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Breakdown of Q.E.D. Vacuum and Luminosity Lifetime of a Heavy Ion Collider*†

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ABSTRACT

During beam crossing at a Relativistic Heavy Ion Collider the peripheral electromagnetic field between the fully stripped ions in colliding bunches is sufficiently intense to induce copious particle production. In this manuscript the production of free pair e^+, e^- leptons and bound-electron plus free e^+ particles are addressed in detail. In particular, both perturbative and non-perturbative approaches to these reactions are addressed. Capturing a produced electron into a bound state of an atom is a charge changing reaction that may effect the useful machine luminosity lifetime. The analogies between strong field effects in heavy ion colliders and the next generation of linear colliders are also discussed.

1. Introduction

At Brookhaven's Relativistic Heavy Ion Collider (RHIC), fully stripped heavy ions will circulate in a collider mode up to beam kinetic energies E_{BEAM}/A of 250 (Z/A) GeV, where Z is the atomic number and A is the atomic mass number of the ions. For the heaviest nuclei at RHIC, i.e., fully stripped $^{197}Au^{79+}$ ions, this corresponds to a beam kinetic energy E_{BEAM}/A of 100 GeV, or a Lorentz gamma parameter of $\gamma = 108$. RHIC has been a construction project at Brookhaven since January 1991, and the first collision of ion beams is expected to take place in the spring of 1997.

In Table I, the major set of parameters for the collider are given[1]. Of particular interest for the discussion in this paper are the luminosity value ($L = 2 \times 10^{26}$ cm^{-2} sec^{-1} for $^{197}Au^{79+}$ beams at top energy), the number of crossing points, the number of bunches per ring, and the number of ions per bunch (10^9 for $^{197}Au^{79+}$ ions).

The primary physics goal of RHIC is the creation and study of a so-called quark-gluon plasma[2]. This unique form of matter is expected to be formed in the *central or near central* collisions of heavy ions at ultra-relativistic energies. The thermodynamic conditions (temperature, energy density, etc.) attained in *central* heavy ion collisions are

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Table I: Major Parameters for the Collider

Energy Range (each beam), Au	7-100 GeV/u
protons	28-250 GeV
Luminosity, Au-Au @ 100 GeV/u & 10 h av.	2×10^{26} $cm^{-2}sec^{-1}$
Operational lifetime Au @ $\gamma > 30$	>10 h
Diamond length	±22 cm rms
Circumference, 4-3/4 C_{AGS}	3833.852 m
Number of crossing points	6
Free space at crossing point	±9 m
Beta @ crossing, horizontal/vertical	6 m
low-beta insertion	2 m
Crossing angle, maximum	7 mrad
Betatron tune, horizontal/vertical	28.824
Transition Energy, γ_T	24.7
Filling mode	Box-Car
No. of bunches/ring	57
No. of Au-ions/bunch	1×10^{9}
Filling time (each ring)	< 1 min
Magnetic Rigidity, $B\rho$: @ injection	96.7 T·m
@ top energy	839.5 T·m
Bending radius, arc dipole	243.241 m
No. of dipoles (180/ring + 12 common)	372
No. of quadrupoles (276 arc + 216 insertion)	492
Dipole field @ 100 GeV/u, Au	3.45 T
Dipole field strength, $\int B d\ell$	32.62 T·m
Dipole current	~5 kA
Dipole yoke length	9.70 m
Quadrupole gradient	71.8 T/m
Arc quadrupole strength, $\int G d\ell / B\rho$	0.09665 m^{-1}
Coil i.d. arc magnets	8 cm
Beam tube i.d.	7.29 cm
Beam separation in arcs	90 cm
Injection kicker strength (95 nsec)	0.132 T·m
Beam dump kicker (1 μsec)	1.2 T·m
Beam stored energy	300 kJ
rf voltage, h=342	400 kV
rf voltage, h=2052	4.5 MV
Acceleration time	1 min

expected to be such that the constituent quarks and gluons of baryons and mesons become deconfined in and a new short lived plasma state. The potential "phases" of both dense nuclear matter and quark matter that may be explored with relativistic heavy ion collisions are shown schematically in Fig. 1.

