48/11/8-91 JS (2)

UNIVERSITY OF WISCONSIN

CENTER FOR PLASMA THEORY AND COMPUTATION

REPORT

Interaction of Bootstrap Current Driven Magnetic Islands

C.C. Hegna and J.D. Callen

Department of Nuclear Engineering & Engineering Physics
University of Wisconsin-Madison
Madison, Wisconsin 53706-1687

October 1991

UW-CPTC 91-16

MADISON, WISCONSIN 53706-1687

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

UW-CPTC--91-16

DE92 002625

Interaction of Bootstrap-Current-Driven Magnetic Islands

C. C. Hegna and J. D. Callen

Departments of Nuclear Engineering and Engineering Physics and Physics

University of Wisconsin

Madison, WI 53706-1687

Abstract

The formation and interaction of fluctuating neoclassical pressure gradient driven magnetic islands is examined. The interaction of magnetic islands produces a stochastic region around the separatrices of the islands. This interaction causes the island pressure profile to be broadened, reducing the island bootstrap current and drive for the magnetic island. A model is presented that describes the magnetic topology as a bath of interacting magnetic islands with low to medium poloidal mode number ($m \cong 3$–30). The islands grow by the bootstrap current effect and damp due to the flattening of the pressure profile near the island separatrix caused by the interaction of the magnetic islands. The effect of this sporadic growth and decay of the islands (*"magnetic bubbling"*) is not normally addressed in theories of plasma transport due to magnetic fluctuations. The nature of the transport differs from statistical approaches to magnetic turbulence since the radial step size of the plasma transport is now given by the characteristic island width. This model suggests that tokamak experiments have relatively short-lived, coherent, long wavelength magnetic oscillations present in the steep pressure-gradient regions of the plasma.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

I. Introduction

The transport of plasma particles and heat across equilibrium magnetic flux surfaces remains one of the most elusive, unsolved problems in fusion plasma physics. It is widely believed that fluctuating magnetic and electric fields are responsible for the observed anomaly in the plasma transport. However, the precise mechanism which causes the fluctuations is not known.

Many theories of anomalous transport consider one of two extremes concerning the magnetic topology of the toroidal confinement system: (1) the magnetic field-lines reside on two-dimensional, toroidal magnetic surfaces; or (2) the existence of ergodic magnetic field-lines that stochastically wander in a three-dimensional volume. For instance, theories that consider electrostatic fluctuations alone (without fluctuating magnetic fields) presume that the magnetic surfaces remain robust in the presence of the electrostatic oscillations, while the transport is caused by $\mathbf{E} \times \mathbf{B}$ drifts across the magnetic surfaces. However, magnetic fluctuations can change the magnetic field topology by causing the formation of magnetic islands on resonant rational surfaces.[1] If the magnetic perturbations are sufficiently large, the islands overlap and the topology is described by stochastic magnetic fields. Radial plasma transport caused by stochastic field-lines occurs because plasma particles will follow field-lines that move off the equilibrium magnetic surfaces.[2,3] Because of rapid transport along the field-lines, stochastic magnetic fields cannot sustain density and pressure gradients perpendicular to the equilibrium surfaces. Although the presumption of strongly overlapping magnetic islands may seem reasonable for the core of reversed field pinch experiments where almost flat pressure profiles are observed, tokamak experiments do not generally exhibit large regions of flat pressure profiles. Therefore, it is natural to assume that if magnetic fluctuations are present in tokamaks, they are not so prominent as to cause global, steady-state, stochastic magnetic fields. We are thus motivated to study an intermediate state of the magnetic topology, where chains of magnetic islands of low to medium mode number ($m \cong 3$–30) are present.

Rebut and co-workers have pointed out the possible importance of magnetic topology on the transport properties of tokamaks.[4–7] They present a model that describes the magnetic configuration as having a collection of magnetic island chains that are imbedded in a steady-state stochastic sea. Anomalous confinement is then explained by having transport along field lines through the island chains that connect different regions of the plasma. A critical feature of the model is the necessity for a steady-state self-sustainment mechanism for the magnetic islands. Most theoretical attempts to account for this self-sustainment rely on destabilizing mechanisms for microtearing modes in the nonlinear magnetic island regime. It has been suggested that a thermal instability is responsible for the islands.[7-9] The thermal instability results from the insulating effect the magnetic field topology has on the plasma within the magnetic island. The differing power delivered to the electrons inside and outside the island causes a temperature profile to arise across the island structure. The difference in electron temperature causes a filamentation of the parallel current density profile through the classical Ohm's Law. Ampere's Law then relates the current perturbation to the island producing magnetic field. Other self-sustainment mechanisms presented by Hugon and Rebut include a different reaction of ions and electrons to the magnetic island due to their differing Larmor radii and the effect of pseudo-gravity as a model for pressure gradient/field-line curvature driven modes.[10] It has also been suggested that the radial electric field which arises to insure ambipolarity, can provide a destabilizing mechanism for microtearing modes due to the electric drift of the electrons.[11,12]

In this work, we will consider magnetic islands produced by the fluctuating bootstrap current. The bootstrap current occurs through a neoclassical mechanism. By including viscous forces in a tokamak, the equilibrium Ohm's Law for the parallel current $J_{\|}$ is written[13,14]

$$\langle \mathbf{E}\cdot\mathbf{B}\rangle = \eta_{\|} \langle J_{\|} B\rangle - \frac{1}{n_e e} \langle \mathbf{B}\cdot\nabla\cdot\overleftrightarrow{\pi}_{\|e}\rangle \ , \tag{1}$$

where $\eta_{\|}$ ($\cong m_e\nu_{ei}/n^2e$) is the Spitzer resistivity, $\overleftrightarrow{\pi}_{\|e}$ is the parallel electron viscous stress tensor and the brackets indicate an average over the equilibrium flux surface. The last term in Eq. (1) can be written $\langle \mathbf{B}\cdot\nabla\cdot\overleftrightarrow{\pi}_{\|e}\rangle = n_e m_e \mu_e U_{\theta e}\langle B^2\rangle$, where μ_e is the electron viscous damping frequency and $U_{\theta e}$ is the poloidal electron flow velocity. The poloidal electron flow is the sum of the poloidal projections of the parallel current and the diamagnetic current caused by the perpendicular pressure gradient. Equation (1) can then be rewritten in a cylindrical approximation

$$J_{\|} \cong \frac{\sigma_{\|}}{1+\sqrt{\varepsilon}} E_{\|} - \frac{\sqrt{\varepsilon}}{1+\sqrt{\varepsilon}} \frac{c}{B_\theta} \frac{dp}{dr} , \tag{2}$$

where ε is the inverse aspect ratio, and the long-mean-free-path version of the frequency for viscous damping has been used, $\mu_e \cong \sqrt{\varepsilon}\,\nu_{ei}$.[13] The first term shows the neoclassical reduction of the electrical conductivity due to the fact that trapped particles cannot carry a parallel current. The second term describing viscous damping of the diamagnetic flow is referred to as the bootstrap current, a current parallel to the equilibrium magnetic field driven by a perpendicular pressure gradient. Experimental observations on toroidal octupoles,[15] tokamaks[16,17] and stellarators[18] suggest that an equilibrium bootstrap current is present.

In addition to driving an equilibrium parallel current, the bootstrap effect also provides access to the pressure-gradient free-energy source for resistive instabilities [for $(dp/dr)(dq/dr) < 0$]. Linear stability theory indicates that the effect of the perturbed bootstrap current produces in collisional plasmas an instability analogous to a resistive interchange mode except that it is more robust since the parallel viscous damping rate allows easier access to the free energy source than magnetic field-line curvature.[14,19] Fitzpatrick has extended the linear instability studies by taking semi-collisional effects into account and found that for devices with shear of order unity the perturbed bootstrap current destabilizes the semi-collisional drift-tearing mode.[20]

Of course, linear theory is only valid if the island produced by the magnetic perturbation is smaller than the linear tearing layer width. When this condition is violated it is more appropriate to implement the nonlinear tearing mode theory of Rutherford.[21] The modification caused by the island bootstrap current on the nonlinear tearing mode has been examined assuming the presence of a single-helicity, coherent magnetic island. The dynamical equation for the island half-width, w_0 is given by[13,22]

$$\frac{4\pi}{\eta_{nc}c^2}\frac{dw_0}{dt} = k_0\Delta' + \sqrt{\varepsilon}\frac{k_1\beta_{pe}L_q/L_p}{w_0}, \tag{3}$$

where η_{nc} is the neoclassical resistivity, k_0 and k_1 are numerical constants of order unity, $\beta_{pe} = 8\pi p_e/B_\theta^2$, $L_q = (d \ln q/dr)^{-1}$, and Δ' is the tearing mode matching parameter computed as the discontinuity in the logarithmic derivative of the vector potential in the ideal exterior region.[23] The electron pressure length scale, $L_p = -(d \ln p_e/dr)^{-1}$, is evaluated outside the island separatrix where toroidal surfaces exists, and the pressure is assumed to be constant within the island separatrix. For low m tearing modes with $\Delta' > 0$, the island initially goes through a regime where neoclassical effects dominate and the island grows as $t^{1/2}$. However, when the island exceeds some threshold width $w_0 \gtrsim w_{th} \cong k_1\sqrt{\varepsilon}\beta_p L_q/k_0 L_p c^2|\Delta'|$, the island grows at the Rutherford rate and saturates due to the large island modifications of Δ'.[24] For discharges with current profiles such that $\Delta' < 0$, the island saturates at $w_0 \cong w_{th}$. Numerical studies of neoclassical tearing instabilities compare well with the analytic theory described above.[25]

In this paper, our basic model topology consists of a bath of magnetic islands with poloidal mode number $m \cong 3$–30. It will be assumed that the islands formed by the magnetic perturbations are larger than the linear tearing layer, so that the nonlinear tearing mode theory used in refs. (13) and (22) to describe the bootstrap current effect can be used. For magnetic perturbations with larger mode number, the relevant radial structure is smaller and the island nature of the mode is less likely to play a role in the dynamics. Since the

density of modes increases dramatically with mode number, perturbations with large mode number can quite easily overlap and a statistical description[26] of the large m-modes would be more appropriate; however, the low m-modes can create relatively robust, coherent magnetic island structures.

