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FINITE β AND NONLOCAL CALCULATION OF COLLISIONLESS AND DISSIPATIVE DRIFT INSTABILITIES

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1. INTRODUCTION

Collisionless and dissipative drift waves, driven by gradients in the plasma density and/or temperatures, are believed to dominate or at least influence the transport properties of a variety of plasma confinement devices. In a study begun in reference to transport in the Field Reversed Configuration (FRC), we have developed a theory of these waves in a high β ($\equiv$ plasma pressure/magnetic pressure) plasma, including the effect of perturbed flow in the direction of the plasma density.

This study was a natural extension of previous calculations; the $\beta = 1$ nature of the FRC makes a proper treatment of high β effects vital to an understanding of that device. In the course of this study we have obtained a comprehensive dispersion relation which shows clearly how the numerical and dissipative drift wave instabilities evolve in wavenumber as β increases. A major finding from this is that the effect of finite β begins to dominate long before $\beta \to 1$; the expansion parameter is $\beta f(k, a_i, K, \omega, L_n)$ where f can be substantially greater than 1, depending on the wavenumber of the wave parallel to the magnetic field (K), the wavenumber parallel to the particle drifts (k), the wave frequency (ω), the strength of the density gradient (L_n), and the ion gyroradius (a_i). The fact that finite β effects can onset for quite small β make this study applicable to confinement schemes such as tokamak in which $\beta \sim$ 1-10% in addition to the natural application to the FRC.

A second surprising fact from the study was that including finite β could result in a compressional flow in the direction of the density gradient, and also a perturbed electric field in that direction, which changes the perturbed orbits.

These effects prove to be lower order in ka_i than the $\beta = 0$ drift effects. That is, finite β effects set in for $\beta << 1$ for modes with $ka_i << 1$.

In this report we derive and quantify the results quoted in the above two paragraphs.

2. ELEMENTS WHICH PARAMETRIZE DRIFT WAVE BEHAVIOR

There are a number of plasma parameters and phenomena which can drive or alter drift wave instabilities. Despite the extensive literature, not all of these parameters and phenomena have been explored. The list of effects includes the following:

Plasma Gradient Drifts:	Drifts proportional to ∇nT are responsible for virtually all drift wave activity and are included in all theories.
Magnetic Gradient Drifts:	These include magnetic curvature effects. They have been modelled in a limited number of examples as a pseudogravity.
Finite Larmor Radius Effects:	In many cases, drift wave growth is of order $(ka_i)^2$. FLR effects are routinely included in drift wave calculations.
Finite Collisionality:	Particle collisions allow cross field transport, but also provide a dissipation which can drive negative energy waves unstable; they are included in calculations of dissipative drift waves.
Finite Beta Effects:	These have been largely ignored, with the notable exception of Ref. 1, which included the electromagnetic component (e-m) of the drift wave introduced by finite β.[1] A subsequent calculation[2] has questioned the existence of drift instability in the finite β regime. The present study obtains

a complete description of the transition from $\beta = 0$ drift instability to higher β instability.

Nonlocal Effects: In this category we combine effects which operate in the direction of the plasma gradients. With few exceptions, previous drift wave theories have been local, in the sense that variations of the perturbed quantities with x, where $n = n(x)$, were neglected, along with perturbed fields E_x. Finite β can introduce an E_x and B_z, through the e-m effect $dB_z/dt = (dE_x/dy - dE_y/dx)c$. In the present study we keep these effects, and show that there is a parameter range in which they can be strong.

3. NONLOCAL, FINITE β, ARBITRARY POLARIZATION DRIFT WAVES

We have derived a general expression for drift waves which retains the effects of finite β, variations in x, where $n_p = n_p(x)$, and electric fields also in the x-direction.