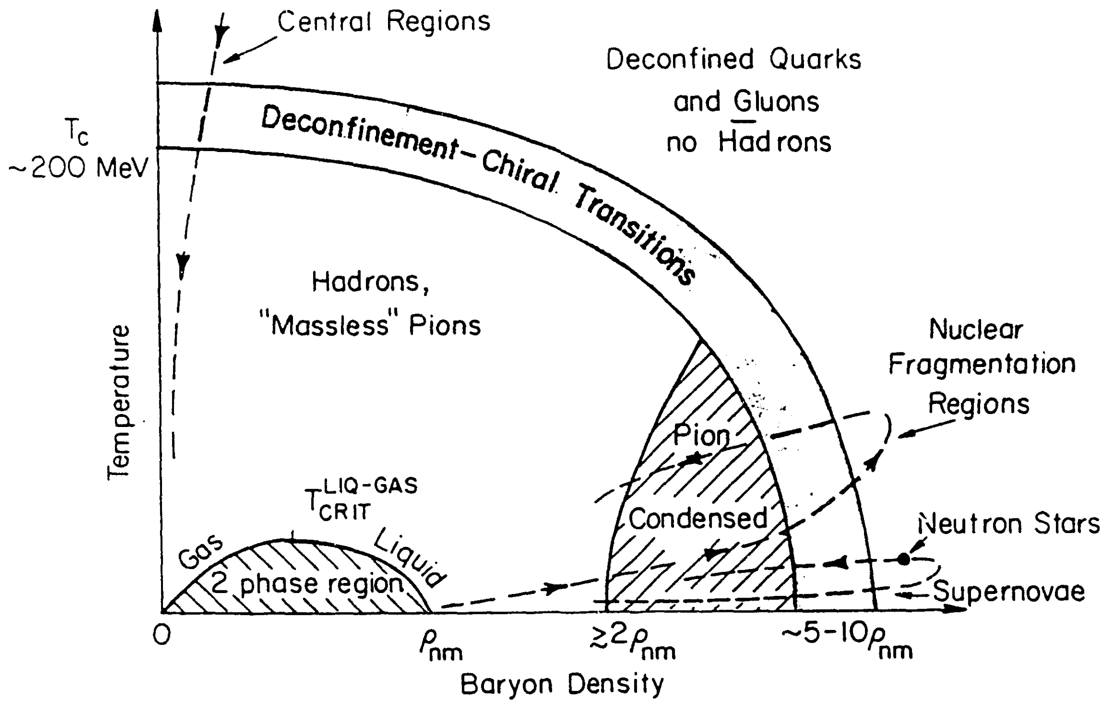

Fig. 1. Schematic phase diagram of both nuclear and quark matter, showing what may be explored with relativistic heavy ion collisions. At a critical temperature and baryon density a phase transition occurs between hadronic matter and quark matter.

2. Peripheral Collisions in Heavy Ion Colliders

Of particular importance to overall accelerator performance and detector design is the physics associated with ***peripheral*** collisions in heavy ion colliders. This physics is associated with the electromagnetic interaction between the heavy ions during bunch crossing. Recent work suggests that detailed understanding of this beam crossing phenomena will require the development of non–perturbative theories of Q.E.D.[3].

For the simplest possible picture, consider an extreme classical approach, where the dominant perpendicular component of the electric field $E_\perp(R_\perp, t)$, due to one ion in the frame of another, is given by

$$E_\perp(R_\perp, t) = \frac{Ze}{R_\perp^2}\gamma_{\text{eff}} f(R_\perp, t) \quad ,$$

where

$$f\left(R_{\perp},t\right)=\left(1+\left(2\Delta t/t\right)^{2}\right)^{-3/2} \quad , \tag{1}$$

and

$$\Delta t=\frac{2R_{\perp}}{\gamma_{\mathrm{eff}}c} \quad .$$

$R_{\perp}$ is the transverse separation of the ions, and γ_{eff} is the effective Lorentz gamma parameter in the frame of one ion ($\gamma_{\mathrm{eff}} = 2\gamma^2$ - 1). For a given $R_{\perp}$ and γ_{eff} value, the Lorentz contracted Coulomb field is seen to be sharply peaked and scaled in magnitude by the charge Ze of the ion. The strength of the electric field may be sufficiently intense to induce considerable production of particles[3] during bunch crossing.