For mode numbers with $m \gtrsim 3$, Δ' is approximately given by $-2m/r_s$ where r_s is the radius of the mode-rational surface $q(r_s) = m/n$; $\Delta' < 0$ describes the stabilizing effect of magnetic field-line bending. The magnetic island dynamics described by Eq. (3), then involves the stabilizing effect of Δ' and the destabilizing effect of the perturbed bootstrap current. Island saturation is achieved by balancing these two terms. The saturated island width is given by

$$w_{0sat} = \sqrt{\varepsilon}\,\frac{k_1\beta_{pe}L_q r_s}{2mk_0L_p} \sim \frac{r_s}{m} \,. \tag{4}$$

If we were to allow for a magnetic island to form at every rational surface and saturate at the value given by Eq. (4), one would conclude that island overlap is inevitable. The density of islands with mode number $m < M$ goes as M^2, and the island width goes as $1/m$. If the product of the density of islands and the island width exceeds some critical value that is approximately one, the islands overlap. Therefore from the theory described above one could conclude that if island widths with large enough M were examined ($M \gtrsim M_{crit} = k_0L_p/\sqrt{\varepsilon}k_1\beta_{pe}r_s$), the islands would always overlap. However, if stochasticity occurs, the perpendicular pressure gradient would disappear and the island destabilization mechanism is lost. On the other hand, recall that Eq. (3) was derived assuming the existence of a single, coherent magnetic island. In order to describe our model topology, we need to take island interactions into account when describing the evolution of a given magnetic island.

The calculation to follow describes the evolution of a given magnetic island in the presence of a bath of other magnetic islands. Since the growth of the magnetic island is caused by the local density and temperature gradients, self-consistent density and temperature profiles in the vicinity of the magnetic island are taken into account. The effect

of a magnetic perturbation of incommensurate helicity on a primary magnetic island is to form a stochastic layer around the separatrix, and particularly the X-point, of the primary island. This causes the local density and temperature profiles near the island to be broadened, thus reducing the driving force for the island growth. If neighboring islands become so wide so that they overlap, the density and temperture profiles become flat; then, the destabilization mechanism is lost. The island goes through a cycle of growth due to the bootstrap current effect and decay due to island interaction causing "radial" transport of plasma density and temperature. The plasma topology is thus envisioned as a collection of these islands that all sporadically grow and decay (*"magnetic bubbling"*).

We note that in order to obtain a completely self-consistent picture of the plasma, an evolution of the global density and temperature profile needs to be determined in addition to the local behavior. However, in this work we will concentrate on the topological aspects of this model since it differs in a number of significant ways from the other magnetic island models descibed above,[4–12,26] and leave transport issues for later work.

In the following section, the magnetic field topology is introduced, as well as a review of the nonlinear Rutherford theory to be used to describe the magnetic island evolution. In section III, a kinetic theory will be used to describe the electron dynamics and compute the current profile in the vicinity of the primary magnetic island. In section IV, an island evolution profile is computed by performing an asymptotic analysis of the perturbed Ampere's Law. A sample example of two interacting magnetic islands is discussed in Section V. We conclude this work with a discussion of this model in Section VI. Finally, a quantitative estimate of fluctuation levels that would be expected in a plasma for a given magnetic island is given in the Appendix.

II. Magnetics

The magnetic fields are described by the flux coordinates Φ, θ, and ϕ, where Φ is the poloidal flux function and serves as a radial variable, θ is the poloidal angle, and ϕ is

the toroidal angle. The equilibrium magnetic field is written $\mathbf{B}_0 = q\nabla\Phi \times \nabla\theta + \nabla\phi \times \nabla\Phi$, where $q = q(\Phi)$ is the inverse rotational transform. Consider magnetic perturbations of the form $\mathbf{B}_1 = \nabla\phi \times \nabla\psi_1$, where

$$\psi_1 = \sum_{m,n} \frac{A_{mn}(\Phi)}{m} \cos(m\theta - n\phi + j_{mn}) . \tag{5}$$

For each magnetic surface with $q = m/n$, a magnetic island forms if the amplitude A_{mn} is non-zero at the resonant surface. In this calculation we will consider the dynamics of a particular island that is resonant with the $q = q_0 = m_0/n_0$ surface. Since every island chain is equivalent up to a radial translation and a rotation due to shear, the full topology can be constructed from the properties of a single magnetic island.[4] It is convenient to work in a helical coordinate system where $\zeta = \phi$, and $\alpha = \theta - \phi/q_0$ is the helical angle that is resonant with the surface $\Phi = \Phi_0$ where $q = q_0$. The magnetic field is now written

$$\mathbf{B} = q\nabla\Phi \times \nabla\alpha - \nabla\zeta \times \nabla\psi , \tag{6}$$

where

$$\psi = \int d\Phi \left(\frac{q}{q_0} - 1\right) - \psi_1 \cong \frac{x^2}{2\Phi_q} - \sum_{m,n} \frac{A_{mn}(\Phi)}{m} \cos\left[m\alpha + \left(\frac{m}{q_0} - n\right)\zeta + j_{mn}\right] , \tag{7}$$

with $\Phi_q = (d \ln q/d\Phi)^{-1}$ evaluated at the resonant surface, and $x = \Phi - \Phi_0$. That part of ψ_1 that is resonant with the $q = q_0$ surface forms a magnetic island. If we use a single harmonic approximation for the resonance at $q = q_0$, $\psi_1 = [A_0(t)/m_0] \cos(m_0\alpha)$, the island half-width w_0 is related to the amplitude of the perturbation by the equation

$$w_0 = 2\sqrt{\frac{A_0\Phi_q}{m_0}} , \tag{8}$$

where the island width is in units of the poloidal flux function.

We now introduce some notation to facilitate the algebraic manipulations to follow. Using the representation for the magnetic field given by Eq. (6), the operator $\mathbf{B}\cdot\nabla$ is written

$$\mathbf{B}\cdot\nabla K = \frac{1}{\mathcal{J}}\left(q\frac{\partial K}{\partial \zeta} - [\psi,K]\right), \tag{9}$$

where $\mathcal{J}$ is the Jacobian for the magnetic coordinates

$$\mathcal{J} = (\nabla\Phi\cdot\nabla\alpha\times\nabla\zeta)^{-1} \cong \frac{r}{B_p} \cong q\frac{R}{B_0}, \tag{10}$$

and a bracket notation is implemented

$$[X,Y] = \frac{\partial X}{\partial\Phi}\frac{\partial Y}{\partial\alpha} - \frac{\partial Y}{\partial\Phi}\frac{\partial X}{\partial\alpha}. \tag{11}$$

This bracket has all the properties of Poisson brackets. Throughout this paper an overbar is used to indicate an average over the angle ζ ($\bar{K} = \oint d\zeta\, K/2\pi$) and $\tilde{K} = K - \bar{K}$.

An asymptotic analysis of the perturbed Ampere's Law results in a dynamical equation for the island. The toroidal component of Ampere's Law can be written in terms of the magnetic flux function as

$$\nabla\cdot(R^{-2}\nabla\psi_1) = \frac{4\pi}{c}\nabla\zeta\cdot\mathbf{J}_1. \tag{12}$$

As in the Rutherford analysis,[21] the matching condition between the island region and the exterior solution is obtained by integrating the appropriate harmonic across the tearing layer. The matching condition is given by

$$\frac{g^{\Phi\Phi}c}{16\sqrt{2}\pi\Phi_q R}\Delta' w = \int_{-1}^{\infty} d\Psi \oint \frac{m_0 d\alpha}{2\pi}\frac{\cos(m_0\alpha)}{[\Psi+\cos(m_0\alpha)]^{1/2}}\bar{J}_{\parallel}, \tag{13}$$

where $\Psi \equiv \psi/\psi_{sx}$, ψ_{sx} is the value of ψ on the separatrix of the island and $g^{\Phi\Phi} \equiv |\nabla\Phi|^2$. The parameter Δ' is the discontinuity in the logarithmic derivative of the exterior vector potential in poloidal flux coordinates and is assumed to be given by $\Delta' = -2m_0/\Phi_0$. The island current is computed from the electron distribution function which is derived in the next section. The ion motion is neoclassically damped at the ion-ion collision frequency[13] and will not add significantly to the island current, so we will neglect its effect.

III. Kinetic Theory

The kinetic theory used here is very similar to that used by Carrera, et al.[22] in their derivation of the bootstrap current modification to the single helicity nonlinear tearing mode. The primary difference in this calculation is in the fluctuating magnetic field where many magnetic harmonics are included. Alternatively, we could use the new Chapman-Enskog-like formalism[27] that has been developed for directly calculating the parallel viscous force in Eq. (1), which leads to the bootstrap current. However, since the kinetic formalism is more conventional, and so that we may deal directly with the density and temperature gradients (rather than indirectly via flows determined through momentum balances), we use the kinetic formalism here.

