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Towards a next-to-leading logarithmic result in $B \to X_s\gamma$

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The calculation of the $O(\alpha_s)$ virtual corrections to the matrix element of the inclusive decay $b \to s\gamma$ is reported. These contributions drastically reduce the large renormalization scale dependence of the leading logarithmic calculation. Combining these results with the preliminary result for the Wilson coefficient $C_7(m_b)$ calculated recently by Chetyrkin, Misiak, and Münz, we estimate the branching ratio to be $BR(B \to X_s\gamma) = (3.25 \pm 0.50) \times 10^{-4}$.

1 Introduction

Rare B meson decays are particularly sensitive to physics beyond the standard model (SM). In order to extract the effects of possible new physics, precise experimental and theoretical work on these decays is required.

The inclusive mode $B \to X_s\gamma$ can be systematically analyzed with help of the expansion in inverse powers of the (heavy) b-quark mass, m_b. At the leading order in such an expansion, the inclusive decay rate is given by the perturbatively calculable free quark decay rate. As the power corrections start at the $1/m_b^2$ level only, we neglect these contributions in this talk and model the decay rate $\Gamma(B \to X_s\gamma)$ by the quark level decay width $\Gamma(b \to s\gamma)$, including perturbative QCD corrections.

The earlier SM computations of the branching ratio for $B \to X_s\gamma$ presented e.g. in refs. [1,2,3,4] are in full agreement with the CLEO measurement [5] $BR(B \to$

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$X_s\gamma) = (2.32 \pm 0.67) \times 10^{-4}$. There are large uncertainties, however, in both the experimental and the theoretical results. In view of the expected increase in the experimental precision, the calculations must be refined correspondingly in order to allow quantitative statements about new physics. So far, only the leading logarithmic QCD corrections of the form $\alpha_s^n \log^n(m_b^2/m_w^2)$ have been resummed completely. A systematic improvement is obviously obtained by calculating all the next-to-leading terms of the form $\alpha_s\, \alpha_s^n \log^n(m_b^2/m_w^2)$.

Before discussing the various steps leading to a next-to-leading result, we briefly review the theoretical framework in which the process $b \to s\gamma(+g)$ is evaluated. Usually one starts form the effective Hamiltonian which is obtained by integrating out the t-quark and the W-boson. Neglecting power (m_b/m_w) suppressed operators of dimension > 6, the effective Hamiltonian reads

$$H_{eff}(b \to s\gamma) = -\frac{4G_F}{\sqrt{2}}\, V_{tb}V_{ts}^* \sum_{i=1}^{8} C_i(\mu)\, O_i(\mu) \quad , \tag{1}$$

where the quantities $C_i(\mu)$ are the Wilson coefficients evaluated at the renormalization scale μ and the O_i are following operators:

$$\begin{aligned}
O_1 &= (\bar{c}_{L\beta}\gamma^\mu b_{L\alpha})\,(\bar{s}_{L\alpha}\gamma_\mu c_{L\beta})\,, \\
O_2 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})\,(\bar{s}_{L\beta}\gamma_\mu c_{L\beta})\,, \\
O_7 &= (e/16\pi^2)\,\bar{s}_\alpha\,\sigma^{\mu\nu}\,(m_b(\mu)R + m_s(\mu)L)\,b_\alpha\; F_{\mu\nu}\,, \\
O_8 &= (g_s/16\pi^2)\,\bar{s}_\alpha\,\sigma^{\mu\nu}\,(m_b(\mu)R + m_s(\mu)L)\,(\lambda^A_{\alpha\beta}/2)\,b_\beta\; G^A_{\mu\nu} \quad .
\end{aligned}$$

As the Wilson coefficients of the penguin induced four-Fermion operators O_3,...,O_6 are very small, we do not explicitly list them here.

From the μ-independence of the effective Hamiltonian, one can derive a renormalization group equation (RGE) for the Wilson coefficients $C_i(\mu)$:

$$\mu\frac{d}{d\mu}C_i(\mu) = \gamma_{ji}\, C_j(\mu) \quad , \tag{2}$$

where the (8×8) matrix γ is the anomalous dimension matrix of the operators O_i. Working to leading-logarithmic precision only, it turns out that it is sufficient to do the matching (at $\mu = m_w$) neglecting QCD corrections; to solve the renormalization group equation using the order α_s anomalous dimension matrix $\gamma^{(0)}$; and to calculate (perturbatively) the matrix elements of the operators O_i at the scale $\mu \approx m_b$, again neglecting QCD corrections. Athough it is clear that the renormalization scale μ should be of the order of m_b, its precise value is not fixed, of course. Following common practice, we vary μ in the range $m_b/2 \le \mu \le 2m_b$. This variation leads to a large ($\pm 25\%$) scale

dependence of the leading logarithmic result. Analytically, the source of the large scale dependence is due to a term of the form $\sim \alpha_s log(\mu^2/m_b^2)$.

