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NUCLEAR MASSES FAR FROM STABILITY: THE INTERPLAY OF THEORY AND EXPERIMENT\*

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**Abstract**

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Mass models seek, by a variety of theoretical approaches, to reproduce the measured mass surface and to predict unmeasured masses beyond it. Subsequent measurements of these predicted nuclear masses permit an assessment of the quality of the mass predictions from the various models. Since the last comprehensive revision of the mass predictions (in the mid-to-late 1970's) over 300 new masses have been reported. Global analyses of these data have been performed by several numerical and graphical methods. These have identified both the strengths and weaknesses of the models. In some cases failures in individual models are distinctly apparent when the new mass data are plotted as functions of one or more selected physical parameters. Several examples will be given. Future theoretical efforts will also be discussed.

**I. Introduction**

A continuing effort among experimentalists who study nuclei far from beta stability is the measurement of the atomic mass surface. As a manifestation of the nuclear force and the nuclear many body system, atomic masses signal important features of nuclear structure on both a macroscopic and microscopic scale. It has thus been a challenge to nuclear theorists to devise models which can reproduce the measured mass surface and to predict successfully the masses of new isotopes. Both the measured mass surface and that beyond it which can be predicted by these models serve as important input to a variety of fundamental and applied problems, e.g., nucleosynthesis calculations, predictions of decay modes of exotic nuclei far from stability, nuclear de-excitation by particle evaporation, decay heat simulations, etc.

Well determined masses of nuclei which lie far from beta stability can provide very sensitive tests of atomic mass models. While a single new mass measurement from one previously uncharacterized isotope carries with it only limited information about the quality of mass predictions from the models, important trends frequently become evident across isotopic sequences or when global comparisons of many new masses are made against the various mass models. It is in this context that a comprehensive and critical assessment of the predictive properties of atomic mass models is presented with the aim of identifying both the successes and failures in the models. A summary of a portion of this effort has been published earlier [HAU84].

**MASTER**

## II. New Masses, Analysis Methods, and Global Comparisons

The last comprehensive update of the atomic mass predictions from nine different models was published in 1976 [MAR76]. Additional predictions from other models appeared in the late 1970's and early 1980's [MON78], [MOL81], [UNO82]. In each case, one of the atomic mass evaluations periodically provided by Wapstra [MAR76], [WAP77], [WAP84], served as the experimentally determined mass data base on which the adjustable parameters of the models were determined. Since the 1975 Wapstra evaluation (which was used in the formulation of many of the models published in 1976) over 300 new mass measurements have been made. An examination of where these new measurements occur in the Chart of Nuclides reveals that they are distributed among almost all the elements at their most neutron-rich or neutron-deficient isotopes. Especially long isotopic sequences of new masses occur in the Na, Rb, and Cs nuclei and in alpha decay chains which originate from  $^{176}\text{Hg}$  and  $^{178}\text{Hg}$ .

It is quite instructive to compare these new measurements (which lie outside the data bases available at the time the various mass models were formulated) with predictions from the models. For such comparisons it is convenient to define  $\Delta = \text{Predicted Mass} - \text{Measured Mass}$ .  $\Delta > 0$  thus denotes cases where the binding energy has been predicted to be too low and conversely,  $\Delta < 0$  corresponds to a prediction of too much nuclear binding. Table 1 summarizes average and root-mean-square deviations for twelve models.

Table 1. Average and Root-Mean-Square Deviations (all energies in keV)

Model <sup>+</sup>	Data Base Used	Old Masses <sup>++</sup> $\langle \Delta \rangle$	New Masses RMS- $\Delta$	New Masses $\langle \Delta \rangle$	New Masses RMS- $\Delta$	RMS Ratio	
M	1971	209	1327	270	-551	1566	1.18
GHT	1975	20	718	276	-478	1096	1.53
SH	1971	-6	718	257	-195	954	1.33
MN	1977	-4	835	213	279	970	1.16
B	1971	-459	1506	121	-768	1772	1.18
BLM	1971	1984	2747	146	1991	3125	1.14
LZ	1975	7	276	268	87	589	2.13
UY	1975	0	393	219	110	1100	2.80
CK	1975	5	312	258	186	1314	4.21
JGK	1975	6	212	271	219	1361	6.42
MS	1975	-7	159	267	-6	695	4.37
JE	1975	0	363	239	24	952	2.62