The electron drift kinetic equation is written

$$\frac{\partial f}{\partial t} + v_{\parallel}\nabla_{\parallel}f + \mathbf{v}_d\cdot\nabla f - ev_{\parallel}E_{\parallel}\frac{\partial f}{\partial W} = C(f). \tag{14}$$

The drift velocity $\mathbf{v}_d$ is written

$$\mathbf{v}_d = c\frac{\mathbf{E}\times\mathbf{B}}{B^2} - \frac{v_{\parallel}}{B}\nabla\times\frac{v_{\parallel}\mathbf{B}}{\Omega_e}\ , \tag{15}$$

where the first term is the $\mathbf{E}\times\mathbf{B}$ drift and the second term is the magnetic drift valid for low beta plasmas (the sum of the ∇B and curvature drift), and Ω_e is the electron gyrofrequency. The curl is computed at fixed energy ($W = m_e v^2/2$) and magnetic moment ($\mu = m_e v_{\perp}^2/2B$). Assuming that the toroidal projection of the covariant representation of the magnetic field dominates, the term $\mathbf{v}_d\cdot\nabla f$ is written using the notation introduced by Eqs. (9) – (11)

$$\mathbf{v}_d\cdot\nabla f = \frac{c}{q}[\varphi,f] + \frac{v_{\parallel}}{qR}[\rho_{\parallel}B_{\zeta},f]\ , \tag{16}$$

where $B_{\zeta} = \mathbf{B}\cdot\partial\mathbf{x}/\partial\zeta$, $\rho_{\parallel} = v_{\parallel}/\Omega_e$, the approximation $qB_{\zeta} = B^2\mathcal{J}$ has been used and the inductive portion of the $\mathbf{E}\times\mathbf{B}$ drift has been neglected. The parallel electric field is written

$$E_{\parallel} = -\nabla_{\parallel}\varphi - \frac{\mathbf{b}}{c}\cdot\frac{\partial\mathbf{A}}{\partial t} = -\frac{1}{qR}(q\partial_{\zeta}\varphi - [\psi,\varphi]) + \frac{1}{cR}\frac{\partial\psi_1}{\partial t}\ , \tag{17}$$

where $\mathbf{b} = \mathbf{B}/|\mathbf{B}|$. The collision operator is taken to be a Lorenz pitch-angle scattering operator (in the ion rest frame)

$$C_0(f) = \nu \frac{\xi}{B} \frac{\partial}{\partial\lambda} (\lambda\xi \frac{\partial f}{\partial\lambda}) \ , \tag{18}$$

where

$$\lambda \equiv \frac{v_\perp^2}{Bv^2} \ , \tag{19a}$$

$$\xi \equiv \sigma(1 - \lambda B)^{1/2} \ , \tag{19b}$$

$$\sigma \equiv \text{sign}\, v_\parallel \ , \tag{19c}$$

$$\nu \equiv 2\nu_e (v_t/v)^3 \ , \tag{19d}$$

v_t is the electron thermal velocity and ν_e is the electron collision frequency.

The electron drift-kinetic equation is now written

$$\frac{\partial f}{\partial t} + \frac{v_\parallel}{qR}(q\partial_\zeta f - [\psi,f]) + \frac{c}{q}[\varphi,f] + \frac{v_\parallel}{qR}[\rho_\parallel B_\zeta,f] - ev_\parallel E_\parallel \frac{\partial f}{\partial V}$$

$$= \nu \frac{\xi}{B} \frac{\partial}{\partial\lambda} (\lambda\xi \frac{\partial f}{\partial\lambda}) \ , \tag{20}$$

where $E_\parallel$ is given by Eq. (18).

A. Orderings

In order to solve the kinetic equation, a perturbation theory is invoked involving two small parameters. As in ref. (22), we use $\gamma \equiv \nu_e/\omega_t$ and $\delta \equiv \rho_{\theta e}/w_0$, where $\rho_{\theta e}$ is the poloidal electron gyroradius, w_0 is the primary island half-width, and $\omega_t = v_t/qR$ is the electron transit frequency around the torus. We order the island half-width $w_0/L_q \sim \gamma$, where $L_q = (d \ln q/dr)^{-1}$. So that the inductive island current and the island bootstrap current enter at the same order in the calculation, we take $\omega_t^{-1}\, \partial/\partial t \sim \delta^2$. This implies that

the electron makes many toroidal orbits before the topology changes. The electrostatic potential is ordered so that the electrostatic and electromagnetic portions of the parallel electric field are comparable. This is accomplished by having $e\varphi/T_e \sim \gamma\delta$. The $\mathbf{E} \times \mathbf{B}$ term in the kinetic equation is comparable to the explicit time-dependent term in the kinetic equation. Therefore, in this calculation, the electrostatic drift does not affect the island dynamics. The somewhat artificial ordering $1/m \sim \gamma$ is used for the poloidal mode number m. This is not a very restrictive assumption; ordering m as unity does not markedly affect the results but complicates the algebra somewhat. The small 1/m ordering has the primary effect of making the motion along the perturbed field-lines larger than the collisionality, so that the electron motion around the island is collisionless. The amplitudes of the non-resonant magnetic harmonics are assumed to be the same order as the resonant harmonic; however, we restrict ourselves to harmonics that are resonant at a magnetic surface not too far from the primary resonance. Formally, we order $\Delta_{mn}/L_q \sim \gamma$, where $\Delta_{mn} = L_q[(m/nq_0) - 1]$ is the distance between the rational surface at $q = m/n$ and $q = q_0$. The effect of the nonresonant harmonics is to produce a stochastic layer around the separatrix of the primary island. If the ratio $w_0/\Delta_{mn} << 1$, the width of the stochastic layer is negligble. Therefore, the nonresonant harmonics that most dramatically affect the primary island are those whose island is close by, $w_0/\Delta_{mn} >> \delta,\gamma$.

The calculation proceeds by expanding the distribution function in a double series, $f = \Sigma_{m,n} f_{mn}\delta^m\gamma^n$. The kinetic equation is then solved analytically assuming $\gamma > \delta$. To $O(\delta^0)$, the spatial distribution and the background Maxwellian nature of the electrons is derived. The $O(\delta)$ corrections introduce collisional terms that will result in the island bootstrap and Ohmic currents.

At this point, we make a parenthetical comment concerning the role of microinstabilities and drift wave effects. If the equilibrium value of the density gradient is used (the density scale length outside and away from the island region), the ratio of the diamagnetic drift frequency to electron transit frequency is $\omega_{*e}/\omega_t \sim \delta$. The assumption $\gamma >$

δ would then correspond to the condition $\nu_e > \omega_{*e}$. We note that linear stability theory of micro-tearing modes in toroidal geometry make the opposite assumption, $\omega_{*e} > \nu_e$.[28] However, in the nonlinear magnetic island regime, the electron distribution function equilibrates on the magnetic surfaces, the density profile flattens, and hence $\omega_{*e} \rightarrow 0$ near the island separatrix.[29,30] In the presence of a significant magnetic island, since the diamagnetic flows and consequent rotation frequency disappear, microinstabilities and drift wave effects play little role in the vicinity of the rational surface. The "macroscopic" island dynamics is then controlled by the neoclassical fluid-like mechanism presented here.

B. Spatial Distribution

To $O(\delta^0)$, the drift kinetic equation is

$$v_{\parallel}\nabla_{\parallel} f_0 = C(f_0) \ . \tag{21}$$

To $O(\delta^0\gamma^0)$, Eq. (21) simplifies to

$$q\partial_\zeta f_{00} - [\psi, f_{00}] = 0 \ . \tag{22}$$

This equation says that to leading order, the electron distribution function equilibrates along the magnetic field-lines. If ψ is made up of the single resonant harmonic, $\psi = \bar{\psi}$ (recall $\bar{K} = \oint d\zeta/2\pi\ K$), Eq. (22) has the solution $f_{00} = f_{00}(\bar{\psi})$. Then, the electron density and temperature profiles become functions of the helical flux surfaces describing the primary magnetic island topology. However when other magnetic harmonics are added the primary magnetic island topology is disturbed. Equation (22) can be separated into a ζ-averaged part and a ζ-dependent part.

$$[\bar{\psi}, \bar{f}_{00}] = -\oint \frac{d\zeta}{2\pi} [\tilde{\psi}, \tilde{f}_{00}] \ , \tag{23}$$

$$(\Delta\zeta)^{-1}\tilde{f}_{00} = [\tilde{\psi}, \bar{f}_{00}] \ , \tag{24}$$

where the operator $(\Delta\zeta)^{-1}$ is defined by

$$(\Delta\zeta)^{-1}K \equiv q\partial_\zeta K - [\bar{\psi},K] - [\tilde{\psi},K] + \oint \frac{d\zeta}{2\pi}[\tilde{\psi},K] \quad , \tag{25}$$

for any function K. Equation (23) describes the deviation from the single-helicity magnetic island topology caused by nonresonant magnetic harmonics.

To understand the effect of the deviation let's consider a number of different magnetic field-line trajectories. Those field-lines that are near the O–point of the primary island are topologically stable, in that trajectories that would normally have elliptic orbits about the O–point continue to execute essentially the same orbit when the other magnetic harmonics are present. Although the precise details of the field-line trajectories may vary, the gross behavior of the orbit is still described by the topological behavior of the single-helicity primary island. Therefore, near the elliptic point of the island $f_{00} = f_{00}(\bar{\psi})$. By a similar argument, those orbits very far from the island separatrix are essentially described the function $\bar{\psi}$.

Where the effect of the nonresonant harmonics is most dramatically felt is near the separatrix of the magnetic island. Small deviations caused by $\tilde{\psi}$ could cause orbits that were trapped inside the island separatrix to be untrapped and vice-versa. Consequently, what we expect near the separatrix of the island is that the primary magnetic island flux surfaces are destroyed; the magnetic field is stochastic. Since stochastic magnetic fields cannot sustain plasma density and temperature gradients, in a region of space around the separatrix, f_{00} is constant.

Summarizing the results of the discussion in the previous paragraphs, the leading order distribution function is described by

$$\bar{f}_{00} = f_{00}(\bar{\psi}) \qquad \text{for } \bar{\psi} < \bar{\psi}_{sx} - |\delta\bar{\psi}| \ , \tag{26a}$$

$$\bar{f}_{00} = \text{constant} \qquad \text{for } \bar{\psi}_{sx} - |\delta\bar{\psi}| < \bar{\psi} < \bar{\psi}_{sx} + |\delta\bar{\psi}| \ , \tag{26b}$$

$$\bar{f}_{00} = f_{00}(\bar{\psi}) \qquad \text{for } \bar{\psi}_{sx} + |\delta\bar{\psi}| < \bar{\psi} \ , \tag{26c}$$

where $\bar{\psi}_{sx}$ is the value of $\bar{\psi}$ on the separatrix. Recalling Eq. (7), $\bar{\psi}_{sx} = A_{m_0n_0}/m_0$.

The value $|\delta\bar{\psi}|$ represents the extent in $\bar{\psi}$ that the magnetic field is stochastic. To estimate its value, we follow the derivation of Rechester and Stix.[31] In their work, the effect of an incommensurate perturbation on a primary magnetic island was examined. It was found that secondary islands formed wherever the Fourier components of the perturbation resonate with the local rotational transform of the primary island. Since the transform goes to zero on the primary island separatrix (indicative of the fact that the separatrix is a singular surface), the secondary islands pile up on either side of the separatrix. By deriving the width of secondary islands and the spacing between secondary islands, an estimate of the extent of the stochastic region can be calculated. In order to keep this work self-contained their derivation is briefly reproduced here.

