A METHOD FOR OBTAINING ELECTRON ENERGY-DENSITY FUNCTIONS FROM LANGMUIR-PROBE DATA USING A CARD-PROGRAMABLE CALCULATOR

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Abstract

This paper presents a method for obtaining electron energy density functions from Langmuir probe data taken in cool, dense plasmas where thin-sheath criteria apply and where magnetic effects are not severe. Noise is filtered out by using regression of orthogonal polynomials. The method requires only a programable calculator (TI-59 or equivalent) to implement and can be used for the most general, nonequilibrium electron energy distribution plasmas. Data from a mercury ion source analyzed using this method are presented and compared with results for the same data using standard numerical techniques.

Acknowledgment

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Introduction

Langmuir probes are among the most elementary plasma diagnostics. Their simplicity renders them useful for obtaining information on plasma properties in a variety of circumstances. Langmuir probes are particularly useful in the cool, dense plasmas of ion sources. There, Debye lengths are much shorter than typical probe radii, which in turn are much smaller than plasma dimensions. Magnetic fields are relatively weak, and ions are much cooler than electrons so that thin-sheath criteria apply to current collection. The plasmas are reasonably isotropic.

It has been shown[1] that electron current, J_e, in the electron-retarding segment of the probe characteristic is related to the electron energy distribution function, $g(\varepsilon)$, by

$$g(\varepsilon) = \frac{4}{neS}\left(\frac{M_e}{2e}\right)^{1/2} \varepsilon^{1/2} \left.\frac{d^2J_e}{dV^2}\right|_{V=\varepsilon} \qquad (1)$$

where

n	=	electron density
e	=	magnitude of electronic charge
S	=	probe surface area
M_e	=	electron mass
ε	=	electron kinetic energy per unit charge (volts)
V	=	electron retarding potential (volts).

If the electron energy distribution function is Maxwellian, i.e., if

$$g(\varepsilon) = \frac{2}{\sqrt{\pi}}\left(\frac{e}{kT_e}\right)^{3/2} \varepsilon^{1/2} \exp\left(\frac{-e\varepsilon}{kT_e}\right) \qquad (2)$$

where k is the Boltzmann constant and T_e is the electron temperature, it is easily shown that the probe electron current is exponential and that

$$\frac{kT_e}{e} = -\left[\frac{d(\ln J_e)}{dV}\right]^{-1}. \qquad (3)$$

For saturation electron current, $J_{e_{sat}}$, the electron density n may then be found from

$$n = \frac{4J_{e_{sat}}}{eS}\left(\frac{\pi M_e}{8kT_e}\right)^{1/2}. \qquad (4)$$

In the non-Maxwellian case, when $g(\varepsilon)$ is more complex than Equation (2), more general equations must be used to obtain electron energy characteristics. Since temperature no longer has its strict meaning, the average electron energy is simply

$$\bar{\varepsilon} = \int_0^\infty \varepsilon\, g(\varepsilon)d\varepsilon. \qquad (5)$$

To find $g(\varepsilon)$, one may use Equation (1) to obtain the function $nSg(\varepsilon)$ if d^2J_e/dV^2 is known over $(0 < V < \infty)$.* The normalalization relation for $g(\varepsilon)$,

$$\int_0^\infty g(\varepsilon)\, d\varepsilon = 1 \qquad (6)$$

may then be used to obtain n and hence $g(\varepsilon)$:

$$nS\, g(\varepsilon) = \frac{4}{e}\left(\frac{M_e}{2e}\right)^{1/2} \varepsilon^{1/2} \left.\frac{d^2J_e}{dV^2}\right|_{V=\varepsilon} \qquad (7)$$

$$g(\varepsilon) = \frac{nS\,g(\varepsilon)}{\int_0^\infty nS\,g(\varepsilon)\,d\varepsilon}. \qquad (8)$$

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* The upper limit need only be a few times $\frac{kT_e}{e}$ for reasonable accuracy.

may be decomposed into a Maxwellian group satisfying Equation (2) and a non-Maxwellian or primary electron group, which is effectively monoenergetic at ε_p, such that for primary electron fraction ν

$$g(\varepsilon) = (1 - \nu) \frac{2}{\pi} \left(\frac{e}{k T_m}\right)^{3/2} \varepsilon^{1/2} \exp\left(\frac{-e\varepsilon}{k T_m}\right) + \nu\delta(\varepsilon - \varepsilon_p) \tag{9}$$

where T_m is the temperature of the Maxwellian electron group and $\delta(\varepsilon-\varepsilon_p)$ is the unit impulse function with units of volts^{-1}.