We consider a slab plasma as shown in Figure 1, where the plasma is infinite and uniform in the z-y plane, $B = B_z\hat{z}$ is the magnetic field, and n is the plasma density. The plasma can be described by the distribution function

$$f_o = f_M\,(v_z,\ v_\perp^2)\ g(\eta)$$

$$\eta \equiv v_y - \int B_z dx \quad .$$

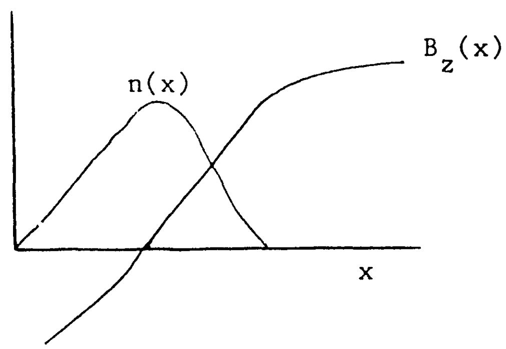

Figure 1. Plasma geometry for drift waves.

A perturbation $\delta E = E(x)e^{iky}e^{iKz}e^{i\omega t}$, $(i\omega/c)\delta B = -\nabla x \delta E$ is applied to the plasma and the response calculated from the Vlasov and Maxwell equations,

$$f_{1\alpha} = - \frac{q}{m} \int dt\ e^{ik \cdot r'} \left(E + \frac{v' \times \delta B}{c}\right) \cdot \nabla_v f_o \quad , \tag{1}$$

$$\nabla \cdot E = 4\pi \sum_\alpha q_\alpha \int f_{1\alpha}\, dv \quad , \tag{2}$$

$$\nabla \times \delta B = \frac{4\pi}{c} \sum_\alpha q_\alpha \int v f_1 dv \quad , \quad \text{(x and z components)} \tag{3}$$

$$\frac{i\omega}{c}\, \delta B = - \nabla \times E \quad ,$$

where the sums are over particle species α, and r', v' are the particle orbits in the unperturbed magnetic field. Making a small Larmor radius approximation ($a_i < L_n = (d\ln n/dx)^{-1}$) the integrand of (1) can be expanded

$$E(x) = E(x) + \frac{\partial E}{\partial x}(x' - x) + \frac{1}{2}\frac{\partial^2 E}{\partial x^2}(x' - x)^2 \quad , \tag{4}$$

and the orbit integrals carried out in the usual way. The result, after much algebra, is

$$a_i^2 \left(1 + \frac{\omega^x}{\omega}\right) \frac{\partial^2 E_y}{\partial x^2} - \frac{k^2 a_i^2 \omega_{ci}}{X\,\omega} \left[W_e - \frac{\omega^x}{\omega}(2 + W_e)\right] \frac{1}{k} \frac{dE_y}{dx} -$$

$$\left\{k^2 \lambda_D^2 + k^2 a_i^2 \left(1 + \frac{\omega^x}{\omega}\right) - \frac{\omega^x}{\omega}\left[1 + \frac{T_e}{T}\right] \frac{\beta}{X} \left[W_e - \frac{\omega^x}{\omega}(2 + W_e)\right]\right\} E_y =$$

$$\left\{K^2 \lambda_D^2 - \left[1 - \frac{\omega^x}{\omega} - \frac{i\nu}{\omega}(1 + W_e)\left(1 - \frac{\omega^x}{\omega}\right)\right] W_e + \right.$$

$$\left.\left[W_e - \frac{\omega^x}{\omega}(2 + W_e)\right] \left(1 - \frac{\omega^x}{\omega}\right) \frac{\beta}{X} W_e\right\} E_z \frac{k}{K} \qquad , \qquad (5)$$

$$\frac{k^2 a_i^2 \omega_{ci}}{X\,\omega} W_e\left(1 - \frac{\omega^x}{\omega}\right) \frac{1}{k} \frac{dE_y}{dx} + \left[\frac{k^2 \lambda_D^2 K^2 c^2}{\omega^2} - \frac{\beta W_e}{X} \frac{\omega^x}{\omega}\left(1 + \frac{T_e}{T_i}\right)\left(1 - \frac{\omega^x}{\omega}\right)\right] E_y =$$

$$\left\{\frac{k^2 \lambda_D^2 K^2 c^2}{\omega^2} - \frac{k^2 \lambda_D^2 K^2 c^2}{\omega^2} \frac{1}{k^2} \frac{d^2}{dx^2} + W_e\left(1 - \frac{\omega^x}{\omega}\right)\left[1 - \frac{i\nu}{\omega}(1 + W_e)\right] + \right.$$

