM 5, 136-139, 1925.

26. Buleev, N.I., "Theoretical Model for Turbulent Transfer in 3-Dimensional Flows: Paper No. 329 Presented at the third UN Int. Conf. on Peaceful Uses of Atomic Energy in Geneva, 1964.

27. Bender, D.J. and Magee, P.M. "Turbulent Heat Transfer in a Rod Bundle" ASME 68 - WA/HT-36., 1969.

28. Bender, D.J. and Magee, P.M. "Turbulent Heat Transfer in a Rod Bundle with Liquid Metal Coolant", GEAP-10052, 1969.

29. Th. Von Karman "Mechanische Ahnlichkeit und Turbulenz" Coll. Works II 337-346, 1931.

30. Cebeci T., Smith, A.M.O., "Analysis of Turbulent Boundary Layers" Academic Press, 1971.

31. Taylor, G.I., "The Transport of Vorticity and Heat Through Fluids in Turbulent Motion" Proceedings of the Royal Society, A Vol. CXXXV pps 685-705, 1932.

32. Ramm, H. and Johannsen, K. "A Phenomenological Turbulence Model and its Application to Heat Transport in Infinite Rod Arrays with Axial Turbulent Flow: J. of Heat Transfer, pps. 231-237. 1975.

33. Kolmogorov, A.N., "Equation of Turbulent Motion of an Incompressible Flud (in Russian) 12V Akad. SSST ser. phys Vol III, No. 1.2, 1942.

34. Prandtl, L. "Uber ein neues formelsystem fur die ausgebildete Turbulenz" Ludwig Prandtl Gesammete Abhandlungen, Springer-Verlag P. 874-887, 1945.

35. Launder, B.E. and Spalding, D.B., "Mathematical Models of Turbulence" Academic Press, 1972.

36. Gosman, A.D.; Pun, W.M.; Runchal, A.K.; Spalding, D.B. and Wolfshtein, M. Heat and Mass Transfer in Recirculating Flows, Academic Press, London, 1969.

37. Launder and Ying, "Prediction of Flow and Heat Transfer in Ducts of Square Cross Section", The Institution of Mech. Eng. Proceedings, Vol. 187, 37/73 pp. 455-461., 1972.

38. Tennekes, H., and Lumley, J.L., "A First Course in Turbulence" The MIT Press, 1972.

39. Wolfshtein, M., Naot D. and Lin, A., "Models of Turbulence" Report NE-746 Ben Gurion University of the Negev, 1974.

40. Davydov, B.I., "On Statistical Dynamics of our Incompressible Turbulent Fluid" Soviet-Physics Doklady Vol. 6, No. 1, pps 10-12, 1961.

41. Harlow, F.H. and Nakayama, A.I. "Transport of Turbulence Decay Rate" Los Alamos Report 3854, 1968.

42. Nee and Korasznay, "Simple Phenomenological Theory of Turbulent Shear Flows" The Physics of Fluids 12, 3 pps 473-484.

43. Cook, J.L.; Nakayama, P.I., "VARR-II A Computer Program for Calculating Time Dependent Turbulent Fluid Flows with Slight Density Variations" WARD-D-0106 Vol. 1, May 1975.

44. Schlichting, H., "Boudary Layer Theory" Translated by J. Kestin McGraw Hill, 1968.

45. Rotta, J.C. Uber eine methode zur berechnung turbuleukr Scherstro mungsfelter, Aerodynamische Versuchsanstalt Gotingen 69A14.

46. Rodi, W. and Spalding, D.B. A Two-Parameter Model of Turbulence and its Application to Free Jet, Warme and Stoffubertragung 3 85, 1970.

47. Ng, K.G. and Spalding, D.B., Turbulence Model for Boundary Layer near Walls, Physics of Fluids, 15 1, 20-30, 1972.

48. Wolfshtein M. "On the Length-Scale-of Turbulence Equation" Israel Journal of Technology 8 1-2, 87-99, 1969.

49. Glushko, G.S. "Differential Equation for the Scale of Turbulence and Calculation of the Turbulent Boundary on a Flat Plate Fluid Mechanics - Soviet Research 1,2 155-162, (1972).

50. Jones, W.P. and Launder, B.D., The Prediction of Laminarization with a Two Equation Model of Turbulence, Int J. Heat Mass Transfer 15 301, 1972.

51. Wilcox, D.C. and Alber I.E., "A Turbulence Model of High Speed Flows" proceedings of Heat Transfer and Fluid Mechanics Institute 14-16 June 1972, Northridge, Calif, Stanford University Press, 1972.

52. Brundrett, E. and Baines, W.D., "The Production and Diffusion of Vorticity in Duct Flow," J. Fluid Mechanics, Vol. 19, pps 375-394, (1964).

53. M. Kh Ibragimov, V.S. Petrishchev, and G.I. Sabelev, Calculation of Secondary Flow in a Turbulent Fluid Stream" Izv An SSSR Mekhanicol Zhidkosti i Gaza vol 4, No. 4, (1969) English translation pps 114-116.

54. D. Naot, A. Shavit and M. Wolfshtein, "Numerical Calculation of Reynolds Stresses in a Square Duct with Secondary Flow" Warme-Stoffubertragung 7(1975) pps 151-161.

55. W.M. Rohsenow, H. Choi, "Heat Mass and Momentum Transfer," Prentice-Hall, Inc. (1961).

56. "55L Laser Doppler Anemometer MARK II" DISA Instruction Manual.

57. Melling, A., "Investigation of Flow in Non Circular Ducts and other Configurations by Laser Doppler Anemometry," University of London, Ph.D. Thesis, 1975.

58. F. Durst, A. Melling and J.H. Whitelaw "Principles and Practice of Laser Doppler Anemometry," Academic Press 1976.

59. Kreid, O.K., "Laser Doppler Velocimeter Measurement in Nonuniform Flow: Error Estimates," Applied Optics 13, No. 8, 1974.

60. Edwards, R.V., Angus, J.D., French, M.J. Dunning, J.W., "Spectral Analysis of the Signal from the Laser Doppler Flowmeter: Time Independent Systems," Journal of Applied Physics 42, No. 2, pp. 837-850, 1971.

61. J.M. Owen and R.H. Rogers "Velocity Biasing in Laser Doppler Anemometers" Proceedings of the LDA-Symposium Copenhagen 1975.

62. George, W.K., Lumley, J.L. "The Laser Doppler velocimeter and its application to the measurement of Turbulence," J. Fluid Mechanics (1973) Vol. 60, part 2, pp. 321-362.

63. Berman, N.S. and Dunning, J.W. "Pipe flow measurements of turbulence and ambiguity using laser-Doppler velocimetry" J. Fluid Mechanics (1973) Vol. 61, part 2, pp. 289-299.

64. George, W.K. "The measurement of turbulence intensities using real time Laser Doppler velocimetry," J. Fluid Mechanics (1974) Vol. 66, part 1, pp. 11-16.

65. George, W.K. "Limitations to Measuring Accuracy Inherent in Laser Doppler Signal" Proceedings of the LDA Symposium Copenhagen 1975.

66. Pao, Y.H. (1965) Structure of Turbulent Velocity on scalar fields at large wavenumbers, Phys. Fluids 8, 1063.

67. D.B. Spalding, B.E. Launder, T.H. Whitelaw, "Turbulence models and their Experimental Verification," The Pennsylvania State University, 1974.

68. A. Quarmby and R. Quirk "Measurements of the radial and tangential eddy difficulties of heat and mass is turbulent flow in a plain tube" Int. J. of Heat and Mass Transfer Vol. 15, No. 11, 1972, pp. 2309-2323.

69. M.K. Ibragimov et al "Calculation of Hydraulic Resistivity Coefficients for Turbulent Fluid Flow in Channels of non-circular cross section" Soviet Atomic Energy Vol. 23, No. 4, pp. 1042-1047.

70. J.G. Bartzis, "Partially Blocked Flow between Parallel Plates," Euratom Internal Report, Ispra, 1975.

71. A.D. Gosman, "Listing of the TEACH-T Computer Program and Sample output," The Pennsylvania State University, August 1975.

72. V.S. Pratap and D.B. Spalding "Fluid Flow and Heat Transfer in three Dimensional Duct Flows" Int. J. Heat Mass. Transfer Vol. 9, pp. 1183-1188, 1976.

73. M. Clark and K.F. Hansen "Numerical Methods in Reactor Analysis," Academic Press, 1964.

74. E. Isaacson and H.B. Keller "Analysis of Numerical Methods" J. Wiley and Sons 1966.

75. DISA Type 55L Laser Doppler Anemometer" Instruction Manual, Disa Information Department.

76. DISA Type 55D35 Rms Voltmeter Instruction and Service Manual, DISA Information Department.

77. Eifler, W. and Nijsing, R., Fundamentals Studies of Fluid Flow and Heat Transfer in Fuel Element Geometries, Report EUR 2193.e, part II, (1965).

78. Eifler, W. and Nijsing, R., Experimental Investigation of Velocity Distribution and Flow Resistance in a Triangular Array of Parallel Rods, Nucl. Eng. Des. 5, (1967), pp. 22-42.

79. Subbotin, V.I. et al., Velocity Field of Turbulent Fluid Flow in a Longitudinal Streamline of Clusters of Rods, AEC-tr-7198 (1971).

80. Trupp, A.C. and Azad, R.S., The Structure of Turbulent Flow in Triangular Array Rod Bundles, Dpt. of Mech. Eng., University of Manitoba, Winnipeg, Canada (1973).

81. Rowe, D.S., Measurement of Turbulent Velocity Intensity and Scale in Rod Bundle Flow Channels, BNWL-1736, UC-80, Battelle, (May 1973).

82. Kjellstrom, B., Studies of Turbulent Flow Parallel to a Rod Bundle of Triangular Array, Report AE-RV-196 AB Atomenergi, Sweden (1971).

83. Hall, Ch. and Svenningsson, P.J., Secondary Flow Velocities in a Rod Bundle of Triangular Array, Report AE-RL-1326, AB Atomenergi, Sweden (1971).

84. Chen, Y.B., "Cooland Mixing in the LMFBR Outlet Plenum" Ph.D. Thesis,MIT, 1977

85. Wang, C.P. "Unified Analysis of Laser Doppler Velocimeter", Journal of Physics E. Scientific Instruments 5, 763-766, 1972.

86. Dubnitshev,Y.N.et al."Laser Doppler Velocimeter", an Optoelectronics Data Processing System", Applied Optics Vol 14, No. 1, Jan. 1955.

87. Maanen, H.R.E. et al. "Reduction of Ambiguity Noise in Laser Doppler Velocimetry by a cross correlation Technique" Proceeding of the LDA Symposium Copenhagen, 1975.

88. Rehme, K. "Anisotropic Eddy Viscosities in the Turbulent Flow Through a Rod Bundle" Symposium on Turbulent Shear Flows University Park, Pennsylvania 1977.

89. Burggraf, O.D., "Analytical and Numerical Studies of Steady Separated Flows," J. Fluid Mechanics 24, 113-151 (1966).

90. Runchal, A.K., "Convergence and Accuracy of Three Finite Difference Schemes for a Two-Dimensional Induction and Convection Problem," Int. Journal of Numerical Methods in Engineering, Vol. 4, 541-550 (1972).

91. Blackburn, D., et al., "Finite Difference Effects in Recirculating Flows," Paper presented to 6th meeting in Liquid Metal Boiling Working Group held at UKAEA, Risley, October 1975.

92. Hirt, C.W., et al., "SOLA-A Numerical Solution Algorithm for Transient Fluid Flows," LA-5852, 1975.

93. Barakat, H.Z., and Clark, T.A., "Analytical and Experimental Study of Transient Laminar Natural Convection Flows in Partially Filled Containers," Proc. 3rd Int. Heat Transfer Conf. 2, 152-162 (1966).

94. Nijsing, R.,Eifler, W. "Temperature Fields in Liquid Metal Cooled Rod Assemblies" Progress in Heat and Mass Transfer Volume, 1973.

95. Launder, B.E. "Progress in the Modeling of Turbulent Transport" the Pennsylvania State University, 1975.

96. Hoffman, G.H., "Improved Form of the low Reynolds Number k-ε turbulence Model" The Physics of Fluids, Vol. 18 No. 3, March 1975.

97. Jones, W.P. and Launder, B.E. "The Calculation of Low Reynolds-Number phenomena with 2-equation model of Turbulence," Int. J. Heat Mass Transfer, Vol. 16, pp. 1119-1130, 1973.

Table 2.1 Summary Table of Measurements in Rod Bundles (Taken from Ref. 1)

Investigator	P/D[#]	L/D_H	$Re \times 10^{-3}$	Velocity		Turb. Intensity			Cross-Correlations		$\frac{\tau_w}{\tau_w}$	Techniques
				Axial	Second.	z	r	θ	$\overline{v'_r v'_z}$	$\overline{v'_\theta v'_z}$		
Eifler, Nijsing [77, 78]	1.05 1.10 1.15	139 90 65	15, 30, 50 15, 30, 50 15, 30, 50	✓ ✓ ✓								Pitot tube
Eifler [19]	1.08	137	10-152	✓								Pitot tube
Subbotin, et al. [79]	1.05 1.10 1.20	154 100 72	18.8-81 18.8-81 18.8-81	✓ ✓ ✓							✓ ✓ ✓	Pitot tube Preston tube
Kjellstrom [82]	1.217	81	149-373	✓	✓	✓	✓	✓	✓		✓	Hot Wire Preston tube
Hall & Svenningsson [83]	1.217	81	270		✓						✓	Hot Wire
Trupp & Azad [80]	1.20 1.35 1.50	51 30 20	12-84 12-84 12-84	✓ ✓ ✓		✓ ✓ ✓	✓ ✓ ✓	✓ ✓ ✓	✓ ✓ ✓		✓ ✓	Hot Wire Pitot Tube Preston Tube
Rowe [81]	1.25* 1.11* 1.25* 1.25** 1.25	85† 117 90 105 138	50-200 50-200 50-200 50-200 50-200	✓ ✓ ✓ ✓ ✓		✓ ✓ ✓ ✓ ✓						LDA
Carajilescov [1]	1.123	77	27	✓		✓						LDA

All data for triangular array except where mentioned otherwise
* Square Array ** Square and triangular channels in same test section
† Based on cross section hydraulic diameter instead of subchannel

Table 2.2

TEST SECTION GEOMETRICAL CHARACTERISTICS

Parameter	Symbol	Size	Units
Rod diameter	D	51.15	mm
Gap	g	6.35	mm
Pitch/Diameter	P/D	1.124	
Unit cell hydraulic diameter	D_H	20.12	mm
Cross section hydraulic diameter	D_H^T	16.375	mm
Unit cell area		6.132×10^{-5}	m^2
Total cross section area		1.346×10^{-3}	m^2

Table 2.3

LDA OPTICAL PARAMETERS

Parameter	Symbol	Value	Units
1. Refractive Indices			
Air	n_o	1.00	
Water	n_w	1.33	
Plexiglas	n_p	1.50	
2. Laser			
Wavelength	λ	0.5145	μ
Beam Diameter	D_o	1.5	mm
3. Optical Unit			
Focal Length	f	120	mm
Beam Separation	2a	40	mm
Half-Angle between Beams			
in air	$\theta/2$	6.64	degrees
in water	$\beta/2$	4.99	degrees
4. Beam Intersection (Fig. 1)			
Diameter of the Beam at Waist	$2b_o$	0.053	mm
Width	$2\sigma_1$	0.053	mm
Height	$2\sigma_2$	0.053	mm
Length	$2\sigma_3$	0.610	mm
5. Calibration factor			
$C = \frac{\lambda}{2n_w \sin \frac{\beta}{2}}$	C	2.224	m/s/MHZ

Table 2.4 Parameter and Error Relations

Quantity	Calculation	Error ($\delta\phi = \left\| \phi_{MEAS} - \phi \right\|$)
v_z, v'_z	v_z, v'_z	$\delta v_z, \delta v'_z$
v_r	$v_1 \cos\theta + v_2 \sin\theta$	$\delta v_r = \delta v_1 \cos\theta + \delta v_2 \sin\theta$
v_θ	$-v_1 \sin\theta + v_2 \cos\theta$	$\delta v = \delta v_1 \sin\theta + \delta v_2 \cos\theta$
v'_r	$(v'^2_1 \cos^2\theta + v'^2_2 \sin^2\theta)^{1/2}$	$\delta v'_r = \frac{v'_1}{v'_r} \cos^2\theta \, \delta v'_1 + \frac{v'_y}{v'_r} \sin^2\theta \, \delta v'_2$
v'_θ	$(v'^2_1 \sin^2\theta + v'^2_2 \cos^2\theta)^{1/2}$	$\delta v'_\theta = \frac{v'_1}{v'_\theta} \sin^2\theta \, \delta v'_1 + \frac{v'_2}{v'_\theta} \cos^2\theta \, \delta v'_2$
k	$\frac{1}{2}(v'^2_1 + v'^2_2 + v'^2_z)$	$\delta k = v'_1 \delta v'_1 + v'_2 \delta v'_2 + v'_z \delta v'_z$
$\overline{v'_r v'_z}$	$\overline{v'_1 v'_z} \cos\theta + \overline{v'_2 v'_z} \sin\theta$	$\delta(\overline{v'_r v'_z}) = \delta(\overline{v'_1 v'_z}) \cos\theta + \delta(\overline{v'_2 v'_z}) \sin\theta$
$\overline{v'_\theta v'_z}$	$-\overline{v'_1 v'_z} \sin\theta + \overline{v'_2 v'_z} \cos\theta$	$\delta(\overline{v'_\theta v'_z}) = \delta(\overline{v'_1 v'_z}) \sin\theta + \delta(\overline{v'_2 v'_z}) \cos\theta$

Re		v_z	v_z'	v_1	v_1'	v_2	v_2'	v_1v_z		$v_2'v_z'$	
								v_1'	v_z'	v_2'	v_z'
9,000	FREQ RANGE (MHZ)	0.5	0.5	0.5	0.5	0.5	0.5	5.0	0.5	0.5	5.0
	SHIFT (MHZ)	0.05	0.05	0.25	0.25	0.25	0.25	1.0	0.05	0.25	1.0
	TRACKER	DISA	DISA	DISA	DISA	DISA	DISA	TSI	DISA	DISA	DISA
26,500	FREQ RANGE	1.5	1.5	1.5/5.0	1.5/5.0	5.0	5.0	5.0	1.5	5.0	5.0
	SHIFT	0.15	0.15	0.7/1.3	0.7/1.3	1.3	1.3	4.0	0.15	0.15	1.0
	TRACKER	DISA	DISA	DISA	DISA	TSI	TSI	TSI	DISA	DISA	TSI
65,000	FREQ RANGE	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
	SHIFT	0.0	0.0	1.3	1.3	1.3	1.3	1.0	0.4	1.0	0.4
	TRACKER	TSI	TSI	TSI	TSI	TSI	TSI	TSI	DISA	TSI	DISA

Table 2.5 Frequency Tracking Arrangement

TABLE 2.6

$(v'_z)_{LAM}$ (in mV) FOR VARIOUS FREQUENCY RANGES

Tracker	Frequency Range	I.F BANDWIDTH				
	ν_{max} (MHZ)	1%	2%	4%	8%	TSI
DISA	0.5	-	60	70	80	
	1.5	22	36	56	81	
	5.0	-	44	-	-	
TSI	5.0					46

Table 3.1 Parameter ζ (Taken from Ref. 39)

Author	ζ Name	ζ Definition	Turbulent Viscocity	
Kolmogorov [3]	Frequency	$f = c\,\frac{k^{1/2}}{\ell}$	$\nu^T = \frac{k}{f} = k^{1/2}\,\ell$	
Rotta [45]	Length-energy product	$L = k\ell$		Duct Flow
Rodi & Spalding [46]	Length-energy product	$L = k\ell$	$\nu^T = k^{1/2}\,\ell$	Free shear flow
Ng & Spalding [67]	Length-energy product	$L = k\ell$	$\nu^T = k^{1/2}\,\ell$	Boundary Layers
Wolfshtein [48]	Length-energy product	$L = k\ell$	$\nu^T = 0.22\ k^{1/2}\,\ell$	One-dimensional flows
Glushko [44]	Length squared	$\lambda = \frac{1}{2}\,\ell^2$	$\nu^T = \alpha\,\ell\,k^{1/2}$ $0.2 < \alpha < 0.8$	High turbulence version of a low turbulence model applied to trans. flat plate boundary layer
Harlow & Nakayama [41]	Dissipation	ε	$\nu^T = 0.2\,\frac{k^2}{\varepsilon} = 1.41\ k^{1/2}\,\ell$	
Jones & Launder [50]	Dissipation	ε	$\nu^T = 0.09\,\frac{k^2}{\varepsilon}$	High turbulence version of a low turbulence model applied to laminarizing boundary layer
Wilcox & Alber [51]	Vorticity squared	Ω^2	$\nu^T = \frac{k}{\Omega} = k^{1/2}\,\ell$	Boundary layers

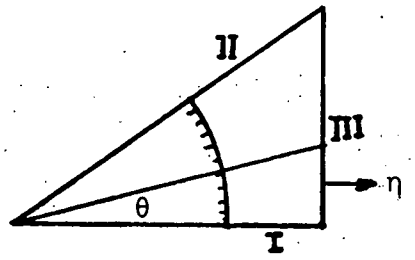
Table 3.2a The Differential Equations

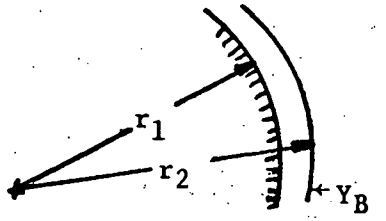
General Form: $\nabla\phi\underline{V} + \Delta T_\phi = \nabla S_\phi + S_\phi^*$

ϕ	T_ϕ		S_ϕ		S_ϕ^*
	$T_{\phi r}$	$T_{\phi\theta}$	$S_{\phi r}$	$S_{\phi\theta}$	
v_r	$-2\nu\frac{\partial v_r}{\partial r}$	$-\frac{\nu_r^{eff}}{r}\frac{\partial v_r}{\partial\theta}$	$-\overline{v_r'^2} - p/\rho$	$+\nu_{r\theta}^{eff}\, r\,\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)$	$\frac{\overline{v_\theta'^2} + \overline{v_\theta'^2}}{r} - \frac{2\nu}{r}\left(\frac{\partial v_\theta}{r\partial\theta} + \frac{v_r}{r}\right)$
rv_θ	$=\nu_{r\theta}^{eff}\, r^2\,\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)$	$-2\nu\,\frac{\partial(rv_\theta)}{r\partial\theta}$	$+\nu_r^{eff}\,\frac{\partial v_r}{\partial\theta}$	$+2\nu v_r - r\overline{v'^2}\frac{rp}{\rho}$	0
v_z	$-\nu_{rz}^{eff}\,\frac{\partial v_z}{\partial r}$	$-\nu_r^{eff}\,\frac{\partial v_z}{r\partial\theta}$	0	0	$-\frac{d(p/\rho)}{dz} = \frac{4\overline{u}^{*2}}{D_H}$
k	$-\frac{\nu_{rz}^{eff}}{\sigma_\kappa}\frac{\partial k}{\partial r}$	$-\frac{\nu_r^{eff}}{\sigma_\kappa}\frac{\partial k}{r\partial\theta}$	0	0	$\nu_{rz}^T\left(\frac{\partial v_z}{\partial r}\right)^2 + \nu_{r\theta}^T\left(\frac{\partial v_z}{r\partial\theta}\right)^2 - \varepsilon$
ε	$-\frac{\nu_{rz}^{eff}}{\sigma_\varepsilon}\frac{\partial\varepsilon}{\partial r}$	$\frac{\nu_{r\theta}^{eff}}{\sigma_\varepsilon}\frac{\partial\varepsilon}{r\partial\theta}$	0	0	$c_{\varepsilon 1}\frac{\varepsilon}{k}\left[\nu_{rz}^T\left(\frac{\partial v_z}{\partial r}\right)^2 + \nu_{r\theta}^T\left(\frac{\partial v_z}{r\partial\theta}\right)^2\right] - c_{\varepsilon 2}\frac{\varepsilon^2}{k}$

Table 3.2b Boundary Conditions

PARAMETER	I	II	III	NEAR WALL REGION	
v_z	$\frac{\partial v_z}{r\partial\theta} = 0$	$\frac{\partial v_z}{r\partial\theta} = 0$	$\frac{\partial v_z}{\partial r} = \frac{\partial v_z}{r\partial\theta} \, tg\theta$	Logarithmic Profile	
v_r	$v_r = 0$	$v_r = 0$	$n \cdot V_{sec} = 0$	$v_r \mid_{wall} = 0$	
v_θ	$v_\theta = 0$	$v_\theta = 0$	$\frac{\partial V_{sec}}{\partial n} = 0$	$v_\theta \mid_{wall} = 0$	
k	$\frac{\partial k}{r\partial\theta} = 0$	$\frac{\partial k}{r\partial\theta} = 0$	$\frac{\partial k}{\partial r} = \frac{\partial k}{r\partial\theta} \, tg\theta$	$k = \frac{\overline{v_r' \, v_z'}}{\sqrt{c\mu}}$	
ε	$\frac{\partial \varepsilon}{r\partial\theta} = 0$	$\frac{\partial \varepsilon}{r\partial\theta} = 0$	$\frac{\partial \varepsilon}{\partial r} = \frac{\partial \varepsilon}{r\partial\theta} \, tg\theta$	1-EQ.	2-EQ.
				$\varepsilon = c_D \frac{k^{3/2}}{\ell}$ Algebraic	$\varepsilon = \frac{u_*}{ky} \, (\overline{v_r' \, v_z'})$





$r_1 = D/2$

$r_2 = r_1 + Y_B$

ϕ	A_e	A_w	A_n	A_s	V_p
V_z k ε rv_θ	$\frac{r_i + r_{i+1}}{2}\Delta\theta$	$\frac{r_i + r_{i-1}}{2}\Delta\theta$	$\frac{r_{i+1} - r_{i-1}}{2}$	$\frac{r_{i+1} - r_{i-1}}{2}$	$r_i\Delta\theta\ \frac{r_{i+1}-r_{i-1}}{2}$
v_r	r_i	r_{i-1}	$r_i - r_{i-1}$	$r_i - r_{i-1}$	$\frac{r_i+r_{i-1}}{2}(r_i-r_{i-1})\Delta\theta$

TABLE 4.1 Control Volume Lengths and Surface

TABLE 4.2: THE DIFFUSION COEFFICIENTS

	$\phi = k, \varepsilon, v_z$	v_r	v_θ
BE	$0.25 \dfrac{r_{i+1}+r_i}{r_{i+1}-r_i} \Delta\theta$	$\dfrac{4r_i}{r_{i+1}-r_{i-1}} \Delta\theta$	$\dfrac{(r_i+r_{i+1})^3}{32r_{i+1}(r_{i+1}-r_i}$
BW	$0.25 \dfrac{r_i+r_{i-1}}{r_i-r_{i-1}}$	$\dfrac{4r_{i-1}}{r_i-r_{i-2}} \Delta\theta$	$\dfrac{(r_i+r_{i-1})^3}{32r_{i-1}(r_i-r_{i-1})}$
BN BS	$0.25 \dfrac{r_{i+1}-r_{i-1}}{r_i \Delta\theta}$	$\dfrac{r_i-r_{i-1}}{2(r_i+r_{i-1}) \Delta\theta}$	$\dfrac{r_{i+1}-r_{i-1}}{\Delta\theta}$
DE	$\dfrac{BE}{\sigma_\phi}\left[\left(\nu_{rz}^{eff}\right)_{i+1,j}+\left(\nu_{rz}^{eff}\right)_{i,j}\right]$	BE · ν	$BE\left[\left(\nu_{r\theta}^{eff}\right)_{i+1,j}+\left(\nu_{r\theta}^{eff}\right)_{i+1,j-1}+\left(\nu_{r\theta}^{eff}\right)_{i,j-1}+\left(\nu_{r\theta}^{eff}\right)_{i,j}\right]$
DW	$\dfrac{BW}{\sigma_\phi}\left[\left(\nu_{rz}^{eff}\right)_{i+1,j}+\left(\nu_{rz}^{eff}\right)_{i,j}\right]$	BW · ν	$BW\left[\left(\nu_{r\theta}^{eff}\right)_{i-1,j}+\left(\nu_{r\theta}^{eff}\right)_{i-1,j-1}+\left(\nu_{r\theta}^{eff}\right)_{i,j-1}+\left(\nu_{r\theta}^{eff}\right)_{i,j}\right]$
DN	$\dfrac{BN}{\sigma_\phi}\left[\left(\nu_{\theta z}^{eff}\right)_{i,j+1}+\left(\nu_{rz}^{eff}\right)_{i,j}\right]$	$BN\left[\left(\nu_{r\theta}^{eff}\right)_{i,j+1}+\left(\nu_{r\theta}^{eff}\right)_{i,j}+\left(\nu_{r\theta}^{eff}\right)_{i-1,j}+\left(\nu_{r\theta}^{eff}\right)_{i-1,j+1}\right]$	BN · ν

TABLE 4.2: THE DIFFUSION COEFFICIENTS (Continued)

	$\phi = k, \varepsilon, v_z$	v_r	v_θ
DS	$\frac{BS}{\sigma_\phi}\left[\left(\nu_{\theta z}^{eff}\right)_{i,j-1} + \left(\nu_{\theta z}^{eff}\right)_{i,j}\right]$	$BS\left[\left(\nu_{r\theta}^{eff}\right)_{i,j-1} + \left(\nu_{r\theta}^{eff}\right)_{i,j} \left(\nu_{r\theta}^{eff}\right)_{i-1,j} + \left(\nu_{r\theta}^{eff}\right)_{i-1,j-1}\right]$	$BS \cdot \nu$
$\hat{\sigma}$	$\hat{0}$	$\hat{0}$	$\hat{1}$

Diffusion Term $DT = \left[DE\left(\frac{r_{i+1}}{r_i}\right)^{\sigma} + DW\left(\frac{r_{i-1}}{r_i}\right)^{\sigma} + DN + DS\right]\phi_{i,j} - DE\ \phi_{i+1,j} - DW\ \phi_{i-1,j} - DN\ \phi_{i,j+1} - DS\ \phi_{i,j-1}$

(Concluded)

TABLE 4.3: THE CONVECTION COEFFICIENTS

Coeff	$\phi = k, \varepsilon, v_z, T_c$	$\phi = v_r$	$\phi = v_\theta$
A_1	$\left[(r_i+r_{i+1})(r_{i+1}-r_i)(v_r)_{i+1,j} + (r_i+r_{i-1})(r_i-r_{i-1})(v_r)_{i,j}\right] \cdot \frac{\Delta\theta}{8}$	$\left\{r_i(r_i-r_{i-1})\left[(v_r)_{i+1,j}+(v_r)_{i,j}\right] + r_{i-1}(r_i-r_{i-1})\left[(v_r)_{i-1}+(v_r)_{i,j}\right]\right\} \cdot \frac{\Delta\theta}{4}$	$\left\{(r_i+r_{i+1})(r_{i+1}-r_i)\left[(v_r)_{i+1,j}+(v_r)_{i+1,j-1}\right] + (r_i+r_{i-1})(r_i-r_{i-1})\right](v_r)_{i,j}+(v_r)_{i,j-1}\Big\} \cdot \frac{\Delta\theta}{16}$
A_2	$\frac{(r_{i+1}-r_{i-1})}{8}\left[(v)_{i,j}+(v)_{i+1}\right]$	$\frac{r_i-r_{i-1}}{8}\left[(v_\theta)_{i,j+1}+(v_\theta)_{i-1,j+1} + (v_\theta)_{i,j}+(v_\theta)_{i-1,j}\right]$	$r_i \frac{r_{i+1}-r_{i-1}}{16}\left[(v_\theta)_{i,j+1}+2(v_\theta)_{i,j}+(v_\theta)_{i,j-1}\right]$
CE	$(\lvert A_1\rvert - A_1)/(r_{i+1}-r_i)$	$(\lvert A_1\rvert - A_1)/(r_{i+1}-r_{i-1})$	$(\lvert A_1\rvert - A_1)\frac{r_{i+1}}{r_{i+1}-r_i}$
CW	$(\lvert A_1\rvert + A_1)/(r_i-r_{i-1})$	$(\lvert A_1\rvert + A_1)/(r_i-r_{i-1})$	$(\lvert A_1\rvert + A_1)\frac{r_{i-1}}{r_{i+1}-r_i}$
CN	$(\lvert A_2\rvert - A_2)$	$\lvert A_2\rvert - A_2$	$\lvert A_2\rvert - A_2$
CS	$\lvert A_2\rvert + A_2$	$\lvert A_2\rvert + A_2$	$\lvert A_2\rvert + A_2$
σ	0	0	1

Convection term $CT = \left[CE\left(\frac{r_i}{r_{i+1}}\right)^\sigma + CW\left(\frac{r_i}{r_{i+1}}\right)^\sigma + CN + CS\right]\phi_{i,j} - CE\,\phi_{i+1,j} - CW\,\phi_{i-1,j} - CN\,\phi_{i,j+1} - CS\,\phi_{i,j-1}$

TABLE 4.4: SOURCE TERMS

ϕ	SU	SP
v_z	$\frac{4\dot{V}_p}{DH}\left(\frac{\tau_w}{\rho}\right)_{avg}$	0
k	$V_p(\nu^T_{rz})_{i,j}\left[\left(\frac{\partial v_z}{\partial r}\right)_{i,j}\right]^2+(\nu^T_{\theta z})_{i,j}\left[\left(\frac{\partial v_z}{r\partial\theta}\right)_{i,j}\right]^2$	$\frac{C_\mu k_{i,j} V_p}{(\nu^T_{rz})_{i,j}}$
ε	$C_1 C_\mu \frac{k_{i,j} V_p}{(\nu^T_{rz})_{i,j}}\left\{(\nu^T_{rz})_{i,j}\left[\left(\frac{\partial v_z}{\partial r}\right)_{i,j}\right]^2+(\nu^T_{\theta z})_{i,j}\left[\left(\frac{\partial v_z}{r\partial\theta}\right)_{i,j}\right]^2\right\}$	$\frac{C_2 C k_{i,j} V_p}{(\nu^T_{rz})_{i,j}}$
v_r	$r_{i-1}\Delta\theta\left[\overline{(v'^2_r)}_{i-1,j}+(p/\rho)_{i-1,j}\right]-r_i\left[\overline{(v'^2_r)}_{i,j}+(p/\rho)_{i,j}\right]$ $+\frac{(\nu^{eff}_{r\theta})_{i,j}+(\nu^{eff}_{r\theta})_{i,j+1}+(\nu^{eff}_{r\theta})+(\nu^{eff}_{r\theta})_{i-1,j}}{8}$ $\cdot (r_i+r_{i-1})\left[\frac{(v_\theta)_{i,j+1}}{r_i}-\frac{(v_\theta)_{i-1,j+1}}{r_{i-1}}\right]$ $-\frac{(\nu^{eff}_{r\theta})_{i,j}+(\nu^{eff}_{r\theta})_{i,j-1}+(\nu^{eff}_{r\theta})_{i-1,j-1}+(\nu^{eff}_{r\theta})_{i-1,j}}{8}$ $\cdot (r_i+r_{i-1})\left[\frac{(v_\theta)_{i,j}}{r_i}-\frac{(v_\theta)_{i-1,j}}{r_{i-1}}\right]$ $+\frac{2}{r_i+r_{i-1}}V_p\left[\frac{\overline{(v'^2_\theta)}_{i,j}+\overline{(v'^2_\theta)}_{i-1,j}}{2}\right]^2$	0

TABLE 4.4: SOURCE TERMS (Continued)

ϕ	SU	SP
v_θ	$\frac{(\nu^{eff}_{r\theta})_{i+1,j}+(\nu^{eff}_{r\theta})_{i+1,j-1}+(\nu^{eff}_{r\theta})_{i,j-1}+(\nu^{eff}_{r\theta})_{i,j}}{4}\frac{r_i+r_{i+1}}{2}\left[(v_r)_{i+1,j}-(v_r)_{i+1,j-1}\right]$ $-\frac{(\nu^{eff}_{r\theta})_{i-1,j}+(\nu^{eff}_{r\theta})_{i-1,j-1}+(\nu^{eff}_{r\theta})_{i,j-1}+(\nu^{eff}_{r\theta})_{i,j}}{4}\frac{r_i+r_{i-1}}{2}\left[(v_r)_{i,j}-(v_r)_{i,j-1}\right]$ $+\frac{r_i(r_{i+1}-r_{i-1})}{2}\left[\overline{(v'^2_\theta)}_{i,j-1}+(p/\rho)_{i,j-1}-\overline{(v'^2_\theta)}_{i,j}-(p/\rho)_{i,j}\right]$	0

Source Term ST = SU - SP ϕ_P

TABLE 4.5 Pressure Coefficients (Eq. 4.68)

QUANTITY	DEFINITION	COMMENTS
DPE	$\frac{r_{i+1} \cdot \Delta\theta}{AP_{i+1,j}} A_e$	A_e is given in Table 4.1 AP is referred to velocity v_r
DPW	$\frac{r_{i-1} \cdot \Delta\theta}{AP_{i,j}} A_w$	A_w is given in Table 4.1 AP is referred to velocity v_r
DPN	$\frac{r_{i+1} - r_{i-1}}{2\, AP_{i,j+1}} A_n$	A_n is given in Table 4.1 AP is referred to velocity v_θ
DPS	$\frac{r_{i+1} - r_{i-1}}{2\, AP_{i,j}} A_s$	A_s is given in Table 4.1 AP is referred to velocity v_θ

TABLE 5.1

CHARACTERISTICS OF THE FULLY DEVELOPED FLOW

Nominal Re	65,000	26,500	9,000
Actual Re	64,810	26,549	8.924
Bulk velocity $v_b^{(**)}$ (m/s)	1.975	1.103	3.67×10^{-1}
Wall shear stress $\frac{\tau_w^{(*)}}{\rho}\left(\frac{m^2}{s^2}\right)$	1.04×10^{-2}	4.12×10^{-3}	6.56×10^{-4}

(*) Calculated from the logarithmic law ($k = 0.42$ $E = 9.8$)

(**) The wall region represented by the logarithmic law

TABLE 5.2

BULK VELOCITIES (m/s) IN THE DEVELOPING REGION

Re \ L/DH	0 (Theoretical)	15	46	113
26,000	0.984	0.997	1.087	1.103
55,000	1.829	1.830	1.95	1.975

TABLE 5.3

y_B^+ SELECTION FOR THE NUMERICAL CALCULATIONS

Author	P/D	Re	y_B^+
Present	1.124	65,000	60
Present	1.124	26,500	45
Present	1.124	9,000	20
Eifler	1.08	52,400	60
Trupp and Azad	1.20	49,000	60
Trupp and Azad	1.35	60,000	60
Kjellstrom	1.217	149,000	100

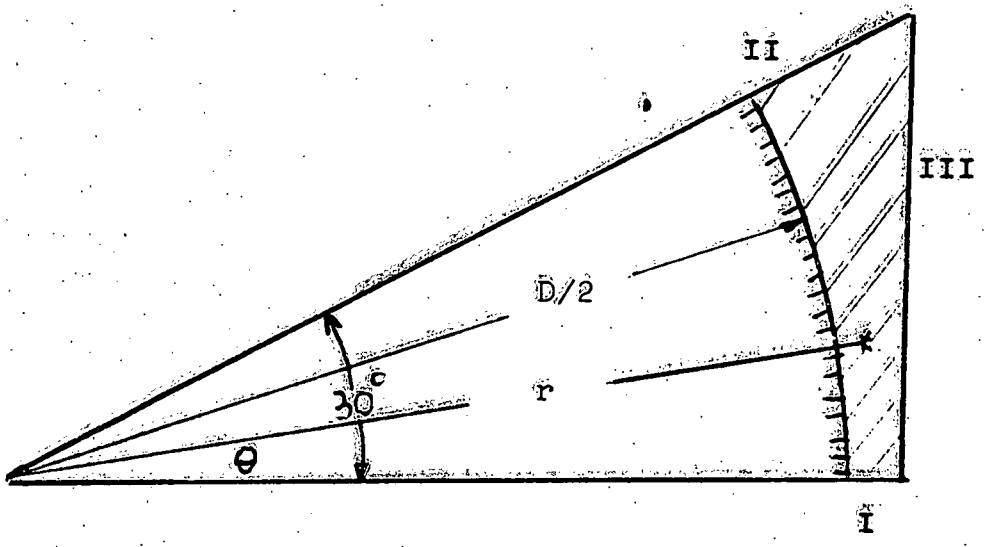


Fig. 1.1 Characteristic triangle for the study of fully developed flow in an infinite triangular array of bare rods

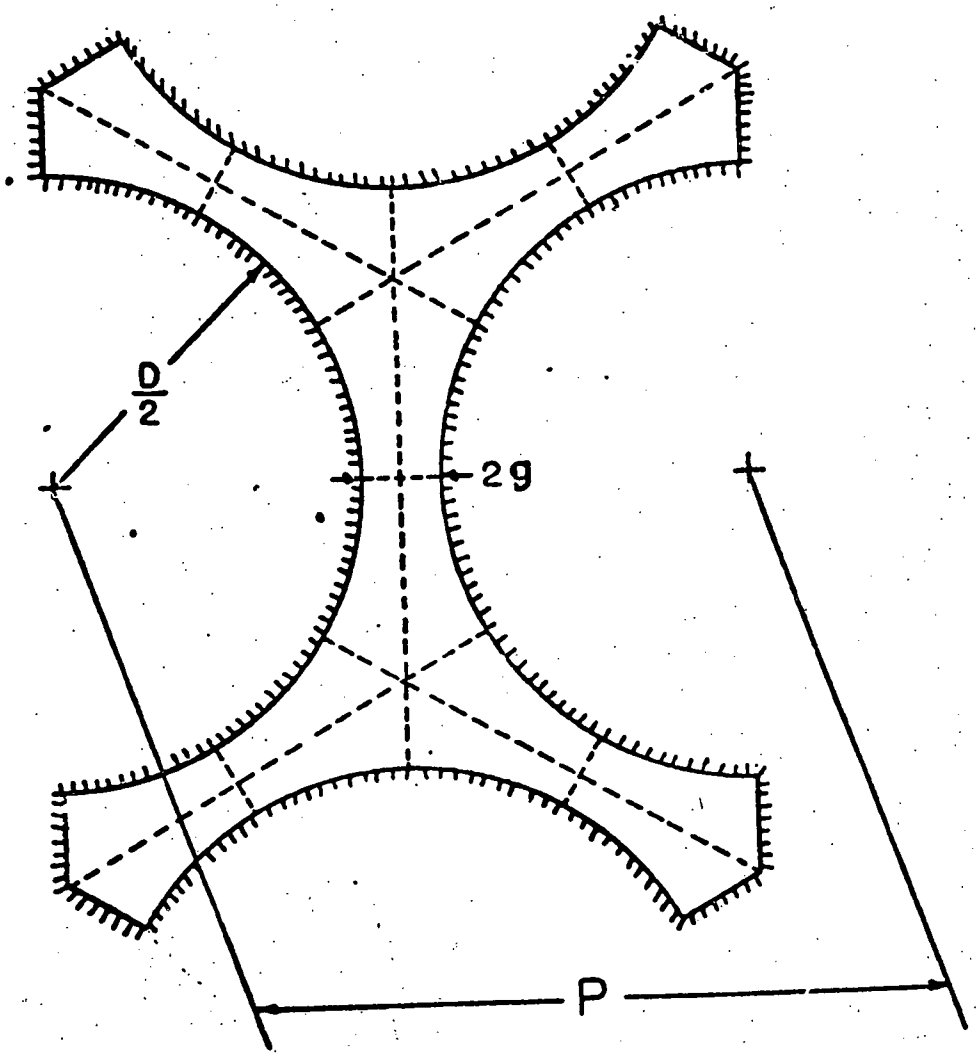


Fig 2.1. Cross Section of the Test Section [1]

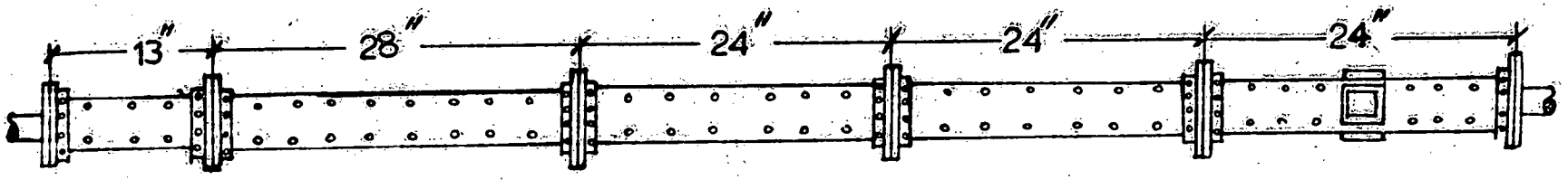


Fig 2.2 Overall View of the Test Section

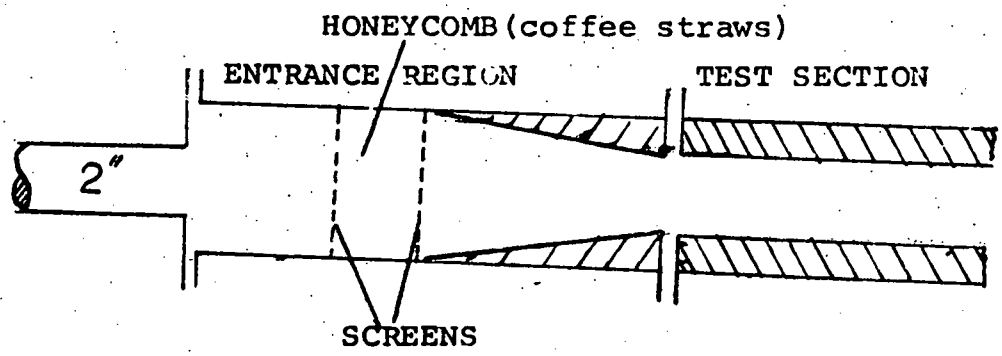


Fig. 2.3 Entrance Region

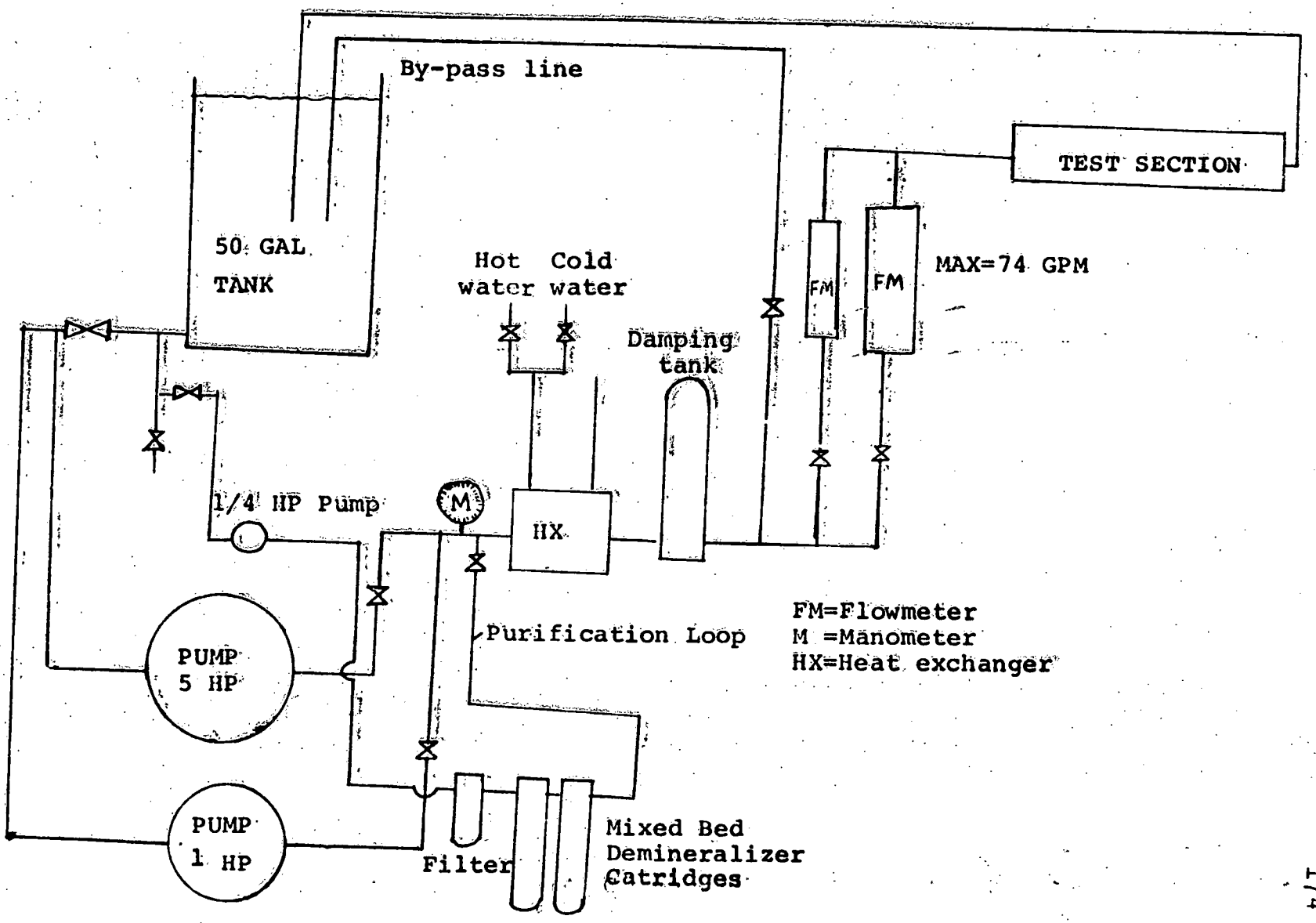


Figure 2.4. The Hydraulic Loop

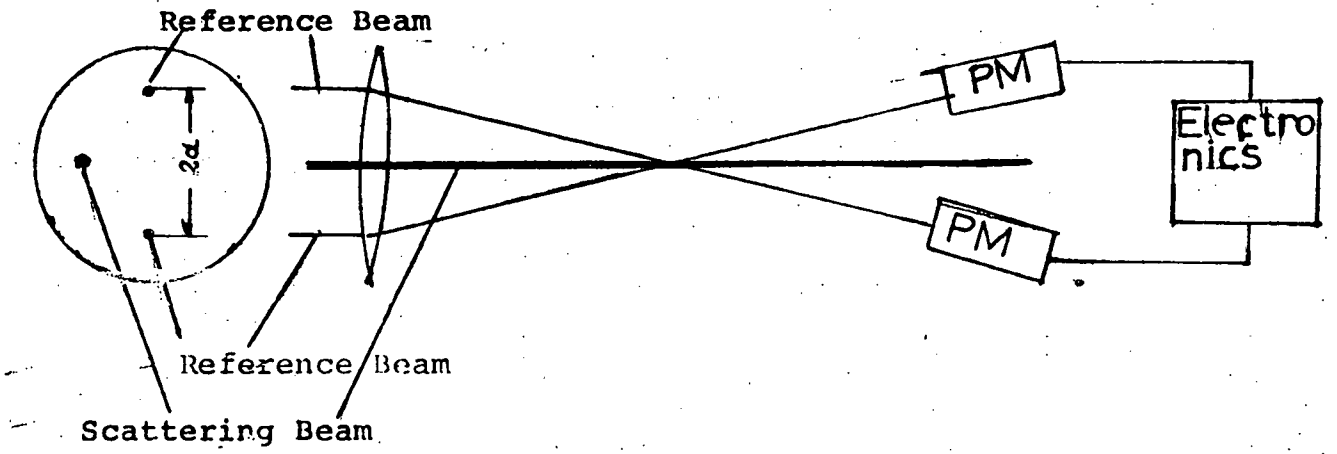
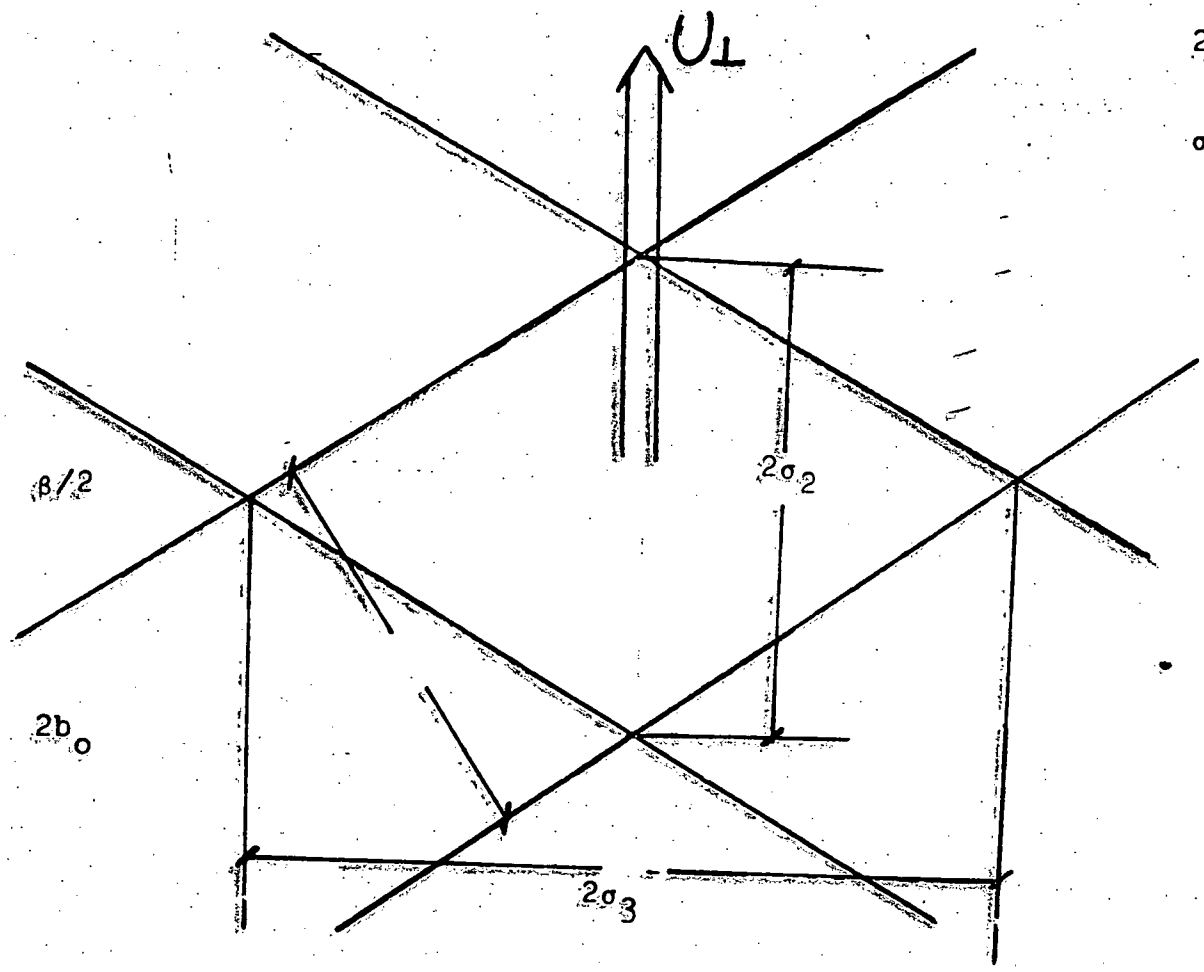


Fig. 2.5 LDA Optical Arrangement

$$2\sigma_1 = 2b_o$$

$$2\sigma_2 = 2\,\frac{2b_o}{\sin\beta}\,\sin\frac{\beta}{2}$$

$$\sigma\sigma_3 = 2\,\frac{2b_o}{\sin\beta}\,\cos\frac{\beta}{2}$$



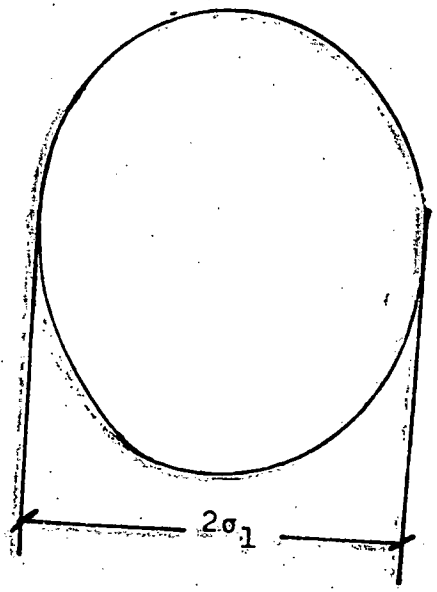


Fig.-2.6. Beam Intersection

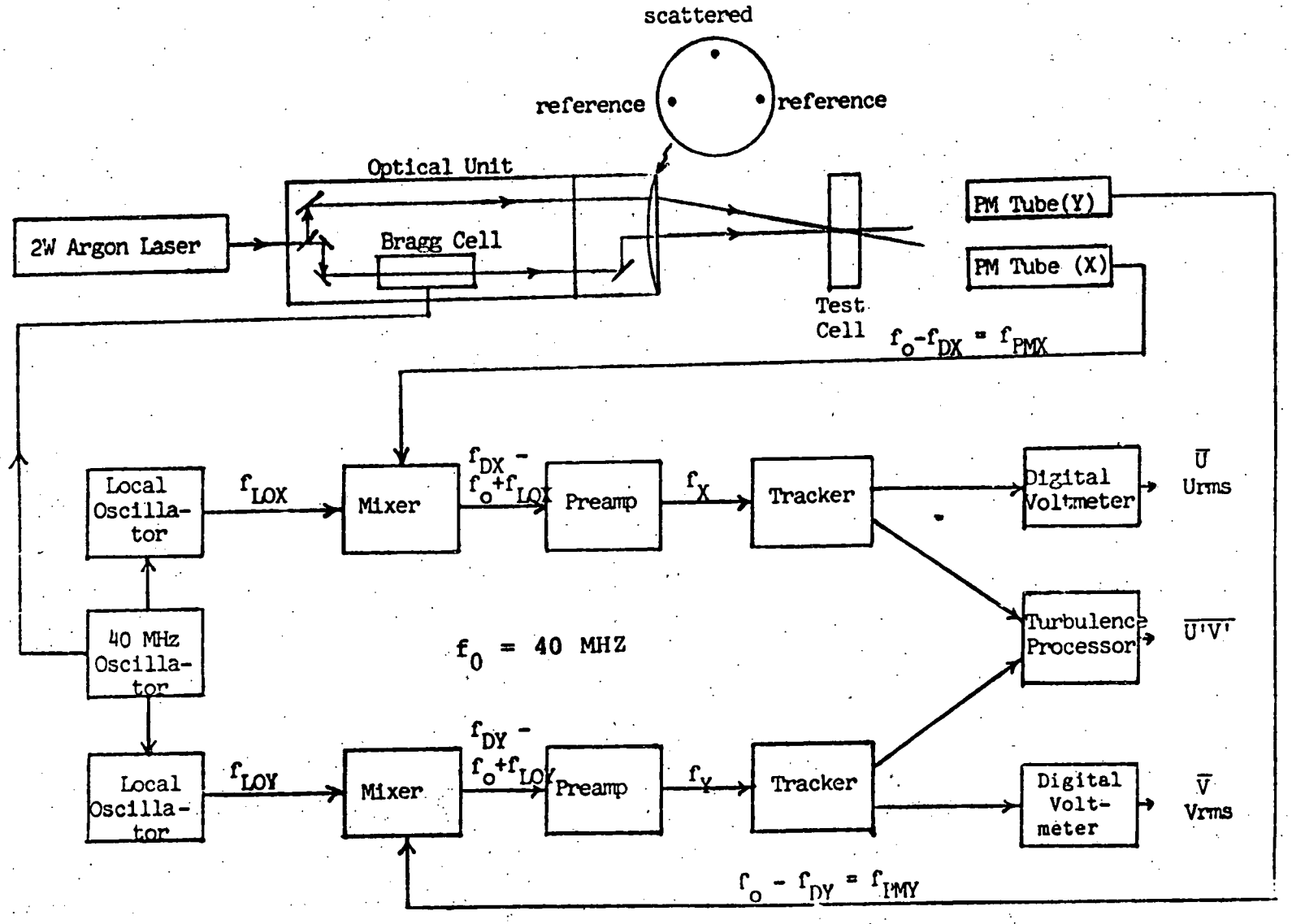


Fig 2.7 Layout of Instruments (Taken from Ref.84)

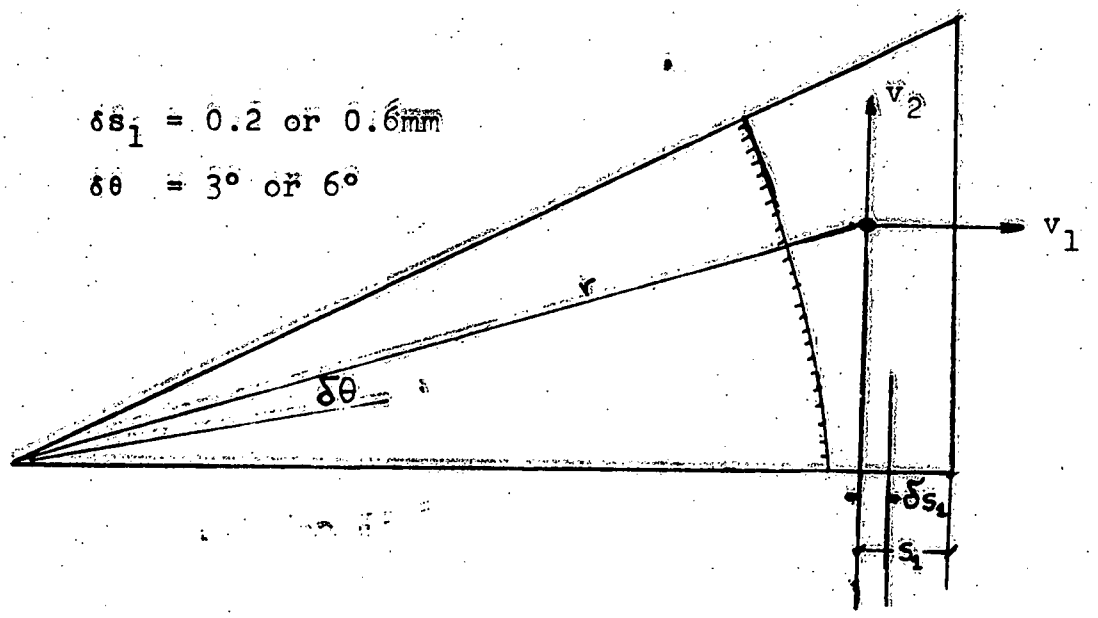
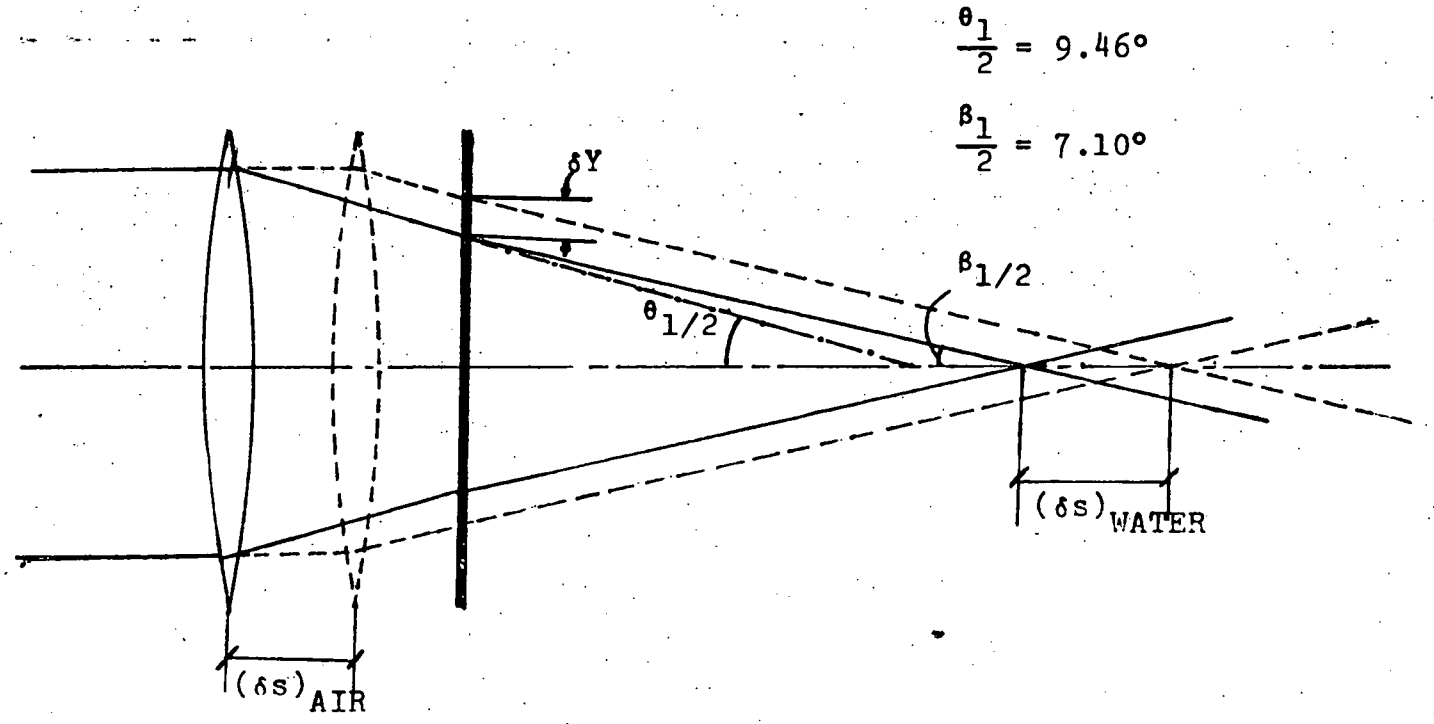


Fig. 2.8. Point selection for measurements



$$\delta Y = (\delta s)_{AIR} tg \frac{\theta 1}{2} = (\delta s)_{WATER} tg \frac{\beta 1}{2}$$

Fig. 2.9. Water Displacements vs Air Displacements

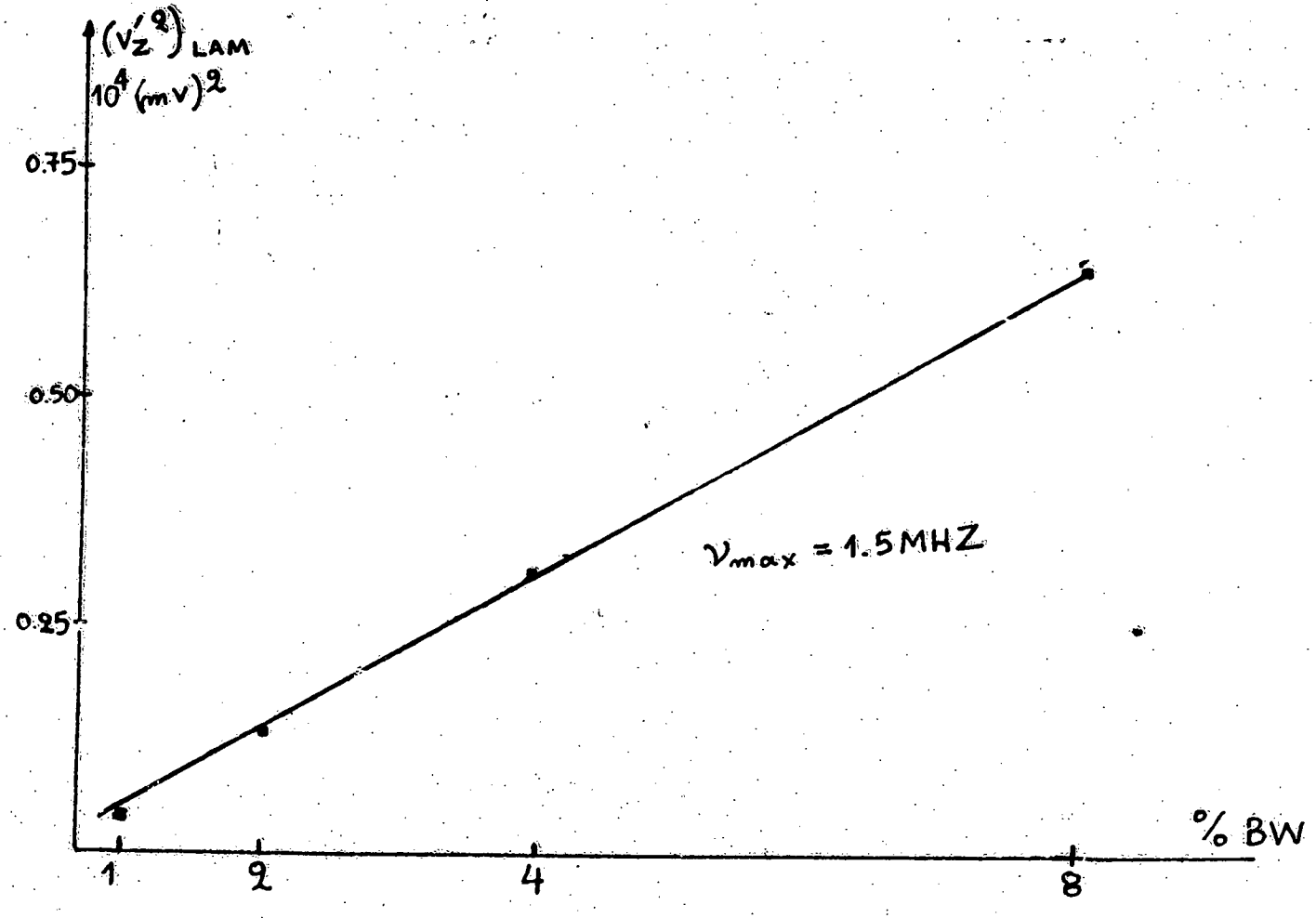


Fig.2.10. $(v_z'^2)_{LAM}$ vs %I.F. BANDWIDTH of the Frequency Tracker

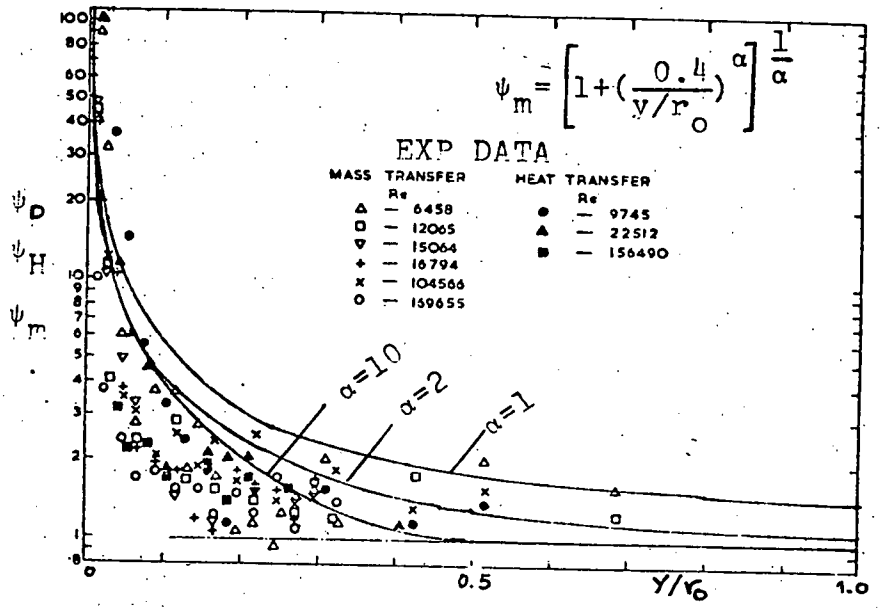


Fig. 3.1. Comparison of ψ_m with Quarmby's data [68]

