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Unification of Yang-Mills Theory and Supergravity in Ten Dimensions

G.F. Chapline

Lawrence Livermore National Laboratory
Livermore, CA 94550

N.S. Manton

Institute for Theoretical Physics
University of California, Santa Barbara
Santa Barbara, CA 93106

ABSTRACT

We show how to generalize the coupling of n=1 super-Maxwell theory and n=1 supergravity in 10-dimensions to the case of a non-abelian gauge group. We find that the supergravity 2-form potential $a_{\mu\nu}$ is coupled to the Yang-Mills gauge potential A_μ via the Chern-Simons 3-form.

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It has recently been demonstrated [1,2] that realistic theories of quarks and leptons and their interactions can be derived by dimensional reduction of E_8 gauge theories defined on certain 10-dimensional spaces. Since the fermions are assigned to the adjoint 248 representation of E_8, these theories would be supersymmetric in flat 10-dimensional spacetime. It happens, however, that the 6-dimensional space of extra dimensions must be a compact coset space [1]. To maintain the supersymmetry of the 10-dimensional gauge theory, it therefore seems necessary to introduce supergravity, which restores supersymmetry as a local invariance [3].

The problem of how to unify supersymmetric Yang-Mills theory and supergravity in 4-dimensions was solved some time ago [4], but to our knowledge a solution for 10-dimensional spacetime has not yet appeared. However, the unification of supersymmetric Maxwell theory and simple (n=1) supergravity in 10-dimensions has been achieved [5], and our results and notation are based on this work. One of the interesting features of the Maxwell-supergravity theory is that a gauge transformation of the Maxwell potential a_μ must be accompanied by a transformation of the 2-form supergravity potential $a_{\mu\nu}$, in order that the Lagrangian be gauge invariant.

In this note we show how to generalize the Maxwell-supergravity couplings to obtain a unification of n=1 supergravity and n=1 supersymmetric Yang-Mills theory in 10-dimensions. The Lagrangian is invariant both under local supersymmetry transformations and local non-abelian gauge transformations. We again find that a gauge transformation must be accompanied by a transformation of the supergravity potential $a_{\mu\nu}$.

The n=1, D=10 supergravity multiplet is $\left\{e^m_\mu,\ \psi_\mu,\ a_{\mu\nu},\ \lambda,\ \phi\right\}$; respectively a zehnbein, Rarita-Schwinger field, 2-form potential, Dirac field, and a scalar. The supergravity Lagrangian is [5,6]:

$$
\begin{aligned}
L_{S.G.} = & -\frac{1}{2}eR(e,\omega(e)) - \frac{1}{2}e\bar{\psi}_\mu\Gamma^{\mu\rho\sigma}D_\rho(\omega(e))\psi_\sigma \\
& -\frac{3}{4}e\phi^{-3/2}f_{\rho\mu\nu}f^{\rho\mu\nu} - \frac{1}{2}e\bar{\lambda}\Gamma^\mu D_\mu(\omega(e))\lambda \\
& -\frac{9}{16}e(\partial_\mu\phi/\phi)^2 - \frac{3}{8}\sqrt{2}e\bar{\psi}_\mu(\not{\partial}\phi/\phi)\Gamma^\mu\lambda \\
& +\frac{\sqrt{2}}{16}e\phi^{-3/4}f_{\alpha\beta\gamma}(\bar{\psi}_\mu\Gamma^{\mu\alpha\beta\gamma\nu}\psi_\nu + 6\bar{\psi}^\alpha\Gamma^\beta\psi^\gamma - \sqrt{2}\bar{\psi}_\mu\Gamma^{\alpha\beta\gamma}\Gamma^\mu\lambda) \\
& + \text{4-fermion couplings.} \qquad (1)
\end{aligned}
$$

The first term is the Einstein Lagrangian, except that the connection ω involves some torsion. e denotes det e^m_μ. The 2-form potential $a_{\mu\nu}$ appears only through the 3-form field $f_{\rho\mu\nu}\equiv\partial_{[\rho}a_{\mu\nu]}$, so the transformation $a_{\mu\nu}\to a_{\mu\nu}+\partial_{[\mu}\Phi_{\nu]}$ has no physical effect. The antisymmetrization, here and elsewhere, is with weight one. Multi-index Γ matrices denote antisymmetrized products of Dirac matrices; e.g. $\Gamma^{\rho\mu\nu}=\Gamma^{[\rho}\Gamma^\mu\Gamma^{\nu]}$. Because of the anti-commutation relations, $\Gamma^{\rho\mu\nu}=\Gamma^\rho\Gamma^\mu\Gamma^\nu$, if ρ, μ, ν are all different.