In a more accurate semi–quantal picture, the moving Coulomb fields act as ensembles of so–called virtual photons. Imposing a cut–off in the range of spectra of these photons via the finite nuclear–size R, results in a crude estimate[4] for the maximum pair masses m that may be created from the beam crossing. This result ($m < R^{-1}\gamma$, where R is the effective charge radius of the heavy ion) gives $m \sim 3$ GeV for $^{197}Au^{79+}$ ions at top RHIC energies. This relatively large value of m also implies considerable production of light mass e^+, e^- pairs may be expected during beam crossing.

It is very important to note that Brookhaven's RHIC will not be the worlds only relativistic heavy ion synchrotron. The large hadron collider (LHC) at CERN is expected to accelerate $^{208}Pb^{82+}$ beams to a beam energy of 3.8 TeV/u or $\gamma \simeq 4100$, and in principle the SSC could accelerate heavy ions such as $^{208}Pb^{82+}$ to a beam energy of 8.0 TeV/u or $\gamma \simeq 8600$. These machines correspond to a maximum value of m given by $m \sim 100$ GeV and $m \sim 250$ GeV respectively. Hence, especially at SSC energies, production of massive particles, e.g. W^+W^- pairs[3], through the peripheral heavy ion electromagnetic field may turn out to be very competitive with more traditional central collisions of electrons or protons. In fact, at SSC energies, it appears[5] that even Higgs production rates from peripheral heavy ion electromagnetic fields are viable at luminosity values of $10^{28}cm^{-2}sec^{-1}$. A detailed discussion of this point, including the important background estimates, are contained in the talk of C. Bottcher[6].

In this paper the discussion is focussed on the most abundant light mass pairs produced by the beam crossing of heavy ions, namely the e^+, e^- pairs. These particles are a potential source of background in detectors at heavy ion colliders. In addition, the produced electron may be captured in a bound orbital of a heavy ion. This charge changing reaction will eventually cause the ion to be lost from the beam, and is a major component in determining the useful luminosity lifetime of a heavy ion collider.[1]

The following section describes calculations using perturbation theory to determine e^+, e^- pair production and the e^+–bound electron mechanism. Our present understanding of non–perturbative effects is then discussed in the subsequent section. Much of the non–perturbative analysis is still the subject of active investigations.

3. Perturbative Calculations

A Monte Carlo event generator named "ELVIRA" has been developed[7] to study the total, single and doubly differential cross section for e^+, e^- free pair production at RHIC bunch crossing. For the particle density expected in a bunch at RHIC it is sufficient to multiply the total e^+, e^- cross section by the machine luminosity value to determine the e^+, e^- pair production rate. For much higher particle densities, such as those found in bunches at a linear collider, it is necessary to average the two body reaction over the beam density profile. The computer program "ELVIRA" can also incorporate this averaging procedure.

The lowest order perturbative diagram corresponding to production of one free e^+, e^- pair is shown in Fig. 2.

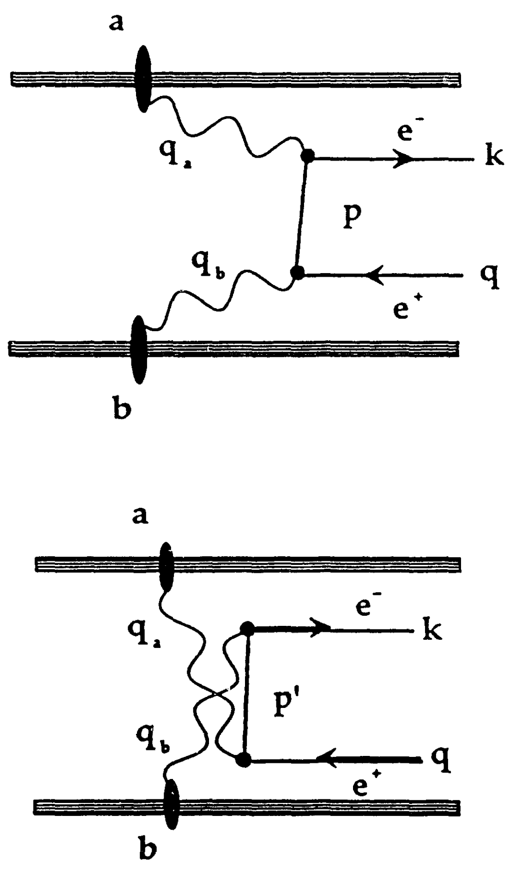

Fig. 2. Schematic representation of the perturbative two- photon diagram, including the exchange contribution. The wavy lines represent the virtual photons.

