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CHIRAL LAGRANGIANS AND THE SSC

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ABSTRACT

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ABSTRACT

In the event that the SSC does not observe any resonances such as a Higgs boson or a techni-rho meson, we would like to know if the SSC can still discover something about the nature of the electroweak symmetry breaking. We will use chiral Lagrangian techniques to address this question and analyze their utility for studying events containing W and Z gauge bosons at the SSC.

1. Introduction

One of the most important questions facing high energy physicists in the next decade is the origin of the electroweak symmetry breaking. In the simplest possible case, the symmetry breaking occurs through the coupling of a complex $SU(2)$ scalar doublet to $SU(2) \times U(1)$ gauge fields. The Higgs mechanism then gives the W and Z gauge bosons their mass, leaving a single physical scalar, the Higgs boson H. The mass of the Higgs boson is, however, a free parameter in the theory and it is necessary to search for it in all mass regimes. The current experimental limit on the mass of the Higgs boson comes from LEP and is $M_H > 55\ GeV$. In building future high energy colliders, it is crucial to know what experimental limits they will be able to obtain on the Higgs mass. It is generally believed that the SSC will be able to observe a Higgs boson with a mass less than about $800\ GeV$.[1]

Another attractive possibility for the breaking of the electroweak symmetry is that of dynamical symmetry breaking. Although no completely satisfactory model exists, these models share some general features, among them the existence of a heavy particle with the quantum numbers of the ρ, usually called the techni-rho, ρ_{TC}[2]. Depending on the details of the model, the SSC will be able to discover a techni-rho up to a mass near $1\ TeV$.

Clearly, the most attractive possibility for electroweak symmetry breaking is the existence of a resonance which can be seen at the SSC. Here we want to ask the pessimistic question: What if there is no hint of a resonance at the SSC? Can we still learn something about the nature of the electroweak symmetry breaking?

In the standard model with a heavy Higgs boson, the $4-W$ gauge boson self coupling grows like M_H^2/M_W^2. Hence such a model contains strongly interacting W bosons. Models with dynamical symmetry breaking are strongly interacting by construction. To analyze theories with strongly interacting W's we recall the example of strongly interacting particles observed experimentally, the pion system. The experimental results from pion scattering are well described through the use of the chiral Lagrangian, which is an expansion in terms of powers of energy[3]. Because of the symmetries of the theory, the low energy behaviour obeys $\pi - \pi$ scattering theorems which were first derived by Weinberg[4].

The low energy theorems can easily be generalized from pions to the longitudinal components of the W and Z gauge bosons. These scattering theorems are valid at low energy, that is $\sqrt{s} \ll \{M_{Res}, 4\pi v\}$ where M_{Res} is the mass of the lowest lying resonance interacting with the W_L's and Z_L's and $v = 246\ GeV$. The lowest order term in an energy expansion of a scattering amplitude, that of $\mathcal{O}(s)$, is completely fixed by the global symmetry of the theory. The next terms in the expansion, however, are quite sensitive to the nature of the symmetry breaking mechanism. It has been suggested that by extracting the energy dependence of longitudinal gauge boson scattering cross sections, it may be possible to differentiate between different mechanisms of electroweak symmetry breaking, even if a resonance is not observed. In Section 2 of this paper, we will pursue the analogy between pions and the W and Z gauge bosons and analyze the utility of the chiral Lagrangian for studying the interactions of longitudinal gauge bosons. We will discuss the phenomenology of attempting to distinguish between models with a Higgs boson and "QCD- like" technicolor models.

In Section 3, we extend our analysis by gauging the chiral Lagrangian with respect to $SU(2) \times U(1)$. This allows us to study the interactions of all polarizations of gauge bosons and to analyze the production of gauge bosons both from vector boson scattering and from quark- antiquark annihilation. The reaction $q\overline{q} \rightarrow W^+W^-$ depends on the three gauge boson vertex which may be affected by higher order terms in the chiral Lagrangian and thus may differ from that of the standard model. Section 4 uses chiral Lagrangian techniques for theories which have larger gauge groups than the standard model. Finally, Section 5 contains some conclusions.

2. Longitudinal Gauge Bosons and the Chiral Lagrangian

2.1. Pion Scattering

We begin this section by reviewing the use of chiral Lagrangians for the study of pion dynamics[3,5]. QCD with two massless quarks has a global $SU(2)_L \times SU(2)_R$ symmetry which is broken by the quark condensate to a custodial $SU(2)_V$. When the symmetry is broken there are three Goldstone bosons formed which are the pions, $\vec{\Pi} = (\pi^{\pm}, \pi^0)$. The pion interactions can be described in terms of the $SU(2)$ field,

$$\Sigma = exp\left(2i\Pi \cdot T/f_\pi\right), \tag{1}$$

where T are $SU(2)$ generators normalized such that $Tr(T_iT_j) = \frac{1}{2}\delta_{ij}$ and $f_\pi = 91\ MeV$. Under $SU(2)_L \times SU(2)_R$ transformations, Σ transforms as,

$$\Sigma \rightarrow L\Sigma R^\dagger \quad . \tag{2}$$

The pion interactions can now be found by an expansion in powers of energy, E^2/Λ^2, where Λ is the scale of chiral symmetry breaking which is presumably of order $1\ GeV$.