<sup>+</sup> M = Myers, GHT = Groote et al., SH = Seeger & Howard, MN = Möller & Nix, B = Bauer, BLM = Beiner et al., LZ = Liran & Zeldes, UY = Uno & Yamada (linear shells), CK = Comay & Kelson, JGK = Jänecke, Garvey-Kelson, MS = Monahan & Serduke, JE = Jänecke & Eynon.

<sup>++</sup> Relative to the 1975 Wapstra masses.

Several significant trends are apparent.  $\langle \Delta \rangle$  values for the 1975 masses are quite small, typically a few kilovolts except for those models

which used the 1971 Wapstra masses. RMS- $\Delta$  values range from 159 to 2747 keV. This spread is a reflection of the degree of conformation of the calculated mass surface to the measured one afforded, in those models with smaller RMS- $\Delta$  values, by the use of increasingly larger numbers of adjustable parameters. When global comparisons are made for all the models to the new masses (up to 276 nuclei) reported since 1975-77 one notes that approximately half of the models display net positive  $\langle \Delta \rangle$  values and the remainder net negative values. RMS- $\Delta$  values for the new masses reveal that all models show poorer fits to these masses than to the 1975 or '77 data base. The last column lists the ratio of the rms-deviations of the new masses to the old ones. The more "fundamental" models (e.g., liquid drop, droplet, simple shell) have larger RMS- $\Delta$  deviations when compared to the old masses than the models based on mass relations or complicated shell corrections. However the comparison also reveals that these simpler approaches show substantially smaller enlargement of rms deviations; models based on mass relations (CK, JGK, and MS) exhibit larger RMS ratios, up to factors of 6.4, which result from progressively poorer predictions for nuclei especially far from stability.

### III. Analysis of Selected Individual Models

A. Seeger and Howard: Figure 1 displays  $\Delta$  values for new masses as a function of neutron number for this model (semiempirical liquid drop plus shell corrections). The solid lines pass through sets of points where  $\Delta$

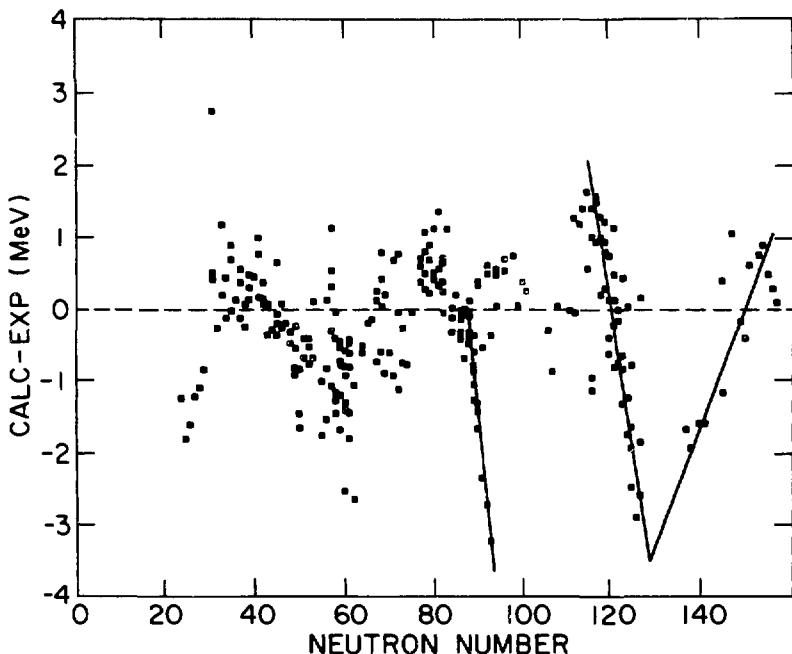


Fig. 1 Delta values of new masses as a function of neutron number for the model for Seeger and Howard.

