



# Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

## Materials & Molecular Research Division

LBL--19653

DE86 001986

Presented at the 3rd International Conference  
on Superconducting Quantum Devices, Berlin,  
Federal Republic of Germany, June 25-28, 1985

APPLICATION OF A DC SQUID TO RF AMPLIFICATION:  
NQR

C. Hilbert, J. Clarke, T. Sleator, and E.L. Hahn

May 1985



This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.

Claude Hilbert and John Clarke

Department of Physics, University of California and  
Materials and Molecular Research Division  
Lawrence Berkeley Laboratory  
Berkeley, California, USA 94720

Tycho Sleator and Erwin L. Hahn

Department of Physics, University of California  
Berkeley, California, USA 94720

Superconducting QUantum Interference Devices (SQUIDs) have been used for more than a decade for the detection of magnetic resonance.<sup>1-10</sup> Until recently, these devices had mostly been confined to operation in the audiofrequency range, so that experiments have been restricted to measurements of resonance at low frequencies,<sup>9-10</sup> or of changes in the static susceptibility of a sample induced by rf irradiation at the resonant frequency.<sup>1-3</sup> However, the recent extension<sup>11</sup> of the operating range of low noise dc SQUIDs to radiofrequencies (rf) allows one to detect magnetic resonance directly at frequencies up to several hundred megahertz. In this paper, we begin by summarizing the properties of dc SQUIDs as tuned rf amplifiers. We then describe first, the development of a SQUID system for the detection of pulsed nuclear quadrupole resonance<sup>12</sup> (NQR) at about 30 MHz and second, a novel technique for observing magnetic resonance in the absence of any externally applied rf fields.

#### dc SQUIDs as Tuned rf Amplifiers

Figure 1 shows a dc SQUID with loop inductance  $L$  coupled to an input circuit consisting of a signal source  $V_i(t)$  of resistance  $R_i$  in series with the input coil of inductance  $L_i$ , a capacitance  $C_i$  and a pick-up coil of inductance  $L_p$ . In our application to magnetic resonance detection,  $V_i(t)$  is the signal induced by the rotat-

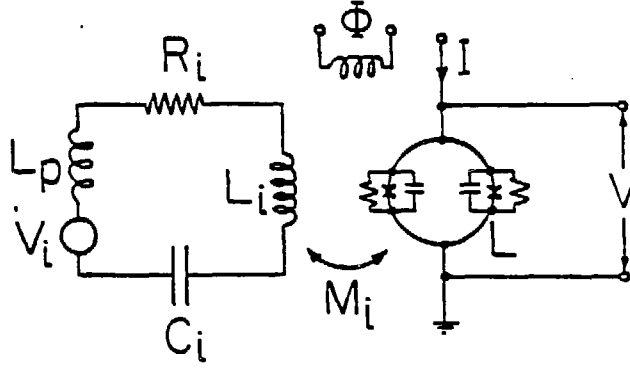


Fig. 1: Configuration of tuned amplifier based on a dc SQUID.

ing magnetization of a sample located inside the pick-up coil. The mutual inductance between the SQUID and the input coil is  $M_i$ , so that the coupling coefficient is  $\alpha^2 = M_i^2 / L L_i$ . We also define an effective coupling coefficient between the SQUID and the input circuit:  $\alpha_e^2 = M_i^2 / L(L_i + L_p)$ . The SQUID is biased with a current  $I$  and a flux  $\Phi$  so that the flux-to-voltage transfer function,  $V_\Phi \equiv \partial V / \partial \Phi$ , is a maximum. In the limit  $Q \gg 1$ , where  $Q$  is the quality factor of the input circuit, it can be shown<sup>11</sup> that the condition for the optimum noise temperature of a dc SQUID with  $\beta \equiv 2LI_0 / \Phi_0 = 1$  is  $R_i = \alpha^2 \omega_0 L_i$  or

$$Q \alpha_e^2 = 1. \quad (1)$$

Here,  $f_0 = \omega_0 / 2\pi$  is the resonant frequency of the input circuit. This condition yields an optimum noise temperature