The first term in the expansion, which is of $\mathcal{O}(E^2)$, has two derivatives and is,

$$\mathcal{L}_2 = \frac{f_\pi^2}{4} Tr\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right). \qquad (3)$$

The absolute normalization of $\mathcal{L}_2$ is completely determined by the requirement that the pions have canonical kinetic energy.

The Lagrangian of Eq. (3) gives $\pi - \pi$ scattering cross sections which are rising with energy and will eventually violate unitarity. To correct this problem, we can include the $\mathcal{O}(E^4)$ terms in the energy expansion; that is, terms with four derivatives,

$$\mathcal{L}_4 = \alpha_1 Tr\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) Tr\left(\partial_\nu \Sigma \partial^\nu \Sigma^\dagger\right) + \alpha_2 Tr\left(\partial_\mu \Sigma \partial_\nu \Sigma^\dagger\right) Tr\left(\partial^\mu \Sigma \partial^\nu \Sigma^\dagger\right) \quad . \qquad (4)$$

The coefficients α_1 and α_2 are *a priori* unknown and contain dynamical information about the model. The sum $\mathcal{L}_2 + \mathcal{L}_4$ is the most general $\mathcal{L}$ consistent with the chiral symmetry. Possible terms involving the pion mass break the chiral symmetry and we neglect them.

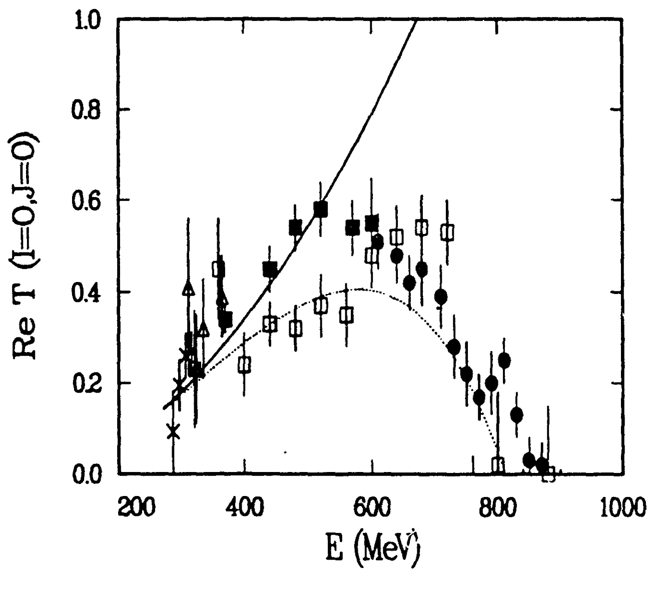

Fig. 1 *Compilation of $\pi - \pi$ scattering data from Ref. [5]. The solid curve is the prediction of the lowest order chiral Lagrangian, Eq. (3), and the dotted line is the best fit using Eqs. (4) and (5).*

In Figure 1, we show a compilation of $\pi - \pi$ scattering data by Donoghue *et. al.* for the $J = 0$, $I = 0$ channel[5]. This figure also shows the prediction obtained using $\mathcal{L}_2$ alone. As advertised, it rises with increasing energy and violates the condition for partial wave unitarity, $| T^{00} |< \frac{1}{2}$, at an energy of about 600 MeV. For comparison, we show the best fit to the data obtained using $\mathcal{L}_2 + \mathcal{L}_4$ which yields[5]:

$$\begin{aligned} \alpha_1 &= -\ .0092 \\ \alpha_2 &= .0080 \quad . \end{aligned} \tag{5}$$

We see that including the E^4 terms in $\mathcal{L}_4$ significantly improves the fit to the data. It also cures the problem of unitarity violation. In the next subsection, we turn to the physical meaning of the α_i and their computation in various models.

2.2. What are α_1 and α_2?

In this subsection, we compute α_1 and α_2 in two models; one of which has a scalar, isoscalar particle (which we will suggestively call H) coupled to the pions and the other with a spin-1 vector, iso-vector particle (the ρ) coupled to the pions.

The most general coupling of a scalar particle to the pions is given by[6],

$$\mathcal{L}_H = g_H Tr\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) H. \tag{6}$$

By integrating out the scalar H and computing the decay width Γ_H of the scalar to pions, it is straightforward to obtain,

$$\begin{aligned} \alpha_1^H &= \frac{4\pi}{3} \frac{\Gamma_H f_\pi^4}{M_H^5} \\ \alpha_2^H &= 0 \quad . \end{aligned} \tag{7}$$

On the other hand, we can compute the contribution to the α_i from a particle with the quantum number of the ρ,

$$\alpha_1^\rho = -\alpha_2^\rho = -12\pi \frac{\Gamma_\rho f_\pi^4}{M_\rho^5} \tag{8}$$

where Γ_ρ is the decay width of the ρ to the pions.

We see that there is a distinctive pattern: scalars have $\alpha_1 > 0$, $\alpha_2 = 0$, while vectors have $\alpha_1 = -\alpha_2$. From the best fit to the pion data, Eq. (5), we see that the data have $\alpha_1 \sim -\alpha_2$ corresponding to the existence of a ρ. It is remarkable that we obtain this result even in the $I = J = 0$ channel which does not couple to the ρ.