$$\left. W_i\left(1 + \frac{\omega^x}{\omega}\right) - \frac{\beta W_e}{X}\left(1 - \frac{\omega^x}{\omega}\right)^2 \left[1 + \frac{i\nu}{\omega}(1 + W_e)\right]\right\} \frac{k}{K} E_z \qquad , \qquad (6)$$

where $\omega^x \equiv \frac{kT}{M\omega_{ci}} \frac{1}{f_o} \frac{\partial f_o}{\partial x}$, W_α is defined below in Eqs. (9)-(10), and $X \equiv 1 + \sum_\alpha \beta_\alpha \left(1 + \frac{\omega^x}{\omega}\right)_\alpha (1 + W_\alpha)$.

The perturbed field E_x has been expressed in terms of E_y, E_z by

$$E_x \simeq \frac{1}{X} \frac{\omega}{\omega^x} \frac{\beta}{ikL_n} \left\{ \left[\frac{2\omega^x}{\omega} + \frac{\omega^x}{\omega} \left(1 + \frac{\omega^x}{\omega}\right) L_n \frac{\partial}{\partial x} \right] E_y - W_e \left(1 - \frac{\omega^x}{\omega}\right) \frac{k}{K} E_z \right\} . \quad (7)$$

Clearly, E_x is not necessarily negligible for finite small β because the parameter which determines the generation of E_x from finite β is not β itself, but

$$\frac{\beta}{kL_n}$$

where $kL_n = ka_i(L_n/a_i)$ can be a small parameter even when $L_n > a_i$.

There is a variety of information contained in Eqs. (5)-(7). One possibility is to solve the differential equation (4th order) as an eigenvalue problem for ω. This requires a specific profile n(x), B(x). We do not attempt this solution, on the grounds that the result would be specific to the FRC and probably not worth the time such a device-specific calculation would require. Two more modest efforts are to:

- Delete all $\partial/\partial x$ and E_x effects and find $\omega(\omega^x, \beta, \nu, ka_i, KL_n)$.
- Write $\partial/\partial x = ik_x$ and find the effect of E_x and harmonic spatial structure in the x-direction.

We discuss these in the next two sections.

4. DRIFT WAVES FOR ARBITRARY β; $E_x = k_x = 0$

Setting $E_x = k_x = 0$, Eqs. (5)-(6) reduce to (ν is the electron collision frequency),

$$k^2 a_i^2 \frac{\omega}{\omega^x}\left(\frac{\omega}{\omega^x} + 1\right)\Bigg[1 + \frac{\beta W_e}{K^2 L_n^{\,2}}\left(\frac{\omega}{\omega^x} - 1\right)\left(\frac{\omega}{\omega^x} - \frac{i\nu}{\omega^x} - \frac{i\nu}{\omega^x} W_e\right) +$$

$$\frac{\beta}{K^2 L_n^{\,2}}\left(\frac{\omega}{\omega^x} + 1\right)\frac{\omega}{\omega^x} W_i\Bigg] = \left(\frac{\omega}{\omega^x} - 1\right)\left(\frac{\omega}{\omega^x} - \frac{i\nu}{\omega^x} - \frac{i\nu}{\omega^x} W_e\right) W_e +$$

$$\frac{\omega}{\omega^x}\left(\frac{\omega}{\omega^x} + 1\right) W_i \quad , \tag{8}$$

where ω^x is the ion drift frequency and the W's are the limit as $\epsilon \to 0$ of

$$W_e = -\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{v e^{-v^2}\, dv}{v + [(\omega - i\nu)/Kv_e] - i\epsilon} \quad , \tag{9}$$

$$W_i = -\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{v e^{-v^2}\, dv}{v + (\omega/Kv_i) - i\epsilon} \quad , \tag{10}$$

with v_e and v_i the thermal velocities.

