Statistical Analyses of Scatterplots to Identify Important Factors in Large-Scale Simulations,1: Review and Comparison of Techniques

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Abstract

Procedures for identifying patterns in scatterplots generated in Monte Carlo sensitivity analyses are described and illustrated. These procedures attempt to detect increasingly complex patterns in scatterplots and involve the identification of (i) linear relationships with correlation coefficients, (ii) monotonic relationships with rank correlation coefficients, (iii) trends in central tendency as defined by means, medians and the Kruskal-Wallis statistic, (iv) trends in variability as defined by variances and interquartile ranges, and (v) deviations from randomness as defined by the chi-square statistic. A sequence of example analyses with a large model for two-phase fluid flow illustrates how the individual procedures can differ in the variables that they identify as having effects on particular model outcomes. The example analyses indicate that the use of a sequence of procedures is a good analysis strategy and provides some assurance that an important effect is not overlooked.

Key Words: Chi-square, correlation coefficient, epistemic uncertainty, interquartile range, Kruskal-Wallis, Latin hypercube sampling, mean, median, Monte Carlo, partial correlation coefficient, rank transform, scatterplot, sensitivity analysis, standardized regression coefficient, subjective uncertainty, top-down correlation, variance

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1. Introduction

Sensitivity analysis is now widely recognized as an essential component of studies based on mathematical modeling (e.g., Refs. 1-6). Here, sensitivity analysis refers to the determination of the effects of uncertain model inputs on model predictions. A number of methods have been proposed for sensitivity analysis, including differential analysis, response surface methodologies, Monte Carlo techniques, and the Fourier amplitude sensitivity test.[7,8,9]

Monte Carlo techniques probably constitute the most widely used approach to sensitivity analysis due to their flexibility, ease of implementation, and conceptual simplicity. When viewed abstractly, a Monte Carlo sensitivity study involves a vector

$$\mathbf{x} = [x_1, x_2, \ldots, x_{nI}] \tag{1}$$

of uncertain model inputs, where each x_i is an uncertain input and nI is the number of such inputs, and a vector

$$\mathbf{y} = \mathbf{f}(\mathbf{x}) = [y_1, y_2, \ldots, y_{nO}] \tag{2}$$

of model predictions, where $\mathbf{f}$ is a function used to represent the model under consideration, each y_j is an outcome of evaluating the model with the input $\mathbf{x}$, and nO is the number of such outcomes. Distributions

$$D_i,\ i = 1, 2, \ldots, nI, \tag{3}$$

are used to characterize the uncertainty in each input x_i, where D_i is the distribution assigned to x_i. Correlations and other relationships between the x_i are also possible.

A sampling procedure such as simple random sampling or Latin hypercube sampling[10] is used to generate a sample

$$\mathbf{x}_k = [x_{1k}, x_{2k}, \ldots, x_{nI,k}],\ k = 1, 2, \ldots, nS, \tag{4}$$

from the population of $\mathbf{x}$'s with the distributions in Eq. (3), where nS is the size of the sample. Evaluation of the model under consideration with the sample elements $\mathbf{x}_k$ in Eq. (4) then creates a sequence of results of the form

$$\mathbf{y}_k = \mathbf{f}(\mathbf{x}_k) = [y_{1k}, y_{2k}, \ldots, y_{nO,k}],\ k = 1, 2, \ldots, nS, \tag{5}$$

where each y_{jk} is a particular outcome of evaluating the model with $\mathbf{x}_k$. The pairs

$$(\mathbf{x}_k, \mathbf{y}_k),\ k = 1, 2, \ldots, nS, \tag{6}$$

constitute a mapping from model input $\mathbf{x}_k$ to model output $\mathbf{y}_k$ that can be explored with various sensitivity analysis techniques to determine how the individual analysis inputs contained in $\mathbf{x}$ (i.e., the x_i's) affect the individual analysis outcomes contained in $\mathbf{y}$ (i.e., the y_j's). Analysis possibilities include regression analysis, correlation analysis, and examination of scatterplots.[8,9,11-17]

Although techniques based on regression analysis and correlation analysis are often successful in identifying the relationships between model input and output embedded in the mapping in Eq. (6), these techniques may fail to identify well-defined, but nonlinear, relationships.[7,14,15,18] If the underlying relationship is nonlinear but monotonic, then a rank transformation will linearize the relationship and result in successful sensitivity analyses with regression-based techniques.[19] However, the underlying relationship can be too complex to be linearized in any simple manner. In these cases, sensitivity analysis techniques are needed that can identify patterns in the mapping in Eq. (6) without recourse to specialized prespecified relationships (e.g., linear or monotonic). The ultimate test of whether or not there is a relationship between an input variable x_i and an output variable y_j lies in determining whether or not the points

$$(x_{ik}, y_{jk}),\ k = 1, 2, \ldots, nS, \tag{7}$$

constitute a random pattern conditional on the marginal distributions for x_i and y_j. This paper will investigate the implications from a sensitivity analysis perspective of a sequence of tests (i.e., hypotheses) for the relationship between x_i and y_j. These hypotheses will run from very specific (i.e., a linear relationship) to quite general (i.e., a nonrandom pattern).

The paper is organized as follows. Example simulation results that will be used to motivate and illustrate the sensitivity analysis procedures are presented in Sect. 2. Then, the analysis procedures are summarized in Sects. 3-7. Specifically, the following five relationships are proposed as the basis for a sequence of sensitivity tests: (i) Linear relationship: $E(y|x) = \beta_0 + \beta_1 x$, where the subscripts have been dropped from y_j and x_i for notational simplicity (Sect. 3); (ii) Monotonic relation: $E[r(y)|r(x)] = \gamma_0 + \gamma_1 r(x)$, where $r(x)$ and $r(y)$ denote the ranks of x and y, respectively (Sect. 4); (iii) Location (central tendency) of y depends on x (Sect. 5); (iv) Variability (spread) of y depends on x (Sect. 6); and (v) y and x are statistically independent: $p(y|x) = p(y)$, where p denotes the density function for y (Sect. 7). Next, the ranking of variable importance and the use of the Iman and Conover[20] top-down correlation procedure to compare variable rankings are discussed in Sects. 8 and 9. Then, examples of the indicated procedures are presented in Sect. 10, and a concluding discussion is given in Sect. 11. A following article discusses Type I and Type II errors and the robustness of analysis outcomes for independent samples.[21]

2. Test Problems

The test problems use results obtained in the 1996 performance assessment (PA) for the Waste Isolation Pilot Plant (WIPP),[18] which was carried out to support the U.S. Department of Energy's (DOE's) application to the U.S. Environmental Protection Agency (EPA) for the certification of the WIPP for the disposal of transuranic waste.[22] In particular, the test problems involve results (Table 1) calculated by the BRAGFLO model (Sect. 4.2, Ref. 18), which was used to represent two phase (i.e., gas and brine) flow in the vicinity of the repository. The BRAGFLO model uses finite difference procedures (Fig. 1) to numerically solve a system of nonlinear partial differential equations (Eqs. 4.2.1 - 4.2.6, Ref. 18) and requires a significant amount of computational resources (e.g., 4 to 5 hours of CPU time on a VAX Alpha with VMS for a single model evaluation).

The 1996 WIPP PA used Latin hypercube sampling to propagate the effects of subjective (i.e., epistemic) uncertainty through the analysis.[18] As a result of guidance given by the EPA,[23] the PA used a Latin hypercube sample (LHS) of size 300 (Sect. 6.3, Ref. 18) from 75 uncertain variables, of which only 27 were used as inputs to the BRAGFLO model in the calculation of the dependent variables in Table 1 (Table 2). To provide a test of the robustness of the uncertainty propagation procedures, the indicated LHS was actually generated as 3 independent samples of size 100 each (Sect. 6.4, Ref. 18). Each of these samples was generated with use of the Iman and Conover restricted pairing technique[24,25] to enforce specified correlations between three pairs of variables (the correlated pairs (*ANHCOMP*, *ANHPRM*) and (*HALCOMP*, *HALPRM*) are used in the calculation of the results in Table 1 and are described in Table 2) and also to ensure that uncorrelated variables had correlations close to zero. The outcome of this sampling was 3 LHSs of size 100 each:

$$R1:\ \mathbf{x}_{1k} = [\mathbf{x}_{1k1}, \mathbf{x}_{1k2}, \ldots, \mathbf{x}_{1k75}], \quad k = 1, 2, \ldots, 100 \tag{8}$$

$$R2:\ \mathbf{x}_{2k} = [\mathbf{x}_{2k1}, \mathbf{x}_{2k2}, \ldots, \mathbf{x}_{2k75}], \quad k = 1, 2, \ldots, 100 \tag{9}$$

$$R3:\ \mathbf{x}_{3k} = [\mathbf{x}_{3k1}, \mathbf{x}_{3k2}, \ldots, \mathbf{x}_{3k75}], \quad k = 1, 2, \ldots, 100, \tag{10}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_{75}]$ corresponds to the 75 uncertain variables indicated in Table 2 and $R1$, $R2$ and $R3$ designate the three replicated (i.e., independently generated) LHSs.

Once the LHSs in Eqs. (8) - (10) were generated, BRAGFLO calculations were performed for a variety of cases (Table 6.9.1, Ref. 18). The two cases considered here are (i) undisturbed (i.e., E0) conditions, and (ii) a drilling intrusion through the lower waste panel at 1000 yr that does not penetrate pressurized brine in the underlying Castile Formation (i.e., E2 conditions or, in the more detailed descriptions given in Ref. 18, E2 conditions with the intrusion occurring at 1000 yr). Results calculated by BRAGFLO are time-dependent. The time-dependent behavior of the results is shown in Fig. 2 for replicate $R1$. For simplicity, the technique comparisons will use the values of the variables at the end points of the individual curves in Fig. 2 (i.e., at 10,000 yr). However, nothing prevents analyses at other times and, in general, sensitivity analyses of time-dependent variables should also be time-dependent (Chapts. 7, 8, Ref. 18).

For perspective and motivation, regression-based results for the variables in Table 1 (obtained in Ref. 18 with the STEP program[26]) are presented in Table 3 for both raw and rank-transformed data. In Table 3, a variable was required to be significant at an α-value of 0.02 to enter a regression model and to remain significant at an α-value of 0.05 to be retained in a regression model, although there were no cases of a variable entering and then being dropped from a regression model. As will be seen, the rank-transformation is often an effective procedure for improving the resolution of regression-based sensitivity analyses. However, as will also be seen, nonmonotonic relationships can result in patterns that cannot be effectively analyzed with rank-transformed data. It is the need to be able to identify such patterns that forms the motivation for this study.

The analyses in Table 3 for repository pressure under undisturbed conditions (*E0:WAS_PRES*) with raw and rank-transformed data are reasonably effective, with (1) R^2 values of 0.82 and 0.81 for raw and rank-transformed data, (2) the same variables selected in both analyses, and (3) only one minor variation in the order of variable selection (i.e., the order of selection of the last two variables in the regression models is reversed). Scatterplots for the first four variables selected in the regression analyses for *E0:WAS_PRES* are presented in Fig. 3. The scatterplots for the first two variables selected in the regression analysis, *WMICDFLG* and *HALPOR*, display well-defined patterns. The pattern for the third variable, *WGRCOR*, is weaker but still detectable. The fourth variable, *ANHPRM*, changes the R^2 values for raw and rank-transformed data by 0.02 and 0.01, respectively, and produces a scatterplot that displays little discernible pattern.

The analyses for cumulative brine inflow from all anhydrite marker beds to the repository under undisturbed conditions (*E0:BRAALIC*) are interesting in that the regression with raw data is not particularly effective (i.e., R^2 = 0.50 at final step of analysis), whereas the regression with rank-transformed data is reasonably successful in accounting for the observed uncertainty (i.e., R^2 = 0.87). Again, examination of scatterplots shows well-defined patterns for the first two variables, *WMICDFLG* and *ANHPRM*, selected in both regression analyses (Fig. 4). Scatterplots for the next two variables, *HALPOR* and *WGRCOR*, selected in the regression analysis with rank-transformed data are also given in Fig. 4. The negative effects of these variables, as indicated by the signs of their standardized regression coefficients, are barely discernible in their scatterplots, with these small effects being consistent with observed changes in R^2 values of 0.05 and 0.02 with the entry of *HALPOR* and *WGRCOR*, respectively, into the regression model. In this example, the regression analyses with both raw and rank-transformed data have identified the two dominant variables, *WMICDFLG* and *ANHPRM*. However, the analysis with raw data in isolation would not be very credible due to its low R^2 value.

The regression analysis with raw data for brine saturation in the lower waste panel after an E2 intrusion (*E2:WAS_SATB*) is quite poor, with the final regression model containing 6 variables but having an R^2 value of only

0.33. The regression analysis with rank-transformed data does somewhat better and results in a final regression model with 6 variables and an R^2 value of 0.61. However, an R^2 value of 0.61 is not particularly reassuring with respect to whether or not all the variables giving rise to the observed uncertainty in *E2:WAS_SATB* have been identified. Additional insights can be obtained by examining scatterplots (Fig. 5). The first two variables identified in the regression analysis with rank-transformed data, *BHPRM* and *WRGSSAT*, show well-defined, and interacting, patterns. In particular, *BHPRM* is the primary determinant of whether or not a high value for *E2:WAS_SATB* occurs; however, given that a high value for *E2:WAS_SATB* occurs, this value is almost completely determined by *WRGSSAT*. Despite the well-defined patterns involving *BHPRM* and *WRGSSAT*, the regression analysis with raw data results in incremental R^2 values of only 0.12 and 0.02 for these two variables, and the regression analysis with rank-transformed data results in incremental R^2 values of only 0.36 and 0.16. The next two variables selected in the regression analysis with rank-transformed data are *ANHPRM* and *HALPOR*. The scatterplot plots for these variables do not show particularly strong patterns, with a stronger pattern actually being shown for the fourth-selected variable, *HALPOR*, than for the third-selected variable, *ANHPRM*. For *E2:WAS_SATB*, the two dominant variables, *BHPRM* and *WRGSSAT*, appear in the regression analyses for both raw and rank-transformed data. However, the R^2 values associated with these regressions (i.e., 0.33 and 0.61) provide little assurance that the dominant variables have been identified. It is only after examination of the associated scatterplots and the development of a physical explanation for the patterns appearing in these plots that some degree of comfort emerges that the dominant variables have indeed been identified.

The final regressions in Table 3 are for pressure in the lower waste panel after an E2 intrusion (*E2:WAS_PRES*). The regression analyses with both raw and rank-transformed data perform very poorly and result in final regression models with R^2 values of only 0.22 and 0.20, respectively. Both regression models select *HALPRM*, *ANHPRM* and *HALPOR*, with the scatterplots for these three variables appearing in Fig. 6. Examination of these scatterplots does not reveal what is giving rise to the observed uncertainty in *E2:WAS_PRES*. In particular, this uncertainty does not appear to arise from either *HALPRM*, *ANHPRM* or *HALPOR* individually or from some form of interaction between these variables. At this point in the analysis reported in Ref. 18, a systematic search was made through the scatterplots for *E2:WAS_PRES* and the remaining variables in Table 2, with this search revealing that the uncertainty in *E2:WAS_PRES* is dominated by *BHPRM* (Fig. 6d). This is disconcerting because the clearly dominant variable was not even identified in the regression with raw or rank-transformed data. In contrast, the analyses for *E2:WAS_SATB* included the dominant variables in the regression models even though the R^2 values were low. As an aside, the interesting pattern involving *E2:WAS_PRES* and *BHPRM* in Fig. 6d results from two phase flow in the borehole connecting the waste panel with overlying formations, with gas typically flowing up the borehole and brine typically flowing down the borehole.[18]

As should be apparent from the regressions in Table 3 and the associated scatterplots in Figs. 3-6, the examination of scatterplots is an important part of sampling-based sensitivity analysis and can reveal patterns that are missed by regression-based procedures. The variables in Table 1 will be used to illustrate a number of procedures for the identification of patterns in scatterplots. These variables were selected to illustrate pattern identification procedures because they constitute a spectrum of analysis possibilities. In particular, regression analysis with both raw and rank-transformed data performs well for *E0:WAS_PRES*; regression analysis with rank-transformed, but not raw, data performs well for *E0:BRAALIC*; regression models with neither raw nor rank-transformed data perform well for *E2:WAS_SATB* but both models still include the two dominant variables; and regression analysis with raw and rank-transformed data fails to identify the dominant variable for *E2:WAS_PRES*.

3. Linear Relation: $y = \beta_0 + \beta_1 x$

The coefficients β_0 and β_1 in a first-order polynomial can be estimated with the well-known ordinary least squares procedure. Specifically, $\hat{\beta}_0$ and $\hat{\beta}_1$ are given by

$$\hat{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}, \tag{11}$$

where

$$\hat{\beta} = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{bmatrix}, \mathbf{X} = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_{nS} \end{bmatrix}, \mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_{nS} \end{bmatrix}$$

and the superscript T denotes matrix transpose.[27] The estimated linear regression model is

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x, \tag{12}$$

with the coefficients $\hat{\beta}_0$ and $\hat{\beta}_1$ deriving from the sampled and calculated values contained in the pairs (x_k, y_k), $k = 1, 2, \ldots, nS$, as indicated in Eq. (11).

The linear correlation coefficient ρ_{xy}, which is also called the Pearson correlation coefficient, provides the most commonly used measure to assess the strength of the linear relationship between x and y in Eq. (12) and is defined by

$$\rho_{xy} = \sigma_{xy} / (\sigma_x \sigma_y), \tag{13}$$

where σ_{xy} denotes the covariance between x and y, and σ_x and σ_y denote the standard deviation of x and y, respectively. In turn, ρ_{xy} is estimated by

$$\hat{\rho}_{xy} = \sum_{k=1}^{nS} (x_k - \bar{x})(y_k - \bar{y}) \Bigg/ \left[\sum_{k=1}^{nS} (x_k - \bar{x})^2\right]^{1/2} \left[\sum_{k=1}^{nS} (y_k - \bar{y})^2\right]^{1/2} \tag{14}$$

where

$$\bar{x} = \sum_{k=1}^{nS} x_k / nS, \; \bar{y} = \sum_{k=1}^{nS} y_k / nS.$$

The quantity $\hat{\rho}_{xy}$ is often called the sample correlation coefficient.

The reason why ρ_{xy}, and hence $\hat{\rho}_{xy}$, provides a measure of the strength of the linear relationship between x and y is not immediately apparent from Eqs. (13) and (14). Rather, this reason is perhaps best understood in the context of the regression model in Eq. (12) with both x and y standardized to variables with a mean of 0 and a standard deviation of 1; that is,

$$\tilde{x}_k = (x_k - \bar{x}) / \hat{\sigma}_x, \; \tilde{y}_k = (y_k - \bar{y}) / \hat{\sigma}_y, \tag{15}$$

where

$$\hat{\sigma}_x = \left[\sum_{k=1}^{nS} (x_k - \bar{x})^2 / (nS - 1) \right]^{1/2}, \hat{\sigma}_y = \left[\sum_{k=1}^{nS} (y_k - \bar{y})^2 / (nS - 1) \right]^{1/2}.$$

Then Eq. (11) yields the regression model

$$(y - \bar{y}) / \hat{\sigma}_y = 0 + \hat{\rho}_{xy} (x - \bar{x}) / \hat{\sigma}_x = \hat{\rho}_{xy} (x - \bar{x}) / \hat{\sigma}_x. \tag{16}$$

Thus, $\hat{\rho}_{xy}$ is the standardized regression coefficient relating x to y. As such, $\hat{\rho}_{xy}$ characterizes the effect that changing x by a fixed fraction of its standard deviation will have on y, with this effect being measured relative to the standard deviation of y.

In addition, the correlation coefficient ρ_{xy}, and hence $\hat{\rho}_{xy}$, provides a measure of the fraction of the variance of y that can be accounted for by x. Again, this is best seen in the context of the regression model in Eq. (12), for which the following identity can be established:[27]

$$\sum_{k=1}^{nS} (y_k - \bar{y})^2 = \sum_{k=1}^{nS} (\hat{y}_k - \bar{y})^2 + \sum_{k=1}^{nS} (\hat{y}_k - y_k)^2. \tag{17}$$

The summation $\sum_k (\hat{y}_k - \bar{y})^2$ represents the part of the variance of y that can be accounted for by $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$, with the result that

$$R^2 = \sum_{k=1}^{nS} (\hat{y}_k - \bar{y})^2 \bigg/ \sum_{k=1}^{nS} (y_k - \bar{y})^2 \tag{18}$$

represents the fraction of the variance of y accounted for by x in a linear approximation to y. The preceding quantity is called the R^2 value or the coefficient of determination for x and y. An R^2 value close to 1 indicates that x can account for most of the uncertainty in y; in contrast, an R^2 value close to 0 indicates that a linear relationship involving x accounts for little of the uncertainty in y.

Like the standardized regression coefficient, the R^2 value can be expressed in terms of $\hat{\rho}_{xy}$. The vector equality in Eq. (11) leads to

$$\hat{\beta}_1 = \sum_{k=1}^{nS} (x_k - \bar{x})(y_k - \bar{y}) \bigg/ \sum_{k=1}^{nS} (x_k - \bar{x})^2 \tag{19}$$

and

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}. \tag{20}$$

Given the preceding representations for $\hat{\beta}_0$ and $\hat{\beta}_1$, some simple algebraic manipulations lead to

$$\sum_{k=1}^{nS} (\hat{y}_k - \bar{y})^2 = \left[\sum_{k=1}^{nS} (x_k - \bar{x})(y_k - \bar{y}) \right]^2 \bigg/ \sum_{k=1}^{nS} (x_k - \bar{x})^2. \tag{21}$$

Hence, from Eqs. (18) and (21),

$$R^2 = \left[\sum_{k=1}^{nS} (x_k - \bar{x})(y_k - \bar{y}) \right]^2 \bigg/ \left[\sum_{k=1}^{nS} (x_k - \bar{x})^2 \right] \left[\sum_{k=1}^{nS} (y_k - \bar{y})^2 \right] = \hat{\rho}_{xy}^2. \tag{22}$$

Thus, the square of the sample correlation coefficient is equal to the fraction of the variance of y that can be accounted for by $\hat{y}$ as defined in Eq. (12), and hence by x under a linear transformation.

The preceding has given two interpretations of the correlation coefficient ρ_{xy}. First, the sample correlation coefficient $\hat{\rho}_{xy}$ can be viewed as the estimated regression coefficient $\hat{\beta}_1$ in Eq. (12) when x and y are standardized to mean 0 and standard deviation 1. Second, $\hat{\rho}_{xy}$ can be viewed as the square root of the R^2 value for the regression model in Eq. (12) (i.e., $\hat{\rho}_{xy}^2 = R^2$). The correlation coefficient can also be viewed as a parameter in a joint normal distribution involving x and y (see Sect. 2.13, Ref. 28); however, this interpretation is not as intuitively appealing as the two involving the regression model in Eq. (12). Moreover, x and y typically do not have normal distributions in sampling-based sensitivity analyses (e.g., see indicated distributions in Table 2).

When $\hat{\rho}_{xy}$ is close to 1 or −1, an almost linear relationship exists between x and y (see definition of $R^2 = \hat{\rho}_{xy}^2$ in Eq. (18)). However, large changes in x may still result in small changes in y if the regression coefficient $\hat{\beta}_1$ in Eq. (12) is small. Indeed, the magnitude $|\hat{\beta}_1|$ of $\hat{\beta}_1$ is not a very informative quantity because $|\hat{\beta}_1|$ depends on the units in which x and y are expressed (e.g., changing the units on x from millimeters to kilometers will have a large effect on $|\hat{\beta}_1|$ but no effect on the underlying physical relationships). For this reason, x and y are often standardized to mean 0 and standard deviation 1. As previously discussed, this standardization results in the equality $\hat{\beta}_1 = \hat{\rho}_{xy}$ and also in $\hat{\beta}_1$ characterizing changes in y normalized to $\hat{\sigma}_y$ relative to changes in x normalized to $\hat{\sigma}_x$.

Although $\hat{\rho}_{xy}^2 \doteq 1$ implies a strong linear dependence between x and y, $\hat{\rho}_{xy} \doteq 0$ cannot be used to infer that no relationship exists between x and y (i.e., that x and y are independent). In particular, zero correlations can occur in the presence of a nonmonotonic relationship between x and y. For example, $\rho_{xy} = 0$ for $y = 1 - x^2$ with $-1 \le x \le 1$ and also for $y = \cos x$ with $0 \le x \le 2\pi$. A more interesting example is given by the scatterplot for *BHPRM* in Fig. 6d. Thus, a linear relationship can be assumed to exist between x and y if $|\hat{\rho}_{xy}|$ is close to 1. Further, linear relationships of lesser strength (i.e., smaller R^2 values) exist for smaller values of $|\hat{\rho}_{xy}|$. For $|\hat{\rho}_{xy}| \doteq 0$, the implication is that no linear relationship exists between x and y.

A significance test can be used to indicate if $\hat{\rho}_{xy}$ appears to be different from 0. For example,

$$t = \hat{\rho}_{xy}(nS-2)^{1/2} / (1-\hat{\rho}_{xy}^2)^{1/2} \tag{23}$$

has a t-distribution with nS–2 degrees of freedom when x, y are uncorrelated and have a bivariate normal distribution (p. 631, Ref. 29). Further,

$$z = \hat{\rho}_{xy}\sqrt{nS} \tag{24}$$

is distributed approximately normally with mean 0 and standard deviation 1 when x and y are uncorrelated, x and y have enough convergent moments (i.e., the tails of their distributions die off sufficiently rapidly), and nS is large (typically > 500) (p. 631, Ref. 29). Then,

$$prob(|r|>|\hat{\rho}_{xy}|) = erfc(|\hat{\rho}_{xy}|\sqrt{nS} / \sqrt{2}), \tag{25}$$

where $prob(|r|>|\hat{\rho}_{xy}|)$ is the probability that random variation would produce a value r for $\hat{\rho}_{xy}$ larger in absolute value than the observed value $\hat{\rho}_{xy}$ and *erfc* is the complementary error function (i.e., $erfc(x) = \left(2/\sqrt{\pi}\right)\int_x^\infty exp(-t^2)dt$) (p. 631, Ref. 29). Significance results obtained with t in Eq. (23) converge to those obtained with z in Eq. (24) as nS

increases. However, as x and y are unlikely to have normal distributions in real analysis problems, results obtained with t and small values of nS should simply be viewed as one form of guidance as to whether or not a linear relationship actually exists between x and y.

If several x_i have scatterplots that appear to have nonzero values for $\hat{\rho}_{x_i y}$, then the relative importance of these x_i can be ordered by the absolute values of $\hat{\rho}_{x_i y}$. This is equivalent to ordering the x_i on the basis of the strength of the linear relationship associated with the pairs (x_{ik}, y_k), $k = 1, 2, \ldots, nS$. This is also equivalent to ordering the x on the basis of p-values obtained from the distributions associated with Eq. (23) or (24), where the p-value designates the probability that a value for $\hat{\rho}_{xy}$ will be obtained that exceeds the observed value for $\hat{\rho}_{xy}$ in absolute value (i.e., $prob(|r| > |\hat{\rho}_{xy}|)$ in Eq. (25)). Actually, the ordering is done on the complements of the p-values because smaller p-values are associated with larger values for $|\hat{\rho}_{x_i y}|$.