Assuming the heavy ions move on straight line trajectories, and the production of an e^+, e^- pair does not change the momentum of the heavy ions, the *total* cross section is given by[8],

$$\sigma_{e^+,e^-} = \frac{1}{(2\beta)^2} \sum_{\sigma_k \sigma_q} \int \frac{d^3k d^3q d^2p_\perp}{(2\pi)^8} \mid A^{(+)}(k,q;\vec{p}_\perp) + A^{(-)}\left(k,q;\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp\right) \mid^2 , \quad (2)$$

where $\vec{q}, \vec{k}$ are the momenta of the positron and electron in the final state, $\vec{p}$ is the intermediate momenta, and σ_q, σ_k run over spins of the positron and electron. The label k is short for quantum numbers $\vec{k}, \sigma_k, S_k$, where $S_k = \pm 1$ for positive and negative energy states. The direct amplitude $A^{(+)}(k, q; \vec{p}_\perp)$ is expressed as a product

$$A^{(+)}(k,q;\vec{p}_\perp) = F\left(\vec{k}_\perp - \vec{p}_\perp : w_a\right) F\left(\vec{q}_\perp - \vec{p}_\perp : w_b\right) T_{kq}\left(\vec{p}_\perp : \beta\right) , \quad (3)$$

where F is the scalar part of the field associated with a heavy ion in momentum space,

$$F(u : w) = \frac{4\pi Z e}{u^2 + (w/\beta\gamma)^2} , \quad (4)$$

and w_a, w_b are the frequencies associated with the field of ions a and b. The amplitude T_{kq} relates intermediate photon lines to outgoing fermion lines and is described in detail in Ref. (8). Equation (4) corresponds to a heavy ion point charge Coulomb force. A finite charge density for the nucleus can also be utilized in ELVIRA.

Calculations show that for $^{197}Au^{79+}$ beams at top RHIC energies, $\sigma_{e^+,e^-} = 33{,}000$ barns and hence, $L\sigma_{e^+,e^-} \simeq 10^7$/sec. In Figs. (3) and (4) the differential cross section for pairs or a single particle

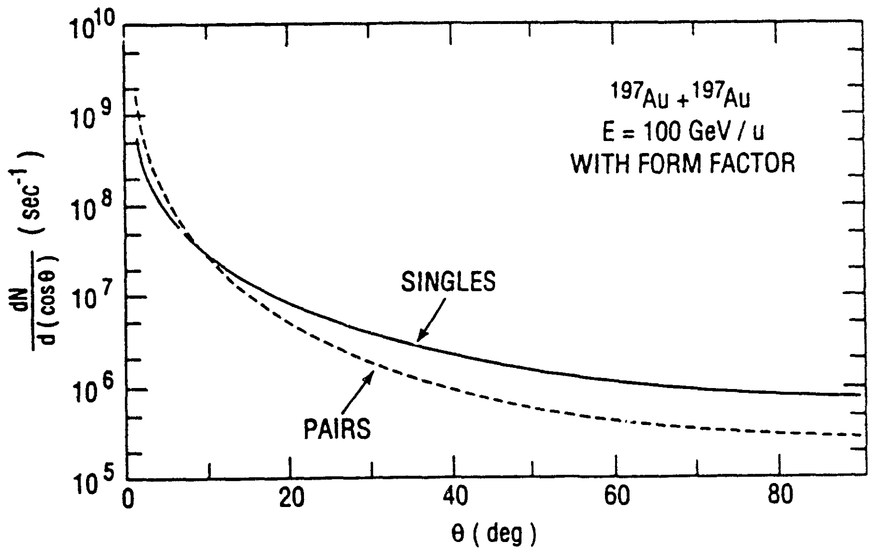

Fig. 3 Plot of dN/d(Cosθ) as a function of the opening angle θ for both pairs and singles and with a nuclear form factor.