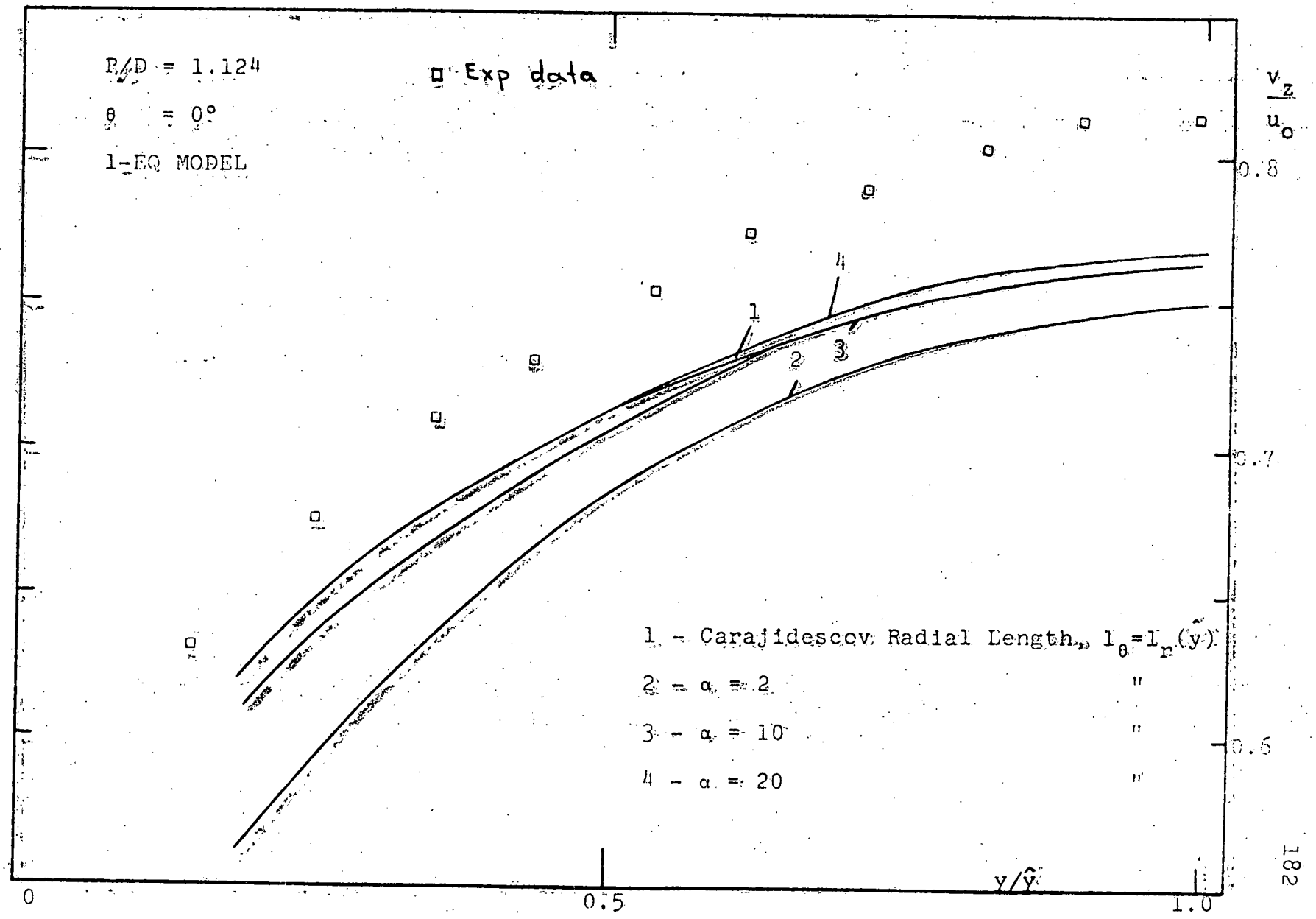


Fig. 3.2. Mixing Lengths Comparisons (1 - Eq. Model)

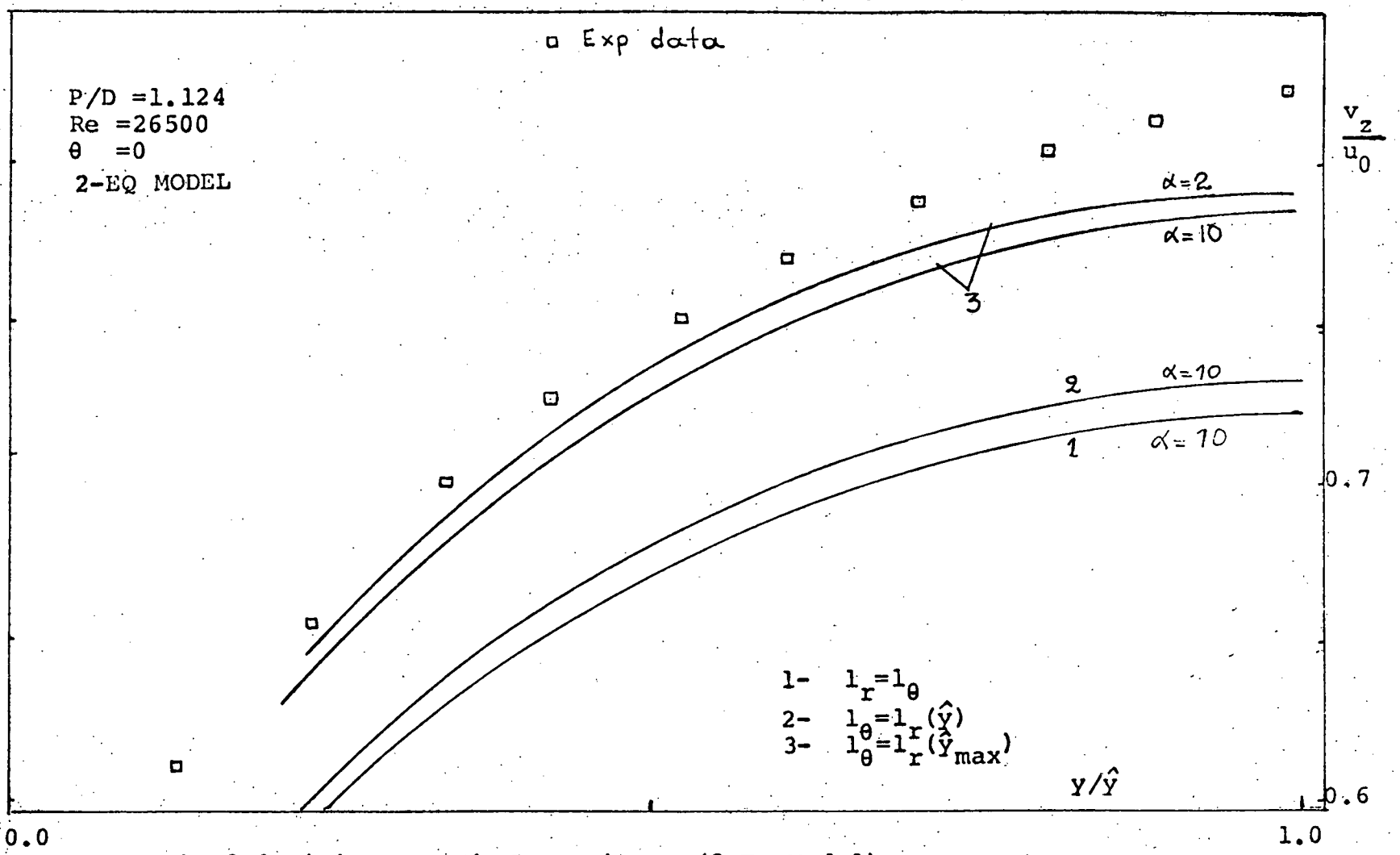


Fig 3.3 Mixing Length Comparisons (2-Eq Model)

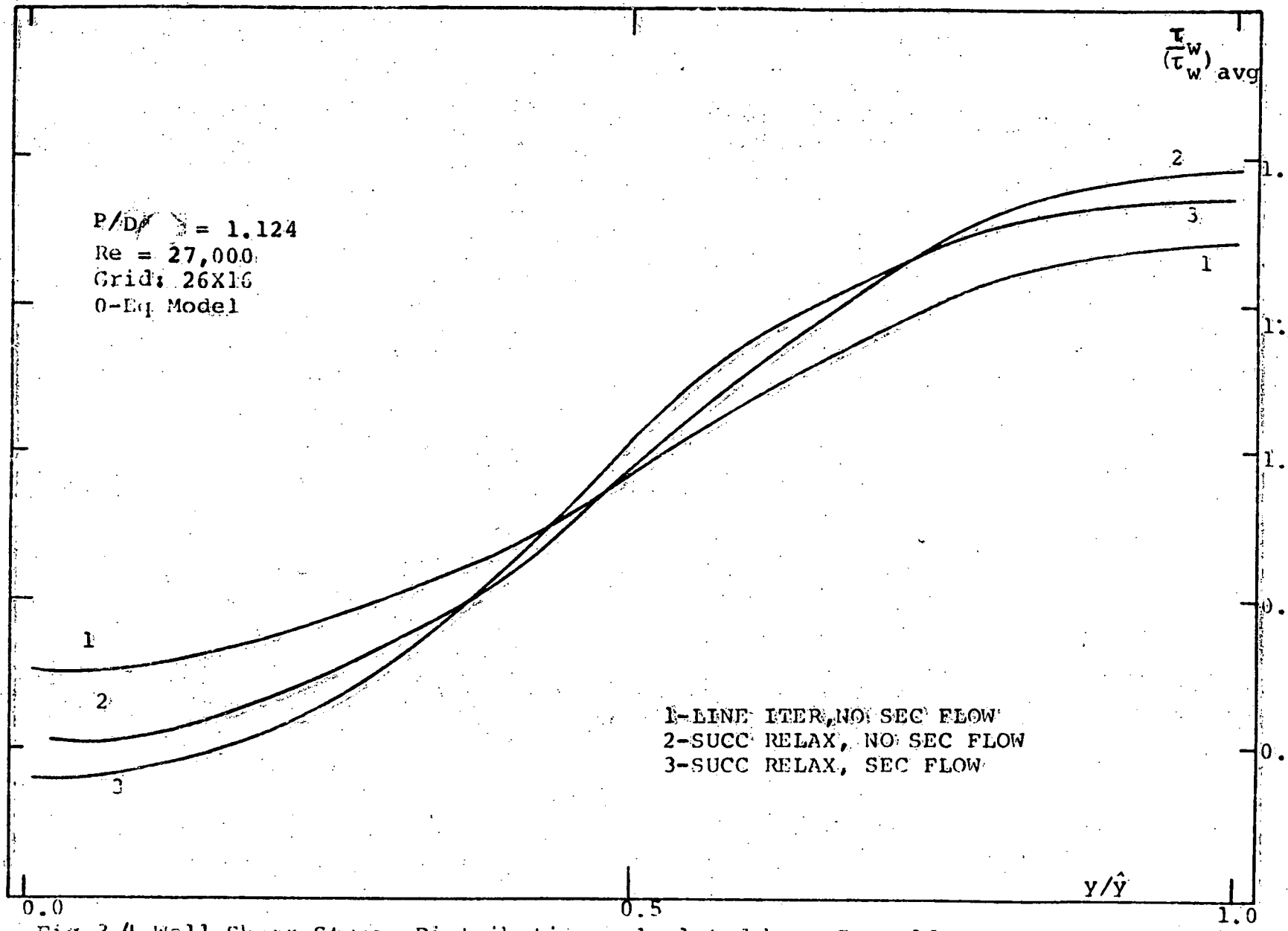


Fig 3.4. Wall Shear Stress Distribution calculated by a Reynolds Number Input

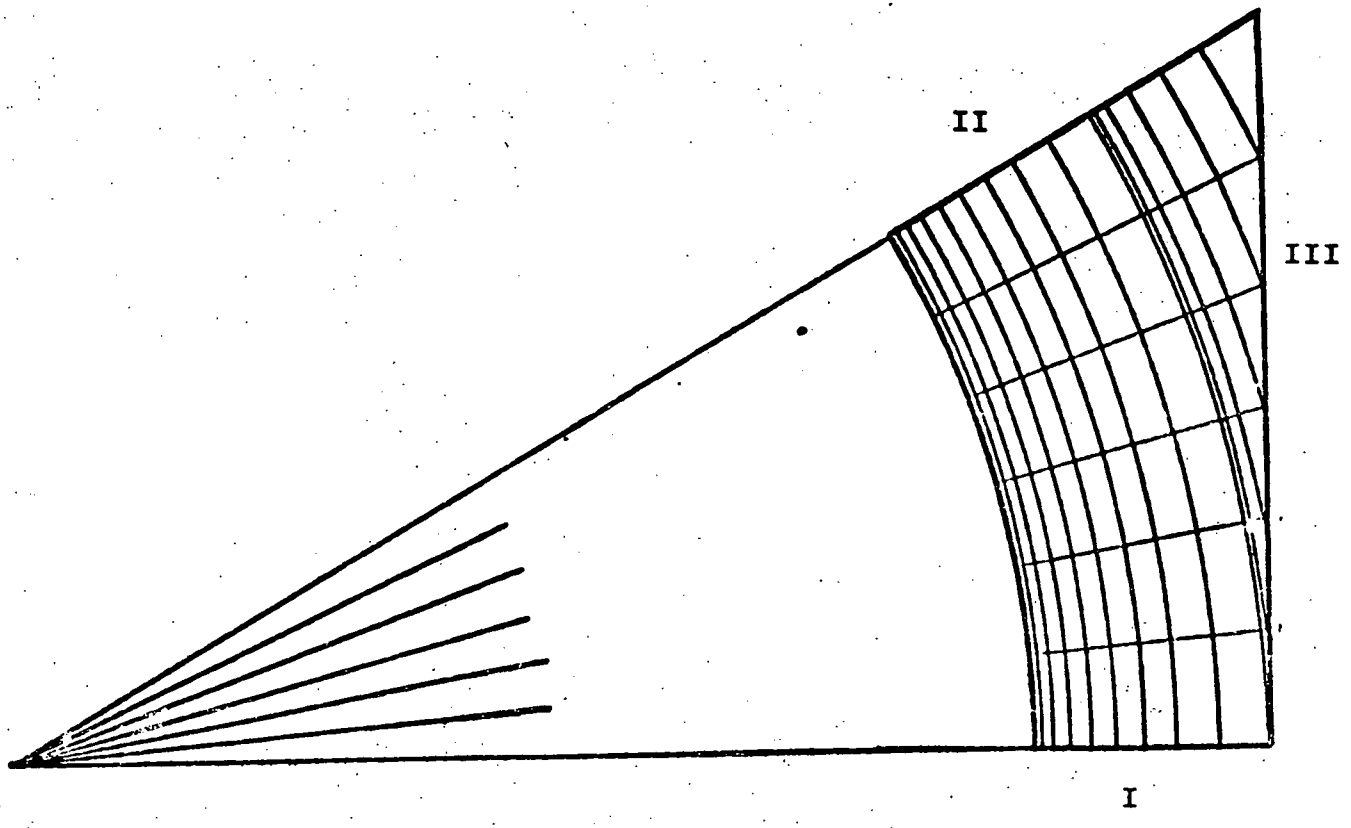


Fig 4.1. Numerical Grid

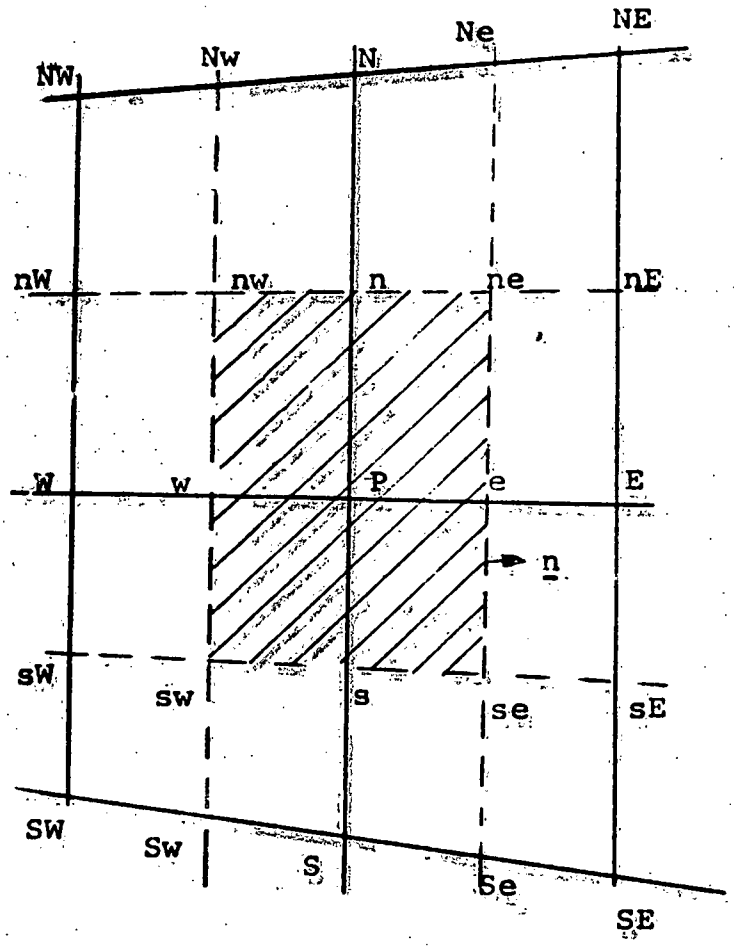


Fig. 4.2. Control Volume

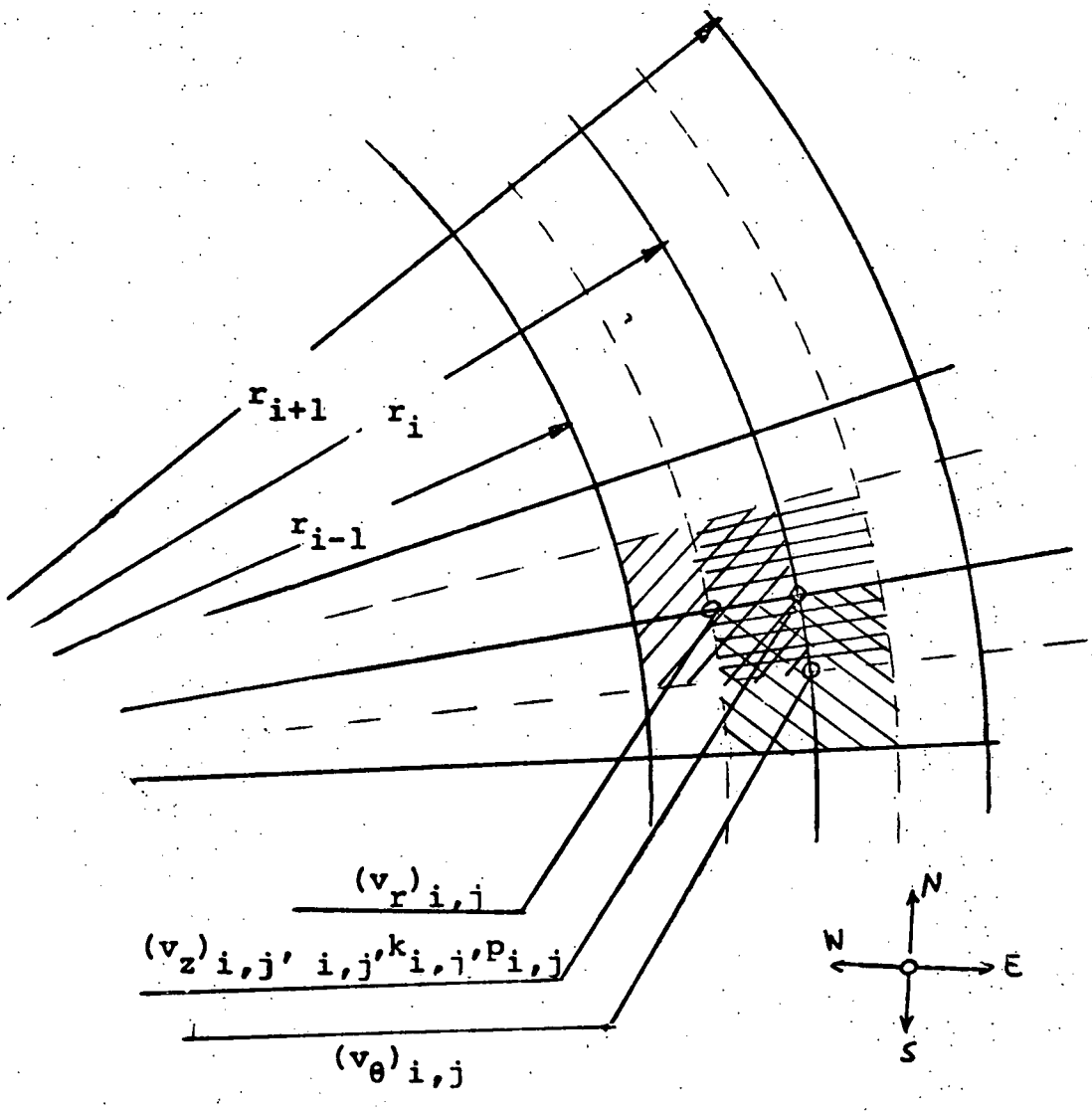


Fig. 4.3. Relative positions of the control volumes at the points P=(i,j)

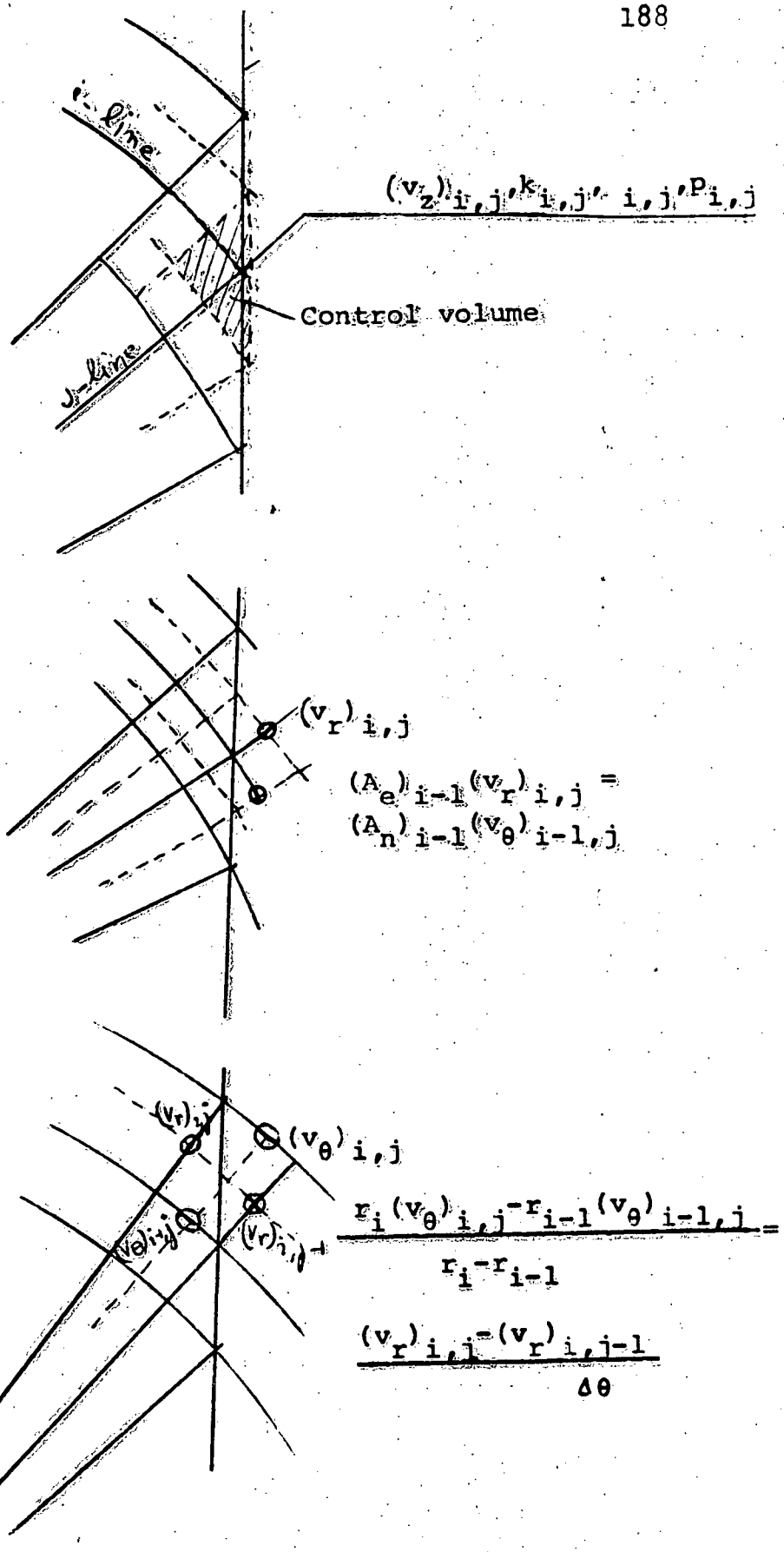


Fig.4.4. Boundary Conditions in Boundary III

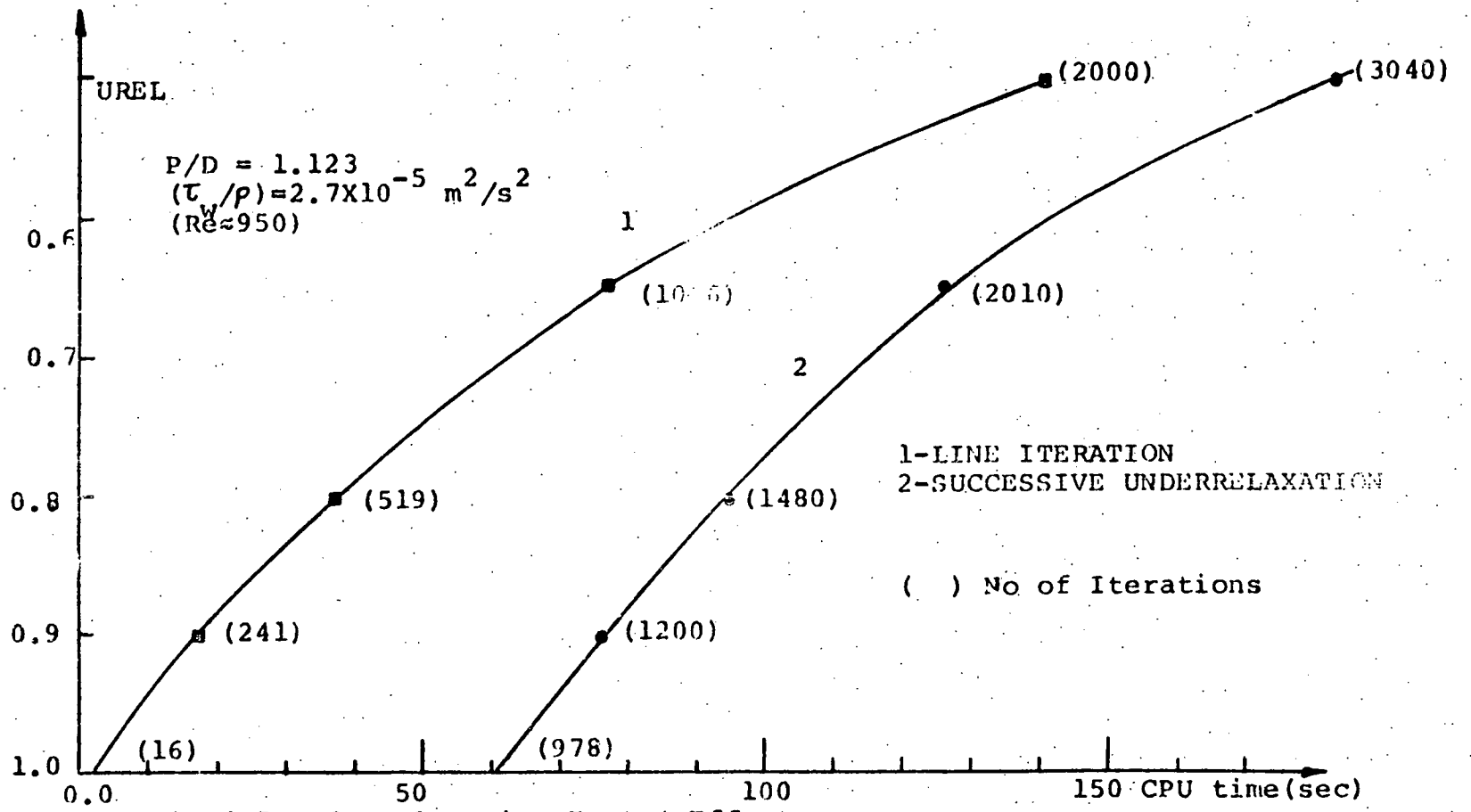


Fig 4.5 Underrelaxation Factor Effect

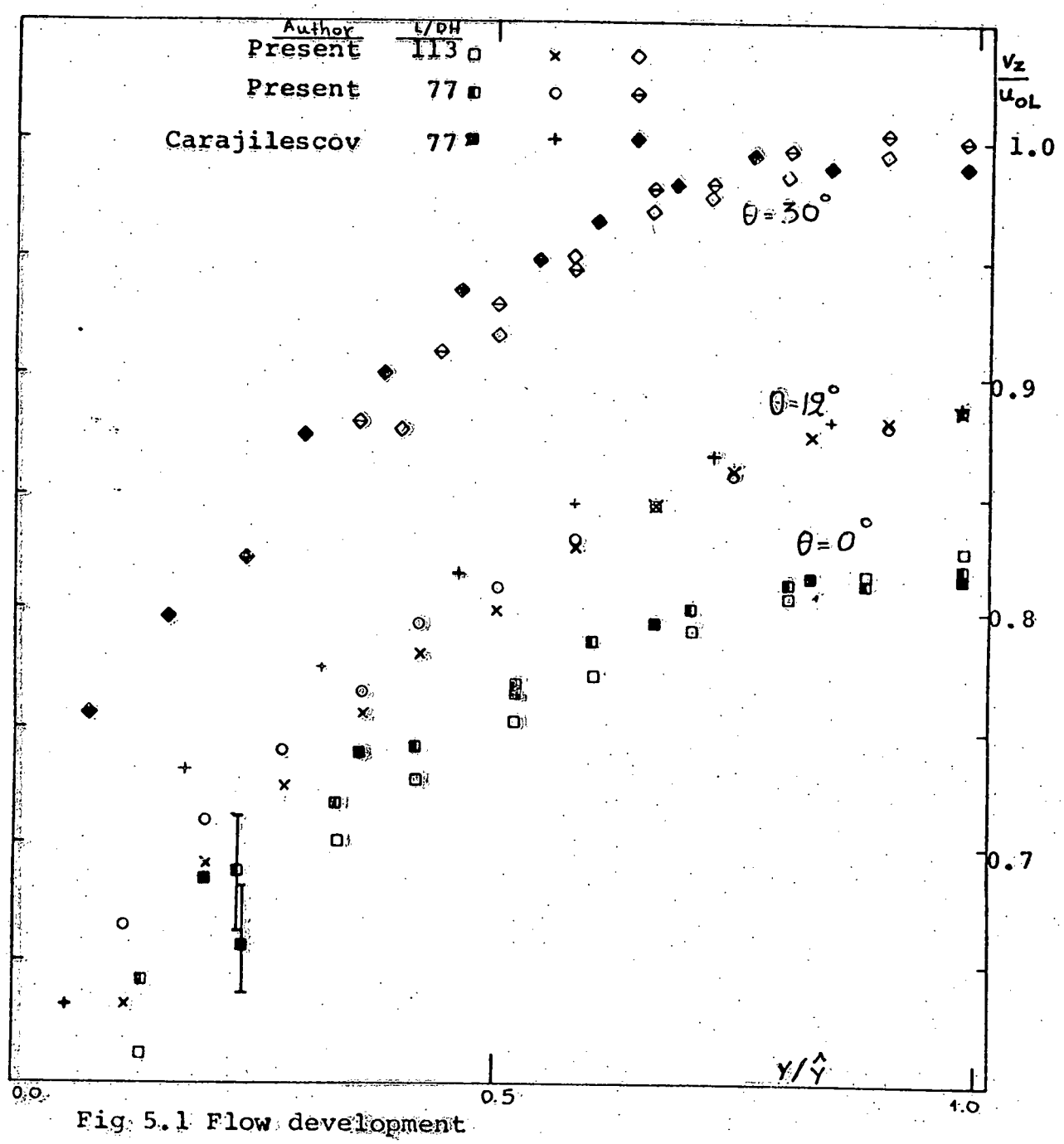


Fig. 5.1 Flow development

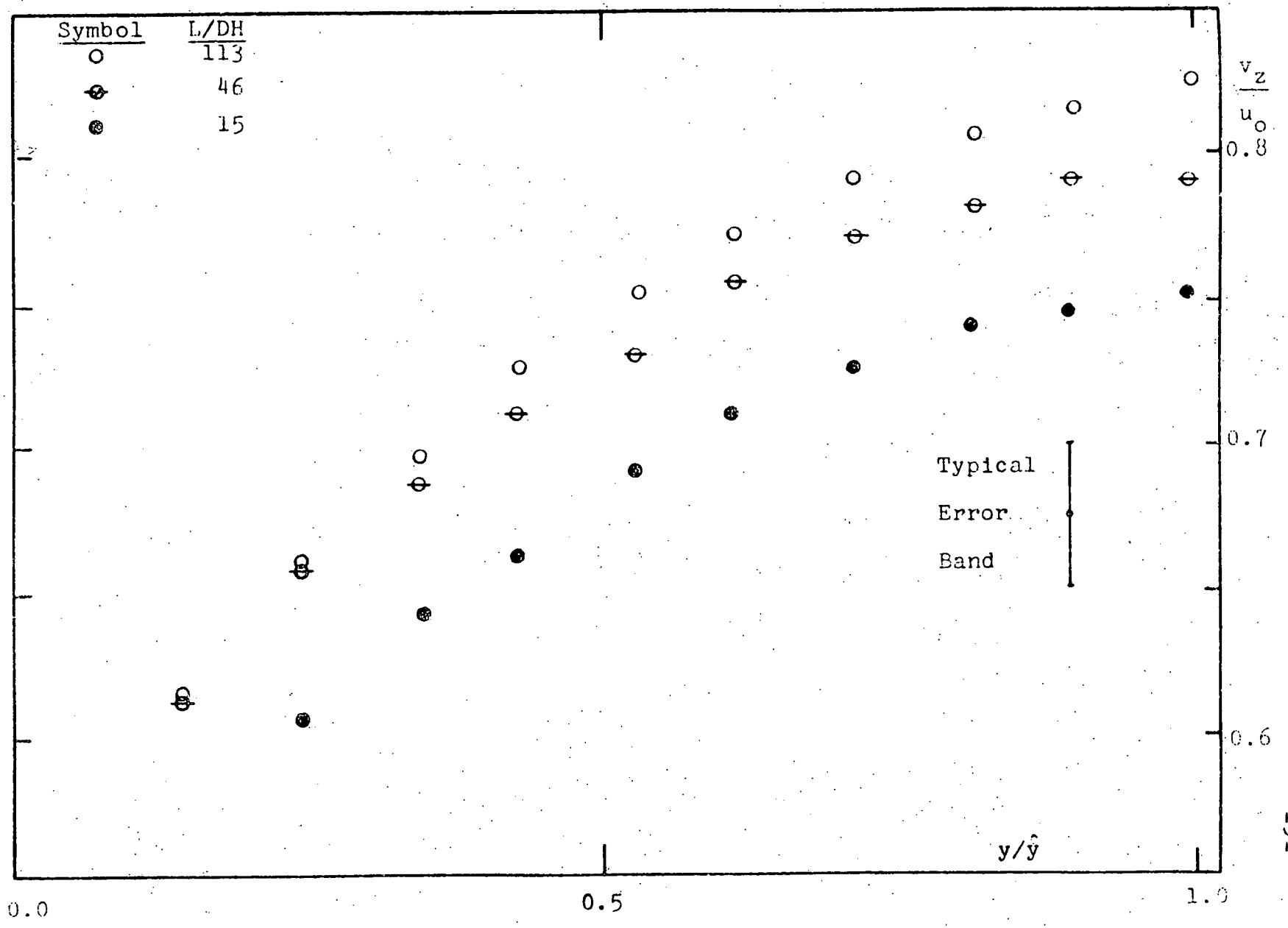


Fig. 5.2a. Axial velocity (Re=23,500, θ=0)

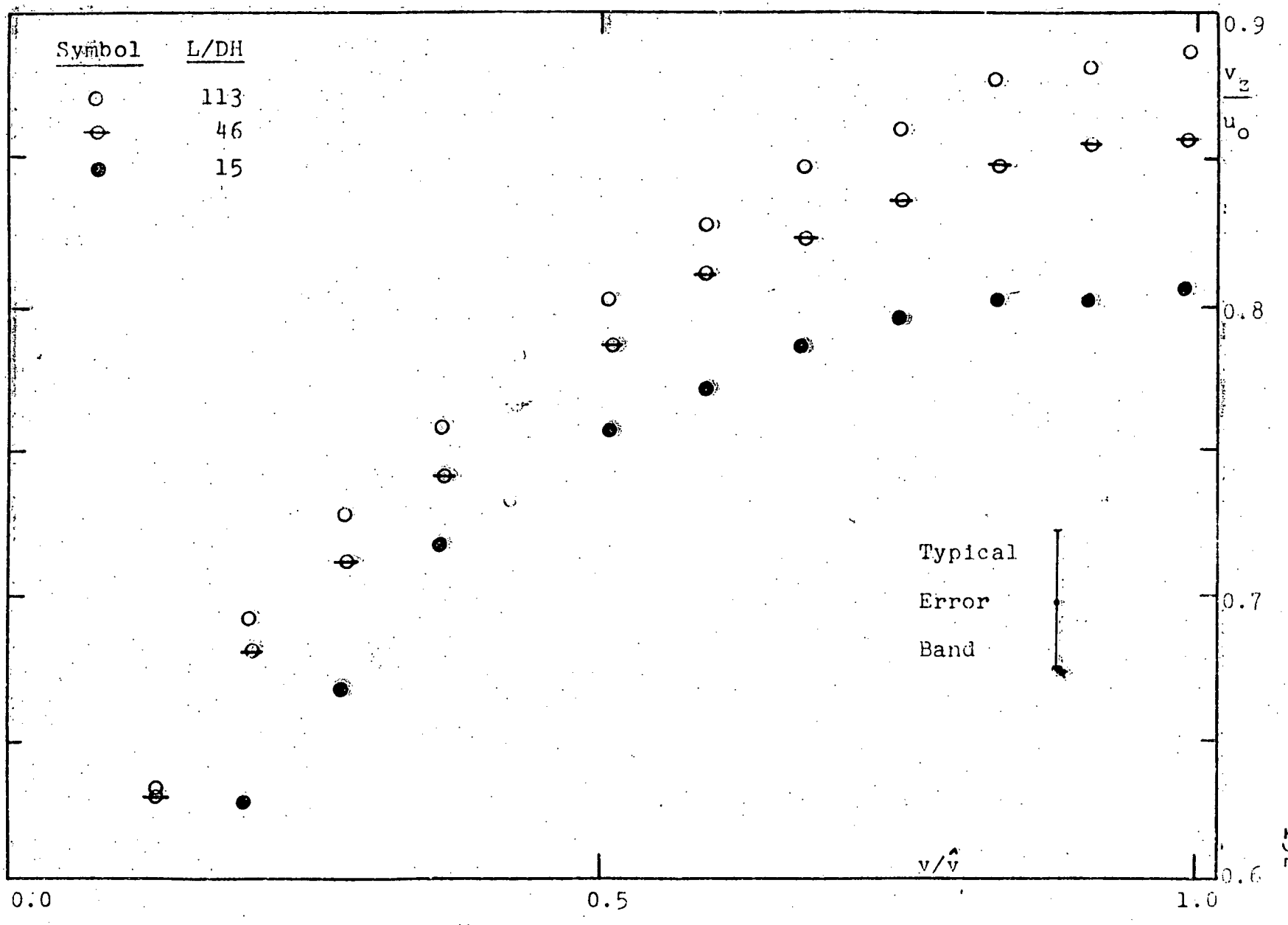


Fig. 5.2b. Axial velocity (Re=26,500, θ=12°)

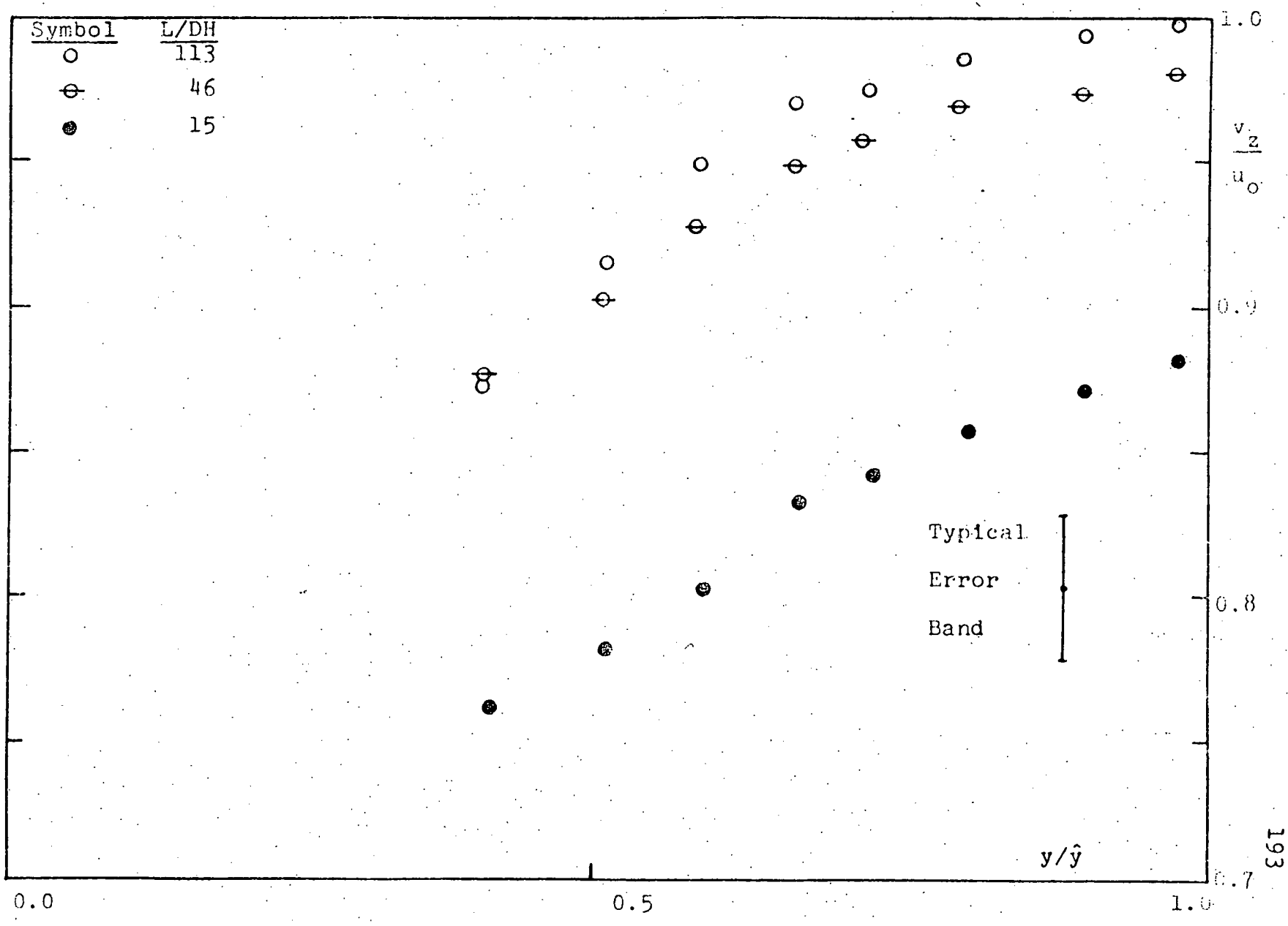


Fig. 5.2c. Axial velocity (Re=26,500, θ=30°)

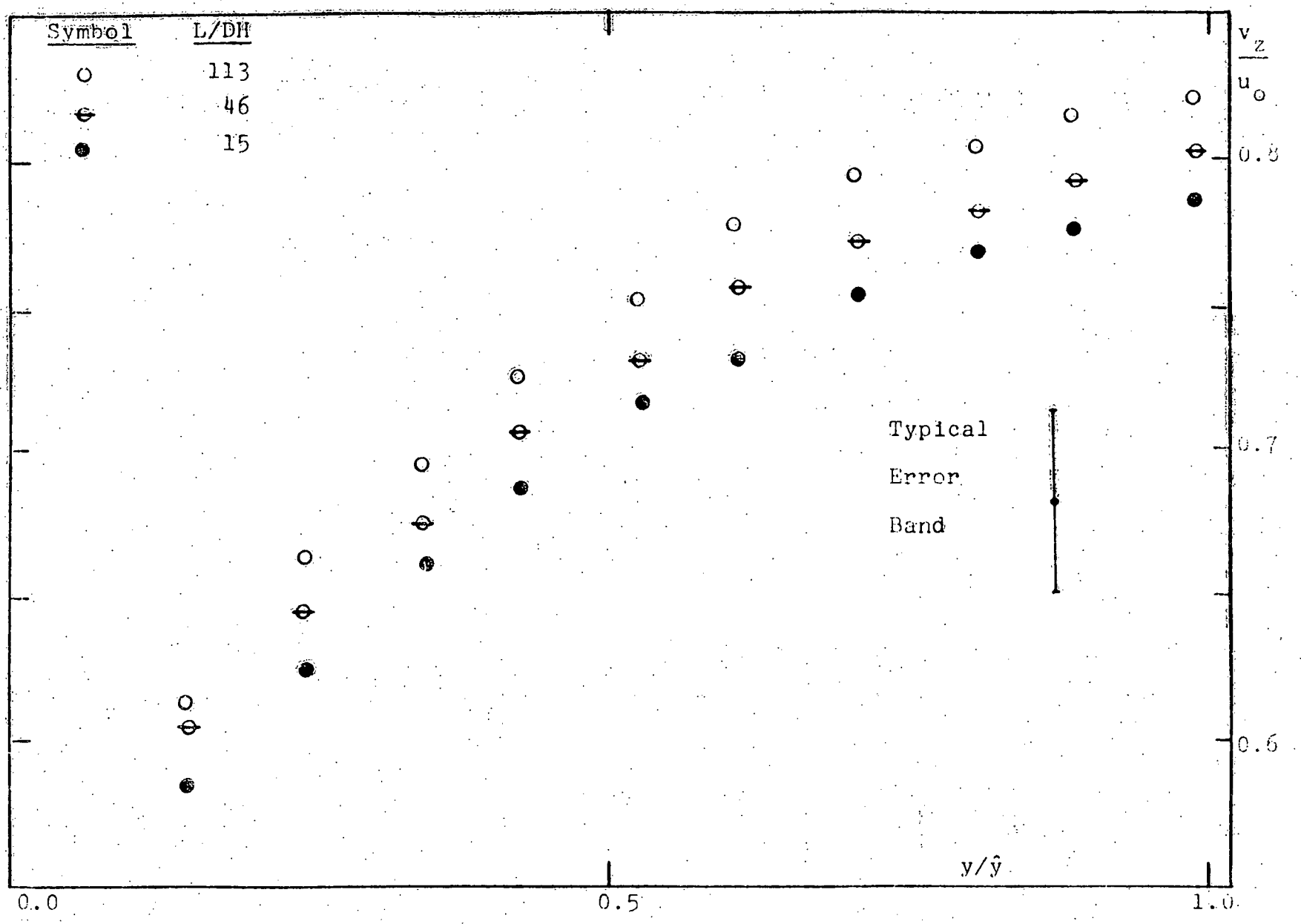


Fig. 5.3a. Axial Velocity (Re=65,000, θ=0°)

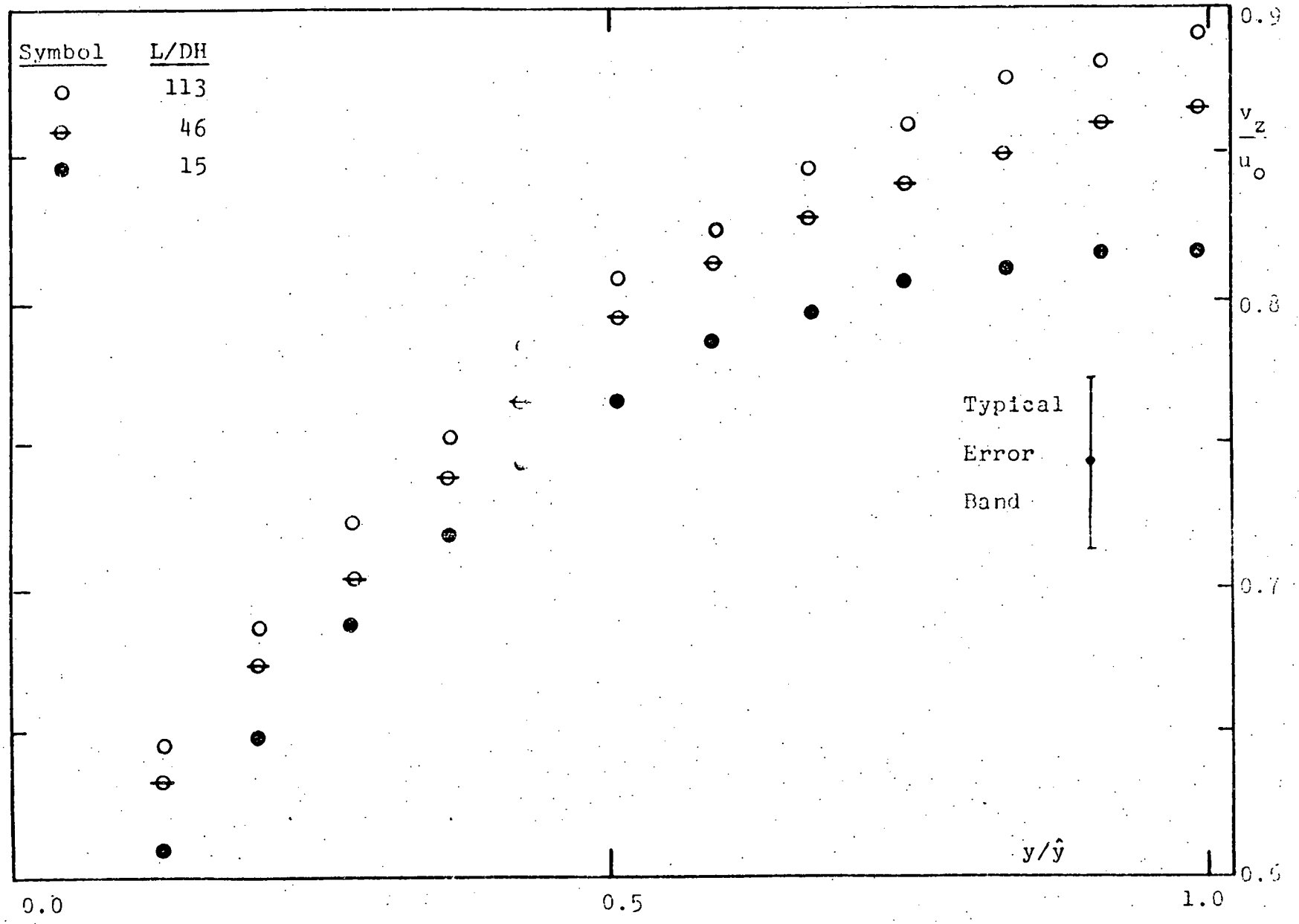


Fig. 5.3b. Axial Velocity (Re=65,000, θ=12°)

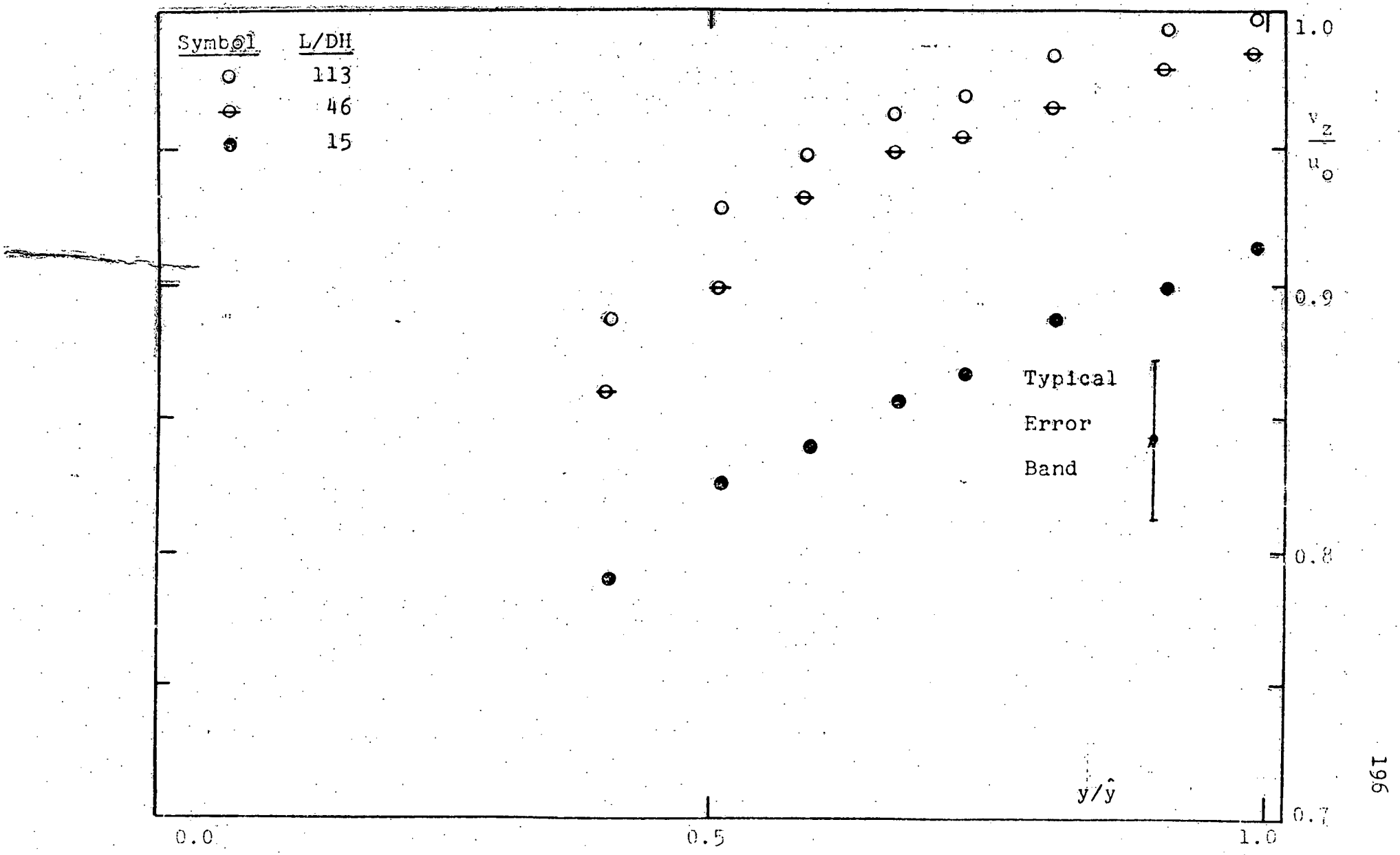


Fig. 5.3c. Normalized Axial Velocity (Re=65,000, θ=30°)

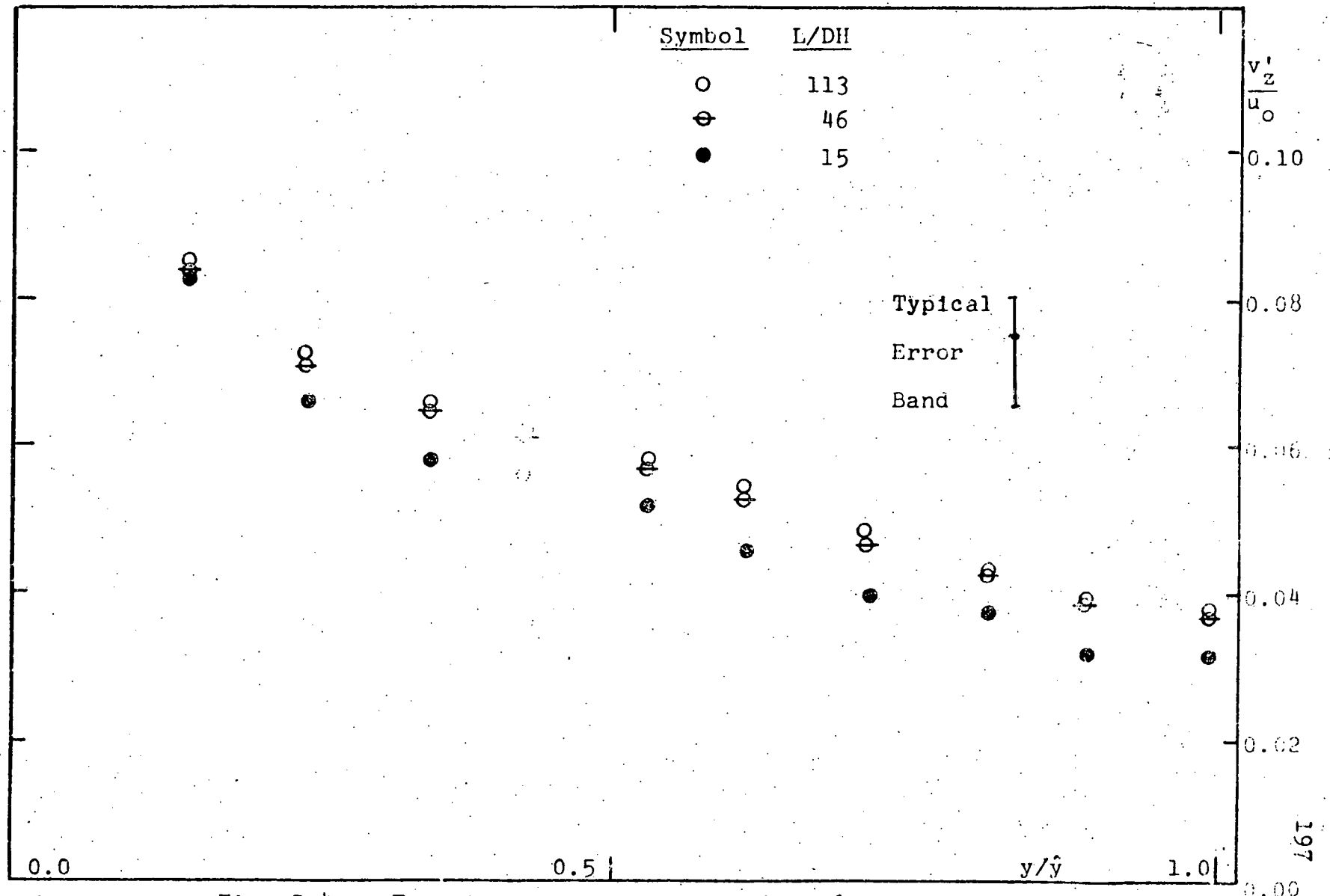


Fig. 5.4a. Turbulent Axial Velocity (Re=26,500, θ=0°)

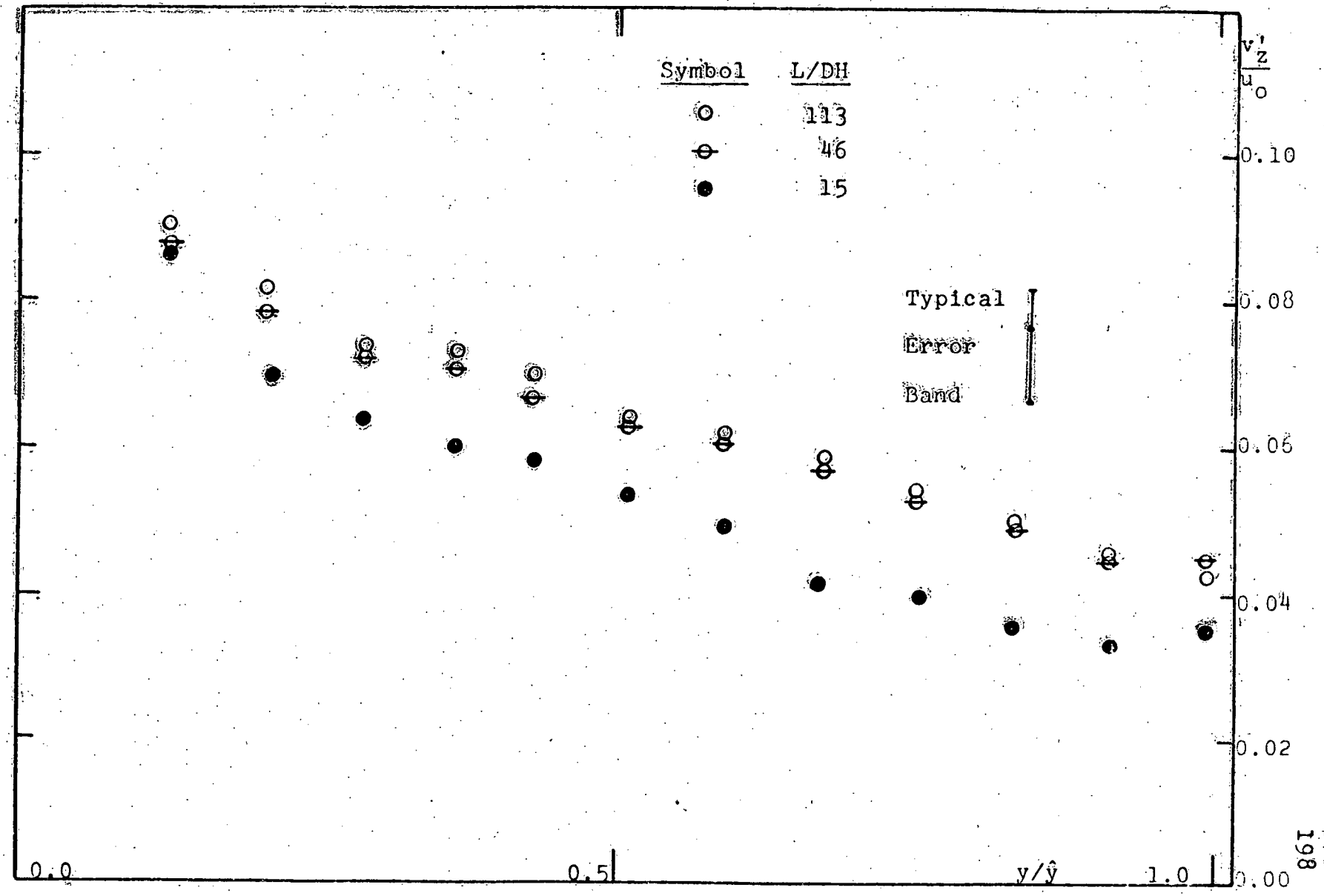


Fig. 5.4b. Turbulent Axial Velocity (Re=26,500, θ=12°)

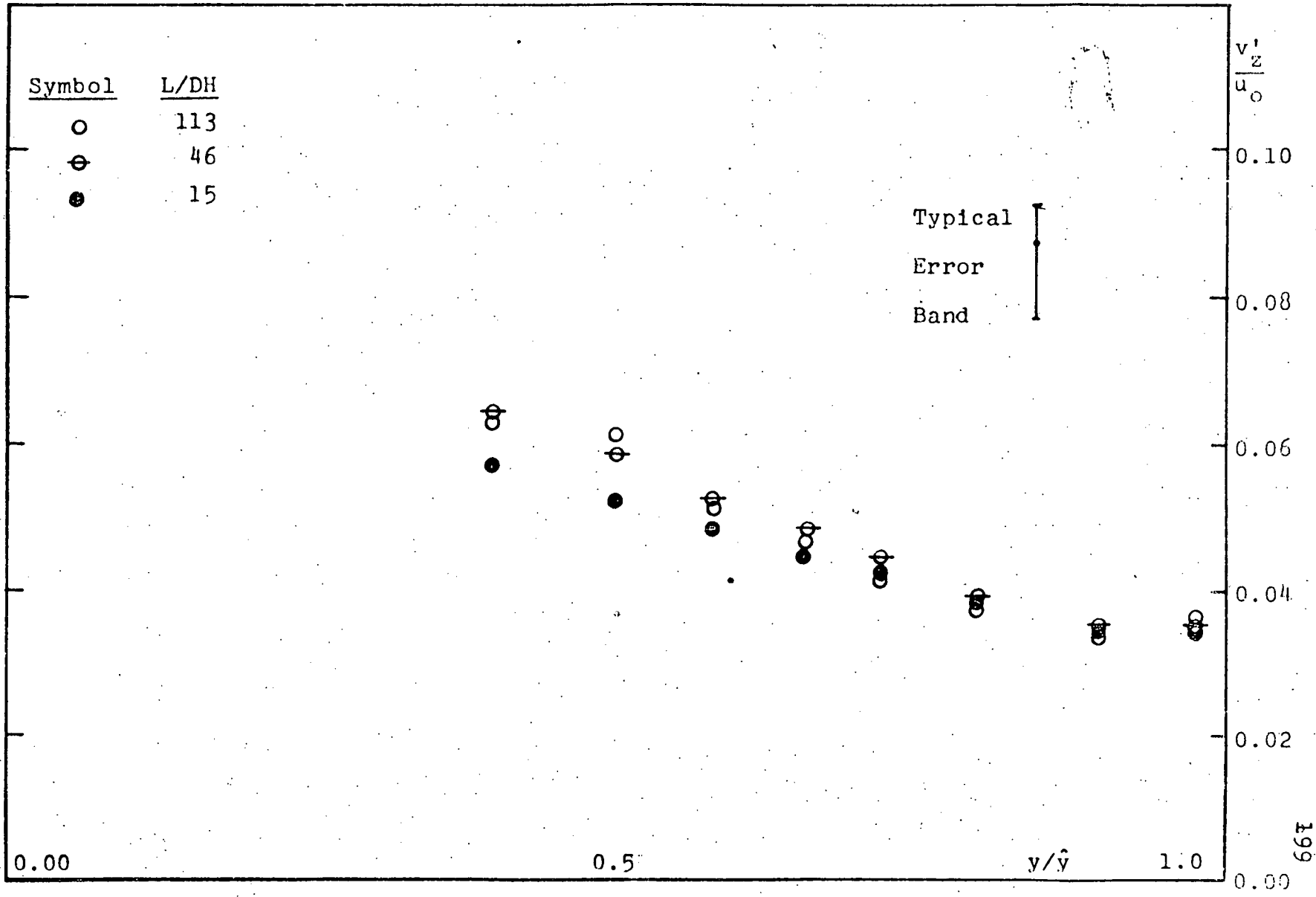


Fig. 5.4c. Turbulent Axial Velocity (Re=26,500, θ=30°)