In the limit $\beta = 0$, $\nu = 0$, and $Kv_i < \omega < Kv_e$, Eq. (8) gives

$$\omega = \omega^x + 2k^2a_i^2/W_e$$

$$= \omega^x \left(1 - 2k^2a_i^2 - i\sqrt{\pi}\, 2k^2a_i^2\omega^x/Kv_e\right) \quad , \tag{11}$$

which is the familiar collisionless branch of the drift wave instability spectrum. When $\beta = 0$, $\nu \neq 0$, $\omega > Kv_e$, Eq. (8) gives

$$\omega = \omega^x + 4k^2a_i^2 \left(\frac{\omega^x}{Kv_e}\right)^2 (\omega^x - i\nu) \quad , \tag{12}$$

which is the drift dissipative branch of the drift wave spectrum. So Eq. (8) extends both the collisionless and dissipative drift waves into the finite β regime.

We first consider the collisionless drift instability. Rewriting Eq. (8) gives

$$\omega = \omega^x + \frac{k^2a_i^2\,(\omega + \omega^x)}{W_e\left[1 - \dfrac{k^2a_i^2\beta}{K^2L_n{}^2}\,\dfrac{\omega}{\omega^x}\,\left(\dfrac{\omega}{\omega^x} + 1\right)\right]} - \frac{W_i\omega^x}{W_e}\left(\frac{\omega}{\omega^x} + 1\right) \quad . \tag{13}$$

This shows that the appropriate "finite-β" parameter is $k^2a_i^2\beta/K^2L_n{}^2$. When this parameter is small, Eq. (13) becomes

$$\omega \equiv \omega^x \left[1 - \frac{2k^2a_i^2 + i\sqrt{\pi}\, 2k^2a_i^2\omega^x/Kv_e}{1 - \frac{2k^2a_i^2}{K^2L_n{}^2}\beta} \right] \quad . \tag{14}$$

This shows the path that the drift instability follows in $k^2a_i^2$ and $K^2L_n{}^2$ parameter space as β increases. From Eq. (13) we see further that as $k^2a_i^2\beta/K^2L_n{}^2$ increases, $\omega/\omega^x < 1$ extends the β range of the collisionless drift instability.

Using a numerical method developed by N. T. Gladd,[3] we have solved Eq. (13) directly for increasing values of β. Figure 2 shows the development of the collisionless drift instability with β. Equation (13) essentially gives ω/ω^x in terms of three parameters, $k^2a_i^2$, $K^2L_n{}^2$, β. In reducing the result to $\mathrm{Im}\omega(\beta)$, we varied ka_i and KL_n as well as β in a manner consistent with the idea that $\omega/Kv_i > 1$ and $W_i < 2k^2a_i^2$ would constrain these parameters. Figure 2 is a qualitative representation of the maximum value of $\mathrm{Im}\omega_i$ for a given β. When $ka_i > 0.7$ or $KL_n > 0.233$, the collisionless drift instability disappeared for all β.

Next we turn to the dissipative drift wave (DDW) branch. Here

$$\frac{\omega}{\omega^x} \simeq 1 + 4k^2a_i^2 \left[\frac{\omega^x}{Kv_e}\right]^2 \left(\frac{\omega}{\omega^x} - \frac{i\nu}{\omega^x}\right) \left[1 - \frac{k^2a_i^2}{K^2L_n^2}\beta\frac{\omega}{\omega^x}\left(\frac{\omega}{\omega^x} + 1\right)\right]^{-1} , \tag{15}$$

where $(\omega^x - i\nu)/Kv_e \gtrsim 1$. This constraint is a severe limit on $k^2a_i^2\beta/K^2L_n{}^2 = (\omega^x/Kv_e)(M/m)^{1/2}\beta$, and finite β quickly forces the mode to $\omega \ll \omega^x$ or to the branch $\omega/\omega^x \cong -1$, both of which are stable. As a practical matter, this means that the DDW would appear unstable only for $\beta < (m_e/m_i)$. However, as β increases the frequency ω/ω^x decreases, until $(\omega - i\nu)/Kv_e < 1$. This mode remains unstable, with

$$\frac{\omega}{\omega^x} \simeq \frac{K^2 L_n{}^2}{\beta k^2 a_i^2} - i \left(\frac{K^2 L_n{}^2}{k^2 a_i^2 \beta}\right)^2 \frac{M}{m} \frac{K^2 L_n{}^2 \omega^{x2}}{\pi \nu^2} \left[\frac{ka_i}{KL_n} \sqrt{\frac{m}{M}} \frac{\nu^2}{\omega^{x2}}\right] \quad . \qquad (16)$$

Figure 3 shows the evolution of the DDW from the $\beta = 0$ limit to larger β, as given by Eq. (16). This drift wave branch is discussed in Ref. 1.