2.3. Finally.....Longitudinal Gauge Boson Scattering

We are now in a position to make the analogy between pions and the longitudinal gauge bosons of the standard model of electroweak interactions. We are aided by the electroweak equivalence theorem [7] which states that at high energy, S-matrix elements involving the longitudinal gauge bosons can be computed to $\mathcal{O}(M_W/E)$ by replacing the external longitudinal gauge bosons by their corresponding Goldstone bosons which we will denote by $w^\pm, z$. This has the important consequence that longitudinal W particles must obey the same low energy theorems as the Goldstone bosons.

We will make the assumption that there are only three Goldstone bosons in the standard model. Since it must contain the $SU(2) \times U(1)$ of the standard model, there are two possibilities for the global symmetry:

$$\begin{aligned} SU(2) \times SU(2) &\rightarrow SU(2)_V \\ SU(2) \times U(1) &\rightarrow U(1) \quad . \end{aligned} \tag{9}$$

In the first case $\rho = 1$ and in the second ρ is a free parameter. Since experimentally ρ is quite close to 1, we will choose the first pattern of symmetry breaking with a custodial $SU(2)_V$. But now the pattern of the global symmetry breaking is EXACTLY the same as for pions. For the case of the electroweak symmetry breaking, the Goldstone fields play the role of the pions in QCD. To lowest order, the only difference is the strength of the coupling, v. We can hence make the analogy[8]:

$$\begin{aligned} f_\pi &\rightarrow v \\ \pi^\pm &\rightarrow w^\pm \\ \pi^0 &\rightarrow z \quad . \end{aligned} \tag{10}$$

The low energy Lagragian for the longitudinal gauge boson interactions is now the same as for pions with the above replacement. This is true no matter what the mechanism of electroweak symmetry breaking. It is straightforward to write down the most general $SU(2) \times SU(2)$ invariant chiral Lagrangian with at most four derivatives,

$$\begin{aligned} \mathcal{L} =& \frac{v^2}{4} Tr \partial_\mu \Sigma \partial^\mu \Sigma^\dagger + \alpha_1 Tr\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) Tr\left(\partial_\nu \Sigma \partial^\nu \Sigma^\dagger\right) \\ &+ \alpha_2 Tr\left(\partial_\mu \Sigma \partial_\nu \Sigma^\dagger\right) Tr\left(\partial^\mu \Sigma \partial^\nu \Sigma^\dagger\right) \quad . \end{aligned} \tag{11}$$

Now, however, $\Sigma = exp(2iw^a T^a/v)$ where $w^a = (w^\pm, z)$. This Lagrangian will be valid in the regime $M_W^2 \ll s \ll 4\pi v^2$, where s is the center-of-mass energy in the Goldstone boson system.

Since we are assuming the custodial $SU(2)_V$ symmetry of the Goldstone boson sector is unbroken, the gauge boson scattering amplitudes are related by crossing and $SU(2)_V$ symmetries and we have,

$$\begin{aligned} \mathcal{A}(W_L^+ W_L^- \rightarrow Z_L Z_L) &\equiv A(s,t,u) \\ \mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) &= A(s,t,u) + A(t,s,u) \\ \mathcal{A}(Z_L Z_L \rightarrow Z_L Z_L) &= A(s,t,u) + A(t,s,u) + A(u,t,s) \\ \mathcal{A}(W_L^\pm Z_L \rightarrow W_L^\pm Z_L) &= A(t,s,u) \\ \mathcal{A}(W_L^\pm W_L^\pm \rightarrow W_L^\pm W_L^\pm) &= A(t,s,u) + A(u,t,s) \quad . \end{aligned} \tag{12}$$

To study the sensitivity of the coefficients α_1 and α_2 to the nature of the spontaneous symmetry breaking, it is convenient to write,

$$\alpha_i(\mu) = \sum_R \alpha_i^R(\mu) + \alpha_i^B(\mu) \quad , \tag{13}$$

where μ is an arbitrary renormalization scale, α_i^R is the contribution from resonances (*cf.* Eqs. (7) and (8)), and α_i^B stands for non-resonant contributions to the scattering. The latter includes possible heavy fermion loops and Goldstone boson loops. (The contributions from heavy fermions are numerically unimportant). The underlying assumption of this approach is that different strongly interacting theories are characterized by different sets of resonances and that the coefficients α_i are dominated by the resonances and not by the non-resonant contributions from α_i^B.

The Lagrangian of Eq. (11) can be used to compute the Goldstone- Goldstone scattering amplitudes to order s^2. There are two types of contributions to this order. The first is a direct coupling that follows from the tree-level Lagrangian. The second is a one- loop correction that must be included at order s^2. The one loop contribution renormalizes the parameters α_1 and α_2. It also gives finite logarithmic corrections that cannot be absorbed into a redefinition of the couplings.