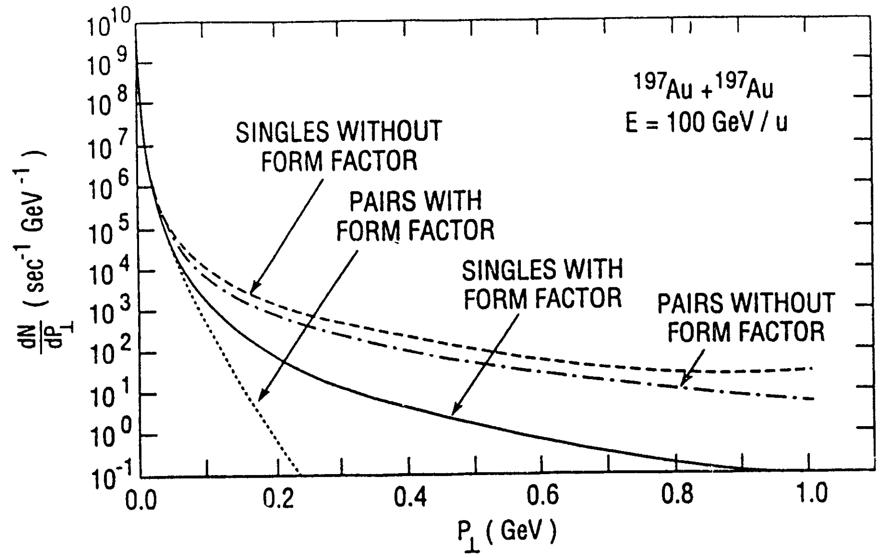

Fig. 4 Comparison of $dN/dp_\perp$ as a function of $p_\perp$, for both pairs and singles, with and without the nuclear form factor.

are shown as a function of opening angle (relative to beam direction) and transverse momentum respectively. The calculations have been normalized to the expected number of particles $N(=L\sigma_{e^+,e^-})$ at $^{197}Au^{79+}$ luminosity values in RHIC. It can be seen that most of the produced particles travel in a forward direction. Also, the transverse momentum distribution of the particles peaks at $p_\perp = 0.0$, and falls rapidly as $p_\perp$ increases. Hence, most of the produced e^+, e^- pairs have very small transverse momentum magnitudes. It is also seen that taking into account the finite charge distribution of a nucleus causes the transverse momentum distribution to fall more rapidly than a point charge nucleus. Further detailed evaluation of the distributions, including dependence on other independent variables and doubly differential cross sections, have been published elsewhere.[7] It is very important to understand the reaction where a produced electron is captured in a bound orbital of one ion. This charge changing reaction will eventually lead to the ion being lost from the beam. A Monte Carlo calculation of the perturbative diagram in Fig. 5 has been carried out.[9] Perturbation theory for this reaction means a one step process that allows the electron to be created from the Q.E.D. vacuum, and captured in a bound state (most often K–shell) of an atom. The calculation involves evaluation of the integral,

$$\sigma_{\rm CAP} = \frac{1}{(2\beta)^2}\sum_{\sigma_q}\int d^3q\,d^2p_\perp\frac{1}{(2\pi)^5}\mid\sum_{S_k}\int d^3k\frac{1}{(2\pi)^3}\tilde{\Phi}_T\left(S_k,\vec{k}\right)B\left(k,q,p_\perp\right)\mid^2, \quad (5)$$

where now $\tilde{\Phi}_T(S_k, \vec{k})$ is the electron bound state wave– function in the collider frame.[9] The result for capture by $^{197}Au^{79+}$ ions is shown in Fig. 6 as a function of the beam γ value.

