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REPORT

Mode Coupling Trigger of Neoclassical Magnetohydrodynamic Tearing Modes in Tokamaks

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Numerical studies of the nonlinear evolution of coupled magnetohydrodynamic — type tearing modes in three-dimensional toroidal geometry with neoclassical effects are presented. The inclusion of neoclassical physics introduces an additional free-energy source for the nonlinear formation of magnetic islands through the effects of a bootstrap current in Ohm's law. The neoclassical tearing mode is demonstrated to be destabilized in plasmas which are otherwise Δ' stable, albeit once a threshold island width is exceeded. A possible mechanism for exceeding or eliminating this threshold condition is demonstrated based on mode coupling due to toroidicity with a pre-existing instability at the q=1 surface.

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The performance gains of the last several years in tokamak fusion plasmas has generated a resurgence in the observation of low helicity magnetic oscillations [1,2]. Often, the onset of such oscillations either cause a violent plasma disruption [3,4] or significantly degrade the plasma confinement [1]. Experimental observations indicate that these instabilities are associated with magnetic reconnection—an interpretation based on the observation of the slow growth of these instabilities, mode numbers which are resonant in the plasma, and the presence of flat-spots in the electron temperature profile about these resonant surfaces [1,2].

One theoretical explanation for such modes is destabilization from the perturbed bootstrap current. Bootstrap currents arise from the viscous damping of the poloidal electron flow. The viscous damping of the portion of the flow produced from the poloidal projection of the diamagnetic current when balanced against electron-ion friction yields a parallel current proportional to the cross-field pressure gradient, i.e., the bootstrap current. In the presence of a magnetic island, the pressure flattens within the island separatrix when parallel transport is fast relative to perpendicular transport. The pressure flattening eliminates the neoclassical bootstrap current within the magnetic island, but a cross-field pressure gradient remains outside the island separatrix. Since the pressure contours deform due to the island formation, a perturbed bootstrap current develops. For an equilibrium with $dp/dq < 0$, where p is the equilibrium pressure and q is the inverse rotational transform, this perturbation produces a destabilizing effect [5,6,8].

The destabilization mechanism is predicated on the assumption that the pressure equilibrates on the modified magnetic surfaces. When the island width is small enough, this is no longer a valid approximation as perpendicular transport mechanisms allow the pressure to cross magnetic surfaces faster than the pressure can equilibrate on the perturbed surfaces. When the magnetic island is smaller than a threshold value, the helical perturbation of the pressure profile about the island is insufficient to destabilize the island. Neoclassical tearing modes therefore require a seed island.

In reference 9, simulation results demonstrated the instability mechanism for neoclassical MHD tearing modes, albeit with an initial condition above the nonlinear island threshold. Experimental evidence suggests that such an initial condition could be stimulated due to the presence of a a 1/1 sawtooth oscillation [10,11] or an edge localized mode (ELM). In this communication, the role of mode coupling to a pre-existing instability at the q=1/1 surface is demonstrated to be a possible mechanism for the creation of the necessary seed island. The simulations can be interpreted in terms of a set of coupled island evolution equations.

Such coupled island evolution equations which incorporate most of the features of neoclassical MHD can be developed by using a nonlinear Rutherford theory [12] amended to include neoclassical effects [13] and expanded to handle mode coupling physics [13–15]. The effects of polarization currents [16] and toroidal shear flow will be ignored, not because they are not potentially important from the experimental standpoint, but rather because they

are not included in the numerical simulations. The result is a set of coupled island evolution equations of the form

$$I_1 \frac{dW_i}{dt} = \Delta_i' + \frac{w_{nc,i} W_i}{W_i^2 + W_d^2} + \sum_j E_{i,j} \frac{W_j^2}{W_i^2} \cos(\phi_{i,j}), \tag{1}$$

where the subscript i refers to a particular island, t has been normalized to the resistive time ($\tau_R = \epsilon^2 R_0^2 \mu_0/\eta$), $I_1 \simeq 0.8227$, ϵ is the inverse aspect ratio, R_0 is the plasma radius, η is the plasma resistivity, Δ' is the stability index from resistive magnetohydrodynamic (MHD) theory [17], $w_{nc,i} = 9.26 \epsilon_s^{0.5} \beta_s'/s_s$ is related to the local bootstrap current,

$$W_d = 5.1 \left(\frac{\chi_\perp}{\chi_{||}}\right)^{0.25} \left(\frac{1}{\epsilon_i s_i n}\right)^{0.5}, \tag{2}$$