Standardized multiple regression coefficients are another popular way of ranking variable importance.[16,30-33] However, when the x_i are independent, the standardized multiple regression coefficient for x_i is equal to $\hat{\rho}_{x_i y}$ and so the two rankings are identical. Specifically, the multiple regression model relating y to the x_i has the form

$$\hat{y} = \hat{\beta}_0 + \sum_{i=1}^{nI} \hat{\beta}_i x_i, \tag{26}$$

where $\hat{\beta}$ has the same functional form as in Eq. (11) with[27]

$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} \hat{\beta}_0 \\ \vdots \\ \hat{\beta}_{nI} \end{bmatrix}, \mathbf{X} = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1nI} \\ \vdots & \vdots & & \vdots \\ 1 & x_{nS} & \cdots & x_{nS,nI} \end{bmatrix}, \mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_{nS} \end{bmatrix}.$$

If the x_{ik}'s have been selected so that the rows of $\mathbf{X}$ are orthogonal (i.e., so that $\mathbf{X}^T\mathbf{X}$ is a diagonal matrix with diagonal elements $d_0, d_1, \ldots, d_{nI}$, which is equivalent to the individual x_i being independent and thus having sample correlations of 0), then

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

$$= \begin{bmatrix} d_0 & 0 & \cdots & 0 \\ 0 & d_1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & d_{nI} \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_{11} & x_{21} & \cdots & x_{nS,1} \\ \vdots & \vdots & & \vdots \\ x_{1nI} & x_{2nI} & \cdots & x_{nSnI} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{nS} \end{bmatrix} \tag{27}$$

and so

$$\hat{\beta}_i = \sum_{k=1}^{nS} x_{ik} y_k / d_k = \sum_{k=1}^{nS} x_{ik} y_k \bigg/ \sum_{k=1}^{nS} x_{ik}^2. \tag{28}$$

Thus, when x_i and y are standardized to mean 0 and standard deviation 1 (see Eq. (15)),

$$\hat{\beta}_i = \sum_{k=1}^{nS} (x_{ik} - \bar{x}_i)(y_k - \bar{y}) \bigg/ \left[\sum_{k=1}^{nS} (x_{ik} - \bar{x}_i)^2 \right]^{1/2} \left[\sum_{k=1}^{nS} (y_k - \bar{y})^2 \right]^{1/2} = \hat{\rho}_{x_i y} \tag{29}$$

and the standardized multiple regression coefficient $\hat{\beta}_i$ and the (sample) correlation coefficient $\hat{\rho}_{x_i y}$ are equal.

Partial correlation coefficients are another popular way of ranking variable importance.[16, 33, 34] However, the partial correlation coefficient is just a special form of the sample correlation coefficient. In particular, if least squares techniques are used to determine the coefficients in

$$\hat{x}_j = \hat{\alpha}_0 + \sum_{\substack{i=1 \\ i \neq j}}^{nI} \hat{\alpha}_i x_i \text{ and } \hat{y} = \hat{\beta}_0 + \sum_{\substack{i=1 \\ i \neq j}}^{nI} \hat{\beta}_i x_i, \tag{30}$$

then the partial correlation coefficient $\hat{p}_{x_j y}$ between x_j and y is the sample correlation $\hat{\rho}_{\hat{x}_j \hat{y}}$ determined for the pairs $(x_{jk} - \hat{x}_{jk}, y_k - \hat{y}_k)$, $k = 1, 2, \ldots, nS$. Thus, $\hat{p}_{x_j y}$ is the sample correlation between x_j and y after a correction has been made for the linear effects of the other x_i.

The following relationship exists between $\hat{p}_{x_j y}$ and the standardized regression coefficient $\hat{\beta}_j$ in Eq. (29):

$$\hat{p}_{x_j y} = \hat{\beta}_j [(1 - R_j^2) / (1 - R_y^2)]^{1/2}, \tag{31}$$

where R_j^2 is the R^2 value that results from regressing x_j on y and the x_i, $i = 1, 2, \ldots, nI$ with $i \neq j$, and R_y^2 is the R^2 value that results from regressing y on the x_i, $i = 1, 2, \ldots, nI$ (Eq. (1), Ref. 35). If the x_i are orthogonal, then

$$R_y^2 = \sum_{i=1}^{nI} R_i^2 = \sum_{i=1}^{nI} \hat{\beta}_i^2 = \sum_{i=1}^{nI} \hat{\rho}_{x_i y}^2, \tag{32}$$

with the first equality following from Eq. (III-74) of Ref. 36, and the second and third equalities following from Eqs. (22) and (29). Thus,

$$\hat{p}_{x_j y} = \hat{\beta}_j \left[(1-\hat{\beta}_j^2) \Big/ \left(1 - \sum_{i=1}^{nI} \hat{\beta}_i^2 \right) \right]^{1/2} = \hat{\rho}_{x_j y} \left[(1-\hat{\rho}_{x_j y}^2) \Big/ \left(1 - \sum_{i=1}^{nI} \hat{\rho}_{x_i y}^2 \right) \right]^{1/2} . \tag{33}$$

Because of the inequality

$$b(1-b^2)^{1/2} > a(1-a^2)^{1/2} \tag{34}$$

for $a^2 + b^2 < 1$ and $a < b$ (Fig. 7), an ordering of variable importance based on $|\hat{p}_{x_j y}|$, $|\hat{\beta}_j|$ or $|\hat{\rho}_{x_j y}|$ produces the same results when the x_i are orthogonal; further, the values for $\hat{\beta}_j$ and $\hat{\rho}_{x_j y}$ will be the same and generally different from $\hat{p}_{x_j y}$.

Due to the conceptual simplicity of the sample correlation coefficient $\hat{\rho}_{xy}$ and its close relationship to standardized regression coefficients and partial correlation coefficients in the presence of orthogonal values for the x_i's, this study will use $\hat{\rho}_{xy}$ to assess the strength of the linear relationship between x and y. In the presence of small deviations from orthogonality (i.e., the existence of small correlations between the x_i), the three measures will still give similar results. However, in the presence of large deviations from orthogonality, the three measures can give quite different, and possibly misleading, indications of the effects of individual variables.

As noted earlier, $\hat{\rho}_{xy} \doteq 0$ should not be interpreted to mean that no relationship exists between x and y. For example,

$$y = \beta_0 x^{\beta_1} \tag{35}$$

results in a low, but nonzero, value for ρ_{xy} even though there is no noise in the relationship between x and y. In this case, a logarithmic transformation will linearize the relationship between x and y. However, such transformations may not exist and, given that they do exist, identifying them is not always easy. For example, logarithmic transformations are not applicable when some of the y values are zero, which is a fairly common analysis situation. One possible transformation of fairly broad applicability is the rank transformation, which is discussed in Sect. 4.

A possible complication in the use of $\hat{\rho}_{xy}$ to identify the existence of a relationship between x and y can be the existence of interactions with other variables. For example, the relationship between y, x_1, and x_2 might be of the form

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2, \tag{36}$$

which can also be expressed as

$$y = \beta_0 + \beta_1[1-(\beta_{12}/\beta_1)x_2]x_1 + \beta_2 x_2$$

$$= \beta_0 + \beta_1 x_1 + \beta_2[1-(\beta_{12}/\beta_2)x_1]x_2 .$$

As long as the variation in x_1 is large relative to the variation in $1-(\beta_{12}/\beta_1)x_2$ or the variation in x_2 is large relative to the variation in $1-(\beta_{12}/\beta_1)x_1$, the fact that x_1 or x_2 does indeed have a significant effect on y should be identified by the corresponding value for $\hat{\rho}_{xy}$. Thus, it is not considered necessary to specifically consider interaction effects to identify important variables, although it is certainly possible to calculate $\hat{\rho}_{xy}$ with $x = x_i x_j$ if desired. Further, use of contingency tables to be discussed later (Sect. 7) allows the identification of nonlinear effects without the assumption of a specific model form.

4. Monotonic Relation: $r(y) = \gamma_0 + \gamma_1 r(y)$

When the relationship between x and y is nonlinear but monotonic, the relationship can be linearized by a rank transformation. Specifically, the pairs (x_k, y_k) are transformed into a new sequence of pairs

$$[r(x_k), r(y_k)],\ k = 1, 2, \ldots, nS, \tag{37}$$

where (i) the smallest value of x_k is assigned a rank of 1 (i.e., $r(x_k) = 1$), the next largest value of x_k is assigned a rank of 2 (i.e., $r(x_k) = 2$), and so up to the largest value of x_k, which is assigned a rank of nS (i.e., $r(x_k) = nS$), (ii) averaged ranks are assigned to equal values of x_k (e.g., if $x_j = x_k$, $x_l \neq x_j$ for $l \neq j, k$, and $p - 1$ observations have values less than x_j, then $r(x_j) = r(x_k) = (p + p + 1)/2$), and (iii) the assignment of the ranks for y (i.e., $r(y_k)$) is accomplished in the same manner as the assignment of ranks for x.

Rank-transformed data can be analyzed in exactly the same manner as discussed in Sect. 3 for untransformed data. In particular, the strength of the linear relationship between the rank-transformed variables in Eq. (37) can be measured with Spearman's rank correlation coefficient for x and y, η_{xy}, which is simply Pearson's correlation coefficient in Eq. (14) calculated on ranks. The test for zero rank correlation uses a table of quantiles for $|\hat{\eta}_{xy}|$ (e.g., Table A10, Ref. 37). For $nS \geq 30$,

$$z = \hat{\eta}_{xy}\sqrt{nS-1} \tag{38}$$

approximately follows the normal distribution for $\eta_{xy} = 0$ (p. 456, Ref. 37), which is very similar to the approximation to the distribution indicated for $\hat{\rho}_{xy}$ in Eq. (24). Thus, similarly to Eq. (25) for $\hat{\rho}_{xy}$,

$$prob(|r| > |\hat{\eta}_{xy}|) = erfc(|\hat{\eta}_{xy}|\sqrt{nS-1}/\sqrt{2}), \tag{39}$$

where $prob(|r| > |\eta_{xy}|)$ is the probability that random variation would produce a value r for η_{xy} larger in absolute value than the observed value $\hat{\eta}_{xy}$.

Regression coefficients and partial correlation coefficients can also be calculated with rank-transformed data as discussed in Sect. 3.[16,33,38-41] As an aside, the form of the regression model after y and the x_i's have been standardized to mean 0 and standard deviation 1 is

$$(y-\bar{y})/\hat{\sigma}_y = \sum_{i=1}^{nI} (\hat{\beta}_i \hat{\sigma}_{x_i} / \hat{\sigma}_y)(x_i - \bar{x}_i)/\hat{\sigma}_{x_i}, \tag{40}$$

where $\hat{\beta}_i$ is the regression coefficient obtained with the original (i.e., nonstandardized) values for y and the x_i's. When rank-transformed data are being used and there are no ties in the y or x_i values, then $\hat{\sigma}_{x_i} = \hat{\sigma}_y$ and so the standardized regression coefficient (i.e., $\hat{\beta}_i \hat{\sigma}_{x_i} / \hat{\sigma}_y$) is the same as the original, nonstandardized coefficient (i.e., $\hat{\beta}_i$). Thus, standardization is automatically accomplished by the use of rank-transformed data as long as there are no ties in the y and x values.

Closely related to Spearman's coefficient is Kendall's τ (pp. 255 - 260, Ref. 37). Because both coefficients give nearly identical significance results, this alternative for identifying monotonic relationships is considered only briefly. Kendall's τ measures the degree of concordance in a set of observations of the form in Eq. (17). The pairs (x_r, y_r) and (x_s, y_s) are said to be concordant if both members of one pair are less than the corresponding members of the other pair (i.e., $x_r < x_s$, $y_r < y_s$ or $x_r > x_s$, $y_r > y_s$). Further, the pairs are said to be discordant if the two members in one pair differ in opposite directions from the corresponding members in the other pair (i.e., $x_r < x_s$, $y_r > y_s$ or $x_r > x_s$, $y_r < y_s$). Kendall's τ is estimated by

$$\hat{\tau}_{xy} = (N_c - N_d)/[nS(nS-1)/2], \tag{41}$$

where N_c is the number of concordant pairs of observations, N_d is the number of discordant pairs of observations, and $nS(nS-1)/2$ is the total number of pairs $\{(x_r, y_r), (x_s, y_s)\}$ of observations. The statistic $\hat{\tau}_{xy}$ has a distribution that is adequately approximated by the normal distribution for sample sizes as small as $nS = 8$. In contrast, larger samples (e.g., $nS \geq 30$) are required for $\hat{\eta}_{xy}$ to approach a normal distribution; fortunately, Monte Carlo sensitivity studies typically use sample sizes larger than $nS = 30$. Because estimates for Spearman's coefficient $\hat{\eta}_{xy}$ and Kendall's $\hat{\tau}_{xy}$ produce similar rankings of monotonicity and $\hat{\eta}_{xy}$ is more intuitively appealing because of its close relationship to Pearson's coefficient $\hat{\rho}_{xy}$, this presentation will use $\hat{\eta}_{xy}$ to identify nonlinear but monotonic relationships in scatterplots.

5. Location of y Dependent on x

Tests for two distinct types of patterns in scatterplots were considered in Sects. 3 and 4, with the Pearson correlation coefficient used to identify linear patterns (Sect. 3) and the Spearman correlation coefficient used to identify nonlinear but monotonic patterns (Sect. 4). This section reviews tests for a broader class of patterns. Specifically, patterns are sought where some measure of central tendency for y changes with changing values for x. Linear and monotonic patterns have this characteristic; however, decidedly nonlinear and nonmonotonic patterns can also have this characteristic (e.g., see the scatterplot for *BHPRM* in Fig. 6).

The approach taken is to divide the values for x (i.e., x_k, $k = 1, 2, \ldots, nS$) into nX classes and then to test to determine if y has a common measure of central tendency across these classes. Thus, x must be defined on at least a nominal scale to permit the definition of the necessary classes. Classic measures of central tendency are the mean or expected value, $E(y)$, and the median, $y_{0.5}$. The mean is a more widely used measure of central tendency but the median is less sensitive to outliers (e.g., see the Princeton robustness study reported in Ref. 43).

Most of the x's under consideration are actually defined on an interval scale (see Table 2), and the required classes are obtained by subdividing the range of x into a sequence of mutually exclusive and exhaustive subintervals containing equal numbers of sampled values (Fig. 8). A few x's are discrete with unequal probabilities for the individual x values (e.g., see *WMICDFLG* in Fig. 3a); for these variables, individual classes are defined for each of the distinct values. However, the optimum definition of the classes is not at all apparent, and in practice, some experimentation may be required to determine an appropriate division of the x values into classes.

For a given variable x and its nX associated classes, the following statistics will be used to identify apparent deviations from a common central tendency: (i) the ANOVA F statistic for equal means, which requires an interval scale for y (Sect. 5.1), (ii) the Kruskal-Wallis test for common locations, which requires an ordinal scale for y (Sect. 5.2), and (iii) the chi-square test for equal medians, which also requires an ordinal scale for y (Sect. 5.3).

5.1 Common Means: ANOVA F Statistic

For notational convenience, let q, $q = 1, 2, \ldots, nX$, designate the individual classes into which the values of x have been divided; let $\mathcal{X}_q$ designate the set such that $k \in \mathcal{X}_q$ only if x_k belongs to class q; and let nX_q equal the number of elements contained in $\mathcal{X}_q$ (i.e., the number of x_k's associated with class q). The ANOVA F test is commonly used to test for equivalence of conditional means:[44]

$$F(nX-1, nS-nX) = \frac{\left[\sum_{q=1}^{nX} nX_q \bar{y}_q^2 - nS\,\bar{y}^2\right] / (nX-1)}{\left[\sum_{q=1}^{nS} y_q^2 - \sum_{q=1}^{nX} nX_q \bar{y}_q^2\right] / (nS-nX)}, \tag{42}$$

where $nX - 1$ and $nS - nX$ are the number of degrees of freedom for the numerator and denominator, respectively, $\bar{y}_q = \sum_{k \in X_q} y_k / nX_q$, and $\bar{y}$ is defined in conjunction with Eq. (14).

If the y values conditional on each class of x values are normally distributed with equal expected values, then the statistic $F(nX - 1, nS - nX)$ in Eq. (42) follows an F distribution with $(nX - 1, nS - nX)$ degrees of freedom. This is the most powerful test for equality of means given that the indicated normality assumptions hold.[44] The probability $prob(F > \hat{F} | \eta_1, \eta_2)$ of exceeding an F statistic of value $\hat{F}$ calculated with (η_1, η_2) degrees of freedom can then be estimated by

$$prob(F > \hat{F} | \eta_1, \eta_2) = I_v(\eta_2 / 2, \eta_1 / 2), \quad v = \eta_2 / (\eta_2 + \eta_1 F), \tag{43}$$

where $I_v(a, b)$ designates the incomplete beta function (p. 222, Ref. 29).

Unfortunately, the y values for each class may not follow a normal distribution. Various goodness of fit tests (e.g., chi-square, Kolmogorov-Smirnov, Cramer-von Mises, Anderson-Darling) can be used to test for normality of the y values (pp. 94 - 95, Ref. 45; Ref. 46). However, the number of observations per class (e.g., 30 or 60 for many of the variables considered in this study) may be too small to provide a powerful test. If a goodness-of-fit test leads to a rejection of the normality hypothesis, then it may be appropriate to apply a normalizing transformation such as the Box-Cox transformation, which includes the logarithmic transformation as a special case (pp. 175 - 185, Ref. 45). Fortunately, the ANOVA F test is robust with respect to deviations from normality (p. 237, Ref. 37). For perspective, Monte Carlo estimates of $prob(F > \hat{F} | \eta_1, \eta_2)$ will be presented in Sect. 10.

5.2 Common Locations: Kruskal-Wallis Test

The Kruskal-Wallis test statistic, $\hat{T}$, is based on rank-transformed data (pp. 229-230, Ref. 37):

$$\hat{T} = \left[\sum_{q=1}^{nX} (R_q^2 / nX_q) - nS(nS+1)^2 / 4\right] / S^2, \tag{44}$$

where

$$R_q = \sum_{k \in X_q} r(y_k), \quad S^2 = \left[\sum_{k=1}^{nS} r(y_k)^2 - nS(nS+1)^2 / 4 \right] / (nS-1)$$

and $r(y_k)$ is defined in conjunction with Eq. (37).

If the y values conditional on each class of x values have the same distribution, then the statistic $\hat{T}$ in Eq. (44) approximately follows a chi-square distribution with $nX - 1$ degrees of freedom (pp. 230 - 231, Ref. 37). Given this approximation, the probability $prob(T > \hat{T} | nX - 1)$ of obtaining a value T that exceeds $\hat{T}$ in the presence of identical y distributions for the individual classes is given by

$$prob(T > \hat{T} | nX - 1) = Q[(nX - 1) / 2, \hat{T} / 2], \tag{45}$$

where $Q\ (a,\ b)$ designates the complement of the incomplete gamma function (p. 215, Ref. 29). A small value for $prob(T > \hat{T} | nC - 1)$ indicates that the y's conditional on individual classes have different distributions and thus, most likely, different means and medians.

5.3 Common Medians: Chi-Square for Contingency Tables

The final possibility considered is that different classes of x values have different median values for y. The chi-square test for contingency tables can be used to test for this situation (pp. 143 - 178, Ref. 37). First, the median $y_{0.5}$ is estimated for all nS observations. Specifically,

$$y_Q = \begin{cases} y_{(Q\,nS)} & \text{if } Q\,nS \text{ is an integer} \\ [y_{([Q\,nS])} + y_{([Q\,nS]+1)}] / 2 & \text{otherwise} \end{cases} \tag{46}$$

where $Q = 0.5$ (Q = 0.25 and 0.75 will be considered in Sect. 6.2) and $y_{(k)}$, $k = 1, 2, \ldots, nS$, denotes the ordering of the y values such that $y_{(k)} \le y_{(k+1)}$ (p. 14, Ref. 47). The individual classes of x values considered in Sects. 5.1 and 5.2 are then further subdivided on the basis of whether y values fall above and below $y_{0.5}$ (Fig. 9). For class q, let nX_{1q} equal the number of y values that exceed $y_{0.5}$, and let nX_{2q} equal the number of y values that are less than or equal to $y_{0.5}$. The result of this partitioning is a $2 \times nX$ contingency table with nX_{rq} observations in each cell. The following statistic can now be defined:

$$\hat{T} = \sum_{q=1}^{nX} \sum_{r=1}^{2} (nX_{rq} - nE_{rq})^2 / nE_{rq}, \tag{47}$$

where

$$nE_{rq} = \left(\sum_{r=1}^{2} nX_{rq} \right) \left(\sum_{q=1}^{nX} nX_{rq} \right) / \, nX$$

and corresponds to the expected number of observations in cell (r, q). If the individual classes of x values, $q = 1, 2, \ldots, nX$, have equal medians, then $\hat{T}$ approximately follows a chi-square distribution with $(nX - 1)(2 - 1) = nX - 1$ degrees of freedom (p. 156, Ref. 37). Thus, the probability of obtaining a value of T that exceeds $\hat{T}$ in the presence of equal medians is given by $prob(T > \hat{T} | nX - 1)$ in Eq. (45). To maintain the validity of the chi-square approximation in the analysis of contingency tables, Conover suggests using a partition in which $nE_{rq} \geq 1$ (p. 156, Ref. 37).

The Kruskal-Wallis rank statistic (Sect. 5.2) also converges to the chi-square statistic with $nX - 1$ degrees of freedom. In a case study (p. 232, Ref. 37), the power of the Kruskal-Wallis test exceeded the power of the median test. We interpret this result as follows: the median test measures only whether observations exceed the common median; it does not measure the extent to which individual observations exceed this median (i.e., nominal versus ordinal scale). Thus, the Kruskal-Wallis test is incorporating more information than the median test.

6. Dispersion of y Dependent on x

In this section, techniques for identifying patterns that involve changes in the dispersion or spread of y with changing values for x are considered. Two measures of dispersion will be considered: the variance σ_y^2, and the interquartile range $y_{0.75} - y_{0.25}$, where $y_{0.75}$ and $y_{0.25}$ represent the 0.75 and 0.25 quantiles of y. The variance is the best known measure of dispersion, and the interquantile range is widely used as a summary of dispersion in box plots.[40, 48] The interquartile range is less sensitive to outliers than the variance, analogous to medians and means. Two test statistics are considered: the ANOVA F statistic with jackknifing for common variances, and the chi-square statistic with contingency tables for common interquartile ranges.

6.1 Common Variances: ANOVA *F* Statistic with Jackknifing

The ANOVA test will use the same classes, $q = 1, 2, \ldots, nX$, of x values introduced in Sect. 5 (Fig. 8). Many procedures exist for testing for common variances: five procedures are summarized in Kleijnen (pp. 225 - 227, Ref. 45), and 56 procedures in Conover et al.[49] Additional discussion is also given in Conover (pp. 239 - 250, Ref. 37), Hamby (pp. 149 - 150, Ref. 9), Piepho[50] and Wludyka and Nelson[51]. Note that common variances can occur even though the associated mean values are different (and vice versa).

For this analysis, a procedure based on jackknifing is used to indicate if different classes of x values have different variances for y. Jackknifing is a general technique for reducing possible bias in estimators and constructing robust confidence intervals.[52, 53] Good results have been obtained with jackknifing in a number of different applications.[17] The procedure operates as follows.

The variance σ_{yq}^2 of y conditional on class q is estimated by

$$\hat{\sigma}_{yq}^2 = \sum_{k \in \mathcal{X}_q} (y_k - \bar{y}_q)^2 / (nX_q - 1) \tag{48}$$

for $q = 1, 2, \ldots, nX$, where $\mathcal{X}_q$, $\bar{y}_q$ and nX_q are defined in conjunction with Eq. (42). Further, an additional nX_q estimators

$$\hat{\sigma}_{yq,-l}^2 = \sum_{\substack{k \in \mathcal{X}_q \\ k \neq l}} (y_k - \bar{y}_{q,-l})^2 / (nX_q - 2) \tag{49}$$

of σ_{yq}^2 are calculated with individual y's (i.e., y_l) omitted from consideration. The values for $\hat{\sigma}_{yq}^2$ and $\hat{\sigma}_{yq,-l}^2$ from Eqs.(48) and (49) can be used to define the so-called pseudo values

$$t_{ql} = nX_q \hat{\sigma}_{yq}^2 - (nX_q - 1)\hat{\sigma}_{yq,-l}^2. \tag{50}$$

For each class of x values, the resultant values for t_{ql} constitute a sample from a population whose expected value is σ_{yq} in the case of common variances (at least if the x's were generated by random sampling). The ANOVA F test can now be used to test for the equality of the means of the variables t_{ql}. Specifically, the F statistic described in Eq. (42) is calculated with the values for t_{ql}, and the corresponding exceedance probability for the resultant F statistic is determined as indicated in Eq. (43). In this application, the jackknife procedure is used to obtain robust confidence interval estimates rather than to reduce bias.

Because variance estimators have long tails to the right, the use of a logarithmic transformation before jackknifing may enhance the capability of the procedure to identify different variances for y. Specifically, t_{ql} in Eq. (50) can be defined by

$$t_{ql} = nX_q \ln(\hat{\sigma}_{yq}^2) - (nX_q - 1)\ln(\hat{\sigma}_{yq,-l}^2), \tag{51}$$

and then the procedures defined in Eqs. (42) and (43) used with this new definition. In this case, the test is for the equality of $\ln(\sigma_{yq}^2)$, which implies equality of σ_{yq}^2.

A related approach is proposed by Archer et al.,[54] who also use the variability of y to assess the importance of factors in large-scale simulation models. Further, they use an ANOVA-like procedure to decompose the total variability of y into main effects, two factor effects, and higher-order interactions among factors. Finally, they apply bootstrapping, which is closely related to jackknifing.[52]

6.2 Common Interquartiles: Chi-Square for Contingency Tables

Another test of variability is based on the previously used partitioning of x into $q = 1, 2, \ldots, nX$ classes, with the hypothesis being that the associated nX interquartile ranges (i.e., $y_{0.75} - y_{0.25}$) are the same (Fig. 10). The quantile values $y_{0.25}$ and $y_{0.75}$ are defined by Eq. (46) with $Q = 0.25$ and 0.75. The individual classes of x values are now divided into subsets of y values that fall within and outside the interquartile range. For class q, let nX_{1q} equal the number of y values that fall within the interquartile range, and nX_{2q} equal the number of y values that fall outside that range. As for the common median test, the result of this partitioning is a $2 \times nX$ contingency table with nX_{rq} observations in each cell. The statistic in Eq. (47) can now be calculated and used with the exceedance probability in Eq. (45). The interquartile test was suggested by the quantile test mentioned in Conover (p. 174, Ref. 37) and, to the best of our knowledge, has not been previously examined in the literature.

7. Distribution of y Dependent on x: Chi-Square for Contingency Tables

The two preceding sections considered procedures for determining if the central tendency of y was dependent on x (Sect. 5) and if the dispersion of y was dependent on x (Sect. 6). In this section, the chi-square test for contingency tables is introduced as a means of determining if the distribution of y is dependent on x (i.e., to determine if y is statistically independent of x).

The test will use the same classes, $q = 1, 2, \ldots, nX$, of x values used in Sects. 5 and 6. Further, y is also divided into classes (Fig. 11). Thus, y must be defined on at least a nominal scale to permit the definition of the necessary classes. For notational convenience, let p, $p = 1, 2, \ldots, nY$, designate the individual classes into which the values of y have been divided; let $\mathcal{Y}_p$ designate the set such that $k \in \mathcal{Y}_p$ only if y_k belongs to class p; and let nY_p equal the number of elements contained in $\mathcal{Y}_p$. Typically, y is defined on at least an ordinal scale, and the classes are defined by ordering the y and then requiring the individual classes to have similar numbers of elements (i.e., the nY_p are approximately equal for $p = 1, 2, \ldots, nY$).

