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Probability of Detection for Cooperative Sensor Systems

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ABSTRACT

In this paper, we quantify how communication increases the effective range of detection of unattended ground sensors. Statistical analysis is used to evaluate the probability of detection for multiple sensors using one, two, and infinite levels of cooperation. Levels of cooperation are defined as the levels of communication between sensors. One level of cooperation means that one sensor passes its state information to several other sensors within a limited communication range, but this information is not passed beyond this range. Two levels of cooperation means that the state information received by this first set of sensors is relayed to another set of sensors within their communication range. Infinite levels of cooperation means that the state information is further percolated out to all sensors within a communicating group. With large numbers of sensors, every sensor will have state information about every other sensor regardless of communication range. With smaller numbers of sensors, isolated groups may form, thus lowering the probability of information transfer.

Keywords: cooperative systems, unattended ground sensors, sensor networks

1. INTRODUCTION

The trend in unattended ground sensors is towards smaller, more intelligent, distributed, self-organizing sensor networks [1]. These cooperative sensor networks rely on communication and distributed signal processing to detect, classify, and locate stationary and moving targets. Instead of all sensors communicating long distances to a centralized sensor fusion processor, the sensor data processing is distributed amongst the sensors, and the communication is localized to the surrounding sensor/processor nodes. Since there is no single point of failure, this distributed network architecture improves fault tolerance in the system. However, the trade-off is that the communication between sensor nodes is more complex. In this paper, we use statistical analysis to address the range of communication design issue for a distributed sensor network.

Recently researchers have been developing distributed signal processing techniques to fuse complementary multi-sensor data from a variety of sources. These techniques are often statistical in nature and can be grouped into either distributed detection or distributed estimation techniques [2-3]. Statistical hypothesis tests using a Neyman-Pearson or Bayesian formulation are examples of distributed detection techniques, while Kalman filtering and information filtering [4-5] are examples of distributed estimation techniques. These techniques can be further categorized by their optimization methods and network topology.

Optimal estimation and selection of sensors are often system design issues that need to be addressed before the system can be fielded. For example, Kalman filtering can be used to give optimal estimates for the linear case, while extended Kalman filtering can be used to give optimal estimates for the well-behaved non-linear case. When performing hypothesis tests, an optimal combination of sensors will minimize the multi-dimensional total probability of error between classes [6]. Genetic algorithms have also been used to determine quasi-optimal sensor combinations where the selection of sensors is based on minimizing a figure-of-merit function [1].

Another system design issue is the topology of the communication network. The topology between the sensor and fusion nodes dictates how these traditionally centralized sensor fusion techniques are distributed. Parallel, serial, and hierarchical networks, with and without feedback, are a few of the different topologies that have been investigated. Often, conditional

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independence of sensor observations is required for these distributed techniques to be valid. This independence may again affect the selection of sensors that are in close proximity to each other.

Regardless of the above mentioned optimal and topological design issues, an interesting observation is that all sensor and fusion nodes in a network typically form a single connected graph. This is independent of the types of sensors and data fusion algorithm. For this reason, the first question to ask is how do we design a self-organizing cooperative sensor network such that all sensors are capable of communicating with one another either directly or indirectly via another node? In particular, we are interested in a surveillance task were it is desirable for all sensors to share information about a detected target. Regardless of the type of sensor or the sensor fusion process, we would like to know the probability that a particular sensor either detects the target or has been informed by another sensor of the target's presence. Most importantly, we would like to be able to design the surveillance task so that we can statistically guarantee that a specified percentage of sensors form a single network around the target. This design problem brings to bear several important questions.

1. How large must the sensing range be?
2. How large must the communication range between sensors be?
3. How many sensors are needed to cover the area where the target is expected to be?

These questions are answered in this paper. The following four sections analyze the probability of detection with zero, one, two, and infinite levels of communication between randomly distributed unattended ground sensors.

2. SINGLE SENSOR WITH NO COOPERATION

Suppose that we want to locate a single target in a circular region of radius G with a sensor with a limited sensing range of radius $0<R<G$ (see Figure 1). The space L will denote the certain event, its elements are the experimental outcomes, and its subsets are events [7]. Let x and y be random variables whose domain is the set L of experimental outcomes. These random variables can also be expressed in polar coordinates r and ϕ.

