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HADRON MASSES IN LATTICE GAUGE THEORIES; THE INCLUSION OF DYNAMICAL FERMIONS*

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Abstract

Hadron masses are calculated on an $8^3 \times 16$ lattice using four flavors of staggered fermion to generate the gauge configurations, but using Wilson fermions to calculate the hadron propagators. The identification of a value of the Wilson hopping parameter with the value of the bare quark mass used in the simulations is discussed.

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1. INTRODUCTION

There have been several talks contributed to this meeting presenting calculations of the hadron spectrum within the full QCD theory, both using staggered fermions and fermions in the Wilson formulation. The staggered formulation is believed to more faithfully represent the chiral properties of the theory, and since it has far fewer degrees of freedom, it is also perhaps easier to implement numerically. However the staggered formulation intrinsically describes four flavors of fermion. Flavor identification itself is also extremely delicate and, in general, may only be made through the use of non-local operators since the four Dirac spinors are constructed from the 16 one component staggered fermion fields at the corners of a 2^4 hypercube. During measurements of the hadron spectrum this drawback is most strongly felt in the contamination of the correlators constructed from local hadronic operators by opposite parity partners, a feature that makes fitting to the data so difficult.

In contrast the Wilson formulation affords perfectly straightforward flavor identification and can be used to describe an arbitrary number of fermion flavors. It is therefore more attractive both for measurements of the hadron spectrum and for the computation of, for example, the weak matrix elements and the matrix elements of the operators relevant to the calculation of hadronic wave functions. The cost, however, is the introduction of a term, characterised by the "r" parameter, that explicitly breaks chiral symmetry, even at vanishing quark mass.

The exercise that I shall present in this talk is an attempt at exploiting the advantages of both formulations, by employing staggered fermions as the dynamical variables used in the generation of the gauge configurations, but calculating the hadronic correlators using fermions in the Wilson formulation. The layout of the rest of this talk is as follows. In section 2 I shall give brief details of the methods used in the calculation. Section 3 will contain our results for the hadron mass spectrum for a variety of values of the bare quark mass and coupling. Section 4 will contain some conclusions and a strategy for further studies.

2. THE GENERATION OF GAUGE CONFIGURATIONS AND CALCULATION OF HADRON PROPAGATORS

I shall begin this section by addressing the question of how we propose identifying a value of the Wilson hopping parameter K with the value of the bare quark mass employed in the generation of the gauge configurations. That there is no straightforward relation between the two at currently accessible values of the coupling and bare quark mass is clear from the different values of the lattice spacing extracted using the ρ mass value in the two formulations.[1] The prescription that we shall adopt here is to tune the hopping parameter so that the pion mass (in units of the lattice spacing) that we obtain using Wilson fermions is the same as that obtained in the simulations purely employing staggered fermions. Our reasons for adopting this procedure are the following. Firstly, to the extent that chiral symmetry is indeed "exact" in nature the physical world corresponds to $m_\pi = 0$; we want to ensure that this condition is satisfied at zero bare quark mass in our calculations. Secondly it is the pion mass that is most sensitive to changes in the value of the bare quark mass or hopping parameter (recall $m_\pi^2 \propto m_q$ whereas $m_\rho \propto m_q$). Finally the pion propagator is the most accurately determined since it is just the sum of the squares of the components of the quark propagator, and indeed bounds all the other hadron propagators from above.

Results for the hadron spectrum purely using staggered fermions, and details of the generation of the gauge configurations, are contained elsewhere.[2] For the purposes of this talk, though, it should be noted that an $8^3 \times 16$ lattice was employed, with periodic boundary conditions in the spatial directions and antiperiodic in the time direction. The configurations were generated using the hybrid algorithm, with four flavors of staggered fermion. The Wilson fermion propagators were calculated with the source at the first time slice using a preconditioned conjugate residual algorithm.[3] Results were obtained at three values of the bare quark mass, $m = 0.1$, 0.05 and 0.025, in units of the lattice spacing. The choice of coupling β is fairly critical in that we want both to be in the scaling region, and to be on the confining side of the finite temperature phase transition.

