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C00-3497-40

ANNUAL PROGRESS REPORT

Contract No: DE-AC02-76ET53032
(formerly EY-76-02-3497)

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Date of Report: August 31, 1980

Report Number: C00-3497-40

ABSTRACT

Research efforts were concentrated on two main topics. One was to obtain a fit between single-mode nonlinear saturation theory and experimental observations on the PR-6 mirror device. A model of this experiment yields good agreement between predictions of the time variation of the fluctuating potential level, the floating potential, the mode wavelength and mode frequency, and observations. The second topic concerned single-mode Landau damping. The previous results of O-Neil-Morales were confirmed, but in much simpler form with no multiple sums. No longer-time corrections of any significant size were uncovered.

REA

DISCLAIMER

INTRODUCTION

During the past year, the major research effort concentrated on fitting the experimental observations on the PR-6 mirror experiment by a single-mode nonlinear saturation theory. This attempt has been largely successful and is now virtually complete. A journal article which gives the details is in preparation.

A second topic concerned the long-time behavior of the O'Neil-Morales solution for nonlinear damping of a single electrostatic wave, or what is essentially equivalent, the saturation by electron trapping of a single unstable mode. During the course of this investigation, we found that the O'Neil-Morales results could be obtained in much simpler form, with no multiple sums over Fourier contributions. This work has been accepted for publication by the Physics of Fluids and will appear in the September issue. Our studies of the longer time behavior were done in considerable detail, but we have not found any nonresonant nonlinear process which is of significant size. For the moment, this work is being set aside, and we will concentrate instead on a study of single-mode time asymptotic limits for parametric instabilities in plasma. This should be of interest for interpretation of recent microwave experiments, and has direct relevance to plasma heating proposals.

Finally, work is underway on extending the single-mode nonlinear theory of the DCLC instability to the finite-beta regime. Such an extension is necessary if one is to attempt to model the experiments carried out on the 2XIIB device at Livermore.

Some details of the two main topics follow:

1. Fitting the PR-6 observations with a single-mode nonlinear theory.

In the previous annual report, we described completion of the analysis of the DCLC instability in a mirror machine, using a model (Model I) which was very unphysical (no sources or mirror losses). While there is no relation between this model and any laboratory mirror device, we did find remarkably satisfactory agreement with simulation studies, although some unexplained discrepancies still persist. This work was published in Phys. Fluids,[1] together with the accompanying simulation paper by Cohen and Maron.[2]

Now we have completed the analysis of a new model (Model II) which does include particles sources and mirror losses, and we find good agreement with the observations on PR-6.[3] The nonlinear method itself is quite similar to that in Model I. In fact, the presence of sources and losses had a negligible effect on the linear stability boundary curves. The second order potential effect was totally different. The induced potential is positive, and much larger than the negative potential determined in Model I. This is entirely as expected, of course, since enhanced scattering of electrons out of the loss cone creates a large positive potential reaction. The third order adjoints are very similar to those obtained in Model I. The final analytic saturation conditions are quite different, having complex coefficients which enable us to solve for the nonlinear frequency shift as well as the saturated level of oscillation.

To fit the PR-6 data, we then assumed that the equilibrium is in the form of a Subtracted Maxwellian,

$$F_0 = \frac{1}{\pi v_h^2(1-\lambda^2)}\left\{e^{-v^2/v_h^2} - e^{-v^2/\lambda^2 v_h^2}\right\}$$

with a fixed v_h^2 but with a time-varying "hole" coefficient λ. Let $t=0$

denote the time at which the instability begins. Read the value of the density at that time, n_i, and take a fixed typical magnetic field value (B=10KG) to obtain an initial value of $(\omega_p/\Omega)_i$. Fix the density scale ε as the reciprocal of the plasma radius $[\varepsilon=(5\text{ cm})^{-1}=0.2\text{ cm}^{-1}]$ and choose an initial ion temperature T_{+i} to obtain both $(\varepsilon a)_i$ and Mv_h^2. This locates an initial point for the plasma on the stability diagram in ω_p/Ω, εa space. Choose λ_i such that the neutral stability curve passes through this point. This also determines the initial values of the wavenumber k_i and frequency ω_i.

