REVIEW COPY

SAND--81-1603C

DE82 005785

Int'l Heat Transfer Conference in Munchen, West Germany, September 6-10, 1982.

CONF-820901--2

SAND 81-1603

MASTER

TWO-DIMENSIONAL ENERGY TRANSFER IN RADIATIVELY PARTICIPATING MEDIA WITH CONDUCTION BY THE P-N APPROXIMATION

Arthur C. Ratzel III*

John R. Howell**

Department of Mechanical Engineering
The University of Texas at Austin

* Graduate student on leave from Sandia Laboratories

** Professor of Mechanical Engineering, Member, ASME

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

MGW

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ABSTRACT

The combined steady state conduction and radiation heat transfer problem for a gray medium within a rectangular enclosure is considered using the differential approximation. The P-1 and P-3 spherical harmonics approximations for the intensity distribution are incorporated in the equation of transfer, and modified Marshak-type boundary conditions are used in the formulation. The enclosure walls are assumed to be isothermal black surfaces.

Two- and five-coupled second order nonlinear partial differential equations are developed for the P-1 and P-3 approximations, respectively, using the energy conservation and moment-of-intensity expressions. The P-1 equations have been numerically solved using finite element techniques, while the P-3 relations have been solved using a finite difference successive-over-relaxation (SOR) algorithm. Two-dimensional temperature profiles and hot wall heat transfer results are presented for square enclosures and different optical width rectangular enclosures for a range of Stark numbers.

1. INTRODUCTION

Heat transfer by simultaneous conduction and radiation in a planar medium capable of absorbing and emitting thermal radiation has received considerable attention owing to its importance in such fields as cryogenics, ablation protection, glass manufacture, and energy conservation. The mathematical difficulties involved in solving such problems are substantial since the descriptive energy relation is a nonlinear integro-differential (or integro-partial differential) equation.

While significant attention has been addressed to one-dimensional problems of combined mode energy transfer, e.g. [1-4], only limited efforts have been directed to multidimensional problems. Typical solution methods include use of the diffusion approximation [5], valid only for optically thick media, and the Hottel zone method [6] for the radiation exchange. In the latter method, an iterative scheme must be applied, with the temperature distribution initially assumed in the radiative computation and then updated when the multimode energy balance is obtained. In addition, the zonal method requires significant subdivision of the volume in order to obtain realistic representations of the temperature field. Multidimensional radiative transfer studies in planar media have also been limited [7-9], with efforts primarily directed to approximation methods. Unfortunately, use of these methods in multimode heat transfer problems would require solution techniques similar to those used with the Hottel zone method and thus would ~~probably~~ be computationally quite lengthy.

An alternative method of solution for coupled conduction and radiation problems, which may be used for multidimensional geometries, incorporates the differential approximation (moment method). In this method, the radiation intensity is approximated by a series of spherical harmonics. This simplification transforms the integral equation of radiative transfer to a series of partial differential equations. The resulting equations and Marshak-type boundary conditions [10] can be solved together with the conservation of energy expression. One obvious advantage of this method is that it simplifies the solution technique, since the partial differential equations may be solved by standard numerical methods.

In this paper, the combined conduction and radiation heat transfer problem for a gray medium within a rectangular enclosure composed of isothermal black walls is considered using the differential approximation. The P-1 and P-3 spherical harmonics approximations for the intensity are used to predict both temperature and heat transfer in the two-dimensional field.

2. FORMULATION

2.1 Background

The problem to be considered is that of two-dimensional heat transfer in a rectangular enclosure composed of black isothermal walls. The intervening medium may absorb and emit radiation and (1) is assumed to have an index of refraction of one; (2) is assumed to be gray and have uniform temperature-independent properties; and (3) may generate energy because of a uniform volumetric source. Figure 1 presents the geometry and defines the nondimensional coordinate systems and temperature boundary conditions. The variables are defined in the Nomenclature. For this problem, the energy equation is expressed as

$$\frac{\partial q_{R_1}}{\partial \tau_1} + \frac{\partial q_{R_3}}{\partial \tau_3} + \frac{\partial q_{c_1}}{\partial \tau_1} + \frac{\partial q_{c_3}}{\partial \tau_3} = \frac{Q'''}{a} \qquad (1)$$

where

$$\tau_i = a x_i \quad (i = 1,3) \qquad (2)$$

substituting for q_{ci}, the conduction energy transfer, and normalizing the geometry by the optical thicknesses in the X_1 and X_3 directions, the following nondimensional energy expression is obtained:

$$\frac{\partial Q_{R_3}}{\partial x} + r\frac{\partial Q_{R_1}}{\partial \eta} = \frac{4N_1}{\tau_L}\left(r^2\frac{\partial^2 \psi}{\partial \eta^2} + \frac{\partial^2 \psi}{\partial x^2}\right) + S \qquad (3)$$

The nondimensional temperature, ψ, and radiative transfer in the i direction, Q_{Ri}, are defined respectively as

$$\psi(x,\eta) = \frac{T(x,\eta)}{T_1} \qquad (4)$$

and

$$Q_{Ri}(x,\eta) = \frac{q_{Ri}(x,\eta)}{e_{b1}} \quad (i=1,3) \qquad (5)$$

It is assumed in this development that surface one will have the highest wall temperature. Introduced in Eq. (3) are two additional terms, the conduction-radiation parameter (Stark number), N_1, and the volumetric generation expression, S, defined below:

$$N_1 = \frac{K a T_1}{4 e_{b1}} \qquad (6)$$

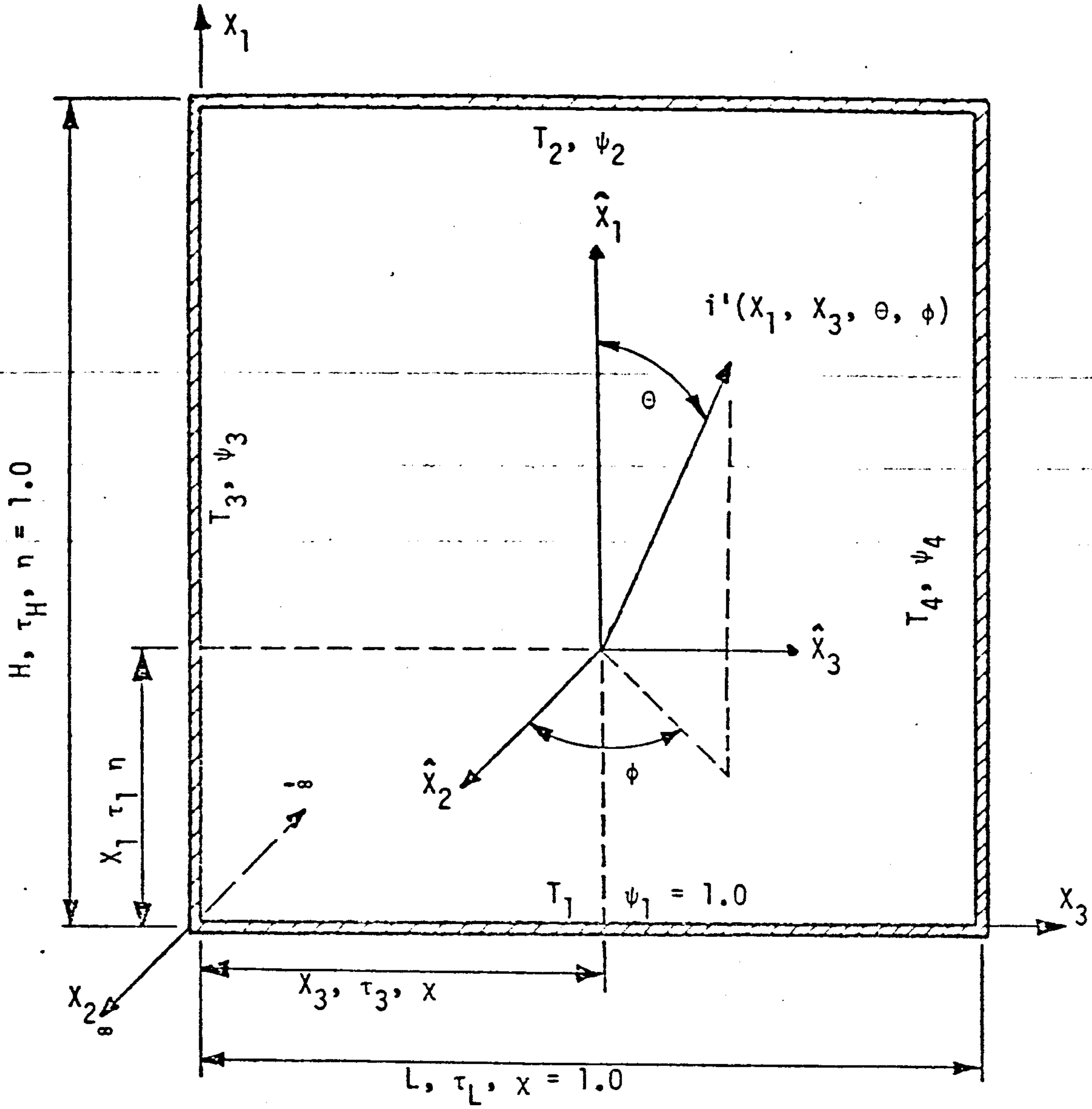


Figure 1: Schematic of the geometry considered in the analysis

$$S = \frac{Q'''L}{e_{b1}} \tag{7}$$

The radiative terms are obtained by integration of the radiative intensity distribution (multiplied by the normal direction cosine) over 4π steradian solid angle.