The partitioning of x and y into nX and nY classes in turn partitions (x, y) into $nX\, nY$ classes (Fig. 11), where (x_k, y_k) belongs to class (q, p) only if x_k belongs to class q of the x values (i.e., $k \in \mathcal{X}_q$) and y_k belongs to class p of the y values (i.e., $k \in \mathcal{Y}_p$). For notational convenience, let O_{pq} denote the set such that $k \in O_{pq}$ only if $k \in \mathcal{X}_q$ (i.e., x_k is in class q of x values) and also $k \in \mathcal{Y}_p$ (i.e., y_k is in class p of y values), and let nO_{pq} equal the number of elements contained in O_{pq}. Further, if x and y are independent, then

$$nE_{pq} = (nY_p / nS)(nX_q / nS)nS = nY_p \, nX_q / nS \tag{52}$$

is an estimate of the expected number of observations (x_k, y_k) that should fall in class (q, p).

The following statistic can be defined:

$$\hat{T} = \sum_{q=1}^{nX} \sum_{p=1}^{nY} (nO_{pq} - nE_{pq})^2 / nE_{pq}, \tag{53}$$

which is the same as the statistic in Eq. (47) except for the upper limit on the inner summation. Asymptotically, $\hat{T}$ follows a chi-square distribution with $(nX-1)$ $(nY-1)$ degrees of freedom when x and y are independent. Thus, the probability of obtaining a value of T that exceeds $\hat{T}$ when x and y are independent is given by $prob(T > \hat{T} | (nX-1)(nY-1))$ in Eq. (45).

Many other measures can also be used to quantify the degree of dependence between two variables x and y: Cramer's contingency coefficient, Pearson's mean-square contingency coefficient, the phi coefficient, and so on (pp. 178-189, Ref. 37). However, these techniques do not offer any advantages over the chi-square contingency table approach already discussed.

8. Identification of Important Variables

The purpose of the statistical procedures under consideration is to identify sampled input variables that have significant effects on individual predicted variables. Conceptually, this is equivalent to identifying scatterplots that exhibit some form of deviation from randomness. Once such scatterplots are identified, the analysts' understanding of the model must be called upon to explain the patterns that appear in these plots.

To provide guidance in examining scatterplots, it is useful to have a numerical way to distinguish between variables that appear to have a substantial effect on a predicted outcome and variables that appear to have little or no effect. For a given statistic, the probability that a larger value would occur due to chance variation provides such a measure (i.e., the probabilities in Eqs. (25), (39), (43), (45)). These probabilities are often called critical values or p-values and designated by $\hat{\alpha}$ or p. A small critical value indicates that under the assumptions of the test, an outcome equal to or greater than the observed value of the statistic is unlikely to occur due to chance. Thus, the implication is that the pattern in the associated scatterplot arose from some underlying relationship between x and y. For a given statistic, the indicated importance of a variable goes up as the value of the corresponding critical value goes down. Thus, an ordering of variables on the basis of the size of their associated critical values provides a way to rank variable importance (i.e., the smaller the critical value, the more important the variable appears to be).

In sensitivity analyses of the type considered in this presentation, the distributions for the sampled input variables typically characterize subjective (i.e., epistemic) uncertainty.[55] Often, the intent of the sensitivity analysis is to identify those variables on which additional research efforts should be expended to reduce the uncertainty in the final outcomes of a large analysis and hence in the decisions based on these outcomes. In this case, the desire may not be to obtain an absolute ranking of variable importance, but rather to prioritize groups of variables for additional research. For example, variables might be divided into the following three groups: Group 1 - important variables that require additional investigation, Group 2 - variables of intermediate importance that may merit additional investigation if time and resources permit, and Group 3 - unimportant variables that do not require additional investigation. One possibility is to define these groups on the basis of critical values (e.g., Group 1 corresponds to variables with $\hat{\alpha} < 0.01$; Group 2 corresponds to variables with $0.01 \leq \hat{\alpha} < 0.05$; and Group 3 corresponds to variables with $0.05 \geq \hat{\alpha}$). However, in practice due to the cost of investigating individual variables, the decision on whether or not to expend resources on the investigation of a particular variable will probably be made on the basis of several considerations rather than solely on the basis of a preselected critical value.

9. Top-Down Correlation

A number of techniques have been described for the identification of relationships between sampled and predicted variables (Sects. 3-7). These techniques will be applied to four predicted variables (Sect. 2). An important question is the extent to which the different techniques agree in their identification of important variables. A tool for assessing such agreement is the top-down correlation introduced by Iman and Conover,[20] which emphasizes agreement/disagreement for the most important variables and places reduced weight on agreement/disagreement for variables of little importance.

The top-down correlation is based on Savage scores:

$$S(h) = \sum_{j=h}^{nI} 1/j, \tag{54}$$

where $S(h)$ is the Savage score of a variable of rank h and nI is the number of ranked variables (Eq. (1)). Thus, the Savage score for the most important variable is $S(1) = 1/1 + 1/2 + \ldots + 1/nI$; the Savage score for the next most important variable is $S(2) = 1/2 + 1/3 + \ldots + 1/nI$; and so on.

Suppose two ranking procedures are under consideration. Further, let h_{1i}, $i = 1, 2, \ldots, nI$, denote the rank for variable x_i obtained with the first procedure, and let h_{2i}, $i = 1, 2, \ldots, nI$, denote the rank for variable x_i obtained with the second procedure. The top-down correlation $\hat{R}_{12}$ for these two tests is defined to be the Pearson correlation coefficient (Eq. (14)) associated with the pairs $[S(h_{1i}), S(h_{2i})]$, $i = 1, 2, \ldots, nI$. That is,

$$\hat{R}_{12} = \left[\sum_{i=1}^{nI} S(h_{1i}) S(h_{2i}) - nI \right] \Big/ [nI - S(1)], \tag{55}$$

with $S(1)$ defined in Eq. (54) and approximately equal to 2.450 + *ln*[*nI* + 0.5)/6.5] for $nI \geq 7$ (Eq. (3), Ref. 20). Large positive values for R_{12} indicate agreement between the two sets of ranks for the most important factors. Exact quantiles for this statistic are given in Iman and Conover (p. 355, Ref. 20; also see Ref. 56). Further,

$$z = \hat{R}_{12} \sqrt{nI - 1} \tag{56}$$

is distributed approximately normally with mean 0 and standard deviation 1 when the two rankings are uncorrelated and *nI* is sufficiently large. Under these conditions,

$$prob\left(|R| > |\hat{R}_{12}|\right) = erfc\left(|\hat{R}_{12}| \sqrt{nI - 2} / \sqrt{2}\right) \tag{57}$$

where $prob\left(|R| > |\hat{R}_{12}|\right)$ is the probability that random variation would produce a value R for $\hat{R}_{12}$ larger in absolute value than the observed value $\hat{R}_{12}$ (p. 631, Ref. 29).

10. Comparison of Procedures for Identification of Important Variables

The following statistics and/or associated tests have been introduced for possible use in the identification of patterns in scatterplots, where the given capital letters will be used to identify the associated procedures in the following discussion: correlation coefficients (CCs, Sect. 3), standardized regression coefficients (SRCs, Sect. 3), partial correlation coefficients (PCCs, Sect. 3), rank correlation coefficients (RCCs, Sect. 4), standardized rank regression coefficients (SRRCs, Sect. 4), partial rank correlation coefficients (PRCCs, Sect. 4), common means (CMNs, Sect. 5.1), common locations (CLs, Sect. 5.2), common medians (CMDs, Sect. 5.3), common variances (CVs, Sect. 6.1), common interquartile ranges (CIQ, Sect. 6.2), and statistical independence (SI, Sect. 7). Further, the following dependent variables with different behaviors have been introduced as examples: *E0:WAS_PRES*, *E0:BRAALIC*, *E2:WAS_SATB*, and *E2:WAS_PRES* (Sect. 2). The results of applying the indicated procedures to these dependent variables are now discussed.

10.1 Repository Pressure under Undisturbed Conditions: y = *E0:WAS_PRES*

The variable y = *E0:WAS_PRES* was included as an example because a linear relationship appears to exist between *E0:WAS_PRES* and several of the sampled variables (Sect. 2). Thus, procedures that can identify linear relationships should work well with *E0:WAS_PRES*, as indeed turned out to be the case (Table 4). In particular, tests based on CCs, RCCs, CMNs, CLs, CMDs and SI identified the same top four variables (i.e., *WMICDFLG HALPOR*, *WGRCOR*, *ANHPRM*) and also assigned these variables the same importance rankings based on p-values.

The scatterplots for these variables show a corresponding decrease in the strength of the relationships with *E0:WAS_PRES* (Fig. 3). For the remaining variables, there was little agreement between the individual procedures, with the *p*-values for the variables with ranks 5 and above typically close to or above 0.1. The only exception to this was for SI, where *ANHBCVGP* was assigned rank 5 with a *p*-value of 0.0194. Based on a visual inspection, there appears to be little difference in the distributions of *E0:WAS_PRES* for the two values of *ANHBCVGP*, although the larger value for *ANHBCVGP* (i.e., the value that implies the van Genuchten-Parker model) may result in fewer small values for *E0:WAS_PRES* (Fig. 12). The tests based on measures of dispersion (i.e., CV, CIQ) performed somewhat differently, with CV indicating no effects for *HALPOR* and *WGRCOR* based on a *p*-value cutoff of 0.1 and CIQ indicating no effect for *WGRCOR* based on the same cutoff.

As discussed in Sect. 3, analyses of variable importance based on CCs, SRCs and PCCs or on RCCs, SRRCs and PRCCs will produce similar results when the input variables (i.e., the x_j's) are uncorrelated. More specifically, CCs and SRCs are equal; RCCs and SRRCs are equal; orderings of variable importance based on CCs, SRCs and PCCs are the same; and orderings of variable importance based on RCCs, SRRCs and PRCCs are the same. The 24 variables used in the calculation of *E0:WAS_PRES* were assumed to be independent, with the Iman and Conover[24] restricted pairing technique being used to assure that the correlations between variables were indeed close to zero (see Footnote b to Table 4). The outcome, as predicted by theory, was that CCs and SRCs were approximately equal, RCCs and SRRCs were approximately equal, rankings based on CCs, SRCs and PCCs were approximately the same, and rankings based on RCCs, SRRCs and PRCCs were approximately the same (Table 5). Approximate correspondence to theory is the best that can be hoped for as the Iman/Conover restricted pairing technique makes the correlations between the sampled variables approximately zero (Table 6) rather than exactly zero.

The large number of procedures under consideration (i.e., CCs, RCCs, CMNs, CLs, CMDs, CVs, CIQs, SI, SRCs, PCCs, SRRCs, PRCCs) can make it difficult to get an overall feeling for the extent to which the individual procedures are agreeing or disagreeing in the identification of important variables. As discussed in Sect. 9, top-down correlation provides a way to compare variable rankings. In particular, top-down correlation gives a compact numeric summary of the comparisons in Tables 4 and 5 (Table 7), with all procedures except for CVs and CIQs showing strong agreement (i.e., top-down correlations close to or equal to one).

The calculation of CMNs, CLs, CMDs, CVs, CIQs and SI in Table 4 was based on the division of the range of the variables under consideration into $nX = 5$ intervals of equal probability. Also, the calculation of SI involved the division of the range of *E0:WAS_PRES* into $nY = 5$ intervals of equal probability. In concept, the outcome of the analysis could be sensitive to the partitioning selected for use (i.e., the values for nX and nY). To check this, the analysis was repeated with $nX = 10$ and $nY = 10$ (Table 8). As comparison of the results obtained with $nX = 5$ and $nX = 10$ shows, some changes in variable rankings did take place. For CMNs, CLs and CMDs with $nX = 5$, *ANHPRM* is the fourth ranked variable with *p*-values of 0.0195, 0.0187 and 0.0663, respectively (Table 4); for the same procedures with $nX = 10$, *ANHPRM* is ranked 4, 4 and 7 with *p*-values of 0.1371, 0.1340 and 0.3398 (Table 8).

For CVs and CIQs, there are some changes in variable ranking (e.g., CV and CIQ assign *SALPRES* ranks of 11 and 6 with p-values of 0.3723 and 0.0868 for $nX = 5$ (Table 4) and rank *SALPRES* third with p-values of 0.0500 and 0.0077 for $nX = 10$ (Table 8)); also, CVs still do not identify an effect for *HALPOR* (ranked 12 with a p-value of 0.3919 for $nX = 5$ and ranked 20 with a p-value of 0.5800 for $nX = 10$), and CIQs still do not identify an effect for *WGRCOR* (ranked 16.5 with a p-value of 0.6626 for $nX = 5$ and ranked 23 with a p-value of 0.9429 for $nX = 10$). For SI, *HALPRM* had a rank of 18 with a p-value of 0.6235 for $nX = 5$ and a rank of 3 with a p-value of 0.0036 for $nX = 10$ (Table 8). Thus, the partitioning in use can have an effect on the variables identified as affecting the y value under consideration. For perspective, the top-down correlations for results obtained with the two griddings are also given in Table 8, with these correlations ranging from 0.854 for (CMN:1 × 5, CMN:1 × 10) to 0.917 for (CIQ:2 × 5, CIQ:2 × 10).

The p-values used to identify important variables in Tables 4, 5 and 8 are calculated with statistical assumptions that are not fully satisfied. For example, in the calculation of p-values for CCs, the sample from the x's consists of three pooled LHSs rather than a random sample (see Eqs. (8) - (10)), and neither the individual x's nor y = *E0:WAS_PRES* has a normal distribution. A Monte Carlo simulation can be used to assess if the use of formal statistical procedures to determine p-values is producing misleading results. Specifically, 10,000 samples of the form

$$(x_k, y_k),\ k = 1, 2, \ldots, 300, \tag{58}$$

can be generated by pairing the 300 values for x (i.e., the 300 values for the particular x under consideration contained in the samples in Eqs. (8) - (10)) with the 300 predicted values for y (i.e., the 300 values for y that resulted from the use of the sample elements in Eqs. (8) - (10)). The specific pairing algorithm was to randomly and without replacement assign an x value to each y value, which is similar to bootstrapping[57] except that the sampling is being performed without replacement. This random assignment was repeated 10,000 times to produce 10,000 samples of the form in Eq. (58).

For a given procedure (i.e., CCs, RCCs, CMNs, CLs, CMDs, CVs, CIQs or SI), each of the 10,000 samples can be used to calculate the value of the statistic used to determine the corresponding p-value. The resulting empirical distribution of the statistic can then be used to estimate the p-value for the statistic actually observed in the analysis. Comparison of the p-value obtained for a given set of statistical assumptions with the p-value obtained from the empirical distribution of the corresponding statistic provides an indication of the robustness of the variable rankings with respect to possible deviations from the assumptions underlying the formal statistical procedures described in Sects. 3-7.

For *E0:WAS_PRES*, the rankings of variable importance with p-values obtained from formal statistical procedures (i.e., CC and RCC in Table 5 and CMN, CL, CMD, CV, CIQ and SI in Table 4) and the ranking of variable importance with p-values obtained from empirical distributions (i.e., CCMC and RCCMC in Table 9 and CMNMC, CLMC, CMDMC, CVMC, CIQMC and SIMC in Table 8) are very similar. The largest difference is in the assignment of tied ranks to the most important variables when the empirical distributions of p-values are used (e.g., use of statistical procedures with CCs results in *WMICDFLG*, *HALPOR* and *WGRCOR* being ranked 1, 2 and 3, and use of the empirical distribution of p-values results in these variables being ranked 2, 2 and 2). The tied ranks with the empirical distributions arise because a sample of size 10,000 was used to generate these distributions, with the result that 0.0001 is the smallest nonzero p-value that can be estimated. In contrast, much smaller nonzero probabilities can be estimated with the formal statistical procedures from Sects. 3-7. Overall, the similarity between the exact (i.e., statistically determined) and empirical p-values in Tables 8 and 9 is quite good, with the two determinations of p-values producing almost identical rankings of variable importance except for the very small (i.e., $< 10^{-4}$) p-values. The associated top-down correlations range from 0.970 for (CMN:1 × 5, CMNMC: 1 × 5) to 0.993 for (CV:1 × 5, CVMC: 1 × 5) (Table 8). For perspective, a top-down correlation of 0.971 results when 24 variables are under consideration, one procedure has ties (i.e., ranks of 2, 2, 2) on the variables assigned ranks of 1, 2, 3 by the other procedure, and identical ranks are assigned to all other variables (e.g., see (CL:2 × 5, CLMC:2 × 5) in Table 8 and (CC, CCMC), (RCC, RCCMC) in Table 9).

Approximate 100 $(1 - \alpha)$% confidence intervals for the empirically determined p-values are given by $p \pm x_{1-\alpha/2}\,[p(1-p)/n]^{1/2}$, where p is the estimated p-value, $n = 10{,}000$ is the sample size in use, and $x_{1-\alpha/2}$ is the $1-\alpha/2$ quantile of the normal distribution (e.g., 1.96 for a 95% confidence interval (pp. 99-100, Method C, Ref. 37)). For example, the approximate 95% confidence interval for $p = 0.0815$ (see *SALPRES* for CCMC in Table 9) is 0.0815 ± 0.0054, with this interval including the statistically determined value of 0.0855. For most procedures, the 95% confidence intervals on the empirically determined p-values include the statistically determined p-values. The results for CVs tend to show less agreement between the formally and empirically estimated p-values than is the case for the other procedures.

A variant of the common means (CMNs) test is to use logarithmically transformed y-values rather than the original untransformed y-values (Sect. 5.1). Possible rationales for such a transformation are to reduce the effects of extreme values on the estimated mean and to transform y into a variable that more closely follows a normal distribution. For y = *E0:WAS_PRES*, the logarithmic transformation has little effect on the outcome of the analysis, with both raw and log-transformed y's resulting in the same assignment of ranks to the top four variables (i.e., *WMICDFLG, HALPOR, WGRCOR, ANHPRM*) (Tables 8, 10).

A variant of the common variances (CVs) test is to use t_{ql} as defined in Eq. (51). The rationale for the use of logarithms in Eq. (51) is to reduce the effects of extreme values and thus produce more stable variance estimators. For y = *E0:WAS_PRES*, use of t_{ql} as defined in Eq. (51) rather than in Eq. (50) had little effect on the outcome of the analysis, with both definitions of t_{ql} resulting in the selection of *WMICDFLG*, *ANHPRM*, *HALPRM* and *WGRCOR* as the top four variables (Tables 8, 10). Further, neither definition results in the identification of the important effects associated with *HALPOR* (Fig. 3b).

10.2 Brine Inflow under Undisturbed Conditions: *y = E0:BRAALIC*

The variable *y = E0:BRAALIC* was included as an example because a nonlinear but monotonic relationship appears to exist between *E0:BRAALIC* and several of the sampled variables (Sect. 2). Thus, procedures that can identify monotonic relationships should work well with *E0:BRAALIC* as indicated by the regressions with raw data ($R^2 = 0.50$) and rank-transformed data ($R^2 = 0.87$) in Table 3. All analysis procedures except CVs identified *ANHPRM* and *WMICDFLG* as the two most important variables, with the variables assigned ranks 1 and 2 changing from test to test (Table 11). The scatterplots for both *ANHPRM* and *WMICDFLG* show strong relationships with *E0:BRAALIC* (Fig. 4a,b). The CVs test assigned rank 2 to *SHPRMCON* with a *p*-value of 0.0426, with this variable also assigned a *p*-value of 0.0057 and a rank of 3 by the CMNs test. No other tests indicated an effect for this variable, which is consistent with the corresponding scatterplot (Fig. 13a). Rank 3 was assigned to *HALPRM* for tests based on RCCs (*p*-value = 0.0014), CLs (*p*-value = 0.0019), CMDs (*p*-value = 0.0050) and SI (*p*-value = 0.0517), with the corresponding scatterplot showing little discernible pattern (Fig. 13b). Rank 4 was assigned to *WGRCOR* by CCs (*p*-value = 0.0048), RCCs (*p*-value = 0.0057), CMNs (*p*-value = 0.0636) and CLs (*p*-value = 0.0427), with the corresponding scatterplot indicating a slight tendency for *E0:BRAALIC* to decrease as *WGRCOR* increases (Fig. 4d). Little discernible pattern appears in the ranks assigned to the remaining variables in Table 11.

Based on knowledge of the model in use, the ordering of variable importance associated with RCCs seems most reasonable, with the signs of the RCCs for the variables ranked 1 through 6 (Table 12) corresponding to the effects that these variables should have on *E0:BRAALIC* (i.e., whether *E0:BRAALIC* should increase or decrease as the corresponding variable increases; see Ref. 18 for a discussion of the underlying physics). The procedures that most closely match the variable rankings obtained with RCCs are based on CLs (top-down correlation (*TDC*) = 0.897), CMDs (*TDC* = 0.913) and SI (*TDC* = 0.838) (Table 13). These are procedures that can be expected to perform reasonably well in the presence of nonlinear but monotonic relations. The top-down correlation for RCCs and CCs is 0.729 (Table 13). Procedures based on measures of dispersion have the poorest agreement with variable rankings based RCCs (i.e., CVs with *TDC* = 0.301, CIQs with *TDC* = 0.531) (Table 13).

Rankings of variable importance based on CCs, SRCs and PCCs are similar, with the rankings based on SRCs and PCCs being identical (Table 12). In like manner, rankings based on RCCs, SRRCs and PRCCs are similar, with the rankings based on SRRCs and PRCCs being identical (Table 12). The associated top down correlations are correspondingly high (i.e., 0.980 for (CC, SRC), (CC, PCC) and 0.912 for (RCC, SRRC), (RCC, PRCC)) (Table 13).

As for *E0:WAS_PRES*, an investigation was carried out to determine if the analysis results obtained for *E0:BRAALIC* are sensitive to the partitioning selected for use (i.e., the values for nX and nY). In particular, the analysis was repeated with nX = 10 for CMs, CLs, CMDs, CVs, CIQs and SI, and nY = 10 for SI (Table 14). As indicated by examination of scatterplots, the two most important variables with respect to *E0:BRAALIC* are *ANHPRM* and *WMICDFLG* (Fig. 4a,b). With the exception of CVs, all tests (i.e., CMNs, CLs, CMDs, CIQs, SI) identified *ANHPRM* and *WMICDFLG* as the two most important variables with grids based on either nX = 5 or nX = 10 (Table 14). After these two variables, there is some jumping around in the rankings assigned to the individual variables, although there is sufficient similarity in the results obtained with nX = 5 and nX = 10 to produce top down correlations that are close to or above 0.9 (Table 14). Scatterplots indicate that, after *ANHPRM* and *WMICDFLG*, none of the remaining variables have a very strong effect on *E0:BRAALIC* (Figs. 4, 13), with the result that the tests are failing to find discernible patterns after these two variables.

The p-values in Tables 11 and 12 are calculated with statistical assumptions that are not fully satisfied. As described in conjunction with Eq. (58), a Monte Carlo procedure can be used to assess if the use of formal statistical procedures to determine p-values is producing misleading results. The p-values based on formal statistical procedures and on Monte Carlo procedures are very similar, with the associated variable rankings having top-down correlations between 0.987 and 0.995 (i.e., CC and RCC in Table 15 and CMN, CL, CMD, CV, CIQ and SI in Table 14). The primary difference is that the most important variables (i.e., *ANHPRM* and *WMICDFLG*) tend to be assigned tied-ranks (i.e., 1.5) in the Monte Carlo simulations because the sample size of 10,000 in use does not allow the estimation of p-values less than 0.0001.

A variant of the common means (CMNs) test is to use logarithmically transformed y-values rather than the original untransformed y-values (Sect. 5.1). Use of both raw and logarithmically transformed variables results in *ANHPRM* and *WMICDFLG* being selected as the two most important variables with respect to *E0:BRAALIC* (Tables 14, 16). Use of logarithmically transformed variables with the CMNs test also results in the identification of *HALPRM* as the third most important variable, with *HALPRM* also assigned a rank of 3 with RCCs, but effectively missed by the CMNs test with raw data (i.e., a p-value of 0.1105 and a rank of 7) (Tables 14, 16). The CMNs test with both raw and logarithmically transformed data assigns rank 4 to *WGRCOR* (Tables 14, 16). Thus, the use of logarithmically transformed data with the CMNs test results in the identification of one possibly important variable (i.e., *WGRCOR*) missed with the use of raw data.

A variant of the common variances (CVs) test is to use t_{ql} as defined in Eq. (51) rather than as defined in Eq. (50). The logarithmic transformation associated with Eq. (51) results in a substantial improvement in that *WMICDFLG* is now identified as an important variable (Table 16); in contrast, *WMICDFLG* was missed with raw data as used in Eq. (50) (Table 14). The associated scatterplot indicates that *WMICDFLG* is a variable that should be identified by any reasonable test (Fig. 4a).

10.3 Repository Saturation under Disturbed Conditions: *y = E2:WAS_SATB*

The variable *y = E2:WAS_SATB* was selected as an example because the regression analyses with raw and rank-transformed data were rather poor (i.e., $R^2 = 0.33$ and 0.61, respectively), although the two most important variables as indicated by scatterplots (i.e., *BHPRM* and *WRGSSAT*) do appear in both regression analyses (Table 3, Fig. 5). Given the strong patterns displayed in the scatterplots for *BHPRM* and *WRGSSAT* and the discernible but less strong patterns associated with *ANHPRM* and *HALPOR* (Fig. 5), procedures that can identify patterns that result from the interaction of two or more variables should work well for *E2:WAS_SATB*. In particular, analyses based on RCCs, CLs, CMDs and SI identified *BHPRM* and *WRGSSAT* as the two most important variables (Table 17). Analyses based on CCs, CMNs and CVs identified *BHPRM* as the most important variable, but did not identify *WRGSSAT* as the second most important variable; in contrast, CIQs identified WRGSSAT as the most important variable and identified *BHPRM* as the third rather than second most important variable (Table 17). Further, CIQs identified *WGRCOR* as the second most important variable (Table 17), which seems to be inconsistent with the weakness of the pattern appearing in the associated scatterplot (Fig. 14) and also the rankings assigned to *WGRCOR* by other procedures (Table 17); this identification may be due to the increased spread in *y* values for large values of *WGRCOR*. The test based on CVs did not identify *WRGSSAT* (i.e., a *p*-value of 0.1750 and a rank of 9) (Table 17).

Given the insights gained from the results of all of the analysis techniques, CCs and RCCs appear to have identified the three dominant variables affecting *E2:WAS_SATB* (i.e., *BHPRM, WRGSSAT, ANHPRM*). However, given the low R^2 values associated with the corresponding regression models with raw and rank-transformed data (Table 3), it would be difficult to place much faith in these identifications without results from tests that are less dependent on linear regression models (i.e., CLs, CMDs, CIQs, SI).

As previously observed for *E0:WAS_PRES* and *E0:BRAALIC* (Tables 5, 12), variable rankings for *E2:WAS_SATB* with CCs, SRCs and PCCs are similar, with SRCs and PCCs producing identical variable rankings (Table 10.15, Ref. 58). A similar pattern also occurs for RCCs, SRRCs and PRCCs (Table 10.15, Ref. 58).