If $e\varepsilon_p >> kT_m$, the probe electron current will be similar to that shown in Figure 1. The linear current component produced by the

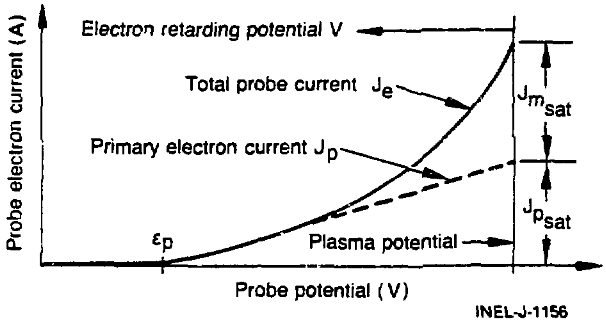

Figure 1. Idealized Langmuir probe trace for a plasma with monoenergetic primary electrons at an energy well above the Maxwellian electron temperature.

primary electrons may be subtracted from the total electron current. Then primary electron density n_p is given in terms of $J_{p_{sat}}$, the primary electron saturation current, by

$$n_p = \frac{4 J_{p_{sat}}}{e S} \left(\frac{M_e}{2e\varepsilon_p}\right)^{1/2} . \tag{10}$$

The remainder of the electron current, that due to Maxwellian electrons, may then be analyzed using Equations (3) and (4).

In the less special case, when the primary electrons are still monoenergetic, but $e\varepsilon_p > kT_m$ as shown in Figure 2, the same technique may be used. However, estimating the slopes can be very difficult.

Beattie[2] has developed a computerized technique for fitting the probe electron current in $(0 \leq V \leq \varepsilon_p)$ to the equation

$$J_e = a_1 - a_2V + a_3\exp(-a_4V). \tag{11}$$

Special Cases

It is possible to avoid dealing directly with the function $g(\varepsilon)$ in the special case when $g(\varepsilon)$

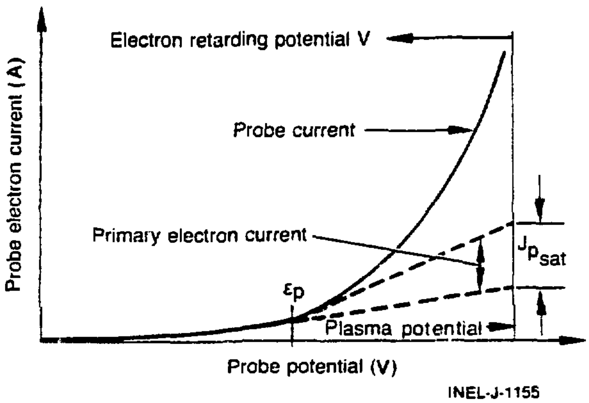

Figure 2. Idealized probe trace for a plasma with primary electron energy near the Maxwellian electron temperatuare.

Here the a_i are experimentally determined constants that contain the parameters n, ν, T_m, and ε_p of Equation (9). If the primary electron energies depart significantly from being monoenergetic, then this method also is insufficient.

General Case

In the general case, which frequently appears in ion source work and where Equations (1) and (5)-(8) must be used, it is critical that a good estimate of $d^2J_e(V)/dV^2$ be obtained. Various electronic schemes have been proposed to accomplish this differentiation during data collection. Because of electrical noise, considerable filtering is needed. This often presents an undesirable complication to experimental apparatus.

Medicus[3] has given a graphical technique for obtaining $g(\varepsilon)$ directly from the probe current characteristic as recorded on an X-Y plotter. This method can be tedious and of somewhat questionable accuracy because it relies on visual estimation of tangent slopes.

Taylor[4] has presented a numerical technique that uses a minicomputer to do polynomial regression of the probe characteristic to the equation

$$J_e = \sum_{i=0}^{l} R_i V^i \tag{12}$$

where R_i are the regression coefficients and l is typically 8 to 10. Equation (12) is then analytically differentiated giving

$$\frac{d^2J_e}{dV^2} = \sum_{i=2}^{l} i(i-1) R_i V^{i-2}. \tag{13}$$

This technique has two inherent limitations. One, which often is acceptable, is that a free exponential must be approximated by a polynomial. The other is that because of limited curvature in a polynomial, a delta function is difficult to represent without going to a very high order. Both limitations are mitigated by increasing l at the expense of longer computational time. Additionally, the polynomial regression of Equation (12) necessitates inversion of large matrices. While this is not difficult, as a rule it requires the capabilities of at least a minicomputer.