$$Q_{Ri}(\chi,\eta) = \int_{\omega=4\pi} \hat{\ell}_i I'(\chi,\eta,\omega)\, d\omega \quad (i=1,3) \tag{8}$$

where $$d\omega = \sin\theta\, d\theta\, d\phi$$

The intensity I' is made nondimensional by the wall one emissive power. The appropriate direction cosines for the η and χ directions are given as

$$\hat{\ell}_1 = \cos\theta \tag{9a}$$

$$\hat{\ell}_3 = \sin\theta \sin\phi \tag{9b}$$

The radiative intensity is obtained from solution of the two-dimensional equation of transfer

$$\hat{\ell}_1 r \frac{\partial I'}{\partial \eta} + \hat{\ell}_3 \frac{\partial I'}{\partial \chi} = \tau_L \left(\psi^4 - I' \right). \tag{10}$$

Eq. (10) and Eqs. (8) and (3) constitute the expressions necessary for solution of the two-dimensional temperature and heat transfer fields for the rectangular enclosure. Note that substitution of Eq. (8) into Eq. (3) results in an integro-partial differential equation which must be solved simultaneously with Eq. (10) to obtain I' and ψ.

2.2 Spherical Harmonics Approximation

To simplify the analysis, the intensity distribution may be represented by using an infinite series of spherical harmonics, as given by Eq. (11) [11].

$$I'(\bar{r},\omega) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} A_\ell^m(\bar{r}) Y_\ell^m(\omega) \tag{11}$$

where $A_\ell^m(\bar{r})$ are position-dependent coefficients and $Y_\ell^m(\omega)$ are normalized spherical harmonics given by

$$Y_\ell^m(\omega) = \left[\frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!} \right]^{1/2} e^{im\phi} P_\ell^m(\cos\theta) \tag{12}$$

Note that $P_\ell^m(\cos\theta)$ are the associated Legendre polynomials of the first kind. Following Cheng [12] and Bayazitoǧlu and Higenyi [13], the intensity distribution is approximated by truncating the series after a finite set of terms, $\ell = 1$ and $\ell = 3$ for the P-1 and P-3 approximations, respectively. The intensity expression is also recast in terms of moments of intensity defined by Eq. (13), by substituting Eq. (11)

<u>Zeroth Moment</u>

$$I_o(\bar{r}) = \int_{\omega=4\pi} I'(\bar{r},\omega)\, d\omega \tag{13a}$$

<u>First Moment</u>

$$I_i(\bar{r}) = \int_{\omega=4\pi} I'(\bar{r},\omega)\, \hat{\ell}_i \, d\omega \quad (i=1,3) \tag{13b}$$

Second Moment

$$I_{ij}(\bar{r}) = \int_{\omega=4\pi} I'(\bar{r},\omega)\,\hat{l}_i\hat{l}_j\,d\omega \qquad (i,j=1,2,3) \tag{13c}$$

...

Nth Moment

$$I_{ij\ldots k}(\bar{r}) = \int_{\omega=4\pi} I'(\bar{r},\omega)\,\hat{l}_i\hat{l}_j\ldots\hat{l}_k\,d\omega \qquad (i,j\ldots k=1,2,3) \tag{13d}$$

into Eq. (13), performing appropriate integrations, and then algebraically solving for the unknown coefficients $A_l^m(\bar{r})$ in terms of the moments. For the two-dimensional geometry considered, the resulting intensity distributions for the P-1 and P-3 approximations are given by Eq. (14) [14].

P-1 Approximation

$$I'(\xi,\eta,\theta,\varphi) = \frac{1}{4\pi}\left[I_0 + 3I_1\cos\theta + 3I_3\sin\theta\sin\varphi\right] \tag{14a}$$

P-3 Approximation

$$\begin{aligned} I'(x,\eta,\theta,\varphi) = \frac{1}{4\pi}\Big[& I_0 + 3I_1\cos\theta + \frac{5}{4}(3I_{11}-I_0)(3\cos^2\theta-1) \\ & + 3I_3\sin\theta\sin\varphi + 15I_{13}\sin\varphi\cos\theta\sin\theta + \frac{7}{4}(5I_{111}-3I_1)(5\cos^3\theta-3\cos\theta) \\ & + \frac{15}{4}(I_0-I_{11}-2I_{33})\cos2\varphi\sin^2\theta + \frac{21}{8}(5I_{311}-I_3)\sin\theta\sin\varphi(5\cos^2\theta-1) \\ & + \frac{105}{4}(I_1-I_{111}-2I_{133})\cos2\varphi\cos\theta\sin^2\theta \\ & + \frac{35}{8}(3I_3-3I_{311}-4I_{333})\sin3\varphi\sin^3\theta\Big] \end{aligned} \tag{14b}$$

The equation of transfer is transformed into a series of moment equations by multiplying Eq. (10) by appropriate direction cosines and integrating over the 4π solid angle. "Closure" conditions [13] are required for the second order moments of the P-1 approximation and for the fourth order moments of the P-3 approximation developed in the moment partial differential equations. These expressions are obtained by substitution of Eq. (14) into appropriate forms of Eq. (13)~~.~~, ~~Eqs.~~ Equations (15) and (16) are the summarized P-1 moment of intensity partial differential equations and closure conditions. To conserve space, the P-3 moment expressions and closure conditions are omitted; the interested reader should consult reference [14] for these details.

P-1 Moment Equations

$$r\frac{\partial I_1}{\partial \eta} + \frac{\partial I_3}{\partial x} = \tau_L\left(4\psi^4 - I_0\right) \tag{15a}$$

$$\frac{\partial I_{11}}{\partial \eta} = -\tau_H I_1 \tag{15b}$$

$$\frac{\partial I_{33}}{\partial x} = -\tau_L I_3 \tag{15c}$$

Closure Conditions

$$I_{11} = I_{33} = \frac{I_0}{3} \tag{16}$$

Eqs. (15) and (16) are combined to reduce the three first-order partial differential equations to one second-order expression.

$$r^2 \frac{\partial^2 I_0}{\partial \eta^2} + \frac{\partial^2 I_0}{\partial x^2} + 3\left(4\psi^4 - I_0\right) = 0 \tag{17}$$

Since the nondimensional moments I_1 and I_3 are equivalent to Q_{R1} and Q_{R3} respectively, the energy equation can be simplified using Eq. (15a) to

$$r^2 \frac{\partial^2 \psi}{\partial \eta^2} + \frac{\partial^2 \psi}{\partial x^2} = \frac{\tau_L^2}{4N_1}\left(4\psi^4 - I_0 - \frac{S}{\tau_L}\right). \tag{18}$$

Eq. (18) is also valid for the P-3 approximation method.

Significant algebraic manipulation to reduce the P-3 moment equations to four coupled second-order partial differential equations in terms of I_0, I_{11}, I_{33}, and I_{13} has been omitted. The four resulting expressions are summarized below [14].

$$\frac{\partial^2 I_{13}}{\partial x^2} + r^2 \frac{\partial^2 I_{13}}{\partial \eta^2} - \frac{7}{3}\tau_L^2 I_{13} + \frac{2}{3} r\left(\frac{\partial^2 I_{11}}{\partial x \partial \eta} + \frac{\partial^2 I_{33}}{\partial x \partial \eta} - \frac{1}{5}\frac{\partial^2 I_0}{\partial x \partial \eta}\right) = 0 \tag{19}$$

$$\frac{\partial^2 I_0}{\partial x^2} + 3r^2 \frac{\partial^2 I_0}{\partial \eta^2} - 5\tau_L^2 I_0 - 5\left(5r^2 \frac{\partial^2 I_{11}}{\partial \eta^2} + \frac{\partial^2 I_{11}}{\partial x^2}\right) - 20r \frac{\partial^2 I_{13}}{\partial x \partial \eta} + 35\tau_L^2 I_{11} - \frac{80}{3}\tau_L^2 \psi^4 = 0 \tag{20}$$

$$\frac{\partial^2 I_{11}}{\partial x^2} + 6r^2 \frac{\partial^2 I_{11}}{\partial \eta^2} - 7\tau_L^2 I_{11} + \frac{\partial^2 I_{33}}{\partial x^2} + 6r \frac{\partial^2 I_{13}}{\partial x \partial \eta} - \frac{1}{5}\left(\frac{\partial^2 I_0}{\partial x^2} + 3r^2 \frac{\partial^2 I_0}{\partial \eta^2}\right) + \frac{28}{3}\tau_L^2 \psi^4 = 0 \tag{21}$$

$$6\frac{\partial^2 I_{33}}{\partial x^2} + r^2\frac{\partial^2 I_{33}}{\partial \eta^2} - 7\tau_L^2 I_{33} + r^2\frac{\partial^2 I_{11}}{\partial \eta^2} + 6r\frac{\partial^2 I_{13}}{\partial x\,\partial \eta}$$
$$-\frac{1}{5}\left(3\frac{\partial^2 I_0}{\partial x^2} + r^2\frac{\partial^2 I_0}{\partial \eta^2}\right) + \frac{28}{3}\tau_L^2\psi^4 = 0 \qquad (22)$$