Top-down correlation provides a formal comparison of the variable rankings obtained with the different procedures (Table 18). A considerable amount of variability exists in the rankings obtained with the different techniques. Rankings based on SI, CVs and CIQs appear to have the least agreement with the rankings obtained with other procedures. Also, rankings based on CVs and CIQs show little agreement (i.e., *TDC*=0.267).

As for *E0:WAS_PRES* and *E0:BRAALIC*, an investigation was carried out for *E2:WAS_SATB* on the effects of using $nX = 10$ rather than $nX = 5$ for CMNs, CLs, CMDs, CVs, CIQs and SI and $nY = 10$ rather than $nY = 5$ for SI (Table 19). The results for the highest ranked variables for the two partitionings were similar, with CMNs, CLs, CMDs, CIQs and SI each identifying the same top 3 variables; however, the identified variables were not necessarily the same from test to test. For CVs, both partitionings yielded the same top two variables but produced different variables with rank 3. After the top three variables, there was often considerable variability in the ranks assigned to the remaining, and typically unimportant, variables. The least agreement between the variable rankings obtained with the two partitionings occurred for SI (*TDC* = 0.746).

Again, *p*-values based on formal statistical procedures and on Monte Carlo procedures are very similar, with the associated rankings having top-down correlations between 0.972 and 0.999 (Tables 19, 20). The primary difference is that the most important variables tend to be assigned tied-ranks in the Monte Carlo simulation (e.g., 1.5 for *BHPRM* and *ANHPRM* for CCs in Table 20).

The use of raw and logarithmically transformed variables with the CMNs test (Sect. 5.1) results in similar rankings of variable importance for *E2:WAS_SATB* (Table 10.19, Ref. 58). Thus, little is gained by the use of logarithmically transformed variables. Similarly, little change in the outcome of the analysis for *E2:WAS_SATB* with CVs took place when t_{ql} as defined in Eq. (51), rather than as in Eq. (50), was used (Table 10.19, Ref. 58).

10.4 Repository Pressure under Disturbed Conditions: *y* = *E2:WAS_PRES*

The variable *y* = *E2:WAS_PRES* was included as an example because regression analyses with raw and rank-transformed data fail to identify the dominant variable *BHPRM* (Sect. 2). Thus, procedures that can identify nonlinear, nonmonotonic relationships should work well with *E2:WAS_PRES*, which turned out to be the case (Table 21). In particular, tests based on CMNs, CLs, CMDs, CVs, CIQs and SI all identified *BHPRM* as the most important variable affecting *E2:WAS_PRES* (Table 21), which is consistent with the strong pattern appearing in the corresponding scatterplot (Fig. 6d). In contrast, tests based on CCs and RCCs failed to identify *BHPRM* as an important variable (i.e., *p*-values of 0.3651 and 0.1704 for CCs and RCCs, respectively) (Table 21). Further, tests based on CMNs, CLs, CMDs, CVs and SI select the variables ranked 2 and 3 from *HALPRM*, *ANHPRM* and *WGRCOR*, while the test based on CIQs assigns ranks 2 and 3 to *WGRCOR* and *SHRGSSAT*, respectively. As indicated by scatterplots, *HALPRM*, *ANHPRM* and *WGRCOR* produce barely discernible patterns (Figs. 6, 15).

Variable rankings for *E2:WAS_PRES* based on CCs, SRCs and PCCs and also on RCCs, SRRCs and PRCCs are the same (Table 10.21, Ref. 58). However, these rankings are misleading because they do not include the dominant variable *BHPRM*.

Due to the failure of CCs and RCCs to identify the dominant variable *BHPRM*, there is less agreement between the variable rankings obtained with the various analysis procedures for *E2:WAS_PRES* than is the case for *E0:WAS_PRES*, *E0:BRAALIC* and *E2:WAS_SATB* (i.e., compare the top-down correlations in Tables 7, 13, 18 and 22). In particular, variable rankings for *E2:WAS_PRES* with CMNs, CLs, CMDs CVs, CIQs and SI are generally similar (Table 22). The exception is the ranking based on CIQs, which shows top-down correlations of 0.429, 0.462 and 0.462 with the rankings obtained with CMNs, CLs and CMDs. Otherwise, the top-down correlations for the variable rankings obtained with CMNs, CLs, CMDs, CVs, CIQs and SI vary between 0.698 and 1.000. In contrast, there is little relationship between the variable rankings obtained with CMNs, CLs, CMDs, CVs, CIQs and SI and with CCs, SRCs, PCCs, RCCs, SRRCs and PRCCs.

An investigation of the effects of using $nX = 10$ rather than $nX = 5$ for CMNs, CLs, CMDs, CVs, CIQs and SI and $nY = 10$ rather than $nY = 5$ for SI was also carried out (Table 23). Each of the indicated procedures with $nX = 5$ and $nX = 10$ identified *BHPRM* as the most important variable. Generally, ranks 2, 3 and sometimes 4 were also assigned to similar variables, although the exact order was not always the same for $nX = 5$ and $nX = 10$. After rank 4, there was considerable variability in the ordering of the variables with $nX = 5$ and $nX = 10$.

The p-values used to identify important variables in Tables 21 and 23 were recalculated with the Monte Carlo procedure described in conjunction with Eq. (58) (Tables 23, 24). The rankings based on analytic determination of p-values and on Monte Carlo determination of p-values are very similar, with the primary difference being the tendency of the Monte Carlo simulation to assign tied ranks to the most important variables.

Use of both raw and logarithmically transformed variables with the CMNs test (Sect. 5.1) results in similar rankings of variable importance for *E2:WAS_PRES* (Table 10.25, Ref. 58). Thus, little is gained in the analysis of *E2:WAS_PRES* with CMNs by the use of logarithmically transformed variables. In contrast, the analysis for *E2:WAS_PRES* with CVs and t_{ql} as defined in Eq. (51) with a logarithmic transformation performed poorly, with the analysis failing to identify the dominant variable *BHPRM* (Table 10.25, Ref. 58). Thus, the use of the logarithmic transformation in Eq. (51) has the potential to improve the performance of the CVs test as it did for *E0:BRAALIC* (Tables 14, 16) and also the potential to degrade performance as is the case for *E2:WAS_PRES*.

11. Discussion

Sensitivity analysis is an essential component of model development, assessment and application. Monte Carlo procedures are widely used in sensitivity studies to develop a mapping (i.e., scatterplot) between uncertain model inputs and associated model results that can then be explored with regression-based techniques. Unfortunately, these techniques sometimes fail to identify important patterns in this mapping because the relationships between model inputs and model results can be too complex to be identified by the linear relationships that most regression analyses are predicated on.

The likelihood of a successful sensitivity analysis can be increased by using a number of different procedures to identify relationships between model inputs and model results. With this strategy, a relationship missed by one procedure may be identified by another procedure. Fortunately, the post-processing of model results for the identification of patterns in scatterplots is relatively inexpensive from a computational perspective; thus, the use of a number of different procedures does not present a significant burden.

This paper describes and illustrates a sequence of procedures for identifying patterns in scatterplots. These procedures are based on attempts to detect increasingly complex patterns in scatterplots and involve the identification of (i) linear relationships with correlation coefficients, (ii) monotonic relationships with rank correlation coefficients, (iii) trends in central tendency as defined by means, medians and the Kruskal-Wallace statistic, (iv) trends in variability as defined by variances and interquantile ranges, and (v) deviations from randomness as defined by the chi-square statistic. As illustrated in a sequence of example analyses with a large model for two-phase fluid flow, the individual procedures can differ in the variables that they identify as having important (significant) effects on particular analysis outcomes. The example results indicate that the use of a sequence of procedures is a good analysis strategy and provides some assurance that an important effect is not overlooked. Based on the experience of this analysis, a possible sequence of tests is correlation coefficients (CCs, Sect. 3), rank correlation coefficients (RCCs, Sect. 4), common locations (CLs, Sect. 5.2) or common medians (CMDs, Sect. 5.3), and statistical independence (SI, Sect. 7).

The procedures under consideration identify patterns in scatterplots that in some sense appear to be nonrandom. However, they provide no explanation for why these patterns exist. Once such patterns are identified, it is the responsibility of the modelers and analysts to develop explanations for them. If such explanations cannot be developed, then the possibility exists that an error is present in the analysis. For this reason, well-designed sensitivity analyses provide both a way to develop insights with respect to the problem under consideration and also a way to check the conceptual and computational implementation of the problem.

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Figure Captions

Fig. 1. Computational (finite difference) grid used in BRAGFLO to represent two phase flow in 1996 WIPP CCA PA subsequent to a drilling intrusion. Same formulation is used in the absence of a drilling intrusion except that regions 1A, 1B and 1C have the same properties as the regions to either side.

Fig. 2. Dependent variables predicted by BRAGFLO model: (2a) pressure in lower waste panel under undisturbed conditions (*E0:WAS_PRES*), (2b) cumulative brine inflow from anhydrite marker beds under undisturbed conditions (*E0:BRAALIC*), (2c) saturation in lower waste panel after an E2 intrusion at 1000 yr (*E2:WAS_SATB*), and (2d) pressure in lower waste panel after an E2 intrusion at 1000 yr (*E2:WAS_PRES*).

Fig. 3. Scatterplots for pressure in lower waste panel under undisturbed (i.e., E0) conditions at 10,000 yr (*E0:WAS_PRES*) versus first four variables selected in stepwise regression analyses with raw and rank-transformed data (Table 3): (3a) *WMICDFLG*, (3b) *HALPOR*, (3c) *WGRCOR*, and (3d) *ANHPRM*.

Fig. 4. Scatterplots for cumulative brine inflow over 10,000 yr from all anhydrite marker beds to repository under undisturbed (i.e., E0) conditions (*E0:BRAALIC*) versus first four variables selected in stepwise regression analysis with rank-transformed data (Table3): (4a) *WMICDFLG*, (4b) *ANHPRM*, (4c) *HALPOR*, and (4d) *WGRCOR*.

Fig. 5. Scatterplots for brine saturation in lower (i.e., intruded) waste panel at 10,000 yr for an E2 intrusion at 1000 yr (*E2:WAS_SATB*) versus first four variables selected in stepwise regression analysis with rank-transformed data (Table 3): (5a) *BHPRM*, (5b) *WRGSSAT*, (5c) *ANHPRM* and (5d) *HALPOR*.

Fig. 6. Scatterplots for pressure in lower waste panel at 10,000 yr with an E2 intrusion into the lower waste panel at 1000 yr (*E2:WAS_PRES*) versus the three variables (*HALPRM*, *ANHPRM*, *HALPOR*) selected in stepwise regression analysis with raw and rank-transformed data (Table 3) and one additional variable (*BHPRM*) identified by examination of scatterplots: (6a) *HALPRM*, (6b) *ANHPRM*, (6c) *HALPOR*, and (6d) *BHPRM*.

Fig. 7. Graph of $d(b, a) = b(1 - b^2)^{1/2} - a(1 - a^2)^{1/2} > 0$ subject to constraints $0 < a < b < 1$, $a^2 + b^2 < 1$.

Fig. 8. Example of the partitioning of the range of x = *HALPOR* into $nX = 5$ classes for y = *E0:WAS_PRES*.

Fig. 9. Example of the partitioning of the range of x = *HALPOR* into $nX = 5$ classes and the range of y = *E0:WAS_PRES* into values above and below the median $y_{0.5}$.

Fig. 10. Example of the partitioning of the range of x = *HALPOR* into $nX = 5$ classes and the range of y = *E0:WAS_PRES* into values inside and outside the interquartile range $[y_{0.25}, y_{0.75}]$.

Fig. 11. Example of the partitioning of the range of x = *HALPOR* into $nX = 5$ classes and the range of y = *E0:WAS_PRES* into $nY = 5$ classes.

Fig. 12. Scatterplot for *E0:WAS_PRES* versus *ANHBCVGP*.

Fig. 13. Scatterplots for *E0:BRAALIC* versus *SHPRMCON* and *HALPRM*.

Fig. 14. Scatterplot for *E2:WAS_SATB* versus *WGRCOR*.

Fig. 15. Scatterplot for *E2:WAS_PRES* versus *WGRCOR*.

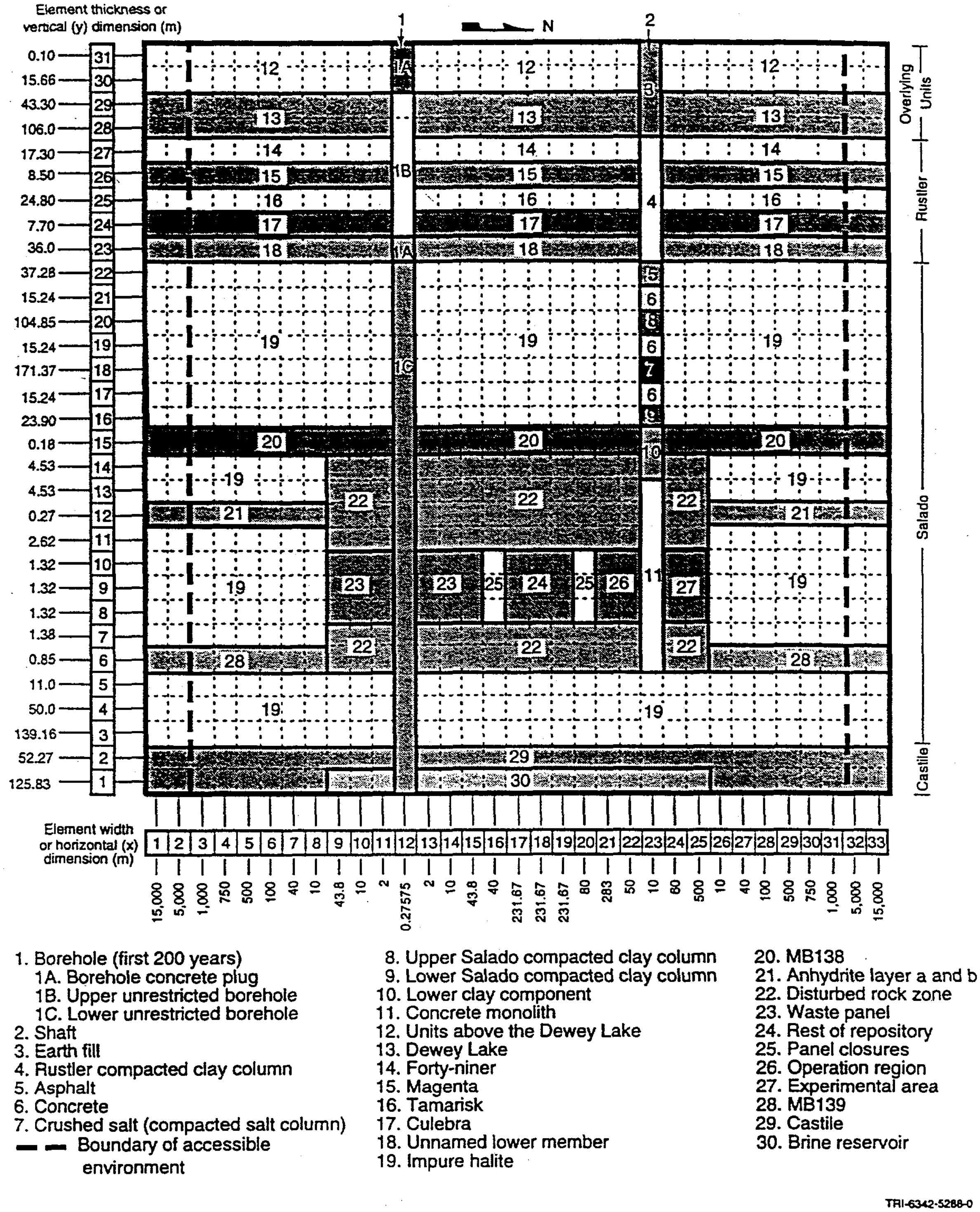
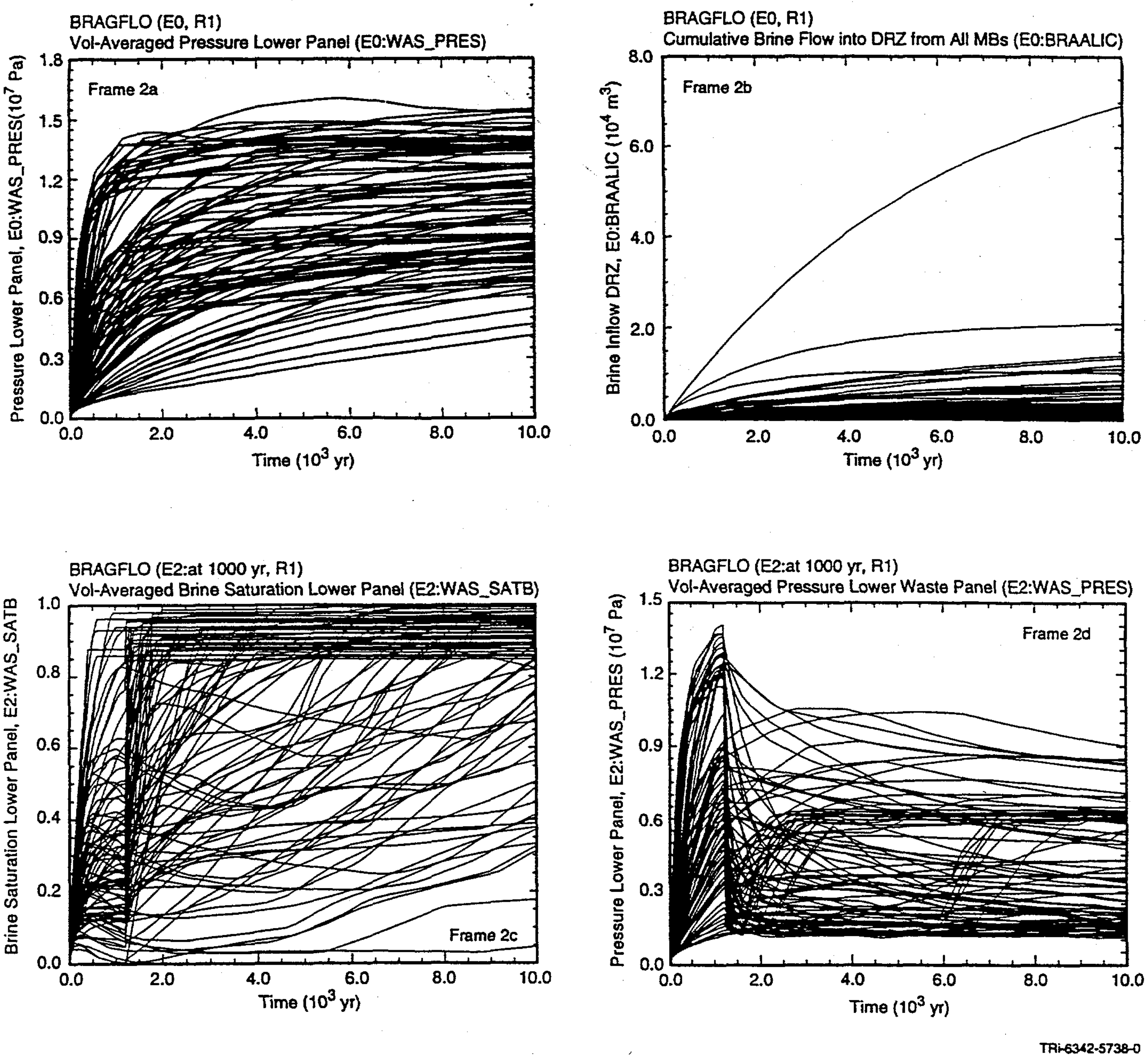


Fig. 1. Computational (finite difference) grid used in BRAGFLO to represent two phase flow in 1996 WIPP CCA PA subsequent to a drilling intrusion. Same formulation is used in the absence of a drilling intrusion except that regions 1A, 1B and 1C have the same properties as the regions to either side.



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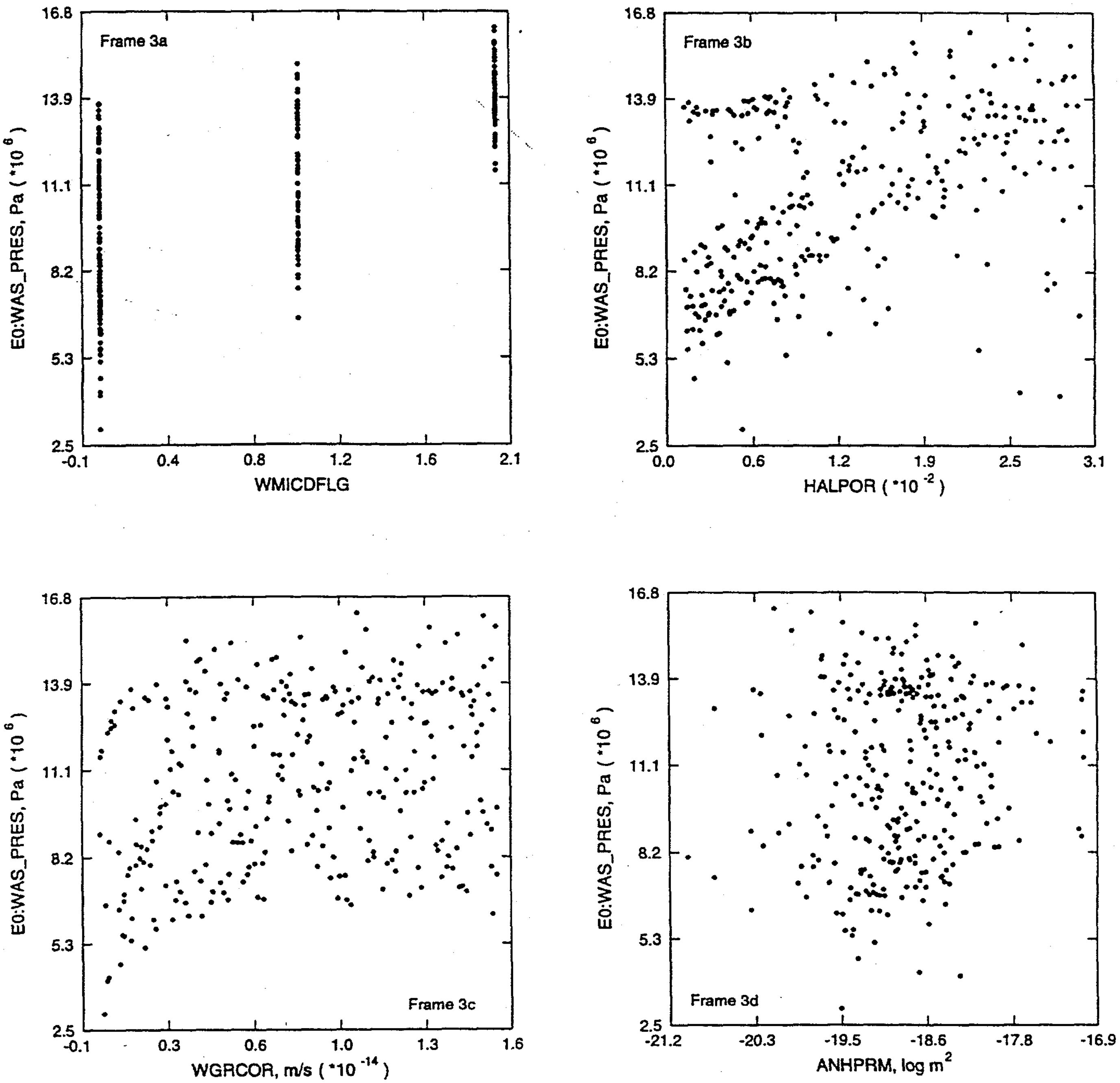

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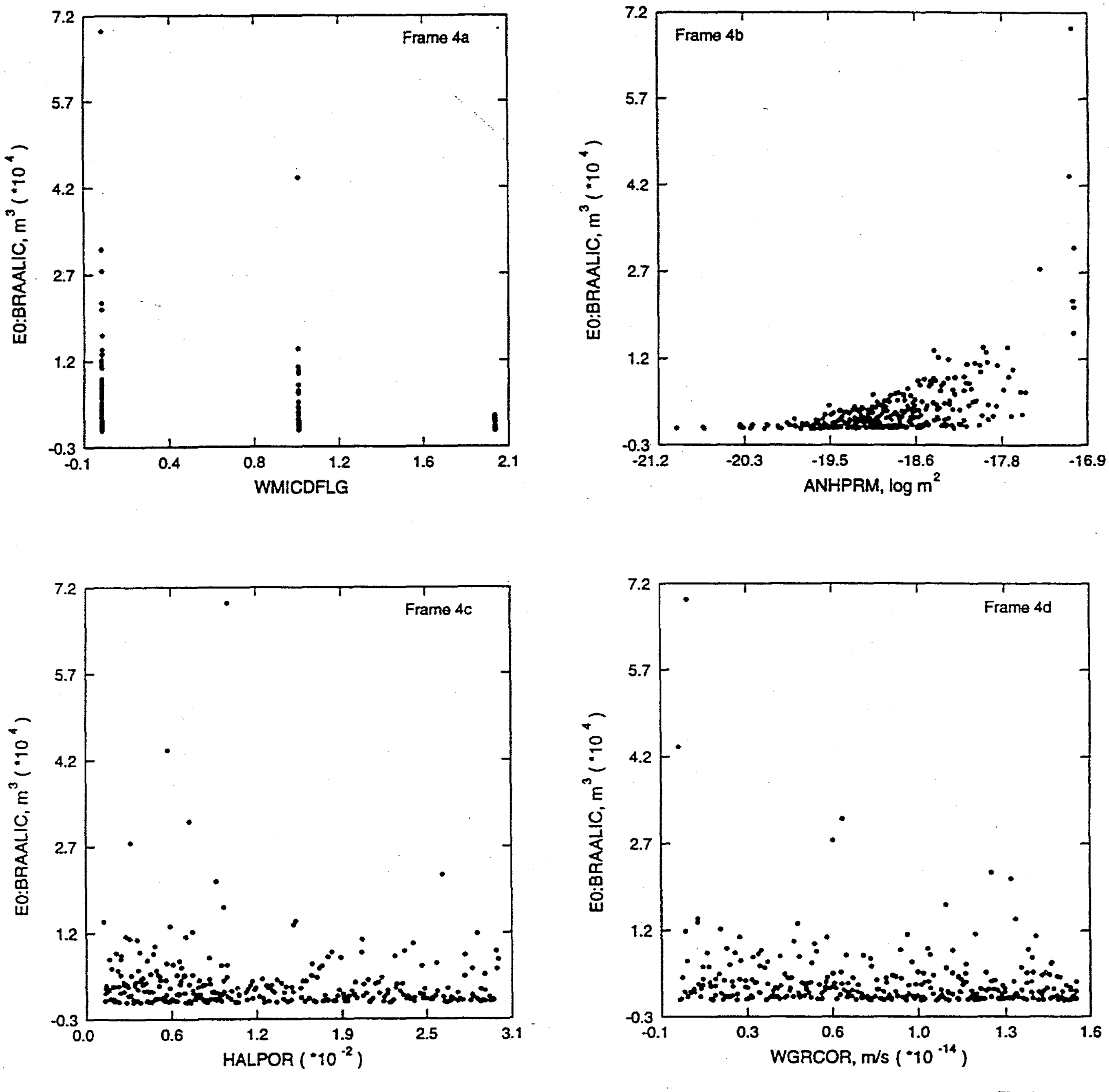
Fig. 3. Scatterplots for pressure in lower waste panel under undisturbed (i.e., E0) conditions at 10,000 yr (*E0:WAS_PRES*) versus first four variables selected in stepwise regression analyses with raw and rank-transformed data (Table 3): (3a) *WMICDFLG*, (3b) *HALPOR*, (3c) *WGRCOR*, and (3d) *ANHPRM*.



