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TITLE DOUBLE-MODE PULSATION

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DOUBLE-MODE PULSATION

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Double mode pulsation is a very pervasive phenomenon in stars all over the Hertzsprung-Russell diagram. In order of increasing radius, examples are: ZZ Ceti stars, the sun, the δ Scuti stars, RR Lyrae variables, the β Cephei variables and those related to them, Cepheids, and maybe even the Mira stars. These many modes have been interpreted as both radial and nonradial modes, but in many cases the actual mode has not been clearly identified. Yellow giants seem to be the most simple pulsators with a large majority of the RR Lyrae variables and Cepheids showing only one pulsation period. We limit this review to those very few cases for classical Cepheids and RR Lyrae variables which display two modes. For these we know many facts about these stars, but the actual cause of the pulsation in two modes simultaneously remains unknown.

Table 1 lists the eleven double-mode Cepheids known at this time. They are all in our galaxy; none have ever been found in even the nearby Magellanic clouds. Intensive searches by Szabados (1977), Pike and Andrews (1979), Henden (1979, 1980) in the northern hemisphere and by Barrell (1982) in the southern hemisphere have not been able to add to this list. It is true that there are strange cases such as HD 161796 recently discussed by Fernie (preprint) where this star seems to show for a time the fundamental radial mode and then later the pure overtone mode. The Table 1 stars are in a well defined class for which we hope to produce complete explanations in the near future.

We see that the double-mode Cepheids all have periods between 2.1 and 6.3 days. With the exception of AX Vel, they all have a larger amplitude for their fundamental mode than for their overtone component. There seems to be no dispute that the two modes are correctly identified, mostly because the period ratio is very close to that expected for these two periods. It is, however, the period ratio that is the cause of most of the puzzle in the stars. All these ratios range from 0.6967 to 0.7105 over a period range of almost a factor of three in the fundamental.

In an extensive program to observe the ten southern hemisphere double-mode Cepheids, Barrell has supplied the mean colors and derived the mean effective temperatures listed in the next to the last column. These are based on Hα data. In the last column, radii, to be discussed later from Balona and Stobie (1979) and Niva and Schmidt (1979) (for TU Cas) are given.

One important interest in the double-mode Cepheids is their number. Stobie (1977) has pointed out that even though there are only a few of these stars known, they constitute a very large fraction of the short period Cepheids. He, and now Barrell in a preprint sent to me to prepare for this review, included in their discussion both type I and type II Cepheids. I feel this is not the correct thing to do. The type II Cepheids are completely different stars, with masses appropriate for population II--about 0.6 solar masses. On the other hand the classical Cepheids, according to

TABLE 1

DOUBLE-MODE CEPHEIDS

Star	P_0 (days)	P_1/P_0	ΔV_0 (mag)	ΔV_1 (mag)	log T_e (Barrell)	$R/R_\odot$
TU Cas	2.1393	0.7097	0.66	0.31	3.804	22.0
U Tr A	2.5684	.7105	.47	.25	3.775	20.2
VX Pup	3.0109	.7092	.46	.33	3.775	47.5
AP Vel	3.1278	.7031	.55	.41	3.770	47.1
BK Cen	3.1752	.7047	.52	.20	3.772	53.7
UZ Cen	3.3343	.7063	.30		3.777	35.8
Y Car	3.6398	.7031	.58	.29	3.771	42.2
AX Vel	3.6731	.7059	.22	.33	3.776	52.9
GZ Car	4.1590	.7052	.16		3.782	46.6
BQ Ser	4.2707	.7053			3.775	
V 367 Sct	6.2931	0.6967	0.5	0.2	3.775	

evolutionary theory have masses like 5, 6, or at most 7 solar masses. In my revision of the Stobie and Barrell data, I get the fractions of Cepheids which are double-mode in the following period ranges as: 1-2 days, 0; 2-3 days, 0.40; 3-4 days, 0.23; 4-5 days, 0.05; 5-6 days, 0.00; and 6-7 days, 0.03. We see that in the period range 2-4 days the double-mode Cepheids comprise about one-third of all the known Cepheids. This may not be too surprising if the pulsation instability strip in the Hertzspring-Russell diagram is very narrow at this low end, and this strip includes the double-mode phenomenon.

Masses of these double-mode Cepheids can be determined by five different methods. Stellar evolution calculations show that the evolution tracks in the Hertsprung-Russell diagram that are in the observed period range correspond to 5-7 solar masses. Becker, Iben, and Tuggle (1977) have given a formula for the stellar mass given the luminosity. Some workers have then used a luminosity from the period-luminosity relation to derive masses. This method does not give the wrong mass, but it is not particularly a wise thing to do because the period-luminosity relation is in itself based on intricate calibrations of the distance scale. We here consider that only for V367 Sct, in the galactic cluster NGC 6649, do we know its luminosity in a reasonably direct way, and therefore we can obtain a mass.