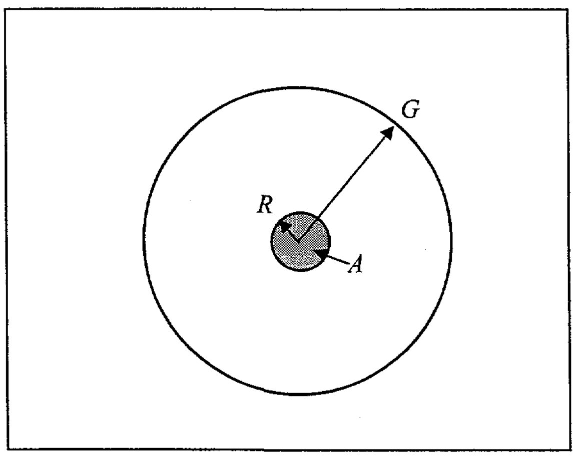

Figure 1. The probability distribution of the target is uniform over the area of certain event. A single sensor with detection range R tries to detect the target.

Assuming a uniform distribution, the probability density function of the target being at a particular r, ϕ position is

$$f(r,\phi) = \begin{cases} \dfrac{1}{\pi G^2} & if \quad 0 \le r \le G, \quad 0 \le \phi \le 2\pi \\ 0 & else \end{cases} \tag{1}$$

For convenience, assume that a sensor is located at the origin. Let A be the set of events ξ such that a sensor detects the target.

$$A = \{\xi : 0 \le r(\xi) \le R\} \tag{2}$$

Assume that the sensor has a probability of detection of one within its limited sensing range R. Then the conditional probability of detecting the target is

$$P(A|r = r', \phi = \phi') = \begin{cases} 1 & \text{if} \quad 0 \le r' \le R, \quad 0 \le \phi' \le 2\pi \\ 0 & \text{else} \end{cases} \tag{3}$$

The total probability of the sensor detecting the target within the region of certain event is

$$\begin{aligned} P(A) &= \int_0^{2\pi}\int_0^{\infty} P(A|r = r', \phi = \phi') f(r', \phi') r' dr' d\phi' \\ &= \int_0^{2\pi}\int_0^{R} \frac{1}{\pi G^2} r' dr' d\phi' \\ &= \frac{\pi R^2}{\pi G^2} \end{aligned} \tag{4}$$

In this case, the probability of a single sensor locating the target is the sensing area divided by the area of certain event. This result is intuitively obvious, and it holds true for a single sensor placed anywhere within the region $0 \le r \le G - R$.

3. MULTIPLE SENSORS WITH ONE LEVEL OF COOPERATION

If N sensors are randomly placed in the area $0 \le r \le G - R$, the probability that all sensors will locate the target is given by

$$P(A_{all}) = (P(A))^N = \left(\frac{R^2}{G^2}\right)^N \tag{5}$$

since each set A is independent of each other. The resulting value is a very small number for large N since the probability of all sensors being within the sensing range of the target is slim.

On the other hand, the probability that at least one sensor detects the target is

$$P(A_{least1}) = 1 - [1 - P(A)]^N = 1 - \left(\frac{G^2 - R^2}{G^2}\right)^N \tag{6}$$

This probability is much larger since only one of the sensors must sense the target.

What is missing from the above expressions is any sort of cooperation. One sensor may have found the target, but no other sensor knows about it. If multi-sensor fusion techniques are to be applied, multiple sensors must know the state of surrounding sensors. Therefore, it is important to not only find the target, but also to relay this information to other members so that multi-sensor fusion techniques can be applied.

