Initial Energy Density of Quark-Gluon Plasma in Relativistic Heavy-Ion Collisions*

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Recently, there is considerable interest in the central rapidity region of highly relativistic heavy-ion collisions.[1-5] Such an interest stems from the possibility of creating hadron matter of high energy density which may exceed the critical energy density for a phase transition between ordinary confined matter and the unconfined quark-gluon plasma.[6] The experimental searches and identification of the quark-gluon plasma may provide a new insight into the question of quark confinement. Furthermore, the creation of a domain of high-energy density may allow one to study matter under unusual conditions such as those which exist in the history of the early universe.

The estimate of the initial energy density is quite uncertain. The initial energy density is nonetheless an important physical quantity. It is one of the factors which determines whether the produced matter can undergo phase transition or not. In Ref. 1 it was assumed that in a central collision a nucleon in one nucleus only makes a single collision with nucleons in the other nucleus. The number of collisions a nucleon suffers in traversing the other nucleus has not been properly treated. The energy density has also been estimated previously by using the color neutralization model of Brodsky et al.[7] However, the color neutralization model gives a central rapidity multiplicity in heavy-ion collision too low by a factor of two.[4] For this reason, we wish to obtain a better estimate of the energy density (in the central rapidity region).

As is well known, a simple Glauber-type multiple collision model can reproduce the total multiplicity and multiplicity plateau near the central rapidity region to within 30%.[8] The simple multiple collision model has an approximate validity as a gross description of the reaction process. We shall adopt a semi-empirical approach. Using the multiple collision model and the thickness function of Glauber,[9] we obtain analytical functional form for all the quantities in question. A single parameter, r_{rms}, is adjusted to fit the experimental central rapidity multiplicity data. The semi-empirical results provide a useful tool to extrapolate to the unknown central rapidity region of heavy-ion collisions.

In the multiple collision model, a nucleon in one nucleus makes many inelastic collisions with nucleons in the other nucleus, the probability of collision being given by the thickness function and the total nucleon-nucleon inelastic cross section. A nucleon may change its identity during its passagae through the other nucleus, but its baryon number remains unchanged. Each nucleon-nucleon or baryon-baryon collision degrades the energies and momenta of the colliding baryons and produces particles outside the nucleus[10] in accordance with the experimental nucleon-nucleon particle production data. Thus, the initial multiplicity distribution shortly after the two nuclei interpenetrate each other comes from nucleon-nucleon collisions in an additive manner. For a given configuration, the degree of this additivity is related to the number of nucleon-nucleon collisions $n(b)$ when the two nuclei pass through each other.

We discuss first the crudest approximation in which one takes the multiplicity distribution to be the same for each nucleon-nucleon or baryon-baryon collision. Then, the initial multiplicity distribution for the collision of nuclei A and B at an impact parameter b is related to the multiplicity distribution for nucleon-nucleon collision dN/dy for the same nucleon-nucleon center-of-mass energy by

$$\frac{dN^{AB}}{dy}(b) = \frac{dN}{dy} n(b). \tag{1}$$

When we average over the impact parameters, the average multiplicity distribution is given by

$$\left\langle \frac{dN^{AB}}{dy} \right\rangle = \frac{dN}{dy} \frac{AB\sigma_{in}}{\sigma_{in}^{AB}}, \tag{2}$$

where σ_{in} is the nucleon-nucleon inelastic cross section and σ_{in}^{AB} is the nucleus-nucleus inelastic cross section.

With a Gaussian form of the thickness function, we obtain the following functional form for the multiplicity distribution

$$\frac{dN^{AB}}{dy}(b) = \frac{dN}{dy} \frac{ABfe}{[1-(1-fe)^{AB}]}, \tag{3}$$

where

$$f = \sigma_{in}/2\pi\beta^2,$$

$$e = \exp[-b^2/2\beta^2],$$

and $\beta^2 = r_{rms}^2 (A^{2/3} + B^{2/3})/3 + \beta_p^2.$

Here, r_{rms} is the root-mean-square radius parameter and $\beta_p = 0.68$ fm is the standard deviation of the nucleon-nucleon thickness function. The functional form of the ratio of the average multiplicity distributions is then

$$R\left(\frac{AB}{pp}, y\right) = \frac{\left\langle \frac{dN^{AB}}{dy} \right\rangle}{\frac{dN}{dy}} = \frac{ABf}{\sum\limits_{i=1}^{AB} [1 - (1-f)^i]/i}. \tag{4}$$

If we mindlessly apply Eqs. (3) and (4) by using the root-mean-squared radius parameter as determined by electron scattering, the theoretical results consistently exceed the experimental values. In order to correct for this

systematic discrepancy, we shall adopt a semi-empirical approach. This consists of assuming the functional form of Eqs. (3) and (4) with its only parameter r_{rms} so chosen as to fit the available central rapidity multiplicity data. We found that r_{rms} = 1.15 fm gives good fits to the experimental central multiplicity data[11-13] of pα, dd, αα, Si+Ag, and Ca+C (Table I). The results of Eqs. (3) and (4) can be used to discuss the collisions of other nuclei.