The radiative heat transfer rates, in terms of I_1 and I_3, are defined for the P-3 approximation below.

$$I_1 = Q_{R_1} = -\frac{1}{\tau_L}\left(r\frac{\partial I_{11}}{\partial \eta} + \frac{\partial I_{13}}{\partial x}\right) \qquad (23a)$$

$$I_3 = Q_{R3} = -\frac{1}{\tau_L}\left(\frac{\partial I_{33}}{\partial x} + r\frac{\partial I_{13}}{\partial \eta}\right) \qquad (23b)$$

Equations (17) and (18) and Eqs. (18) ~~, (19), (20), (21), and~~ through (22) constitute the governing expressions necessary for solution of the two-dimensional, rectangular geometry conduction-radiation problem using the P-1 and P-3 intensity approximations, respectively.

2.3 Boundary Conditions

In incorporating the spherical harmonics approximation for the intensity, four boundary conditions for I_0 and for I_0, I_{11}, I_{33}, and I_{13} are required for the P-1 and P-3 approximations, respectively. (Boundary conditions for the nondimensional temperature, ψ, have previously been specified by Figure 1.) Expressions for the moment boundary conditions are obtained by finding the intensity leaving the boundary surfaces

with $f_w = \frac{1}{\pi}\psi_i^4 \quad (i=1,2,3,4) \qquad (25)$ ~~(24)~~

Switch this expression with the one on the next page. They are out of order and incorporating the modified Marshak boundary conditions [10, 15], defined below

$$\int_{\omega=2\pi} I'_w(\bar{r},\omega)\, P_l^m(\cos\theta)\, d\omega = \int_{\omega=2\pi} f_w\, P_l^m(\cos\theta)\, d\omega \qquad (24)$$

Application of Eq. (24) results in a set of two boundary condition equations for the P-1 approximation and a set of eight equations for the P-3 approximation. These are summarized in the appendix. Reference [14] provides further detail on the boundary condition development.

Solution Method

Two numerical methods were used to solve the sets of coupled nonlinear partial differential equations [14]. A finite element computer program, TWODEPEP [16, 17], developed and issued by International Mathematical and Statistical Libraries, Inc., was used for the solution of the two coupled P-1 expressions. The finite element used is the standard six-node triangle with quadratic basis functions, and the algebraic equations developed are solved by a damped Newton's method. Eighty-eight triangles were symmetrically positioned in the field for all P-1 analyses, yielding an estimated error of the order 10^{-3} for the functions ψ and I_0.

The TWODEPEP program was not used for the P-3 equations since the additional equations required out-of-core storage. Hence, estimated computer running time was expected to be a factor of ten to twenty greater than the P-1 computation times of 45 to 90 seconds on the Cyber 170/750 computer system used at The University of Texas at Austin.

Instead, the five coupled equations were solved using standard elliptic equation successive-over-relaxation (SOR) techniques [18]. In the SOR method, the five elliptic equations were solved independently with four of the variables being treated as knowns during each individual SOR computation. This procedure essentially linearized the moment of intensity expressions. To expedite the computational process, the P-1 approximation solution or a previously obtained P-3 result~~s~~ from a similar problem~~s~~ ~~were~~ was used to initiate the SOR algorithm. All finite difference computational molecules [18] used in the formulation were of second-order accuracy, and spatial increment sizes were fixed by

OK P.

THIS TERM IS USED BY AMES Ref [18] TO DEFINE THE FINITE DIFFERENCE EQUATIONS

$$\tau_L \Delta x_3 \le 0.025 \tag{26}$$

and

$$\frac{r \Delta x_3}{\Delta x_1} \le 1.0 \tag{27}$$

yielding estimated errors of the order 10^{-3}. Typical computation times for aspect ratios less than three ranged between 70 and 300 seconds, depending upon choice of initial data and grid array size. For the aspect ratio of five, running time approached 600 seconds on the Cyber 170/750.

3. RESULTS

Solutions for a range of Stark numbers and aspect ratios were considered, holding the optical depth τ_H fixed at one. In this way, problems in the range of optically thin to optically thick in the χ

direction could be considered with moderate optical thickness in the η direction. Two wall temperature combinations, summarized below, were considered in the analysis.

No Volumetric Source

$$\Psi_1 = 1.0 \tag{28a}$$

$$\Psi_i = 0.5 \quad (i = 2,3,4) \tag{28b}$$

Volumetric Source Included

$$\Psi_i = 1.0 \quad (i = 1,2,3,4) \tag{29}$$

Figure 2 compares results from the P-1 and P-3 approximation analyses, for a square enclosure (τ_H = 1.0). There is close agreement of the centerline nondimensional temperatures for Stark numbers of 0.1 and greater. Below N_1 = 0.1, radiative transfer dominates and the P-1 approximation would be expected to be less accurate. Previous two-dimensional radiative studies [19] have indicated the close agreement of the P-3 approximation method with results reported by Modest [9]. Figure 3 presents nondimensional combined mode (total) heat transfer from the hot wall of the enclosure, defined by Eq. (30). Again, second-order accurate computational Kok molecules were used for all derivatives.

$$Q_{T_1} = I_1 - \left(\frac{4N_1}{\tau_H}\right)\frac{\partial\Psi}{\partial\eta} \tag{30}$$

These results compare well for higher Stark numbers, but deviate as N_1 approaches zero. In purely radiative problems [19], the P-1 heat transfer rates are much greater than the Hottel zone and Modest results. While P-3 approximation heat transfer results also tended to be higher, they more closely matched the trends of the other solution methods [19].