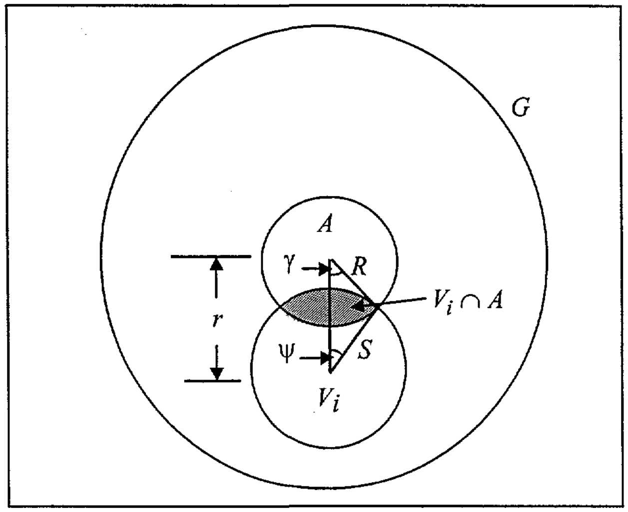

Figure 2. The target is assumed to be at the origin. The probability that sensor i learns about the target from another sensor inside the circle of radius R is a function of the overlapping area of communication.

Consider two sensors as shown in Figure 2. Let V_i be the set of sensors which detect sensor *i*. Assume that the communication range has radius $S>R$ where $R+2S<G$. Also assume that the probability of communicating with a sensor is uniform throughout the communication range. Then the conditional probability of communicating with sensor *i* is

$$P(V_i \mid x = x'-x_i, y = y'-y_i) = \begin{cases} 1 & if\,(x'-x_i)^2 + (y'-y_i)^2 \le S^2 \\ 0 & else \end{cases} \tag{7}$$

In this expression, a more realistic model would take into account free-space electromagnetic wave propagation including multi-pathing. This is beyond the scope of this initial analysis.

The set of sensors that detect the target and can communicate with sensor *i* is $V_i \cap A$, and the conditional probability of single sensor which detects a target and sensor *i* is

$$P(V_i \cap A \mid x = x_i, y = y_i) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(V_i \mid x = x'-x_i, y = y'-y_i)P(A \mid x = x', y = y')f(x', y')dy'\,dx'$$
$$= \begin{cases} \dfrac{\pi R^2}{\pi G^2} & if\,0 \le r_i \le S-R \\ \dfrac{Q_1}{\pi G^2} & if\,S-R \le r_i \le S \\ \dfrac{Q_2}{\pi G^2} & if\,S \le r_i \le R+S \\ 0 & else \end{cases} \tag{8}$$

where

$$Q_1 = S^2(\psi - 0.5\sin 2\psi) + R^2(\pi - \gamma + 0.5\sin 2\gamma) \tag{9}$$

$$Q_2 = \psi S^2 + \gamma R^2 - rS\sin\psi \tag{10}$$

$$\psi = \cos^{-1}\left(\frac{r^2 + S^2 - R^2}{2rS}\right) \tag{11}$$

$$\gamma = \sin^{-1}\left(\frac{S\sin\psi}{R}\right) \tag{12}$$

The variables Q_1 and Q_2 are the overlapping areas of communication between sets A and V_i (see Figure 2).

Let V be the set of sensors that detect *any* of the N-1 sensors. The total probability of any sensor that detects the target and sensor i is

$$P(V \cap A) = 1 - \left[1 - P(V_i \cap A)\right]^{N-1} \tag{13}$$

since $\overline{V \cap A} = \bigcap_{i=1}^{N-1} \overline{(V_i \cap A)}$ where bar denotes a negation.

If B is the set of sensors which did not detect the target, but did find another sensor that detected the target, then

$$B = \overline{A} \cap (V \cap A) \ . \tag{14}$$

This means that the inner circle of radius R is excluded. Therefore,

$$P(B \mid r = r', \phi = \phi') = \begin{cases} 1 - \left(1 - \dfrac{\pi R^2}{\pi G^2}\right)^{N-1} & \text{if } R \le r_i \le S - R \\ 1 - \left(1 - \dfrac{Q_1}{\pi G^2}\right)^{N-1} & \text{if } S - R \le r_i \le S \\ 1 - \left(1 - \dfrac{Q_2}{\pi G^2}\right)^{N-1} & \text{if } S \le r_i \le R + S \\ 0 & \text{else} \end{cases} \tag{15}$$