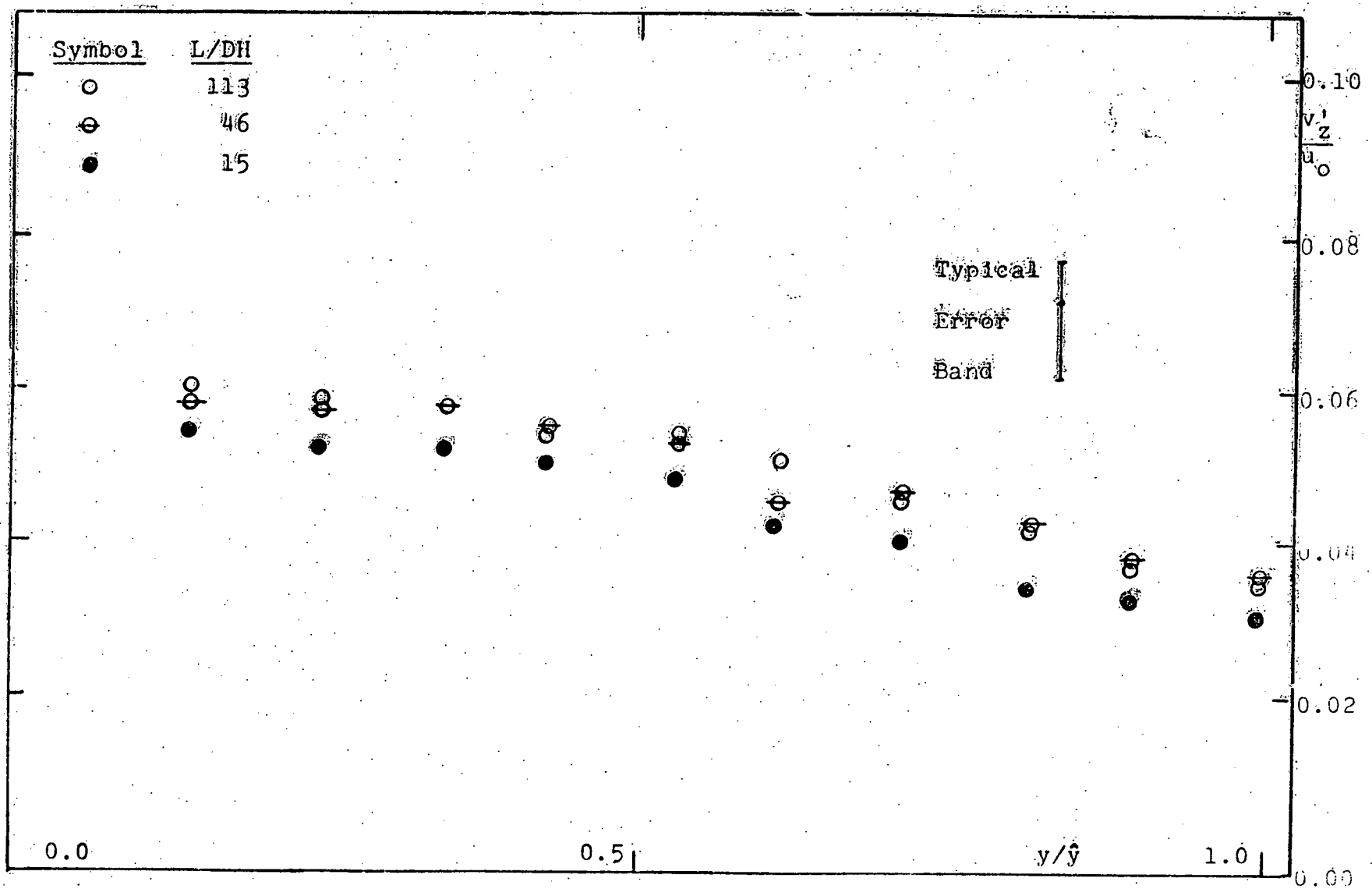


Fig. 5.5a. Turbulent Axial Velocity (Re=65,000, θ=0°)

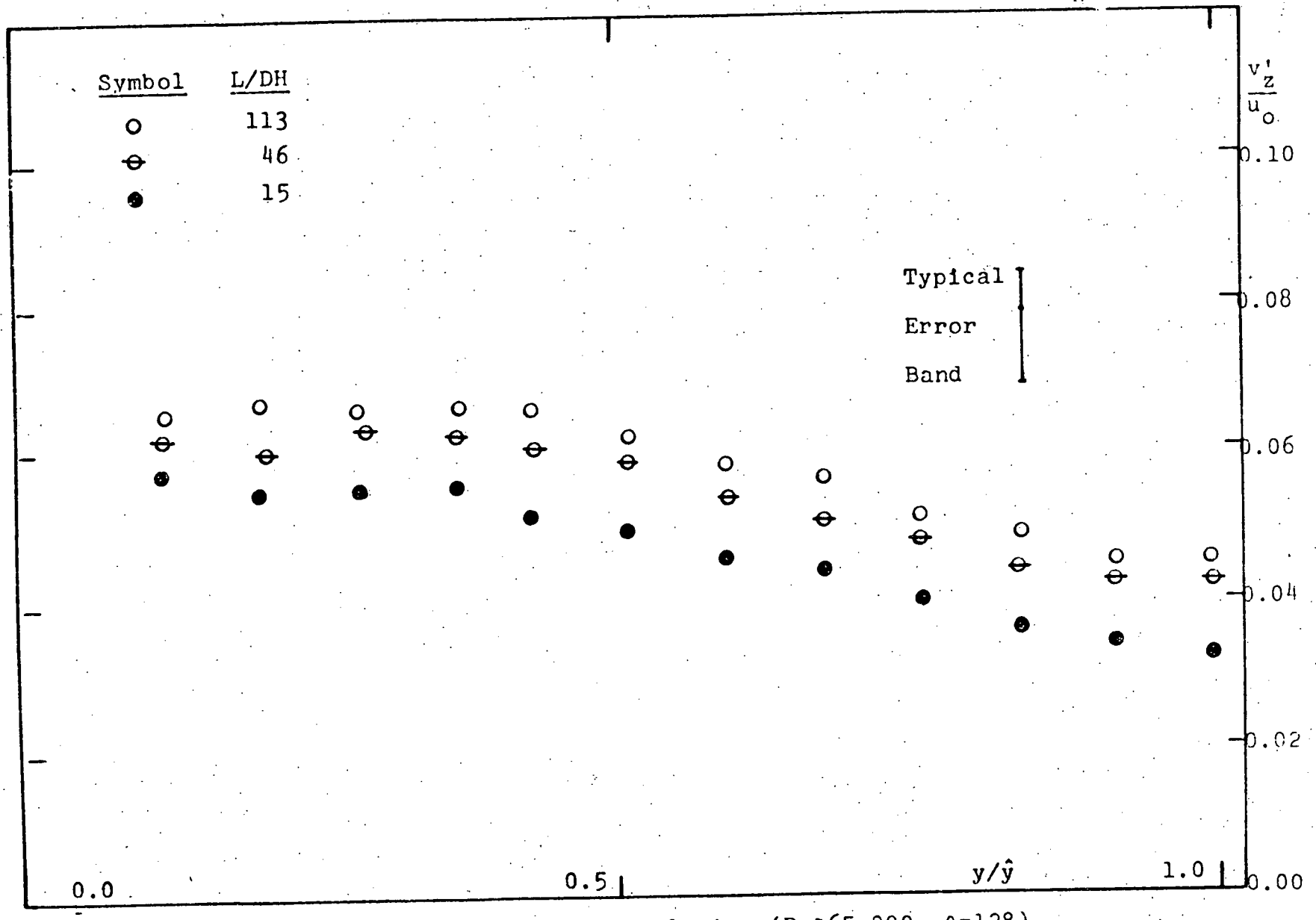


Fig. 5.5b. Turbulent Axial Velocity (Re≈65,000, θ=12°)

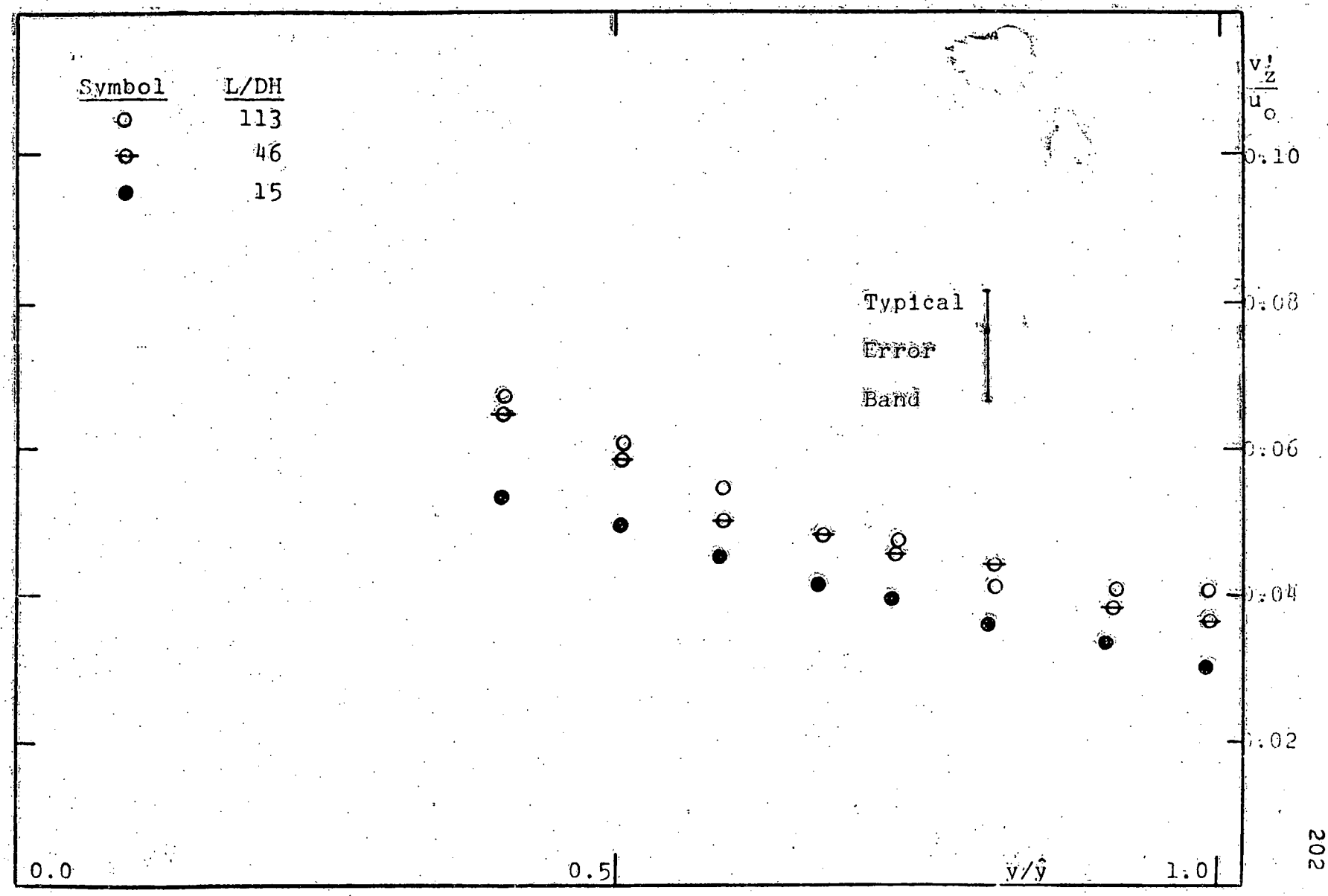


Fig. 5.5c. Turbulent Axial Velocity (Re=65,000, θ=30°)

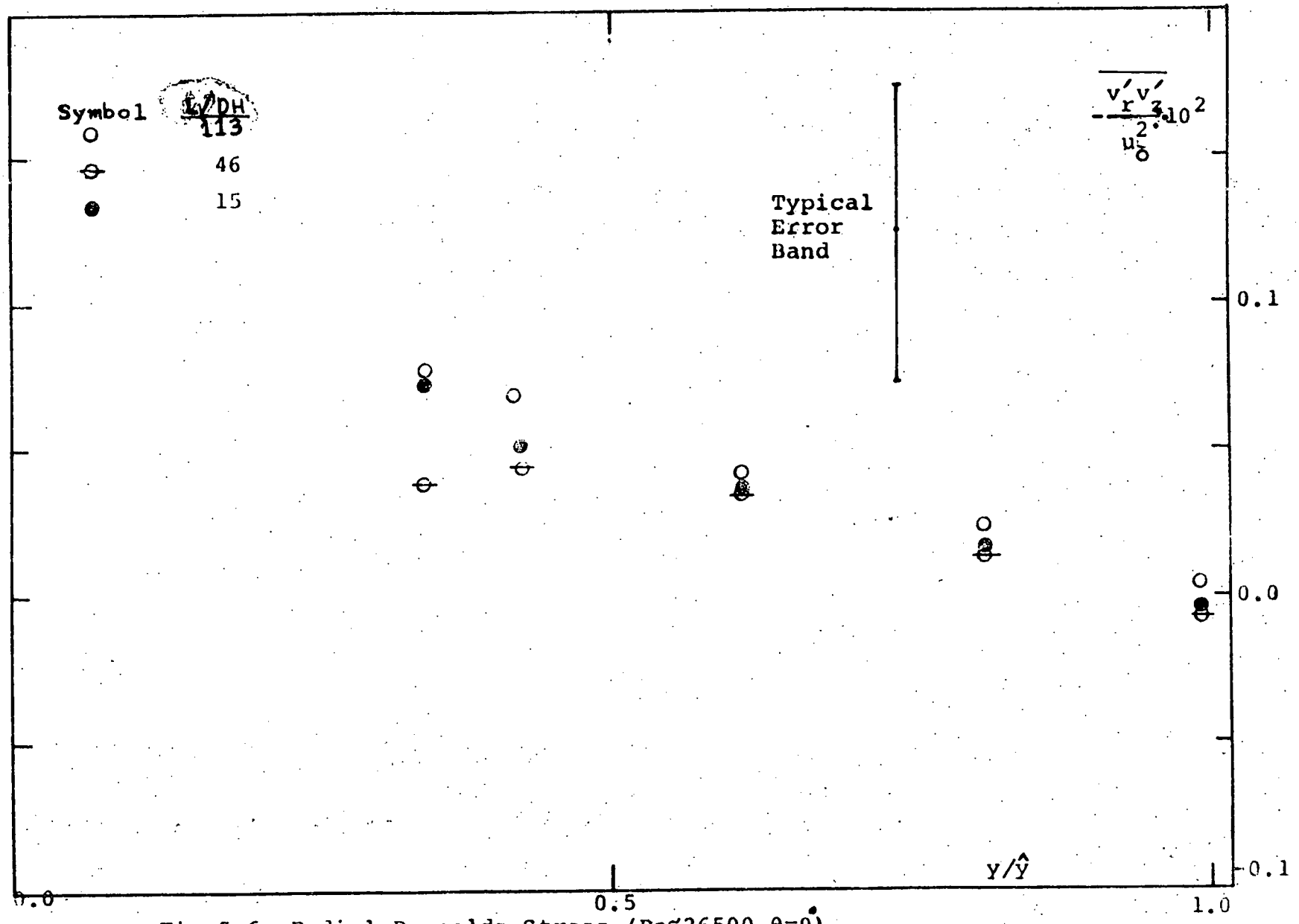


Fig 5.6a Radial Reynolds Stress (Re≈26500, θ=0)

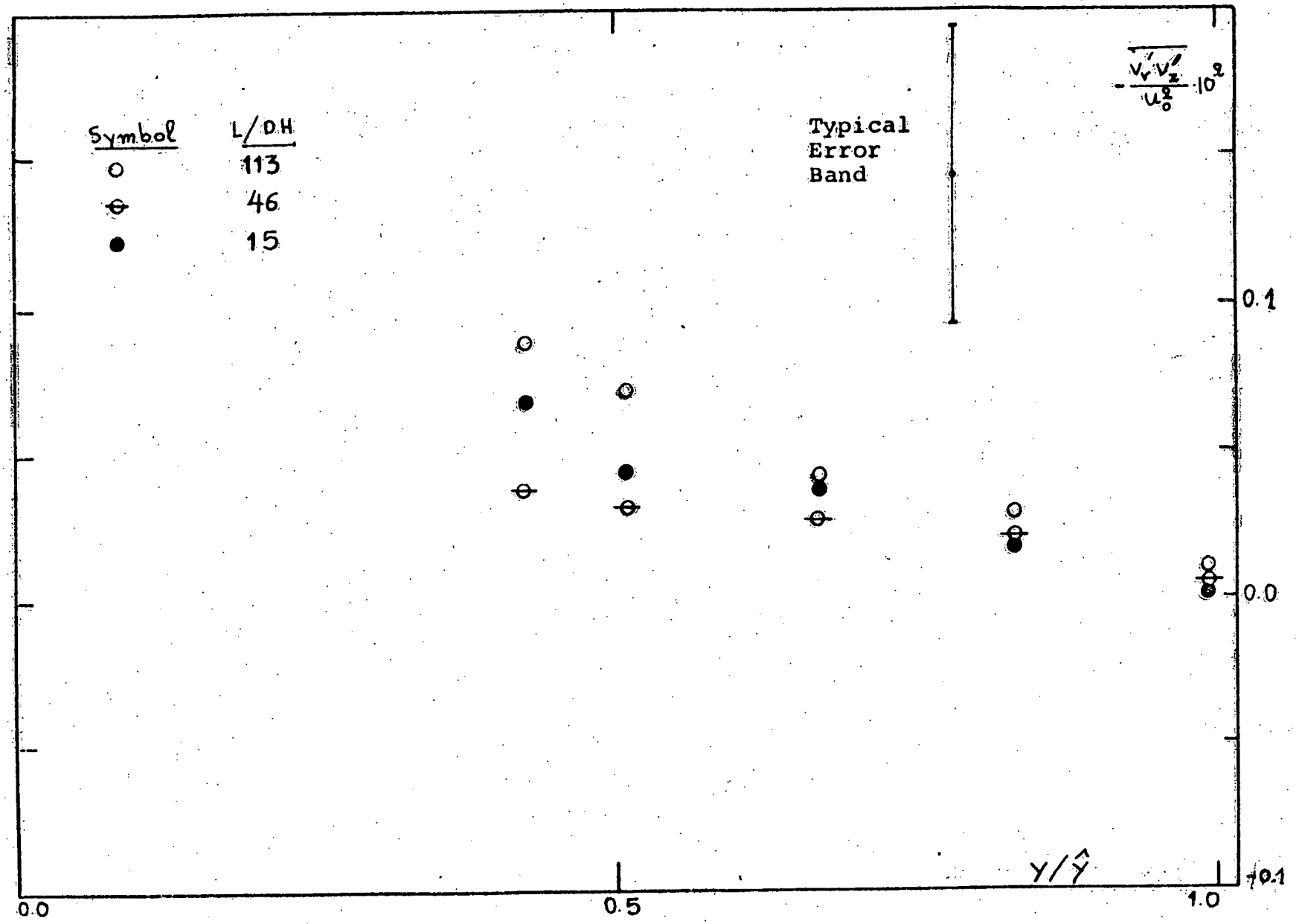


Fig 5.6b. Radial Reynolds Stress (Re≈26500,θ=12)

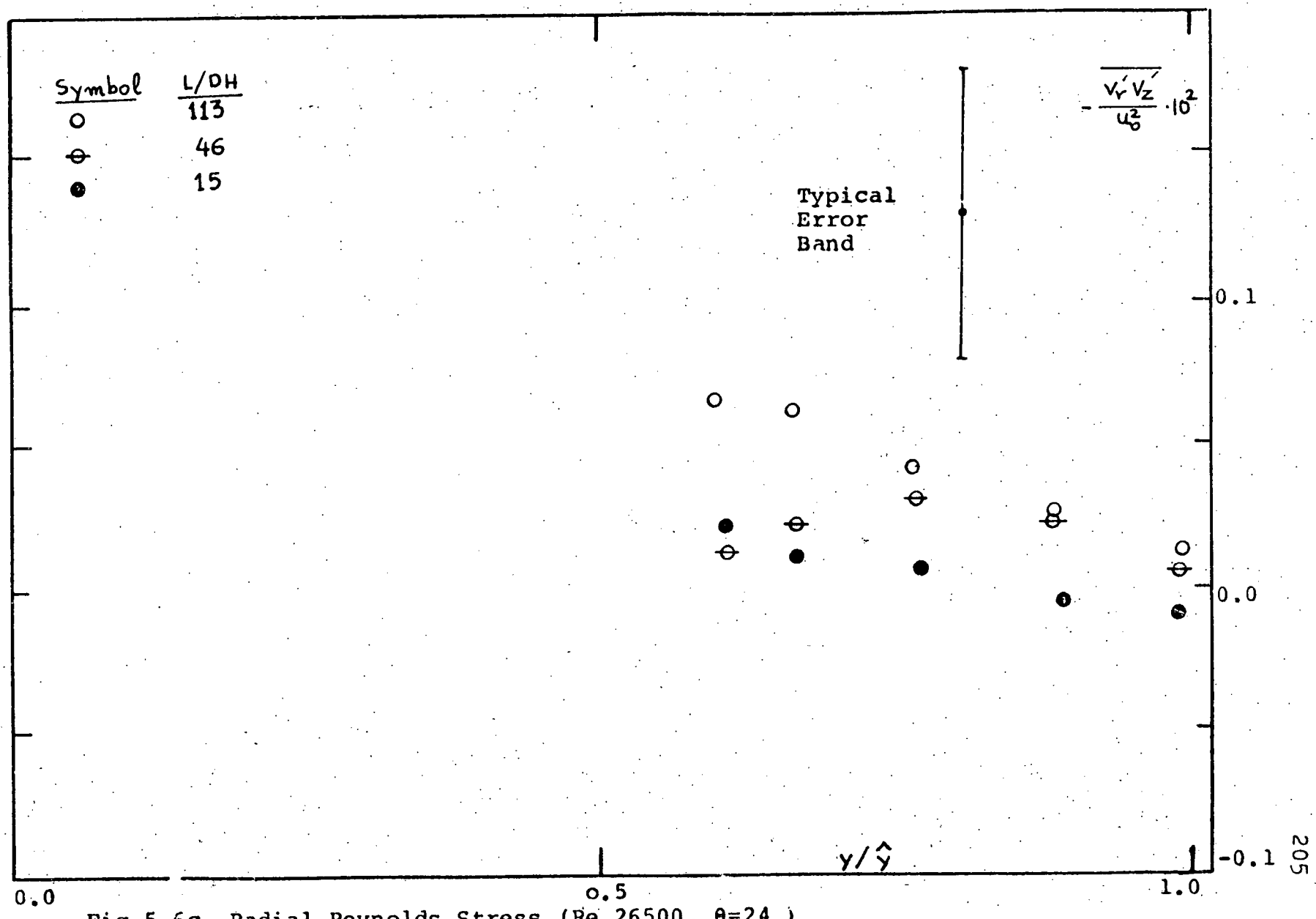


Fig 5.6c. Radial Reynolds Stress (Re 26500, θ=24)

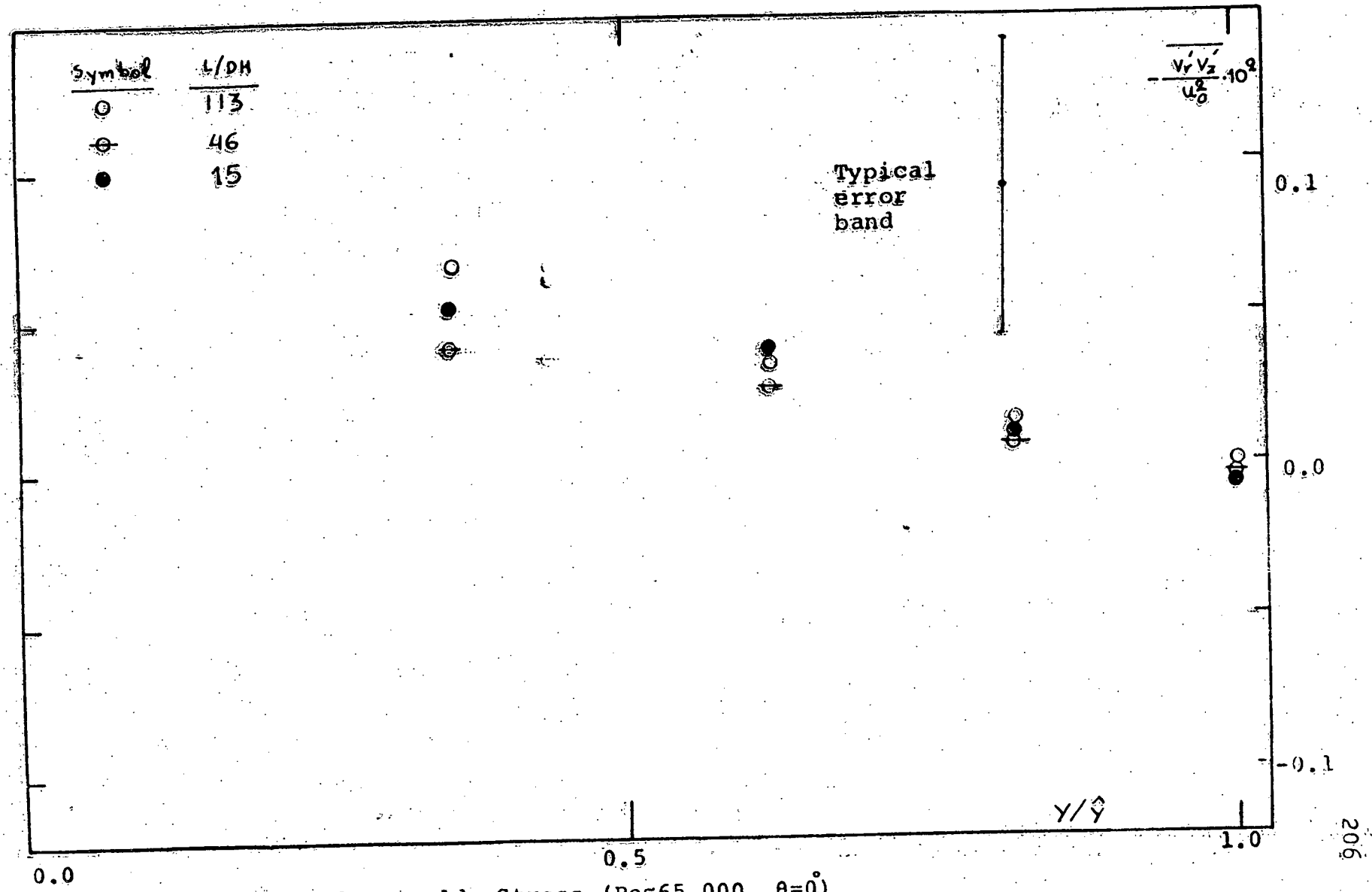


Fig 5.7a. Radial Reynolds Stress (Re≈65,000 ,θ=0°)

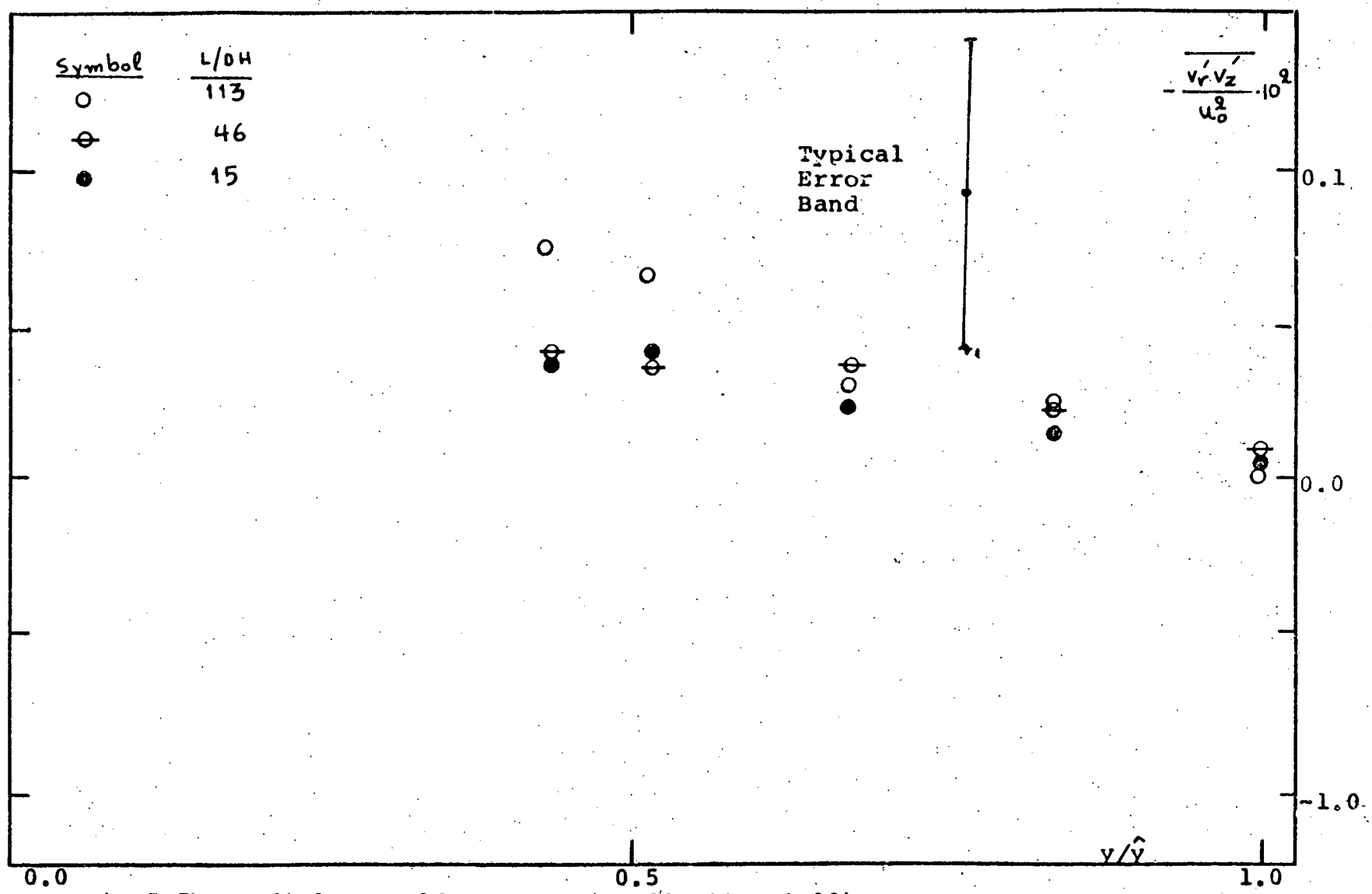


Fig 5.7b. Radial Reynolds Stress (Re≈65,000 , θ=12)

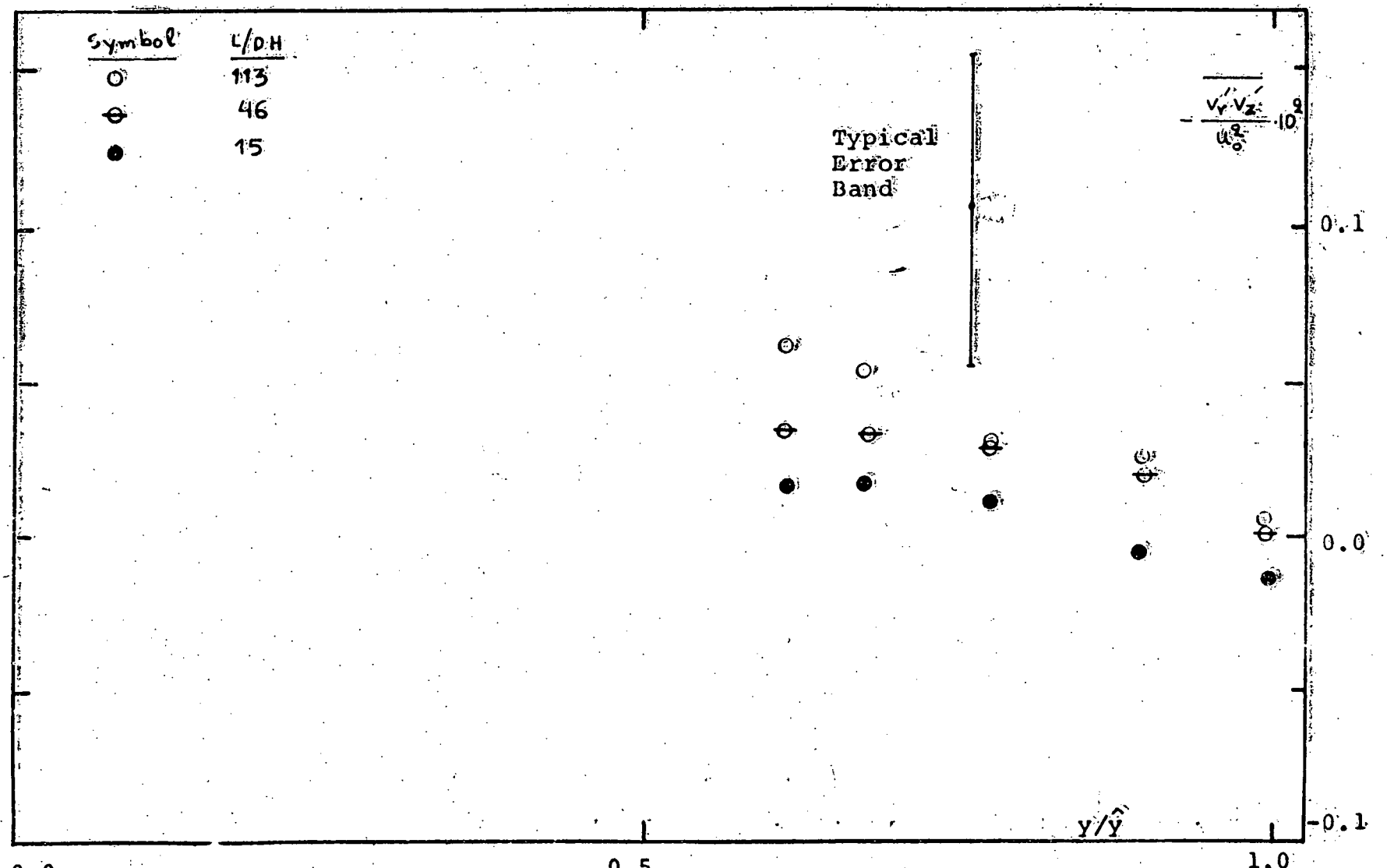


Fig 5.7c. Radial Reynolds Stress(Re≈65,000 ,θ=24)

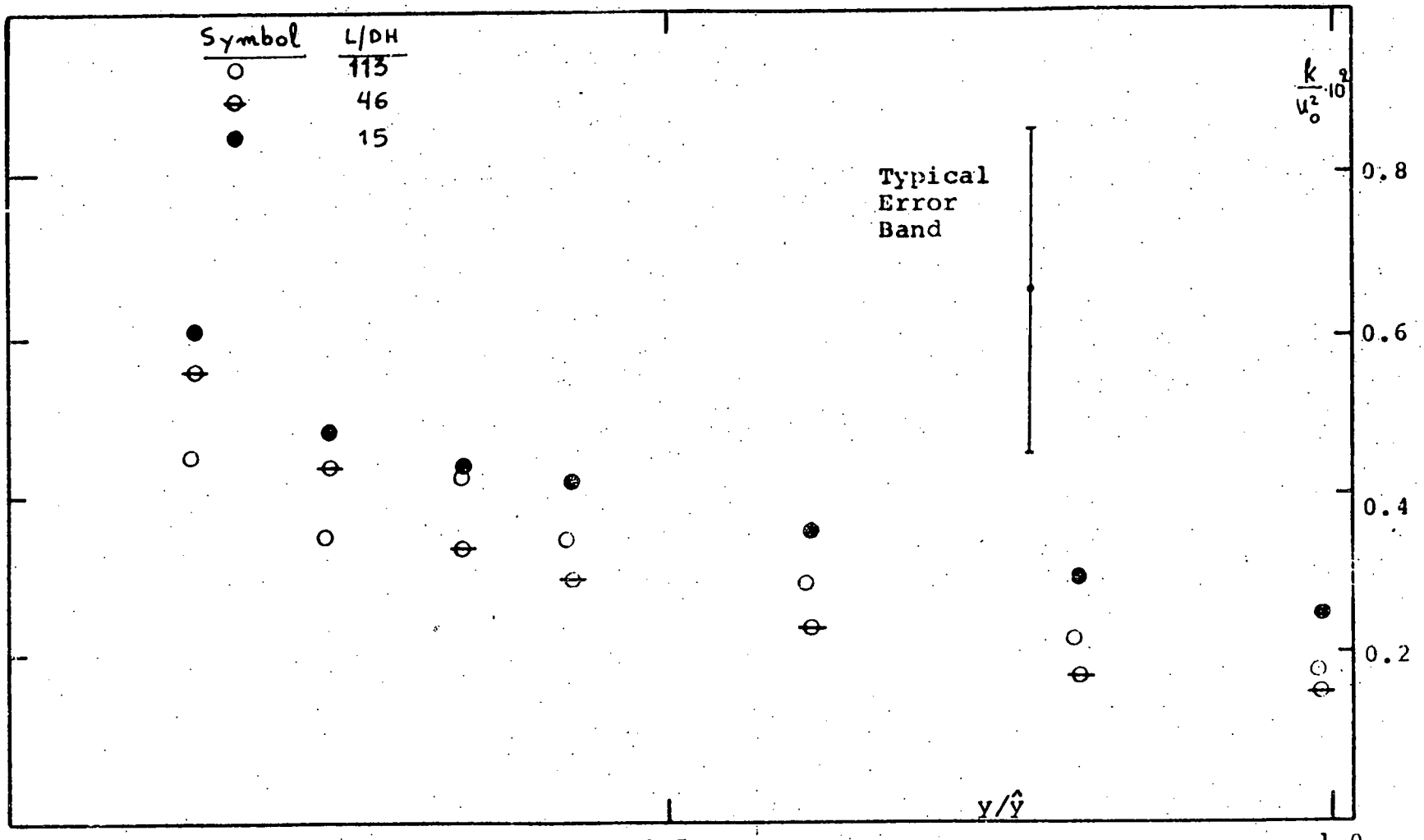


Fig 5.8a. Turbulent Kinetic Energy (Re≈26,500,θ=0°)

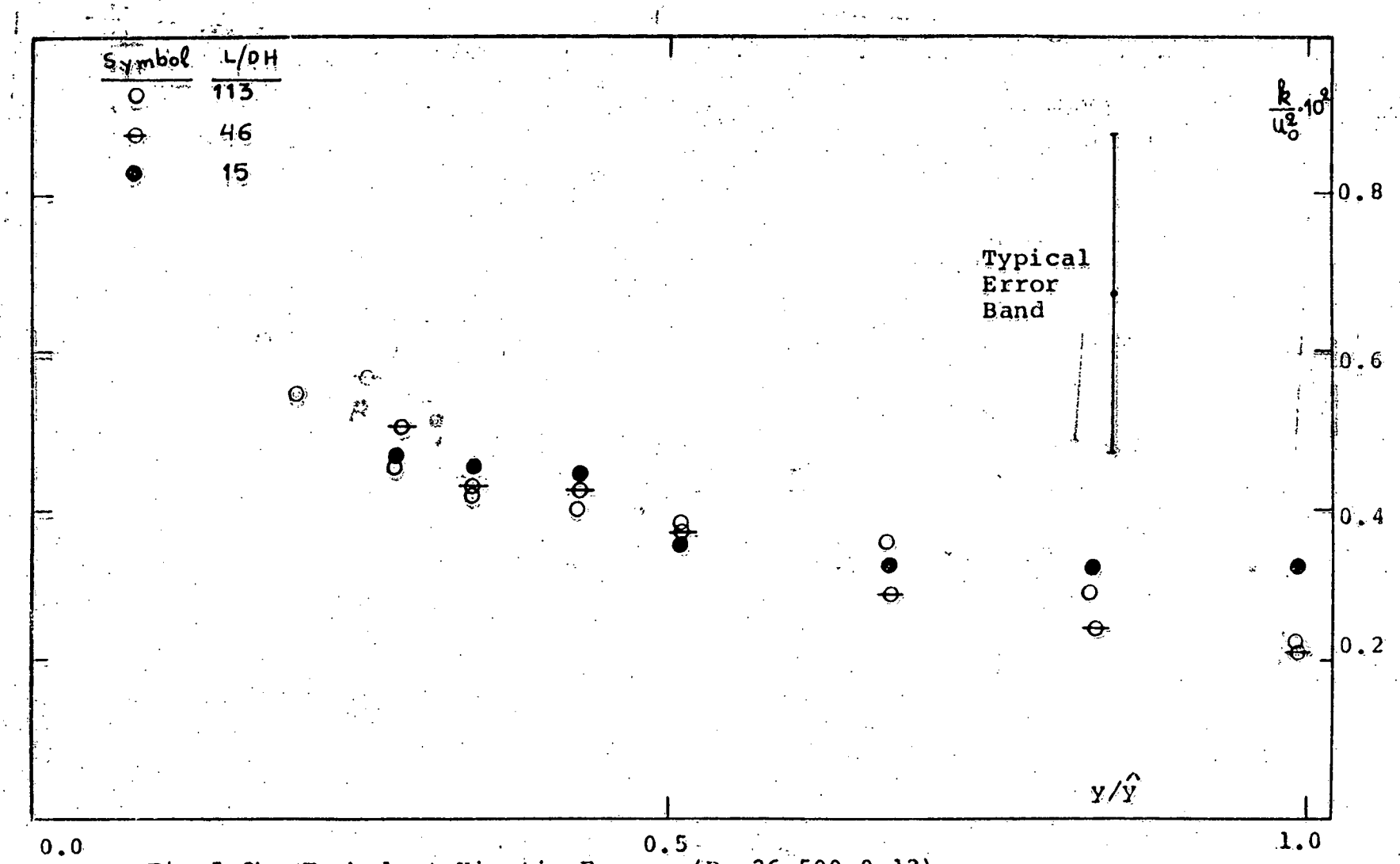


Fig 5.8b. Turbulent Kinetic Energy (Re=26,500 θ=12)

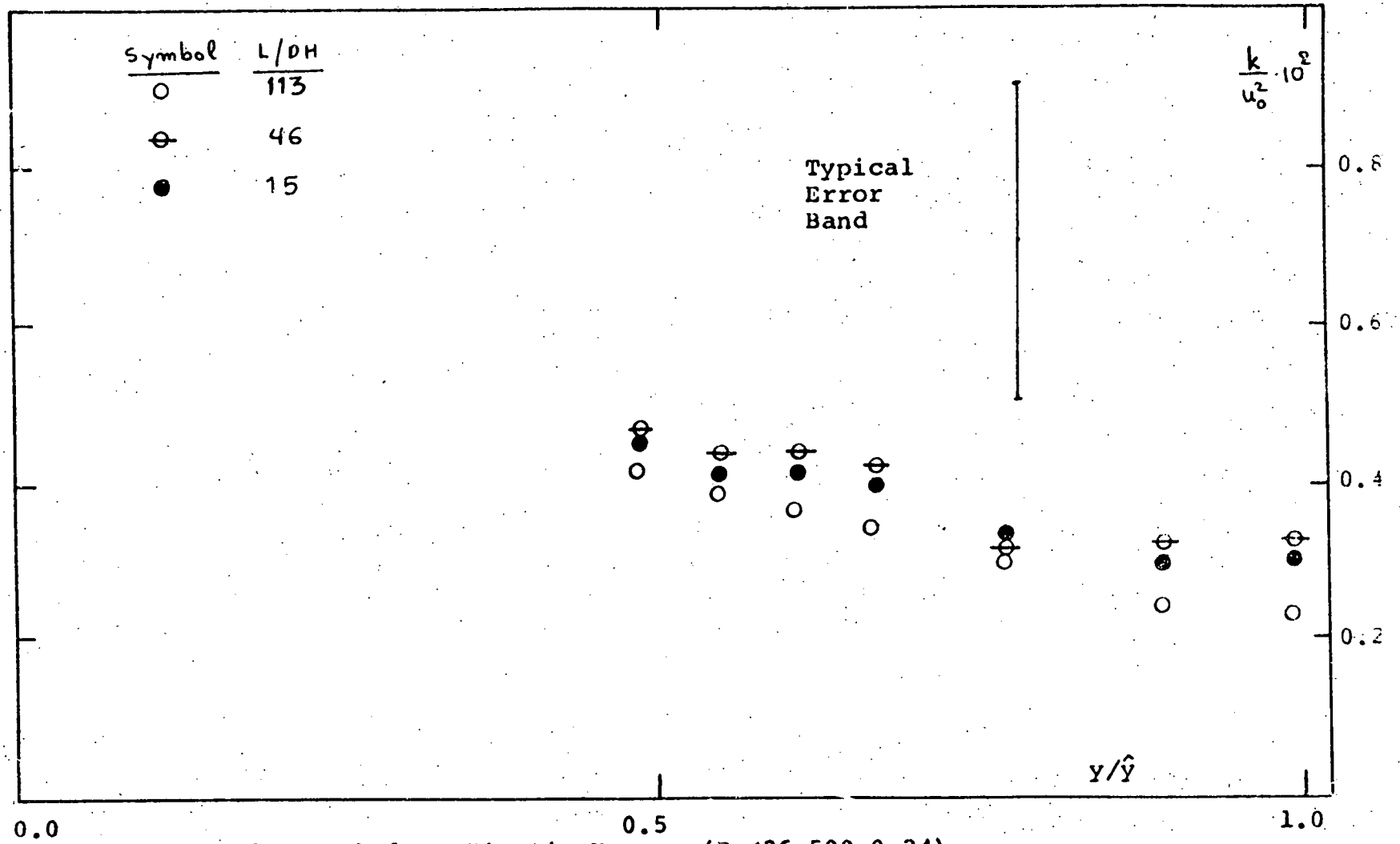


Fig 5.8c. Turbulent Kinetic Energy (Re≈26,500,θ=24)

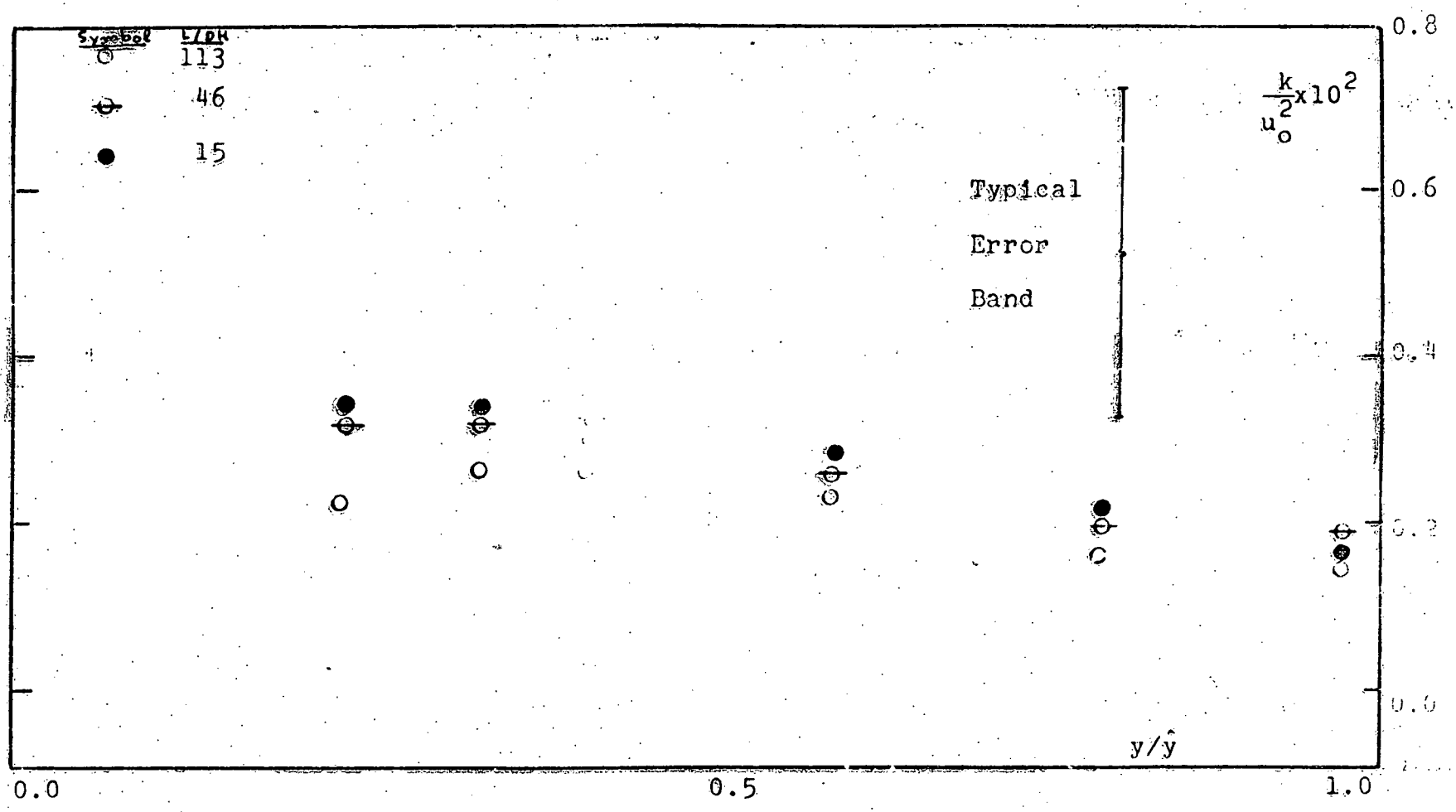


Fig. 5.9a. Turbulent Kinetic Energy (Re=65,000, θ=0°)

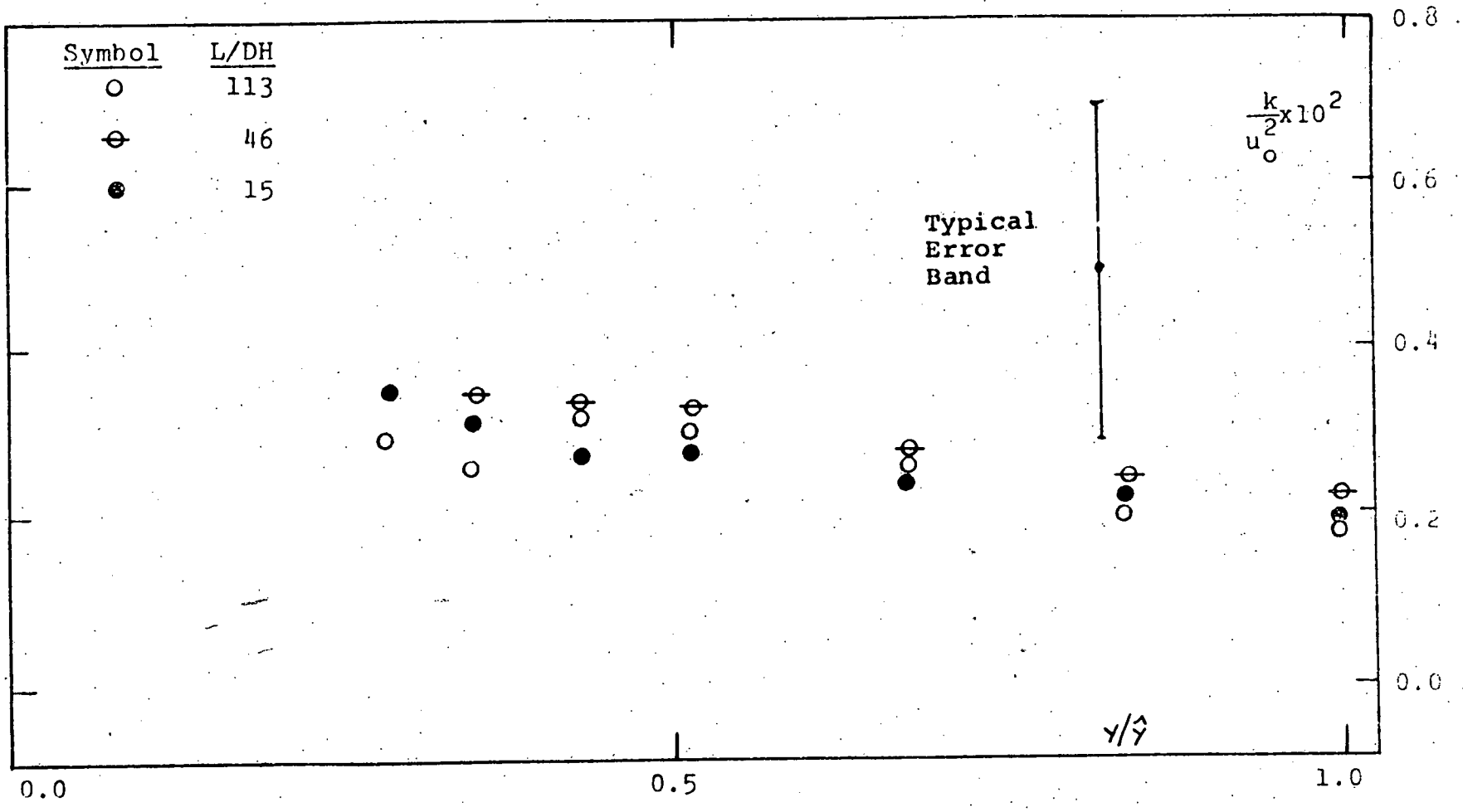


Fig. 5.9b. Turbulent Kinetic Energy (Re=65,000, θ=12°)

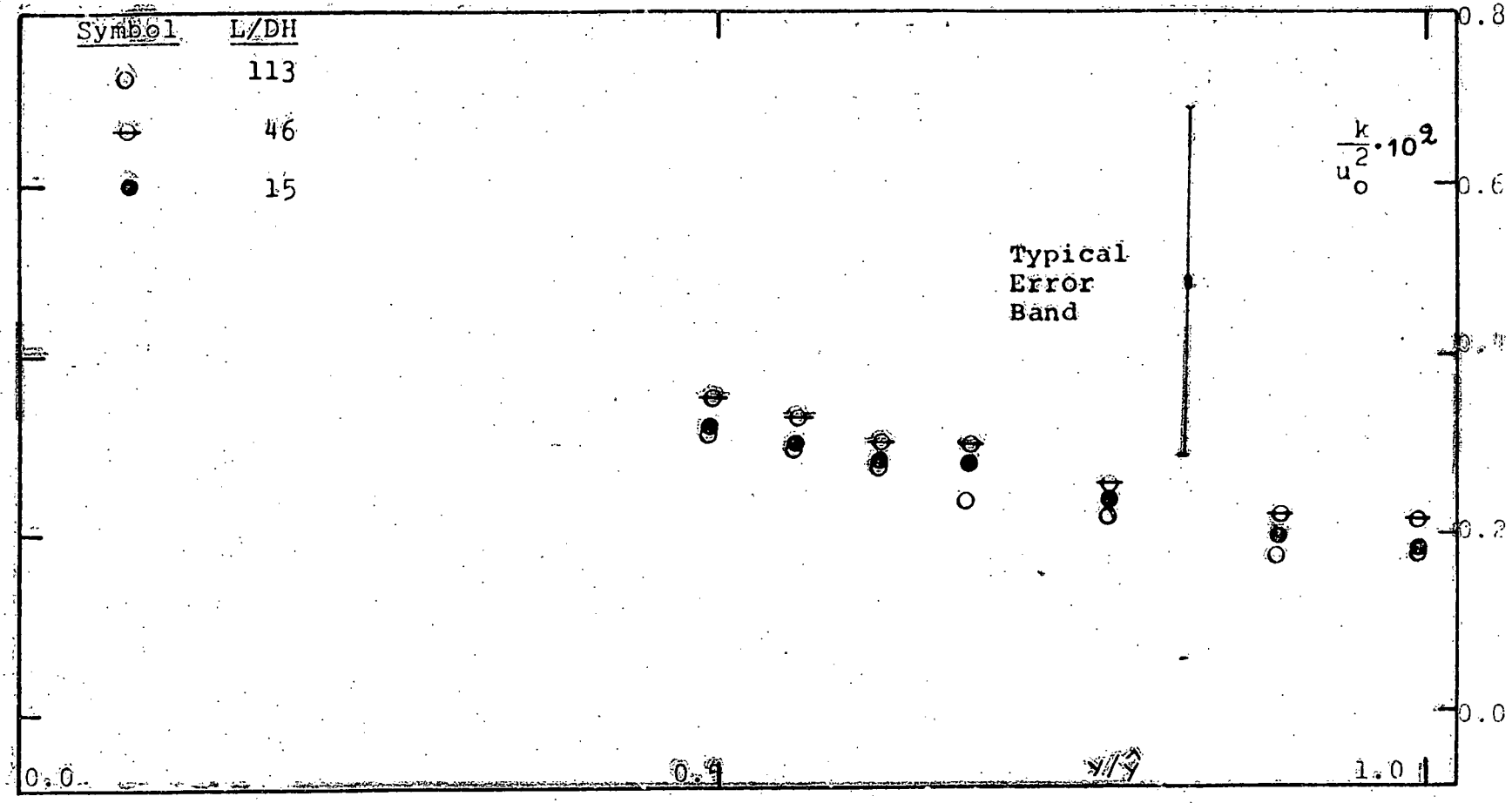


Fig. 5.9c. Turbulent Kinetic Energy (Re≈65,000, θ=24°)

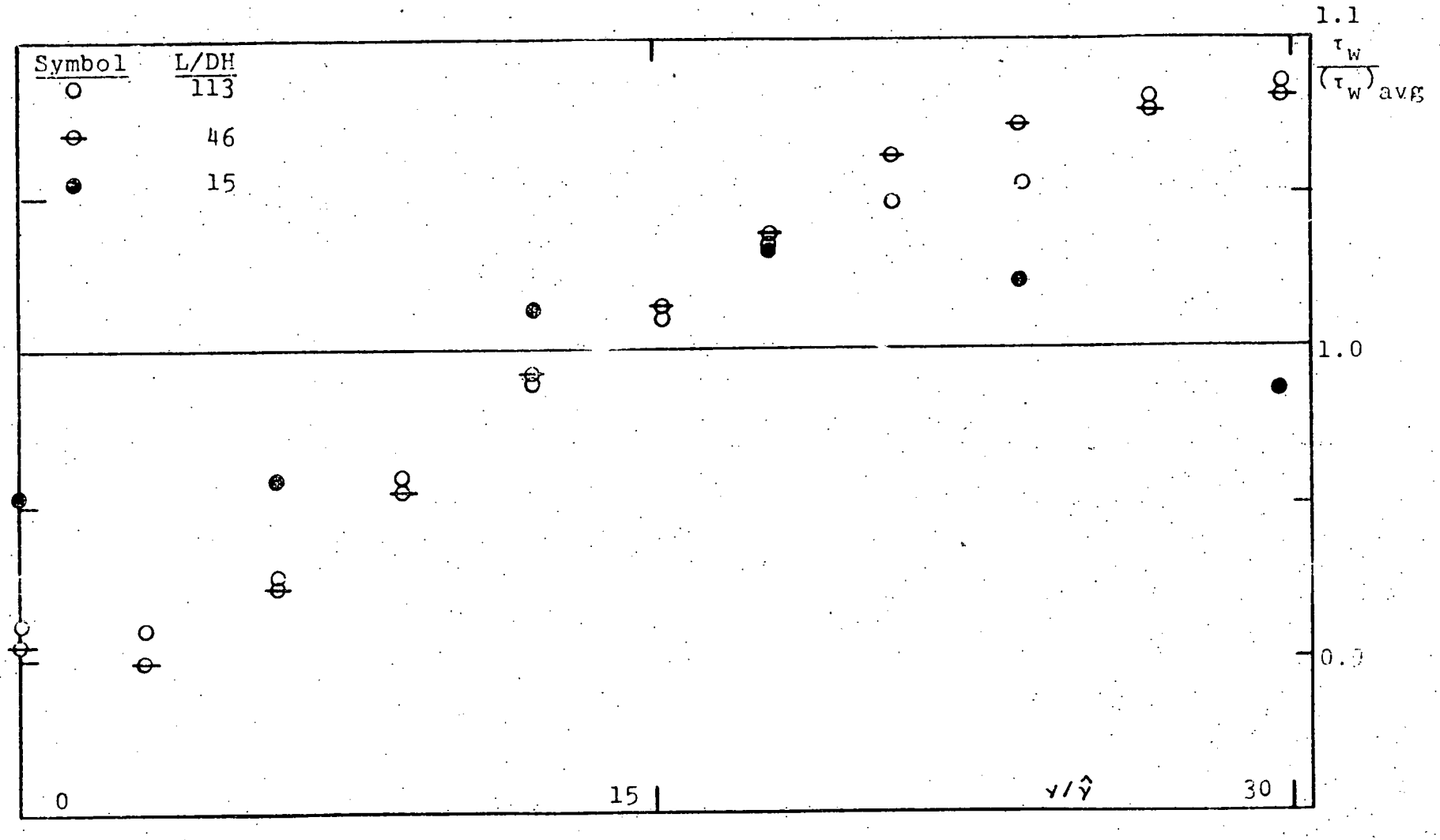


Fig. 5.10. Wall Shear Stress Distribution as Predicted by the Logarithmic Law (Re=26,500)

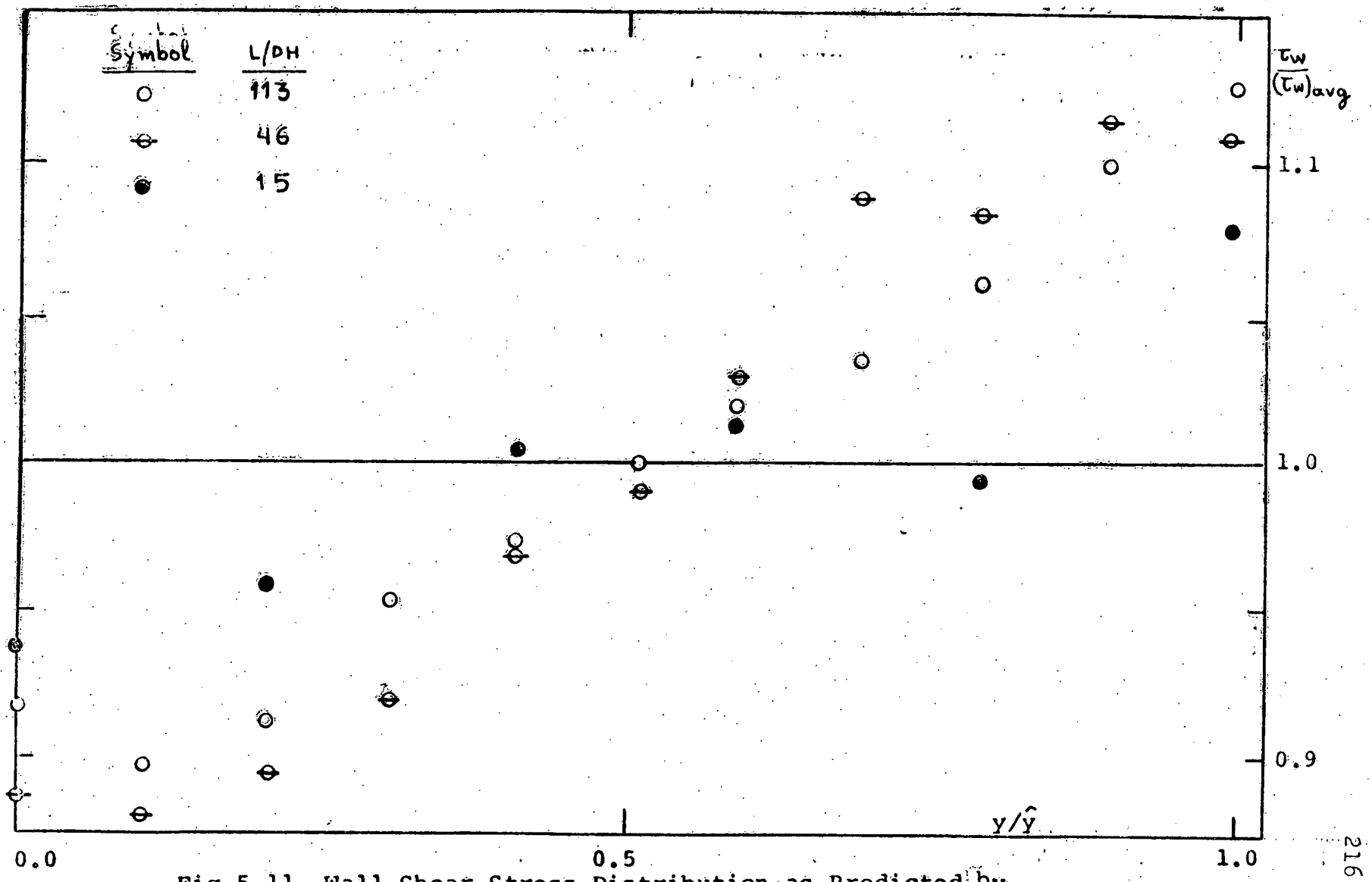


Fig 5.11. Wall Shear Stress Distribution as Predicted by the Logarithmic Law (Re≈65,000)

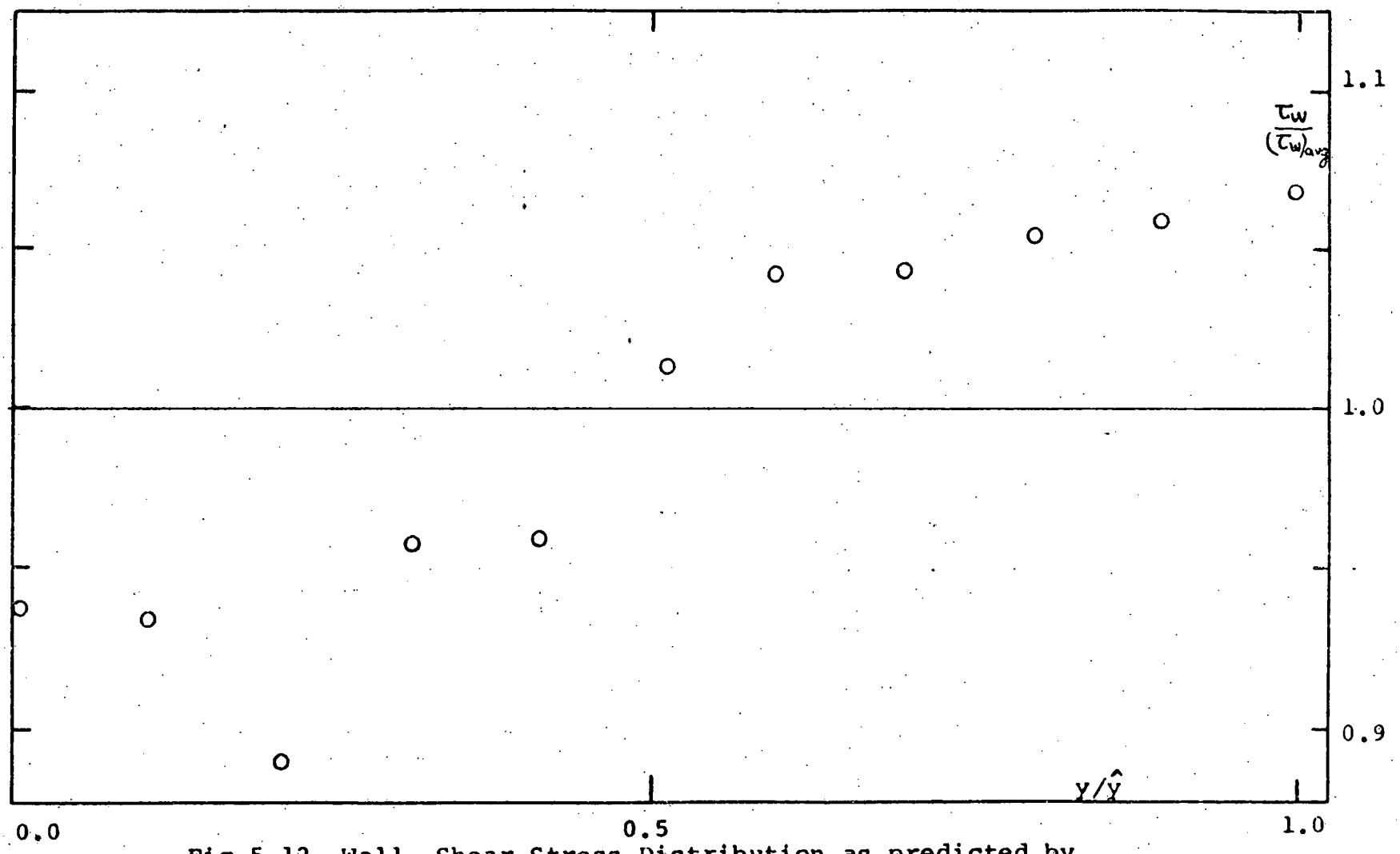


Fig 5.12. Wall Shear Stress Distribution as predicted by the Logarithmic Law (Re≈9,000)