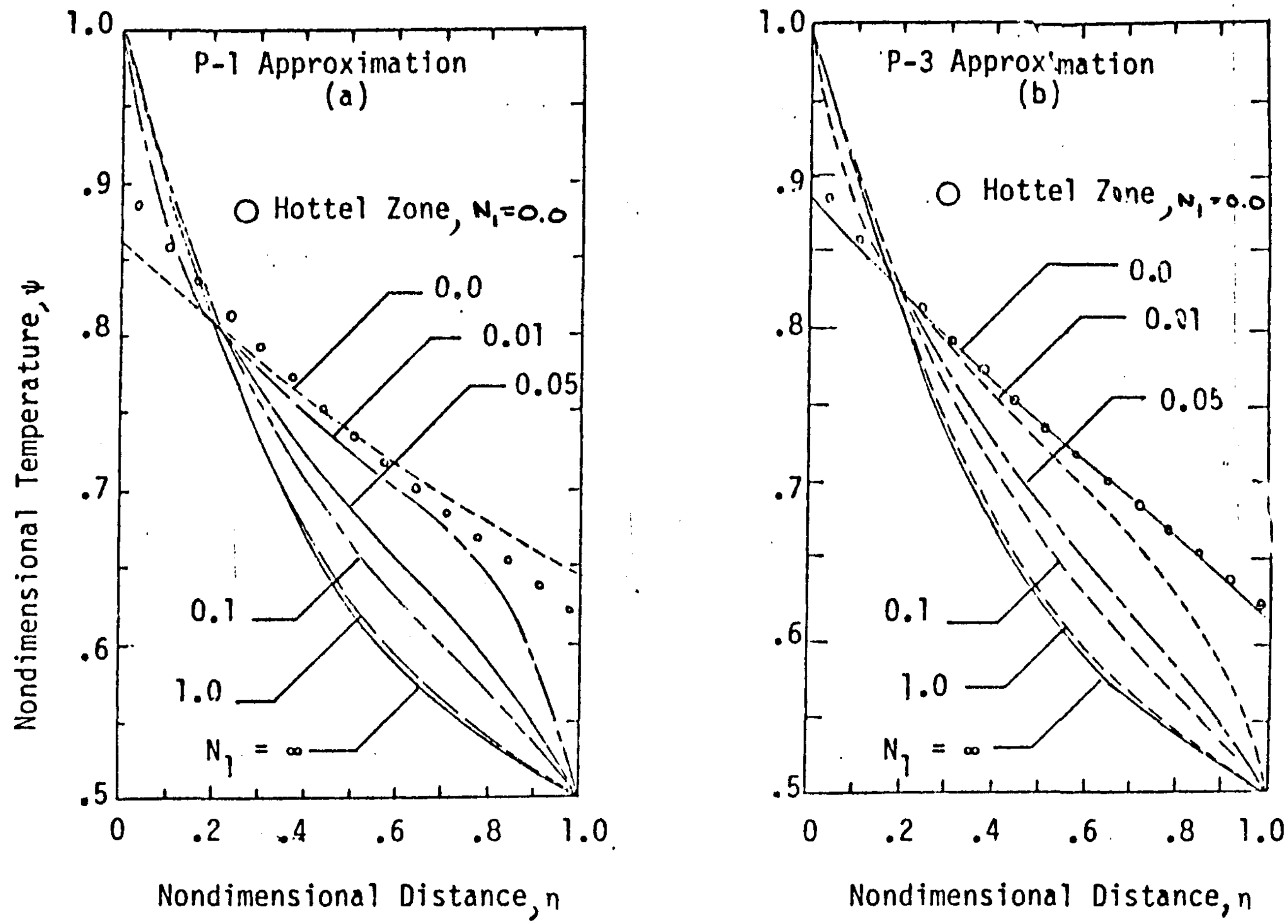
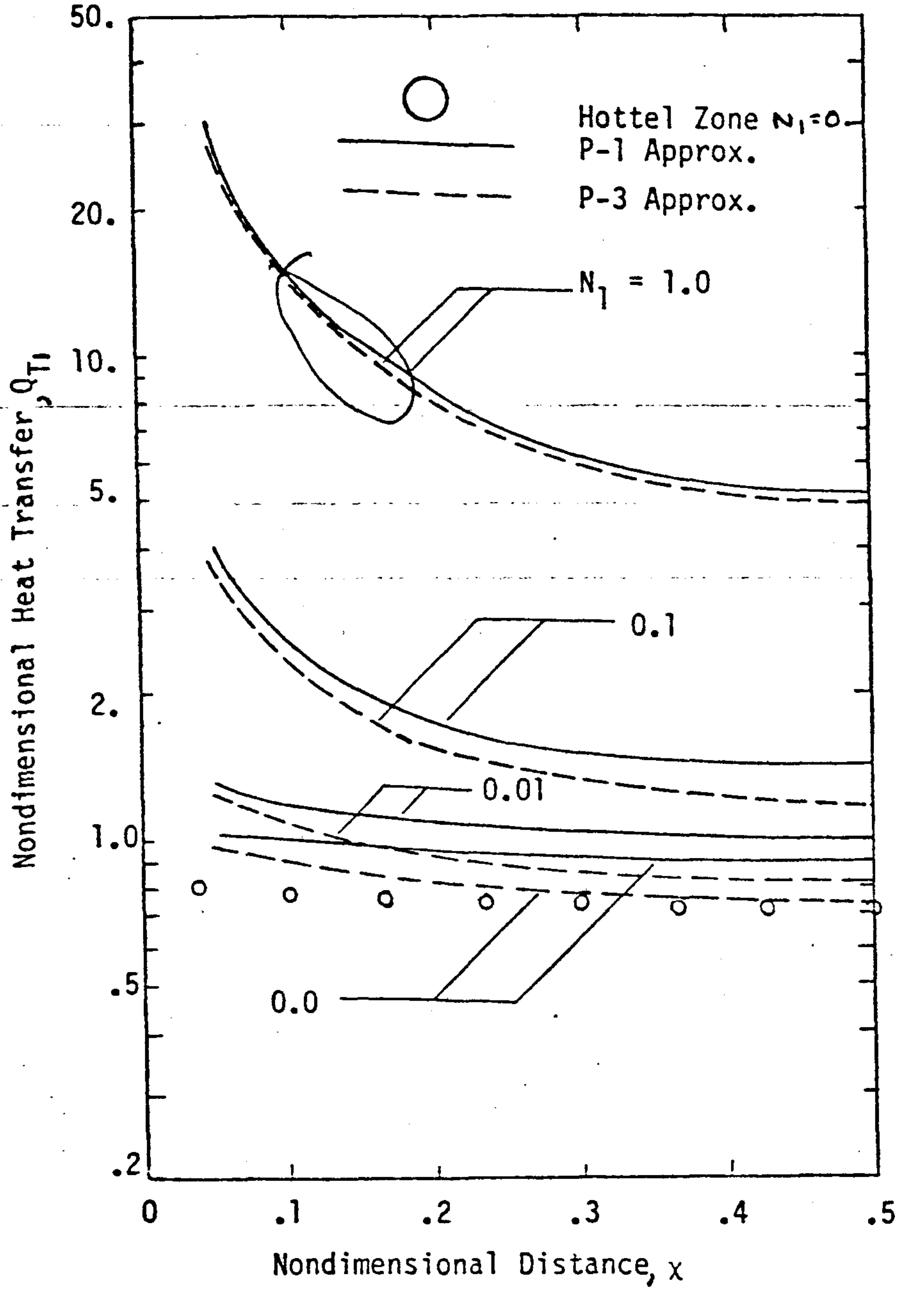


Figure 2: Comparable nondimensional centerline temperature profiles for the P-1 and P-3 approximation methods in a square enclosure: $\tau_H = 1.0$.



THESE CURVES HAVE SINCE BEEN REDRAWN TO CORRECT THE DRAFTSMAN'S LICENSE

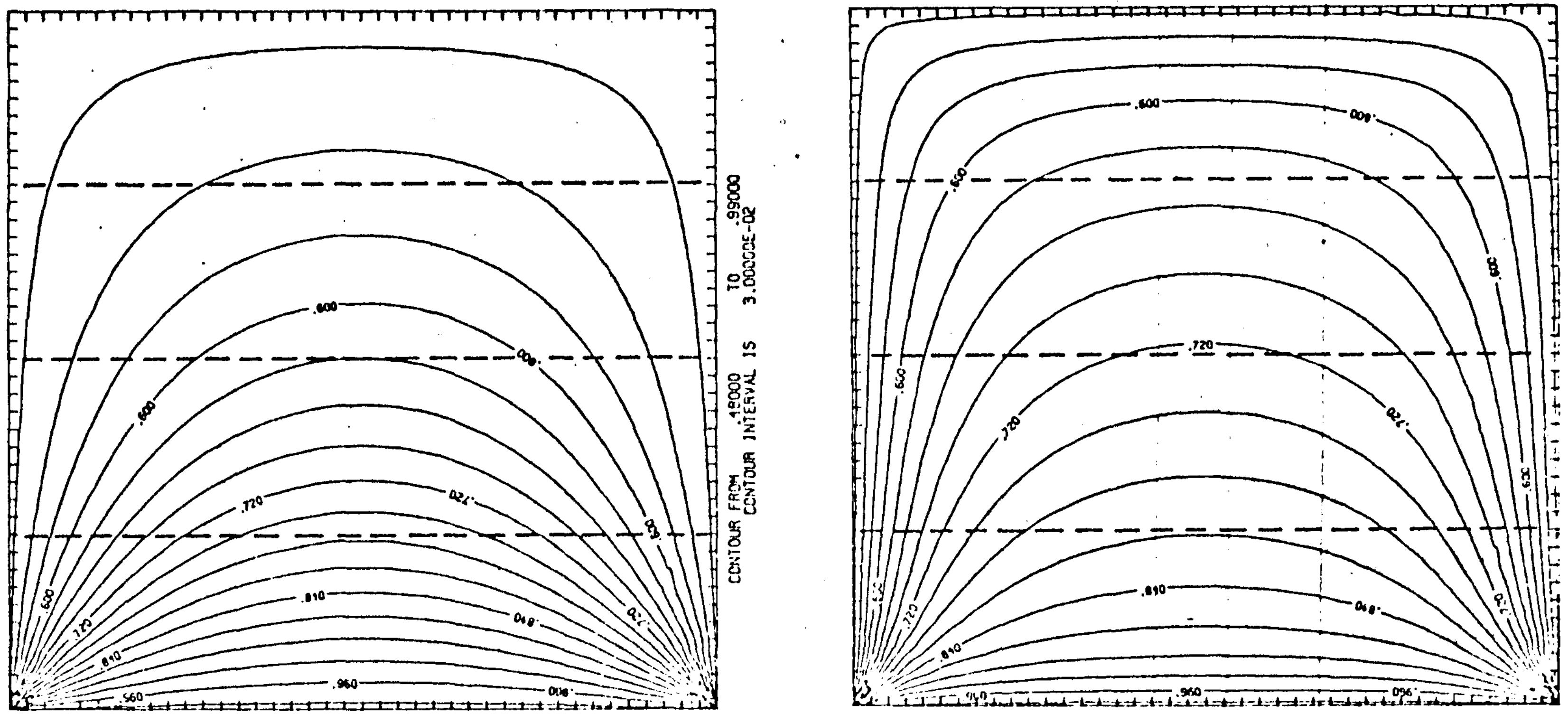
Figure 3: Comparative P-1 and P-3 approximation results for the total nondimensional hot wall heat transfer in a square enclosure: ~~(~~$\tau_H = 1.0$~~)~~

Note that the comparative Hottel results presented in Figures 2 and 3 were obtained using 225 volume and 60 surface elements [21]. Figure 4 displays two nondimensional temperature fields for the P-3 approximation. P-1 contour plots of these cases follow the same trends and have thus been omitted.

