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CALIBRATION OF A NEUTRON LOG IN PARTIALLY SATURATED MEDIA IV: EFFECTS OF SONDE-WALL GAP

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Abstract

A gap between a neutron sonde and the wall of a borehole can have a significant effect on the observed count rate. We have experimentally determined this effect to be linear with gaps as large as 2.5 cm. The count rate is given by

$$N_N = K_0 + K_1 g$$

where K_0 is the count rate that would be observed at zero gap, and g is the gap. The parameters K_0 and K_1 are dependent on both water (ie. hydrogen) content and bulk density. In many situations failure to correct the count rate for this gap effect can result in a significant degradation in the accuracy of the water content calculated from the count rate.

In a dry borehole, K_1 is small at zero formation water content, and increases with formation water content. In a water-filled borehole, K_1 is large at zero formation water content, and tends to decrease with increasing formation water content, becoming zero, as of course it must, if the formation is pure water. The absolute value of K_1 increases with increasing density.

We have determined a representation for K_0 and K_1 from experimental data. This representation can be used to adjust the count rate at a given gap to equal its zero-gap value. The accuracy of the zero gap equation can then be recovered.

Introduction

The measurement of formation water content (hydrogen index) with the epithermal neutron log is now the regular practice at the Nevada Test Site (NTS). As a result there is a need for the regular calibration of neutron sondes. A new set of algorithms has been developed to calibrate sondes in both large and small holes and in both partially and fully saturated media. The data for sonde calibration are generated at the LLNL Hydrogen Content Test Facility at NTS. A set of computer codes implement these algorithms and allow for the calibration of a new sonde in a more or less automatic fashion.

The neutron sonde is mainly sensitive to the "hydrogen index". This index is equal to the volume fraction of fresh water that would contain the same amount of hydrogen. Although complex computer simulations are used to design sondes, in most cases empirical calibration curves are used to relate the neutron count rate in an individual sonde to the hydrogen index. These issues are discussed in [2]. Each sonde is thus a separate experiment and we will not seek to determine a universal set of calibration parameters.

The Hydrogen Content Test Facility

The Hydrogen Content Test Facility consists of 12 stations, where each station is made up of 15 aluminum cells. A cell is a box 2 m high with 30 cm square top and bottom faces. The cells are clamped together in a 5x3 array to create a station. A station provides a unique combination of water content and bulk density; the values of these combinations are given in Table 1 . A large (ie. > 2 m diameter) dry hole is simulated by placing the sonde on the external face of a center cell. A small (ie. < 30 cm) dry hole is simulated by removing an internal cell and clamping the sonde to a wall of one of the remaining cells. The small hole may be filled with water to simulate a water filled hole. A detailed description of this facility is given in [1]. The sonde is a commercial single-detector epithermal neutron sonde to which we have attached additional shielding in an attempt to reduce hole size effects. The sonde is suspended from a crane mounted in a high bay. Metal shims are placed between the sonde and the face of a cell to simulate the effect of a gap between the sonde and a borehole wall.

Table 1 Values of Hydrogen Index and Bulk Density for HCTF

station	I_H	ρ
1	0.000	0.89
2	0.000	2.10
3	0.000	2.63
4	0.125	0.99
5	0.126	1.51
6	0.127	2.32
7	0.364	1.26
8	0.308	2.05
9	0.367	2.58
10	0.480	1.90
11	0.730	1.60
12	0.960	1.07

Calibration at Zero Gap

We seek a relation between the water content I_H, bulk density ρ and neutron count rate N_N; later we will compensate for the effects of gap. Logging industry experience suggests the appropriate transformation for the count rate is $y = \log(N_N)$. We need to specify the dependence of I_H on ρ and y.

$$I_H = F(y, \rho, \beta)$$

where β is a vector of parameters to be determined from data. A set of desirable attributes for F are as follows.

- The function fits the data closely in the regions of interest

- F is linear in the unknown parameters
- F is parsimonious in the number of parameters and terms
- Only the values of the parameters change for different sondes
- F and the derivatives of F are smooth and "well behaved"

A quadratic form involving y, ρ and $y\rho$ has most of these desirable properties. A candidate for F is

$$I_H = \beta_0 + \beta_1 y + \beta_2 y^2 + \beta_3 \rho + \beta_4 \rho^2 + \beta_5 \rho y + \beta_6 \rho y^2 + \beta_7 y \rho^2 + \beta_8 \rho^2 y^2$$

This function is not parsimonious; it seems likely that some of the terms could be dropped and still maintain an adequate fit. In order to determine which terms to drop we use the "all possible subsets" approach, where the fit is evaluated for all possible inclusions and exclusions of the terms in the expression. If the constant term β_0 is always retained then there are $2^8-1=255$ subsets, each subset containing the constant and one or more terms. For each subset the parameters are determined by the minimization of the maximum absolute error-- the l-∞ norm-- rather than the least squares approach.