values are linear with neutron number  $N$ . The more obvious of these is the rapidly falling trend which begins near  $N = 114$  and continues to  $N = 126$ , followed by a reversal that extends to  $N = 150$ . Another trend starts for points with  $N = 82$  and extends for approximately ten neutron numbers. The correlation of these effects with neutron shell closures at  $N = 82$  and  $126$  may be understood by examination of the treatment of the microscopic shell corrections in the Seeger and Howard model [SEE75]. Without invoking an ad hoc enlargement of the  $N = 126$  shell gap by 3 MeV (which tapered smoothly and symmetrically to zero at  $N = 108$  and  $N = 144$ ) it was not possible to obtain simultaneously the proper single particle level ordering and a good fit to the known (1971) mass surface. The  $N = 82$  gap is affected to second order by this prescription. While an optimized fit to the 1971 masses was obtained in this way, the trend in the predictions of masses of nearly all new isotopes (since 1971) with  $N = 114$  to  $150$  and some new masses for isotopes with  $N = 82$  to  $92$  is clear evidence that the procedure described above will not work satisfactorily for these nuclei which lie further from stability.

B. Jänecke, Garvey-Kelson: The value of 6.42 in the RMS Ratio column in Table 1 for this model is a result of a small number of very poorly predicted masses for nuclei quite far from stability. Closer to stability this model (and similar ones, e.g., CK and MS) provide excellent predictions with  $\Delta$  values usually within  $\pm 1.5$  MeV. A useful way to illustrate this feature in these models is by plotting  $\Delta$  values as a function of how far each isotope is from the valley of beta stability. For this purpose, the quantity  $N - Z - (0.4A^2)/(200 + A)$  gives the difference in neutron numbers of the isotope of interest and that of the isotope nearest the stability line of the same element. Positive values of this quantity correspond to neutron-rich nuclei, negative values to proton-rich nuclei, and values near zero represent nuclei close to the stability line. Figure 2 displays  $\Delta$  values versus number of neutrons from stability for this model. Many points cluster about the dashed horizontal  $\Delta = 0$  line but there is a clear trend that shows that proton-rich nuclei are not bound enough and neutron-rich nuclei are too well bound. Use of this model (and the others of similar type) for calculations of  $r$ -process nucleosynthesis will therefore introduce a strong bias that results from the prediction of the location of the neutron drip line too close to stability. As shown here the trend is approximately proportional to  $T_z^3$  and is a reflection of need for correction terms [JAN84] in the transverse Garvey-Kelson mass relationship on which these models are based.

C. Jänecke and Eynon: This model, which involves the solution of inhomogeneous third order partial difference equations, represents one approach that aims to correct the deficiencies of type noted above in models that employ (homogeneous) mass relations. In particular, the introduction of an inhomogeneous term is meant to account for variations in the effective neutron-proton residual interaction as a function of nucleon number and neutron excess. It is therefore instructive to compare the quality of mass predictions from this model to those which derive their mass predictions from solutions of homogeneous partial difference equations. Figure 3 shows a plot of  $\Delta$  values versus neutrons from stability for the Jänecke and Eynon model. Two features are immediately apparent: (1) the largest  $\Delta$  values are considerably reduced -- note the