$\epsilon_i = \epsilon \rho_i$, $s_i = \rho_i q_i' q_i^{-1}$, $\beta_i' = -\beta_0 q_i^2 \epsilon_i^{-2} p_i'$. Here, $\chi_\perp$ and $\chi_{||}$ are the perpendicular and parallel pressure diffusivities respectively, the mode is resonant at $\rho \equiv r/a = \rho_i \equiv r_i/a$ where $q_i = m_i/n_i$, p_i' is the pressure gradient evaluated at the resonant surface normalized to the pressure on-axis, Δ_i' is the tearing mode matching parameter, and β_0 is the normalized pressure on axis. (Note that the introduction of β_0 is strictly for convenience when making comparisons with the later numerical simulations, the real dependency is on the unnormalized local pressure gradient.) The third term in Eq. (1) describes the perturbed bootstrap current which is typically destabilizing in tokamaks. The novel aspect of neoclassical instabilities is that magnetic islands can occur even in the limit of resistive MHD tearing stability, $\Delta' < 0$. Finally, the $E_{i,j} W_j^2$ represents either coupling to another mode which has an island width of W_j or could also represent the width of a vacuum magnetic island due to a field error. The inclusion of this term is analogous to the model used for double tearing modes [14] and can be developed by application of E-matrix theory [13,15]. The factor $E_{i,j}$ is the magnitude of the coupling between W_i and W_j and in the absence of rotation is expected to be order the inverse aspect ratio [13,15]. In principle, as the summation indicates it is possible for many such couplings to exist, but in general they are limited to the coupling of poloidal modes with the same toroidal mode number (e.g., 1/1, 2/1, 3/1, etc.) and also the nonlinear couplings (1/1, 2/2, 3/3). The factor of $\phi_{i,j}$ is a phase factor between two islands and in general may be a function of time (differential island rotation), but for the numerical simulations of this paper can be set to zero.

In the absence of the fourth term (the mode coupling contribution), this model is the Fitzpatrick model, which is essentially the equivalent of the Qu and Callen model [5] or the Carrera model *et al.* [6] in the limit $\chi_{\|}/\chi_{\perp} \to \infty$, i.e., where the pressure completely equilibrates on each flux surface. The nonlinear saturation of a standard Δ' tearing mode can also be included, by replacing the Δ' with $\Delta_0'(1 - W_{sat}/W_i)$, where W_{sat} is the saturated island width. However, in the case of neoclassical tearing modes, the Δ' will in general be negative and vary little during the island growth.

The dynamics of the island evolution model can be summarized in the simple phase space diagram of Figure 1 which assumes $\Delta' < 0$ and a sufficiently large bootstrap current so that the neoclassical mode has a region of instability (i.e., $w_{nc,i} > 2W_d|\Delta'|$.) Under the assumption that W_j is small (or weak coupling), the model predicts a small island is produced with a magnitude of W_{driven}. This implies that the generated seed island is insufficient to perturb the bootstrap current and that the mode never "enters" the neoclassical regime, which begins at the threshold condition W_{thresh} and which would then lead to the saturated island of width W_{sat}.

However, if the magnitude of W_j is allowed to increase (e.g., it represents an unstable mode), the generated seed island of width W_i will become of sufficient size so that the mode passes into the neoclassical regime. In terms of the phase space diagram this represents a situation when the minimum of dW/dt located between W_{driven} and W_{thresh} equals zero. An appropriate criterion can be derived by identification of this point (and also assuming the saturation is well separated from the threshold vicinity), and is given by

$$W_j^2 > \frac{4}{27}\frac{W_d^4(|\Delta_i'|)^3}{w_{nc,i}^2 E_{i,j}}. \tag{3}$$

Note, that this criterion is on the magnitude of the "unstable" mode and not on the magnitude of the "driven" seed island.

As this criterion reflects, the "unstable" mode must achieve an amplitude such that a sufficiently large seed island is driven. This sufficient size is dependent on four features evaluated at the resonant surface of the "driven" island. Each of these quantities has the potential to vary in time and are represented by, a large bootstrap current (w_{nc}), a large coupling coefficient ($E_{i,j}$), a large ratio of parallel to perpendicular transport (w_d), and also in some sense a small magnitude of Δ', though in the case of the latter it should be emphasized that the expectation is that Δ' is expected to be nearly constant and negative. At this point, it should be reiterated that this model lacks the very important feature of rotational shear which presumably could significantly decrease the magnitude of coupling between the various harmonics and hence the magnitude of any generated seed island [14,7].