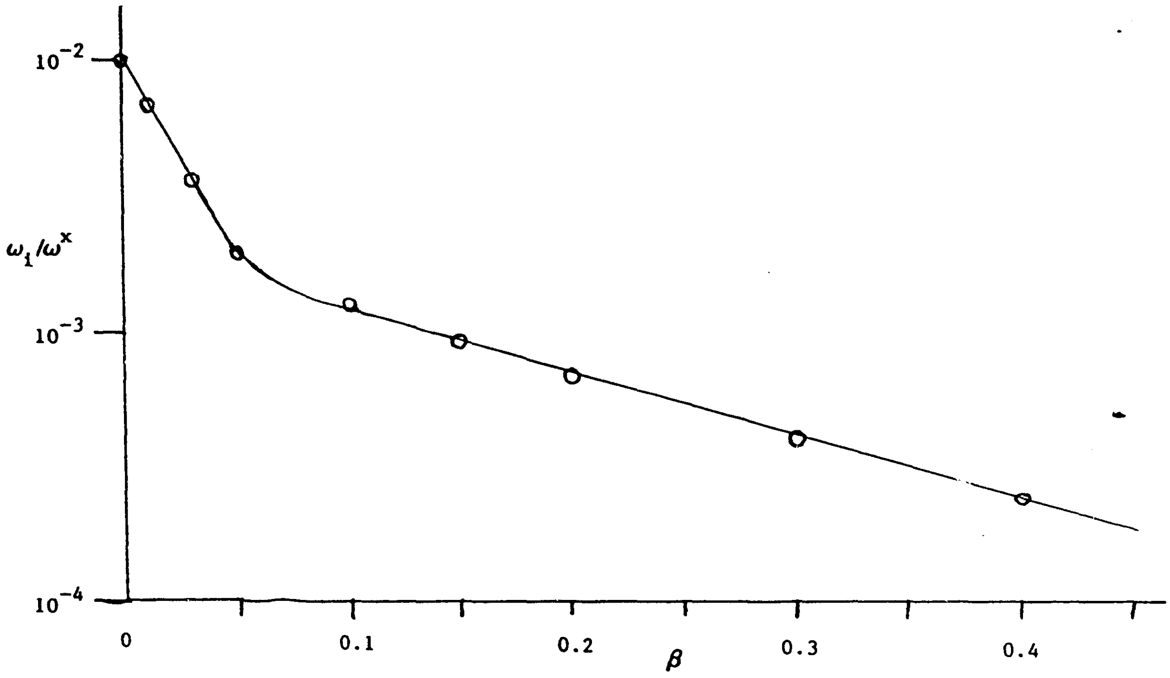

Figure 2. ω_i/ω^X vs. β for the collisionless drift instability.