To advance in time, we use an energy argument due to Marx,[4] which enables us to find $\bar{E}_+(t)$, which is then related to $\lambda^2(t)[\bar{E}_+=T_{+i}(\lambda^2+3/2)]$. The only "fudge-factor" in the entire argument is the assumption that $(1/n)(dn/dt)=-p\nu_s$ where ν_s is the classical Spitzer drag and p is adjusted to give a best fit. We require a $p\cong 10$ which is in general agreement with the observation of Kanaev and Yushmanov on the observed density decay rate.

Once we know how to advance λ, n, T_+, T_- in time, we then know where the plasma is on the stability diagram relative to the neutral stability curve. This then yields the nonlinearly saturated values of the fluctuating potential, the floating potential and the nonlinear frequency shift, as well as the instantaneous value of the dominant-mode's frequency and wavelength.

The results show good agreement with the behavior observed in Ref. 3. The rise in floating potential and fluctuating potential is close to that observed. The mode wavelength is of the right order of magnitude and increases in time, as observed. The frequency is not far from observation. The best fit is for $T_+\cong 400$ ev which is considerably higher than that quoted in Ref. 3. However, some recent measurements by Kanaev[5] indicating the presence of cold gas until just before onset of the instability, also make it plausible that the actual ion energy may be considerably higher than 150 ev. Improved models which would give a more natural fit to the density decay, as well, and might make the use of our single "fudge-factor" unnecessary are known to us. It is not clear if the effort required to extend the previous calculation would be worthwhile, however.

Details are in a manuscript being prepared for submission to Physics of Fluids, and in the Ph.D. thesis of Richard C. Myer.

2. Simplified Form of the O'Neil-Morales Trapping Solution.

We have carried out the O'Neil-Morales[6,7] expansion method for determining the time behavior of single mode Landau damping. Their method relates the complex nonlinear frequency shift to an integral of the difference between the nonlinear and linear distribution functions, taken over those velocities in the immediate vicinity of the phase velocity of the wave (resonant region). We found that it was possible to carry out some of the integrals directly, without having to resort to Fourier expansions. This then enabled us to express the final answer in a closed analytic form which requires only a single integral. The results in Refs. 6 and 7 required the performance of one or two multiple sums in addition to the same integral. Thus, our form of the result is much more amenable to numerical evaluation or analytic consideration. The result just described has been accepted for publication[8] and will appear in the September issue of Physics of Fluids. A preprint is attached to this report.

We also spent considerable time evaluating the non-resonant contributions to the nonlinear frequency shift. It was known that these contributions were linear in nature until a time $t_1 \cong (\omega_p \tau)\tau$. Here τ is the trapping time and ω_p the plasma frequency. Since $\omega_p \tau \gg 1$, $t_1 \gg \tau$. We had hoped that the onset of nonlinear behavior of the numerically superior nonresonant electrons might change the character of the solution for times $t \geq t_1$. As far as we can determine, the effects are much too small to be of significance. We may return to these effects, later, but for now are suspending further work on the problem.

3. Personnel

FACULTY -

Albert Simon (MAS and Physics) - Dr. Simon has devoted 20% of his time to this project during the academic year, and full time for one and one-half summer months. He supervised three graduate students, Richard C. Myer, Thierry DeWandre and Robert Ferraro.

GRADUATE RESEARCH ASSISTANTS -

(a) Richard C. Myer (MAS Department) - Mr. Myer, who completed his undergraduate studies at Stony Brook, has virtually completed his dissertation on the nonlinear saturation of the drift cyclotron loss-cone instability. He accepted a position as a plasma physicist at TRW in Los Angeles and is there now.

(b) Thierry DeWandre (MAS Department) - Mr. DeWandre, who completed his undergraduate and master's level studies at the Catholic University of Louvain, Belgium, is now beginning his fourth year at Rochester. He is now working on the application of nonlinear saturation theories to parametric instabilities in plasma.

