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Supersymmetry Breaking in Superstring Theory by Gaugino Condensation and its Phenomenology

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Supersymmetry Breaking in Superstring Theory by Gaugino Condensation and its Phenomenology[*]

(Ph.D. Dissertation, University of California at Berkeley)

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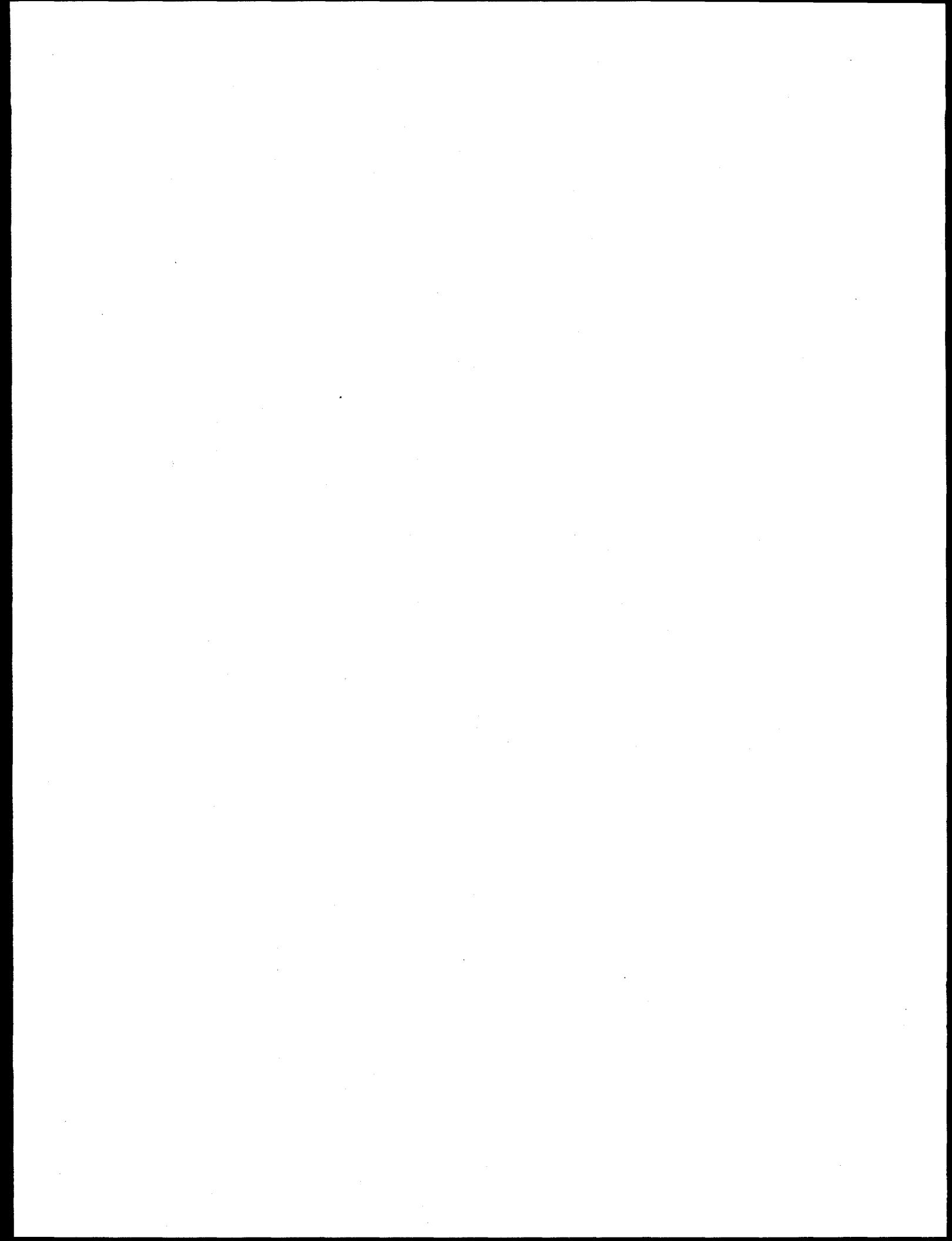
Abstract

Weakly-coupled heterotic string is known to have problems of dilaton/moduli stabilization, supersymmetry breaking (by hidden-sector gaugino condensation), gauge coupling unification, QCD axion, as well as cosmological problems involving dilaton/moduli and axion. We study these problems by adopting the point of view that they arise mostly due to our limited calculational power, little knowledge of the full vacuum structure, and an inappropriate treatment of gaugino condensation. It turns out that these problems can be solved or are much less severe after a more consistent and complete treatment.

There are two kinds of non-perturbative effects in our construction of string effective field theory: the field-theoretical non-perturbative effects of gaugino condensation (with an important constraint ignored in the past) and the stringy non-perturbative effects conjectured by S. Shenker, which are best described using the linear multiplet formalism. Stringy non-perturbative corrections to the Kähler potential are invoked to stabilize the dilaton at a value compatible with a weak coupling regime. Modular invariance is ensured through the Green-Schwarz counterterm and string threshold corrections which, together with hidden matter condensation, lead to moduli stabilization at the self-dual point where the *vev*'s of moduli's F components vanish. In the vacuum, supersymmetry is broken at a realistic scale with vanishing cosmological constant. As for soft supersymmetry breaking, our model always leads to a dilaton-dominated scenario. For the strong CP problem, the model-independent axion has the right properties to be the QCD axion. Furthermore, there is a natural hierarchy between the dilaton/moduli mass and the gravitino mass, which could solve both the cosmological moduli problem and the cosmological problem of the model-independent axion.

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Chapter 1

Preamble

How the electroweak symmetry is broken is one of the fundamental questions of particle physics. In the standard model, the scalar Higgs doublet acquires a non-vanishing vacuum expectation value (*vev*), and therefore breaks the electroweak symmetry. However, the field-theoretical loop corrections to the masses of scalar particles are quadratically divergent. Therefore, the scale of electroweak symmetry breaking is in fact unstable against radiative corrections, and how the very large hierarchy between the Planck scale and the scale of electroweak symmetry breaking is generated remains a mystery. Currently, weak scale supersymmetry [1] is the most promising solution this hierarchy problem. Supersymmetric theories are free from quadratic divergences due to delicate cancellations between boson and and fermion loop corrections, and therefore can stabilize the hierarchy between the Planck scale and the electroweak scale. However, supersymmetry itself does not explain the origin of the electroweak scale. Furthermore, supersymmetry introduces new particles (*i.e.*, supersymmetric partners of the standard model particles.) Therefore, as a requirement of particle phenomenology, supersymmetry must be broken and the resulting theory is a supersymmetric extension of the standard model with supersymmetry *softly* broken at the electroweak scale. The experimental search for supersymmetric partners is very important to our understanding of electroweak symmetry breaking. It will also shed light on the mechanism of supersymmetry breaking as well as the physics at (and possibly above) the scale where supersymmetry is broken. On the other hand, constructing a realistic scheme of supersymmetry breaking remains one of the big challenges to supersymmetry phenomenology.

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Although it is possible, without knowing the details of the supersymmetry breaking mechanism, to parametrize the effects of softly broken supersymmetry in an effective description, yet it involves a huge numbers of unknown parameters and thus makes phenomenological analyses highly intractable. It is therefore desirable to have a realistic supersymmetry breaking scheme which predicts all the soft supersymmetry breaking parameters in terms of only a few parameters.

It is well known that superstring theory offers, according to the above consideration, the most powerful scheme of supersymmetry phenomenology. More precisely, all the *parameters* appearing in the effective description of the superstring are in principle determined by the dynamics of superstring alone, *i.e.*, by the *vev*'s of certain fields (*e.g.*, the string dilaton and moduli.) Besides, the most compelling reason to study superstring theory is the fact that it is the only known candidate theory of quantum gravity. However, at the perturbative level the superstring has many vacua parametrized by flat directions (*e.g.*, the string dilaton and moduli) which will be lifted only after non-perturbative effects are included[1]. Even with the recent progress of string duality, there is still little knowledge of these non-perturbative effects and hence how the above powerful feature of superstring theory is realized. Earlier attempts to study the phenomenology of superstrings [2] have either ignored the non-perturbative effects responsible for stabilizing the string dilaton/moduli or relied on the racetrack model[2] [3], and therefore their results may not be reliable. It

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[1]It is very possible that the same non-perturbative effects are also responsible for supersymmetry breaking.

[2]As will be discussed later, the racetrack model suffers from a negative cosmological constant

is believed and will be shown in the following chapters that it is possible to draw reliable predictions from superstrings only after the relevant non-perturbative effects are fully taken into account.

Our study of superstring phenomenology contains two kinds of non-perturbative effects: the stringy non-perturbative effects generated above the string scale, and the field-theoretical non-perturbative effects of gaugino condensation generated by strongly-interacting gauge groups below the string scale. As for stringy non-perturbative effects, they have always been ignored in the past. The existence of significant stringy non-perturbative effects was first conjectured by S.H. Shenker [4]. The recent development of string duality has provided further evidence [5, 6] for Shenker's conjecture. It was first noticed by T. Banks and M. Dine that significant stringy non-perturbative effects could have interesting implications [7]. Here we will study in detail the phenomenological implications of stringy non-perturbative effects using the linear multiplet formalism of superstring effective theory. It was first pointed out in [8] that the field-theoretical limit of weakly-coupled heterotic string theory should be described using the linear multiplet formalism rather than the chiral multiplet formalism. A similar point of view has also been emphasized by other authors [9, 10, 11]. Furthermore, our study represents a concrete and elegant realization of this viewpoint. As we shall see in Chapter 2, in the linear multiplet formalism the string coupling is the linear multiplet L which is the natural parametrization of stringy physics. On the other hand, the coupling of string effective field theory

problem as well as an un-naturalness problem.

is $L/(1 + f(L))$ which is the natural parametrization of field-theoretical effects; it is modified in the presence of stringy effects $f(L)$. Therefore, the linear multiplet formalism naturally distinguishes stringy effects from field-theoretical effects, and it is this feature that makes the incorporation of stringy effects with the effective field theory simple and transparent. This advantage of the linear multiplet formalism is very crucial to our study where both stringy and field-theoretical non-perturbative effects are considered. As we will see, stringy non-perturbative effects do play an important role in stabilizing the string dilaton/moduli and in breaking super-symmetry via the field-theoretical non-perturbative effects of gaugino condensation [12, 13, 14].

As for the field-theoretical non-perturbative effects, gaugino condensation has always played a unique role: at low energy, the strong dilaton-Yang-Mills interaction leads to gaugino condensation which not only breaks supersymmetry spontaneously but also generates a non-perturbative potential which may eventually stabilize the dilaton[3]. In the scheme of gaugino condensation the stabilization of string dilaton/moduli and the breaking of supersymmetry are therefore unified in the sense that they are two aspects of a single non-perturbative phenomenon. Furthermore, gaugino condensation has its own important phenomenological motivations: gaugino condensation occurs in the hidden sector of a generic string model [15, 16]; it can break supersymmetry at a sufficiently small scale and may induce viable soft

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[3]In general there is also matter condensation which generates a non-perturbative potential for string moduli.

supersymmetry breaking effects in the observable sector through gravity and/or an anomalous U(1) gauge interaction [17]. However, although gaugino condensation has been studied since 1982, it still has several long-standing problems in the context of superstrings. Firstly, superstring phenomenology based on the scheme of gaugino condensation has been long plagued by the infamous dilaton runaway problem [7, 16]. That is, (assuming that the tree-level Kähler potential of the dilaton is a good approximation) one generally finds that the supersymmetric vacuum with vanishing coupling constant and no gaugino condensation is the only stable minimum in the weak-coupling regime. Secondly, modular invariance is a very important property of superstring. However, most of the studies of gaugino condensation had neither complete nor correct treatments of modular invariance. As we shall see, a fully modular invariant treatment of gaugino condensation has non-trivial phenomenological implications. Thirdly, in the past the gaugino condensate has always been described by an *unconstrained* chiral superfield U which corresponds to the bound state of $\mathcal{W}^{\alpha}\mathcal{W}_{\alpha}$ in the underlying theory. It was pointed out recently that U should be a *constrained* chiral superfield [18, 19, 20, 21] due to the constrained superspace geometry of the underlying Yang-Mills theory:

$$U = -(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)V,$$

$$\bar{U} = -(\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - 8R^{\dagger})V, \tag{1.1}$$

where V is an unconstrained vector superfield. Fourthly, superstring phenomenology based on gaugino condensation suffers from several cosmological problems such

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as the cosmological moduli problem [22] and the cosmological bound on the invisible axion [23]. These cosmological problems either destroy the successful nucleosynthesis or overclose the universe.

These formidable problems might make one think that the weakly-coupled heterotic string theory is in grave danger. On the other hand, these problems are not unrelated to one another because the superstring has a highly constrained and predictive framework. As we shall see, in fact these problems arise from our poor understanding of non-perturbative string dynamics as well as incorrect/incomplete treatments of superstring phenomenology in the past. Once we know how to proceed in the right direction, these problems turn out to be solved or much less serious. For the first problem, we emphasize the advantage of using the linear multiplet formalism and show that stringy non-perturbative effects may stabilize the dilaton at a value compatible with a weak coupling regime [12, 13]. For the second and the third problems, full modular invariance is ensured through the Green-Schwarz term and string threshold corrections, and the constraint on the gaugino condensate U is explicitly solved using the linear multiplet formalism [12, 13, 14]. They do lead to unique predictions of superstring theory about supersymmetry breaking, the compactification scale, and axion physics[4]. For example, string moduli are stabilized at the self-dual point, and therefore they do not participate in supersymmetry breaking because the vev's of moduli's F terms vanish [14]. This is certainly a de-

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[4]These unique predictions were unknown in the past due to the aforementioned first three problems.

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sirable feature in consideration of flavor changing neutral current (FCNC) because non-vanishing *vev*'s of moduli's F terms generically lead to non-universal contributions to the soft supersymmetry breaking parameters. For the fourth problem, let's recall the standard lore of superstring phenomenology which tells us that, based on a very naive order-of-magnitude estimate, string dilaton and moduli gain from supersymmetry breaking masses of order of the gravitino mass. Since the gravitino mass is of order of the electroweak scale, these small masses of the dilaton and moduli lead to the cosmological moduli problem. On the other hand, our model is realistic enough for us to discuss these issues based on actual computations rather than educated guesses: it turns out that the string dilaton and moduli are in fact much heavier than the gravitino, which may be sufficient to solve the cosmological moduli problem [24]. Furthermore, the large entropy produced by the decays of the heavy moduli in our model will dilute the axion density and therefore raise the cosmological bound on the axion decay constant. As we shall see, this could solve the cosmological problem of the invisible axion.

Finally, let's make a brief comment on how the recent development of string duality might affect the status of weakly-coupled heterotic string theory. There have been claims in the literature in favor of the strongly-coupled heterotic string theory by arguing that it is unlikely that the weakly-coupled heterotic string theory can solve the dilaton runaway problem. However, the recent observation of string dualities actually implies that the strong coupling limit of heterotic string theory, which can be described by another weakly-coupled theory (*i.e.*, M-theory compact-

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ified on $R^{10} \times S^1/Z_2$ [25]), is plagued by a similar runaway problem [26]. Therefore, there seem to be only two logical options for solving the runaway problem: either a truly non-perturbative heterotic string theory which does not allow a weakly-coupled description, or a weakly-coupled theory (*i.e.*, the weakly-coupled heterotic string theory or the strong coupling limit of heterotic string theory). So far the first option remains a remote possibility.[5] On the other hand, as for the second option both the weakly-coupled heterotic string theory and the strong coupling limit of heterotic string theory certainly deserve further study[6]. As mentioned before, it is our purpose here to show that the weakly-coupled heterotic string theory could solve the dilaton runaway problem as well as lead to a satisfactory phenomenology [24].

In Chapter 2, a simple string orbifold model with a hidden E_8 gauge group and no hidden matter is used to illustrate the studies of the linear multiplet formalism, the incorporation of stringy non-perturbative effects, static gaugino condensation, and the dilaton runaway problem. In Chapter 3, we give the motivations for studying dynamical gaugino condensation, and then show that static gaugino condensation is indeed the appropriate low-energy effective description of dynamical

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[5]Some recent attempts at a non-perturbative formulation of heterotic string theory can be found in [27].

[6]Although recently there is an argument of coupling unification preferring the strong coupling limit of heterotic string theory to the weakly-coupled heterotic string theory [28], it involves assumptions that are not true generically. For example, it is assumed in [28] that the compactification volume V_{comp} is of order M_{GUT}^{-6}, where M_{GUT} is the grand unification scale. However, in our model the moduli associated with compactification are stabilized at the self-dual point, and therefore the argument of [28] is not valid.

gaugino condensation. In Chapter 4, we extend our previous studies to a generic string orbifold model. The resulting model is generic and realistic enough, and we are therefore in a position to address several important phenomenological issues. In Chapter 5, we discuss phenomenological issues such as the dilaton and moduli masses, axion physics, soft supersymmetry breaking parameters, gauge coupling unification, as well as cosmological issues.

Chapter 2

The Stringy Story of Gaugino Condensation

2.1 Introduction

Constructing a realistic scheme of supersymmetry breaking is one of the big challenges to supersymmetry phenomenology. However, in the context of superstring phenomenology, there are actually more challenges. As is well known, a very powerful feature of superstring phenomenology is that all the *parameters* of the model are in principle dynamically determined by the *vev*'s of certain fields. One of these important fields is the string dilaton whose *vev* determines the gauge coupling constants. On the other hand, how the dilaton is stabilized is outside the reach of perturbation theory since the dilaton's potential remains flat to all order in perturbation theory according to the non-renormalization theorem. Therefore, understanding how the dilaton is stabilized (*i.e.*, how the gauge coupling constants are determined) is of no less significance than understanding how supersymmetry is broken. Gaugino condensation has been playing a unique role in these issues: Gaugino condensation not only breaks supersymmetry but also generates a non-perturbative dilaton potential which may eventually stabilize the dilaton. Furthermore, gaugino condensation has its own important phenomenological motivations [15, 16, 17]. Unfortunately, this beautiful scheme of gaugino condensation has been long plagued by the infamous dilaton runaway problem [7, 16]. (The recent observation of string dualities further implies that the strong-coupling regime is plagued by a similar runaway problem [26].) Only a few solutions to the dilaton runaway problem have been proposed. Assuming the scenario of two or more gaugino condensates, the racetrack

model stabilizes the dilaton and breaks supersymmetry with a more complicated dilaton superpotential generated by multiple gaugino condensation [3]. However, stabilization of the dilaton in the racetrack model requires a delicate cancellation between the contributions from different gaugino condensates, which is not very natural. Furthermore, it has a large and negative cosmological constant when supersymmetry is broken. The other solutions generically require the presence of an additional source of supersymmetry breaking (*e.g.*, a constant term in the superpotential) [16, 29]. It is therefore fair to say that there is no satisfactory solution so far.

Recently, there have been several new developments and insights in superstring phenomenology. It is our purpose to show that these new ingredients play important roles in the above issues and can eventually lead to a promising solution. One of these new ingredients is the linear multiplet formalism of superstring effective theories [8, 9, 10]: Among the massless string modes, a real scalar (dilaton), an antisymmetric tensor field (the Kalb-Ramond field) and their supersymmetric partners can be described either by a chiral superfield S or by a linear multiplet L, which is known as the chiral-linear duality [30]. By definition, the linear multiplet L is a vector superfield that satisfies the following constraints [30]:

$$-(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)L = 0,$$

$$-(\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - 8R^{\dagger})L = 0. \qquad (2.1)$$

The lowest component of L is the dilaton field ℓ, and its *vev* is related to the gauge

coupling constant as follows[1]: $g^2(M_s) = 2\langle \ell \rangle$, where M_s is the string scale [31, 32]. Although the chiral-linear duality is obvious at tree level, it becomes obscure when quantum effects are included. Although scalar-2-form field strength duality, which is contained in chiral-linear duality, has been shown to be preserved in perturbation theory [33], the situation is less clear in the presence of non-perturbative effects, which are important in the study of gaugino condensation. It has recently been shown [18, 20] that gaugino condensation can be formulated directly using a linear multiplet for the dilaton. Although a formal equivalence between the chiral and linear multiplet formalisms has been shown [20], the content of the resulting chiral-linear duality transformation is in general very complicated. If there is an elegant description of gaugino condensates in the context of superstring effective theories, it may be simple in only one of these formalisms, but not in both. Therefore, a pertinent issue is: which formalism is better? Here we will construct the effective theory of gaugino condensation directly in the linear multiplet formalism without referring to the chiral multiplet formalism. There is reason to believe that the linear multiplet formalism is in fact more appropriate. The stringy reason for choosing the linear multiplet formalism is that the precise field content of the linear multiplet appears in the massless string spectrum, and $\langle L \rangle$ plays the role of string loop expansion parameter. Therefore, string information is more naturally encoded in the linear multiplet formalism of string effective theory. Furthermore, as we will see

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[1]However, as we shall see in Section 2.2.2, this identification of gauge coupling constant in terms of $\langle \ell \rangle$ will be modified in the presence of stringy non-perturbative effects [4].

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in Chapter 2, stringy effects are believed to be important in the stabilization of the dilaton and supersymmetry breaking by gaugino condensation; therefore, it is more appropriate to study these issues in the linear multiplet formalism.

The other new ingredient concerns the effective description of gaugino condensation. In the known models of gaugino condensation using the chiral superfield representation for the dilaton, the gaugino condensate has always been described by an *unconstrained* chiral superfield U which corresponds to the bound state of $\mathcal{W}^\alpha \mathcal{W}_\alpha$ in the underlying theory. It was pointed out recently that U should be a *constrained* chiral superfield [18, 19, 20, 21] due to the constrained superspace geometry of the underlying Yang-Mills theory:

$$U = -(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R)V,$$

$$\bar{U} = -(\mathcal{D}^\alpha\mathcal{D}_\alpha - 8R^\dagger)V, \tag{2.2}$$

where V is an unconstrained vector superfield. Furthermore, in the linear multiplet formalism the linear multiplet L and the constrained U, $\bar{U}$ nicely merge into an unconstrained vector superfield V [18], and therefore the effective Lagrangian can elegantly be described by V alone.

The third new ingredient is the stringy non-perturbative effect conjectured by S.H. Shenker [4]. It is further argued in [7] that the Kähler potential can in principle receive significant stringy non-perturbative corrections although the superpotential cannot generically. Significant stringy non-perturbative corrections to the Kähler potential imply that the usual dilaton runaway picture is valid only in the weak-

coupling regime; as pointed out in [7], these corrections may naturally stabilize the dilaton.[2]

In the next section we describe the linear multiplet formalism of string effective Yang-Mills theory, whose effective theory below the condensation scale is constructed and analyzed in Section 2.3. It is then shown in Section 2.4 that supersymmetry is broken and the dilaton is stabilized in a large class of models of static gaugino condensation. Here we use the Kähler superspace formulation [34] of supergravity, suitably extended to incorporate the linear multiplet [35].

2.2 The Linear Multiplet Formalism

2.2.1 Effective Yang-Mills Theory from Superstring

In the realm of superstring effective Yang-Mills theory, there are two important ingredients, namely, the symmetry group of modular transformations and the linear multiplet. In order to make the discussion as explicit as possible in this chapter, we consider here orbifolds with gauge group[3] $E_8 \otimes E_6 \otimes U(1)^2$, which have been studied most extensively in the context of modular symmetries [31, 32, 36]. They contain three untwisted $(1,1)$ moduli T^I, $I = 1, 2, 3$, which transform under SL(2,Z) as

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[2]Choosing a specific form for possible non-perturbative corrections to the Kähler potential, [48] has discussed the possibility of stabilizing the dilaton in a model of gaugino condensation using chiral superfield representation for the dilaton. However, neither the issue of modular anomaly cancellation nor the constraint (2.2) was taken into account.

[3]As for phenomenological consideration, it is more desirable to discuss a generic orbifold. Such a non-trivial generalization will be made in Chapter 4.

follows:

$$T^I \rightarrow \frac{aT^I - ib}{icT^I + d}, \quad ad - bc = 1, \quad a, b, c, d \in \mathbb{Z}. \tag{2.3}$$

The corresponding Kähler potential is

$$G = \sum_I g^I + \sum_A \exp(\sum_I q_A^I g^I)|\Phi^A|^2 + \mathcal{O}(|\Phi|^4), \tag{2.4}$$

where $g^I = -\ln(T^I + \bar{T}^I)$, and the modular weights q_A^I depend on the particular matter field Φ^A as well as on the modulus T^I. However, it is well known that the effective theory obtained from the massless truncation of superstring is *not* invariant under the modular transformations (2.3) at one loop [37, 38]. Counterterms, that correspond to the result of integrating out massive modes, have to be added to the effective theory in order to restore modular invariance since string theory is known to be modular invariant to all orders of the loop expansion [39]. Two types of such counterterms have been discussed in the literature [31, 36, 38], the so-called f-type counterterms (*i.e.*, string threshold corrections) and the Green-Schwarz counterterm. The Green-Schwarz counterterm, which is analogous to the Green-Schwarz anomaly cancellation mechanism in D=10, is naturally implemented with the linear multiplet formalism [30]. In Chapters 2 and 3 we consider only those orbifolds in which the full modular anomaly is cancelled by the Green-Schwarz counterterm alone (*i.e.*, orbifolds with universal modular anomaly cancellation), and more generic orbifolds with both types of counterterms present will be considered in Chapter 4. Indeed, an orbifold has universal modular anomaly cancellation unless its modulus T^I corresponds to an internal plane which is left invariant under some

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orbifold group transformations, which may happen only if an $N=2$ supersymmetric twisted sector is present [40]. Therefore, a large class of orbifolds, including the Z_3 and Z_7 orbifolds, is under consideration in this chapter.

The antisymmetric tensor field of superstring theories undergoes Yang-Mills gauge transformations. In the effective theory, it can be incorporated into a gauge invariant vector superfield L, the so-called modified linear multiplet, coupled to the Yang-Mills degrees of freedom as follows:

$$-(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)L \;=\; (\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)\Omega \;=\; \sum_a \mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)^a,$$

$$-(\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^\dagger)L \;=\; (\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^\dagger)\Omega \;=\; \sum_a \mathrm{Tr}(\mathcal{W}_{\dot{\alpha}}\mathcal{W}^{\dot{\alpha}})^a, \qquad (2.5)$$

where Ω is the Yang-Mills Chern-Simons superform. The summation extends over the indices a numbering simple subgroups of the full gauge group. The modified linear multiplet L contains the linear multiplet as well as the Chern-Simons superform, and its gauge invariance is ensured by imposing appropriate transformation properties for the linear multiplet. The generic lagrangian describing the linear multiplet coupled to supergravity and matter in the presence of Yang-Mills Chern-Simons superform is [31]:

$$K \;=\; \ln L + g(L) + G,$$

$$\mathcal{L} \;=\; \int \mathrm{d}^4\theta\, E\, \{-2 + f(L)\} + \int \mathrm{d}^4\theta\, E\, \{bL\sum_I g^I\}, \qquad (2.6)$$

$$b \;=\; \frac{C}{8\pi^2} = \frac{2}{3}b_0, \qquad (2.7)$$

where L is the modified linear multiplet and $C = 30$ is the Casimir operator in

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the adjoint representation of E_8. b_0 is the E_8 one-loop β-function coefficient. The first term of $\mathcal{L}$ is the superspace integral which yields the kinetic actions for the linear multiplet, supergravity, matter and Yang-Mills fields. The second term in (2.6) is the Green-Schwarz counterterm, which is "minimal" in the sense of [31]. Furthermore, arbitrariness in the two functions $g(L)$ and $f(L)$ is reduced by the requirement that the Einstein term in $\mathcal{L}$ be canonical. Under this constraint, $g(L)$ and $f(L)$ are related to each other by the following first-order differential equation [35]:

$$L\frac{\mathrm{d}g(L)}{\mathrm{d}L} = -L\frac{\mathrm{d}f(L)}{\mathrm{d}L} + f(L), \tag{2.8}$$

The complete component lagrangian of (2.6) with the tree-level Kähler potential (i.e., $g(L) = 0$ and $f(L) = 0$) has been presented in [9] based on the Kähler superspace formulation. Similar studies have also been performed in the superconformal formulation of supergravity [8, 10]. In the following, we are interested in the effective lagrangian of (2.6) below the condensation scale.

2.2.2 Stringy Effects versus Field-Theoretical Effects

In this section we would like to illustrate how stringy effects are naturally incorporated with the superstring effective field theory using the linear multiplet formalism. Consider again the effective field theory defined at the string scale M_s. The quantum corrections, $g(L)$ and $f(L)$, to the tree-level Kähler potential of (2.6) are naturally interpreted as stringy effects. Indeed, in the context of superstring L plays the role of string loop expansion parameter (i.e., the string coupling), and therefore stringy effects are naturally parametrized by L. Although perturbative

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contributions to $g(L)$ and $f(L)$ are generically small, yet, as first pointed out by Shenker [4], there can be significant stringy non-perturbative contributions. It is then interesting to ask how the usual relation between the dilaton ℓ and the gauge coupling constant of the effective field theory, $g^2(M_s) = 2\langle \ell \rangle$, might get modified in the presence of stringy effects? It is straightforward to compute the gauge coupling constant at the string scale, $g(M_s)$, defined by the effective field theory (2.6) as follows:

$$g^2(M_s) = \left\langle \frac{2\ell}{1 + f(\ell)} \right\rangle. \tag{2.9}$$

Indeed, the presence of stringy effects do affect the usual interpretation of the gauge coupling constant of the effective field theory in terms of the string dilaton. More precisely, the linear multiplet formalism naturally distinguishes stringy effects from field-theoretical effects; that is, ℓ is the natural parametrization of stringy effects and $\langle 2\ell/(1 + f(\ell)) \rangle$ is on the other hand the natural parametrization of field-theoretical effects. Therefore, the linear multiplet formalism of superstring effective field theory has the advantage of incorporating stringy effects with the effective field theory in a simple and transparent manner. As mentioned before, this unique feature of linear multiplet formalism is crucial to our study here, since stringy non-perturbative effects do play an important role in the stringy story of gaugino condensation.

On the other hand, in the chiral multiplet formalism where the string dilaton is described by a chiral superfield S chiral superfield ($s = S|_{\theta = \bar{\theta} = 0}$), S has to be re-defined order by order in perturbation, which is clear from the perturbative chiral-linear duality. Furthermore, in the chiral multiplet formalism there is no

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clear distinction between stringy effects and field-theoretical effects; more precisely, we always have from the chiral multiplet formalism of the superstring effective field theory $g^2(M_s) = \langle 2/(s+\bar{s}) \rangle$ even when stringy effects are included. One may also derive this result by a duality transformation from the linear multiplet formalism (2.6) to the corresponding chiral multiplet formalism of (2.6). It has been shown [31] that $1/(S+\bar{S})$ corresponds to $L/(1+f)$ through this duality transformation, and therefore the interpretations of $g^2(M_s)$ in both formalisms are consistent with each other. In conclusion, we emphasize the advantage of using the linear multiplet formalism over the chiral multiplet formalism in telling the stringy story of gaugino condensation. More evidence of this advantage will be discovered in the following sections.

2.2.3 Low-Energy Effective Degrees of Freedom

Below the condensation scale at which the gauge interaction becomes strong, the effective lagrangian of the Yang-Mills sector can be described by a composite chiral superfield U, which corresponds to the chiral superfield $\mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)$ of the underlying theory. (We consider here gaugino condensation of a simple gauge group.) The scalar component of U is naturally interpreted as the gaugino condensate. It was pointed out only recently that the composite field U is actually a constrained chiral superfield [19, 20, 21]. The constraint on U can be seen most clearly through the constrained superspace geometry of the underlying Yang-Mills theory. As a consequence of this constrained geometry, the chiral superfield $\mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)$ and its

hermitian conjugate $\text{Tr}(\mathcal{W}_{\dot\alpha}\mathcal{W}^{\dot\alpha})$ satisfy the following constraint:

$$(\mathcal{D}^\alpha \mathcal{D}_\alpha - 24R^\dagger)\text{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) - (\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 24R)\text{Tr}(\mathcal{W}_{\dot\alpha}\mathcal{W}^{\dot\alpha}) = \text{total derivative.} \quad (2.10)$$

(2.10) has a natural interpretation in the context of a 3-form supermultiplet, and indeed $\text{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)$ can be interpreted as the degrees of freedom of the 3-form field strength [41]. The explicit solution to the constraint (2.10) has been presented in [21], and it allows us to identify the constrained chiral superfield $\text{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)$ with the chiral projection of an unconstrained vector superfield L:

$$\text{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) = -(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R)L,$$

$$\text{Tr}(\mathcal{W}_{\dot\alpha}\mathcal{W}^{\dot\alpha}) = -(\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^\dagger)L. \quad (2.11)$$

Below the condensation scale, the constraint (2.10) is replaced by the following constraint on U and $\bar U$:

$$(\mathcal{D}^\alpha \mathcal{D}_\alpha - 24R^\dagger)U - (\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 24R)\bar U = \text{total derivative.} \quad (2.12)$$

Similarly, the solution to (2.12) allows us to identify the constrained chiral superfield U with the chiral projection of an unconstrained vector superfield V:

$$U = -(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R)V,$$

$$\bar U = -(\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^\dagger)V. \quad (2.13)$$

(2.13) is the explicit constraint on U and $\bar U$.

In fact, the constraint on U and $\bar U$ enters the linear multiplet formalism of gaugino condensation very naturally. As described in Section 2.2.1, the linear multiplet

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formalism of supersymmetric Yang-Mills theory is described by a gauge-invariant vector superfield L which satisfies

$$-(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)L = (\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)\Omega = \text{Tr}(\mathcal{W}^{\alpha}\mathcal{W}_{\alpha}),$$

$$-(\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - 8R^{\dagger})L = (\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - 8R^{\dagger})\Omega = \text{Tr}(\mathcal{W}_{\dot{\alpha}}\mathcal{W}^{\dot{\alpha}}). \tag{2.14}$$

For the linear multiplet formalism of the superstring effective lagrangian below the condensation scale, (2.14) is replaced by

$$-(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)V = U,$$

$$-(\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - 8R^{\dagger})V = \bar{U}, \tag{2.15}$$

where U is the gaugino condensate chiral superfield, and V contains the linear multiplet as well as the "fossil" Chern-Simons superform. In view of (2.15), it is clear that the constraint on U and $\bar{U}$ arises naturally in the linear multiplet formalism of gaugino condensation. Furthermore, the low-energy degrees of freedom (*i.e.*, the linear multiplet and the gaugino condensate) are nicely merged into a single vector superfield V, and therefore the linear multiplet formalism of gaugino condensation can elegantly be described by V alone in the context of superstring. The detailed construction of the effective lagrangian for the vector superfield V will be presented in the next section.