Large Dry Holes

Table 2 shows zero gap data on sonde 145, taken under conditions to simulate a large dry hole. Table 3 shows the results of minimax fits on all possible subsets. The number of terms (column 1) varies from 1 to 8, the number of possible subsets for the indicated number of terms is given in column 2.

Table 2 Zero Gap Count Rate Data
for a Typical Sonde (Large Dry Hole)

station	count rate
1	289.3333
2	313.3333
3	264.9167
4	177.1667
5	151.8333
6	130.0833
7	66.6667
8	70.7500
9	49.8333
10	44.9167
11	31.7500
12	28.5833

The smallest maximum absolute residual for all the possible subsets is given in column 3. For example when the number of terms is 4, there are 70 possible subsets of the 8 terms. The smallest maximum absolute residual of the 70 fits is .0342; recall for each of the 70 fits the values of the parameters are such as to minimize the maximum absolute residual.

Table 3 Results of All Possible Subsets Minimax Regressions

terms	subsets	error
1	8	0.1851
2	28	0.0885
3	56	0.0636
4	70	0.0342
5	56	0.0317
6	28	0.0266
7	8	0.0220
8	1	0.0220

Figure 1 is a plot of the maximum residual against the number of terms, the minimum residual of course occurs when the full 8 terms are used. However, we see from this plot that no more than 4 terms is appropriate because additional terms produce only a marginal improvement in the fit.

The variables associated with the residual 0.0342 corresponding to a 4 term model are: y, y^2, ρ, $y\rho$. The expression corresponding to this choice of variables is

$$I_H = \beta_0 + \beta_1 y + \beta_2 y^2 + \beta_3 \rho + \beta_4 y\rho$$

If a weighting factor is assigned to each station we can enhance the accuracy of the calibration in the plausible range $0.0 \le I_H \le 0.5$. Since the grain densities at NTS are never greater than 3.5 g/cc we constrain ρ such that $1.0 \le \rho \le 3.5(1-I_H) + I_H$. We have used minimax regression to discover the appropriate functional form. Now we will estimate the parameters by weighted least squares, where the weights are selected to optimize the accuracy of the calibration in the plausible range at the expense of the accuracy elsewhere.

Table 4 Zero Gap Calibration for a Typical Sonde (Large Dry Hole)

I_H	K_0	ρ	calc. I_H	error
0.	3492.61	0.89	0.0399	-0.0399
0.	3765.69	2.10	0.0014	-0.0014
0.	3222.25	2.63	0.0014	-0.0014
0.1250	2119.93	0.99	0.1208	0.0042
0.1260	1831.89	1.51	0.1282	-0.0022
0.1270	1575.11	2.32	0.1206	0.0064
0.3640	783.31	1.26	0.3768	-0.0128
0.3080	828.06	2.05	0.3027	0.0053
0.3670	581.91	2.58	0.3780	-0.0110
0.4800	530.32	1.90	0.4676	0.0124
0.7300	377.22	1.60	0.6334	0.0966
0.9600	323.99	1.07	0.7541	0.2059

Table 4 shows the results of a zero gap calibration for sonde 145 under conditions simulating a large dry hole. The raw count rate is replaced by the quantity K_0 which is in API units and differs slightly from the observed count at zero gap; K_0 is explained in more detail in the section dealing with the effects of gap. The fit is excellent in the plausible range below a water content of 0.63. There the water content differs by 14 percent from 0.73, the station value. The prediction at the highest water content,0.96, is worse; the error is 21 percent. However, there is little interest in measurements at this high end. The error of the zero gap fit is from 3 to 5 percent for water contents in the more plausible range.

Figure 2 is a plot of the calibration equation given by

$$I_H = 6.253 - 1.357y + 0.073y^2 - 0.275\rho + .030\rho y$$

The constraint $\rho < 3.5(1-I_H) + I_H$ is incorporated into the plot. This surface is of course valid only for the case of zero gap.

Small Dry Holes

Table 5 shows the results of a calibration for a small hole using the same function as the large hole case. We note the overall fit is superior, which suggests perhaps fewer terms would suffice.

Table 5 Zero Gap Calibration for a Typical Sonde (Small Dry Hole)

I_H	K_0	ρ	calc.I_H	error
0.	4509.93	0.89	0.0540	−0.0540
0.	7455.84	2.10	0.0007	−0.0007
0.	7414.70	2.63	0.0004	−0.0004
0.1250	3176.71	0.99	0.1228	0.0022
0.1260	3083.88	1.51	0.1266	−0.0006
0.1270	3050.78	2.32	0.1238	0.0032
0.3640	1454.70	1.26	0.3721	−0.0081
0.3080	1698.12	2.05	0.3035	0.0045
0.3670	1392.58	2.58	0.3749	−0.0079
0.4800	1134.47	1.90	0.4722	0.0078
0.7300	762.25	1.60	0.6768	0.0532
0.9600	651.57	1.07	0.7743	0.1857

Small Wet Holes

Table 6 shows the results of a calibration for a small water-filled hole, again using the same function as the large dry hole case.