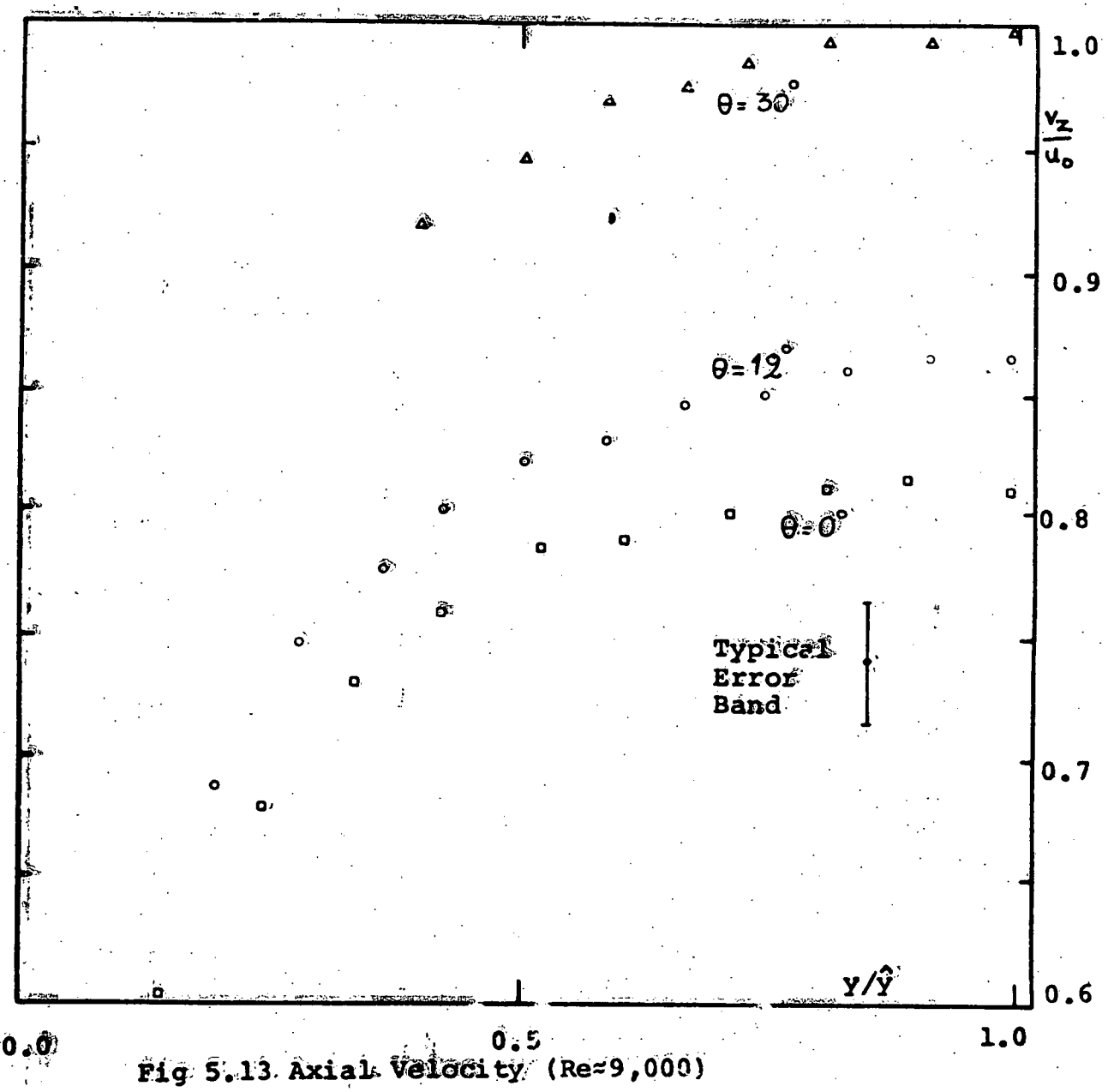


Fig 5.13 Axial Velocity (Re=9,000)

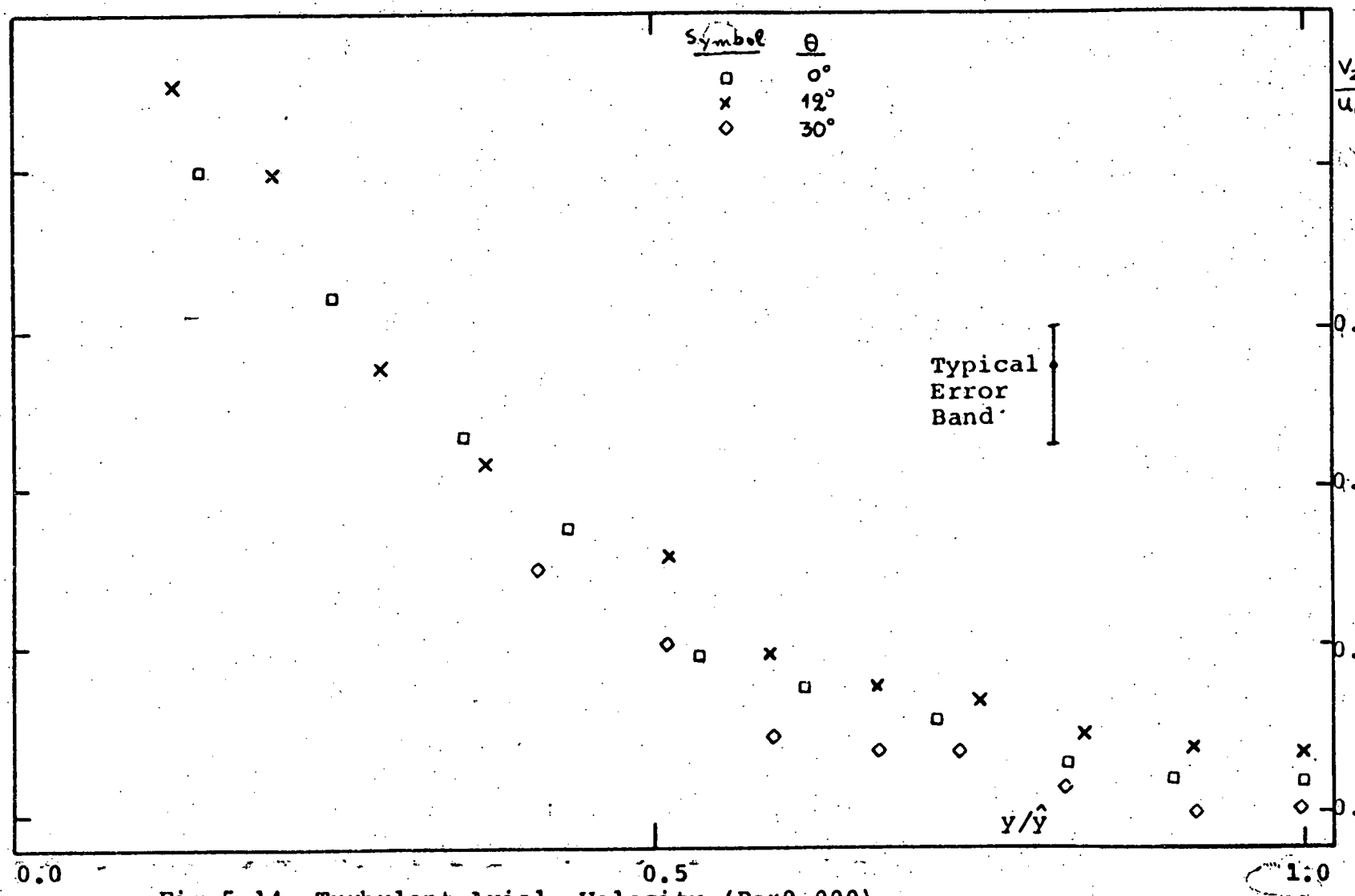


Fig 5.14. Turbulent Axial Velocity (Re≈9,000)

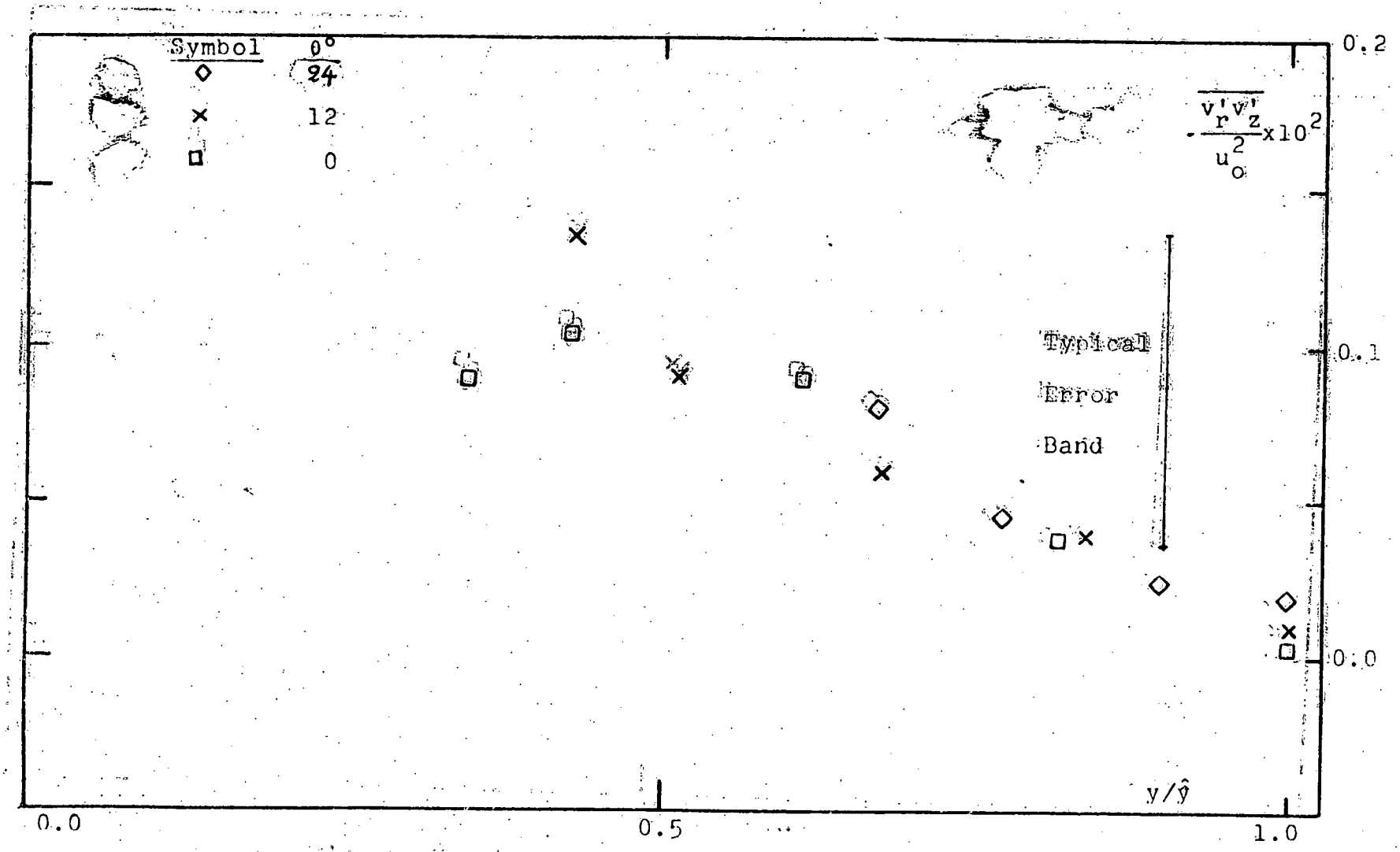


Fig. 5.15. Radial Shear Stress (Re=9,000)

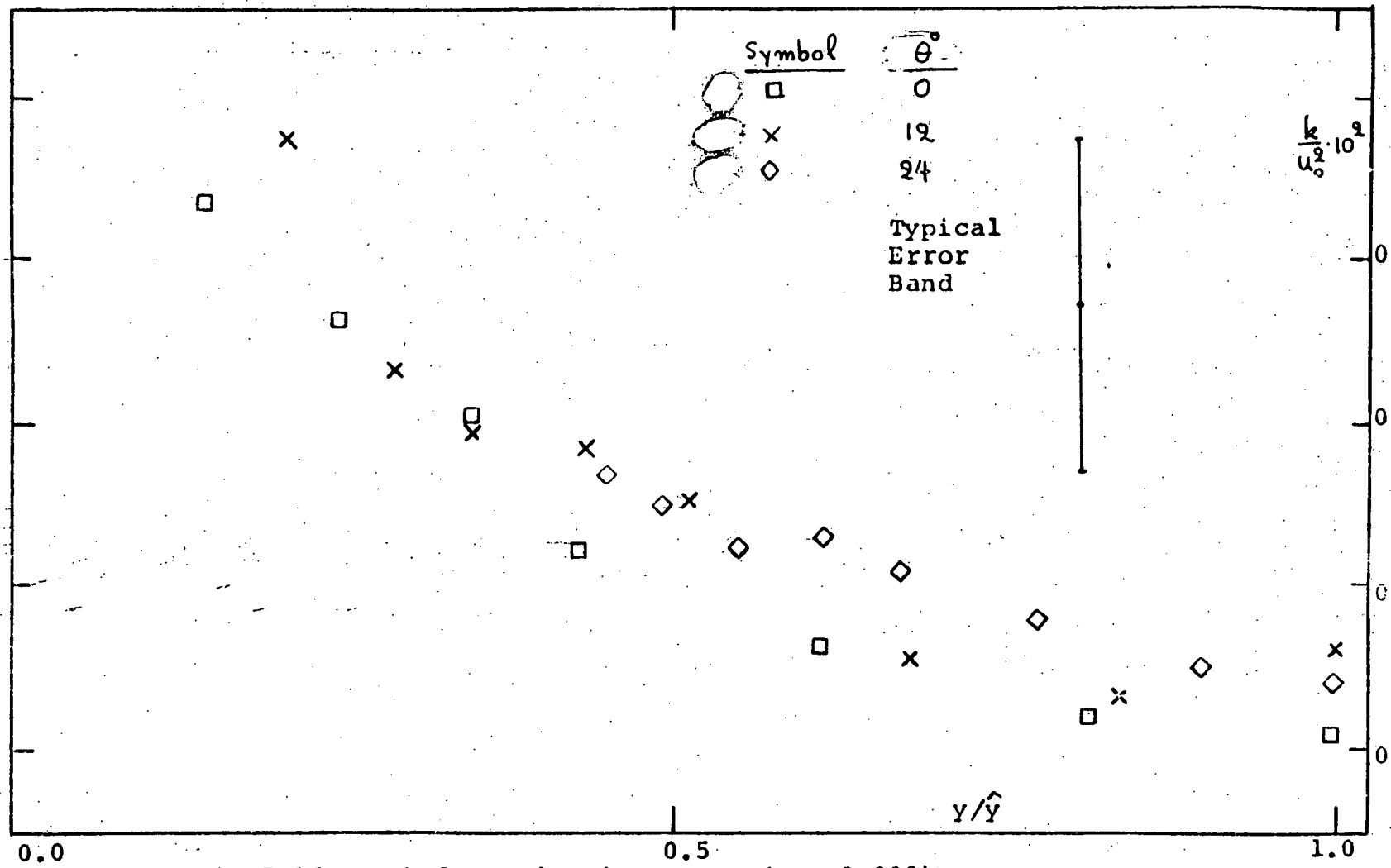


Fig.5.16. Turbulent Kinetic Energy (Re≈ 9,000)

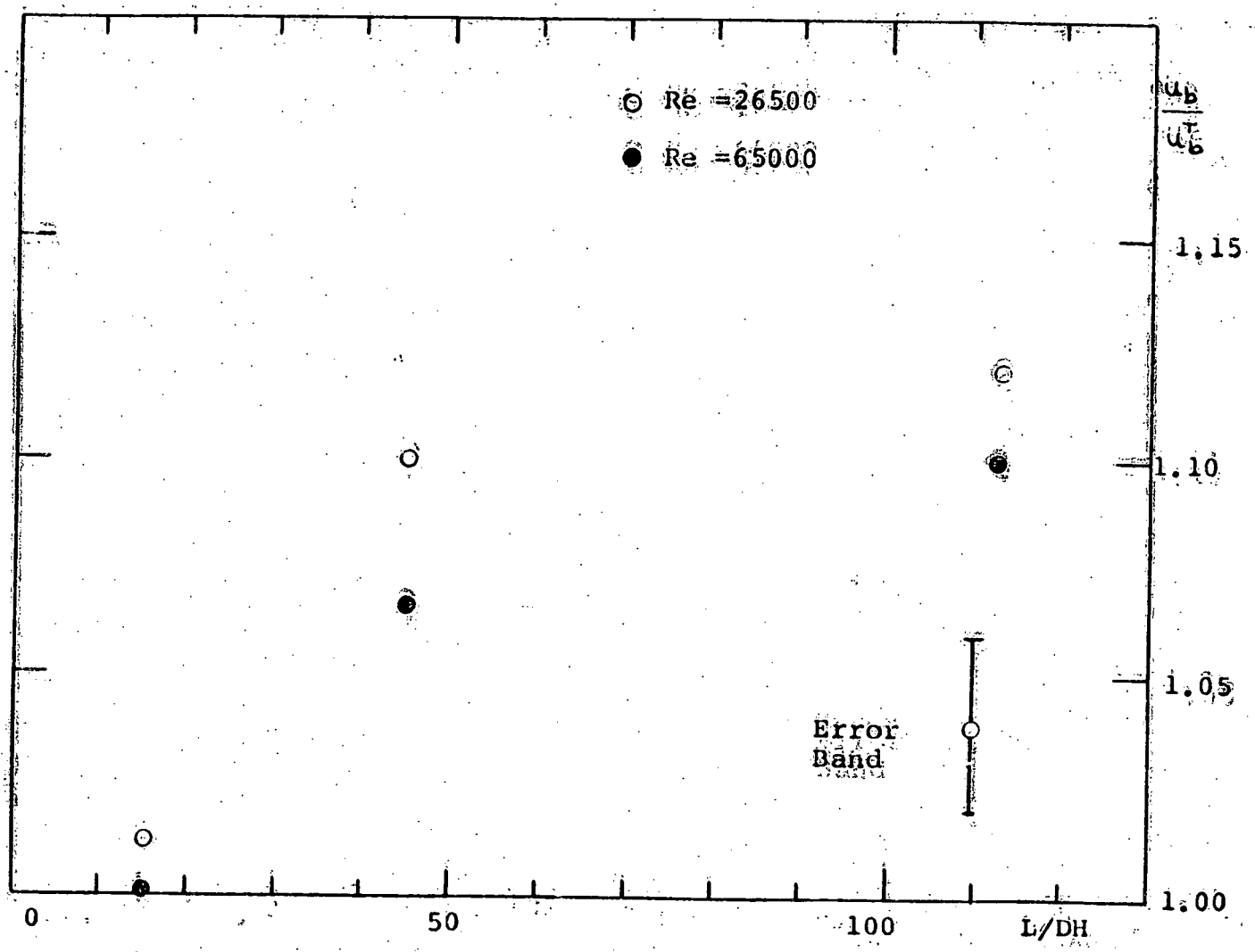


Fig 5.17. Flow Split

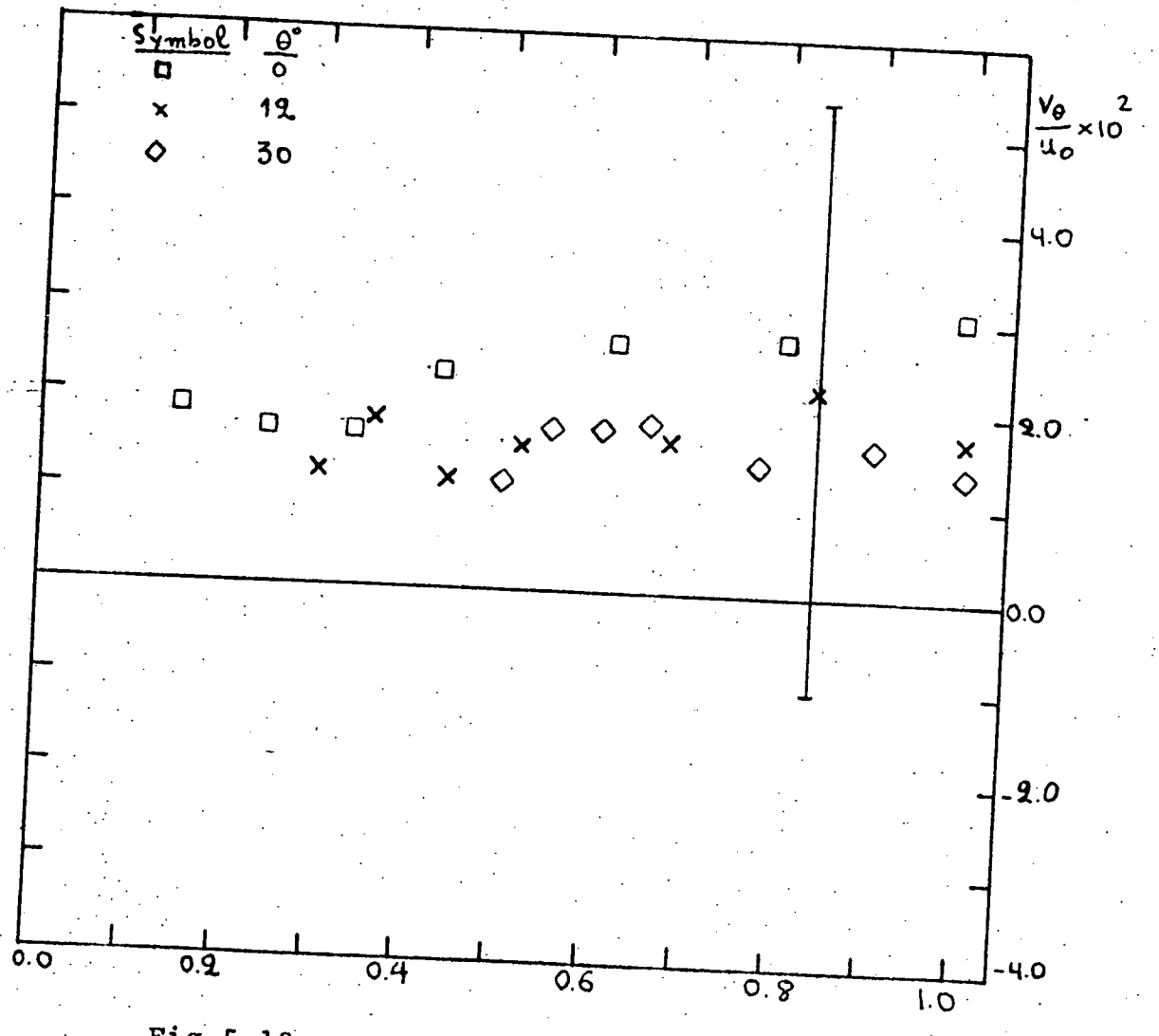


Fig 5.18. v_θ Velocity

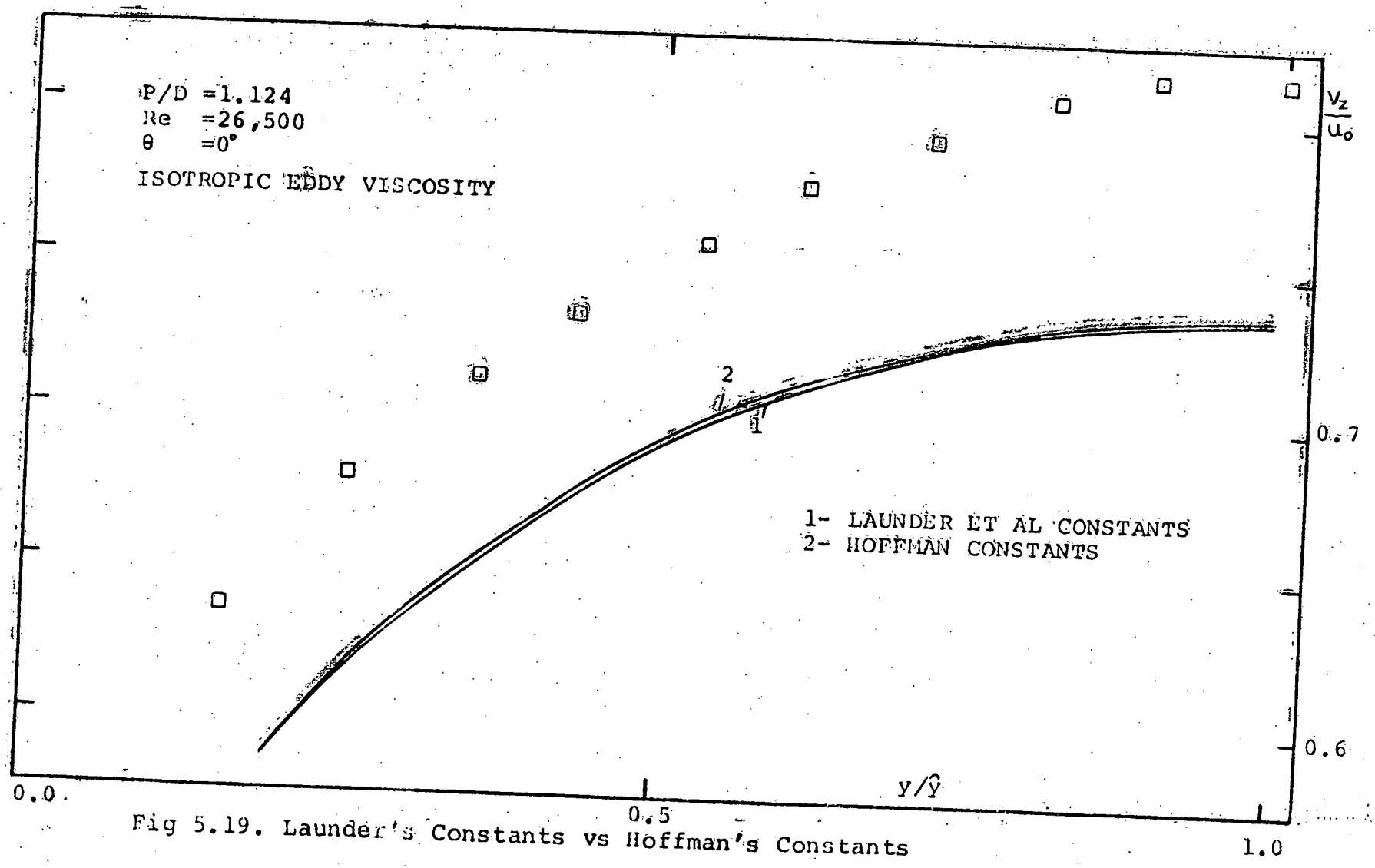


Fig 5.19. Launder's Constants vs Hoffman's Constants

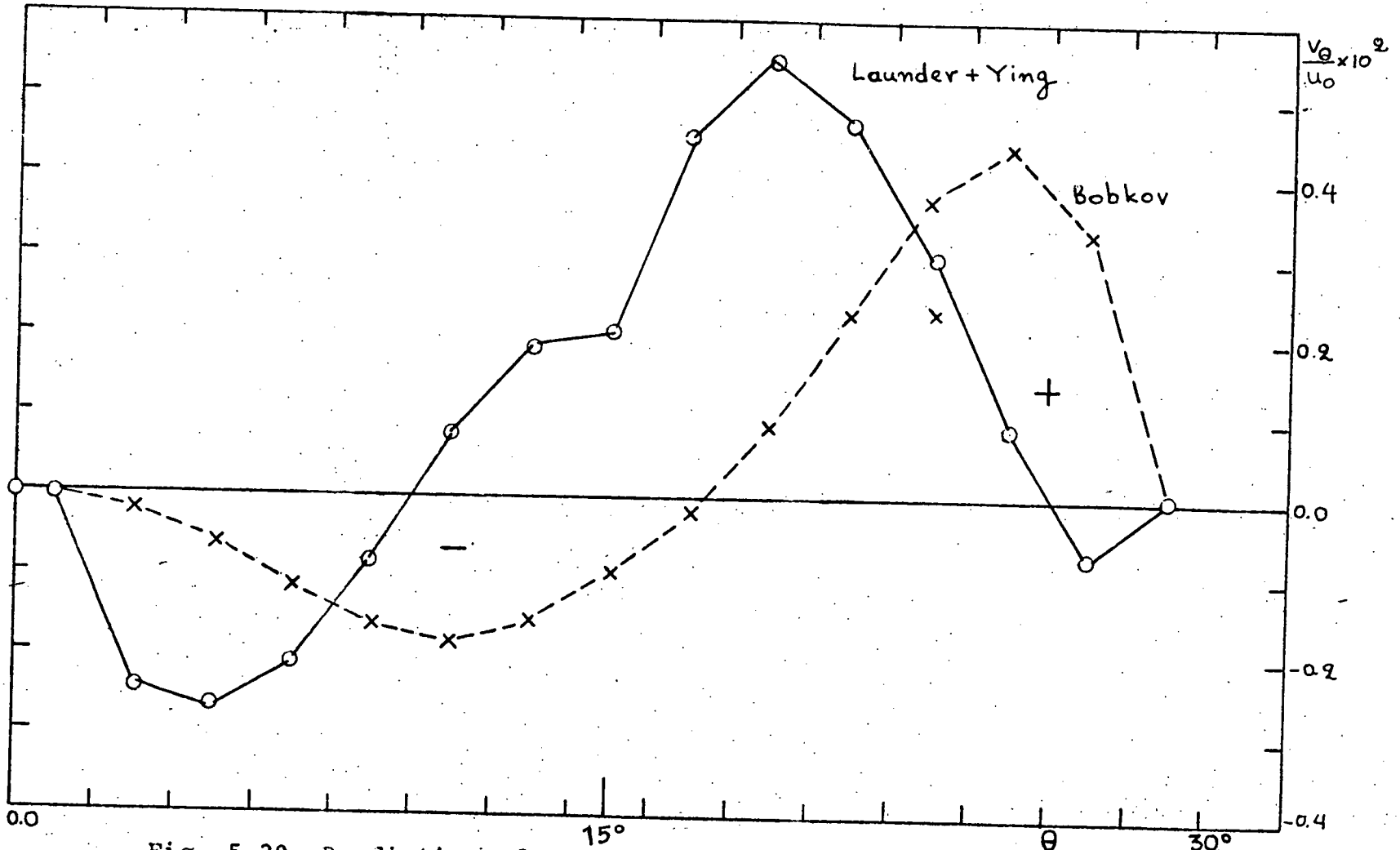


Fig. 5.20. Prediction of v_θ Velocity at $y=y_B$

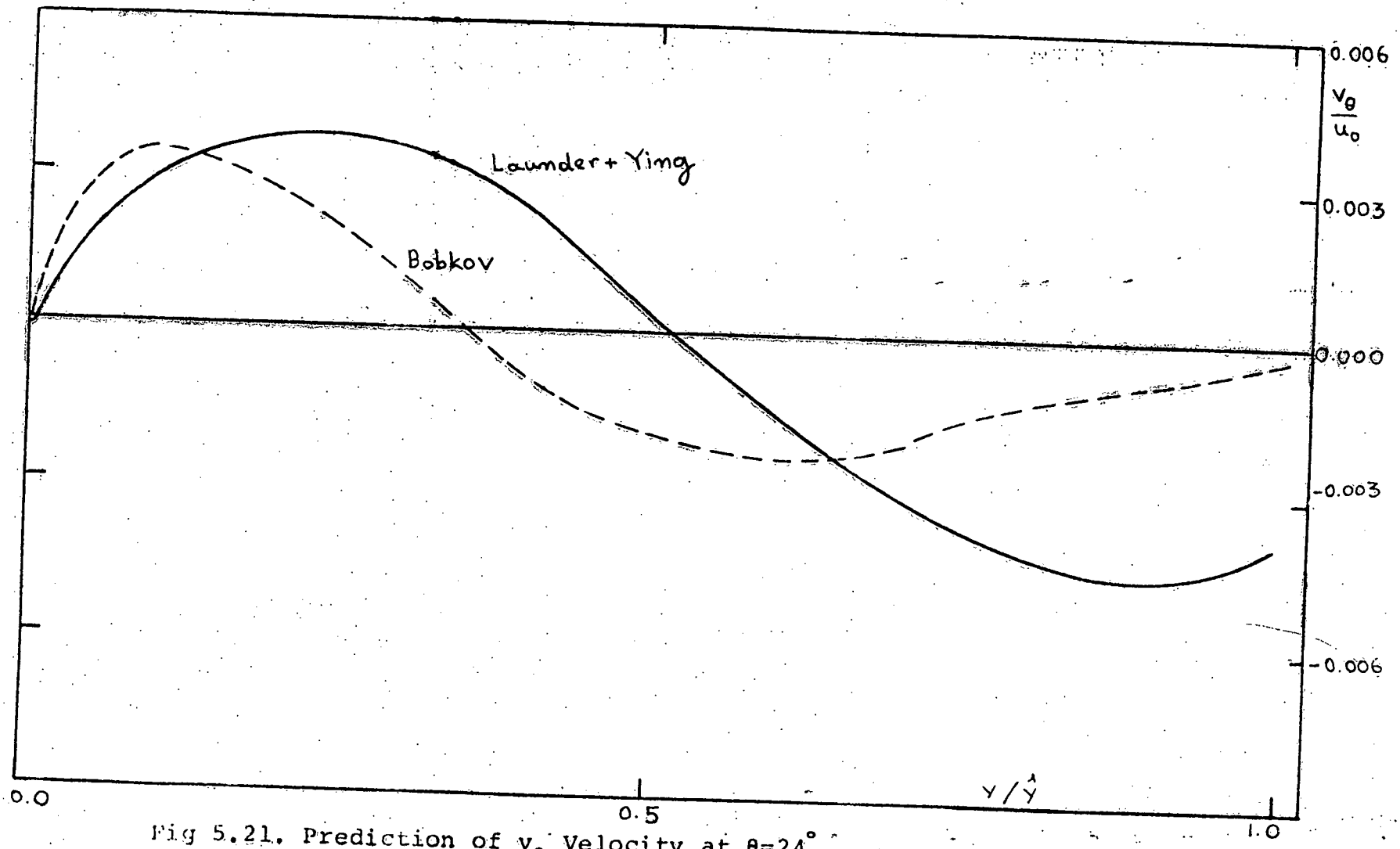


Fig 5.21. Prediction of v_θ Velocity at $\theta=24°$

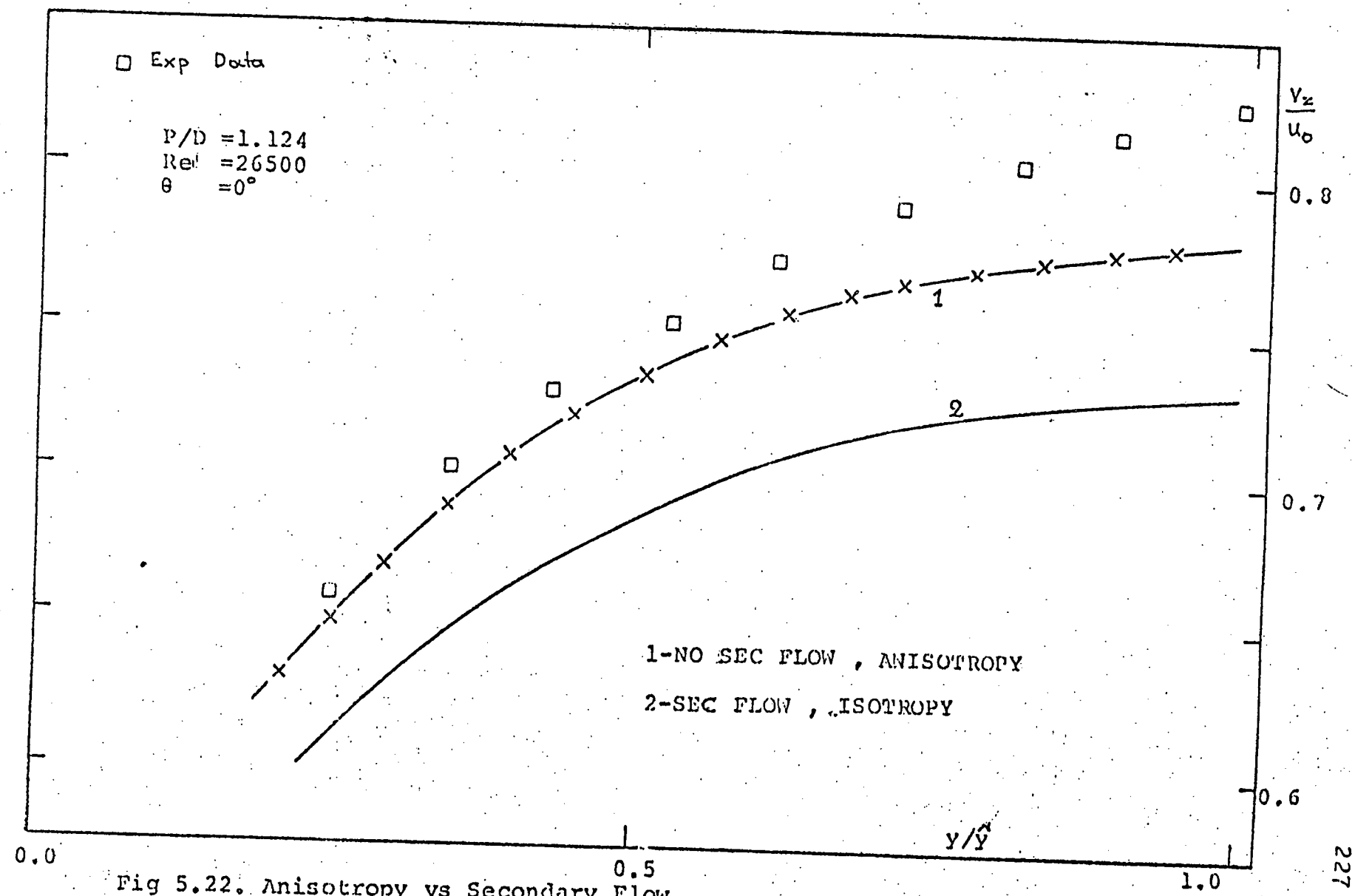


Fig 5.22. Anisotropy vs Secondary Flow

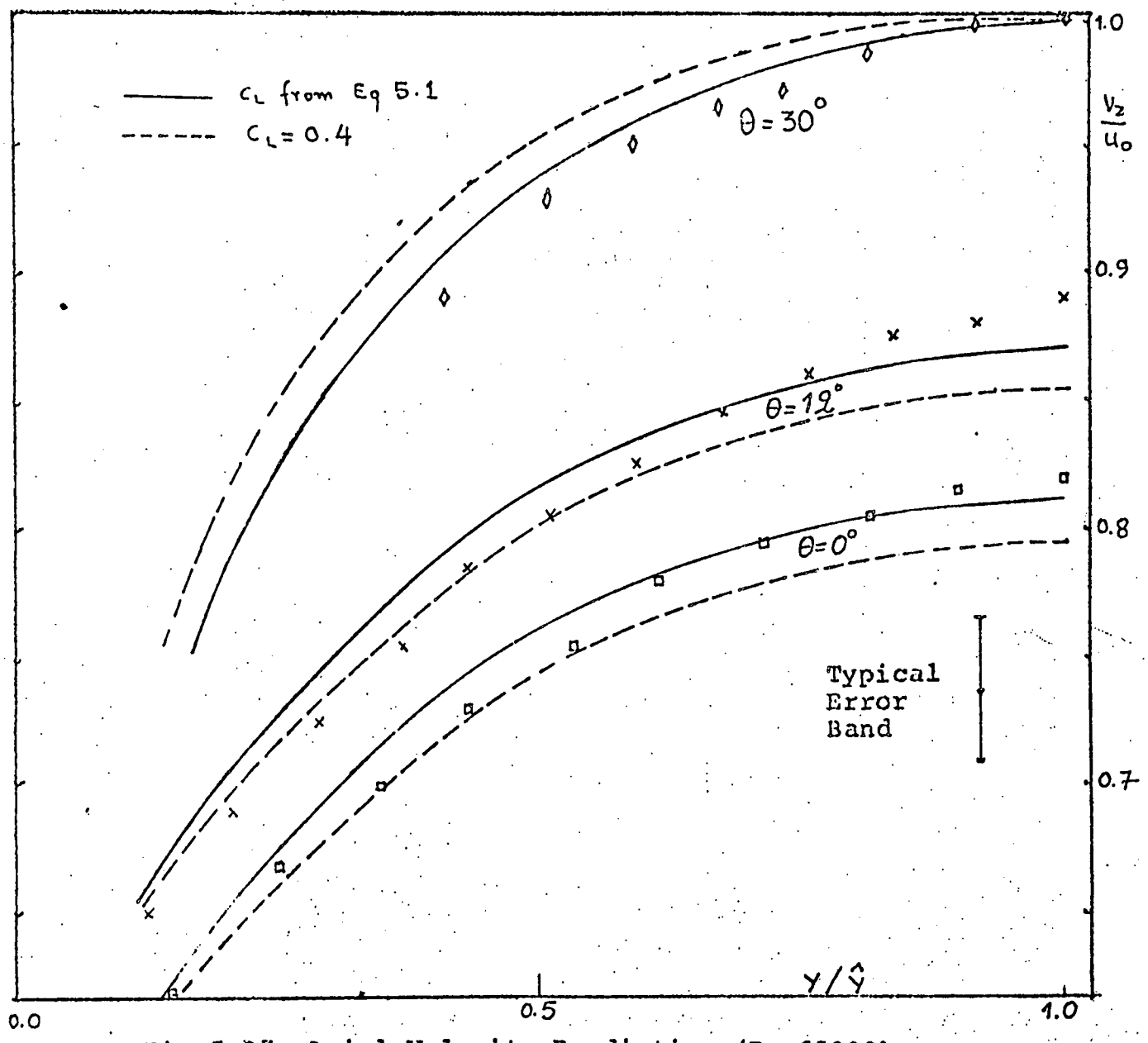


Fig 5.23. Axial Velocity Prediction (Re≈65000)

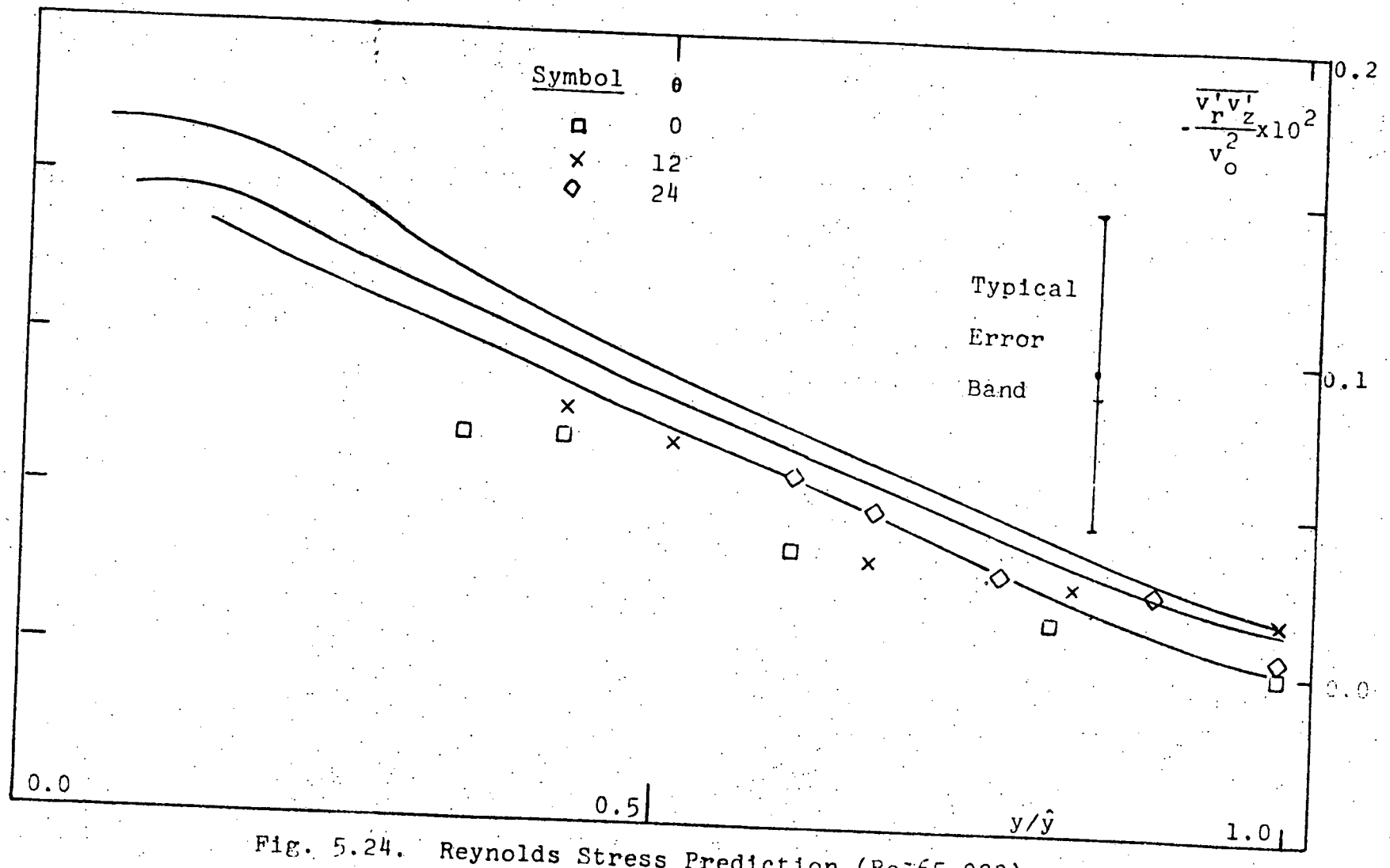


Fig. 5.24. Reynolds Stress Prediction (Re=65,000)

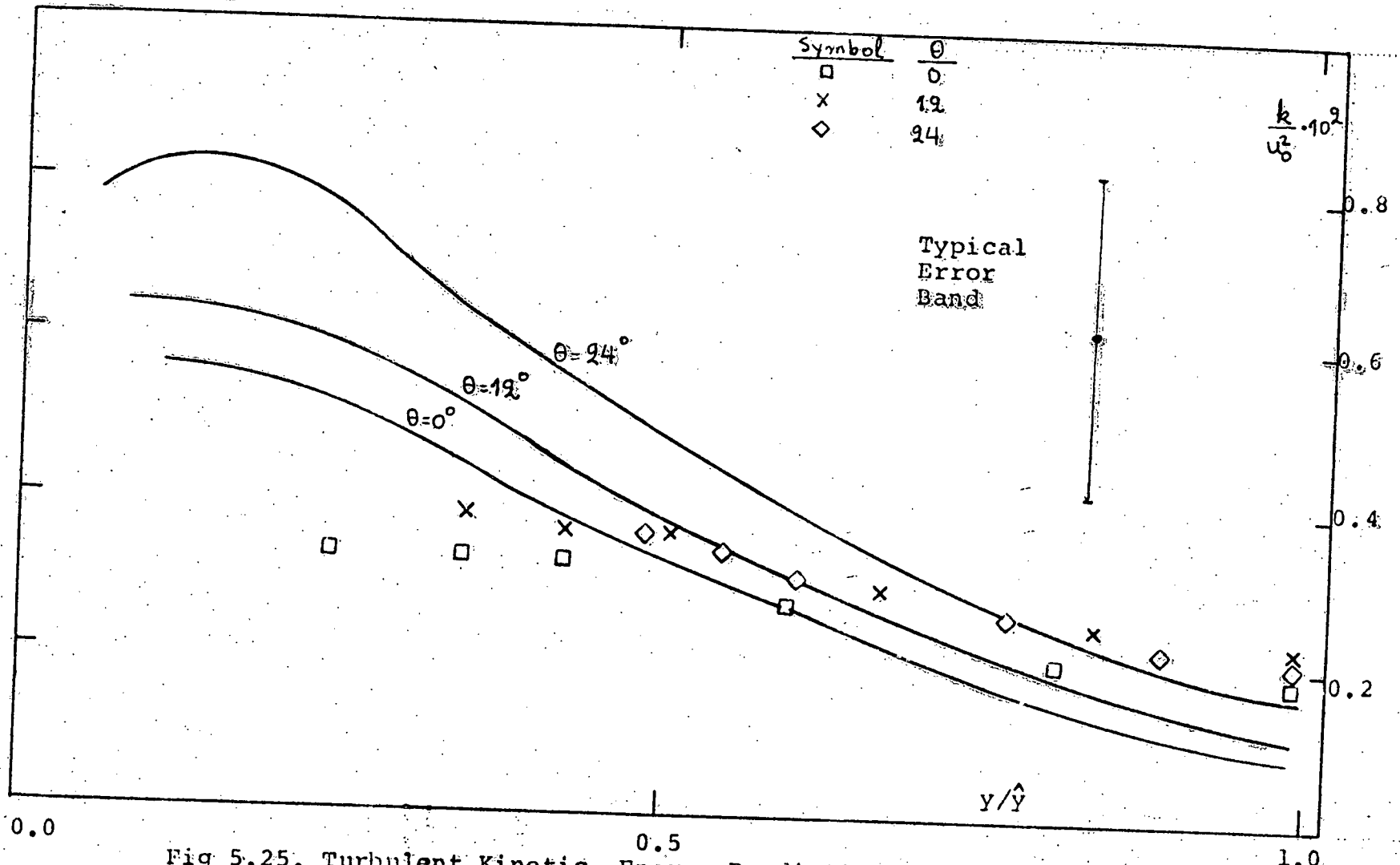


Fig 5.25. Turbulent Kinetic Energy Predictions (Re≈65000)

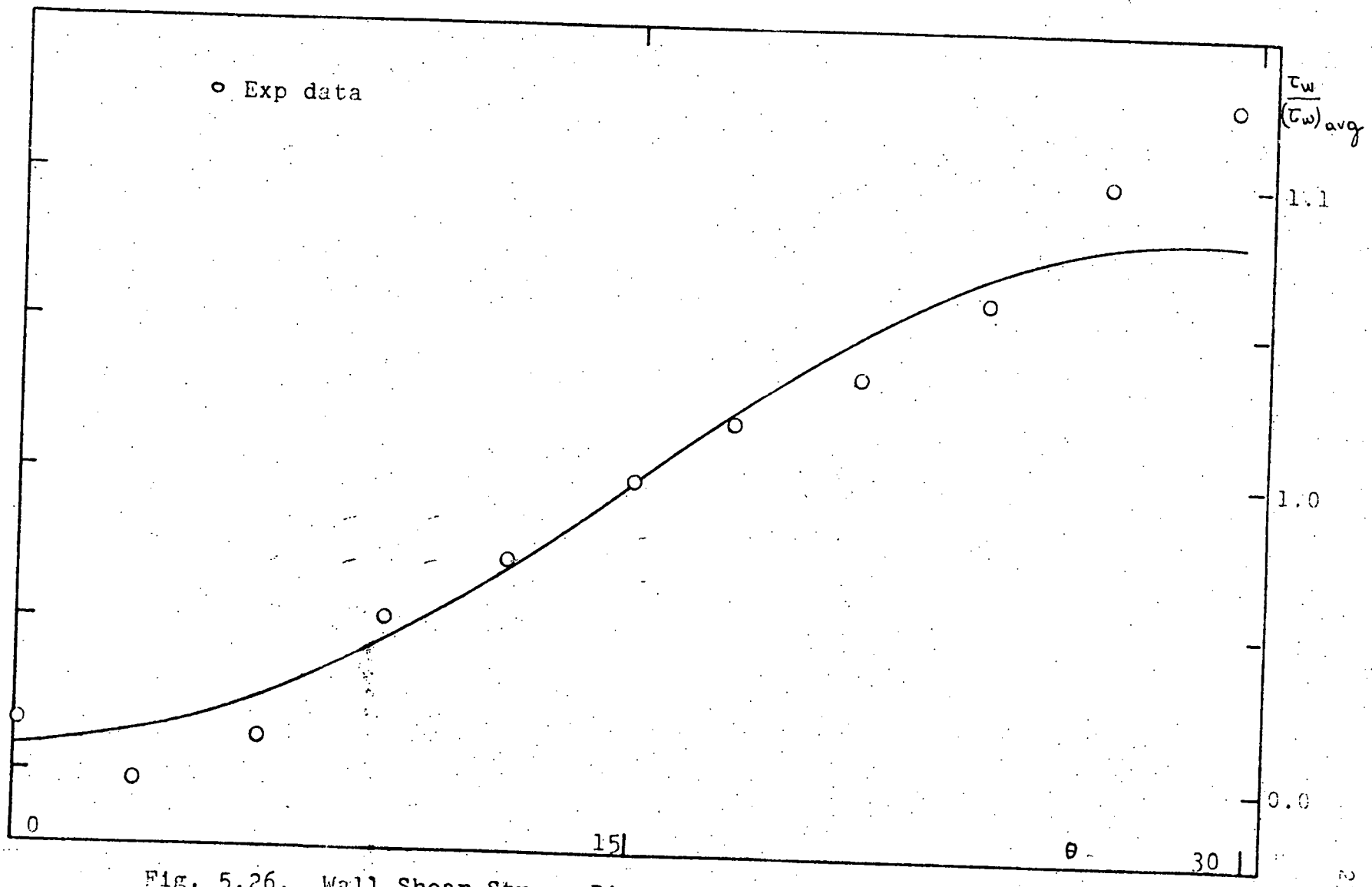


Fig. 5.26. Wall Shear Stress Distribution Prediction (Re=65,000)

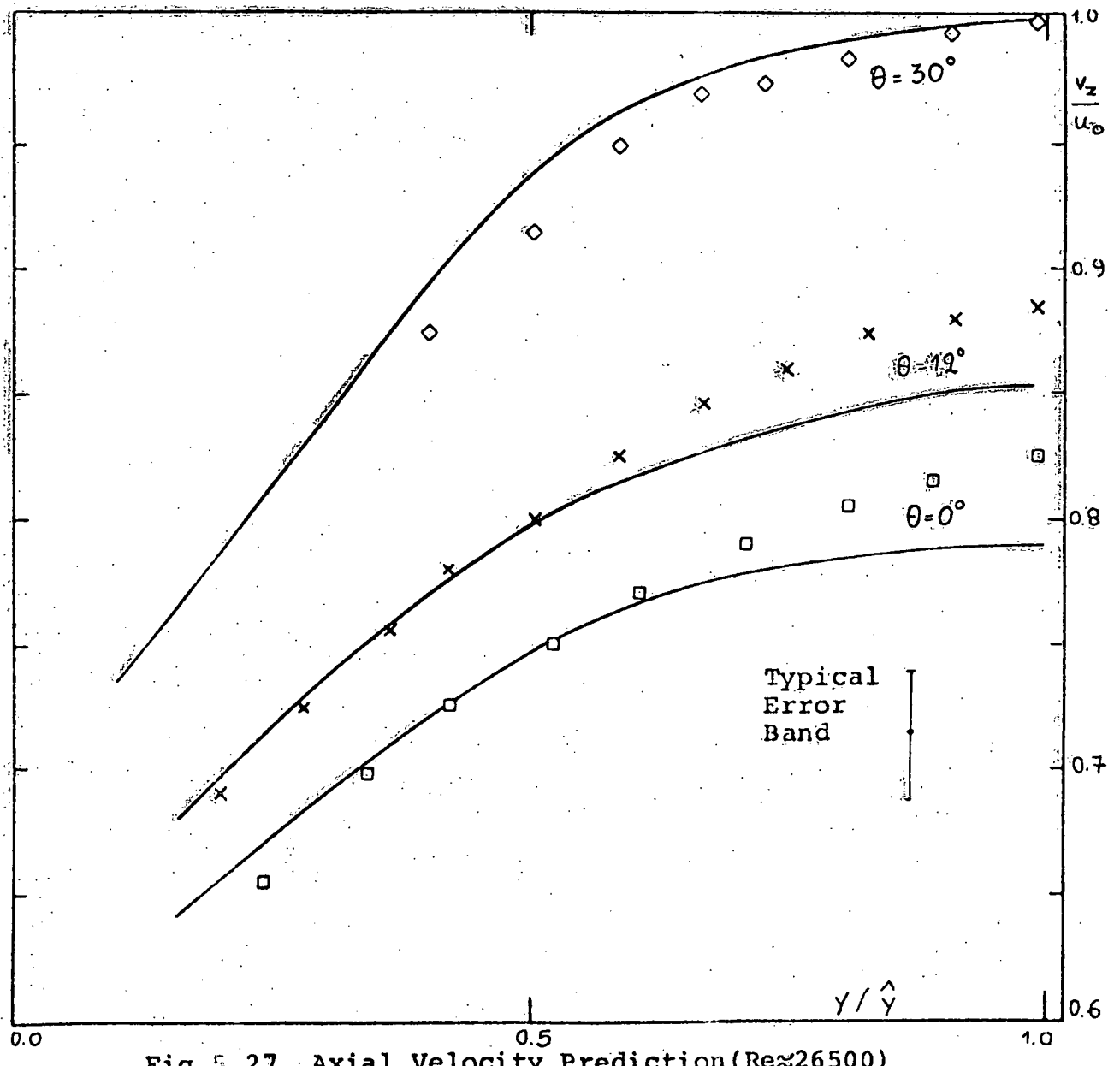


Fig 5.27. Axial Velocity Prediction (Re≈26500)

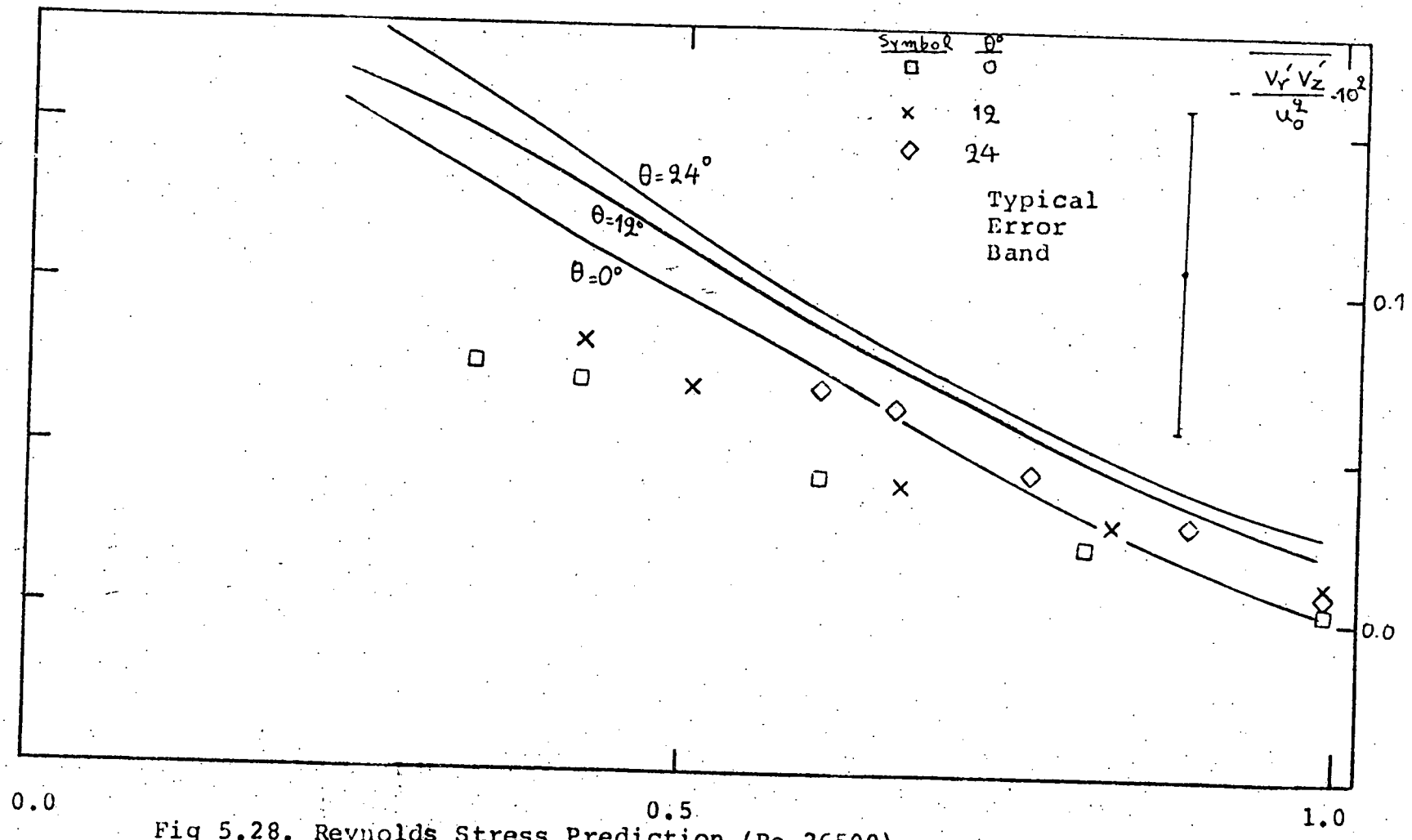


Fig 5.28. Reynolds Stress Prediction (Re 26500)

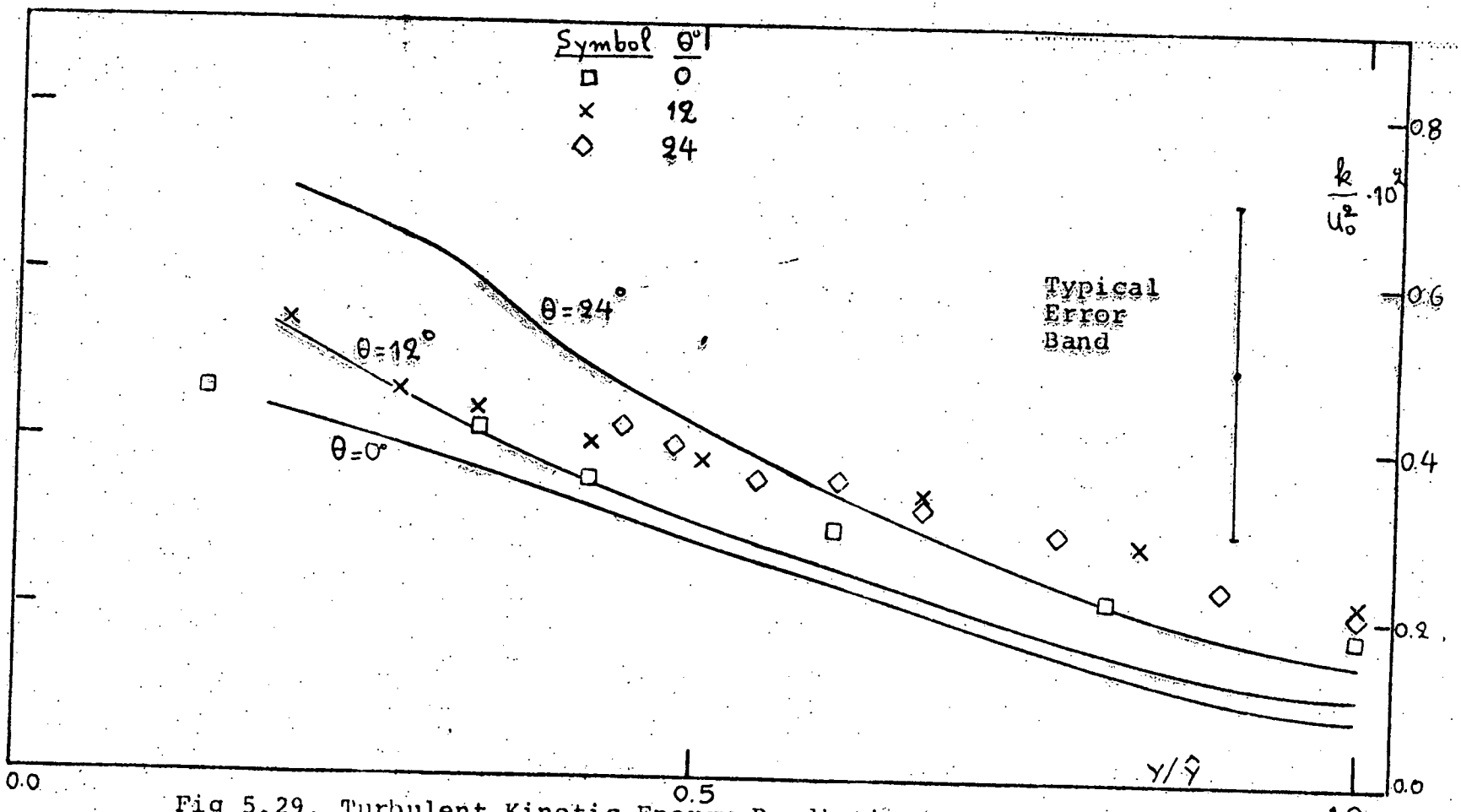


Fig 5.29. Turbulent Kinetic Energy Prediction(Re 26500)

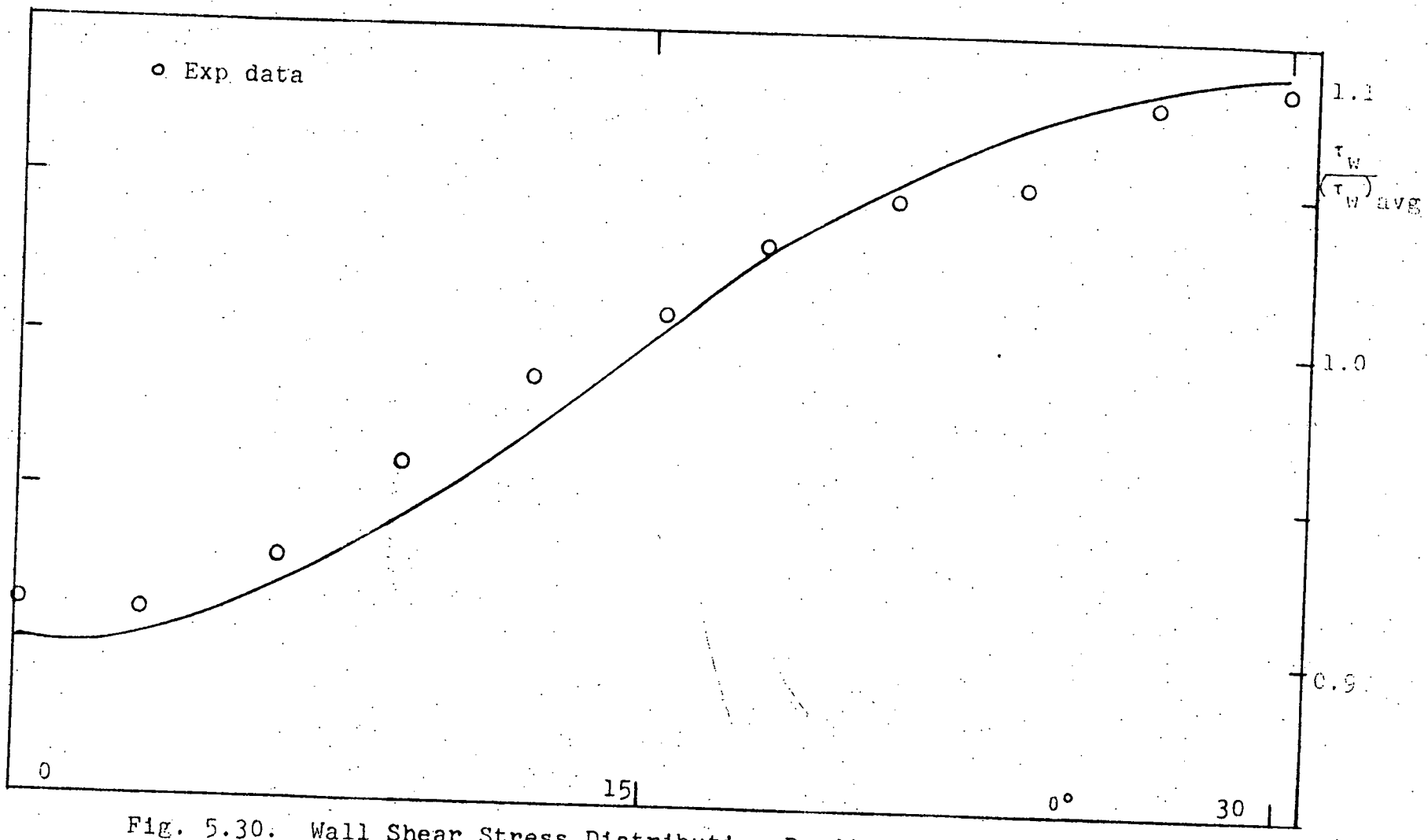


Fig. 5.30. Wall Shear Stress Distribution Prediction (Re=26,500)

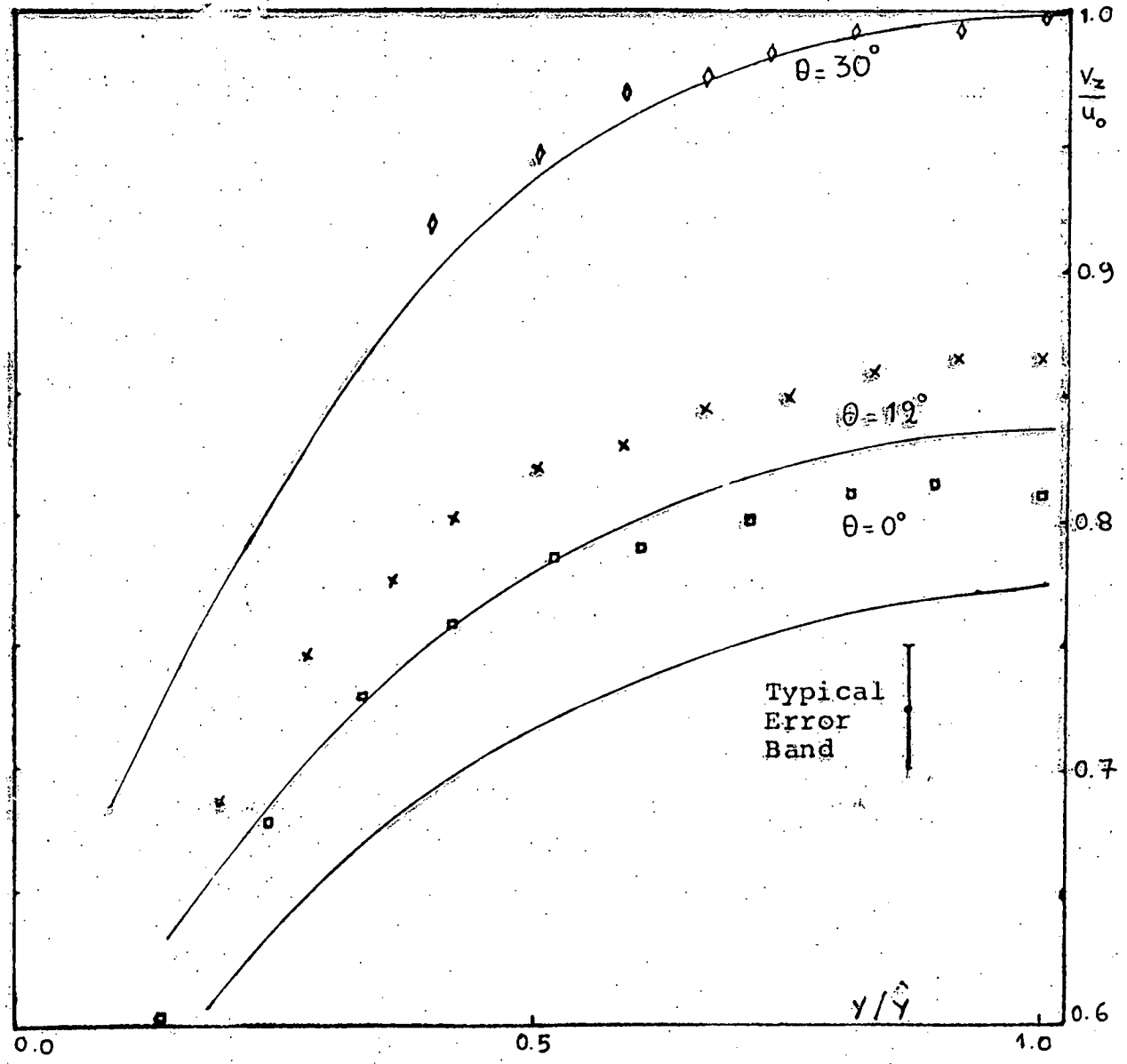


Fig 5.31. Axial Velocity Prediction (Re≈9000)