Cox (1980) shows that a theoretical mass can be derived using only the well known period and an even approximate T_e value. The Barrell data allow us to get a very accurate mass because she has given accurate T_e values. This theoretical mass results from using four equations discussed by Cox (1979) for the four unknowns: the radius, the luminosity, the pulsation constant Q, and the theoretical mass. Since the theoretical stellar evolution mass-luminosity relation is one of the four equations, and since it has a strong effect, the theoretical masses are usually very close to the evolution masses.

The well-known pulsation mass can be found for those cases where the luminosity and the effective surface temperature is known. For our double-mode Cepheids again only V367 Sct qualifies for this mass determination.

The beat or double-mode mass discussed first by Petersen (1973) is based on only the observed period and period ratio. In the temperature range which can be very wide, there is a unique curve for a given mass and composition in the Π_1/Π_0 versus Π_0 (Petersen) diagram. Thus, the use of the period data gives a mass called the beat mass. It is this mass which is very low compared to the other masses.

The last type of mass we discuss here uses the Wesselink radius determined by Balona and Stobie and by Niva and Schmidt. As is typical, the Wesselink mass is not very accurate, because the mass from the period mean density relation goes as the cube of the rather uncertain radius.

These five kinds of mass are listed in Table 2. The most reliable ones are the theoretical masses, of course assuming that there is no major error in the pulsation theory constant Q or in the evolution theory mass-luminosity relation. Actually, recent calculations by Matraka, Wassermann, and Weigert (1982) and by Becker and Cox (1982) show that the masses derived from the Becker, Iben, and Tuggle mass-luminosity formula are a bit large, about 15-20 percent. For the theoretical masses we also give the corresponding Q values in days. The beat masses from a Petersen diagram are the one given next. Finally, the rather uncertain Wesselink masses and the Q value are listed. The conclusion at this point is that there seems to remain the long standing beat Cepheid mass anomaly.

TABLE 2

DOUBLE-MODE CEPHEID MASSES

Star	M_{EV}	M_T	Q_T	M_Q	M_B	M_W	Q_W
TU Cas	-	4.8	0.0366	-	1.4	3.2	0.0374
U Tr A	-	4.8	0.0371	-	1.6	2.0	0.0397
VX Pup	-	5.1	0.0375	-	1.8	17	0.0378
AP Vel	-	5.1	0.0376	-	1.6	15	0.0376
BK Cen	-	5.1	0.0376	-	1.7	24	0.0393
UZ Cen	-	5.3	0.0377	-	1.7	5.8	0.0376
Y Car	-	5.4	0.0379	-	1.8	7.9	0.0374
AX Vel	-	5.5	0.0380	-	2.0	16	0.0379
GZ Car	-	5.9	0.0382	-	2.0	8.3	0.0378
BQ Ser	-	5.8	0.0383	-	2.0	-	
V 367 Sct	6.9	6.9	0.0394	5.0	2.3	-	-

The radii used for the Wesselink masses in Table 2 are given in Table 1. One can see that there is considerable uncertainty in some of these values because the masses obtained when using them are very anomolous. Theoretical masses given in Table 2 indicate radii that range from 25 to 56 solar radii for, respectively, TU Cas and V367 Sct. At least there is no major discrepancy between the theoretical and Wesselink radii as there might be if the masses were only one-third of the theoretical masses.

Combining luminosities from the Sandage and Tammann (1969) period luminosity relation with the newly observed surface effective temperatures, Barrell has produced the Hertzsprung-Russell diagram given as Figure 1. The most surprising thing about this diagram is that the double-mode Cepheids are all at an almost unique surface effective temperature. It is very suggestive that the double-mode phenomenon is related to the transition betweeı e fundamental and first overtone pulsation modes. Unfortunately, we still ›nnot produce nonlinear calculations which show this pulsation in both modes imultaneously at least over long periods of time.

Figure 1 is actually the theoretical Hertzsprung-Russell diagram of King, Cox, Eilers, and Davey (1973) and the observational one reported King, Hansen, Ross, and Cox (1973). The cross-hatched region is probably not relevant to our current discussion. There is indeed a discrepancy between the observed double-mode Cepheids and the fundamental blue edge both from the observational and the theoretical viewpoints. Well established Cepheids occur at bluer positions in this diagram, for example, SU Cas, EV Sct, and CV Mon, and they seem to be in the fundamental pulsation mode. The distance from the F blue edge and the existance of bluer fundamental mode pulsators apparently precludes for these Cepheids the explanation that there is some mode switching. The Pel and Lub (1978) data, however, seem to support the concept of mode switching since there few cases are even bluer.

