INSTITUTE FOR SPACE STUDIES

SUPERNOVAE, NEUTRON STARS, AND TWO KINDS OF NEUTRINO

H. Y. CHIU

American Physical Society
1962 Summer Meeting in the West
Seattle, Washington
August 27-29, 1962

GODDARD SPACE FLIGHT CENTER
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

DISCLAIMER

SUPERNOVAE, NEUTRON STARS, AND TWO KINDS OF NEUTRINO*

H. Y. CHIU**

The Institute for Space Studies

New York 27, N. Y.

and

Physics Department, Columbia University

New York 27, N. Y.

* Paper presented as an invited lecture at the 1963 Spring Meeting of the American Physical Society held at Washington, D. C. This version is not intended for publication. A fuller version is under preparation.

** NRC-NAS Senior Research Associate

Today I would like to discuss the role played by neutrinos in the core of a star which has undergone a supernova explosion. Related to this problem are some controversial questions, such as the existence of neutron stars, the Schwarzschild singularity in general relativity, and the meaning of the conservation of baryons in the neighborhood of a Schwarzschild singularity. We shall also discuss the problem of detection of neutron stars.

Inside a star nuclear reactions take place, in which hydrogen is converted into heavier elements. Elements inside a star thus undergo thermonuclear evolution. Thermonuclear evolution will cease when the nuclear binding energy per particle reaches a maximum, which occurs at the element iron 56, and at a temperature of 4×10^9 °K. Figure 1 shows the evolution of a star which has completed the thermonuclear evolution cycle. First hydrogen is converted into helium, then helium into carbon, and then carbon into magnesium, then to silicon, and then to iron. Each conversion process occurs at a different temperature. The gradual increase of the temperature is caused by a gravitational contraction.

At a temperature of 4×10^9 °K, the state of maximum binding energy per nucleon is reached, and these elements also come into statistical equilibrium. Further increase of temperature will only shift the state of equilibrium towards lower elements. It turns out that, at a temperature of 8×10^9 °K, the equilibrium state is suddenly shifted from iron into helium. Since this disintegration absorbs energy, gravitational collapse on to the core occurs at a very rapid rate and the core collapses. When the core collapses, the envelope will also contract and be heated adiabatically.

The rate of nuclear reactions is a very steep function of temperature. If they are expressed in the form of a power law, the rate is proportional to something like the 20th power of the temperature. Hence a sudden change in the temperature of the nuclear fuel rich region will cause it to explode. Therefore, while the core collapses, the envelope will explode. This is the version of the supernova collapse theory presented by Hoyle, Fowler, and the Burbidges.

At $T \sim 6\times10^9$ ^oK, kinetic energy is comparable to the rest energy of electrons. Electron pairs are created in thermodynamic equilibrium with radiation. The electron pairs can annihilate into neutrinos:

$$e^- + e^+ \rightarrow \nu_e + \bar{\nu}_e \qquad (1)$$

causing the star to lose energy rapidly. The rate at which energy is dissipated is rather large, of the order of 10^{23} ergs/cm^3-sec, at a temperature of 6×10^9 ^oK. Only a negligible fraction of the neutrinos thus created is reabsorbed by the star, and practically all the energy is lost. This rate is to be compared with the energy density of a star, which is around

10^{24} ergs/cm^3 at a density of 10^6 g/cm^3. The star will have to contract by a good fraction of its radius in a second or two — and this rapid rate precludes any static equilibrium state for the star. The star contracts with a rate very much near the free fall rate.

No matter what the collapse mechanism is, we want to know answers to the following two questions:

(1) What happens to the envelope?

(2) What happens to the core?

Hoyle, Fowler, Colgate, Cameron, and many others have done a great deal of work regarding the expanding envelope. The general result is: fast neutron capture reactions will proceed in the envelope, a shock wave will be generated, the envelope will be expelled, and cosmic rays will be produced. It will be a source of energetic electrons, high energy cosmic rays, radiowaves, and heavy elements. We shall not consider it here.

What happens to the core has only been speculated in the past — perhaps because there is no astronomical evidence as to what might be left as the core and also the physics there is not very clear. It was first speculated that white dwarfs are remnants of such an explosion. White dwarfs are stars of high density, of around 10^6 g/cm^3.

In the following, we shall show that white dwarfs are not expected to be remnants of the core of a supernova.

Moreover, the number of white dwarfs is much too large to be accounted for by the total number of supernova explosions.

From what we shall discuss below, we conclude that the most plausible alternative for the remnant of the core will be neutron stars.