The total probability of a sensor not detecting the target, but communicating with a sensor that did detect the target is

$$\begin{aligned} P(B) &= \int_0^{2\pi}\int_0^{\infty} P(B \mid r = r', \phi = \phi') f(r', \phi') r'\, dr'\, d\phi' \\ &= \frac{1}{G^2}\left(1 - \left(1 - \frac{\pi R^2}{\pi G^2}\right)\right)\left((S-R)^2 - R^2\right) + \frac{2}{G^2}\int_{S-R}^{S}\left(1 - \left(1 - \frac{Q_1}{\pi G^2}\right)\right) r'\, dr' + \frac{2}{G^2}\int_{S}^{S+R}\left(1 - \left(1 - \frac{Q_2}{\pi G^2}\right)\right) r'\, dr' \end{aligned} \tag{16}$$

Figure 3 shows how one level of cooperation can increase the probability that a sensor is alerted of a target. The curves show how the probability of detection increases as the number of sensors (the y-axis) increases and as the range of detection and communication (the three curves) increase. For simplicity, $R=S$ in this figure. The left most side of each curve is the probability of detection with no communication as discussed in the previous section. One level of cooperation increases the effective range of detection to $R+S$ as the number of sensors approaches infinity. Notice that the top curve shows that 210 randomly distributed sensors would be required for 90% of the sensors to detect the target either through direct sensing or one level of communication.

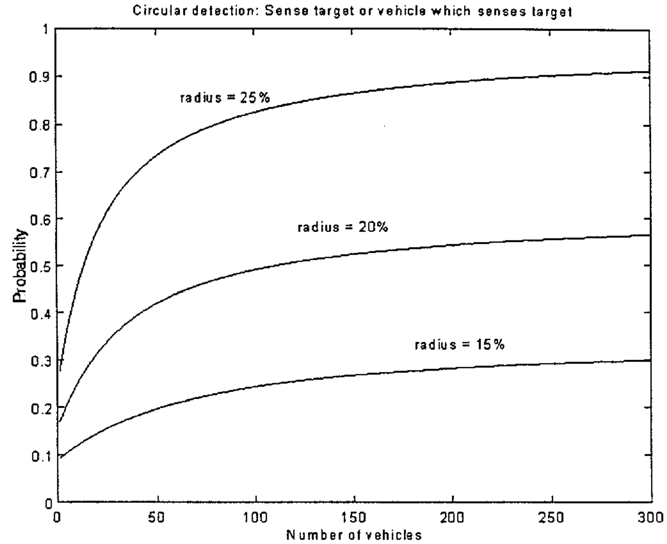

Figure 3. Single level of cooperation. Sensor senses a target or a sensor that senses a target. Communication and sensor range are equal.

4. MULTIPLE SENSORS WITH TWO LEVELS OF COOPERATION

This section extends the analysis to another level of cooperation. Let C be the set of sensors which are not in A and not in B, but communicate with a sensor which communicates with a sensor which detects the target. Let C_1 be the set of sensors in C where $R \le r \le R + S$, and C_2 be the set of sensors in C where $R + S \le r \le R + 2S$. Sets C_1 and C_2 are mutually exclusive; therefore, we can add their total probabilities.

For $R \le r \le R + S$ (see Figure 4),

$$P(V_i \cap B \mid r = r, \phi = \phi) = \frac{2}{\pi G^2} \int_0^{\beta} \int_{R_{\min}}^{R_{\max}} P(B \mid r = r', \phi = \phi')r'\, dr'\, d\phi' \tag{17}$$

where

$$R_{\min} = \min\left[R, r\cos\phi' - \sqrt{S^2 - r^2\left(1 - \cos^2\phi'\right)}\right] \tag{18}$$

$$R_{\max} = \max\left[R + S, r\cos\phi' + \sqrt{S^2 - r^2\left(1 - \cos^2\phi'\right)}\right] \tag{19}$$