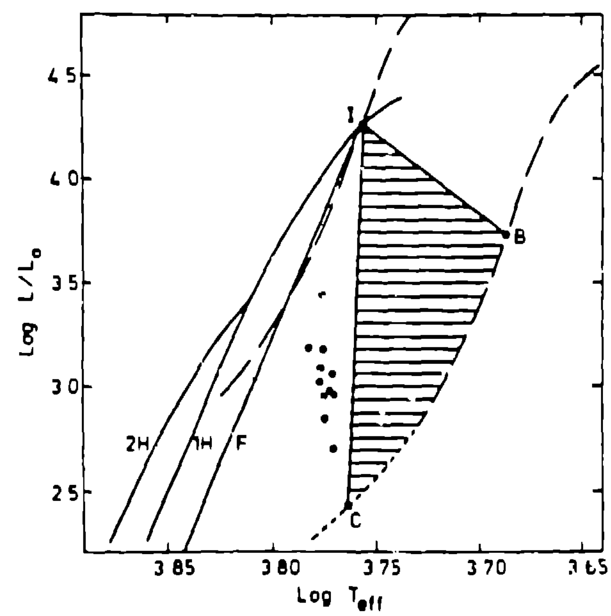

Figure 5. The theoretical fundamental (F), first overtone (1H) and second overtone (2H) blue edges of KCED (solid lines) are shown in the T–L plane together with the empirical Instability Strip of KHRC (long-dashed lines and dot-shading). The approximate extent of the predicted complex-mode behaviour region is shown by the cross hatched area – the empirical red edge of the Strip was extrapolated to define point C. The beat Cepheids are shown by dots, the open circle represents TU Cas.

Fig. 1. The Barrell Hertzsprung-Russell diagram in the region of the double-mode Cepheids.

One more thing that Barrell has done is to try to obtain composition data for the double-mode Cepheids. By looking at the iron lines in the spectra, it appears that the log of the iron abundance relative to the sun is -0.21 with an error of 0.33. This means that the iron abundance is about 0.62 solar with a factor of two possible error.

For the double-mode Cepheids, all seems to indicate that the stars are normal blue-looping yellow giants except their period ratios. Four ideas to reconcile these low period ratios--0.70 rather than the expected 0.74-- have been proposed. Cox, Deupree, King, and Hodson (1979) have suggested that a surface layer enhanced in helium would change the structure of the outer layers to appear less concentrated. This would increase the periods of all modes with the fundamental mode being increased the most. Thus, the fundamental to overtone period ratio would be decreased as required to accord with the observations. The enhancement in helium needed is up to a mass fraction of 0.65 over the outer envelope down to a temperature of 250,000K. This is about 10^{-3} of the mass of the Cepheid.

Stothers (1979) has suggested that the period ratios of these double-mode Cepheids could be reduced to near the observed level if there is a rather strong magnetic field in the surface layers. This field would produce a pressure comparable to that from the gas. This magnetic field would be tangled by the convection and therefore not be too easily observed as a uniform field on the surface. The weak fields seen in some Cepheids gives support to this idea even though the observed fields seem to be much smaller than required to reduce the period ratios.

Recently Simon (preprint) has proposed that the cause of the large period ratios in the double-mode Cepheids is an incorrect opacity for the elements heavier than helium. An opacity increase by a factor of two in the temperature range between about 100,000K and 1,000,000K changes the structure enough so that the observed low period ratios are predicted. In spite of this proposed reasonable solution to the double-mode mass problem, there does not seem to be any reason for the opacities to be this wrong. If there is a problem, however, it would be in the elements C, N, O, and Ne. One nice feature of this idea is that it would also change the period range for bumps to occur in the longer period Cepheids to that observed, that is 7-11 days. Another good feature is that the RR Lyrae variables seem to have the correct beat masses, as we will see later. If the opacity of elements heavier than hydrogen and helium are the problem, it would have a very small effect on the low Z population II RR Lyrae stars.

One problem with the magnetic field and increased opacity ideas is the observation that the period ratios are almost constant over the entire period range of the double-mode Cepheids. The enhanced helium model can predict the correct zero slope in the Petersen diagram, but the results to date for these two more recent ideas show the normal negative slope, though the period ratios are at least in the proper range. It is conceivable that this problem can be rectified however, for both the magnetic field and opacity increase only .

Cox (1980) at the last Goddard Meeting suggested that there might be an admixture of nonradial modes which are not recognized but able to distort our ideas about the pulsation modes seen. There is some support to this idea because there are some unstable nonradial modes with high ℓ values that are known. Just exactly how this interaction might occur is to date unknown.

We now turn to the other class of variable star that shows simple double-mode behavior, the RR Lyrae variables. A field star AQ Leo was discovered to have two modes by Jerzykiewicz and Wenzel (1977). Cox, King, and Hodson (1980) found that from pulsation theory the mass was 0.65 solar mass. In a recent preprint, Jerzykiewicz, Schult, and Wenzel have now shown that this star has a color that places it between the fundamental and overtone pulsators in the Hertzsprung-Russell diagram. This indicates that the star is in a stage of evolution where it is switching from the fundamental to the overtone or vice versa. The theoretical switching timescale of about 150 years seems too long for the observations to show now, but over the last 20 years it seems that if anything there has been a switch from the overtone to the fundamental.

Sandage, Katem, and Sandage (1981) have given photographic photometry data for many stars in the RR Lyrae instability strip of the globular cluster M15. They note that the Bailey c type variables with periods between 0.38 and 0.43 days show erratic behavior which might be attributed to double-mode pulsation. Cox, Hodson, and Clancy (in press) have studied these stars and find ten to be indeed pulsating in both the fundamental and overtone simultaneously with a period ratio of 0.746 within 0.001. They then use a Petersen type diagram of period ratio versus fundamental mode period to obtain a mass of 0.65 solar mass. These ten double-mode RR Lyrae variables are listed in Table 3. The best case, V31, is shown in Figure 2 where both modes are plotted after prewhitening with the other mode, its harmonics, and any mode interaction terms.