In order to compute $|\delta\bar{\psi}|$ it is convenient to transform from the magnetic coordinates Φ, α to the coordinates $\bar{\psi}$, v by

$$\bar{\psi} = \bar{\psi}_{sx} \left[2\frac{x^2}{w_0^2} - \cos(m_0\alpha)\right] , \tag{27}$$

$$\frac{dv}{d\alpha} = \frac{m_0 w_0}{x} . \tag{28}$$

Surfaces of constant $\bar{\psi}$ represent the magnetic surfaces of the primary magnetic island. The value $\bar{\psi} = \bar{\psi}_{sx}$ refers to the separatrix of the primary island ; the value $\bar{\psi} = -\bar{\psi}_{sx}$ is the O–point. The angle-like coordinate v labels the location on a magnetic surface. The magnetic field given by Eqs. (6) and (7) can be separated into a "coherent" part, representing the effect of the primary island, and a remainder, representing the perturbations with incommensurate helicities. Using the transformation given by Eqs. (27) and (28), the "coherent" part of the magnetic field is given by

$$\mathbf{B}_c = \frac{q_0\Phi_q}{m_0 w_0} \nabla\bar{\psi} \times \nabla v - \nabla\zeta \times \nabla\bar{\psi} , \tag{29}$$

where w_0 is the primary island half-width given by Eq. (8) and $\Phi_q = (d \ln q/d\Phi)^{-1}$ at $q = q_0$. Note that the condition $\mathbf{B}_c \cdot \nabla\bar{\psi} = 0$ is satisfied. The remaining part of the magnetic field is written

$$\mathbf{B}_1 = \nabla v \times \nabla\zeta \, \frac{x}{m_0 w_0} \sum_{m,n} A_{mn} \sin\left[m\alpha + \left(\frac{m}{q_0} - n\right)\zeta + j_{mn}\right], \tag{30}$$

where x and α are functions of $\bar{\psi}$ and v as defined by (27) and (28).

The thickness of the stochastic region outside the primary separatrix can now be computed. As pointed out in ref. (31), a similar calculation can be performed for the thickness of the stochastic layer inside the separatrix. To within a factor of approximately 2, the thicknesses inside and outside the separatrix are the same.

Outside the separatrix, it is convenient to use a new flux surface label, k (≤ 1), given by

$$k^2 = \frac{2\bar{\psi}_{sx}}{\bar{\psi} + \bar{\psi}_{sx}} = \left[\frac{x^2}{w_0^2} + \sin^2(m_0\alpha/2)\right]^{-1}, \tag{31}$$

where k = 1 now labels the separatrix and a new angle variable is introduced, $u = v/2k$. By integrating Eq. (28), we get the expressions $m_0\alpha/2 = \mathrm{am}(u,k)$, $\sin(m_0\alpha/2) = \mathrm{sn}(u,k)$, and $x = (w_0/k)\,\mathrm{dn}(u,k)$, where sn, cn, and dn are the Jacobian elliptic functions and am is the inverse of the elliptic integral of the first kind. The complete elliptic integral of the first kind is K(k). The angle variable u increases by 2K(k) as $m_0\alpha$ goes from $-\pi$ to π. Therefore, outside the separatrix, the functions are periodic in the variable $\pi u/K(k)$.

Now suppose that an asymmetry is introduced by a single "extra" magnetic perturbation. The perturbing magnetic field from Eq. (30) is written

$$\mathbf{B}_1 = \nabla u \times \nabla\zeta \, \frac{2}{m_0} \mathrm{dn}(u,k)\, A_{mn} \sin\left[m\alpha + \left(\frac{m}{q_0} - n\right)\zeta + j_{mn}\right]. \tag{32}$$

This magnetic perturbation produces a magnetic island at the q = m/n surface with an island half-width given by $w_{mn} = 2\sqrt{A_{mn}\Phi_q/m}$. We now seek an approximate form for the magnetic surfaces near the primary magnetic island. This is analogous to finding the

solution to the distribution function from Eqs. (23)–(25) near the separatrix. The magnetic surfaces are labeled by $\chi = \bar{\chi}(k) + \tilde{\chi}(k,u,\zeta)$. The function $\tilde{\chi}$ is expanded in a Fouries series

$$\tilde{\chi} = \sum_{\mu,\nu} \chi_{\mu\nu}(k) \exp\left[i\mu\frac{\pi u}{K(k)} + i\nu\,\zeta\left(\frac{m}{q_0} - n\right)\right] . \tag{33}$$

The magnetic differential equation is written $\mathbf{B}_c\cdot\nabla\tilde{\chi} = -\mathbf{B}_1\cdot\nabla\bar{\chi}$. The nonlinear term $\mathbf{B}_1\cdot\nabla\tilde{\chi}$ has been neglected. This assumption is analogous to dropping the last two terms in Eq. (25) and allows us to ignore nonlinear coupling that would produce nonzero amplitudes for $\chi_{\mu\nu}$ with $\nu \neq 1$. The perturbing magnetic field is expanded in a Fourier series

$$\left(\frac{x}{w_0}\right) A_{mn} \sin\left[m\alpha + \left(\frac{m}{q_0} - n\right)\zeta + j_{mn}\right]$$

$$= \sum_{\mu,\nu} C_{\mu\nu}(k) \exp\left[i\mu\frac{\pi u}{K(k)} + i\nu\,\zeta\left(\frac{m}{q_0} - n\right)\right] . \tag{34}$$

From the magnetic differential equation, we obtain

$$i[\nu - \mu\Omega(k)]\,\chi_{\mu\nu} = \frac{w_0}{\Delta_{mn}}\frac{1}{nq_0}\frac{d\bar{\chi}}{d\bar{\psi}}\,C_{\mu\nu} , \tag{35}$$

where the local rotational transform for the primary island region is defined by

$$\Omega(k) = \frac{\pi}{2kK(k)}\frac{n_0 w_0}{n\Delta_{mn}} , \tag{36}$$

and $\Delta_{mn} = \Phi_q[(m/nq_0) - 1]$. There is a resonance wherever $k = k_{\mu\nu}$, where $\Omega(k_{\mu\nu}) = \nu/\mu$. By picking $d\bar{\chi}/dk = (k - k_{\mu\nu})$ in the vicinity of $k = k_{\mu\nu}$, Eq. (35) can be integrated to find the approximate magnetic surfaces near $k = k_{\mu\nu}$

$$\chi \cong \frac{1}{2}(k - k_{\mu\nu})^2 + C(k)\sum_{+,-} C_{\mu\nu}(k) \exp\left[i\mu\frac{\pi u}{K(k)} + i\nu\,\zeta\left(\frac{m}{q_0} - n\right)\right] , \tag{37}$$

where the sum is over μ, ν and $-\mu$, $-\nu$,

$$C(k) = \frac{ik^3}{4\bar{\psi}_{sx}}\frac{w_0}{\Delta_{mn}q_0 n}\frac{1}{\mu\Omega'} , \tag{38}$$

and $\Omega' = d\Omega/dk$ at $k = k_{\mu\nu}$. Equation (37) is the characteristic magnetic potential to model magnetic island formation. At each value of the local rotational transform where $\Omega = \nu/\mu$ secondary islands form. Since $\Omega \rightarrow 0$ as the separatrix is approached, an infinite number of islands appear for $\nu = 1$.

In order to find the amplitude of the secondary island, the coefficients $C_{\mu\nu}$ need to be determined. They are given by

$$C_{\mu\nu} = \frac{A_{mn}}{4ikK(k)} \int_0^{2K} du \; dn(u,k)\, e^{-i\mu u\pi/K} \left[\delta_{\nu,1} e^{i(m\alpha + j_{mn})} - \delta_{\nu,-1} e^{-i(m\alpha + j_{mn})}\right] , \quad (39)$$

where $\alpha = \alpha(k,u)$. The integral is now evaluated assuming the ratio $\Delta \equiv |n_0 w_0/\Delta_{mn} n|$ is small and the ratio $M \equiv m/m_0$ is integer. Following the asymptotic treatment outlined in detail in ref. (31), the coefficient $C_{\mu\nu}$ can be computed by deforming the contour and expanding the integral around the uppermost singular points. Equations (36)–(38) can be computed to give a secondary island amplitude in k-space, δw_k, given by

$$\delta w_k^2 = \frac{w_{mn}^2}{w_0^2} \left(\frac{2}{k}\right)^{2(M-1)} \frac{M}{(2M)!} \frac{1}{(dK/dk)} \frac{e^{-K'/\Delta}}{\Delta^{(2M-1)}} , \quad (40)$$

where $K' = K(\sqrt{1-k^2})$ and w_{mn} is the amplitude of the magnetic island at the $q = m/n$ surface assuming Φ_q at $q = q_0$ equals Φ_q at $q = m/n$.

The secondary island widths are nonanalytic functions of Δ, where Δ is essentially the ratio of the primary island half-width to the distance between rational surfaces. When Δ is small, the secondary islands are not important, and therefore the assumption of only keeping magnetic harmonics with $w_0/\Delta_{mn} >> \delta,\gamma$ is justified. The secondary islands overlap when the spacing between the islands determined by $\Delta k = \nu/\mu^2\Omega'$ equals δw_k. Using the asymptotic approximation $K' = \pi/2$ as $k \rightarrow 1$, $|\delta\bar{\psi}|$ is given by

$$|\delta\bar{\psi}| \cong \frac{w_{mn}^2}{4\Phi_q} \frac{1}{(2M-1)!} \frac{e^{-\pi/2\Delta}}{(\Delta/2)^{(2M+1)}} . \quad (41)$$

In this calculation it has been assumed that there are only two incommensurate helicities, while the model topology we would like to examine has a bath of interacting magnetic islands. For each magnetic harmonic a $|\delta\bar{\psi}|$ can be computed. Because Eq. (41) shows a dramatic dependence on the parameter Δ, the actual width of the stochastic layer around the primary island separatrix is likely dominated by the magnetic harmonic with the largest Δ, and the overlap of secondary islands produced by two different magnetic perturbations is negligible as long as the two Δ's are different in magnitude.