Estimates for the masses were extracted by performing a χ^2 fit to the data, with the errors being obtained using a "Jackknife" technique[4]. Because of the relatively small lattice size it was found necessary to include two masses in the fitting function for all of the particles under consideration. For each of the π, ρ and proton, mass measurements were made using two operators, defined as follows

$$\pi_1 = \bar{\psi}\gamma_5\psi; \quad \pi_2 = \bar{\psi}\gamma_4\gamma_5\psi$$

$$\rho_1 = \bar{\psi}\gamma_i\psi; \quad \rho_2 = \bar{\psi}\gamma_4\gamma_i\psi \qquad (i = 1, 2, 3)$$

$$[\text{proton}_1]_\tau = \left[u^a C\gamma_R d^b\right]\left[\gamma_L u^c\right]_\tau \epsilon^{abc}$$

$$[\text{proton}_2]_\tau = \left[u^a C\gamma_R d^b\right]\left[\gamma_R u^c\right]_\tau \epsilon^{abc}.$$

3. RESULTS FOR THE HADRON SPECTRUM

As stated above, our aim in determining K was to ensure that the pion mass obtained from the operator π_1 with propagators constructed from Wilson fermions agreed (to within errors) with that obtained purely using staggered fermions. In practice we obtained mass estimates at each value of m and β over a range of values of K for a limited number of configurations. It was then possible to tune to the correct value by noting that m_π^2 is approximately linear in K^{-1} at fixed bare quark mass m. Though this procedure proved relatively economical in computer time at the higher two masses, the quality of the data at $m = 0.025$ was such that a considerable number of configurations had to be studied. However it is possible to employ a considerably less stringent termination criterion in the conjugate residual algorithm if it is only the pion correlator that is to be calculated than if all the particle correlators are to be constructed. Table 1 shows the various parameters that were used in the spectrum calculation, together with the corresponding pion masses.

I shall now turn to the determination of the remaining particle masses, beginning with some general comments. Firstly it should be noted that most of the fits were obtained by starting at the third or fourth time slice; to include earlier time slices would, of course, improve the statistical errors on the masses, but at the cost of greater systematic errors arising from the finite size of the lattice. Secondly even though a very large number of configurations were analysed the errors on the masses at $m = 0.025$ are quite large; this is unfortunately just a result of the considerable noise at time slices distant from the source. Table 2 shows the mass measurements obtained at the three values of the bare quark mass.

Despite the large uncertainties in the masses at the lowest value of the quark mass, the consistency in the fits for the π, ρ and proton using the two different operators is encouraging, as is the substantial Δ to proton mass splitting. The fits obtained for the A_1 and the scalar at $m = 0.05$ and $m = 0.025$ unfortunately must be considered unreliable as the signals are overwhelmed by the noise at time slices more distant than the sixth; the result is likely to be a considerable overestimation of the masses. Perhaps the best benchmark for mass spectrum calculations is the plot of m_p/m_ρ against m_π/m_ρ, which is shown in Fig. 1. Though the trend of the measurements is disappointing, it should be emphasized that we are indeed working on a small lattice, and that the finite size effects are greatest on the proton, particularly at the lowest value of the bare quark mass.

4. CONCLUSIONS

The method that has been outlined above should provide an economical way of including the effects of fermion loops in the Wilson formulation of lattice fermions. It is clear that the present results are subject to substantial systematic errors arising from the finite size of the lattice, and one of the first tasks should be to repeat the exercise on a somewhat larger lattice. We also aim to apply this technique to take account of the effects of fermion loops in the matrix elements of the operators relevant to the calculation of hadronic wave functions.[5]

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TABLE 1

The various parameters used in the simulations are listed. Columns 5 and 6 show the pion mass obtained purely using staggered fermions and using a mixture of staggered and Wilson fermions respectively, as discussed in the text.

m_s	β	# of Configurations	K	$m_{\pi\|sf}$	$m_{\pi\|wf}$
0.1	5.4	21	0.1525	0.837(2)	0.837(10)
0.05	5.2	25	0.1646	0.613(2)	0.613(20)
0.025	5.15	57	0.1681	0.454(2)	0.439(30)

TABLE 2

The measurements of the particle masses at each of the three values of the bare quark mass are shown.