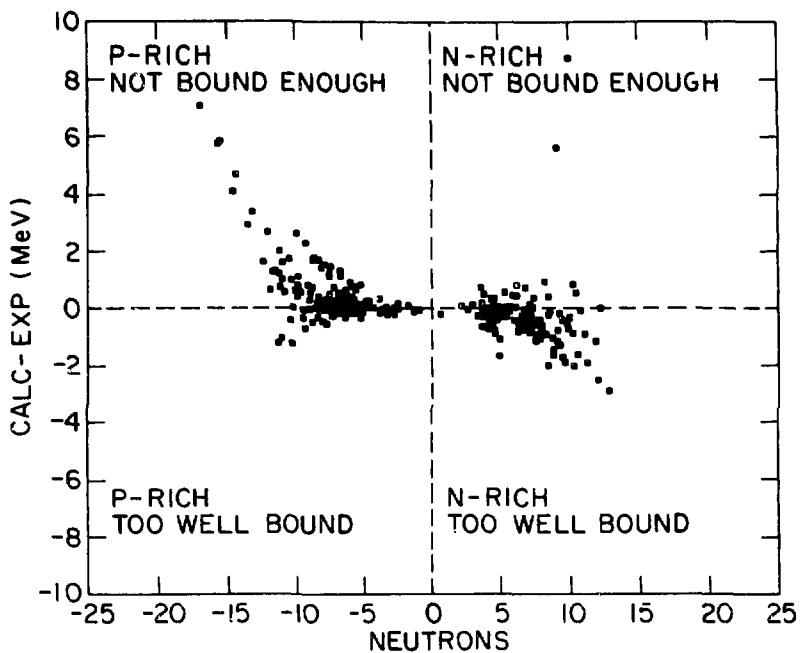


Fig. 2 Delta values for new masses as a function of neutrons from stability for the model of Jänecke, Garvey-Kelson.

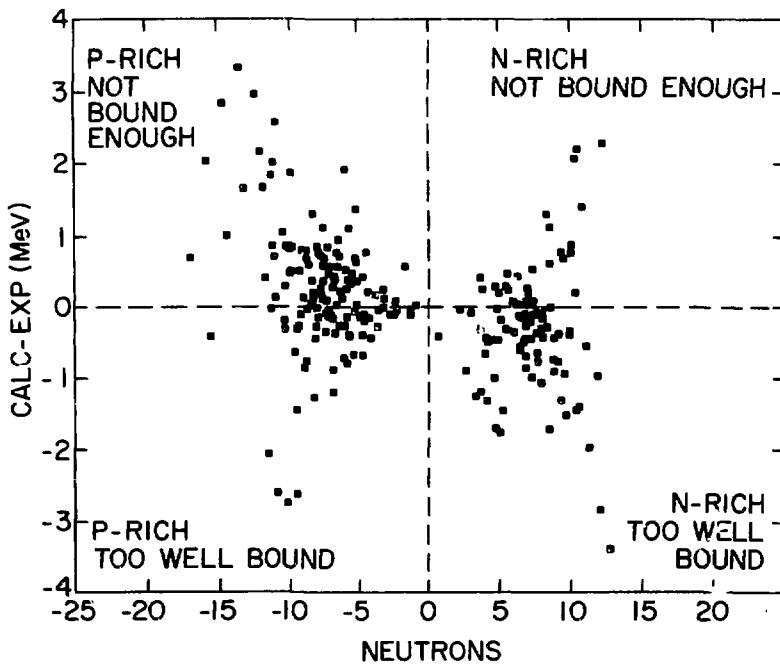


Fig. 3 Delta values for new masses as a function of neutrons from stability for the model of Jänecke and Eynon.

change of a factor of 2.5 in the vertical scale relative to Figure 2; and (2) points scatter more uniformly into the four quadrants of the plot, suggesting that the isospin dependence has been more successfully treated. One expects, therefore, that this model would be more suitable for use as input for nucleosynthesis calculations or in other applications requiring more reliable predictions far off the stability line.

#### IV. Summary and Future Directions

The analysis methods described here have highlighted some of the systematic features in the predictive properties of several of the commonly used atomic mass models. Additional understanding of these features and the availability of many new atomic masses for isotopes far from the stability line will serve as a basis for improving the models. The need clearly exists for a comprehensive revision and update of the mass predictions. A project, coordinated by the author, has been started to accomplish this. It is expected that new sets of mass predictions from a number of groups may be available late in 1986.

#### V. References

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