Now that the spatial distribution of f_{00} has been clarified, the kinetic equation to $O(\delta^0\gamma^1)$ can be examined.

$$v_{\parallel}\nabla_{\parallel} f_{01} = C(f_{00}) \ . \tag{42}$$

The left-hand-side of the equation can be annihilated by averaging over the magnetic surfaces of the primary magnetic island. Recall that in the region of $\bar{\psi}$–space described by Eqs. (26a) and (26c), the $\bar{\psi}$ surfaces are assumed to be robust. In that region of space, an average over a flux surface can be defined by

$$\langle K\rangle = \frac{\oint d\alpha \ \bar{K} / \partial_{\Phi}\bar{\psi}(\bar{\psi},\alpha)}{\oint d\alpha \ 1 / \partial_{\Phi}\bar{\psi}(\bar{\psi},\alpha)} \ , \tag{43}$$

for any function K. Flux surface averaged quantities have the property that $\mathbf{B}\cdot\nabla \langle K\rangle = 0$. In the region around the separatrix that is stochastic, it is demanded that the distribution function f_{01} be constant, so in that region $\nabla_{\parallel} f_{01} = 0$ identically. Using Eq. (43) and the restriction demanded by the stochastic region, Eq. (42) can be satisfied by letting f_{00} be a Maxwellian where the density and temperature profiles have the spatial structure given in Eq. (26).

C. Island Current

In this section, the electron current in the vicinity of the primary magnetic island is computed. Writing the kinetic equation to order $O(\delta^1\gamma^0)$, we obtain

$$v_{\|}\nabla_{\|} f_{10} + \frac{v_{\|}}{qR}[\rho_{\|}B_{\zeta},f_{00}] = 0. \tag{44}$$

The quantity $\rho_{\|}B_{\zeta}$ to leading order is a function of the poloidal angle $\theta = \alpha + \zeta/q_0$. Therefore $[\rho_{\|}B_{\zeta},f_{00}] = -\partial\alpha(\rho_{\|}B_{\zeta})\partial_{\Phi}f_{00} = -q_0\partial\zeta(\rho_{\|}B_{\zeta}\partial_{\Phi}f_{00})$. Since $\partial_{\zeta} \sim O(1)$, Eq. (43) has the solution

$$f_{10} = \rho_{\|}B_{\zeta}\partial_{\Phi}f_{00} + g_{10}\ , \tag{45}$$

where g_{10} has the same spatial structure as given by Eq. (26), $g_{10} = g_{10}(\bar{\psi})$ for $\bar{\psi} < \bar{\psi}_{sx} - |\delta\bar{\psi}|$, and $\bar{\psi} > \bar{\psi}_{sx} + |\delta\bar{\psi}|$ and constant otherwise.

In order to find an equation for g_{10}, we go to $O(\gamma^1\delta^1)$ in the kinetic equation. After averaging over ζ, which amounts to a bounce average, we find the equation

$$\frac{eq}{cT_e}\frac{\partial\bar{\psi}_1}{\partial t} - [\bar{\psi}, \bar{f}_{11} - (e\bar{\varphi}/T_e)f_{00}] - \oint\frac{d\zeta}{2\pi}[\tilde{\psi}, \tilde{f}_{11} - (e\tilde{\varphi}/T_e)f_{00}]$$

$$= \frac{qRB_{\zeta}}{\Omega_e}\partial_{\Phi}f_{00}\,\bar{\xi}^{-1}\,C(\bar{\xi}) + \frac{qR}{v_{\|}}C(g_{10})\,. \tag{46}$$

After inverting the collision operator and demanding the spatial constraint on g_{10} described after Eq. (45), we find

$$g_{10} = -\sigma v\{\frac{B_{\zeta}}{\Omega_e}<\partial_{\Phi}f_{00}> + (\frac{v}{v_t})^3\frac{ef_{00}}{R_0cT_e}<\partial_t\bar{\psi}_1>\}I(\lambda)\ , \tag{47}$$

where

$$I(\lambda) = \Theta(\lambda_c - \lambda)\int_{\lambda}^{\lambda_c}\frac{B_0 d\lambda}{2|\bar{\xi}|}\ , \tag{48}$$

Θ is the Heaviside step function, B_0 and R_0 are the magnetic field and major radius on the magnetic axis, $\lambda_c = 1/B_0(1 + \varepsilon)$, and ε is the inverse aspect ratio. The flux surface averaging operator is clear as it pertains to the first term in Eq. (47) since $\partial_{\Phi}f_{00}$, given by

$$\frac{\partial f_{00}}{\partial\Phi} = \{\frac{1}{n}\frac{\partial n}{\partial\Phi} + (\frac{m_e v^2}{2T_e} - \frac{3}{2})\frac{1}{T_e}\frac{\partial T_e}{\partial\Phi}\}\,f_{00}\ , \tag{49}$$

is zero in the stochastic region, where the average over a flux surface is not defined. However, the flux surface average of $\partial_t\bar{\psi}_1$ is not defined in the stochastic region. The resolution of this problem is postponed until the quasineutrality condition is discussed.

By multiplying f_{10} by $ev_{\|}$ and integrating over velocity space, the lowest order parallel island current is computed,

$$j_{\|e} = \frac{1}{\eta_n Rc} <\partial_t\bar{\psi}_1> - 1.46\sqrt{\varepsilon}\frac{cnT_eB\zeta}{B}\{\frac{<\partial_\Phi n>}{n} + \frac{<\partial_\Phi T_e>}{T_e}\} , \qquad (50)$$

where $\eta_n = (0.51m_e\nu_{ei}/ne^2)(1 - 1.95\sqrt{\varepsilon})^{-1}$ is the neoclassical resistivity, with the $\sqrt{\varepsilon}$ indicating the trapped particle effect. The second term in Eq. (50) is the island bootstrap current. Physically, the origin of the bootstrap term comes from the banana orbits of the trapped electrons interacting with the island pressure gradient. This produces a diamagnetic current. Viscosity between the current-carrying untrapped electrons and the immobile trapped electrons cause the diamagnetic current to be enhanced; the passing electrons drift in the same direction as the trapped electrons as a result of collisions. The neoclassical mechanism enhances the diamagnetic response to the magnetic island pressure gradient. If we had used the new Chapman-Enskog-like formalism[27] for directly calculating the parallel viscous force, the $\sqrt{\varepsilon}$ in Eq. (50) would be replaced by $\mu_e/\nu_e \sim \sqrt{\varepsilon}/(1+\nu_{*e})$, the ratio of the viscous damping rate and the collisional friction. Also, the electron density and temperature gradients would be replaced by an overall plasma pressure gradient plus temperature gradient terms with collisionality-regime-dependent coefficients.[32]

We conclude this section by considering constraints on the parallel current resulting from quasineutrality. Self-consistency demands the condition $\nabla\cdot\mathbf{J} = 0$ from Ampere's law. We neglect here the effects of inertia[21] and interchange modes. Resistive interchange modes can drive magnetic islands in the Rutherford regime;[33] however, for the "banana" collisionality regime considered here, the island bootstrap current is larger. Under these assumptions, the island current must satisfy the condition $\nabla_{\|} j_{\|} = 0$. This is precisely the

condition that describes the leading order spatial distribution of the electron distribution function. By arguments similar to those used to describe f_{00} by Eq. (26), $j_{\|}$ is given by

$$j_{\|} = j_{\|}(\bar{\psi}) \qquad \text{for } \bar{\psi} < \bar{\psi}_{sx} - |\delta\bar{\psi}| \ , \tag{51a}$$

$$j_{\|} = \text{constant} \qquad \text{for } \bar{\psi}_{sx} - |\delta\bar{\psi}| < \bar{\psi} < \bar{\psi}_{sx} + |\delta\bar{\psi}| \ , \tag{51b}$$

$$j_{\|} = j_{\|}(\bar{\psi}) \qquad \text{for } \bar{\psi}_{sx} + |\delta\bar{\psi}| < \bar{\psi} \ , \tag{51c}$$

Physically, what is occuring is that the electrostatic potential near the primary island responds so that the current resides on the constant–$\bar{\psi}$ contours of the magnetic island, where the contours are robust, just as the plasma density and temperature profiles are similarly constrained. However, in the stochastic region, the electron current flattens due to rapid transport along the stochastic field–lines. Note that this is not quite the Rutherford requirement on the electron current in the nonlinear regime of the tearing mode. This difference will result in a correction to the Rutherford integral of order $(|\delta\bar{\psi}|/\bar{\psi}_{sx})^2$, but will not have a dramatic effect on the overall result.

D. Island Density and Temperature Profiles

To construct the density and electron temperature profiles, assume that there are particle and heat sources in the plasma interior and that diffusion processes are present.[22,33-35] The density and temperature profiles in the vicinity of the magnetic island are constructed by demanding that the flux of particles and heat are continuous. Inside the island separatrix the profiles are assumed flat and, as previously discussed, in the stochastic layer around the separatrix the profiles are also flat. Very far from the island the profiles are required to satisfy the asymptotic statements $dn/d\bar{\psi} \rightarrow dn/d\Phi|_{eq}(q/q_0 - 1)^{-1}$ and $dT_e/d\bar{\psi} \rightarrow dT_e/d\Phi|_{eq}(q/q_0 - 1)^{-1}$, where $dn/d\Phi|_{eq}$ and $dT_e/d\Phi|_{eq}$ are the equilibrium values of the density and electron temperature gradients, respectively, in the exterior region. We note that the exterior values of the profiles are assumed to be given. Although the interior

profiles of the density and temperature are accounted for self-consistently, the exterior values are not. From these conditions, the profiles are required to be

$$\frac{dn}{d\bar{\psi}} = \frac{dn}{d\Phi}\Big|_{eq} \frac{\Phi_q}{w_0} \frac{\Theta\{\bar{\psi} - (\bar{\psi}_{sx} + |\delta\bar{\psi}|)\}}{I_0} , \tag{52a}$$

$$\frac{dT_e}{d\bar{\psi}} = \frac{dT_e}{d\Phi}\Big|_{eq} \frac{\Phi_q}{w_0} \frac{\Theta\{\bar{\psi} - (\bar{\psi}_{sx} + |\delta\bar{\psi}|)\}}{I_0} , \tag{52b}$$

where

$$I_0 = \frac{\Phi_q}{w_0} \oint \frac{m_0 d\alpha}{2\pi} \partial_\Phi \bar{\psi}(\bar{\psi},\alpha) = \oint \frac{m_0 d\alpha}{2\pi} \sqrt{\frac{\Psi + \cos(m_0\alpha)}{2}} , \tag{53}$$

and $\Psi \equiv \bar{\psi}/\bar{\psi}_{sx}$. The island bootstrap current is now written

$$j_{\|b} = -1.46\sqrt{\varepsilon}\frac{cnT_eB_\zeta}{B} \left(\frac{1}{n}\frac{dn}{d\Phi}\Big|_{eq} + \frac{1}{T_e}\frac{dT_e}{d\Phi}\Big|_{eq}\right) \frac{\Theta\{\bar{\psi} - (\bar{\psi}_{sx} + |\delta\bar{\psi}|)\}}{I_0 I_2} , \tag{54}$$

where

$$I_2 = \oint \frac{m_0 d\alpha}{2\pi} \sqrt{\frac{2}{\Psi + \cos(m_0\alpha)}} . \tag{55}$$