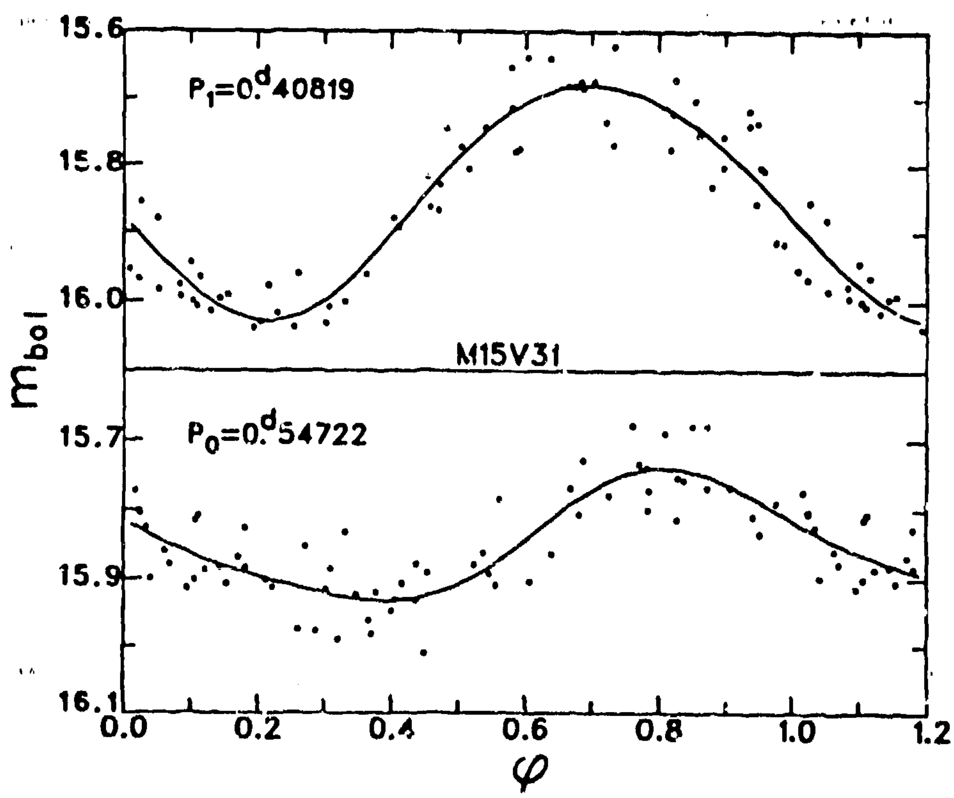

Fig. 2. The two pulsation modes of the RR Lyrae variable V31 in M15 are displayed. Each mode has been isolated from the double-mode behavior by prewhitening by the other mode and any mode iteration terms.

It is of great interest to know if double-mode RR Lyrae variables can be found in other globular clusters, especially for one in the Oosterhoff group I with shorter period RR Lyrae variables. Two have been found in M3, and they are also listed in Table 3. Finally, the single case for M68 discussed by Andrews (1980) is also given.

TABLE 3

Number	P_1(d)(SKS)	log T_e(K)(SKS)	P_1(d)	P_1/P_0	A_1(mag)	A_1/A_0	σ(mag)	log $L/L_\odot$	log T_e(K)	$R/R_\odot$
				M15 Double-Mode Variables						
39	0.389568	3.818	0.389573	0.7450	0.397	2.4	0.046	1.78	3.848	5.2
41	0.391743	3.847	0.391761	0.7545	0.349	2.8	0.056	1.79	-	-
61	0.399660	3.850	0.400065	0.7472	0.426	1.7	0.068	1.78	3.846	5.3
III-5	0.399864	3.847	0.404615	0.7628	0.343	2.2	0.059	-	-	-
26	0.402243	3.825	0.402275	0.7408	0.327	3.4	0.050	1.78	3.844	5.4
30	0.405976	3.833	0.405977	0.7454	0.331	2.0	0.047	1.78	3.844	5.4
58	0.407669	(3.854)	0.407685	0.7468	0.298	1.4	0.057	1.76	3.838	5.3
31	0.408231	3.840	0.408191	0.7459	0.347	1.8	0.043	1.79	3.846	5.4
53	0.414161	3.847	0.414165	0.7457	0.327	1.9	0.056	1.79	3.844	5.4
17	0.428872	3.822	0.428872	0.7462	0.321	1.7	0.048	1.78	3.836	5.6
				M3 Double-Mode Variables						
68	0.355973			0.7450	.45	1.1				
87	0.3575			0.745						
				M68 Double-Mode Variable						
3	0.39074			0.7416						

The Peterson diagram for several masses in the observed period and surface effective temperature ranges is given in Figure 3. This is the figure that is published by Cox, Hodson, and Clancy (1982). The M15 stars are marked by the symbol X, with V31 circled. The two shorter period M3 stars are overlapped at the + sign at a period of 0.48 day. The M68 case is almost coincident with an M15 star. The very low case for M15 is moved up to the rest of the M15 stars if newer data by Filippenko and Simon (1981) is used. From inspection of the diagram, it appears that the M15 RR Lyrae stars have a mass of 0.65 solar mass, the M3 stars, a mass of 0.55 solar mass, and the sole M68 star, a mass of perhaps 0.60 solar mass. Maybe the difference between the Oosterhoff groups is a difference of mass, with the Oosterhoff group II having the higher mass of 0.65 solar mass.