2.3 Gaugino Condensation in Superstring Effective Theory

2.3.1 A Simple Model

Constructing the linear multiplet formalism of gaugino condensation requires the specification of two functions of the vector superfield V, namely, the superpotential and the Kähler potential. In the linear multiplet formalism, there is no classical superpotential [19], and the quantum superpotential originates from the non-perturbative effects of gaugino condensation. This non-perturbative superpotential, whose form was dictated by the anomaly structure of the underlying theory, was first obtained by Veneziano and Yankielowicz [42, 43, 44, 45]. The details of its generalization to the case of matter coupled to $N=1$ supergravity in the Kähler superspace formulation has been presented in [46], and the superpotential term in the Lagrangian reads:

$$\int \mathrm{d}^4\theta \, \frac{E}{R} \, e^{K/2} W_{VY} = \int \mathrm{d}^4\theta \, \frac{E}{R} \frac{1}{8} bU \ln(e^{-K/2}U/\mu^3),$$

$$\int \mathrm{d}^4\theta \, \frac{E}{R^\dagger} \, e^{K/2} \bar{W}_{VY} = \int \mathrm{d}^4\theta \, \frac{E}{R^\dagger} \frac{1}{8} b\bar{U} \ln(e^{-K/2}\bar{U}/\mu^3), \qquad (2.16)$$

where $U = -(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)V$ is the constrained gaugino condensate chiral superfield with Kähler weight 2, and μ is a constant with dimension of mass that is left undetermined by the method of anomaly matching.

As for the Kähler potential for V, there is little knowledge beyond tree level. The best we can do at present is to treat all physically reasonable Kähler potentials

on the same footing and to look for possible general features and/or interesting special cases. In particular, we are interested in a specific class of Kähler potentials where there are significant stringy non-perturbative corrections as pointed out in [4, 7]. Before discussing this general analysis, it is instructive to examine a simple yet un-realistic linear multiplet model for gaugino condensation defined as follows [19]:

$$K = \ln V + G,$$

$$\mathcal{L}_{eff} = \int d^4\theta\, E\left\{-2 + bVG\right\} + \int d^4\theta\, \frac{E}{R}\, e^{K/2} W_{VY} + \int d^4\theta\, \frac{E}{R^{\dagger}}\, e^{K/2} \bar{W}_{VY},$$

$$G = -\sum_I \ln(T^I + \bar{T}^I). \tag{2.17}$$

This simple model describes the effective theory for (2.6) below the condensation scale, where the Kähler potential of V assumes its tree-level form. It is a "static" model of gaugino condensation in the sense that no kinetic term for U is included. From the viewpoint of the anomaly structure, static as well as dynamical models of gaugino condensation are interesting in their own right. However, as will be discussed in Chapter 3, dynamical models rather than static models generically occur in the context of superstrings. Dynamical models of gaugino condensation in the linear multiplet formalism [18, 20] have been studied less extensively. On the other hand, as will also be shown in Chapter 3, after integrating out the heavy modes the static model of gaugino condensation is proven to be the appropriate effective description for the dynamical model[4]. Therefore, in Chapter 2 we will

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[4]Unlike studies using the chiral multiplet formalism in the past, proving such a connection

concentrate on static models of gaugino condensation, and there will be no loss of generality.

With $U = -(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)V$ and $\bar{U} = -(\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - 8R^{\dagger})V$, we can rewrite the superpotential terms of $\mathcal{L}_{eff}$ as a single D term by superspace partial integration. For example, for any chiral superfield X of zero Kähler weight:

$$\frac{1}{8}\int d^4\theta \, \frac{E}{R}U_a \ln X + \text{h.c.} = \int d^4\theta \, EV_a \ln(X\bar{X})$$
$$-\partial_m\left(\int d^4\theta \, \frac{E\ln X}{8R}\mathcal{D}_{\dot{\alpha}}V_a E^{\dot{\alpha}m} + \text{h.c.}\right), \qquad (2.18)$$

where $E^{\dot{\alpha}m}$ is an element of the supervielbein, and the total derivative on the right hand side contains the chiral anomaly ($\propto \partial_m B^m \simeq F^a_{mn}\tilde{F}_a^{mn}$) of the F term on the left hand side. Therefore, up to a total derivative, the simple model (2.17) can be rewritten as follows:

$$K = \ln V + G,$$

$$\mathcal{L}_{eff} = \int d^4\theta \, E\{-2 + bVG + bV\ln(e^{-K}\bar{U}U/\mu^6)\}. \qquad (2.19)$$

In (2.19), the modular anomaly cancellation by the Green-Schwarz counterterm is transparent [19]. The Green-Schwarz counterterm bVG and the superpotential D term $bV\ln(e^{-K}\bar{U}U/\mu^6)$ are *not* modular invariant separately, but their sum is modular invariant, which ensures the modular invariance of the full theory. In fact, the Green-Schwarz counterterm cancels the T^I moduli-dependence of the superpotential completely. This is a unique feature of the linear multiplet formalism, and, as

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between static and dynamical gaugino condensation is much more non-trivial in the linear multiplet formalism with the constraint on U incorporated consistently.

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we will see later, has interesting implications for the moduli-dependence of physical quantities.

Throughout this paper only the bosonic and gravitino parts of the component lagrangian are presented, since we are interested in the vacuum configuration and the gravitino mass. In the following, we enumerate the definitions of bosonic component fields of the vector superfield V.

$$\ell = V|_{\theta=\bar{\theta}=0},$$

$$\sigma^m_{\alpha\dot{\alpha}} B_m = \frac{1}{2}[\mathcal{D}_\alpha, \mathcal{D}_{\dot{\alpha}}]V|_{\theta=\bar{\theta}=0} + \frac{2}{3}\ell\sigma^a_{\alpha\dot{\alpha}} b_a,$$

$$u = U|_{\theta=\bar{\theta}=0} = -(\bar{\mathcal{D}}^2 - 8R)V|_{\theta=\bar{\theta}=0},$$

$$\bar{u} = \bar{U}|_{\theta=\bar{\theta}=0} = -(\mathcal{D}^2 - 8R^\dagger)V|_{\theta=\bar{\theta}=0},$$

$$D = \frac{1}{8}\mathcal{D}^\beta(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\beta V|_{\theta=\bar{\theta}=0}$$

$$= \frac{1}{8}\mathcal{D}_{\dot{\beta}}(\mathcal{D}^2 - 8R^\dagger)\mathcal{D}^{\dot{\beta}}V|_{\theta=\bar{\theta}=0}, \qquad (2.20)$$

where

$$-\frac{1}{6}M = R|_{\theta=\bar{\theta}=0}, \quad -\frac{1}{6}\bar{M} = R^\dagger|_{\theta=\bar{\theta}=0}, \quad -\frac{1}{3}b_a = G_a|_{\theta=\bar{\theta}=0} \qquad (2.21)$$

are the auxiliary components of supergravity multiplet. It is convenient to write the lowest components of $\mathcal{D}^2 U$ and $\bar{\mathcal{D}}^2\bar{U}$ as follows:

$$-4F_U = \mathcal{D}^2 U|_{\theta=\bar{\theta}=0}, \quad -4\bar{F}_{\bar{U}} = \bar{\mathcal{D}}^2\bar{U}|_{\theta=\bar{\theta}=0}. \qquad (2.22)$$

$(F_U - \bar{F}_{\bar{U}})$ can be explicitly expressed as follows:

$$(F_U - \bar{F}_{\bar{U}}) = 4i\nabla^m B_m + u\bar{M} - \bar{u}M. \qquad (2.23)$$

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The expression for $(F_U + \bar{F}_{\bar{U}})$ contains the auxiliary field D. The bosonic components of T^I and $\bar{T}^I$ are

$$t^I = T^I|_{\theta=\bar{\theta}=0}, \quad -4F^I = \mathcal{D}^2 T^I|_{\theta=\bar{\theta}=0},$$

$$\bar{t}^I = \bar{T}^I|_{\theta=\bar{\theta}=0}, \quad -4\bar{F}^I = \bar{\mathcal{D}}^2 \bar{T}^I|_{\theta=\bar{\theta}=0}. \tag{2.24}$$

We leave the details of constructing the component lagrangian for this simple model (in the Kähler superspace formulation) to Section 2.3.2, and present here only the scalar potential obtained from eliminating the auxiliary fields in the boson Lagrangian given in (2.46) below:

$$V_{pot} = \frac{1}{16e^2\ell}(1 + 2b\ell - 2b^2\ell^2)\mu^6 e^{-1/b\ell}. \tag{2.25}$$

Eq. (2.25) agrees with the result obtained in [18], where the model defined by (2.17) was studied for the case of a single modulus using the superconformal formulation of supergravity.

However, this simple model is not viable. As expected, the weak-coupling limit $\ell = 0$ is always a minimum. As shown in Fig. 2.1, the scalar potential starts with $V_{pot} = 0$ at $\ell = 0$, first rises and then falls without limit as ℓ increases. Therefore, V_{pot} is unbounded from below, and this simple model has no well-defined vacuum. This may be somewhat surprising because the model defined by (2.17) superficially appears to be of the no-scale type: the Green-Schwarz counterterm, that destroys the no-scale property of chiral models and destabilizes the potential, is cancelled here by quantum effects that induce a potential for the condensate. However the resulting quantum contribution to the Lagrangian (2.19), $bV \ln(U\bar{U}/V)$, has an

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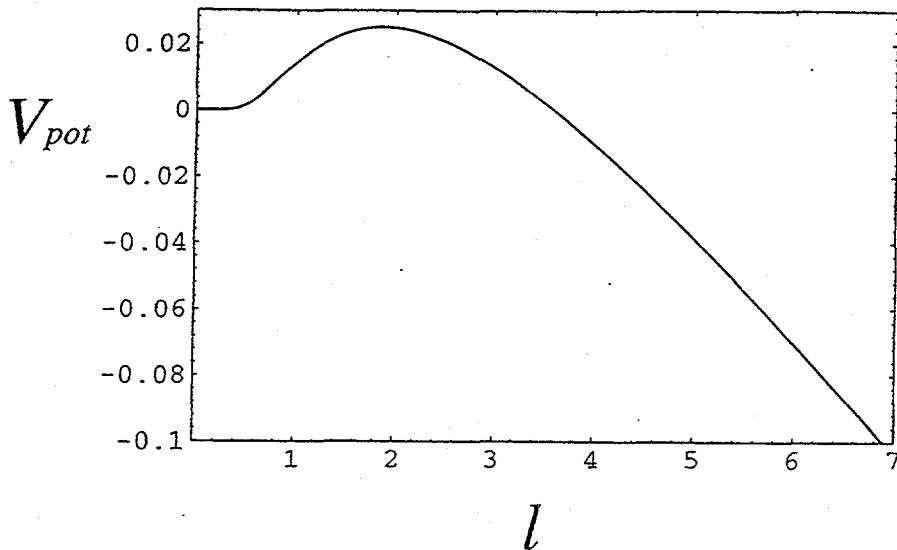
Figure 2.1: The scalar potential V_{pot} (in reduced Planck units) is plotted versus the dilaton ℓ. $\mu=1$.

implicit T^I-dependence through the superfield U due to its nonvanishing Kähler weight: $w(U) = 2$. This implicit moduli-dependence is a consequence of the anomaly matching condition, and parallels the construction of the effective theory in the chiral multiplet formalism [41, 42, 43, 44] which is also not of the no-scale form once the Green-Schwarz counterterm is included.

If we take a closer look at (2.25), it is clear that the unboundedness of V_{pot} in the strong-coupling limit $\ell \to \infty$ is caused by a term of two-loop order: $-2b^2\ell^2$. This observation strongly suggests that the underlying reason for unboundedness is our poor control over the model in the strong-coupling regime. The form of the superpotential W_{VY} is completely fixed by the underlying anomaly structure. However the Kähler potential is much less constrained, and the choice (2.17) cannot be expected

to be valid in the strong-coupling regime where the non-perturbative contributions should not be ignored. We conclude that the unboundedness shown in Fig. 2.1 simply reflects the importance of non-perturbative contributions to the Kähler potential. In particular, it is natural to expect that the stringy non-perturbative effects conjectured by Shenker [4, 7] are the non-perturbative contributions to the Kähler potential ignored in this simple model. In the absence of a better knowledge of the exact Kähler potential, we will consider models with generic Kähler potentials in the following sections.

2.3.2 General Static Model

In this section, we show how to construct the component lagrangian for generic linear multiplet models of static gaugino condensation in the Kähler superspace formulation. Further computational details can be found in [9, 34]. Although our results can probably be rephrased in the chiral multiplet formalism, the equivalent chiral multiplet formalism are expected to be rather complicated because of the constraint on the gaugino condensate chiral superfield U. Quite generally we do not expect a simple ansatz in one formalism to appear simple in the other.

As suggested in Section 2.3.1, we extend the simple model in (2.17) to linear multiplet models of static gaugino condensation with generic Kähler potentials defined as follows:

$$K = \ln V + g(V) + G,$$

$$\mathcal{L}_{eff} = \int d^4\theta\, E\,\{(-2 + f(V)) + bVG + bV\ln(e^{-K}\bar{U}U/\mu^6)\}. \qquad (2.26)$$

For convenience, we also write $\ln V + g(V) \equiv k(V)$. $g(V)$ and $f(V)$ represent quantum corrections to the tree-level Kähler potential. Here we have chosen to keep the Kähler potential under discussion as generic as possible. However, as suggested by [7], stringy non-perturbative corrections to the Kähler potential are probably the most important non-perturbative corrections. And, as we have discussed in detail in Section 2.2.2, such stringy non-perturbative corrections can be nicely parametrized by $g(V)$ and $f(V)$ using the linear multiplet formalism. According to (2.8), $g(V)$ and $f(V)$ are unambiguously related to each other by the following first-order differential equation:

$$V\frac{dg(V)}{dV} = -V\frac{df(V)}{dV} + f, \qquad (2.27)$$

$$g(V = 0) = 0 \quad \text{and} \quad f(V = 0) = 0. \qquad (2.28)$$

The boundary condition of $g(V)$ and $f(V)$ at $V = 0$ (the weak-coupling limit) is fixed by the tree-level Kähler potential. Before trying to specify $g(V)$ and $f(V)$, it is reasonable to assume for the present that $g(V)$ and $f(V)$ are arbitrary but bounded.

In the construction of the component field lagrangian, we use the chiral density multiplet method [34], which provides us with the locally supersymmetric generalization of the F term construction in global supersymmetry. The chiral density multiplet r and its hermitian conjugate r̄ for the generic model in (2.26) are:

$$\text{r} = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\{(-2 + f(V)) + bVG + bV\ln(e^{-K}\bar{U}U/\mu^6)\},$$

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$$\overline{r} = -\frac{1}{8}(\mathcal{D}^2 - 8R^\dagger)\{(-2 + f(V)) + bVG + bV\ln(e^{-K}\overline{U}U/\mu^6)\}. \qquad (2.29)$$

In order to obtain the component lagrangian $\mathcal{L}_{eff}$, we need to work out the following expression

$$\frac{1}{e}\mathcal{L}_{eff} = -\frac{1}{4}\mathcal{D}^2\mathrm{r}|_{\theta=\bar{\theta}=0} + \frac{i}{2}(\bar{\psi}_m\bar{\sigma}^m)^\alpha \mathcal{D}_\alpha \mathrm{r}|_{\theta=\bar{\theta}=0}$$

$$- (\bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n + \bar{M})\mathrm{r}|_{\theta=\bar{\theta}=0} + \text{h.c.} \qquad (2.30)$$

An important point in the computation of (2.30) is the evaluation of the component field content of the Kähler supercovariant derivatives, a rather tricky process. The details of this computation have by now become general wisdom and we can to a large extent rely on the existing literature [47]. In particular, the Lorentz transformation and the Kähler transformation are incorporated in a very similar way in the Kähler superspace formulation, and the Lorentz connection as well as the so-called Kähler connection A_M are incorporated into the Kähler supercovariant derivatives in a concise and constructive way. The Kähler connection A_M is not an independent field but rather expressed in terms of the Kähler potential K as follows:

$$A_\alpha = \frac{1}{4}E_\alpha{}^M\partial_M K, \quad A_{\dot{\alpha}} = -\frac{1}{4}E_{\dot{\alpha}}{}^M\partial_M K, \qquad (2.31)$$

$$\sigma^a_{\alpha\dot{\alpha}}A_a = \frac{3}{2}i\sigma^a_{\alpha\dot{\alpha}}G_a - \frac{1}{8}i[\mathcal{D}_\alpha,\mathcal{D}_{\dot{\alpha}}]K. \qquad (2.32)$$

In order to extract the explicit form of the various couplings, we choose to write out explicitly the vectorial part of the Kähler connection and keep only the Lorentz connection in the definition of covariant derivatives when we present the component

expressions. In the following, we give the lowest component of the vectorial part of the Kähler connection $A_m|_{\theta=\bar{\theta}=0}$ for our generic static model.

$$A_m = e_m{}^a A_a + \frac{1}{2}\psi_m{}^\alpha A_\alpha + \frac{1}{2}\bar{\psi}_{m\dot{\alpha}}A^{\dot{\alpha}}. \tag{2.33}$$

$$
\begin{aligned}
A_m|_{\theta=\bar{\theta}=0} &= -\frac{i}{4\ell}(\ell g_\ell + 1)B_m + \frac{i}{6}(\ell g_\ell - 2)e_m{}^a b_a \\
&\quad + \sum_I \frac{1}{4(t^I + \bar{t}^I)}(\nabla_m \bar{t}^I - \nabla_m t^I).
\end{aligned} \tag{2.34}
$$

$$
\begin{aligned}
g_\ell &= \frac{dg(V)}{dV}\Big|_{\theta=\bar{\theta}=0}, \quad g_{\ell\ell} = \frac{d^2 g(V)}{dV^2}\Big|_{\theta=\bar{\theta}=0}, \\
f_\ell &= \frac{df(V)}{dV}\Big|_{\theta=\bar{\theta}=0}, \quad f_{\ell\ell} = \frac{d^2 f(V)}{dV^2}\Big|_{\theta=\bar{\theta}=0}.
\end{aligned} \tag{2.35}
$$

Another hallmark of the Kähler superspace formulation are the chiral superfield X_α and the antichiral superfield $\bar{X}^{\dot{\alpha}}$. They arise in complete analogy with usual supersymmetric abelian gauge theory except that now the corresponding vector superfield is replaced by the Kähler potential:

$$
\begin{aligned}
X_\alpha &= -\frac{1}{8}(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)\mathcal{D}_\alpha K, \\
\bar{X}^{\dot{\alpha}} &= -\frac{1}{8}(\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^\dagger)\mathcal{D}^{\dot{\alpha}}K.
\end{aligned} \tag{2.36}
$$

In the computation of (2.30), we need to decompose the lowest components of the following six superfields: X_α, $\bar{X}^{\dot{\alpha}}$, $\mathcal{D}_\alpha R$, $\mathcal{D}^{\dot{\alpha}}R^\dagger$, $(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot{\alpha}}\bar{X}^{\dot{\alpha}})$ and $(\mathcal{D}^2 R + \bar{\mathcal{D}}^2 R^\dagger)$ into component fields. This is done by solving the following six simple algebraic

equations:

$$\left(V\frac{dg}{dV}+1\right)\mathcal{D}_\alpha R + X_\alpha = \Xi_\alpha, \qquad (2.37)$$

$$3\mathcal{D}_\alpha R + X_\alpha = -2(\sigma^{cb}\epsilon)_{\alpha\varphi}T_{cb}{}^{\varphi}. \qquad (2.38)$$

$$\left(V\frac{dg}{dV}+1\right)\mathcal{D}^{\dot\alpha}R^\dagger + \bar{X}^{\dot\alpha} = \bar{\Xi}^{\dot\alpha}, \qquad (2.39)$$

$$3\mathcal{D}^{\dot\alpha}R^\dagger + \bar{X}^{\dot\alpha} = -2(\bar\sigma^{cb}\epsilon)^{\dot\alpha\dot\varphi}T_{cb\dot\varphi}. \qquad (2.40)$$

$$\left(V\frac{dg}{dV}+1\right)(\mathcal{D}^2 R + \bar{\mathcal{D}}^2 R^\dagger) + (\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot\alpha}\bar{X}^{\dot\alpha}) = \Delta, \qquad (2.41)$$

$$3(\mathcal{D}^2 R + \bar{\mathcal{D}}^2 R^\dagger) + (\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot\alpha}\bar{X}^{\dot\alpha}) = -2R_{ba}{}^{ba} + 12 G^a G_a$$

$$+ 96 R R^\dagger. \qquad (2.42)$$

The identities (2.38), (2.40) and (2.42) arise solely from the structure of Kähler superspace. (2.38) and (2.40) involve the torsion superfields $T_{cb}{}^{\varphi}$ and $T_{cb\dot\varphi}$, which in their lowest components contain the curl of the Rarita-Schwinger field. The identities (2.37), (2.39) and (2.41) arise directly from the definitions of X_α, $\bar{X}^{\dot\alpha}$, $(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot\alpha}\bar{X}^{\dot\alpha})$, and therefore they depend on the Kähler potential explicitly. Computing X_α, $\bar{X}^{\dot\alpha}$ and $(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot\alpha}\bar{X}^{\dot\alpha})$ according to (2.36) defines the contents of Ξ_α, $\bar{\Xi}^{\dot\alpha}$ and Δ respectively. In the following, we present the component field expressions of the lowest components of Ξ_α, $\bar{\Xi}^{\dot\alpha}$ and Δ.

$$\frac{i}{2}(\bar\psi_m\bar\sigma^m)^\alpha\Xi_\alpha|_{\theta=\bar\theta=0} - \frac{i}{2}\bar{\Xi}_{\dot\alpha}(\bar\sigma^m\psi_m)^{\dot\alpha}|_{\theta=\bar\theta=0}$$

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$$= -\frac{1}{8\ell}(\ell g_\ell + 1)(\bar{u} + \frac{4}{3}\ell\bar{M})(\psi_m \sigma^{mn}\psi_n)$$

$$-\frac{1}{8\ell}(\ell g_\ell + 1)(u + \frac{4}{3}\ell M)(\bar{\psi}_m \bar{\sigma}^{mn}\bar{\psi}_n)$$

$$+\frac{i}{4\ell}(\ell g_\ell + 1)(\eta^{mn}\eta^{pq} - \eta^{mq}\eta^{np})(\bar{\psi}_m \bar{\sigma}_n \psi_p)\nabla_q \ell$$

$$+\frac{i}{6}(\ell g_\ell + 1)\epsilon^{mnpq}(\bar{\psi}_m \bar{\sigma}_n \psi_p)e_q{}^a b_a$$

$$-\frac{i}{4\ell}(\ell g_\ell + 1)\epsilon^{mnpq}(\bar{\psi}_m \bar{\sigma}_n \psi_p)B_q$$

$$-\frac{1}{4}(\mathcal{D}^a\mathcal{D}^\alpha k)\psi_{a\alpha}|_{\theta=\bar{\theta}=0} - \frac{1}{4}\bar{\psi}_{a\dot{\alpha}}(\mathcal{D}^a\mathcal{D}^{\dot{\alpha}} k)|_{\theta=\bar{\theta}=0}. \qquad (2.43)$$

The way $\Xi_\alpha|_{\theta=\bar{\theta}=0}$ and $\bar{\Xi}^{\dot{\alpha}}|_{\theta=\bar{\theta}=0}$ are presented in (2.43) will be useful for the computation of (2.30).

$$\Delta|_{\theta=\bar{\theta}=0}$$

$$= -\frac{1}{\ell^2}(\ell^2 g_{\ell\ell} - 1)\nabla^m \ell \nabla_m \ell + \frac{1}{\ell^2}(\ell^2 g_{\ell\ell} - 1)B^m B_m$$

$$+4\sum_I \frac{1}{(t^I + \bar{t}^I)^2}\nabla^m \bar{t}^I \nabla_m t^I - \frac{4}{9}(\ell^2 g_{\ell\ell} - \ell g_\ell - 2)\bar{M}M$$

$$+\frac{4}{9}(\ell^2 g_{\ell\ell} + 2\ell g_\ell + 1)b^a b_a - 4\sum_I \frac{1}{(t^I + \bar{t}^I)^2}\bar{F}^I F^I$$

$$-\frac{4}{3\ell}(\ell^2 g_{\ell\ell} + \ell g_\ell)B^m e_m{}^a b_a - \frac{1}{2\ell}(\ell g_\ell + 1)(F_U + \bar{F}_{\bar{U}})$$

$$-\frac{1}{6\ell}(2\ell^2 g_{\ell\ell} - \ell g_\ell - 3)(u\bar{M} + \bar{u}M) - \frac{1}{4\ell^2}(\ell^2 g_{\ell\ell} - 1)\bar{u}u$$

$$+2\nabla^m \nabla_m k - (\mathcal{D}^a\mathcal{D}^\alpha k)\psi_{a\alpha}|_{\theta=\bar{\theta}=0} - \bar{\psi}_{a\dot{\alpha}}(\mathcal{D}^a\mathcal{D}^{\dot{\alpha}} k)|_{\theta=\bar{\theta}=0}. \qquad (2.44)$$

It is unnecessary to decompose the last two terms in (2.43) and in (2.44) because they eventually cancel with one another.

Eqs.(2.31–44) describe the key steps involved in the computation of (2.30).

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The rest of it is standard and will not be detailed here. In the following, we present the component field expression of $\mathcal{L}_{eff}$ as the sum of the bosonic part $\mathcal{L}_B$ and the gravitino part $\mathcal{L}_{\tilde{G}}$ as follows.[5]

$$\mathcal{L}_{eff} = \mathcal{L}_B + \mathcal{L}_{\tilde{G}}. \tag{2.45}$$

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}_B &= -\frac{1}{2}\mathcal{R} - \frac{1}{4\ell^2}(1 + \ell g_\ell)\nabla^m\ell\,\nabla_m\ell \\
&+ \frac{1}{4\ell^2}(\ell g_\ell + 1)B^m B_m - (1 + b\ell)\sum_I \frac{1}{(t^I + \bar{t}^I)^2}\nabla^m\bar{t}^I\,\nabla_m t^I \\
&+ \frac{1}{9}(\ell g_\ell - 2)\bar{M}M - \frac{1}{9}(\ell g_\ell - 2)b^a b_a \\
&+ (1 + b\ell)\sum_I \frac{1}{(t^I + \bar{t}^I)^2}\bar{F}^I F^I \\
&+ \frac{1}{8\ell}\{1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6) + 2b\ell\}(F_U + \bar{F}_{\bar{U}}) \\
&- \frac{1}{8\ell}\{1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6) + \frac{2}{3}b\ell(\ell g_\ell + 1)\}(u\bar{M} + \bar{u}M) \\
&- \frac{1}{16\ell^2}(1 + 2b\ell)(1 + \ell g_\ell)\bar{u}u \\
&- \frac{i}{2}b\ln(\frac{\bar{u}}{u})\nabla^m B_m - \frac{i}{2}b\sum_I \frac{1}{(t^I + \bar{t}^I)}(\nabla^m\bar{t}^I - \nabla^m t^I)B_m. \tag{2.46}
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}_{\tilde{G}} &= \frac{1}{2}\epsilon^{mnpq}(\bar{\psi}_m\bar{\sigma}_n\nabla_p\psi_q - \psi_m\sigma_n\nabla_p\bar{\psi}_q) \\
&- \frac{1}{8\ell}\{1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6)\}\bar{u}\,(\psi_m\sigma^{mn}\psi_n) \\
&- \frac{1}{8\ell}\{1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6)\}u\,(\bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n)
\end{aligned}
$$

---

[5]Only the bosonic and gravitino parts of the component field expressions are presented here.

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$$-\frac{1}{4}(1+b\ell)\sum_I \frac{1}{(t^I + \bar{t}^I)} \epsilon^{mnpq}(\bar{\psi}_m \bar{\sigma}_n \psi_p)(\nabla_q \bar{t}^I - \nabla_q t^I)$$

$$+\frac{i}{4\ell}(1+b\ell)(1+\ell g_\ell)(\eta^{mn}\eta^{pq} - \eta^{mq}\eta^{np})(\bar{\psi}_m \bar{\sigma}_n \psi_p)\nabla_q \ell$$

$$-\frac{i}{4}b\ell(\eta^{mn}\eta^{pq} - \eta^{mq}\eta^{np})(\bar{\psi}_m \bar{\sigma}_n \psi_p)\nabla_q \ln(\bar{u}u)$$

$$+\frac{1}{4}b\ell\,\epsilon^{mnpq}(\bar{\psi}_m \bar{\sigma}_n \psi_p)\nabla_q \ln(\frac{\bar{u}}{u}). \tag{2.47}$$

For completeness, we also give the definitions of covariant derivatives:

$$\nabla_m \ell = \partial_m \ell, \quad \nabla_m t^I = \partial_m t^I, \quad \nabla_m \bar{t}^I = \partial_m \bar{t}^I,$$

$$\nabla_m \psi_n{}^\alpha = \partial_m \psi_n{}^\alpha + \psi_n{}^\beta \omega_{m\beta}{}^\alpha, \quad \nabla_m \bar{\psi}_{n\dot{\alpha}} = \partial_m \bar{\psi}_{n\dot{\alpha}} + \bar{\psi}_{n\dot{\beta}} \omega_m{}^{\dot{\beta}}{}_{\dot{\alpha}}. \tag{2.48}$$

To proceed further, we need to eliminate the auxiliary fields from $\mathcal{L}_{eff}$ through their equations of motion. The equation of motion of the auxiliary field $(F_U + \bar{F}_U)$ is

$$f + 1 + b\ell \ln(e^{-k}\bar{u}u/\mu^6) + 2b\ell = 0. \tag{2.49}$$

Eq. (2.49) implies that in static models of gaugino condensation the auxiliary field $\bar{u}u$ is expressed in terms of dilaton ℓ. The equations of motion of F^I, $\bar{F}^I$ and the auxiliary fields b^a, M, $\bar{M}$ of the supergravity multiplet are (if $\ell g_\ell - 2 \neq 0$)

$$F^I = 0, \quad \bar{F}^I = 0,$$

$$b^a = 0,$$

$$M = \frac{3}{4}bu, \quad \bar{M} = \frac{3}{4}b\bar{u}. \tag{2.50}$$

Now we are left with only one auxiliary field to eliminate, where this auxiliary field can be either $i\ln(\bar{u}/u)$ or B_m. This corresponds to the fact that there are two ways

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to perform duality transformation. If we take $i\ln(\bar{u}/u)$ to be auxiliary, its equation of motion is

$$\nabla_q\{\, B^q \;-\; \frac{i}{2}\ell\,\epsilon^{mnpq}(\bar{\psi}_m\bar{\sigma}_n\psi_p)\,\} \;=\; 0, \qquad (2.51)$$

which ensures that $\{B^q - \frac{i}{2}\ell\,\epsilon^{mnpq}(\bar{\psi}_m\bar{\sigma}_n\psi_p)\}$ is dual to the field strength of an antisymmetric tensor [18]. The term $B^m B_m$ in the lagrangian $\mathcal{L}_{eff}$ thus generates a kinetic term of this antisymmetric tensor field and its coupling to the gravitino. The other way to perform the duality transformation is to treat B_m as an auxiliary field by rewriting the term $-\frac{i}{2}b\ln(\bar{u}/u)\nabla^m B_m$ in $\mathcal{L}_{eff}$ as $\frac{i}{2}bB^m\nabla_m\ln(\bar{u}/u)$, and then to eliminate B_m from $\mathcal{L}_{eff}$ through its equation of motion as follows:

$$\begin{aligned}
B_m \;=\; & -i\frac{b\ell^2}{(\ell g_\ell + 1)}\nabla_m\ln(\frac{\bar{u}}{u}) \\
& + i\frac{b\ell^2}{(\ell g_\ell + 1)}\sum_I{}' \frac{1}{(t^I + \bar{t}^I)}(\,\nabla_m\bar{t}^I - \nabla_m t^I\,).
\end{aligned} \qquad (2.52)$$

The terms $B^m B_m$ and $\frac{i}{2}bB^m\nabla_m\ln(\bar{u}/u)$ in $\mathcal{L}_{eff}$ will generate a kinetic term for $i\ln(\bar{u}/u)$. It is clear that $i\ln(\bar{u}/u)$ plays the role of the pseudoscalar dual to B_m in the lagrangian obtained from the above after a duality transformation. With (2.49–52), it is then trivial to eliminate the auxiliary fields from $\mathcal{L}_{eff}$. The physics of $\mathcal{L}_{eff}$ will be investigated in the following sections.

2.3.3   Gaugino Condensate and the Gravitino Mass

Hidden-sector gaugino condensation has been a very attractive scheme [15, 16] for supersymmetry breaking in the context of superstring. However, before we can make any progress in superstring phenomenology, two important questions must

be answered: is the dilaton stabilized, and is supersymmetry broken? Past analyses have generally found that, in the absence of a second source of supersymmetry breaking, the dilaton is destabilized in the direction of vanishing gauge coupling constant (the so-called runaway dilaton problem) and supersymmetry is unbroken. To address the above questions in generic linear multiplet models of gaugino condensation, we first show how the three issues of supersymmetry breaking, gaugino condensation and dilaton stabilization are reformulated, and how they are interrelated, by examining the explicit expressions for the gravitino mass and the gaugino condensate. A detailed investigation of the vacuum will be presented in Section 2.4.