It is assumed that the stochastic zone around the separatrix is symmetric about $x = 0$. This symmetry is lost if the primary island overlaps with a neighboring magnetic island. We can model island overlap by eliminating the density and electron temperature gradient on the side of the separatrix that the overlap occurs. The matching of the profiles is then enforced on either side of the two island structure, while the profiles are assumed flat all the way across both island regions. Island overlap essentially cuts the island bootstrap current for each island in half since the density and temperature gradient is only present outside the combination of the two magnetic islands, but not between them.

IV. Magnetic Island Dynamics

As in all tearing mode problems, the matching condition between the island region and the exterior solution is obtained by integrating the appropriate harmonic of the perturbed Ampere's law across the tearing layer. The matching condition is given by Eq. (13) where the usual constant–$\bar{\psi}$ and single harmonic approximations are made for the evolution of the single magnetic island. Inserting the parallel current from Eqs. (50) and (54), we obtain the equation

$$\frac{cg^{\Phi\Phi}}{16\pi R\Phi_q} w_0 \Delta' = \frac{1}{\eta_n cR} \frac{1}{m_0} \frac{dA_0}{dt} F_0(|\delta\bar{\psi}|/\bar{\psi}_{sx})$$

$$+ 1.46 \sqrt{\varepsilon}\, cnT_e R_0 \left(\frac{1}{n} \frac{dn}{d\Phi}\Big|_{eq} + \frac{1}{T_e} \frac{dT_e}{d\Phi}\Big|_{eq}\right) F_1(|\delta\bar{\psi}|/\bar{\psi}_{sx}) \ , \quad (56)$$

where F_0 and F_1 are dimensionless functions of the quantity $(|\delta\bar{\psi}|/\bar{\psi}_{sx}) \equiv \delta\Psi$. As $\delta\Psi \to 0$, Eq. (56) can be rewritten in the form given by Eq. (3), where F_0 and F_1 become the constants $F_0 = 1/2k_0 \cong 0.8$ and $F_1 = k_1/2.92k_0 \cong 0.8$. The deviation caused by $\delta\Psi$ (due to the presence of a nearby island of incommensurate helicity) on F_0 has the approximate form

$$F_0 = 1/2k_0 + \int_{1-\delta\Psi}^{1+\delta\Psi} d\Psi \oint \frac{m_0\alpha}{2\pi} \cos(m_0\alpha) \sqrt{\frac{2}{\Psi+\cos(m_0\alpha)}} \{G(1+\delta\Psi) - G(\Psi)\} \ , \quad (57)$$

where

$$G(K) = \frac{\oint d\alpha \cos(m_0\alpha) \sqrt{\frac{2}{K+\cos(m_0\alpha)}}}{\oint d\alpha \sqrt{\frac{2}{K+\cos(m_0\alpha)}}} \ . \quad (58)$$

Asymptotically, one can show that the remaining integral goes as $(\delta\Psi)^2$ as $\delta\Psi \to 0$. Since this is small when $\delta\Psi$ is small, we will ignore its effect in what follows. A finite value of $\delta\Psi$ has a more profound effect on the second integral F_1,

$$F_1 = \int_{1+\delta\Psi}^{\infty} d\Psi \frac{1}{I_0(\Psi)\, I_2(\Psi)} \oint \frac{m_0 d\alpha}{2\pi} \sqrt{\frac{2}{\Psi+\cos(m_0\alpha)}} \ , \quad (59)$$

where $I_0(\Psi)$ and $I_2(\Psi)$ are given by (53) and (55). Treating Ψ as large under the integral, F_1 is given by the asymptotic function

$$F_1 \cong \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+\delta\Psi}} \,. \tag{60}$$

The actual value for F_1 as $\delta\Psi \rightarrow 0$, is approximately 0.8; the asymptotic function is reasonably accurate with respect to its numerical value. After collecting the information from Eqs. (56) – (60), the magnetic island evolution is given by

$$\frac{4\pi}{\eta_n c^2} \frac{1}{g^{\Phi\Phi}} \frac{dw_0}{dt} = 0.6\,\Delta' + \sqrt{\varepsilon}\frac{1.4\beta_{pe}L_q(L_n^{-1} + L_{T_e}^{-1})}{\sqrt{w_0^2 + 4\Phi_q|\delta\bar{\psi}|}} \,, \tag{61}$$

where $L_q = (d \ln q/d\Phi)^{-1}|\nabla\Phi|$, $L_n = -(d \ln n/d\Phi)^{-1}|\nabla\Phi|$, and $L_{T_e} = -(d \ln T_e/d\Phi)^{-1}|\nabla\Phi|$. The extra metric factor $g^{\Phi\Phi}$ in the first term results from using the extent in the poloidal flux function as the unit for measuring the island width. In the absence of the island bootstrap current, given by the last term in Eq. (61), the Rutherford nonlinear island growth[21] is reproduced.

The primary effect of island interactions comes in through the island bootstrap current term. Physically, the effect of an incommensurate magnetic perturbation on the primary magnetic island is to flatten the local density and tempertaure profile in the vicinity of the separatrix through the creation of the stochastic layer. So instead of the profiles being flattened over the island width, the presence of the stochastic layer causes the density and temperature profiles to be flattened over a slightly larger region. Recall from Eq. (41) that $|\delta\bar{\psi}|$ is a function of the nonresonant magnetic harmonics, so that the evolution of a given magnetic island w_0 depends on the island separation parameter Δ for all the other magnetic islands. In this way all the magnetic islands are coupled together. In order to describe a topology with a bath of magnetic islands, one could describe the evolution of any particular island with Eq. (61) while taking all the island interactions into account

through $|\delta\bar{\psi}|$, which however will be dominated by the island that most nearly overlaps with the fundamental island w_0.

V. Example - Two Coupled Magnetic Islands

Since the interactions are usually dominated by pairs of nearly overlapping islands, as a simple example of this theory, let's consider the evolution of two interacting magnetic islands. Each island is governed by Eq. (61). The matching parameter Δ' is assumed to be negative, representing the stabilizing effect of field-line bending. Islands form at the $q = m_0/n_0$ and $q = m_1/n_1$ surfaces. Therefore $\Delta_0' = -2m_0/r_0|\nabla\Phi|$ and $\Delta_1' = -2m_1/r_1|\nabla\Phi|$, where $q(r_0) = m_0/n_0$ and $q(r_1) = m_1/n_1$. The island dynamical equations are

$$\frac{dw_0}{dt} = -m_0 + \frac{Q_0}{\sqrt{w_0^2 + w_1^2 h_0}} , \tag{62}$$

$$\left(\frac{r_1}{r_0}\right)^2 \frac{dw_1}{dt} = -m_1 + \frac{Q_1}{\sqrt{w_1^2 + w_0^2 h_1}} , \tag{63}$$

where

$$Q_{0,1} = \sqrt{\varepsilon}\, 1.1\beta_{pe} L_q (L_n^{-1} + L_{T_e}^{-1}) , \tag{64}$$

is evaluated at the appropriate rational surfaces, and the coupling functions are given by

$$h_0 = \left(\frac{r_1}{r_0}\right)^2 \frac{1}{\Gamma(2M)} \frac{e^{-\pi/2\Delta_0}}{(\Delta_0/2)^{(2M+1)}} , \tag{65}$$

$$h_1 = \left(\frac{r_0}{r_1}\right)^2 \frac{1}{\Gamma(2/M)} \frac{e^{-\pi/2\Delta_1}}{(\Delta_1/2)^{(2/M+1)}} , \tag{66}$$

where $\Delta_0 = |n_0 w_0/n_1\Delta_{mn}|$, $\Delta_1 = |n_1 w_1/n_0\Delta_{mn}|$, and $M = m_1/m_0$. Time has been normalized to the timescale $4\pi r_0^2/1.2\eta_n c^2$ and the island half-widths w_0 and w_1 are normalized to $r_0|\nabla\Phi|$ and $r_1|\nabla\Phi|$, respectively. In the absence of island coupling (h_0 and $h_1 \rightarrow 0$), the islands grow and independently saturate at $w_0 = Q_0/m_0$ and $w_1 = Q_1/m_1$.

Let's consider two limits. In the first limit, suppose the inequality $Q_0 r_0/m_0 + Q_1 r_1/m_1 < \Delta_{mn}$ is satisfied. Here, the saturated island half-widths in the uncoupled limit are small enough that they would not overlap. When island coupling is restored, the islands saturate at a slightly smaller half-width. The saturated half-widths can be computed by solving the equations

$$w_0^2 + w_1^2 h_0(w_0) = (Q_0/m_0)^2 , \tag{67}$$

$$w_1^2 + w_0^2 h_1(w_1) = (Q_1/m_1)^2 . \tag{68}$$

Treating h_0 and h_1 as small, $w_0 \cong (Q_0/m_0) - (m_0/Q_0)(Q_1/m_1)^2 h_0(Q_0/m_0)$, and $w_1 \cong (Q_1/m_1) - (m_1/Q_1)(Q_0/m_0)^2 h_1(Q_1/m_1)$. So in this limit the magnetic island dynamics do not change significantly from when the single-helicity theory is used.