$$T_N(f_0) = 7\omega_0 L T / R, \quad (2)$$

a dynamic range in the bandwidth of the tuned circuit

$$D = 6Q / (T / 1K), \quad (3)$$

and a power gain at resonance

$$G(f_0) = V_\Phi / \omega_0. \quad (4)$$

In practice, the inductive and capacitive coupling of the SQUID to the input circuit produces changes in the impedance<sup>13</sup>

$$\Delta R_i = -\omega_0^2 M_i V_\phi C_p L_i \quad (5)$$

and

$$\Delta L_i = -\alpha^2 (L/\mathcal{L}) L_i, \quad (6)$$

where  $C_p$  is the parasitic capacitance between the SQUID loop and the input coil, and  $\mathcal{L}$  is the dynamic input inductance of the SQUID. These changes must be taken into account in the design of the amplifier.

The dc SQUIDS used in this work are planar, thin-film devices<sup>11</sup> coupled to a spiral input coil with a coupling coefficient  $\alpha^2 = 0.6$ . Their measured performance at 100 MHz and at a bath temperature of 4.2K is typically:  $T_N = 1.7K$ ,  $D = 1.5Q$  and  $G = 20$  dB. The input circuit can be tuned to any frequency from about 1 MHz to 300 MHz.

#### dc-SQUID System for Detection of Pulsed NQR

The most widely used technique for the detection of magnetic resonance involves the study of the free induction decay of nuclear signals after the application of a rf-pulse to the sample.<sup>14</sup> Figure 2 is a schematic of our SQUID-based system for the detection of NQR. Rf pulses are amplified and coupled into a cold transmitter coil via an impedance matching circuit. The sample is located inside a pick-up coil that is connected in series with an identical oppositely-wound coil. Both coils are actually situated inside the transmitter coil, and can be adjusted to minimize their inductive coupling to the transmitter coil. In addition, a grounded Faraday shield between the transmitter and pick-up coils minimizes capacitive coupling. An optimum balance of about 3 parts in  $10^5$  is possible. The gradiometer-like configuration reduces the current induced into the input circuit by the rf pulses. After the rf pulse is turned off, the precessing magnetization of the sample induces a signal voltage across the pick-up coil. The pick-up coils are connected in series with an air-capacitor  $C_i$  (adjustable from the top of the cryostat), the input coil  $L_i$  of the SQUID, and