Table 6 Zero Gap Calibration for a Typical Sonde (Small Wet Hole)

I_H	K_0	ρ	calc. I_H	error
0.	3506.39	0.89	0.0379	-0.0379
0.	4877.60	2.10	0.0014	-0.0014
0.	4289.26	2.63	0.0015	-0.0015
0.1250	1940.83	0.99	0.1232	0.0018
0.1260	1731.07	1.51	0.1219	0.0041
0.1270	1385.85	2.32	0.1214	0.0056
0.3640	629.87	1.26	0.3806	-0.0166
0.3080	645.66	2.05	0.3083	-0.0003
0.3670	425.50	2.58	0.3798	-0.0128
0.4800	409.73	1.90	0.4589	0.0211
0.7300	232.27	1.60	0.7023	0.0277
0.9600	202.27	1.07	0.8273	0.1327

Effects of Gap in Dry Holes

A gap between a neutron sonde and the wall of the borehole can have a significant effect on the observed count rate. This is illustrated in Figure 3 where we see API count rate plotted against gap for sonde 145, calibrated for large dry holes. At each station a line is fit to the data by linear least squares, these lines are superimposed on the data in Figure 3. We note the following: increasing gap causes the count to increase, a straight line is a good fit to the data at all stations, and both the slope and intercept of each line are different for each station. We can model the count rate N_N by the expression

$$N_N = K_0 + K_1 g$$

where K_0 is the count rate that would be observed at zero gap and g is the gap. The parameters K_0 and K_1 are dependent on both hydrogen index and bulk density. In fact the zero gap count rate $K_0(\rho, I_H)$ is just the inverse of our previously determined zero gap calibration function. If an observed count rate at a non-zero gap could be adjusted to the count rate that would be observed if the gap were zero the the hydrogen index could be determined by the previous zero gap formula. The adjustment is given by

$$K_0(\rho, I_H) = N_N - K_1(\rho, I_H) g$$

We need to find a reasonable representation for $K_1(\rho, I_H)$ from the available data. There are twelve values of K_1 and the associated quantities ρ and I_H. We have what is known as the *scattered data* problem, which is defined as follows. Construct a smooth (at least continuous first partial derivatives) bivariate function $F(x,y)$ which takes on prescribed values, $F(x_k, y_k) = f_k$, $k = 1, 2, \ldots$ n. The points (x_k, y_k) are not assumed to satisify any particular spacing conditions, hence the term *scattered*. The scattered data problem is much more difficult in two dimensions than in one dimension and many authors have addressed this problem see [3]. For dry holes we have chosen a technique known as Hardy's multiquadric, see [4].

Hardy's method is a *global* method, meaning the interpolant is dependent on all the data points, and addition or deletion of a data point or the change in one of the coordinates of a data point will propagate throughout the domain of definition. Global methods are based on the following concept. For each point in the domain (x_k, y_k) choose a function $G_k(x,y)$ and then determine coefficients A_k such that

$$F(x,y) = \sum_k A_k G_k(x,y)$$

interpolates the data. The function $G_k(x,y)$ is known as a *basis* function.

The basis function used for Hardy's Multiquadric is the upper hyperbolid, given by

$$G_k = ((x-x_k)^2 + (y-y_k)^2 + r^2)^{1/2}$$

where r is a parameter that determines the semi-axis of the hyperbola. We have found this technique works well for both large and small dry holes.

Table 7 Values of K_1 for a Typical Sonde (Large Dry Hole)

station	I_H	ρ	K_1
1	0.000	0.89	139.1366
2	0.000	2.10	91.4527
3	0.000	2.63	328.3218
4	0.125	0.99	481.0872
5	0.126	1.51	615.7723
6	0.127	2.32	597.7886
7	0.364	1.26	635.3373
8	0.308	2.05	661.4060
9	0.367	2.58	591.4852
10	0.480	1.90	599.8184
11	0.730	1.60	558.4402
12	0.960	1.07	416.3153

In Table 7 we have values of K_1 for sonde 145 calibrated for a large dry hole. The scattered points in the domain are the values of bulk density and hydrogen content for each station. Figure 4 shows a set of level curves for the interpolated $K_1(\rho, I_H)$ surface. This function is parameterized by the 12 values of A_k and r. It is not critical that this surface be extremely accurate because K_1 is only used to adjust the observed count to be approximately the count at zero gap. The interpolated value of K_1 need not be more accurate than the gap measurement in a borehole.

The hydrogen index is determined by a solution of the equation

$$I_H = \beta_0 + \beta_1 y + \beta_2 y^2 + \beta_3 \rho + \beta_4 y \rho$$

where

$$y = \ln(N_N - K_1(\rho, I_H) g)$$

The solution must be obtained by iteration since I_H appears on both sides of the equation. We have found Newton's method to work quite well.

Effects of Gap in Wet Holes

Figure 5 shows a plot of API count rate against gap for sonde 23 calibrated for small wet holes. We note an increasing gap causes the count rate to *decrease*, in contrast to the dry hole. The slope and intercept vary from station to station, and the effect is still linear in gap. Table 8 shows the values of the slope for each station.