The simulation results presented in this paper are based on the *neofar* code which uses a set of model equations based on neoclassical reduced MHD [27]. The details of these equations are described elsewhere [19]. In the simulations, two illustrative cases will be considered: the case of 2/2 & 3/2 harmonics and the case of 1/1, 2/1, 3/2, & 2/2 harmonics. In both cases, many additional harmonics and couplings are neglected which is made possible due to the spectral decomposition in the poloidal and toroidal directions. In principle such an approximation is not egregious, because such couplings are deemed to be lower order. The same equilibrium is used for both cases and the equilibrium harmonics are also allowed to evolve. The equilibrium is arranged so that both the 2/2 and 1/1 harmonics are unstable at the q=1 surface, though the 1/1 is manifestly more unstable. Also the pressure gradient is arranged so that it is zero inside the q=1.27 surface so that neoclassical effects will not modify the dynamics of harmonics resonant at the q=1 surface. The pressure and q-profiles used in the simulation as a function of the flux coordinate ρ are presented in Figure 2.

The results for the first case are illustrated in Figure 3. In this case and in the absence of neoclassical effects, the 2/2 mode grows in amplitude and saturates due to the quasilinear flattening of the equilibrium current gradient in the vicinity of it's rational surface. In this case the 3/2 harmonic is driven to a finite amplitude due to mode coupling and saturates at a small amplitude relative to the 2/2. The novel part of the simulations is that the introduction of neoclassical effects causes the 3/2 harmonic to switch (once it reaches a sufficient amplitude) from being driven by the 2/2 harmonic to one driven by the perturbed bootstrap current. However, to achieve this result the bootstrap current term has been artificially enhanced by a factor of 100. This is likely not an inherent flaw in the simulations, but rather is the result of an inability to simulate experimentally relevant ratio's of $\chi_{||}/\chi_{\perp}$. At the threshold condition, increasing the magnitude of the bootstrap current by a factor of 100 is nominally equivalent to increasing the ratio of $\chi_{||}/\chi_{\perp}$ by a factor of 10^4.

Some of this difficulty can be overcome by considering a more robust instability as illustrated in Figure 4 where the 1/1, 2/1, 3/2, and 2/2 harmonics are included in the simulations. However, the numerical model of the 1/1 dynamics is not strictly correct in this neoclassical reduced MHD model, principally because the model cannot reproduce a sawtooth crash. (Resistivity is a constant across the entire plasma extent.) Nonetheless, the coupling which it produces should still be correct. To artificially produce a sawtooth crash, additional dissipation is added in the Ohm's law on the q=1 resonant modes once the 1/1 harmonic reaches an amplitude of 10^{-4} at it's resonant surface). The effect of many sawtooth cycles coupled to very large differences in the growth rate of the 1/1 and the reconnection rate at the coupled surfaces are not considered here, but are probably also important.

As illustrated in Figure 3 and in the absence of neoclassical effects, the robust growth of the 1/1 drives the 2/1 harmonic due to the toroidal mode coupling, but also nonlinearly drives the 2/2 harmonic which in turn poloidally couples to the 3/2 harmonic. At the sawtooth crash, both the 1/1 and 2/2 modes dramatically decrease as expected from the additional artificial dissipation. Subsequently the 2/1 and 3/2 harmonics lose their drive mechanism and also begin to decay.

The more interesting case is when neoclassical effects alter the dynamics of the 2/1 and 3/2 harmonics after the sawtooth crash. (Note that in the first half of the simulation the neoclassical effects were not on for convenience.) In both cases, the two harmonics are left in a state above their respective neoclassical thresholds and therefore they continue to grow in amplitude.

In conclusion, a possible trigger mechanism for neoclassical MHD tearing modes based on the $1/1$ mode has been demonstrated in a toroidal geometry but in the absence of rotation. The dynamics of the island evolution can be interpreted on the basis of coupled island evolution equations. These island evolution equations are used to formulate a criterion, Eq. (3), on the magnitude of the $1/1$ mode for the triggering of neoclassical MHD tearing modes.

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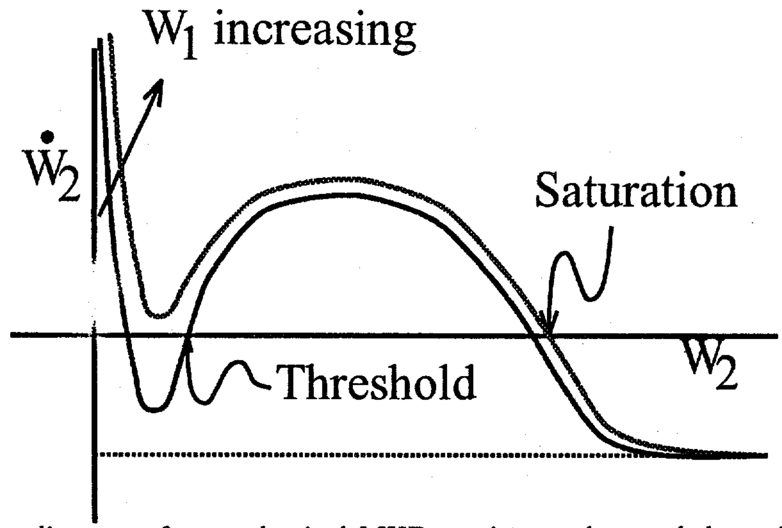

FIG. 1. The phase diagram of a neoclassical MHD tearing mode coupled toroidally to another unstable mode illustrates that when the coupling is weak, a threshold exists for the neoclassical tearing mode, but when the coupling is strong, the threshold is eliminated.