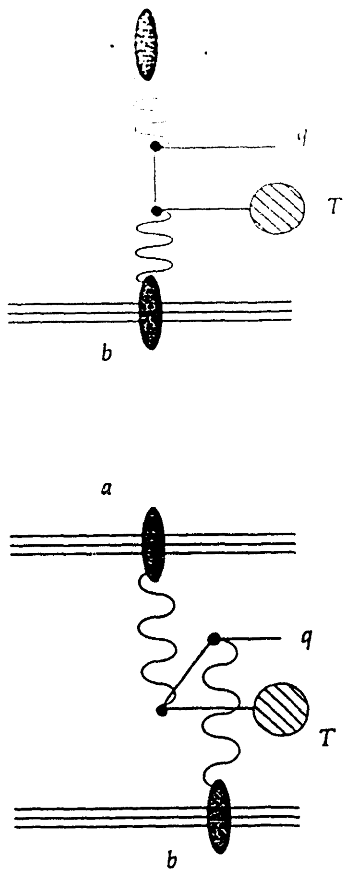

Fig. 5 Schematic representation of the perturbative capture mechanism, including the exchange contribution.

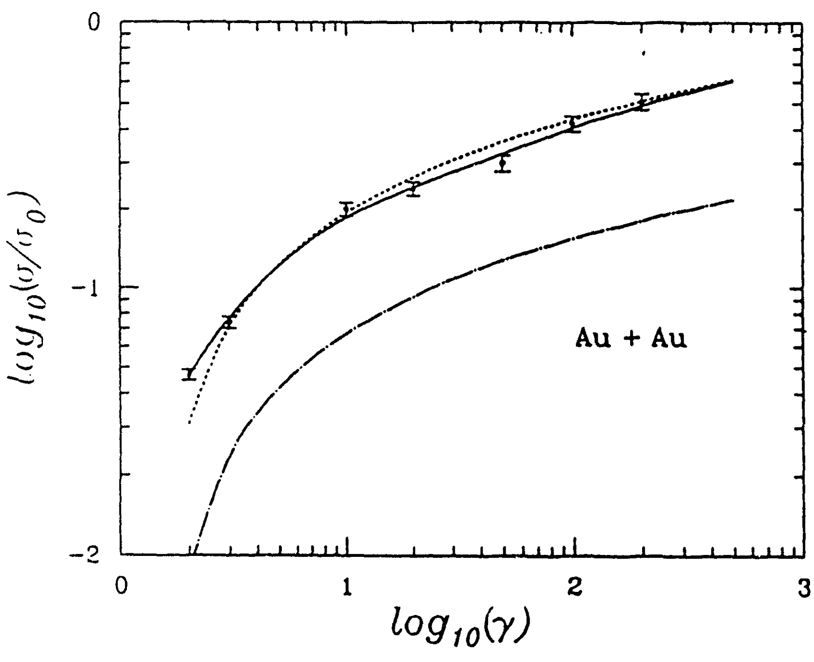

Fig. 6 Scaled capture cross section for Au + Au. The scaling factor σ_o = 164.7 b. Full line and dashed line correspond respectively to Monte Carlo evaluation of Eq. (5), and an approximation to this described in Ref. (9). The chain dashed line is the Weizsächer–Williams method.

For RHIC energies σ_{CAP} = 72 barns, and thus for 6 intersection points in the collider, and the machine bunch and luminosity parameters shown in Fig. 1, a 14 hour luminosity half life can be obtained.[1] (This value also includes smaller losses from other sources such as beam-gas, central collisions and electromagnetic decay of excited nuclei.)

Analysis of both the γ and Z dependence for free pair production and capture indicate $\sigma_{e^+,e^-} \sim Z^4\ (\ln\ \gamma)^3$ and $\sigma_{CAP} \sim Z^x\ \ln\gamma$ respectively, where x is between 6 and 7 for Z values between 20 and 80.

4. Non–Perturbative Electrodynamics

From Landau's perturbative formula for the free one–pair production probability $P_1(b)$ at impact parameter b,

$$P_1\,(b) = \frac{14}{9\pi^2}\left(Z^2\alpha^2\right)\ln^2\left(\frac{\gamma_{\text{eff}}\lambda\!\!\!^{-}_c}{2b}\right)\left(\frac{\lambda\!\!\!^{-}_c}{b}\right)^2, \tag{6}$$

where $\lambda\!\!\!^{-}_c$ is the reduced Compton wavelength of an electron (386 fm), we get

$$\gamma \geq e^{3\pi/\left(2\sqrt{14}\alpha^2 Z^2\right)} \quad , \tag{7}$$

for $P_1\,(b) \geq 1$ at $b = \lambda\!\!\!^{-}_c$. Hence for $^{197}Au^{79+}$ ions, $P_1(b{=}\lambda\!\!\!^{-}) \geq 1$ when $\gamma \geq 44.2$. This estimate clearly indicates that simple perturbative Q.E.D. is not sufficient to describe electrodynamic phenomena during beam crossing at RHIC, and higher order processes such as multiple pair production are expected to be present.