Further analysis of the Hertzsprung-Russell diagram for these M15 stars shows that the best fit is for a helium mass fraction of 0.28. This puts the blue and red edges where they are shown in Figure 4. This blue edge calculated by Cox, Hodson, and Clancy agrees well with that calculated by many others such as Tuggle and Iben (1972). The red edge is more controversial, being based on the Deupree (1977) calculations of two dimensional time dependent convection. Such red edges are now being verified however by new Stellingwerf (1982) results. The Baker and Gough (1979) and Gonzi and

Osaki (1980) linear theory results give red edges much cooler and do not agree so well with the observations. Figure 4 constrains all the M15 RR Lyrae variables to have a mass of 0.65 solar mass, but the change in plotted position which arises from a change of 0.05 solar mass is indicated. Lines of constant period in the fundamental mode as well as the lines of constant period ratio are also shown.

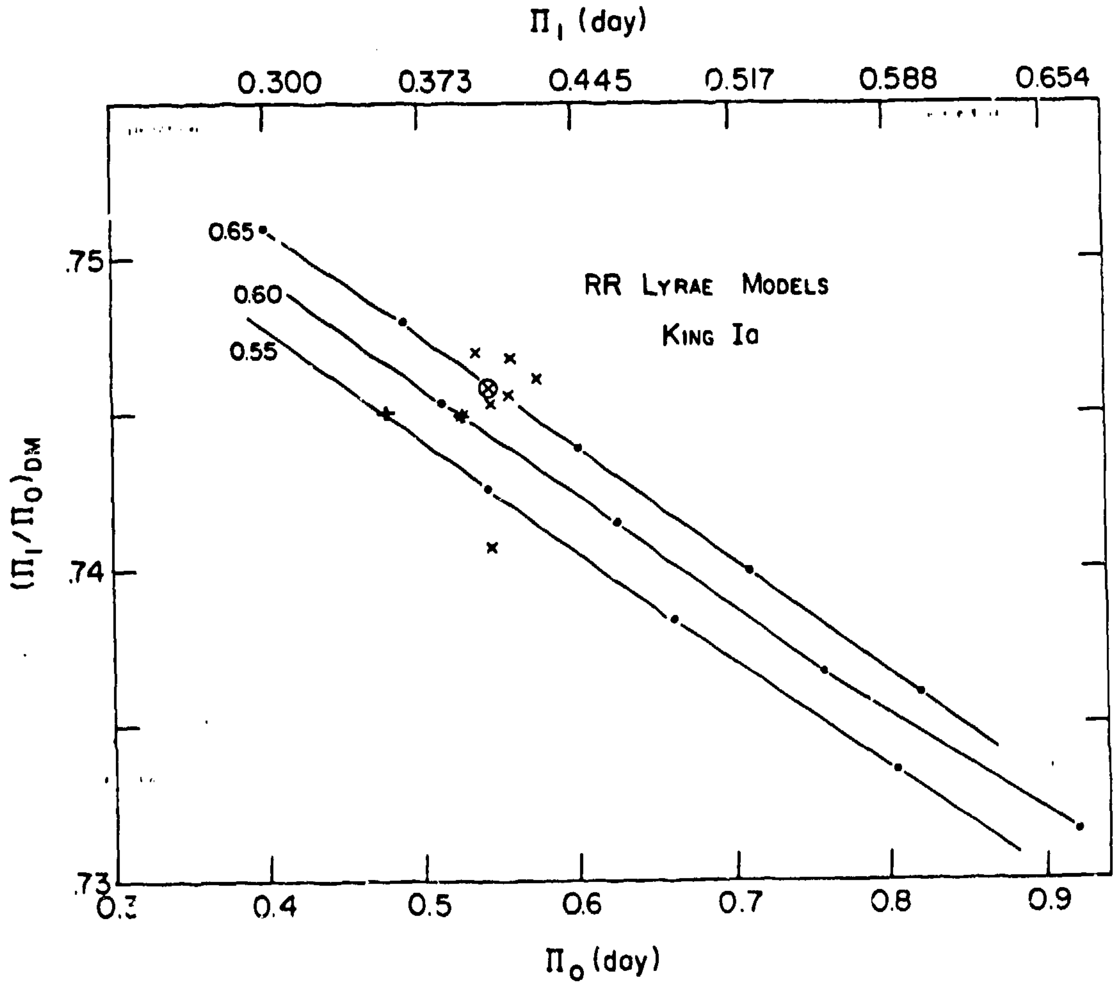

Fig. 3. The Petersen diagram of period ratio versus period for the double-mode RR Lyrae variables. The M15 prints are given as X's, the two M3 points by a single + sign, and the sole M68 double-mode variable lies among the M15 points at a period of 0.53 day.