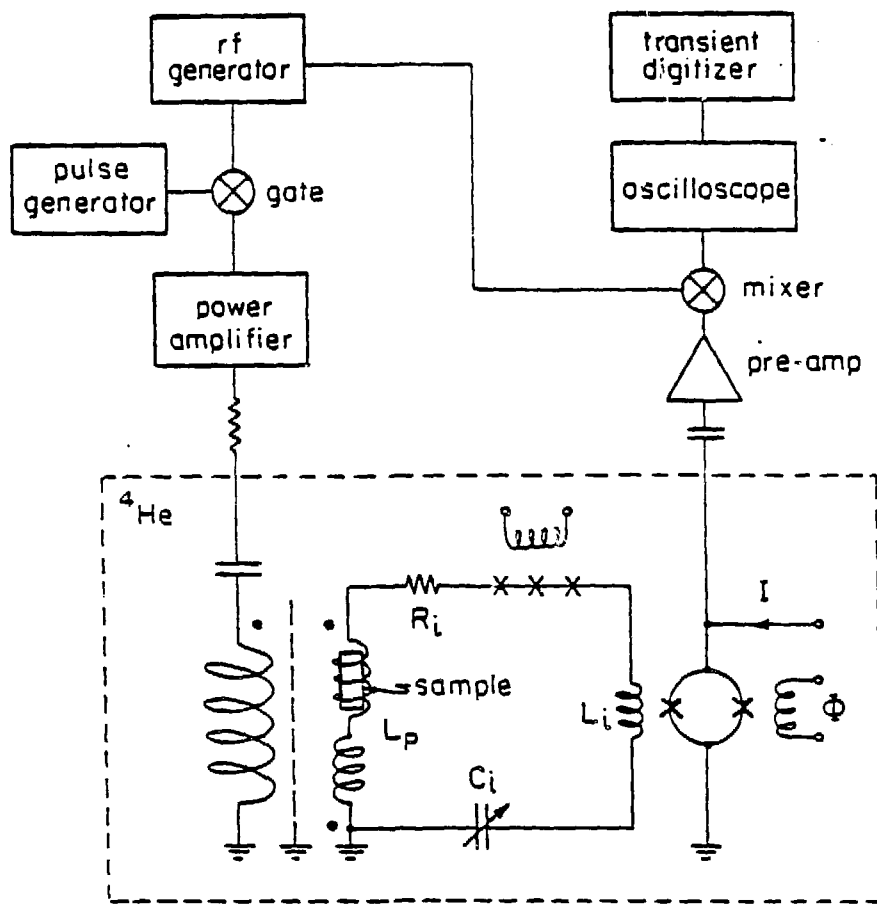


Fig. 2: Schematic layout for SQUID-based detection of NQR.

a series array of 20 Josephson tunnel junctions. The resistor  $R_i$  represents contact resistance and losses in the capacitor. The SQUID is enclosed in a superconducting Nb shield, and its output is matched to a low noise, room temperature amplifier. The amplified signal is mixed down by a double balanced mixer with a reference supplied by the rf generator. The mixed-down signal is passed through a low-pass filter and observed on an oscilloscope, and, after digitizing, stored in a computer for further analysis or averaging.

A novel feature of the input circuit is the series array of twenty  $10 \mu\text{m} \times 10 \mu\text{m}$ , Nb-NbOx-PbIn Josephson tunnel junctions. Each junction has a critical current of about  $4 \mu\text{A}$ , and a hysteretic current-voltage characteristic with a resistance of about  $50 \Omega$  at biases above the sum of the energy gaps. For signal currents below the critical current, the array has zero resistance. On the

other hand, the relatively large current induced by each rf pulse causes the junctions to switch rapidly to the resistive state with a total resistance of about 1 k $\Omega$ . Thus, the array acts as a Q-spoiler, not only providing additional protection for the SQUID but, more importantly, reducing the ring-down time of the tuned circuit after the end of the rf pulse. The quality factor  $Q$  is about 1/2 with the junctions in the resistive state. The switching threshold of the Q-spoiler, that is, the critical current of the junctions, can be varied by means of a static magnetic field applied parallel to the plane of the films.

We tested our NQR detector using approximately 0.32 cm<sup>3</sup> of powdered NaClO<sub>3</sub>. The filling factor, referred to both pick-up coils, was 0.13. At 4.2K <sup>35</sup>Cl nuclei exhibit a quadrupole resonance at 30.6856 MHz. The spin-lattice relaxation time,  $T_1$ , was reduced to about 20 min at 4.2K by  $\gamma$ -ray irradiation; the spin-spin relaxation time,  $T_2$ , was 240  $\mu$ s. The inductance  $L_1$  was 5.6 nH, the combined inductance  $L_p$  of the pick-up coils was 2.5  $\mu$ H, and the effective coupling coefficient  $\alpha_e^2$  was thus about 10<sup>-3</sup>. Most of the tests were performed with  $R_i = 0.2 \Omega$ , yielding a  $Q$  of 2,500 at the <sup>35</sup>Cl resonant frequency. Thus,  $Q\alpha_e^2$  was of the order of unity, as required for optimum operation of the amplifier [Eq. (1)]. At 4.2K with a  $Q$  of 2,500, the overall system noise temperature, including the Nyquist noise from  $R_i$ , was  $6 \pm 1$ K.

Figure 3 illustrates the mixed-down signal that follows an rf pulse. The rf signal and the resonant frequency of the tuned circuit were at the resonance of the <sup>35</sup>Cl nuclei. Figure 3 shows a single oscilloscope trace of the free induction decay following an initial nuclear spin tipping angle of  $2 \times 10^{-4}$  rad. The initial rms voltage induced across the pick-up coil was about 20 nV in a bandwidth of 10 kHz. By measuring the spin tipping angle required to obtain a signal-to-noise ratio of unity, we determined that the minimum number of <sup>35</sup>Cl spins observable with a single pulse in a bandwidth of 10 kHz was about  $2 \times 10^{16}$ . This is equivalent to about  $2 \times 10^{16}$  nuclear Bohr magnetons.