The explicit expression for the gaugino condensate in terms of the dilaton ℓ is determined by (2.49):

$$\bar{u}u = \frac{1}{e^2}\ell\mu^6 e^{g-(f+1)/b\ell}. \tag{2.53}$$

With $g(\ell)=0$ and $f(\ell)=0$, we recover the result of the simple model (2.17) [18]. For generic models, the dilaton dependence of the gaugino condensate involves $g(\ell)$ and $f(\ell)$ which represent stringy non-perturbative corrections to the tree-level Kähler potential. Recall that in the linear multiplet formalism the gauge coupling of the superstring effective field theory is $g^2(M_s) = \langle 2\ell/(1+f(\ell))\rangle$. Therefore, it is easy to see that the dependence on the gauge coupling constant $g(M_s)$ of the gaugino condensate is indeed consistent with the usual results obtained by the renormalization group equation arguments. According to our assumption of boundedness for $g(\ell)$ and $f(\ell)$ (especially at $\ell=0$ where following (2.28) we have the boundary conditions $g(\ell=0)=0$ and $f(\ell=0)=0$), $\ell=0$ is the only pole of $g-(f+1)/b\ell$.

Therefore, we can draw a simple and clear relation between $\langle \bar{u}u \rangle$ and $\langle \ell \rangle$: gauginos condense (*i.e.*, $\langle \bar{u}u \rangle \neq 0$) if and only if the dilaton is stabilized (*i.e.*, $\langle \ell \rangle \neq 0$.) Note that this conclusion does not depend on the details of the quantum corrections g and f.

Another physical quantity of interest is the gravitino mass $m_{\tilde{G}}$ which is the natural order parameter measuring supersymmetry breaking. The expression for $m_{\tilde{G}}$ follows directly from $\mathcal{L}_{\tilde{G}}$.

$$m_{\tilde{G}} = \frac{1}{4}b\sqrt{\langle \bar{u}u \rangle}, \tag{2.54}$$

where we have used (2.49). This expression for the gravitino mass is simple and elegant even for generic linear multiplet models of static gaugino condensation. From the viewpoint of superstring effective theories, an interesting feature of (2.54) is that the gravitino mass $m_{\tilde{G}}$ contains no explicit dependence on the modulus T^I, which provides a direct relation between $m_{\tilde{G}}$ and $\langle \bar{u}u \rangle$. This feature can be traced to the fact that the Green-Schwarz counterterm cancels the T^I dependence of the superpotential completely, a unique feature of the linear multiplet formalism. As we will see in Section 4.5, this unique feature is still true even in a generic string orbifold model. We recall that in the chiral multiplet formalism of gaugino condensation – without the condition (2.12) – that have been studied previously (with or without the Green-Schwarz cancellation mechanism), $m_{\tilde{G}}$ always involves a moduli-dependence, and therefore the relation between supersymmetry breaking (*i.e.*, $m_{\tilde{G}} \neq 0$) and gaugino condensation (*i.e.*, $\langle \bar{u}u \rangle \neq 0$) remains undetermined un-

til the true vacuum can be found. By contrast, in generic linear multiplet models of gaugino condensation, there is a simple and direct relation, Eq.(2.54): supersymmetry is broken (*i.e.*, $m_{\tilde{G}} \neq 0$) if and only if gaugino condensation occurs ($\langle \bar{u}u \rangle \neq 0$). We wish to emphasize that the above features of the linear multiplet model are unique in the sense that they are simple only in the linear multiplet model. This is related to the fact pointed out in Sections 2.1 and 2.2.3 that, once the constraint (2.12) on the condensate field U is imposed, the chiral counterpart of the linear multiplet model is in general very complicated, and it is more natural to work in the linear multiplet formalism. Our conclusion of this section is best illustrated by the following diagram:

$$
\boxed{\begin{array}{c} \text{Supersymmetry} \\ \text{Breaking} \end{array}} \iff \boxed{\begin{array}{c} \text{Gaugino} \\ \text{Condensation} \end{array}} \iff \boxed{\begin{array}{c} \text{Stabilized} \\ \text{Dilaton} \end{array}}
$$
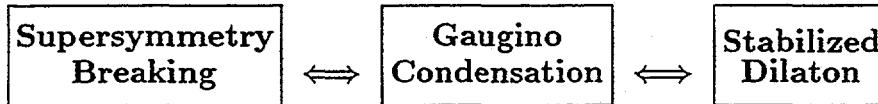
The equivalence among the above three issues is obvious. Therefore, in the following section, we only need to focus on one of the three issues in the investigation of the vacuum, for example, the issue of dilaton stabilization.

2.4 Supersymmetry Breaking and Stabilization of the Dilaton

As argued in Section 2.3.1, non-perturbative contributions to the Kähler potential should be introduced to cure the unboundedness problem of the simple model (2.17). In the context of the generic model of static gaugino condensation (2.26), it is therefore interesting to address the question as to how the simple model (2.17)

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should be modified in order to obtain a viable theory (*i.e.*, with V_{pot} bounded from below). We start with the scalar potential V_{pot} arising from (2.46) after solving for the auxiliary fields (using (2.49), (2.50) and (2.52)). Recalling that (2.27) yields the identity $1 + \ell g_\ell = 1 + f - \ell f_\ell$, we obtain

$$V_{pot} = \frac{1}{16e^2\ell} \{ (1 + f - \ell f_\ell)(1 + b\ell)^2 - 3b^2\ell^2 \} \mu^6 e^{g - (1+f)/b\ell}, \qquad (2.55)$$

which depends only on the dilaton ℓ. The necessary and sufficient condition for V_{pot} to be bounded from below is

$$f - \ell f_\ell \geq -\mathcal{O}(\ell e^{1/b\ell}) \quad \text{for} \quad \ell \to 0, \qquad (2.56)$$

$$f - \ell f_\ell \geq 2 \qquad \text{for} \quad \ell \to \infty. \qquad (2.57)$$

It is clear that condition (2.56) is not at all restrictive, and therefore has no nontrivial implication. On the contrary, condition (2.57) is quite restrictive; in particular the simple model (2.17) violates this condition. Condition (2.57) not only restricts the possible forms of the function f in the strong-coupling regime but also has important implications for dilaton stabilization and for supersymmetry breaking. To make the above statement more precise, let us revisit the unbounded potential of Fig. 2.1, with the tree-level Kähler potential defined by $g(V) = f(V) = 0$. Adding physically reasonable corrections $g(V)$ and $f(V)$ (constrained by (2.56–57)) to this simple model should not qualitatively alter its behavior in the weak-coupling regime. Therefore, as in Fig. 2.1, the potential of the modified model in the weak-coupling regime starts with $V_{pot} = 0$ at $\ell = 0$, first rises and then falls as ℓ increases. On the other hand, adding $g(V)$ and $f(V)$ completely alters the strong-coupling behavior

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of the original simple model. As guaranteed by condition (2.57), the potential of the modified model in the strong-coupling regime is always bounded from below, and in most cases rises as ℓ increases. Joining the weak-coupling behavior of the modified model to its strong-coupling behavior therefore strongly suggests that its potential has a non-trivial minimum (at $\ell \neq 0$). Furthermore, if this non-trivial minimum is global, then the dilaton is stabilized. We conclude that not only does (2.56–57) tell us how to modify the theory, but a large class of theories so modified have naturally a stabilized dilaton (and therefore broken supersymmetry by the argument of Section 2.3.3). In view of the fact that there is currently little knowledge of the exact Kähler potential, the above conclusion, which applies to generic Kähler potentials subject to (2.56–57), is especially important to the search for supersymmetry breaking and dilaton stabilization[6]. As discussed in Sections 2.1 and 2.2.2, the most interesting physical implication of this conclusion is that it is actually stringy non-perturbative effects that stabilize the dilaton and allow dynamical supersymmetry breaking via the field-theoretical non-perturbative effect of gaugino condensation. Furthermore, (2.57) can be interpreted as the necessary condition for stringy non-perturbative effects to stabilize the dilaton.[7]

Here we use a simple example only to illustrate the above important argument. A more detailed discussion of possible stringy non-perturbative corrections will be

[6]Similar points of view was advocated in [48] using the chiral multiplet formalism. However, neither modular invariance nor the important constraint (2.12) was considered in [48].

[7]In the presence of significant stringy non-perturbative effects, (2.57) could have implications for gauge coupling unification. This is considered in the study of multi-gaugino and matter condensation [14].
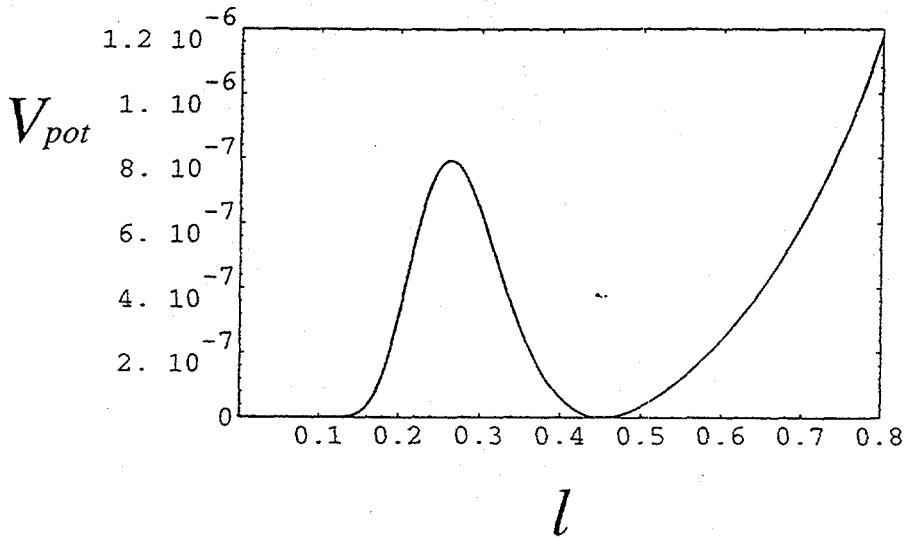
Figure 2.2: The scalar potential V_{pot} (in reduced Planck units) is plotted versus the dilaton ℓ. $A = 6.92$, $B = 1$ and $\mu = 1$.

given in Chapters 4 and 5 where a generic and phenomenologically viable model is presented. Consider $f(V) = Ae^{-B/V}$, where A and B are constants to be determined by the non-perturbative dynamics. The regulation conditions (2.56–57) require $A \geq 2$. In Fig. 2.2, V_{pot} is plotted versus the dilaton ℓ, where $A = 6.92$, $B = 1$ and $\mu = 1$. Fig. 2.2 has two important features. Firstly, V_{pot} of this modified theory is indeed bounded from below, and the dilaton is stabilized. Therefore, we obtain supersymmetry breaking, gaugino condensation and dilaton stabilization in this example. The gravitino mass is $m_{\tilde{G}} = 7.6 \times 10^{-5}$ in reduced Planck units. Secondly, the *vev* of dilaton is stabilized at the phenomenologically interesting range ($\langle \ell \rangle = 0.45$ in Fig. 2.2). The above features involve no unnaturalness since they are insensitive to A. Furthermore, the dilaton is naturally stabilized in a weak

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coupling regime if B is of order one. Fig. 2.2 is a nice realization of the argument in the preceding paragraph. It should be contrasted with the racetrack model where at least three gaugino condensates and large numerical coefficients are needed in order to achieve similar results. Besides, the racetrack model has a serious phenomenological problem of having a large negative cosmological constant. We can also consider possible stringy non-perturbative contributions to the Kähler potential suggested in [4]. It turns out that we obtain the same general features as those of Fig. 2.2. This is not surprising since, as argued in the preceding paragraph, the important features that we find in Fig. 2.2 are common to a large class of models. More such discussions will be presented in Chapters 4 and 5 in conjunction with other issues.

Note that the value of the cosmological constant is irrelevant to the arguments presented here and in Section 2.3.3. In other words, the generic model (2.26) suffers from the usual cosmological constant problem, although we can find a fine-tuned subset of models whose cosmological constants vanish. For example, the cosmological constant of Fig. 2.2 vanishes by fine tuning A. It remains an open question as to whether or not the cosmological constant problem could be resolved within the context of the linear multiplet formalism of gaugino condensation if the exact Kähler potential were known.

2.5 Concluding Remarks

We have presented a concrete example of a solution to the infamous runaway dilaton problem, within the context of local supersymmetry and the linear multiplet formalism for the string dilaton. We considered models for a static condensate that reflect the modular anomaly of the effective field theory while respecting the exact modular invariance of the underlying string theory. The simplest such model [18, 19] has a nontrivial potential that is, however, unbounded in the direction of strong coupling. Including stringy non-perturbative corrections [4, 7] to the Kähler potential for the dilaton, the potential is stabilized, allowing a vacuum configuration in which condensation occurs and supersymmetry is broken. This is in contrast to previous analyses, based on the chiral multiplet formalism for the dilaton, in which supersymmetry breaking with a bounded vacuum energy was achieved only by introducing an additional source of supersymmetry breaking, such as a constant term in the superpotential [16, 29, 46].

In further contrast to most of the models studied using the chiral multiplet formalism, supersymmetry breaking arises from a nonvanishing vacuum expectation value of the auxiliary field associated with the dilaton rather than the moduli: roughly speaking, in the dual chiral multiplet formalism, $\langle F_S \rangle \neq 0$ rather than $\langle F^I \rangle \neq 0$. That is, only the dilaton participates in supersymmetry breaking (the so-called dilaton-dominated scenario.) As we shall see in Chapter 4, this unique feature is in fact true in generic string orbifold models, which therefore has non-

trivial implications for FCNC. As a consequence, gaugino masses and A terms are generated at tree level. Although scalar masses are still protected at tree level by a Heisenberg symmetry [49], they will be generated at one loop by renormalizable interactions[8]. For the model considered here, the hierarchy (about five orders of magnitude) between the Planck scale and the gravitino mass is insufficient to account for the observed scale of electroweak symmetry breaking. Of course, this is completely due to the large gauge content of the hidden E_8 gauge group under consideration in this chapter, and will certainly be improved when a generic string model with a product of smaller hidden gauge groups $\mathcal{G} = \Pi_a \mathcal{G}_a$. In that case, we will have to generalize the studies of this chapter by considering multiple gaugino condensation as well as hidden matter condensation. Another unsatisfactory feature of the model presented in Chapter 2 is that, according to (2.55), the moduli T^I remain flat directions of the scalar potential, and therefore the *vev* of t^I is undetermined. Fortunately, this is a feature belonging only to string models with hidden E_8 gauge group and no hidden matter. As we shall see in Chapter 4, in a generic string model where multiple gaugino condensation as well as hidden matter condensation occurs naturally, hidden matter condensation together with string threshold corrections[9] generates a non-perturbative potential for the moduli T^I. Furthermore, the moduli are therefore stabilized at the self-dual point. The generalization of our formalism to generic string orbifold models, including models

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[8]The situation is more complicated in a generic orbifold model, and will be discussed in Chapter 5.

[9]Both are required by modular invariance.

without universal anomaly cancellation, will be presented in Chapter 4.

As mentioned before, we have only dealt with generic models of static gaugino condensation in this chapter, but in the context of supergravity or superstrings it can be shown that models of dynamical gaugino condensation rather than models of static gaugino condensation occur. Therefore, in the next chapter we will answer two questions: first, we show how to construct generic models of dynamical gaugino condensation using the linear multiplet formalism. Secondly, we study how the models of dynamical gaugino condensation are connected to the models of static gaugino condensation, and show that static gaugino condensation is indeed the appropriate effective description of dynamical gaugino condensation and therefore justify the use of static gaugino condensation in Chapter 2. Notice that the Kalb-Ramond field (or the model-independent axion, in the dual description) remains massless in the static models considered here. It has recently been shown in the context of global supersymmetry [20, 18] that an axion mass term is naturally generated in models of dynamical gaugino condensation. Again, as we shall see in Chapter 3, one of the axions does get a very large mass through dynamical gaugino condensation in the context of local supersymmetry. On the other hand, after this very heavy axion is integrated out, the resulting axion content is in fact the same as that of static gaugino condensation, and we are still left with a massless model-independent axion. Furthermore, we will show in Chapters 4 and 5 that this model-independent axion axion will pick up a very small mass through multiple gaugino condensation. It can escape the cosmological bound on the axion decay

constant and it has the desirable properties to be the candidate for the QCD axion.

Chapter 3

Dynamical Gaugino
Condensation

3.1 Introduction

In Chapter 2, we have studied models of static gaugino condensation using the linear multiplet formalism. As mentioned before, one of the major motivations for studying models of dynamical gaugino condensation is the observation that kinetic terms of the gaugino condensate naturally arise from field-theoretical loop corrections [19] as well as from classical string corrections [50]. For example, the relevant field-theoretical one-loop correction has been computed using the chiral multiplet formalism [19, 51]:

$$\mathcal{L}_{one-loop} \ni \frac{N_G}{128\pi^2} \int d^4\theta\, E\, (S + \bar{S})^2 \, (\mathcal{W}^\alpha \mathcal{W}_\alpha) \, (\mathcal{W}_{\dot{\alpha}} \mathcal{W}^{\dot{\alpha}}) \ln \Lambda^2, \qquad (3.1)$$

where Λ is the effective cut-off and N_G is the number of gauge degrees of freedom. Therefore, the confined theory using the linear multiplet formalism should contain a term which corresponds to (3.1):

$$\mathcal{L}_{eff} \ni \int d^4\theta\, E\, \frac{\bar{U}U}{V^2}, \qquad (3.2)$$

as well as higher-order corrections $\left(\bar{U}U/V^2\right)^2$, $\left(\bar{U}U/V^2\right)^3$, $\cdots$. These D terms are corrections to the Kähler potential, and will generate the kinetic terms for the gaugino condensate U. An interesting interpretation of these corrections is that they are S-duality invariant in the sense defined by Gaillard and Zumino [52]. This S-duality, which is an SL(2,R) symmetry among elementary fields, is a symmetry of the equations of motion only of the dilaton-gauge-gravity sector in the limit of vanishing gauge coupling constants. The implication of this S-duality for gaugino condensation has recently been studied in [19] using the chiral multiplet formalism.

For studies of gaugino condensation in the past where the important constraint (2.12) was not included, the connection between static and dynamical gaugino condensation is very easy to see and trivial: static gaugino condensation is just the low-energy limit of dynamical gaugino condensation after the gaugino condensate is integrated out. However, it certainly becomes a non-trivial issue once the constraint (2.12) is included, and it is necessary to settle this issue in order to justify the use of static gaugino condensation in the context of superstrings or supergravity. Therefore, in this chapter we would like to study generic models of dynamical gaugino condensation. In Section 3.2, the field component Lagrangian for the generic model of dynamical gaugino condensation is constructed, and its vacuum structure is analyzed. In Section 3.3, the S-dual models of dynamical gaugino condensation are studied. In particular, we show that the model of static gaugino condensation is the appropriate effective description for the model of dynamical gaugino condensation and its implications.

3.2 Generic Model of Dynamical Gaugino Condensation

It will be shown in this section how to construct the component field Lagrangian for the generic model of dynamical gaugino condensation using the Kähler superspace formulation of supergravity [34, 35]. Similar to Chapter 2, we consider here string orbifold models with gauge groups $E_8 \otimes E_6 \otimes U(1)^2$, three untwisted (1,1) moduli T^I ($I = 1, 2, 3$) [31, 32, 36], and universal modular anomaly cancellation [40] (e.g., the Z_3 and Z_7 orbifolds). The confined E_8 hidden sector is described by

the following generic model of a single dynamical gaugino condensate U with Kähler potential K:

$$K = \ln V + g(V, \bar{U}U) + G,$$

$$\mathcal{L}_{eff} = \int d^4\theta\, E \left\{ \left(-2 + f(V, \bar{U}U) \right) + bVG \right\} + \left\{ \int d^4\theta\, \frac{E}{R} e^{K/2} W_{V\Upsilon} + \text{h.c.} \right\},$$

$$G = -\sum_I \ln(T^I + \bar{T}^I), \tag{3.3}$$

where $U = -(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)V$, $\bar{U} = -(\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - 8R^{\dagger})V$. We also write $\ln V + g(V, \bar{U}U) \equiv k(V, \bar{U}U)$. The term $\left(-2 + f(V, \bar{U}U) \right)$ of $\mathcal{L}_{eff}$ is the superspace integral which yields the kinetic actions for the linear multiplet, supergravity, matter, and gaugino condensate. The term bVG is the Green-Schwarz counterterm [31] which cancels the full modular anomaly here. $b = C/8\pi^2 = 2b_0/3$, and $C = 30$ is the Casimir operator in the adjoint representation of E_8. b_0 is the E_8 one-loop β-function coefficient. $g(V, \bar{U}U)$ and $f(V, \bar{U}U)$ represent the quantum corrections to the tree-level Kähler potential. $g(V, \bar{U}U)$ and $f(V, \bar{U}U)$ are taken to be arbitrary but bounded here. The dynamical model (3.3) is the straightforward generalization of the static model (2.26) by including the $\bar{U}U$ dependence in the Kähler potential. Using superspace partial integration (2.18), up to a total derivative we can also rewrite (3.3) as a single D term:

$$K = \ln V + g(V, \bar{U}U) + G,$$

$$\mathcal{L}_{eff} = \int d^4\theta\, E \left\{ \left(-2 + f(V, \bar{U}U) \right) + bVG + bV\ln(e^{-K}\bar{U}U/\mu^6) \right\}. \tag{3.4}$$

Only the bosonic and gravitino parts of the component field Lagrangian will be

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presented here. In the following, for convenience and completeness we enumerate the definitions of the bosonic component fields:

$$\ell = V|_{\theta=\bar{\theta}=0},$$

$$\sigma^m_{\alpha\dot{\alpha}} B_m = \frac{1}{2}[\mathcal{D}_\alpha, \mathcal{D}_{\dot{\alpha}}]V|_{\theta=\bar{\theta}=0} + \frac{2}{3}\ell\sigma^a_{\alpha\dot{\alpha}}b_a,$$

$$u = U|_{\theta=\bar{\theta}=0} = -(\bar{\mathcal{D}}^2 - 8R)V|_{\theta=\bar{\theta}=0},$$

$$\bar{u} = \bar{U}|_{\theta=\bar{\theta}=0} = -(\mathcal{D}^2 - 8R^\dagger)V|_{\theta=\bar{\theta}=0},$$

$$-4F_U = \mathcal{D}^2 U|_{\theta=\bar{\theta}=0}, \quad -4\bar{F}_U = \bar{\mathcal{D}}^2\bar{U}|_{\theta=\bar{\theta}=0},$$

$$D = \frac{1}{8}\mathcal{D}^\beta(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\beta V|_{\theta=\bar{\theta}=0}$$

$$= \frac{1}{8}\mathcal{D}_{\dot{\beta}}(\mathcal{D}^2 - 8R^\dagger)\mathcal{D}^{\dot{\beta}}V|_{\theta=\bar{\theta}=0},$$

$$t^I = T^I|_{\theta=\bar{\theta}=0}, \quad -4F^I = \mathcal{D}^2 T^I|_{\theta=\bar{\theta}=0},$$

$$\bar{t}^I = \bar{T}^I|_{\theta=\bar{\theta}=0}, \quad -4\bar{F}^I = \bar{\mathcal{D}}^2\bar{T}^I|_{\theta=\bar{\theta}=0}, \tag{3.5}$$

where $b_a = -3G_a|_{\theta=\bar{\theta}=0}$, $M = -6R|_{\theta=\bar{\theta}=0}$, $\tilde{M} = -6R^\dagger|_{\theta=\bar{\theta}=0}$ are the auxiliary components of the supergravity multiplet. $(F_U - \bar{F}_U)$ can be expressed as follows:

$$(F_U - \bar{F}_U) = 4i\nabla^m B_m + u\tilde{M} - \bar{u}M, \tag{3.6}$$

and $(F_U + \bar{F}_U)$ contains the auxiliary field D. We also write $Z \equiv \bar{U}U$, and its bosonic component $z \equiv Z|_{\theta=\bar{\theta}=0} = \bar{u}u$.

The construction of component field Lagrangian using chiral density multiplet method [34] has been detailed in Chapter 2, and therefore only the key steps are presented here. The chiral density multiplet r and its hermitian conjugate $\bar{r}$ for the

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generic model (3.3) are:

$$\mathbf{r} = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\left\{\left(-2 + f(V,\bar{U}U)\right) + bVG + bV\ln(e^{-K}\bar{U}U/\mu^6)\right\},$$

$$\bar{\mathbf{r}} = -\frac{1}{8}(\mathcal{D}^2 - 8R^\dagger)\left\{\left(-2 + f(V,\bar{U}U)\right) + bVG + bV\ln(e^{-K}\bar{U}U/\mu^6)\right\}, \qquad (3.7)$$

and the component field Lagrangian $\mathcal{L}_{eff}$ is the same as (2.30). The $A_m|_{\theta=\bar{\theta}=0}$ for the generic model (3.3) is:

$$
\begin{aligned}
A_m|_{\theta=\bar{\theta}=0} = {} & -\frac{i}{4\ell}\cdot\frac{(1+\ell g_\ell)}{(1-zg_z)}B_m + \frac{i}{6}\left[\frac{(1+\ell g_\ell)}{(1-zg_z)} - 3\right]e_m{}^a b_a \\
& + \frac{1}{4(1-zg_z)}\sum_I \frac{1}{(t^I + \bar{t}^I)}(\nabla_m \bar{t}^I - \nabla_m t^I) \\
& - \frac{zg_z}{4(1-zg_z)}\nabla_m \ln\left(\frac{\bar{u}}{u}\right). \qquad (3.8)
\end{aligned}
$$

The following are the simplified notations for partial derivatives of g:

$$g_\ell \equiv \frac{\partial g(\ell,z)}{\partial \ell}, \quad g_z \equiv \frac{\partial g(\ell,z)}{\partial z}, \qquad (3.9)$$

and similarly for other functions.

We need to decompose the lowest components of the following six superfields: X_α, $\bar{X}^{\dot{\alpha}}$, $\mathcal{D}_\alpha R$, $\mathcal{D}^{\dot{\alpha}}R^\dagger$, $(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot{\alpha}}\bar{X}^{\dot{\alpha}})$ and $(\mathcal{D}^2 R + \bar{\mathcal{D}}^2 R^\dagger)$ into component fields, where

$$X_\alpha = -\frac{1}{8}(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)\mathcal{D}_\alpha K,$$

$$\bar{X}^{\dot{\alpha}} = -\frac{1}{8}(\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^\dagger)\mathcal{D}^{\dot{\alpha}}K,$$

$$(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot{\alpha}}\bar{X}^{\dot{\alpha}}) = -\frac{1}{8}\mathcal{D}^2\bar{\mathcal{D}}^2 K - \frac{1}{8}\bar{\mathcal{D}}^2 \mathcal{D}^2 K - \mathcal{D}^{\alpha\dot{\alpha}}\mathcal{D}_{\alpha\dot{\alpha}}K$$

$$- G^{\alpha\dot{\alpha}}[\mathcal{D}_\alpha, \mathcal{D}_{\dot{\alpha}}]K + 2R^\dagger\bar{\mathcal{D}}^2 K + 2R\mathcal{D}^2 K$$

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$$-(\mathcal{D}^\alpha G_{\alpha\dot\alpha} - 2\mathcal{D}_{\dot\alpha}R^\dagger)\mathcal{D}^{\dot\alpha}K$$

$$+(\mathcal{D}^{\dot\alpha}G_{\alpha\dot\alpha} + 2\mathcal{D}_\alpha R)\mathcal{D}^\alpha K. \tag{3.10}$$

This is done by solving the following six algebraic equations:

$$\left(1 + V\frac{\partial g}{\partial V}\right)\mathcal{D}_\alpha R + \left(1 - Z\frac{\partial g}{\partial Z}\right)X_\alpha \;=\; \Xi_\alpha, \tag{3.11}$$

$$3\mathcal{D}_\alpha R + X_\alpha \;=\; -2(\sigma^{cb}\epsilon)_{\alpha\varphi}T_{cb}{}^\varphi. \tag{3.12}$$

$$\left(1 + V\frac{\partial g}{\partial V}\right)\mathcal{D}^{\dot\alpha}R^\dagger + \left(1 - Z\frac{\partial g}{\partial Z}\right)\bar X^{\dot\alpha} \;=\; \bar\Xi^{\dot\alpha}, \tag{3.13}$$

$$3\mathcal{D}^{\dot\alpha}R^\dagger + \bar X^{\dot\alpha} \;=\; -2(\bar\sigma^{cb}\epsilon)^{\dot\alpha\dot\varphi}T_{cb\dot\varphi}. \tag{3.14}$$

$$\left(1 + V\frac{\partial g}{\partial V}\right)(\mathcal{D}^2 R + \bar{\mathcal{D}}^2 R^\dagger) + \left(1 - Z\frac{\partial g}{\partial Z}\right)(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot\alpha}\bar X^{\dot\alpha}) \;=\; \Delta, \tag{3.15}$$

$$3(\mathcal{D}^2 R + \bar{\mathcal{D}}^2 R^\dagger) + (\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot\alpha}\bar X^{\dot\alpha}) \;=\; -2R_{ba}{}^{ba} + 12G^\alpha G_\alpha$$

$$+ 96RR^\dagger. \tag{3.16}$$

The computation of (3.10) defines the contents of Ξ_α, $\bar\Xi^{\dot\alpha}$ and Δ. Eqs. (3.8–16) describe the key steps in the computations of (2.30). In the following sections, several important issues of this construction will be discussed.

3.2.1 Canonical Einstein Term

In order to have the correctly normalized Einstein term in $\mathcal{L}_{eff}$, an appropriate constraint should be imposed on the generic model (3.3). Therefore, it is shown below how to compute the Einstein term for (3.3). According to (3.3), the following

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are those terms in $\mathcal{L}_{eff}$ that will contribute to the Einstein term:

$$\frac{1}{e}\mathcal{L}_{eff} \ni \frac{1}{4}\left[2 - f + \ell f_\ell - b\ell(1 + \ell g_\ell)\right](\mathcal{D}^2 R + \bar{\mathcal{D}}^2 R^\dagger)|_{\theta=\bar{\theta}=0}$$

$$+ \frac{1}{32}\left[z f_z + b\ell(1 - z g_z)\right]\left(\frac{1}{\bar{u}}\mathcal{D}^2\bar{\mathcal{D}}^2\bar{U} + \frac{1}{u}\bar{\mathcal{D}}^2\mathcal{D}^2 U\right)|_{\theta=\bar{\theta}=0}. \qquad (3.17)$$

Note that the terms $\mathcal{D}^2\bar{\mathcal{D}}^2\bar{U}$ and $\bar{\mathcal{D}}^2\mathcal{D}^2 U$ are related to $\mathcal{D}^\alpha X_\alpha$ and $\mathcal{D}_{\dot{\alpha}}\bar{X}^{\dot{\alpha}}$ through the following identities:

$$\mathcal{D}^2\bar{\mathcal{D}}^2\bar{U} = 16\mathcal{D}^\alpha\mathcal{D}_\alpha\bar{U} + 64i G^a\mathcal{D}_a\bar{U} - 48\bar{U}G^a G_a + 48i\bar{U}\mathcal{D}^a G_a$$

$$- 8\bar{U}\mathcal{D}^\alpha X_\alpha + 16 R^\dagger\bar{\mathcal{D}}^2\bar{U} + 8(\mathcal{D}^\alpha G_{\alpha\dot{\alpha}})(\mathcal{D}^{\dot{\alpha}}\bar{U}).$$

$$\bar{\mathcal{D}}^2\mathcal{D}^2 U = 16\mathcal{D}^\alpha\mathcal{D}_a U - 64i G^a\mathcal{D}_a U - 48 U G^a G_a - 48i U\mathcal{D}^a G_a$$

$$- 8U\mathcal{D}_{\dot{\alpha}}\bar{X}^{\dot{\alpha}} + 16 R\mathcal{D}^2 U - 8(\mathcal{D}^{\dot{\alpha}}G_{\alpha\dot{\alpha}})(\mathcal{D}^\alpha U). \qquad (3.18)$$

The contributions of $(\mathcal{D}^2 R + \bar{\mathcal{D}}^2 R^\dagger)|_{\theta=\bar{\theta}=0}$ and $(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot{\alpha}}\bar{X}^{\dot{\alpha}})|_{\theta=\bar{\theta}=0}$ to the Einstein term are obtained by solving (3.15–16):

$$(\mathcal{D}^2 R + \bar{\mathcal{D}}^2 R^\dagger)|_{\theta=\bar{\theta}=0} \ni -\frac{2(1 - z g_z)}{(2 - \ell g_\ell - 3 z g_z)}R_{ba}{}^{ba}|_{\theta=\bar{\theta}=0}.$$

$$(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot{\alpha}}\bar{X}^{\dot{\alpha}})|_{\theta=\bar{\theta}=0} \ni +\frac{2(1 + \ell g_\ell)}{(2 - \ell g_\ell - 3 z g_z)}R_{ba}{}^{ba}|_{\theta=\bar{\theta}=0}. \qquad (3.19)$$

By combining (3.17–19), it is straightforward to show that the Einstein term in $\mathcal{L}_{eff}$ is correctly normalized if and only if the following constraint is imposed:

$$(1 + z f_z)(1 + \ell g_\ell) = (1 - z g_z)(1 - \ell f_\ell + f), \qquad (3.20)$$

which is a first-order partial differential equation. From now on, the study of the generic model (3.3) always assumes the constraint (3.20). (3.20) will be useful in

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simplifying the expression of $\mathcal{L}_{eff}$, and it turns out to be convenient to define h as follows:

$$
\begin{aligned}
h &\equiv \frac{(1 + zf_z)}{(1 - zg_z)}, \\
&= \frac{(1 - \ell f_\ell + f)}{(1 + \ell g_\ell)}.
\end{aligned}
\tag{3.21}
$$

Furthermore, the partial derivatives of h satisfy the following consistency condition:

$$
(h - \ell h_\ell)(zg_z - 1) + zh_z(1 + \ell g_\ell) + 1 = 0. \tag{3.22}
$$

Eqs. (3.21–22) will also be very useful in simplifying the expression of $\mathcal{L}_{eff}$. Notice that $h = 1$ for generic models of static gaugino condensation, and (3.20) is reduced to (2.27). We will show in Section 3.3.2 how to construct physically interesting solutions for this partial differential equation (3.20).