In the Maxwell-supergravity theory, one defines a new 3-form field

$$F^{(Ab)}_{\rho\mu\nu} \equiv f_{\rho\mu\nu} - \frac{\kappa}{\sqrt{2}}a_{[\rho}f_{\mu\nu]} \qquad (2)$$

to effect the coupling of $a_{\mu\nu}$ to the gauge potential a_μ. *Here* $f_{\mu\nu}=\partial_\mu a_\nu-\partial_\nu a_\mu$ and κ is the 10-dimensional gravitational coupling constant. $F^{(Ab)}_{\rho\mu\nu}$ is gauge invariant if one postulates that a gauge transformation $\delta a_\mu=\partial_\mu\Lambda$ is accompanied by a transformation $\delta a_{\mu\nu}=\frac{\kappa}{\sqrt{2}}\Lambda f_{\mu\nu}$. The Maxwell-supergravity Lagrangian is obtained from the pure supergravity Lagrangian by adding a number of manifestly gauge invariant terms and making the replacement $f_{\rho\mu\nu}\to F^{(Ab)}_{\rho\mu\nu}$ [see ref 5].

Now consider the supersymmetry transformation

$$\delta a_\mu=\tfrac{1}{2}\Omega_\mu, \tag{3a}$$

and the part of the supersymmetry transformation of $a_{\mu\nu}$ which involves a_μ:

$$\delta a_{\mu\nu}=\frac{\kappa}{\sqrt{2}}\Omega_{[\mu}a_{\nu]}. \tag{3b}$$

Here $\Omega_\mu\equiv\phi^{3/8}\bar{\varepsilon}\Gamma_\mu\chi$, where χ is the Dirac field of the supersymmetric Maxwell theory and ε is the supersymmetry parameter. Under these transformations the change in the field strength $F^{(Ab)}_{\rho\mu\nu}$ is

$$\delta F^{(Ab)}_{\rho\mu\nu}=-\frac{\kappa}{\sqrt{2}}\Omega_{[\rho}f_{\mu\nu]}. \tag{4}$$

Since the r.h.s. of (4) is gauge invariant, we expect that both the non-abelian generalization of $F^{(Ab)}_{\rho\mu\nu}$ and its order κ supersymmetry variation should be gauge invariant too.

One might guess that to generalize the D=10 Maxwell-supergravity theory to the non-abelian case one would simply introduce traces wherever products of fields carrying an internal symmetry index appear, replace derivatives $\partial_\mu\cdot$ by covariant derivatives $D_\mu\cdot=\partial_\mu\cdot+g[A_\mu,\cdot]$, and replace the Maxwell field tensor $f_{\mu\nu}$ by its non-abelian counterpart $F_{\mu\nu}\equiv\partial_\mu A_\nu-\partial_\nu A_\mu+g[A_\mu,A_\nu]$. However, this is not quite correct. The appropriate generalization of $F^{(Ab)}_{\rho\mu\nu}$ in the Yang-Mills case is

$$F^{(YM)}_{\rho\mu\nu}\equiv f_{\rho\mu\nu}-\frac{\kappa}{\sqrt{2}}\chi_{\rho\mu\nu}, \qquad (5)$$

where $\chi_{\rho\mu\nu}$ is the Chern-Simons 3-form:

$$\chi_{\rho\mu\nu}\equiv\mathrm{Tr}(A_{[\rho}F_{\mu\nu]}-\frac{2}{3}gA_{[\rho}A_\mu A_{\nu]}).$$

It has the property that

$$\partial_{[\sigma}\chi_{\rho\mu\nu]}=\frac{1}{2}\mathrm{Tr}(F_{[\sigma\rho}F_{\mu\nu]}),$$

where the r.h.s. is the second Chern form. In the abelian case one has

$$\partial_{[\sigma}a_\rho f_{\mu\nu]}=\frac{1}{2}f_{[\sigma\rho}f_{\mu\nu]},$$

which shows that $a_{[\rho}f_{\mu\nu]}$ is the abelian Chern-Simons form. It is therefore plausible that $\chi_{\rho\mu\nu}$ should be the non-abelian generalization of $a_{[\rho}f_{\mu\nu]}$.