For the classical Cepheid case, the double-mode phenomenon poses two problems, the period ratio and the actual cause of the double-mode pulsation. While neither problem is understood, at least for the double-mode RR Lyrae case the problem is only to learn the cause of the pulsation. Stellingwerf (1975) showed that if one can get simultaneous instability of full-amplitude fundamental and overtone modes toward each other, then obviously the equilibrium situation will be pulsation in both modes at the same time. This situation has not yet been demonstrated in realistic models; one full amplitude mode may decay to the other, but the other is stable against any mode switch.

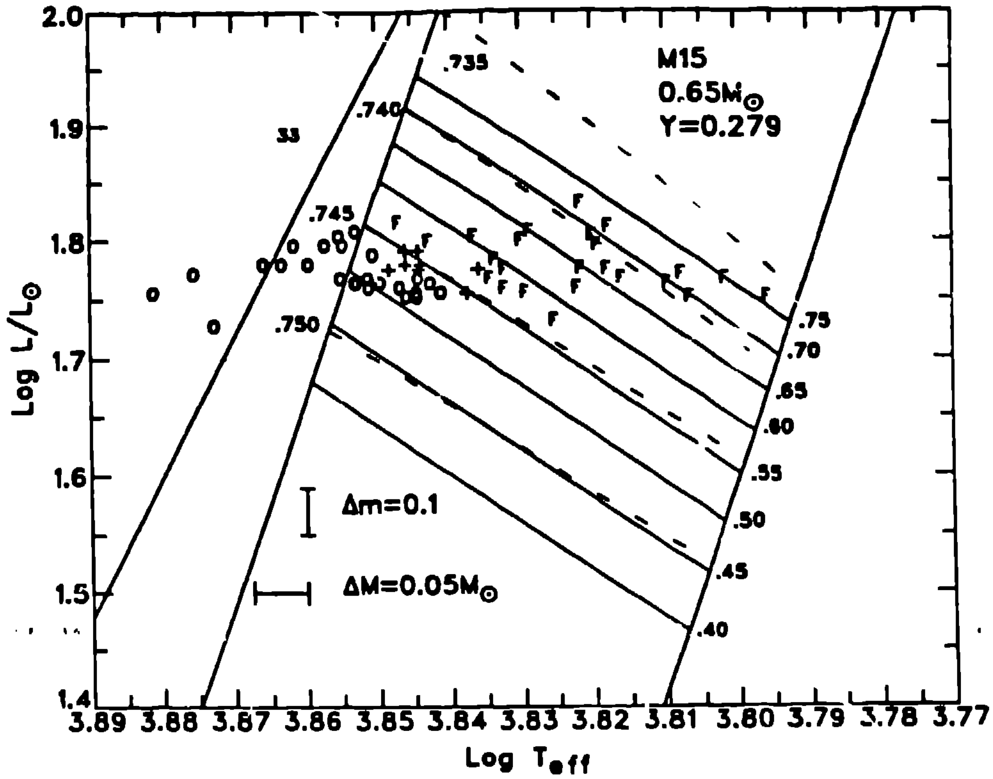

Fig. 4. The theoretical H-R diagram for 0.65 $M_\odot$ models. M15 RR Lyrae stars are plotted assuming the mean luminosity of them is log $L/L_\odot$ = 1.78. Fundamental (F) overtone (O) and double-mode (+) stars as marked.

Simon, Cox, and Hodson (1980) have calculated nonlinear Cepheid models that start out in two modes at once. They both grow as linear theory predicts until, at some threshold amplitude, one of the modes begins to dominate. The other mode decays in time giving ultimately a pure mode. Cases where either the fundamental or the overtone can dominate are displayed, but never can both occur at the same time. These authors consider what might happen if the limiting amplitude is not large enough to defeat the presence of the other mode. In that case, mode switching will occur, or at least a tendency to do so will exist. If the switch is complete to a level for the now dominant mode which can defeat the original mode, then the mode switching is complete. This is case a in the Figure 5, where the suppression amplitude of the fundamental is larger than the limiting fundamental amplitude. Case b is the opposite case where the fundamental mode cannot be suppressed by the overtone amplitude and the switch to the fundamental is complete. The case where both modes are strong enough to defeat the other can result in the much discussed either-or modal behavior. That is case c. Finally in case d, neither mode can get enough strength to suppress the other, and a compromise mixed mode situation exists.

Regev and Buchler (1981) and Buchler and Regev (1981) have developed a simple system of equations which represent in a crude way both the double-mode pulsation of two modes and the energy equation. The results of their work is that with certain parameters, the mode switch can be followed. In another case with a different set of parameters a wandering around the intersection point in Figure 5 case d is seen.