Using the relationship between dN^{AB}/dy and the energy density ε as given by Ref. 1, we can calculate the energy density in the central rapidity region. The energy density $\varepsilon(b)$ in the collision of nuclei A and B with an impact parameter b is given by

$$\frac{\varepsilon(b)}{\varepsilon_0} = \frac{AB\, T(b)\sigma_{in}}{\{1-[1-T(b)\sigma_{in}]^{AB}\}\mathscr{A}(b)} \quad (5)$$

where $T(b)$ is the normalized thickness function and can be approximated by

$$T(b) = \exp[-b^2/2\beta^2]/2\pi\beta^2,$$

Table I

Nuclei		$(dN^{AB}/dy)/(dN/dy)$	
A	B	exp	Theory
p	p	1	1
p	α	1.18 ± 0.07	1.19
d	d	1.4 ± 0.10	1.42
α	α	1.74 ± 0.09	1.71
Si	Ag	~ 95	96.0
Ca	C	~ 25	27.1

$\mathscr{A}(b)$ is the transverse overlap area of the two nuclei, and ε_0 is a bombarding-energy-dependent unit of energy density. At the time of 1 fm/c after the nuclei collide with each other, particles are produced. The energy density unit ε_0 at that time is 0.84 GeV/fm³ for $\sqrt{s}/A$ = 11.8 GeV, 1.14 GeV/fm³ for $\sqrt{s}/A$ = 31.4 GeV, and 1.80 GeV/fm³ for $\sqrt{s}/A$ = 270 GeV. We calculate the initial energy density for head-on collisions of two equal nuclei. In Fig. 1 we show this quantity as a function of A. The numerical result can be parameterized as
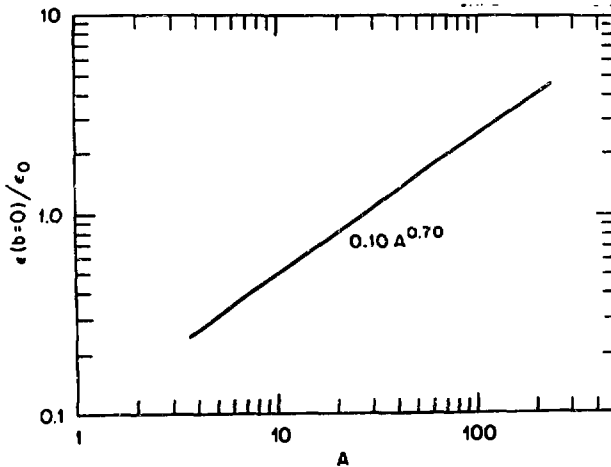

Figure 1

$$\frac{\varepsilon(b=0)}{\varepsilon_0} = 0.10\, A^{0.709}$$

which goes approximately as $A^{2/3}$, as expected from Eq. (5). We also calculate the energy density in units of ε_0 as a function of the impact parameter for the collision of ^{238}U on ^{238}U. The results are shown in Fig. 2. The energy density is about 5 units of ε_0. It oscillates as a function of the impact parameter with an amplitude of about 0.5 ε_0. Our energy density estimates are greater than those of the previous estimates[1,4] as the detailed treatments are different. At an incident energy of $\sqrt{s}/A$ = 30 GeV per nucleon,

this corresponds to an energy density of about 5 GeV/fm^3 which exceeds the energy density of 2 GeV/fm^3 estimated to be the critical energy density for a phase transition.[4]
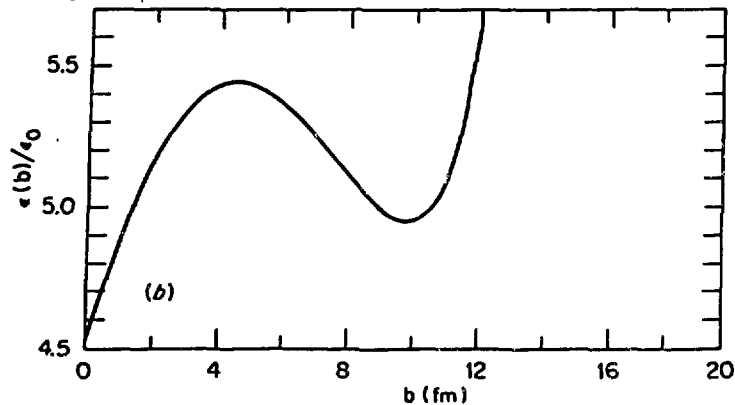

Figure 2

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