(c) Robert D. Ferraro (Physics Department) - Mr. Ferraro, who completed his undergraduate studies at Cornell, is beginning his third year of graduate studies. He is working on the extension of the DCLC theory to the finite-beta regime and comparison with 2XIIB.

Other Federal Support. None of the personnel working on this contract have other federal support.

REFERENCES

1. R.C. Myer and Albert Simon, Phys. Fluids 23, 963 (1980).

2. B.I. Cohen and N. Maron, Phys. Fluids 23, 974 (1980).

3. B.I. Kanaev and E.E. Yushmanov, Sov. Physics JETP, 40, 290 (1975).

4. K.D. Marx, Phys. Fluids 13, 1355 (1970).

5. B.I. Kanaev, Nuclear Fusion 19, 347 (1979).

6. T.M. O'Neil, Phys. Fluids 8, 2255 (1965).

7. G.J. Morales and T.M. O'Neil, Phys. Rev. Lett. 28, 417 (1972).

8. Thierry Dewandre and Albert Simon, to be published, Phys. Fluids, Sept. 1980.

PREVIOUS PUBLICATIONS UNDER THIS CONTRACT

1. A. Simon, "Nonlinear Transport in Weakly Ionized Plasma," Phys. Fluids 11, 1181 (1968).

2. A. Simon, "Convection of Weakly Ionized Plasma in a Non-uniform Magnetic Field," Phys. Fluids 11, 1186 (1968).

3. J.J. Kim and A. Simon, "Nonlinear Theory of the ExB Instability," Phys. Fluids 12, 895 (1969).

4. A. Simon and J.N. Shiau, "Quasilinear Theory of the Positive Column Spiral Instability," Phys. Fluids 12, 2630 (1969).

5. J.N. Shiau and A. Simon, "Nonlinear Theory of the Positive Column Spiral Instability, " Phys. Fluids 13, 2569 (1970).

6. A.M. Sleeper and A. Simon, "Nonlinear Theory of the Drift-Dissipative Instability," Phys. Fluids 13, 2757 (1970).

7. Albert Simon and C.I. Weng, "Quasilinear Theory of the Mirror Flute Instability," Phys. Fluids 15, 2341 (1972).

8. V.G. Abhyankar and A. Simon, "Linear Threshold of the Buneman-Farley Instability," Phys. Fluids 17, 261 (1974).

9. Donald A. Monticello and Albert Simon, "Nonlinear Theory of the Collisional Drift-Wave Instability," Phys. Fluids 17, 791 (1974).

10. V.G. Abhyankar and A. Simon, "Stability Boundaries for Temperature and Density Gradient Flute Modes in Mirror Machines," Phys. Fluids 17, 2040 (1974).

11. A. Simon, L. Gross and P.J. Catto, "Shear Stability Boundaries for Collisionless Drift Modes in Finite Geometry," Phys. Fluids 18, 231 (1975).

12. M.N. Rosenbluth and P.J. Catto, "An Improved Calculation of Critical Magnetic Shear in an Inhomogeneous Plasma," Nucl. Fusion 15, 573 (1975).

13. W.M. Tang, C.S. Liu, M.N. Rosenbluth, P.J. Catto and J.D. Callen, "Finite Beta and Resonant Electron Effects of Trapped Electron Instabilities," Nucl. Fusion 16, 191 (1976).

14. A. Simon and M.N. Rosenbluth, "Single-Mode Saturation of the Bump-on-Tail Instability: Immobile Ions," Phys. Fluids 19, 1567 (1976).

15. P.J. Catto, K.T. Tsang, J.D. Callen and W.M. Tang, "Resant Electron Effects on Trapped Electron Instabilities," Phys. Fluids 19, 1596 (1976).

16. A. Simon, "Single-Mode Saturation of the Bump-on-Tail Instability: II. Mobile Ions," Phys. Fluids 20, 79 (1977).

PREVIOUS PUBLICATIONS UNDER THIS CONTRACT (Cont.)