Figure 4 illustrates the effect of the Josephson Q-spoiler on the ring-down of a tuned circuit ( $Q = 100$ ) after the rf pulses. The

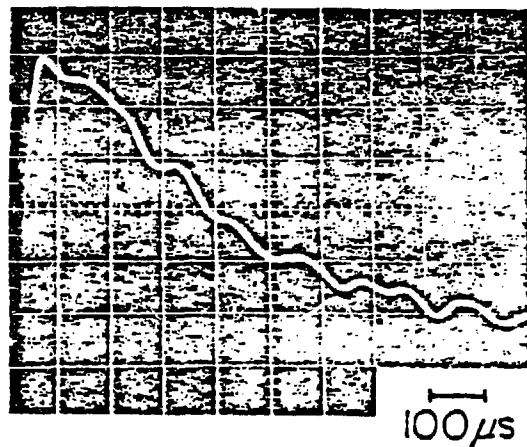


Fig. 3: Oscilloscope traces of free-induction decay of  $^{35}\text{Cl}$  for a small signal ( $\Phi \ll \Phi_c$ ).

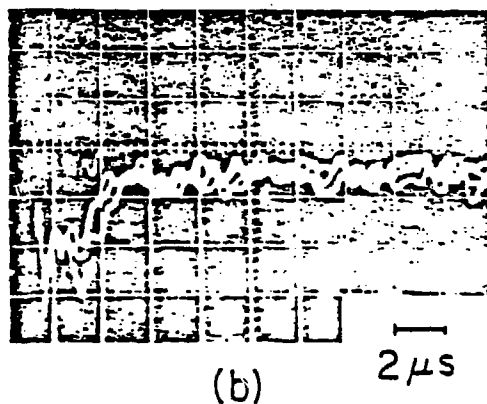
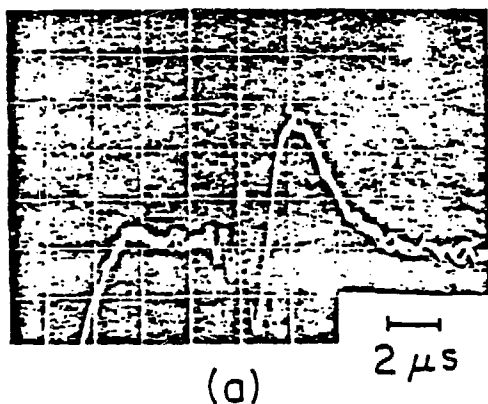


Fig. 4: Oscilloscope traces showing ring-down of tuned circuit following rf pulse, with  $Q = 100$  (a) without Q-spoiler, and (b) with Q-spoiler.

voltage ring-down time of the circuit  $\tau$  was about  $1 \mu\text{s}$  at  $30.5 \text{ MHz}$ . The photographs show the mixed-down output signal, with the traces triggered at the end of the rf pulse. Figure 4(a) shows the response to a rf pulse corresponding to a peak-to-peak magnetic field of  $1.5 \text{ mT}$  at the sample, in the absence of the Q-spoiler. Figure 4(b) shows the response to the same rf-pulse in the presence of a Q-spoiler with the critical current of the junctions reduced by a magnetic field: The overall recovery time has been reduced from  $17 \mu\text{s}$  to about  $4 \mu\text{s}$ .

To conclude this section, we note several advantages of this system over conventional systems with room temperature amplifiers: (i) an



improvement in voltage resolution of 1 or 2 orders of magnitude, due in part to the low noise temperature of the SQUID amplifier, and in part to the high Q that the Q-spoiler allows one to use; (ii) the elimination of any amplifier dead-time; (iii) the minimization of losses in the tuned circuit; and (iv) a wide tuning range from about 1 to 300 MHz.