$$\beta = \cos^{-1}\left(\frac{S}{2r}\right) \tag{20}$$

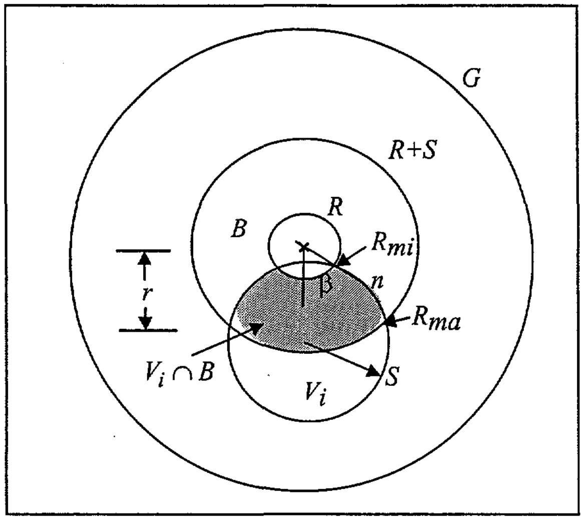

Figure 4. Sensor i is between $R \le r \le R + S$. Gray region is the area where sensor i can communicate with another sensor which does not detect the target, but communicates with another sensor that does.

Since $C_1 = \overline{B} \cap \overline{\left(\bigcap_{i=1}^{N-2} \overline{(V_i \cap B)}\right)}$,

$$P(C_1 \mid r = r, \phi = \phi) = [1 - P(B \mid r = r, \phi = \phi)][1 - (1 - P(V_i \cap B \mid r = r, \phi = \phi))^{N-2}] \quad (21)$$

and the total probability is

$$P(C_1) = \frac{2\pi}{\pi G^2} \int_R^{R+S} P(C_1 | r = r, \phi = \phi) r dr \quad (22)$$

For $R + S \le r \le R + 2S$ (see Figure 5),

$$P(V_i \cap B \mid r = r, \phi = \phi) = \frac{2}{\pi G^2} \int_0^{\beta} \int_{R_{\min}}^{R+S} P(B \mid r = r', \phi = \phi') r' dr' d\phi' \quad (23)$$

where

$$R_{\min} = r \cos\phi' - \sqrt{(R+S)^2 + r^2(1 - \cos^2 \phi')} \quad (24)$$

$$\beta = \cos^{-1}\left(\frac{(S+R)^2 + r^2 - S^2}{2(S+R)r}\right) \quad (25)$$

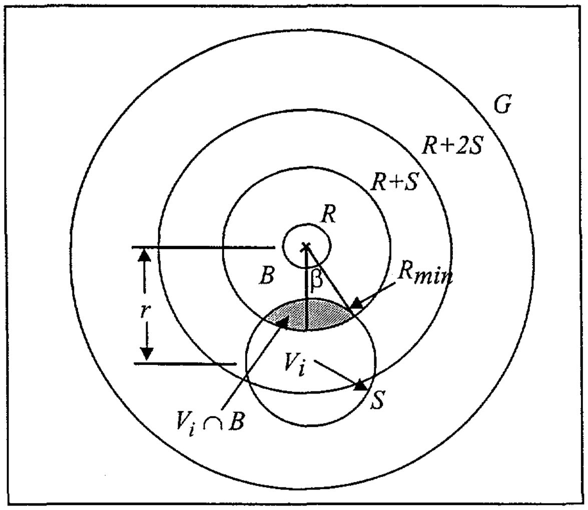

Figure 5. Sensor i is between $R+S \le r \le R+2S$*. Gray region is the area where sensor i can communicate with another sensor which does not detect the target, but communicates with another sensor that does.*

Since $C_2 = \overline{\left(\bigcap_{i=1}^{N-2} \overline{(V_i \cap B)} \right)}$,

$$P(C_2 | r = r, \phi = \phi) = \left[1 - \left(1 - P(V_i \cap B | r = r, \phi = \phi)\right)^{N-2} \right] \tag{26}$$

and the total probability is

$$P(C_2) = \frac{2\pi}{\pi G^2} \int_{R+S}^{R+2S} P(C_2 | r = r, \phi = \phi) r dr \tag{27}$$

Finally, since C_1 and C_2 are mutually exclusive,

$$P(C) = P(C_1) + P(C_2) \tag{28}$$

Figure 6 shows how two level of cooperation can increase the probability that a sensor is alerted of a target. Again, the curves show how the probability of detection increases as the number of sensors (the y-axis) increases and as the range of detection and communication (the three curves) increase. Comparing these results with those in Figure 3, we see that only 24 sensors are needed to achieve a 90 percent probability of detection with two levels of cooperation, as opposed to 210 sensors with one level of cooperation. Also, two levels of cooperation increase the effective range of detection to $R+2S$ as the number of sensors approaches infinity.