Recent work on non–perturbative electrodynamics for e^+, e^- pair production has shown[10] that under the dynamical conditions present in a heavy ion collider that a large class of higher order multiple pair production diagrams can be resummed.[10] Under the extremely good assumptions of ignoring the interaction between the produced e^+, e^- particles, and assuming the heavy ions move on unperturbed straight line trajectories, the higher order N– pair production probability $P_b(N)$ may be written in terms of the lowest order result[10] for one pair production as a Poisson expression,

$$P_b\,(N) = \frac{\left[P_1\,(b)\right]^N e^{-P_1(b)}}{N!} \quad . \tag{8}$$

Multiplying Eq. (8) by N, summing over N, and integrating over all impact parameters gives

$$\sigma_{e^+,e^-} = \sum_N N\sigma^N_{e^+,e^-} \quad , \tag{9}$$

where $\sigma^N_{e^+,e^-}$ is the non–perturbative cross section for producing N e^+, e^- pairs per interaction. The Poisson form of Eq. (8) immediately leads to the result that the cross section calculated with the lowest order Feynman diagram for one pair production, can be reinterpreted as the *average* cross sections for N–pair production in a probability conserving non–perturbative approach. In this way the product $L\sigma_{e^+,e^-}$ represents the *average* number of pairs per interaction. This is an extremely useful result, and once again its utility follows directly from the Poisson form in Eq. (8).

In Fig. (7) the non perturbative N-pair probability $P_N(b)$ is plotted for $^{208}Pb^{82+}$ beams at the LHC. It can be seen that multiple pair probabilities fall rapidly beyond the Compton wavelength, however, the average number of pairs per interaction is calculated to be 5.2.