3.2.2   Component Field Lagrangian with Auxiliary Fields

Once the issue of canonical Einstein term is settled, it is straightforward to compute $\mathcal{L}_{eff}$ according to (3.6–13). The rest of it is standard and will not be detailed here. Because the component construction of supergravity is well known for its complexity, here we try our best to minimize irrelevant details. However, two important aspects of this construction using the linear multiplet formalism are worth emphasizing: how to solve the constraint (2.12) and how to perform a duality transformation for the vector component B_m of V. As we shall see, they have non-trivial implications for the axions. Therefore, first we present the component Lagrangian with auxiliary fields, and in the next section we show how to perform

a duality transformation for B_m. In the following, we present the component field expression of $\mathcal{L}_{eff}$ as the sum of the bosonic Lagrangian $\mathcal{L}_B$ and the gravitino Lagrangian $\mathcal{L}_{\tilde{G}}$.

$$\mathcal{L}_{eff} = \mathcal{L}_B + \mathcal{L}_{\tilde{G}}. \tag{3.23}$$

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}_B ={}& -\frac{1}{2}\mathcal{R} - \frac{1}{4\ell^2}(h - \ell h_\ell)(1 + \ell g_\ell)\nabla^m\ell\,\nabla_m\ell \\
&+ \frac{1}{2\ell}zh_z(1 + \ell g_\ell)\,\nabla^m\mathrm{ln}(\bar{u}u)\,\nabla_m\ell \\
&+ \frac{u}{4\bar{u}}h_z\cdot zg_z\frac{(2 - zg_z)}{(1 - zg_z)}\nabla^m\bar{u}\,\nabla_m\bar{u} \\
&- \frac{1}{2}h_z\left[\frac{(2 - zg_z)}{(1 - zg_z)} - zg_z\right]\nabla^m\bar{u}\,\nabla_m u \\
&+ \frac{\bar{u}}{4u}h_z\cdot zg_z\frac{(2 - zg_z)}{(1 - zg_z)}\nabla^m u\,\nabla_m u \\
&- \frac{zh_z}{2(1 - zg_z)}\sum_I\frac{1}{(t^I + \bar{t}^I)}(\nabla^m\bar{t}^I - \nabla^m t^I)\,\nabla_m\mathrm{ln}\left(\frac{\bar{u}}{u}\right) \\
&+ \frac{zh_z}{4(1 - zg_z)}\sum_{I,J}\frac{1}{(t^I + \bar{t}^I)(t^J + \bar{t}^J)}\nabla^m\bar{t}^I\,\nabla_m\bar{t}^J \\
&- \frac{1}{2}\sum_{I,J}\left[2(h + b\ell)\delta_{IJ} + \frac{zh_z}{(1 - zg_z)}\right]\frac{\nabla^m\bar{t}^I\,\nabla_m t^J}{(t^I + \bar{t}^I)(t^J + \bar{t}^J)} \\
&+ \frac{zh_z}{4(1 - zg_z)}\sum_{I,J}\frac{1}{(t^I + \bar{t}^I)(t^J + \bar{t}^J)}\nabla^m t^I\,\nabla_m t^J \\
&+ \frac{(2 - \ell g_\ell - 3zg_z)}{9(1 - zg_z)}b^a b_a \\
&+ \frac{(1 + \ell g_\ell)}{4\ell^2(1 - zg_z)}B^m B_m \\
&+ \frac{i}{2\ell}\left[h + b\ell - \frac{1}{(1 - zg_z)}\right]B^m\,\nabla_m\mathrm{ln}\left(\frac{\bar{u}}{u}\right)
\end{aligned}
$$

$$-\frac{i}{2\ell}\left[h + b\ell - \frac{1}{(1-zg_z)}\right]\sum_I \frac{(\nabla^m \bar{t}^I - \nabla^m t^I)}{(t^I + \bar{t}^I)}B_m$$

$$+4h_z(1 - zg_z)(\nabla^m B_m)^2$$

$$-2ih_z\left[1 - zg_z - \frac{1}{3}(1 + \ell g_\ell)\right](u\bar{M} - \bar{u}M)\nabla^m B_m$$

$$-\frac{1}{4}h_z\left[1 - zg_z - \frac{2}{3}(1 + \ell g_\ell)\right](u\bar{M} - \bar{u}M)^2$$

$$-\frac{1}{9}\left[3 + (\ell h_\ell - h)(1 + \ell g_\ell)\right]\bar{M}M$$

$$-\frac{1}{8\ell}\left[\begin{array}{c}1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6) \\ + \frac{2}{3}(\ell h_\ell + b\ell)(1 + \ell g_\ell)\end{array}\right](u\bar{M} + \bar{u}M)$$

$$+\frac{1}{4}h_z(1 - zg_z)(F_U + \bar{F}_{\bar{U}})^2$$

$$+\left\{\begin{array}{c}\frac{1}{8\ell}\left[1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6)\right] \\ + \frac{1}{4\ell}(\ell h_\ell + b\ell)(1 - zg_z) \\ - \frac{1}{6}h_z(1 + \ell g_\ell)(u\bar{M} + \bar{u}M)\end{array}\right\}(F_U + \bar{F}_{\bar{U}})$$

$$+(h + b\ell)\sum_I \frac{1}{(t^I + \bar{t}^I)^2}\bar{F}^I F^I$$

$$-\frac{1}{16\ell^2}(\ell h_\ell + h + 2b\ell)(1 + \ell g_\ell)\bar{u}u. \tag{3.24}$$

$$\frac{1}{e}\mathcal{L}_{\tilde{G}} = \frac{1}{2}\epsilon^{mnpq}(\bar{\psi}_m\bar{\sigma}_n\nabla_p\psi_q - \psi_m\sigma_n\nabla_p\bar{\psi}_q)$$

$$-\frac{1}{8\ell}\left[1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6)\right]\bar{u}(\psi_m\sigma^{mn}\psi_n)$$

$$-\frac{1}{8\ell}\left[1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6)\right]u(\bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n)$$

$$-\frac{1}{4}(h + b\ell)\sum_I \frac{1}{(t^I + \bar{t}^I)}\epsilon^{mnpq}(\bar{\psi}_m\bar{\sigma}_n\psi_p)(\nabla_q\bar{t}^I - \nabla_q t^I)$$

$$+\frac{i}{4\ell}(h+b\ell)(1+\ell g_\ell)(\eta^{mn}\eta^{pq}-\eta^{mq}\eta^{np})(\bar{\psi}_m\bar{\sigma}_n\psi_p)\nabla_q\ell$$

$$-\frac{i}{4}[(1-zg_z)(h+b\ell)-1](\eta^{mn}\eta^{pq}-\eta^{mq}\eta^{np})(\bar{\psi}_m\bar{\sigma}_n\psi_p)\nabla_q\ln(\bar{u}u)$$

$$+\frac{1}{4}(h-1+b\ell)\,\epsilon^{mnpq}(\bar{\psi}_m\bar{\sigma}_n\psi_p)\nabla_q\ln\left(\frac{\bar{u}}{u}\right).\tag{3.25}$$

The bosonic Lagrangian $\mathcal{L}_B$ contains usual auxiliary fields and the vector field B_m which is dual to an axion. The details of this duality and the structure of $\mathcal{L}_B$ will be discussed in the following sections. The gravitino Lagrangian $\mathcal{L}_{\tilde{G}}$ is in its simplest form. An important physical quantity in $\mathcal{L}_{\tilde{G}}$ is the gravitino mass $m_{\tilde{G}}$ which is the natural order parameter measuring supersymmetry breaking. The expression of $m_{\tilde{G}}$ follows directly from $\mathcal{L}_{\tilde{G}}$:

$$m_{\tilde{G}}=\left\langle\left|\frac{1}{8\ell}\left[1+f+b\ell\ln(e^{-k}\bar{u}u/\mu^6)\right]u\right|\right\rangle.\tag{3.26}$$

3.2.3 Duality Transformation of B_m

As pointed out in [18, 21], the constraint (2.12) allows us to interpret the degrees of freedom of U as those of a 3-form supermultiplet, and the vector field B_m is dual to a 3-form Γ^{npq}. Since a 3-form is dual to a 0-form in four dimensions, B_m is also dual to a pseudoscalar a. In this section, we show explicitly how to rewrite the B_m part of $\mathcal{L}_B$ in terms of the dual description using a. According to (3.24), the B_m terms in $\mathcal{L}_B$ are:

$$\frac{1}{e}\mathcal{L}_B\;\ni\;+\frac{(1+\ell g_\ell)}{4\ell^2(1-zg_z)}B^mB_m$$

$$+\frac{i}{2\ell}\left[h+b\ell-\frac{1}{(1-zg_z)}\right]B^m\nabla_m\ln\left(\frac{\bar{u}}{u}\right)$$

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$$-\frac{i}{2\ell}\left[h + b\ell - \frac{1}{(1 - zg_z)}\right]\sum_I \frac{(\nabla^m \bar{t}^I - \nabla^m t^I)}{(t^I + \bar{t}^I)}B_m$$

$$-2ih_z\left[1 - zg_z - \frac{1}{3}(1 + \ell g_\ell)\right](u\bar{M} - \bar{u}M)\nabla^m B_m$$

$$+4h_z(1 - zg_z)(\nabla^m B_m)^2. \tag{3.27}$$

They are described by the following generic Lagrangian of B_m:

$$\frac{1}{e}\mathcal{L}_{B_m} = \alpha B^m B_m + \beta \nabla^m B_m + \zeta^m B_m + \tau(\nabla^m B_m)^2. \tag{3.28}$$

To find the dual description of $\mathcal{L}_{B_m}$, consider the following Lagrangian $\mathcal{L}_{Dual}$.

$$\frac{1}{e}\mathcal{L}_{Dual} = \alpha B^m B_m + \beta \nabla^m B_m + \zeta^m B_m + a\nabla^m B_m - \frac{1}{4\tau}a^2. \tag{3.29}$$

In $\mathcal{L}_{Dual}$, the auxiliary field a acts like a Lagrangian multiplier, and its equation of motion is:

$$a = 2\tau\nabla^m B_m. \tag{3.30}$$

Therefore, $\mathcal{L}_{B_m}$ follows directly from $\mathcal{L}_{Dual}$ using (3.30). On the other hand, we can treat the B_m in $\mathcal{L}_{Dual}$ as auxiliary, and write down the equation of motion for B_m as follows:

$$B_m = \frac{1}{2\alpha}\left(\nabla_m a + \nabla_m \beta - \zeta_m\right). \tag{3.31}$$

Eliminating B_m from $\mathcal{L}_{Dual}$ through (3.31) and then performing a field re-definition $a \Rightarrow a - \beta$, we obtain the Lagrangian $\mathcal{L}_a$ of a:

$$\frac{1}{e}\mathcal{L}_a = -\frac{1}{4\alpha}\left(\nabla^m a - \zeta^m\right)\left(\nabla_m a - \zeta_m\right) - \frac{1}{4\tau}\left(a - \beta\right)^2. \tag{3.32}$$

Therefore, $\mathcal{L}_a$ is the dual description of $\mathcal{L}_{B_m}$ in terms of a which is interpreted as an axion. Notice that dynamical gaugino condensation naturally generates a mass term

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for the axion a which corresponds to the appearance of non-vanishing $(\nabla^m B_m)^2$ in the dual description. The fact that a is massive in dynamical gaugino condensation has already been observed in [18, 20]. On the other hand, the $(\nabla^m B_m)^2$ term vanishes in static gaugino condensation (*i.e.*, $h_z = 0$ in (3.27)), and it is found that the model-independent axion dual to B_m is either massless or very light [12, 14, 18, 20]. This issue of axion mass seems to be a contradiction because we expect static gaugino condensation to be the appropriate effective description of dynamical gaugino condensation; the resolution is the following: In comparison with static gaugino condensation (*e.g.*, [12, 14]), dynamical gaugino condensation contains one more axionic degree of freedom a, and indeed a is very massive (*e.g.*, compared to the dilaton mass). As will be shown in Section 3.3.1, after integrating out this massive axion a, the resulting axionic contents of dynamical gaugino condensation are identical to those of static gaugino condensation. Therefore, at low energy we are always left with a massless or very light model-independent axion.

According to (3.27–28) and (3.32), the $\mathcal{L}_{eff}$ defined by (3.23–25) is rewritten in the dual description as follows:

$$\mathcal{L}_{eff} = \mathcal{L}_{kin} + \mathcal{L}_{pot} + \mathcal{L}_{\tilde{G}}, \tag{3.33}$$

where $\mathcal{L}_{kin}$ and $\mathcal{L}_{pot}$ refer to the kinetic part and the non-kinetic part of the bosonic Lagrangian respectively. $\mathcal{L}_{\tilde{G}}$ is defined by (3.25).

$$\frac{1}{e}\mathcal{L}_{kin} = -\frac{1}{2}\mathcal{R} - \frac{1}{4\ell^2}(h - \ell h_\ell)(1 + \ell g_\ell)\nabla^m \ell \, \nabla_m \ell$$

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$$-\frac{(1-zg_z)}{(1+\ell g_\ell)}\ell^2\nabla^m a\,\nabla_m a + \frac{1}{2\ell}zh_z(1+\ell g_\ell)\,\nabla^m\ln(\bar{u}u)\,\nabla_m\ell$$

$$+\,i\frac{(1-zg_z)}{(1+\ell g_\ell)}\left[h + b\ell - \frac{1}{(1-zg_z)}\right]\ell\nabla^m a\,\nabla_m\ln\!\left(\frac{\bar{u}}{u}\right)$$

$$-\,i\frac{(1-zg_z)}{(1+\ell g_\ell)}\left[h + b\ell - \frac{1}{(1-zg_z)}\right]\sum_I\frac{(\nabla^m\bar{t}^I - \nabla^m t^I)}{(t^I+\bar{t}^I)}\ell\nabla_m a$$

$$+\frac{1}{4}\left\{\begin{array}{l} zh_z\cdot zg_z\frac{(2-zg_z)}{(1-zg_z)} \\[2mm] +\frac{(1-zg_z)}{(1+\ell g_\ell)}\left[h + b\ell - \frac{1}{(1-zg_z)}\right]^2 \end{array}\right\}\frac{1}{\bar{u}^2}\nabla^m\bar{u}\,\nabla_m\bar{u}$$

$$-\frac{1}{2}\left\{\begin{array}{l} zh_z\left[\frac{(2-zg_z)}{(1-zg_z)}-zg_z\right] \\[2mm] +\frac{(1-zg_z)}{(1+\ell g_\ell)}\left[h + b\ell - \frac{1}{(1-zg_z)}\right]^2 \end{array}\right\}\frac{1}{\bar{u}u}\nabla^m\bar{u}\,\nabla_m u$$

$$+\frac{1}{4}\left\{\begin{array}{l} zh_z\cdot zg_z\frac{(2-zg_z)}{(1-zg_z)} \\[2mm] +\frac{(1-zg_z)}{(1+\ell g_\ell)}\left[h + b\ell - \frac{1}{(1-zg_z)}\right]^2 \end{array}\right\}\frac{1}{u^2}\nabla^m u\,\nabla_m u$$

$$-\frac{1}{2}\left\{\begin{array}{l} \frac{zh_z}{(1-zg_z)} \\[2mm] +\frac{(1-zg_z)}{(1+\ell g_\ell)}\left[h + b\ell - \frac{1}{(1-zg_z)}\right]^2 \end{array}\right\}\sum_I\frac{(\nabla^m\bar{t}^I - \nabla^m t^I)}{(t^I+\bar{t}^I)}\nabla_m\ln\!\left(\frac{\bar{u}}{u}\right)$$

$$+\frac{1}{4}\left\{\begin{array}{l} \frac{zh_z}{(1-zg_z)} \\[2mm] +\frac{(1-zg_z)}{(1+\ell g_\ell)}\left[h + b\ell - \frac{1}{(1-zg_z)}\right]^2 \end{array}\right\}\sum_{I,J}\frac{\nabla^m\bar{t}^I\,\nabla_m\bar{t}^J}{(t^I+\bar{t}^I)(t^J+\bar{t}^J)}$$

$$-\frac{1}{2}\sum_{I,J}\left\{\begin{array}{l} 2(h+b\ell)\delta_{IJ} + \frac{zh_z}{(1-zg_z)} \\[2mm] +\frac{(1-zg_z)}{(1+\ell g_\ell)}\left[h + b\ell - \frac{1}{(1-zg_z)}\right]^2 \end{array}\right\}\frac{\nabla^m\bar{t}^I\,\nabla_m t^J}{(t^I+\bar{t}^I)(t^J+\bar{t}^J)}$$

$$+\frac{1}{4}\left\{\begin{array}{l} \frac{zh_z}{(1-zg_z)} \\[2mm] +\frac{(1-zg_z)}{(1+\ell g_\ell)}\left[h + b\ell - \frac{1}{(1-zg_z)}\right]^2 \end{array}\right\}\sum_{I,J}\frac{\nabla^m t^I\,\nabla_m t^J}{(t^I+\bar{t}^I)(t^J+\bar{t}^J)}. \qquad (3.34)$$

$$\frac{1}{e}\mathcal{L}_{pot} = \frac{h_z(1+\ell g_\ell)^2}{36(1-zg_z)}(u\bar{M}-\bar{u}M)^2$$

$$-\frac{1}{9}\left[3 + (\ell h_\ell - h)(1 + \ell g_\ell)\right]\bar{M}M$$

$$-\frac{1}{8\ell}\left[\begin{array}{c} 1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6) \\ + \frac{2}{3}(\ell h_\ell + b\ell)(1 + \ell g_\ell) \end{array}\right](u\bar{M} + \bar{u}M)$$

$$-\frac{i}{4}\left[1 - \frac{(1 + \ell g_\ell)}{3(1 - zg_z)}\right]a(u\bar{M} - \bar{u}M)$$

$$+\frac{1}{4}h_z(1 - zg_z)(F_U + \bar{F}_{\bar{U}})^2$$

$$+\left\{\begin{array}{c} \frac{1}{8\ell}\left[1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6)\right] \\ + \frac{1}{4\ell}(\ell h_\ell + b\ell)(1 - zg_z) \\ - \frac{1}{6}h_z(1 + \ell g_\ell)(u\bar{M} + \bar{u}M) \end{array}\right\}(F_U + \bar{F}_{\bar{U}})$$

$$+(h + b\ell)\sum_I \frac{1}{(t^I + \bar{t}^I)^2}\bar{F}^I F^I$$

$$-\frac{1}{16\ell^2}(\ell h_\ell + h + 2b\ell)(1 + \ell g_\ell)\bar{u}u$$

$$-\frac{\bar{u}u}{16zh_z(1 - zg_z)}a^2. \tag{3.35}$$

The $b^a b_a$ term has been eliminated by its equation of motion, $b^a = 0$, and $\mathcal{L}_{kin}$ is in its simplest form. Note that the kinetic terms of those axionic degrees of freedom a, $i\ln(\bar{u}/u)$ and $i(\bar{t}^I - t^I)$ are more complicated, which essentially reflects the non-trivial constraint (2.12) satisfied by U and $\bar{U}$. An important issue is the structure of $\mathcal{L}_{pot}$, and it will be discussed in the next section.

3.2.4 The Scalar Potential

It is straightforward to solve the equations of motion for the auxiliary fields b^a, F^I, $\bar{F}^I$, M, $\bar{M}$ and $(F_U + \bar{F}_{\bar{U}})$ respectively as follows:

$$b^a = 0,$$

$$F^I = 0, \quad \bar{F}^I = 0,$$

$$M = -\frac{3}{8\ell}\left[1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6)\right]u - \frac{3iu}{4}a,$$

$$\bar{M} = -\frac{3}{8\ell}\left[1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6)\right]\bar{u} + \frac{3i\bar{u}}{4}a,$$

$$(F_U + \bar{F}_{\bar{U}}) = \frac{(\ell h_\ell - h)}{4zh_z}\left[1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6)\right]\frac{\bar{u}u}{\ell}$$

$$-\frac{(\ell h_\ell + b\ell)}{2zh_z}\cdot\frac{\bar{u}u}{\ell}. \tag{3.36}$$

Note that $\langle|M|\rangle = 3m_{\tilde{G}}$ because $\langle a \rangle = 0$ always. To obtain the scalar potential, the auxiliary fields are eliminated from $\mathcal{L}_{eff}$ defined by (3.33), and $\mathcal{L}_{eff}$ is then rewritten as follows:

$$\frac{1}{e}\mathcal{L}_{eff} = \frac{1}{e}\mathcal{L}_{kin} - V_{pot} + \frac{1}{e}\mathcal{L}_{\tilde{G}}, \tag{3.37}$$

where V_{pot} is the scalar potential. $\mathcal{L}_{kin}$ and $\mathcal{L}_{\tilde{G}}$ are defined by (3.34) and (3.25) respectively.

$$V_{pot} = \frac{1}{16}(\ell h_\ell + h + 2b\ell)(1 + \ell g_\ell)\frac{\bar{u}u}{\ell^2}$$

$$+ \frac{1}{64zh_z(1 - zg_z)}\left\{\begin{array}{l} 1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6) \\ \\ +2(\ell h_\ell + b\ell)(1 - zg_z) \end{array}\right\}^2 \frac{\bar{u}u}{\ell^2}$$

$$-\frac{(2 - \ell g_\ell - 3zg_z)}{64(1 - zg_z)}\left[1 + f + b\ell\ln(e^{-k}\bar{u}u/\mu^6)\right]^2\frac{\bar{u}u}{\ell^2}$$

$$+\frac{(h - \ell h_\ell - 3zh_z)\bar{u}u}{16zh_z}a^2. \tag{3.38}$$

Several interesting aspects of V_{pot} can be uncovered. Firstly, there is always a trivial vacuum with $\langle V_{pot}\rangle = 0$ in the specific weak-coupling limit defined as follows:

$$\ell \to 0, \quad z \to \frac{1}{e^2}\ell\mu^6 e^{-1/b\ell} \to 0, \quad \text{and} \quad g(\ell, z),\ f(\ell, z) \to 0. \tag{3.39}$$

Note that quantum corrections to the Kähler potential, g and f, should vanish in this limit. As expected, this is consistent with the well-known runaway behavior of the dilaton near the weak-coupling limit.

To proceed further, in the following of this section we only study V_{pot} in the $z \ll 1$ regime. Since a physically interesting model of dynamical gaugino condensation should predict a small scale of condensation (i.e., $\langle z\rangle \ll 1$), there is no loss of generality in this choice. Note that in the $z \ll 1$ regime we have $h \approx 1$, $\ell h_\ell \approx 0$, $zh_z \approx 0$ and $zg_z \approx 0$ up to small corrections that depend on z. The structure of V_{pot} can be analyzed as follows: The only axion-dependent term in V_{pot} is the effective axion mass term, the last term in V_{pot}. In order to avoid a tachyonic axion, the sign of the effective axion mass term must be positive. Therefore, the absence of a tachyonic axion requires $zh_z > 0$, which is the first piece of information about the $\bar{U}U$-dependence of the dynamical model. Furthermore, $\langle a\rangle = 0$ always, and therefore the last term in V_{pot} is of no significance in discussing the vacuum structure. Because of $zh_z > 0$, the second term in V_{pot} is always positive. The signs of

the first term and the third term in V_{pot} remain undetermined in general; however, near the weak-coupling limit the first term is positive and the third term is negative (which is expected because the third term is the contribution of auxiliary fields M and $\bar{M}$). Notice that the second term in V_{pot} contains a factor $1/zh_z$ ($1/zh_z \gg 1$), and therefore it is the dominant contribution to V_{pot} except near the path γ defined by $\left\{ 1 + f + b\ell \ln(e^{-k}\bar{u}u/\mu^6) + 2(\ell h_\ell + b\ell)(1 - zg_z) \right\} = 0$. Hence, the vacuum always sits close to the path γ. This observation will be essential to the following discussion of vacuum structure.

The second piece of information about the $\bar{U}U$-dependence of the dynamical model can be obtained as follows. For $0 < \ell < \infty$, the first term and the third term in V_{pot} vanish in the limit $z \to 0$ generically. If h_z has a pole at $z = 0$, then the second term in V_{pot} also vanishes for $z \to 0$ and $0 < \ell < \infty$. Therefore, for those dynamical models whose h_z has a pole at $z = 0$, there exists a continuous family of degenerate vacua (parametrized by $\langle \ell \rangle$) with $\langle z \rangle = 0$ (no gaugino condensation), $m_{\tilde{G}} = 0$ (unbroken supersymmetry) and $\langle V_{pot} \rangle = 0$. In other words, in the vicinity of $z = 0$ those models always exhibit runaway of z toward the degenerate vacua at $z = 0$ which do not have the desired physical features; whether those models may possess other non-trivial vacuum or not is outside the scope of this simple analysis.

On the other hand, the dynamical models whose h_z has no pole at $z = 0$ are much more interesting. If h_z has no pole at $z = 0$, then $V_{pot} \to \infty$ for $z \to 0$ and $0 < \ell < \infty$. Therefore, these dynamical models exhibit no runaway of z toward $z = 0$ except for the weak-coupling limit (3.39). Furthermore, the equation of

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motion for z is

$$1 + f + b\ell \ln(e^{-k}\bar{u}u/\mu^6) + 2(\ell h_t + b\ell)(1 - zg_z) = 0 + \mathcal{O}(zh_z). \qquad (3.40)$$

Impose (3.40), and from (3.26) we have the gravitino mass $m_{\tilde{G}} = \frac{1}{4}b\langle |u|\rangle + \mathcal{O}\left(z^{3/2}h_z\right)$. To the lowest order, it is identical to the $m_{\tilde{G}}$ of static gaugino condensation, (2.54); therefore, similar to Section 2.3 we can argue that *supersymmetry is broken if and only if the dilaton is stabilized* for dynamical gaugino condensation. In fact, for dynamical models whose h_z has no pole at $z = 0$, it can be shown that they are effectively described by static gaugino condensation of Chapter 2. As pointed out in Section 3.1, kinetic terms of the gaugino condensate U naturally arise in generic string models, where these terms are S-duality invariant and correspond to corrections $\bar{U}U/V^2$, $\left(\bar{U}U/V^2\right)^2$, $\cdots$ to the Kähler potential. This interesting class of S-dual dynamical gaugino condensation obviously belongs to dynamical models whose h_z has no pole at $z = 0$ discussed here. In Section 3.3, S-dual dynamical gaugino condensation will be studied in detail.

3.3 S-Dual Model of Dynamical Gaugino Condensation

As discussed in Section 3.1, we consider in this section models of dynamical gaugino condensation where the kinetic terms for gaugino condensate arise from the S-dual loop corrections defined by (3.2). More precisely, we consider the following dynamical model:

$$K = \ln V + g(V, X) + G,$$

$$\mathcal{L}_{eff} \;=\; \int d^4\theta\, E\left\{ (-2 + f(V,X)) + bVG + bV\ln(e^{-K}\bar{U}U/\mu^6) \right\}, \qquad (3.41)$$

$$\left(2 + X\frac{\partial f}{\partial X}\right)\left(1 - V\frac{\partial g}{\partial V}\right) = \left(2 - X\frac{\partial g}{\partial X}\right)\left(1 - f + V\frac{\partial f}{\partial V}\right). \qquad (3.42)$$

For convenience, we have written the S-dual combination $(\bar{U}U)^{\frac{1}{2}}/V$ as a vector superfield X, and therefore its lowest component $x = X|_{\theta=\bar{\theta}=0}$ is $x = (\bar{u}u)^{\frac{1}{2}}/\ell = \sqrt{z}/\ell$. Eq. (3.42) guarantees the correct normalization of the Einstein term. $g(V,X)$ and $f(V,X)$ satisfy the boundary condition in the weak-coupling limit defined by (3.39). We also assume that $g(V,X)$ and $f(V,X)$ have the following power-series representations[1] in terms of X^2:

$$g(V,X) \;\equiv\; g^{(0)}(V) + g^{(1)}(V)\cdot X^2 + g^{(2)}(V)\cdot X^4 + \cdots.$$

$$f(V,X) \;\equiv\; f^{(0)}(V) + f^{(1)}(V)\cdot X^2 + f^{(2)}(V)\cdot X^4 + \cdots. \qquad (3.43)$$

Furthermore, $g^{(n)}(V)$ and $f^{(n)}(V)$ $(n \geq 0)$ are assumed to be arbitrary but bounded here. The interpretation of each term in (3.43) is obvious: As has been discussed in Section 2.2.2, in the linear multiplet formalism $g^{(0)}(V)$ and $f^{(0)}(V)$ are to be identified as stringy (non-perturbative) corrections to the Kähler potential. $g^{(n)}(V)\cdot X^{2n}$ and $f^{(n)}(V)\cdot X^{2n}$ $(n \geq 1)$ are therefore S-dual loop corrections to the Kähler potential in the presence of stringy (non-perturbative) effects.

It is also more convenient to use the coordinates (ℓ, x) instead of (ℓ, z) for the field configuration space. The component field expressions constructed in Section 3.2 can easily be rewritten in the new coordinates (ℓ, x) according to the

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[1]It should be noted that one can actually start with a more generic dynamical model by considering more generic $g(V,X)$ and $f(V,X)$, and the discussions of Section 3.3 remain valid.

following rules:

$$\ell g_\ell \quad \to \quad \ell g_\ell - x g_x$$

$$z g_z \quad \to \quad \frac{1}{2} x g_x, \tag{3.44}$$

where

$$g_\ell \equiv \frac{\partial g(\ell, x)}{\partial \ell}, \quad g_x \equiv \frac{\partial g(\ell, x)}{\partial x} \tag{3.45}$$

on the right-hand side of (3.44) are to be understood as partial derivatives in the coordinates (ℓ, x). The scalar potential of this generic model follows directly from (3.38):

$$
\begin{aligned}
V_{pot} =\ & \frac{1}{16}(1 + \ell g_\ell - x g_x)(h + \ell h_\ell - x h_x + 2b\ell)x^2 \\[2mm]
& + \frac{1}{16 x h_x (2 - x g_x)} \left\{ \begin{array}{l} 1 + f + b\ell \ln(e^{-k}\bar{u}u/\mu^6) \\[2mm] + (2 - x g_x)(\ell h_\ell - x h_x + b\ell) \end{array} \right\}^2 x^2 \\[2mm]
& - \frac{(4 - 2\ell g_\ell - x g_x)}{64(2 - x g_x)}\left[1 + f + b\ell \ln(e^{-k}\bar{u}u/\mu^6) \right]^2 x^2 \\[2mm]
& + \frac{(2h - 2\ell h_\ell - x h_x)\bar{u}u}{16 x h_x} a^2. \tag{3.46}
\end{aligned}
$$

The kinetic terms also follow directly from (3.34). The absence of a tachyonic axion requires $x h_x > 0$.

3.3.1 Effective Description of Dynamical Gaugino Condensation

As discussed in Section 3.1, one of the major motivations for studying dynamical gaugino condensation is to understand how static gaugino condensation

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could emerge as the effective description of dynamical gaugino condensation after all the heavy modes belonging to dynamical gaugino condensation are integrated out. Unlike studies in the past where the important constraint (2.12) on the gaugino condensate chiral superfield U is ignored, proving the above connection is certainly non-trivial. From this point of view, our construction in Section 3.2 can be regarded as efforts to solve (2.12) in the context of dynamical gaugino condensation using the linear multiplet formalism, and the above connection is actually obvious after (2.12) is explicitly solved. In order to make the following discussion as explicit as possible, in this section we choose to study S-dual dynamical gaugino condensation. However, we would like to emphasize that our discussion is actually valid for any dynamical model whose h_z has no pole at $z = 0$.

Firstly, the axionic contents of dynamical gaugino condensation are a, $i\ln(\bar{u}/u)$ and $i(\bar{t}^I - t^I)$. Since a physically interesting model of dynamical gaugino condensation should predict a small scale of condensation (i.e., $\langle x \rangle \ll 1$), it is clear from (3.46) that generally the condensate x and the axion a are much heavier than the other fields, and therefore should be integrated out. It is straightforward to integrate out a and x through their equations of motion: The equation of motion for a is $a = 0$. The equation of motion for x is:

$$1 + f + b\ell \ln(e^{-k}\bar{u}u/\mu^6) + (2 - xg_x)(\ell h_t - xh_x + b\ell) = 0 + \mathcal{O}(x^2). \quad (3.47)$$

(3.47) can be re-written in a more instructive form:

$$x^2 = \frac{\mu^6}{e^2\ell} e^{g^{(0)} - (1+f^{(0)})/b\ell} + \mathcal{O}(x^4), \quad (3.48)$$

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where we have used the fact that $g \approx g^{(0)}$, $f \approx f^{(0)}$, $h \approx 1$, $\ell g_\ell \approx \ell g_\ell^{(0)}$, $\ell f_\ell \approx \ell f_\ell^{(0)}$, $\ell h_\ell \approx 0$, $x g_x \approx 0$, $x f_x \approx 0$ and $x h_x \approx 0$ up to corrections of order $\mathcal{O}(x^2)$. The (bosonic) effective Lagrangian, $\mathcal{L}_{eff} = \mathcal{L}_{kin} - e V_{pot}$, of the dynamical model (3.34,46) after integrating out a and x is as follows:

$$
\begin{aligned}
\frac{1}{e} \mathcal{L}_{kin} = & -\frac{1}{2} \mathcal{R} - \frac{1}{4\ell^2} \left(1 + \ell g_\ell^{(0)} \right) \nabla^m \ell \, \nabla_m \ell \\
& - (1 + b\ell) \sum_I \frac{1}{(t^I + \bar{t}^I)^2} \nabla^m \bar{t}^I \, \nabla_m t^I + \frac{1}{4\ell^2} \left(1 + \ell g_\ell^{(0)} \right) \tilde{B}^m \tilde{B}_m \\
& + \mathcal{O}(x^2),
\end{aligned} \tag{3.49}
$$

where

$$
\begin{aligned}
\tilde{B}_m \equiv & -i \frac{b\ell^2}{\left(1 + \ell g_\ell^{(0)} \right)} \nabla_m \ln(\frac{\bar{u}}{u}) \\
& + i \frac{b\ell^2}{\left(1 + \ell g_\ell^{(0)} \right)} \sum_I \frac{1}{(t^I + \bar{t}^I)} (\nabla_m \bar{t}^I - \nabla_m t^I).
\end{aligned} \tag{3.50}
$$

$$
V_{pot} = \frac{1}{16 e^2 \ell} \left\{ \left(1 + f^{(0)} - \ell f_\ell^{(0)} \right) (1 + b\ell)^2 - 3 b^2 \ell^2 \right\} \mu^6 e^{g^{(0)} - (1 + f^{(0)})/b\ell} + \mathcal{O}(x^4). \tag{3.51}
$$

Furthermore, (3.42) leads to $\ell g_\ell^{(0)} = f^{(0)} - \ell f_\ell^{(0)}$ to the lowest order in x^2.