We begin by considering the values of α_1 and α_2 that one would obtain in a theory with a very massive Higgs boson, $M_H^2 \gg s$. In the perturbative regime of the standard model with a Higgs bosons one has loop diagrams with gauge and Higgs bosons that contribute terms of $\mathcal{O}(s^2/v^4)$ to the scattering amplitudes. These terms can be extracted from the low energy limit of the one loop calculation of vector boson scattering performed in Ref. [9]. We find the complete result to $\mathcal{O}(s^2/v^4)$,

$$\begin{aligned} A(s,t,u) =& \frac{s}{v^2} + \frac{4}{v^4}\Big(2\alpha_1(\mu)s^2 + \alpha_2(\mu)(t^2+u^2)\Big) \\ &+ \frac{1}{16\pi^2 v^4}\left\{-\frac{1}{12}\Big(3t^2+u^2-s^2\Big)\log\Big(-\frac{t}{\mu^2}\Big)\right. \\ &\left. -\frac{1}{12}\Big(3u^2+t^2-s^2\Big)\log\Big(-\frac{u}{\mu^2}\Big) - \frac{s^2}{2}\log\Big(-\frac{s}{\mu^2}\Big)\right\} \end{aligned} \tag{14}$$

with $\alpha_i(\mu) = \alpha_i^B(\mu) + \alpha_i^H$, α_i^H given by Eq. (7) and $\alpha_i^B(\mu)$ given by

$$\begin{aligned} \alpha_1^B(\mu) =& \frac{1}{4}\left[-\frac{1}{16\pi^2}\left(\frac{9\pi}{4\sqrt{3}} - \frac{37}{9}\right) - \frac{1}{48\pi^2}\log\left(\frac{\mu}{M_H}\right)\right] \\ \alpha_2^B(\mu) =& \frac{1}{4}\left[-\frac{1}{16\pi^2}\left(\frac{2}{9}\right) - \frac{2}{48\pi^2}\log\left(\frac{\mu}{M_H}\right)\right] \quad . \end{aligned} \tag{15}$$

The non-logarithmic terms depend on the regularization prescription, and as given above correspond to the mass renormalization of Ref. [9]. The numerical results are quite sensitive to the non-logarithmic terms which is one way to see why the procedure only works when the coefficients are dominated by the tree-level exchange of resonances.

To analyze the coefficients due to a model with dynamical symmetry breaking it will suffice to rescale the best fit results from low-energy pion physics, to mimic a "QCD-like" technicolor theory. By taking the best fit from Eq. (5) and doing this rescaling, Dobado and Herrero[11] find(with μ in GeV),

$$\begin{aligned} \alpha_1^\rho(\mu) =& \frac{1}{4}\left[-.011 - \frac{1}{48\pi^2}\log\left(\frac{f_\pi \mu}{v}\right)\right] \\ \alpha_2^\rho(\mu) =& \frac{1}{4}\left[-.0046 - \frac{2}{48\pi^2}\log\left(\frac{f_\pi \mu}{v}\right)\right] \end{aligned} \tag{16}$$

where we have assumed the gauge group of the technicolor theory is $SU(3)$. It is trivial to enlarge the gauge group by rescaling the argument of the logarithm in Eq. (16). It is interesting to note that the values obtained by rescaling QCD approximately satisfy the relation $\alpha_1 = -\alpha_2$ at $\mu = 1\ TeV$.

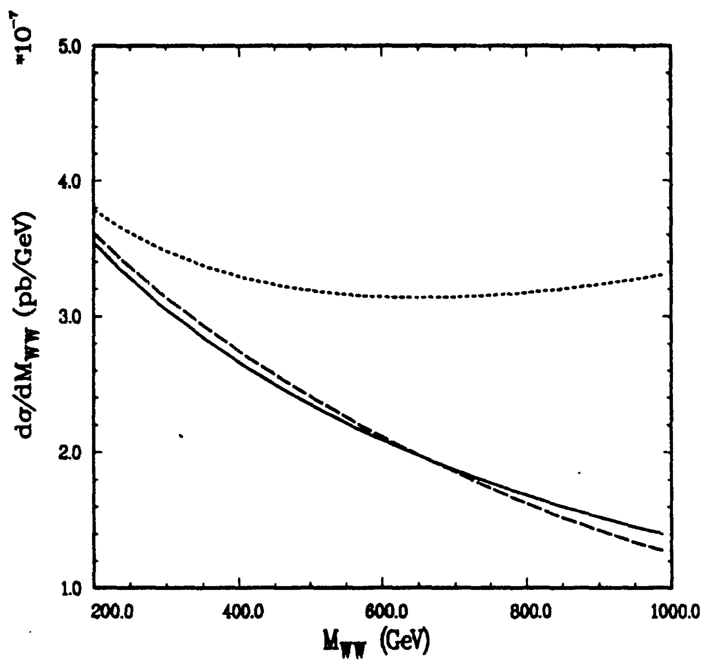

Fig. 2. *$d\sigma/dM_{WW}$ for $W_L^+W_L^-$ production from vector boson fusion in pp collisions at $\sqrt{S} =$ 40 TeV. The solid line is the low energy result, the dotted line includes the $\mathcal{O}(s^2)$ terms for a 2 TeV Higgs boson, and the dashed line has the $\mathcal{O}(s^2)$ terms for a "QCD"-like model.*