Figures 5 and 6 present P-3 approximation results for the centerline temperature profiles of rectangular enclosures (τ_H = 1.0) for Stark numbers of 1.0 and 0.01. Comparative "exact" solutions for the limiting one-dimensional results [2] show that the centerline temperatures are less affected by the sidewall temperature as the aspect ratio increases. Table 1 shows the variation of centerline hot wall heat transfer and average hot wall heat transfer for the different aspect ratios ~~geometries~~. The increase in the average nondimensional heat transfer compared with centerline heat transfer occurs because of the lower bounding sidewall temperatures. The spatial variation of heat transfer ~~curves~~ from the hot wall for these cases follows the trends presented in Figure 3 and are omitted.

For the results presented, all corner point nondimensional temperatures were fixed at the arithmetic average of the adjacent walls to avoid corner point discontinuities. The heat transfer from the hot wall at the corners is thus "fictitious", and has been omitted from Figure 3, although it is included in all energy balances. Comparative two-dimensional isotherms for aspect ratios of 0.5, 1.0, and 5.0 and a Stark number of 0.01 are given in Figures 7a, 4b, and 7b, respectively.

As a final example of the application of the differential approximation to combined mode problems in two-dimensional fields, a uniform volume source of S = 4.0 is considered in a square enclosure



(a) $N_1 = 1.0$

(b) $N_1 = 0.01$

Figure 4: P-3 approximation nondimensional temperature distributions in a square enclosure:
($\tau_H = 1.0$)

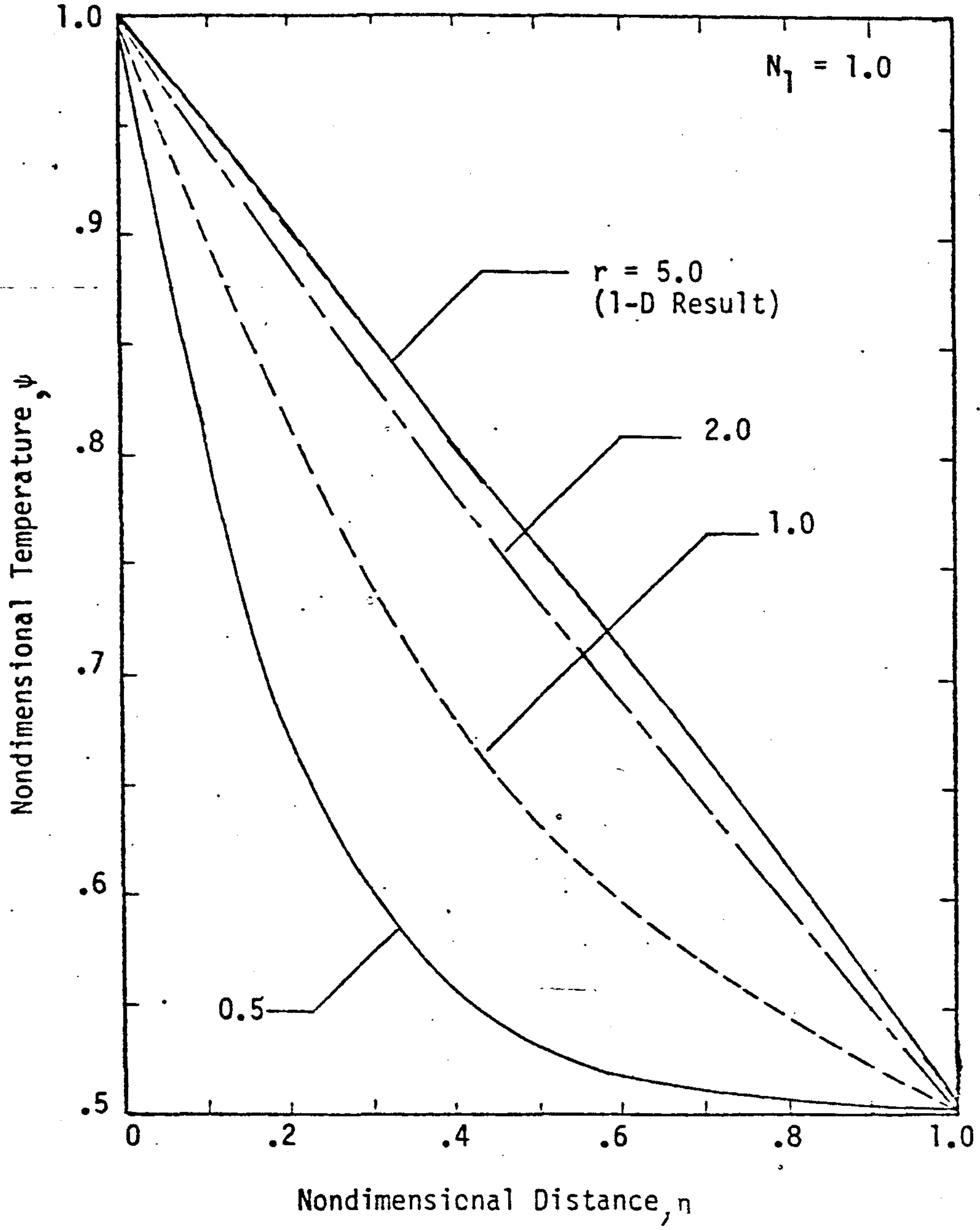


Figure 5: P-3 approximation results for nondimensional centerline temperature distributions with aspect ratios varied; ~~(τ_H = 1.0), and~~ N_1 = 1.0