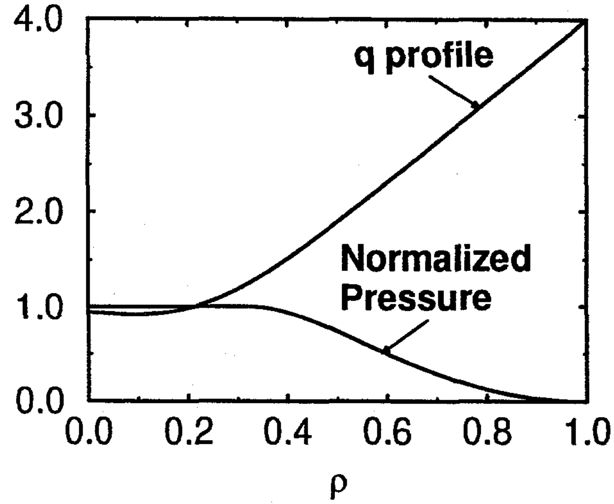

FIG. 2. Equilibrium pressure and q-profiles are arranged so that modes resonant at the q=1 surface are stable to neoclassical effects and modes resonant at q=3/2 and q=2/1 surfaces are stable to resistive (Δ') MHD effects.

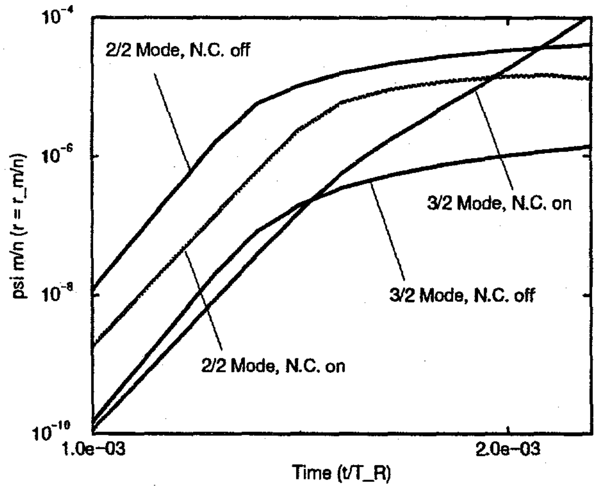

FIG. 3. In the absence of neoclassical effects (N.C. off) the 3/2 harmonic saturates at a finite amplitude due to mode coupling with the 2/2 harmonic. The 2/2 mode is Δ' unstable and saturates due to quasilinear flattening of the equilibrium current gradient. The inclusion of neoclassical effects (N.C. on) from the perturbed bootstrap current eliminates the previous saturated state.

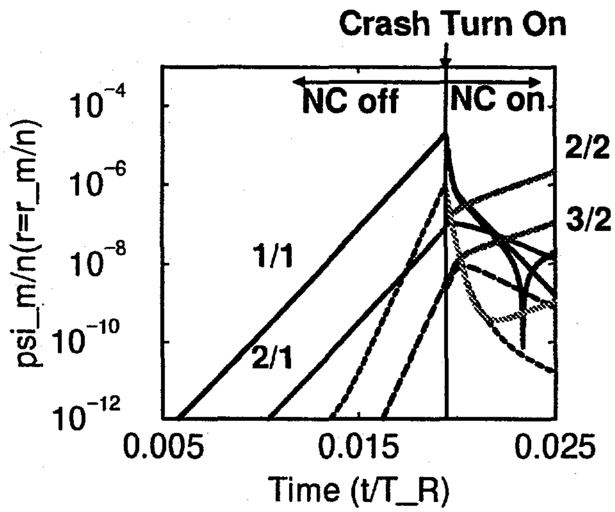

FIG. 4. An unstable 1/1 harmonic drives islands at the q=2/1 and q=3/2 surfaces, which can reach sufficient size to be above the neoclassical threshold. In the absence of neoclassical effects these modes decay. The 1/1 sawtooth crash is initiated by additional additional dissipation to the q=1 resonant harmonics at the crash time.