There is, however, an important difference between $\pi - \pi$ scattering and longitudinal gauge boson scattering. Whereas $\pi - \pi$ scattering can be studied directly, the experiments involving longitudinal gauge bosons are done with protons. To find the longitudinal gauge boson scattering cross sections, we must integrate the Goldstone boson cross sections with the distribution functions of the gauge bosons in the proton. Since these distribution functions are rapidly falling with increasing s, we do not see the rising cross sections as we did for $\pi - \pi$ scattering. In Fig. 2, we show the expectations for a 2-TeV standard model Higgs boson and for a "QCD"-like model using the $\mathcal{O}(s^2)$ terms in the chiral Lagrangian and including all loop effects. These cross sections use the relevant values for $\alpha_1(\mu)$ and $\alpha_2(\mu)$ for the two cases. These curves include the contributions of $w^\pm$ and z Goldstone boson loops to α_i^B and are explicitly independent of the renormalization scale μ. The curves for rescaled QCD are almost indistinguishable from those of the low energy theorems, while the 2 TeV Higgs case slightly increases the cross sections. It seems unlikely that it will be possible to distinguish between various models on the basis of such small differences. (Of course, the differences between the models would appear to be larger if we had extended the curves of Fig. 2 to higher energy. Unfortunately, in that regime partial wave unitarity is violated and the predictions of chiral perturbation theory are no longer valid.)

2.4. The Worst Case Scenario

The full range of low energy physics can be surveyed by scanning over α_1 and α_2, generating a set of models that are consistent for energies below the scale at which partial wave unitarity is violated. Since unitarity violation signals the onset of new physics (such as resonances), in this section we shall search for models in which the violation is pushed to the highest possible energy. Models of this sort represent a worst case scenario in the sense that there are no resonances and the only signal is perhaps a small change in the rate of longitudinal gauge boson production. A study of this type has been performed by Bagger, Dawson, and Valencia using the chiral Lagrangian to $\mathcal{O}(s^2)$[12].

Our practical definition of unitarity will be that for an isospin-I spin-J partial wave T_{IJ} we have $Re(T_{IJ} < \frac{1}{2})$. Using this condition, it is not hard to show that the leading contribution to the T_{00} partial wave violates unitarity at about $1.2\ TeV$. This bound, however, can be extended by a judicious choice of α_1 and α_2. Scanning the (α_1, α_2) parameter space, we find that the values,

$$\begin{aligned} \alpha_1 &= -.00167 \\ \alpha_2 &= .00147 \end{aligned} \tag{17}$$

measured at a renormalization scale $\mu = 1500\ GeV$, delay the unitarity breakdown as much as possible, until $2.0\ TeV$. It is interesting to note that it is not possible to push the scale of unitarity breakdown beyond this point.

To get a rough idea of the number of gauge boson events which such a model would predict we compute the number of longitudinally polarized gauge boson pairs produced from vector boson scattering in 1 SSC year with $|\ y_V\ | < 2.5$ and $1.5\ TeV\ <$

$M_{VV} < 2.0\, TeV$. In the W^+W^-, $W^\pm Z$, and ZZ channels we find 290, 670, and 90 events respectively. The background can be estimated by considering the lowest order cross sections for $q\bar{q} \to W^+W^-$, $W^\pm Z$, and ZZ. With the same restrictions, we found 790, 560, and 150 background events[12]. Naively, this looks hopeless. The one possibility would seem to be if the polarization of the outgoing gauge bosons could be measured since vector boson scattering produces predominantly longitudinal gauge bosons, while $q\bar{q}$ annihilation yields mostly transverse gauge bosons.

As has been emphasized by many authors [13], the W^+W^+ channel may be more fruitful. This is because the background arises at higher order and requires gluon exchange. If we require that the W^+ decay to e or μ, we estimate in Ref. [12] that there would be 4.6 leptonic signal events with 3.5 background events per year.

While one cannot take this model too seriously as a realistic alternative to the standard model, it is a logical possibility. The signal from such a model would be exceedingly hard to detect, but it does indicate the sensitivity that might be required to detect the mechanism for weak symmetry breaking at the SSC.

3. Gauging the the Chiral Lagrangian

We can extend our analysis to include transversely polarized gauge bosons and also the three and four gauge boson vertices by gauging the chiral Lagrangian with respect to $SU(2) \times U(1)$. This is simply done for the terms involving only Σ fields:

$$\begin{aligned}\mathcal{L} \to & \frac{v^2}{4} Tr\left(D_\mu \Sigma D^\mu \Sigma^\dagger\right) + \alpha_1 Tr\left(D_\mu \Sigma D^\mu \Sigma^\dagger\right) Tr\left(D_\nu \Sigma D^\nu \Sigma^\dagger\right) \\ & + \alpha_2 Tr\left(D_\mu \Sigma D_\nu \Sigma^\dagger\right) Tr\left(D^\mu \Sigma D^\nu \Sigma^\dagger\right) \quad ,\end{aligned} \tag{18}$$

where

$$D^\mu \Sigma = \partial^\mu \Sigma - i\left(g\vec{W}^\mu \cdot \vec{\tau} + g' B^\mu Y\right)\Sigma + i\Sigma g' B^\mu \left(Y + \tau_3\right) \quad , \tag{19}$$

where g and g' are the $SU(2)$ and $U(1)$ couplings respectively, τ_i are the $SU(2)$ generators normalized such that $Tr(\tau_i \tau_j) = \frac{1}{2}\delta_{ij}$, and Y is the $U(1)$ hypercharge normalized such that the electromagnetic charge Q is $Q = Y + \tau_3$. In the unitary gauge, $\Sigma = 1$, it is easy to see that the above Lagrangian gives the correct W mass, $M_W = \frac{1}{2} g v$. If we do not assume a custodial $SU(2)_V$ then there is an additional term of $\mathcal{O}(E^2)$,

$$\mathcal{L}' = \frac{g^2}{4} \beta_1 v^2 \left[Tr\left(\tau_3 \Sigma^\dagger D_\mu \Sigma\right)\right]^2 \quad . \tag{20}$$