17. P.J. Catto and K.T. Tsang, "Linearized Gyro-Kinetic Equation with Collisions," Phys. Fluids 20, 396 (1977).

18. A. Simon and L.S. Gross, "Nonlinear Saturation of the Collisionless Drift Instability," Phys. Fluids 20, 946 (1977).

19. A. Simon and S. Migliuolo, "Nonlinear Saturation of Two Unstable Modes: Survival Competition," Phys. Fluids 20, 1855 (1977).

20. K.T. Tsang, J.D. Callen and P.J. Catto, "Collisional Effects on Trapped Electron Instabilities," Phys. Fluids 20, 2113 (1977).

21. K.T. Tsang and P.J. Catto, "Analytic Solutions of the Two-Dimensional Eigenvalue Problem for the Trapped-Electron Instability in Tokamaks," Phys. Rev. Lett. 39, 1664 (1977).

22. P.J. Catto, "Adiabatic Modifications to Plasma Turbulence Theories," Phys. Fluids 21, 147 (1978).

23. P.J. Catto and K.T. Tsang, "Trapped Electron Instability in Tokamaks: Analytic Solution of the Two-Dimensional Eigenvalue Problem," Phys. Fluids 21, 1381 (1978).

24. K.T. Tsang, P.J. Catto, J.D. Whitson and J. Smith, "Absolute Universal Instability is Not Universal," Phys. Rev. Lett. 40, 327 (1978).

25. P.J. Catto, "Linearized Gyrokinetics," Plasma Physics 20, 719 (1978).

26. S. Migliuolo and A. Simon, "Nonlinear Saturation of the Dissipative Trapped Electron Instability," Phys. Fluids 21, 2218 (1978).

LIST OF PUBLICATIONS (Sept. 1979 - Aug. 1980)

A. WORK COMPLETED PREVIOUSLY, BUT PUBLISHED THIS YEAR

Title	Authors	Journal/Status	Percentage DOE Support
Nonlinear Saturation of the Drift Cyclotron Loss-Cone Instability	R.C. Myer & A. Simon	Phys. Fluids 23, 963 (1980)	100%
Nonlinear Ion Cyclotron Waves in Mirror Machines	B.I. Cohen & 19 others	8th International Conference on Plasma Physics & Controlled Nuclear Fusion Research, Brussels, July 1980.	100%

B. WORK COMPLETED THIS YEAR, OR CONTINUING

Title	Authors	Journal/Status	Percentage DOE Support
Time Behavior of Nonlinear Plasma Oscillations	Thierry DeWandre and Albert Simon	Phys Fluids (accepted for publication)	100%
Nonlinear Saturation of the Drift Cyclotron Loss-Cone Instability: II. Comparison with PR-6 Data.	R.C. Myer and Albert Simon	In preparation. to be submitted to Phys. Fluid.	100%

THESES COMPLETED UNDER THIS CONTRACT

1. John J. Kim, "The Second Order Shifts in Frequency and Amplitude in the Non-Linear Theory of ExB Instability," Physics Dept., 1969.

2. Jeng-Nan Shiau, "Small Amplitude Nonlinear Theory of the Positive Column Spiral Instability," MAS Department, 1969.

3. Arthur M. Sleeper, "The Drift Dissipative Instability," Physics Dept., 1970.

4. Cheng-I Weng, "Nonlinear Theory of the Mirror Flute Instability," MAS Dept., 1972.

5. Donald A. Monticello, "Nonlinear Theory of the Collisional Drift Wave Instability," Physics Dept., 1973.

6. Vishwas Govind Abhyankar,"Density and Temperature Gradient Flute-Mode Instabilities," MAS Dept., 1974.

7. Leonard Gross, "Nonlinear Theory of the Collisionless Drift Instability," Applied Physics Program, 1975.

8. Stefano Migliuolo, "Nonlinear Saturation of the Dissipative Trapped Electron Instability," Physics Dept., 1977.

9. Richard C. Myer, "Nonlinear Saturation of the Drift Cyclotron Loss-Cone Instability," MAS Dept., 1980.

Time behavior of Nonlinear Plasma Oscillations

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We analyze the time behavior of a single weakly damped electron mode. We obtain analytic expressions for the time-dependent mode damping and frequency shift equivalent to those of O'Neil and Morales and O'Neil, but in much simpler form since there are no sums.