Table 8 Values of K_1 for Typical Sonde (Small Wet Hole)

station	I_H	ρ	K_1
1	0.000	0.89	-1157.1679
2	0.000	2.10	-2794.2032
3	0.000	2.63	-2913.5449
4	0.125	0.99	-1244.8737
5	0.126	1.51	-1130.3120
6	0.127	2.32	-743.1316
7	0.364	1.26	-266.2252
8	0.308	2.05	-240.4899
9	0.367	2.58	-142.0189
10	0.480	1.90	-121.4028
11	0.730	1.60	-23.9282
12	0.960	1.07	-10.3671

For wet holes Hardy's Multiquadric fails to interpolate the slope parameter K_1 in some regions of the (I_H, ρ) domain. The slope changes so rapidly that the algorithm "overshoots" and the interpolent can become positive. A positive value for K_1 leads to serious difficulties in the iteration for the calculation for I_H. We use a modified version of another global interpolation technique known as Shepard's method. The basis function is based on the inverse distance. In general Shepard's method is less accurate than Hardy's, however for wet holes it avoids the overshoot problem. Figure 6 shows the level curves of a Shepard's interpolation of the slope data from sonde 23 calibrated for small wet holes. Note these level curves are not as smooth as those for dry holes.

Validation

A necessary condition for the validity of our algorithm is that it should approximate the hydrogen index of the stations given the count rate, gap, wet holes. The calculated hydrogen index does match the station's hydrogen index as well as the zero gap calibration; compare errors in Table 9 to the errors in table 6. The quantity K_0 is not exactly the measured API count rate at zero gap; its value is affected by the count rates at other gaps. Note that because the algorithm does not change the count associated with a zero gap, the counts in Table 9 with zero gap do not exactly match the values of K_0 as shown in Table 6.

Table 9 does not prove the procedure will accurately estimate water content in a borehole-- we would expect the algorithm to reproduce the input data-- but it does demonstrate that the interpolation algorithm has reasonable behavior.

Table 9 Validity Test of Count Adjustment Algorithm for a Typical Sonde Calibrated for Small Wet Holes

I_H	calc. I_H	error	count	gap	ρ	adj. count
0.	0.0356	-0.0356	3577.23	0.	0.89	3577.23
0.	0.0421	-0.0421	2948.31	0.3750	0.89	3384.07
0.	0.0361	-0.0361	2691.82	0.7500	0.89	3562.21
0.	0.0021	-0.0021	4769.49	0.	2.10	4769.49
0.	0.0010	-0.0010	4584.08	0.1250	2.10	4933.35
0.	0.0008	-0.0008	4267.84	0.2500	2.10	4966.39
0.	0.0007	-0.0007	3947.60	0.3750	2.10	4995.43
0.	0.0013	-0.0013	3497.00	0.5000	2.10	4894.07
0.	0.0023	-0.0023	2994.21	0.6250	2.10	4740.47
0.	0.0016	-0.0016	2737.49	0.7500	2.10	4833.07
0.	0.0021	-0.0021	2320.88	0.8750	2.10	4765.67
0.	0.0006	-0.0006	2205.86	1.0000	2.10	5000.06
0.	0.0012	-0.0012	4330.11	0.	2.63	4330.11
0.	0.0018	-0.0018	3135.79	0.3750	2.63	4228.34
0.	0.0013	-0.0013	2124.14	0.7500	2.63	4309.27
0.1250	0.1197	0.0053	1979.01	0.	0.99	1979.01
0.1250	0.1288	-0.0038	1416.01	0.3750	0.99	1882.80
0.1250	0.1214	0.0036	1026.99	0.7500	0.99	1960.57
0.1260	0.1209	0.0051	1741.63	0.	1.51	1741.63
0.1260	0.1236	0.0024	1291.26	0.3750	1.51	1715.12
0.1260	0.1214	0.0046	888.73	0.7500	1.51	1736.39
0.1270	0.1273	-0.0003	1339.10	0.	2.32	1339.10
0.1270	0.1147	0.0123	1348.49	0.1250	2.32	1441.90
0.1270	0.1132	0.0138	1268.48	0.2500	2.32	1455.60
0.1270	0.1223	0.0047	1099.09	0.3750	2.32	1377.98
0.1270	0.1270	-0.0000	969.53	0.5000	2.32	1341.10
0.1270	0.1249	0.0021	893.31	0.6250	2.32	1357.84
0.1270	0.1225	0.0045	818.80	0.7500	2.32	1376.55
0.1270	0.1255	0.0015	702.51	0.8750	2.32	1352.81
0.1270	0.1154	0.0116	689.24	1.0000	2.32	1436.12

Table 9 (contd.) Validity Test of Count Adjustment Algorithm for a Typical Sonde Calibrated for Small Wet Holes