Since this term violates the custodial $SU(2)_V$, the coefficient is related to the ρ parameter, $\rho = 1 - 2g^2\beta_1$.

To $\mathcal{O}(E^4)$ there are 13 CP conserving terms involving gauge fields and Goldstone bosons which were first written down by Longhitano for the $SU(2) \times U(1)$ case[14]. Most of these terms violate the custodial $SU(2)_V$ symmetry and are presumably small. In principle, these coefficients are all free parameters although one can attempt to calculate them in specific models. For example, Longhitano has computed the leading logarithmic contribution to the various α_i due to a heavy Higgs boson.

It is straightforward to use Longhitano's Lagrangian (and the Feynman rules he supplies) to compute the amplitudes for various longitudinal gauge boson scattering amplitudes. We will present two illustrative examples.

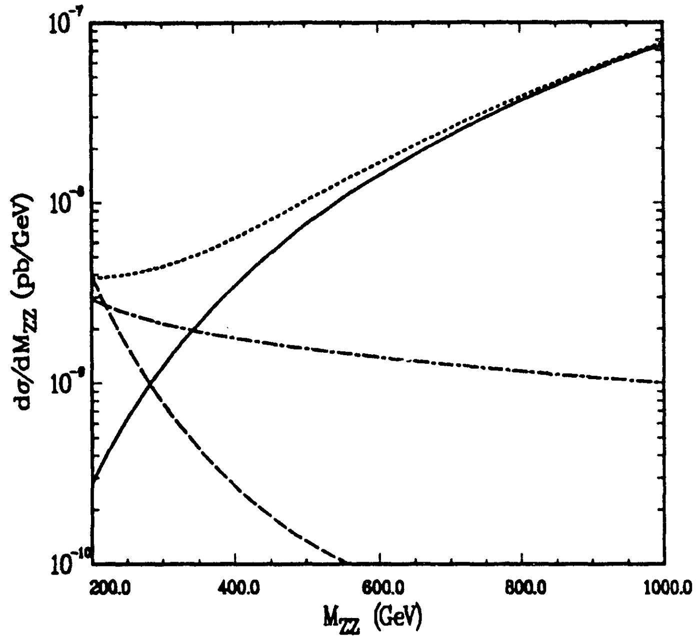

Fig. 3 *$d\sigma/dM_{ZZ}$ for producing Z pairs from the reaction $pp \to ZZ \to ZZ$. The dotted (dashed) line is the total rate for producing 2 longitudinal (transverse) Z's. The dot-dashed line is the rate for 1 transverse and 1 longitudinal Z. The solid line contains only the contribution from $Z_L Z_L \to Z_L Z_L$.*

In Fig. 3, we show the cross section for the vector boson fusion process $pp \to ZZ \to ZZ$ at the SSC, $\sqrt{S} = 40\ TeV$. This figure has α_1 and α_2 given in Eq. (17), with all the other coefficients of the chiral Lagrangian set to zero and shows the various polarization states. From the figure we can clearly see the cross section for producing two longitudinally polarized gauge bosons rising with increasing s as predicted by the low energy theorems (s is the center-of-mass energy in the boson-boson scattering system). The first lesson to be learned from this is that at high energy the only contribution which is numerically important is that where all the bosons are longitudinally polarized. This contribution of course can be obtained just from the Goldstone boson scattering amplitudes with the use of the equivalence theorem as in the previous section. So as far as longitudinal gauge boson production is concerned, gauging the chiral Lagrangian has absolutely no effect. Fig. 3 also shows the cross sections for producing one transverse and one longitudinal gauge boson and for producing two transversely polarized gauge bosons (we have summed over the different transverse polarizations.) This graph reinforces our intuition that only the longitudinal gauge bosons are important in vector boson scattering.

It is interesting to contrast Fig. 3 with what we would find in the standard model with a heavy Higgs boson. In that case, the longitudinal modes dominate only near the Higgs pole, while away from the pole the transverse modes dominate. This is of course because the longitudinal cross section no longer rises with s, but is cut off by the Higgs resonance. It is important to remember that Fig. 3 (and indeed all the results of chiral perturbation theory) are only valid below the scale of the lightest resonance.