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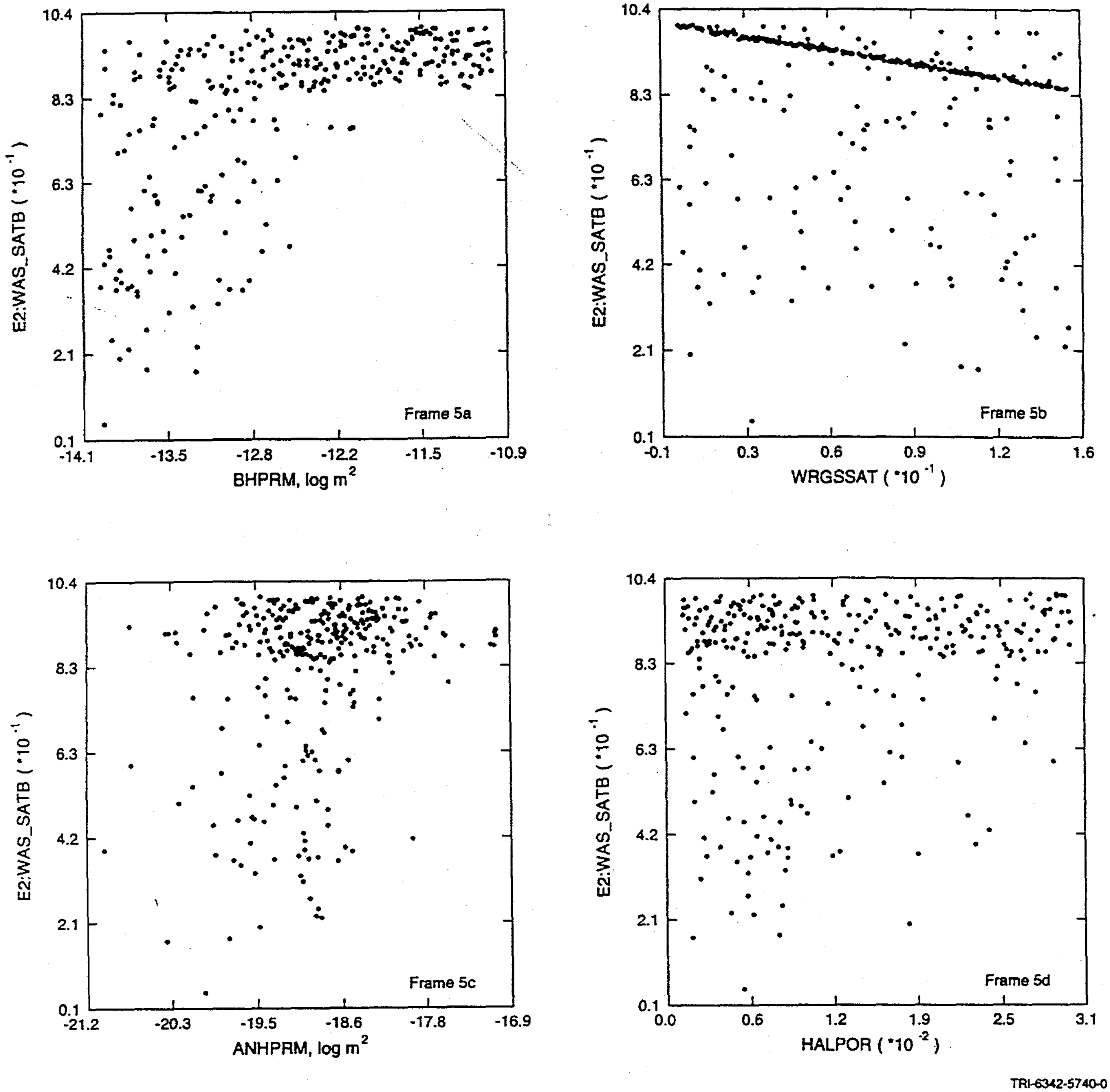

Fig. 5. Scatterplots for brine saturation in lower (i.e., intruded) waste panel at 10,000 yr for an E2 intrusion at 1000 yr (*E2:WAS_SATB*) versus first four variables selected in stepwise regression analysis with rank-transformed data (Table 3): (5a) *BHPRM*, (5b) *WRGSSAT*, (5c) *ANHPRM* and (5d) *HALPOR*.

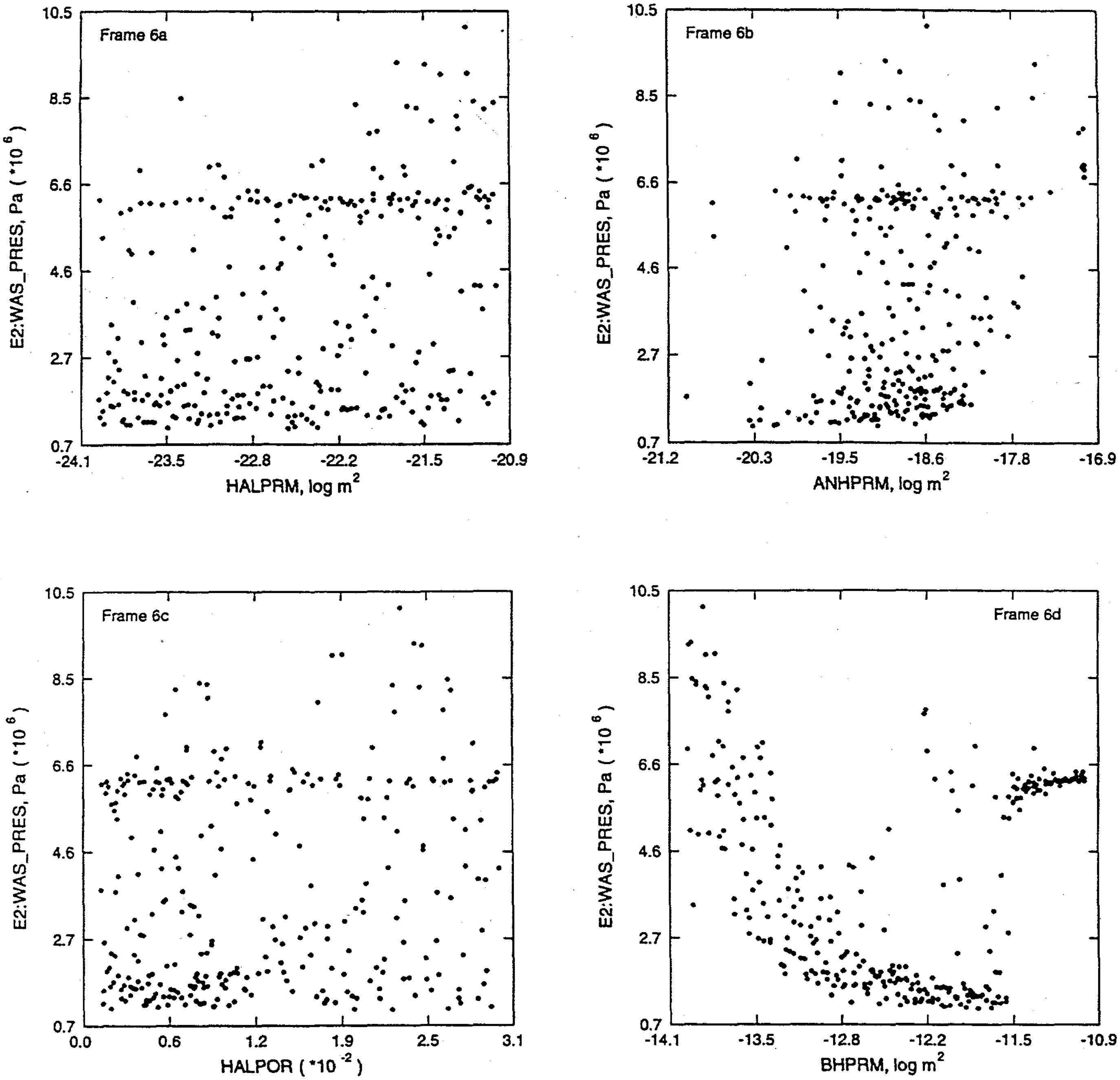

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Fig. 6. Scatterplots for pressure in lower waste panel at 10,000 yr with an E2 intrusion into the lower waste panel at 1000 yr (*E2:WAS_PRES*) versus the three variables (*HALPRM*, *ANHPRM*, *HALPOR*) selected in stepwise regression analysis with raw and rank-transformed data (Table 3) and one additional variable (*BHPRM*) identified by examination of scatterplots: (6a) *HALPRM*, (6b) *ANHPRM*, (6c) *HALPOR*, and (6d) *BHPRM*.

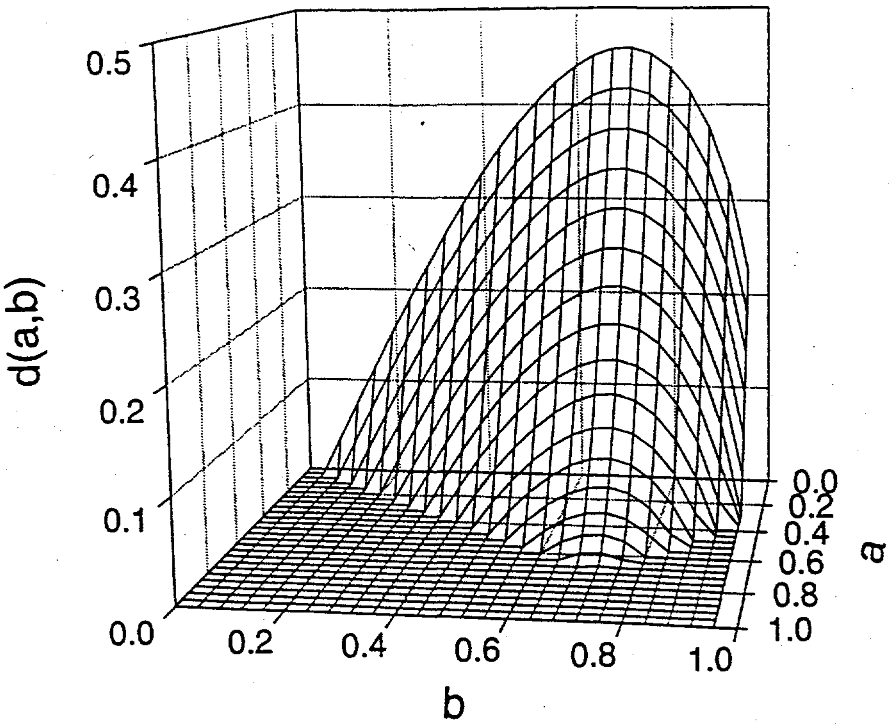

Fig. 7. Graph of $d(b, a) = b(1 - b^2)^{1/2} - a(1 - a^2)^{1/2} > 0$ subject to constraints $0 < a < b < 1$, $a^2 + b^2 < 1$.

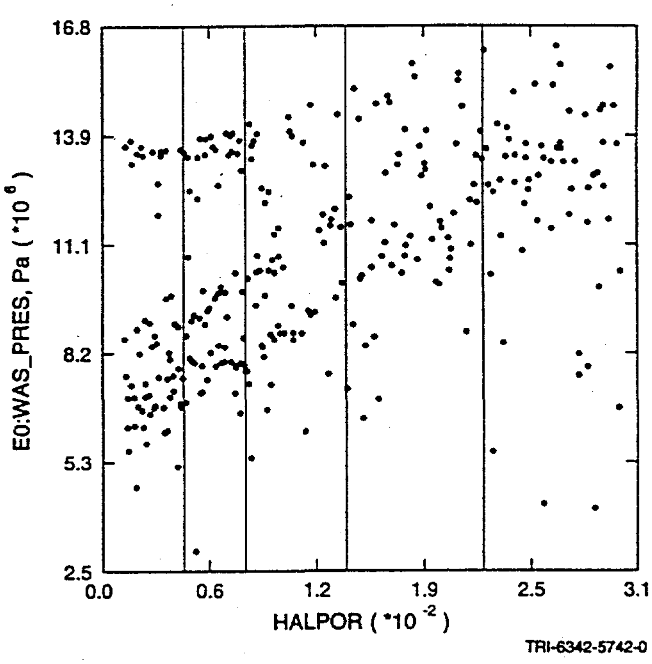

Fig. 8. Example of the partitioning of the range of $x = HALPOR$ into $nX = 5$ classes for $y = E0{:}WAS\_PRES$.

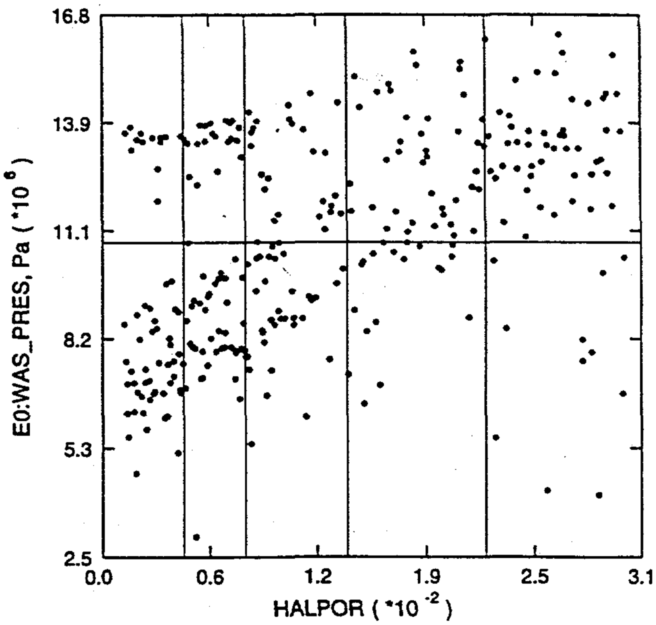

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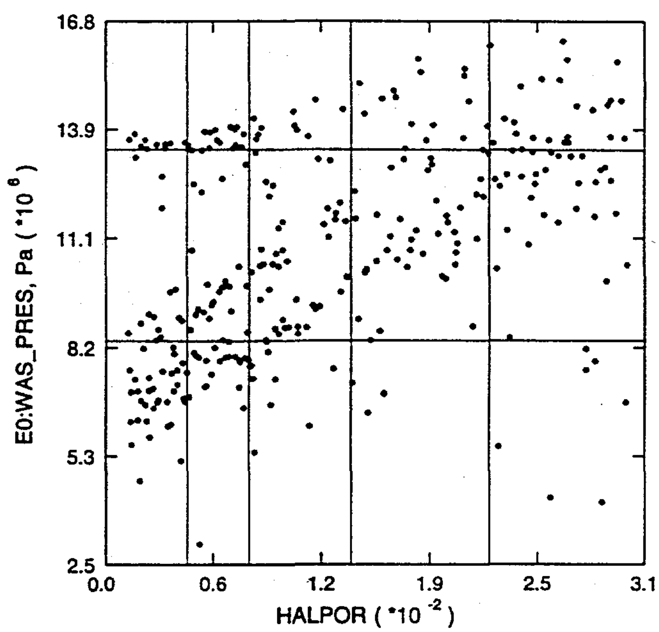
Fig. 9. Example of the partitioning of the range of x = *HALPOR* into nX = 5 classes and the range of y = *E0:WAS_PRES* into values above and below the median $y_{0.5}$.



TRI-6342-5744-0

Fig. 10. Example of the partitioning of the range of x = *HALPOR* into nX = 5 classes and the range of y = *E0:WAS_PRES* into values inside and outside the interquartile range $[y_{0.25}, y_{0.75}]$.

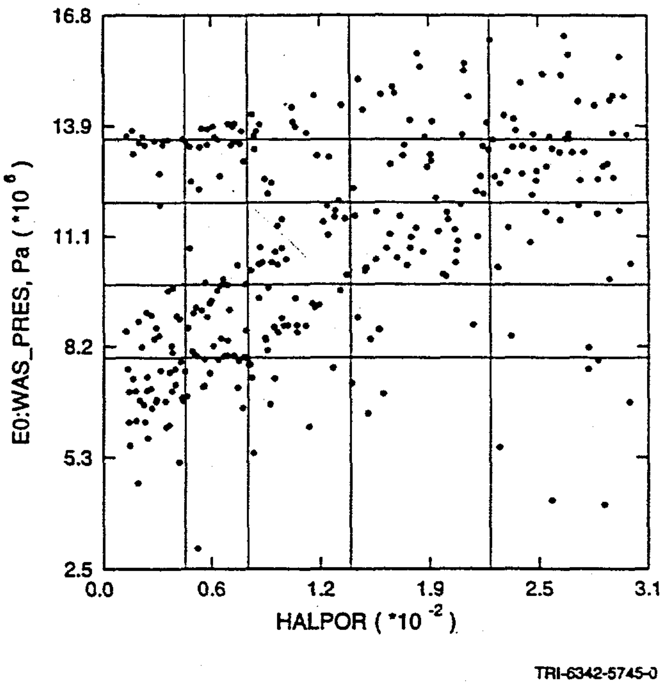


Fig. 11. Example of the partitioning of the range of x = *HALPOR* into $nX = 5$ classes and the range of y = *E0:WAS_PRES* into $nY = 5$ classes.

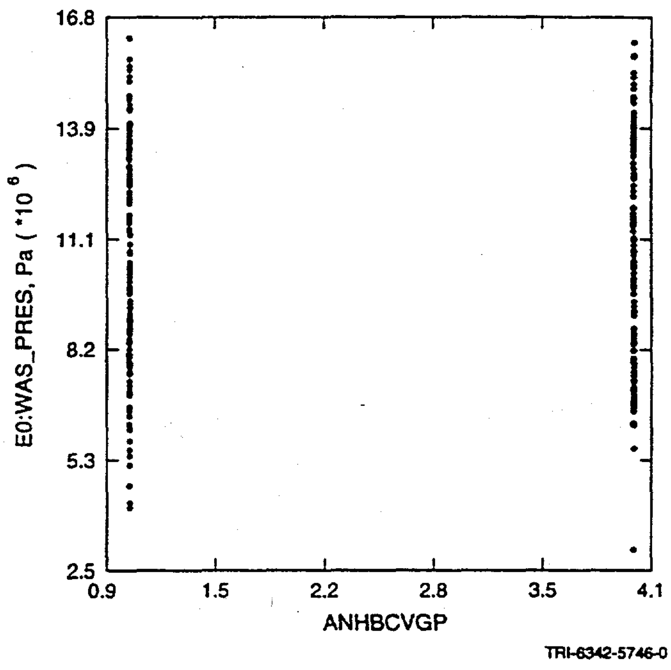


Fig. 12. Scatterplot for *E0:WAS_PRES* versus *ANHBCVGP*.

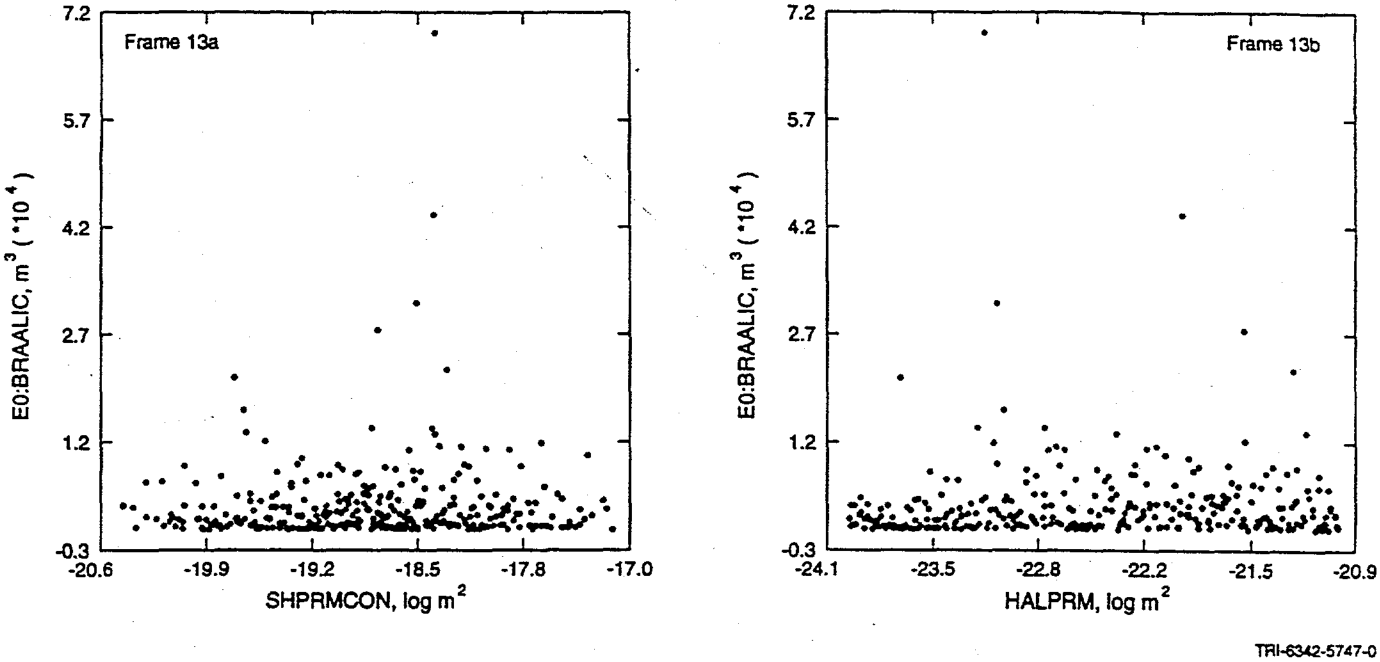


Fig. 13. Scatterplots for *E0:BRAALIC* versus *SHPRMCON* and *HALPRM*.

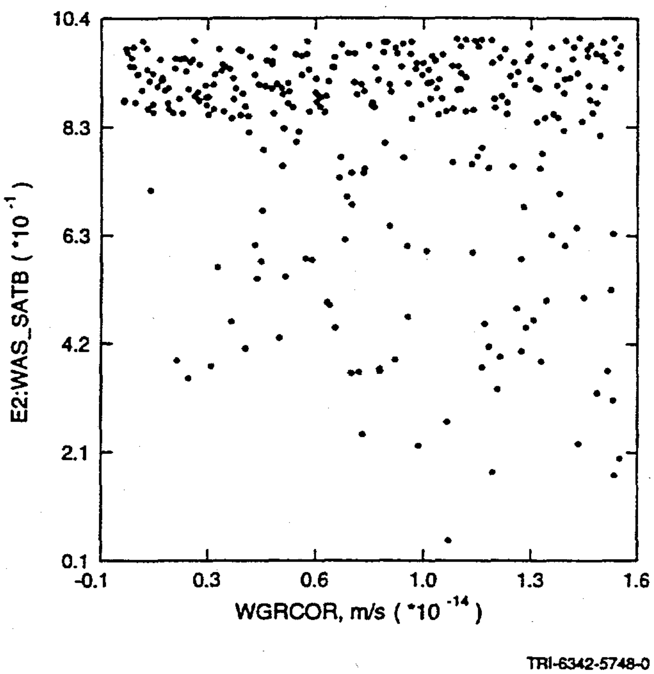


Fig. 14. Scatterplot for *E2:WAS_SATB* versus *WGRCOR*.

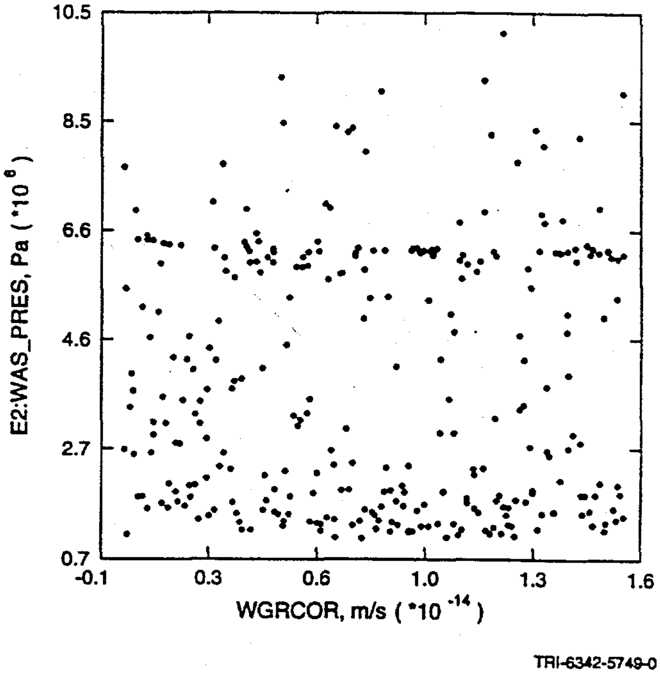


Fig. 15. Scatterplot for *E2:WAS_PRES* versus *WGRCOR*.

Table 1. Definition of Dependent Variables Predicted by BRAGFLO Model Selected for Use in Comparison of Statistical Procedures for Identification of Patterns in Scatterplots

E0:WAS_PRES—Pressure (Pa) in lower repository waste panel (region 23, Fig. 1) at 10,000 yr under undisturbed (i.e., E0) conditions. Number of sampled variables: 26 (Table 2).

E0:BRAALIC—Cumulative brine inflow (m^3) to vicinity of repository over 10,000 yr from anhydrite marker beds (regions 20, 21, 28, Fig. 1) under undisturbed (i.e., E0) conditions. Same sampled variables as *E0:WAS_PRES*.

E2:WAS_SATB—Brine saturation (dimensionless) in lower repository waste panel (region 23, Fig. 1) at 10,000 yr after a drilling intrusion through the lower waste panel at 1000 yr that does not penetrate pressurized brine in the underlying Castile Formation (i.e., an E2 intrusion). Same sampled variables as *E0:WAS_PRES* plus *BHPRM* (Table 2).

E2:WAS_PRES—Pressure (Pa) in lower repository waste panel (region 23, Fig. 1) at 10,000 yr after a drilling intrusion through the lower waste panel at 1000 yr that does not penetrate pressurized brine in the underlying Castile Formation (i.e., an E2 intrusion). Same sampled variables as *E2:WAS_SATB*.

Table 2. Uncertain Variables Used as Input to BRAGFLO in the Calculation of the Dependent Variables in Table 1 (see Table 5.2.1, Ref. 18, and App. PAR, Ref. 22, for additional information and a discussion of all 75 variables included in the LHS)

ANHBCEXP—Brooks-Corey pore distribution parameter for anhydrite (dimensionless). Distribution: Student's with 5 degrees of freedom. Range: 0.491 to 0.842. Mean, Median: 0.644.

ANHBCVGP—Pointer variable for selection of relative permeability model for use in anhydrite. Distribution: Discrete with 60% 0, 40% 1. Value of 0 implies Brooks-Corey model; value of 1 implies van Genuchten-Parker model.

ANHCOMP—Bulk compressibility of anhydrite (Pa^{-1}). Distribution: Student's with 3 degrees of freedom. Range: 1.09×10^{-11} to 2.75×10^{-10} Pa^{-1}. Mean, Median: 8.26×10^{-11} Pa^{-1}. Correlation: −0.99 rank correlation[23] with *ANHPRM*. Variable 21 in LHS.