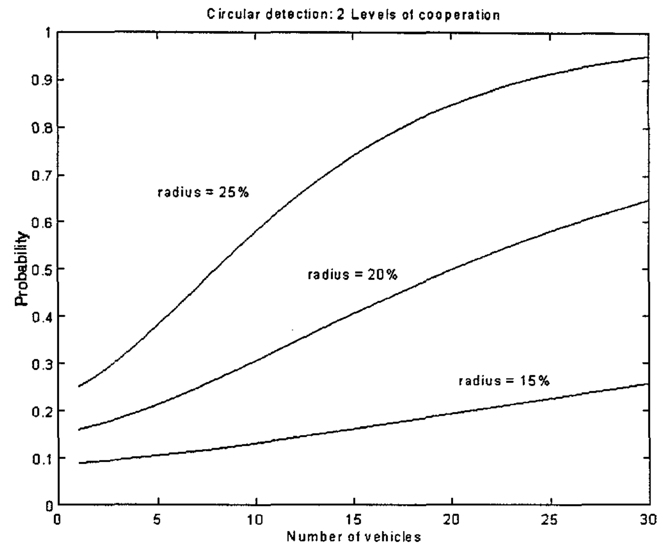

Figure 6. Two levels of cooperation. Sensor detects a target, or communicates with a sensor that detects a target, or communicates with a sensor that communicates with a sensor that detects a target. Communication and sensor range are equal.

5. MULTIPLE SENSORS WITH INFINITE COOPERATION

It is possible to extend the above analysis for over two levels of cooperation; however, the computational intensity of the numerical integration becomes enormous. Therefore, in this section, we take a different approach. Instead of determining the probability that all sensors communicate with one another, we determine the probability that sensors are isolated from one another.

With infinite cooperation, we are interested in the case where every sensor passes on the information it receives to every other sensor within range. If a target is detected by one sensor, all other sensors will know about it unless there are islands of isolated sensors that are separated from the others.

Let A_d be the area of detection/communication, assuming they are equal. Let A_c be the area of certain event (area of probability 1 detection). The probability of detecting a target is $P_d = \frac{A_d}{A_c}$. The probability of not detecting a target is $1 - P_d$. Given N randomly placed sensors, the number of sensors that detect the target is NP_d. The number of sensors that do not detect the target is $N(1 - P_d)$. Outside of the detection region A_d, the probability of a single sensor not communicating with any of the other N-1 sensors is $(1 - P_d)^{N-1}$. The probability of communicating with at least one other sensor is

$$P_1 = \left(1 - (1 - P_d)^{N-1}\right). \tag{29}$$

Therefore, the probability of detecting the target or communicating with at least one other sensor is

$$P_{T1} = P_d + (1 - P_d)\left(1 - (1 - P_d)^{N-1}\right) \tag{30}$$

Note that this expression goes to one as N goes to infinity. While this expression takes into account single isolated sensors, it does not take into account two or more sensors that are isolated. Therefore, the probability of infinite detection is less than the above expression by the probability of isolation for two or more sensors.

The probability that two sensors communicate with each other but do not communicate with any other sensor is

$$P_2 = \left[1 - (1 - P_d)^{N-1}\right](P_d)(1 - \alpha P_d)^{N-2} \tag{31}$$

where $\alpha \geq 1$ represents the increase in area spanned by two sensors which communicate with each other. If $\alpha = 1$, the two sensors are on top of each other. The variable α should be a distribution, but for simplicity, we consider α to be a single scalar value that is the average of the distribution. The first term in brackets is the probability of communicating with at least one other sensor. The other two terms are the probability that the two sensors do not communicate with a third sensor.