Objective

The objective of this study was to develop a method for reducing Langmuir probe data with the following additional features:

- Capability of handling general, non-Maxwellian distributions
- Capability of smoothing out inherent electrical and digitization noise without losing the resolution needed to see a narrow primary electron spike
- Suitability for use with small, card-programable hand calculators to accommodate the experimenter who does not have ready access to a larger computer.

Approach

Two physical observations are made first. One is that even though there may be a reasonably large fraction of non-Maxwellian electrons present in the plasma, the majority of electrons, in ion sources at least, tend to be in thermal equilibrium. Hence the probe trace will generally approximate an exponential function in the electron-retarding region of the probe characteristic or trace. This is illustrated in Figure 3 where the distributions of Figure 3(a) yield idealized probe traces as shown in Figure 3(b).

The second observation is that the feature of a probe trace that indicates the presence of a primary electron spike is the sharp curvature or slope discontinuity suggested in Figures 1 and 2. The information contained in such an irregularity can be easily preserved only by piecewise smoothing.

These observations suggest that the probe electron current may be most easily analyzed if separated into three parts: one an approximating exponential current and the other two residual or remainder currents, one on either side (in retarding potential V) of any sharp curvature or slope discontinuity. Figure 4 illustrates this separation. If the exponential current J_{ex} is subtracted from J_e, what remains contains all the information about the non-Maxwellain electrons as well as all the noise. Assuming there is at most one primary electron spike at ε_p in $g(\varepsilon)$, and choosing that value of V as the separation point between J_h and J_l (see Figure 4), J_h and J_l will be smooth functions and can be successfully approximated by an appropriate polynomial.

Smoothing Technique

The technique chosen for smoothing is polynomial regression using the orthogonal polynomials[5]

$$P_{nm}(x) = \sum_{i=0}^{m} (-1)^i \binom{m}{i}\binom{m+i}{i} \frac{x^{(i)}}{n^{(i)}}$$

$$3\text{-}5 \quad x,m = 0, 1, 2, \ldots, n \quad . \qquad (14)$$

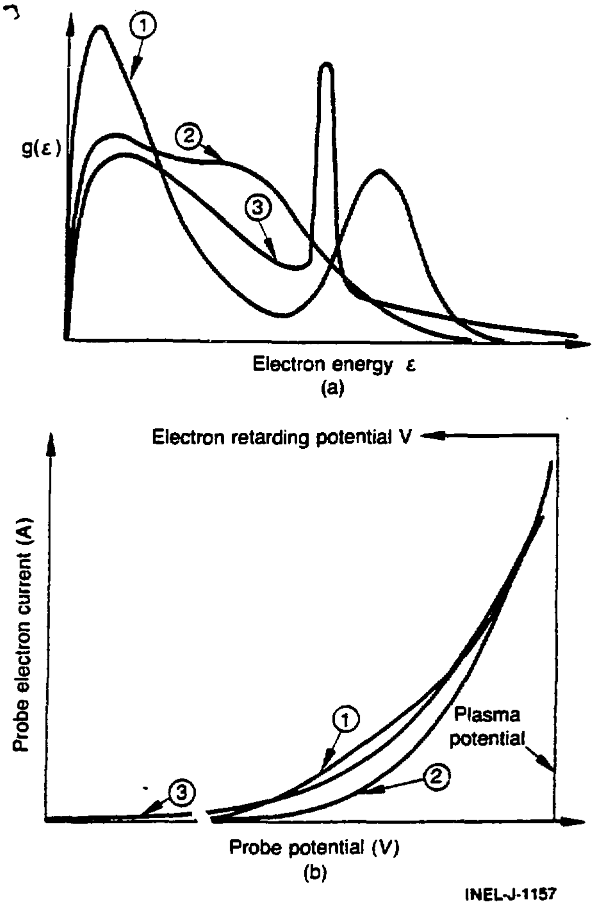

Figure 3. Various electron energy distribution functions (a) give different idealized Langmuir probe traces (b).