In the following discussion the rotation of a star is ignored. Many people will, of course, object that this cannot be done. They will claim that because of the conservation of angular momentum, a rotating body cannot be compressed to densities comparable to neutron stars, which is 10^{14} g/cm^3, the same as nuclear density. I do not agree with this kind of argument. First, stars do not rotate as a rigid body; there are differential rotations. This we know even for the case of the sun whose structure is relatively simple. Second, if a star has an extended envelope, like a red giant, then most of its angular momentum will be distributed in the envelope. During an explosion stage, the envelope is blown away, and most of the angular momentum will be carried away by the envelope.

For the above reasons, we shall not worry about the angular momentum. We shall first see if neutron stars can be formed. Second, we shall find ways of detecting them.

We now consider the type II supernova, which are explosions of fairly massive stars. Hoyle and Fowler assigned to an average type

II supernova a mass of around 30 times the mass of the sun, which is around 2×10^{33} g. In their model, the mass of the core is taken to be around 20 $M_\odot$ and the rest makes up the envelope. The central density at the time of collapse is around 10^6 g/cm^3. The relation between the density and temperature of the star is roughly

$$\rho \propto T^3$$

assuming no energy is lost during the dynamical collapse. If a substantial amount of stellar energy is removed rapidly, then the actual temperature we shall obtain is much less than the temperature given by this simple equation.

Using this relation, if we find the relaxation time for dissipating stellar energy by neutrino emission processes is too short compared with the collapse time, we may conclude that the above equation is not valid and the star will be much colder than it would be without neutrino emission process.

We note incidentally that since the temperature considered may come as high as 10^{12} °K, the creation of π and μ meson pairs must also be considered, in addition to electron pair formation.

To consider this problem, we divide the neutrino processes into two groups, of which one deals with the neutrino associated with the μ meson, and the other deals with the production of electron neutrinos. This distinction is important because a recent

Table 1. Neutrino Production Processes

Electron-Neutrino Processes		$C = \frac{\sigma v}{c} / G^2 P_\nu^2$
e-1	$e^- + p \rightarrow n + \nu_e$	$\frac{5.5}{\pi}$
e-2	$e^+ + n \rightarrow p + \bar{\nu}_e$	$\frac{5.5}{\pi}$
e-3	$e^- + e^+ \rightarrow \nu_e + \bar{\nu}_e$	$\frac{4}{3\pi}$
	$\gamma + e^\pm \rightarrow e^\pm + \nu_e + \bar{\nu}_e$	$\frac{4}{9} \frac{\alpha}{\pi^2} \ln \frac{E}{mc^2}$
μ Neutrino Processes		
	$\mu^- + \mu^+ \rightarrow \nu_\mu + \bar{\nu}_\mu$	$\frac{4}{3\pi}$
	$\mu^\pm \rightarrow e^\pm + \nu_e + \nu_\mu$	$\frac{1}{\tau} = 0.45 \times 10^6\ \mathrm{sec}^{-1}$
	$\mu^- + p \rightarrow n + \nu_\mu$	$\frac{5.5}{\pi}$
	$\mu^+ + n \rightarrow p + \bar{\nu}_\mu$	$\frac{5.5}{\pi}$
	$\pi^\pm \rightarrow \mu^\pm + \nu_\mu$	$\frac{1}{\tau} = 0.39 \times 10^8\ \mathrm{sec}^{-1}$

experiment by Danby, Schwartz et al has demonstrated that they are different. It turns out that the most important μ-neutrino process is from the decay of π mesons. All electron neutrino processes seem to be equally important. Table 1 lists all neutrino processes considered together with their cross-sections.

In Table 2, we tabulate the temperature, density of the star, and the relaxation time for cooling by neutrinos. We find the curious phenomena that the relaxation time first decreases, then increases again. This is due to the following fact: although the rate of emission of neutrinos increases with temperature, when the stellar matter is opaque to neutrinos, they will be temporarily contained inside the star and then the action of the Pauli Principle will inhibit the further emission of neutrinos. When this occurs, the rate of dissipation of stellar energy is governed not by the production rate, but the rate at which neutrinos can leak out of the star. Hence the neutrino luminosity of a star becomes small as the neutrino opacity increases.

For electron neutrinos, the minimum occurs at a temperature of around 4×10^{10} °K. For μ neutrinos the minimum occurs somewhere around $T \sim 6 \times 10^{11}$ °K. Both rates are large enough to dissipate stellar energy in a time very short compared with the time of dynamical collapse. Hence, when the collapse takes place, it will take place as if the gas is very cold, essentially absolute zero the purpose of stellar structure theory.