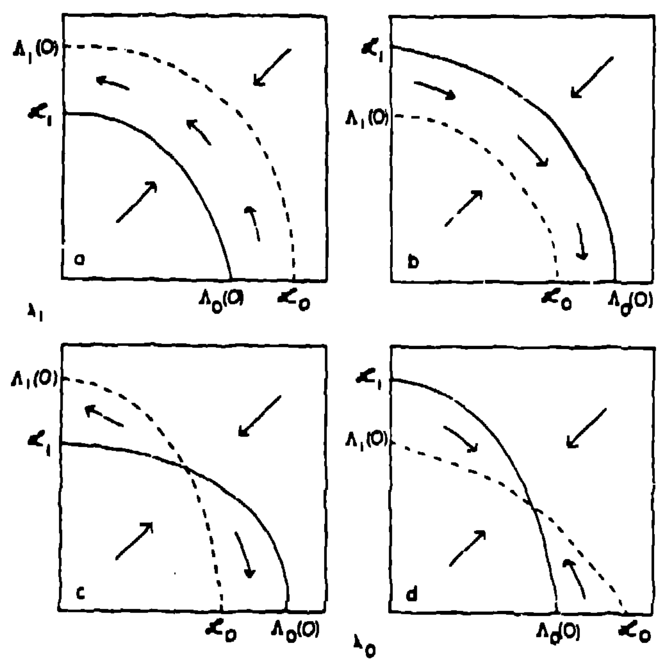

FIG. 2.—Schematic representations of the functions $\Lambda_0(\lambda_1)$ (*solid curves*) and $\Lambda_1(\lambda_0)$ (*dashed curves*) for the four categories (*a–d*) given in the text. In each case, the abscissa λ_0 measures the amount of F, and the ordinate λ_1 the amount of 1O. The single-mode limiting amplitudes, $\Lambda_0(0)$ and $\Lambda_1(0)$, and suppression amplitudes, $\mathscr{L}_0$ and $\mathscr{L}_1$, have been indicated. Arrows show how system points evolve in the amplitude-amplitude space.

Fig. 5. The Simon, Cox, and Hodson diagrams here show the behavior of modes depending on their amplitudes. Cases a and b give pure modes. Case c is the either-or case. Double-mode pulsation can occur if one has the situation of case d.

There has been considerable discussion about whether the basic cause of double-mode pulsation is a resonance between two of the naturally occurring, or normal, modes. This is a follow-on from the rather successful Simon and Schmidt (1976) thought that the bump Cepheids have a resonance between the fundamental mode and the second overtone which causes the observed light and velocity curve bumps. Papers by Simon (1979), Petersen (1979, 1980), and Takeuti and Aikawa (1980 and preprint) discuss whether the resonance could be between the fundamental, the first overtone, and the third. Numerically there is a resonance, but Simon, Cox, and Hodson (1980) were not able to show that the double mode behavior is likely for resonant model envelopes.

An idea, pursued for this conference, to find the cause of double-mode pulsation is discussed by Hodson and Cox. This can be seen in Figure 6, which is a somewhat schematic version of a diagram first made by Stellingwerf. For RR Lyrae variables, the linear theory growth rates are plotted versus effective temperature of models. Here it is assumed that the mass of the RR Lyrae variable is 0.65 solar mass, has a population II composition, and has a luminosity of 60 suns. The overtone blue edge (1HBE) of the instability strip, the fundamental mode blue edge (FBE), a transition line (TL), and the red edge (RE) from the Deupree work are labelled. The two other solid lines are the stability of full amplitude solutions, that is, they give the rate of growth of the fundamental in the full amplitude solution (F in 1H) and the growth of the overtone in the fundamental full amplitude solution (1H in F). The transition line is where a blueward evolving fundamental mode star would switch to the overtone because the overtone wants to grow at this and hotter temperatures. We show that the overtone is always stable at all effective temperatures, as a series of RR Lyrae calculations by Simon, and reported by Cox show. As just discussed, and as Stellingwerf has emphasized, the stability of a mode is assured if its amplitude is large enough. If the amplitude is decreased by some mechanism such as turbulent viscosity damping, then the amplitude may not exceed the suppression amplitude, and mode switching might result.

Figure 6 indicates with its dashed lines what can happen for reduced amplitudes. These lines must start from the two blue edges because for those points the full amplitude solutions must have zero amplitude just as linear theory assumes. Reduction of the pulsation amplitude at cooler temperatures to the right in the diagram would mean that the nonlinear stability results would approach the linear ones. If this rotation can be large enough, it is possible that between the FBE and the TL both modes would be unstable against decay to the other mode and double-mode behavior could be found. As drawn in Figure 6, the region where there is instability in both nonlinear modes, and therefore double-mode pulsation, is very narrow. This does not match the observations in M15 as shown in Figure 4. One possible solution to this problem is to make the unconventional proposal that the helium mass fraction differs from star to star in M15, so that this double-mode behavior can occur in an effective temperature range much larger than Figure 6 indicates.