Under an infinitesimal gauge transformation

$$\delta A_\mu=D_\mu\Lambda, \qquad (6)$$

the Chern-Simons form $X_{\rho\mu\nu}$ changes. However if we postulate an accompanying transformation

$$\delta a_{\mu\nu}=\sqrt{2}\kappa \mathrm{Tr}(\Lambda\partial_{[\mu}A_{\nu]}), \tag{7}$$

then $\delta F^{(YM)}_{\rho\mu\nu}=0$. We show below that $F^{(YM)}_{\rho\mu\nu}$ is also invariant under finite gauge transformations, which is non-trivial in the non-abelian case. The supersymmetry transformations analogous to (3a) and (3b) are

$$\delta A_{\mu}=\frac{1}{2}\Omega_{\mu}, \quad \delta a_{\mu\nu}=\frac{\kappa}{\sqrt{2}}\mathrm{Tr}(\Omega_{[\mu}A_{\nu]}), \tag{8}$$

where Ω_{μ} is the same as before, except that χ is now a Dirac field in the adjoint representation of the gauge group. Under these transformations

$$\delta F^{(YM)}_{\rho\mu\nu}=-\frac{\kappa}{\sqrt{2}}\mathrm{Tr}\,(\Omega_{[\rho}F_{\mu\nu]}). \tag{9}$$

The r.h.s. of (9) is gauge invariant and has the same form as (4) with $F_{\mu\nu}$ replacing $f_{\mu\nu}$.

These results suggest that the Lagrangian unifying Yang-Mills theory and supergravity for D=10 can be obtained from the Maxwell-supergravity theory by replacing $F^{(Ab)}_{\rho\mu\nu}$ by $F^{(YM)}_{\rho\mu\nu}$ in addition to making the usual changes from an abelian to a non-abelian gauge theory mentioned before. The supersymmetry transformation laws in the Yang-Mills case should differ from those in the abelian case only to the extent that $F^{(Ab)}_{\rho\mu\nu}$ is replaced by $F^{(YM)}_{\rho\mu\nu}$ and $f_{\mu\nu}$ by $F_{\mu\nu}$. The proposed Lagrangian and transformations are

$$L = L_{S.G.}\text{(with } F_{\rho\mu\nu} \text{ replaced by } F^{(YM)}_{\rho\mu\nu})$$

$$-\frac{1}{4}e\phi^{-3/4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

$$-\frac{1}{2}e\mathrm{Tr}(\bar{\chi}\not{D}(\hat{\omega})\chi)$$

$$-\frac{1}{8}\kappa e\phi^{-3/8}\mathrm{Tr}(\bar{\chi}\Gamma^{\mu}\Gamma^{\rho\sigma}(F_{\rho\sigma}+\hat{F}_{\rho\sigma}))(\psi_{\mu}+\frac{1}{12}\sqrt{2}\Gamma_{\mu}\lambda)$$

$$+\frac{1}{16}\sqrt{2}\kappa e\phi^{-3/4}\mathrm{Tr}(\bar{\chi}\Gamma^{\alpha\beta\gamma}\chi)\hat{F}^{(YM)}_{\alpha\beta\gamma}$$

$$-\frac{1}{16\mathrm{x}96}\sqrt{2}\kappa^{2}e\mathrm{Tr}(\bar{\chi}\Gamma_{\alpha\beta\gamma}\chi)\bar{\psi}_{\mu}(4\Gamma^{\alpha\beta\gamma}\Gamma^{\mu}+3\Gamma^{\mu}\Gamma^{\alpha\beta\gamma})\lambda$$

$$-\frac{1}{512}\kappa^{2}e\mathrm{Tr}(\bar{\chi}\Gamma_{\alpha\beta\gamma}\chi)\bar{\lambda}\Gamma^{\alpha\beta\gamma}\lambda; \qquad (10)$$

$$\delta e^{m}_{\mu} = \frac{1}{2}\bar{\varepsilon}\Gamma^{m}\psi_{\mu}, \quad \delta\phi = -\frac{1}{3}\sqrt{2}\,\bar{\varepsilon}\,\lambda\phi,$$