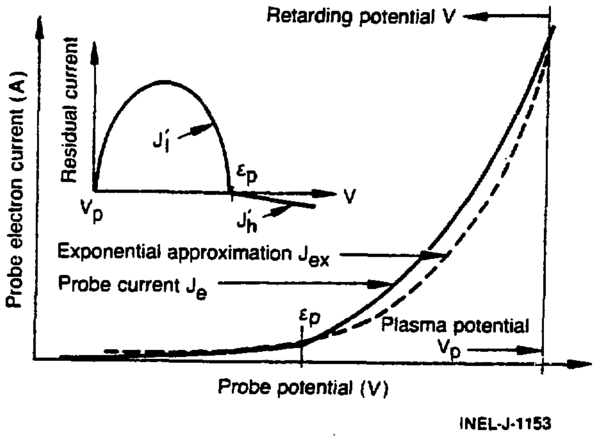

Figure 4. Decomposition of probe electron current into an approximating exponential and two residual currents (inset), one above (J_h) and one below (J_l) the irregularity in the probe trace due to primary electrons at energy εp.

The polynomials $P_{nm}(x)$ are m^{th} order polynomials in x, with the property that

$$\sum_{x=0}^{n} P_{nj}(x)\, P_{nk}(x) = 0, \; j \neq k. \qquad (15)$$

An analytic function y(x) can be approximated by the sum

$$y(x) \approx a_o P_{no}(x) + a_1 P_{n1}(x) + \ldots + a_m P_{nm}(x). \qquad (16)$$

The a_i here are regression coefficients given for n + 1 equally spaced data points in the independent variable x by

$$a_i = \frac{\sum_{x=0}^{n} y(x)\, P_{ni}(x)}{\sum_{x=0}^{n} P_{ni}^{2}(x)}, \; i = 0, 1, 2, \ldots, m. \qquad (17)$$

The orthogonality property of $P_{nm}(x)$ alleviates the need to solve simultaneous equations; hence it is practical to program the solution in the limited capacity of a programable calculator. The only requirement is that the data points J_e be equally spaced in V.

Differentiation

The second derivative of the smoothed residual electron current J' can be obtained by differentiating Equation (16) once the a_i are known. However, it is more convenient to use simple central second differences to approximate it according to

$$\frac{d^2 J'}{d\, V^2} = \frac{J'(V + \Delta V) + J'(V - \Delta V) - 2\, J'(V)}{(\Delta V)^2} \qquad (18)$$

where it is assumed that the residual electron currents J' are smoothed values separated in V by ΔV. At the slope discontinuity the second derivative is taken to be the difference in slopes on either side divided by ΔV. This specification is needed because the smoothing functions don't generally give the same value for J' at the breakpoint ϵ_p.

Other Considerations

Plasma potential is assumed to have been determined by ordinary means such as the double probe technique[6,7] or by locating the intersection of extensions of the currents from electron- and ion-retarding regions[8] as illustrated by Figure 5. Note that Figure 5 is a semilogarithmic plot of probe electron current.

The choice of the separation point ϵ_p is an important one. Obviously if there is no slope discontinuity or sharp curvature in $J_e(V)$, then there is no great advantage to dividing the residual current about ϵ_p. On the other hand, if there is an irregularity at some point other than

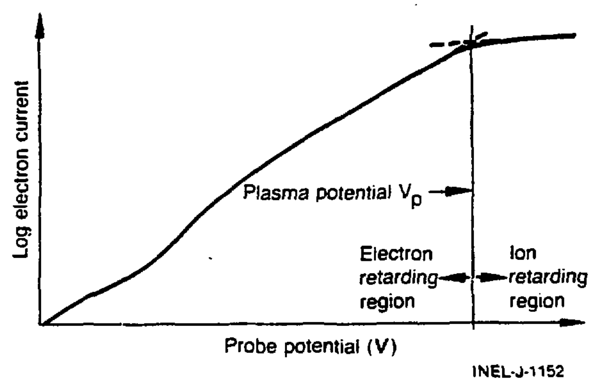

Figure 5. A frequently used technique for determining plasma potential from a single probe trace in moderate-to-dense plasmas.

ϵ_p, the smoothing process (which is intended to filter out point-to-point departures from simple curves) will mask it, and its information will be lost.

As suggested by Figure 3, it is not always easy to determine where such a division point should be. This process can be materially aided by semilogarithmic presentation of the raw data. Semilogarithmic plots of the probe trace curves often show a sharp curvature or slope discontinuity, which would require separating the residual currents. The current from a purely Maxwellian electron population, which in the absence of ion current would appear as a straight line in a semilogarithmic plot, would be entirely exponential, leaving no residual currents to deal with once the ion current was removed.