The second example is W^+W^- pair production. This process occurs both through gauge boson fusion and through quark- antiquark annihilation. As seen in the previous section, in the vector boson scattering process it is only the longitudinal gauge bosons which are numerically important. Their interactions can be found using the electroweak equivalence theorem. However, W^+W^- pairs can also be produced by $q\bar{q}$ annihilation. The s-channel Z and γ exchange graphs involve the 3 gauge boson vertices and hence are affected by the $\mathcal{O}(E^4)$ terms of the chiral Lagrangian, as first pointed out by Holdom and Terning[15]. Unfortunately, the 3-gauge boson vertices depend on a different combinations of coefficients in the chiral Lagrangian than do the 4-gauge boson vertices. The process $q\bar{q} \to W^+W^-$ has a sensitive cancellation between diagrams. In the high energy limit in which we neglect terms of $\mathcal{O}(M_Z^2/s)$, the s-channel Z and γ exchange diagrams give an amplitude $\mathcal{A}_s \sim s/M_W^2$ while the t-channel diagram is proportional to $\mathcal{A}_t \sim -s/M_W^2$. When the diagrams are combined, the contributions which grow like s cancel and the amplitude respects perturbative unitarity. However, if we change the three gauge boson vertex from that of the standard model, this cancellation is no longer exact. At high energy, of course, the cancellation must be restored, but at intermediate energy it is possible to have a regime where the amplitude grows like s. Ahn *et. al.* call this phenomena "delayed unitarity cancellation"[16]. For large enough values of the $\mathcal{O}(E^4)$ coefficients in the chiral Lagrangian this effect can be numerically important.[15,17]

4. Beyond the Standard Model

It is in extentions of the standard model that the real power of the chiral Lagrangian approach becomes apparent[18]. Chiral Lagrangians provide a systematic and model-independent approach to electroweak symmetry breaking. If future colliders have insufficient energy to produce any new resonances which may be associated with the electroweak symmetry breaking, then the chiral Lagrangian may yield important information in the low energy region below the scale of the new resonances.

In this section, we will make the simplifying assumption that whatever physics breaks the electroweak symmetry, it has a global chiral symmetry group $G_L \times G_R$ which is spontaneously broken to the diagonal subgroup G. There are then $dim\ G$ Goldstone bosons Π^A which can be parameterized by the matrix $\Sigma = exp(2i\Pi^A T^A/v)$, where the T^A are now the generators of G (normalized such that $Tr(T^A T^B) = \frac{1}{2}\delta_{AB}$) and $v = 246\ GeV$. Of course, three of the Goldstone bosons are exactly massless and become the longitudinal components of the $W^\pm$ and Z gauge bosons. The remaining $dim\ G-3$ Goldstone bosons must acquire mass; they are known as pseudo-Goldstone bosons.

Once the embedding of the $SU(3)\times SU(2)\times U(1)$ standard model gauge group in the chiral symmetry group $G_L \times G_R$ is specified all of the various interactions can be computed. To define the embeddings, we first construct the matrices $X^\alpha = X^{\alpha A}T^A$, $X^a = X^{aA}T^A$ and $X = X^A T^A$ which generate $SU(3)$, $SU(2)$, and $U(1)$, respectively. The embedding is then defined by the covariant derivative,

$$D_\mu \Sigma = \partial_\mu \Sigma - ig_s G^\alpha_\mu [X^\alpha, \Sigma] - igW^a_\mu X^a \Sigma + ig' B_\mu \Sigma X \tag{21}$$

where G^α_μ is the $SU(3)$ color gauge field, and W^a_μ and B_μ are the $SU(2)\times U(1)$ gauge bosons. The gauge couplings explicitely break the global symmetry group $G_L \times G_R$.

The self-interactions of the Goldstone bosons and their interactions with the standard model gauge fields are given by the chiral Lagrangian. The Lagrangian is non-renormalizable, but it makes sense as an effective theory for energies $s \leq 4\pi v^2$. As before, to lowest order in the energy expansion, the effective Lagrangian is given by

$$\mathcal{L} = \frac{v^2}{4} Tr D_\mu \Sigma D^\mu \Sigma^\dagger \tag{22}$$

where the covariant derivative is now given in Eq. (21). The mass term for the Pseudo-Goldstone bosons is extremely model dependent, so we will neglect possible mass effects.

In a general model there will be pseudo-Goldstone bosons which are not color singlets. As an example we will compute the contribution of these colored particles to longitudinal vector boson production via gluon fusion. For concreteness we shall present our results in terms of the Farhi-Susskind[19] model which has $G = SU(8)$, so there are 63 Goldstone bosons. The embeddings are such that there is one color octet and two color triplets of weak -triplet pseudo Goldstone bosons.

We will compute the amplitude for producing two longitudinally polarized Z_L bosons or a $W^+_L W^-_L$ pair due to loops of colored pseudo-scalar bosons[20]. The relevant interactions can be found from Eqs. (21) and (22). We work in the high energy limit and use the equivalence theorem to replace the longitudinal vector bosons by the corresponding Goldstone bosons. Since the global symmetry group is always larger than $SU(2)\times SU(2)$, the effective Lagrangian has a custodial $SU(2)_V$ symmetry which guarantees that the two amplitudes are identical, $\mathcal{A}(gg \to Z_L Z_L) = \mathcal{A}(gg \to W^+_L W^-_L)$ The amplitude for $g^\alpha_\mu(q_1)g^\beta_\nu(q_2) \to z(p_1)z(p_2)$ is required by gauge invariance to have the form

$$\begin{aligned}\mathcal{A} =\Big\{ &\tilde{A}(s,t,u)\Big(-\frac{s}{2}g_{\mu\nu} + q_{2\mu}q_{1\nu}\Big)\\ &+ \tilde{B}(s,t,u)\Big(-\frac{ut}{2}g_{\mu\nu} - sp_{1\mu}p_{1\nu} - tq_{2\mu}p_{1\nu} - up_{1\mu}q_{1\nu}\Big)\Big\}\epsilon^\mu(q_1)\epsilon^\nu(q_2)\delta_{\alpha\beta}\quad .\end{aligned} \tag{23}$$