Figure 6 shows that the overtone mode is always stable at full amplitude for all effective temperatures in the instability strip. At a somewhat smaller amplitude, however, it may be that there is a small region in the middle of the strip where the overtone is unstable to a switch to the fundamental. The interesting thing is that if this overtone is always locked into its mode, evolution in the cooler or redward direction would give very red Bailey c type variables. Since these variables are not observed in any confirmed cases, it appears that RR Lyrae variables evolve only blueward in the instability strip. This has important implications for low mass stellar evolution on the horizontal branch.

To this theoretical discussion we should add the observational evidence on mode switching. Hodson, Stellingwerf, and Cox (1979) found that TU Cas has a decreasing overtone amplitude from magnitude observations over a 57 year time span. This decay was verified by Niva (1979) using radial velocity observations since 1917. On the other hand, Faulkner and Shobbrook (1979) discovered that the overtone in the other very short period double-mode Cepheid U TrA seems to be growing. These mode changes are at about the theoretically predicted rates, but they do conflict with the fact that many of the short period Cepheids have the two modes as if they were stuck there in their evolution. The apparently more common case is documented by Madore, Stobie, and Van den Bergh (1978) who found that the longest period double-mode Cepheid V367 Sct seems to have had no change in amplitude over a period of 50 years.

For more details on observational aspects of double-mode pulsation one should refer to the excellent recent review by Stobie (1980). Also the other excellent review at that time by Cox (1980) includes some more theoretical ideas about the cause of the anomalous period ratios and the causes of double-mode pulsation.

At present the status of this research on double-mode pulsation leads to the following conclusions: 1) Double-mode Cepheids and RR Lyrae variables have masses, radii, and luminosities normal for their evolution. 2) Classical Cepheid period ratios require some composition, magnetic field or opacity influences. 3) Double-mode behavior is likely not mode switching but nevertheless it seems to require both F and 1H modes to be unstable to each other at or near the transition line in the Hertzsprung diagram.

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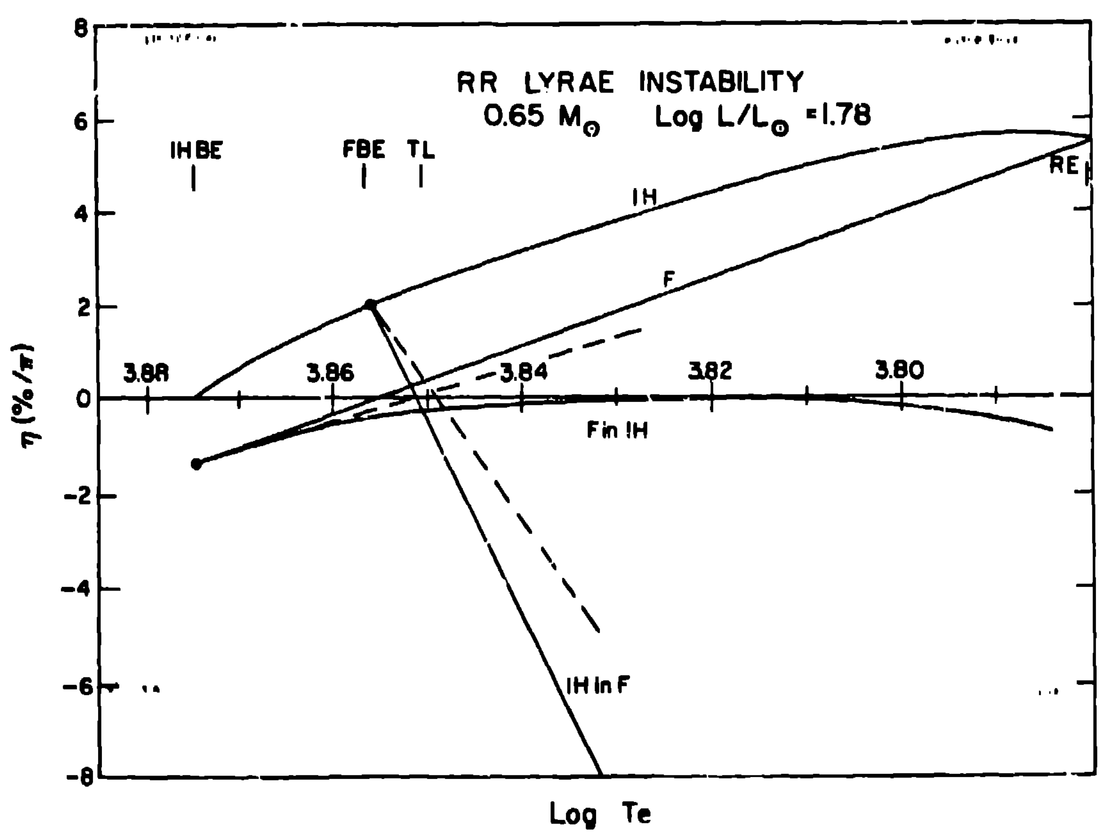

Fig. 6. Growth rates versus effective temperature is plotted for both linear and nonlinear pulsations of RR Lyrae variables.

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