$$\delta a_{\mu\nu} = \frac{1}{4}\sqrt{2}\phi^{3/4}(\bar{\varepsilon}\Gamma_{\mu}\psi_{\nu}-\bar{\varepsilon}\,\Gamma_{\nu}\psi_{\mu}-\frac{1}{2}\sqrt{2}\,\bar{\varepsilon}\,\Gamma_{\mu\nu}\lambda)+\frac{1}{2}\sqrt{2}\kappa\phi^{3/8}\bar{\varepsilon}\Gamma_{[\mu}\mathrm{Tr}(\chi A_{\nu]}),$$

$$\delta\lambda = -\frac{3}{8}\sqrt{2}(\hat{\not{D}}\phi/\phi)\varepsilon+\frac{1}{8}\phi^{-3/4}\Gamma^{\alpha\beta\gamma}\varepsilon\hat{F}^{(YM)}_{\alpha\beta\gamma}+\frac{1}{12\mathrm{x}36}\sqrt{2}\kappa\mathrm{Tr}(\bar{\chi}\Gamma^{\alpha\beta\gamma}\chi)\Gamma_{\alpha\beta\gamma}\varepsilon,$$

$$\delta\psi_{\mu} = D_{\mu}(\hat{\omega}(e,\psi))\varepsilon+\frac{1}{32}\sqrt{2}\phi^{-3/4}(\Gamma_{\mu}{}^{\alpha\beta\gamma}-9\delta^{\alpha}_{\mu}\Gamma^{\beta\gamma})\varepsilon\hat{F}^{(YM)}_{\alpha\beta\gamma}$$

$$-\frac{1}{16\mathrm{x}32}(\Gamma_{\mu}{}^{\alpha\beta\gamma}-5\delta^{\alpha}_{\mu}\Gamma^{\beta\gamma})\varepsilon\bar{\lambda}\Gamma_{\alpha\beta\gamma}\lambda$$

$$+\frac{1}{96}\sqrt{2}\left[(\bar{\psi}_{\mu}\Gamma_{mn}\lambda)\Gamma^{mn}\varepsilon+(\bar{\lambda}\Gamma_{mn}\varepsilon)\Gamma^{mn}\psi_{\mu}+2(\bar{\psi}_{\mu}\lambda)\varepsilon-2(\bar{\lambda}\varepsilon)\psi_{\mu}+4(\bar{\psi}_{\mu}\Gamma_{m}\varepsilon)\Gamma^{m}\lambda\right]$$

$$-\frac{1}{256}\kappa\mathrm{Tr}(\bar{\chi}\Gamma^{\alpha\beta\gamma}\chi)(\Gamma_{\mu\alpha\beta\gamma}-5g_{\mu\alpha}\Gamma_{\beta\gamma})\varepsilon,$$

$$\delta A_{\mu} = \frac{1}{2}\phi^{3/8}\bar{\varepsilon}\Gamma_{\mu}\chi,$$

$$\delta\chi = -\frac{1}{4}\phi^{-3/8}\Gamma^{mn}\hat{F}_{mn}\varepsilon$$

$$+\frac{1}{64}\sqrt{2}\kappa\left[3(\bar{\lambda}\chi)\varepsilon-\frac{3}{2}(\bar{\lambda}\Gamma^{\alpha\beta}\chi)\Gamma_{\alpha\beta}\varepsilon-\frac{1}{24}(\bar{\lambda}\Gamma^{\alpha\beta\gamma\delta}\chi)\Gamma_{\alpha\beta\gamma\delta}\varepsilon\right]. \qquad (11)$$

The caret symbol over a field denotes the supercovariant generalization of that field. For the exact formula, see ref. 5.

Our proof that the Lagrangian (10) is supersymmetric depends on showing that all terms occuring in the variation of the Lagrangian under a supersymmetry transformation have essentially the same gauge covariant form in the non-abelian and abelian cases. Since in the abelian theory the net variation vanishes [5], the same will be true in the non-abelian theory. Because the Lagrangian and supersymmetry transformations, when expressed in terms of $F^{(Ab)}_{\rho\mu\nu}$ and $F^{(YM)}_{\rho\mu\nu}$ respectively, have the same form in both cases, it is clear that at the algebraic level the variations of the Lagrangian do too. However, this argument may not be sufficient since many terms in the variation of L involve partial derivatives of the field variations, and it is not obvious that these occur in the required gauge covariant combinations.