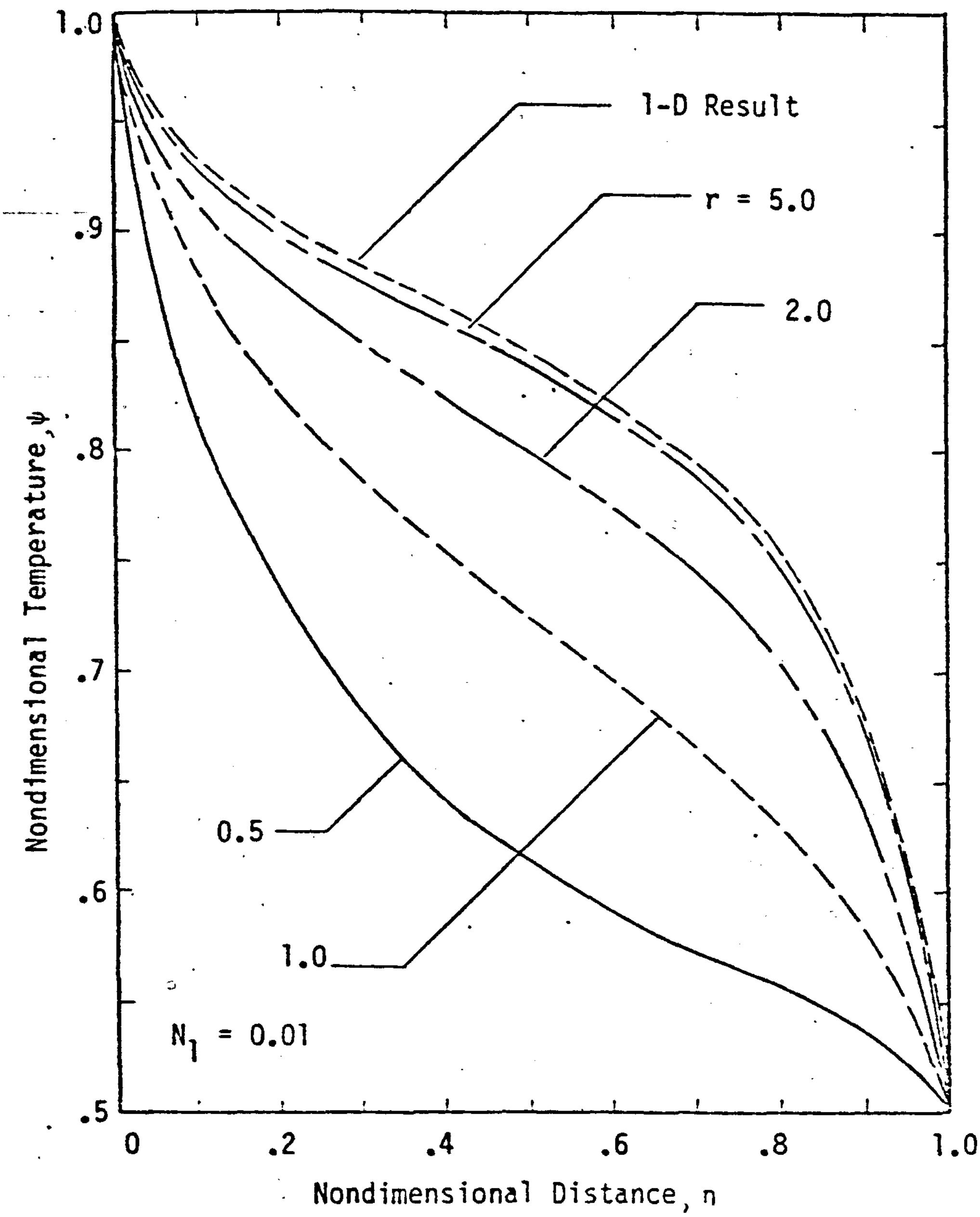


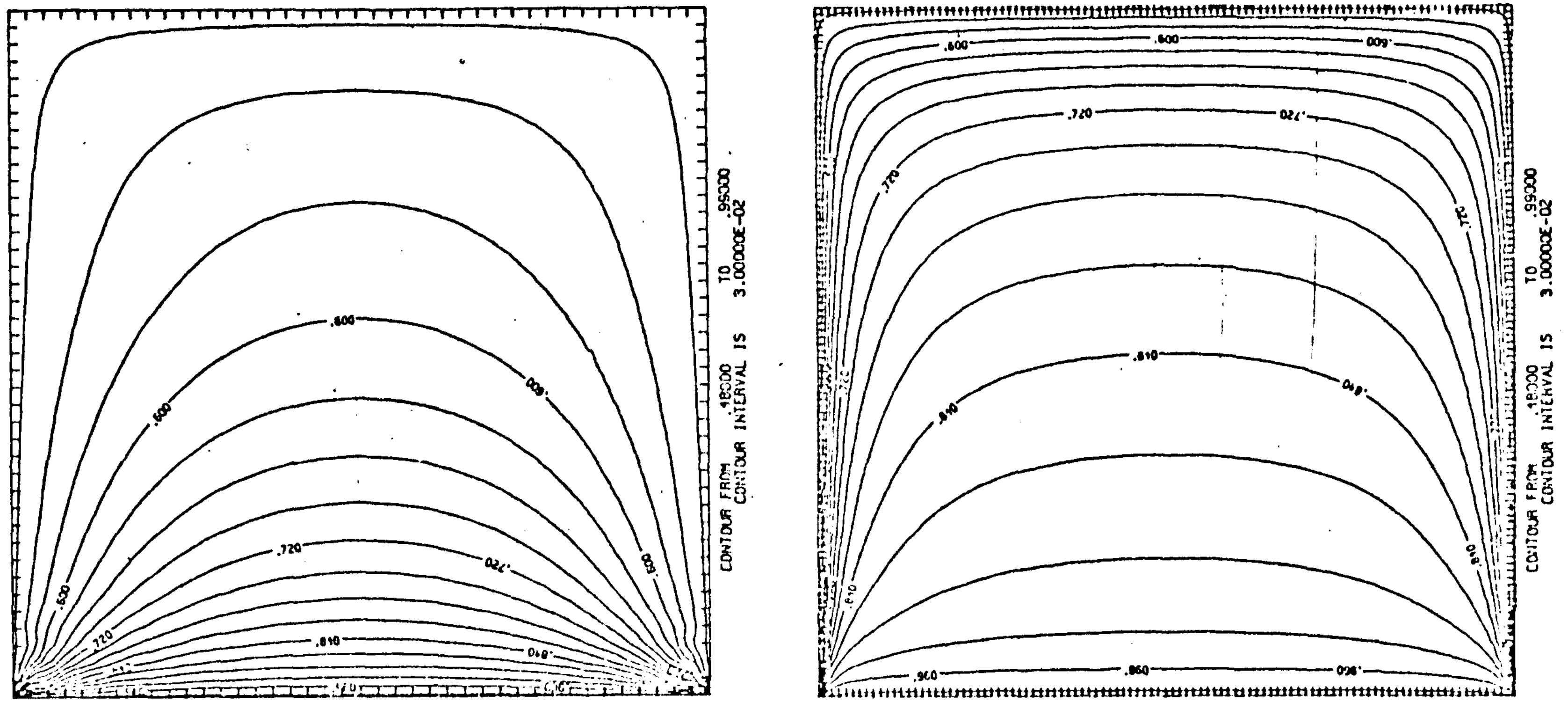
Figure 6: P-3 approximation results for nondimensional centerline temperature distributions with aspect ratios varied, $\tau_H = 1.0$, ~~and~~ $N_1 = 0.01$

Table 1 Heat transfer results for different aspect ratio rectangular enclosures where $\tau_H = 1.0$

Stark Number	Aspect Ratio	Nondimensional Centerline Heat Flux*		Nondimensional Average Heat Flux*	
		P-1	P-3	P-1	P-3
	0.5	9.852	9.328	17.80	17.17
	1.0	5.203	4.877	11.57	10.75
1.0	2.0	3.155	3.032	6.811	6.797
	5.0	2.640	2.592	4.582	4.270
	∞	2.615	2.580	2.615	2.580
			(2.572)**		(2.572)
	0.5	2.053	1.791	2.820	2.564
	1.0	1.418	1.194	2.063	1.830
0.1	2.0	1.026	0.888	1.450	1.342
	5.0	0.827	0.788	1.110	1.005
	∞	0.809	0.776	0.809	0.776
			(0.7694)		(0.7694)
	0.5	1.263	1.035	1.317	1.100
	1.0	1.019	0.818	1.094	0.923
0.01	2.0	0.779	0.667	0.891	0.779
	5.0	0.625	0.587	0.730	0.658
	∞	0.595	0.572	0.595	0.572
			(0.5675)		(0.5675)

*Results given for $\varepsilon_{wi} = 1.0$, $\psi_{w1} = 1.0$, $\psi_{wi} = 0.5$ (i=2,3,4), S = 0.0

**Numbers in parentheses are exact one-dimensional results taken from Reference [2].



(a) r = 0.5

(b) r = 5.0

Figure 7: P-3 approximation nondimensional temperature distributions for rectangular enclosures; ~~(~~τ_H = 1.0~~)~~, ~~and~~ N_1 = 0.01

with equal temperature walls. Figure 8 gives the centerline temperature distribution for a range of Stark numbers for the P-1 and P-3 approximations. The trends are similar to those noted in one-dimensional conduction and radiation studies using the spherical harmonics expansions [4]; i.e., the P-1 approximation underpredicts the temperature field near the interior of the medium as the Stark number decreases. Similarly, it was found that the P-1 approximation underpredicts the wall heat transfer near the centerline of the enclosure in purely radiative studies [19] when volume source terms are included. The higher order approximation has been found to approach more closely exact one-dimensional work and zonal two-dimensional work and is recommended for Stark numbers below 1.0. Figure 9 presents typical isotherms for S = 4.0 with Stark numbers of 0.0 and 0.1. These contour plots are provided to show the "no-slip" condition at the wall when conduction is included. For the purely radiative case, the gas temperature discontinuity at the wall results in circular isotherms.

4. CONCLUSIONS

The differential approximation, using the P-1 and P-3 spherical harmonics approximations for the intensity distribution, has been shown to be useful for obtaining temperature and heat transfer results in combined conduction and radiation problems in rectangular geometries. ~~The formulation of the governing partial differential equations and boundary conditions for the moments of intensity for the two approximation methods has been summarized, and the solution techniques~~