#### Thermal Noise NQR and Nuclear Spin Fluctuations

The fact that dc SQUID amplifiers can have a noise temperature comparable with or lower than the bath temperature makes possible a new means of detecting magnetic resonance. The sample is placed inside the inductor  $L_p$  of a series tuned circuit (Fig. 1), and the spectral density of the current fluctuations in the circuit is measured. In the absence of a sample, the resistance  $R_i$  produces a Nyquist current noise with a Lorentzian power spectrum. The technique consists of detecting and analyzing the departures from the Lorentzian lineshape introduced by the sample. We assume that this sample can be characterized by a complex susceptibility  $\chi(\omega) = \chi'(\omega) - j\chi''(\omega)$ . The impedance of the pick-up coil is modified by the sample to<sup>14</sup>

$$Z_p' = j\omega L_p' = j\omega L_p(1 + 4\pi\chi\xi) = j\omega(L_p + L_s) + R_s, \quad (7)$$

where  $\xi$  is the filling factor. The presence of the spin inductance  $L_s = 4\pi\chi' L_p \xi$  shifts the resonance of the circuit, while the spin resistance  $R_s = 4\pi\chi'' \omega L_p \xi$  modifies the damping and acts as a source of Nyquist noise.

We can compute the Nyquist noise of the spins in terms of the microscopic parameters of the sample. For all practical purposes in this experiment, the nuclear quadrupole moments interacting with local electric field gradients can be analyzed as though they were magnetic dipoles of spin 1/2 interacting with an external magnetic field  $B_0$ . In this case the splitting of the ground and excited states is  $\hbar\omega_s$ , where  $\omega_s/2\pi = \gamma B_0/2\pi$  is the spin frequency and  $\gamma$  is the gyromagnetic ratio. At a spin temperature  $T_s$ , the equilibrium magnetization of the sample is:<sup>14</sup>

$$M_0 = (n\gamma\hbar/2)\tanh(\hbar\omega_s/2k_B T_s), \quad (8)$$

where  $n$  is the number of nuclei per unit volume. Assuming a Lorentzian lineshape for the nuclear resonance, we can write<sup>14</sup>

$$\chi''(\omega) = \gamma T_2 M_0 / 2 [1 + (\omega - \omega_s)^2 T_2^2], \quad (9)$$

where  $T_2$  is the spin-spin relaxation time.

The general expression for the spectral density of Nyquist voltage noise produced by the spin resistance  $R_s$  is given by:

$$S_V^S(\omega) = (\hbar\omega/\pi) R_s \coth(\hbar\omega/2k_B T_s). \quad (10)$$

Combining Eqs. (8)-(10) with  $R_s = 4\pi\chi''\omega L_p \xi$  and neglecting terms of order  $(\omega - \omega_s)/\omega_s \ll 1$ , we obtain

$$S_V^S(\omega) = \xi L_p n \hbar^2 \gamma^2 \omega_s^2 T_2 / [1 + (\omega - \omega_s)^2 T_2^2]. \quad (11)$$

Hence, the spectral density of the spin fluctuations is centered at the resonance frequency, with an amplitude that is, remarkably, independent of temperature.

Our experiments were performed on 0.63 cm<sup>3</sup> of powdered NaClO<sub>3</sub>, with a filling factor of about 35%. The <sup>35</sup>Cl nucleus has a NQR transition frequency of 30.6856 MHz, while  $T_2 = 240$   $\mu$ s. We performed two separate experiments. In the first, the spins were allowed to reach thermal equilibrium with the helium bath ( $T = T_s$ ), which was at 1.5K. The spin-lattice relaxation time  $T_1$  had been reduced to 20 min by  $\gamma$ -ray irradiation. In the thermal limit  $\hbar\omega \ll k_B T$ , the spectral density of the current noise in the input circuit is

$$S_I(\omega) = \frac{(2/\pi)\hbar\gamma T[R_i + R_s(\omega)]}{[R_i + R_s(\omega)]^2 + \{\omega[L_i + L_s(\omega) + L_p] - 1/\omega C_i\}^2}. \quad (12)$$