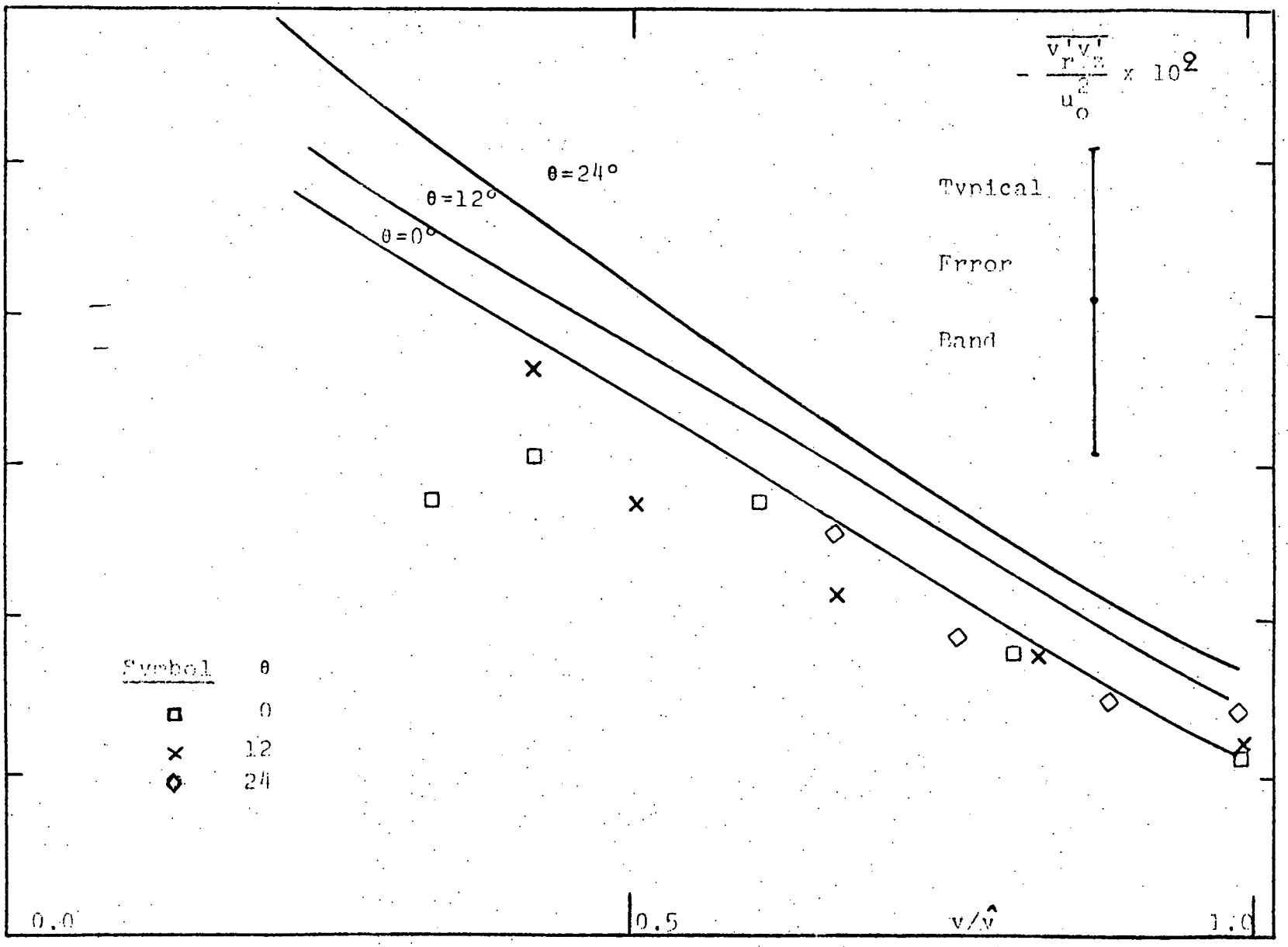


Fig. 5.32. Reynolds Stress Prediction (Re≈9,000)

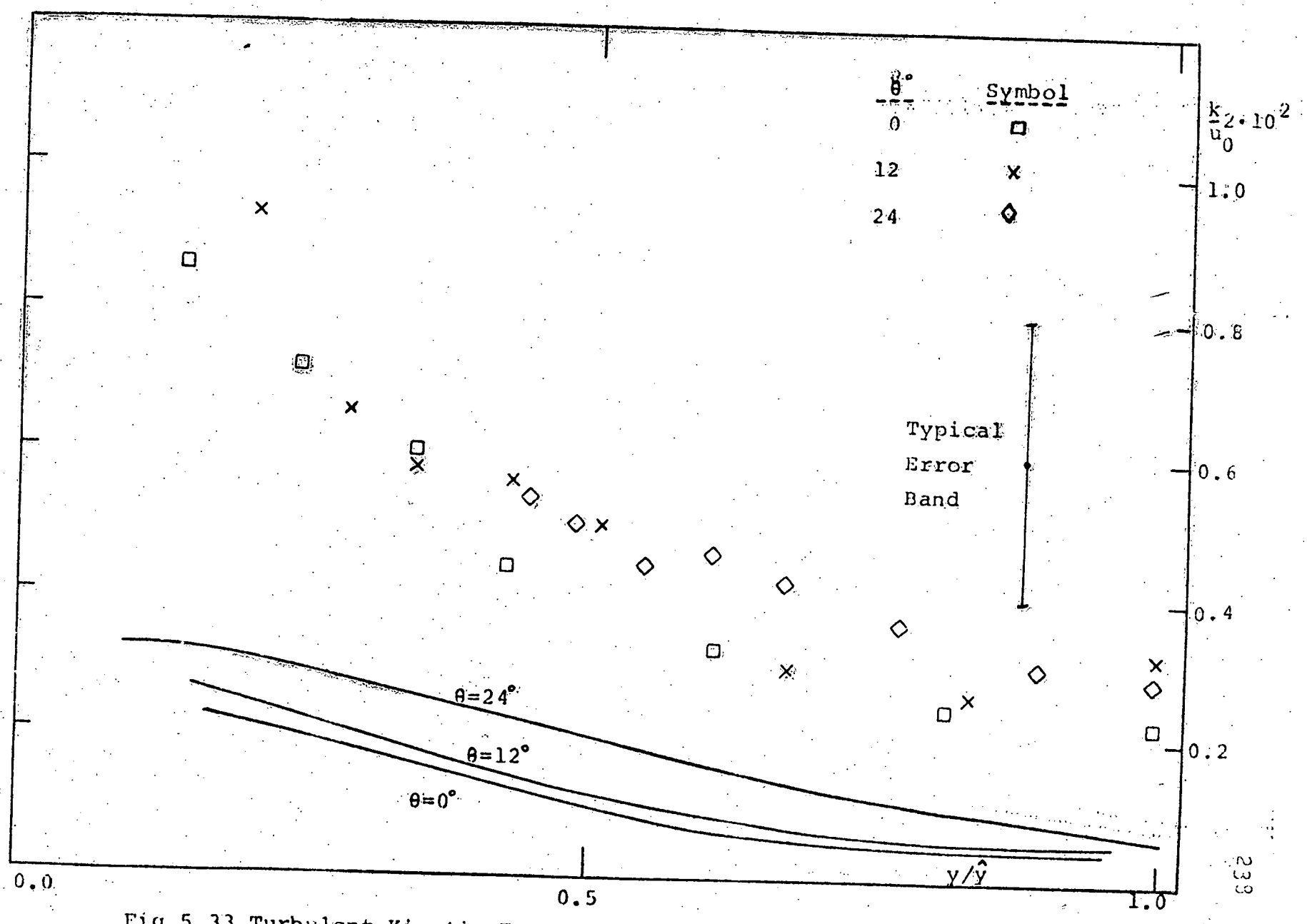


Fig.5.33. Turbulent Kinetic Energy (Re≈9,000)

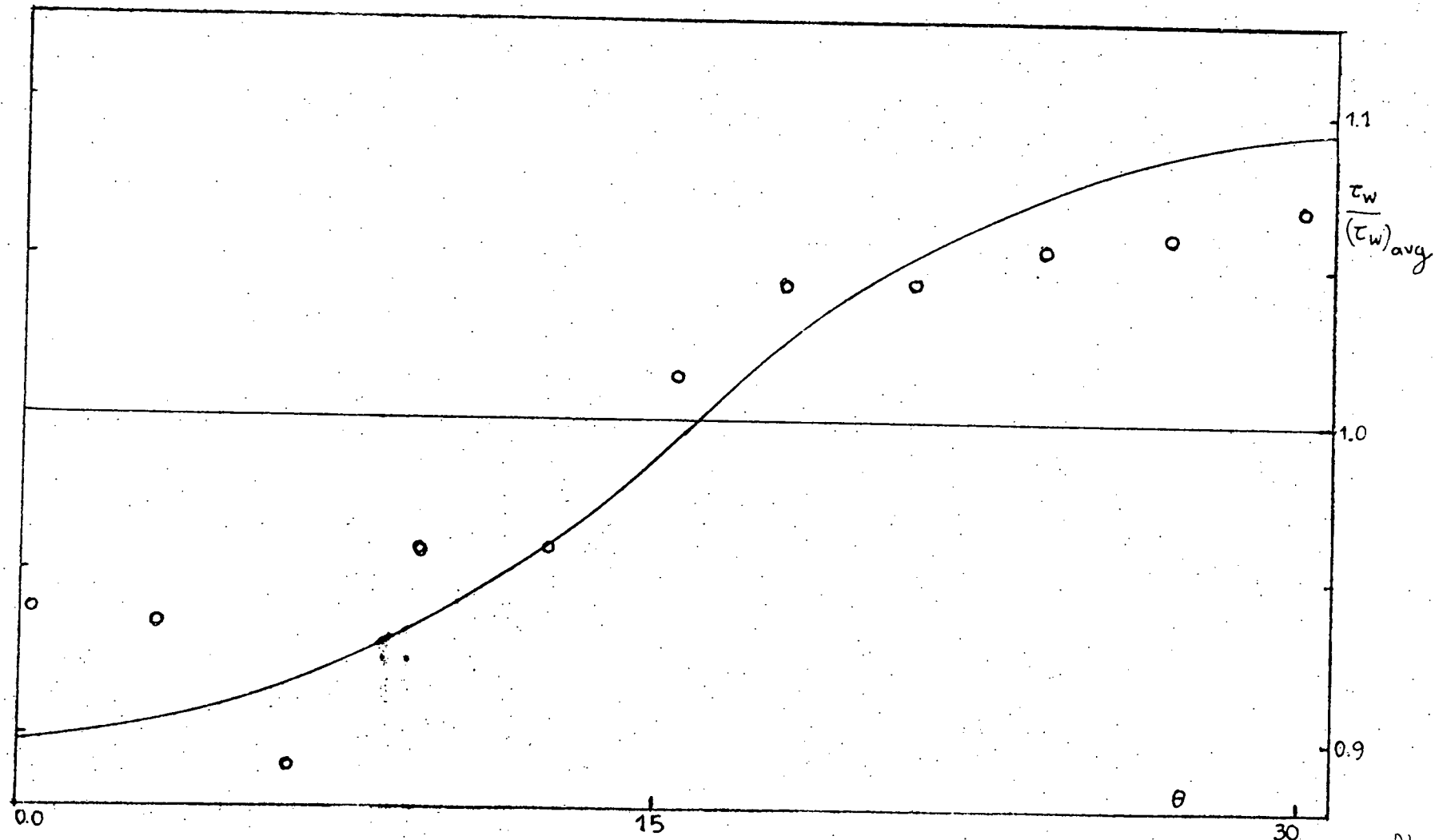


Fig 5.34. Wall Shear Stress Distribution Prediction (Re≈9000)

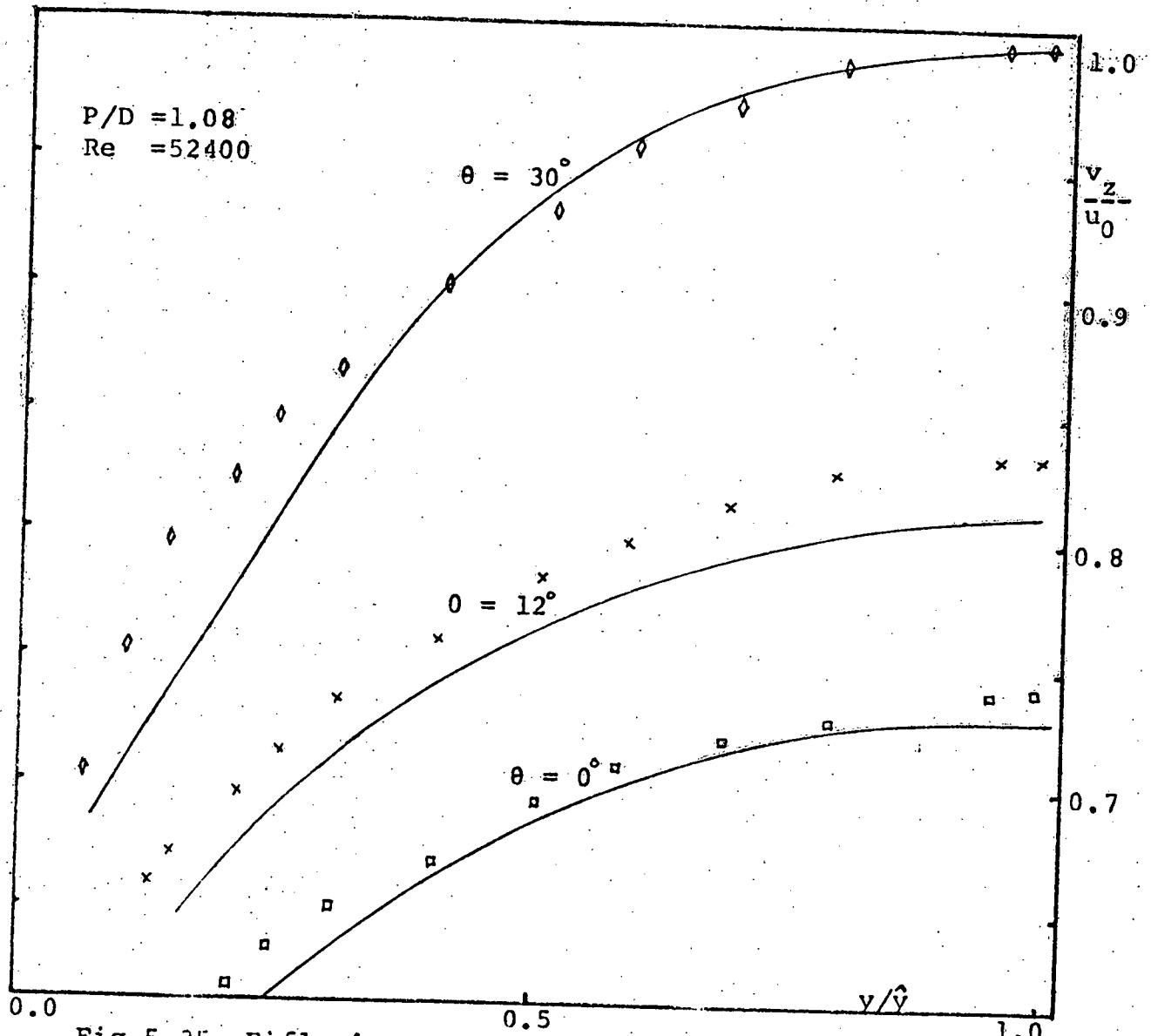


Fig 5.35. Eifler's Data Prediction

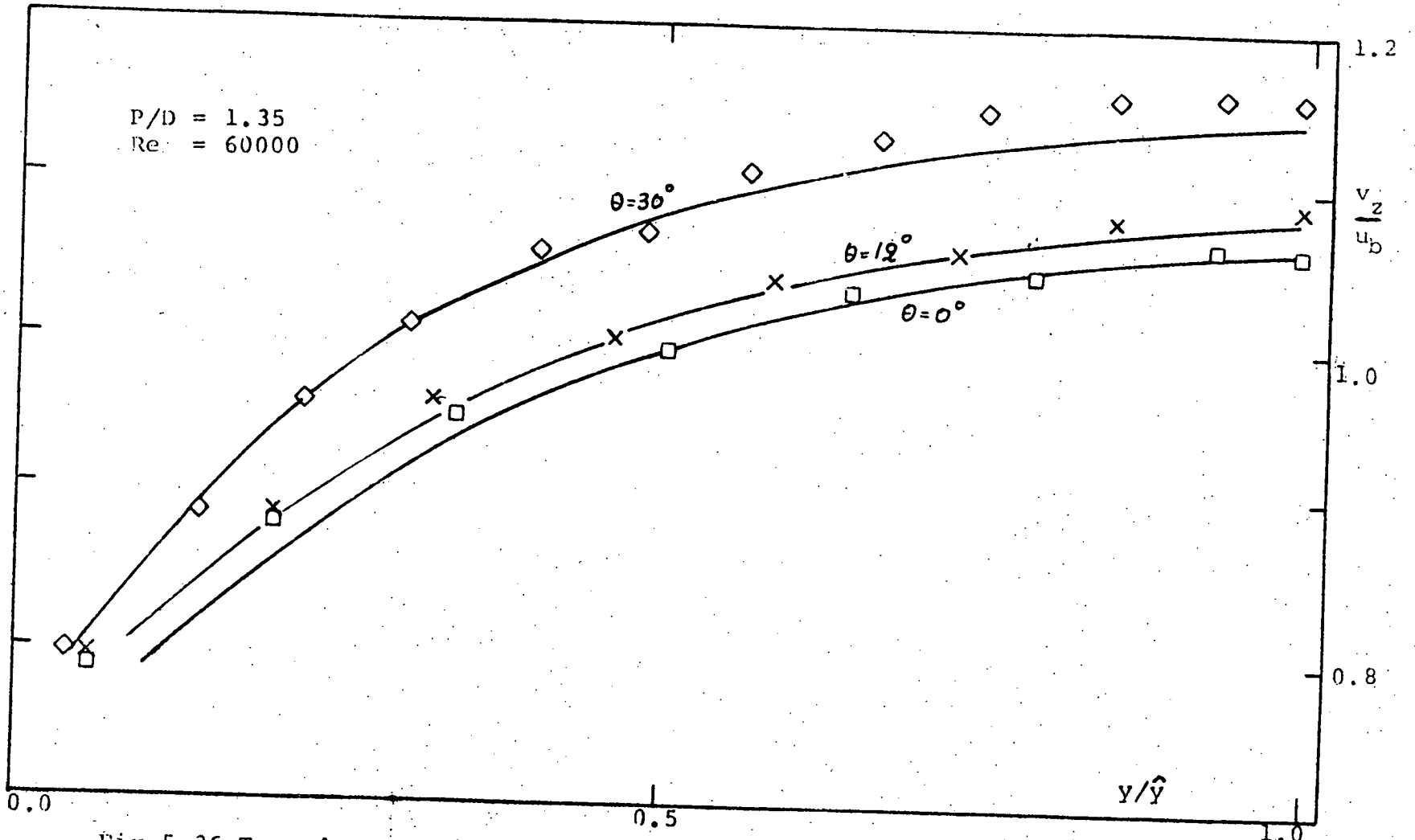


Fig 5.36 Trupp's Data Prediction

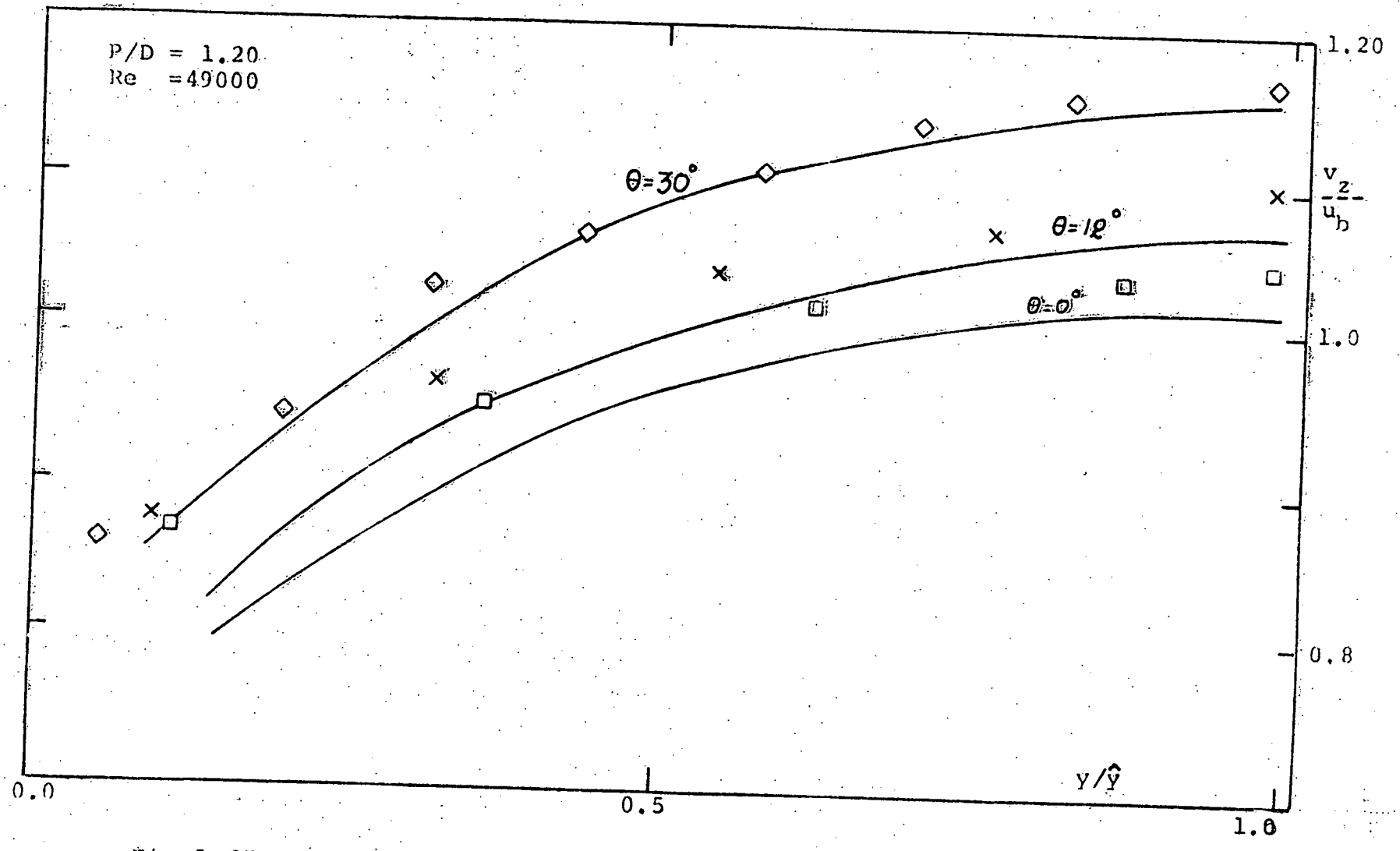


Fig 5.37. Trupp's Data Prediction

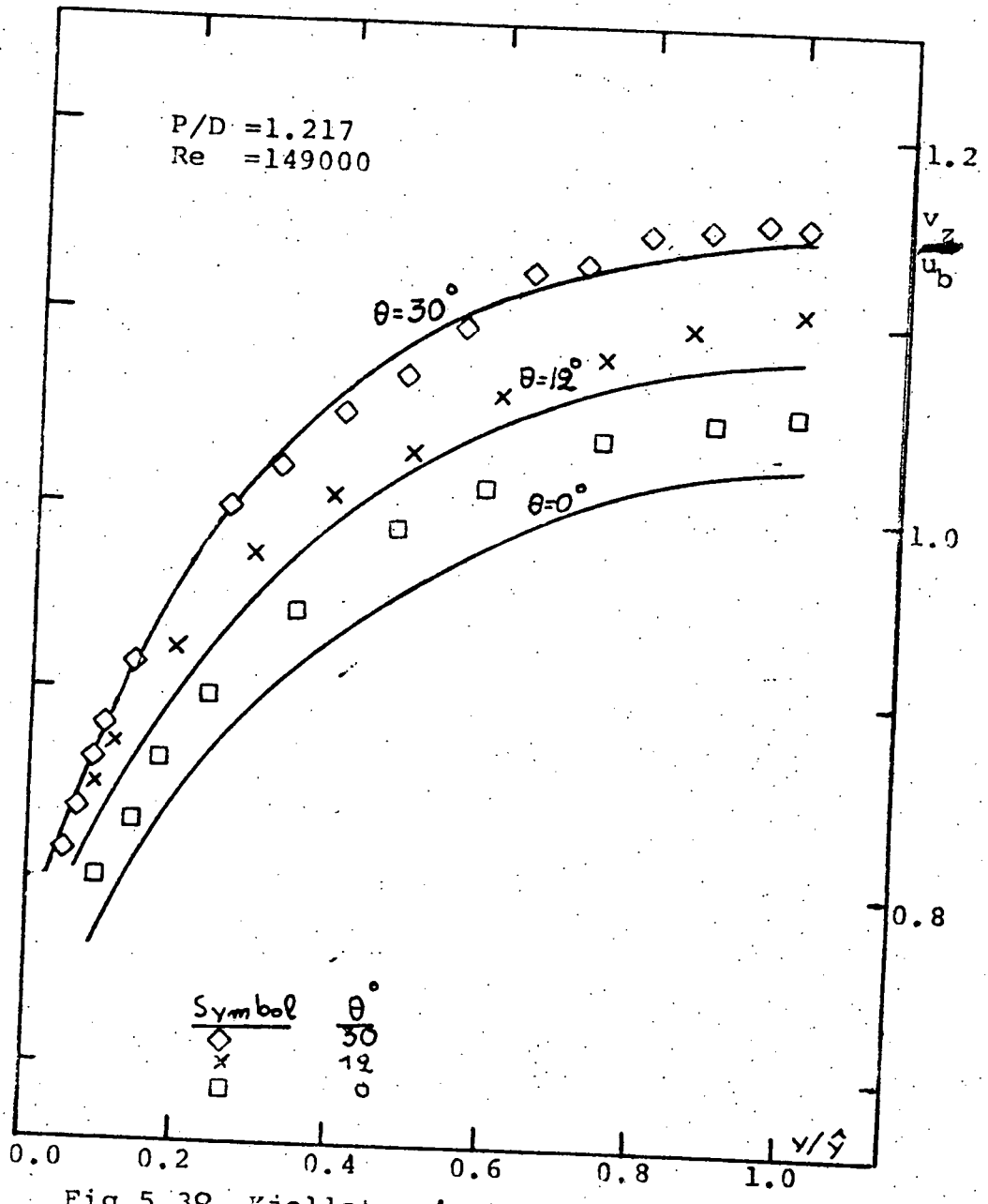


Fig 5.38. Kjellstrom's Data Prediction

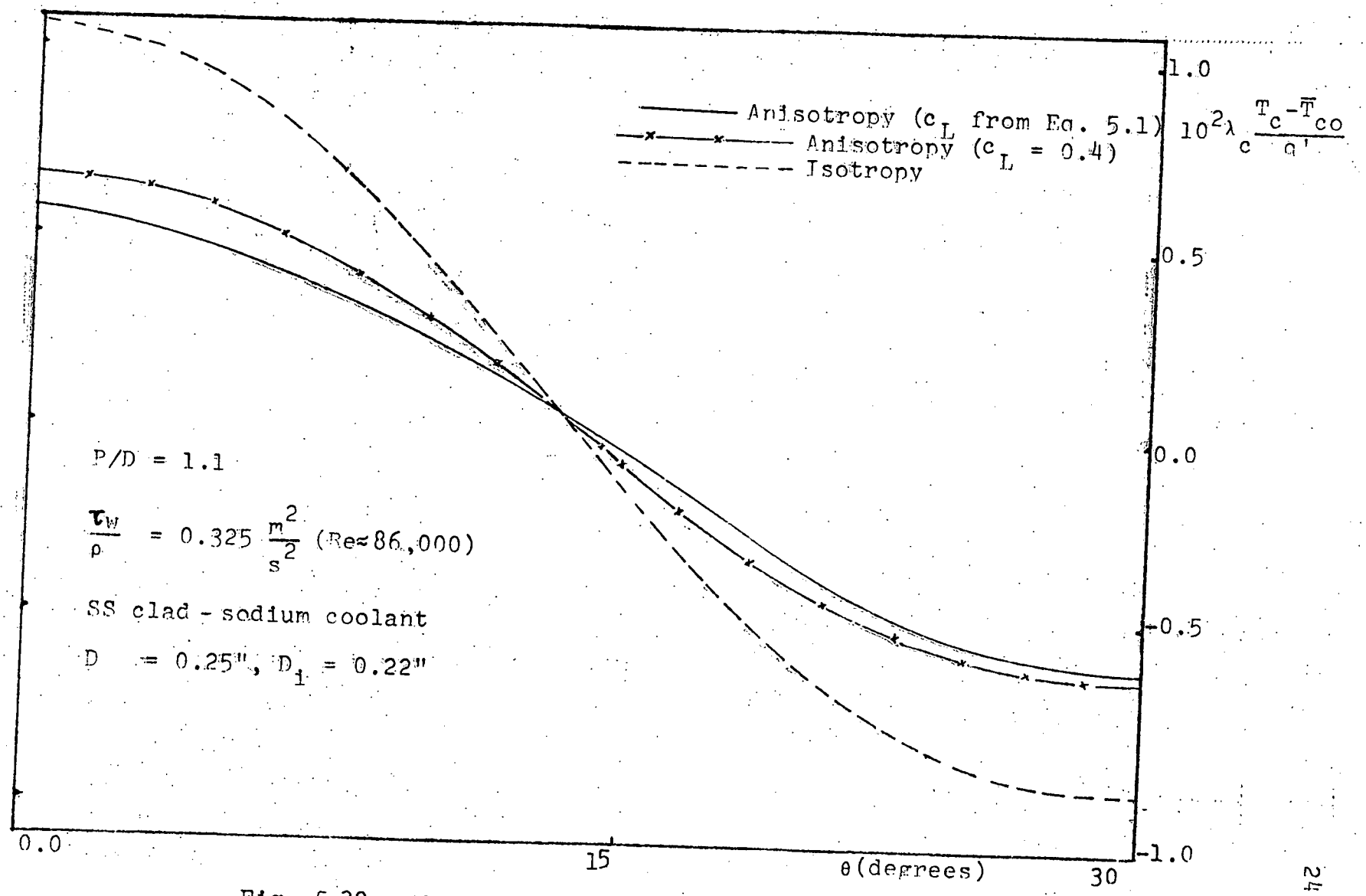


Fig. 5.39. Outside clad temperature difference

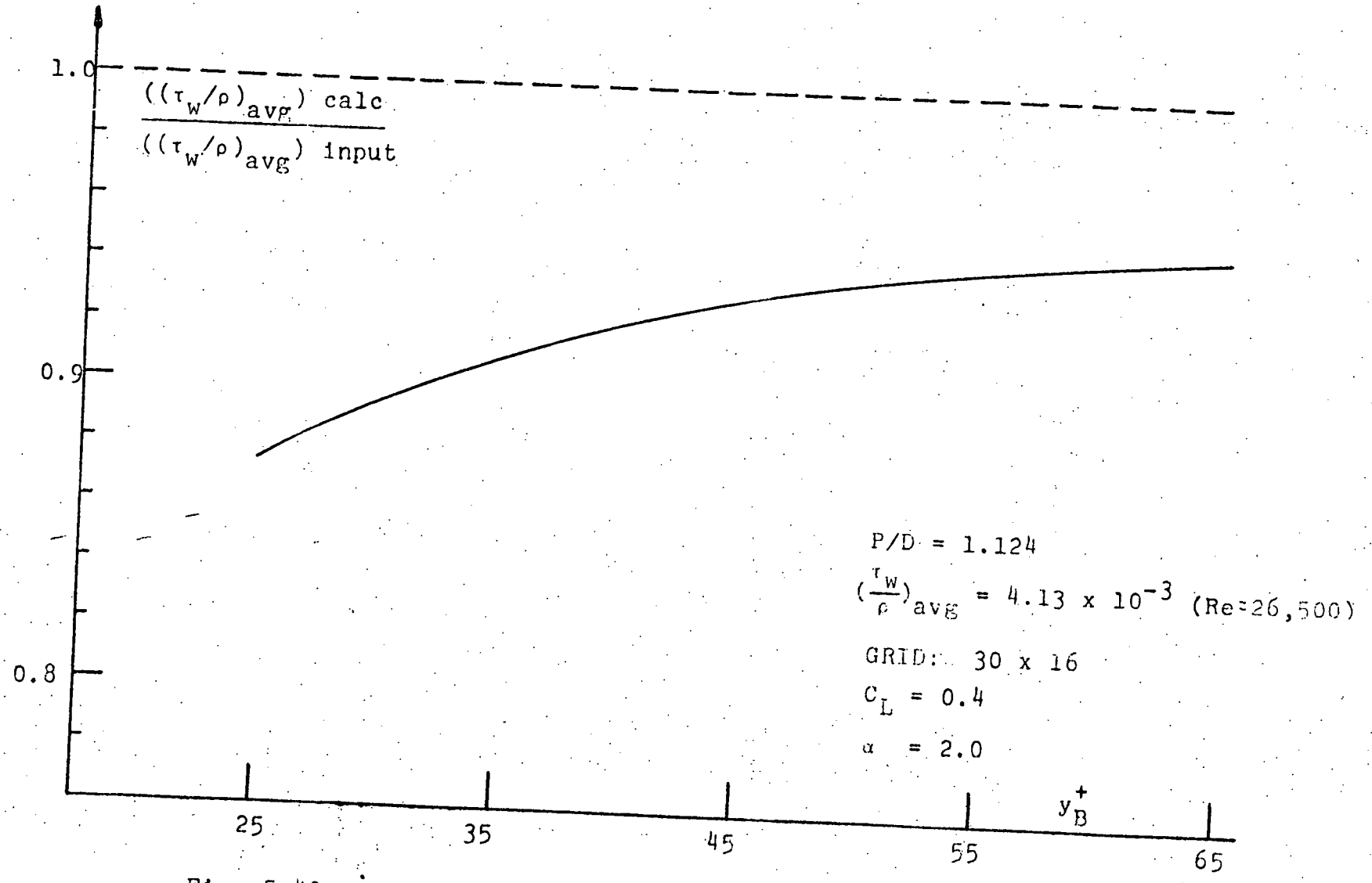


Fig. 5.40. Calculated to input average wall shear stress vs. y_B^+

APPENDIX A

THE FINITE DIFFERENCE FORM OF THE CONVECTION TERM IN FLUID FLOW

A.1 Introduction

It is known that a general way of solving for any specific quantity ϕ defined within a moving fluid is to obtain a finite difference equation from the corresponding differential equation and solve it numerically. Usually the time-independent finite difference equation has the form

$$\begin{aligned}\phi_{i,j,k} = {} & c_{i+1,j,k}\,\phi_{i+1,j,k} + c_{i,j+1,k}\,\phi_{i,j+1,k} + \\ & c_{i,j,k+1}\,\phi_{i,j,k+1} + c_{i-1,j,k}\,\phi_{i-1,j,k} + \\ & c_{i,j-1,k}\,\phi_{i,j-1,k} + c_{i,j,k-1}\,\phi_{i,j,k-1} + \\ & S_{i,j,k}.\end{aligned} \tag{A.1}$$

The system of Eq. (A.1) is often elliptic and has to be solved iteratively. On the other hand, the system is non-linear in most cases since the coefficients c_{ij} can change from iteration to iteration.

Of course the major concern is to use numerical techniques which unconditionally guarantee the convergence of Eq. (A.1). However, due to the complexity of the non-linear effects such numerical techniques are difficult to find.

One way to approach the problem is to try to guarantee the convergence of Eq. (A.1), assuming it is linear and then find ways to cope with the non-linearity. In such a case the first question to be asked is 'Is the

system of linear equations corresponding to Eq. (A.1) convergent?" This, of course, depends on the type of coefficients the $c_{i,j}$'s are.

One idea widely used [36] is that if the $c_{i,j,k}$'s obey the condition

$$|c_{i+1,j,k}| + |c_{i,j+1,k}| + |c_{i,j,k+1}| + |c_{i-1,j,k}| + |c_{i,j-1,k}| + |c_{i,j,k-1}| \leq 1 \qquad (A.2)$$

then convergence is guaranteed by a number of iteration techniques [73].

The only term in equations of the type (A.1) that create difficulties within the restriction (A.2) are the convection terms.

In the remainder of this Appendix, discussion will be limited to the convective term approximations.

Consider an orthogonal system of coordinates (x_i, x_j, x_k) and a fluid with density steady with time. Without loss of generality we will limit our discussion to the particular direction, e.g., the x_i direction.

Integrating over the control volume shown in Fig. A.1 the convection term $(CT)_i$ in x_i direction of a physical quantity, ϕ, is given by

$$(CT)_i = (\rho A)_e \, v_{ie} \, \phi_e - (\rho A)_w \, v_{iw} \, \phi_w \qquad (A.3)$$

where A is the flow per unit velocity.

v_i is the velocity along the direction i.

The natural way of approximating ϕ_e and ϕ_w as we can see from Fig. 4.1 is

$$\phi_e = \frac{\phi_E + \phi_P}{2} + O(\Delta X_e^2)$$

and

$$\phi_w = \frac{\phi_W + \phi_P}{2} + O(\Delta X_w^2) \qquad (A.4)$$

This is the central difference scheme (CDS). It can be easily shown that such a scheme does not always obey condition (A.2). It has been found that such an approximation has led to instabilities [89], [90].

To provide for an approximation obeying condition (A.2) the "up-wind difference scheme" (UDS) has been introduced [11] and has been widely used, e.g., [36], [54], [1]. In this case

$$\phi_e = \begin{cases} \phi_P + O(\frac{\Delta X_e}{2}) & \text{when } v_{ie} > 0 \\ \phi_E + O(\frac{\Delta X_e}{2}) & \text{when } v_{ie} < 0 \end{cases} \qquad (A.5)$$

and similarly for ϕ_w.

Such an approximation although it obeys condition (A.2) is less accurate than the CDS by one degree of accuracy. This means that obviously a higher number of points will be required to get accurate enough results. However, in some cases the relatively low degree (1st order) of accuracy in the UDS might lead to inaccurate results [91].

One way to overcome these drawbacks of the above two schemes is to use some mixed scheme of central differencing and upwind differencing. The codes SOLA [92] and VARR-II [43] introduce an input parameter such that

$0 \leq \alpha \leq 1$

when $\alpha = 0$ the central difference scheme is used

$\alpha = 1$ the upwind difference scheme is used.

The code TEACH [71] used the following mixed scheme. In case the diffusion is the dominant mechanism, use the central difference scheme, whereas in case the convection mechanism is dominant, use the upwind difference scheme. Even with these improvements there remain cases where UDS has to be used.

In our work we have introduced the Upwind Derivative Difference Scheme (UDDS) which is second order accurate and obeys condition (A.2).

1.2 The Upwind Derivative Difference Scheme

The second order approximation of ϕ_e and ϕ_w can be written as follows

$$\phi_e = \phi_P + \left(\frac{\partial \phi}{\partial x_i}\right)_P \frac{\Delta X_e^2}{2} + O\left(\frac{\Delta X_e^2}{4}\right)$$

$$\phi_w = \phi_P - \left(\frac{\partial \phi}{\partial x_i}\right)_P \frac{\Delta X_w}{2} + O\left(\frac{\Delta X_w^2}{4}\right) \qquad (A.6)$$

then

$$(CT)_i = \phi_P \left[(\rho A)_e v_{ie} - (\rho A)_w v_{iw}\right] + \left[(\rho A)_e v_{ie} \Delta X_e + (\rho A)_w v_{iw} \Delta X_w\right] \left(\frac{\partial \phi}{\partial x_i}\right)_P \qquad (A.7)$$

The contribution of the first term in the right hand side of Eq. (A.7) is zero due to the continuity condition. Thus the net contribution of the convection term in the i direction is

$$(CT')_i = (\rho A)_e v_{ie} \Delta X_e + (\rho A)_w v_{iw} \Delta X_w \left(\frac{\partial \phi}{\partial x_i}\right)_P \qquad (A.8)$$

let

$$C_i = (\rho A)_e v_{ie} \Delta X_e + (\rho A)_w v_{iw} \Delta X_x \qquad (A.9)$$

then

$$(CT')_i = C_i \left(\frac{\partial \phi}{\partial x_i}\right)_P \qquad (A.10)$$

we further approximate

$$\left(\frac{\partial \phi}{\partial x_i}\right)_P = \begin{cases} \dfrac{\phi_P - \phi_W}{\Delta X_w} + O(\Delta X_w) & \text{if} \quad C_i > 0 \\ \\ \dfrac{\phi_E - \phi_P}{\Delta X_e} + O(\Delta X_e) & \text{if} \quad C_i < 0 \end{cases} \qquad (A.11)$$

Under approximation (A.11), Eq. (A.10) can be written as

$$(CT')_i = (CE + CW)\phi_P - CE\phi_E - CW\phi_W \qquad (A.12)$$

where

$$CE = 0.5 \frac{|C_i| - C_i}{\Delta X_e}$$

$$CW = 0.5 \frac{|C_i| + C_i}{\Delta X_w} \qquad (A.13)$$

and CE, CW are positive numbers.

Thus, Eqs. (A.12) and (A.13) obey condition (A.2) and are second order accurate and therefore superior to the UDS from an accuracy point of view. With respect now to the rate of convergence in the actual non-linear problems, our experience of work in secondary flows in infinite triangular arrays show that the effect in the range of convergence seems to be minor.

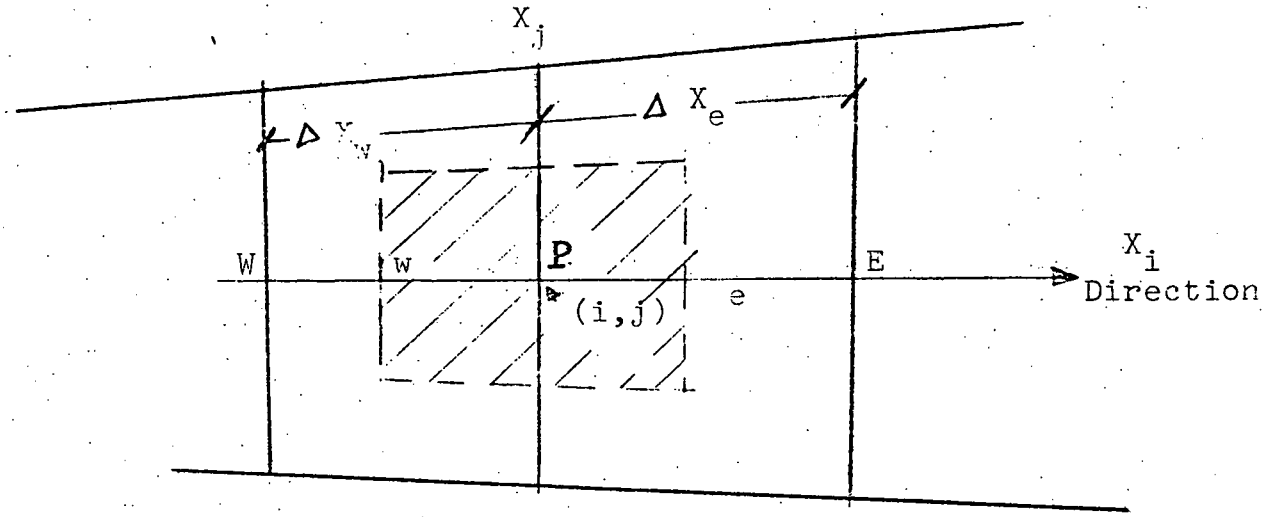


Fig. A.1. Discretization in (X_i, X_j) plane

APPENDIX B

HEAT TRANSFER CALCULATIONS

B.1 General Equations

B.1.1 The Coolant Region

The time smoothed heat transfer equation [ ] assuming time-independent properties can also be put in the general form of Eq. (3.67) i.e.:

$$\Delta \underline{V}\, T_c + \Delta \tilde{\underline{q}}_c = S^* \tag{B.1}$$

where T_c is the coolant temperature

$\underline{V}$ is the velocity vector

$$\tilde{\underline{q}}_c = (\tilde{q}_{cr}, \tilde{q}_{c\theta}) \tag{B.2a}$$

$$\tilde{q}_{cr} = - \alpha_H \frac{\partial T_c}{\partial r} + \overline{T_c' v_r'} \tag{B.2b}$$

$$\tilde{q}_{c\theta} = - \alpha_H \frac{\partial T_c}{r \partial \theta} + \overline{T_c' v_\theta'} \tag{B.2c}$$

α_H is the heat diffusivity

$$S^* = - \frac{1}{\rho c_p} \frac{4}{D_H} q_o'' \frac{\bar{v}_z}{u_b} \tag{B.3}$$

is the coolant density

c_p is the coolant specific heat

q_o'' is the average heat flux in the outside surface of the clad given by the relation

$$\bar{q}''_o = \frac{q'}{\pi D} \tag{B.4}$$

and q' is the Linear Heat Generation ratio (heat generation rate per unit length).

$\overline{T'_c v'_r}$ and $\overline{T'_c v'_\theta}$ are the turbulent diffusion terms.

A widely used approximation for modeling the turbulent diffusion terms is to treat them similarly to the turbulent momentum diffusion terms. Thus in our case we have

$$\overline{T'_c V'_r} = - \frac{\nu^T_{rz}}{\sigma_T} \frac{\partial T_c}{\partial r} \tag{B.5a}$$

$$\overline{T'_c V'_\theta} = - \frac{\nu^T_{r\theta}}{\sigma_T} \frac{\partial T_c}{r \partial \theta} \tag{B.5b}$$

where σ_T is the heat diffusitivity to momentum diffusivity ratio and in general can be a function of position and flow characteristics.

B.1.2 The Clad Region

The clad heat transfer equation also can be put in the general form of Equation (3.67) with only the diffusion term as a non zero term i.e.

$$\nabla \tilde{\underline{q}}_{c\ell} = 0 \tag{B.6}$$

where

$$\tilde{\underline{q}}_{c\ell} = (\tilde{q}_{c\ell r}, \tilde{q}_{c\ell\theta}) \tag{B.7a}$$

$$\tilde{q}_{c\ell} = -\lambda_{c\ell} \frac{\partial T_{c\ell}}{\partial r} \qquad \text{(B.7b)}$$

$$\tilde{q}_{c\ell\theta} = -\lambda_{c\ell} \frac{\partial T_c}{r \partial \theta} \qquad \text{(B.7c)}$$

and $\lambda_{c\ell}$ is the clad heat conductivity.

B.1.3 The Boundary Conditions

The boundary conditions for all the symmetry lines are written as

$$\frac{\partial T_c}{\partial n}, \frac{\partial T_{c\ell}}{\partial n} = 0 \text{ for all symmetry lines} \qquad \text{(B.8)}$$

($\underline{n}$ is the unit vector perpendicular to a symmetry line)

The boundary conditions differ at the walls where temperature gradients are developed due to heat generation in the fuel.

B.1.3.1 Outside Clad Surface

In this case the obvious boundary conditions are temperature and heat flux continuity i.e.

$$T_c = T_{c\ell\,\ell} \qquad \text{(B.9)}$$

$$-\lambda_{c\ell} \frac{\partial T_{c\ell}}{\partial r} = -\lambda_c \frac{\partial T_c}{\partial r} \qquad \text{(B.10)}$$

where λ_c is the thermal conductivity of the coolant.

B.1.3.2 Inside Clad Surface

For the inside clad region we assume heat flux uniformities i.e.

$$q''_i = - \lambda_{c\ell} \frac{\partial T_c}{\partial r} = \text{const} = \frac{q'}{\pi D_i} \qquad \text{(B.11)}$$

where D_i is the inside clad diameter

B.1.3.3 Temperature Level

Since the boundary conditions for this problem are temperature gradient conditions, the resultant solution will yield the relative temperature distribution between points but not the absolute temperature levels. To obtain the absolute temperature levels, a constant temperature is algebraically subtracted from the temperature of every point in the field. This constant temperature is obtained as the difference between the calculated temperature and a prescribed temperature at a given point, the prescribed temperature being known from physical considerations.

B.2 The Finite Difference Equations

Since Equations (B.1) and (B.6) have the general form of Equation (3.67) the procedure of obtaining finite difference equations for coolant and clad temperature is quite similar to that of flow calculations.

The control volume of Figure 4.2 is also used here and it is identical with that used for the axial velocity v_z.

The equations have also the form (4.8d) i.e.

$$DT + CT = ST \qquad (B.12)$$

where DT is the diffusion term given by the relation

$$DT = \tilde{q}_{re}A_e - \tilde{q}_{rw}A_w + \tilde{q}_{\theta n}A_n - \tilde{q}_{\theta s}A_s \qquad B.13)$$

CT is the convection term given by the relation

$$CT = \begin{cases} A_e V_{re} T_{ce} - A_w V_{rw} T_{cw} + A_n V_{\theta n} T_{cn} - A_s V_{\theta s} T_{cs} & \text{for } T_c \\ 0 \text{ for } T_{c\ell} \end{cases}$$

and ST is the source term given by the relation

$$ST = \begin{cases} -\frac{1}{\rho c_p}\frac{4}{D_H} q''_o \frac{1}{u_b}(v_z)_p & \text{for } T_c \\ 0 & \text{for } T_{c\ell} \end{cases}$$

A_e, A_w, A_n, A_s are given in Table 4.1.

B.2.1 The Diffusion Term

B.2.1.1 Coolant Region

Taking equations (B.2), finite-differencing them and applying them to Equation (B.13) we end up with

$$DT = (DE + DW + DN + DS)T_{c,i,j} - DE \cdot T_{c,i+1,j} - DW \cdot T_{c,i-1,j} - DN \cdot T_{c,i,j+1} - DS \cdot T_{c,i,j-1} \qquad (B.16)$$

where

$$DE = BE \left\{ 2\alpha_H + \frac{1}{\sigma_T} \left| \left(\nu^T_{rz}\right)_{i,j} + \left(\nu^T_{rz}\right)_{i+1,j} \right| \right\} \quad \text{(B.17a)}$$

$$DW = BW \left\{ 2\alpha_H + \frac{1}{\sigma T} \left| \left(\nu^T_{rz}\right)_{i,j} + \left(\nu^T_{rz}\right)_{i-1,j} \right| \right\} \quad \text{(B.17b)}$$

$$DN = BN \left\{ 2\alpha_H + \frac{1}{\sigma T} \left| \left(\nu^T_{\theta z}\right)_{i,j} + \left(\nu^T_{\theta z}\right)_{i+1,j} \right| \right\} \quad \text{(B.17c)}$$

$$DS = BS \left\{ 2\alpha_H + \frac{1}{\sigma T} \left| \left(\nu^T_{\theta z}\right)_{i,j} + \left(\nu^T_{\theta z}\right)_{i,j-1} \right| \right\} \quad \text{(B.17d)}$$

BE, BW, BN, BS are given from Table 4.2.

B.2.1.2 Clad Region

Working as in the previous section and taking into consideration Equations (B.7) we find finally

$$DT = (DE + DW + DN + DS)T_{c\ell,i,j} - DE \cdot T_{c\ell,i+1,j} - DWT_{c\ell,i-1,j} - DN \cdot T_{c\ell,i,j+1} - DST_{c\ell,i,j-1} \quad \text{(B.18)}$$

where

$$\begin{aligned} DE &= 2BE\ \lambda_{c\ell} \\ DW &= 2 \cdot BW\ \lambda_{c\ell} \\ DN &= 2 \cdot BN\ \lambda_{c\ell} \\ DS &= 2 \cdot BS\ \lambda_{c\ell} \end{aligned} \quad \text{(B.19)}$$

where

BE, BW, BN, BS are given in Table 4.2.

B.2.2 The Convection Term

For the clad temperature there is no convection term.

For the coolant temperature the convection term is identical in form with that for axial velocity v_z, i.e.

$$CT = [CE + CW + CN + CS]\, T_{c,i,j} - CE \cdot T_{c,i+1,j} - CW \cdot T_{c,i-1,j} - CN \cdot T_{c,i,j+1} - CS \cdot T_{c,i,j-1} \quad \text{(B.20)}$$

where

CE, CW, CN, CS are given by Table 4.3.

B.2.3 The Source Term

For the clad temperature there is no source term.

For the coolant temperature the source term is given by (B.15).

B.2.4 General Form of FDE

Equation (B.12) can be written taking into consideration Equations (B.16), (B.17), (B.18), (B.19), and (B.20) as

$$AP \cdot \phi_P = AE \cdot \phi_E + AN\phi_N + AS\phi_S + AS\phi_S + SP \quad \text{(B.21)}$$

where

$$AE = CE + DE$$
$$AW = DW + DW \qquad (B.22)$$
$$AN = DN + DN$$
$$AS = CS + DS$$

For clad temperature CE = CW = CN = CS = ST = 0

For the coolant temperature CE, CW, CN, CS, DE, DW, DN, DS, ST are given from Table 4.3 and Equations (B.17), (B.19), and (B.15).

For the clad temperature DE, DW, DN, DS are given by Equation (B.19).

B.2.5 Numerical Solution

It is evident that Equation (B.22) has the same form as the general FDE for flow calculations (4.44). Thus we can apply the same numerical techniques we applied for flow calculations i.e. the Line Iterations scheme (4.32) or the successive relaxation method (§ 4.3.3). Although the computer code built in the present work has both options present calculations are performed with Line Iteration Technique.

APPENDIX C

EXPERIMENTAL DATA

In this appendix the experimental grid and the quantities measured are tabulated.

GRID FOR AXIAL VELOCITY MEASUREMENT

I=	1	2	3	4	5	6	7	8	9	10	11	12	TETA
J													
11	0.0	4.092E-01	4.547E-01	5.001E-01	5.455E-01	5.910E-01	6.364E-01	6.819E-01	7.273E-01	7.728E-01	8.182E-01	9.637E-01	30.000
10	0.0	3.459E-01	3.962E-01	4.465E-01	4.969E-01	5.472E-01	5.975E-01	6.478E-01	6.981E-01	7.484E-01	7.987E-01	8.491E-01	27.000
9	0.0	2.759E-01	3.316E-01	3.873E-01	4.430E-01	4.987E-01	5.544E-01	6.101E-01	6.658E-01	7.215E-01	7.772E-01	8.329E-01	24.000
8	0.0	2.613E-01	3.229E-01	3.844E-01	4.460E-01	5.076E-01	5.691E-01	6.307E-01	6.922E-01	7.538E-01	8.153E-01	8.769E-01	21.000
7	0.0	1.858E-01	2.545E-01	3.223E-01	3.901E-01	4.578E-01	5.256E-01	5.934E-01	6.611E-01	7.289E-01	7.957E-01	8.645E-01	18.000
6	0.0	1.103E-01	1.845E-01	2.586E-01	3.327E-01	4.069E-01	4.810E-01	5.552E-01	6.293E-01	7.034E-01	7.776E-01	8.517E-01	15.000
5	0.0	1.162E-01	1.965E-01	2.769E-01	3.572E-01	4.376E-01	5.179E-01	5.983E-01	6.786E-01	7.590E-01	8.393E-01	9.196E-01	12.000
4	0.0	1.404E-01	2.263E-01	3.123E-01	3.983E-01	4.842E-01	5.702E-01	6.561E-01	7.421E-01	8.281E-01	9.140E-01	1.000E 00	9.000
3	0.0	9.504E-02	1.855E-01	2.760E-01	3.665E-01	4.570E-01	5.475E-01	6.380E-01	7.285E-01	8.190E-01	9.095E-01	1.000E 00	6.000
2	0.0	1.585E-01	2.523E-01	3.436E-01	4.393E-01	5.327E-01	6.262E-01	7.196E-01	8.131E-01	9.065E-01	1.000E 00	0.0	3.000
1	0.0	1.496E-01	2.441E-01	3.386E-01	4.331E-01	5.276E-01	6.220E-01	7.165E-01	8.110E-01	9.055E-01	1.000E 00	0.0	0.0

I=	13	14	15	TETA
J				
11	9.091E-01	9.546E-01	1.000E 00	3.000E 01
10	8.994E-01	9.497E-01	1.000E 00	2.700E 01
9	8.886E-01	9.443E-01	1.000E 00	2.400E 01
8	9.364E-01	1.000E 00	0.0	2.100E 01
7	9.322E-01	1.000E 00	0.0	1.800E 01
6	9.259E-01	1.000E 00	0.0	1.500E 01
5	1.000E 00	0.0	0.0	1.200E 01
4	0.0	0.0	0.0	9.000E 00
3	0.0	0.0	0.0	6.000E 00
2	0.0	0.0	0.0	3.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 26,500 , 65,000
LENGTH/HYDRAULIC DIAMETER= 113 , 46

GRID FOR AXIAL VELOCITY MEASUREMENT

I=	1	2	3	4	5	6	7	8	9	10	11	12	TETA
J													
6	0.0	4.092E-01	4.547E-01	5.001E-01	5.455E-01	5.910E-01	6.364E-01	6.819E-01	7.273E-01	7.728E-01	8.182E-01	8.637E-01	30.000
5	0.0	2.759E-01	3.316E-01	3.873E-01	4.430E-01	4.987E-01	5.544E-01	6.101E-01	6.658E-01	7.215E-01	7.772E-01	8.329E-01	24.000
4	0.0	1.868E-01	2.545E-01	3.223E-01	3.901E-01	4.578E-01	5.256E-01	5.934E-01	6.611E-01	7.289E-01	7.967E-01	8.645E-01	18.000
3	0.0	1.162E-01	1.965E-01	2.769E-01	3.572E-01	4.376E-01	5.179E-01	5.983E-01	6.786E-01	7.590E-01	8.393E-01	9.196E-01	12.000
2	0.0	9.504E-02	1.855E-01	2.760E-01	3.665E-01	4.570E-01	5.475E-01	6.380E-01	7.285E-01	8.190E-01	9.095E-01	1.000E 00	6.000
1	0.0	1.496E-01	2.441E-01	3.386E-01	4.331E-01	5.276E-01	6.220E-01	7.165E-01	8.110E-01	9.055E-01	1.000E 00	0.0	0.0

I=	13	14	15	TETA
J				
6	9.091E-01	9.546E-01	1.000E 00	3.000E 01
5	8.886E-01	9.443E-01	1.000E 00	2.400E 01
4	9.322E-01	1.000E 00	0.0	1.800E 01
3	1.000E 00	0.0	0.0	1.200E 01
2	0.0	0.0	0.0	6.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 26,500 , 65,000
LENGTH/HYDRAULIC DIAMETER= 15

GRID FOR AXIAL VELOCITY MEASUREMENT

I=	1	2	3	4	5	6	7	8	9	10	11	12	TETA
J													
11	0.0	4.092E-01	4.547E-01	5.001E-01	5.455E-01	5.910E-01	6.364E-01	6.819E-01	7.273E-01	7.728E-01	8.182E-01	8.637E-01	30.000
10	0.0	3.459E-01	3.962E-01	4.465E-01	4.969E-01	5.472E-01	5.975E-01	6.478E-01	6.981E-01	7.484E-01	7.987E-01	8.491E-01	27.000
9	0.0	3.316E-01	3.873E-01	4.430E-01	4.987E-01	5.544E-01	6.101E-01	6.658E-01	7.215E-01	7.772E-01	8.329E-01	8.886E-01	24.000
8	0.0	2.613E-01	3.229E-01	3.844E-01	4.460E-01	5.076E-01	5.691E-01	6.307E-01	6.922E-01	7.538E-01	8.153E-01	8.769E-01	21.000
7	0.0	1.868E-01	2.545E-01	3.223E-01	3.901E-01	4.578E-01	5.256E-01	5.934E-01	6.611E-01	7.289E-01	7.967E-01	8.645E-01	18.000
6	0.0	1.845E-01	2.586E-01	3.327E-01	4.069E-01	4.810E-01	5.552E-01	6.293E-01	7.034E-01	7.776E-01	8.517E-01	9.259E-01	15.000
5	0.0	1.162E-01	1.965E-01	2.769E-01	3.572E-01	4.376E-01	5.179E-01	5.983E-01	6.786E-01	7.590E-01	8.393E-01	9.196E-01	12.000
4	0.0	1.404E-01	2.263E-01	3.123E-01	3.983E-01	4.842E-01	5.702E-01	6.561E-01	7.421E-01	8.281E-01	9.140E-01	1.000E 00	9.000
3	0.0	9.504E-02	1.855E-01	2.760E-01	3.665E-01	4.570E-01	5.475E-01	6.380E-01	7.285E-01	8.190E-01	9.095E-01	1.000E 00	6.000
2	0.0	1.589E-01	2.523E-01	3.458E-01	4.393E-01	5.327E-01	6.262E-01	7.196E-01	8.131E-01	9.065E-01	1.000E 00	0.0	3.000
1	0.0	1.496E-01	2.441E-01	3.386E-01	4.331E-01	5.276E-01	6.220E-01	7.165E-01	8.110E-01	9.055E-01	1.000E 00	0.0	0.0

I=	13	14	15	TETA
J				
11	9.091E-01	9.546E-01	1.000E 00	3.000E 01
10	8.994E-01	9.497E-01	1.000E 00	2.700E 01
9	9.443E-01	1.000E 00	0.0	2.400E 01
8	9.384E-01	1.000E 00	0.0	2.100E 01
7	9.322E-01	1.000E 00	0.0	1.800E 01
6	1.000E 00	0.0	0.0	1.500E 01
5	1.000E 00	0.0	0.0	1.200E 01
4	0.0	0.0	0.0	9.000E 00
3	0.0	0.0	0.0	6.000E 00
2	0.0	0.0	0.0	3.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 8924.
LENGTH/HYDRAULIC DIAMETER= 113

AXIAL VELOCITY/MAX VELOCITY

J	I= 1	2	3	4	5	6	7	8	9	10	11	12	TETA
11	0.0	8.771E-01	9.096E-01	9.173E-01	9.400E-01	9.504E-01	9.592E-01	9.712E-01	9.789E-01	9.861E-01	9.885E-01	9.928E-01	30.000
10	0.0	8.489E-01	8.693E-01	8.861E-01	9.017E-01	9.197E-01	9.306E-01	9.429E-01	9.544E-01	9.694E-01	9.796E-01	9.825E-01	27.000
9	0.0	8.010E-01	8.225E-01	8.429E-01	8.633E-01	8.825E-01	8.993E-01	9.149E-01	9.293E-01	9.377E-01	9.544E-01	9.568E-01	24.000
8	0.0	7.818E-01	8.038E-01	8.266E-01	8.477E-01	8.681E-01	8.849E-01	9.017E-01	9.149E-01	9.233E-01	9.321E-01	9.448E-01	21.000
7	0.0	7.242E-01	7.638E-01	7.866E-01	8.106E-01	8.341E-01	8.525E-01	8.705E-01	8.873E-01	9.041E-01	9.146E-01	9.221E-01	18.000
6	0.0	6.427E-01	7.026E-01	7.410E-01	7.674E-01	7.945E-01	8.165E-01	8.405E-01	8.580E-01	8.724E-01	8.897E-01	8.986E-01	15.000
5	0.0	6.307E-01	6.906E-01	7.283E-01	7.554E-01	7.814E-01	8.043E-01	8.297E-01	8.456E-01	8.620E-01	8.765E-01	8.849E-01	12.000
4	0.0	6.391E-01	6.871E-01	7.206E-01	7.511E-01	7.748E-01	7.998E-01	8.177E-01	8.345E-01	8.453E-01	8.566E-01	8.597E-01	9.000
3	0.0	6.699E-01	6.899E-01	6.859E-01	7.196E-01	7.444E-01	7.698E-01	7.914E-01	8.110E-01	8.259E-01	8.362E-01	8.448E-01	6.000
2	0.0	6.139E-01	6.643E-01	6.988E-01	7.266E-01	7.530E-01	7.775E-01	7.974E-01	8.106E-01	8.221E-01	8.305E-01	0.0	3.000
1	0.0	6.106E-01	6.595E-01	6.978E-01	7.266E-01	7.530E-01	7.746E-01	7.980E-01	8.032E-01	8.163E-01	8.273E-01	0.0	0.0

J	I= 13	14	15	TETA
11	9.962E-01	9.983E-01	1.000E 00	3.000E 01
10	9.916E-01	9.952E-01	9.969E-01	2.700E 01
9	9.698E-01	9.727E-01	9.789E-01	2.400E 01
8	9.496E-01	9.537E-01	0.0	2.100E 01
7	9.293E-01	9.336E-01	0.0	1.800E 01
6	9.072E-01	9.122E-01	0.0	1.500E 01
5	8.878E-01	0.0	0.0	1.200E 01
4	0.0	0.0	0.0	9.000E 00
3	0.0	0.0	0.0	6.000E 00
2	0.0	0.0	0.0	3.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 26549.
LENGTH/HYDRAULIC DIAMETER= 1.13

TURB AXIAL VELOCITY/MAX VELOCITY

I=	1	2	3	4	5	6	7	8	9	10	11	12	TET
J													
11	0.0	6.579E-02	6.334E-02	6.236E-02	5.670E-02	5.350E-02	5.102E-02	4.654E-02	4.354E-02	4.001E-02	3.849E-02	3.798E-02	30.000
10	0.0	7.068E-02	6.921E-02	6.701E-02	6.383E-02	6.089E-02	5.843E-02	5.498E-02	5.201E-02	4.804E-02	4.504E-02	4.203E-02	27.000
9	0.0	7.555E-02	7.312E-02	7.068E-02	6.921E-02	6.579E-02	6.334E-02	5.990E-02	5.695E-02	5.473E-02	5.003E-02	4.604E-02	24.000
8	0.0	7.677E-02	7.555E-02	7.312E-02	6.970E-02	6.726E-02	6.334E-02	5.966E-02	5.695E-02	5.300E-02	4.854E-02	4.604E-02	21.000
7	0.0	8.285E-02	7.799E-02	7.555E-02	7.190E-02	6.970E-02	6.628E-02	6.211E-02	5.843E-02	5.473E-02	4.903E-02	4.754E-02	18.000
6	0.0	9.982E-02	7.498E-02	7.799E-02	7.312E-02	7.190E-02	6.799E-02	6.211E-02	5.892E-02	5.646E-02	5.102E-02	4.804E-02	15.000
5	0.0	9.740E-02	8.164E-02	7.361E-02	7.312E-02	6.872E-02	6.383E-02	6.136E-02	5.843E-02	5.201E-02	4.903E-02	4.504E-02	12.000
4	0.0	8.528E-02	7.434E-02	7.019E-02	6.726E-02	6.579E-02	6.138E-02	5.794E-02	5.448E-02	4.854E-02	4.479E-02	4.354E-02	9.000
3	0.0	9.498E-02	7.555E-02	7.068E-02	6.603E-02	6.457E-02	6.236E-02	5.843E-02	5.399E-02	4.754E-02	4.102E-02	4.153E-02	6.000
2	0.0	8.285E-02	7.214E-02	6.701E-02	6.481E-02	6.138E-02	5.744E-02	5.251E-02	4.504E-02	4.077E-02	3.900E-02	0.0	3.000
1	0.0	8.528E-02	7.312E-02	6.701E-02	6.334E-02	5.966E-02	5.597E-02	4.854E-02	4.354E-02	3.925E-02	3.798E-02	0.0	0.0

I=	13	14	15	TETA
J				
11	3.722E-02	3.722E-02	3.722E-02	3.000E 01
10	3.976E-02	3.849E-02	3.798E-02	2.700E 01
9	4.354E-02	4.304E-02	4.153E-02	2.400E 01
8	4.404E-02	4.329E-02	0.0	2.100E 01
7	4.554E-02	4.454E-02	0.0	1.800E 01
6	4.554E-02	4.404E-02	0.0	1.500E 01
5	4.354E-02	0.0	0.0	1.200E 01
4	0.0	0.0	0.0	9.000E 00
3	0.0	0.0	0.0	6.000E 00
2	0.0	0.0	0.0	3.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 26549.
LENGTH/HYDRAULIC DIAMETER= 113.