The total probability in Equation (30) is thus reduced by the probability in Equation (31).

$$\begin{aligned} P_{T2} &= P_d + (1 - P_d)\left\{\left[1 - (1 - P_d)^{N-1}\right] - P_2\right\} \\ &= P_d + (1 - P_d)\left[1 - (1 - P_d)^{N-1}\right]\left[1 - (P_d)(1 - P_d)^{N-2}\right] \end{aligned} \tag{32}$$

Extending this analysis to 3 sensors, the probability that 3 sensors communicate with each other but do not communicate with any other sensors is

$$P_3 = \left[1 - (1 - P_d)^{N-1}\right]\left[1 - (P_d)(1 - \alpha P_d)^{N-2}\right](P_d)^2(1 - (2\alpha - 1)P_d)^{N-3} \tag{33}$$

The first term in brackets is the probability of communicating with at least one other sensor. The second term in brackets is the probability of communicating with a second sensor. The last two terms are the probability that the three sensors do not communicate with a fourth sensor.
Subtracting Equation (33) from Equation (32), the total probability is

$$P_{T3} = P_d + (1 - P_d)\left[1 - (1 - P_d)^{N-1}\right]\left[1 - (P_d)(1 - P_d)^{N-2}\right]\left[1 - (P_d)^2(1 - (2\alpha - 1)P_d)^{N-3}\right] \tag{34}$$

Continuing for *N-1* sensors, the lower bound ($\alpha = 1$) on the total probability for infinite cooperation becomes

$$P_{TN-2} = P_d + (1 - P_d)\prod_{n=0}^{\left\lfloor \frac{N-2}{2} \right\rfloor}\left[1 - (P_d)^n(1 - P_d)^{N-n-1}\right] \tag{35}$$

Since $\alpha = 1$, this expression assumes that isolated sensors will be coincident with each other. Since in reality $1 < \alpha < 1.5$, the total probability for infinite cooperation is larger.

Figure 7 illustrates how infinite levels of cooperation can increase the probability that a sensor is alerted of a target. Comparing these results with those in Figure 6, we see that only 9 sensors are needed to achieve a 90 percent probability of detection with infinite levels of cooperation, as opposed to 24 sensors with two levels of cooperation. Also notice that regardless of the size of the sensing/communication range, the probability of detection always goes to one as the number of sensors goes to infinity.

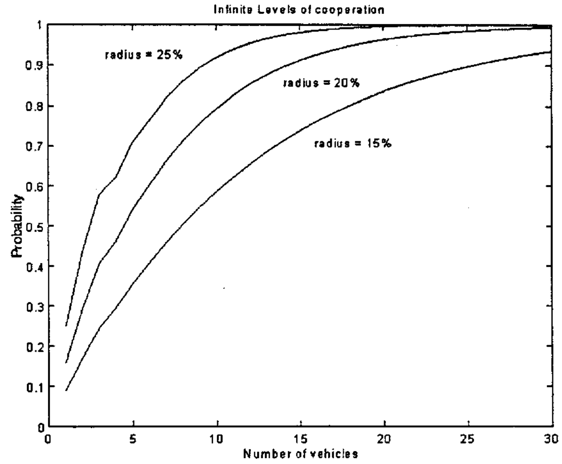

Figure 7. Lower bound on infinite levels of cooperation. Sensor either detects a target or communicates with any other sensor, and is not a group of one or more isolated sensors.

6. CONCLUSION

In this paper, we have used statistical analysis to determine the probability of detection of distributed unattended ground sensors with limited sensing and communication range. The analysis assumes that the probability distribution of the sensors and the target are uniform. Without cooperation, the probability that a sensor detects the target is the sensing area over the area of certain event. With one level of cooperation (detect the target or communicate with another sensor which detects a target), the probability of detection increases with the number of sensors. Also, the effective detection range doubles as the number of sensors go to infinity. With two levels of cooperation (detect the target or communicate with another sensor which detects a target or communicate with another sensor which communicates with another which detects the target), the probability of detection increases even faster. The detection range triples as the number of sensors go to infinity. With infinite levels of cooperation and infinite numbers of sensors, the effective detection range covers the area of certain event regardless of the actual sensing radius. In the future, this analysis will be used to determine the communication and sensing range and the number of sensors necessary to perform a variety of missions.

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