It has been objected that the gravitational red shift will
inish the actual amount of energy dissipated. However, the
shift is never very large. When the gravitational red shift
rge, the star approaches a singular situation which we
n discuss.

ron Stars

ler, and Salpeter, have independently investigated this
me time ago. They found that in the density regime
^{13} g/cm^3, no equilibrium configuration could exist for
This is because of the inverse beta process which rapidly
s the number of electrons. The Fermi energy of the
s at a density of 10^9 g/cm^3 is around 10 Mev, which is
e to the energy difference between isobars. Therefore,
eta processes will take place. As a result the total
f electrons decreases as the density increases. Eventually
l dissolve into free neutrons and neutrons will not
te substantially to the pressure until the density is
m^3. Whatever calculation one makes, one finds in the
regime $10^9 - 10^{12}$ g/cm^3 no stable star can exist. If we
tar with a density in this range, a slight touch with one's
will cause it to collapse to a density of 10^{13} g/cm^3.
se neutrons cannot decay, because the energy of decaying

trons is only 1.8 mev, and electronic states are occupied an energy larger than this energy; the exact value depends ly on the temperature. The decay of neutrons in a neutron is forbidden by the Pauli exclusion principle.

e static structure of neutron stars have been studied even e early days of nuclear physics. Among the contributors ppenheimer and Volkhoff, J. A. Wheeler, and A.G.W. Cameron. le 3). They used the equation of stellar structure as given e general relativity theory and they have tried every kind of tion of state, including that for an incompressible gas. onclusion they obtained are all the same: No matter what to the equation of state of the neutrons, the center of on stars becomes gravitationally singular when the mass of tar reaches one solar mass. By singular, I mean the density mes infinite, and the time metric $g_{44} = 0$. The result $g_{44} = 0$ ther serious. It means the following thing: a photon emitted e center will be infinitely red-shifted by the time it reaches stant observer, and hence cannot be seen. Also, at the singular the gravitational potential energy of any particle is equal s rest energy. Hence the net energy of any particle is . That means that it takes no energy to create a pair of cles — any kind of particle.

en the mass of a neutron star exceeds the critical value, does not exist any static equilibrium configuration for tar.

TABLE 3

General Relativistic Treatment of

Neutron Star Models

Model	Equation of State	Critical Mass
Oppenheimer-Volkhoff	Perfect Fermi Gas	0.76 $M_\odot$
K. Schwarzschild	Incompressible fluid at nuclear repulsive core density	~ 1 $M_\odot$
A.G.W. Cameron	Skyrme computation, according to many body theory	$\sim 1 \rightarrow 3$ $M_\odot$

At the critical mass $g_{44} = 0$ at the center and pressure becomes infinite. For a point mass

$$g_{44} = \left(1 - \frac{2GM}{rc^2}\right) = \left(1 - \frac{r_o}{r}\right)$$

$$r_o = 2.6 \text{ km for } M = 1\ M_\odot$$

$$r_o = 10^{-53} \text{ cm for Proton}$$

In our case we started with 20 solar masses before the explosion. Only a fraction is blown away; what happens to the rest of the mass? Since there does not exist any equilibrium configuration, will the collapse take place indefinitely?

Oppenheimer and Snyder take this attitude. They argue that the whole star will fall through the singularity whose radius is defined in Table 3, and thus will disappear from the rest of the universe. They found out that the disappearance from the world will be approached only asymptotically. Hence this is consistent with the idea that there is no equilibrium configuration. To a local observer who willingly volunteers to fall with the star, however, it will take a finite time and for the case of the sun the time is rather short — around one day.

Once the fall is nearly completed, we have no way of telling where the star is, since all photons and gravitons and neutrinos leaving the star will be infinitely red shifted so that its gravitational effect cannot be felt. However the star may behave within the singularity, to us the star is forever lost, together with its 10^{54} ergs of energy and its 10^{57} nucleons. To external observers, this disappearance not only does not conserve nucleon numbers, but also does not conserve the energy.

Wheeler takes another attitude. He argued that when the mass of the star is very near the critical mass, if one adds just little drop of matter to the star, carefully extracting every bit of energy, the net increase of the mass of the star as observed by a distant observer on its gravitational pull is much less than the mass added. The energy of the drop of matter is converted into energy — into a kind of energy that can escape. This means neutrinos, radiation, and gravitons. In his picture, nucleons just dissolve into energy, but the energy itself, at least, is conserved.