2 Steps to a next-to-leading result

In order to get the next-to-leading logarithmic result for the branching ratio, one has to improve the Wilson coeffients at the scale $\mu \approx m_b$ and in addition one has to work out the $O(\alpha_s)$ corrections to the matrix elements for $b \to s\gamma$. The improved Wilson coefficients are obtained in two steps: First, the matching at the scale $\mu = m_W$ has to be calculated including order α_s corrections [6]. Second, the RGE step down to the scale $\mu \approx m_b$ has to be done using the order α_s^2 anomalous dimension matrix $\gamma^{(1)}$. This second step is the hardest one, because some entries of the anomalous dimension matrix (like γ_{27}) have to be extracted from 3-loop diagrams [7]! The calculation of the order α_s corrections to the matrix element $b \to s\gamma$ involves the bremsstrahlung process $b \to s\gamma g$ and virtual corrections to $b \to s\gamma$. While the bremsstrahlung corrections (together with those virtual corrections which cancel the infrared and collinear singularities) have been worked out earlier [1,2,8], Greub, Hurth, and Wyler have worked out the virtual corrections completely [9]. Technically, the most difficult part was the calculation of the order α_s corrections to the contribution from the operator O_2; the corresponding 2-loop diagrams are shown in ref. [9]. Using the Mellin-Barnes representation of certain progagator type denominators, we could write the result M_2 of the 2-loop diagrams in the form

$$M_2 = c_0 + \sum_{n,m} c_{nm} \left(\frac{m_c^2}{m_b^2}\right)^n \log^m \left(\frac{m_c^2}{m_b^2}\right) \quad , \tag{3}$$

with $n = 1, 2, 3, 4, ...$ and $m = 0, 1, 2, 3$. The coefficients c_0 and c_{nm} are pure numbers, i.e., independent of any parameters like $m_b, m_c, ...$. Note, in particular, that there is no naked $\log(m_c^2/m_b^2)$ term present in eq. (3). So the limit $m_c \to 0$ of M_2 exists.

3 Preliminary results for the branching ratio $BR(B \to X_s\gamma)$

Summing up, in order to get the next-to-leading logarithmic result for $BR(B \to X_s\gamma)$, one has to know both, the $O(\alpha_s)$ matrix elements and the next-to-leading order Wilson coefficients [a] at $\mu \approx m_b$. Only the combination of these two ingredients is independent of the renormalization scheme. It turns out that in the naive dimensional scheme (NDR) with $\overline{\rm MS}$ subtraction, the correction to

[a] In fact, it is sufficient to know only $C_7(\mu)$ to next-to-leading precision.

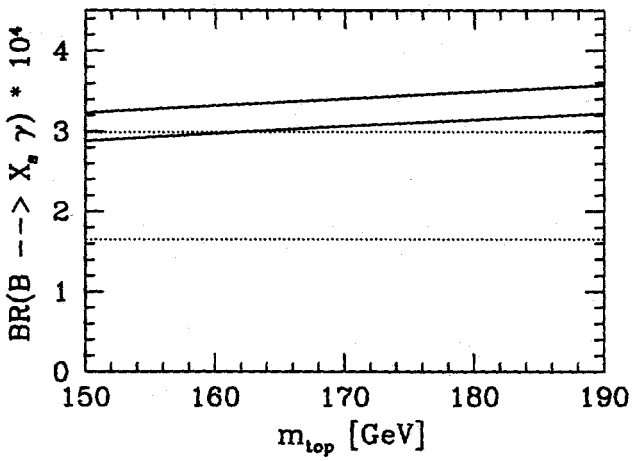

Figure 1: Branching ratio for $B \to X_s\gamma$ as a function of m_t. The upper (lower) solid curve is for $\mu = m_b/2$ ($\mu = 2m_b$). The dotted curves show the CLEO $1-\sigma$ bounds[5]. The other input parameters are taken at their central values.

$C_7(m_b)$ is small[7]. Therefore, a good approximation for $BR(B \to X_s\gamma)$ is obtained by using the leading value for $C_7(m_b)$ in the numerical evaluation of the matrix elements, as presented in[9]. While the μ dependence was about $\pm 25\%$ in the leading logarithmic calculation (varying μ between $m_b/2$ and $2m_b$), it gets drastically reduced to $\pm 6\%$ when taking systematically into account the virtual corrections to the matrix elements. The term $\sim \alpha_s log(m_b^2/\mu^2)$, which caused the large scale dependence of the leading logarithmic result, is cancelled by the $O(\alpha_s)$ virtual corrections to the matrix element. In Fig. 1 we show the remaining μ-dependence as a function of the top quark mass m_t; all the other input parameters are taken at their central values. Combining the uncertainties in m_t and μ ($m_t = (170 \pm 15)$ GeV; $m_b/2 \leq \mu \leq 2m_b$) leads to an error of about 9% in the branching ratio. Besides that, there are other errors to be taken into account, stemming from the uncertainties in $\alpha_s(m_Z)$, the semileptonic branching ratio, and the ratio m_c/m_b. Taking $\alpha_s(m_Z) = (0.117 \pm 0.006)$, $BR_{sl} = (10.4 \pm 0.4)\%$, and $m_c/m_b = (0.29 \pm 0.02)$, one obtains an extra error of about $\pm 12\%$ [2]. To conclude, we end up with a preliminary prediction for the branching ratio $BR(B \to X_s\gamma) = (3.25 \pm 0.30 \pm 0.40) \times 10^{-4}$, where the first error is due to the (μ, m_t) variation and the second error due to the other input parameters. Adding the errors in quadrature, we get $BR(B \to X_s\gamma) = (3.25 \pm 0.50) \times 10^{-4}$.

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