Particle	$m = 0.1$	$m = 0.05$	$m = 0.025$
π_1	0.84(1)	0.61(2)	0.44(3)
π_2	0.82(2)	0.61(2)	0.50(6)
ρ_1	0.90(1)	0.74(5)	0.64(4)
ρ_2	0.90(2)	0.75(4)	0.65(12)
A_1	0.97(12)	1.34(17)	1.23(12)
Scalar	1.01(12)	1.48(22)	1.50(26)
proton_1	1.48(5)	1.31(6)	1.23(18)
proton_2	1.47(7)	1.31(10)	1.20(22)
Δ	1.53(5)	1.51(4)	1.45(12)

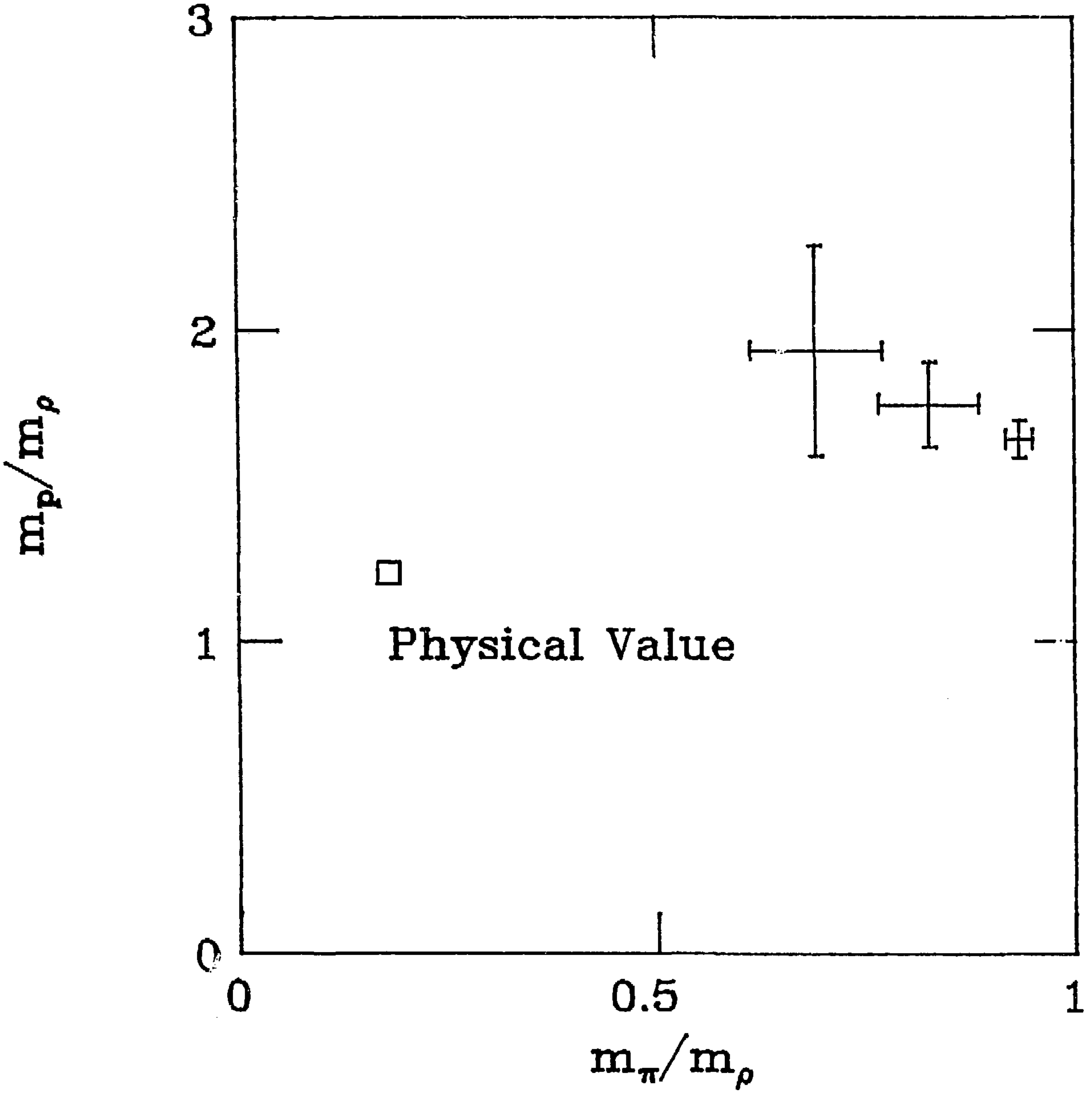

Fig. 1: The ratios of m_p/m_ρ against m_π/m_ρ shown at $m = 0.1$, $m = 0.05$ and $m = 0.025$.