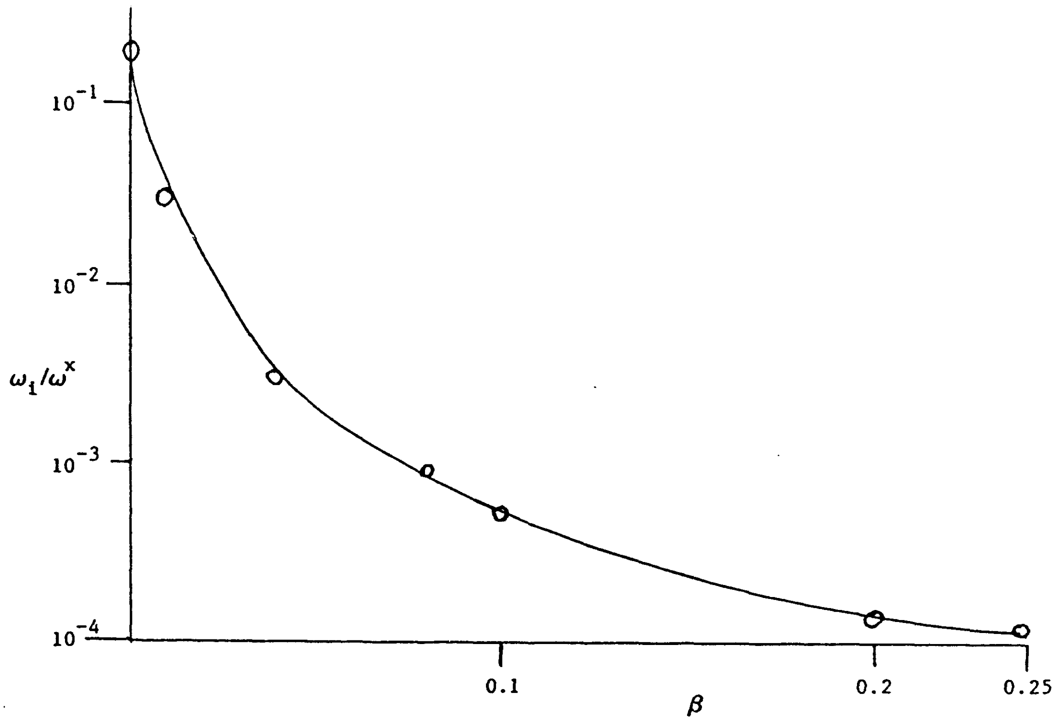

Figure 3. ω/ω^X vs. β for the DDW.

5. NONLOCAL EFFECTS AT $\beta \neq 0$

When the E_x and k_x terms are retained in the general derivation of high β drift waves, an interesting feature is apparent from Eqs. (15)-(16). A new parameter,

$$\frac{k_x}{k_y}\frac{\omega_{ci}}{\omega} \quad , \tag{17}$$

competes with terms of order 1. While we have not yet explored the consequence of this new parameter, the physics of its appearance is clear, as follows.

The perturbed charge density in the drift wave is determined by the perturbed velocities,

$$i(\omega + kV_D)\, n_1 = - V_{1x}\frac{dn_o}{dx} - n_o\nabla \cdot V_1 \quad . \tag{18}$$

Because for low frequency waves V_{1x} is the same for electrons and ions to order $k^2a_i^2$, and $dn_{oe}/dx = dn_{oi}/dx$, the RHS of Eq. (17) is $o(k^2a_i^2)$ when only E_y perturbations are included. The k_x and E_x terms produce a δB_z, which gives a $V_D \times \delta B_z$ contribution to δV_x which is opposite for electrons and ions. This leads to

$$\sum_{i,e} n_o\nabla \cdot V_1 = \frac{\omega}{\omega_{ci}} kE_y + \frac{kV_d}{\omega}\left(\frac{d}{dx} E_y - ikE_x\right) \quad ,$$

$$\sum_{i,e} V_x \cdot \frac{dn_o}{dx} = \frac{k^2a_i^2E_yc}{B} + \frac{i\omega}{\omega_c}\left[\frac{E_xc}{B} + \frac{kc}{\omega}\frac{E_xV_d}{B_o}\right] + \frac{ik_xV_d}{\omega_c}\frac{cE_y}{B} \quad ,$$

where $E_x \sim (\beta/kL_n)E_y = \beta(a_i/L_n)(1/ka_i)E_y$ shows that the contributions from k_x, E_x can be substantial even for $\beta << 1$. The implications of this ordering of β, ka_i, a_i/L_n will be explored in the continuation of the present Grant period.

ACKNOWLEDGMENT

We thank Dr. N. T. Gladd for providing us with a set of algorithms which enabled us to solve Eq. (8) for a variety of parameters, leading to Figures 2 and 3. He also helped us in the use of these algorithms, and it is a pleasure to acknowledge his expert assistance.

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