PACS numbers: 52

The classic papers of O'Neil[1] and Morales and O'Neil[2] correctly predict the time behavior of a single weakly damped mode for times of order τ, where τ is the trapping time. For times beyond this range, their calculation must be extended to higher order terms in $(\omega_p\tau)^{-1}$ to include nonlinear behavior of nonresonant electrons. We exclude "sideband effects" by restrictive boundary conditions. While studying these higher order terms, we have obtained greatly simplified forms of the existing results for the time behavior of the single mode damping and frequency shift. These expressions, since they contain no sums, may constitute a more useful analytic description of the time behavior than has been available previously, even for $t \simeq \tau$. Similar expressions can readily be obtained for the higher order terms.

Consider a single electron wave in a collisionless plasma with fixed ions, in the limit $\omega_p \tau >> 1$ and $\gamma_\ell \tau << 1$. The complex wave-frequency follows by using the "subtraction method" of Morales and O'Neil[2], which relates the difference between nonlinear and linear frequencies, $\delta\omega(t) \equiv (\omega - \omega_\ell)$, to the difference between nonlinear and linear distribution functions $(f - f^\ell)$. Thus

$$\delta\omega(t) = i \frac{4\pi en}{kE_k} \left(\frac{\partial\varepsilon}{\partial\omega}\bigg|_{\omega_\ell}\right)^{-1} \int_{-\infty}^{\infty} dv \int_{-\lambda/2}^{\lambda/2} \frac{dx}{\lambda}\, e^{-ikx} \left[f(x,v,t) - f^\ell(x,v,t)\right]. \quad (1)$$

For $t \le \tau$, the region where $f(x,v,t)$ differs significantly from $f^\ell(x,v,t)$ is confined to the neighborhood of the linear phase velocity v_p. Hence, one may expand $(f - f^\ell)$ around v_p and keep only the lowest order terms. The integration is performed by changing variables, from (x,v) to (ξ,κ) [see Ref. 1, Eqs. (14) and 15)], and then, for $\kappa^2 < 1$, to the variables (u,κ) where

$$du = d\xi\,(1 - \kappa^2 \sin^2\xi)^{-1/2} \tag{2}$$

The integrals in u-space may then be carried out directly[3]. A similar transform is used for $\kappa^2 > 1$.

To lowest order in $(\omega_p\tau)^{-1}$, we find the damping coefficient $\gamma(t)$,

$$\gamma(t) = \mathrm{Im}\,\delta\omega^{(1)} + \gamma_\ell$$

$$= (64/\pi^2)\gamma_\ell \int_0^1 dk \left\{ \kappa^{-5}\left[(K-E)\frac{cn(z)}{sn(z)} - KZ(z)\frac{dn(z)}{sn^2(z)}\right] + \kappa \left[(K-E)\frac{dn(y)}{sn(y)} - KZ(y)\frac{cn(y)}{sn^2(y)}\right]\right\} \tag{3}$$

where $z = (t/\kappa\tau)$, $y = (t/\tau)$, $\gamma_\ell = (\pi/2)\ \omega_\ell\ (\omega_p/k)^2\ (\partial f_0/\partial v)_{v=v_p}$.

Here, γ_ℓ is the linear Landau damping; sn, cn, and dn are Jacobian elliptic functions; K and E are the complete elliptic integrals of the first and second kind, respectively, and Z is Jacobi's zeta function[4]. The modulus of these functions is κ. This damping coefficient expression is entirely equivalent to the original O'Neil result, but in much simpler form since there is no sum. We also find

$$\Gamma(t) \equiv \int_0^t \gamma(t')\,dt'$$

$$= (64/\pi^2)\,\gamma_\ell\tau \int_0^1 d\kappa \left\{ \kappa^{-4}\left[E - K\,dn(z) + \frac{cn(z)}{sn(z)}KZ(z)\right] + \kappa\left[E - (1-\kappa^2+\kappa^2 cn(y))\,K + \frac{dn(y)}{sn(y)}KZ(y)\right]\right\} \tag{4}$$