Instead, we use the general result that if L depends on a number of fields ϕ_i then

$$\delta L = \frac{\delta L}{\delta \phi_i}\, \delta\phi_i, \tag{12}$$

where $\frac{\delta L}{\delta \phi_i} = 0$ would be a classical field equation. A number of integrations by parts are necessary to obtain (12) and thereby eliminate all derivatives of $\delta\phi_i$. With the exception of $\delta\psi_\mu$ all the field variations are algebraic in ε, so this formalism is convenient for comparing the non-abelian and abelian cases.

It is clear from eqs. (10) and (11) that almost all terms in the field equations and field variations will be manifestly covariant and of the same form in the abelian and non-abelian cases. The possible exceptions are those terms arising from variations of $F^{(YM)}_{\rho\mu\nu}$ (or $F^{(Ab)}_{\rho\mu\nu}$) with respect to A_μ (or a_μ) and $a_{\mu\nu}$. To see what happens for these terms, consider the simplified Lagrangian

$$\tilde{L} = -\frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{3}{4}F^{(YM)}_{\rho\mu\nu}F^{(YM)\rho\mu\nu} \qquad (13)$$

and the partial variations δA_μ and $\delta a_{\mu\nu}$ given by (8). In this case, eq. (12) becomes

$$\delta\tilde{L} = \frac{1}{2}\mathrm{Tr}\left\{(D_\mu F^{\mu\rho} + \frac{3\kappa}{\sqrt{2}}F^{(YM)\rho\mu\nu}F_{\mu\nu})\Omega_\rho\right\}, \qquad (14)$$

which is covariant; the non-covariant terms proportional to $(\partial_\rho F^{(YM)\rho\mu\nu})\Omega_\mu A_\nu$ which occur in the separate variations with respect to A_μ and $a_{\mu\nu}$ cancel. Eq. (14) can also be derived by using eq. (9) to compute the variation of the second term in (13), and then non-covariant terms never appear.

The abelian case is quite analogous. If

$$\tilde{L} = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{3}{4}F^{(Ab)}_{\rho\mu\nu}F^{(Ab)\rho\mu\nu}$$

and δa_μ and $\delta a_{\mu\nu}$ are given by (3a) and (3b) then

$$\delta\tilde{L} = \frac{1}{2}(\partial_\mu f^{\mu\rho} + \frac{3\kappa}{\sqrt{2}}F^{(Ab)\rho\mu\nu}f_{\mu\nu})\Omega_\rho,$$

which is of the same form as (14). This result extends easily to the full Lagrangian and supersymmetry transformations and completes our argument that the supersymmetry of the Maxwell-supergravity theory implies the supersymmetry of the Yang-Mills generalization, eq. (10).

One remaining question about the Yang-Mills-supergravity theory is whether the Lagrangian is invariant under finite gauge transformations. The answer is yes, as can be seen as follows. Under a finite gauge transformation, A_μ changes to

$$A^h_\mu = h^{-1}A_\mu h + \frac{1}{g}h^{-1}\partial_\mu h$$

and the Chern-Simons form $\chi_{\rho\mu\nu}$ changes to

$$\chi^h_{\rho\mu\nu} = \chi_{\rho\mu\nu} - \frac{2}{g}\partial_{[\rho}\mathrm{Tr}(\partial_\mu h h^{-1}A_{\nu]})$$
$$- \frac{2}{3g^2}\mathrm{Tr}\left[(\partial_{[\rho}h)h^{-1}(\partial_\mu h)h^{-1}(\partial_{\nu]}h)h^{-1}\right].$$

Since both $\partial_{[\sigma}\chi^h_{\rho\mu\nu]}$ and $\partial_{[\sigma}\chi_{\rho\mu\nu]}$ are equal to the gauge invariant quantity $\frac{1}{2}\mathrm{Tr}(F_{[\sigma\rho}F_{\mu\nu]})$, $\chi^h_{\rho\mu\nu} - \chi_{\rho\mu\nu}$ is a closed 3-form. If the 10-dimensional spacetime has the topology of $\mathbb{R}^{10}$, then $\chi^h_{\rho\mu\nu}-\chi_{\rho\mu\nu}$ is also exact; i.e. there exists a 2-form $J_{\mu\nu}$ such that

$$\chi^h_{\rho\mu\nu} = \chi_{\rho\mu\nu} + \partial_{[\rho}J_{\mu\nu]}. \qquad (15)$$

If we postulate that

$$a^h_{\mu\nu} = a_{\mu\nu} + \frac{\kappa}{\sqrt{2}}J_{\mu\nu}, \qquad (16)$$

then the 3-form $F^{(YM)}_{\rho\mu\nu}$ and hence the complete Lagrangian (10) is gauge invariant.