ANHPRM—Logarithm of anhydrite permeability (m^2). Distribution: Student's with 5 degrees of freedom. Range: −21.0 to −17.1 (i.e., permeability range is 1×10^{-21} to $1 \times 10^{-17.1}$ m^2). Mean, Median: −18.9. Correlation : −0.99 rank correlation with *ANHCOMP*.

ANRBRSAT—Residual brine saturation in anhydrite (dimensionless). Distribution: Student's with 5 degrees of freedom. Range: 7.85×10^{-3} to 1.74×10^{-1}. Mean, Median: 8.36×10^{-2}.

ANRGSSAT—Residual gas saturation in anhydrite (dimensionless). Distribution: Student's with 5 degrees of freedom. Range: 1.39×10^{-2} to 1.79×10^{-1}. Mean, median: 7.71×10^{-2}.

BHPRM—Logarithm of borehole permeability (m^2). Distribution: Uniform. Range: −14 to −11 (i.e., permeability range is 1×10^{-14} to 1×10^{-11} m^2). Mean, median: −12.5.

HALCOMP—Bulk compressibility of halite (Pa^{-1}). Distribution: Uniform. Range: 2.94×10^{-12} to 1.92×10^{-10} PA^{-1}. Mean, median: 9.75×10^{-11} Pa^{-1}, 9.75×10^{-11} Pa^{-1}. Correlation: −0.99 rank correlation with *HALPRM*.

HALPOR—Halite porosity (dimensionless). Distribution: Piecewise uniform. Range: 1.0×10^{-3} to 3×10^{-2}. Mean, median: 1.28×10^{-2}, 1.00×10^{-2}.

HALPRM—Logarithm of halite permeability (m^2). Distribution: Uniform. Range: −24 to −21 (i.e., permeability range is 1×10^{-24} to 1×10^{-21} m^2). Mean, median: −22.5, −22.5. Correlation: −0.99 rank correlation with *HALCOMP*.

SALPRES—Initial brine pressure, without the repository being present, at a reference point located in the center of the combined shafts at the elevation of the midpoint of MB 139 (Pa). Distribution: Uniform. Range: 1.104×10^7 to 1.389×10^7 Pa. Mean, median: 1.247×10^7 Pa, 1.247×10^7 Pa.

SHBCEXP—Brooks-Corey pore distribution parameter for shaft (dimensionless). Distribution: Piecewise uniform. Range: 0.11 to 8.10. Mean, median: 2.52, 0.94.

SHPRMASP—Logarithm of permeability (m^2) of asphalt component of shaft seal (m^2). Distribution: Triangular. Range: −21 to −18 (i.e., permeability range is 1×10^{-21} to 1×10^{-18} m^2). Mean, mode: −19.7, −20.0.

Table 2. (Continued)

SHPRMCLY—Logarithm of permeability (m^2) for clay components of shaft. Distribution: Triangular. Range: –21 to –17.3 (i.e., permeability range is 1×10^{-21} to $1 \times 10^{-17.3}$ m^2). Mean, mode: –18.9, –18.3.

SHPRMCON—Same as *SHPRMASP* but for concrete component of shaft seal for 0 to 400 yr. Distribution: Triangular. Range: –17.0 to –14.0 (i.e., permeability range is 1×10^{-17} to 1×10^{-14} m^2). Mean, mode: -15.3, -15.0.

SHPRMDRZ—Logarithm of permeability (m^2) of DRZ surrounding shaft. Distribution: Triangular. Range: –17.0 to –14.0 (i.e., permeability range is 1×10^{-17} to 1×10^{-14} m^2). Mean, mode: –15.3, –15.0.

SHPRMHAL—Pointer variable (dimensionless) used to select permeability in crushed salt component of shaft seal at different times. Distribution: Uniform. Range: 0 to 1. Mean, mode: 0.5, 0.5. A distribution of permeability (m^2) in the crushed salt component of the shaft seal is defined for each of the following time intervals: [0, 10 yr], [10, 25 yr], [25, 50 yr], [50, 100 yr], [100, 200 yr], [200, 10000 yr]. *SHPRMHAL* is used to select a permeability value from the cumulative distribution function for permeability for each of the preceding time intervals with result that a rank correlation of 1 exists between the permeabilities used for the individual time intervals.

SHRBRSAT—Residual brine saturation in shaft (dimensionless). Distribution: Uniform. Range: 0 to 0.4. Mean, median: 0.2, 0.2.

SHRGSSAT—Residual gas saturation in shaft (dimensionless). Distribution: Uniform. Range: 0 to 0.4. Mean, median: 0.2, 0.2.

WASTWICK—Increase in brine saturation of waste due to capillary forces (dimensionless). Distribution: Uniform. Range: 0 to 1. Mean, median: 0.5, 0.5.

WFBETCEL—Scale factor used in definition of stoichiometric coefficient for microbial gas generation (dimensionless). Distribution: Uniform. Range: 0 to 1. Mean, median: 0.5, 0.5.

WGRCOR—Corrosion rate for steel under inundated conditions in the absence of CO_2 (m/s). Distribution: Uniform. Range: 0 to 1.58×10^{-14} m/s. Mean, median: 7.94×10^{-15} m/s, 7.94×10^{-15} m/s.

WGRMICH—Microbial degradation rate for cellulose under humid conditions (mol/kg•s). Distribution: Uniform. Range: 0 to 1.27×10^{-9} mol/kg•s. Mean, median: 6.34×10^{-10} mol/kg•s, 6.34×10^{-10} mol/kg•s.

WGRMICI—Microbial degradation rate for cellulose under inundated conditions (mol/kg•s). Distribution: Uniform. Range: 3.17×10^{-10} to 9.51×10^{-9} mol/kg•s. Mean, median: 4.92×10^{-9} mol/kg•s, 4.92×10^{-9} mol/kg•s.

WMICDFLG—Pointer variable for microbial degradation of cellulose. Distribution: Discrete, with 50% 0, 25% 1, 25% 2. WMICDFLG = 0, 1, 2 implies no microbial degradation of cellulose, microbial degradation of only cellulose, microbial degradation of cellulose, plastic and rubber.

WRBRNSAT—Residual brine saturation in waste (dimensionless). Distribution: Uniform. Range: 0 to 0.552. Mean, median: 0.276, 0.276.

WRGSSAT—Residual gas saturation in waste (dimensionless). Distribution: Uniform. Range: 0 to 0.15. Mean, median: 0.075, 0.075.

Table 3. Stepwise Regression Analyses with Raw and Rank-Transformed Data with Pooled Results from Replicates R1, R2 and R3 (i.e., for a total of 300 observations) for Output Variables *E0:WAS_PRES*, *E0:BRAALIC*, *E2:WAS_SATB* and *E2:WAS_PRES* at 10,000 yr

	Raw Data, *E0:WAS_PRES*			Rank-Transformed Data, *E0:WAS_PRES*		
Step[a]	Variable[b]	SRC[c]	R^{2d}	Variable[b]	SRRC[e]	R^{2d}
1	*WMICDFLG*	0.72	0.51	*WMICDFLG*	0.71	0.52
2	*HALPOR*	0.47	0.73	*HALPOR*	0.45	0.73
3	*WGRCOR*	0.25	0.79	*WGRCOR*	0.23	0.79
4	*ANHPRM*	0.13	0.81	*ANHPRM*	0.11	0.80
5	*SHRGSSAT*	0.07	0.81	*SALPRES*	0.07	0.80
6	*SALPRES*	0.06	0.82	*SHRGSSAT*	0.06	0.81

	Raw Data, *E0:BRAALIC*			Rank-Transformed Data, *E0:BRAALIC*		
Step	Variable	SRC	R^2	Variable	SRRC	R^2
1	*ANHPRM*	0.56	0.32	*WMICDFLG*	–0.66	0.43
2	*WMICDFLG*	–0.31	0.42	*ANHPRM*	0.59	0.75
3	*WGRCOR*	–0.16	0.45	*HALPOR*	–0.16	0.80
4	*WASTWICK*	–0.15	0.47	*WGRCOR*	–0.15	0.82
5	*ANHBCEXP*	–0.12	0.49	*HALPRM*	0.14	0.85
6	*HALPOR*	–0.10	0.50	*SALPRES*	0.12	0.86
7				*WASTWICK*	–0.10	0.87

	Raw Data, *E2:WAS_SATB*			Rank-Transformed Data, *E2:WAS_SATB*		
Step	Variable	SRC	R^2	Variable	SRRC	R^2
1	*BHPRM*	0.37	0.12	*BHPRM*	0.59	0.36
2	*ANHPRM*	0.30	0.21	*WRGSSAT*	–0.40	0.52
3	*HALPOR*	0.21	0.25	*ANHPRM*	0.23	0.57
4	*WGRCOR*	–0.19	0.29	*HALPOR*	0.13	0.59
5	*WRGSSAT*	–0.15	0.31	*SHPRMHAL*	–0.12	0.60
6	*WMICDFLG*	–0.14	0.33	*WGRCOR*	–0.10	0.61

	Raw Data, *E2:WAS_PRES*			Rank-Transformed Data, *E2:WAS_PRES*		
Step	Variable	SRC	R^2	Variable	SRRC	R^2
1	*HALPRM*	0.37	0.14	*HALPRM*	0.36	0.13
2	*ANHPRM*	0.24	0.20	*ANHPRM*	0.24	0.19
3	*HALPOR*	0.14	0.22	*HALPOR*	0.14	0.20

[a] Steps in stepwise regression analysis.

[b] Variables listed in order of selection in regression analysis with *ANHCOMP* and *HALCOMP* excluded from entry into regression model because of –0.99 rank correlation within the pairs (*ANHPRM*, *ANHCOMP*) and (*HALPRM*, *HALCOMP*).

[c] Standardized regression coefficients (SRCs) in final regression model.

[d] Cumulative R^2 value with entry of each variable into regression model.

[e] Standardized rank regression coefficients (SRRCs) in final regression model.

Table 4. Comparison of Variable Rankings with Different Analysis Procedures[a] for y = *E0:WAS_PRES*, the Variables in Table 2[b] and a Maximum of Five Classes of Values for Each Variable (i.e., nX =5)[c]

Variable[d] Name	CC Rank	CC p-Val	RCC Rank	RCC p-Val	CMN: 1×5 Rank	CMN: 1×5 p-Val	CL: 1×5 Rank	CL: 1×5 p-Val
WMICDFLG	1.0	0.0000	1.0	0.0000	1.0	0.0000	1.0	0.0000
HALPOR	2.0	0.0000	2.0	0.0000	2.0	0.0000	2.0	0.0000
WGRCOR	3.0	0.0000	3.0	0.0000	3.0	0.0000	3.0	0.0000
ANHPRM	4.0	0.0241	4.0	0.0268	4.0	0.0195	4.0	0.0187
SALPRES	5.0	0.0855	5.0	0.0664	13.0	0.6283	13.0	0.5672
WGRMICI	17.0	0.7753	20.0	0.8940	23.0	0.9705	23.0	0.9649
SHPRMCON	18.0	0.7878	18.0	0.8618	10.0	0.4099	11.0	0.4878
ANHBCVGP	20.0	0.8084	15.0	0.7686	18.0	0.8062	16.0	0.7686

Variable Name	CMD: 2×5 Rank	CMD: 2×5 p-Val	CV: 1×5 Rank	CV: 1×5 p-Val	CIQ: 2×5 Rank	CIQ: 2×5 p-Val	SI: 5×5 Rank	SI: 5×5 p-Val
WMICDFLG	1.0	0.0000	1.0	0.0000	1.0	0.0000	1.0	0.0000
HALPOR	2.0	0.0000	12.0	0.3919	2.0	0.0000	2.0	0.0000
WGRCOR	3.0	0.0025	4.0	0.1244	16.5	0.6626	3.0	0.0003
ANHPRM	4.0	0.0663	2.0	0.0042	3.0	0.0007	4.0	0.0049
SALPRES	9.0	0.4932	11.0	0.3723	6.0	0.0868	21.0	0.7554
WGRMICI	24.0	0.9702	24.0	0.8900	5.0	0.0595	13.0	0.3239
SHPRMCON	6.0	0.2674	5.0	0.1287	4.0	0.0244	7.0	0.1487
ANHBCVGP	14.0	0.6442	13.0	0.4752	24.0	1.0000	5.0	0.0194

[a] Table includes only variables that had a p-value less than 0.1 for at least one of the procedures under consideration although the variable rankings for a specific procedure are based on the p-values obtained for that procedure for all 24 variables included in the analysis (See Footnote b).

[b] Table 2 contains 27 variables but *BHPRM* was not used in the calculation of E0 results (i.e., *E0:WAS_PRES* and *E0:BRAALIC*) and the variables in the pairs (*ANHPRM*, *ANHCOMP*), (*HALPRM*, *HALCOMP*) have a –0.99 rank correlation. As a result, *BHPRM*, *ANHCOMP* and *HALCOMP* were not included in the analysis, which resulted in 24 variables (i.e., x's) for analysis with each procedure.

[c] Variables *ANHBCVGP*, *WMICDFLG* in Table 2 are discrete with 2, 3 levels, respectively; for these variables, nX = 2, 3. Also, nY = 5 for SI.

[d] Variables are listed in the table based on their ordering with the p-values obtained for CCs; thus, the listed rankings for CCs will monotonically increase, which will not in general be the case for the other procedures.

Table 5. Comparison of Variable Rankings with Correlation Coefficients, Standardized Regression Coefficients and Partial Correlation Coefficients with Raw and Rank Transformed Data for y = E0:WAS_PRES

Variable[a] Name	*p*-Val	CC Rank	CC Value	SRC Rank	SRC Value	PCC Rank	PCC Value
WMICDFLG	0.0000	1.0	0.7124	1.0	0.7234	1.0	0.8642
HALPOR	0.0000	2.0	0.4483	2.0	0.4651	2.0	0.7469
WGRCOR	0.0000	3.0	0.2762	3.0	0.2460	3.0	0.5113
ANHPRM	0.0241	4.0	0.1302	4.0	0.1277	4.0	0.2953
SALPRES	0.0855	5.0	0.0993	6.0	0.0639	6.0	0.1526

Variable[b] Name	*p*-Val	RCC Rank	RCC Value	SRRC Rank	SRRC Value	PRCC Rank	PRCC Value
WMICDFLG	0.0000	1.0	0.7229	1.0	0.7207	1.0	0.8564
HALPOR	0.0000	2.0	0.4521	2.0	0.4511	2.0	0.7256
WGRCOR	0.0000	3.0	0.2608	3.0	0.2303	3.0	0.4739
ANHPRM	0.0268	4.0	0.1280	4.0	0.1093	4.0	0.2476
SALPRES	0.0664	5.0	0.1062	5.0	0.0723	5.0	0.1667

[a] Comparison based on variables that had a *p*-value less than 0.1 for CCs. Ranks based on values for CCs. SRCs, PCCs in column "VALUE".

[b] Comparison based on variables that had a *p*-value less than 0.1 for RCCs. Ranks based on values for RCCs. SRRCs, PRCCs in column "VALUE".

Table 6. Correlations with Raw and Rank Transformed Data between *WMICDFLG*, *HALPOR*, *WGRCOR*, *ANHPRM* and *SALPRES*

Raw Data

	WMICDFLG	*HALPOR*	*WGRCOR*	*ANHPRM*
HALPOR	–0.035			
WGRCOR	0.027	0.022		
ANHPRM	0.001	–0.004	0.013	
SALPRES	0.056	–0.007	0.001	–0.012

Rank-Transformed Data

	WMICDFLG	*HALPOR*	*WGRCOR*	*ANHPRM*
HALPOR	–0.008			
WGRCOR	0.031	0.014		
ANHPRM	0.018	0.005	0.021	
SALPRES	0.053	–0.010	0.001	0.004

Table 7. Top-Down Correlation Matrix for Variable Rankings with Different Analysis Procedures for $y = E0{:}WAS\_PRES$, Variables included in Table 4[a] and a Maximum of Five Classes of x values (i.e., $nX = 5$)

Top-Down Correlation Matrix

RCC	0.982										
CMN	0.972	0.981									
CL	0.972	0.981	1.000								
CMD	0.972	0.981	1.000	1.000							
CV	0.731	0.740	0.769	0.769	0.769						
CIQ	0.860	0.831	0.872	0.872	0.872	0.705					
SI	0.946	0.967	0.972	0.972	0.972	0.720	0.839				
SRC	0.986	0.996	0.967	0.967	0.967	0.719	0.824	0.963			
PCC	0.986	0.996	0.967	0.967	0.967	0.719	0.824	0.963	1.000		
SRRC	0.996	0.986	0.963	0.963	0.963	0.715	0.840	0.951	0.995	0.995	
PRCC	0.996	0.986	0.963	0.963	0.963	0.715	0.840	0.951	0.995	0.995	1.000
	CC	RCC	CMN	CL	CMD	CV	CIQ	SI	SRC	PCC	SRRC

Top-Down Correlation Matrix p Values

RCC	0.005										
CMN	0.005	0.005									
CL	0.005	0.005	0.004								
CMD	0.005	0.005	0.004	0.004							
CV	0.026	0.025	0.021	0.021	0.021						
CIQ	0.011	0.014	0.011	0.011	0.011	0.031					
SI	0.006	0.005	0.005	0.005	0.005	0.028	0.013				
SRC	0.005	0.004	0.005	0.005	0.005	0.029	0.015	0.005			
PCC	0.005	0.004	0.005	0.005	0.005	0.029	0.015	0.005	0.004		
SRRC	0.004	0.005	0.005	0.005	0.005	0.029	0.013	0.006	0.004	0.004	
PRCC	0.004	0.005	0.005	0.005	0.005	0.029	0.013	0.006	0.004	0.004	0.004
	CC	RCC	CMN	CL	CMD	CV	CIQ	SI	SRC	PCC	SRRC

[a] Variable rankings used in calculation of top-down correlation are based on only the 8 variables included in Table 4. Specifically, each procedure was used to rank these 8 variables from 1 to 8 (i.e., p-values for CCs, RCCs, CMNs, CLs, CMDs, CVs, CIQs, SI; absolute values of coefficients for SRCs, PCCs, SRRCs, PRCCs); then, top-down correlations were calculated on these rankings.

Table 8. Comparison of Variable Rankings for $y = E0{:}WAS\_PRES$ Obtained with a Maximum of Five Classes of x Values (i.e., $nX = 5$) and Analytic Determination of p-values with Variable Rankings Obtained with (i) a Maximum of Ten Classes of x values (i.e., $nX = 10$) and Analytic Determination of p-values and (ii) a Maximum of Five Classes of x values (i.e., $nX = 5$) and Monte Carlo Determination of p-values

Variable[a] Name	CMN: 1 × 5[b] Rank	*p*-Val	CMN: 1 × 10[c] Rank	*p*-Val	CMNMC: 1 × 5[d] Rank	*p*-Val
WMICDFLG	1.0	0.0000	1.0	0.0000	2.0	0.0000
HALPOR	2.0	0.0000	2.0	0.0000	2.0	0.0000
WGRCOR	3.0	0.0000	3.0	0.0000	2.0	0.0000
ANHPRM	4.0	0.0195	4.0	0.1371	4.0	0.0214
SHPRMASP	5.0	0.1439	11.0	0.5087	5.0	0.1495
WRBRNSAT	6.0	0.1506	6.0	0.1947	6.0	0.1526
SHRGSSAT	7.0	0.2488	15.0	0.7062	7.0	0.2497
ANRBRSAT	8.0	0.3034	18.0	0.7693	8.0	0.3027
HALPRM	9.0	0.4097	8.0	0.4092	10.0	0.4060
SHPRMCON	10.0	0.4099	12.0	0.5115	9.0	0.4053
SHRBRSAT	11.0	0.4325	10.0	0.4560	11.0	0.4239
WFBETCEL	12.0	0.5694	7.0	0.4034	12.0	0.5645
SALPRES	13.0	0.6283	20.0	0.8300	13.0	0.6378
ANHBCEXP	14.0	0.7116	16.0	0.7465	14.0	0.7035
WASTWICK	15.0	0.7490	22.0	0.8444	15.0	0.7446
ANRGSSAT	16.0	0.7521	13.0	0.6511	16.0	0.7483
SHPRMHAL	17.0	0.7661	14.0	0.6734	17.0	0.7699
ANHBCVGP	18.0	0.8062	19.0	0.8062	18.0	0.7997
SHBCEXP	19.0	0.8100	21.0	0.8342	19.0	0.8099
WRGSSAT	20.0	0.8358	5.0	0.1542	20.0	0.8377
SHPRMCLY	21.0	0.8601	9.0	0.4218	21.0	0.8625
SHPRMDRZ	22.0	0.8726	17.0	0.7562	22.0	0.8755
WGRMICI	23.0	0.9705	23.0	0.9606	23.0	0.9717
WGRMICH	24.0	0.9975	24.0	0.9919	24.0	0.9973
TDC[e]	1.000		0.854		0.970	

Variable Name	CL: 1 × 5 Rank	*p*-Val	CL: 1 × 10 Rank	*p*-Val	CLMC: 1 × 5 Rank	*p*-Val
WMICDFLG	1.0	0.0000	1.0	0.0000	2.0	0.0000
HALPOR	2.0	0.0000	2.0	0.0000	2.0	0.0000
WGRCOR	3.0	0.0000	3.0	0.0002	2.0	0.0000
ANHPRM	4.0	0.0187	4.0	0.1340	4.0	0.0212
SHPRMASP	5.0	0.1237	9.0	0.4376	5.0	0.1277
WRBRNSAT	6.0	0.2042	7.0	0.2838	6.0	0.2053
ANRBRSAT	7.0	0.2710	16.0	0.7391	7.0	0.2710
SHRGSSAT	8.0	0.3153	17.0	0.7495	8.0	0.3167
HALPRM	9.0	0.3923	6.0	0.2725	9.0	0.3901
SHRBRSAT	10.0	0.4625	12.0	0.5456	10.0	0.4575
SHPRMCON	11.0	0.4878	11.0	0.4655	11.0	0.4852
WFBETCEL	12.0	0.5194	8.0	0.3728	12.0	0.5153
SALPRES	13.0	0.5672	20.0	0.8266	13.0	0.5817
SHPRMHAL	14.0	0.6945	13.0	0.5517	14.0	0.6996
SHBCEXP	15.0	0.7390	21.0	0.8301	15.0	0.7399
ANHBCVGP	16.0	0.7686	19.0	0.7686	16.0	0.7654
ANHBCEXP	17.0	0.7703	18.0	0.7594	17.0	0.7658
ANRGSSAT	18.0	0.8272	15.0	0.7298	18.0	0.8209
WASTWICK	19.0	0.8318	22.0	0.8443	19.0	0.8292
WRGSSAT	20.0	0.8826	5.0	0.2088	20.0	0.8839
SHPRMDRZ	21.0	0.8897	14.0	0.7065	21.0	0.8937
SHPRMCLY	22.0	0.9032	10.0	0.4426	22.0	0.9062
WGRMICI	23.0	0.9649	23.0	0.9691	23.0	0.9663
WGRMICH	24.0	0.9865	24.0	0.9894	24.0	0.9839
TDC[e]	1.000		0.861		0.971	

Variable Name	CMD: 2 × 5 Rank	*p*-Val	CMD: 2 × 10 Rank	*p*-Val	CMDMC: 2 × 5 Rank	*p*-Val
WMICDFLG	1.0	0.0000	1.0	0.0000	1.5	0.0000
HALPOR	2.0	0.0000	2.0	0.0000	1.5	0.0000
WGRCOR	3.0	0.0025	3.0	0.0124	3.0	0.0018
ANHPRM	4.0	0.0663	7.0	0.3398	4.0	0.0690
SHPRMASP	5.0	0.2427	14.0	0.6302	5.0	0.2401
SHPRMCON	6.0	0.2674	9.0	0.3725	6.0	0.2718
ANRBRSAT	7.0	0.3386	18.5	0.7532	7.0	0.3329
HALPRM	8.0	0.3883	8.0	0.3614	8.0	0.3967
SALPRES	9.0	0.4932	18.5	0.7532	9.0	0.5058
WRBRNSAT	10.0	0.5037	13.0	0.6163	10.0	0.5180
WRGSSAT	11.0	0.5249	5.0	0.1596	11.0	0.5223
SHRGSSAT	12.0	0.6151	23.0	0.9281	13.0	0.6050
ANHBCEXP	13.0	0.6387	11.0	0.5075	14.0	0.6224
ANHBCVGP	14.0	0.6442	16.0	0.6442	12.0	0.5685
SHRBRSAT	15.0	0.6868	15.0	0.6441	15.0	0.6950
SHPRMDRZ	16.5	0.7358	10.0	0.4311	17.0	0.7283
WFBETCEL	16.5	0.7358	17.0	0.7265	16.0	0.7169
SHPRMCLY	18.0	0.7847	6.0	0.3293	18.0	0.7659
SHPRMHAL	19.0	0.8325	4.0	0.1177	19.0	0.8357
SHBCEXP	21.0	0.9197	22.0	0.9114	22.0	0.9093
ANRGSSAT	21.0	0.9197	12.0	0.5887	20.0	0.9082
WASTWICK	21.0	0.9197	20.0	0.8729	21.0	0.9085
WGRMICH	23.0	0.9554	21.0	0.8930	23.0	0.9439
WGRMICI	24.0	0.9702	24.0	0.9835	24.0	0.9664
TDC	1.000		0.768		0.986	

Variable Name	CV: 1 × 5 Rank	*p*-Val	CV: 1 × 10 Rank	*p*-Val	CVMC: 1 × 5 Rank	*p*-Val
WMICDFLG	1.0	0.0000	1.0	0.0000	1.0	0.0000
ANHPRM	2.0	0.0042	2.0	0.0172	2.0	0.0031
HALPRM	3.0	0.1184	4.0	0.0844	4.0	0.1095
WGRCOR	4.0	0.1244	6.0	0.1173	3.0	0.1094
SHPRMCON	5.0	0.1287	5.0	0.0929	5.0	0.1201
SHRGSSAT	6.0	0.1466	15.0	0.4691	7.0	0.1411
ANHBCEXP	7.0	0.1539	7.0	0.1928	6.0	0.1393
SHPRMASP	8.0	0.1612	8.0	0.2953	8.0	0.1517
SHPRMCLY	9.0	0.3102	9.0	0.3614	9.0	0.2957
SHBCEXP	10.0	0.3221	11.0	0.4091	10.0	0.3049
SALPRES	11.0	0.3723	3.0	0.0500	11.0	0.3564
HALPOR	12.0	0.3919	20.0	0.5800	12.0	0.3817
ANHBCVGP	13.0	0.4752	16.0	0.4752	13.0	0.4800
WGRMICH	14.0	0.5612	13.0	0.4415	14.0	0.5502
SHPRMDRZ	15.0	0.6067	21.0	0.8635	15.0	0.5942
WASTWICK	16.0	0.6185	10.0	0.3697	16.0	0.6053
WRGSSAT	17.0	0.6398	14.0	0.4670	17.0	0.6237
WRBRNSAT	18.0	0.6632	17.0	0.5542	18.0	0.6588
ANRGSSAT	19.0	0.6761	12.0	0.4391	19.0	0.6666
WFBETCEL	20.0	0.7531	23.0	0.9435	20.0	0.7443
SHRBRSAT	21.0	0.8197	18.0	0.5606	21.0	0.8109
SHPRMHAL	22.0	0.8340	24.0	0.9844	22.0	0.8224
ANRBRSAT	23.0	0.8378	19.0	0.5700	23.0	0.8330
WGRMICI	24.0	0.8900	22.0	0.9219	24.0	0.8854
TDC	1.000		0.892		0.993	