AXIAL VELOCITY/MAX VELOCITY

I=	1	2	3	4	5	6	7	8	9	10	11	12	TETA
J													
11	0.0	8.425E-01	8.969E-01	9.089E-01	9.235E-01	9.329E-01	9.441E-01	9.544E-01	9.619E-01	9.664E-01	9.736E-01	9.760E-01	30.000
10	0.0	8.441E-01	8.657E-01	8.873E-01	8.993E-01	9.096E-01	9.223E-01	9.329E-01	9.448E-01	9.528E-01	9.616E-01	9.650E-01	27.000
9	0.0	8.065E-01	8.285E-01	8.513E-01	8.698E-01	8.801E-01	8.969E-01	9.089E-01	9.237E-01	9.329E-01	9.408E-01	9.472E-01	24.000
8	0.0	7.794E-01	8.070E-01	8.307E-01	8.489E-01	8.657E-01	8.796E-01	8.969E-01	9.106E-01	9.168E-01	9.261E-01	9.314E-01	21.000
7	0.0	7.266E-01	7.578E-01	7.842E-01	8.082E-01	8.285E-01	8.446E-01	8.633E-01	8.813E-01	8.875E-01	8.974E-01	9.050E-01	18.000
6	0.0	6.487E-01	6.990E-01	7.290E-01	7.578E-01	7.842E-01	8.070E-01	8.261E-01	8.405E-01	8.552E-01	8.691E-01	8.265E-01	15.000
5	0.0	6.131E-01	6.811E-01	7.146E-01	7.415E-01	7.674E-01	7.890E-01	8.129E-01	8.249E-01	8.367E-01	8.465E-01	8.534E-01	12.000
4	0.0	6.131E-01	6.787E-01	7.074E-01	7.338E-01	7.574E-01	7.736E-01	7.938E-01	8.086E-01	8.193E-01	8.288E-01	8.341E-01	9.000
3	0.0	5.786E-01	6.403E-01	6.775E-01	7.050E-01	7.276E-01	7.530E-01	7.679E-01	7.861E-01	7.986E-01	8.060E-01	8.114E-01	6.000
2	0.0	6.139E-01	6.571E-01	6.882E-01	7.132E-01	7.348E-01	7.564E-01	7.705E-01	7.842E-01	7.926E-01	7.952E-01	0.0	3.000
1	0.0	6.115E-01	6.564E-01	6.875E-01	7.122E-01	7.343E-01	7.554E-01	7.707E-01	7.806E-01	7.904E-01	7.918E-01	0.0	0.0

I=	13	14	15	TETA
J				
11	9.794E-01	9.799E-01	9.806E-01	3.000E 01
10	9.700E-01	9.736E-01	9.760E-01	2.700E 01
9	9.544E-01	9.566E-01	9.578E-01	2.400E 01
8	9.329E-01	9.353E-01	0.0	2.100E 01
7	9.089E-01	9.113E-01	0.0	1.800E 01
6	8.832E-01	8.849E-01	0.0	1.500E 01
5	8.578E-01	0.0	0.0	1.200E 01
4	0.0	0.0	0.0	9.000E 00
3	0.0	0.0	0.0	6.000E 00
2	0.0	0.0	0.0	3.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 26163.
LENGTH/HYDRAULIC DIAMETER= 46

TURB AXIAL VELOCITY/MAX VELOCITY

I=	1	2	3	4	5	6	7	8	9	10	11	12	TETA
J													
11	0.0	6.579E-02	6.211E-02	5.966E-02	5.720E-02	5.350E-02	4.978E-02	4.854E-02	4.404E-02	4.153E-02	3.976E-02	3.773E-02	30.00
10	0.0	7.068E-02	6.824E-02	6.579E-02	6.285E-02	5.966E-02	5.720E-02	5.473E-02	5.102E-02	4.729E-02	4.479E-02	4.228E-02	27.00
9	0.0	7.434E-02	7.312E-02	6.824E-02	6.701E-02	6.457E-02	6.089E-02	5.843E-02	5.350E-02	5.226E-02	4.679E-02	4.504E-02	24.00
8	0.0	7.555E-02	7.190E-02	7.068E-02	6.701E-02	6.285E-02	6.089E-02	5.744E-02	5.473E-02	5.102E-02	4.604E-02	4.479E-02	21.00
7	0.0	8.289E-02	7.677E-02	7.190E-02	6.946E-02	6.701E-02	6.383E-02	5.990E-02	5.843E-02	5.275E-02	4.878E-02	4.729E-02	18.00
6	0.0	8.771E-02	8.285E-02	7.677E-02	7.190E-02	6.946E-02	6.750E-02	6.408E-02	5.990E-02	5.473E-02	5.152E-02	4.754E-02	15.00
5	0.0	8.892E-02	7.921E-02	7.312E-02	7.019E-02	6.726E-02	6.383E-02	6.040E-02	5.621E-02	5.251E-02	4.854E-02	4.454E-02	12.00
4	0.0	8.285E-02	7.068E-02	6.946E-02	6.579E-02	6.211E-02	6.040E-02	5.597E-02	5.176E-02	4.804E-02	4.354E-02	4.253E-02	9.00
3	0.0	9.013E-02	7.555E-02	6.824E-02	6.579E-02	6.187E-02	5.892E-02	5.646E-02	5.152E-02	4.604E-02	4.153E-02	4.001E-02	6.00
2	0.0	9.526E-02	7.190E-02	6.457E-02	6.236E-02	5.843E-02	5.350E-02	4.854E-02	4.429E-02	4.001E-02	3.900E-02	0.0	3.00
1	0.0	8.285E-02	7.190E-02	6.457E-02	6.162E-02	5.720E-02	5.201E-02	4.604E-02	4.253E-02	3.925E-02	3.798E-02	0.0	0.0

I=	13	14	15	TETA
J				
11	3.594E-02	3.492E-02	3.594E-02	3.000E 01
10	4.001E-02	3.900E-02	3.696E-02	2.700E 01
9	4.253E-02	4.228E-02	4.102E-02	2.400E 01
8	4.404E-02	4.329E-02	0.0	2.100E 01
7	4.479E-02	4.354E-02	0.0	1.800E 01
6	4.479E-02	4.404E-02	0.0	1.500E 01
5	4.429E-02	0.0	0.0	1.200E 01
4	0.0	0.0	0.0	9.000E 00
3	0.0	0.0	0.0	6.000E 00
2	0.0	0.0	0.0	3.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 26163.
LENGTH/HYDRAULIC DIAMETER= 46

AXIAL VELOCITY/MAX VELOCITY

I=	1	2	3	4	5	6	7	8	9	10	11	12	IET
J													
6	0.0	7.625E-01	7.722E-01	7.830E-01	7.914E-01	8.010E-01	8.091E-01	8.312E-01	8.417E-01	8.470E-01	8.505E-01	8.645E-01	30.00
5	0.0	7.130E-01	7.278E-01	7.458E-01	7.638E-01	8.211E-01	7.926E-01	8.165E-01	8.225E-01	8.365E-01	8.432E-01	8.469E-01	24.00
4	0.0	6.559E-01	6.859E-01	7.106E-01	7.386E-01	7.602E-01	7.926E-01	8.058E-01	8.168E-01	8.285E-01	8.345E-01	8.417E-01	18.00
3	0.0	5.935E-01	6.293E-01	6.655E-01	7.170E-01	7.341E-01	7.554E-01	7.703E-01	7.866E-01	7.960E-01	8.017E-01	8.083E-01	12.00
2	0.0	5.210E-01	5.911E-01	6.293E-01	6.703E-01	6.930E-01	7.163E-01	7.357E-01	7.513E-01	7.604E-01	7.676E-01	7.681E-01	6.00
1	0.0	5.470E-01	6.091E-01	6.427E-01	6.609E-01	6.914E-01	7.115E-01	7.285E-01	7.400E-01	7.470E-01	7.506E-01	0.0	0.0

I=	13	14	15	IETA
J				
6	8.724E-01	8.777E-01	8.825E-01	3.000E 01
5	8.585E-01	8.659E-01	8.710E-01	2.400E 01
4	8.417E-01	8.475E-01	0.0	1.800E 01
3	8.062E-01	0.0	0.0	1.200E 01
2	0.0	0.0	0.0	6.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 24018.
LENGTH/HYDRAULIC DIAMETER= 15

TURB AXIAL VELOCITY/MAX VELOCITY

I=	1	2	3	4	5	6	7	8	9	10	11	12	TET-
J													
6	0.0	5.621E-02	5.473E-02	5.251E-02	5.102E-02	4.903E-02	4.854E-02	4.554E-02	4.354E-02	4.203E-02	3.976E-02	3.849E-02	30.000
5	0.0	6.579E-02	6.457E-02	6.211E-02	5.917E-02	5.843E-02	5.597E-02	5.350E-02	5.152E-02	4.779E-02	4.604E-02	4.304E-02	24.000
4	0.0	7.068E-02	6.824E-02	6.334E-02	6.211E-02	5.843E-02	5.473E-02	5.275E-02	4.978E-02	4.354E-02	4.052E-02	3.543E-02	18.000
3	0.0	8.771E-02	7.068E-02	6.457E-02	6.089E-02	5.892E-02	5.473E-02	5.003E-02	4.354E-02	4.102E-02	3.722E-02	3.594E-02	12.000
2	0.0	9.013E-02	7.068E-02	6.138E-02	5.843E-02	5.670E-02	5.251E-02	4.704E-02	4.253E-02	3.900E-02	3.594E-02	3.492E-02	6.000
1	0.0	8.285E-02	6.701E-02	5.843E-02	5.720E-02	5.300E-02	4.754E-02	4.153E-02	3.849E-02	3.389E-02	3.337E-02	0.0	0.0

I=	13	14	15	TETA
J				
6	3.594E-02	3.594E-02	3.466E-02	3.000E 01
5	4.228E-02	3.976E-02	3.798E-02	2.400E 01
4	3.696E-02	3.594E-02	0.0	1.800E 01
3	3.071E-02	0.0	0.0	1.200E 01
2	0.0	0.0	0.0	6.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 24016.
LENGTH/HYDRAULIC DIAMETER= 15

AXIAL VELOCITY/MAX VELOCITY

I=	1	2	3	4	5	6	7	8	9	10	11	12	TETA
J													
11	0.0	8.939E-01	9.098E-01	9.315E-01	9.402E-01	9.547E-01	9.628E-01	9.669E-01	9.723E-01	9.751E-01	9.864E-01	9.905E-01	30.000
10	0.0	8.445E-01	8.704E-01	8.885E-01	9.016E-01	9.175E-01	9.306E-01	9.456E-01	9.592E-01	9.642E-01	9.705E-01	9.823E-01	27.000
9	0.0	7.978E-01	8.205E-01	8.445E-01	8.645E-01	8.817E-01	8.994E-01	9.148E-01	9.279E-01	9.293E-01	9.479E-01	9.455E-01	24.000
8	0.0	7.743E-01	7.947E-01	8.218E-01	8.450E-01	8.676E-01	8.826E-01	9.012E-01	9.143E-01	9.225E-01	9.238E-01	9.461E-01	21.000
7	0.0	7.208E-01	7.534E-01	7.693E-01	8.024E-01	8.350E-01	8.554E-01	8.704E-01	8.867E-01	9.007E-01	9.134E-01	9.157E-01	18.000
6	0.0	6.591E-01	7.026E-01	7.316E-01	7.675E-01	7.942E-01	8.264E-01	8.418E-01	8.613E-01	8.740E-01	8.830E-01	8.867E-01	15.000
5	0.0	6.510E-01	6.904E-01	7.276E-01	7.584E-01	7.865E-01	8.087E-01	8.286E-01	8.481E-01	8.613E-01	8.767E-01	8.844E-01	12.000
4	0.0	6.520E-01	6.877E-01	7.239E-01	7.525E-01	7.761E-01	7.960E-01	8.191E-01	8.336E-01	8.490E-01	8.572E-01	8.676E-01	9.000
3	0.0	5.934E-01	6.469E-01	6.806E-01	7.176E-01	7.454E-01	7.647E-01	7.879E-01	8.033E-01	8.268E-01	8.323E-01	8.486E-01	6.000
2	0.0	6.247E-01	6.677E-01	6.981E-01	7.298E-01	7.579E-01	7.811E-01	7.978E-01	8.146E-01	8.223E-01	8.327E-01	0.0	3.000
1	0.0	6.238E-01	6.716E-01	7.033E-01	7.316E-01	7.575E-01	7.811E-01	7.983E-01	8.073E-01	8.150E-01	8.241E-01	0.0	0.0

I=	13	14	15	TETA
J				
11	9.959E-01	9.955E-01	1.000E 00	3.000E 01
10	9.878E-01	9.932E-01	1.000E 00	2.700E 01
9	9.678E-01	9.746E-01	9.787E-01	2.400E 01
8	9.547E-01	9.610E-01	0.0	2.100E 01
7	9.325E-01	9.370E-01	0.0	1.800E 01
6	9.066E-01	9.143E-01	0.0	1.500E 01
5	8.921E-01	0.0	0.0	1.200E 01
4	0.0	0.0	0.0	9.000E 00
3	0.0	0.0	0.0	6.000E 00
2	0.0	0.0	0.0	3.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 64899.
LENGTH/HYDRAULIC DIAMETER= 113

TURBULENT AXIAL VELOCITY / MAX VEL

I=	1	2	3	4	5	6	7	8	9	10	11	12	TETA
J													
11	0.0	6.876E-02	6.454E-02	6.265E-02	5.981E-02	5.601E-02	5.266E-02	5.024E-02	4.684E-02	4.439E-02	4.241E-02	4.241E-02	30.000
10	0.0	7.483E-02	7.297E-02	6.923E-02	6.689E-02	6.359E-02	5.981E-02	5.696E-02	5.314E-02	5.024E-02	4.782E-02	4.635E-02	27.000
9	0.0	7.623E-02	7.530E-02	7.390E-02	7.157E-02	6.923E-02	6.548E-02	6.076E-02	5.791E-02	5.410E-02	5.073E-02	4.830E-02	24.000
8	0.0	7.530E-02	7.390E-02	7.250E-02	7.063E-02	6.829E-02	6.548E-02	6.171E-02	5.744E-02	5.410E-02	5.073E-02	4.830E-02	21.000
7	0.0	7.250E-02	7.297E-02	7.250E-02	7.157E-02	6.923E-02	6.689E-02	6.218E-02	5.791E-02	5.553E-02	5.217E-02	4.928E-02	18.000
6	0.0	6.923E-02	7.017E-02	7.063E-02	6.923E-02	6.829E-02	6.736E-02	6.265E-02	5.981E-02	5.601E-02	5.314E-02	4.879E-02	15.000
5	0.0	6.829E-02	6.923E-02	6.783E-02	6.689E-02	6.689E-02	6.454E-02	5.981E-02	5.744E-02	5.266E-02	4.928E-02	4.635E-02	12.000
4	0.0	6.548E-02	6.642E-02	6.454E-02	6.359E-02	6.359E-02	6.076E-02	5.744E-02	5.314E-02	4.830E-02	4.635E-02	4.439E-02	9.000
3	0.0	6.218E-02	6.265E-02	6.218E-02	6.218E-02	5.886E-02	5.886E-02	5.505E-02	5.024E-02	4.684E-02	4.340E-02	4.141E-02	6.000
2	0.0	6.076E-02	6.076E-02	5.981E-02	5.744E-02	5.505E-02	5.169E-02	4.782E-02	4.340E-02	4.141E-02	3.840E-02	0.0	3.000
1	0.0	6.076E-02	5.886E-02	5.744E-02	5.601E-02	5.410E-02	5.073E-02	4.782E-02	4.291E-02	3.941E-02	3.789E-02	0.0	0.0

I=	13	14	15	TETA
J				
11	4.141E-02	4.141E-02	4.141E-02	3.000E 01
10	4.537E-02	4.291E-02	4.291E-02	2.700E 01
9	4.684E-02	4.586E-02	4.537E-02	2.400E 01
8	4.635E-02	4.537E-02	0.0	2.100E 01
7	4.733E-02	4.586E-02	0.0	1.800E 01
6	4.684E-02	4.537E-02	0.0	1.500E 01
5	4.537E-02	0.0	0.0	1.200E 01
4	0.0	0.0	0.0	9.000E 00
3	0.0	0.0	0.0	6.000E 00
2	0.0	0.0	0.0	3.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 64,895
LENGTH/HYDRAULIC DIAMETER= 113

AXIAL VELOCITY/MAX VELOCITY

I=	1	2	3	4	5	6	7	8	9	10	11	12	TETA
J													
11	0.0	8.641E-01	8.866E-01	9.010E-01	9.140E-01	9.370E-01	9.428E-01	9.505E-01	9.559E-01	9.622E-01	9.689E-01	9.748E-01	30.000
10	0.0	8.416E-01	8.605E-01	8.821E-01	8.933E-01	9.104E-01	9.280E-01	9.370E-01	9.464E-01	9.500E-01	9.599E-01	9.644E-01	27.000
9	0.0	8.042E-01	8.177E-01	8.479E-01	8.686E-01	8.866E-01	9.032E-01	9.185E-01	9.338E-01	9.392E-01	9.442E-01	9.518E-01	24.000
8	0.0	7.763E-01	8.038E-01	8.281E-01	8.101E-01	8.749E-01	8.888E-01	9.055E-01	9.145E-01	9.248E-01	9.334E-01	9.388E-01	21.000
7	0.0	7.156E-01	7.444E-01	7.786E-01	8.002E-01	8.312E-01	8.533E-01	8.758E-01	8.839E-01	8.974E-01	9.073E-01	9.118E-01	18.000
6	0.0	6.481E-01	6.872E-01	7.237E-01	7.561E-01	7.867E-01	8.065E-01	8.357E-01	8.515E-01	8.654E-01	8.740E-01	8.843E-01	15.000
5	0.0	6.355E-01	6.769E-01	7.066E-01	7.412E-01	7.687E-01	7.988E-01	8.168E-01	8.348E-01	8.425E-01	8.542E-01	8.614E-01	12.000
4	0.0	6.260E-01	6.625E-01	7.025E-01	7.318E-01	7.561E-01	7.777E-01	7.966E-01	8.114E-01	8.222E-01	8.344E-01	8.438E-01	9.000
3	0.0	5.779E-01	6.283E-01	6.629E-01	6.964E-01	7.214E-01	7.426E-01	7.651E-01	7.840E-01	7.979E-01	8.101E-01	8.213E-01	6.000
2	0.0	6.053E-01	6.472E-01	6.796E-01	7.111E-01	7.396E-01	7.552E-01	7.727E-01	7.871E-01	7.975E-01	8.074E-01	0.0	3.000
1	0.0	6.076E-01	6.463E-01	6.787E-01	7.075E-01	7.313E-01	7.552E-01	7.732E-01	7.844E-01	7.943E-01	8.011E-01	0.0	0.0

I=	13	14	15	TETA
J				
11	9.820E-01	9.847E-01	9.860E-01	3.000E 01
10	9.734E-01	9.793E-01	9.829E-01	2.700E 01
9	9.599E-01	9.640E-01	9.658E-01	2.400E 01
8	9.451E-01	9.451E-01	0.0	2.100E 01
7	9.199E-01	9.172E-01	0.0	1.800E 01
6	8.893E-01	8.933E-01	0.0	1.500E 01
5	8.677E-01	0.0	0.0	1.200E 01
4	0.0	0.0	0.0	9.000E 00
3	0.0	0.0	0.0	6.000E 00
2	0.0	0.0	0.0	3.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 64005.
LENGTH/HYDRAULIC DIAMETER= 46

TURB AXIAL VELOCITY/MAX VELOCITY

I=	1	2	3	4	5	6	7	8	9	10	11	12	TETA
J													
11	0.0	6.595E-02	6.218E-02	5.886E-02	5.601E-02	5.024E-02	5.121E-02	4.830E-02	4.635E-02	4.537E-02	4.439E-02	4.091E-02	30.000
10	0.0	7.110E-02	6.876E-02	6.548E-02	6.218E-02	5.934E-02	5.601E-02	5.266E-02	5.024E-02	4.830E-02	4.684E-02	4.340E-02	27.000
9	0.0	7.204E-02	6.829E-02	6.736E-02	6.548E-02	6.171E-02	5.886E-02	5.505E-02	5.169E-02	4.879E-02	4.733E-02	4.488E-02	24.000
8	0.0	6.736E-02	6.829E-02	6.689E-02	6.454E-02	6.076E-02	5.601E-02	5.314E-02	4.976E-02	4.782E-02	4.537E-02	4.390E-02	21.000
7	0.0	6.736E-02	6.829E-02	6.689E-02	6.454E-02	6.265E-02	6.029E-02	5.553E-02	5.217E-02	5.362E-02	4.537E-02	4.439E-02	18.000
6	0.0	6.454E-02	6.454E-02	6.689E-02	6.548E-02	6.454E-02	6.076E-02	5.744E-02	5.410E-02	5.024E-02	4.684E-02	4.488E-02	15.000
5	0.0	6.218E-02	6.171E-02	6.265E-02	6.218E-02	6.123E-02	5.839E-02	5.505E-02	5.121E-02	4.830E-02	4.488E-02	4.390E-02	12.000
4	0.0	5.886E-02	5.744E-02	5.886E-02	5.981E-02	5.696E-02	5.601E-02	5.266E-02	4.830E-02	4.488E-02	4.340E-02	4.241E-02	9.000
3	0.0	5.791E-02	5.601E-02	5.601E-02	5.791E-02	5.696E-02	5.505E-02	5.314E-02	4.928E-02	4.537E-02	4.291E-02	4.500E 00	6.000
2	0.0	5.505E-02	6.031E 01	5.744E-02	5.744E-02	5.457E-02	5.217E-02	4.830E-02	4.488E-02	4.091E-02	3.890E-02	0.0	3.000
1	0.0	5.744E-02	5.649E-02	5.696E-02	5.505E-02	5.217E-02	4.537E-02	4.635E-02	4.241E-02	3.941E-02	3.687E-02	0.0	0.0

I=	13	14	15	TETA
J				
11	3.840E-02	3.687E-02	3.687E-02	3.000E 01
10	4.141E-02	4.091E-02	3.789E-02	2.700E 01
9	4.340E-02	4.191E-02	4.191E-02	2.400E 01
8	4.291E-02	4.340E-02	0.0	2.100E 01
7	4.390E-02	4.390E-02	0.0	1.800E 01
6	4.390E-02	4.488E-02	0.0	1.500E 01
5	4.340E-02	0.0	0.0	1.200E 01
4	0.0	0.0	0.0	9.000E 00
3	0.0	0.0	0.0	6.000E 00
2	0.0	0.0	0.0	3.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 64005.
LENGTH/HYDRAULIC DIAMETER= 46

AXIAL VELOCITY/MAX VELOCITY

I= J	1	2	3	4	5	6	7	8	9	10	11	12	TETA
6	0.0	7.997E-01	8.258E-01	8.330E-01	8.416E-01	8.497E-01	8.632E-01	8.641E-01	8.704E-01	8.857E-01	8.920E-01	8.983E-01	30.000
5	0.0	7.201E-01	7.381E-01	7.579E-01	7.768E-01	7.957E-01	8.101E-01	8.177E-01	8.326E-01	8.407E-01	8.488E-01	8.591E-01	24.000
4	0.0	6.746E-01	6.989E-01	7.187E-01	7.493E-01	7.709E-01	7.885E-01	8.029E-01	8.123E-01	8.177E-01	8.240E-01	8.357E-01	18.000
3	0.0	6.130E-01	6.535E-01	6.904E-01	7.214E-01	7.471E-01	7.673E-01	7.858E-01	7.966E-01	8.065E-01	8.114E-01	8.159E-01	12.000
2	0.0	5.690E-01	6.179E-01	6.526E-01	6.823E-01	7.088E-01	7.345E-01	7.538E-01	7.705E-01	7.813E-01	7.912E-01	7.979E-01	6.000
1	0.0	5.905E-01	6.319E-01	6.652E-01	6.926E-01	7.201E-01	7.396E-01	7.570E-01	7.723E-01	7.763E-01	7.858E-01	0.0	0.0

I= J	13	14	15	TETA
6	9.028E-01	9.122E-01	9.167E-01	3.000E 01
5	8.690E-01	8.753E-01	8.776E-01	2.400E 01
4	8.375E-01	8.474E-01	0.0	1.800E 01
3	8.191E-01	0.0	0.0	1.200E 01
2	0.0	0.0	0.0	6.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 60125.
LENGTH/HYDRAULIC DIAMETER= 15

TURB AXIAL VELOCITY/MAX VELOCITY

I=	1	2	3	4	5	6	7	8	9	10	11	12	TETA
J													
6	0.0	5.505E-02	5.024E-02	5.024E-02	4.830E-02	4.635E-02	4.439E-02	4.340E-02	4.091E-02	4.782E-02	3.687E-02	3.378E-02	30.000
5	0.0	6.454E-02	6.359E-02	6.123E-02	5.981E-02	5.744E-02	5.505E-02	5.266E-02	5.121E-02	5.024E-02	4.537E-02	4.340E-02	24.000
4	0.0	6.454E-02	6.454E-02	6.218E-02	6.265E-02	5.981E-02	5.505E-02	5.410E-02	5.024E-02	4.830E-02	4.439E-02	4.291E-02	18.000
3	0.0	5.981E-02	5.791E-02	5.791E-02	5.649E-02	5.362E-02	5.121E-02	4.782E-02	4.439E-02	4.041E-02	3.789E-02	3.585E-02	12.000
2	0.0	5.744E-02	5.505E-02	5.266E-02	5.314E-02	5.217E-02	5.024E-02	4.733E-02	4.241E-02	3.789E-02	3.482E-02	3.273E-02	6.000
1	0.0	5.601E-02	5.505E-02	5.457E-02	5.217E-02	5.024E-02	4.537E-02	4.291E-02	3.789E-02	3.533E-02	3.273E-02	0.0	0.0

I=	13	14	15	TETA
J				
6	3.482E-02	3.167E-02	3.061E-02	3.000E 01
5	4.141E-02	3.789E-02	3.738E-02	2.400E 01
4	3.890E-02	3.789E-02	0.0	1.800E 01
3	3.378E-02	0.0	0.0	1.200E 01
2	0.0	0.0	0.0	6.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 60125.
LENGTH/HYDRAULIC DIAMETER= 15.

TURB AXIAL VELOCITY/MAX VELOCITY

I=	1	2	3	4	5	6	7	8	9	10	11	12	TETA
J													
6	0.0	5.505E-02	5.024E-02	5.024E-02	4.830E-02	4.635E-02	4.439E-02	4.340E-02	4.091E-02	4.782E-02	3.687E-02	3.378E-02	30.000
5	0.0	6.454E-02	6.359E-02	6.123E-02	5.981E-02	5.744E-02	5.505E-02	5.266E-02	5.121E-02	5.024E-02	4.537E-02	4.340E-02	24.000
4	0.0	6.454E-02	6.454E-02	6.218E-02	6.265E-02	5.981E-02	5.505E-02	5.410E-02	5.024E-02	4.830E-02	4.439E-02	4.291E-02	18.000
3	0.0	5.981E-02	5.791E-02	5.791E-02	5.649E-02	5.362E-02	5.121E-02	4.782E-02	4.439E-02	4.041E-02	3.789E-02	3.585E-02	12.000
2	0.0	5.744E-02	5.505E-02	5.266E-02	5.314E-02	5.217E-02	5.024E-02	4.733E-02	4.241E-02	3.789E-02	3.482E-02	3.273E-02	6.000
1	0.0	5.601E-02	5.505E-02	5.457E-02	5.217E-02	5.024E-02	4.537E-02	4.291E-02	3.789E-02	3.533E-02	3.273E-02	3.0	0.0

I=	13	14	15	TETA
J				
6	3.482E-02	3.167E-02	3.061E-02	3.000E 01
5	4.141E-02	3.789E-02	3.738E-02	2.400E 01
4	3.890E-02	3.789E-02	0.0	1.800E 01
3	3.378E-02	0.0	0.0	1.200E 01
2	0.0	0.0	0.0	6.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 60125.
LENGTH/HYDRAULIC DIAMETER= 15

AXIAL VELOCITY/MAX VELOCITY

I= J	1	2	3	4	5	6	7	8	9	10	11	12	TETA
11	0.0	9.186E-01	9.351E-01	9.458E-01	9.575E-01	9.741E-01	9.811E-01	9.797E-01	9.894E-01	9.906E-01	9.953E-01	9.976E-01	30.000
10	0.0	8.797E-01	8.939E-01	9.104E-01	9.222E-01	9.363E-01	9.434E-01	9.528E-01	9.599E-01	9.642E-01	9.741E-01	9.788E-01	27.000
9	0.0	8.561E-01	8.726E-01	8.939E-01	8.986E-01	9.127E-01	9.257E-01	9.316E-01	9.387E-01	9.514E-01	9.571E-01	9.592E-01	24.000
8	0.0	7.995E-01	8.278E-01	8.526E-01	8.675E-01	8.833E-01	8.962E-01	9.080E-01	9.127E-01	9.245E-01	9.340E-01	9.328E-01	21.000
7	0.0	7.264E-01	7.818E-01	8.137E-01	8.349E-01	8.467E-01	8.703E-01	8.774E-01	8.892E-01	8.986E-01	9.080E-01	9.127E-01	18.000
6	0.0	6.840E-01	7.571E-01	7.925E-01	8.160E-01	8.337E-01	8.476E-01	8.573E-01	8.703E-01	8.797E-01	8.844E-01	8.892E-01	15.000
5	0.0	5.495E-01	6.863E-01	7.500E-01	7.795E-01	8.014E-01	8.233E-01	8.335E-01	8.455E-01	8.538E-01	8.618E-01	8.660E-01	12.000
4	0.0	5.873E-01	6.934E-01	7.429E-01	7.783E-01	7.958E-01	8.090E-01	8.208E-01	8.318E-01	8.389E-01	8.420E-01	8.439E-01	9.000
3	0.0	4.788E-01	6.321E-01	7.075E-01	7.500E-01	7.736E-01	7.906E-01	8.050E-01	8.177E-01	8.224E-01	8.243E-01	8.278E-01	6.000
2	0.0	6.014E-01	6.840E-01	7.323E-01	7.623E-01	7.863E-01	7.972E-01	8.054E-01	8.125E-01	8.165E-01	8.196E-01	0.0	3.000
1	0.0	6.014E-01	6.816E-01	7.311E-01	7.642E-01	7.858E-01	7.948E-01	8.019E-01	8.113E-01	8.151E-01	8.137E-01	0.0	0.0

I= J	13	14	15	TETA
11	9.991E-01	1.001E 00	1.CC0E 00	3.000E 01
10	9.835E-01	9.858E-01	9.858E-01	2.700E 01
9	9.658E-01	9.670E-01	0.0	2.400E 01
8	9.387E-01	9.399E-01	0.0	2.100E 01
7	9.175E-01	9.151E-01	0.0	1.800E 01
6	8.915E-01	0.0	0.0	1.500E 01
5	8.679E-01	0.0	0.0	1.200E 01
4	0.0	0.0	0.0	9.000E 00
3	0.0	0.0	0.0	6.C00E 00
2	0.0	0.0	0.0	3.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 8924.
LENGTH/HYDRAULIC DIAMETER= 11.3

TURB AXIAL VELOCITY/MAX VELOCITY

I=	1	2	3	4	5	6	7	8	9	10	11	12	TETA
J													
11	0.0	7.364E-02	6.880E-02	6.394E-02	6.052E-02	5.167E-02	5.414E-02	4.919E-02	4.919E-02	4.670E-02	4.419E-02	4.166E-02	30.000
10	0.0	8.088E-02	7.606E-02	6.880E-02	6.637E-02	6.637E-02	6.150E-02	5.414E-02	5.537E-02	5.414E-02	5.068E-02	4.794E-02	27.000
9	0.0	8.569E-02	7.606E-02	7.364E-02	6.880E-02	6.637E-02	6.150E-02	5.906E-02	5.611E-02	5.414E-02	5.192E-02	4.919E-02	24.000
8	0.0	1.001E-01	8.569E-02	7.847E-02	7.364E-02	7.122E-02	6.637E-02	6.028E-02	5.783E-02	5.611E-02	5.217E-02	5.167E-02	21.000
7	0.0	1.215E-01	1.048E-01	9.049E-02	7.847E-02	7.606E-02	6.880E-02	6.394E-02	5.955E-02	5.660E-02	5.365E-02	5.217E-02	18.000
6	0.0	1.215E-01	1.048E-01	9.049E-02	8.088E-02	7.122E-02	6.880E-02	6.028E-02	5.759E-02	5.414E-02	5.217E-02	4.919E-02	15.000
5	0.0	1.334E-01	1.215E-01	9.767E-02	8.569E-02	7.847E-02	7.364E-02	6.028E-02	5.736E-02	5.537E-02	5.068E-02	4.919E-02	12.000
4	0.0	1.263E-01	1.144E-01	9.288E-02	8.088E-02	7.122E-02	6.028E-02	5.709E-02	5.990E-02	5.167E-02	4.919E-02	4.794E-02	9.000
3	0.0	1.263E-01	1.239E-01	1.025E-01	8.569E-02	7.485E-02	6.394E-02	5.660E-02	5.365E-02	4.919E-02	4.670E-02	4.544E-02	6.000
2	0.0	1.227E-01	1.048E-01	9.049E-02	7.485E-02	6.272E-02	5.660E-02	5.266E-02	4.869E-02	4.544E-02	4.494E-02	0.0	3.000
1	0.0	1.227E-01	1.072E-01	8.809E-02	7.606E-02	6.101E-02	5.709E-02	5.291E-02	4.670E-02	4.469E-02	4.419E-02	0.0	0.0

I=	13	14	15	TETA
J				
11	4.166E-02	4.166E-02	4.039E-02	3.000E 01
10	4.670E-02	4.544E-02	4.745E-02	2.700E 01
9	4.869E-02	4.919E-02	0.0	2.400E 01
8	5.043E-02	4.919E-02	0.0	2.100E 01
7	5.043E-02	5.018E-02	0.0	1.800E 01
6	4.994E-02	0.0	0.0	1.500E 01
5	4.919E-02	0.0	0.0	1.200E 01
4	0.0	0.0	0.0	9.000E 00
3	0.0	0.0	0.0	6.000E 00
2	0.0	0.0	0.0	3.000E 00
1	0.0	0.0	0.0	0.0

REYNOLDS NUMBER= 8924.
LENGTH/HYDRAULIC DIAMETER= 11.3

MEASURED TURBULENT QUANTITIES

θ	$y/\hat{y}$	v_r'/u_o (x10^2)	v_θ'/u_o (x10^2)	k/u_o^2 (x10^3)	$-\frac{\overline{v_r'v_z'}}{u_o^2}$ (x10^4)
0	0.055				
	0.150				
	0.244	3.66	4.04	7.23	
	0.339	4.12	5.08	6.02	
	0.433	4.04	4.14	4.56	9.21
	0.622	3.70	4.43	3.30	10.79
	0.811	3.66	3.84	2.50	4.12
	1.000	3.53	3.74	2.30	0.95
6	0.095				
	0.186				
	0.276	3.40	4.13	6.68	
	0.367	3.92	4.33	3.37	11.70
	0.457	4.04	4.23	4.60	8.76
	0.638	3.66	4.23	3.17	5.82
	0.819	3.71	4.04	2.71	2.99
	1.000	3.72	4.23	2.62	0.63
12	0.197				
	0.277				
	0.357	4.21	5.04	5.83	
	0.438	4.48	5.56	5.62	13.61
	0.518	4.42	5.36	5.09	9.11
	0.679	3.68	4.12	3.17	6.19
	0.839	4.03	3.75	2.80	4.16
	1.000	3.96	4.94	3.22	1.41
18	0.322	4.86	5.37	6.72	
	0.390	4.46	4.85	5.25	
	0.458	4.91	4.82	5.26	
	0.526	4.86	5.37	4.99	10.73
	0.593	4.69	4.87	4.33	8.71
	0.729	4.37	5.01	3.82	5.67
	0.864	4.48	5.02	3.62	3.34
	1.000	4.26	4.58	3.21	1.86
24	0.443	4.86			
	0.499	4.47	5.35	5.05	
	0.554	4.91	4.83	4.57	
	0.610	4.86	5.48	4.62	4.13
	0.666	4.69	5.16	4.22	8.22
	0.777	4.37	4.91	3.72	4.83
	0.899	4.48	4.12	3.03	2.966
	1.000	4.25	4.08	2.95	2.039

RE = 9,000
L/DH = 113

MEASURED TURBULENT QUANTITIES

θ	$y/\hat{y}$	v'_r/u_o (x10^2)	v'_θ/u_o (x10^2)	k/u_o^2 (x10^3)	$-\frac{\overline{v'_r v'_z}}{u_o^2}$ (x 10^4)
0	0.055				
	0.150	4.27			
	0.244	4.45			
	0.339	4.60	4.06	4.28	8.07
	0.433	4.32	3.67	3.76	7.94
	0.622	3.93	4.06	3.16	4.78
	0.811	3.57	3.67	2.30	2.69
	1.000	3.41	3.41	1.92	0.83
6	0.095				
	0.186	4.42			
	0.276	4.47			
	0.367	4.59	3.24	4.77	6.4
	0.457	4.31	3.55	3.73	3.1
	0.638	3.93	3.93	3.32	2.61
	0.819	3.75	3.67	2.56	0.93
	1.000	3.79	3.02	2.07	
12	0.197	4.91	3.86	5.43	
	0.277	4.74	3.77	4.66	
	0.357	4.65	3.47	4.48	
	0.438	4.60	3.47	4.12	8.68
	0.518	4.42	3.77	3.92	7.48
	0.679	4.30	3.95	3.49	4.41
	0.839	4.08	3.89	2.84	3.14
	1.000	3.89	3.05	2.22	1.44
18	0.322	4.78	4.37	5.08	
	0.390	4.76	3.92	4.60	
	0.458	4.57	3.89	4.34	
	0.526	4.62	3.89	4.12	7.84
	0.593	4.36	3.63	3.63	7.06
	0.729	4.83	4.14	3.59	4.38
	0.864	4.15	3.80	2.78	3.56
	1.000	3.86	3.22	2.30	1.42
24	0.443	4.63	3.98	4.36	
	0.499	4.62	3.78	4.04	
	0.554	4.36	3.62	3.70	
	0.610	4.49	4.15	3.74	7.214
	0.666	4.45	4.04	3.50	6.680
	0.777	4.54	4.06	3.16	4.638
	0.899	4.05	3.64	2.47	3.008
	1.000	3.83	3.39	2.21	1.429

RE = 26,500
L/DH = 113

MEASURED TURBULENT QUANTITIES

θ	$y/\hat{y}$	v_r'/u_o (x10^2)	v_θ'/u_o (x10^2)	k/u_o^2 (x10^3)	$-\frac{\overline{v_r'v_z'}}{u_o^2}$ (x 10^4)
0	0.055				
	0.150	4.76	4.75	5.7	
	0.244	4.08	4.65	4.5	
	0.339	3.54	4.07	3.56	4.41
	0.433	3.59	3.46	3.15	4.52
	0.622	3.46	3.59	2.6	4.04
	0.811	2.92	3.33	1.88	1.61
	1.000	2.62	3.59	1.71	-0.03
6	0.095				
	0.186	4.54	4.76	5.02	
	0.276	3.84	4.19	3.95	
	0.367	3.73	4.96	4.10	3.3
	0.457	3.59	3.59	3.21	3.42
	0.638	3.07	3.95	2.85	2.55
	0.819	2.79	4.41	2.42	1.36
	1.000	3.22	4.85	2.50	-0.027
12	0.197				
	0.277	4.77	5.17	5.15	
	0.357	4.21	4.42	4.32	
	0.438	4.23	4.84	4.33	3.67
	0.518	4.20	4.31	3.85	3.34
	0.679	3.23	3.83	2.85	3.86
	0.839	3.17	4.04	2.50	2.25
	1.000	3.61	3.95	2.34	0.64
18	0.322				
	0.390	4.59	5.03	4.74	
	0.458	4.43	4.53	4.26	
	0.526	4.38	4.92	4.21	2.86
	0.593	4.35	4.72	3.86	3.08
	0.729	3.51	4.74	3.13	3.43
	0.864	3.12	4.51	2.63	2.91
	1.000	3.21	4.31	2.40	0.96
24	0.443				
	0.499	4.71	5.35	4.63	
	0.554	4.43	4.87	4.02	
	0.610	4.54	4.54	3.77	1.96
	0.666	4.30	5.11	3.66	2.46
	0.777	3.38	4.26	2.65	3.43
	0.899	3.27	4.22	2.33	2.57
	1.000	3.23	3.83	2.10	0.98

RE = 26,500
L/DH = 46

MEASURED TURBULENT QUANTITIES

θ	$y/\hat{y}$	v_r'/u_o (x10^2)	v_θ'/u_o (x10^2)	k/u_o^2 (x10^3)	$-\frac{\overline{v_r'v_z'}}{u_o^2}$ (x10^4)
0	0.055				
	0.150	4.97		6.18	
	0.244	4.65	5.70	4.95	
	0.339	4.87	5.70	4.52	7.32
	0.433	4.76	5.50	4.28	5.07
	0.622	4.87	5.40	3.77	3.67
	0.811	4.54	5.08	3.06	1.55
	1.000	4.42	4.87	2.74	-0.03
6	0.095				
	0.186	5.38			
	0.276	4.67	5.18	4.31	
	0.367	4.97	4.87	4.16	5.65
	0.457	4.87	5.18	4.14	5.14
	0.638	4.76	5.08	3.53	4.87
	0.819	5.08	5.08	3.34	1.43
	1.000	4.87	5.18	3.14	0.17
12	0.197	5.47			
	0.277	5.27	4.88	4.67	
	0.357	4.98	5.18	4.44	
	0.438	5.37	4.99	4.43	6.94
	0.518	5.16	5.67	3.92	4.34
	0.679	4.67	5.06	3.32	2.5
	0.839	5.07	4.98	3.22	2.00
	1.000	5.18	5.19	3.36	0.41
18	0.322	5.42			
	0.390	5.22	6.28	5.26	
	0.458	5.11	5.36	4.45	
	0.526	5.50	5.50	4.52	4.56
	0.593	6.16	5.84	5.99	2.88
	0.729	4.94	5.53	3.70	1.69
	0.864	5.04	5.54	3.43	1.25
	1.000	5.16	5.27	3.37	0.13
24	0.443	5.11			
	0.499	5.31	5.79	4.79	
	0.554	4.89	5.38	4.21	
	0.610	5.22	5.36	4.23	2.06
	0.666	5.31	5.38	4.18	1.38
	0.777	4.81	5.03	3.48	0.71
	0.899	4.63	4.55	3.00	-0.27
	1.000	5.04	4.90	3.19	-0.97

RE = 26,500
L/DH = 15

MEASURED TURBULENT QUANTITIES

θ	$y/\hat{y}$	v_r'/u_o (x10^2)	v_θ'/u_o (x10^2)	k/u_o^2 (x10^3)	$-\frac{\overline{v_r'v_z'}}{u_o^2}$ (x10^4)
0	0.055				
	0.150	3.34			
	0.244	3.59			
	0.339	3.85	2.98	2.80	7.26
	0.433	3.49	3.23	2.67	7.20
	0.622	3.39	2.87	2.24	3.82
	0.811	3.02	2.92	1.78	1.84
	1.000	2.92	2.70	1.50	0.12
6	0.095				
	0.186	3.42			
	0.276	3.42			
	0.367	3.69	3.29	3.12	6.04
	0.457	3.39	3.39	2.83	5.67
	0.638	3.38	3.08	2.53	2.65
	0.819	2.92	2.76	1.88	1.95
	1.000	2.97	3.54	1.60	-0.025
12	0.197	3.16			
	0.277	3.37			
	0.357	3.27			
	0.438	3.67	3.16	3.36	7.68
	0.518	3.43	3.15	3.12	6.68
	0.679	3.47	3.10	2.70	3.06
	0.839	3.07	2.87	2.07	2.69
	1.000	3.06	2.72	1.85	0.298
18	0.322	3.32			
	0.390	3.41	2.67	3.43	
	0.458	3.45	3.03	3.40	
	0.526	3.54	3.04	3.27	7.20
	0.593	3.37	3.16	2.96	5.95
	0.729	3.51	3.17	2.63	3.33
	0.864	3.145	2.80	2.07	2.37
	1.000	2.96	2.83	1.87	0.66
24	0.443	3.33			
	0.499	3.47	2.87	3.35	
	0.554	3.42	3.11	3.16	
	0.610	3.45	3.03	2.86	6.21
	0.666	3.29	2.87	2.60	5.41
	0.777	3.37	3.06	2.24	3.08
	0.899	3.17	2.84	1.94	2.77
	1.000	3.07	2.56	1.80	0.672

RE = 65,000
L/DH = 113

MEASURED TURBULENT QUANTITIES

θ	$y/\hat{y}$	v'_r/u_o (x10^2)	v'_θ/u_o (x10^2)	k/u_o^2 (x10^3)	$-\frac{\overline{v'_r v'_z}}{u_o^2}$ (x10^4)
0	0.055				
	0.150	3.84			
	0.244	4.11			
	0.339	3.79	4.11	3.21	4.37
	0.433	4.32	4.06	3.29	3.98
	0.622	3.95	3.4	2.64	2.61
	0.811	3.23	3.57	2.07	0.80
	1.000	3.23	3.51	1.83	-0.312
6	0.095				
	0.186	3.87			
	0.276	4.00	3.79	3.11	
	0.367	4.06	4.32	3.46	4.06
	0.457	4.21	3.95	3.32	3.91
	0.638	4.10	3.74	3.08	2.70
	0.819	3.52	3.78	2.76	1.13
	1.000	3.51	3.73	3.26	-0.07
12	0.197				
	0.277	4.12			
	0.357	4.32	4.17	3.76	
	0.438	4.20	3.81	3.51	4.02
	0.518	4.00	3.82	3.43	3.63
	0.679	4.04	3.75	2.85	3.70
	0.839	3.79	3.79	2.46	2.66
	1.000	3.53	3.88	2.33	1.16
18	0.322				
	0.390	4.27	4.27	3.94	
	0.458	4.30	4.08	3.75	
	0.526	4.14	3.92	3.47	4.24
	0.593	4.37	4.10	3.39	3.61
	0.729	3.78	3.69	2.6	3.30
	0.864	4.03	3.82	2.54	3.07
	1.000	3.51	3.52	2.21	1.38
24	0.443				
	0.499	4.27	3.84	3.62	
	0.554	4.30	4.11	3.47	
	0.610	4.14	3.79	3.02	4.24
	0.666	4.44	4.10	3.11	3.62
	0.777	3.78	3.88	2.62	3.29
	0.899	4.03	3.72	2.39	3.075
	1.000	3.52	3.73	2.20	1.375

RE = 65,000
L/DH = 46

MEASURED TURBULENT QUANTITIES

θ	$y/\hat{y}$	v'_r/u_o (x10^2)	v'_θ/u_o (x10^2)	k/u_o^2 (x10^3)	$-\frac{v'_r v'_z}{u_o^2}$ (x10^4)
0	0.055				
	0.150	4.42	4.63	3.64	
	0.244	4.16	4.58	3.45	
	0.339	4.22	4.84	3.57	5.65
	0.433	4.05	4.42	3.18	6.67
	0.622	3.89	4.63	2.87	4.23
	0.811	3.78	3.84	2.21	1.33
	1.000	3.50	3.57	1.80	-0.025
6	0.095				
	0.186	4.16			
	0.276	4.06	4.58	3.28	
	0.367	3.91	4.57	3.24	4.26
	0.457	3.85	4.52	3.14	4.71
	0.638	3.52	4.31	2.68	3.43
	0.819	3.41	4.05	2.13	1.6
	1.000	3.57	3.78	1.92	0.23
12	0.197				
	0.277	4.17	4.56	3.66	
	0.357	4.06	4.50	3.36	
	0.438	3.90	3.90	2.97	4.08
	0.518	4.04	3.80	2.87	4.05
	0.679	3.70	4.19	2.56	2.96
	0.839	3.61	4.44	2.37	1.76
	1.000	3.63	3.78	1.95	0.636
18	0.322				
	0.390	3.87	4.53	3.73	
	0.458	4.06	4.82	3.66	
	0.526	4.65	4.87	3.78	3.0
	0.593	3.58	4.48	3.13	3.51
	0.729	3.61	4.35	2.78	2.31
	0.864	3.50	4.29	2.47	0.96
	1.000	3.55	4.24	2.26	-0.148
24	0.443				
	0.449	3.83			
	0.554	3.80	4.20	3.15	
	0.610	3.74	4.77	2.82	1.83
	0.666	3.84	3.06	2.89	1.84
	0.777	3.42	4.94	2.41	1.08
	0.899	3.30	3.78	2.13	-0.32
	1.000	3.19	3.67	1.90	-1.02

RE = 65,000
L/DH = 15

APPENDIX D

FLOW SPLIT-EQUAL PRESSURE DROP TEST

Let us take the well known formula of pressure drop

$$\Delta p = f \frac{L}{D_H} \frac{\rho u_b^2}{2}$$

Assuming equal pressure drops for all the subchannels and applying above relation for the central characteristic triangle and the total test section we find finally

$$\frac{f}{f^T} \frac{u_b^2}{(u_b^T)^2} = \frac{D_H^T}{D_H}$$

where D_H^T is the test section hydraulic diameter and f^T is the test section friction factor.

Assuming

$$f \sim Re^{-m}, \quad f^T \sim (Re^T)^{-m}$$

we end up with the relation

$$\frac{u_b}{u_b^T} = \left(\frac{D_H}{D_H^T}\right)^{\frac{1+m}{2-m}}$$

From Table 2.2 D_H=20.12 mm and D_H^T=16.375 mm.

For m=0.20 $\quad \frac{u_b}{u_b^T} = 1.147$

For m=0.25 $\quad \frac{u_b}{u_b^T} = 1.158$

APPENDIX E

COMPUTER PROGRAMS

In order to solve the finite difference equations for heat and flow two computer codes were written.

The main symbols used in the above codes are explained in Table E1.

The computer program structure for flow calculation is shown in Tables D2 and D3.

The structure of the computer programs for heat transfer calculations is shown in Table D4.

Listings of both computer codes are given at the end of this appendix.

TABLE E1

SYMBOLS FOR COMPUTER CODES

Symbol	Explanation
ALHGR	Linear heat generation rate (q')
CK	Von karman constant
CMU	Constant c_μ
CA1	Constant c_1
CA2	Constant c_2
C1	Constant $c_{\varepsilon 1}$
C2	Constant $c_{\varepsilon 2}$
CONDC	Coolant heat conductivity (λ_c)
CONCL	Clad heat conductivity (λ_{cl})
CSH	Coolant specific heat (C_p)
DENSIT	Coolant density (ρ)
DI	Clad Inner diameter (D_i)
DIA	Rod Diameter (D)
DH	Hydraulic Diameter (D_H)
ED	Energy dissipation
IM	No. of radial points in the clad
IMAX	Number of radial points in the coolant
IN	Total number of radial points in the coolant
(IMON,JMON)	Point for detailed print out (per iteration)
(IREF,JREF)	Point where pressure is preset to zero
JN	No. of tangential points
MAXIT	Maximum number of iterations
PIT	Pitch
PITR	Pitch ratio

PSI	$1/\sigma_T$
RE	Reynolds number
TE	Turbulent kinetic energy (k)
TWAVE, TALAV	Average shear stress
TT	Temperature
U	Axial velocity v_z
USTAR	Friction velocity u*
URF___	Underrelaxation factor (UREL)
V1	Radial velocity (v_r)
V2	Tangential velocity (v_θ)
VB, VAVG	Bulk velocity
X1	Radial spacing
X2	Tangential spacing
YB	y_B^+

TABLE E2

COMPUTER PROGRAM STRUCTURE FOR FLOW CALCULATIONS

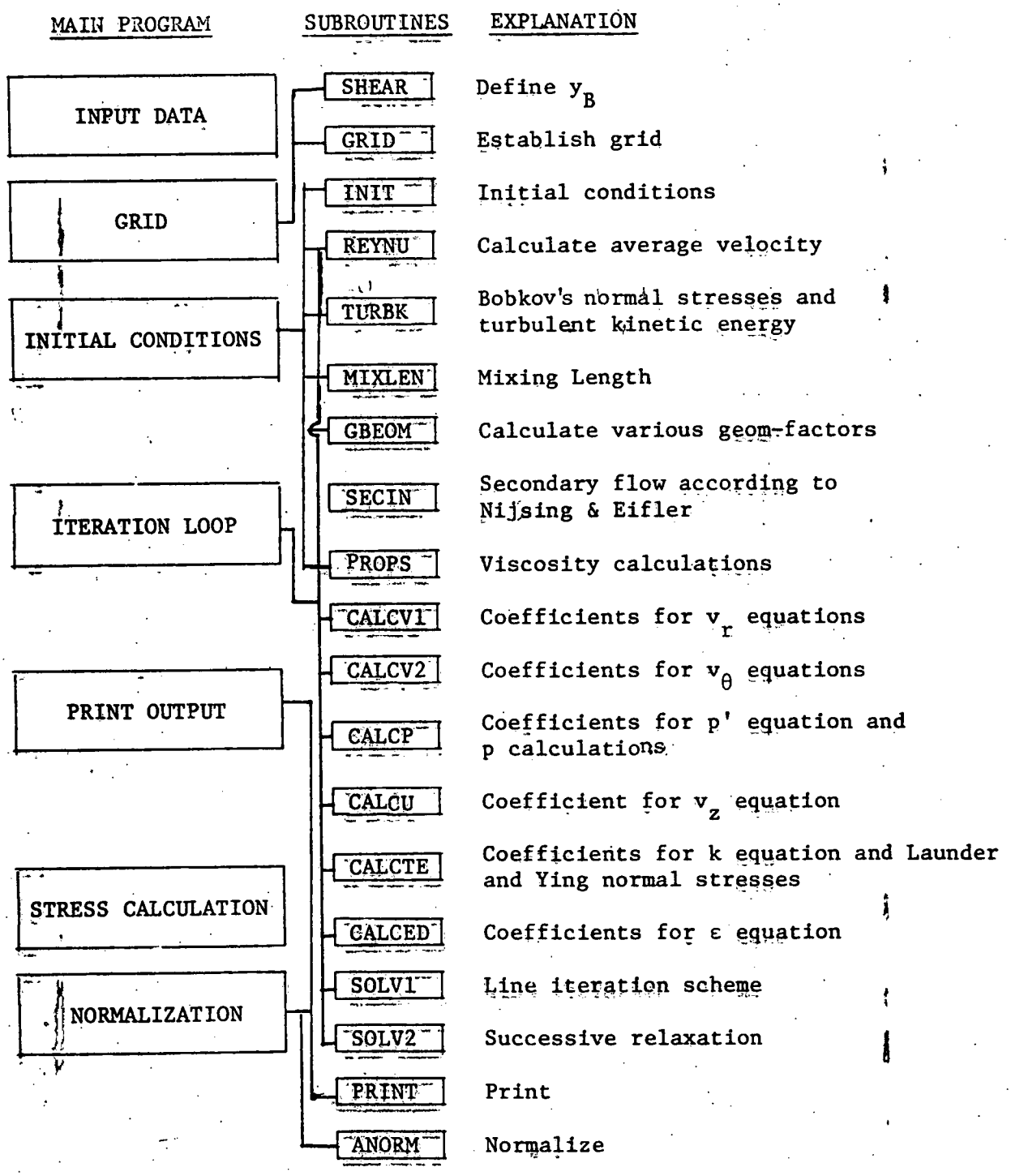


TABLE E3

ITERATION LOOP FOR FLOW CALCULATIONS

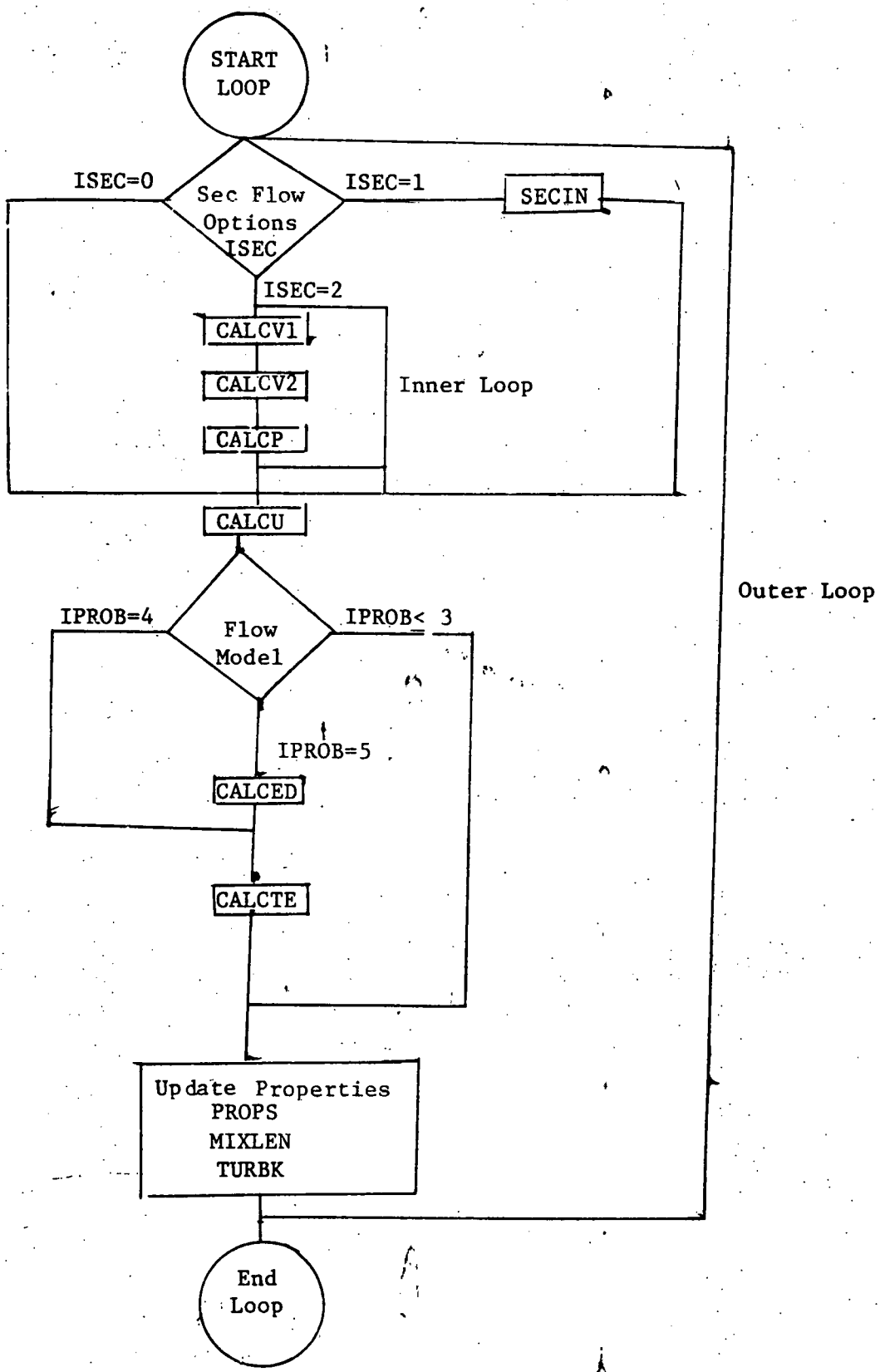
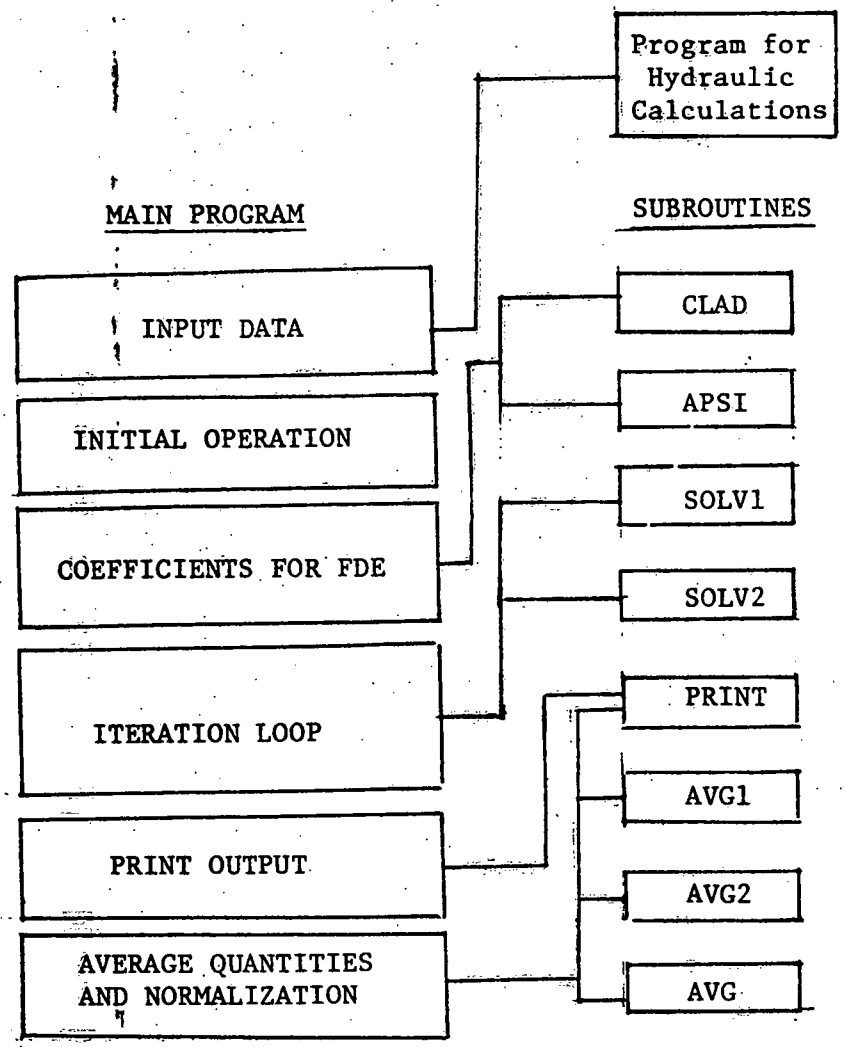