We have so far treated the neutron star as a classical object. It has no built-in mechanism to remember how many nucleons it has been given. It remembers the mass only as energy, a property of the gravitational field. Given a neutron star whose mass is equal to the critical mass, we pump in energy and dissolve the star from its gravitational binding. When this is completed, we get mass back. Then we carry away these particles to gravitational field-free space. We then measure how much mass is there. The mass we obtained in this way is a constant times the critical mass, independent of the way the neutron star is formed, indicating that the neutron star has no memory of its nucleon number.

An analogous situation exists in electromagnetism but there quantum theory resolves the difficulty. Consider a nucleus of charge Z greater than 137. The binding energy of its K-shell electrons will be greater than $2mc^2$, the rest energy of an electron pair. Spontaneous creations of electron pairs can take place. However, once a pair is created, the positron will escape and the electron will fill up a quantum state. The spontaneous creation process will continue as long as there are quantum states available. The total number of bound electrons is limited. One cannot have a nucleus with an indefinitely large number of bound electrons.

In the case of a neutron star, which is a gravitationally bound object, this is not so simple. First, if quantum theory comes in at all, it will not come in in a way as simple as that for a nucleus with charge $Z > 137$. The system is too large. Its dimension is much larger than its de Broglie wave length. Second, the possibility of anti-gravity is practically ruled out by the very precise experiment of Eötvös as pointed out by Schiff. Third, if any ground state exists at all, it will have an energy so low that it does not mean anything to us because the object is so big. These are the difficulties we deal with.

Astronomically there are some indications that neutron stars exist. From the rate of occurence of Type II supernova in other galaxies, we estimate in our galaxy that there are around 10^7 neutron stars, of which around 100 neutron stars are within 100 light years from the sun. Zwicky recently reported the discovery of two nearby objects which show large proper motion and which have extremely high surface temperature and small radius. The radius is of the order of 100 km. The radius of a neutron star, discounting the possible existence of an envelope, is of the order of 20 km. We do not know the properties of the envelope of a neutron star, but it is not beyond reason that it may be quite extended. In general it is expected that these neutron stars will appear to be bluer and also dimmer than white dwarfs. So far no theory concerning the envelope of such a condensed star exists.

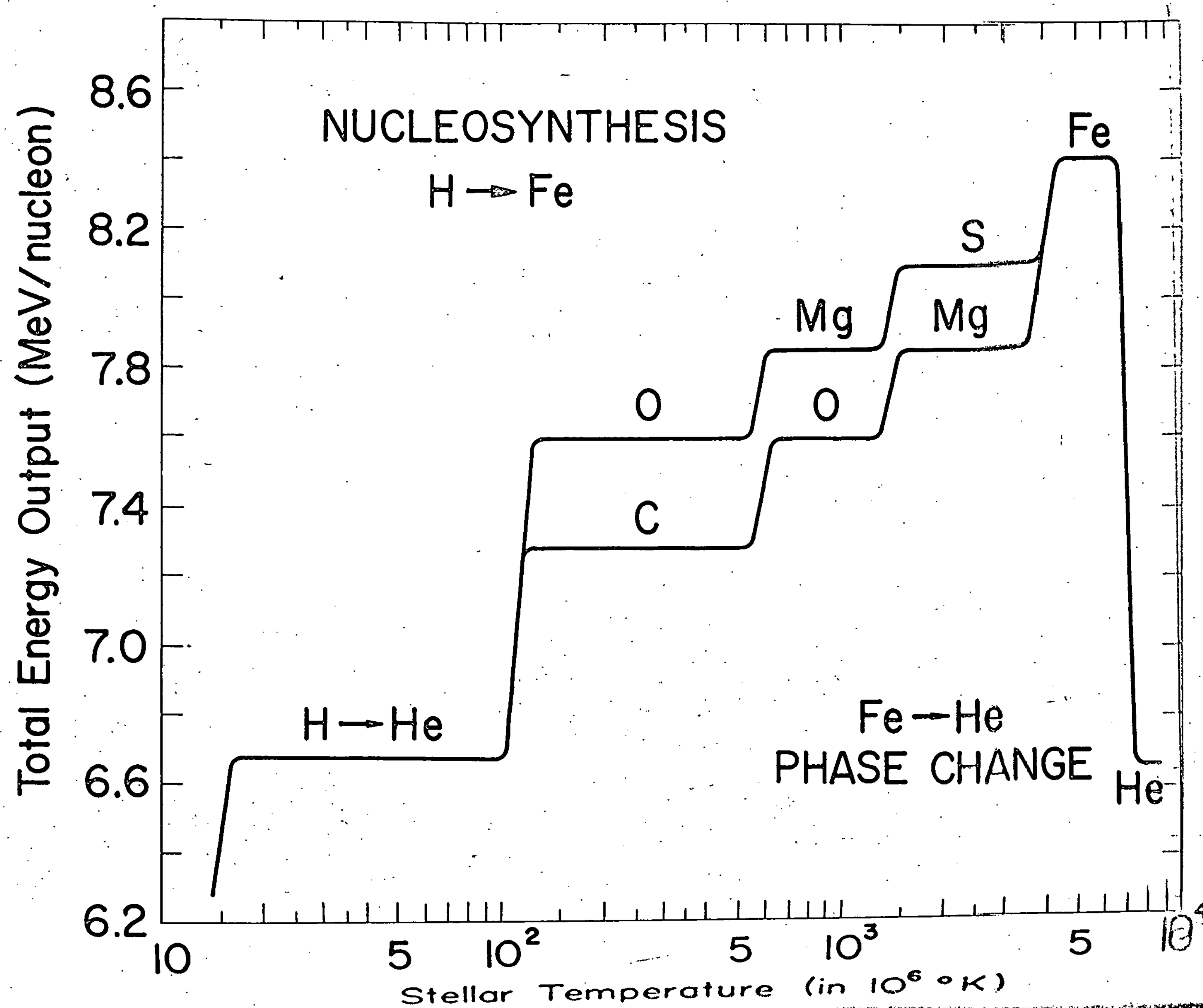
In conclusion we may state the following: there definitely exists a contradiction or paradox between the theory of general relativity, and the theory of elementary particles. While the theory of elementary particles has not been developed to a degree such that we may trust all of its predictions, it is certainly founded on extremely sound experimental basis. As for general relativity, the theory is founded on extremely sound theoretical reasoning, but its experimental verification has so far been limited to only second order. Perhaps in the center of a neutron star, where the two theories meet, lies the unified theory long looked for.