The discussion thus far has dealt with idealized curves and fairly simple distribution functions. A number of complicating factors tend to cloud the results. These include such things as the tendency for the J and V axes to become nonrectilinear, the variation in work function over the probe surface area, secondary emissions from the probe, probe insulator contamination, noise in the plasma, electrical noise in the probe circuitry, plasma perturbation by the probe, and magnetic effects. The present method does not address these issues.

Ion current has also been neglected until now. In the electron-retarding region of the probe characteristic, ion current is saturated. Two factors tend to make it negligible in probe data reduction using the present method. The first is that it is much smaller in magnitude than electron current in most cases, due to the mass disparity between electrons and ions. The second is that it tends to be a very weak function of potential in the electron-retarding region, usually approximating a first-order increase (decrease in J_e) with electron-retarding potential V. When the second derivative is taken, there is little or no contribution from this constituent. Hence it may be ignored. It is useful, however, to shift the entire J_e curve to lie on the positive side of the V axis.

The exponential current, J_{ex}, is defined by the values of J_e at ϵ_p and at one other location, usually one close to plasma potential. Difficulties are encountered should $J(\epsilon_p)$ be negative. Again, the effect of this shift vanishes when the differentiation occurs.

With the second derivative of the residual currents obtained as described above, the total second derivative is the sum

$$\frac{d^2 J_e}{d\ V^2} = \frac{d^2 J'}{d\ V^2} + B^2 A \exp(-BV) \qquad (19)$$

where A and B are the constants of the exponential approximation

$$J_{ex} = A \exp(-BV) \qquad (20)$$

determined by the two chosen points as described. The distribution function $g(\epsilon)$ is then obtained from of Equations (1) and (5).

Results

The foregoing process has been implemented on a card-programable calculator. Figures 6, 7, and 8 show the calculated distribution function data for three idealized probe curves. The

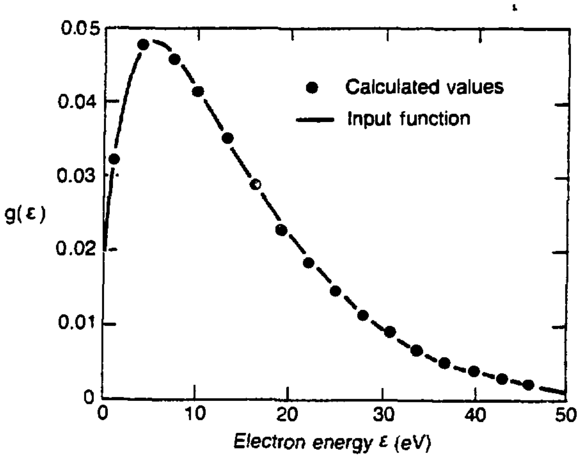

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Figure 6. Energy density function for purely Maxwellian electrons.

distributions shown as inputs were converted to probe traces by numerically integrating Equation (1) twice and plotting. Normal digitization noise was introduced as data were then extracted from the plots and entered into the calculator for analysis. Fifth-order smoothing was used in data reduction. Except for the very sharp curvatures associated with the quasi-monoenergetic primary electron spike in Figure 7, fifth-order smoothing appears to be adequate. Even there, the area under the input curve and calculated point representations are very nearly the same, suggesting the same density of primary electrons in each case. Choosing higher-order smoothing or spacing data points closer would give a more accurate representation of their energy spread.

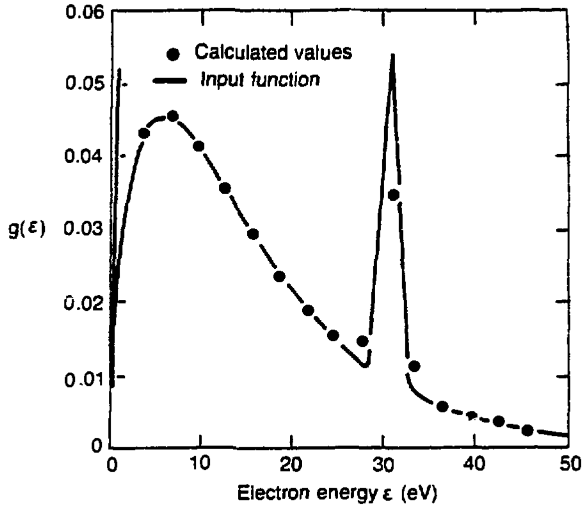

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Figure 7. Energy density function for Maxwellian electrons and monoenergetic primary electrons.