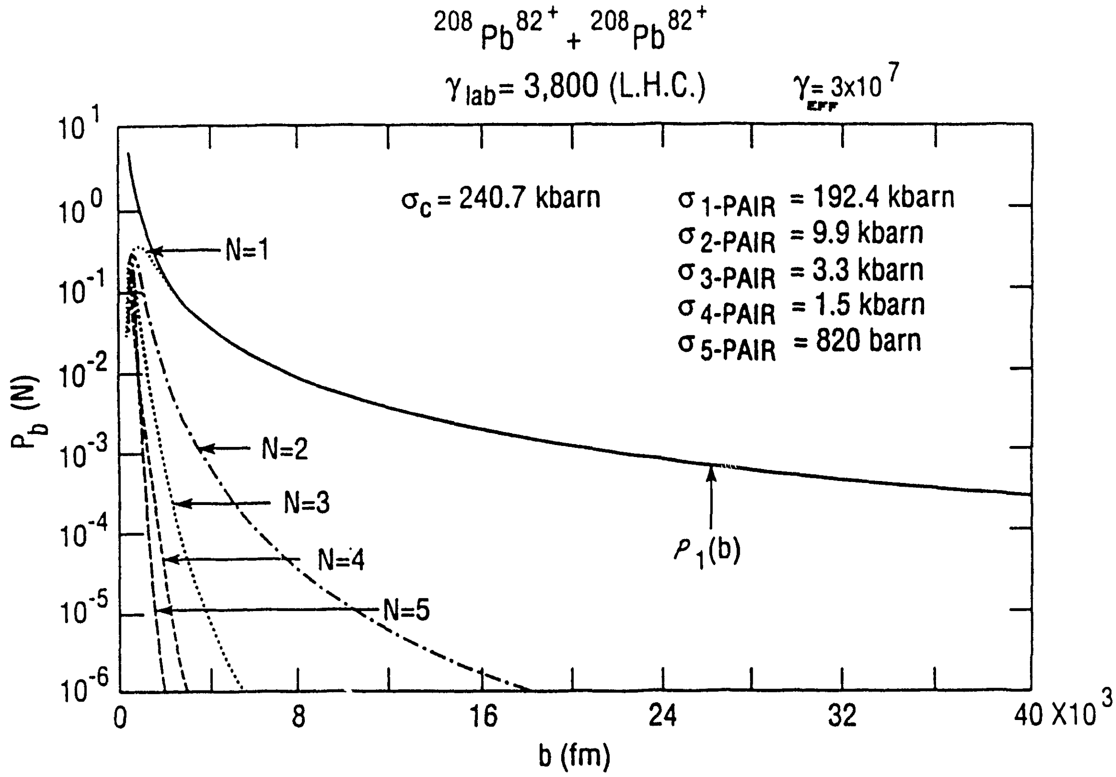

Fig. 7 Probability distribution for N-pair production at LHC energies as a function of impact parameter b. $P_1(b)$ is the lowest order perturbative result corresponding to Eq. (2). $P_b(N)$ is the Poisson form for N-pair production.

Non-perturbative effects on the e^- capture cross section are important when estimating the heavy ion beam luminosity lifetime. Recent studies show[11] that we expect the largest deviation from perturbation theory at smaller impact parameters. It has also been recently deduced that the general ***non-perturbative*** cross section for capture $\sigma'_{\rm CAP}$ scales as[11],

$$\sigma'_{\rm CAP} = A \ln \gamma_{\rm eff} + B' \tag{10}$$

where A and B have been shown to be independent of $\gamma_{\rm eff}$ when $\gamma_{\rm eff} >> 10$. Perturbation theory for capture scales as[9, 11]

$$\sigma_{\rm CAP} = A \ln \gamma_{\rm eff} + B \tag{11}$$

where the coefficient A in Eqs. (10) and (11) are indeed identical[11] and it has been deduced $B' > B$. Calculations are under way to determine $\sigma'_{\rm CAP}$ via a so-called non- perturbative coupled channels approach. The scaling in Eq. (10) also tells us that experimental measurements of σ'_{CAP} at AGS or CERN fixed target energies would be sufficient to determine σ'_{CAP} at RHIC or higher energy machines.

5. Discussion

This talk has addressed some of the new and unusual electromagnetic phenomena associated with bunch crossing at an ultra–relativistic heavy ion collider. The pair production problem, which is a potential source of background in detectors, appears well understood in both the perturbative and non–perturbative domain. The capture problem is sound in the perturbative region and considerable progress has been made in understanding general features of the energy scaling in the non–perturbative domain. Numerical evaluation of σ'_{CAP} is in progress at this time using a coupled–channels approach.

The usefulness of peripheral heavy ion collisions to induce large mass resonance production, i.e., W^+, W^- or Higgs, appears promising. However, the usefulness of this mechanism from a machine operation point of view is directly coupled to the expected luminosity lifetime via the non–perturbative capture cross section.

The importance of electromagnetic processes in designing linear colliders has also been discussed at this meeting.[12] In this regard the close physics parallels between future linear colliders and present day heavy ion synchrotrons is interesting. Putting Z = 1 in Eq. (7) shows that for linear e^+, e^- colliders we may expect deviations from simple perturbation theory for 2–body collisions only when $\gamma \geq e^{140^2}$! Non–perturbative effects in linear colliders may only be expected when considering coherent pair production effects between *bunches* of charged electrons and positrons. This is analogous to the two body heavy ion collision where non–perturbative multi–pair production effects arise from coherence between the charged protons in the nucleus. Simple scaling arguments indicate that for values of $\gamma \geq (b/\lambdabar_c)^{1/2}$ we may expect coherent pair production from electron bunches. (This scaling is also readily derived from Eq. (6) for $b \neq \lambdabar_c$. The exponential is unity for linear collider bunch dimensions). Substituting in the expected bunch dimensions at the TLC gives $(b/\lambdabar_c)^{1/2} = 500$. Hence strong coherent multi–pair production may be expected at the TLC. For SLC bunch dimensions $(b/\lambdabar_c)^{1/2} \simeq 5 \times 10^3$, and hence it is borderline of coherent multi–pair production is present here.

The expectation of coherent multi–pair production from TLC bunches suggests that some of the technology and insight developed to investigate these phenomena in heavy ion colliders could be quite useful in investigating these effects in the next generation of linear colliders.

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