The other limit is given by $Q_0 r_0/m_0 + Q_1 r_1/m_1 > \Delta_{mn}$. Here, the saturated island half-widths in the uncoupled limit would overlap. Recall, that we model the onset of the overlap by eliminating the pressure gradient on the side of the island where the overlap occurs. This is accomplished by letting $Q_{0,1} \rightarrow Q_{0,1}/2$. Note the implicit assumption throughout this entire analysis is that the electrons react faster than the magnetic islands when the topology changes. Island overlap results when the separatrices of the two islands touch. The electrons can then quickly equilibrate over the region with the two island chains in it. As far as the magnetic islands are concerned, this happens faster than the islands can react. If the inequality $Q_0 r_0/m_0 + Q_1 r_1/m_1 > 2\Delta_{mn}$ is satisfied, the islands saturate with overlapping separatrices. The pressure gradient on just one side of the separatrix is sufficient to sustain the island. If the islands satisfy the inequality $\Delta_{mn} < Q_0 r_0/m_0 + Q_1 r_1/m_1 < 2\Delta_{mn}$, no saturation is possible. Before the separatrices touch, the pressure gradient is sufficiently strong to overcome the stabilizing effects of Δ'. Once the separatrices touch, the pressure term is greatly reduced very rapidly (via parallel motion along the stochastic field lines), and Δ' causes island suppression. Once the islands shrink to the point where the pressure gradient between the islands can recover, the islands start to

grow slowly (on the island magnetic reconnection time scale) and this cycle of slow growth and abrupt decay continues with the island sizes staying not too far away from overlap conditions.

Of course, the example of this section is not meant to be representative of what is occuring in a tokamak. It is merely used as a tool to illustrate the phenomenology of the island interaction. A more accurate description of the magnetic topology would entail tracking many different islands as they grow and decay. The density of islands is such that the magnetic islands will, on the average, interact because of barely touching separatrices, so no set of saturated islands as described by Eqs. (67) and (68) would exist. Island separatrices touching requires a minimum electron poloidal beta, approximately given by $\beta_{pe} \gtrsim \Delta_{mn} m_0 L_p / 2 r_0 L_q \sqrt{\varepsilon}$, for a given distance between rational surfaces Δ_{mn}. However, since the minimum Δ_{mn} scales as $\Delta_{mn} \sim 1/n^2 q' \sim q^2/m^2 q'$, this implies the minimum β_{pe} scales as 1/m. In addition, we would not expect the islands to have the simple cyclic motion described above since any particular island will interact with more than one island over any extended period of time. A steady state of many overlapping magnetic islands will not be reached since the overlap of islands does not allow density or electron temperature gradients, so the island growth mechanism would be lost. Thus *"magnetic bubbling"* topology is described as a bath of magnetic islands that all sporadically grow slowly due to the neoclassical pressure gradient effect and decay abruptly due to island interaction.

VI. Discussion

The interaction of cross-field density and temperature gradients with the drift motion of particles in toroidal geometry causes a parallel current in long mean-free-path plasmas. This current is called the bootstrap current. Fluctuating bootstrap currents can cause the formation of magnetic islands at rational surfaces in resistive plasmas. The modification caused by island bootstrap currents on the nonlinear evolution of the single-

helicity tearing mode has been computed previously.[13,22] In this work, we have extended this theory to include the interaction of neighboring bootstrap-current-driven magnetic islands. The island coupling is taken into account by self-consistently constructing the plasma density and temperature profiles in the vicinity of the magnetic island. The introduction of a magnetic perturbation of incommensurate helicity causes the formation of a stochastic region around the separatrix of the magnetic island. The stochastic region flattens the island density and temperature profiles around the separatrix, reducing the island bootstrap current and the magnetic island drive. If neighboring magnetic islands overlap, the plasma profiles are flattened across the overlapping island region and the free energy source for the islands is halved.

The model that has been presented describes the magnetic configuration as a bath of interacting magnetic islands with poloidal number $m \cong 3\text{-}30$. The magnetic island dynamics are assumed to obey a nonlinear tearing mode evolution that includes the effect of the neoclassical pressure gradient and island interactions. In toroidal geometry, island dynamics are also coupled in the exterior region of the tearing mode through the construction of the matching parameter Δ';[36] however for the islands with mode number $m \gtrsim 3$, Δ' is usually stabilizing and given by $-2m/r$. The island dynamics is described by sporadic slow growth and rapid decay of the island width due to the destabilizing effect of the island bootstrap current and the stabilizing effect of the island interactions.

The model presented here is similar to the model topology introduced by Rebut and co-workers.[4–7] However, our model differs in a number of ways. Whereas both pictures of the topology contain chains of magnetic islands, the stochastic region in our model is mostly restricted to the regions about the island separatrices. In between the islands, during the time between island interactions, robust toroidal magnetic surfaces exist, while the topology of refs. (4)–(7) has the magnetic islands embedded in a self-sustained, steady-state, stochastic sea. The existence of good toroidal magnetic surfaces allows the plasma to have "radial" density and temperature gradients. These gradients, through the neoclassical

mechanism described above, cause the formation of the magnetic islands. In contrast, the Rebut et al. model relies on a thermal instability (or other a combination of other mechanisms) driving microtearing modes which requires a minimum threshold value of ∇T_e for self-sustainment. Most importantly, the topology given in our model is dynamic. Each island obeys an evolution equation which describes its island growth and decay, unlike the static topology of refs. (4)–(7), where a steady island self-sustainment mechanism is required. Recently, Kadomtsev has introduced a plasma transport picture that involves magnetic islands in the toroidal equilibrium.[37] This model has magnetic islands whose island widths are much smaller than the ion gyroradius, whereas our model examines magnetic islands whose widths are typically many ion gyroradii.

The model presented here suggests that low to medium mode number, coherent magnetic structures could be present in tokamak plasmas. The magnetic structures are stationary in the plasma frame (the equilibrium radial electric field causes an $\mathbf{E} \times \mathbf{B}$ toroidal rotation in the lab frame), have radial extents typically a few centimeters, and have large correlation lengths along field-lines. The "bubbling" nature of the model topology suggests that these islands are short-lived, with lifetimes ($\Delta t \sim 4\pi w^2/6\eta c^2$) typically $\lesssim$ 3 ms for say 3 cm islands. The magnitude of the magnetic perturbations we are considering are typically $\tilde{b}_r/B_0 \cong 10^{-4}$–$10^{-5}$. The density and temperature fluctuations associated with these magnetic perturbation satisfy a mixing length relation $\tilde{n}/n_0 \cong 0.26\ \Delta r/L_n$ (see Appendix), where $\Delta r = 2w$ is the full island width, and $\tilde{n}$ is defined as the peak-to-peak density fluctuation amplitude. Thus for islands with full island width 3 cm and $L_n = 100$ cm, the density and temperature fluctuations arising due to magnetic island formation are quite small, $\tilde{n}/n_0 \cong \tilde{T}/T_0 \cong 8 \times 10^{-3}$.

In order to completely describe the plasma dynamics, transport coefficients from this model should be computed. The nature of the transport is different from statistical approaches to magnetic turbulence,[26] where the growth and decay of imbedded, coherent magnetic islands is usually not addressed. The radial step size of the plasma transport is

given here by the characteristic island width. In the guise of drift-wave fluctuations, a form for the electron heat diffusivity, $\chi_e \sim \gamma w^2$, has been suggested,[38] where w is the island width and γ is the fluctuation "birth-death" rate. In this transport process electrons (and ions) are fixed to the field-lines, and the plasma transport occurs because the field-lines diffuse. This prediction probably underestimates the amount of plasma transport for the present model since electrons can travel very quickly along the combined island and stochastic regions of the field-line structure, and an island overlap causes "lumps" of particles and electron and ion heat to be transferred from one island chain to the next. The combined effects of collisions and magnetic fluctuations near the island structure probably enhance this transport; however we are not yet prepared to suggest a form for the transport coefficients for this *"magnetic bubbling"* situation.

Finally, we note that an extension of the present theory could be used to understand the interaction of plasma pressure induced magnetic islands in stellarator equilibria.[34,35] It has been suggested that resistive interchange modes may be responsible for magnetic island formation and magnetic stochasticity in Heliotron–E.[39] In a fashion similiar to the one described here, the effect of the island interaction would reduce the effect of bad curvature and Pfirsch-Schlüter currents in the island vicinity by flattening the island pressure profile in the stochastic field region near the separatrix.

Acknowledgments

The authors would like to acknowledge useful discussions with N. Mattor. J. D. C. would like to acknowledge useful discussions with M. Hugon and "spiritual motivation" by P. H. Rebut. This work is supported by the U. S. DOE under grant no. DE-FG02-86ER53218.

Appendix: Density and Temperature Fluctuation Amplitudes

In the model topology presented here, the islands are fixed in the frame of the plasma. What would be observed in the laboratory then is the magnetic structure as the toroidally rotating plasma carries it by. Associated with these helical magnetic perturbations of the axisymmetric equilibrium are density and temperature fluctuations. Some direct measurements of magnetic fluctuations can be made with magnetic probes[40] (in the plasma edge) and heavy ion beam probes.[41] Measurements of density and temperature fluctuations with long perpendicular wavelengths ($k_\perp \rho_i < 1$) are relevant to the present model. (For instance, beam emission spectroscopy measurements of density fluctuations can probe the small $k_\perp \rho_i$ regime.[42]) A mixing length argument is often invoked to relate the amplitude of the fluctuations to the radial correlation length. In this Appendix we compute the relationship between the radial correlation distance, (in this model, the full width of the magnetic island), and the density and temperature fluctuation amplitude.