NEUTRINO LOSS RATES IN A NEUTRON STAR

TEMPERATURE °K	DENSITY g/cm³	ENERGY DENSITY ergs/cm³	NEUTRINO PRODUCTION RATE		MEAN FREE PATH cm		RADIUS OF STAR	RELAXATION TIME sec.	
			ν_e	ν_μ	ν_e	ν_μ		ν_e	ν_μ
10^{10}	10^7	1.6×10^{26}	10^{26}		10^{10}		10^9	1	
	4×10^7	10^{27}	3×10^{27}		1.6×10^9		6.3×10^8	3×10^{-1}	
	1.6×10^8	6.3×10^{27}	2×10^{29}	4×10^{12}	2.5×10^8	> Star	4×10^8	3×10^{-2}	
	6.3×10^8	4×10^{28}	1.3×10^{31}	7×10^{21}	4×10^7	> Star	2.5×10^8	3×10^{-3}	
	2.5×10^9	2.5×10^{29}	10^{33}	9×10^{28}	6.3×10^6	> Star	1.6×10^8	0.1	3
10^{11}	10^{10}	1.6×10^{30}	EQ.	10^{31}	10^6	> Star	10^8	0.3	0.1
	4×10^{10}	10^{31}	EQ.	2×10^{34}	1.6×10^5	> Star	6.3×10^7	0.5	5×10^{-4}
	1.6×10^{11}	6.3×10^{31}	EQ.	2×10^{36}	2.5×10^4	> Star	4×10^7	1	3×10^{-5}
	6.3×10^{11}	4×10^{32}	EQ.	5×10^{37}	4×10^3	> Star	2.5×10^7		10^{-5}
	2.5×10^{12}	2.5×10^{33}	EQ.	EQ.	6×10^2	~ Star	1.6×10^7		10^{-4}
10^{12}	10^{13}	1.6×10^{34}	EQ.	EQ.		~ Star	10^7		10^{-3}

FIGURE CAPTION

Figure 1. The evolution of elements inside a star. The elements are characterized by their binding energy which is plotted as a function of temperature. Iron-helium disintegration occurs at around 7×10^9 °K.



NUCLEOSYNTHESIS
H → Fe
Total Energy Output (MeV/nucleon)
8.6
8.2
7.8
7.4
7.0
6.6
6.2
Fe
S
Mg
Mg
O
O
C
H → He
Fe → He
PHASE CHANGE
He
10
5
10²
5
10³
5
10⁴
Stellar Temperature (in 10⁶ °K)