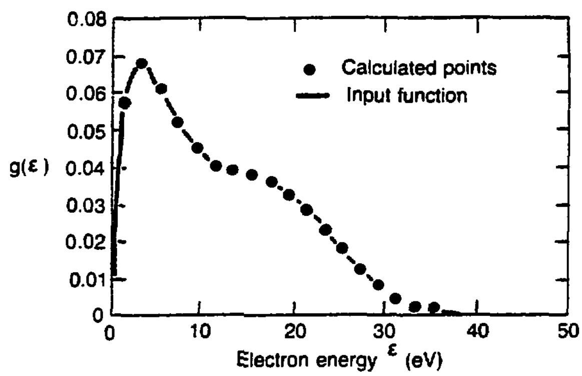

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Figure 8. Energy density function for non-Maxwellian electrons.

Figure 9 shows several distribution functions obtained from actual probe traces taken in an operating mercury ion source. These traces were taken at different locations within the source under steady state operating conditions. Differences in electron density and energy characteristics are plainly evident.

For comparison, the probe traces whose distributions are shown in Figure 9 were analyzed using Beattie's method (Reference 2). Results are listed in Table 1 together with some corresponding parameters obtained using the present method. Agreement is reasonably good.

Conclusions

The method presented here for reducing Langmuir probe data is one of many available. It has the advantages of requiring only a card-programable calculator for implementation and of enabling the experimenter to deal successfully

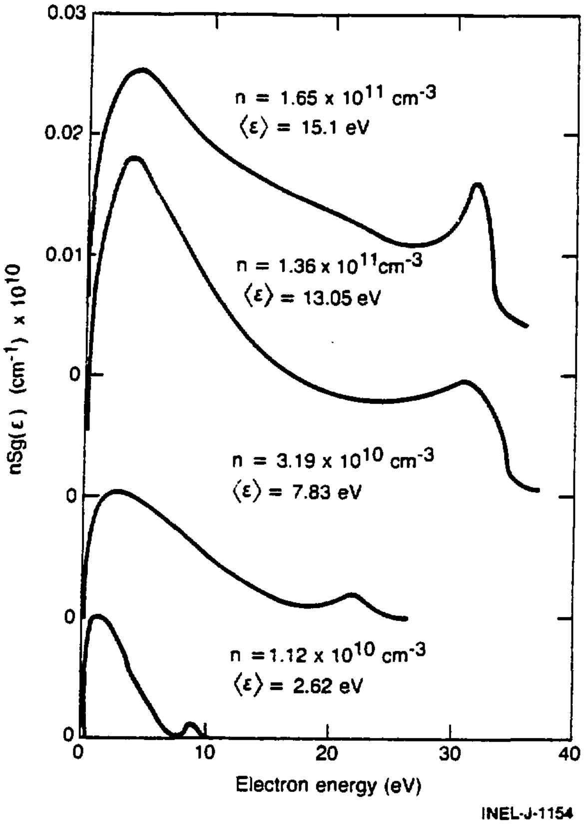

Figure 9. Comparison of calculated electron energy density functions from measurements at four locations in an operating mercury ion source.

with non-Maxwellian electron energy distributions, including those containing spikes of primary electrons. While it has inherent limitations, it has considerable versatility, and its accuracy appears to be satisfactory considering the uncertainties of other aspects in an experimental system.

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Table 1. Comparison of Parameters Calculated for Four Traces Using Different Methods

Beatties Method	Trace 1	Trace 2	Trace 3	Trace 4
Plasma potential (V)	38	40	41	45
Maxwellian electron temperature (eV)	7.0	7.3	4.7	3.9
Maxwellian electron density (cm^{-3})*	1.84 + 11	1.26 + 11	3.68 + 10	1.04 + 10
Primary electron energy (eV)	30.9	32.3	25.2	-
Primary electron density (cm^{-3})*	2.40 + 10	1.48 + 10	9.73 + 8	0
Total electron density (cm^{-3})*	2.09 + 11	1.40 + 11	3.77 + 10	1.04 + 10
Average electron energy (eV)	12.8	13.2	7.5	5.8
Present Method				
Plasma potential (V)	37	38	39	40
Apparent Maxwellian temperature (eV)	7.2	7.0	5.0	2.4
Primary electron energy (eV)	32	31	22	9
Total electron density (cm^{-3})*	1.65 + 11	1.36 + 11	3.19 + 10	1.12 + 10
Average electron energy (eV)	15.1	13.0	7.8	2.5

* a.a + bb implies a.a x 10^{bb}.