The amplitude vanishes at tree level. The one-loop amplitude is finite, and is given by

$$\begin{aligned}\tilde{A}(s,t,u) =&T(R)\Big(\frac{\alpha_s}{\pi v^2}\Big)\\ \tilde{B}(s,t,u) =&0\end{aligned} \tag{24}$$

where $T(R) = 1/2$ for each color triplet and $T(R) = 3$ for each color octet.

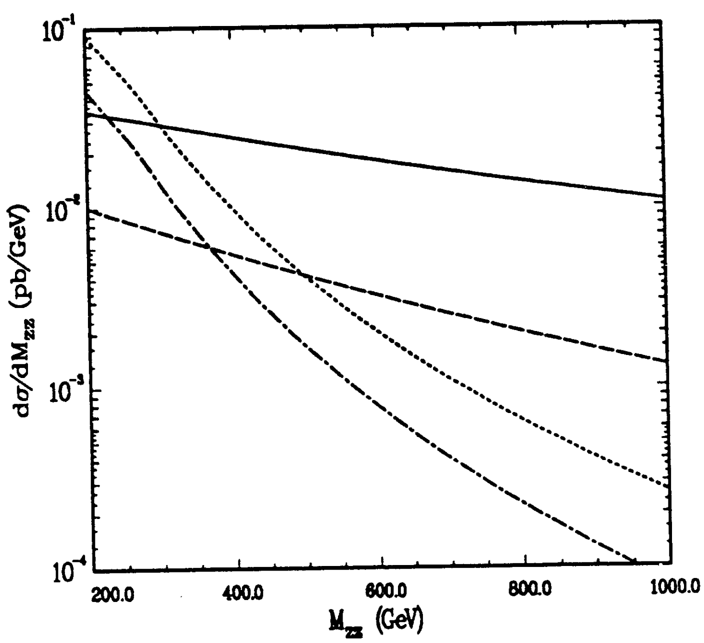

Fig. 4 $d\sigma/dM_{ZZ}$ *for pp collisions with* $|\, y_Z \,| < 2.5$. *The solid (dashed) curve gives the contribution to* $gg \to Z_L Z_L$ *from a loop of color octet, weak-triplet pseudo-Goldstone bosons at* $\sqrt{S} = 40\ TeV$ ($\sqrt{S} = 16\ TeV$). *The dotted (dot-dashed) line shows the contribution from* $q\bar{q} \to ZZ$ *at* $\sqrt{S} = 40\ TeV$ ($\sqrt{S} = 16\ TeV$).

In Fig. 4, we see that the contribution of colored pseudo-Goldstone bosons to Z boson pair production can be very large. In the Farhi- Susskind model, it dominates the standard model contribution from $q\bar{q} \to ZZ$ for $M_{ZZ} > 300\ GeV$. This results from the large octet color factor and from the fact that the gluon luminosity is much larger than that for quarks. We did not include a mass for the pseudo-Goldstone bosons, so the figure is valid only for $M_{ZZ} \geq 2\mu$, where μ is the pseudo-Goldstone boson mass. Below threshold, the curves are extremely sensitive to the precise form of the mass matrix. From this figure we see that at future hadron colliders such as the SSC or the LHC, the contributions from colored pseudo-scalars are potentially large and dominate the standard model contribution.

5. Conclusions and Outlook

We have studied high energy scattering of longitudinal gauge bosons using the equivalence theorem and chiral perturbation theory to $\mathcal{O}(s^2)$. A strongly interacting electroweak symmetry breaking sector could produce light resonances which should be seen by the next generation of colliders. It could also happen that there are no relatively light resonances (with mass less than $\sim 1.5\ TeV$). It is in this case that our analysis becomes relevant since the scattering amplitudes should show deviations from the low energy theorems that depend on the nature of electroweak symmetry breaking. However, if the strongly interacting theory that gives rise to the effective Lagrangian generates resonances that are too heavy (above $\sim 2.5\ TeV$), the deviations from the low energy theorems will be dominated by non-resonant scattering (like Goldstone boson loops) making this analysis less sensitive to the nature of electroweak symmetry breaking. The difference between models is quite small for vector boson scattering.

Unless one is able to measure the polarization of the final W and Z gauge bosons, the contribution from $q\bar{q}$ annihilation dominates over that from vector boson fusion. However, this contribution is also sensitive to higher order terms in the chiral Lagrangian. Much work still need to be done to perform a systematic analysis of the chiral Lagrangian and its affect on gauge boson pair production. However, the power of the approach is clear in that it is valid for any mechanism of spontaneous symmetry breaking.

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