In comparison with static gaugino condensation studied in Chapter 2, it is clear that the effective Lagrangian of dynamical gaugino condensation after integrating out the heavy fields are indeed identical to the Lagrangian of the static model,

(2.46), to the lowest order in x^2. Note that, in (3.51), the $\mathcal{O}(x^4)$ terms do not depend on the remaining axionic degrees of freedom (i.e., $i \ln(\bar{u}/u)$ and $i(\bar{t}^I - t^I)$), and therefore these remaining axions are massless as they should be in static gaugino condensation[2] [12]. In conclusion, after integrating out the heavy modes the axions left in the effective theory of dynamical gaugino condensation are identical to those of static gaugino condensation. Consistently there is always a massless (or very light in multiple gaugino condensation [14]) model-independent axion. According to the equation of motion for x, (3.48), $x^2 \ll 1$ actually holds for any value of ℓ. It implies that only the lowest-order terms of (3.49) and (3.51) are important, and, as we have expected and now prove here, the static model of gaugino condensation is indeed the appropriate effective description of the dynamical model. This proof therefore justifies the use of static gaugino condensation in Chapter 2.

This proof also implies that the necessary and sufficient condition for V_{pot} of dynamical gaugino condensation to be bounded from below is exactly the same as that of static gaugino condensation (2.57),

$$ f^{(0)} - \ell f_\ell^{(0)} \geq 2 \quad \text{for} \quad \ell \to \infty, \tag{3.52} $$

which depends only on stringy non-perturbative effects $g^{(0)}$ and $f^{(0)}$. (3.52) does not depend on the details of S-dual loop corrections, and therefore it holds for generic S-dual dynamical models. Furthermore, (3.52) implies that only stringy non-perturbative effects are important in stabilizing the dilaton, and therefore allowing

---

[2]As pointed out in [14] as well as in Chapter 4 here, these axionic degrees of freedom naturally acquire different masses in scenarios of multiple gaugino condensation.

supersymmetry breaking via gaugino condensation. S-dual loop corrections play no role in this issue, and S-dual loop corrections alone cannot stabilize the dilaton. As discussed in Section 2.4, (3.52) can also be interpreted as the necessary condition for the dilaton to be stabilized.

3.3.2 Solving for Dynamical Gaugino Condensation

In the previous section, the dynamical model of gaugino condensation is analyzed through its effective Lagrangian after integrating out the heavy modes. One can also analyze the dynamical model directly, and obtain the same conclusion. Here, we would like to present a typical example of dynamical gaugino condensation as a concrete supplement to the analysis of Section 3.3.1. Solving for dynamical gaugino condensation is generically difficult due to the partial differential equation, (3.20) or (3.42), which guarantees the correct normalization of the Einstein term. On the other hand, only those solutions of (3.20) which are of physical interest deserve study. Therefore, in the following we show explicitly how to construct the solution for the interesting S-dual model of dynamical gaugino condensation defined by (3.41–43). In order to simplify the presentation but leave the generality of our conclusion unaffected, we choose a specific form for $f(V, X)$ in the following discussion: $f(V, X) = f^{(0)}(V) + \varepsilon X^2$, where ε is a constant and $|\varepsilon|$ is in principle a small number because X-dependent terms arise from loop corrections. In this restricted solution space, (3.42) together with the boundary condition (3.39) can be re-expressed as an infinite number of ordinary differential equations with

appropriate boundary conditions (evaluated at $\theta = \bar{\theta} = 0$) as follows:

$$\ell g_\ell^{(0)} = f^{(0)} - \ell f_\ell^{(0)}.$$

$$\ell g_\ell^{(1)} - \left(1 - f^{(0)} + \ell f_\ell^{(0)}\right) g^{(1)} = -\varepsilon \cdot \ell g_\ell^{(0)} + 2\varepsilon.$$

$$\ell g_\ell^{(n)} - n \left(1 - f^{(0)} + \ell f_\ell^{(0)}\right) g^{(n)} = -\varepsilon \cdot \ell g_\ell^{(n-1)} - \varepsilon(n-1)g^{(n-1)},$$

$$\text{for} \quad n \geq 2. \tag{3.53}$$

The associated boundary conditions in the weak-coupling limit are:

$$g^{(0)}(\ell = 0) = 0, \quad f^{(0)}(\ell = 0) = 0,$$

$$g^{(1)}(\ell = 0) = -2\varepsilon,$$

$$g^{(n)}(\ell = 0) = -\frac{2}{n}\varepsilon^n \quad \text{for} \quad n \geq 2. \tag{3.54}$$

Therefore, $g(V, X)$ is unambiguously[3] related to $f(V, X)$ in this interesting solution space.

Firstly, notice that the boundedness of $g^{(n)}$ and $f^{(n)}$ can be guaranteed if (3.52) is satisfied. Therefore, the solution defined by (3.53–54)[4] exists at least for viable dynamical models in the sense of (3.52). Secondly, $g^{(n)}$ is suppressed by a small factor $|\varepsilon|^n$, which is obvious from (3.53–54). Therefore, the solution defined by (3.53–54) converges for $x^2 < \mathcal{O}\left(1/\varepsilon\right)$. Since a physically interesting model of gaugino condensation should predict a small scale of condensation (*i.e.*, $\langle x^2 \rangle \ll 1$), this

---

[3]In fact, there is one free parameter β involved due to the fact that $g_\ell^{(n)}(\ell = 0)$ is not well-defined in (4.15); this ambiguity can be parametrized by $g_\ell^{(n)}(\ell = 0) = n\varepsilon^{n-1}\beta$. We take $\beta = 0$ here.

[4]The generalization to generic $f(V, X)$ is straightforward.
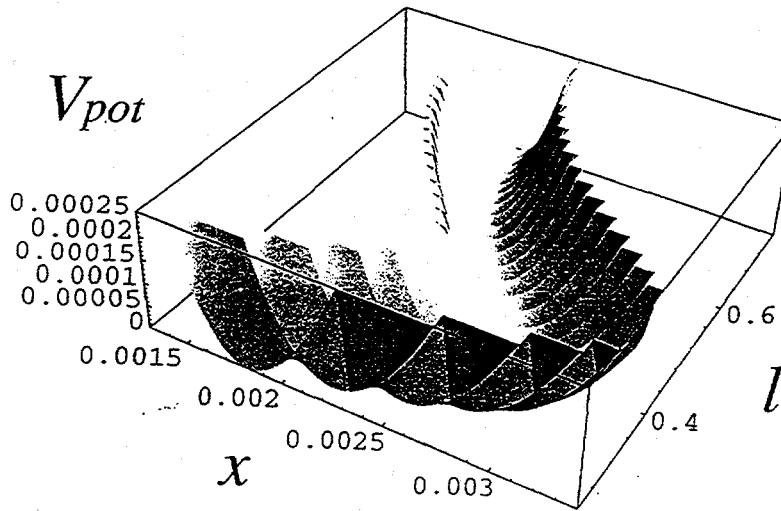
Figure 3.1: The scalar potential V_{pot} (in reduced Planck units) is plotted versus ℓ and x. $A = 6.8$, $B = 1$, $\varepsilon = -0.1$ and $\mu = 1$. (The rippled surface of V_{pot} is simply due to discretization of the ℓ-axis.)

solution does cover the regime of physical interest.[5]

(3.52) is the necessary condition for stringy non-perturbative effects to stabilize the dilaton. By looking into the details of the scalar potential, it can also be argued [12] that stringy non-perturbative corrections to the Kähler potential may naturally stabilize the dilaton if (3.52) is satisfied. In the following, the solution defined by (3.53–54) is used to construct a typical realization of this argument. Furthermore, it is the typical feature of this example rather than the specific form of $g(V, X)$ and $f(V, X)$ assumed in this example that we want to emphasize. In

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[5]This solution can in principle be extended into the $x^2 > \mathcal{O}(1/\varepsilon)$ regime using the method of characteristics.
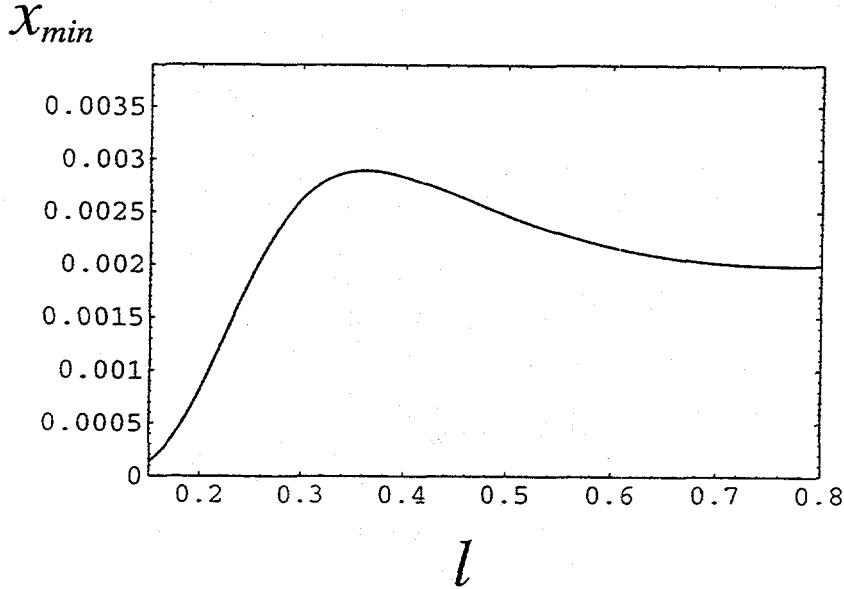
X_{min}

Figure 3.2: $x_{min}(\ell)$ is plotted versus ℓ for Figure 3.1.

Fig. 3.1, the scalar potential V_{pot} is plotted versus ℓ and x for an example with

$f(V, X) = f^{(0)}(V) + \varepsilon X^2$ and $f^{(0)}(V) = A \cdot e^{-B/V}$. There is a non-trivial vacuum

with the dilaton stabilized at $\langle \ell \rangle = 0.52$, x stabilized at $\langle x \rangle = \langle \sqrt{\bar{u}u}/\ell \rangle = 0.0024$,

and (fine-tuned) vanishing vacuum energy $\langle V_{pot} \rangle = 0$. Supersymmetry is broken

at the vacuum and the gravitino mass $m_{\tilde{G}} = 4 \times 10^{-4}$ in reduced Planck units. To

uncover more details of dilaton stabilization in Fig. 3.1, a cross section of V_{pot} is

presented in Fig. 3.3. More precisely, with the value of ℓ fixed, V_{pot} is minimized

only with respect to x; the location of this minimum is denoted as $(\ell, x_{min}(\ell))$.

The path defined by $(\ell, x_{min}(\ell))$ is shown in Fig. 3.2. The cross section of V_{pot}

is obtained by making a cut along $(\ell, x_{min}(\ell))$; that is, the cross section of V_{pot}

is defined as $V'_{pot}(\ell) \equiv V_{pot}(\ell, x_{min}(\ell))$. Fig. 3.3 shows that the dilaton is indeed

stabilized at $\langle \ell \rangle = 0.52$. Therefore, we have presented a concrete example with

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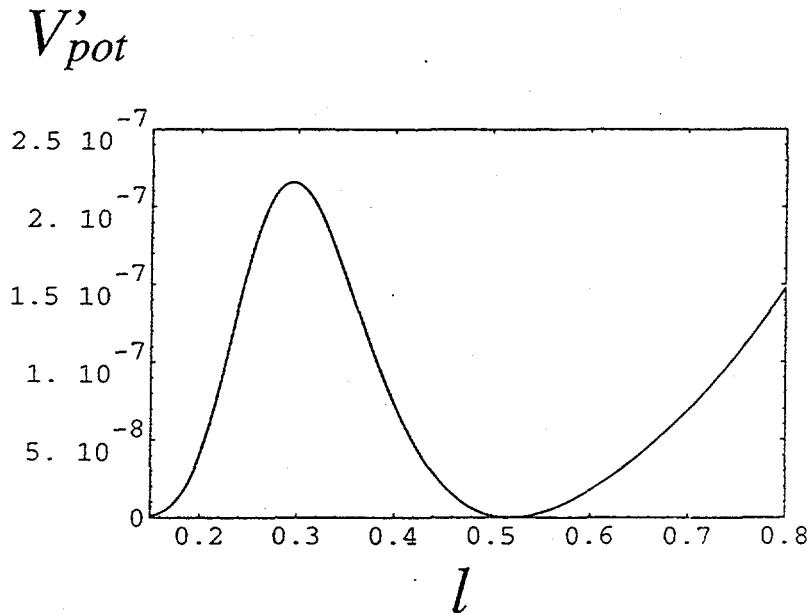
$$V'_{pot}$$

Figure 3.3: The cross section of the scalar potential, $V'_{pot}(\ell) \equiv V_{pot}(\ell, x_{min}(\ell))$ (in reduced Planck units), is plotted versus ℓ for Figure 3.1.

stabilized dilaton, broken supersymmetry, and (fine-tuned) vanishing cosmological constant. As pointed out in Sections 2.1 and 2.5, this is in contrast with condensate models studied previously [3, 16, 28] which either need the assistance of an additional source of supersymmetry breaking or have a large and negative cosmological constant problem.

3.4 Concluding Remarks

The field component Lagrangian for the linear multiplet formalism of generic dynamical gaugino condensation is constructed and studied. A major conclusion of this chapter is that static gaugino condensation is indeed the appropriate effective description of dynamical gaugino condensation after the heavy modes are integrated

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out. Some issues about the axions are also clarified. This justifies our studies in Chapter 2, and allows us to use static gaugino condensation in constructing more realistic models in Chapter 4.

Chapter 4

Gaugino and Matter Condensation in Generic String Models

4.1 Introduction

It was recently shown how to formulate gaugino condensation using the linear multiplet [8, 30] formalism for the dilaton superfield, both in global supersymmetry [18, 20] and in the superconformal formulation of supergravity [18]. Using the Kähler superspace formulation of supergravity [34, 35], which we use throughout this study, it was subsequently shown [19] how to include the Green-Schwarz term for a string model with a pure Yang-Mills E_8 hidden sector. In this case there are no moduli-dependent threshold corrections and there is a single constant – the E_8 Casimir C – that governs both the Green-Schwarz counterterm and the coupling renormalization. This model of gaugino condensation has been studied in detail in Chapters 2 and 3, where it was found that the dilaton can be stabilized at a phenomenologically acceptable value with broken supersymmetry if stringy non-perturbative corrections [4, 7] to the Kähler potential are included. However, the model studied in Chapters 2 and 3 has several drawbacks from the viewpoint of phenomenology. As discussed in Section 2.5, due to the large gauge content of E_8 a sufficiently large gauge hierarchy is not generated. Furthermore, the string moduli T^I remain flat directions. As we have pointed out, these unsatisfactory features belong only to the specific string model with with a pure Yang-Mills E_8 hidden sector, and therefore are not generic at all. As we will see, in a generic string model the hidden sector contains a product of smaller gauge groups. Therefore, a large enough gauge hierarchy could be generated naturally. Furthermore, a generic string

model contains hidden matter, and together with string threshold corrections the hidden matter condensation lifts the flat directions associated with the moduli.

Consider a generic string model whose hidden sector gauge group is a product of simple groups: $\mathcal{G} = \prod_a \mathcal{G}_a$. One immediate difficulty is the following: since we need to describe several gaugino condensates $U_a \simeq \text{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)_a$ and each gaugino condensate U_a is constrained by (2.12) separately, therefore according to (2.13) we need to introduce several vector superfields V_a. However, since the theory has a single dilaton ℓ, it must be identified with the lowest component of $V = \sum_a V_a$. What should we do with the other components $\ell_a = V_a|_{\theta = \bar{\theta} = 0}$? We will see that, in our description, these are non-propagating degrees of freedom which actually do not appear in the Lagrangian. Similarly only one antisymmetric tensor field (also associated with $V = \sum_a V_a$) is dynamical. This allows us to generalize our approach to the case of multiple gaugino condensation.

Let us stress that the goal is very different from the so-called "racetrack" ideas [3] where resorting to multiple gaugino condensation is necessary in order to get supersymmetry breaking. Here supersymmetry is broken already for a single gaugino condensate. Indeed, we will see that the picture which emerges from multiple gaugino condensation (complete with threshold corrections and Green-Schwarz mechanism) is very different from the standard "racetrack" description: indeed, the scalar potential is largely dominant by the condensate with the largest one-loop beta-function coefficient.

To be more precise, we generalize in this chapter the Lagrangian (2.26) studied

in Chapter 2 to string models with arbitrary hidden sector gauge groups and with three untwisted (1,1) moduli T^I. We take the Kähler potential for the effective theory at the condensation scale to be:

$$K = k(V) + \sum_I g^I, \quad g^I = -\ln(T^I + \bar{T}^I), \quad V = \sum_{a=1}^n V_a, \tag{4.1}$$

where the V_a are vector superfields and n is the number of (asymptotically free) nonabelian gauge groups $\mathcal{G}_a$ in the hidden sector:

$$\mathcal{G}_{\text{hidden}} = \prod_{a=1}^n \mathcal{G}_a \otimes U(1)^m. \tag{4.2}$$

We will take $\mathcal{G}_{\text{hidden}}$ to be a subgroup of E_8. In general, there will be hidden matter associated with the hidden sector gauge groups.

We introduce both gaugino condensate superfields U_a and hidden matter condensate superfields Π^α that are non-propagating:

$$U_a \simeq \text{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)_a, \quad \Pi^\alpha \simeq \prod_A \left(\Phi^A\right)^{n_\alpha^A}, \tag{4.3}$$

where $\mathcal{W}_a$ and Φ^A are the gauge and matter chiral superfields, respectively. The matter condensate Π^α is a chiral superfield of Kähler weight $w = 0$, while the gaugino condensate U_a associated with gauge subgroup $\mathcal{G}_a$ is a chiral superfield of Kähler weight $w = 2$, and is identified with the chiral projection of V_a:

$$U_a = -(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R)V_a, \quad \bar{U}_a = -(\mathcal{D}^\alpha\mathcal{D}_\alpha - 8R^\dagger)V_a. \tag{4.4}$$

We are thus introducing n scalar fields $\ell_a = V_a|_{\theta=\bar\theta=0}$. However only one of these is physical, namely $\ell = \sum_a \ell_a$; the others do not appear in the effective component Lagrangian constructed below.

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The effective Lagrangian for multiple gaugino condensation is constructed and analyzed in Sections 4.2–4.5. In an appendix we discuss a parallel construction using the chiral supermultiplet representation for the dilaton and unconstrained chiral supermultiplets for the gaugino condensates in order to illustrate the differences between the two approaches and the significance of including the constraints (4.4).

4.2   Construction of the Effective Lagrangian

We adopt the following superfield Lagrangian:

$$\mathcal{L}_{eff} = \mathcal{L}_{KE} + \mathcal{L}_{GS} + \mathcal{L}_{th} + \mathcal{L}_{VY} + \mathcal{L}_{pot}, \qquad (4.5)$$

where

$$\mathcal{L}_{KE} = \int d^4\theta\, E\,[-2 + f(V)], \quad k(V) = \ln V + g(V), \qquad (4.6)$$

is the kinetic energy term for the dilaton, chiral and gravity superfields. The functions $f(V), g(V)$ parameterize stringy nonperturbative effects. According to (2.8), they are related by the following first-order differential equation:

$$V\frac{dg(V)}{dV} = -V\frac{df(V)}{dV} + f, \qquad (4.7)$$

which ensures that the Einstein term has canonical form [12]. In the classical limit $g = f = 0$; we therefore impose the boundary condition at the weak-coupling limit:

$$g(V = 0) = 0 \quad \text{and} \quad f(V = 0) = 0. \qquad (4.8)$$

Two counterterms are introduced to cancel the modular anomaly [31], namely the Green-Schwarz counterterm [37, 38]:

$$\mathcal{L}_{GS} = b \int d^4\theta \, EV \sum_I g^I, \quad b = \frac{C}{8\pi^2}, \tag{4.9}$$

and the term induced by string loop corrections [36]:

$$\mathcal{L}_{th} = -\sum_{a,I} \frac{b_a^I}{64\pi^2} \int d^4\theta \, \frac{E}{R} U_a \ln \eta^2(T^I) + \text{h.c.} \tag{4.10}$$

The parameters

$$b_a^I = C - C_a + \sum_A \left(1 - 2q_I^A\right) C_a^A, \quad C = C_{E_8}, \tag{4.11}$$

vanish for orbifold compactifications with no $N = 2$ supersymmetry sector [40]. Here C_a and C_a^A are quadratic Casimir operators in the adjoint and matter representations, respectively. q_I^A are the modular weights of the matter superfields Φ^A of the underlying hidden sector. The term

$$\mathcal{L}_{VY} = \sum_a \frac{1}{8} \int d^4\theta \, \frac{E}{R} U_a \left[b_a' \ln(e^{-K/2} U_a / \mu^3) + \sum_\alpha b_a^\alpha \ln \Pi^\alpha \right] + \text{h.c.}, \tag{4.12}$$

where μ is a mass parameter naturally of order one in reduced Planck units (which we will set to unity hereafter), is the generalization to supergravity [43, 44] of the Veneziano-Yankielowicz superpotential term generated by condensation, including [54] the gauge invariant composite matter fields Π^α introduced in eq. (4.3) (one can also take linear combinations of such gauge invariant monomials that have the same modular weight). Finally

$$\mathcal{L}_{pot} = \frac{1}{2} \int d^4\theta \, \frac{E}{R} e^{K/2} W(\Pi^\alpha, T^I) + \text{h.c.} \tag{4.13}$$

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is a superpotential for the hidden matter condensates Π^α that respects the symmetries of the superpotential $W(\Phi^A, T^I)$ of the underlying theory.

The coefficients b'_a and b^α_a in (4.12) are dictated by the chiral and conformal anomalies of the underlying field theory. Under modular transformations, we have:

$$T^I \quad \rightarrow \quad \frac{aT^I - ib}{icT^I + d}, \quad ad - bc = 1, \quad a, b, c, d \in \mathbb{Z},$$

$$g^I \quad \rightarrow \quad g^I + H^I + \bar{H}^I, \quad H^I = \ln(icT^I + d),$$

$$\Phi^A \quad \rightarrow \quad e^{-\sum_I H^I q^A_I} \Phi^A,$$

$$\lambda_a \quad \rightarrow \quad e^{-\frac{i}{2}\sum_I \mathrm{Im}H^I} \lambda_a, \quad \chi^A \rightarrow e^{\frac{1}{2}\sum_I (i\mathrm{Im}H^I - 2q^A_I H^I)} \chi^A, \quad \theta \rightarrow e^{-\frac{i}{2}\sum_I \mathrm{Im}H^I} \theta,$$

$$U_a \quad \rightarrow \quad e^{-i\sum_I \mathrm{Im}H^I} U_a, \quad \Pi^\alpha \rightarrow e^{-\sum_I H^I q^\alpha_I} \Pi^\alpha,$$

$$q^\alpha_I \quad = \quad \sum_A n^A_\alpha q^A_I. \tag{4.14}$$

The field-theoretical loop corrections to the effective Yang-Mills Lagrangian from orbifold compactification have been determined [31, 32] using supersymmetric regularization procedures that ensure a supersymmetric form for the modular anomaly. Matching the variation under (4.14) of that contribution to the Yang-Mills Lagrangian with the variation of the effective Lagrangian (4.12) we require

$$\delta\mathcal{L}_{VY} = -\frac{1}{64\pi^2} \sum_{a,I} \int d^4\theta \frac{E}{R} U_a \left[C_a - \sum_{A,I} C^A_a \left(1 - 2q^A_I\right) \right] H^I + \mathrm{h.c.}, \tag{4.15}$$

which implies

$$b'_a + \sum_{\alpha,A} b^\alpha_a n^A_\alpha q^A_I = \frac{1}{8\pi^2} \left[C_a - \sum_A C^A_a \left(1 - 2q^A_I\right) \right] \quad \forall\ I. \tag{4.16}$$

In the flat space limit where the reduced Planck mass[1] $M'_P \to \infty$, under a canonical scale transformation

$$\lambda \to e^{\frac{3}{2}\sigma}\lambda, \quad U \to e^{3\sigma}U, \quad \Phi^A \to e^\sigma\Phi^A, \quad \Pi^\alpha \to e^{\sum_A n_\alpha^A \sigma}\Pi^\alpha, \quad \theta \to e^{-\frac{1}{2}\sigma}\theta,$$

we have the standard trace anomaly as determined by the β-functions:

$$\delta\mathcal{L}_{eff} = \frac{1}{64\pi^2}\sigma\sum_a \int d^4\theta \frac{E}{R}U_a\left(3C_a - \sum_A C_a^A\right) + \text{h.c.} + \mathcal{O}(M'^{-1}_P), \qquad (4.17)$$

which requires

$$3b'_a + \sum_{\alpha,A} b_a^\alpha n_\alpha^A = \frac{1}{8\pi^2}\left(3C_a - \sum_A C_a^A\right) + \mathcal{O}(M'^{-1}_P). \qquad (4.18)$$

Eqs. (4.16) and (4.18) are solved by [54] (up to $\mathcal{O}(M'^{-1}_P)$ corrections)

$$b'_a = \frac{1}{8\pi^2}\left(C_a - \sum_A C_a^A\right),$$

$$\sum_{\alpha,A} b_a^\alpha n_\alpha^A q_I^A = \sum_A \frac{C_a^A}{4\pi^2}q_I^A, \quad \sum_{\alpha,A} b_a^\alpha n_\alpha^A = \sum_A \frac{C_a^A}{4\pi^2}. \qquad (4.19)$$

Note that the above arguments do not completely fix $\mathcal{L}_{eff}$ since we can *a priori* add chiral and modular invariant terms of the form:

$$\Delta\mathcal{L} = \sum_{a,\alpha} b'_{a\alpha}\int \mathrm{d}^4\theta EV_a \ln\left(e^{\sum_I q_I^\alpha g^I}\Pi^\alpha\bar\Pi^\alpha\right). \qquad (4.20)$$

For specific choices of the $b'_{a\alpha}$ the matter condensates Π^α can be eliminated from the effective Lagrangian. However the resulting component Lagrangian has a linear dependence on the unphysical scalar fields $\ell_a - \ell_b$, and their equations of motion impose physically unacceptable constraints on the moduli supermultiplets. To ensure that $\Delta\mathcal{L}$ contains the fields ℓ_a only through the physical combination $\sum_a \ell_a$,

---

[1]The reduced Planck mass $M'_P = M_P/\sqrt{8\pi}$, where M_P is the Planck mass.

we have to impose $b'_{a\alpha} = b'_\alpha$ independent of a. If these terms were added, the last condition in (4.19) would become

$$\sum_{\alpha,A} b^\alpha_a n^A_\alpha + \sum_A b'_\alpha n^A_\alpha = \sum_A \frac{C^A_a}{4\pi^2}. \qquad (4.21)$$

We shall not include such terms here.

Combining (4.11) with (4.19) gives $b^I_a = 8\pi^2 \left(b - b'_a - \sum_\alpha b^\alpha_a q^\alpha_I\right)$. Combining the terms (4.6)–(4.13) by superspace partial integration (2.18), the "Yang-Mills" part of the Lagrangian (4.5) can be expressed – up to a total derivatives that we drop in the subsequent analysis – as a modular invariant D term:

$$\mathcal{L}_{eff} = \int d^4\theta\, E\left(-2 + f(V) + \sum_a V_a \left\{ b'_a \ln(\bar{U}_a U_a/e^g V) + \sum_\alpha b^\alpha_a \ln\left(\Pi^\alpha_r \bar{\Pi}^\alpha_r\right) \right.\right.$$
$$\left.\left. - \sum_I \frac{b^I_a}{8\pi^2} \ln\left[(T^I + \bar{T}^I)\, |\eta^2(T^I)|^2\right] \right\}\right) + \mathcal{L}_{pot}, \qquad (4.22)$$

where

$$\Pi^\alpha_r = \prod_A (\Phi^A_r)^{n^A_\alpha} = e^{\sum_I q^\alpha_I g^I/2} \Pi^\alpha, \quad \Phi^A_r = e^{\sum_I q^A_I g^I/2} \Phi^A, \qquad (4.23)$$

is a modular invariant field composed of elementary fields that are canonically normalized in the vacuum. The interpretation of this result in terms of renormalization group running will be discussed below. We have implicitly assumed affine level-one compactification. The generalization to higher affine levels is trivial.

The construction of the component field Lagrangian obtained from (4.22) parallels that given in Section 2.3.2 for the case $\mathcal{G} = E_8$. Since the superfield Lagrangian is a sum of F terms that contain only spinorial derivatives of the superfield V_a, and the Green-Schwarz and kinetic terms that contain V_a only through the sum V, the

unphysical scalars ℓ_a appear in the component Lagrangian only through the physical dilaton ℓ. The result for the bosonic Lagrangian is:

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}_B &= -\frac{1}{2}\mathcal{R} - (1+b\ell)\sum_I \frac{1}{(t^I + \bar{t}^I)^2}\left(\partial^m \bar{t}^I\,\partial_m t^I - \bar{F}^I F^I\right) \\
&\quad -\frac{1}{16\ell^2}(1+\ell g_\ell)\left[4\left(\partial^m\ell\,\partial_m\ell - B^m B_m\right) + \bar{u}u - 4e^{K/2}\ell\left(W\bar{u} + u\bar{W}\right)\right] \\
&\quad +\frac{1}{9}(\ell g_\ell - 2)\left[\bar{M}M - b^m b_m - \frac{3}{4}\left\{\bar{M}\left(\sum_b b_b' u_b - 4We^{K/2}\right) + \text{h.c.}\right\}\right] \\
&\quad +\frac{1}{8}\sum_a\left\{\frac{f+1}{\ell} + b_a'\ln(e^{2-K}\bar{u}_a u_a) + \sum_\alpha b_a^\alpha \ln(\pi^\alpha\bar{\pi}^\alpha)\right. \\
&\quad\quad \left. + \sum_I\left[bg^I - \frac{b_a^I}{4\pi^2}\ln|\eta(t^I)|^2\right]\right\}\left(F_a - u_a\bar{M} + \text{h.c.}\right) \\
&\quad -\frac{1}{16\ell}\sum_a\left[b_a'(1+\ell g_\ell)\bar{u}u_a - 4\ell u_a\left(\sum_\alpha b_a^\alpha\frac{F^\alpha}{\pi^\alpha} + (b_a' - b)\frac{F^I}{2\mathrm{Re}t^I}\right) + \text{h.c.}\right] \\
&\quad +\frac{i}{2}\sum_a\left[b_a'\ln(\frac{u_a}{\bar{u}_a}) + \sum_\alpha b_a^\alpha\ln(\frac{\pi^\alpha}{\bar{\pi}^\alpha})\right]\nabla^m B_m^a - \frac{b}{2}\sum_I\frac{\partial^m \mathrm{Im}t^I}{\mathrm{Re}t^I}B_m, \\
&\quad +\sum_{I,a}\frac{b_a^I}{16\pi^2}\left[\zeta(t^I)\left(2iB_a^m\nabla_m t^I - u_a F^I\right) + \text{h.c.}\right] \\
&\quad +e^{K/2}\left[\sum_I F^I\left(W_I + K_I W\right) + \sum_\alpha F^\alpha W_\alpha + \text{h.c.}\right], \tag{4.24}
\end{aligned}
$$

where

$$
\zeta(t) = \frac{1}{\eta(t)}\frac{\partial\eta(t)}{\partial t}, \quad \eta(t) = e^{-\pi t/12}\prod_{m=1}^\infty\left(1 - e^{-2m\pi t}\right),
$$

$$
\ell = V|_{\theta=\bar{\theta}=0},
$$

$$
\sigma_{\alpha\dot\alpha}^m B_m^a = \frac{1}{2}[\mathcal{D}_\alpha, \mathcal{D}_{\dot\alpha}]V_a|_{\theta=\bar{\theta}=0} + \frac{2}{3}\ell_a\sigma_{\alpha\dot\alpha}^m b_m, \quad B^m = \sum_a B_a^m,
$$

$$
u_a = U_a|_{\theta=\bar{\theta}=0} = -(\bar{\mathcal{D}}^2 - 8R)V_a|_{\theta=\bar{\theta}=0}, \quad u = \sum_a u_a,
$$

$$
\bar{u}_a = \bar{U}_a|_{\theta=\bar{\theta}=0} = -(\mathcal{D}^2 - 8R^\dagger)V_a|_{\theta=\bar{\theta}=0}, \quad \bar{u} = \sum_a \bar{u}_a,
$$

$$-4F^a = \mathcal{D}^2 U_a|_{\theta=\bar\theta=0}, \quad -4\bar F^a = \bar{\mathcal{D}}^2\bar U_a|_{\theta=\bar\theta=0}, \quad F_U = \sum_a F^a,$$

$$\pi^\alpha = \Pi^\alpha|_{\theta=\bar\theta=0} \quad \bar\pi^\alpha = \bar\Pi^\alpha|_{\theta=\bar\theta=0}$$

$$-4F^\alpha = \mathcal{D}^2\Pi^\alpha|_{\theta=\bar\theta=0}, \quad -4\bar F^\alpha = \bar{\mathcal{D}}^2\bar\Pi^\alpha|_{\theta=\bar\theta=0},$$

$$t^I = T^I|_{\theta=\bar\theta=0}, \quad -4F^I = \mathcal{D}^2 T^I|_{\theta=\bar\theta=0},$$

$$\bar t^I = \bar T^I|_{\theta=\bar\theta=0}, \quad -4\bar F^I = \bar{\mathcal{D}}^2\bar T^I|_{\theta=\bar\theta=0}, \tag{4.25}$$

b_m and $M = (\bar M)^\dagger = -6R|_{\theta=\bar\theta=0}$ are auxiliary components of the supergravity multiplet [34]. Notice that $\zeta(t)$ defined in (4.25) is related to the Einstein function $\hat G_2(t)$ [53] as follows: $\hat G_2(t) = -\pi\left(1 + 4\zeta(t)\mathrm{Re}t\right)/\mathrm{Re}t$. For $n = 1$, $u_a = u$, etc., (4.24) reduces to (2.46) of Section 2.3.2.