Table 8. (Continued)

Variable Name	CIQ: 2 × 5 Rank	CIQ: 2 × 5 *p*-Val	CIQ: 2 × 10 Rank	CIQ: 2 × 10 *p*-Val	CIQMC: 2 × 5 Rank	CIQMC: 2 × 5 *p*-Val
WMICDFLG	1.0	0.0000	1.0	0.0000	1.5	0.0000
HALPOR	2.0	0.0000	2.0	0.0000	1.5	0.0000
ANHPRM	3.0	0.0007	4.0	0.0112	3.0	0.0005
SHPRMCON	4.0	0.0244	5.0	0.1005	4.0	0.0279
WGRMICI	5.0	0.0595	7.0	0.1719	5.0	0.0565
SALPRES	6.0	0.0868	3.0	0.0077	6.0	0.0893
SHPRMHAL	7.0	0.1801	10.0	0.2993	7.0	0.1729
SHPRMDRZ	8.0	0.1801	14.0	0.4944	8.0	0.1789
WGRMICH	9.0	0.2548	9.0	0.2133	9.0	0.2547
SHPRMASP	10.0	0.3232	8.0	0.1849	10.0	0.3172
SHRGSSAT	11.0	0.3232	13.0	0.4559	11.0	0.3317
SHBCEXP	12.0	0.5249	17.0	0.6441	12.0	0.5281
WASTWICK	13.0	0.5467	12.0	0.4559	13.0	0.5356
WFBETCEL	14.0	0.5918	20.5	0.8729	14.0	0.5948
SHPRMCLY	15.0	0.6387	6.0	0.1426	15.0	0.6264
WGRCOR	16.5	0.6626	23.0	0.9429	16.0	0.6746
WRBRNSAT	16.5	0.6626	20.5	0.8729	17.0	0.6814
HALPRM	18.5	0.6868	11.0	0.3725	19.0	0.7063
ANHBCEXP	18.5	0.6868	16.0	0.6163	18.0	0.7021
ANRGSSAT	20.0	0.7113	18.0	0.6718	20.0	0.7120
WRGSSAT	21.0	0.8557	15.0	0.5887	21.0	0.8508
ANRBRSAT	22.0	0.9197	22.0	0.8930	23.0	0.9122
SHRBRSAT	23.0	0.9554	19.0	0.7265	24.0	0.9426
ANHBCVGP	24.0	1.0000	24.0	1.0000	22.0	0.9010
TDC	1.000		0.917		0.987	

Variable Name	SI: 5 × 5 Rank	SI: 5 × 5 *p*-Val	SI: 10 × 10 Rank	SI: 10 × 10 *p*-Val	SIMC: 5 × 5 Rank	SIMC: 5 × 5 *p*-Val
WMICDFLG	1.0	0.0000	1.0	0.0000	1.5	0.0000
HALPOR	2.0	0.0000	2.0	0.0000	1.5	0.0000
WGRCOR	3.0	0.0003	4.0	0.0073	3.0	0.0003
ANHPRM	4.0	0.0049	5.0	0.0128	4.0	0.0038
ANHBCVGP	5.0	0.0194	8.0	0.1271	5.0	0.0178
WRGSSAT	6.0	0.1229	12.0	0.2786	6.0	0.1196
SHPRMCON	7.0	0.1487	6.0	0.0326	7.0	0.1529
WASTWICK	8.0	0.1850	21.5	0.8743	8.0	0.1829
SHBCEXP	9.0	0.2436	17.0	0.6767	9.0	0.2441
SHPRMHAL	10.0	0.2518	10.0	0.2028	10.0	0.2540
SHPRMASP	11.0	0.2601	11.0	0.2623	11.0	0.2673
SHPRMDRZ	12.0	0.3142	7.0	0.1129	12.0	0.3205
WGRMICI	13.0	0.3239	16.0	0.6363	13.0	0.3252
ANHBCEXP	14.0	0.3438	9.0	0.1768	14.0	0.3472
WFBETCEL	15.0	0.3856	21.5	0.8743	15.0	0.3905
SHRBRSAT	16.0	0.4299	15.0	0.5527	16.0	0.4308
ANRBRSAT	17.0	0.4765	20.0	0.7701	17.0	0.4725
HALPRM	18.0	0.6235	3.0	0.0036	18.0	0.6307
SHRGSSAT	19.0	0.6482	19.0	0.7525	19.0	0.6587
WRBRNSAT	20.0	0.6849	18.0	0.7343	20.0	0.6981
SALPRES	21.0	0.7554	13.0	0.3310	21.0	0.7662
SHPRMCLY	22.0	0.9265	23.0	0.9348	22.0	0.9305
WGRMICH	23.0	0.9437	24.0	0.9709	23.0	0.9429
ANRGSSAT	24.0	0.9763	14.0	0.5316	24.0	0.9791
TDC	1.000		0.812		0.988	

[a] Twenty-four (24) variables included in analysis; see Footnote b to Table 4.

[b] Variable rankings obtained with a maximum of five classes of x values (i.e., $nX = 5$) and analytic determination of p-values.

[c] Variable rankings obtained with a maximum of ten classes of x values (i.e., $nX = 10$) and analytic determination of p-values.

[d] Variable rankings obtained with a maximum of five classes of x values (i.e., $nX = 5$) and Monte Carlo determination of p-values.

[e] Top-down correlation with variable rankings obtained with a maximum of five classes of x values (i.e., $nX = 5$) and analytic determination of p-values.

Table 9. Comparison of Variable Rankings for $y = E0{:}WAS\_PRES$ Obtained with Correlation Coefficients (CCs, RCCs) and Analytic Determination of p-values with Rankings Obtained with Monte Carlo Determination of p-values

Variable[a] Name	CC[b] Rank	CC[b] p-Val	CCMC[c] Rank	CCMC[c] p-Val
WMICDFLG	1.0	0.0000	2.0	0.0000
HALPOR	2.0	0.0000	2.0	0.0000
WGRCOR	3.0	0.0000	2.0	0.0000
ANHPRM	4.0	0.0241	4.0	0.0236
SALPRES	5.0	0.0855	5.0	0.0815
SHRGSSAT	6.0	0.1553	6.0	0.1551
WASTWICK	7.0	0.2163	7.0	0.2200
SHRBRSAT	8.0	0.2226	8.0	0.2222
ANHBCEXP	9.0	0.2369	9.0	0.2379
WFBETCEL	10.0	0.2770	10.0	0.2832
SHPRMCLY	11.0	0.5213	11.0	0.5264
HALPRM	12.0	0.5767	12.0	0.5761
SHPRMASP	13.0	0.6041	13.0	0.6192
WRBRNSAT	14.0	0.6444	14.0	0.6465
SHBCEXP	15.0	0.6831	15.0	0.6875
ANRBRSAT	16.0	0.7237	16.0	0.7236
WGRMICI	17.0	0.7753	17.0	0.7772
SHPRMCON	18.0	0.7878	18.0	0.7878
SHPRMDRZ	19.0	0.7925	19.0	0.7990
ANHBCVGP	20.0	0.8084	20.0	0.8016
WRGSSAT	21.0	0.8251	21.0	0.8279
ANRGSSAT	22.0	0.8834	22.0	0.8879
WGRMICH	23.0	0.9291	23.0	0.9247
SHPRMHAL	24.0	0.9474	24.0	0.9459
TDC		0.971		

Variable Name	RCC Rank	RCC p-Val	RCCMC Rank	RCCMC p-Val
WMICDFLG	1.0	0.0000	2.0	0.0000
HALPOR	2.0	0.0000	2.0	0.0000
WGRCOR	3.0	0.0000	2.0	0.0000
ANHPRM	4.0	0.0268	4.0	0.0250
SALPRES	5.0	0.0664	5.0	0.0634
SHRGSSAT	6.0	0.2322	6.0	0.2335
WFBETCEL	7.0	0.2408	7.0	0.2469
WASTWICK	8.0	0.2726	8.0	0.2758
SHRBRSAT	9.0	0.3068	9.0	0.3056
SHPRMASP	10.0	0.4201	10.0	0.4291
ANHBCEXP	11.0	0.4383	11.0	0.4352
WRBRNSAT	12.0	0.5519	12.0	0.5581
HALPRM	13.0	0.6412	13.0	0.6419
SHPRMCLY	14.0	0.6812	14.0	0.6848
ANHBCVGP	15.0	0.7686	15.0	0.7654
SHBCEXP	16.0	0.8486	16.0	0.8501
SHPRMDRZ	17.0	0.8599	17.0	0.8596
SHPRMCON	18.0	0.8618	18.0	0.8644
SHPRMHAL	19.0	0.8710	19.0	0.8785
WGRMICI	20.0	0.8940	20.0	0.8934
WGRMICH	21.0	0.9576	21.0	0.9559
WRGSSAT	22.0	0.9848	22.0	0.9863
ANRBRSAT	23.0	0.9964	23.0	0.9973
ANRGSSAT	24.0	0.9991	24.0	0.9990
TDC		0.971		

a Twenty-four (24) variables included in analysis; see Footnote b to Table 4.

b Variable rankings obtained with analytic determination of p-values.

c Variable rankings obtained with Monte Carlo determination of p-values.

d Top-down correlation between variable rankings obtained with analytic and Monte Carlo determination of p-values.

Table 10. Exceedance Probabilities (i.e., *p*-values) for Common Mean and Common Variance Tests Calculated with Use of Logarithms[a] for y = E0:WAS_PRES, the Variables in Table 2[b] and a Maximum of Five Classes of Values for Each Variable (i.e., $nX = 5$)[c]

Variable Name	CMN: Log,1 × 5 Rank	*p*-Val	CMNMC: Log,1 × 5 Rank	*p*-Val
WMICDFLG	1.0	0.0000	2.0	0.0000
HALPOR	2.0	0.0000	2.0	0.0000
WGRCOR	3.0	0.0000	2.0	0.0000
ANHPRM	4.0	0.0085	4.0	0.0112

Variable Name	CV: Log,1 × 5 Rank	*p*-Val	CVMC: Log,1 × 5 Rank	*p*-Val
WMICDFLG	1.0	0.0000	1.0	0.0000
ANHPRM	2.0	0.0151	2.0	0.0100
WGRCOR	3.0	0.1051	3.0	0.0672
HALPRM	4.0	0.1116	4.0	0.0786

a Log y_k instead of y_k in Eq. (42) for common means (CMNs) and t_{ql} as defined in Eq. (51) rather than as defined in Eq. (50) for common variances (CVs); for each test; table contains variables with *p*-values less than 0.1.

b See Footnote b, Table 4.

c See Footnote c, Table 4.

Table 11. Comparison of Variable Rankings with Different Analysis Procedures[a] for $y = E0{:}BRAALIC$, the Variables in Table 2[b] and a Maximum of Five Classes of x Values (i.e., $nX = 5$)[c]

Variable Name[d]	CC Rank	CC p-Val	RCC Rank	RCC p-Val	CMN: 1 × 5 Rank	CMN: 1 × 5 p-Val	CL: 1 × 5 Rank	CL: 1 × 5 p-Val
ANHPRM	1.0	0.0000	2.0	0.0000	1.0	0.0000	2.0	0.0000
WMICDFLG	2.0	0.0000	1.0	0.0000	2.0	0.0000	1.0	0.0000
WASTWICK	3.0	0.0045	6.0	0.0405	6.0	0.1062	16.0	0.4411
WGRCOR	4.0	0.0048	4.0	0.0057	4.0	0.0636	4.0	0.0427
ANHBCEXP	5.0	0.0095	15.0	0.6490	13.0	0.4467	19.0	0.7146
WFBETCEL	6.0	0.0555	8.0	0.2131	5.0	0.0732	10.0	0.2299
WRBRNSAT	7.0	0.0615	11.0	0.4046	11.0	0.3483	12.0	0.2889
HALPOR	8.0	0.0934	5.0	0.0087	19.0	0.5960	7.0	0.1431
HALPRM	11.0	0.2593	3.0	0.0014	7.0	0.1105	3.0	0.0019
SHPRMDRZ	12.0	0.2910	22.0	0.8392	21.0	0.6935	5.0	0.1060
SHPRMCON	14.0	0.3369	12.0	0.4170	3.0	0.0057	11.0	0.2394
SHRGSSAT	18.0	0.4767	14.0	0.5371	14.0	0.5044	9.0	0.2139
WGRMICI	21.0	0.5809	17.0	0.6663	20.0	0.6466	21.0	0.8966
SHRBRSAT	23.0	0.7329	10.0	0.2767	22.0	0.6946	6.0	0.1174

Variable Name	CMD: 2 × 5 Rank	CMD: 2 × 5 p-Val	CV: 1 × 5 Rank	CV: 1 × 5 p-Val	CIQ: 2 × 5 Rank	CIQ: 2 × 5 p-Val	SI: 5 × 5 Rank	SI: 5 × 5 p-Val
ANHPRM	2.0	0.0000	1.0	0.0078	1.0	0.0000	2.0	0.0000
WMICDFLG	1.0	0.0000	13.0	0.4046	2.0	0.0000	1.0	0.0000
WASTWICK	15.5	0.5467	8.0	0.2961	10.0	0.3883	15.0	0.5246
WGRCOR	5.0	0.0231	5.0	0.2125	13.5	0.6868	11.0	0.3644
ANHBCEXP	21.0	0.8088	7.0	0.2321	22.5	0.9554	21.0	0.7776
WFBETCEL	15.5	0.5467	6.0	0.2194	8.0	0.3084	13.0	0.4186
WRBRNSAT	13.0	0.3883	12.0	0.3851	7.0	0.2942	14.0	0.4186
HALPOR	4.0	0.0155	20.0	0.5416	6.0	0.2805	4.0	0.0698
HALPRM	3.0	0.0050	11.0	0.3596	24.0	0.9702	3.0	0.0517
SHPRMDRZ	6.0	0.0306	24.0	0.7101	22.5	0.9554	7.0	0.2202
SHPRMCON	10.0	0.2674	2.0	0.0426	17.0	0.8325	8.0	0.2436
SHRGSSAT	8.0	0.0504	17.0	0.5177	3.0	0.0628	6.0	0.2056
WGRMICI	18.0	0.6868	22.0	0.6096	4.0	0.0780	9.0	0.2863
SHRBRSAT	7.0	0.0362	18.0	0.5347	5.0	0.1395	5.0	0.1917

[a, b, c, d] See Footnotes a, b, c, d to Table 4.

Table 12. Comparison of Variable Rankings with Correlation Coefficients, Standardized Regression Coefficients and Partial Correlation Coefficients with Raw and Rank Transformed Data for $y = E0{:}BRAALIC$

Variable[a] Name	p-Val	CC Rank	CC Value	SRC Rank	SRC Value	PCC Rank	PCC Value
ANHPRM	0.0000	1.0	0.5655	1.0	0.5568	1.0	0.6317
WMICDFLG	0.0000	2.0	−.3210	2.0	−.2931	2.0	−.3878
WASTWICK	0.0045	3.0	−.1639	4.0	−.1451	4.0	−.2075
WGRCOR	0.0048	4.0	−.1628	3.0	−.1669	3.0	−.2370
ANHBCEXP	0.0095	5.0	−.1497	5.0	−.1155	5.0	−.1663
WFBETCEL	0.0555	6.0	−.1105	8.0	−.0757	8.0	−.1098
WRBRNSAT	0.0615	7.0	−.1080	9.0	−.0733	9.0	−.1065
HALPOR	0.0934	8.0	−.0969	6.0	−.0993	6.0	−.1435

Variable[b] Name	p-Val	RCC Rank	RCC Value	SRRC Rank	SRRC Value	PRCC Rank	PRCC Value
WMICDFLG	0.0000	1.0	−.6521	1.0	−.6533	1.0	−.8787
ANHPRM	0.0000	2.0	0.5804	2.0	0.5937	2.0	0.8619
HALPRM	0.0014	3.0	0.1850	5.0	0.1443	5.0	0.3817
WGRCOR	0.0057	4.0	−.1598	4.0	−.1509	4.0	−.3963
HALPOR	0.0087	5.0	−.1518	3.0	−.1539	3.0	−.4031
WASTWICK	0.0405	6.0	−.1185	7.0	−.0948	7.0	−.2617

a, b See Footnotes a, b to Table 5.

Table 13. Top Down Correlation Matrix for Variable Rankings with Different Analysis Procedures for $y = E0{:}BRAALIC$, Variables included in Table 11[a] and a Maximum of Five Classes of x values (i.e., $nX = 5$)[b]

RCC	0.729										
CMN	0.841	0.721									
CL	0.589	0.897	0.626								
CMD	0.573	0.913	0.606	0.971							
CV	0.623	0.301	0.820	0.199	0.157						
CIQ	0.581	0.531	0.584	0.526	0.556	0.285					
SI	0.455	0.838	0.531	0.908	0.952	0.072	0.651				
SRC	0.980	0.728	0.839	0.618	0.612	0.630	0.588	0.476			
PCC	0.980	0.728	0.839	0.618	0.612	0.630	0.588	0.476	1.000		
SRRC	0.711	0.912	0.679	0.808	0.877	0.242	0.618	0.817	0.751	0.751	
PRCC	0.711	0.912	0.679	0.808	0.877	0.242	0.618	0.817	0.751	0.751	1.000
	CC	RCC	CMN	CL	CMD	CV	CIQ	SI	SRC	PCC	SRRC

[a] Same as Footnote a to Table 7 except for use of 14 variables from Table 11.

[b] See Table 10.10, Ref. 58, for top-down correlation matrix p-values.

Table 14. Comparison of Variable Rankings for $y = E0{:}BRAALIC$ Obtained with a Maximum of Five Classes of x Values (i.e., $nX = 5$) and Analytic Determination of p-values with Variable Rankings Obtained with (i) a Maximum of Ten Classes of x values (i.e., $nX = 10$) and Analytic Determination of p-values and (ii) a Maximum of Five Classes of x values (i.e., $nX = 5$) and Monte Carlo Determination of p-values (see Table 10.11, Ref. 58, for omitted results)

Variable[a]	CMN: 1 × 5[b]		CMN: 1 × 10[c]		CMNMC: 1 × 5[d]	
Name	Rank	p-Val	Rank	p-Val	Rank	p-Val
ANHPRM	1.0	0.0000	1.0	0.0000	1.5	0.0000
WMICDFLG	2.0	0.0000	2.0	0.0000	1.5	0.0000
SHPRMCON	3.0	0.0057	4.0	0.0655	3.0	0.0036
WGRCOR	4.0	0.0636	5.0	0.0723	4.0	0.0506
WFBETCEL	5.0	0.0732	10.0	0.2163	5.0	0.0572
WASTWICK	6.0	0.1062	6.0	0.1085	6.0	0.0856
HALPRM	7.0	0.1105	12.0	0.4030	7.0	0.0961
SHBCEXP	8.0	0.1140	3.0	0.0120	8.0	0.0995
			...			
ANRGSSAT	23.0	0.7033	23.0	0.9300	24.0	0.7447
SHPRMHAL	24.0	0.7056	21.0	0.7932	23.0	0.7421
TDC[e]	1.000		0.891		0.987	

Variable	CL: 1 × 5		CL: 1 × 10		CLMC: 1 × 5	
Name	Rank	p-Val	Rank	p-Val	Rank	p-Val
WMICDFLG	1.0	0.0000	1.0	0.0000	1.5	0.0000
ANHPRM	2.0	0.0000	2.0	0.0000	1.5	0.0000
HALPRM	3.0	0.0019	3.0	0.0052	3.0	0.0013
WGRCOR	4.0	0.0427	8.0	0.2368	4.0	0.0438
SHPRMDRZ	5.0	0.1060	4.0	0.0206	5.0	0.1095
SHRBRSAT	6.0	0.1174	6.0	0.1781	6.0	0.1166
HALPOR	7.0	0.1431	13.0	0.5392	7.0	0.1427
SHBCEXP	8.0	0.1524	5.0	0.0441	8.0	0.1532
			...			
SHPRMHAL	23.0	0.9367	21.0	0.8705	24.0	0.9392
SHPRMCLY	24.0	0.9385	19.0	0.7203	23.0	0.9387
TDC	1.000		0.941		0.987	

Variable	CMD: 2 × 5		CMD: 2 × 10		CMDMC: 2 × 5	
Name	Rank	p-Val	Rank	p-Val	Rank	p-Val
WMICDFLG	1.0	0.0000	1.0	0.0000	1.5	0.0000
ANHPRM	2.0	0.0000	2.0	0.0000	1.5	0.0000
HALPRM	3.0	0.0050	3.0	0.0089	3.0	0.0040
HALPOR	4.0	0.0155	8.0	0.1596	4.0	0.0169
WGRCOR	5.0	0.0231	7.0	0.1271	5.0	0.0221
SHPRMDRZ	6.0	0.0306	4.0	0.0215	6.0	0.0275
SHRBRSAT	7.0	0.0362	5.0	0.0282	7.0	0.0347
SHRGSSAT	8.0	0.0504	9.0	0.1849	8.0	0.0486
			...			
SHPRMHAL	23.5	0.9702	20.5	0.8514	24.0	0.9672
WRGSSAT	23.5	0.9702	10.0	0.4311	23.0	0.9658
TDC	1.000		0.919		0.987	

Variable	CV: 1 × 5		CV: 1 × 10		CVMC: 1 × 5	
Name	Rank	p-Val	Rank	p-Val	Rank	p-Val
ANHPRM	1.0	0.0078	1.0	0.0010	1.0	0.0000
SHPRMCON	2.0	0.0426	7.0	0.2704	2.0	0.0058
SHBCEXP	3.0	0.1463	2.0	0.0329	3.0	0.0774
ANRBRSAT	4.0	0.1994	5.0	0.1188	4.0	0.1278
WGRCOR	5.0	0.2125	3.0	0.0995	5.0	0.1424
WFBETCEL	6.0	0.2194	13.0	0.4615	6.0	0.1455
ANHBCEXP	7.0	0.2321	4.0	0.1165	7.0	0.1697
WASTWICK	8.0	0.2961	6.0	0.2503	8.0	0.2450
			...			
ANRGSSAT	23.0	0.6631	23.0	0.6481	23.0	0.9119
SHPRMDRZ	24.0	0.7101	19.0	0.5875	24.0	0.9791
TDC	1.000		0.870		0.995	

Variable	CIQ: 2 × 5		CIQ: 2 × 10		CIQMC: 2 × 5	
Name	Rank	p-Val	Rank	p-Val	Rank	p-Val
ANHPRM	1.0	0.0000	1.0	0.0000	1.5	0.0000
WMICDFLG	2.0	0.0000	2.0	0.0000	1.5	0.0000
SHRGSSAT	3.0	0.0628	4.0	0.0856	3.0	0.0628
WGRMICI	4.0	0.0780	6.0	0.1719	4.0	0.0757
SHRBRSAT	5.0	0.1395	13.0	0.5341	5.0	0.1382
HALPOR	6.0	0.2805	3.0	0.0235	6.0	0.2710
WRBRNSAT	7.0	0.2942	8.0	0.2803	7.0	0.2917
WFBETCEL	8.0	0.3084	15.5	0.6441	9.0	0.3089
			...			
ANHBCEXP	22.5	0.9554	23.5	0.9892	22.0	0.9472
HALPRM	24.0	0.9702	10.0	0.4071	24.0	0.9751
TDC	1.000		0.869		0.987	

Variable	SI: 5 × 5		SI: 10 × 10		SIMC: 5 × 5	
Name	Rank	p-Val	Rank	p-Val	Rank	p-Val
WMICDFLG	1.0	0.0000	1.0	0.0000	1.5	0.0000
ANHPRM	2.0	0.0000	2.0	0.0000	1.5	0.0000
HALPRM	3.0	0.0517	7.0	0.2313	3.0	0.0514
HALPOR	4.0	0.0698	6.0	0.2028	4.0	0.0706
SHRBRSAT	5.0	0.1917	8.0	0.2786	5.0	0.1898
SHRGSSAT	6.0	0.2056	14.0	0.5738	6.0	0.2058
SHPRMDRZ	7.0	0.2202	9.0	0.2955	7.0	0.2221
SHPRMCON	8.0	0.2436	3.0	0.0814	8.0	0.2566
			...			
SALPRES	23.0	0.9326	18.0	0.7870	23.0	0.9354
ANRGSSAT	24.0	0.9537	24.0	0.9846	24.0	0.9561
TDC	1.000		0.748		0.988	

a, b, c, d, e See Footnotes a, b, c, d, e to Table 8.

Table 15. Comparison of Variable Rankings for $y = E0{:}BRAALIC$ Obtained with Correlation Coefficients (CCs, RCCs) and Analytic Determination of *p*-values with Rankings Obtained with Monte Carlo Determination of *p*-values (see Table 10.12, Ref. 58, for omitted results)

Variable[a] Name	CC[b] Rank	CC[b] *p*-Val	CCMC[c] Rank	CCMC[c] *p*-Val
ANHPRM	1.0	0.0000	1.5	0.0000
WMICDFLG	2.0	0.0000	1.5	0.0000
WASTWICK	3.0	0.0045	3.0	0.0022
WGRCOR	4.0	0.0048	4.0	0.0029
ANHBCEXP	5.0	0.0095	5.0	0.0115
WFBETCEL	6.0	0.0555	6.0	0.0528
WRBRNSAT	7.0	0.0615	7.0	0.0585
HALPOR	8.0	0.0934	8.0	0.0947
		...		
SHRBRSAT	23.0	0.7329	23.0	0.7371
SHPRMHAL	24.0	0.7958	24.0	0.8000
TDC[d]		0.987		

Variable Name	RCC Rank	RCC *p*-Val	RCCMC Rank	RCCMC *p*-Val
WMICDFLG	1.0	0.0000	1.5	0.0000
ANHPRM	2.0	0.0000	1.5	0.0000
HALPRM	3.0	0.0014	3.0	0.0009
WGRCOR	4.0	0.0057	4.0	0.0044
HALPOR	5.0	0.0087	5.0	0.0084
WASTWICK	6.0	0.0405	6.0	0.0401
SALPRES	7.0	0.1107	7.0	0.1105
WFBETCEL	8.0	0.2131	8.0	0.2107
		...		
WGRMICH	23.0	0.8513	23.0	0.8479
SHPRMHAL	24.0	0.8619	24.0	0.8632
TDC		0.988		

a, b, c, d See Footnotes a, b, c, d in Table 9.

Table 16. Exceedance Probabilities (i.e., *p*-values) for Common Mean and Common Variance Tests Calculated with Use of Logarithms[a] for $y = E0{:}BRAALIC$, the variables in Table 2,[b] and a Maximum of Five Classes of Values for Each Variable (i.e., $nX = 5$)[c]

Variable Name	CMN: Log,1×5 Rank	CMN: Log,1×5 *p*-Val	CMNMC: Log,1×5 Rank	CMNMC: Log,1×5 *p*-Val
WMICDFLG	1.0	0.0000	1.5	0.0000
ANHPRM	2.0	0.0000	1.5	0.0000
HALPRM	3.0	0.0022	3.0	0.0022
WGRCOR	4.0	0.0284	4.0	0.0286
SHPRMDRZ	5.0	0.0967	5.0	0.1029

Variable Name	CV: Log,1×5 Rank	CV: Log,1×5 *p*-Val	CVMC: Log,1×5 Rank	CVMC: Log,1×5 *p*-Val
ANHPRM	1.0	0.0000	1.0	0.0000
WMICDFLG	2.0	0.0002	2.0	0.0064
SHPRMCON	3.0	0.0019	3.0	0.0403
SHBCEXP	4.0	0.0130	4.0	0.1104
WASTWICK	5.0	0.0144	5.0	0.1140
ANRBRSAT	6.0	0.0189	6.0	0.1341
ANHBCEXP	7.0	0.0290	7.0	0.1699
WRBRNSAT	8.0	0.0304	8.0	0.1711
WFBETCEL	9.0	0.0754	9.0	0.2968
WGRMICH	10.0	0.0930	10.0	0.3384

a, b, c See Footnotes a, b, c to Table 10.