When the input circuit is tuned to the spin frequency ( $f_0 = f_s$ ), Eq. (12) reduces approximately to  $2k_B T/\pi[R_i + R_s(\omega)]$  on resonance, so that the effect of the spins is to produce a "dip" in the spectral density. A representative example of our experimental data, averaged over 3 hours, is shown in Fig. 5. The <sup>35</sup>Cl resonant fre-

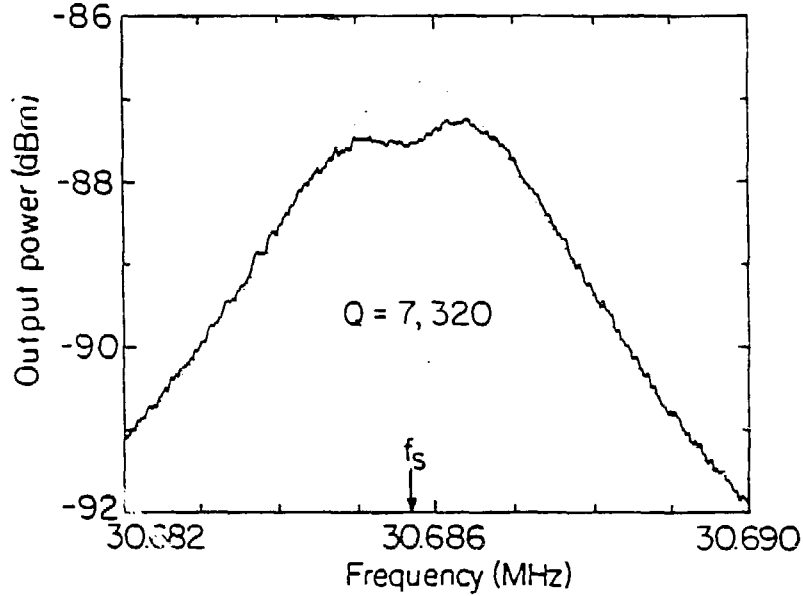


Fig. 5: Spectral density of noise current in the presence of a  $\text{NaClO}_3$  sample in thermal equilibrium at 1.5K.

quency  $f_s$  measured in a separate, pulsed NQR experiment at the same temperature is indicated by an arrow. We observe a dip in the spectral density centered at  $f_s$ , which arises from the peak in  $R_s(\omega)$  at  $f_s$ . We have fitted the data to Eq. (12), allowing  $Q$ ,  $R_s$ ,  $f_s$ , and  $T_2$  to vary. We find  $f_s = 30.68565$  MHz and  $\Delta f_s = 1/\pi T_2 = 1.3$  kHz, in excellent agreement with the values measured separately, while  $Q = 7,320$  and  $R_s/R_i = 0.14$ . The dotted line in Fig. 5 indicates the response we would expect from this fit in the absence of the sample.

In the second experiment we use a non-irradiated sample with a  $T_1$  of about 1 week. We equalize the spin populations of the ground and excited states by applying a continuous rf signal at frequency  $f_s$  for a few minutes. For zero population difference, the net magnetization, susceptibility and spin resistance  $R_s$  are zero, while the spin temperature  $T_s$  is infinite. However, because of the temperature independence of Eq. (11),  $R_s T_s$  remains constant, equal to the value in the equilibrium state. After the rf signal is removed, the population difference remains zero for a time short compared with  $T_1$ . Thus, the spectral density of the current becomes