Delete

See comment on page 26

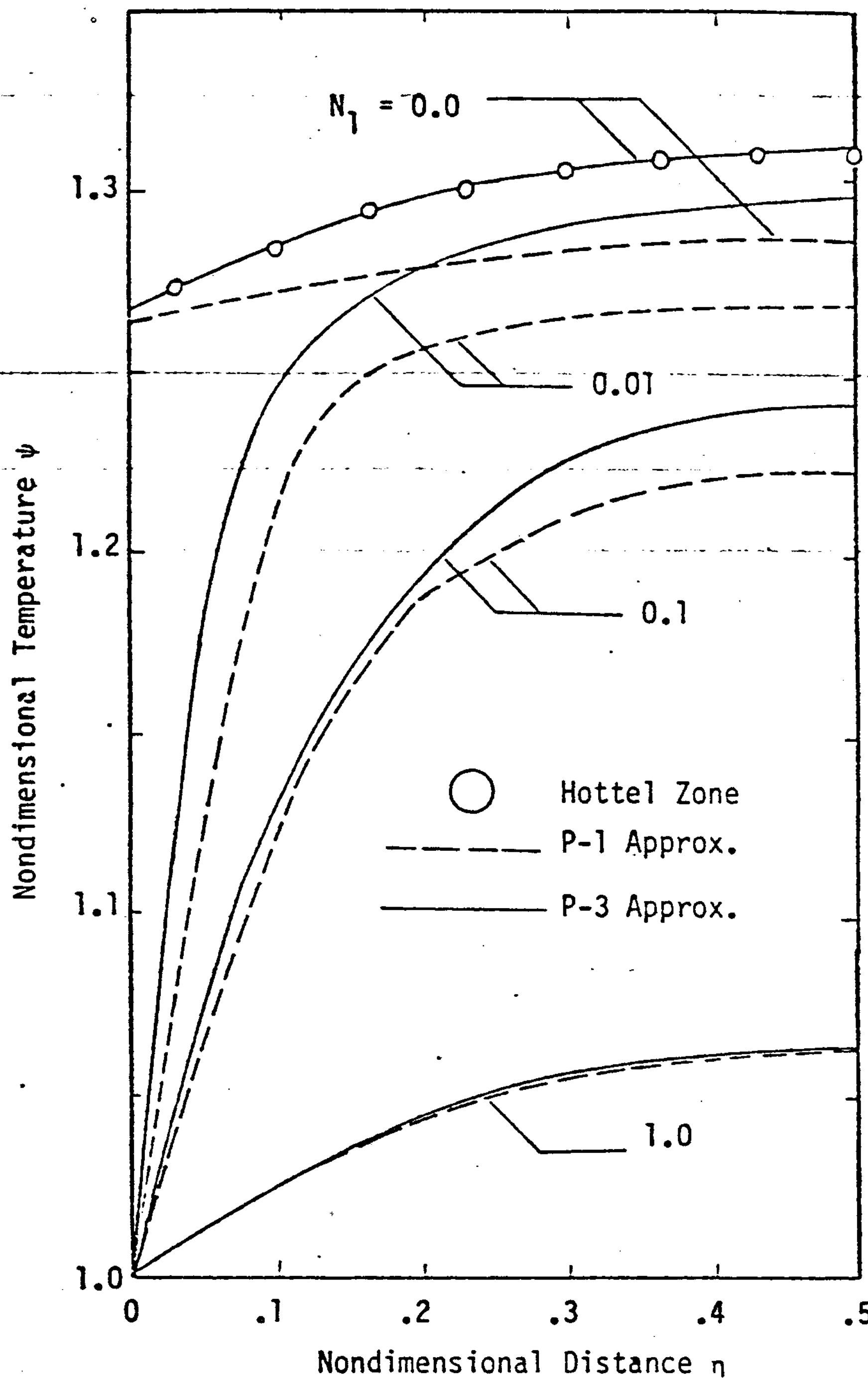
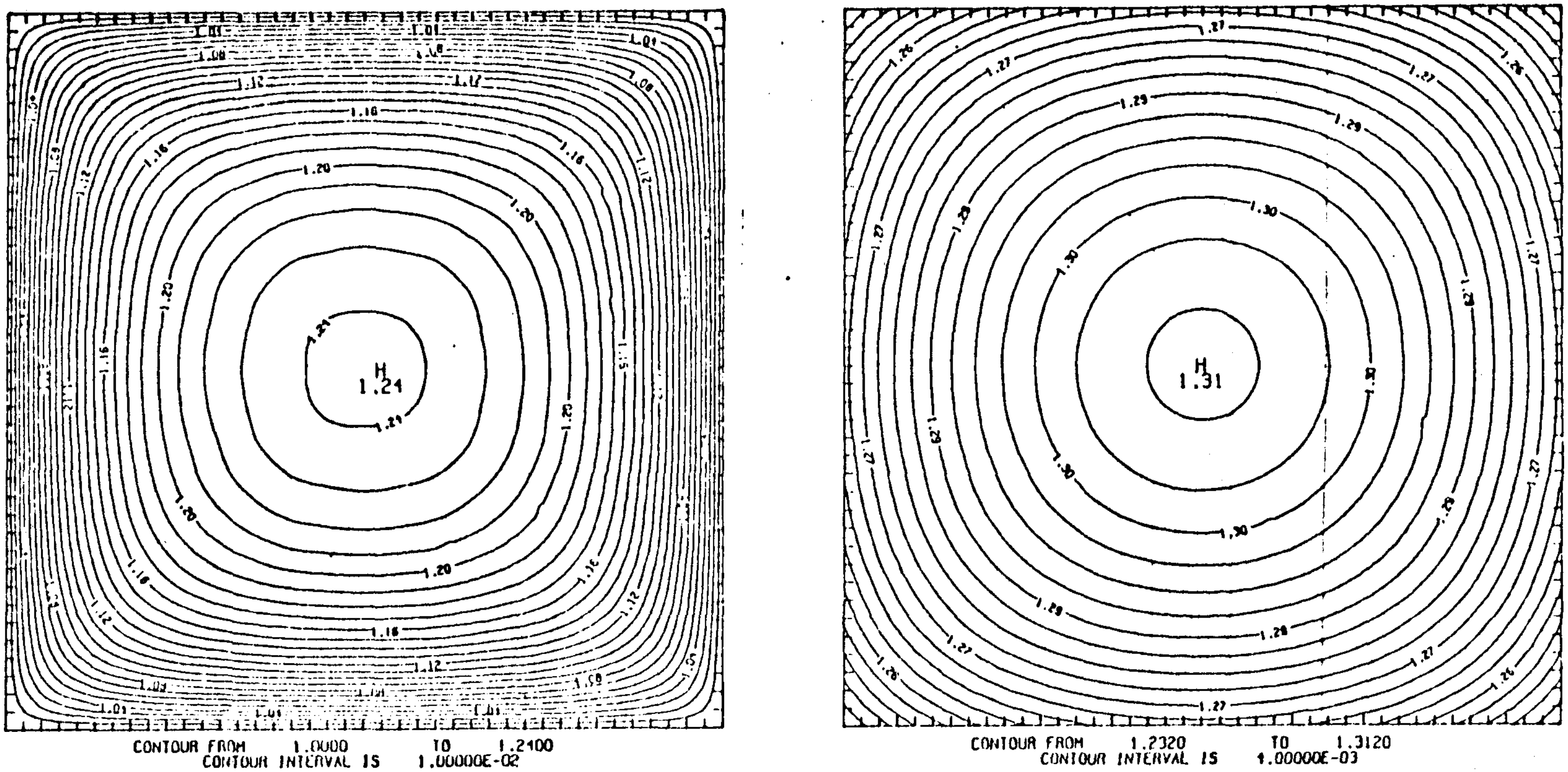


Figure 8: Comparative nondimensional center-line temperature distributions in a square enclosure ($\tau_H = 1.0$) for the P-1 and P-3 approximation methods.



(a) $N_1 = 0.1$

(b) $N_1 = 0.0$

Figure 9: P-3 approximation nondimensional temperature results for the square enclosure; ~~with~~ ($\tau_H = 1.0$), ~~with~~ and a ~~volumetric source~~ $S = 4.0$

Delete

used are discussed. Representative examples have been presented for square and rectangular enclosures for a range of Stark numbers, and volumetric generation has been considered for a square geometry. In general, the P-1 and P-3 approximation methods have been found to yield similar results for $N_1 \geq 1.0$, but for radiation dominated problems, ($N_1 < 1.0$), the P-3 approximation method is preferred. The P-1 approximation tends to overpredict hot surface heat transfer rates when there is no volumetric generation and to underpredict the medium temperatures if generation is included. Although no "exact" two-dimensional results are available for comparison, the differential methods are believed to be accurate for predicting temperature profiles and heat transfer rates. Purely radiative two-dimensional P-3 approximation results [19] have been shown to compare favorably with Hottel zone [20] and Modest [9] solutions, and limiting cases for large optical width enclosures have approached exact one-dimensional combined-mode solutions. This paper is believed to represent the first published results for the coupled two-dimensional problem in a rectangular enclosure.

This is more a summary than a conclusion and I would delete it.