TABLE E4

COMPUTER PROGRAM STRUCTURE FOR THERMAL CALCULATIONS



```
C     COMPUTER PROGRAM FOR FLOW CALCULATIONS                                    0001
C                                                                               0002
C        I N S T R U C T I O N S                                                0003
C                                                                               0004
C     IPILE=0     REYNOLDS NO INPUT                                           0005
C     IPILE=1     AVG WALL SHEAR STRESS INPUT                                 0006
C                                                                               0007
C        IPROB.EQ.1: LAMINAR FLOW                                               0008
C        IPROB.GT.1:  TURBULENT FLOW                                            0009
C        IPROB.EQ.3; 0-EQUATION MODEL                                           0010
C        IPROB.EQ.4; 1-EQUATION MODEL                                           0011
C        IPROB.EQ.5; 2-EQUATION MODEL                                           0012
C                                                                               0013
C      ISEC=0;  NO SECONDARY FLOW                                               0014
C      ISEC=1;  NIJSING-EIFLER SECONDARY FLOW                                   0015
C      ISEC=2;  SECONDARY FLOW                                                  0016
C*****************************************************************************0017
C                                                                               0018
C     ITURB=1      NORMAL STRESSES FROM BOBKOV MODEL                          0019
C     ITURB=2      NORMAL STRESSES FROM LAUNDER YING MODEL                    0020
C                                                                               0021
C     IPR=1       ISOTROPIC VISCOCITY                                         0022
C     IPR=2       NON-ISOTROPIC VISCOSITY                                     0023
C                                                                               0024
C     ILEN=1      CARAJILESCOV RADIAL LENGTH                                  0025
C     ILEN=2      PRESENT RADIAL LENGTH                                       0026
C                                                                               0027
C     ICONV=0     UDDS SCHEME                                                 0028
C     ICONV=1     UDS SCHEME                                                  0029
C                                                                               0030
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                                   0031
      DIMENSION V1P(30,16),V2P(30,16),V3P(30,16),T23(30,16),T13(30,16)          0032
      COMMON/HD/HEDU(9),HEDV1(9),HEDV2(9),HEDP(9),HEDK(9),HEDD(9),              0033
     1  HEDM(9),HEDL(9),HEDN(9),HEDPR(9),HEDV1P(9),HEDV2P(9),HEDV3P(9),         0034
     2 HEDT13(9),HEDT23(9)                                                      0035
      COMMON/ALEN/CL,RL,IPR,ILEN                                                0036
```

```
      COMMON/BB/JS                                                      0037
      COMMON/BBB/NVIS                                                   0038
      COMMON/ATURB/DVS(30,16),CA,CA1,CA2,CB,ITURB                       0039
      COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),      0040
     / ANG(16),ANGM(16),VBP(16)                                         0041
      COMMON/GEOMD/DV1E(30),DV1W(30),DV1N(30),CV1E(30),CV1W(30)         0042
      COMMON/GEOME/DV2E(30),DV2W(30),DV2N(30),CV2E(30),CV2W(30),ARPN(30)0043
      COMMON                                                            0044
     1/UVEL/URFU,RESU,NSWPU                                             0045
     1/V1VEL/URFV1,RESV1,NSWPV1                                         0046
     1/V2VEL/URFV2,RESV2,NSWPV2                                         0047
     1/PCOR/RESORM,URFP,RESPP,NSWPP,IPREF,JPREF                         0048
     1/TEN/URFK,RESK,NSWPK                                              0049
     1/TDIS/URFE,RESE,NSWPD                                             0050
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),   0051
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)           0052
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),0053
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                         0054
     1/AGRID/R(30),X1(30),X2(16)                                        0055
     1/STRESS/T11(30,16),T22(30,16)                                     0056
     1/GEOM/BE(30),BN(30),BW(30),BS(30),VP(30),VP1(30)                  0057
     1/GEOMA/DPE(30,16),DPW(30,16),DPN(30,16),DPS(30,16)                0058
     1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB            0059
     1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED                       0060
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),PR,YB,TWAVE                  0061
     1/COEF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SU(30,16) 0062
      EQUIVALENCE (T11(1),V1P(1)),(T22(1),V2P(1)),(DPE(1),V3P(1)),      0063
     1 (DPW(1),T13(1)),(DPN(1),T23(1))                                  0064
      LOGICAL INCALU,INCALL,INCALP,INPRO,INCALK,INCALD,INCALM,          0065
     1                INCLV1,INCLV2,ICLT11,ICLT22,ICLT13,ICLT23,ICLT33  0066
C                                                                       0067
C***  PROBLEM DEFINITION                                                0068
      READ(5,700) IPILE,IPROB,ISEC,ILEN,IPR,ITURB                       0069
C***GRID                                                                0070
      READ(5,700) IN,JN,IT,JT                                           0071
      READ(5,701)YB                                                     0072
```

```
C***    NUMERICAL PROCEDURE                                                    0073
      READ(5,700) NSOLU,ITR,NSWPV1,NSWPV2,NSWPP,NSWPU,NSWPK,NSWPD             0074
C***   PROBLEM DATA                                                           0075
      IF(IPILE.EQ.0) READ(5,701) PIT,DIA,VISCOS,RE                           0076
      IF(IPILE.EQ.1) READ(5,701) PIT,DIA,VISCOS,TWAVE                        0077
C***  NO ITERATIONS AND PRINT-OUT INFORMATION                                 0078
      READ(5,700) MAXIT,INDPRI,ISTRES,INORM                                  0079
C***TITLES                                                                   0080
      READ(5,010)HFDU,HEDV1,HEDV2,HEDP,HEDK,HEDD,HEDL,                       0081
     1 HEDN,HEDPR,HEDV1P,HEDV2P,HEDV3P,HEDT23,HEDT13                         0082
      IF(IPROB.EQ.1) ISEC=0                                                  0083
      IF(IPROB.EQ.1) ISTRES=0                                                0084
      IF(IPROB.EQ.1) YB=0.0                                                  0085
      IF(IPROB.EQ.2) IPR=3                                                   0086
      IF(IPROB.EQ.3) ITURB=1                                                 0087
C***  PRINT INPUT DATA                                                       0088
      WRITE(6,700)IPILE,IPROB,ISEC,ILEN,IPR,ITURB                            0089
      WRITE(6,700)IN,JN,IT,JT                                                0090
      WRITE(6,701)YB                                                         0091
      WRITE(6,700)NSOLU,ITR,NSWPV1,NSWPV2,NSWPP,NSWPU,NSWPK,NSWPD            0092
      IF(IPILE.EQ.0) WRITE(6,701)PIT,DIA,VISCOS,RE                           0093
      IF(IPILE.EQ.1) WRITE(6,701)PIT,DIA,VISCOS,TWAVE                        0094
      WRITE(6,700)MAXIT,INDPRI,ISTRES,INORM                                  0095
      WRITE(6,010) HEDU,HEDV1,HEDV2,HEDP,HEDK,HEDD,HEDL,                     0096
     1 HEDN,HEDPR,HEDV1P,HEDV2P,HEDV3P,HEDT23,HEDT13                         0097
      PITR=PIT/DIA                                                           0098
C***  INITIAL OPERATIONS                                                     0099
      INUM=0                                                                 0100
      IVIS=1                                                                 0101
      IF(IPROB.EQ.2) IVIS=2                                                  0102
      ICONV=0                                                                0103
      JJL=1                                                                  0104
      RAT=1.0                                                                0105
      NVIS=2                                                                 0106
      INOR=1                                                                 0107
      NITER=0                                                                0108
```

```
      IND=1                                                          0109
      RESU=0.0                                                       0110
      RESK=0.0                                                       0111
      RESE=0.0                                                       0112
      RESV1=0.0                                                      0113
      RESV2=0.0                                                      0114
      RESPP=0.0                                                      0115
      RESORM=0.0                                                     0116
      URFV2=1.0                                                      0117
      URFV1=1.0                                                      0118
      URFP=1.0                                                       0119
      URFU=1.0                                                       0120
      URFK=1.0                                                       0121
      URFE=1.0                                                       0122
      URFVIS=1.0                                                     0123
      CC=1.0D-06                                                     0124
      ND=500                                                         0125
      IF(NSOLU.EQ.0)  ND=60                                          0126
      INM=IN-1                                                       0127
      JNM=JN-1                                                       0128
      IL=2                                                           0129
      IF(IPROB.EQ.1)  IL=1                                           0130
      ILP=IL+1                                                       0131
C***  TURBULENT CONSTANTS                                            0132
      CMU=0.09                                                       0133
      CK=0.42                                                        0134
      E=9.8                                                          0135
      CD=1.0                                                         0136
      C1=1.44                                                        0137
      C2=1.92                                                        0138
      CA1=1.5                                                        0139
      CA2=0.4                                                        0140
      CL=0.4+DEXP(-17.*(PITR-1.))*1.62                               0141
      RL=10.0                                                        0142
      PRTE=1.0                                                       0143
      CVIS=0.18                                                      0144
```

```
      CDIS=0.38                                                          0145
      IF(IPROB.EQ.4) CMU=CVIS*CDIS                                       0146
      PRED=CK*CK       /(C2-C1)/(CMU**.5)                                0147
      CA=2.*(2.-8.*CA2)/11./CA1                                          0148
      CB=2.*(9.*CA2+6.)/CA1/33.                                          0149
      IMON=5                                                             0150
      JMON=5                                                             0151
      IREF=2                                                             0152
      JREF=2                                                             0153
C                                                                        0154
C                                                                        0155
C***  HYDRAULIC PARAMETERS OF THE FLOW                                   0156
C*** COMPUTE CHANNEL AREA                                                0157
      ATOT=PIT*PIT/8./DSQRT(3.0D0)-DIA*DIA*3.14159/48.                   0158
C*** WETTED PERIMETER                                                    0159
      PW=DIA*3.14159/12.                                                 0160
C*** HYDRAULIC DIAMETER                                                  0161
      DH=4.*ATOT/PW                                                      0162
C*** AVERAGE VELOCITY                                                    0163
      IF(IPILE.EQ.0)VAVG=VISCOS*RE/DH                                    0164
C***  INITIAL VALUES FOR SHEAR STRESS                                    0165
      CALL SHEAR                                                         0166
C-----GRID                                                               0167
      CALL GRID(RAT)                                                     0168
C-----DEPENDENT VARIABLE SELECTION                                       0169
      INCALU=.TRUE.                                                      0170
      INCLV1=.TRUE.                                                      0171
      INCLV2=.TRUE.                                                      0172
      INCALK=.TRUE.                                                      0173
      INCALD=.TRUE.                                                      0174
      INCALL=.TRUE.                                                      0175
      INCALP=.TRUE.                                                      0176
      INCALM=.TRUE.                                                      0177
      INPRO=.TRUE.                                                       0178
      ICLT11=.FALSE.                                                     0179
      ICLT22=.FALSE.                                                     0180
```

```
      ICLT33=.FALSE.                                                    0181
      ICLT13=.FALSE.                                                    0182
      ICLT23=.FALSE.                                                    0183
      IF(ISEC.EQ.0) INCLV1=.FALSE.                                      0184
      IF(ISEC.EQ.0) INCLV2=.FALSE.                                      0185
      IF(ISEC.LE.1) INCALP=.FALSE.                                      0186
      IF(IPROB.EQ.1) INCALL=.FALSE.                                     0187
      IF(IPROB.EQ.1) INPRO=.FALSE.                                      0188
      IF(IPROB.LE.4) INCALD=.FALSE.                                     0189
      IF(IPROB.LE.2) INCALK=.FALSE.                                     0190
      IF(ISTRES.EQ.0) GO TO 49                                          0191
      IF(IPROB.EQ.2) GO TO 9                                            0192
      ICLT11=.TRUE.                                                     0193
      ICLT22=.TRUE.                                                     0194
      ICLT33=.TRUE.                                                     0195
9     CONTINUE                                                          0196
      ICLT13=.TRUE.                                                     0197
      ICLT23=.TRUE.                                                     0198
49    CONTINUE                                                          0199
C                                                                       0200
C***  INITIAL OPERATIONS                                                0201
C                                                                       0202
C-----CALCULATE GEOMETRICAL QUANTITIES AND SET VARIABLES TO ZERO        0203
C*** SET VALUES IN STORE TO ZERO                                        0204
      DO 30 J=1,JT                                                      0205
      DO 30 I=1,IT                                                      0206
      U(I,J)=0.0                                                        0207
      V1(I,J)=0.0                                                       0208
      V2(I,J)=0.0                                                       0209
      P(I,J)=0.0                                                        0210
      PP(I,J)=0.0                                                       0211
      TE(I,J)=0.0                                                       0212
      ED(I,J)=0.0                                                       0213
      AL(I,J)=0.0                                                       0214
      VIS(I,J)=VISCOS                                                   0215
      PRUT(I,J)=1.0                                                     0216
```

```
      T11(I,J)=0.0                                                      0217
      T22(I,J)=0.0                                                      0218
      DPE(I,J)=0.0                                                      0219
      DPW(I,J)=0.0                                                      0220
      DPN(I,J)=0.0                                                      0221
      DPS(I,J)=0.0                                                      0222
      DVS(I,J)=0.0                                                      0223
30    CONTINUE                                                          0224
      IF(IPROB.GT.1) CALL INIT                                          0225
      IF(IPROB.GT.1) GO TO 129                                          0226
      DO 77 J=1,JN                                                      0227
      IH=IMAX(J)                                                        0228
      DO 77 I=2,IH                                                      0229
77    U(I,J)=VB                                                         0230
129   CONTINUE                                                          0231
      IF(IPROB.GT.1) CALL PROPS(IVIS)                                   0232
      CALL BGEOM                                                        0233
      IND=0                                                             0234
      IF(IPROB.GT.2) CALL TURBK                                         0235
C-----INITIAL OUTPUT                                                    0236
      WRITE(6,210)                                                      0237
      IF(IPILE.EQ.0) WRITE(6,220) VAVG                                  0238
      IF(IPILE.EQ.0) WRITE(6,230) RE                                    0239
      IF(IPILE.EQ.1) WRITE(6,270) TWAVE                                 0240
      WRITE(6,250) VISCOS                                               0241
      WRITE(6,240) PITR                                                 0242
      IF(INCLV2) CALL PRINT(1,1,V2,HEDV2)                               0243
      IF(INCLV1) CALL PRINT(1,1,V1,HEDV1)                               0244
      IF(INCALP) CALL PRINT(1,1,P,HEDP)                                 0245
      IF(INCALU) CALL PRINT(1,1,U,HEDU)                                 0246
      IF(INCALK) CALL PRINT(1,1,TE,HEDK)                                0247
      IF(INCALD) CALL PRINT(1,1,ED,HEDD)                                0248
      IF(INCALL) CALL PRINT(1,1,AL,HEDL)                                0249
      IF(IPROB.GT.1) CALL PRINT(1,1,VIS,HEDN)                           0250
      IF(IPROP.GT.1) CALL PRINT(1,1,PRUT,HEDPR)                         0251
      IF(IPILE.EQ.0) WRITE(6,100) PR,TALAV,DH,VAVG,VB                   0252
```

```
      IF(IPILE.EQ.1) WRITE(6,106) FR,VB,TWAVE,TALAV                    0253
      WRITE (6,200)                                                    0254
      DO 23 J=1,JN                                                     0255
      XST=X1(IL)*USTAR(J)/VISCOS                                       0256
      WRITE (6,313) J,RTAL(J),USTAR(J),XST                             0257
23    CONTINUE                                                         0258
      UNORM=VB                                                         0259
      TENORM=TWAVE                                                     0260
      ENORM=4.0*TENORM*TWAVE**(0.5D0)/DH                               0261
      V2NORM=0.1*VB                                                    0262
      N11=0                                                            0263
      U1=U(IMON,JMON)                                                  0264
C                                                                      0265
C***        ITERATION LOOP                                             0266
CHAPTER 3 3 3 3 3 3 3 ITERATION LOOP 3 3 3 3 3 3 3 3 3                 0267
      WRITE(6,310) IMON,JMON                                           0268
  300 NITER=NITER+1                                                    0269
C-----UPDATE MAIN DEPENDENT VARIABLES                                  0270
      IF(ISEC.EQ.0) GO TO 46                                           0271
      IF(ISEC.EQ.1) CALL SECIN                                         0272
      IF(ISEC.EQ.1) GO TO 46                                           0273
      DO 6 N=1,ITR                                                     0274
      CALL CALCV2                                                      0275
      CALL CALCV1                                                      0276
      CALL CALCP                                                       0277
6     CONTINUE                                                         0278
46    CONTINUE                                                         0279
      CALL CALCU                                                       0280
      IF(IPROB.LE.3) GO TO 36                                          0281
      CALL CALCTE                                                      0282
      IF(IPROB.EQ.5) CALL CALCED                                       0283
36    CONTINUE                                                         0284
C-----UPDATE FLUID PROPERITIES                                         0285
      IF(IPROB.EQ.3) CALL TURBK                                        0286
      IF(IPROB.GT.3.AND.ISEC.EQ.2) CALL TURBK                          0287
      IF(ILEN.GE.2) CALL MIXLEN                                        0288
```

```
      IF(INPRO) CALL PROPS(IVIS)                                        0289
C-----INTERMEDIATE OUTPUT                                               0290
      RESU=RESU/UNORM                                                   0291
      RESK=RESK/TENORM                                                  0292
      RESE=RESE/ENORM                                                   0293
      IF(ISEC.GT.0) RESV2=RESV2/V2NORM                                  0294
      IF(ISEC.GT.0) RESV1=RESV1/V2NORM                                  0295
      WRITE(6,311) NITER,RESU,RESV1,RESV2,RESORM,RESK,RESE              0296
     1            ,U(IMON,JMON),V1(IMON,JMON),V2(IMON,JMON),            0297
     1   P(IMON,JMON),TE(IMON,JMON),ED(IMON,JMON)                       0298
      IF(MOD(NITER,INDPRI).NE.0) GO TO 301                              0299
      IF(INCLV2) CALL PRINT(1,1,V2,HEDV2)                               0300
      IF(INCLV1) CALL PRINT(1,1,V1,HEDV1)                               0301
      IF(INCALP) CALL PRINT(1,1,P,HEDP)                                 0302
      IF(INCALU) CALL PRINT(1,1,U,HEDU)                                 0303
      IF(INCALK) CALL PRINT(1,1,TE,HEDK)                                0304
      IF(INCALD) CALL PRINT(1,1,ED,HEDD)                                0305
      IF(IPILE.EQ.0) WRITE(6,100) FR,TALAV,DH,VAVG,VB                   0306
      IF(IPILE.EQ.1) WRITE(6,106) FR,VB,TWAVE,TALAV                     0307
      WRITE (6,200)                                                     0308
      DO 33 J=1,JN                                                      0309
      XST=X1(IL)*USTAR(J)/VISCOS                                        0310
      WRITE (6,313) J,RTAL(J),USTAR(J),XST                              0311
33    CONTINUE                                                          0312
      WRITE(6,310) IMON,JMON                                            0313
  301 CONTINUE                                                          0314
C-----TERMINATION TESTS                                                 0315
      IF(NITER.EQ.MAXIT) GO TO 302                                      0316
      ERR=DMAX1(RESU,RESK,RESE,RESV2)                                   0317
      IF(ERR.GT.CC) GO TO 300                                           0318
      IF((NITER-N11).LT.ND) GO TO 300                                   0319
      N11=NITER                                                         0320
      RSU=(U(IMON,JMON)-U1)/U(IMON,JMON)                                0321
      U1=U(IMON,JMON)                                                   0322
      IF(RSU.LT.1.0E-03) GO TO 300                                      0323
  302 CONTINUE                                                          0324
```

```
C                                                                       0325
C***    FINAL OUTPUT                                                    0326
C                                                                       0327
      IF(INCLV2) CALL PRINT(1,1,V2,HEDV2)                               0328
      IF(INCLV1) CALL PRINT(1,1,V1,HEDV1)                               0329
      IF(INCALP) CALL PRINT(1,1,P,HEDP)                                 0330
      IF(INCALU) CALL PRINT(1,1,U,HEDU)                                 0331
      IF(INCALK) CALL PRINT(1,1,TE,HEDK)                                0332
      IF(INCALD) CALL PRINT(1,1,ED,HEDD)                                0333
      IF(INCALL) CALL PRINT(1,1,AL,HEDL)                                0334
      IF(IPROB.GT.1) CALL PRINT(1,1,VIS,HEDN)                           0335
      IF(IPROB.GT.1) CALL PRINT(1,1,PRUT,HEDPR)                         0336
      IF(ISTPES.EQ.0) GO TO 45                                          0337
      IF(IPROB.GT.2) CALL TURBK                                         0338
      DO 51 J=1,JN                                                      0339
      IH=IMAX(J)                                                        0340
      DO 51 I=2,IH                                                      0341
      H1=1.0                                                            0342
      H2=1.0                                                            0343
      IF(I.EQ.2) H2=0.0                                                 0344
      IF(I.EQ.IH) GO TO 63                                              0345
      DU1= ((U(I+1,J)-U(I,J))*H1+(U(I,J)-U(I-1,J))*H2)/((R(I+1)-R(I))*H1 0346
     / +(R(I)-R(I-1))*H2)                                               0347
      GO TO 67                                                          0348
63    DU1= (U(I,J)-U(I-1,J))/(R(I)-R(I-1))                              0349
67    CONTINUE                                                          0350
      T13(I,J)= +(VIS(I,J)-VISCOS)*DU1                                  0351
      V3P(I,J)=2.*TE(I,J)-T11(I,J)-T22(I,J)                             0352
51    CONTINUE                                                          0353
      DO 61 J=1,JN                                                      0354
      IH=IMAX(J)                                                        0355
      DO 61 I=2,IH                                                      0356
      T23(I,J)=0.0                                                      0357
      IF(J.EQ.1.OR.J.EQ.JN) GO TO 61                                    0358
      H1=1.0                                                            0359
      H2=1.0                                                            0360
```

```
      IF(I.EQ.IH) H2=0.0                                                0361
      DU2= ((U(I,J+1)-U(I,J))*H1+(U(I,J)-U(I,J-1))*H2)/X2(2)/(H2+H1)    0362
      T23(I,J)=+(VIS(I,J)-VISCOS)/PRUT(I,J)*DU2                         0363
61    CONTINUE                                                          0364
C***   RMS VALUES                                                       0365
      DO 52 J=1,JN                                                      0366
      IH=IMAX(J)                                                        0367
      DO 52 I=IL,IH                                                     0368
      V1P(I,J)=DSQRT(T11(I,J))                                          0369
      V2P(I,J)=DSQRT(T22(I,J))                                          0370
      V3P(I,J)=DSQRT(V3P(I,J))                                          0371
52    CONTINUE                                                          0372
      IF(ICLT11) CALL PRINT(2,1,V1P,HEDV1P)                             0373
      IF(ICLT22) CALL PRINT(2,1,V2P,HEDV2P)                             0374
      IF(ICLT33) CALL PRINT(2,1,V3P,HEDV3P)                             0375
      IF(ICLT13) CALL PRINT(2,1,T13,HEDT13)                             0376
      IF(ICLT23) CALL PRINT(2,1,T23,HEDT23)                             0377
45    CONTINUE                                                          0378
      IF(IPILE.EQ.0) WRITE(6,100) FR,TALAV,DH,VAVG,VB                   0379
      IF(IPILE.EQ.1) WRITE(6,106) FR,VB,TWAVE,TALAV                     0380
      IF(IPILE.EQ.1) RE=VB*DH/VISCOS                                    0381
      WRITE(6,230) RE                                                   0382
      WRITE (6,200)                                                     0383
      DO 13 J=1,JN                                                      0384
      XST=X1(IL)*USTAR(J)/VISCOS                                        0385
      WRITE (6,313) J,RTAL(J),USTAR(J),XST                              0386
13    CONTINUE                                                          0387
      IF(INORM.EQ.0) GO TO 47                                           0388
C***  NORMALIZE                                                         0389
      WRITE(6,499)                                                      0390
      DO 64 J=1,JN                                                      0391
      IH=IMAX(J)                                                        0392
      DO 64 I=1,IH                                                      0393
      AL(I,J)=AL(I,J)/X1(IH)                                            0394
64    CONTINUE                                                          0395
      UO=U(IN,JN)                                                       0396
```

```
      USAV=TALAV**0.5                                                    0397
      GO TO (71,72,73,74),INORM                                          0398
71    FV=1./UO                                                           0399
      WRITE(6,504)                                                       0400
      CALL ANORM(1,INORM,ISTRES,ICURVE,FV)                               0401
      GO TO 47                                                           0402
72    FV=1./VB                                                           0403
      WRITE(6,500)                                                       0404
      CALL ANORM(1,INORM,ISTRES,ICURVE,FV)                               0405
      GO TO 47                                                           0406
73    FV=1./USAV                                                         0407
      WRITE(6,501)                                                       0408
      CALL ANORM(2,INORM,ISTRES,ICURVE,FV)                               0409
      GO TO 47                                                           0410
74    FV=1.                                                              0411
      WRITE(6,502)                                                       0412
      CALL ANORM(3,INORM,ISTRES,ICURVE,FV)                               0413
47    CONTINUE                                                           0414
48    CONTINUE                                                           0415
C-----FORMAT STATEMENTS                                                  0416
499   FORMAT(1H1,20X,' **********NORMALIZED MIXING LENGTH   ********')   0417
504   FORMAT(1H1,20X,' **********NORMALIZED RESULTS  BY MAX VEL ****')   0418
500   FORMAT(1H1,20X,' **********NORMALIZED RESULTS  BY VB  ********')   0419
501   FORMAT(1H1,20X,' **********NORMALIZED RESULTS  BY USAV********')   0420
502   FORMAT(1H1,20X,' **********NORMALIZED RESULTS  BY USTAR(J)*****')  0421
  010 FORMAT(9A4)                                                        0422
700   FORMAT(10I5)                                                       0423
701   FORMAT(8D10.3)                                                     0424
  210 FORMAT(1H0,47X,51HKASE T1 FULLY DEVELOPED FLOW IN A BARE ROD BUNDL 0425
     1E  ,////)                                                          0426
  220 FORMAT(//1H0,15X,21H BULK FLUID VELOCITY ,T60,1H=,3X,1PD11.3)      0427
  230 FORMAT(1H0,15X,16HREYNOLDS NUMBER ,T60,1H=,3X,1PD11.3)             0428
  240 FORMAT(1H0,15X,15HPITCH    RATIO ,T60,1H=,3X,1PD11.3)              0429
  250 FORMAT(1H0,15X,18HLAMINAR VISCOSITY ,T60,1H=,3X,1PD11.3)           0430
270   FORMAT(//1H0,15X,21H AVE SHEAR STRESS/RO ,T60,1H=,3X,1PD11.3)      0431
  310 FORMAT(1H0,4HITER,4X,106HI------------ABSOLUTE RESIDUAL SOURCE SUM 0432
```

```
     1S-------------I       I-------FIELD VALUES AT MONITORING LOCATION,          0433
     2     1H(,I2,1H,,I2,1H),9H---------I/2X,2HNO,6X,4HUMOM,6X,4HV1MO,6X,       0434
     34HV2MO,6X,4HMASS,6X,4HTKIN,6X,4HDISP,10X,1HU,9X,2HV1,8X,2HV2,8X,1H        0435
     4P,9X,1HK,9X,1HD)                                                          0436
  311 FORMAT(1H ,I3,4X,1P6D10.3,3X,1P6D10.3)                                    0437
  402 FORMAT(///5X,1HI,7X,5HXU(I),6X,10HS.S.COEFF.)                             0438
  403 FORMAT(/5X,I2,3X,2(1PD11.3))                                              0439
100   FORMAT(31H0FLOW INFORMATION CALCULATED IS//                              0440
     110X,50HFR,      FRICTION FACTOR.........................=,1PD15.6/       0441
     210X,50HTALAV,   AVERAGE WALL SHEAR STRESS/ROREF.........=,1PD15.6/       0442
     310X,50HDH,      HYDRAULIC DIAMETER.....................=,1PD15.6/       0443
     410X,50HVAVG,    BULK VELOCITY..........................=,1PD15.6/       0444
     510X,50HVB,      COMPUTED BULK VELOCITY.................=,1PD15.6)       0445
106   FORMAT(31H0FLOW INFORMATION CALCULATED IS//                              0446
     110X,50HFR,      FRICTION FACTOR.........................=,1PD15.6/       0447
     410X,50HVB       BULK VELOCITY..........................=,1PD15.6/       0448
     210X,50HTWAVE,   AVERAGE WALL SHEAR STRESS/ROREF.........=,1PD15.6/       0449
     510X,50HTALAV,   COMPUTED AVE SHEAR STRESS/ROREF.........=,1PD15.6)       0450
200   FORMAT(1H130X,35HDISTRIBUTION OF WALL SHEAR STRESSES///                  0451
     /5X,1HJ,10X,9HTAL/TALAV,10X,5HUSTAR//)                                   0452
313   FORMAT(4X,I2,8X,D11.4,6X,1PD11.3,6X,1PD11.3/)                            0453
      STOP                                                                     0454
      END                                                                      0455
      SUBROUTINE  SHEAR                                                        0456
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                                  0457
      COMMON                                                                   0458
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),       0459
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                                0460
     1/AGRID/R(30),X1(30),X2(16)                                               0461
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE                         0462
     1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB                   0463
C*************************************************************               0464
C   SUBROUTINE FOR COMPUTATION OF WALL FRICTION BY IBRAGIMOV METHOD            0465
C*************************************************************               0466
C                                                                              0467
      IF(IPRO.EQ.1) GO TO 10                                                   0468
```

```
C*** COMPUTE YAV                                                        0469
      YAV=0.5*(3./3.1416*PIT*DLOG(3.D0)=DIA)                            0470
C*** COMPUTE FRICTION FACTOR, FR                                        0471
      X=ATOT/YAV**2                                                     0472
      XK=X**0.25*PIT*(1./DSQRT(3.D0)-0.5)/YAV                           0473
      XX=-0.021*XK**3                                                   0474
      RFR=(.58+.42*DEXP(XX))*(1.+.1*(2.*YAV/DIA+1.)**(4./3.))           0475
      IF(IPILE.EQ.1) GO TO 2                                            0476
      FCIR=0.046*RE**(-.2)                                              0477
      FR=RFR*FCIR                                                       0478
C*** COMPUTE DISTRIBUTION OF TAL AND USTAR                              0479
      TALAV=0.5*FR*VAVG**2                                              0480
      TWAVE=TALAV                                                       0481
      GO TO 3                                                           0482
2     REP=1./1.8                                                        0483
      BAS=TWAVE*(DH**0.2)*(VISCOS**(-0.2))/RFR/0.023                    0484
      VAVG=BAS**REP                                                     0485
      RE=VAVG*DH/VISCOS                                                 0486
      FCIR=0.046*RE**(-.2)                                              0487
      FR=RFR*FCIR                                                       0488
      TALAV=TWAVE                                                       0489
3     CONTINUE                                                          0490
      AC=3.85*PIT/(X**0.8*YAV)                                          0491
      CC=(AC/2.)**0.5*3.1416/6.                                         0492
      C=1.-6./3.1416*(3.1416/2./AC)**0.5*DEXP(-AC)*DERF(CC)             0493
      C=1./C                                                            0494
      B=7.7/(X**0.8*YAV)                                                0495
      CC=3.1416/6.*(0.25*B*PIT)**0.5                                    0496
      CD=-0.5*B*(PIT-DIA)                                               0497
      CI=6./(3.14159*B*PIT)**0.5*DEXP(CD)*DERF(CC)                      0498
      C=1./(1.-CI)                                                      0499
      DTETA=3.1416/6./DFLOAT(JNM)                                       0500
      DO 1 J=1,JN                                                       0501
      TETA=DTETA*DFLOAT(J-1)                                            0502
      YT=0.5*(PIT/DCOS(TETA)-DIA)                                       0503
      BY=-B*YT                                                          0504
```

```
      RTAL(J)=C*(1.-DEXP(BY))                                                    0505
      USTAR(J)=(TALAV*RTAL(J))**0.5                                              0506
1     CONTINUE                                                                   0507
      RETURN                                                                     0508
10    CONTINUE                                                                   0509
      IF(IPILE.EQ.1) GO TO 30                                                    0510
      FR=16/RE                                                                   0511
      TALAV=0.5*FR*VAVG**2                                                       0512
      TWAVE=TALAV                                                                0513
      GO TO 40                                                                   0514
30    TALAV=TWAVE                                                                0515
      RE=TWAVE*(DH/VISCOS)**2/8.                                                 0516
      VAVG=RE*VISCOS/DH                                                          0517
      FR=16./RE                                                                  0518
40    CONTINUE                                                                   0519
      DO 20 J=1,JN                                                               0520
      RTAL(J)=1.0                                                                0521
20    USTAR(J)=DSQRT(TALAV)                                                      0522
      VB=VAVG                                                                    0523
      RETURN                                                                     0524
      END                                                                        0525
      END                                                                        0526
      SUBROUTINE GRID(RAT)                                                       0527
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                                    0528
      COMMON                                                                     0529
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),         0530
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                                  0531
     1/AGRID/R(30),X1(30),X2(16)                                                0532
     1/FLUPR/UREVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB                    0533
     1/WALLP/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE                          0534
C*************************************************************                0535
C   SUBROUTINE TO COMPUTE THE GRID COORDINATES                                 0536
C*************************************************************                0537
C                                                                              0538
C*** CIRCUMFERENTIAL COORDINATES, X2(J)                                        0539
      DTETA=3.14159/6./DFLOAT(JNM)                                             0540
```

```
      X2(1)=0.                                                          0541
      DO 1 J=2,JN                                                       0542
1     X2(J)=X2(J-1)+DTETA                                               0543
C***  RADIAL COORDINATES, X1(I)                                         0544
      X1(1)=0.0                                                         0545
      GAP=0.5*(PIT-DIA)                                                 0546
      M=IN-JN+JJL                                                       0547
      MN=M-1                                                            0548
C***  ESTIMATE WALL SUBLAYER THICKNESS                                  0549
      XB= YB*VISCOS/DSQRT(TALAV)                                        0550
      X1(IL)=XB                                                         0551
      X1(M)=GAP                                                         0552
      DX1=(GAP-XB)/DFLOAT(M-IL)                                         0553
      IF(RAT.GT.1.)DX1=(GAP-XB)*(RAT-1.)/(RAT**(M-IL)-1.)               0554
      DO 20 I=ILP,MN                                                    0555
20    X1(I)=X1(I-1)+DX1*RAT**(I-IL-1)                                   0556
      JJP=JJL+1                                                         0557
      DO 3 J=JJP,JN                                                     0558
      TETA=X2(J)                                                        0559
3     X1(M+J-JJL)= PIT/2./DCOS(TETA)-DIA/2.                             0560
C***  RADIAL DISTANCES, R(I)                                            0561
      DO 4 I=1,IN                                                       0562
4     R(I)=0.5*DIA+X1(I)                                                0563
      DO 5 J=1,JN                                                       0564
      IMAX(J)=M+J-JJL                                                   0565
      IF(J.LE.JJL) IMAX(J)=M                                            0566
5     CONTINUE                                                          0567
      DO 7 I=1,M                                                        0568
7     JMIN(I)=1                                                         0569
      MP=M+1                                                            0570
      DO 8 I=MP,IN                                                      0571
8     JMIN(I)=JMIN(I-1)+1                                               0572
C***  PRINT OUT COORDINATES,                                            0573
      WRITE(6,101) (X1(I),I=1,IN)                                       0574
      WRITE(6,102) (X2(J),J=1,JN)                                       0575
      WRITE(6,103) (IMAX(J),J=1,JN)                                     0576
```

```
      WRITE(6,104)  (JMIN(I),I=1,IN)                                    0577
101   FORMAT(25HODISTANCES IN DIRECTION-1/(1H 4D25.8))                  0578
102   FORMAT(25HODISTANCES IN DIRECTION-2/(1H 4D25.8))                  0579
103   FORMAT(25HOVALUES OF IMAX          /(1H 4I4))                     0580
104   FORMAT(25HOVALUES OF JMIN          /(1H 4I4))                     0581
      RETURN                                                            0582
      END                                                               0583
      SUBROUTINE  INIT                                                  0584
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                           0585
      COMMON/ALEN/CL,RL,IPR,ILEN                                        0586
      COMMON                                                            0587
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),   0588
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)          0589
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),0590
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                         0591
     1/AGRID/R(30),X1(30),X2(16)                                        0592
     1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB            0593
     1/STRESS/T11(30,16),T22(30,16)                                     0594
     1/WALLP/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE                  0595
     1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED                       0596
C*****************************************************************    0597
C   SUBROUTINE FOR COMPUTATION OF INITIAL CONDITIONS                    0598
C*****************************************************************    0599
C                                                                       0600
C*** INITIAL CONDITIONS FOR AXIAL VELOCITY                              0601
      U0=14.7                                                           0602
      DO 3 J=1,JN                                                       0603
      IH=IMAX(J)                                                        0604
      BK=0.387*(1.+0.05*R(IH)/R(1))                                     0605
      TT=   DEXP(-1.26*DSQRT(R(IH)/R(1)))*10.                           0606
      C=45.*(1.-0.34*45./14.7+0.039*(45./14.7)**2)-DLOG(45.0D0)/BK      0607
      DO 3 I=2,IH                                                       0608
      XS=USTAR(J)*X1(I)/VISCOS                                          0609
      IF(XS.GT.45.0) GO TO 4                                            0610
      US=XS*(1.-0.34*XS/14.7+0.039*(XS/14.7)**2)                        0611
      GO TO 3                                                           0612
```

```
    4 Y=X1(I)/X1(IH)                                                    0613
      US=DLOG(YS)/BK+C*DLOG(TT*(2.-Y)/2./(1.+(TT-1.)*(1.-Y)**2))/BK     0614
3     U(I,J)=USTAR(J)*US                                                0615
C***  CORRECT VALUES                                                    0616
      CALL REYNO                                                        0617
      VB=VAVG                                                           0618
C*** COMPUTE MIXING LENGTH                                              0619
      CALL MIXLEN                                                       0620
C*** INITIAL CONDITIONS FOR TKE ASSUMING BOBKOV'S CORRELATION           0621
      CALL TURBK                                                        0622
      IF(IPROB.EQ.5) GO TO 60                                           0623
      GO TO 50                                                          0624
60    DO 20 J=1,JN                                                      0625
      IH=IMAX(J)                                                        0626
      DO 20 I=IL,IH                                                     0627
20    ED(I,J)=CDIS*TE(I,J)**1.5/AL(I,J)                                 0628
50    CONTINUE                                                          0629
      RETURN                                                            0630
      END                                                               0631
      SUBROUTINE TURBK                                                  0632
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                           0633
      DIMENSION FP(30,16),FT(30,16),FZ(30,16)                           0634
      COMMON/ATURB/DVS(30,16),CA,CA1,CA2,CB,ITURB                       0635
      COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),      0636
     / ANG(16),ANGM(16),VBP(16)                                         0637
      COMMON                                                            0638
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),   0639
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)          0640
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),0641
     2  ISEC,IPILE,JJL,IGONV,NSOLU,INUM,JL,JLP                         0642
     1/AGRID/R(30),X1(30),X2(16)                                        0643
     1/STRESS/T11(30,16),T22(30,16)                                     0644
     1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB            0645
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE                  0646
C                                                                       0647
C***  TURBULENT KINETIC ENERGY AND NORMAL STRESS CALCULATIONS           0648
```

```
C                                                                       0649
      FV=1.-VAVG/U(IN,JN)                                               0650
      IF(IND.EQ.1) GO TO 1                                              0651
      GO TO (15,45),ITURB                                               0652
1     CONTINUE                                                          0653
      DO 10 J=1,JN                                                      0654
      IH=IMAX(J)                                                        0655
      DO 10 I=IL,IH                                                     0656
      ETA=X1(I)/X1(IH)                                                  0657
      YZ=-1.48*ETA                                                      0658
      YT=-1.17*ETA                                                      0659
      YR=-0.77*ETA                                                      0660
      FP(I,J)=0.40*DEXP(YR)                                             0661
      FT(I,J)=0.60*DEXP(YT)                                             0662
      FZ(I,J)=0.95*DEXP(YZ)                                             0663
10    CONTINUE                                                          0664
15    CONTINUE                                                          0665
      DO 20 J=1,JN                                                      0666
      IH=IMAX(J)                                                        0667
      DO 20 I=IL,IH                                                     0668
      T11(I,J)=(FP(I,J)*FV*U(I,J))**2                                   0669
      T22(I,J)=(FT(I,J)*FV*U(I,J))**2                                   0670
      IF(IND.EQ.0.AND.IPROB.NE.3) GO TO 20                              0671
      T33      =(FZ(I,J)*FV*U(I,J))**2                                  0672
      TE(I,J)=0.5*(T11(I,J)+T22(I,J)+T33)                               0673
20    CONTINUE                                                          0674
      RETURN                                                            0675
45    CONTINUE                                                          0676
      CALL MODSTR                                                       0677
      RETURN                                                            0678
      END                                                               0679
      SUBROUTINE MIXLEN                                                 0680
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                           0681
      COMMON/ALEN/CL,RL,IPR,ILEN                                        0682
      COMMON                                                            0683
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),   0684
```

```
     2  ED(30,16),AL(30,16),VLS(30,16),GEN(30,16),PRUT(30,16)                  0685
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),       0686
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                                 0687
     1/AGRID/R(30),X1(30),X2(16)                                               0688
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),ER,YB,TWAVE                         0689
     1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED                              0690
     1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,PIA,DH,ATOT,RE,VAVG,VB                   0691
C*************************************************************               0692
C   SUBROUTINE TO COMPUTE THE MIXING LENGTH                                   0693
C*************************************************************               0694
      DO 10 J=1,JN                                                           0695
      IH=IMAX(J)                                                             0696
      DO 10 I=2,IH                                                           0697
      IF((ILEN.GT.1) GO TO 20                                                0698
      ETA=X1(I)/X1(IH)                                                       0699
      IF(ETA.GT.0.44) GO TO 15                                               0700
      AL(I,J)=X1(I)                                                          0701
      GO TO 10                                                               0702
15    CT=3.14159/.38*(ETA-0.44)                                              0703
      AL(I,J)  =(0.44+0.066*DSIN(CT))*X1(IH)                                 0704
      GO TO 10                                                               0705
20    CONTINUE                                                               0706
      IF(I.EQ.2) GO TO 11                                                    0707
      IF(I.EQ.IH) GO TO 12                                                   0708
      H1=(R(I-1)-R(I))/(R(I+1)-R(I))                                         0709
      H2=1./H1                                                               0710
      DU1=((U(I+1,J)-U(I,J))*H1+(U(I,J)-U(I-1,J))*H2)/((R(I+1)-R(I))*H1+     0711
     /  (R(I)-R(I-1))*H2)                                                    0712
      GO TO 13                                                               0713
12    DU1=(U(I,J)-U(I-1,J))/(R(I)-R(I-1))                                    0714
      GO TO 13                                                               0715
11    DU1=(U(3,J)-U(2,J))/(R(3)-R(2))                                        0716
13    CONTINUE                                                               0717
      IF(ILEN.EQ.2) AL(I,J)=(DU1/USTAR(J)*CK)**RL + (1./CL/X1(IH))**RL       0718
      AR=1./RL                                                               0719
      AL(I,J)=1./AL(I,J)**AR                                                 0720
```

```
10    CONTINUE                                                                0721
      RETURN                                                                  0722
      END                                                                     0723
      SUBROUTINE  SECIN                                                       0724
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                                 0725
      COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),            0726
     / ANG(16),ANGM(16),VBP(16)                                               0727
      COMMON                                                                  0728
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),         0729
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)                 0730
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),      0731
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                               0732
     1/AGRID/R(30),X1(30),X2(16)                                              0733
     1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB                  0734
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE                        0735
C*************************************************************              0736
C   SUBROUTINE TO COMPUTE STREAM FUNCTION USING NIJSING APPROXIMATION         0737
C*************************************************************              0738
      PI=3.14159                                                              0739
C***  V2-VELOCITY                                                            0740
      DO 100 J=3,JNM                                                          0741
      IH=IMAX(J)-1                                                            0742
      DU=(USTAR(J)  -USTAR(J-1))/X2(2)                                        0743
      DO 100 I=2,IH                                                           0744
      ANGLE=PI*2.  *X1(I)/(X1(IH)+X1(IH-1))                                   0745
100   V2(I,J)=0.628*DU*DCOS(ANGLE)                                            0746
C***  V1-VELOCITY                                                            0747
      DO 102 J=2,JNM                                                          0748
      IH=IMAX(J)-1                                                            0749
      DO 101 I=2,IH                                                           0750
101   V1(I+1,J)=(ARW(I)*V1(I,J)+ARN(I)*(V2(I,J)-V2(I,J+1)))/ARE(I)            0751
102   CONTINUE                                                                0752
      CALL MODV1                                                              0753
      RETURN                                                                  0754
      END                                                                     0755
      SUBROUTINE  REYNU                                                       0756
```

```
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                                  0757
      COMMON                                                                   0758
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),          0759
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)                 0760
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),       0761
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                                0762
     1/AGRID/R(30),X1(30),X2(16)                                              0763
     1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB                  0764
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE                        0765
C*************************************************************              0766
C    SUBROUTINE FOR COMPUTATION OF BULK VELOCITY, VB                          0767
C*************************************************************              0768
C                                                                             0769
      AV=0.0                                                                  0770
      ETETA=X2(2)                                                             0771
      DO 20 J=1,JN                                                            0772
C***  INTERNAL POINTS                                                         0773
      TAV=0.0                                                                 0774
      IH=IMAX(J)                                                              0775
      IF(IPROB.EQ.1) GO TO 40                                                 0776
      TAV=VISCOS*(R(1)*25./2.+125.*VISCOS/USTAR(J)/3.)                        0777
      A1=U(IL,J)/USTAR(J)-5.                                                  0778
      A2=U(IL,J)/USTAR(J)+5.                                                  0779
      YS=X1(IL)*USTAR(J)/VISCOS                                               0780
      A1=A1/DLOG(YS/5.0D0)                                                    0781
      A2=(A2-A1*DLOG(5.0D0*YS))/2.                                            0782
      AI1=              A2*(YS-5.)+A1*(YS*(DLOG(YS)-1.)-5.*(DLOG(5.0D0)-1.    0783
     )))                                                                      0784
      AI2= A1*(YS**2-25.)/2. +A2*(YS**2*( DLOG(YS)-0.5)/2.-25.*( DLOG(5.      0785
     /0D0)-0.5)/2.)                                                           0786
      TAV=TAV+VISCOS*(AI1*R(1)+VISCOS/USTAR(J)*AI2)                           0787
40    CONTINUE                                                                0788
      DO 10 I=ILP,IH                                                          0789
      DAV=(R(I)*U(I,J)    +R(I-1)*U(I-1,J)    )*(R(I)-R(I-1))/2.              0790
   10 TAV=TAV+DAV                                                             0791
      TAV=TAV*X2(2)                                                           0792
```

```
      IF(J.EQ.1.OR.J.EQ.JN)  TAV=0.5*TAV                                0793
20    AV=AV+TAV                                                         0794
C***  CALCULATE BULK VELOCITY                                           0795
      VB=AV/ATOT                                                        0796
      IF(VB.EQ.0.0)  VB=VAVG                                            0797
      IF(IPILE.EQ.1.AND.IND.EQ.0)  GO TO 35                             0798
C***  NORMALIZE VELOCITY DISTRIBUTION                                   0799
      F=VAVG/VB                                                         0800
      DO 39 J=1,JN                                                      0801
      IH=IMAX(J)                                                        0802
      DO 39 I=2,IH                                                      0803
39    U(I,J)=F*U(I,J)                                                   0804
35    CONTINUE                                                          0805
      RETURN                                                            0806
      END                                                               0807
      SUBROUTINE  BGEOM                                                 0808
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                           0809
      COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),      0810
     / ANG(16),ANGM(16),VBP(16)                                         0811
      COMMON/GEOMD/DV1E(30),DV1W(30),DV1N(30),CV1E(30),CV1W(30)         0812
      COMMON/GEOME/DV2E(30),DV2W(30),DV2N(30),CV2E(30),CV2W(30),ARPN(30)  0813
      COMMON                                                            0814
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),  0815
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                         0816
     1/AGRID/R(30),X1(30),X2(16)                                        0817
     1/GEOM/BE(30),BN(30),BW(30),BS(30),VP(30),VP1(30)                  0818
C*******************************************************************  0819
C    SUBROUTINE FOR CALCULATION OF GEOM FACTORS                         0820
C*******************************************************************  0821
      DTETA=X2(2)                                                       0822
C***  COMPUTE BE,BW,BN,BS AND VP                                        0823
      VP(IN)=DTETA*R(IN)*(R(IN)-R(INM))/2.                              0824
      DX1(1)=X1(2)                                                      0825
      RM(1)=0.5*(R(2)+R(1))                                             0826
      DO 21 I=2,INM                                                     0827
      BE(I)=.25*DTETA*(R(I+1)+R(I))/(R(I+1)-R(I))                       0828
```

```
      BW(I)=.25*DTETA*(R(I-1)+R(I))/(R(I)-R(I-1))                                0829
      BN(I)=.25*(R(I+1)-R(I-1))/R(I)/DTETA                                       0830
      BS(I)=BN(I)                                                                0831
      VP1(I)   =(R(I)**2-R(I-1)**2)*DTETA/2.                                     0832
      VP(I)=DTETA*(R(I+1)-R(I-1))* R(I)/2.                                       0833
      RM(I)=0.5*(R(I)+R(I+1))                                                    0834
      ARE(I)=RM(I)*DTETA                                                         0835
      ARW(I)=RM(I-1)*DTETA                                                       0836
      ARN(I)=0.5*(R(I+1)-R(I-1))                                                 0837
      ARP(I)=R(I)*DTETA                                                          0838
      DX1(I)=R(I+1)-R(I)                                                         0839
      DV1E(I)=2.*ARP(I)/ARN(I)                                                   0840
      DV1N(I)=2.*(R(I)-R(I-1))/(R(I)+R(I-1))/DTETA                               0841
      CV1E(I)=DX1(I-1)/4./ARN(I)                                                 0842
      ARPN(I)=ARN(I)*R(I)                                                        0843
      DV2E(I)=2.                *(R(I)+R(I+1))**2/4./R(I+1)*BE(I)                0844
      DV2W(I)=          2.      *(R(I)+R(I-1))**2/4./R(I-1)*BW(I)                0845
      DV2N(I)=4.*R(I)*BN(I)                                                      0846
      CV2E(I)=R(I+1)/4./DX1(I)                                                   0847
      CV2W(I)=R(I-1)/4./DX1(I-1)                                                 0848
      IF(I.EQ.2) GO TO 21                                                        0849
      DV1W(I)=2.*ARP(I-1)/ARN(I-1)                                               0850
      CV1W(I)=DX1(I-1)/4./ARN(I-1)                                               0851
21    CONTINUE                                                                   0852
      ANG(1)=0.0                                                                 0853
      DO 22 J=2,JN                                                               0854
      I=IMAX(J)                                                                  0855
      ANG(J)=DTAN(X2(J))                                                         0856
      ANGM(J)=DTAN(X2(J)-0.5D0*X2(2))                                            0857
      VBP(J)=((R(I+1)+R(I))*DSIN(X2(2))-(R(I)+R(I-1))*X2(2))*(R(I)+R(I-1         0858
     /))/8.                                                                      0859
22    CONTINUE                                                                   0860
      VBP(1)=0.0                                                                 0861
      VBP(JN)=RM(INM)*(R(IN)*DSIN(X2(2)/2.)-RM(INM)*X2(2)/2.)                    0862
      VP(IN)=VBP(JN)                                                             0863
      RETURN                                                                     0864
```

```
      END                                                                0865
      SUBROUTINE PROPS(IVIS)                                             0866
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                            0867
      COMMON                                                             0868
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),    0869
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)           0870
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30), 0871
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                          0872
     1/AGRID/R(30),X1(30),X2(16)                                        0873
     1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB            0874
     1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED                       0875
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),PR,YB,TWAVE                  0876
      COMMON/ALEN/CL,RL,IPR,ILEN                                        0877
C                                                                       0878
C*** VISCOCITY AND VISCOCITY RATIO CALCULATIONS                         0879
C                                                                       0880
C***   VISCOCITY RATIO                                                  0881
      IF(IND.EQ.0) GO TO 11                                             0882
      CTAN=0.154                                                        0883
      GO TO (13,15,17),IPR                                              0884
13    CONTINUE                                                          0885
      DO 14 J=1,JN                                                      0886
      IH=IMAX(J)                                                        0887
      DO 14 I=1,IH                                                      0888
14    PRUT(I,J)=1.0                                                     0889
      GO TO 11                                                          0890
15    CONTINUE                                                          0891
      DO 16 J=1,JN                                                      0892
      IH=IMAX(J)                                                        0893
      DO 16 I=1,IH                                                      0894
16    PRUT(I,J)=AL(I,J)/AL(IN,JN)                                       0895
      GO TO 11                                                          0896
17    CONTINUE                                                          0897
      DO 10 J=1,JN                                                      0898
      IH=IMAX(J)                                                        0899
      AK=1.+X1(IH)/R(1)                                                 0900
```

```
      BK=0.407+0.02*X1(IH)/R(1)                                          0901
      C=30.*(1.-0.34*30./14.7+0.039*(30./14.7)**2)-DLOG(30.0D0)/BK       0902
      B=(1.82-1.3*X1(IH)/R(1))/(1.+0.14*(DLOG10(RE)-5.))                 0903
      DO 10 I=IL,IH                                                      0904
      Y=1.-X1(I)/X1(IH)                                                  0905
      A2=(2.*AK-Y*(AK-1.))/(AK-Y*(AK-1.))/(AK+1.)                        0906
      PRUT(I,J)=AK*(B+Y**2)*(1.-Y**2)*A2/2./(1.+B)/CTAN                  0907
10    CONTINUE                                                           0908
11    CONTINUE                                                           0909
C                                                                        0910
C***  RADIAL VISCOSITY                                                   0911
      GO TO (1,2),IVIS                                                   0912
1     CONTINUE                                                           0913
      DO 100 J=1,JN                                                      0914
      IH=IMAX(J)                                                         0915
      DO 100 I=IL,IH                                                     0916
      VISOLD=VIS(I,J)                                                    0917
      IF(IPROB.LT.5) ED(I,J)=CDIS*TE(I,J)**1.5/AL(I,J)                   0918
      IF(ED(I,J).EQ.0.) GO TO 102                                        0919
      VIS(I,J)=          TE(I,J)**2*CMU/ED(I,J)+VISCOS                   0920
      GO TO 101                                                          0921
  102 VIS(I,J)=VISCOS                                                    0922
C-----UNDER-RELAX VISCOSITY                                              0923
101   VIS(I,J)=URFVIS*VIS(I,J)+(1.-URFVIS)*VISOLD                        0924
  100 CONTINUE                                                           0925
      RETURN                                                             0926
C                                                                        0927
2     CONTINUE                                                           0928
      DO 200 J=1,JN                                                      0929
      IH=IMAX(J)                                                         0930
      DO 200 I=IL,IH                                                     0931
200   VIS(I,J)=CTAN*PRUT(I,J)*X1(IH)*USTAR(J)                            0932
      RETURN                                                             0933
      END                                                                0934
      SUBROUTINE CALCV1                                                  0935
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                            0936
```

```
C                                                                       0937
      COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),      0938
     / ANG(16),ANGM(16),VBP(16)                                         0939
      COMMON/GEOMD/DV1E(30),DV1W(30),DV1N(30),CV1E(30),CV1W(30)         0940
      COMMON/ATURB/DVS(30,16),CA,CA1,CA2,CB,ITURB                       0941
      COMMON                                                            0942
     1/V1VEL/URFV1,RESV1,NSWPV1                                         0943
     1/PCOR/RESORM,URFP,RESPP,NSWPP,IPREF,JPREF                         0944
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),   0945
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)           0946
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),0947
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                          0948
     1/AGRID/R(30),X1(30),X2(16)                                        0949
     1/GEOM/BE(30),BN(30),BW(30),BS(30),VP(30),VP1(30)                  0950
     1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB            0951
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE                  0952
     1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED                       0953
     1/STRESS/T11(30,16),T22(30,16)                                     0954
     1/GEOMA/DPE(30,16),DPW(30,16),DPN(30,16),DPS(30,16)                0955
     1/COEF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SU(30,16) 0956
C                                                                       0957
C***V1 VELOCITY                                                         0958
C                                                                       0959
      DTETA=X2(2)                                                       0960
      VISE=VISCOS                                                       0961
      VISW=VISCOS                                                       0962
      DO 100 J=2,JNM                                                    0963
      IH=IMAX(J)                                                        0964
      DO 101 I=3,IH                                                     0965
C-----CALCULATE DIFFUSION COEFFICIENTS                                  0966
      IF(I.EQ.IH) VIS(I,J-1)=VIS(I,J)                                   0967
      VISN=(VIS(I,J)+VIS(I,J+1)+VIS(I-1,J+1)  +VIS(I-1,J))*0.25         0968
      VISS=(VIS(I,J)+VIS(I,J-1)+VIS(I-1,J-1)  +VIS(I-1,J))*0.25         0969
      DE=VISE*DV1E(I)                                                   0970
      DW=VISW*DV1W(I)                                                   0971
      DN=VISN*DV1N(I)                                                   0972
```

```
      DS=VISS*DV1N(I)                                                    0973
C---- CALCULATE CONVECTION COEFFICIENTS                                 0974
      V1E=(V1(I,J)*DX1(I)          +V1(I+1,J)*DX1(I-1))/(DX1(I)+DX1(I-1) 0975
     /)                                                                  0976
      V1W=(V1(I,J)*DX1(I-2)+V1(I-1,J)*DX1(I-1))/(DX1(I-2)+DX1(I-1))      0977
      V2N=(V2(I,J+1)+V2(I-1,J+1))/2.                                     0978
      V2S=(V2(I,J)  +V2(I-1,J))/2.                                       0979
      CE=ARP(I)*V1E                                                      0980
      CW= ARP(I-1)*V1W                                                   0981
      CN=DX1(I-1)*V2N                                                    0982
      CS=DX1(I-1)*V2S                                                    0983
      SMP=CN-CS+CE-CW                                                    0984
C---- ASSEMBLE MAIN COEFFICIEMTS                                        0985
      IF(ICONV.EQ.1) GO TO 21                                            0986
      A1=CE+CW                                                           0987
      CE=(DABS(A1)-A1)*CV1E(I)                                           0988
      CW=            (DABS(A1)+A1)*CV1W(I)                               0989
      A2P=(CN+CS)/2.                                                     0990
      CN=(DABS(A2P)-A2P)/2.                                              0991
      CS=(DABS(A2P)+A2P)/2.                                              0992
      GO TO 22                                                           0993
21    CE=0.5*(DABS(CE)-CE)                                               0994
      CW=0.5*(DABS(CW)+CW)                                               0995
      CN=0.5*(DABS(CN)-CN)                                               0996
      CS=0.5*(DABS(CS)+CS)                                               0997
22    CONTINUE                                                           0998
      AE(I,J)=CE+DE                                                      0999
      AW(I,J)=CW+DW                                                      1000
      AN(I,J)=CN+DN                                                      1001
      AS(I,J)=CS+DS                                                      1002
      AP(I,J)=AE(I,J)+AW(I,J)+AN(I,J)+AS(I,J)                            1003
C---- CALCULATE  SOURCE TERMS                                           1004
      TTP=(T22(I,J)+T22(I-1,J))/2.                                       1005
      S1=TTP*VP1(I)/RM(I-1)                                              1006
      S2=ARP(I-1)*(T11(I-1,J)+P(I-1,J))-ARP(I)*(T11(I,J)+P(I,J))         1007
      S31=      VISN*RM(I-1)       *(V2(I,J+1)/R(I)-V2(I-1,J+1)/R(I-1))  1008
```

```
      S32=-    VISS*RM(I-1)       *(V2(I,J)/R(I)-V2(I-1,J)/R(I-1))                1009
      SU(I,J)=S1+S2+S31+S32                                                       1010
101   CONTINUE                                                                    1011
      I=IH+1                                                                      1012
      AP(I,J)=1.0                                                                 1013
      AE(I,J)=0.0                                                                 1014
      AW(I,J)=0.0                                                                 1015
      AN(I,J)=0.0                                                                 1016
      AS(I,J)=0.0                                                                 1017
      IF(J.EQ.JNM) GO TO 104                                                      1018
      SU(I,J)=ARN(I-1)*V2(I-1,J)/ARE(I-1)                                         1019
      GO TO 100                                                                   1020
104   SU(I,J)=0.0                                                                 1021
100   CONTINUE                                                                    1022
      DO 300 J=2,JNM                                                              1023
      IH=IMAX(J)-1                                                                1024
      DO 301 I=2,IH                                                               1025
      DPE(I,J)=ARP(I+1)/AP(I+1,J)                                                 1026
      DPE(I,J)=DPE(I,J)*URFV1                                                     1027
301   CONTINUE                                                                    1028
      IH=IMAX(J)                                                                  1029
      DO 302 I=3,IH                                                               1030
      DPW(I,J)=ARP(I-1)/AP(I,J)                                                   1031
      DPW(I,J)=DPW(I,J)*URFV1                                                     1032
302   CONTINUE                                                                    1033
300   CONTINUE                                                                    1034
      DO 350 J=2,JNM                                                              1035
      IH=IMAX(J)+1                                                                1036
      DO 350 I=3,IH                                                               1037
      AP(I,J)=AP(I,J)/URFV1                                                       1038
      SU(I,J)=SU(I,J)+(1.-URFV1)*AP(I,J)*V1(I,J)                                  1039
350   CONTINUE                                                                    1040
C                                                                                 1041
C***SOLUTION OF THE DIFFERENCE EQUATION                                           1042
C                                                                                 1043
      RESV1=0.0                                                                   1044
```

```
      DO 400 N=1,NSWPV1                                                 1045
      IF(NSOLU.EQ.0) CALL SOLV1(3,2,JNM,1,V1,RES)                       1046
      IF(NSOLU.EQ.1) CALL SOLV2(3,2,JNM,1,V1,RES)                       1047
      IF(RES.GT.RESV1) RESV1=RES                                        1048
      CALL MODV1                                                        1049
400   CONTINUE                                                          1050
      RETURN                                                            1051
      END                                                               1052
      SUBROUTINE CALCV2                                                 1053
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                           1054
      COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),      1055
     / ANG(16),ANGM(16),VBP(16)                                         1056
      COMMON/GEOME/DV2E(30),DV2W(30),DV2N(30),CV2E(30),CV2W(30),ARPN(30)1057
      COMMON/ATURB/DVS(30,16),CA,CA1,CA2,CB,ITURB                       1058
      COMMON                                                            1059
     1/V2VEL/URFV2,RESV2,NSWPV2                                         1060
     1/PCOR/RESORM,URFP,RESPP,NSWPP,IPREF,JPREF                         1061
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),   1062
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)          1063
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),1064
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                         1065
     1/AGRID/R(30),X1(30),X2(16)                                        1066
     1/GEOM/BF(30),BN(30),BW(30),BS(30),VP(30),VP1(30)                  1067
     1/GEOMA/DPE(30,16),DPW(30,16),DPN(30,16),DPS(30,16)                1068
     1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,CIA,DH,ATOT,RE,VAVG,VB            1069
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE                  1070
     1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED                       1071
     1/STRESS/T11(30,16),T22(30,16)                                     1072
     1/COEF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SU(30,16) 1073
                                                                        1074
C                                                                       1075
C***   V2 VELOCITY                                                      1076
C                                                                       1077
      DTETA=X2(2)                                                       1078
      VISN=VISCOS                                                       1079
      VISS=VISCOS                                                       1080
      VISP=VISCOS
```

```
      DO 100 J=3,JNM                                                    1081
      IH=IMAX(J)-1                                                      1082
      DO 101 I=2,IH                                                     1083
C-----CALCULATE DIFFUSION COEFFICIENTS                                  1084
      IF(I.EQ.IH) VIS(I+1,J-1)=VIS(I+1,J)                               1085
      VISE=(VIS(I+1,J)+VIS(I+1,J-1)+VIS(I,J-1)+VIS(I,J))/4.             1086
      VISW=(VIS(I-1,J)+VIS(I-1,J-1)+VIS(I,J-1)+VIS(I,J))/4.             1087
      DE=DV2E(I)*VISE                                                   1088
      DW=DV2W(I)*VISW                                                   1089
      DN=DV2N(I)*VISN                                                   1090
      DS=DV2N(I)*VISS                                                   1091
C---- CALCULATE CONVECTION COEFFICIENTS                                 1092
      V1E=(V1(I+1,J-1) + V1(I+1,J))/2.                                  1093
      V1W=(V1(I,J)+V1(I,J-1))/2.                                        1094
      IF(J.NE.JN) V2N=(V2(I,J)+V2(I,J+1))/2.                            1095
      IF(J.EQ.JN) V2N=0.0                                               1096
      V2S=(V2(I,J)+V2(I,J-1))/2.                                        1097
      IF(J.EQ.2) V2S=0.0                                                1098
      CE=ARE(I)*V1E                                                     1099
      CW=ARW(I)*V1W                                                     1100
      CN=ARN(I)*V2N                                                     1101
      CS=ARN(I)*V2S                                                     1102
      SMP=CN-CS+CE-CW                                                   1103
C---- ASSEMB,E MAIN COEFFICIEMTS                                        1104
      IF(ICONV.EQ.1) GO TO 21                                           1105
      A1=CE*DX1(I)+CW*DX1(I-1)                                          1106
      CE=(DABS(A1)-A1)*CV2E(I)                                          1107
      CW=(DABS(A1)+A1)*CV2W(I)                                          1108
      A2=CN+CS                                                          1109
      CN=(DABS(A2)-A2)*R(I)/4.                                          1110
      CS=(DABS(A2)+A2)*R(I)/4.                                          1111
      GO TO 22                                                          1112
21    CE=0.5*(DABS(CE)-CE)*R(I+1)                                       1113
      CW=0.5*(DABS(CW)+CW)*R(I-1)                                       1114
      CN=0.5*(DABS(CN)-CN)*R(I)                                         1115
      CS=0.5*(DABS(CS)+CS)*R(I)                                         1116
```

```
22     CONTINUE                                                                  1117
       AE(I,J)=CE+DE                                                             1118
       AW(I,J)=CW+DW                                                             1119
       AN(I,J)=CN+DN                                                             1120
       IF(J.EQ.JN)  AN(I,J)=0.0                                                  1121
       AS(I,J)=CS+DS                                                             1122
       IF(J.EQ.2)  AS(I,J)=0.0                                                   1123
       AP(I,J)=(DE*R(I+1)+DW*R(I-1))/R(I)+R(I)*(CE/R(I+1)+CW/R(I-1))             1124
      /   +AN(I,J)+AS(I,J)                                                       1125
       IF(J.EQ.2)  AP(I,J)=AP(I,J)+2.*(CS+DS)                                    1126
       IF(J.EQ.JN)AP(I,J)=AP(I,J)+2.*(CN+DN)                                     1127
C----- CALCULATE  SOURCE TERMS                                                  1128
       S1=-ARPN(I)                          *(T22(I,J)+P(I,J)-T22(I,J-1)-P(I,J-1))  1129
       S3=VISI*RM(I)*                    (V1(I+1,J)-V1(I+1,J-1))                 1130
     )     -(V1(I,J)-V1(I,J-1))*VISW*RM(I-1)                                    1131
       SU(I,J)=S1+S3                                                             1132
101    CONTINUE                                                                  1133
       I=IH+1                                                                    1134
       AP(I,J)=1.0                                                               1135
       AE(I,J)=0.0                                                               1136
       AN(I,J)=0.0                                                               1137
       AS(I,J)=0.0                                                               1138
       IF(J.EQ.JNM.OR.J.LE.JJL) GO TO 104                                        1139
       SU(I,J)=DX1(I-1)*(V1(I,J)-V1(I,J-1))/ARP(I)                               1140
       AW(I,J)=R(I-1)/R(I)                                                       1141
       GO TO 100                                                                 1142
104    AW(I,J)=0.0                                                               1143
       SU(I,J)=0.0                                                               1144
100    CONTINUE                                                                  1145
C                                                                                1146
       JNMM=JN-2                                                                 1147
       DO 300 J=2,JNMM                                                           1148
       IH=IMAX(J)                                                                1149
       DO 300 I=2,IH                                                             115
       DPN(I,J)=ARPN(I)/AP(I,J+1)                                                115
       DPN(I,J)=DPN(I,J)*URFV2                                                   115
```

```
300   CONTINUE                                                          1153
      DO 301 J=3,JNM                                                    1154
      IH=IMAX(J)-1                                                      1155
      DO 301 I=2,IH                                                     1156
      DPS(I,J) = ARPN(I)/AP(I,J)                                        1157
      DPS(I,J)=DPS(I,J)*URFV2                                           1158
301   CONTINUE                                                          1159
      DO 350 J=3,JNM                                                    1160
      IH=IMAX(J)                                                        1161
      DO 350 I=2,IH                                                     1162
      AP(I,J)=AP(I,J)/URFV2                                             1163
      SU(I,J)=SU(I,J)+(1.-URFV2)*AP(I,J)*V2(I,J)                        1164
350   CONTINUE                                                          1165
C                                                                       1166
C***SOLUTION OF THE DIFFERENCE EQUATION                                 1167
C                                                                       1168
      RESV2=0.0                                                         1169
      DO 400 N=1,NSWPV2                                                 1170
      IF(NSOLU.EQ.0) CALL SOLV1(2,3,JNM,-1,V2,RES)                      1171
      IF(NSOLU.GE.1) CALL SOLV2(2,3,JNM,-1,V2,RES)                      1172
      IF(RES.GT.RESV2) RESV2=RES                                        1173
      CALL MODV2                                                        1174
400   CONTINUE                                                          1175
      RETURN                                                            1176
      END                                                               1177
      SUBROUTINE CALCP                                                  1178
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                           1179
      COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),      1180
     / ANG(16),ANGM(16),VBP(16)                                         1181
      COMMON                                                            1182
     1/PCOR/RESORM,URFP,RESPP,NSWPP,IPREF,JPREF                         1183
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),   1184
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)           1185
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30), 1186
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                         1187
     1/AGRID/R(30),X1(30),X2(16)                                        1188
```

```
      1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB                    1189
      1/GEOM/BE(30),BN(30),BW(30),BS(30),VP(30),VP1(30)                          1190
      1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE                          1191
      1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED                               1192
      1/COEF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SU(30,16)         1193
      1/STRESS/T11(30,16),T22(30,16)                                             1194
      1/GEOMA/DPE(30,16),DPW(30,16),DPN(30,16),DPS(30,16)                        1195
                                                                                 1196
C                                                                                1197
C                                                                                1198
C***  PRESSURE                                                                   1199
                                                                                 1200
      DTETA=X2(2)                                                                1201
      RESORM=0.0                                                                 1202
      RESPP=0.0                                                                  1203
      DO 100 J=2,JNM                                                             1204
      IH=IMAX(J)                                                                 1205
      DO 101 I=2,IH                                                              1206
C---  CALCULATE COEFFICIENTS                                                     1207
      AE(I,J)=ARE(I)*DPE(I,J)                                                    1208
      AW(I,J)=ARW(I)*DPW(I,J)                                                    1209
      AN(I,J)=ARN(I)*DPN(I,J)                                                    1210
      AS(I,J)=ARN(I)*DPS(I,J)                                                    1211
      AP(I,J)=AE(I,J)*R(I)/R(I+1)+AW(I,J)*R(I)/R(I-1)+AN(I,J)+AS(I,J)            1212
C***  COMPUTE SOURCE TERMS                                                       1213
      CN=     V2(I,J+1)*ARN(I)                                                   1214
      CS=     V2(I,J)   *ARN(I)                                                  1215
      CE=     V1(I+1,J)*ARE(I)                                                   1216
      CW=     V1(I,J)*ARW(I)                                                     1217
      SMP=CN-CS+CE-CW                                                            1218
      SU(I,J)=-SMP                                                               1219
      IF(I.EQ.IH)  SU(I,J)=CW-CN                                                 1220
C---  COMPUTE SUM OF ABSOLUTE MASS SOURCES                                       1221
      RESORM=RESORM+DABS(SMP)                                                    1222
101   CONTINUE                                                                   1223
100   CONTINUE                                                                   1224
C---  SOLUTION OF THE DIFFERENCE EQUATIONS
```

```
      DO 400 N=1,NSWPP                                                  1225
      IF(NSOLU.EQ.0) CALL SOLV1(2,2,JNM,0,PP,RES)                       1226
      IF(NSOLU.EQ.1) CALL SOLV2(2,2,JNM,0,PP,RES)                       1227
      IF(RES.GT.RESPP) RESPP=RES                                        1228
      CALL MODP                                                         1229
400   CONTINUE                                                          1230
C                                                                       1231