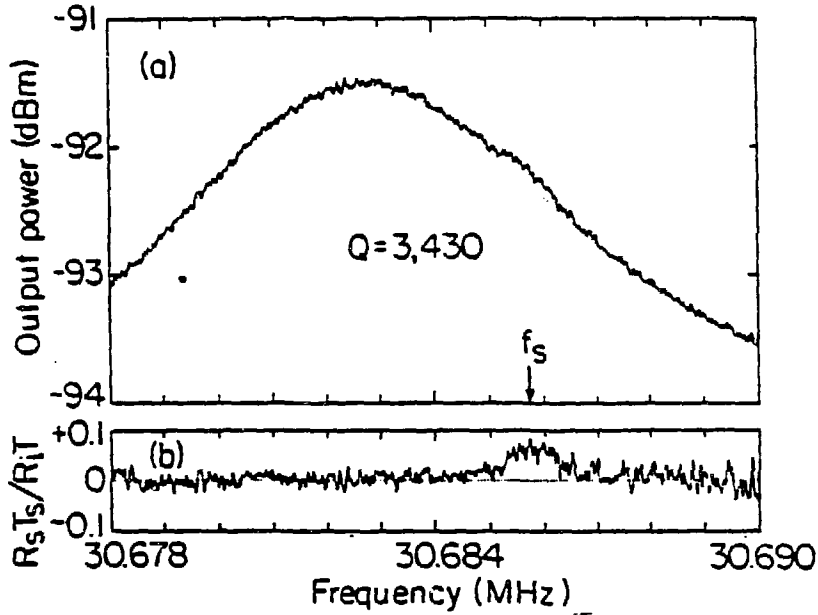


Fig. 6: Spectral density of (a) noise current in the presence of a saturated  $\text{NaClO}_3$  sample ( $T_s \rightarrow \infty$ ), and (b) nuclear spin noise of  $\text{NaClO}_3$  sample obtained from (a).

$$S_I(\omega) = \frac{(2/\pi)k_B[TR_i + T_s R_s(\omega)]}{R_i^2 + [\omega(L_i + L_p) - 1/\omega C_i]^2} . \quad (13)$$

Thus, we expect to observe a "bump" in the spectral response due to the nonequilibrium contribution  $T_s R_s$  to the noise.

An example of our data, averaged over 7 hours, is shown in Fig. 6(a), with the spin resonant frequency indicated with an arrow. Fitting the data with the values of  $f_s$  and  $\Delta f_s$  obtained in the first experiment, we find  $Q = 3,430$  and  $R_s T_s / R_i T = 0.06$ . The dotted line indicates the expected spectral density in the absence of a sample. Figure 6(b) shows the excess noise observed in Fig. 6(a) due to the spins. The bump at  $f_s$  represents the first observation of nuclear spin noise, and arises from the decay of the nonequilibrium state via spontaneous emission into the circuit.

## Acknowledgement

This work is supported partially by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division of the U.S. Department of Energy under contract No. DE-AC03-76SF00098 and partially by the National Science Foundation under contract DMR83-08082.

## References

1. Hirschkoff, E.C., O.G. Symko, L.L. Vant-Hull, J.C. Wheatley. 1970. J. Temp. Phys. 2, 653.
2. Bishop, J.H., E.C. Hirschkoff, J.C. Wheatley. 1971. J. Low Temp. Phys. 5, 607.
3. Day, E.P. 1972. Phys. Rev. Lett. 29, 540.
4. Meredith, D.J., G.R. Pickett, O.G. Symko. 1973. J. Low Temp. Phys. 13, 607.
5. Webb, R.A. 1977. Rev. Sci. Instrum. 48, 1585.
6. Chamberlin, R.V., L.A. Moberly, O.G. Symko. 1979. J. Low Temp. Phys. 35, 337.
7. Suzuki, H., Y. Higashino, T. Ohtsuka. 1980. J. Low Temp. Phys. 41, 449.
8. Pickens, K.S., D.I. Bolef, M.R. Holland, R.K. Sundfors. 1984. Phys. Rev. B 30, 3644.
9. Ehnholm, G.J., J.P. Ekstrom, M.T. Lojonen, J.K. Soini. 1979. Cryogenics 19, 673.
10. Wennberg, A.K.M., L.J. Friedman, H.M. Bozler. 1984. Physics 126 B+C, 265.
11. Hilbert, C., J. Clarke. 1983. Appl. Phys. Lett. 43, 694; 1985. IEEE Trans. Magn. 21, 1029; 1985. J. Low Temp. Phys. 61, 3/4.
12. Bloom, M., E.L. Hahn, B. Herzog. 1965. Phys. Rev. 97, 1699.
13. Hilbert, C., J. Clarke. 1984. Appl. Phys. Lett. 45, 799.
14. See for instance, Slichter, C.P. 1984. Principles of Magnetic Resonance. Springer, New York.

## **DISCLAIMER**

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.