The equations of motion for the auxiliary fields $b_m, M, F^I, F^a + \bar F^a$ and F^α give, respectively:

$$b_m = 0, \quad M = \frac{3}{4}\left(\sum_a b'_a u_a - 4W e^{K/2}\right),$$

$$F^I = \frac{\mathrm{Re}t^I}{2(1+b\ell)}\left\{\sum_a \bar u_a\left[(b - b'_a) + \frac{b^I_a}{2\pi^2}\zeta(\bar t^I)\mathrm{Re}t^I\right] - 4e^{K/2}\left(2\mathrm{Re}t^I\bar W_I - \bar W\right)\right\},$$

$$\bar u_a u_a = \frac{\ell}{e^2}e^{g - (f+1)/b'_a\ell - \sum_I b^I_a g^I/8\pi^2 b'_a}\prod_I |\eta(t^I)|^{b^I_a/2\pi^2 b'_a}\prod_\alpha (\pi^\alpha_r\bar\pi^\alpha_r)^{-b^\alpha_a/b'_a}, \quad \pi^\alpha_r = \Pi^\alpha_r|_{\theta=\bar\theta=0},$$

$$0 = \sum_a b^\alpha_a u_a + 4\pi^\alpha e^{K/2}W_\alpha \quad \forall\ \alpha. \tag{4.26}$$

Using these, the Lagrangian (4.24) reduces to

$$\frac{1}{e}\mathcal{L}_B = -\frac{1}{2}\mathcal{R} - (1+b\ell)\sum_I \frac{\partial^m\bar t^I\,\partial_m t^I}{(t^I + \bar t^I)^2} - \frac{1}{4\ell^2}(1 + \ell g_\ell)\left(\partial^m\ell\,\partial_m\ell - B^m B_m\right)$$

$$- \sum_a\left(b'_a\omega_a + \sum_\alpha b^\alpha_a\phi^\alpha\right)\nabla^m B^a_m - \frac{b}{2}\sum_I \frac{\partial^m\mathrm{Im}t^I}{\mathrm{Re}t^I}B_m$$

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$$+i\sum_{I,a}\frac{b_a^I}{8\pi^2}\left[\zeta(t^I)B_a^m\nabla_m t^I - \text{h.c.}\right] - V_{pot},$$

$$
\begin{aligned}
V_{pot} = {} & \frac{(1+\ell g_\ell)}{16\ell^2}\left\{\bar{u}u + \ell\left[\bar{u}\left(\sum_a b_a' u_a - 4e^{K/2}W\right) + \text{h.c.}\right]\right\} \\
& + \frac{1}{16(1+b\ell)}\sum_I\left|\sum_a u_a\left(b - b_a' + \frac{b_a^I}{2\pi^2}\zeta(t^I)\text{Re}t^I\right) - 4e^{K/2}\left(2\text{Re}t^I W_I - W\right)\right|^2 \\
& + \frac{1}{16}\left(\ell g_\ell - 2\right)\left|\sum_b b_b' u_b - 4We^{K/2}\right|^2,
\end{aligned}
\tag{4.27}
$$

where we have introduced the notation

$$u_a = \rho_a e^{i\omega_a}, \quad \pi^\alpha = \eta^\alpha e^{i\phi^\alpha}, \tag{4.28}$$

and

$$2\phi^\alpha = -i\ln\left(\frac{\sum_a b_a^\alpha u_a \bar{W}_\alpha}{\sum_a b_a^\alpha \bar{u}_a W_\alpha}\right) \quad \text{if} \quad W_\alpha \neq 0. \tag{4.29}$$

To go further we have to be more specific. Assume[2] that for fixed α, $b_a^\alpha \neq 0$ for only one value of a. For example, we allow no representations (n,m) with both n and $m \neq 1$ under $\mathcal{G}_a \otimes \mathcal{G}_b$. Then $u_a = 0$ unless $W_\alpha \neq 0$ for every α with $b_a^\alpha \neq 0$. We therefore assume that $b_a^\alpha \neq 0$ only if $W_\alpha \neq 0$.

Since the Π^α are gauge invariant operators, we may take W linear in Π:

$$W(\Pi, T) = \sum_\alpha W_\alpha(T)\Pi^\alpha, \quad W_\alpha(T) = c_\alpha \prod_I [\eta(T^I)]^{2(q_I^\alpha - 1)}, \tag{4.30}$$

where $\eta(T)$ is the Dedekind function. If there are gauge singlets M^i with modular weights q_I^i, then the constants c_α are replaced by modular invariant functions:

$$c_\alpha \to w_\alpha(M, T) = c_\alpha \prod_i (M^i)^{n_i^\alpha} \prod_I [\eta(T^I)]^{2n_i^\alpha q_I^i}.$$

---

[2] For, e.g., $\mathcal{G} = E_6 \otimes SU(3)$, we take $\Pi \simeq (27)^3$ of E_6 or $(3)^3$ of $SU(3)$.

In addition if some M^i have gauge invariant couplings to vector-like representations of the gauge group

$$W(\Phi, T, M) \ni c_{iAB} M^i \Phi^A \Phi^B \prod_I [\eta(T^I)]^{2(q_I^A + q_I^B + q_I^i)},$$

one has to introduce condensates $\Pi^{AB} \simeq \Phi^A \Phi^B$ of dimension two, and corresponding terms in the effective superpotential:

$$W(\Pi, T, M) \ni c_{iAB} M^i \Pi^{AB} \prod_I [\eta(T^I)]^{2(q_I^A + q_I^B + q_I^i)}.$$

Since the M^i are unconfined, they cannot be absorbed into the composite fields Π. The case with only vector-like representations has been considered in [54]. To simplify the present discussion, we ignore this type of coupling and assume that the composite operators that are invariant under the gauge symmetry (as well as possible discrete global symmetries) are at least trilinear in the nonsinglets under the confined gauge group. We further assume that there are no continuous global symmetries–such as a flavor $SU(N)_L \otimes SU(N)_R$ whose anomaly structure has to be considered [54]. With these assumptions the equations of motion (4.26) give, using $\sum_\alpha b_a^\alpha q_I^\alpha + b_a^I / 8\pi^2 = b - b_a'$,

$$\rho_a^2 = e^{-2b_a'/b_a} e^K e^{-(1+f)/b_a \ell - b \sum_I g^I / b_a} \prod_I |\eta(t^I)|^{4(b-b_a)/b_a} \prod_\alpha |b_a^\alpha / 4c_\alpha|^{-2b_a^\alpha/b_a},$$

$$\pi_r^\alpha = -e^{-\frac{1}{2}[k + \sum_I (1 - q_I^\alpha) g^I]} \frac{b_a^\alpha}{4W_\alpha} u_a, \quad b_a \equiv b_a' + \sum_\alpha b_a^\alpha. \tag{4.31}$$

Note that promoting the second equation above to a superfield relation, and substituting the expression on the right hand side for Π in (4.22) gives

$$\mathcal{L}_{eff} = \int d^4\theta \, E \left(-2 + f(V) + \sum_a V_a \left\{ b_a \ln(\bar{U}_a U_a / e^g V) \right. \right.$$

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$$-\sum_\alpha b_a^\alpha \ln\left(e^{\sum_I g^I(1-q_I^\alpha)} |4W_\alpha/b_a^\alpha|^2\right)$$

$$-\sum_I \frac{b_a^I}{8\pi^2} \ln\left[\left(T^I + \bar{T}^I\right) |\eta^2(T^I)|^2\right]\Bigg\}\Bigg) + \mathcal{L}_{pot}. \tag{4.32}$$

It is instructive to compare this result with the effective Yang-Mills Lagrangian found [31, 32] by matching field-theoretical and string loop calculations. Making the identifications $V \to L$, $U_a \to \text{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)_a$, the effective Lagrangian at scale μ obtained from those results can be written as follows:

$$\mathcal{L}_{eff}^{YM}(\mu) = \int \mathrm{d}^4\theta \, E\Bigg(-2 + f(V) + \sum_a V_a\Bigg\{\frac{1}{8\pi^2}\left(C_a - \frac{1}{3}\sum_A C_a^A\right)\ln\left[\frac{M_s^6 g_s^{-4}}{\mu^6 g_a(\mu)^{-4}}\right]$$

$$-\frac{1}{4\pi^2}\sum_A C_a^A \ln\left[g_s^{\frac{2}{3}} Z_A(M_s)/g_a^{\frac{2}{3}}(\mu)Z_A(\mu)\right]$$

$$-\sum_I \frac{b_a^I}{8\pi^2} \ln\left[\left(T^I + \bar{T}^I\right) |\eta^2(T^I)|^2\right]\Bigg\}\Bigg), \tag{4.33}$$

with $M_s^2 \approx g_s^2 \approx 2\langle\ell\rangle$ ($g_s \equiv g(M_s)$) in the string perturbative limit, $f(V) = g(V) = 0$. The first term in the brackets in (4.32) can be identified with the corresponding term (4.33) provided

$$\sum_\alpha b_a^\alpha = \frac{1}{12\pi^2}\sum_A C_a^A, \qquad b_a = \frac{1}{8\pi^2}\left(C_a - \frac{1}{3}\sum_A C_a^A\right). \tag{4.34}$$

In fact, this constraint follows from (4.19) if the Π^α are all of dimension three, which is consistent with the fact that only dimension-three operators survive in the superpotential in the limit $M'_P \to \infty$. Then b_a is proportional to the β-function for $\mathcal{G}_a$, and $\langle\rho_a\rangle \approx \langle|\lambda_a^\alpha \lambda_{a\alpha}|\rangle$ has the correct exponential suppression factor for a small gauge coupling constant as expected by a RGE analysis. In the absence of (stringy) nonperturbative corrections to the Kähler potential ($f(V) = g(V) = 0$),

$2\langle V |_{\theta=\bar\theta=0}\rangle = 2\langle \ell \rangle = g_s^2 = M_s^2$ is the string scale in reduced Planck units and also the gauge coupling at that scale [31, 32]. Therefore, the argument of the logarithm in (4.33),

$$\left\langle \frac{\bar{U}_a U_a}{V} \right\rangle^{1/3} \approx \frac{\langle |\lambda_a^\alpha \lambda_{a\alpha}| \rangle^{2/3}}{g_s^{2/3}} = \frac{\langle |\lambda_a^\alpha \lambda_{a\alpha}| \rangle^{2/3}}{M_s^2 g_s^{-4/3}}, \qquad (4.35)$$

gives the exact two-loop result for the coefficient of C_a in the renormalization group running from the string scale to the appropriate condensation scale [31, 32, 46]. The relation between $\langle \pi^\alpha \rangle$ and $\langle u_a \rangle$, and hence the appearance of the gaugino condensate as the effective infra-red cut-off for massless matter loops, is related to the Konishi anomaly [55]. The matter loop contributions have additional two-loop corrections involving matter wave-function renormalization [51, 56, 57, 58]:

$$\frac{\partial \ln Z_A(\mu)}{\partial \ln \mu^2} = -\frac{1}{32\pi^2}\left[\ell e^g \sum_{BC} e^{\sum_I g^I \left(1 - q_I^A - q_I^B - q_I^C\right)} Z_A^{-1}(\mu) Z_B^{-1}(\mu) Z_C^{-1}(\mu) |W_{ABC}|^2 \right.$$

$$\left. -4 \sum_a g_a^2(\mu) C_2^a(R_A) \right] + \mathcal{O}(g^4) + \mathcal{O}(\Phi_A^2), \qquad (4.36)$$

where $C_2^a(R_A) = (\dim\mathcal{G}_a/\dim R_A)C_a^A$, R_A is the representation of $\mathcal{G}_a$ on Φ_A. The boundary condition on Z_A [31] is $Z_A(\mu_s) = (1 - p_A \ell)^{-1}$, where p_A is the coefficient of $e^{\sum_I q_I^A g^I} |\Phi^A|^2$ in the Green-Schwarz counterterm of the underlying theory: $V = \sum_I g^I + p_A e^{\sum_I q_I^A g^I} |\Phi^A|^2 + \mathcal{O}(|\Phi^A|^4)$. The second line of (4.32) can be interpreted as a rough parameterization of the second line of (4.33).

In the following analysis, we retain only dimension three operators in the superpotential, and do not include any unconfined matter superfields in the effective

condensate Lagrangian. The potential V_{pot} takes the form:

$$V_{pot} = \frac{1}{16\ell^2} \sum_{a,b} \rho_a \rho_b \cos \omega_{ab} R_{ab}(t^I), \quad \omega_{ab} = \omega_a - \omega_b,$$

$$R_{ab} = (1 + \ell g_\ell)(1 + b_a \ell)(1 + b_b \ell) - 3\ell^2 b_a b_b + \frac{\ell^2}{(1 + b\ell)} \sum_I d_a(t^I) d_b(\bar{t}^I),$$

$$d_a(t^I) = b - b'_a + \frac{b_a^I}{2\pi^2} \zeta(t^I) \mathrm{Re}t^I - \sum_\alpha b_a^\alpha \left[1 - 4(q_I^\alpha - 1)\zeta(t^I)\mathrm{Re}t^I \right]$$

$$= (b - b_a)\left(1 + 4\zeta(t^I)\mathrm{Re}t^I \right)$$

$$= -(b - b_a) \frac{\mathrm{Re}t^I}{\pi} \hat{G}_2(t^I). \tag{4.37}$$

Note that $d_a(t^I) \propto F^I \propto \hat{G}_2(t^I)\mathrm{Re}t^I$ vanishes at the self-dual point $t^I = 1$, where $\zeta(t^I) = -1/4$, $\hat{G}_2(t^I) = 0$, $\eta(t^I) \approx 0.77$. For $\mathrm{Re}t^I \gtrsim 1$ we have, to a very good approximation, $\zeta(t^I) \approx -\pi/12$, $\eta(t^I) \approx e^{-\pi t/12}$. Note that also ρ_a – and hence the potential V_{pot} – vanishes in the limits of large and small radii; from (4.31) we have

$$\lim_{t^I \to \infty} \rho_a^2 \sim (2\mathrm{Re}t^I)^{(b-b_a)/b_a} e^{-\pi(b-b_a)\mathrm{Re}t^I/3b_a},$$

$$\lim_{t^I \to 0} \rho_a^2 \sim (2\mathrm{Re}t^I)^{(b_a-b)/b_a} e^{-\pi(b-b_a)/3b_a\mathrm{Re}t^I}, \tag{4.38}$$

where the second line follows from the first by the duality invariance of ρ_a^2. So there is potentially a "runaway moduli problem". However, as will be shown in Section 4.4, the moduli are stabilized at a physically acceptable vacuum, namely the self-dual point.

4.3 Axion Content of the Effective Theory

Next we consider the axion states of the effective field theory. If all $W_\alpha \neq 0$, the equations of motion for ω_a obtained from (4.27) read:

$$\frac{\partial \mathcal{L}}{\partial \omega_a} = -b'_a \nabla^m B^a_m - \frac{1}{2} \sum_{\alpha,b} b^\alpha_b \left(\frac{b^\alpha_a u_a}{\sum_c b^\alpha_c u_c} + \text{h.c.} \right) \nabla^m B^b_m - \frac{\partial V_{pot}}{\partial \omega_a} = 0. \qquad (4.39)$$

These give, in particular,

$$\sum_a \frac{\partial \mathcal{L}}{\partial \omega_a} = -\sum_a b_a \nabla^m B^a_m = 0. \qquad (4.40)$$

The one-forms B^a_m are *a priori* dual to 3-forms:

$$B^a_m = \frac{1}{2} \epsilon_{mnpq} \left(\frac{1}{3!4} \Gamma^{npq}_a + \partial^n b^{pq}_a \right), \qquad (4.41)$$

where Γ^{npq}_a and b^{pq}_a are 3-form and 2-form potentials, respectively; (4.41) assures the constraints (2.10) for $\text{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) \to U_a$; explicitly

$$(\mathcal{D}^\alpha \mathcal{D}_\alpha - 24 R^\dagger) U_a - (\mathcal{D}_{\dot\alpha} \mathcal{D}^{\dot\alpha} - 24R) \bar{U}_a = -2i\,{}^*\Phi_a = -\frac{2i}{3!} \epsilon_{mnpq} \partial^m \Gamma^{npq}_a = -16i \nabla^m B^a_m.$$
$$(4.42)$$

We obtain

$$-b'_a{}^*\Phi_a - \frac{1}{2} \sum_{\alpha,b} b^\alpha_b \left(\frac{b^\alpha_a u_a}{\sum_c b^\alpha_c u_c} + \text{h.c.} \right) {}^*\Phi_b = 8\frac{\partial V_{pot}}{\partial \omega_a}, \quad \sum_a b_a {}^*\Phi_a = 0. \qquad (4.43)$$

If $\Gamma^{npq} \neq 0$, b^{pq} can be removed by a gauge transformation $\Gamma^{npq} \to \Gamma^{npq} + \partial^{[n} \Lambda^{pq]}$. Thus

$$B^a_m = \frac{1}{2nb_a} \epsilon_{mnpq} \partial^n \tilde{b}^{pq} + \frac{1}{3!8} \epsilon_{mnpq} \Gamma^{npq}_a, \quad \sum_a b_a \Gamma^{npq}_a = 0, \quad \tilde{b}^{pq} = \sum_a b_a b^{pq}_a, \qquad (4.44)$$

and we have the additional equations of motion:

$$\frac{\delta}{\delta \tilde{b}_{pq}} \mathcal{L}_B = 0, \quad \left(\frac{1}{b_a} \frac{\delta}{\delta \Gamma_{npq}^a} - \frac{1}{b_b} \frac{\delta}{\delta \Gamma_{npq}^b} \right) \mathcal{L}_B = 0, \quad \frac{\delta}{\delta \phi} \mathcal{L}_B \equiv \frac{\partial \mathcal{L}_B}{\partial \phi} - \nabla^m \left(\frac{\partial \mathcal{L}_B}{\partial (\nabla^m \phi)} \right),$$

(4.45)

which are equivalent, respectively, to

$$\epsilon_{mnpq} \sum_a \frac{1}{b_a} \nabla^n \frac{\delta}{\delta B_a^m} \mathcal{L}_B = 0, \quad \left(\frac{1}{b_a} \frac{\delta}{\delta B_a^m} - \frac{1}{b_b} \frac{\delta}{\delta B_b^m} \right) \mathcal{L}_B = 0, \qquad (4.46)$$

with

$$
\begin{aligned}
\frac{1}{e} \frac{\delta}{\delta B_m^a} \mathcal{L}_B &= \frac{(1 + \ell g_\ell)}{2\ell^2} B^m + b_a' \partial^m \omega_a + \frac{1}{2} \sum_{\alpha, b} b_a^\alpha \left(\frac{b_b^\alpha u_b}{\sum_c b_c^\alpha u_c} + \text{h.c.} \right) \partial^m \omega_b \\
&+ \sum_\alpha b_a^\alpha \left[\partial^m \ell \frac{\partial \phi^\alpha}{\partial \ell} + \sum_I \left(\partial^m t^I \frac{\partial \phi^\alpha}{\partial t^I} + \text{h.c.} \right) \right] \\
&+ i \sum_{a, I} \frac{b_a^I}{8\pi^2} \left[\zeta(t^I) \partial^m t^I - \text{h.c.} \right] - \frac{b}{2} \sum_I \frac{\partial^m \text{Im} t^I}{\text{Re} t^I}.
\end{aligned}
$$

(4.47)

Combining these with (4.39) and the equations of motion for ℓ and t^I, one can eliminate B_m^a to obtain the equations of motion for an equivalent scalar-axion Lagrangian.

Again, these equations simplify considerably if we assume that for fixed α, $b_a^\alpha \neq 0$ for only one value of a. In this case, (4.39) reduces to

$$\nabla^m B_m^a = -\frac{1}{b_a} \frac{\partial V}{\partial \omega_a}, \qquad (4.48)$$

and we have

$$\frac{\partial \phi^\alpha}{\partial \ell} = 0, \quad \frac{\partial \phi^\alpha}{\partial t^I} = i\zeta(t^I)\left(q_I^\alpha - 1\right), \qquad (4.49)$$

if we restrict the potential to terms of dimension three with no gauge singlets M^i.

Using $\sum_\alpha b^\alpha_a (q^\alpha_I - 1) + b^I_a/8\pi^2 = b - b_a$ gives:

$$\frac{1}{e}\frac{\delta}{\delta B^a_m}\mathcal{L}_B = \frac{(1+\ell g_\ell)}{2\ell^2}B^m + b_a\partial^m\omega_a + i\sum_I\left\{\partial^m t^I\left[\zeta(t^I)(b-b_a) + \frac{b}{4\mathrm{Re}t^I}\right] - \mathrm{h.c.}\right\}$$

$$\approx \frac{(1+\ell g_\ell)}{2\ell^2}B^m + b_a\partial^m\omega_a + \sum_I\partial^m\mathrm{Im}t^I\left[(b-b_a)\frac{\pi}{6} - \frac{b}{2\mathrm{Re}t^I}\right], \qquad (4.50)$$

where the last line corresponds to the approximation $\zeta(t^I) \approx -\pi/12$. In the following we illustrate these equations using specific cases.

4.3.1 Single Gaugino Condensate

As we have seen in Section 2.3.2, for the case of a single gaugino condensate there is an axion $\omega = \omega_a + (\pi/6)(b/b_a - 1)\sum_I\mathrm{Im}t^I$ that has no potential, and, setting

$$B^m_a = \frac{1}{2}\epsilon^{mnpq}\partial_n b_{pq} = -\frac{2\ell^2}{(1+\ell g_\ell)}\left(b_a\partial^m\omega - \frac{b}{2}\sum_I\frac{\partial^m\mathrm{Im}t^I}{\mathrm{Re}t^I}\right), \qquad (4.51)$$

the equations of motion derived from (4.27) are equivalent to those of the effective bosonic Lagrangian:

$$\frac{1}{e}\mathcal{L}_B = -\frac{1}{2}\mathcal{R} - (1+b\ell)\sum_I\frac{\partial^m\bar{t}^I\partial_m t^I}{(t^I+\bar{t}^I)^2} - \frac{1}{4\ell^2}(1+\ell g_\ell)\partial^m\ell\,\partial_m\ell - V(\ell,t^I,\bar{t}^I)$$

$$-\frac{\ell^2}{(1+\ell g_\ell)}\left(b_a\partial^m\omega - \frac{b}{2}\sum_I\frac{\partial^m\mathrm{Im}t^I}{\mathrm{Re}t^I}\right)\left(b_a\partial_m\omega - \frac{b}{2}\sum_I\frac{\partial_m\mathrm{Im}t^I}{\mathrm{Re}t^I}\right). \qquad (4.52)$$

4.3.2 Two Gaugino Condensates: $b_1 \neq b_2$

Making the approximation $\eta(t) \approx e^{-\pi t/12}$, the Lagrangian (4.27) can be written as follows:

$$\frac{1}{e}\mathcal{L}_B = -\frac{1}{2}\mathcal{R} - (1+b\ell)\sum_I\frac{\partial^m\bar{t}^I\partial_m t^I}{(t^I+\bar{t}^I)^2} - \frac{1}{4\ell^2}(1+\ell g_\ell)(\partial^m\ell\,\partial_m\ell - B^m B_m)$$

$$-\omega\nabla^m\tilde{B}_m - \omega'\nabla^m B_m - \frac{b}{2}\sum_I\frac{\partial^m\mathrm{Im}t^I}{\mathrm{Re}t^I}B_m - V_{pot}, \qquad (4.53)$$

where

$$\omega = \frac{b_1\omega_1 - b_2\omega_2}{b_1 - b_2} - \frac{\pi}{6}\sum_I \mathrm{Im}t^I, \quad \omega' = -\frac{\omega_{12}}{\beta} + \frac{b\pi}{6}\sum_I \mathrm{Im}t^I,$$

$$\beta = \frac{b_1 - b_2}{b_1 b_2}, \quad \tilde{B}^m = \sum_a b_a B_a^m. \tag{4.54}$$

We have

$$\omega_1 = \omega + \frac{\pi}{6}\sum_I \mathrm{Im}t^I + \frac{1}{b_1}\left(\omega' - \frac{b\pi}{6}\sum_I \mathrm{Im}t^I\right),$$

$$\omega_2 = \omega + \frac{\pi}{6}\sum_I \mathrm{Im}t^I + \frac{1}{b_2}\left(\omega' - \frac{b\pi}{6}\sum_I \mathrm{Im}t^I\right),$$

$$\frac{\partial V_{pot}}{\partial\omega_1} = -\frac{\partial V_{pot}}{\partial\omega_2} = \frac{\partial V_{pot}}{\partial\omega_{12}}. \tag{4.55}$$

Then taking ω, ω' and t^I as independent variables, the equations of motion for ω and ω' are:

$$\nabla^m \tilde{B}_m = 0, \quad \tilde{B}_m = \frac{1}{2}\epsilon_{mnpq}\partial^n \tilde{b}^{pq},$$

$$\nabla^m B_m = \frac{1}{8}{}^*\Phi = \beta\frac{\partial V}{\partial\omega_{12}}, \quad B_m = \frac{1}{3!8}\epsilon_{mnpq}\Gamma^{npq}. \tag{4.56}$$

Substituting the first of these into the Lagrangian (4.53), we see that the axion ω and the three-form $\tilde{B}_m$ drop out because they appear only linearly in the Lagrangian; hence they play the role of Lagrange multipliers. The equation of motion for $\tilde{b}_{mn}$ implies the constraint on the phase ω as follows:

$$\nabla_m\partial^m\omega = 0. \tag{4.57}$$

The equations of motion for $\mathrm{Im}t^I$ and Γ_{mnp} read:

$$0 = \nabla_m\left[(1+b\ell)\frac{\partial^m\mathrm{Im}t^I}{2\left(\mathrm{Re}t^I\right)^2} + \frac{b}{2\mathrm{Re}t^I}B^m\right] - i\left(\frac{\partial V}{\partial t^I} - \text{h.c.}\right) - \frac{b\pi}{48}{}^*\Phi,$$

$$0 = \frac{(1+\ell g_t)}{2\ell^2}B^m + \partial^m \omega' - \frac{b}{2}\sum_I \frac{\partial^m \mathrm{Im}t^I}{\mathrm{Re}t^I}, \tag{4.58}$$

and the equivalent bosonic Lagrangian is:

$$\frac{1}{e}\mathcal{L}_B = -\frac{1}{2}\mathcal{R} - (1+b\ell)\sum_I \frac{\partial^m \bar{t}^I \partial_m t^I}{(t^I + \bar{t}^I)^2} - \frac{1}{4\ell^2}(1+\ell g_t)\,\partial^m \ell\,\partial_m \ell$$

$$-\frac{\ell^2}{(1+\ell g_t)}\left(\partial^m \omega' - \frac{b}{2}\sum_I \frac{\partial^m \mathrm{Im}t^I}{\mathrm{Re}t^I}\right)\left(\partial_m \omega' - \frac{b}{2}\sum_I \frac{\partial_m \mathrm{Im}t^I}{\mathrm{Re}t^I}\right)$$

$$-V_{pot}(\ell, t^I, \bar{t}^I, \omega_{12}). \tag{4.59}$$

As in Section 4.3.1, there is a single dynamical axion ω' – or, via a duality transformation, $^*\Phi$ – but there is now a potential for the axion in the multi-condensate case.

4.3.3 General Case

We introduce n linearly independent vectors $\tilde{B}_m, B_m, \hat{B}_m^i$, $i = 1\ldots n-2$, and decompose the B_a^m as follows:

$$B_a^m = a_a \tilde{B}^m + c_a B^m + \sum_i d_a^i \hat{B}_i^m, \quad \hat{B}_i^m = \sum_a e_i^a B_a^m. \tag{4.60}$$

Then

$$\sum_a \left[b_a \omega_a + (b-b_a)\frac{\pi}{6}\sum_I \mathrm{Im}t^I\right]\nabla_m B_a^m = \omega \nabla_m \tilde{B}^m + \omega'\nabla_m B^m + \sum_i \omega^i \nabla_m \hat{B}_i^m,$$

$$\omega_a = \omega + \frac{\pi}{6}\sum_I \mathrm{Im}t^I + \frac{1}{b_a}\left(\omega' - \frac{b\pi}{6}\sum_I \mathrm{Im}t^I\right) + \sum_i \frac{e_i^a}{b_a}\omega^i, \tag{4.61}$$

and the Lagrangian can be written as in (4.53) with an additional term:

$$\frac{1}{e}\mathcal{L}_B \to \frac{1}{e}\mathcal{L}_B - \sum_i \omega^i \nabla_m \hat{B}_i^m, \tag{4.62}$$

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The equations of motion for the phases ω, ω' and ω^i are:

$$\nabla_m \tilde{B}^m = -\frac{\partial V_{pot}}{\partial \omega} = -\sum_a \frac{\partial V_{pot}}{\partial \omega_a} = 0,$$

$$\nabla_m B^m = -\frac{\partial V_{pot}}{\partial \omega'} = -\sum_a \frac{1}{b_a} \frac{\partial V_{pot}}{\partial \omega_a} = \frac{1}{2} \sum_{ab} \beta_{ab} \frac{\partial V_{pot}}{\partial \omega_{ab}} = \frac{1}{8} {}^* \Phi, \quad \beta_{ab} \equiv \frac{b_a - b_b}{b_a b_b}$$

$$\nabla_m \hat{B}_i^m = -\frac{\partial V_{pot}}{\partial \omega^i} = -\sum_a \frac{e_i^a}{b_a} \frac{\partial V_{pot}}{\partial \omega_a} = \frac{1}{8} {}^* \Phi_i, \tag{4.63}$$

and the equations for $\Gamma^i_{mnp} = 8\epsilon_{mnpq} \hat{B}_i^q$ give $\partial^m \omega^i = 0$. Hence

$$\omega_{ab} = -\beta_{ab} \left(\omega' - \frac{b\pi}{6} \sum_I \text{Im} t^I \right) + \theta_{ab}, \quad \theta_{ab} = \text{constant.} \tag{4.64}$$

Therefore, as in the two-condensate case of Section 4.3.2, there is one dynamical axion with a potential. The dual bosonic Lagrangian is the same as (4.59), with $V_{pot} = V_{pot}(\ell, t^I, \bar{t}^I, \omega_{ab})$.

4.4 The Effective Potential

The potential (4.37) can be written in the form

$$V_{pot} = \frac{1}{16\ell^2} (v_1 - v_2 + v_3),$$

$$v_1 = (1 + \ell g_\ell) \left| \sum_a (1 + b_a \ell) u_a \right|^2, \quad v_2 = 3\ell^2 \left| \sum_a b_a u_a \right|^2,$$

$$v_3 = \frac{\ell^2}{(1 + b\ell)} \sum_I \left| \sum_a d_a(t^I) u_a \right|^2 = 4\ell^2 (1 + b\ell) \sum_I \left| \frac{F^I}{\text{Re} t^I} \right|^2. \tag{4.65}$$

In the strong coupling limit

$$\lim_{\ell \to \infty} V_{pot} = (\ell g_\ell - 2) \left| \sum_a b_a u_a \right|^2, \tag{4.66}$$

giving the exactly same condition on the functions f, g as (2.57) to assure bound-edness of the scalar potential. Therefore (2.57), the necessary condition for stringy

non-perturbative effects to stabilize the dilaton, is indeed true in general. Note however that if $v_1 = v_3 = 0$ has a solution with $v_2 \neq 0$, the vacuum energy is always negative. $v_3 = 0$ is solved by $t^I = 1$, *i.e.* the self-dual point. As explained below, this is the only nontrivial minimum if the cosmological constant is fine-tuned to vanish. In the case of two condensates, there is no solution to $v_1 = 0$, $v_2 \neq 0$, for $f \geq 0$, and the cosmological constant can be fine-tuned to vanish, as will be illustrated below in a toy example. More generally, the scalar potential V_{pot} is dominated by the gaugino condensate with the largest one-loop β-function coefficient, so the general case is qualitatively very similar to the single condensate case, and it appears that positivity of the scalar potential can always be imposed. Otherwise, one would have to appeal to another source of supersymmetry breaking to cancel the cosmological constant, such as a fundamental 3-form potential [21, 41] whose field strength is dual to a constant that has been previously introduced in the superpotential [16], and/or an anomalous $U(1)$ gauge symmetry [17].

In the following we study Z_3-inspired toy models with E_6 and/or $SU(3)$ gauge groups in the hidden sector, and $3N_f$ matter superfields [59] in the fundamental representation f. Asymptotic freedom requires $N_{27} \leq 3$ and $N_3 \leq 5$. For a true Z_3 orbifold there are no moduli-dependent threshold corrections: $b_a^I = 0$. In this case, universal anomaly cancellation determines the average value of the matter modular weights in these toy models as: $\langle\, 2q_I^{27} - 1 \,\rangle = 2/N_{27}$, $\langle\, 2q_I^3 - 1 \,\rangle = 18/N_3$. In some models Wilson line breaking of the hidden sector E_8 generates vector-like representations that could acquire masses above the condensation scale, so

that the universal anomaly cancellation sum rule is not saturated by light states alone. In this case the q_f^α no longer drop out of the equations, so some of the above formulae would be slightly modified. In addition, one would have to include threshold effects [32], unless the masses of the heavy states are pushed to the string scale. Here we assume for simplicity that the sum rule is saturated by the light states. Denoting the fundamental matter fields by $\Phi_f^{I\alpha}$, $\alpha = 1, \ldots, N_f$, the hidden matter condensates can be constructed as

$$\Pi_f^\alpha = \prod_{I=1}^3 \Phi_f^{I\alpha}, \quad b_{E_6}^\alpha = \frac{3}{4\pi^2}, \quad b_{SU(3)}^\alpha = \frac{1}{8\pi^2},$$

where gauge indices have been suppressed.