I_H	calc. I_H	error	count	gap	ρ	adj. count
0.3640	0.3825	−0.0185	626.06	0.	1.26	626.06
0.3640	0.3773	−0.0133	536.44	0.3750	1.26	636.81
0.3640	0.3810	−0.0170	427.60	0.7500	1.26	628.98
0.3080	0.3064	0.0016	650.21	0.	2.05	650.21
0.3080	0.3116	−0.0036	547.66	0.3750	2.05	637.92
0.3080	0.3069	0.0011	468.57	0.7500	2.05	648.95
0.3670	0.3733	−0.0063	434.81	0.	2.58	434.81
0.3670	0.3898	−0.0228	355.95	0.3750	2.58	411.70
0.3670	0.3745	−0.0075	325.96	0.7500	2.58	433.07
0.4800	0.4532	0.0268	416.84	0.	1.90	416.84
0.4800	0.4480	0.0320	407.45	0.1250	1.90	423.54
0.4800	0.4621	0.0179	374.95	0.2500	1.90	405.83
0.4800	0.4647	0.0153	356.52	0.3750	1.90	402.61
0.4800	0.4653	0.0147	340.50	0.5000	1.90	401.89
0.4800	0.4676	0.0124	322.64	0.6250	1.90	399.13
0.4800	0.4639	0.0161	311.31	0.7500	1.90	403.63
0.4800	0.4372	0.0428	319.55	0.8750	1.90	437.89

Effects of Noise

When logging a borehole, count rate, gap, and density are always measured with some error. The count rate is noisy because it is derived from a Poisson random process; the smaller the integration time over which counts are accumulated the larger the variance of the measurement.

The noise in N_N, ρ, and g "feeds thru" the algorithm and causes two types of errors, bias and variability. Bias means the average of the estimated I_H over many repeat runs of the log is never equal to the true value of I_H no matter how many times the log is repeated. Variability describes the fluctuations in the estimated I_H over many repeat logs of the same borehole.

We can write the hydrogen index as

$$I_H = H(c,\rho,g)$$

where for notational convenience we let $c = N_N$. The hydrogen index is a function H of the three variables c,ρ,g. The variance of I_H is given by (see [5])

$$\mathrm{var}\{I_H\} = \left(\frac{\partial H}{\partial c}\right)^2 \mathrm{var}\{c\} + \left(\frac{\partial H}{\partial \rho}\right)^2 \mathrm{var}\{\rho\} + \left(\frac{\partial H}{\partial g}\right)^2 \mathrm{var}\{g\}$$

The partial derivatives are evaluated at the expected (average) values of c, ρ, g. If the variance of c, ρ, and g were all zero, the accuracy of the estimated hydrogen index would be limited only by model error as indicated in Tables 4, 5 and 6 and the count adjustment algorithm.

Even though we lack an explicit expression for the function H its partial derivatives can be evaluated as follows. For notational convenience let $W = I_H$. Then we can write

$$W = H(c,\rho,g) = F(c,\rho,g,W)$$

Where F is a function of 4 variables and F is given by

$$F(c,\rho,g,W) = \beta_0 + \beta_1 \ln(c-K_1 g) + \beta_2(\ln(c-K_1 g))^2 + \beta_3\rho + \beta_4\rho\ln(c-K_1 g)$$

The partial derivatives of H can now be expressed in terms of the partial derivatives of F as follows.

$$\frac{\partial H}{\partial c} = \frac{F_c}{1 - F_W}$$

$$\frac{\partial H}{\partial \rho} = \frac{F_\rho}{1 - F_W}$$

$$\frac{\partial H}{\partial g} = \frac{F_g}{1 - F_W}$$

The partial derivative of F with respect to variable g is given by

$$F_g = \frac{-K_1(\beta_1 + 2\beta_2 \ln(c-K_1 g) + \beta_4\rho)}{c-K_1 g}$$

The partial derivative of F with respect to variable W is given by

$$F_W = \frac{-(\beta_1 + 2\beta_2 \ln(c-K_1 g) + \beta_4\rho)}{c-K_1 g}\frac{\partial K_1}{\partial W} g$$

The last two equations together with the partial derivative of K_1 with respect to I_H are sufficient for the calculation of the uncertainty in the estimated hydrogen index caused by uncertainty in the gap measurement. For example suppose we have a borehole with $\rho = 2.05$, a one standard deviation uncertainty in the gap measurement of .05 (ie. varg$\{g\} = .05^2$) and an average gap of 0.25 in. . Under these assumptions the standard error of the estimated hydrogen index (square root of var$\{I_H\}$) is less than 0.01 for small wet holes.

Conclusions

Most neutron sondes do not have the capability to measure gap. If one can estimate the mean value of the gap (perhaps as a function of depth) and its standard deviation, one can use the method we have described to estimate the uncertainty in the measured water content caused by the absence of the gap correction. A caliper log might be used to estimate plausible values of gap. Thus a log user can decide whether a gap correction is important in a specific situation. For example, in a 30-cm dry hole, in a medium in which the hydrogen index is 0.125 (i.e., 12.5 water-filled porosity), a gap of 1.25 cm, if uncorrected, would cause an error (expressed as hydrogen index) of 0.03 in the measured water content and the same error if the true value is 0.37. In a water-filled hole, the error is 0.04 for a true value of 0.125, and 0.05 for a true value of 0.37.