C---  CORRECT VELOCITIES                                                1232
C                                                                       1233
      DO 500 J=2,JNM                                                    1234
      IH=IMAX(J)                                                        1235
      DO 500 I=3,IH                                                     1236
500   V1(I,J)=V1(I,J)+DPW(I,J)*(PP(I-1,J)-R(I)/R(I-1)*PP(I,J))          1237
      DO 501 J=2,JN                                                     1238
      IH=IMAX(J)-1                                                      1239
      DO 501 I=2,IH                                                     1240
501   V2(I,J)=V2(I,J)+DPS(I,J)*(PP(I,J-1)-PP(I,J))                      1241
C---   PRESSURES WITH PROVISION OF UNDER RELAXATION                     1242
      PPREF=PP(IPREF,JPREF)                                             1243
      DO 502 J=1,JN                                                     1244
      IH=IMAX(J)                                                        1245
      DO 503 I=2,IH                                                     1246
      P(I,J)=P(I,J)+URFP*(PP(I,J)-PPREF)                                1247
      PP(I,J)=0.0                                                       1248
503   CONTINUE                                                          1249
502   CONTINUE                                                          1250
      RETURN                                                            1251
      END                                                               1252
      SUBROUTINE CALCU                                                  1253
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                           1254
      COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),      1255
     / ANG(16),ANGM(16),VBP(16)                                         1256
      COMMON                                                            1257
     1/UVEL/URFU,RESU,NSWPU                                             1258
     1/PCOR/RESORM,URFP,RESPP,NSWPP,IPREF,JPREF                         1259
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),   1260
```

```
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)                  1261
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),       1262
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                                 1263
     1/AGRID/R(30),X1(30),X2(16)                                               1264
     1/GEOM/BE(30),BN(30),BW(30),BS(30),VP(30),VP1(30)                         1265
     1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB                   1266
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE                         1267
     1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED                              1268
     1/COEF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SU(30,16)        1269
C                                                                              1270
C***  AXIAL VELOCITY                                                           1271
C                                                                              1272
      DELTA=X2(2)                                                              1273
      CE=0.0                                                                   1274
      CN=0.0                                                                   1275
      CW=0.0                                                                   1276
      CS=0.0                                                                   1277
      CP=0.0                                                                   1278
      DO 100 J=2,JNM                                                           1279
      IH=IMAX(J)                                                               1280
      DO 101 I=ILP,IH                                                          1281
C-----CALCULATE DIFFUSION COEFFICIENTS                                         1282
      IF(I.EQ.IH.AND.J.LE.JJL) GO TO 103                                       1283
      DW=(VIS(I,J)+VIS(I-1,J))*BW(I)                                           1284
      DN=(VIS(I,J)/PRUT(I,J)+VIS(I,J+1)/PRUT(I,J+1))*BN(I)                     1285
      IF(I.EQ.IH) GO TO 104                                                    1286
      DE=(VIS(I,J)+VIS(I+1,J))*BE(I)                                           1287
      DS=(VIS(I,J)/PRUT(I,J)+VIS(I,J-1)/PRUT(I,J-1))*BS(I)                     1288
      GO TO 105                                                                1289
104   DE=0.0                                                                   1290
      DS=0.0                                                                   1291
105   CONTINUE                                                                 1292
      IF(ISEC.EQ.0) GO TO 150                                                  1293
      V1E=V1(I+1,J)                                                            1294
      V1W=V1(I,J)                                                              1295
      V2N=V2(I,J+1)                                                            1296
```

```
      V2S=V2(I,J)                                                       1297
      CE=ARE(I)*V1E                                                     1298
      CW=ARW(I)*V1W                                                     1299
      CN=ARN(I)*V2N                                                     1300
      CS=ARN(I)*V2S                                                     1301
      SMP=CN-CS+CE-CW                                                   1302
C-----ASSEMBLE MAIN COEFFICIENTS                                        1303
      IF(ICONV.EQ.1) GO TO 21                                           1304
      IF(I.EQ.IH) GO TO 23                                              1305
      A1=CE*DX1(I)+CW*DX1(I-1)                                          1306
      CE=(DABS(A1)-A1)/DX1(I)/4.                                        1307
      CW=(DABS(A1)+A1)/DX1(I-1)/4.                                      1308
      A2=CN+CS                                                          1309
      CN=(DABS(A2)-A2)/4.                                               1310
      CS=(DABS(A2)+A2)/4.                                               1311
      GO TO 150                                                         1312
21    CN=0.5*(DABS(CN)-CN)                                              1313
      CW=0.5*(DABS(CW)+CW)                                              1314
      IF(I.EQ.IH) GO TO 24                                              1315
      CE=0.5*(DABS(CE)-CE)                                              1316
      CS=0.5*(DABS(CS)+CS)                                              1317
      GO TO 150                                                         1318
24    CE=0.0                                                            1319
      CS=0.0                                                            1320
      GO TO 150                                                         1321
23    CE=0.0                                                            1322
      CS=0.0                                                            1323
      CW=CW/2.                                                          1324
      CN=-CN/2.                                                         1325
150   CONTINUE                                                          1326
      AE(I,J)=CE+DE                                                     1327
      AW(I,J)=CW+DW                                                     1328
      AN(I,J)=CN+DN                                                     1329
      AS(I,J)=CS+DS                                                     1330
      AP(I,J)=AE(I,J)+AW(I,J)+AN(I,J)+AS(I,J)                           1331
      AP(I,J)=(AP(I,J)+CP)/URFU                                         1332
```

```
      SU(I,J)=4.*TWAVE*VP(I)/DH                                          1333
      IF(I.EQ.IH)  SU(I,J)=4.*TWAVE*VBP(J)/DH                            1334
      SU(I,J)=SU(I,J)+CP*U(I,J)+(1.-URFU)*AP(I,J)*U(I,J)                 1335
  101 CONTINUE                                                           1336
      GO TO 100                                                          1337
103   CONTINUE                                                           1338
      TETA=X2(J)                                                         1339
      CC=ANG(J)     *(R(IH)-R(IH-1))/R(IH)/DTETA                         1340
      AE(IH,J)=0.0                                                       1341
      AN(IH,J)=CC/(1.+CC)                                                1342
      AW(IH,J)=1./(1.+CC)                                                1343
      AS(IH,J)=0.0                                                       1344
      AP(IH,J)=1.0                                                       1345
      SU(IH,J)=0.0                                                       1346
  100 CONTINUE                                                           1347
C                                                                        1348
C***SOLUTION OF THE DIFFERENCE EQUATION                                  1349
C                                                                        1350
      RESU=0.0                                                           1351
      DO 400 N=1,NSWPU                                                   1352
      CALL REYNU                                                         1353
      ISTART=3                                                           1354
      IF(IPROB.EQ.1)  ISTART=2                                           1355
      IF(NSOLU.EQ.0)  CALL SOLV1(ISTART,2,JNM,0,U,RES)                   1356
      IF(NSOLU.GE.1)  CALL SOLV2(ISTART,2,JNM,0,U,RES)                   1357
      IF(RES.GT.RESU)  RESU=RES                                          1358
      CALL MODU                                                          1359
400   CONTINUE                                                           1360
      RETURN                                                             1361
      END                                                                1362
      SUBROUTINE CALCTE                                                  1363
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                            1364
      COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),       1365
     / ANG(16),ANGM(16),VBP(16)                                          1366
      COMMON/ATURB/DVS(30,16),CA,CA1,CA2,CB,ITURB                        1367
      COMMON                                                             1368
```

```
     1/TEN/URPK,RESK,NSWPK                                                          1369
     1/PCOR/RESORM,URPP,RESPP,NSWPP,IPREF,JPREF                                     1370
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),               1371
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)                       1372
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),            1373
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                                      1374
     1/AGRID/R(30),X1(30),X2(16)                                                    1375
     1/STRESS/T11(30,16),T22(30,16)                                                 1376
     1/GEOM/BE(30),BN(30),BW(30),BS(30),VP(30),VP1(30)                              1377
     1/FLUPR/UREVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB                        1378
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE                              1379
     1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED                                   1380
     1/COEF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SU(30,16)             1381
C                                                                                   1382
C***  TURBULENT KINETIC ENERGY                                                      1383
C                                                                                   1384
      DTETA=X2(2)                                                                   1385
      CE=0.0                                                                        1386
      CN=0.0                                                                        1387
      CW=0.0                                                                        1388
      CS=0.0                                                                        1389
      CP=0.0                                                                        1390
      DO 100 J=2,JNM                                                                1391
      IH=IMAX(J)                                                                    1392
      DO 101 I=ILP,IH                                                               1393
C-----CALCULATE DIFFUSION COEFFICIENTS                                              1394
      IF(I.EQ.IH.AND.J.LE.JJL) GO TO 103                                            1395
      DW=(VIS(I,J)+VIS(I-1,J))*BW(I)/PRTE                                           1396
      DN=(VIS(I,J)/PRUT(I,J)+VIS(I,J+1)/PRUT(I,J+1))*BN(I)/PRTE                     1397
      IF(I.EQ.IH) GO TO 104                                                         1398
      DE=(VIS(I,J)+VIS(I+1,J))*BE(I)/PRTE                                           1399
      DS=(VIS(I,J)/PRUT(I,J)+VIS(I,J-1)/PRUT(I,J-1))*BS(I)/PRTE                     1400
      GO TO 105                                                                     1401
104   DE=0.0                                                                        1402
      DS=0.0                                                                        1403
105   CONTINUE                                                                      1404
```

```
      IF(ISPEC.EQ.0) GO TO 150                                          1405
      V1E=V1(I+1,J)                                                     1406
      V1W=V1(I,J)                                                       1407
      V2N=V2(I,J+1)                                                     1408
      V2S=V2(I,J)                                                       1409
      CE=ARE(I)*V1E                                                     1410
      CW=ARW(I)*V1W                                                     1411
      CN=ARN(I)*V2N                                                     1412
      CS=ARN(I)*V2S                                                     1413
      SMP=CN-CS+CE-CW                                                   1414
C-----ASSEMBLE MAIN COEFFICIENTS                                        1415
      IF(ICONV.EQ.1) GO TO 21                                           1416
      IF(I.EQ.IH) GO TO 23                                              1417
      A1=CE*DX1(I)+CW*DX1(I-1)                                          1418
      CE=(DABS(A1)-A1)/DX1(I)/4.                                        1419
      CW=(DABS(A1)+A1)/DX1(I-1)/4.                                      1420
      A2=CN+CS                                                          1421
      CN=(DABS(A2)-A2)/4.                                               1422
      CS=(DABS(A2)+A2)/4.                                               1423
      GO TO 150                                                         1424
21    CN=0.5*(DABS(CN)-CN)                                              1425
      CW=0.5*(DABS(CW)+CW)                                              1426
      IF(I.EQ.IH) GO TO 24                                              1427
      CE=0.5*(DABS(CE)-CE)                                              1428
      CS=0.5*(DABS(CS)+CS)                                              1429
      GO TO 150                                                         1430
24     CE=0.0                                                           1431
      CS=0.0                                                            1432
      GO TO 150                                                         1433
23    CE=0.0                                                            1434
      CS=0.0                                                            1435
      CW=CW/2.                                                          1436
      CN=-CN/2.                                                         1437
150   CONTINUE                                                          1438
      AE(I,J)=CE+DE                                                     1439
      AW(I,J)=CW+DW                                                     1440
```

```
      AN(I,J)=DN+CN                                                     1441
      AS(I,J)=DS+CS                                                     1442
      SP  =    -CD*CMU*TE(I,J)*VP(I)/VIS(I,J)                          1443
      IF(I.EQ.IH)  SP      =SP      *VBP(J)/VP(I)                      1444
      AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP                        1445
      AP(I,J)=(AP(I,J)+CP)/URFK                                         1446
C-----SOURCE TERMS                                                      1447
      IF(I.EQ.IH) GO TO 160                                             1448
      H1=(R(I-1)-R(I))/(R(I+1)-R(I))                                    1449
      H2=1./H1                                                          1450
      DU1=((U(I+1,J)-U(I,J))*H1+(U(I,J)-U(I-1,J))*H2)/(R(I+1)-R(I-1))   1451
      DU2=(U(I,J+1)-U(I,J-1))/R(I)/DTETA/2.                             1452
      GO TO 170                                                         1453
160   DU1=(U(I,J)-U(I-1,J))/(R(I)-R(I-1))                               1454
      DU2=(U(I,J+1)-U(I,J))/R(I)/DTETA                                  1455
170   CONTINUE                                                          1456
      GEN(I,J)=(VIS(I,J)*DU1**2+VIS(I,J)/PRUT(I,J)*DU2**2)              1457
      SU(I,J)=GEN(I,J)*VP(I)                                            1458
      IF(I.EQ.IH)  SU(I,J)=SU(I,J)*VBP(J)/VP(I)                         1459
      SU(I,J)=SU(I,J)+CP*TE(I,J)+(1.-URFK)*AP(I,J)*TE(I,J)              1460
      IF(ITURB.EQ.1)  GO TO 101                                         1461
      T11(I,J)=TE(I,J)*(2.*(1.-1./CA1)/3.+(CB*GEN(I,J)+CA*VIS(I,J)*DU1**1462
     /2)/ED(I,J))                                                       1463
      T22(I,J)=TE(I,J)*(2.*(1.-1./CA1)/3.+(CB*GEN(I,J)+CA*VIS(I,J)*DU2**1464
     /2/PRUT(I,J))/ED(I,J))                                             1465
  101 CONTINUE                                                          1466
      GO TO 100                                                         1467
103   CONTINUE                                                          1468
      TETA=X2(J)                                                        1469
      CC=ANG(J)     *(R(IH)-R(IH-1))/R(IH)/DTETA                        1470
      AE(IH,J)=0.0                                                      1471
      AN(IH,J)=CC/(1.+CC)                                               1472
      AW(IH,J)=1./(1.+CC)                                               1473
      AS(IH,J)=0.0                                                      1474
      SU(IH,J)=0.0                                                      1475
      AP(IH,J)=AN(IH,J)+AW(IH,J)                                        1476
```

```
      IF(ITURB.EQ.1) GO TO 100                                          1477
      T11(IH,J)=AN(IH,J)*T11(IH,J+1)+AW(IH,J)*T11(IH-1,J)               1478
      T22(IH,J)=AN(IH,J)*T22(IH,J+1)+AW(IH,J)*T22(IH-1,J)               1479
  100 CONTINUE                                                          1480
C                                                                       1481
C***SOLUTION OF THE DIFFERENCE EQUATION                                 1482
C                                                                       1483
      RESK=0.0                                                          1484
      DO 400 N=1,NSWPK                                                  1485
      IF(NSOLU.EQ.0) CALL SOLV1(3,2,JNM,0,TE,RES)                       1486
      IF(NSOLU.EQ.1) CALL SOLV2(3,2,JNM,0,TE,RES)                       1487
      IF(RES.GT.RESK) RESK=RES                                          1488
      CALL MODTE                                                        1489
400   CONTINUE                                                          1490
      RETURN                                                            1491
      END                                                               1492
      SUBROUTINE CALCED                                                 1493
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                           1494
      COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),      1495
     / ANG(16),ANGM(16),VBP(16)                                         1496
      COMMON                                                            1497
     1/TDIS/URFE,RESE,NSWPD                                             1498
     1/PCOR/RESORM,URFP,RESPP,NSWPP,IPREF,JPREF                         1499
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),   1500
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)           1501
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30), 1502
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                          1503
     1/AGRID/R(30),X1(30),X2(16)                                        1504
     1/GEOM/BP(30),BN(30),BW(30),BS(30),VP(30),VP1(30)                  1505
     1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB            1506
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE                  1507
     1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED                       1508
     1/COEF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SU(30,16) 1509
C                                                                       1510
C***  ENERGY DISSIPATION                                                1511
C                                                                       1512
```

```
      DTETA=X2(2)                                                        1513
      CE=0.0                                                             1514
      CN=0.0                                                             1515
      CW=0.0                                                             1516
      CS=0.0                                                             1517
      CP=0.0                                                             1518
      DO 100 J=2,JNM                                                     1519
      IH=IMAX(J)                                                         1520
      DO 101 I=ILP,IH                                                    1521
C-----CALCULATE DIFFUSION COEFFICIENTS                                   1522
      IF(I.EQ.IH.AND.J.LE.JJL) GO TO 103                                 1523
      DW=(VIS(I,J)+VIS(I-1,J))*BW(I)/PRED                                1524
      DN=(VIS(I,J)/PRUT(I,J)+VIS(I,J+1)/PRUT(I,J+1))*BN(I)/PRED          1525
      IF(I.EQ.IH) GO TO 104                                              1526
      DE=(VIS(I,J)+VIS(I+1,J))*BE(I)/PRED                                1527
      DS=(VIS(I,J)/PRUT(I,J)+VIS(I,J-1)/PRUT(I,J-1))*BS(I)/PRED          1528
      GO TO 105                                                          1529
104   DE=0.0                                                             1530
      DS=0.0                                                             1531
105   CONTINUE                                                           1532
      IF(ISEC.EQ.0) GO TO 150                                            1533
      V1E=V1(I+1,J)                                                      1534
      V1W=V1(I,J)                                                        1535
      V2N=V2(I,J+1)                                                      1536
      V2S=V2(I,J)                                                        1537
      CE=ARE(I)*V1E                                                      1538
      CW=ARW(I)*V1W                                                      1539
      CN=ARN(I)*V2N                                                      1540
      CS=ARN(I)*V2S                                                      1541
      SMP=CN-CS+CE-CW                                                    1542
C-----ASSEMBLE MAIN COEFFICIENTS                                         1543
      IF(ICONV.EQ.1) GO TO 21                                            1544
      IF(I.EQ.IH) GO TO 23                                               1545
      A1=CE*DX1(I)+CW*DX1(I-1)                                           1546
      CE=(DABS(A1)-A1)/DX1(I)/4.                                         1547
      CW=(DABS(A1)+A1)/DX1(I-1)/4.                                       1548
```

```
      A2=CN+CS                                                          1549
      CN=(DABS(A2)-A2)/4.                                               1550
      CS=(DABS(A2)+A2)/4.                                               1551
      GO TO 150                                                         1552
21    CN=0.5*(DABS(CN)-CN)                                              1553
      CW=0.5*(DABS(CW)+CW)                                              1554
      IF(I.EQ.IH) GO TO 24                                              1555
      CE=0.5*(DABS(CE)-CE)                                              1556
      CS=0.5*(DABS(CS)+CS)                                              1557
      GO TO 150                                                         1558
24    CE=0.0                                                            1559
      CS=0.0                                                            1560
      GO TO 150                                                         1561
23    CE=0.0                                                            1562
      CS=0.0                                                            1563
      CW=CW/2.                                                          1564
      CN=CN/2.                                                          1565
150   CONTINUE                                                          1566
      AE(I,J)=CE+DE                                                     1567
      AW(I,J)=CW+DW                                                     1568
      AN(I,J)=DN+CN                                                     1569
      AS(I,J)=DS+CS                                                     1570
      SP    =          =C2*VP(I)*CMU*TE(I,J)/VIS(I,J)                 1571
      IF(I.EQ.IH) SP     =SP     *VBP(J)/VP(I)                        1572
      AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP                        1573
      AP(I,J)=(AP(I,J)+CP)/URFE                                         1574
C-----SOURCE TERMS                                                      1575
      SU(I,J)=          +C1*CMU*GEN(I,J)*VP(I)       *TE(I,J)/VIS(I,J)  1576
      IF(I.EQ.IH) SU(I,J)=SU(I,J)*VBP(J)/VP(I)                          1577
      SU(I,J)=SU(I,J)+CP*ED(I,J)+(1.-URFE)*AP(I,J)*ED(I,J)              1578
  101 CONTINUE                                                          1579
      GO TO 100                                                         1580
103   CONTINUE                                                          1581
      TETA=X2(J)                                                        1582
      CC=ANG(J)     *(R(IH)-R(IH-1))/R(IH)/DTETA                        1583
      AE(IH,J)=0.0                                                      1584
```

```
      AN(IH,J)=CC/(1.+CC)                                                    1585
      AW(IH,J)=1./(1.+CC)                                                    1586
      AS(IH,J)=0.0                                                           1587
      SU(IH,J)=0.0                                                           1588
      AP(IH,J)=AN(IH,J)+AW(IH,J)                                             1589
  100 CONTINUE                                                               1590
C                                                                            1591
C***SOLUTION OF THE DIFFERENCE EQUATION                                      1592
C                                                                            1593
      RESE=0.0                                                               1594
      DO 400 N=1,NSWPD                                                       1595
      IF(NSOLU.EQ.0) CALL SOLV1(3,2,JNM,0,ED,RES)                            1596
      IF(NSOLU.EQ.1) CALL SOLV2(3,2,JNM,0,ED,RES)                            1597
      IF(RES.GT.RESE) RESE=RES                                               1598
      CALL MODED                                                             1599
400   CONTINUE                                                               1600
      RETURN                                                                 1601
      END                                                                    1602
      SUBROUTINE SOLV1(ISTART,JSTART,JEND,NSOL,PHI,RES)                      1603
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                                1604
      DIMENSION PHI(30,16),A(30),B(30),C(30),D(30)                           1605
      COMMON                                                                 1606
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),     1607
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                              1608
     1/AGRID/R(30),X1(30),X2(16)                                            1609
     1/COEF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SU(30,16)     1610
C                                                                            1611
C**SOLUTION BY MATRIX FACTORIZATION ALONG RADIAL LINE                        1612
C                                                                            1613
      IS=ISTART-1                                                            1614
      A(IS)=0.0                                                              1615
      RES=0.0                                                                1616
C-----COMMENCE W-E SWEEP                                                     1617
      DO 100 J=JSTART,JEND                                                   1618
      C(IS)=PHI(IS,J)                                                        1619
      IH=IMAX(J)                                                             1620
```

```
      IF(NSOL.EQ.1)IH=IH+1                                              1621
      DO 101 I=ISTART,IH                                                1622
C-----ASSEMBLE TDMA COEFFICIENTS                                        1623
      A(I)=AE(I,J)                                                      1624
      B(I)=AW(I,J)                                                      1625
      IF(J.EQ.2.AND.NSOL.GE.0) GO TO 10                                 1626
      IF(J.EQ.JNM.AND.NSOL.GE.0) GO TO 11                               1627
      IF(J.EQ.JN) GO TO 14                                              1628
      IF(J.EQ.1) GO TO 15                                               1629
      C(I)=AN(I,J)*PHI(I,J+1)+AS(I,J)*PHI(I,J-1)+SU(I,J)                1630
      D(I)=AP(I,J)                                                      1631
      GO TO 12                                                          1632
10    C(I)=AN(I,J)*PHI(I,J+1)                          +SU(I,J)         1633
      D(I)=AP(I,J)-AS(I,J)                                              1634
      GO TO 12                                                          1635
11    C(I)=                      +AS(I,J)*PHI(I,J-1)+SU(I,J)            1636
      D(I)=AP(I,J)-AN(I,J)                                              1637
      GO TO 12                                                          1638
14    C(I)=AS(I,J)*PHI(I,J-1)+SU(I,J)                                   1639
      D(I)=AP(I,J)                                                      1640
      GO TO 12                                                          1641
15    C(I)=AN(I,J)*PHI(I,J+1)+SU(I,J)                                   1642
      D(I)=AP(I,J)                                                      1643
12    CONTINUE                                                          1644
C-----CALCULATE COEFFICIENTS OF RECURRENCE FORMULA                      1645
      TERM=1./(D(I)-B(I)*A(I-1))                                        1646
      A(I)=A(I)*TERM                                                    1647
      C(I)=(C(I)+B(I)*C(I-1))*TERM                                      1648
101   CONTINUE                                                          1649
C-----OBTAIN NEW PHI'S                                                  1650
      DO 102 II=ISTART,IH                                               1651
      I=IH-II+ISTART                                                    1652
      Z=PHI(I,J)                                                        1653
      PHI(I,J)=A(I)*PHI(I+1,J)+C(I)                                     1654
      RS=DABS(Z-PHI(I,J))                                               1655
      IF(RS.GT.RES) RES=RS                                              1656
```

```
102   CONTINUE                                                                1657
  100 CONTINUE                                                                1658
      RETURN                                                                  1659
      END                                                                     1660
      SUBROUTINE SOLV2(ISTART,JSTART,JEND,NSOL,PHI,RES)                       1661
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                                 1662
      DIMENSION PHI(30,16),A(30),B(30),C(30),D(30),PHO(30,16)                 1663
      COMMON                                                                  1664
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),      1665
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                               1666
     1/AGRID/R(30),X1(30),X2(16)                                             1667
     1/COEF/AP(30,16),AN(30,16),AS(30,16),AE(30,16),AW(30,16),SU(30,16)      1668
C                                                                            1669
C**SOLUTION BY POINT ITERATION                                               1670
C                                                                            1671
      IF(INUM.EQ.0) GO TO 650                                                1672
      DO 651 J=1,JN                                                          1673
      DO 651 I=1,IN                                                          1674
      PHO(I,J)=PHI(I,J)                                                      1675
651   CONTINUE                                                               1676
650   CONTINUE                                                               1677
      RES=0.0                                                                1678
      DO 201 J=JSTART,JEND                                                   1679
      IH=IMAX(J)                                                             1680
      IF(NSOL.EQ.1) IH=IH+1                                                  1681
      DO 202 I=ISTART,IH                                                     1682
      IF(J.EQ.1) GO TO 10                                                    1683
      DDS=PHI(I,J-1)                                                         1684
      IF(INUM.EQ.1) DDS=PHO(I,J-1)                                           1685
      GO TO 11                                                               1686
10    DDS=0.0                                                                1687
11    CONTINUE                                                               1688
      IF(J.EQ.JN) GO TO 14                                                   1689
      ANUM=AE(I,J)*PHI(I+1,J)+AW(I,J)*PHI(I-1,J)+AN(I,J)*PHI(I,J+1)     +    1690
     / AS(I,J)*DDS          +SU(I,J)                                        1691
      GO TO 15                                                               1692
```

```
14    ANUM=AE(I,J)*PHI(I+1,J)+AW(I,J)*PHI(I-1,J)+AS(I,J)*DDS+SU(I,J)
15    CONTINUE
      IF(AP(I,J).EQ.0.0) GO TO 202
      Z=PHI(I,J)
      PHI(I,J)=ANUM/AP(I,J)
      RS=DABS(Z-PHI(I,J))
      IF(RS.GT.RES) RES=RS
202   CONTINUE
201   CONTINUE
      RETURN
      END
      SUBROUTINE PROMOD
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)
      DIMENSION UV(16)
      COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),
     / ANG(16),ANGM(16),VBP(16)
      COMMON/ATURB/DVS(30,16),CA,CA1,CA2,CB,ITURB
      COMMON
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),
     2  ISEC,IPILE,JJL,ICCNV,NSOLU,INUM,JL,JLP
     1/AGRID/R(30),X1(30),X2(16)
     1/FLUPR/URFVIS,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VAVG,VB
     1/STRESS/T11(30,16),T22(30,16)
     1/TURB/CD,CMU,C1,C2,CK,E,CVIS,CDIS,PRTE,PRED
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE
C
C***   BOUNDARY CONDITIONS
C
C
      ENTRY MODU
C
C***  WALL
C***  COMPUTE USTAR
      IF(IPROB.EQ.1) GO TO 114
```

```
      H1=(R(IL+2)-R(IL))/(R(IL+1)-R(IL))/(R(IL+2)-R(IL+1))                    1729
      H2=(R(IL+1)-R(IL))/(R(IL+2)-R(IL))/(R(IL+2)-R(IL+1))                    1730
      H=(R(IL+2)+R(IL+1)-2.*R(IL))/(R(IL+1)-R(IL))/(R(IL+2)-R(IL))            1731
      DO 52 J=2,JNM                                                           1732
      DO 53 L=1,100                                                           1733
      B3=1./CK/X1(IL)                                                         1734
      YS=USTAR(J)*X1(IL)/VISCOS                                               1735
      A3=DLOG(E*YS)/CK                                                        1736
      C4=(H1*U(IL+1,J)    -H2*U(IL+2,J)    )/(B3+H*A3)                        1737
      IF(DABS((USTAR(J)-C4)/C4).LT.1.0D-5) GO TO 54                           1738
      USTAR(J)=C4                                                             1739
      YS=USTAR(J)*X1(IL)/VISCOS                                               1740
53    CONTINUE                                                                1741
54    CONTINUE                                                                1742
      U(IL,J)=USTAR(J)*DLOG(E*YS)/CK                                          1743
C***   NEAR WALL REYNOLDS STRESS                                              1744
      UV(J)=R(1)/R(IL)*USTAR(J)**2 -2.*(R(IL)**2-R(1)**2)/R(IL)/DH*TWAVE      1745
52    CONTINUE                                                                1746
      GO TO 106                                                               1747
114   CONTINUE                                                                1748
      DO 49 J=2,JNM                                                           1749
49    USTAR(J)=DSQRT(VISCOS*U(2,J)/X1(2))                                     1750
106   CONTINUE                                                                1751
      USTAR(1)=USTAR(2)                                                       1752
      USTAR(JN)=USTAR(JNM)                                                    1753
C*** COMPUTE NEW FRICTION FACTOR                                              1754
      S=0.5*(USTAR(1)**2+USTAR(JN)**2)                                        1755
      DO 20 J=2,JNM                                                           1756
20    S=S+USTAR(J)**2                                                         1757
      TALAV=S/DFLOAT(JNM)                                                     1758
      FR=2.*TALAV/VB**2                                                       1759
      IF(IPILE.EQ.0) TWAVE=TALAV                                              1760
C*** WALL SHEAR STRESS DISTRIBUTION                                           1761
      DO 30 J=1,JN                                                            1762
      RTAL(J)=USTAR(J)**2/TALAV                                               1763
30    CONTINUE                                                                1764
```

```
C***  SOUTH BOUNDARY                                                    1765
      IH=IMAX(1)                                                        1766
      DO 11 I=IL,IH                                                     1767
11    U(I,1)=U(I,2)                                                     1768
C***  NORTH BOUNDARY                                                    1769
      DO 12 I=2,INM                                                     1770
12    U(I,JN)=U(I,JNM)                                                  1771
      U(IN,JN)=U(INM,JN)                                                1772
      RETURN                                                            1773
C                                                                       1774
      ENTRY MODTE                                                       1775
C                                                                       1776
C***  WALL                                                              1777
      IF(IPROB.EQ.3) GO TO 72                                           1778
      DO 71 J=2,JNM                                                     1779
71    TE(IL,J)=UV(J)/CHU**0.5                                           1780
72    CONTINUE                                                          1781
C***  SOUTH BOUNDARY                                                    1782
      IH=IMAX(1)                                                        1783
      DO 41 I=IL,IH                                                     1784
41    TE(I,1)=TE(I,2)                                                   1785
C***  NORTH BOUNDARY                                                    1786
      DO 42 I=2,INM                                                     1787
42    TE(I,JN)=TE(I,JNM)                                                1788
      TE(IN,JN)=TE(INM,JN)                                              1789
      RETURN                                                            1790
C                                                                       1791
      ENTRY MODED                                                       1792
C                                                                       1793
C***  WALL                                                              1794
      DO 83 J=2,JNM                                                     1795
83    ED(IL,J)=UV(J)*USTAR(J)/CK/X1(IL)                                 1796
C***  SOUTH BOUNDARY                                                    1797
      IH=IMAX(1)                                                        1798
      DO 81 I=IL,IH                                                     1799
81    ED(I,1)=ED(I,2)                                                   1800
```

```
C***  NORTH BOUNDARY                                                    1801
      DO 82 I=2,INM                                                     1802
82    ED(I,JN)=ED(I,JNM)                                                1803
      ED(IN,JN)=ED(INM,JN)                                              1804
      RETURN                                                            1805
C                                                                       1806
      ENTRY MODV1                                                       1807
C                                                                       1808
C***  SOUTH BOUNDARY                                                    1809
      IH=IMAX(1)                                                        1810
      DO 121 I=2,IH                                                     1811
121   V1(I,1)=V1(I,2)                                                   1812
      DO 122 I=1,INM                                                    1813
122   V1(I,JN)=V1(I,JNM)                                                1814
      V1(IN,JN)=V1(INM,JN)                                              1815
      RETURN                                                            1816
C                                                                       1817
      ENTRY MODV2                                                       1818
      RETURN                                                            1819
C                                                                       1820
      ENTRY MODP                                                        1821
C                                                                       1822
      IH=IMAX(1)                                                        1823
      DO 93 I=2,IH                                                      1824
93    PP(I,1)=PP(I,2)                                                   1825
      DO 92 I=2,INM                                                     1826
92    PP(I,JN)=PP(I,JNM)                                                1827
      PP(IN,JN)=PP(INM,JN)                                              1828
      RETURN                                                            1829
C                                                                       1830
      ENTRY MODSTR                                                      1831
C                                                                       1832
      IH=IMAX(1)                                                        1833
      DO 301 I=2,IH                                                     1834
      T22(I,1)=T22(I,2)                                                 1835
301   T11(I,1)=T11(I,2)                                                 1836
```

```
      DO 302 I=2,INM                                                    1837
      T11(I,JN)=T11(I,JNM)                                              1838
      T22(I,JN)=T22(I,JNM)                                              1839
302   T11(IN,JN)=T11(IN,JNM)                                            1840
      T22(IN,JN)=T22(IN,JNM)                                            1841
      RETURN                                                            1842
      END                                                               1843
      SUBROUTINE PRINT(ISTART,JSTART,PHI,HEAD)                          1844
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                           1845
      DIMENSION PHI(30,16),HEAD(9),STORE(50)                            1846
      DIMENSION F(7),F4(11)                                             1847
      COMMON                                                            1848
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),1849
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                         1850
     1/AGRID/R(30),X1(30),X2(16)                                        1851
      DATA F/4H(1H ,4H,A4,,4HI3, ,4H11I ,4H10, ,4H7X, ,                 1852
     14HA6) /                                                           1853
      DATA F4/4H 1I ,4H 2I ,4H 3I ,4H 4I ,4H 5I ,4H 6I ,                1854
     1        4H 7I ,4H 8I ,4H 9I ,4H10I ,4H11I /                       1855
      DATA HI,HY/4H  I=,4H Y =/                                         1856
C                                                                       1857
C***   PRINT OUT SUBROUTINE                                             1858
C                                                                       1859
      ISKIP=1                                                           1860
      JSKIP=1                                                           1861
      WRITE(6,110)HEAD                                                  1862
      ISTA=ISTART-12                                                    1863
  100 CONTINUE                                                          1864
      ISTA=ISTA+12                                                      1865
      IEND=ISTA+11                                                      1866
      IEND=MIN0(IN,IEND)                                                1867
      F(4)=F4(IEND-ISTA)                                                1868
      WRITE(6,F) HI, (I,I=ISTA,IEND,ISKIP), HY                          1869
      WRITE(6,112)                                                      1870
      DO 101 JJ=JSTART,JN,JSKIP                                         1871
      J=JSTART+JN-JJ                                                    1872
```

```
      IH=IMAX(J)                                                        1873
      DO 120 I=ISTA,IEND                                                1874
      A=PHI(I,J)                                                        1875
      IF(DABS(A).LT.1.D-20)  A=0.0                                      1876
120   STORE(I)=A                                                        1877
      WRITE(6,113) J,(STORE(I),I=ISTA,IEND,ISKIP),X2(J)                 1878
101   CONTINUE                                                          1879
C-----------------------------------------------                        1880
      IF(IEND.LT.IN)GO TO 100                                           1881
      RETURN                                                            1882
  110 FORMAT(1H0,20(2H*-),7X,9A4,7X,20(2H-*))                           1883
  112 FORMAT(3H  J)                                                     1884
  113 FORMAT(1H ,I3,1P12D10.2,0PF7.3)                                   1885
      END                                                               1886
      SUBROUTINE ANORM(L,INORM,ISTRES,ICURVE,FV)                        1887
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                           1888
      DIMENSION V1P(30,16),V2P(30,16),V3P(30,16),T23(30,16),T13(30,16)  1889
      COMMON/HD/HEDU(9),HEDV1(9),HEDV2(9),HEDP(9),HEDK(9),HEDD(9),      1890
     1  HEDM(9),HEDL(9),HEDN(9),HEDPR(9),HEDV1P(9),HEDV2P(9),HEDV3P(9),  1891
     2 HEDT13(9),HEDT23(9)                                              1892
      COMMON                                                            1893
     1/VAR/U(30,16),V1(30,16),V2(30,16),P(30,16),PP(30,16),TE(30,16),   1894
     2  ED(30,16),AL(30,16),VIS(30,16),GEN(30,16),PRUT(30,16)          1895
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30), 1896
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                         1897
     1/AGRID/R(30),X1(30),X2(16)                                        1898
     1/STRESS/T11(30,16),T22(30,16)                                     1899
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE                  1900
     1/GEOMA/DPE(30,16),DPW(30,16),DPN(30,16),DPS(30,16)                1901
      EQUIVALENCE (T11(1),V1P(1)),(T22(1),V2P(1)),(DPE(1),V3P(1)),      1902
     1 (DPW(1),T13(1)),(DPN(1),T23(1))                                  1903
      FF=FV                                                             1904
      DO 54 J=1,JN                                                      1905
      IH=IMAX(J)                                                        1906
      IF(L.EQ.3)  FF=FV/USTAR(J)                                        1907
      FVV=FF**2                                                         1908
```

```
      DO 54 I=1,IH                                                      1909
      U(I,J)=U(I,J)*FF                                                  1910
      IF(IPROB.GT.2)  TE(I,J)=TE(I,J)*FVV                               1911
      IF(ISTRES.EQ.0)  GO TO 54                                         1912
      V1P(I,J)=V1P(I,J)*FF                                              1913
      V2P(I,J)=V2P(I,J)*FF                                              1914
      V3P(I,J)=V3P(I,J)*FF                                              1915
      T13(I,J)=T13(I,J)*FVV                                             1916
      T23(I,J)=T23(I,J)*FVV                                             1917
54    CONTINUE                                                          1918
      CALL PRINT(1,1,U,HEDU)                                            1919
      IF(IPROB.GT.2)  CALL PRINT(1,1,TE,HEDK)                           1920
      IF(ISTRES.EQ.0)  GO TO 15                                         1921
      CALL PRINT(2,1,V1P,HEDV1P)                                        1922
      CALL PRINT(2,1,V2P,HEDV2P)                                        1923
      CALL PRINT(2,1,V3P,HEDV3P)                                        1924
      CALL PRINT(2,1,T13,HEDT13)                                        1925
      CALL PRINT(2,1,T23,HEDT23)                                        1926
15    CONTINUE                                                          1927
      RETURN                                                            1928
      END                                                               1929
```

```
C   I N S T R U C T I O N S                                                    0001
C                                                                              0002
C     IWAL=0        TEMPERATURE BOUNDARY CONDITIONS                          0003
C     IWAL=1        FLUX BOUNDARY CONDITIONS                                 0004
C                                                                            0005
C     IREG=1        COOLANT ONLY                                             0006
C     IREG=2        COOLANT+CLAD                                             0007
C                                                                            0008
C     IPSI=1        HEAT TO MOMENTUM RATIO EQUAL TO ONE                      0009
C                                                                            0010
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                                0011
      DIMENSION RTW(16),ANU(16),TT(41,16)                                    0012
      COMMON/HD/HEDU(9),HEDV1(9),HEDV2(9),HEDP(9),HEDK(9),HEDD(9),           0013
     1  HEDM(9),HEDL(9),HEDN(9),HEDPR(9),HEDV1P(9),HEDV2P(9),HEDV3P(9),      0014
     3 HEDTC(9),HEDPSI(9),HEDTCL(9)                                          0015
      COMMON/BB/JS                                                           0016
      COMMON/GEOMC/RM(30),ARE(30),ARW(30),ARN(30),ARP(30),DX1(30),           0017
     / ANG(16),ANGM(16),VBP(16)                                              0018
      COMMON/CCLAD/CONCL,DI,FLUXI,FLUCL(16),TCL(10,16),IM,IMN                0019
      COMMON                                                                 0020
     1/TEMPC/URFTC,RESTC,NSWTC                                               0021
     1/VAR/U(30,16),V1(30,16),V2(30,16),VIS(30,16),PRUT(30,16),              0022
     2 TC(30,16),PSI(30,16)                                                  0023
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),     0024
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                               0025
     1/AGRID/R(30),X1(30),X2(16)                                             0026
     1/GEOM/BE(30),BN(30),BW(30),BS(30),VP(30)                               0027
     1/FLUPR/CONDC,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VB,TREF,PRAND            0028
     1/WALLF/TALAV,USAV,RTAL(16),USTAR(16),FR,YB,TWAVE,FLUC(16)              0029
     1/COEF/AP(41,16),AN(41,16),AS(41,16),AE(41,16),AW(41,16),SU(41,16)      0030
      LOGICAL INCALU,INCLV1,INCLV2,INPRO                                     0031
      INCALU=.TRUE.                                                          0032
      INCLV1=.TRUE.                                                          0033
      INCLV2=.TRUE.                                                          0034
      INPRO=.TRUE.                                                           0035
      N1=41                                                                  0036
```

```
      N2=30                                                                    0037
      N3=16                                                                    0038
      READ (15)U,V1,V2,VIS,PRUT,USTAR,RTAL,TWAVE,X1,X2,R,RM,ARE,ARW,ARN        0039
      READ (15)ARP,DX1,ANG,VBP,BE,BW,BN,BS,DH,ATOT,RE,VB,PIT,DIA,PITR          0040
      READ (15)IN,JN,JJL,IPROB,ISEC,INM,JNM,IL,ILP,IPILE,IMAX                  0041
      READ (15) VISCOS                                                         0042
      VAVG=VB                                                                  0043
      IF(ISEC.EQ.0) INCLV1=.FALSE.                                             0044
      IF(ISEC.EQ.0) INCLV2=.FALSE.                                             0045
      NITER=0                                                                  0046
      PI=3.14159                                                               0047
      IND=1                                                                    0048
      RESTC=0.0                                                                0049
      IL=2                                                                     0050
      DIETA=X2(2)                                                              0051
      CE=0.0                                                                   0052
      CN=0.0                                                                   0053
      CW=0.0                                                                   0054
      CS=0.0                                                                   0055
C***    NUMERICAL PROCEDURE                                                    0056
      READ(5,700) NSOLU,INUM,NSWTC,INDPRI,IMON,JMON,INORM,IWAL,IREF,JREF       0057
      READ(5,700) IPSI                                                         0058
      READ(5,700) IREG,IM                                                      0059
      READ(5,701) CC,URFTC,FACTOR                                              0060
      IF(IREG.GT.1) READ(5,701) CONCL,DI                                       0061
C***   PROBLEM DATA                                                            0062
      READ(5,701)ALHGR,CONDC,DENSIT,CSH,TREF                                   0063
C***   PLOT CURVES                                                             0064
      READ(5,700) ICURVE,ICURVS,JS                                             0065
C***TITLES                                                                     0066
      READ(5,010)HEDU,HEDV1,HEDV2,HEDP,HEDK,HEDD,HEDL,                         0067
     1 HEDN,HEDPR,HEDV1P,HEDV2P,HEDV3P,HEDTC,HEDPSI,HEDTCL                     0068
C***  PRINT INPUT DATA                                                         0069
      WRITE(6,700)NSOLU,INUM,NSWTC,INDPRI,IMON,JMON,INORM,IWAL,IREF,JREF       0070
      WRITE(6,700)IPSI                                                         0071
      WRITE(6,700)IREG,IM                                                      0072
```

```
      WRITE(6,701)CC,URFTC,FACTOR                                       0073
      IF(IREG.GT.1) WRITE(6,701)CONCL,DI,DIA                            0074
      WRITE(6,701)ALHGR,CONDC,DENSIT,CSH,TREF                           0075
      WRITE(6,700)ICURVE,ICURVS,JS                                      0076
      WRITE(6,010) HEDU,HEDV1,HEDV2,HEDP,HEDK,HEDD,HEDL,                0077
     1 HEDN,HEDPR,HEDV1P,HEDV2P,HEDV3P,HEDTC,HEDPSI,HEDTCL              0078
      WRITE(6,210)                                                      0079
      WRITE(6,240) PITR                                                 0080
      WRITE(6,270) TWAVE                                                0081
      WRITE(6,220) VB                                                   0082
      WRITE(6,230) RE                                                   0083
      IF(INCALU) CALL PRINT(1,1,U,HEDU,IN,N2,N3)                        0084
      IF(INCLV1) CALL PRINT(1,1,V1,HEDV1,IN,N2,N3)                      0085
      IF(INCLV2) CALL PRINT(1,1,V2,HEDV2,IN,N2,N3)                      0086
      CALL PRINT(1,1,VIS,HEDN,IN,N2,N3)                                 0087
      IF(IPROB.GT.1) CALL PRINT(1,1,PRUT,HEDPR,IN,N2,N3)                0088
      WRITE (6,200)                                                     0089
      ITR=1                                                             0090
      IT=IM+IN                                                          0091
      IMP=IM+1                                                          0092
      IMN=IM-1                                                          0093
      DO 29 J=1,JN                                                      0094
      XST=X1(IL)*USTAR(J)/VISCOS                                        0095
      WRITE (6,313) J,RTAL(J),USTAR(J),XST                              0096
29    CONTINUE                                                          0097
C                                                                       0098
C***   PARAMETERS AND CONTROL INDICES                                   0099
C                                                                       0100
C***  INITIAL OPERATIONS                                                0101
      DIFFC=CONDC/DENSIT/CSH                                            0102
      PRAND=VISCOS/DIFFC                                                0103
      PE=RE*PRAND                                                       0104
C                                                                       0105
C-----CALCULATE GEOMETRICAL QUANTITIES AND SET VARIABLES TO ZERO         0106
      IF(IREG.GT.1) FLUXI=ALHGR/PI/DI                                   0107
      FLUX=ALHGR/PI/DIA                                                 0108
```

```
      CALL APSI(PSI,IPSI)                                                0109
      DO 30 J=I,JN                                                       0110
      FLUC(J)=FLUX                                                       0111
      IF(IREG.GT.1) FLUCL(J)=FLUXI                                       0112
      DO 32 I=1,IT                                                       0113
32    TT(I,J)=0.0                                                        0114
30    CONTINUE                                                           0115
      CALL PRINT(IM,1,TT,HEDTC,IT,N1,N3)                                 0116
      CALL PRINT(1,1,TT,HEDTCL,IM,N1,N3)                                 0117
      CALL PRINT(1,1,PSI,HEDPSI,IN,N2,N3)                                0118
      TWI=TREF                                                           0119
      TW=TC(1,1)                                                         0120
      DO 257 J=1,JN                                                      0121
      IH=IMAX(J)                                                         0122
      DO 257 I=1,IH                                                      0123
      VIS(I,J)=(VIS(I,J)-VISCOS)*PSI(I,J)                                0124
 257  CONTINUE                                                           0125
      DO 6 I=2,INM                                                       0126
6     VP(I)=DTETA*(R(I+1)-R(I-1))* R(I)/2.                               0127
      IND=0                                                              0128
C-----INITIAL OUTPUT                                                     0129
      CALL PRINT(1,1,VIS,HEDN,IN,N2,N3)                                  0130
      TCNORM=TREF                                                        0131
C                                                                        0132
      DO 100 J=2,JNM                                                     0133
      IH=IMAX(J)                                                         0134
      DO 101 I=2,IH                                                      0135
C-----CALCULATE DIFFUSION COEFFICIENTS                                   0136
      IF(I.EQ.IH.AND.J.LE.JJL) GO TO 103                                 0137
      DW=(2.*DIFFC+VIS(I,J)+VIS(I-1,J))*BW(I)                            0138
      DN=(2.*DIFFC+VIS(I,J)/PRUT(I,J)+VIS(I,J+1)/PRUT(I,J+1))*BN(I)      0139
      IF(I.EQ.IH) GO TO 104                                              0140
      DE=(2.*DIFFC+VIS(I,J)+VIS(I+1,J))*BE(I)                            0141
      DS=(2.*DIFFC+VIS(I,J)/PRUT(I,J)+VIS(I,J-1)/PRUT(I,J-1))*BS(I)      0142
      GO TO 105                                                          0143
104   DE=0.0                                                             0144
```

```
      DS=0.0                                                            0145
105   CONTINUE                                                          0146
      IF(ISEC.EQ.0) GO TO 150                                           0147
      V1E=V1(I+1,J)                                                     0148
      V1W=V1(I,J)                                                       0149
      V2N=V2(I,J+1)                                                     0150
      V2S=V2(I,J)                                                       0151
      CE=ARE(I)*V1E                                                     0152
      CW=ARW(I)*V1W                                                     0153
      CN=ARN(I)*V2N                                                     0154
      CS=ARN(I)*V2S                                                     0155
C-----ASSEMBLE MAIN COEFFICIENTS                                        0156
      IF(I.EQ.IH) GO TO 23                                              0157
      A1=CE*DX1(I)+CW*DX1(I-1)                                          0158
      CE=(DABS(A1)-A1)/DX1(I)/4.                                        0159
      CW=(DABS(A1)+A1)/DX1(I-1)/4.                                      0160
      A2=CN+CS                                                          0161
      CN=(DABS(A2)-A2)/4.                                               0162
      CS=(DABS(A2)+A2)/4.                                               0163
      GO TO 150                                                         0164
23    CE=0.0                                                            0165
      CS=0.0                                                            0166
      CW=CW/2.                                                          0167
      CN=-CN/2.                                                         0168
150   CONTINUE                                                          0169
      AE(I,J)=CE+DE                                                     0170
      AW(I,J)=CW+DW                                                     0171
      AN(I,J)=CN+DN                                                     0172
      AS(I,J)=CS+DS                                                     0173
      AP(I,J)=AE(I,J)+AW(I,J)+AN(I,J)+AS(I,J)                           0174
      SU(I,J)=ALHGR*U(I,J)/VB/DENSIT/ATOT/CSH/12.*VP(I)                 0175
      IF(I.EQ.IH) SU(I,J)=SU(I,J)*VBP(J)/VP(I)                          0176
      SU(I,J)=-SU(I,J)                                                  0177
  101 CONTINUE                                                          0178
      GO TO 100                                                         0179
103   CONTINUE                                                          0180
```

```
      TETA=X2(J)                                                        0181
      CC=ANG(J)    *(R(IH)-R(IH-1))/R(IH)/DTETA                         0182
      AE(IH,J)=0.0                                                      0183
      AN(IH,J)=CC/(1.+CC)                                               0184
      AW(IH,J)=1./(1.+CC)                                               0185
      AS(IH,J)=0.0                                                      0186
      AP(IH,J)=1.0                                                      0187
      SU(IH,J)=0.0                                                      0188
  100 CONTINUE                                                          0189
      IF(IWAL.EQ.0) GO TO 261                                           0190
      DO 262 J=2,JNM                                                    0191
      AP(1,J)=1.0                                                       0192
      AE(1,J)=1.0                                                       0193
      AW(1,J)=0.0                                                       0194
      AN(1,J)=0.0                                                       0195
      AS(1,J)=0.0                                                       0196
      SU(1,J)=FLUX*X1(2)/CONDC                                          0197
262   CONTINUE                                                          0198
261   CONTINUE                                                          0199
C                                                                       0200
      DX=X1(2)                                                          0201
      IF(IREG.GT.1) CALL CLAD(X2,CONDC,DIA,DX)                          0202
C***SOLUTION OF THE DIFFERENCE EQUATION                                 0203
C                                                                       0204
      ISTART=1                                                          0205
      IF(IWAL.EQ.0) ISTART=2                                            0206
C***      ITERATION LOOP                                                0207
  300 NITER=NITER+1                                                     0208
      DO 837 II=1,ITR                                                   0209
      IF(NSOLU.EQ.0) CALL SOLV1(ISTART,2,JNM,TT,RES)                    0210
      IF(NSOLU.EQ.1) CALL SOLV2(ISTART,2,JNM,TT,RES,URFTC)              0211
      IH=IMAX(1)                                                        0212
      DO 273 I=1,IH                                                     0213
273   TT(I,1)=TT(I,2)                                                   0214
      DO 274 I=1,INM                                                    0215
274   TT(I,JN)=TT(I,JNM)                                                0216
```

```
      TT(IN,JN)=TT(INM,JN)                                              0217
837   CONTINUE                                                          0218
C-----INTERMEDIATE OUTPUT                                               0219
      RESTC=RES/TCNORM                                                  0220
      DELTA=TT(IREF,JREF)-TREF                                          0221
      TTT=TT(IMON,JMON)-DELTA                                           0222
      WRITE(6,313) NITER,RESTC,TTT                                      0223
      IF(MOD(NITER,INDPRI).NE.0) GO TO 301                              0224
      IF(IWAL.EQ.0) GO TO 381                                           0225
      DELTA=TT(IREF,JREF)-TREF                                          0226
      DO 382 J=1,JN                                                     0227
      IH=IMAX(J)                                                        0228
      DO 382 I=1,IH                                                     0229
      TT(I,J)=TT(I,J)-DELTA                                             0230
382   CONTINUE                                                          0231
381   CONTINUE                                                          0232
      CALL PRINT(IM,1,TT,HEDTC,IT,N1,N3)                                0233
      CALL PRINT(1,1,TT,HEDTCL,IM,N1,N3)                                0234
  301 CONTINUE                                                          0235
C-----TERMINATION TESTS                                                 0236
      IF(NITER.EQ.NSWTC) GO TO 302                                      0237
      IF(RESTC.GT.CC) GO TO 300                                         0238
302   CONTINUE                                                          0239
C*** BOUNDARY VALUES                                                    0240
      IF(IWAL.EQ.0) GO TO 281                                           0241
      DELTA=TT(IREF,JREF)-TREF                                          0242
      DO 282 J=1,JN                                                     0243
      IH=IMAX(J)                                                        0244
      DO 282 I=1,IH                                                     0245
      TT(I,J)=TT(I,J)-DELTA                                             0246
282   CONTINUE                                                          0247
281   CONTINUE                                                          0248
      CALL PRINT(IM,1,TT,HEDTC,IT,N1,N3)                                0249
      CALL PRINT(1,1,TT,HEDTCL,IM,N1,N3)                                0250
      DO 610 J=1,JN                                                     0251
      IF(IREG.GT.1) FLUCL(J)=CONCL*(TT(1,J)-TT(2,J))                    0252
```

```
610   FLUC(J)=CONDC*(TT(IM,J)-TT(IMP,J))                                0253
      CALL AVG2(FLUX,FLUC,JN,JNM)                                       0254
      IF(IREG.GT.1) CALL AVG2(FLUXI,FLUCL,JN,JNM)                       0255
 611  CONTINUE                                                          0256
      CALL AVG(ATOT,TB,TT,U,VB,IN,INM,JN,JNM,IMAX,IM)                   0257
      CALL AVG1(TW,TT,JN,JNM,IM)                                        0258
      IF(IREG.GT.1) CALL AVG1(TWI,TT,JN,JNM,1)                          0259
      DO 612 J=1,JN                                                     0260
      RTW(J)=CONDC*(TT(IM,J)-TW)/ALHGR                                  0261
      ANU(J)=FLUC(J)*DH/CONDC/(TT(IM,J)-TB)                             0262
612   CONTINUE                                                          0263
      CALL AVG2(ANUA,ANU,JN,JNM)                                        0264
      DO 613 J=1,JN                                                     0265
      ANU(J)=ANU(J)/ANUA                                                0266
      IF(IREG.GT.1) FLUCL(J)=FLUCL(J)/FLUXI                             0267
613   FLUC(J)=FLUC(J)/FLUX                                              0268
      WRITE(6,800) ALHGR,FLUX,TB,TW,RE,PRAND,PE,ANUA,TWI,FLUXI          0269
      WRITE(6,820)                                                      0270
      DO 815 J=1,JN                                                     0271
      IF(IREG.EQ.1) WRITE(6,313) J,RTW(J),FLUC(J),ANU(J)                0272
      IF(IREG.EQ.2) WRITE(6,313) J,RTW(J),FLUC(J),ANU(J),FLUCL(J)       0273
815   CONTINUE                                                          0274
      IF(INORM.EQ.0) STOP                                               0275
      WRITE(6,615)                                                      0276
      DO 614 J=1,JN                                                     0277
      IH=IMAX(J)                                                        0278
      DO 614 I=IM,IH                                                    0279
      TT(I,J)=TT(I,J)/TB                                                0280
614   CONTINUE                                                          0281
      CALL PRINT(IM,1,TT,HEDTC,IT,N1,N3)                                0282
      CALL PRINT(1,1,TT,HEDTCL,IM,N1,N3)                                0283
  010 FORMAT(9A4)                                                       0284
700   FORMAT(10I5)                                                      0285
701   FORMAT(8D10.3)                                                    0286
  210 FORMAT(1H0,47X,51HHEAT TRANSFER CALCULATIONS IN A BARE ROD BUNDLE 0287
     1   ,/////)                                                        0288
```

```
  220 FORMAT(//1HO,15X,21H BULK FLUID VELOCITY ,T60,1H=,3X,1PD11.3)                0289
  230 FORMAT(1HO,15X,16HREYNOLDS NUMBER ,T60,1H=,3X,1PD11.3)                       0290
  240 FORMAT(1HO,15X,15HPITCH    RATIO ,T60,1H=,3X,1PD11.3)                        0291
  250 FORMAT(1HO,15X,18HLAMINAR VISCOSITY ,T60,1H=,3X,1PD11.3)                     0292
270   FORMAT(//1HO,15X,21H AVE SHEAR STRESS/RO ,T60,1H=,3X,1PD11.3)               0293
  310 FORMAT(1HO,4HITER,4X,106HI------------ABSOLUTE RESIDUAL SOURCE SUM          0294
     1S-------------I       I-------FIELD VALUES AT MONITORING LOCATION,         0295
     2     1H(,I2,1H,,I2,1H),9H--------I/2X,2HNO,6X,4HUMOM,6X,4HV1MO,6X,        0296
     34HV2MO,6X,4HMASS,6X,4HTKIN,6X,4HDISP,10X,1HU,9X,2HV1,8X,2HV2,8X,1H         0297
     4P,9X,1HK,9X,1HD)                                                           0298
  311 FORMAT(1H ,I3,4X,1P6D10.3,3X,1P6D10.3)                                     0299
  402 FORMAT(///5X,1HI,7X,5HXU(I),6X,10HS.S.COEFF.)                              0300
  403 FORMAT(/5X,I2,3X,2(1PD11.3))                                               0301
200   FORMAT(1H1,30X,35HDISTRIBUTION OF WALL SHEAR STRESSES///                   0302
     /5X,1HJ,10X,9HTAL/TALAV,10X,5HUSTAR//)                                      0303
313   FORMAT(4X,I4,8X,D11.4,6X,1PD11.3,6X,1PD11.3,6X,1PD11.3/)                   0304
615   FORMAT(1H1,' NORMALIZATION OF THE RESULTS')                                0305
800   FORMAT(31HOHEAT INFORMATION CALCULATED IS//                                0306
     110X,50HALHGR,  LINEAR HEAT GENERATION RATIO.............=,1PD15.6/        0307
     210X,50HFLUX,   OUTSIDE CLAD HEAT FLUX...................=,1PD15.6/        0308
     310X,50HTB,     COOLANT BULK TEMPERATURE.................=,1PD15.6/        0309
     410X,50HTW,     AVERAGE CLAD TEMPERATURE.................=,1PD15.6/        0310
     510X,50HRE,     REYNOLDS NUMBER..........................=,1PD15.6/        0311
     610X,50HPRAND,  PRANTDL NUMBER...........................=,1PD15.6/        0312
     710X,50HPE,     PECLET NUMBER............................=,1PD15.6/        0313
     810X,50HANUA,   AVERAGE NUSSELT NUMBER...................=,1PD15.6/        0314
     810X,50HTWI     AVERAGE INNER CLAD TEMPERATURE...........=,1PD15.6/        0315
     810X,50HFLUXI   AVERAGE INNER CLAD FLUX..................=,1PD15.6)        0316
820   FORMAT(1H1,30X,31HDISTRIBUTION OF HEAT PARAMETERS///                       0317
     /5X,1HJ,10X,15HWALL TEMP RATIO,5X,14HOUT FLUX RATIO,5X,8HLOCAL NU,         0318
     15X,10HINNER FLUX/)                                                         0319
      STOP                                                                       0320
      END                                                                        0321
      SUBROUTINE APSI(PSI,IPSI)                                                  0322
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                                    0323
      DIMENSION PSI(30,16)                                                       0324
```

```
      COMMON
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP
     1/AGRID/R(30),X1(30),X2(16)
     1/FLUPR/CONDC,VISCOS,DENSIT,PIT,DIA,DH,ATOT,RE,VB,TREF,PRAND
C
C SUBROUTINE FOR CALCULATION OF HEAT TO MOMENTUM EDDY DIFFUSIVITY RATIO
C
      GO TO (1,2),IPSI
 1    CONTINUE
      DO 10 J=1,JN
      DO 10 I=1,IN
10    PSI(I,J)=1.0
      RETURN
C
2     CONTINUE
      RETURN
      END
      SUBROUTINE CLAD(X2,CONDC,DIA,DX)
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)
      DIMENSION X2(16),R(10)
      COMMON/CCLAD/CONCL,DI,FLUXI,FLUCL(16),TCL(10,16),IM,IMN
      COMMON
     1/TEMPC/URFTC,RESTC,NSWTC
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP
     1/COEF/AP(41,16),AN(41,16),AS(41,16),AE(41,16),AW(41,16),SU(41,16)
C
      IN=IN+IM-1
      INM=IN-1
      IMN=IM-1
      DTETA=X2(2)
      DO 1 J=1,JN
      IMAX(J)=IMAX(J)+IM-1
      IH=IMAX(J)
      MP=IH-IM
```

```
      DO 1 II=1,MP                                                      0361
      I=IH-II+1                                                         0362
      AE(I,J)=AE(I-IMN,J)                                               0363
      AW(I,J)=AW(I-IMN,J)                                               0364
      AN(I,J)=AN(I-IMN,J)                                               0365
      AS(I,J)=AS(I-IMN,J)                                               0366
      AP(I,J)=AP(I-IMN,J)                                               0367
      SU(I,J)=SU(I-IMN,J)                                               0368
    1 CONTINUE                                                          0369
      R(1)=DI/2.                                                        0370
      R(IM)=DIA/2.                                                      0371
      DR=(DIA-DI)/2./DFLOAT(IMN)                                        0372
      DO 10 I=2,IMN                                                     0373
   10 R(I)=R(I-1)+DR                                                    0374
      DO 20 J=2,JNM                                                     0375
      AP(1,J)=1.0                                                       0376
      AE(1,J)=1.0                                                       0377
      AW(1,J)=0.0                                                       0378
      AN(1,J)=0.0                                                       0379
      AS(1,J)=0.0                                                       0380
      SU(1,J)=FLUXI*DR/CONCL                                            0381
      DO 21 I=2,IMN                                                     0382
      BE    =.25*DTETA*(R(I+1)+R(I))/(R(I+1)-R(I))                      0383
      BW    =.25*DTETA*(R(I-1)+R(I))/(R(I)-R(I-1))                      0384
      BN    =.25*(R(I+1)-R(I-1))/R(I)/DTETA                             0385
      BS=BN                                                             0386
      AE(I,J)=2.*CONCL*BE                                               0387
      AW(I,J)=2.*CONCL*BW                                               0388
      AN(I,J)=2.*CONCL*BN                                               0389
      AS(I,J)=2.*CONCL*BS                                               0390
      SU(I,J)=0.0                                                       0391
      AP(I,J)=AE(I,J)+AW(I,J)+AN(I,J)+AS(I,J)                           0392
   21 CONTINUE                                                          0393
      AE(IM,J)=CONDC/CONCL*DR/DX                                        0394
      AW(IM,J)=1.0                                                      0395
      AN(IM,J)=0.0                                                      0396
```

```
      AS(IM,J)=0.0                                                      0397
      SU(IM,J)=0.0                                                      0398
      AP(IM,J)=AE(IM,J)+AW(IM,J)                                        0399
20    CONTINUE                                                          0400
      RETURN                                                            0401
      END                                                               0402
      SUBROUTINE AVG(ATOT,PB,PH,U,VB,IN,INM,JN,JNM,IMAX,IM)             0403
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                           0404
      DIMENSION PH(41,16),U(30,16),IMAX(16)                             0405
      COMMON                                                            0406
     1/AGRID/R(30),X1(30),X2(16)                                        0407
C*******************************************************************  0408
C    SUBROUTINE FOR COMPUTATION OF BULK TEMPERATURE                     0409
C*******************************************************************  0410
      IMN=IM-1                                                          0411
      IMP=IM+1                                                          0412
      AV=0.0                                                            0413
      DO 22 J=1,JN                                                      0414
      TAV=0.0                                                           0415
      IH=IMAX(J)                                                        0416
      DO 12 I=IMP,IH                                                    0417
      DAV=(R(I-IMN)*U(I-IMN,J)     +R(I-IM)*U(I-IM,J)          )*(R(I-I 0418
     1MN)-R(I-IMN-1))/2.                                                0419
 12    TAV=TAV+DAV                                                      0420
      TAV=TAV*X2(2)                                                     0421
      IF(J.EQ.1.OR.J.EQ.JN) TAV=0.5*TAV                                 0422
22    AV=AV+TAV                                                         0423
      VB=AV/ATOT                                                        0424
C                                                                       0425
      AV=0.0                                                            0426
      DTETA=X2(2)                                                       0427
      DO 20 J=1,JN                                                      0428
      TAV=0.0                                                           0429
      IH=IMAX(J)                                                        0430
      DO 10 I=IMP,IH                                                    0431
      DAV=(R(I-IMN)*PH(I,J)*U(I-IMN,J)+R(I-IM)*PH(I,J)*U(I-IM,J))*(R(I-I 0432
```

```
     1MN)-R(I-IMN-1))/2.                                                    0433
   10 TAV=TAV+DAV                                                           0434
      TAV=TAV*X2(2)                                                         0435
      IF(J.EQ.1.OR.J.EQ.JN) TAV=0.5*TAV                                     0436
20    AV=AV+TAV                                                            0437
C*** CALCULATE BULK TEMPERATURE                                            0438
      PB=AV/ATOT/VB                                                         0439
      RETURN                                                                0440
      END                                                                   0441
      SUBROUTINE AVG1(PB,PH,JN,JNM,I)                                       0442
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                               0443
      DIMENSION PH(41,16)                                                   0444
      S=0.5*(PH(I,1)+PH(I,JN))                                              0445
      DO 1 J=2,JNM                                                          0446
 1    S=S+PH(I,J)                                                           0447
      PB=S/DFLOAT(JNM)                                                      0448
      RETURN                                                                0449
      END                                                                   0450
      SUBROUTINE AVG2(PB,PH,JN,JNM)                                         0451
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                               0452
      DIMENSION PH(16)                                                      0453
      S=0.5*(PH(1)+PH(JN))                                                  0454
      DO 1 J=2,JNM                                                          0455
 1    S=S+PH(J)                                                             0456
      PB=S/DFLOAT(JNM)                                                      0457
      RETURN                                                                0458
      DEBUG SUBCHK                                                          0459
      END                                                                   0460
      SUBROUTINE SOLV2(ISTART,JSTART,JEND,PHI,RES,UREL)                     0461
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                               0462
      DIMENSION PHI(41,16),A(41),B(41),C(41),D(41),PHO(41,16)               0463
      COMMON                                                                0464
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),    0465
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                             0466
     1/AGRID/R(30),X1(30),X2(16)                                            0467
     1/COEF/AP(41,16),AN(41,16),AS(41,16),AE(41,16),AW(41,16),SU(41,16)     0468
```

```
C                                                                       0469
C**SOLUTION BY POINT ITERATION                                          0470
C                                                                       0471
      IF(INUM.EQ.0) GO TO 650                                           0472
      DO 651 J=1,JN                                                     0473
      DO 651 I=1,IN                                                     0474
      PHO(I,J)=PHI(I,J)                                                 0475
651   CONTINUE                                                          0476
650   CONTINUE                                                          0477
      RES=0.0                                                           0478
      DO 201 J=JSTART,JEND                                              0479
      IH=IMAX(J)                                                        0480
      DO 202 I=ISTART,IH                                                0481
      DDS=PHI(I,J-1)                                                    0482
      IF(INUM.EQ.1) DDS=PHO(I,J-1)                                      0483
      IF(I.EQ.1) GO TO 10                                               0484
      ANUM=AE(I,J)*PHI(I+1,J)+AW(I,J)*PHI(I-1,J)+AN(I,J)*PHI(I,J+1)    +0485
     / AS(I,J)*DDS          +SU(I,J)                                    0486
      GO TO 12                                                          0487
10    ANUM=AE(I,J)*PHI(I+1,J)+SU(I,J)                                   0488
12    CONTINUE                                                          0489
      IF(AP(I,J).EQ.0.0) GO TO 202                                      0490
      Z=PHI(I,J)                                                        0491
      PHI(I,J)=ANUM/AP(I,J)                                             0492
      RS=DABS(Z-PHI(I,J))                                               0493
      IF(RS.GT.RES) RES=RS                                              0494
202   CONTINUE                                                          0495
201   CONTINUE                                                          0496
      RETURN                                                            0497
                                                                        0498
      END                                                               0499
      SUBROUTINE SOLVI(ISTART,JSTART,JEND,PHI,RES)                      0500
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                           0501
      DIMENSION PHI(41,16),A(41),B(41),C(41),D(41)                      0502
      COMMON                                                            0503
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30),0504
```

```
      2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                                0505
      1/COEF/AP(41,16),AN(41,16),AS(41,16),AE(41,16),AW(41,16),SU(41,16)       0506
C                                                                              0507
C**SOLUTION BY MATRIX FACTORIZATION ALONG RADIAL LINE                          0508
C                                                                              0509
      A(1)=0.0                                                                 0510
      C(1)=0.0                                                                 0511
      RES=0.0                                                                  0512
      DO 100 J=JSTART,JEND                                                     0513
      IH=IMAX(J)                                                               0514
      DO 101 I=ISTART,IH                                                       0515
      A(I)=AE(I,J)                                                             0516
      B(I)=AW(I,J)                                                             0517
      IF(J.EQ.2) GO TO 10                                                      0518
      IF(J.EQ.JNM) GO TO 11                                                    0519
      C(I)=AN(I,J)*PHI(I,J+1)+AS(I,J)*PHI(I,J-1)+SU(I,J)                       0520
      D(I)=AP(I,J)                                                             0521
      GO TO 12                                                                 0522
10    C(I)=AN(I,J)*PHI(I,J+1)                          +SU(I,J)                0523
      D(I)=AP(I,J)-AS(I,J)                                                     0524
      GO TO 12                                                                 0525
11    C(I)=                    +AS(I,J)*PHI(I,J-1)+SU(I,J)                     0526
      D(I)=AP(I,J)-AN(I,J)                                                     0527
12    CONTINUE                                                                 0528
      IF(I.EQ.1) GO TO 101                                                     0529
      TERM=1./(D(I)-B(I)*A(I-1))                                               0530
      A(I)=A(I)*TERM                                                           0531
      C(I)=(C(I)+B(I)*C(I-1))*TERM                                             0532
101   CONTINUE                                                                 0533
      DO 102 II=ISTART,IH                                                      0534
      I=IH-II+ISTART                                                           0535
      Z=PHI(I,J)                                                               0536
      PHI(I,J)=A(I)*PHI(I+1,J)+C(I)                                            0537
      RS=DABS(Z-PHI(I,J))                                                      0538
      IF(RS.GT.RES) RES=RS                                                     0539
102   CONTINUE                                                                 0540
```

```
  100 CONTINUE                                                          0541
      RETURN                                                            0542
      END                                                               0543
      SUBROUTINE PRINT(ISTART,JSTART,PHI,HEAD,IE,IK,JK)                 0544
      IMPLICIT REAL*8(A-H,O-Z), INTEGER (I-N)                           0545
      DIMENSION PHI(IK,JK),HEAD(9),STORE(50)                            0546
      DIMENSION F(7),F4(11)                                             0547
      COMMON                                                            0548
     1/ALL/IT,JT,IN,JN,INM,JNM,IL,ILP,IPROB,IGRID,IMAX(16),IND,JMIN(30), 0549
     2  ISEC,IPILE,JJL,ICONV,NSOLU,INUM,JL,JLP                         0550
     1/AGRID/R(30),X1(30),X2(16)                                        0551
      DATA F/4H(1H ,4H,A4,,4HI3, ,4H111 ,4H10, ,4H7X, ,                 0552
     1 4HA6) /                                                          0553
      DATA F4/4H 1I ,4H 2I ,4H 3I ,4H 4I ,4H 5I ,4H 6I ,                0554
     1        4H 7I ,4H 8I ,4H 9I ,4H10I ,4H11H /                       0555
      DATA HI,HY/4H  I=,4H Y =/                                         0556
C                                                                       0557
C***    PRINT OUT SUBROUTINE                                            0558
C                                                                       0559
      ISKIP=1                                                           0560
      JSKIP=1                                                           0561
      WRITE(6,110)HEAD                                                  0562
      ISTA=ISTART-12                                                    0563
  100 CONTINUE                                                          0564
      ISTA=ISTA+12                                                      0565
      IEND=ISTA+11                                                      0566
      IEND=MINO(IE,IEND)                                                0567
      F(4)=F4(IEND-ISTA)                                                0568
      WRITE(6,F) HI, (I,I=ISTA,IEND,ISKIP), HY                          0569
      WRITE(6,112)                                                      0570
      DO 101 JJ=JSTART,JN,JSKIP                                         0571
      J=JSTART+JN-JJ                                                    0572
      IH=IMAX(J)                                                        0573
      DO 120 I=ISTA,IEND                                                0574
      A=PHI(I,J)                                                        0575
      IF(DABS(A).LT.1.D-20) A=0.0                                       0576
```

```
120     STORE(I)=A                                                        0577
        WRITE(6,113) J,(STORE(I),I=ISTA,IEND,ISKIP),X2(J)                 0578
101     CONTINUE                                                          0579
C-------------------------------------------------                        0580
        IF(IEND.LT.IE)GO TO 100                                           0581
        RETURN                                                            0582
  110 FORMAT(1HO,20(2H*-),7X,9A4,7X,20(2H-*))                             0583
  112 FORMAT(3H  J)                                                       0584
  113 FORMAT(1H ,I3,1P12D10.2,0PF7.3)                                     0585
        END                                                               0586
```