In the analysis of the models described below, we assume – for obvious phenomenological reasons – that the vacuum energy vanishes at the minimum $\langle\, V_{pot}\,\rangle = 0$. Thus we solve the following equations:

$$V_{pot} = \frac{\partial V_{pot}}{\partial x} = 0, \quad x = \ell, t^I, \omega_a. \tag{4.67}$$

For $x = \ell, t^I$, we have

$$\frac{\partial \rho_a}{\partial x} = \frac{1}{2}\left(A_x + \frac{1}{b_a}B_x\right)\rho_a, \quad B_\ell = \frac{(1 + \ell g_\ell)}{\ell^2}, \quad B_I = \frac{b}{2\mathrm{Re}t^I}\left[1 + 4\zeta(t^I)\mathrm{Re}t^I\right],$$

$$\frac{\partial V_{pot}}{\partial x} = \left(A_x - \frac{2}{\ell}\delta_{x\ell}\right)V_{pot} + \frac{1}{16\ell^2}\sum_{ab}\rho_a\rho_b\cos\omega_{ab}\left(\frac{B_x}{b_a}R_{ab} + \frac{\partial}{\partial x}R_{ab}\right)$$

$$= \frac{1}{16\ell^2}\sum_{ab}\rho_a\rho_b\cos\omega_{ab}\left(\frac{B_x}{n}\sum_c\beta_{ca}R_{ab} + \frac{\partial}{\partial x}R_{ab}\right)$$

$$+ \left(A_x - \frac{2}{\ell}\delta_{x\ell} + \frac{B_x}{n}\sum_a\frac{1}{b_a}\right)V_{pot}, \tag{4.68}$$

where β_{ab} is defined in (4.63). By assumption, the last term in (4.68) vanishes in the vacuum. Note that the self-dual point, $d_a(t^I) = B_I = 0$, $t^I = 1$, is always

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a solution to the minimization equations for t^I. It is the only solution for the single condensate case. For the multi-condensate case, if we restrict our analysis to the (relatively) weak coupling region, $\ell < 1/b_-$, where b_- is the smallest β-function coefficient, the scalar potential V_{pot} is dominated by the gaugino condensate with the largest β-function coefficient b_+ : $V_{pot} \approx \rho_+^2 R_{++}/16\ell^2$. Moreover, since $\pi b/3b_a > 1$, the scalar potential V_{pot} is always dominated by this term for $\mathrm{Re} t^I > 1$ (c.f. Eq. (4.38)), so the only minimum for $\mathrm{Re} t^I > 1$ is $\mathrm{Re} t^I \to \infty$, $\rho_a \to 0$. By duality the only minimum for $\mathrm{Re} t^I < 1$ is $\mathrm{Re} t^I \to 0$, $\rho_a \to 0$, so the self-dual point is the only nontrivial solution. Since our scalar potential is always dominated by one gaugino condensate, the picture is very different from the "race-track" models studied previously [3].

At the self-dual point with $V_{pot} = 0$, we have

$$
\begin{aligned}
\frac{\partial^2 V_{pot}}{\partial (t^I)^2} &\approx \frac{1}{32\ell^2} \sum_{ab} \rho_a \rho_b \cos \omega_{ab} \left(\frac{\pi^2}{9} \frac{\ell^2}{(1+b\ell)} (b - b_a)(b - b_b) - \frac{b\pi}{6n} \sum_c \beta_{ca} R_{ab} \right) \\
&\approx \frac{\rho_+^2}{32} \left(\frac{\pi^2}{9} \frac{(b - b_+)^2}{(1+b\ell)} - \frac{b\pi}{6n\ell^2} \sum_c \beta_{c+} R_{++} \right).
\end{aligned}
\tag{4.69}
$$

Positivity of the potential requires $R_{++} \geq 0$, and $\beta_{c+} \leq 0$ by definition, so the extremum at the self-dual point with $V_{pot} = 0$, $\rho_+ \neq 0$ is a true minimum. In practice, the last term is negligible, and the normalized moduli squared mass is:

$$
m_{t^I}^2 \approx \left\langle \frac{1}{4} \frac{(b - b_+)^2}{(1+b\ell)^2} \rho_+^2 \right\rangle.
\tag{4.70}
$$

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4.4.1 Single Gaugino Condensate with Hidden Matter

In this case $\beta_{ab} = 0$, and the minimization equations for t^I require

$$\frac{\partial}{\partial t^I} \left| 1 + 4\zeta(t^I)\text{Re}t^I \right|^2 = 0,$$

which is solved by $1 + 4\zeta(t^I)\text{Re}t^I = 0$, $t^I = 1$. Then $v_3 = F^I = 0$, and the scalar potential V_{pot} is qualitatively the same as in the E_8 case studied in Chapter 2 – except for the fact that here the string moduli are stabilized at the self-dual point. (Note however that if $\beta_{ab} = 0$ one can choose the $b'_{a\alpha}$ in (4.20) such that the matter condensates drop out of the effective Lagrangian; then $R_{a\alpha}$ is independent of the moduli which remain undetermined.) The quantitative difference from the E_8 case is the value of the β-function coefficient: $b_{E_6} = (12 - 3N_{27})/8\pi^2$, $b_{SU(3)} = (6 - N_3)/16\pi^2$. As in Chapter 2, two possible choices for the function f are $f = Ae^{-B/V}$ [7] and $f = A_p(\sqrt{V})^{-p}e^{-B/\sqrt{V}}$ [4], where we fine tune the parameter A (or A_p) to get a vanishing cosmological constant.

Attention has been drawn to the leading correction for small coupling that is of the form $f = Ae^{-B/\sqrt{V}}$ [4]. If we restrict f to this form, we have to require a rather large value for the parameter A: $A \simeq 40$ in order to cancel the cosmological constant. On the other hand, the important feature of f here is its behaviour in the strong coupling regime; if f contains terms of the form $Ae^{-B/V^{\frac{n}{2}}}$, the strong coupling limit will be dominated by the term with the largest value of n. In the numerical analysis we take $f = Ae^{-B/V}$; adding to this a term of the form $f = A'e^{-B'/\sqrt{V}}$ will not significantly affect the analysis. We find that the *vev* of ℓ is insensitive to the

content of the hidden sector; it is completely determined by stringy non-perturbative effects, provided a potential for ℓ is generated by the strongly coupled hidden Yang-Mills sector. More specifically, taking $f = Ae^{-B/V}$ we find that $\langle V_{pot} \rangle = 0$ requires $A \approx e^2 \approx 7.4$, and the dilaton is stabilized at a value $\langle \ell \rangle \approx B/2$. Taking $B = 1$ gives $\langle \ell \rangle \approx 0.5$, $\langle f(\ell) \rangle \approx 1$, and the squared gauge coupling at the string scale is $g_s^2 = \langle 2\ell/(1 + f) \rangle \approx 0.5$. If instead we use $f = Ae^{-B/\sqrt{V}}$, the corresponding numbers are $A \approx 2e^3 \approx 40$, $\langle \ell \rangle \approx B^2/9$, $g_s^2 \approx 2B^2/27$. Therefore, the *vev* of the dilaton ℓ completely determined by stringy non-perturbative effects, and the dilaton is naturally stabilized at a weak coupling regime if, for example, the parameter B in the function f considered here is of order one.

One may look more closely at the second choice which is a genuine stringy nonperturbative effect[3]. Taking for illustrative purposes $f = \left(A_0 + A_1/\sqrt{\ell}\right) e^{-B/\sqrt{\ell}}$, where the condition (4.66) or (2.57) requires A_0 to be larger than 2, one finds a realistic minimum for $\mathcal{O}(1)$ values of the parameters: $B\langle \ell \rangle^{-1/2} \approx 1.1$ to 1.3, $A_0 \approx 2.7$ to 5.3 and $A_1 \approx -3.1$ to -4.6. Therefore, the previous problem of a rather large value of A $(A \approx 40)$ for $f = Ae^{-B/\sqrt{V}}$ does not exist in general. From now on we take $f = Ae^{-1/V}$ in the numerical analysis, but notice that the major conclusions of the analysis apply to more generic choices for f.

The scalar potential V_{pot} for $\mathcal{G}_a = E_6$, $N_{27} = 1$, is plotted in Figures 4.1–

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[3]We do not consider here the case where the coefficient B in the exponent is moduli-dependent [6]. Such stringy nonperturbative contributions would perturb the moduli ground state away from the self-dual point. However, one has to worry about the problem of modular invariance for this type of stringy nonperturbative contributions [60]
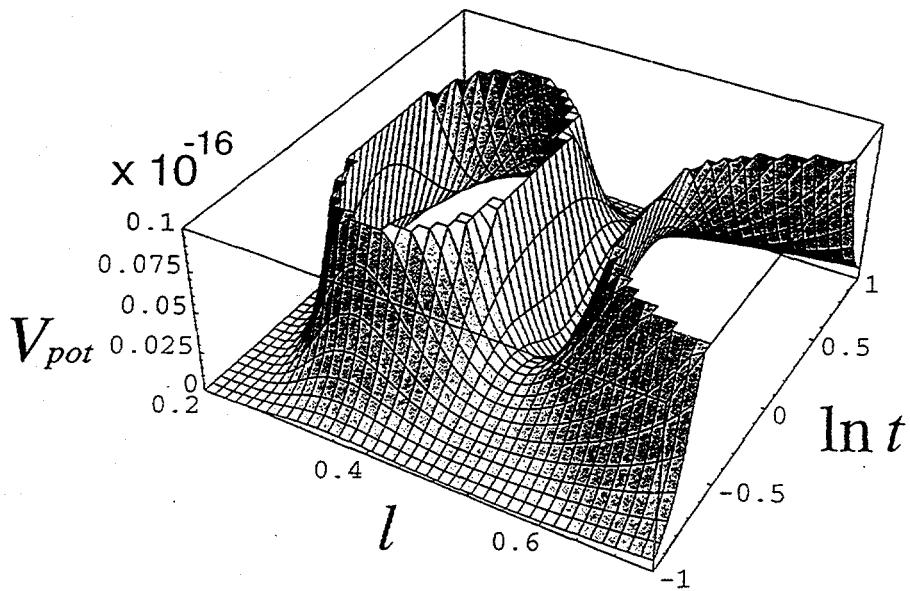
Figure 4.1: The scalar potential V_{pot} (in reduced Planck units) is plotted versus ℓ and $\ln t$.

4.3. Fig. 4.1 shows the scalar potential in the $\ell, \ln t$ plane, where we have set $t^I = t$, $\mathrm{Im}\, t = 0$; with this choice of variables the T-duality invariance of the scalar potential is manifest. Fig. 4.2 shows the scalar potential V_{pot} for ℓ at the self-dual point $t^I = 1$, and Fig. 4.3 shows the scalar potential for $\ln t$ with ℓ fixed at its *vev*.

The qualitative features of the scalar potential are independent of the content of the hidden sector. Fixing A in each case by the condition $\langle V_{pot} \rangle = 0$, we find for $\mathcal{G}_a = E_6$

$$A = \begin{cases} 7.324 \\ 7.359 \,, \\ 7.381 \end{cases} \quad \langle \ell \rangle = \begin{cases} 0.502 \\ 0.501 \approx g_s^2, \\ 0.500 \end{cases} \quad \text{for} \quad N_{27} = \begin{cases} 1 \\ 2 \,. \\ 3 \end{cases} \quad (4.71)$$

For $\mathcal{G}_a = SU(3)$, $N_3 = 1$, we find $A = 7.383$, $\langle \ell \rangle = 0.500 \approx g_s^2$. As will be discussed in Section 4.5, the scale of supersymmetry breaking in this case is far too
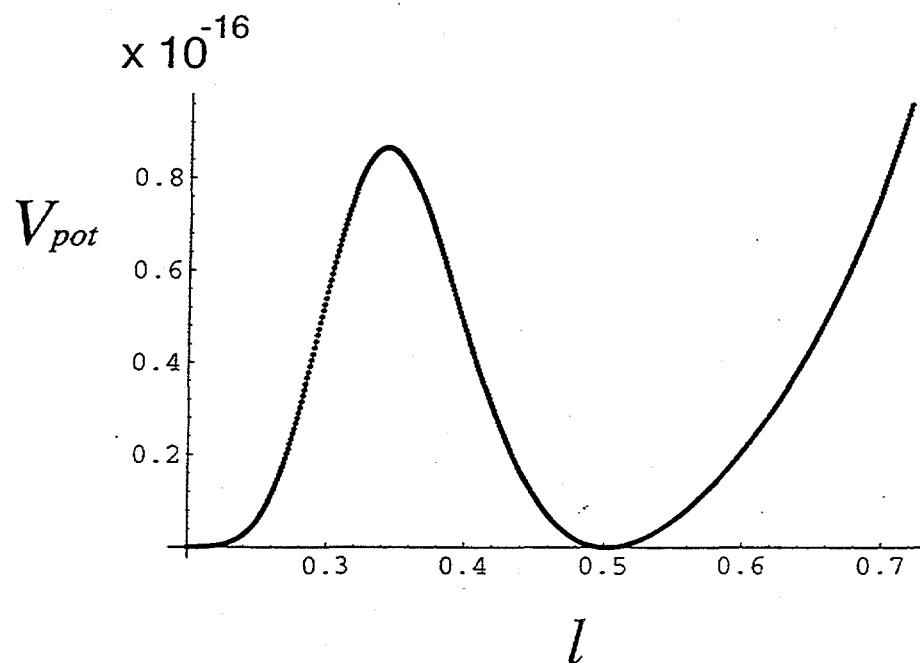
Figure 4.2: The scalar potential V_{pot} (in reduced Planck units) is plotted versus ℓ with $t^I = 1$ (the self-dual point).
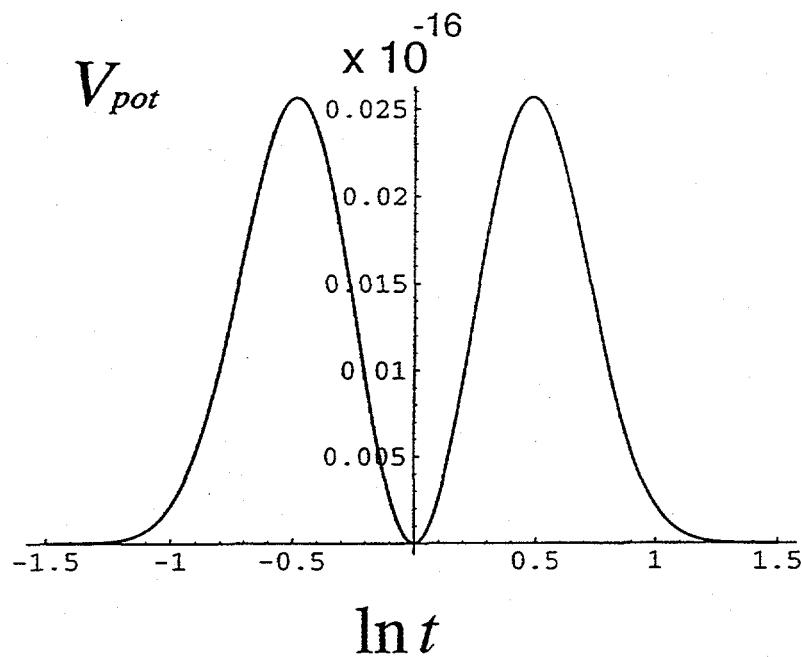

Figure 4.3: The scalar potential V_{pot} (in reduced Planck units) is plotted versus $\ln t$ with $\ell = \langle \ell \rangle$.

small, and further decreases with increasing N_3.

4.4.2 Two Gaugino Condensates

We have

$$\frac{\partial V_{pot}}{\partial \omega_1} = -\frac{\partial V_{pot}}{\partial \omega_2} = -\rho_1 \rho_2 R_{12} \sin \omega_{12},$$

$$\sum_{abc} \beta_{ca} \rho_a \rho_b R_{ab} \cos \omega_{ab} = \beta_{21} \left(\rho_1^2 R_{11} - \rho_2^2 R_{22} \right). \tag{4.72}$$

Minimization with respect to ω_1 requires either $\langle \sin \omega_{12} \rangle = 0$ or $\langle R_{12} \rangle = 0$. Identifying $b_1 = b_+$, $b_2 = b_-$, positivity of the scalar potential requires $R_{11} \geq 0$, which in turn implies $R_{12} > 0$, so the extrema in ω are at $\sin \omega_{12} = 0$, with $\cos \omega_{12} = -1$ $(+1)$ corresponding to minima (maxima):

$$\frac{\partial^2 V_{pot}}{\partial \omega_{12}^2} = -\rho_1 \rho_2 R_{12} \cos \omega_{12}, \quad m_{\omega_{12}}^2 = \left\langle \frac{3 b_+^2 \beta_{12}^2 R_{12}}{2(1 + b_+ \ell)^2} \rho_1 \rho_2 \right\rangle. \tag{4.73}$$

There is also a small $\mathrm{Im} t^I$-ω_{12} mixing. Note that while in contrast to the single condensate case, the dynamical axion is no longer massless, its mass is exponentially suppressed relative to the gravitino mass by a factor $\sim \langle \rho_2/\rho_1 \rangle^{1/2}$. Therefore, in generic string models there is only one very light axion[4] (i.e., the model-independent axion). As will be discussed in Chapter 5, this very light axion has the right properties to be the QCD axion [61].

For $\mathcal{G} = E_6 \otimes SU(3)$, the potential is dominated by the E_6 gaugino condensate, and the results are the same as in (4.71). The only other gauge groups in the

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[4]As discussed in Section 3.3.1, this statement is true in the context of both static and dynamical gaugino condensation, where the former is the effective description of the latter.

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restricted set considered here that are subgroups of E_8 are $\mathcal{G} = [SU(3)]^n$, $n \leq 4$; these cannot generate sufficient supersymmetry breaking.

4.5   Supersymmetry Breaking

The pattern and scale of supersymmetry breaking are determined by the *vev*'s of the F components of the chiral superfields. From the equations of motion for π^α and ρ_a we obtain, at the self-dual point $\langle F^I \rangle = 0$:

$$\langle F^\alpha \rangle = \frac{(1 + \ell g_\ell)}{4\ell^2 b_a} \pi^\alpha \left(\bar{u} + \ell \sum_b b_b \bar{u}_b \right) \approx \frac{3b_+^2}{4b_a} \pi^\alpha \bar{u}_+ (1 + \ell b_+)^{-1}, \quad b_a^\alpha \neq 0,$$

$$\langle F^a + \bar{F}^a \rangle = \frac{1}{4\ell^2 b_a}(1 + \ell g_\ell)(1 + \ell b_a) \left[u_a \left(\bar{u} + \ell \sum_b b_b \bar{u}_b \right) + \text{h.c.} \right]$$

$$\approx \frac{3b_+^2}{4b_a} \frac{1 + \ell b_a}{1 + \ell b_+} \left(u_a \bar{u}_+ + \bar{u}_a u_+ \right), \tag{4.74}$$

where the approximations on the right hand sides are exact for a single gaugino condensate. The dominant contribution is from the gaugino condensate with the largest β-function coefficient:

$$\langle F^+ + \bar{F}^+ \rangle = \frac{3\rho_+^2 b_+}{2}. \tag{4.75}$$

It has been known for some time that, if the dominant supersymmetry breaking effects come from the dilaton rather than the moduli, the soft supersymmetry breaking parameters are naturally flavor blind, and non-universal squark and slepton masses that could induce unacceptably large flavor-changing neutral currents (FCNC) could be thereby avoided [62]. Therefore, the fact that the F^I vanish in the vacuum is a desirable feature for phenomenology. And it should be empha-

sized that this unique feature is just the natural consequence of modular invariance and a correct treatment of gaugino condensation in string theory. In other words, a modular invariant treatment of gaugino condensation in string theory naturally leads to the phenomenologically desirable dilaton-dominated supersymmetry breaking scenario, which is very impressive! However, as we will see in Chapter 5, the dilaton-dominated supersymmetry breaking scenario is not always free from the FCNC problem, which means the the analysis of dilaton-dominated scenario in the past [2, 62] is oversimplified. In fact, possible non-universal couplings of the matter superfields to the Green-Schwarz counterterm could induce non-universal squark and slepton masses. More discussion of this problem will be given in Chapter 5.

Another important parameter for soft supersymmetry breaking in the observable sector is the gravitino mass $m_{\tilde{G}}$. The derivation of the gravitino part of the Lagrangian again parallels the construction in Section 2.3.2. The gravitino mass $m_{\tilde{G}}$ is determined by the term:

$$
\mathcal{L}_{mass}(\psi) = -\frac{1}{8}\psi^m \sigma_{mn} \psi^n \sum_a \bar{u}_a \left\{ \frac{1+f}{\ell} + b'_a \ln(e^{2-K}\bar{u}_a u_a) + \sum_\alpha b_a^\alpha \ln(\pi^\alpha \bar{\pi}^\alpha) \right.
$$
$$
\left. + \sum_I \left[bg^I - \frac{b_a^I}{4\pi^2} \ln|\eta(t^I)|^2 \right] \right\} - e^{K/2}\bar{W}\psi^m \sigma_{mn}\psi^n + \text{h.c.}, \quad (4.76)
$$

giving, when the equations of motion (4.26) are imposed,

$$
m_{\tilde{G}} = \frac{1}{3}\langle |M| \rangle = \frac{1}{4} \langle |\sum_a b'_a u_a - 4e^{K/2}W| \rangle = \frac{1}{4} \langle |\sum_a b_a u_a| \rangle \approx \frac{1}{4} b_+ \langle \rho_+ \rangle. \quad (4.77)
$$

The scale of supersymmetry breaking is governed by the *vev* (4.31) of the gaugino condensate with the largest β-function coefficient. This includes the usual

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suppression factor $\langle \rho_a \rangle \propto e^{-1/b_a g_s^2}$, where $g_s^2 = \langle 2\ell/(1+f) \rangle$ is the effective squared coupling constant at the string scale. However, there are also other important parameters that determine the scale of the hierarchy between the supersymmetry breaking scale and the Planck scale. The dependence on the string moduli provides a second exponential suppression factor:

$$\langle \rho_a \rangle \propto \langle \prod_I |\eta(t^I)|^{2(b-b_a)/b_a} \rangle = |\eta(1)|^{6(b-b_a)/b_a} \approx e^{-\pi(b-b_a)/2b_a}. \qquad (4.78)$$

On the other hand, the numerical factor $\prod_\alpha |b_\alpha^\alpha/4c_\alpha|^{-b_\alpha^\alpha/b_a}$ generates an exponential enhancement if $c_\alpha \sim 1$. This is the largest numerical uncertainty in our analysis. *A priori*, c_α is related to the Yukawa couplings of matter fields in the hidden sector. However, there is an arbitrary normalization factor in the definition of Π^α. If the hidden-sector Yukawa couplings were known, it might be possible to estimate c_α by a matching condition for the *vev*'s of the second lines of (4.32) and (4.33). In our numerical analysis, we have set $c_\alpha = 1$. Then, if the hidden gauge group with the largest β-function coefficient is $\mathcal{G}_+ = E_6$ with $3N_{27}$ matter chiral superfields in the fundamental representation, we obtain:

$$m_{\tilde{G}} = \begin{cases} 1.1 \times 10^{-9} \\ 3.3 \times 10^{-11} \\ 1.65 \times 10^{-15} \end{cases} \quad \text{for} \quad N_{27} = \begin{cases} 1 \\ 2\,, \\ 3 \end{cases} \qquad (4.79)$$

in reduced Planck units. For $\mathcal{G}_+ = SU(3)$ with three matter chiral superfields in the fundamental representation, we obtain an unacceptably large gauge hierarchy: $m_{\tilde{G}} = 2.2 \times 10^{-32}$; $m_{\tilde{G}}$ decreases rapidly as N_3 increases, *i.e.* as the β-function coefficient decreases.

4.6 Concluding Remarks

In the class of models studied here, the introduction of a parameterization for stringy nonperturbative contributions to the Kähler potential for the dilaton generically allows a stable vacuum at a nontrivial, phenomenologically acceptable point in the dilaton/moduli space. In particular, when we impose the constraint that the cosmological constant vanishes, we find that in the linear multiplet formalism, the string moduli t^I are stabilized at the self-dual point, and their associated F components vanish in the vacuum, which results in a phenomenologically desirable dilaton-dominated supersymmetry breaking scenario. This striking feature of string phenomenology is in fact just the consequence of modular invariance and a correct treatment of gaugino condensation[5]. Therefore, in this sense the experimental search for a dilaton-dominated supersymmetry breaking scenario can be regarded as an indirect test of the modular invariance of superstring theory.

A salient feature of our formalism is that there is little qualitative difference between a single condensate and a multi-condensate scenario. For several gaugino condensates with equal (or very similar) β-function coefficients, the scalar potential reduces to that of the single gaugino condensate case, except that there may be flat directions. If $b_1 = b_2 = \cdots b_k$, then at the self-dual point $\rho_a/\rho_1 = \zeta_a = $ constant and the potential vanishes identically in the direction $\sum_{a=1}^k \zeta_a e^{i\omega_a} = 0$, $\rho_{a>k} = 0$.

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[5]As discussed in the appendix, an incomplete/incorrect treatment of gaugino condensation and/or modular invariance is the reason why this unique feature of string phenomenology has been ignored in the past.

This always has a solution if $\zeta_a = 1$, in which case the flat direction preserves supersymmetry and there is no barrier between this solution and the interesting, supersymmetry breaking solution. For several gaugino condensates with different β-function coefficients, the scalar potential is dominated by the gaugino condensate(s) with the largest β-function coefficient, and the result is essentially the same as in the single gaugino condensate case, except that a very small mass is generated for the dynamical (model-independent) axion. In all cases, stringy nonperturbative corrections to the dilaton Kähler potential are required to stabilize the dilaton. This picture is very different from previously studied "racetrack" models [3] where dilaton stabilization is achieved through cancellations among different gaugino condensates with similar β-function coefficients. The qualitative difference between an E_8 hidden sector and one with a product gauge group is the presence of hidden matter; in the E_8 case there is no hidden matter and the scalar potential is independent of the moduli, which therefore remain undetermined in the classical vacuum of the effective condensate theory. More phenomenological discussions of the model constructed in this chapter will be presented in Chapter 5.

4.7   Appendix: Chiral Multiplet Formalism

There has been interest in the question as to whether the linear and chiral multiplet formalisms are equivalent at the quantum level. They are presumably equivalent in the sense that technically we may always perform a duality transformation at the superfield level on the Lagrangian (4.5) so as to recast it entirely in terms of chiral

supermultiplets. The resulting effective Lagrangian should be the chiral multiplet formalism with the gaugino condensates constrained by (2.12), and it is apt to be rather complicated.

The string phenomenology that we have constructed and studied so far is quite different from the "conventional" string phenomenology in several aspects. Besides the aforementioned linear-chiral duality question, the "conventional" string phenomenology is different from ours in the sense that the constraint (2.12) on gaugino condensates has always been ignored, and usually the treatment of modular invariance is incomplete or incorrect in the "conventional" study of string phenomenology. Therefore, a more practical question that we address in this appendix is the extent to which our studies in Sections 4.1–4.6 can be reproduced if one takes as a starting point the usual chiral multiplet formalism for the dilaton with the gaugino condensates represented by *unconstrained* chiral superfields (*i.e.*, the "conventional" approach), and modular invariance is ensured through the Green-Schwarz mechanism and string threshold corrections. In particular, we would like to know how an incorrect treatment of gaugino condensation (*i.e.*, a treatment without the constraint (2.12) on gaugino condensates) might have affected our understanding of string phenomenology in the past.

In the chiral multiplet formalism, the Green-Schwarz counterterm appears as a correction to the Kähler potential, which we take to be

$$K(S, T^I) = \ln(L) + \tilde{g}(L) + \sum_I g^I, \quad L^{-1} = S + \bar{S} - b \sum_I g^I, \qquad (4.80)$$

where $\tilde{g}$ is the correction from stringy nonperturbative effects in the chiral multiplet formalism[6]. Modular invariance of the Yang-Mills Lagrangian at the quantum level is assured by the transformation property of S under (4.14):

$$S \to S + b \sum_I H^I, \tag{4.81}$$

and modular covariance of the Kähler potential $(K \to K + \sum_I (H^I + \bar{H}^I))$ requires that it depend on S only through the vector superfield L defined in (4.81). We introduce static gaugino and matter condensate superfields U_a and Π^α as before, but now the gaugino condensate chiral superfield

$$U_a = e^{K/2} H_a^3 \tag{4.82}$$

is not constrained by the constraint (2.12) or (4.42) because H_a is taken to be an *unconstrained* chiral superfield in the treatment here. (This is what has always been done in the conventional study of string phenomenology.) We construct the superpotential in analogy to (4.5), using the standard approach of Veneziano and Yankielowicz:

$$W_{tot} = W_{cond} + W(\Pi), \tag{4.83}$$

where $W(\Pi)$ is the same as in (4.30), and

$$W_{cond} = W_C + W_{VY} + W_{th}, \quad W_C = \frac{1}{4} S \sum_a H_a^3,$$

$$W_{VY} = \frac{1}{4} \sum_a H_a^3 \left(3b'_a \ln H_a + \sum_\alpha b_a^\alpha \ln \Pi^\alpha \right),$$

---

[6]Notice that the vector superfield L here is simply a convenient notation for $(S + \bar{S} - b \sum_I g^I)^{-1}$. It should not be confused with the L used in the linear multiplet formalism.

$$W_{th} \quad = \quad \frac{1}{4} \sum_{a,I} \frac{b_a^I}{8\pi^2} H_a^3 \ln[\eta^2(T^I)], \tag{4.84}$$

where W_C represents the classical contribution of gaugino condensation. H_a^3 transforms in the same way as U_a under rigid chiral and conformal transformations, and the anomaly matching conditions give the same constraints on the coefficients b's as in Section 4.2. Then it is straightforward to check that, under the modular transformation (4.14) with $H_a \to e^{-\sum_I H^I/3}$, we have $W_{cond} \to e^{-\sum_I H^I/3} W_{cond}$, as required by modular invariance of the Lagrangian. Summing the various contributions, the superpotential for H_a can be written in the following form:

$$W_{cond} = \frac{1}{4} \sum_a b_a' H_a^3 \ln \left\{ e^{S/b_a'} H_a^3 \prod_\alpha (\Pi^\alpha)^{b_a^\alpha/b_a'} \prod_I [\eta(T^I)]^{-b_a^I/4\pi^2 b_a'} \right\}. \tag{4.85}$$

The bosonic Lagrangian takes the standard form:

$$\begin{aligned}
\mathcal{L}_B \quad = \quad &-\frac{1}{2}\mathcal{R} - \frac{1}{3}M\bar{M} + K_{i\bar{m}} \left(F^i \bar{F}^{\bar{m}} - \partial_\mu z^i \partial^\mu z^{\bar{m}} \right) \\
&+ e^{K/2} \left[F^i \left(W_i + K_i W \right) - \bar{M}W + \text{h.c.} \right],
\end{aligned} \tag{4.86}$$

where $Z^i = S, T^I, H_a, \Pi^\alpha$, $z^i = Z^I|_{\theta=\bar\theta=0}$. In our static model $K_{i\bar{m}}, K_i = 0$ for $Z^i, Z^m = H_a, \Pi^\alpha$, and the equations of motion for F^i give $W_i = 0$ for these fields. This determines the chiral superfields H_a, Π^α as holomorphic functions of S, T^I. Making the same restrictions on $W(\Pi)$ and the b_a^α as in Section 4.2, we obtain:

$$H_a^3 \quad = \quad e^{(2n+1)i\pi(b_a'-b_a)/b_a - b_a'/b_a} e^{-S/b_a} \prod_I [\eta(T^I)]^{2(b-b_a)/b_a} \prod_\alpha |b_a^\alpha/4c_\alpha|^{-b_a^\alpha/b_a},$$

$$\Pi^\alpha \quad = \quad -\frac{b_a^\alpha}{4c_\alpha} H_a^3 \prod_I [\eta(T^I)]^{-2(q_I^\alpha-1)}, \quad b_a^\alpha \neq 0. \tag{4.87}$$

As in (4.31), the correct dependence of the gaugino condensates on the squared gauge coupling constant $\langle 2/\mathrm{Re}s \rangle$, $s = S|_{\theta=\bar\theta=0}$, is recovered. Note however that,

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in contrast to (4.31), the phases of gaugino condensate here are quantized once $\text{Im}s$ is fixed at its *vev.* Using these results gives

$$W_{tot} = W(S, T^I) = -\frac{1}{4} \sum_a b_a H_a^3. \tag{4.88}$$

The scalar potential V_{pot} is determined in the standard way after eliminating the remaining auxiliary fields through their equations of motion:

$$M = -3e^{K/2}W, \quad \bar{F}^{\bar{m}} = -e^{K/2}K^{i\bar{m}}\left(W_i + K_i W\right), \quad Z^i = S, T^I,$$

$$V_{pot}(s, t^I, \bar{t}^I) = e^K \left[K^{i\bar{m}}\left(W_i + K_i W\right)\left(\bar{W}_{\bar{m}} + K_{\bar{m}}\bar{W}\right) - 3|W|^2 \right]. \tag{4.89}$$

The inverse Kähler metric for the Kähler potential (4.81) is:

$$K^{IJ} = \frac{4(\text{Re}t^I)^2}{(1 - bK_s)}\delta^{IJ}, \quad K^{I\bar{s}} = -\frac{2b\text{Re}t^I}{(1 - bK_s)},$$

$$K^{s\bar{s}} = \frac{1 - bK_s + 3b^2 K_{s\bar{s}}}{K_{s\bar{s}}(1 - bK_s)}, \tag{4.90}$$

and the scalar potential V_{pot} reduces to

$$V_{pot} = \frac{e^K}{1 - bK_s}\left\{ K_{s\bar{s}}^{-1}\left(1 - bK_s + 3b^2 K_{s\bar{s}}\right)|W_s + K_s W|^2 + 4\sum_I \left(\text{Re}t^I\right)^2 |W_I + K_I W|^2 \right.$$

$$\left. -2b\left[\left(\bar{W}_s + K_s\bar{W}\right)\sum_I \text{Re}t^I\left(W_I + K_I W\right) + \text{h.c.}\right]\right\} - 3e^K|W|^2. \tag{4.91}$$

We have

$$-2\text{Re}t^I\left(W_I + K_I W\right) = -\sum_a \frac{1}{4b_a}\left[1 - bK_s - \frac{b - b_a}{b_a}\text{Re}t^I\zeta(t^I)\right]H_a^3,$$

$$W_s + K_s W = \sum_a \frac{1}{4b_a^2}\left(1 - K_s b_a\right)H_a^3, \tag{4.92}$$

and the scalar potential can be written in the following form:

$$V_{pot} = \frac{e^K}{16(1 - bK_s)}\sum_{ab}|h_a h_b|^3 \cos\omega_{ab}R_{ab}, \tag{4.93}$$

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where here ω_a is the phase of $h_a^3 = H_a^3|_{\theta=\bar{\theta}=0}$, ω_{ab} is defined as before, and

$$
R_{ab} = b_a b_b f_{ab}(\ell) + (b - b_a)(b - b_b) \sum_I |1 + 4\mathrm{Re}t^I \zeta(t^I)|^2, \quad \ell = L|_{\theta=\bar{\theta}=0},
$$

$$
f_{ab}(\ell) = (1 - bK_s) \left[\frac{(1 - b_a K_s)(1 - b_a K_s)}{b_a b_b K_{s\bar{s}}} - 3 \right]. \tag{4.94}
$$

In the absence of stringy nonperturbative effects, $K_s = -\ell$, $K_{s\bar{s}} = \ell^2$, $f_{ab} \to -2b\ell$ as $\ell \to \infty$, and the scalar potential V_{pot} is unstable in the strong coupling direction, as expected. A positive definite scalar potential requires that $f_{++}(\ell)$ be positive semi-definite where, as before, b_+ is the largest b_a. Note that the perturbative expression for $f_{aa}(\ell)$ is negative for $b_a \ell > 1.4$, while in the linear multiplet formalism the corresponding expression is negative only for $b_a \ell > 2.4$, so stringy nonperturbative effects are required to be more important in the *unconstrained* chiral multiplet formalism[7] here. If there is only one gaugino condensate, the self-dual point for the moduli is again a minimum, but $\langle F^I \rangle \neq 0$. In the general case, the minimization equations for the moduli read:

$$
\frac{\partial V_{pot}}{\partial t^I} = \frac{e^K}{16(1 - bK_s)} \sum_{ab} |h_a h_b|^3 \cos\omega_{ab} \left(\frac{2b}{n} \zeta(t^I) \sum_c \beta_{ca} R_{ab} + \frac{\partial}{\partial t^I} R_{ab} \right)
$$
$$
+ \left(A + \frac{2b}{n} \zeta(t^I) \sum_a \frac{1}{b_a} \right) V_{pot}, \tag{4.95}
$$

where β_{ab} is defined as in (4.63). Again imposing $\langle V_{pot} \rangle = 0$, the minimum is shifted slightly away from the self-dual point if some $\beta_{ab} \neq 0$.