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References

1. Hearst, J. R, et al. *Calibration of a Neutron Log in Partially Saturated Media, Part II: Error Analysis*, Trans. Soc. Prof. Well Log Analysts, 22nd Annual Logging Symposium, Mexico City.

2. Hearst, J. R, Nelson, P. H., *Well Logging for Physical Properties*, Mc Graw Hill (To be Published).

3. Schumaker, L. L. "Fitting Surfaces to Scattered Data", in *Approximation Theory II* pp 203-268, Lorentz, G. G., et al. eds. Academic Press, New York, 1966 .

4. Hardy,R. L., *The Application of Multiquadric Equations and Point Mass Anomaly Models to Crustal Movement Studies* NOAA TR NOS 76 NGS 11, 1978 .

5. Kendall, M., Stuart, A. *Advanced Theory of Statistics* Macmillan Publishing Co. 4th ed. 1976 .

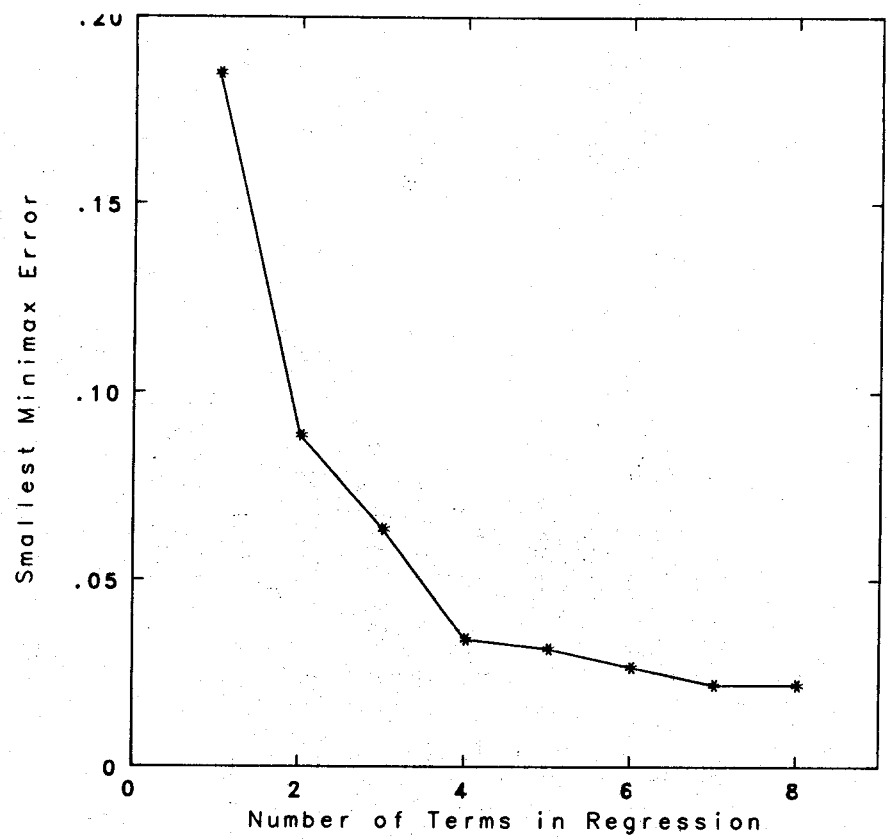

FIGURE 1 MINIMUM RESIDUAL VS NUMBER OF TERMS

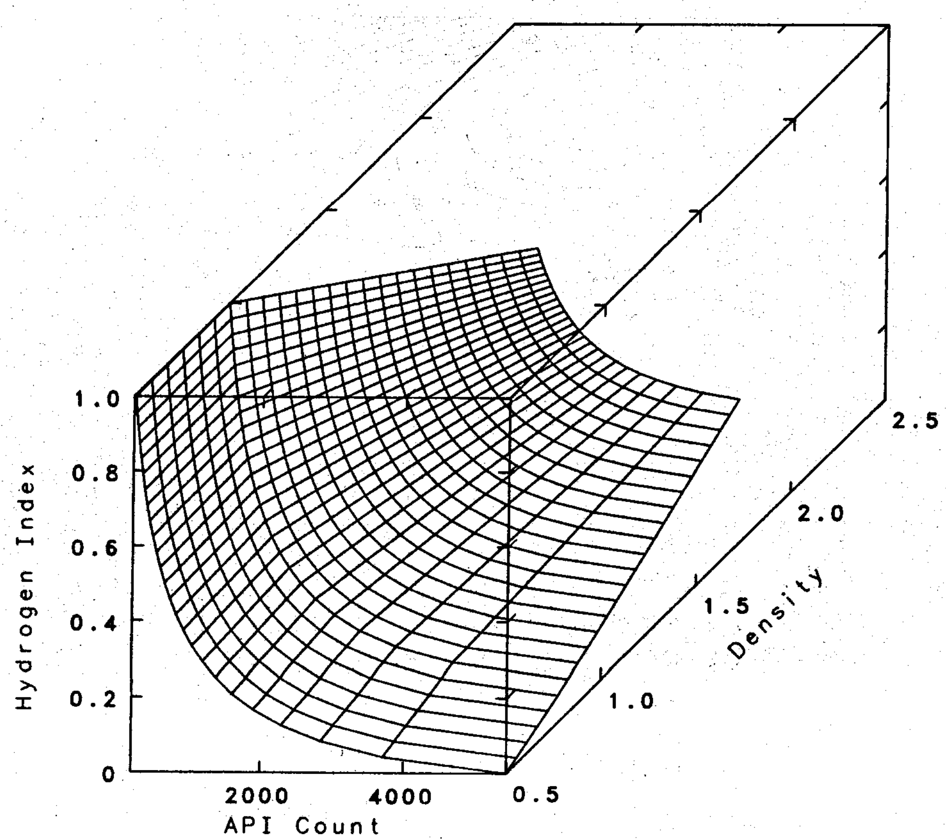

FIGURE 2 ZERO GAP CALIBRATION SURFACE — LARGE DRY HOLES