The asymptotic result

$$\Gamma(\infty) = (64/\pi)\ \gamma_\ell\ \tau \int_0^1 d\kappa \quad \left\{\kappa^{-4}\ [\frac{E}{\pi} - \frac{\pi}{4K}] + \frac{\kappa}{\pi}\ [E + (\kappa^2-1)K]\right\} \tag{5}$$

follows readily and is identical to that of O'Neil. It may be obtained by isolating the secular term of the integral $\int_0^t \Gamma(t')dt'$. The first part of the $\gamma(t)$, $\Gamma(t)$, and $\Gamma(\infty)$ integrals represents the untrapped electron contribution while the second part represents that of the trapped electrons.

To second order, we find the frequency shift

$$\mathrm{Re}\ \delta\omega^{(2)}(t) = -(8/\pi\tau)\ (\omega_p/k)^3\ (\partial^2 f_o/\partial v^2)_{v=v_p}$$

$$\times \int_0^1 d\kappa \quad \left\{\kappa^{-4}\ [4E - (8-\kappa^2)\ K + Q(z)\ \kappa^{-2}]\right.$$

$$\left. + \ \kappa \quad [4E - 4(1+\kappa^2)\ K + K + Q\ (y)]\right\} \quad , \tag{6}$$

$$\text{where } Q(u) = 8\ \mathrm{sn}^{-2}u\ [K - E - KZ(u) \ \ \mathrm{cn}(u)\ \mathrm{dn}(u)\ \mathrm{sn}^{-1}(u)]\ . \tag{7}$$

Again, this must be entirely equivalent to the Morales - O'Neil result, but in much simpler form since now there are no multiple sums. We also find

$$\int_0^t \mathrm{Re}\ \delta\omega^{(2)}(t')dt' = -(8/\pi\tau)\ (\omega_p/k)^3\ (\partial^2 f_o/\partial v^2)_{v=v_p}$$

$$\times \int_0^1 dk \ \left\{\kappa^{-4}\ \left(t\ [2(E-K) + \kappa^2\ K]^2\ (\kappa^2 K)^{-1} + R(z)\ \kappa^{-1}\right)\right.$$

$$\left. + \ \kappa \quad \left[(t\ (2E-K)^2\ K^{-1} + R(y)\right]\right\} \ , \tag{8}$$

where $$R(u) = 4\tau \left[EZ(u) - (K-E)\frac{cn(u)\,dn(u)}{sn(u)} + KZ(u)\frac{cn^2(u)}{sn^2(u)}\right] \tag{9}$$

The asymptotic shift

$$\mathrm{Re}\,\delta\omega^{(2)}(t \to \infty) = (8/\pi\tau)\,(\omega_p/k)^3\,(\partial^2 f_o/\partial v^2)_{v=v_p}$$
$$\times \int_0^1 d\kappa \left\{ \frac{[2(E-K) + \kappa^2 K]^2}{\kappa^6 K} + \kappa\,\frac{(2E-K)^2}{K} \right\} \tag{10}$$

is identical to the Morales - O'Neil result. Again, the first part of the integrals in Eqs. (6), (8) and (10) represents the untrapped electron contribution while the second part represents the trapped electron contribution.

It is a pleasure to acknowledge the assistance of Dr. George Morales, who made available to us unpublished notes, and useful conversations with Drs. Tom O'Neil and Guy Laval.

This work was supported by the United States Department of Energy, under contract EY-76-S-02-3497.

REFERENCES

[1]T.M. O'Neil, Phys. Fluids 8, 2255(1965).

[2]G.J. Morales and T.M. O'Neil, Phys. Rev. Lett. 28, 417(1972).

[3]P.F. Byrd and M.D. Friedman, Handbook of Elliptic Integrals for Engineers and Scientists, 2nd ed. (Springer-Verlag, Berlin, Heidelberg, New York, 1971).

[4]ibid, p.33.