Outside the island separatrix, the density distribution is given by the relation

$$n(\bar{\psi}) = \frac{dn}{d\Phi}\Big|_{eq} \frac{\Phi_q}{w_0} \int d\bar{\psi} \, \frac{1}{I_0} \, , \tag{A1}$$

where we have used Eq. (52) to describe the density gradient near the island, and I_0 is given by Eq. (53). We note that a similar relation holds for the island temperature distribution, and the discussion to follow describing the density fluctuation also describes the temperature fluctuations. Making the asymptotic approximation $I_0 \cong \sqrt{\Psi/2}$, Eq. (A1) is integrated to yield

$$n(\bar{\psi}) = n(\bar{\psi}_{sx}) \pm \frac{dn}{d\Phi}\Big|_{eq} \frac{w_0}{\sqrt{2}} \left\{ \sqrt{(\bar{\psi}/\bar{\psi}_{sx})} - 1 \right\} \, , \tag{A2}$$

where $n(\bar{\psi}_{sx})$ is the density on the separatrix and the sign is determined by the side of the magnetic island on which the observation is made. Now suppose that the density profile is

observed at a particular radial position (a fixed x in our notation) in a given toroidal angle plane. The density would appear to oscillate since the plasma is constrained to be constant on the (helically distorted) magnetic island flux surfaces. The peak-to-peak amplitude of the fluctuation at a given $x = \bar{x}$ would then be given by

$$\frac{\tilde{n}}{n_0} \equiv \frac{n_{max} - n_{min}}{n_0} \cong \frac{w_0}{\Phi_n} \frac{1}{\sqrt{2}} \left\{ \sqrt{\frac{2\bar{x}^2}{w_0^2} + 1} - \sqrt{\frac{2\bar{x}^2}{w_0^2} - 1} \right\} , \qquad \text{(A3)}$$

where n_{max} and n_{min} are the maximum and minimum values of the density, respectively at $x = \bar{x}$. Recall that w_0 is the island half-width in units of the poloidal flux function, so the radial correlation length is given by $\Delta r \equiv 2w_0/|\nabla\Phi|$. The largest value of $\tilde{n}/n_0$ occurs when $\bar{x} = w_0$, in which case (A3) is given by

$$\frac{\tilde{n}}{n_0} = \frac{\Delta r}{L_n} \frac{\sqrt{3} - 1}{2\sqrt{2}} \cong 0.26 \frac{\Delta r}{L_n} , \qquad \text{(A4)}$$

where $L_n = \Phi_n/|\nabla\Phi|$. We repeat for emphasis that this discussion holds for either density or electron temperature fluctuations. Futhermore, this relation is not restricted to describing only the model presented in this paper, but should describe density or temperature fluctuations caused by any magnetic island larger than the resistive[23] or stochastic layer [cf., Eq.(41)] thicknesses.

References

1. M. N. Rosenbluth, R. Z. Sagdeev, J. B. Taylor, and G. M. Zaslavski, Nucl . Fusion **6**, 297 (1966).
2. A. B. Rechester and M. N. Rosenbluth, Phys. Rev. Lett. **40**, 38 (1978).
3. B. B. Kadomtsev and O. Pogutse in *Plasma Physics and Controlled Nuclear Fusion Research 1984* , London (IAEA, Vienna, 1985), Vol. 2, p. 69.
4. P. H. Rebut, M. Brusati, M. Hugon, and P. Lallia in *Plasma Physics and Controlled Nuclear Fusion Research 1986*, Kyoto (IAEA, Vienna, 1987), Vol. 2, p. 187.
5. P. H. Rebut and M. Brusati, Plasma Phys. Contr. Fusion **28**, 1[illegible] (1986).
6. P. H. Rebut, P. P. Lallia, and M. L. Watkins in *Plasma Physics and Controlled Nuclear Fusion Research 1988*, Nice (IAEA, Vienna, 1989), Vol. 2, p. 191.
7. P. H. Rebut and M. Hugon in *Plasma Physics and Controlled Nuclear Fusion Research 1984*, London (IAEA, Vienna, 1985), Vol. 2, p. 197.
8. R. B. White and F. Romanelli, Phys. Fluids B **1**, 977 (1989).
9. X. Garbet, F. Mourges, and A. Samain, Plasma Phys. Contr. Fusion **32**, 917 (1990).
10. M. Hugon and P. H. Rebut in *Plasma Physics and Controlled Nuclear Fusion Research 1990*, Washington (IAEA, Vienna, 1991), in press.
11. A. Samain, Plasma Phys. Contr. Fusion **26**, 731 (1984).
12. X. Garbet, F. Mourges, and A. Samain, Plasma Phys. Contr. Fusion **30**, 343 (1988); X. Garbet, F. Mourges, and A. Samain, Plasma Phys. Contr. Fusion **32**, 131 (1990).

13. J. D. Callen, W. X. Qu, K. D. Siebert, B. A. Carreras, K. C. Shaing, and D. A. Spong in *Plasma Physics and Controlled Nuclear Fusion Research 1986*, Kyoto (IAEA, Vienna, 1987), Vol. 2, p. 157.

14. J. D. Callen and K. C. Shaing, Phys. Fluids **28**, 1845 (1985); K. C. Shaing and J. D. Callen, Phys. Fluids **28**, 1859 (1985).

15. M. C. Zarnstorff and S. C. Prager, Phys. Fluids **29**, 298 (1986).

16. M. C. Zarnstorff, M. G. Bell, M. Bitter, R. J. Goldston, B. Grek, R. J. Hawryluk, K. Hill, D. Johnson, D. McCune, H. Park, A. Ramsey, G. Taylor, and R. Wieland, Phys. Rev. Lett. **60**, 1306 (1988).

17. G. A. Navratil, R. A. Gross, M. E. Mauel, S. A. Sabbagh, M. G. Bell, R. Bell, N. Bretz, R. Budny, M. S. Chance, E. D. Fredrickson, R. Hatcher, R. J. Hawryluk, A. C. Janos, S. C. Jardin, D. L. Jassby, D. McCune, K. M. McGuire, S. S. Medley, D. Mueller, M. Okayabashi, D. K. Owens, H. K. Park, B. C. Stratton, E. J. Synakowski, G. Taylor, R. M. Wieland, M. C. Zarnstorff, J. Kesner, E. S. Marmar, J. L. Terry, P. A. Politzer, L. L. Lao, G. D. Porter, C. D. Challis, M. S. Chu, A. D. Turnbull, E. J. Strait, D. Wroblewski, K. Matsuda, and the DIII-D Team in *Plasma Physics and Controlled Nuclear Fusion Research 1990*, Washington (IAEA, Vienna, 1991), Vol. 1, p. 209.

18. M. Murakami, B. A. Carreras, L. R. Baylor, G. L. Bell, T. S. Bigelow, A. C. England, J. C. Glowienka, H. C. Howe, T. C. Jernigan, D. K. Lee, V. E. Lynch, C. H. Ma, D. A. Rasmussen, J. S. Tolliver, M. R. Wade, J. B. Wilgen, and W. R. Wing, Phys. Rev. Lett. **66**, 707 (1991).

19. T. S. Hahm, Phys. Fluids **31**, 3709 (1988).

20. R. Fitzpatrick, Phys. Fluids B **2**, 2636 (1990).

21. P. H. Rutherford, Phys. Fluids **16**, 1903 (1973).

22. R. Carrera, R. D. Hazeltine, and M. Kotschenreuther, Phys. Fluids **29**, 899 (1986); also W. X. Qu and J. D. Callen, University of Wisconsin Report UWPR 85-5, October 1985 (unpublished).

23. H. P. Furth, J. Kileen, and M. N. Rosenbluth, Phys. Fluids **6**, 459 (1963).

24. R. B. White, D. A. Monticello, M. N. Rosenbluth, and B. V. Waddell, Phys. Fluids **20**, 800 (1977).

25. D. A. Spong, B. A. Carreras, K. C. Shaing, J. D. Callen, and L. Garcia, Bull. Am. Phys. Soc. **33**, 2012 (1988).

26. O. J. Kwon, P. H. Diamond, H. Biglari, Phys. Fluids B **2**, 291 (1990).

27. J. D. Callen, Z. Chang, and J. P. Wang, *Proceedings of the 17th European Conference on Controlled Fusion and Plasma Heating* (European Physical Society, Geneva, 1990), Vol. 14B Part II, p. 853; J. P. Wang and J. D. Callen (to be published).

28. J. W. Connor, S. C. Cowley, and R. J. Hastie, Plasma Phys. Contr. Fusion **32**, 799 (1990).

29. D. Biskamp, Nucl. Fusion **19**, 777 (1979).

30. B. D. Scott, A. B. Hassam. and J. F. Drake, Phys. Fluids **28**, 275 (1985).

31. A. B. Rechester and T. H. Stix, Phys. Rev. Lett. **36**, 587 (1976).

32. F. L. Hinton and R. D. Hazeltine, Rev. Mod. Phys. **48**, 239 (1976).

33. M. Kotschenreuther, R. D. Hazeltine, and P. J. Morrison, Phys. Fluids **28**, 294 (1985).

34. J. R. Cary and M. Kotschenreuther, Phys. Fluids **28**, 1392 (1985).

35. C. C. Hegna and A. Bhattacharjee, Phys. Fluids B **1**, 392 (1989).

36. J. W. Connor, S. C. Cowley, R. J. Hastie, T. C. Hender, A. Hood, and T. J. Martin, Phys. Fluids **31**, 577 (1988); J. W. Connor, R. J. Hastie, and J. B. Taylor, Phys. Fluids B **3**, 1532 (1991).

37. B. B. Kadomtsev, Nucl. Fusion **31**, 1301 (1991).

38. J. D. Callen, Phys. Rev. Lett. **39**, 1540 (1977).

39. Y. Nakamura, M. Wakatani, C. C. Hegna, and A. Bhattacharjee, Phys. Fluids B **2**, 2528 (1990).

40. P. Pribyl, J. Liberati, B. Wells, R. Taylor, and G. Tynan, Bull. Am. Phys. Soc. **35**, 1947 (1990).

41. T. P. Crowley, Bull. Am. Phys. Soc. **35**, 2128 (1990).

42. S. F. Paul, R. J. Fonck, and D. Roberts, Bull. Am. Phys. Soc. **35**, 2091 (1990); S. F. Paul and R. J. Fonck, Rev. Sci. Instrum. **61**, 3496 (1990); R. J. Fonck, P. A. Dupperex, and S. F. Paul, Rev. Sci. Instrum **61**, 3487 (1990).

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

END

DATE FILMED

12/05/91