The effective Lagrangian constructed using the linear multiplet formalism – like the string and field-theoretical loop-corrected Yang-Mills Lagrangian [31, 32] – de-

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[7] *Unconstrained* chiral multiplet formalism means the chiral multiplet formalism without the constraint (2.12) or (4.42).

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pends only on the variables t^I and the modular invariant field ℓ, so the Lagrangian is invariant under modular transformations on the t^I alone. In contrast, the effective Lagrangian constructed using this *unconstrained* chiral multiplet formalism has an explicit s–dependence which accounts for the fact that the self-dual point is not the minimum. The *unconstrained* chiral multiplet construction forces a holomorphic coefficient for the interpolating superfield for the Yang-Mills composite superfield $U \simeq \text{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)$, and hence cannot faithfully reflect the non-holomorphic contribution from the Green-Schwarz counterterm. This is again related to the fact that the *unconstrained* chiral multiplet construction does not account for the constraint (2.12) or (4.42) which has to be satisfied by the gaugino condensate superfields. Our analysis in this appendix explicitly explains why in the past the study of string phenomenology using the *unconstrained* chiral multiplet formalism has not been able to predict moduli stabilization at the self-dual point and therefore a dilaton-dominated supersymmetry breaking scenario.

Chapter 5

Phenomenology of Weakly-Coupled Superstring

5.1 Introduction

In Chapter 4, we have constructed string models which include supersymmetry broken at a realistic scale, a stabilized dilaton, moduli fields with couplings respecting modular invariance and a vanishing cosmological constant. We believe that it is sufficiently realistic to allow for a discussion of many phenomenological issues associated with supersymmetry breaking, moduli physics and axion physics based on actual computations rather than educated guesses[1]. Needless to say, we have no miraculous solution for either dilaton stabilization or the vanishing of the cosmological constant. Although these are incorporated in the model by fixing some parameters (only the second constraint requires fine tuning), the model is still predictive enough in many respects. In Sections 5.2 and 5.3, we comment on several problems associated with string moduli and axion. In particular, these analyses are quite insensitive to the details of the string models, and therefore the conclusions are fairly model-independent. In Section 5.4, we study the pattern of soft supersymmetry breaking parameters. As expected, the conclusions of this section are sensitive to the details of the specific string model under consideration. In Section 5.5, we comment on gauge coupling unification in the presence of significant stringy non-perturbative effects. In order to make the presentation transparent, in most sections we start with the known results and problems of string phenomenol-

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[1]As we shall see, several such educated guesses about string phenomenology which have been regarded as standard turn out to be inappropriate according to our actual computations.

ogy studied in the past[2]. We then present the results obtained from the realistic model constructed in Chapter 4. In particular, we emphasize how the standard lore of string phenomenology is modified within our model, and how the problems of string phenomenology could naturally be solved by these important modifications[3].

5.2 Moduli Physics

At the perturbative level, the dilaton and moduli are are flat directions of the potential, and they are lifted only through non-perturbative effects. It is often argued that the non-perturbative effects which break supersymmetry also lift these flat directions. As we have learned from the standard lore of string phenomenology, a naive oder-of-magnitude estimate concludes that string dilaton and moduli have masses of order (or no larger than) the gravitino mass [22, 63], where the natural scale of gravitino mass is about 1 TeV. Obviously, these light dilaton and moduli fields with couplings suppressed by the Planck scale could lead to serious cosmological problems. A rough estimate for the decay rate Γ of string dilaton or moduli is at most

$$\Gamma \sim \frac{m^3}{8\pi M'^2_P},\tag{5.1}$$

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[2] As discussed in the appendix of Chapter 4 and elsewhere, these studies in the past are based on the *unconstrained* chiral multiplet formalism.

[3] As we have seen and shall see, many so-called problems of weakly-coupled string phenomenology known in the past are not really problems of weakly-coupled string phenomenology itself. In fact, they are mostly due to our limited calculational power in string theory, little knowledge of its true vacuum structure, and an incorrect/inappropriate treatment of gaugino condensation.

where m is the mass of string dilaton or moduli, $M'_P = M_P/\sqrt{8\pi}$ is the reduced Planck scale and M_P is the Planck scale. This slow decay rate is the source of cosmological problems. That is, relic dilaton and moduli produced in the very early universe survive to a dangerously late epoch. With the slow decay rate (5.1), they result in a low reheat temperature T_R [22, 64]:

$$T_R \sim 5 \left(\frac{m}{\text{TeV}} \right)^{3/2} \ \text{keV}. \tag{5.2}$$

Such a low reheat temperature is inconsistent with successful nucleosynthesis unless $m \geq \mathcal{O}(3) \times 10^4$ GeV (if $T_R \geq \mathcal{O}(1)$ MeV is required.) According to the standard lore of string phenomenology, $m \geq \mathcal{O}(3) \times 10^4$ GeV would imply an un-naturally large gravitino mass, which is not acceptable. This is the so-called cosmological moduli problem [22, 64, 65], where the Polonyi problem is an earlier version of this problem in the context of spontaneously broken supergravity [66]. In order to solve the cosmological moduli problem, there have been attempts at a hierarchy between moduli and squark masses [65, 67]; however, none of them is realistic. There are also possible cosmological solutions to the cosmological moduli problem, such as a weak scale inflation [64].

Now, let's leave the standard lore of string phenomenology and turn to the realistic model constructed in Chapter 4. One can easily extract from the scalar potential the masses of the dilaton and of the moduli, which are particularly relevant for cosmology. According to (4.70), one finds the mass of the moduli m_{t^I} as follows:

$$m_{t^I} \approx \left\langle \frac{1}{2} \frac{(b - b_+)}{(1 + b\ell)} \rho_+ \right\rangle. \tag{5.3}$$

where ρ_+ is the hidden-sector gaugino condensate with the largest one-loop β-function coefficient b_+. As for the mass of the dilaton m_d, one finds:

$$m_d \sim \frac{1}{b_+^2} m_{\tilde{G}}.$$ (5.4)

According to (4.77), the gravitino mass is: $m_{\tilde{G}} \approx \frac{1}{4} b_+ \langle \rho_+ \rangle$. In generic string models b/b_+ and $1/b_+^2$ are naturally large numbers, and therefore in contrast to the standard lore of string phenomenology our model has a natural hierarchy between the dilaton/moduli and squark/slepton masses. More precisely, in order to generate a realistic hierarchy of order $m_{\tilde{G}} \approx 10^{-15} M_P' \approx 10^3$ GeV, it is required that $b/b_+ \approx 10$ for the string models under consideration. (Such an example has been presented in Section 4.5.) In this case, $m_{t^I} \approx 20 m_{\tilde{G}} \approx 20$ TeV and $m_d \sim 10^3 m_{\tilde{G}} \approx 10^3$ TeV (where $m_{\tilde{G}} \approx 1$ TeV.) This natural hierarchy between the dilaton/moduli and squark/slepton masses could be sufficient to solve the cosmological moduli problem.

One may wonder why the mass of dilaton is particularly large in our model. In fact, this specific feature has to do with the cancellation of the cosmological constant. In our model, it is implicitly assumed that the mechanism which breaks supersymmetry is also responsible for the cancellation of the cosmological constant, which is the minimal and most economical assumption[4]. With this assumption, $\langle V_{pot} \rangle = 0$ leads to $\langle 1 + \ell g_\ell \rangle \approx 3 b_+^2 \langle \ell^2 \rangle$. According to (4.27), the kinetic term of dilaton contains the small factor $\langle 1 + \ell g_\ell \rangle$, which therefore leads to an enhancement of the mass of dilaton. On the other hand, there is so far very little insight about

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[4]In our model, positivity of the scalar potential can always be imposed. One thus does not need to appeal to another source of supersymmetry breaking to cancel the cosmological constant.

how the cosmological constant problem should be solved. It is possible that there are other sources which could contribute to the cancellation of cosmological constant. However, a detailed analysis of these more complicated scenarios is beyond the scope of our study here. We wish to emphasize that, even if $\langle 1 + \ell g_t \rangle$ might turn out to be, for example, an $\mathcal{O}(1)$ number in some other more complicated solutions to the cosmological constant problem, the natural hierarchy between the dilaton/moduli and squark/slepton masses still exists as long as gaugino condensation is the major source of supersymmetry breaking; in this case we have $m_t \approx 20 m_{\tilde{G}} \approx 20$ TeV and $m_d \sim (1/b_+) \, m_{\tilde{G}} \sim 30 m_{\tilde{G}} \approx 30$ TeV.

5.3 Axion Physics

The invisible axion is an elegant solution to the strong CP problem. In string theory, there seem to be many such axion candidates. However, as for the weakly-coupled superstring, it has been argued that QCD cannot be the dominant contribution to the potential of any string axion [68], and therefore none of the string axions is qualified for the QCD axion. For the string model-independent axion, it is usually argued (again using the *unconstrained* chiral multiplet formalism) that the model-independent axion cannot be the QCD axion due to both stringy non-perturbative effects (of order e^{-c/g_s} for the superpotential of dilaton) and non-perturbative dynamics of the hidden sector which breaks the Peccei-Quinn symmetry [7, 68]. For string axions associated with the T^I moduli, Peccei-Quinn symmetries are significantly broken by world-sheet instanton effects [68]. On the other hand, we have

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emphasized that the constraint (2.12) on gaugino condensates, which has been ignored in the above arguments, has non-trivial effects on axion physics. Furthermore, stringy non-perturbative effects are most naturally described by the linear multiplet formalism. As we shall see, in the realistic model constructed in Chapter 4 where both stringy non-perturbative effects and hidden-sector gaugino condensation are fully included using the linear multiplet formalism, the model-independent axion does have the right features to be the QCD axion. The resolution for the stringy non-perturbative contribution, e^{-c/g_s}, to the superpotential of the dilaton is simple and impressive: as argued in [7, 68] using the chiral multiplet formalism, it seems plausible that there should be significant $e^{-c\sqrt{S}}$ contributions to the superpotential of dilaton, leading to the QCD axion problem raised by Banks and Dine [68]. On the other hand, in the linear multiplet formalism of string effective theory where the dilaton is represented by a vector superfield L, it is simply impossible to write down any L-dependent contribution ($e.g.$, $e^{-c/\sqrt{L}}$) to the superpotential -- a constraint coming from holomorphy. Therefore, in the linear multiplet formalism the QCD axion problem of Banks and Dine [68] is resolved in an elegant way, and one should re-examine the attractive possibility of the string model-independent axion being the QCD axion in this framework.

For any of the string axions to solve the strong CP problem, there is also a cosmological constraint. Cosmological considerations require the decay constant F_a of the invisible axion to lie between 10^{10} GeV and 10^{12} GeV (the so-called axion window [23, 69]). The upper bound on the axion decay constant, $F_a \leq 10^{12}$ GeV,

is due to the requirement that the energy density of the coherent oscillations of the axion be less than the critical density of the universe [23]. However, in superstring theory the axion decay constant F_a is naturally of order the Planck scale, and therefore the cosmological upper bound on F_a is seriously violated. Although it was shown by Choi and Kim [70] that the decay constant F_a of the model-independent axion in the weakly-coupled heterotic string theory actually is $M_P'/16\pi^2 \approx 10^{16}$ GeV, this is still much larger than the cosmological upper bound. On the other hand, cosmological constraints could be quite scheme-dependent; for example, it has been pointed out that the entropy production due to the decays of massive particles dilutes the axion density and therefore raise the upper bound on F_a [71]. Based on the above idea Kawasaki, Moroi and Yanagida [72] have proposed a refined scenario where the Polonyi fields of supergravity models are natural candidates for entropy production. The new cosmological upper bound on F_a in this scheme is:

$$F_a \leq 5 \times 10^{15} \left(\frac{m_\phi}{10 \text{ TeV}}\right)^{-3/4} \quad \text{GeV}, \tag{5.5}$$

where m_ϕ is the mass of the Polonyi field. In order to keep successful primordial nucleosynthesis in this scheme, m_ϕ should be larger than about 10 TeV. With $m_\phi \approx$ 10 TeV, $F_a \leq 5 \times 10^{15}$ GeV and therefore the string model-independent axion is almost consistent with this new upper bound. However, $m_\phi \geq 10$ TeV seems unnatural according to the standard lore of string phenomenology where one expects $m_\phi \approx m_{\tilde{G}} \approx 1$ TeV. On the contrary, the cosmological scenario of Kawasaki *et al* naturally occurs in our model constructed in Chapter 4. As discussed in Section 5.2,

in our model there is a natural hierarchy between the moduli and gravitino masses $(m_{t^I} \approx 20 m_{\tilde{G}} \approx 20~\mathrm{TeV})$, and therefore the decays of moduli serve the purpose of raising the cosmological upper bound on F_a to a value consistent with the F_a of string model-independent axion. This natural hierarchy is indeed a desirable feature of our model since it not only could solve the cosmological moduli problem but also keeps the energy density of the oscillations of string model-independent axion from overclosing the universe.

One particularly interesting aspect of our model constructed using the linear multiplet formalism of gaugino condensation in Chapter 4 is axion physics. Pseudoscalar fields are the phases ω_a of the condensates and the so-called model-independent axion which is dual to the fundamental antisymmetric tensor field. The latter couples in a universal way to the $F^{a\mu\nu}\tilde{F}^a_{\mu\nu}$ term of each gauge subgroup. If again we look at the dynamical model with one E_8 gaugino condensate in Chapter 3, we find that out of the two possible pseudoscalar the condensate phase is very heavy whereas the string model-independent axion remains massless. This is obviously the supersymmetric counterpart of what happens with the scalars. If we allow for more than one gaugino condensate, the model-independent axion acquires a very small mass[5] (typically exponentially suppressed relative to the gravitino mass by a factor of order $\langle \rho_2/\rho_1 \rangle^{1/2}$ in the two-condensate case according to (4.73)). Furthermore, as we have seen in Section 5.2, the axions associated with the T^I moduli get masses

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[5] Higher-dimension operators might give extra contributions to the mass of this axion. However, these contributions can be argued to be negligible using discrete R symmetry [7].

of order $20\, m_{\tilde{G}}$. Therefore, we are always left with only one very light axion, the model-independent axion, and it has the right properties to be the QCD axion. Remember that there are two kinds of non-perturbative effects in our model (*i.e.*, the field-theoretical non-perturbative effects of hidden-sector gaugino condensation constrained by (2.12) and stringy non-perturbative effects), and they are best described using the linear multiplet formalism. In contrast to the argument against the string model-independent axion as the QCD axion [68] in the presence of both stringy non-perturbative effects and non-perturbative dynamics of the hidden sector using the *unconstrained* chiral multiplet formalism, in our model the model-independent axion can indeed be the QCD axion. As explained before, the reason why the model-independent axion has the desirable features in the linear multiplet formalism are a correct treatment of gaugino condensation and the fact that such stringy non-perturbative effects of dilaton are actually forbidden in the superpotential due to holomorphy. As for the decay constant F_a of the model-independent axion in our model, there is an additional reduction factor of $\langle 2\ell^2(1 + \ell g_\ell)\rangle^{1/2}$ compared to the result obtained by Choi and Kim [70]. As discussed in Section 5.2, this reduction factor comes from the fact that the kinetic term of dilaton in (4.27) contains the small factor $\langle 1+\ell g_\ell\rangle \approx 3b_+^2\langle \ell^2\rangle$ when $\langle V_{pot}\rangle = 0$ is imposed. More precisely, this reduction factor is about $\langle 2\ell^2(1 + \ell g_\ell)\rangle^{1/2} \approx \langle \sqrt{6}b_+\ell^2\rangle \approx 1/50$ if the gravitino mass is about 1 TeV. Besides the fact that the cosmological scenario of Kawasaki *et al* naturally occurs in our model, this reduction in the model-independent axion's decay constant is certainly desirable from the viewpoint of the cosmological upper

bound on F_a. Indeed, with this reduction factor the axion decay constant in our model is $F_a \approx 2 \times 10^{14}$ GeV, which is truly consistent with the upper bound on F_a ($\approx 5 \times 10^{15}$ GeV) imposed by the scenario of Kawasaki *et al.*

5.4 Soft Supersymmetry Breaking Parameters

In contrast to the studies of moduli and axion, the analysis of soft supersymmetry breaking parameters is much more sensitive to the very details of a string model. Unfortunately, our current knowledge of string models is still limited. Although in the following we will try to discuss soft supersymmetry breaking parameters in a model-independent way whenever it is possible, yet it should be kept in mind that our analysis cannot cover all the interesting possibilities and therefore should not be regarded as final.

It is straightforward to compute the soft supersymmetry breaking terms, that are generated at the condensation scale $\mu_{cond} \approx \langle \rho_+ \rangle^{1/3}$, for our model constructed in Chapter 2. The gaugino masses m_{λ_b} are:

$$m_{\lambda_b} = -\left\langle \frac{g_b^2(\mu_{cond})}{8\ell^2}(1 + \ell g_\ell) \sum_a (1 + b_a \ell) \bar{u}_a \right\rangle \approx -\frac{3}{8} \frac{g_b^2(\mu_{cond}) b_+^2}{1 + b_+ \langle \ell \rangle} \langle \bar{u}_+ \rangle. \quad (5.6)$$

Notice that the expression of gaugino masses contains the small factor $\langle 1 + \ell g_\ell \rangle$ discussed at the end of Section 5.2, and therefore gaugino masses are suppressed by b_+^2 after $\langle V_{pot} \rangle = 0$ is imposed. Therefore, it is possible that this suppression of gaugino masses could be relieved in models with a more complicated mechanism of cosmological constant cancellation.

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The soft terms in the scalar potential are sensitive to the – as yet unknown – details of matter-dependent contributions to the Green-Schwarz counterterm and string threshold corrections. We neglect the former[6], and write the Green-Schwarz counterterm as follows:

$$V_{GS} = b \sum_I g^I + \sum_A p_A e^{\sum_I q_A^I g^I} |\Phi^A|^2 + \mathcal{O}(|\Phi^A|^4), \tag{5.7}$$

where the Φ^A are gauge nonsinglet chiral superfields, the q_A^I are their modular weights, and the full Kähler potential reads

$$K = k(V) + \sum_I g^I + \sum_A e^{\sum_I q_A^I g^I} |\Phi^A|^2 + \mathcal{O}(|\Phi^A|^4). \tag{5.8}$$

Under these assumptions, the scalar masses and cubic "A terms" are given, respectively, by the following:

$$m_A^2 = \frac{1}{16} \left\langle \left| \sum_a u_a \frac{(p_A - b_a)}{(1 + p_A \ell)} \right|^2 \right\rangle \approx \frac{1}{16} \left\langle \frac{(p_A - b_+)^2}{(1 + p_A \ell)^2} \rho_+^2 \right\rangle,$$

$$V_A(\phi) = \frac{1}{4} e^{K/2} \sum_{a,A} \bar{u}_a \phi^A W_A(\phi) \left[\frac{p_A - b_a}{1 + p_A \ell} + b_a - (1 + \ell g_\ell) \frac{1 + b_a \ell}{3 \ell} \right] + \text{h.c.}$$

$$\approx \frac{1}{4} e^{K/2} \bar{u}_+ \left[\sum_A \frac{p_A - b_+}{1 + p_A \ell} \phi^A W_A(\phi) + \frac{3 b_+}{1 + b_+ \ell} W(\phi) \right] + \text{h.c.}, \tag{5.9}$$

where $\phi = \Phi|_{\theta=\bar{\theta}=0}$ and $W(\Phi)$ is the cubic superpotential for chiral matter superfields. Note that the squared scalar masses are always positive. As concluded in Section 4.6, we find in our model that moduli t^I are stabilized at the self-dual point and their associated $\langle F^I \rangle$ vanish in the vacuum, which results in a dilaton-dominated supersymmetry breaking scenario. According to (5.9), both the scalar

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[6]If string threshold corrections are determined by a holomorphic function, they cannot contribute to the scalar masses.

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masses and A terms are indeed independent of their modular weights by virtue of the fact that $\langle F^I \rangle = 0$. For the FCNC constraints, this feature of dilaton-dominated scenario is a potential advantage over a moduli-dominant scenario. In the past, it was generally believed that a dilaton-dominated scenario results in universal soft supersymmetry breaking parameters due to the universality of dilaton couplings [62]. However, here we wish to stress that the above statement did not take into account the matter-dependent contributions to the Green-Schwarz counterterm, and therefore a dilaton-dominated scenario does not guarantee universal soft supersymmetry breaking parameters. It is clear from the computations of our dilaton-dominated scenario in (5.9) that soft supersymmetry breaking parameters are universal – and unwanted flavor-changing neutral currents are thereby suppressed – if the matter couplings (p_A) to the Green-Schwarz counterterm are also universal. Unfortunately, so far there is little knowledge of p_A's; therefore, the best we can do right now is to study the consequences of several seemingly reasonable choices of p_A's. One possibility is that p_A's are universal; thus we have universal soft supersymmetry breaking parameters and in this case A terms in (5.9) reduce to

$$V_A(\phi) \approx \frac{3}{4} e^{K/2} \bar{u}_+ \frac{p_A \left(1 + 2b_+ \ell\right) - b_+^2 \ell}{(1 + p_A \ell)(1 + b_+ \ell)} W(\phi) + \text{h.c.} \equiv A e^{K/2} W(\phi) + \text{h.c.}. \quad (5.10)$$

For example, if the Green-Schwarz counterterm is simply independent of the matter fields Φ^A (*i.e.*, $p_A = 0$), we have $m_A = m_{\tilde{G}}$, $A \approx 2m_\lambda$. As for choices of non-universal p_A's, a possibility is that the Green-Schwarz counterterm depends only on the radii R_I of the three compact tori that determine the untwisted-sector part of

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the Kähler potential (5.8):

$$K = k(V) - \sum_I \ln(2R_I^2) + \mathcal{O}(|\Phi_{\text{twisted}}^A|^2),$$

where $2R_I^2 = T^I + \bar{T}^I - \sum_A |\Phi_I^A|^2$ in string units. In this case, $p_A = b$ for the untwisted chiral superfields Φ_I^A, and $p_A = 0$ for the twisted chiral superfields Φ_{twisted}^A. The untwisted scalars have masses comparable to the moduli masses: $m_{\text{untwisted}} = m_t/2 \approx A/3$. Finally, we note that if $b_+ \approx b/10 \approx 1/30$, gaugino masses are suppressed relative to the gravitino mass at the condensation scale $\mu_{cond} \sim 10^{-4} M_P'$: $m_\lambda \sim m_{\text{twisted}}/40$. If there is a sector with $p_A = b$ and a Yukawa coupling of order one involving $SU(3)$ (anti-) triplets (e.g., $\bar{D}DN$, where N is a standard model singlet), its two-loop contribution to gaugino masses [73] can be more important than the standard one-loop contribution, generating a physical mass for gluinos that is well within experimental bounds for $m_{\tilde{G}} \approx 1$ TeV. Such a coupling could also generate a *vev* for N, thus breaking possible additional $U(1)$'s at a scale ~ 10 TeV. The phenomenologically required μ term of the MSSM may also be generated by the *vev* of a Standard Model gauge singlet or by one of the other mechanisms that have been proposed in the literature [74].

In contrast to the case of universal p_A's, for the case of non-universal p_A's one has to worry about the FCNC problem. Scenarios in which the sparticles of the first two generations have masses as high as 20 TeV have in fact been proposed [75] to solve the FCNC problem. However, it has recently been pointed out that such scenarios may suffer from a negative scalar top mass squared driven by two-

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loop renormalization group evolution [76][7]. Clearly, a better understanding of the matter dependence of the Green-Schwarz counterterm is required to make precise predictions for soft supersymmetry breaking. Nevertheless our model suggests soft supersymmetry breaking patterns that may differ significantly from those generally assumed in the context of the MSSM. Phenomenological constraints such as current limits on sparticle masses, gauge coupling unification and a charge and color invariant vacuum can be used to restrict the allowed values of p_A's as well as the low-energy spectrum of the string effective field theory. To conclude, we would like to stress that the model presented above is certainly not final and some of the results obtained, especially on the low-energy sector of the theory, may receive modifications. Possible sources of modification are the presence of an anomalous $U(1)$ symmetry [17] or a constant term in the superpotential that breaks modular invariance [77, 78].

5.5 Gauge Coupling Unification

String non-perturbative corrections necessary to stabilize the dilaton could make significant corrections to the unification of gauge couplings. The functions $f(\ell)$ and $g(\ell)$ introduced above and the threshold corrections whose form is dictated by T duality invariance contribute as follows to the value of couplings at unification:

$$g_a^{-2}(M_s) \;=\; g_s^{-2} + \frac{C_a}{8\pi^2}\ln(\lambda e) - \frac{1}{16\pi^2}\sum_I b_a^I \ln(t^I + \bar{t}^I)|\eta^2(t^I)|^2, \qquad (5.11)$$

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[7]We thank Hitoshi Murayama for pointing out this problem to us.

$$g_s^{-2} = \frac{1+f}{2\ell}, \quad M_s^2 = \lambda g_s^2 M_P^2, \tag{5.12}$$

with

$$\lambda = \frac{1}{2} e^{g-1}(1+f) \tag{5.13}$$

Let us note however that this parameter is worth $1/(2e) \approx 0.18$ in the perturbative case and $e^{-1.65} \approx 0.19$ in the one gaugino condensate model.

Let us take this opportunity to clarify two confusing statements in the literature about gauge coupling unification in weakly-coupled superstring. Firstly, we stress that the dependence on the radii moduli T^I does not allow an interpretation of the unification scale as the inverse radius of compactification. While the result (5.11) has been derived only for orbifold compactifications, its large T^I limit is consistent with the behavior found in the large T^I limit of Calabi-Yau compactification. (Note that in our model moduli are stabilized at the self-dual point, therefore far from this limit.) Secondly, it is often stated that one can determine from the low-energy values of gauge couplings the precise value of the gauge coupling unification scale to be 3×10^{16} GeV. We think that this is a misleading statement since most string models constructed so far that hold a claim for being realistic include new forms of matter which perturb the evolution of the gauge couplings at some intermediate threshold [79].

5.6 Concluding Remarks

As discussed in Chapter 1, the weakly-coupled heterotic string theory is known to have problems with dilaton/moduli stabilization, supersymmetry breaking, gauge coupling unification, QCD axion, as well as cosmological problems involving dilaton/moduli and axion. In the literature some of these problems are often treated as evidence against the weakly-coupled heterotic string theory. However, it is actually hard to say whether these problems are inherent to the weakly-coupled heterotic string theory or they simply reflect our ignorance of important string dynamics. Furthermore, some of these problems will probably re-appear even in the study of the strong-coupling limit of the heterotic string theory. In this work we study these problems by adopting the point of view that they arise mostly due to our limited calculational power, little knowledge of of the full vacuum structure, and an inappropriate treatment of gaugino condensation. Indeed, after a careful review one finds that the phenomenological studies of the weakly-coupled heterotic string theory in the literature contain several essential flaws. It is therefore of utmost importance to correct these flaws and then re-examine the problems of weakly-coupled heterotic string theory. In conclusion, three essential changes to the standard lore of string phenomenology have to be made. The first essential change is about the effective field theory of the weakly-coupled heterotic string. It is emphasized that the linear multiplet formalism rather than the chiral multiplet formalism is the appropriate framework for the effective field theory of the weakly-coupled heterotic string. The

second essential change is the inclusion of possible stringy non-perturbative effects in addition to the usual field-theoretical non-perturbative effects produced by gaugino condensation. The third essential change is an improved treatment of gaugino condensation by including the constraint (2.12). As discussed in Chapter 2, the last two changes are most naturally implemented using the linear multiplet formalism. Finally, notice that full modular invariance is always maintained in our construction. This is important because modular invariance is supposed to be an exact quantum symmetry of closed string theory [80].

In Chapters 2–4, the linear multiplet formalism with the aforementioned features is constructed for an E_8 model as well as a generic orbifold model. It is particularly transparent in this framework to realize how the dilaton can be stabilized by stringy non-perturbative contributions to the Kähler potential.[8] Furthermore, supersymmetry can be broken at a realistic scale once the dilaton is stabilized. As for the moduli, they are always stabilized at their self-dual points where the moduli actually do not contribute to supersymmetry breaking – a beautiful consequence of modular invariance and a correct treatment of gaugino condensation. Phenomenologically, we always have a dilaton-dominated scenario of supersymmetry breaking. The fact that the compactification moduli are stabilized at the self-dual points also invalidates the Newton's constant (or gauge coupling unification) argument of Witten against the weakly-coupled heterotic string theory. As for the masses of moduli,

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[8]Of course, still we don't know how to calculate these stringy non-perturbative effects. However, the point is that these effects are at least under good control here.

in contrast to the standard lore of string phenomenology a careful analysis reveals that there is a natural hierarchy between moduli and gravitino masses. It is not difficult to see how this hierarchy arises: in a generic orbifold model with realistic supersymmetry breaking scale, there is already a natural hierarchy between the E_8 β-function coefficient b (associated with the Green-Schwarz counterterm) and the b_a of the largest hidden gauge subgroup ($b/b_+ \approx 10$). Such a hierarchy between moduli and gravitino masses has important cosmological consequences. As discussed in Chapter 5, it not only could solve the cosmological moduli problem but also keeps the energy density of the oscillations of the string model-independent axion from overclosing the universe. As for the strong CP problem, there is always only one very light axion (the model-independent axion) in our model, and it does have the right features to be the QCD axion in contrast to the conclusion of Banks and Dine [68]. The difference between our result and that of Banks and Dine has to do with our improved treatment of gaugino condensation and a non-renormalization theorem associated with the linear multiplet which is unique to the linear multiplet formalism. In conclusion, it is fair to say that these problems of the weakly-coupled heterotic string theory can be solved or are much less severe.

As expected, the origin of the cosmological constant remains a mystery here although it is indeed under better control and the cosmological constant can be fine tuned to zero in our treatment. Again, a final resolution of this problem might have to wait for a complete understanding of superstring dynamics. The other unsettled issue in this work is the soft supersymmetry breaking pattern. Although our model

always predicts a dilaton-dominated scenario of supersymmetry breaking, yet in contrast to the standard lore of string phenomenology we point out that whether a dilaton-dominated scenario predicts universal soft supersymmetry breaking parameters actually depends on whether the matter couplings to the Green-Schwarz counterterm are universal. To settle this issue, a better understanding of the matter dependence of the Green-Schwarz counterterm for generic string models is certainly required; it deserves further studies and could lead to a rich phenomenology. Another potential problem of this work is that the gaugino masses might be too small. Whether this is a serious problem or not can be very model-dependent, especially in the context of superstrings where one generically encounters scenarios beyond the MSSM. In conclusion, we emphasize that this work is certainly not final, and it is very important to understand more about the non-perturbative aspects of superstrings, realistic string model building and the phenomenology. After a careful re-examination of the aforementioned problems of the weakly-coupled heterotic string theory, it is also hoped that those misunderstandings of the current status of weakly-coupled heterotic string theory in the literature are clarified by this work.

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