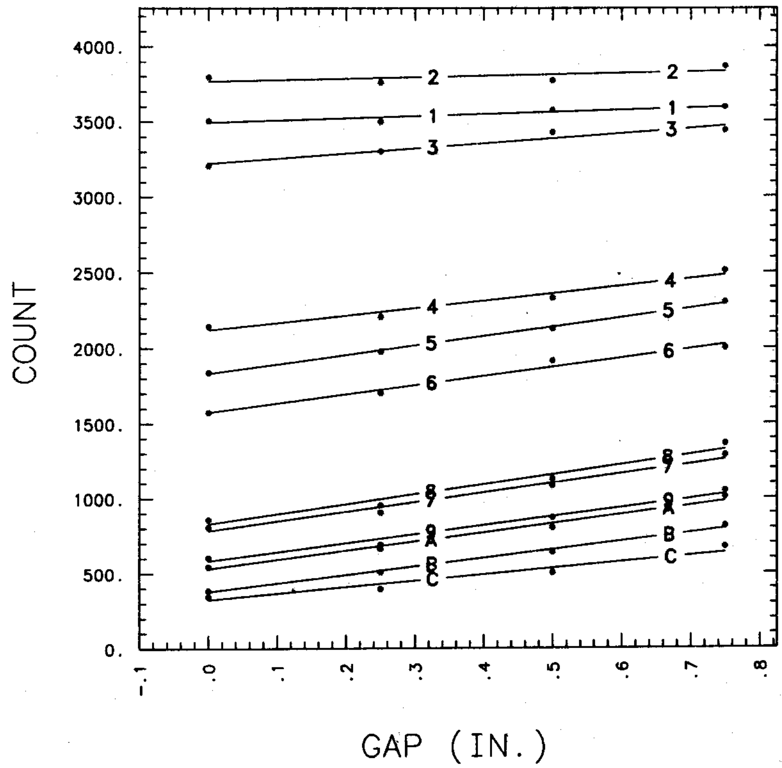

FIGURE 3 COUNT RATE VS GAP — LARGE DRY HOLES

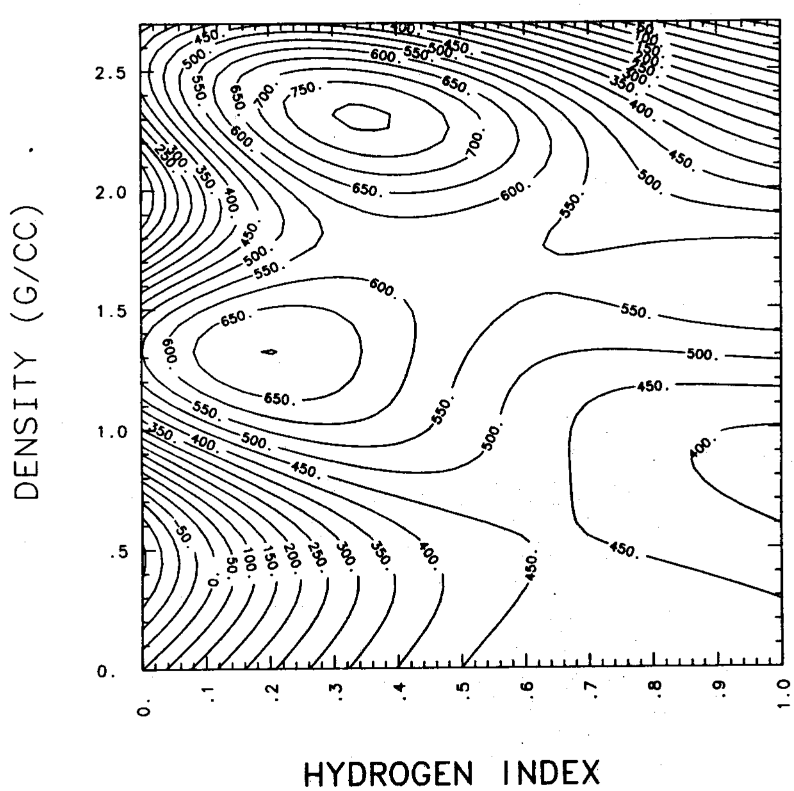

FIGURE 4 LEVEL CURVES FOR K1 INTERPOLATION — LARGE DRY HOLES

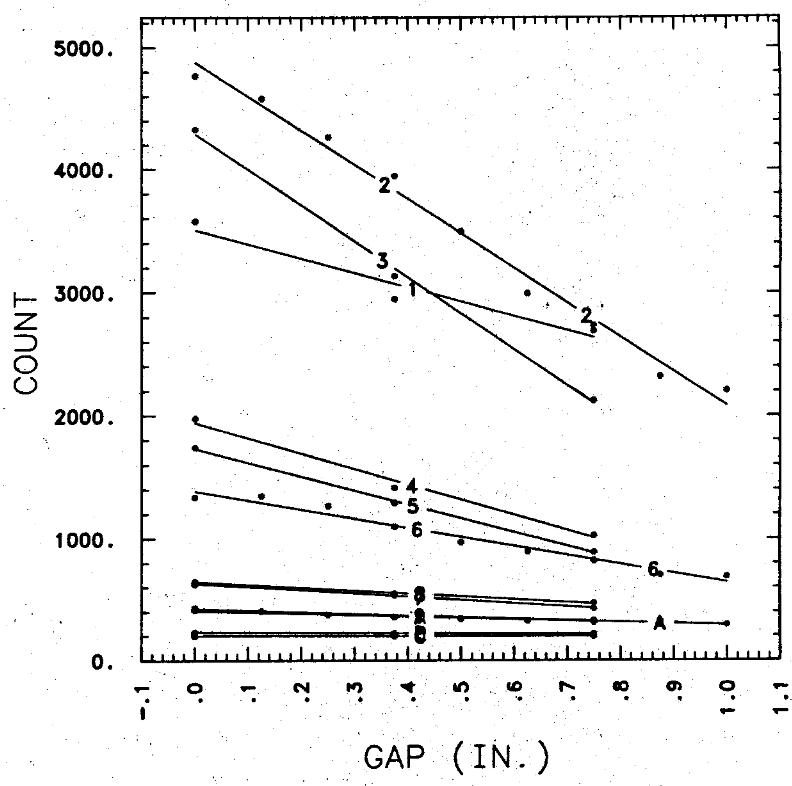

FIGURE 5 COUNT RATE VS GAP — SMALL WET HOLES

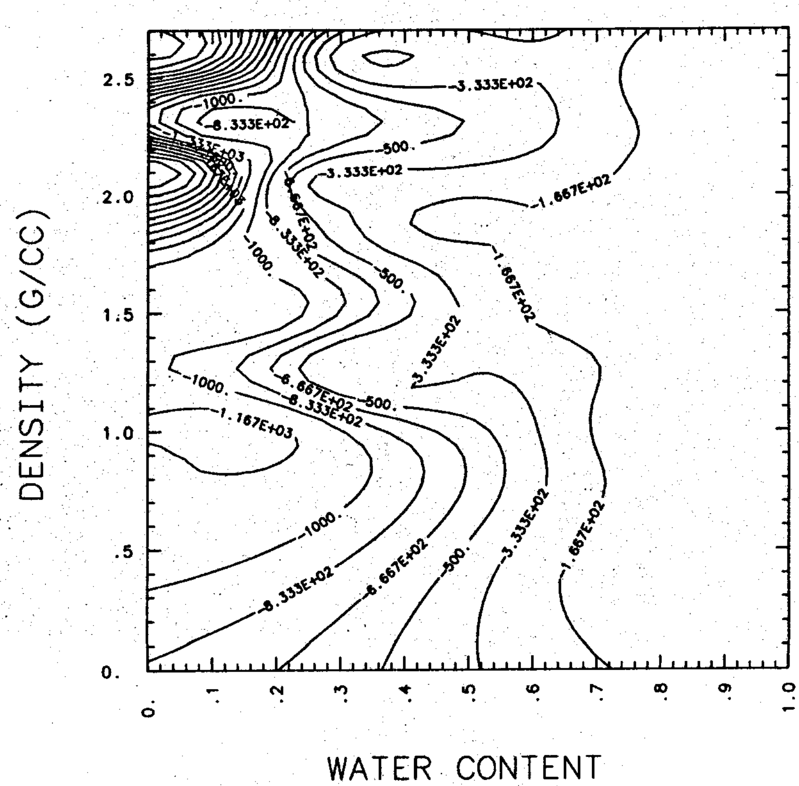

FIGURE 6 LEVEL CURVES FOR K1 INTERPOLATION — SMALL WET HOLES

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