5. APPENDIX: BOUNDARY CONDITIONS

P-1 Approximation

$$I_0: \quad \pm \frac{\partial I_0}{\partial \eta} + \frac{3}{2}\tau_H I_0 + 6\tau_H \psi_i^4 = 0 \tag{27a}$$

$$\pm \frac{\partial I_0}{\partial x} + \frac{3}{2}\tau_L I_0 + 6\tau_L \psi_i^4 = 0 \tag{27b}$$

P-3 Approximation

$$I_0: \quad \pm\frac{\partial I_0}{\partial\eta} - \frac{35}{48}\tau_H I_0 + 35\tau_H\left(\frac{5}{16}I_{11} - \frac{1}{3}\psi_i^4\right) \mp 5\left(2\frac{\partial I_{11}}{\partial\eta} + \frac{1}{r}\frac{\partial I_{13}}{\partial x}\right) = 0 \quad (28a)$$

$$\pm\frac{\partial I_0}{\partial x} - \frac{35}{48}\tau_L I_0 + 35\tau_L\left(\frac{5}{16}I_{33} - \frac{1}{3}\psi_i^4\right) \mp 5\left(2\frac{\partial I_{33}}{\partial x} + r\frac{\partial I_{13}}{\partial\eta}\right) = 0 \quad (28b)$$

$$I_{11}: \quad \pm\frac{\partial I_{11}}{\partial\eta} - \frac{15}{16}\tau_H I_{11} + \tau_H\left(2\psi_i^4 - \frac{3}{16}I_0\right) \pm \frac{\partial I_{13}}{\partial x} = 0 \quad (29a)$$

$$\pm\frac{\partial I_{11}}{\partial x} - \frac{35}{16}\tau_L I_{11} + \frac{7}{2}\tau_L\left(\frac{I_0}{16} - \frac{5}{16}I_{33} + \psi_i^4\right) \pm\left(\frac{\partial I_{33}}{\partial x} + 3r\frac{\partial I_{13}}{\partial\eta} - \frac{1}{5}\frac{\partial I_0}{\partial x}\right) = 0 \quad (29b)$$

$$I_{33}: \quad \pm\frac{\partial I_{33}}{\partial\eta} - \frac{35}{16}\tau_H I_{33} + \frac{7}{2}\tau_H\left(\frac{I_0}{16} - \frac{5}{16}I_{11} + \psi_i^4\right) \pm\left(\frac{\partial I_{11}}{\partial\eta} + \frac{3}{r}\frac{\partial I_{13}}{\partial x} - \frac{1}{5}\frac{\partial I_0}{\partial x}\right) = 0 \quad (30a)$$

$$\pm\frac{\partial I_{33}}{\partial x} - \frac{15}{16}\tau_L I_{33} + \tau_L\left(2\psi_i^4 - \frac{3}{16}I_0\right) \pm r\frac{\partial I_{13}}{\partial\eta} = 0 \quad (30b)$$

$$I_{13}: \quad \pm\frac{\partial I_{13}}{\partial\eta} - \frac{35}{24}\tau_H I_{13} \pm \frac{1}{3r}\left(\frac{\partial I_{33}}{\partial x} + \frac{\partial I_{11}}{\partial x} - \frac{1}{5}\frac{\partial I_0}{\partial x}\right) = 0 \quad (31a)$$

$$\pm\frac{\partial I_{13}}{\partial x} - \frac{35}{24}\tau_L I_{13} \pm \frac{r}{3}\left(\frac{\partial I_{33}}{\partial\eta} + \frac{\partial I_{11}}{\partial\eta} - \frac{1}{5}\frac{\partial I_0}{\partial\eta}\right) = 0 \quad (31b)$$

Equation numbers with (a) designate equations for walls one and two, and those with (b) designate expressions for walls three and four. Positive signs are used for walls one and three, and negative signs are used for walls two and four.

move to ~~front~~ of report (right after abstract)

NOMENCLATURE

a	absorption coefficient
e_{b1}	blackbody emissive power of wall 1
f_w	~~right hand side of intensity~~ defined by Eq. (25)
H	height of rectangular enclosure
i'	radiative intensity distribution
I'	nondimensional radiative intensity distribution
I'_w	nondimensional intensity leaving enclosure surface
I_0	nondimensional zeroth moment of intensity
I_i	nondimensional first moment of intensity (i=1,3)
I_{ij}	nondimensional second moment of intensity (i,j=1,3)
I_{ijk}	nondimensional third moment of intensity (i,j,k=1,3)
k	thermal conductivity
$\hat{\ell}_i$	direction cosine (i=1,3)
L	width of rectangular enclosure
N_1	conduction-radiation parameter (Stark number); Eq. (6)
Q'''	uniform volumetric source
q_{ci}	conduction heat transfer rate in ith coordinate direction (i=1,3)
Q_{ci}	nondimensional ith direction conduction heat transfer rate (i=1,3)
q_{Ri}	radiative heat transfer rate in ith coordinate direction (i=1,3)
Q_{Ri}	nondimensional ith direction radiative heat transfer rate (i=1,3)

Q_{T1}	nondimensional total heat transfer rate from wall 1
r	aspect ratio (=L/H)
$\bar{r}$	vector position in medium
S	nondimensional volumetric source; Eq. (7)
T	temperature
T_i	temperature of wall i (i=1,2,3,4)
X_i	position coordinate i (i=1,2,3)
ΔX_i	finite difference step size in i direction (i=1,3)

Greek Symbols

η	normalized position in X_1 coordinate direction
θ	elevation angle ($0 \leq \theta \leq \pi$)
τ_i	nondimensional optical position in coordinate direction i (i=1,3)
τ_H	optical depth in X_1 direction
τ_L	optical depth in X_3 direction
ϕ	azimuthal angle ($0 \leq \phi \leq 2\pi$)
χ	normalized position in X_3 coordinate direction
ψ, ψ_w	nondimensional temperature (of wall)
ω	solid angle

REFERENCES

1. Viskanta, R., and Grosh, R. J., Heat Transfer by Simultaneous Conduction and Radiation in an Absorbing Medium, J. Heat Transfer, vol. 84, no. 1, pp. 63-72, 1962.

2. Crosbie, A. L., and Viskanta, R., Interaction of Heat Transfer by Conduction and Radiation in a Nongray Planar Medium, Wärme-und Stouffübertragung, vol. 4, pp. 205-212, 1971.

3. Yuen, W. W., and Wong, L. W., Heat Transfer by Conduction and Radiation in a One-Dimensional Absorbing, Emitting, and Anisotropically Scattering Medium, J. Heat Transfer, vol. 12, no. 2, pp. 303-307, 1980.

4. Ratzel, A. C., and Howell, J. R., Heat Transfer by Conduction and Radiation in One-Dimensional Planar Medium Using the Differential Approximation, ASME paper 81-HT-72, August 1980.

5. Deissler, R. G., Diffusion Approximation for Thermal Radiation in Gases with Jump Boundary Condition, J. Heat Transfer, vol. 86, no. 2, pp. 240-246, 1964.

6. Hottel, H. C., and Cohen, E. S., Radiant Heat Exchange in a Gas-filled Enclosure: Allowance for Nonuniformity of Gas Temperature, AIChE J., vol. 4, no. 1, pp. 3-14, 1958.

7. Cheng, P., Exact Solutions and Differential Approximations for Multi-Dimensional Radiative Transfer in Cartesian Coordinate Configurations, Thermal Control and Radiation, Progress in Astronautics and Aeronautics, C. L. Tien, ed., vol. 31, pp. 269-308, 1972.

8. Glatt, L., and Olfe, D. B., Radiative Equilibrium of a Gray Medium in a Rectangular Enclosure, J. Quan. Spec., vol. 13, no. 9, pp. 881-895, 1973.

9. Modest, M. F., Radiative Equilibrium in a Rectangular Enclosure Bounded by Gray Walls, J. Quan. Spec., vol. 15, no. 6, pp. 445-461, 1975.

10. Marshak, R. E., Note on the Spherical Harmonic Method as Applied to the Milne Problem for a Sphere, Phys. Rev., vol. 71, pp. 443-446, 1947.

11. Jeans, J. H., The Equations of Radiative Transfer of Energy, Monthly Notices of Royal Astronomical Society, vol. 78, pp. 28-36, 1917.

12. Cheng, P., Dynamics of a Radiating Gas with Application to Flow over a Wavy Wall, AIAA J., vol. 4, no. 2, pp. 238-245, 1966.

13. Bayazitoglu, Y., and Higenyi, J., Higher Order Differential Equations of Radiative Transfer: P-3 Approximation, AIAA J., vol. 17, no. 4, pp. 424-431.

14. Ratzel, A. C., Differential Approximation for Solution of one-and Two-Dimensional Radiation and Conduction Energy Transfer in Gray Participating Media, Ph.D. dissertation, The University of Texas at Austin, December 1981.

15. Davison, B., [redacted], Neutron Transport Theory, Oxford, Clarendon Press, 1958.

16. TWODEPEP Users Manual, International Mathematical and Statistical Libraries Manual, IMSL TDP-0003, Houston, September 1981.

17. Sewell, G., A Finite Element Program with Automatic, User-Controlled Mesh Grading, Proc. 3d IMACS Int. Sym. on Computer Methods for Partial Differential Equations, New Brunswick, NJ., pp. 8-10, 1979.

18. Ames, W. F., Numerical Methods for Partial Differential Equations, New York, Academic Press, 1977.

19. Ratzel, A. C., and Howell, J. R., Two-dimensional Radiation in Absorbing-Emitting-Scattering Media Using the P-N Approximation, Submitted to ASME/AIAA Thermophysics and Heat Transfer Meeting, St. Louis, 1982.

20. Larsen, M., Hottel Zone Code, developed for Ph.D. research at The University of Texas at Austin, August 1981.