Table 17. Comparison of Variable Rankings with Different Analysis Procedures[a] for y = *E2:WAS_SATB*, the Variables in Table 2[b] and a Maximum of Five Classes of x Values (i.e., $nX = 5$)[c]

Variable[d] Name	CC Rank	CC *p*-Val	RCC Rank	RCC *p*-Val	CMN: 1 × 5 Rank	CMN: 1 × 5 *p*-Val	CL: 1 × 5 Rank	CL: 1 × 5 *p*-Val
BHPRM	1.0	0.0000	1.0	0.0000	1.0	0.0000	1.0	0.0000
ANHPRM	2.0	0.0000	3.0	0.0001	2.0	0.0000	3.0	0.0001
HALPOR	3.0	0.0006	5.0	0.0269	4.0	0.0124	12.0	0.3437
WGRCOR	4.0	0.0017	6.0	0.1446	6.0	0.0296	11.0	0.3179
WRGSSAT	5.0	0.0081	2.0	0.0000	5.0	0.0143	2.0	0.0000
WMICDFLG	6.0	0.0214	7.0	0.1745	7.0	0.0317	10.0	0.2824
WGRMICH	7.0	0.0838	8.0	0.1842	3.0	0.0021	4.0	0.0059
SHPRMHAL	8.0	0.0996	4.0	0.0225	10.0	0.1586	8.0	0.1528
WRBRNSAT	11.0	0.2350	13.0	0.4950	8.0	0.0801	6.0	0.0270
ANRBRSAT	15.0	0.6402	20.0	0.6645	19.0	0.7070	13.0	0.3977
SHPRMCLY	21.0	0.9020	16.0	0.6137	11.0	0.1743	7.0	0.0972
SHPRMCON	23.0	0.9478	19.0	0.6549	9.0	0.1149	5.0	0.0202

Variable Name	CMD: 2 × 5 Rank	CMD: 2 × 5 *p*-Val	CV: 1 × 5 Rank	CV: 1 × 5 *p*-Val	CIQ: 2 × 5 Rank	CIQ: 2 × 5 *p*-Val	SI: 5 × 5 Rank	SI: 5 × 5 *p*-Val
BHPRM	1.0	0.0000	1.0	0.0000	3.0	0.0054	2.0	0.0000
ANHPRM	3.0	0.0003	2.0	0.0000	4.0	0.0628	3.0	0.0002
HALPOR	23.5	0.8557	3.0	0.0011	6.0	0.1324	7.0	0.1328
WGRCOR	13.0	0.5037	5.0	0.0067	2.0	0.0019	6.0	0.1010
WRGSSAT	2.0	0.0000	9.0	0.1750	1.0	0.0000	1.0	0.0000
WMICDFLG	8.0	0.2187	6.0	0.0114	15.0	0.5134	8.0	0.1542
WGRMICH	4.0	0.0130	4.0	0.0050	7.0	0.2311	5.0	0.0564
SHPRMHAL	6.0	0.0218	7.0	0.1122	11.0	0.4628	10.0	0.2278
WRBRNSAT	12.0	0.3883	8.0	0.1749	25.0	0.9825	9.0	0.2128
ANRBRSAT	17.0	0.6868	25.0	0.9798	8.0	0.2674	4.0	0.0495
SHPRMCLY	7.0	0.1627	21.0	0.7874	16.0	0.5467	16.0	0.5739
SHPRMCON	5.0	0.0206	22.0	0.8224	10.0	0.3546	12.0	0.4075

[a, c, d] See Footnotes a, b, c to Table 4

[b] Same as Footnote b to Table 4 except that *BHPRM* is used in the calculation of E2 results (i.e., *E2:WAS_SATB* and *E2:WAS_PRES*) and so was included as an independent variable, which resulted in 25 variables (i.e., x's) for analysis with each procedure.

Table 18. Top Down Correlation Matrix for Variable Rankings with Different Analysis Procedures for y = *E2:WAS_SATB*, Variables included in Table 17[a] and a Maximum of Five Classes of x values (i.e., $nX = 5$)[b]

	CC	RCC	CMN	CL	CMD	CV	CIQ	SI	SRC	PCC	SRRC
RCC	0.851										
CMN	0.919	0.790									
CL	0.643	0.815	0.781								
CMD	0.640	0.840	0.763	0.982							
CV	0.947	0.742	0.950	0.609	0.602						
CIQ	0.490	0.631	0.422	0.494	0.503	0.267					
SI	0.551	0.727	0.561	0.702	0.706	0.363	0.840				
SRC	0.988	0.844	0.902	0.646	0.647	0.926	0.530	0.557			
PCC	0.988	0.844	0.902	0.646	0.647	0.926	0.530	0.557	1.000		
SRRC	0.876	0.989	0.812	0.806	0.830	0.762	0.655	0.732	0.859	0.859	
PRCC	0.876	0.989	0.812	0.806	0.830	0.762	0.655	0.732	0.859	0.859	1.000

[a] Same as Footnote a to Table 7 except for use of 12 variables from Table 17.

[b] See Table 10.16, Ref. 58, for top-down correlation matrix p-values.

Table 19. Comparison of Variable Rankings for $y = E2{:}WAS\_SATB$ Obtained with a Maximum of Five Classes of x Values (i.e., $nX = 5$) and Analytic Determination of p-values with Variable Rankings Obtained with (i) a Maximum of Ten Classes of x values (i.e., $nX = 10$) and Analytic Determination of p-values and (ii) a Maximum of Five Classes of x values (i.e., $nX = 5$) and Monte Carlo Determination of p-values (see Table 10.17, Ref. 58, for omitted results)

Variable[a] Name	CMN: 1 × 5[b] Rank	*p*-Val	CMN: 1 × 10[c] Rank	*p*-Val	CMNMC: 1 × 5[d] Rank	*p*-Val
BHPRM	1.0	0.0000	1.0	0.0000	1.5	0.0000
ANHPRM	2.0	0.0000	2.0	0.0000	1.5	0.0000
WGRMICH	3.0	0.0021	3.0	0.0053	3.0	0.0022
HALPOR	4.0	0.0124	6.0	0.0546	4.0	0.0116
WRGSSAT	5.0	0.0143	8.0	0.1113	5.0	0.0146
WGRCOR	6.0	0.0296	5.0	0.0343	6.0	0.0294
WMICDFLG	7.0	0.0317	4.0	0.0317	7.0	0.0320
WRBRNSAT	8.0	0.0801	9.0	0.1416	8.0	0.0791
...						
ANHBCVGP	24.0	0.8920	23.0	0.8920	25.0	0.8930
ANRGSSAT	25.0	0.8929	20.0	0.7163	24.0	0.8913
TDC[e]	1.000		0.962		0.988	

Variable Name	CL: 1 × 5 Rank	*p*-Val	CL: 1 × 10 Rank	*p*-Val	CLMC: 1 × 5 Rank	*p*-Val
BHPRM	1.0	0.0000	1.0	0.0000	1.5	0.0000
WRGSSAT	2.0	0.0000	2.0	0.0000	1.5	0.0000
ANHPRM	3.0	0.0001	3.0	0.0008	3.0	0.0001
WGRMICH	4.0	0.0059	5.0	0.0289	4.0	0.0056
SHPRMCON	5.0	0.0202	6.0	0.0963	5.0	0.0165
WRBRNSAT	6.0	0.0270	4.0	0.0132	6.0	0.0240
SHPRMCLY	7.0	0.0972	11.0	0.3016	7.0	0.0932
SHPRMHAL	8.0	0.1528	8.0	0.1902	8.0	0.1521
...						
ANHBCVGP	24.0	0.9133	24.0	0.9133	22.0	0.9125
SHRGSSAT	25.0	0.9424	7.0	0.1763	25.0	0.9418
TDC	1.000		0.930		0.988	

Variable Name	CMD: 2 × 5 Rank	*p*-Val	CMD: 2 × 10 Rank	*p*-Val	CMDMC: 2 × 5 Rank	*p*-Val
BHPRM	1.0	0.0000	2.0	0.0000	1.5	0.0000
WRGSSAT	2.0	0.0000	1.0	0.0000	1.5	0.0000
ANHPRM	3.0	0.0003	3.0	0.0035	3.0	0.0001
WGRMICH	4.0	0.0130	6.0	0.0856	4.0	0.0135
SHPRMCON	5.0	0.0206	7.0	0.1538	5.0	0.0207
SHPRMHAL	6.0	0.0218	5.0	0.0669	6.0	0.0227
SHPRMCLY	7.0	0.1627	15.0	0.5075	7.0	0.1639
WMICDFLG	8.0	0.2187	9.0	0.2187	8.0	0.2133
...						
ANHBCEXP	23.5	0.8557	25.0	0.9558	23.0	0.8523
SHBCEXP	25.0	0.9825	16.0	0.6163	25.0	0.9827
TDC	1.000		0.835		0.988	

Variable Name	CV: 1 × 5 Rank	*p*-Val	CV: 1 × 10 Rank	*p*-Val	CVMC: 1 × 5 Rank	*p*-Val
BHPRM	1.0	0.0000	1.0	0.0000	1.5	0.0000
ANHPRM	2.0	0.0000	2.0	0.0000	1.5	0.0000
HALPOR	3.0	0.0011	5.0	0.0059	3.0	0.0022
WGRMICH	4.0	0.0050	3.0	0.0043	4.0	0.0082
WGRCOR	5.0	0.0067	4.0	0.0058	5.0	0.0100
WMICDFLG	6.0	0.0114	6.0	0.0114	6.0	0.0140
SHPRMHAL	7.0	0.1122	7.0	0.0765	7.0	0.1208
WRBRNSAT	8.0	0.1749	17.0	0.6414	8.0	0.1806
...						
SHPRMDRZ	24.0	0.8702	20.0	0.7694	24.0	0.8727
ANRBRSAT	25.0	0.9798	25.0	0.9997	25.0	0.9794
TDC	1.000		0.909		0.988	

Variable Name	CIQ: 2 × 5 Rank	*p*-Val	CIQ: 2 × 10 Rank	*p*-Val	CIQMC: 2 × 5 Rank	*p*-Val
WRGSSAT	1.0	0.0000	1.0	0.0000	1.0	0.0000
WGRCOR	2.0	0.0019	2.0	0.0021	2.0	0.0012
BHPRM	3.0	0.0054	3.0	0.0063	3.0	0.0055
ANHPRM	4.0	0.0628	6.0	0.2622	4.0	0.0670
SHRBRSAT	5.0	0.1257	7.5	0.2803	5.0	0.1209
HALPOR	6.0	0.1324	4.0	0.1481	6.0	0.1317
WGRMICH	7.0	0.2311	14.0	0.4814	7.0	0.2359
ANRBRSAT	8.0	0.2674	15.0	0.5075	8.0	0.2613
...						
SHPRMDRZ	24.0	0.9197	11.0	0.3838	24.0	0.9152
WRBRNSAT	25.0	0.9825	25.0	0.9761	25.0	0.9777
TDC	1.000		0.872		0.999	

Variable Name	SI: 5 × 5 Rank	*p*-Val	SI: 10 × 10 Rank	*p*-Val	SIMC: 5 × 5 Rank	*p*-Val
WRGSSAT	1.0	0.0000	1.0	0.0000	2.0	0.0000
BHPRM	2.0	0.0000	2.0	0.0000	2.0	0.0000
ANHPRM	3.0	0.0002	3.0	0.0058	2.0	0.0000
ANRBRSAT	4.0	0.0495	19.0	0.8034	4.0	0.0451
WGRMICH	5.0	0.0564	12.0	0.4276	5.0	0.0568
WGRCOR	6.0	0.1010	9.5	0.3310	6.0	0.0963
HALPOR	7.0	0.1328	9.5	0.3310	7.0	0.1271
WMICDFLG	8.0	0.1542	7.0	0.2502	8.0	0.1508
...						
SHPRMDRZ	24.0	0.9489	25.0	0.9612	24.0	0.9458
SHBCEXP	25.0	0.9537	17.0	0.7701	25.0	0.9574
TDC	1.000		0.746		0.972	

a Twenty-five (25) variables included in analysis; see Footnote b to Table 17.

b, c, d, e See Footnotes b, c, d, e to Table 8.

Table 20. Comparison of Variable Rankings for $y = E2{:}WAS\_SATB$ Obtained with Correlation Coefficients (CCs, RCCs) and Analytic Determination of p-values with Rankings Obtained with Monte Carlo Determination of p-values (see Table 10.18, Ref. 58, for omitted results)

Variable[a] Name	CC[b] Rank	CC[b] p-Val	CCMC[c] Rank	CCMC[c] p-Val
BHPRM	1.0	0.0000	1.5	0.0000
ANHPRM	2.0	0.0000	1.5	0.0000
HALPOR	3.0	0.0006	3.0	0.0004
WGRCOR	4.0	0.0017	4.0	0.0020
WRGSSAT	5.0	0.0081	5.0	0.0088
WMICDFLG	6.0	0.0214	6.0	0.0227
WGRMICH	7.0	0.0838	7.0	0.0844
SHPRMHAL	8.0	0.0996	8.0	0.0998
		...		
SHPRMDRZ	24.0	0.9823	24.0	0.9824
SHBCEXP	25.0	0.9943	25.0	0.9943
TDC[d]		0.988		

Variable Name	RCC Rank	RCC p-Val	RCCMC Rank	RCCMC p-Val
BHPRM	1.0	0.0000	2.0	0.0000
WRGSSAT	2.0	0.0000	2.0	0.0000
ANHPRM	3.0	0.0001	2.0	0.0000
SHPRMHAL	4.0	0.0225	4.0	0.0207
HALPOR	5.0	0.0269	5.0	0.0287
WGRCOR	6.0	0.1446	6.0	0.1480
WMICDFLG	7.0	0.1745	7.0	0.1750
WGRMICH	8.0	0.1842	8.0	0.1885
		...		
HALPRM	24.0	0.9544	24.0	0.9569
WASTWICK	25.0	0.9832	25.0	0.9834
TDC			0.972	

a Twenty-five (25) variables included in analysis; see Footnote b to Table 17.

b, c, d See Footnotes b, c, d to Table 9.

Table 21. Comparison of Variable Rankings with Different Analysis Procedures[a] for y = *E2:WAS_PRES*, the Variables in Table 2[b] and a Maximum of Five Classes of Values for each Variable (i.e., nX =5)[c]

Variable[d]	CC		RCC		CMN: 1 × 5		CL: 1 × 5	
Name	Rank	*p*-Val	Rank	*p*-Val	Rank	*p*-Val	Rank	*p*-Val
HALPRM	1.0	0.0000	1.0	0.0000	2.0	0.0000	2.0	0.0000
ANHPRM	2.0	0.0000	2.0	0.0000	3.0	0.0002	3.0	0.0000
HALPOR	3.0	0.0090	3.0	0.0184	5.0	0.0415	5.0	0.0940
SHPRMDRZ	6.0	0.1684	9.0	0.2417	13.0	0.4281	12.0	0.3131
ANHBCEXP	7.0	0.1786	8.0	0.2373	4.0	0.0405	4.0	0.0602
BHPRM	10.0	0.3651	6.0	0.1704	1.0	0.0000	1.0	0.0000
SHRGSSAT	14.0	0.5958	12.0	0.3948	25.0	0.9511	23.0	0.7738
ANRBRSAT	19.0	0.7133	14.0	0.4378	7.0	0.1513	7.0	0.1304
WGRCOR	20.0	0.7676	17.0	0.6560	17.0	0.5428	9.0	0.2242

Variable	CMD: 2 × 5		CV: 1 × 5		CIQ: 2 × 5		SI: 5 × 5	
Name	Rank	*p*-Val	Rank	*p*-Val	Rank	*p*-Val	Rank	Value
HALPRM	2.0	0.0000	2.0	0.0014	11.0	0.4530	2.0	0.0002
ANHPRM	3.0	0.0007	24.0	0.9251	12.0	0.4628	4.0	0.0049
HALPOR	5.0	0.0700	7.0	0.1410	18.0	0.6151	11.0	0.3142
SHPRMDRZ	17.0	0.6868	4.0	0.0298	13.0	0.5037	17.0	0.6111
ANHBCEXP	4.0	0.0595	16.0	0.5178	19.5	0.6868	14.0	0.4414
BHPRM	1.0	0.0000	1.0	0.0000	1.0	0.0000	1.0	0.0000
SHRGSSAT	22.0	0.8325	14.0	0.3905	3.0	0.0289	5.0	0.0698
ANRBRSAT	6.0	0.0823	22.0	0.7194	4.0	0.0739	10.0	0.2518
WGRCOR	14.5	0.5249	3.0	0.0296	2.0	0.0130	3.0	0.0002

a, b, c, d See Footnotes a, b, c, d to Table 17.

Table 22. Top Down Correlation Matrix for Variable Rankings with Different Analysis Procedures for y = *E2:WAS_PRES*, Variables included in Table 23[a] and a Maximum of Five Classes of x values (i.e., $nX = 5$)[b]

RCC	0.967										
CMN	0.398	0.577									
CL	0.378	0.567	0.997								
CMD	0.378	0.567	0.997	1.000							
CV	0.097	0.230	0.698	0.706	0.706						
CIQ	–.427	–.248	0.429	0.462	0.462	0.715					
SI	0.144	0.342	0.798	0.826	0.826	0.850	0.816				
SRC	0.990	0.975	0.423	0.408	0.408	0.072	–.438	0.149			
PCC	0.990	0.975	0.423	0.408	0.408	0.072	–.438	0.149	1.000		
SRRC	0.967	1.000	0.577	0.567	0.567	0.230	–.248	0.342	0.975	0.975	
PRCC	0.967	1.000	0.577	0.567	0.567	0.230	–.248	0.342	0.975	0.975	1.000
	CC	RCC	CMN	CL	CMD	CV	CIQ	SI	SRC	PCC	SRRC

[a] Same as Footnote a to Table 7 except for use of 9 variables from Table 21.

[b] See Table 10.22, Ref. 58, for top-down correlation matrix p-values.

Table 23. Comparison of Variable Rankings for $y = E2{:}WAS\_PRES$ Obtained with a Maximum of Five classes of x Values (i.e., $nX = 5$) and Analytic Determination of p-values with Variable Rankings Obtained with (i) a Maximum of Ten Classes of x values (i.e., $nX = 10$) and Analytic Determination of p-values and (ii) a Maximum of Five Classes of x values (i.e., $nX = 5$) and Monte Carlo Determination of p-values (see Table 10.23, Ref. 58, for omitted results)

Variable[a] Name	CMN: 1 × 5[b] Rank	*p*-Val	CMN: 1 × 10[c] Rank	*p*-Val	CMNMC: 1 × 5[d] Rank	*p*-Val
BHPRM	1.0	0.0000	1.0	0.0000	1.5	0.0000
HALPRM	2.0	0.0000	2.0	0.0000	1.5	0.0000
ANHPRM	3.0	0.0002	3.0	0.0000	3.0	0.0002
ANHBCEXP	4.0	0.0405	9.0	0.2063	4.0	0.0419
HALPOR	5.0	0.0415	7.0	0.1914	5.0	0.0438
ANHBCVGP	6.0	0.1130	4.0	0.1130	6.0	0.1072
ANRBRSAT	7.0	0.1513	15.0	0.3538	7.0	0.1513
SHBCEXP	8.0	0.1773	10.0	0.2147	8.0	0.1733
...						
WFBETCEL	24.0	0.9015	6.0	0.1751	24.0	0.8972
SHRGSSAT	25.0	0.9511	24.0	0.7887	25.0	0.9555
TDC[e]	1.000		0.805		0.988	

Variable Name	CL: 1 × 5 Rank	*p*-Val	CL: 1x10 Rank	*p*-Val	CLMC: 1 × 5 Rank	*p*-Val
BHPRM	1.0	0.0000	1.0	0.0000	2.0	0.0000
HALPRM	2.0	0.0000	3.0	0.0000	2.0	0.0000
ANHPRM	3.0	0.0000	2.0	0.0000	2.0	0.0000
ANHBCEXP	4.0	0.0602	10.0	0.2585	4.0	0.0625
HALPOR	5.0	0.0940	13.0	0.3454	5.0	0.0972
ANHBCVGP	6.0	0.1099	6.0	0.1099	6.0	0.1031
ANRBRSAT	7.0	0.1304	11.0	0.2851	7.0	0.1312
SHBCEXP	8.0	0.1919	12.0	0.2878	8.0	0.1887
...						
WFBETCEL	24.0	0.8482	14.0	0.3540	24.0	0.8468
SHRBRSAT	25.0	0.9199	20.0	0.6008	25.0	0.9230
TDC	1.000		0.828		0.972	

Variable Name	CMD: 2 × 5 Rank	*p*-Val	CMD: 2 × 10 Rank	*p*-Val	CMDMC: 2 × 5 Rank	*p*-Val
BHPRM	1.0	0.0000	1.0	0.0000	1.5	0.0000
HALPRM	2.0	0.0000	2.0	0.0001	1.5	0.0000
ANHPRM	3.0	0.0007	3.0	0.0012	3.0	0.0005
ANHBCEXP	4.0	0.0595	8.0	0.2288	4.0	0.0583
HALPOR	5.0	0.0700	5.0	0.1596	5.0	0.0718
ANRBRSAT	6.0	0.0823	14.5	0.4311	6.0	0.0827
WMICDFLG	7.0	0.2187	7.0	0.2187	7.0	0.2133
SHPRMASP	8.0	0.2942	9.0	0.2451	8.0	0.2932
...						
WFBETCEL	24.5	0.9197	17.0	0.4814	24.0	0.9080
WRGSSAT	24.5	0.9197	22.0	0.6993	25.0	0.9091
TDC	1.000		0.800		0.987	

Variable Name	CV: 1 × 5 Rank	*p*-Val	CV: 1 × 10 Rank	*p*-Val	CVMC: 1 × 5 Rank	*p*-Val
BHPRM	1.0	0.0000	1.0	0.0012	1.0	0.0000
HALPRM	2.0	0.0014	2.0	0.0201	2.0	0.0021
WGRCOR	3.0	0.0296	3.0	0.0491	3.0	0.0278
SHPRMDRZ	4.0	0.0298	5.0	0.0799	4.0	0.0280
ANHBCVGP	5.0	0.1173	6.0	0.1173	5.0	0.1184
WMICDFLG	6.0	0.1393	8.0	0.1393	8.0	0.1383
HALPOR	7.0	0.1410	10.0	0.1817	7.0	0.1347
SHRBRSAT	8.0	0.1453	15.0	0.3933	6.0	0.1311
...						
ANHPRM	24.0	0.9251	22.0	0.7599	24.0	0.9175
SALPRES	25.0	0.9938	25.0	0.9958	25.0	0.9940
TDC	1.000		0.824		0.995	

Variable Name	CIQ: 2 × 5 Rank	*p*-Val	CIQ: 2 × 10 Rank	*p*-Val	CIQMC: 2 × 5 Rank	*p*-Val
BHPRM	1.0	0.0000	1.0	0.0000	1.0	0.0000
WGRCOR	2.0	0.0130	5.0	0.0565	2.0	0.0132
SHRGSSAT	3.0	0.0289	2.0	0.0163	3.0	0.0277
ANRBRSAT	4.0	0.0739	4.0	0.0308	4.0	0.0704
SHRBRSAT	5.0	0.2093	17.0	0.5075	5.0	0.2055
WASTWICK	6.0	0.2427	9.0	0.2451	6.0	0.2431
SHPRMASP	7.0	0.2805	6.0	0.1719	7.0	0.2721
WRBRNSAT	8.0	0.2942	13.0	0.3838	8.0	0.2973
...						
SALPRES	24.0	0.8889	20.5	0.6993	24.0	0.8946
SHPRMCON	25.0	0.9702	25.0	0.9865	25.0	0.9764
TDC	1.000		0.754		0.999	

Variable Name	SI: 5 × 5 Rank	*p*-Val	SI: 10 × 10 Rank	*p*-Val	SIMC: 5 × 5 Rank	*p*-Val
BHPRM	1.0	0.0000	1.0	0.0000	1.5	0.0000
HALPRM	2.0	0.0002	4.0	0.0082	1.5	0.0000
WGRCOR	3.0	0.0002	2.0	0.0028	3.0	0.0002
ANHPRM	4.0	0.0049	3.0	0.0032	4.0	0.0033
SHRGSSAT	5.0	0.0698	22.0	0.8482	5.0	0.0699
SHBCEXP	6.0	0.1010	15.0	0.3495	6.0	0.0989
WGRMICI	7.0	0.1985	11.0	0.1646	7.0	0.2013
ANHBCVGP	8.0	0.2427	14.0	0.3398	8.0	0.2380
...						
SHPRMHAL	24.0	0.9064	24.0	0.8863	24.0	0.9102
SHPRMCON	25.0	0.9898	20.0	0.5316	25.0	0.9933
TDC	1.000		0.735		0.988	

a Twenty-five (25) variables included in analysis; see Footnote b to Table 17.

b, c, d, e See Footnotes b, c, d, e to Table 8.

Table 24. Comparison of Variable Rankings for $y = E2{:}WAS\_PRES$ Obtained with Correlation Coefficients (CCs, RCCs) and Analytic Determination of p-values with Rankings Obtained with Monte Carlo Determination of p-values (see Table 10.24, Ref. 58, for omitted results)

Variable Name[a]	CC[b] Rank	CC[b] p-Val	CCMC[c] Rank	CCMC[c] p-Val
HALPRM	1.0	0.0000	1.5	0.0000
ANHPRM	2.0	0.0000	1.5	0.0000
HALPOR	3.0	0.0090	3.0	0.0098
ANHBCVGP	4.0	0.1123	4.0	0.1072
SHPRMASP	5.0	0.1606	5.0	0.1610
SHPRMDRZ	6.0	0.1684	6.0	0.1670
ANHBCEXP	7.0	0.1786	7.0	0.1795
WGRMICI	8.0	0.1905	8.0	0.1827
		...		
SHPRMCON	24.0	0.9794	24.0	0.9798
ANRGSSAT	25.0	0.9891	25.0	0.9897
TDC[d]			0.988	

Variable Name	RCC Rank	RCC p-Val	RCCMC Rank	RCCMC p-Val
HALPRM	1.0	0.0000	1.5	0.0000
ANHPRM	2.0	0.0000	1.5	0.0000
HALPOR	3.0	0.0184	3.0	0.0194
ANHBCVGP	4.0	0.1099	4.0	0.1031
WGRMICI	5.0	0.1477	5.0	0.1444
BHPRM	6.0	0.1704	6.0	0.1746
SHPRMASP	7.0	0.1946	7.0	0.1896
ANHBCEXP	8.0	0.2373	8.0	0.2404
		...		
SHBCEXP	24.0	0.9389	24.0	0.9367
SHPRMCON	25.0	0.9918	25.0	0.9918
TDC			0.988	

a Twenty-five (25) variables included in analysis; see Footnote b to Table 17.

b, c, d See Footnotes b, c, d to Table 9.