It is customary to assume that all fields in a gauge theory decay *rapidly at spatial infinity in which case* $\mathbf{R}^{10}$ *is effectively* compactified to $S^9 x\mathbf{R}^1$, and gauge transformations h(x) tend rapidly to 1 at infinity. With this topology $\chi^h_{\rho\mu\nu}-\chi_{\rho\mu\nu}$ is still exact, since the relevant cohomology group $H_3(S^9)$ is trivial. The interpretation of this result is that when $h\to 1$ at infinity, $J_{\mu\nu}$ can be chosen to vanish sufficiently rapidly at infinity that its surface integrals there are zero.

Suppose now that h(x) is a gauge transformation homotopic to the identity. Then there exists a continuous family of gauge transformations $\tilde{h}(x,\tau)$, $0\le\tau\le 1$, such that $\tilde{h}(x,0)=1$ and $\tilde{h}(x,1)=h(x)$. In this case we can give an explicit formula for $J_{\mu\nu}$:

$$J_{\mu\nu} = -\frac{2}{g}\mathrm{Tr}(\partial_{[\mu}hh^{-1}A_{\nu]}) + \frac{2}{g^2}\int_0^1 \mathrm{Tr}\left[\partial_{[\mu}\tilde{h}\tilde{h}^{-1}\frac{d}{d\tau}(\partial_{\nu]}\tilde{h}\tilde{h}^{-1})\right]d\tau. \qquad (17)$$

This formula was derived from a more general formula of Chern's [7]. If $h(x)=1+g\Lambda(x)$, with Λ infinitesimal, then the natural choice $\tilde{h}(x,\tau)=1+g\tau\Lambda(x)$ gives $J_{\mu\nu}=2\mathrm{Tr}(A_{[\mu}\partial_{\nu]}\Lambda)$. Substituting this expression into (16) leads to a transformation law for $a_{\mu\nu}$ which differs from (7) only by a physically *irrelevant curl*. If the gauge transformation is not homotopic to the identity then formula (17) cannot be used; however $J_{\mu\nu}$ still exists and formulae (15) and (16) give the transformation laws for $\chi_{\rho\mu\nu}$

and $a_{\mu\nu}$.

To summarize, we have generalized the Maxwell-supergravity theory in 10-dimensions and obtained the Yang-Mills-supergravity theory. The appearance of the Chern-Simons 3-form is the most novel feature. This form has appeared before in gauge theories, notably in the analysis of the axial anomaly [8], and also as a mass term in 3-dimensional Yang-Mills theory [9].

It is conceivable that Chern-Simons forms, or some generalization to include fermionic fields, may play a much larger role in supergravity theories. Recall that part of the 11-dimensional supergravity Lagrangian is $Y \equiv F \wedge F \wedge A$, where A is a 3-form potential and $F=dA$ [10]. Y is a Chern-Simons form, because in dimension 12 or greater $dY = F \wedge F \wedge F$, and integrals of $F \wedge F \wedge F$ over closed 12-manifolds are topological invariants.

It would be most interesting to consider the dimensional reduction of the Yang-Mills-supergravity theory to 4-dimensions. If one used the conventional method, where all fields are simply assumed to be independent of the six extra coordinates, one should obtain an N=4 supergravity theory coupled to six N=4 vector multiplets as well as to an N=4 Yang-Mills multiplet. This theory has been predicted to be finite to all orders in perturbation theory by Kallosh [11]. On the other hand, one could use the more sophisticated dimensional reduction procedure [12], where the extra dimensions form a compact coset space. This would extend the models discussed in refs. 1 and 2 to include gravity. The resulting 4-dimensional theories would have less than N=4 supersymmetry and might be more realistic phenomenologically.

It is interesting that the present theory has the same zero mass particle content in 10-dimensions as the theory of interacting unoriented supersymmetric strings [13]. However, our theory is not the zero slope limit of the string theory and the dimensional reduction just referred to can be carried out without assuming that the scale size of the space of extra dimensions is extremely small.

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