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Caustic Rings of Dark Matter

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Abstract

It is shown that the infall of collisionless dark matter onto isolated galaxies produces a series of caustic rings in the halo dark matter distribution. The properties of these caustics are investigated. It is found in particular that the density profile of the caustic behaves as the inverse distance to the ring. Bumps in the rotation curve of NGC 3198 are interpreted as due to caustic rings of dark matter.

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There are compelling reasons to believe that the dominant component of the dark matter of the universe is non-baryonic collisionless particles [1]. The leading candidates are axions, WIMPs and massive neutrinos. The word "collisionless" signifies that the particles are so weakly interacting that they have moved purely under the influence of gravity since their decoupling at a very early time (of order 10^{-4} sec for axions, of order 1 sec for neutrinos and WIMPs). In the limit where the primordial velocity dispersion of the particles is neglected, they all lie on the same 3-dim. 'sheet' in 6-dim. phase-space. Their phase-space evolution must obey Liouville's theorem. This implies that the 3-dim. sheet cannot tear and hence that it satisfies certain topological constraints.

Let us assume that collisionless dark matter (CDM) exists. Usually, CDM means 'cold dark matter' (e.g., axions or WIMPs) but because massive neutrinos ($m_\nu \gg 10^{-7}$ eV) behave, from the point of view of this paper, in a similar way we include them in our definition as well. Because their phase-space sheet cannot tear, CDM particles must be present everywhere in space, including specifically intergalactic space. The space density may be reduced by stretching of the phase-space sheet but it cannot vanish. Moreover, the average space density is recovered as soon as the average is taken over distances larger than the distance CDM may have locally moved away from perfect Hubble flow. In a region which is sparsely populated with galaxies, this distance is much smaller than the distance between galaxies. The implication is that isolated galaxies are surrounded by unseen CDM and hence, because of gravity, CDM keeps falling onto isolated galaxies continuously from all directions. If the galaxy merges with other galaxies to form a cluster, infall onto the galaxy gets shut off because of lack of material but infall onto the cluster continues assuming that the cluster is itself isolated. In an open universe ($\Omega < 1$), the infall process eventually turns off because the universe becomes very dilute. However, even if our own universe is open, we are far from having reached the turn-off time.

It has been shown [2] that, under a wide range of circumstances, CDM infall onto an isolated galaxy produces a halo whose density falls off like $\frac{1}{r^2}$ for large r, where r is the distance to the galactic center. Such a halo implies a flat galactic rotation curve [3]. It has

also been shown [4] that the angular momentum carried by the CDM particles has the effect of depleting the inner halo and of making the halo contribution to the rotation velocity vanish at $r \to 0$, thereby introducing an effective core radius. Both the approximately flat rotation curves and the presence of effective core radii are consistent with observation. The focus of the present paper is the appearance of ring caustics in the halo distribution caused by CDM infall. The existence of these caustics was noted in ref. [4] but their properties were left unexplored.

Consider then the infall of CDM onto an isolated galaxy. Let's first neglect the velocity dispersion of the infalling particles. In practice it is sufficient that the velocity dispersion of the infalling matter is much smaller than the rotation velocity of the galaxy but it is convenient to consider the extreme case of zero velocity dispersion first. Consider the time evolution of all CDM particles which are about to fall onto the galaxy for the first time in their history at time t. For the sake of definiteness, we may consider all particles which have zero radial velocity ($\dot{r} = 0$) for the first time then. Such particles are said to be at their 'first turnaround'; they were receding from the galaxy as part of the general Hubble flow before t and will be falling onto the galaxy just after t. These particles form a closed surface, enclosing the galaxy, hereafter called the turnaround 'sphere' at time t. (The present turnaround sphere of the Milky Way galaxy has a radius of order 2 Mpc.) The turnaround sphere at time t falls through the central parts of the galaxy at a time of order $2t$. Particles falling through the galactic disk (assuming the galaxy is a spiral) get scattered by an angle $\Delta\theta \sim 10^{-3}$ by the gravitational fields of various inhomogeneities such as molecular clouds, globular clusters and stars [5]. However most particles carry too much angular momentum to reach the luminous parts of the galaxy and are scattered much less. Because the scattering is small, the particles on the turnaround sphere at time t, after falling through the galaxy, form a new sphere which reaches its maximum radius R' at some time t'. The radius R' at the second turnaround is smaller than the radius R at the first turnaround because the galaxy has grown by infall in the meantime. The sphere continues oscillating in this way although it gets progressively fuzzier because of scattering off inhomogeneities in the galaxy.

3

From a topological viewpoint, each time the sphere falls through the galaxy it turns itself inside out and hence there is a ring of special space-time points associated with each fall through the galaxy. This ring may be defined as the locus of points which are inside the sphere last. Fig. I illustrates this. It shows successive 2-dim. cross-sections of a sphere falling in and out. The cusps in frames (d) and (e) are located at the intersections of the ring with the plane of the figure. Because CDM falls in continuously, the ring in space-time associated with one infalling sphere is in fact a persistent feature in space. For an arbitrary angular momentum distribution on the turnaround sphere, the ring is a closed loop of arbitrary shape. However, if the angular momentum distribution is dominated by a smooth component which carries *net* angular momentum, the ring resembles a circle. If there is no angular momentum at all, the ring reduces to a point at the galactic center. As we will soon see, the ring is the location of a caustic with strong density contrast. There is one caustic ring due to CDM particles falling through the galaxy for the first time, a caustic ring of smaller radius due to particles falling through for the second time, a yet smaller ring due to particles falling through for the third time, and so on. The caustic rings move slowly in space, generally in an outward direction, as the properties of the infalling CDM particles (in particular, their turnaround radius and angular momentum distribution) change.

Let's obtain the density profile of the caustic in a particular case of axial symmetry about the z-axis and parity (z→-z). The symmetry assumptions will force the caustic to lie on a circle in the z=0 plane. However the density profile near the caustic is expected to be independent of these assumptions because it is determined by the local properties of the CDM composing it at the time, as opposed to the global symmetry properties of the system. Consider then the time evolution of CDM particles initially located with uniform density on a turn-around sphere of radius R and with the angular momentum per unit mass distribution $\vec{\ell}(\hat{r}) = \omega \vec{r} \times (\hat{z} \times \vec{r})$ where $\vec{r} = R\hat{r}$. Thus the turnaround sphere is assumed to be initially rotating about the z-axis with angular velocity ω as if it were a rigid body. Let's assume further that the particles fall into the spherically symmetric potential: $V(r) = v_{rot}^2 \ln(R/r)$, which is such that it produces a flat rotation curve with rotation velocity v_{rot}. One readily

4

finds that a continuous flow of particles falling through under the stated conditions produces a density distribution which has a caustic ring in the $z = 0$ plane of radius a given by:

$$a \, v_{rot}\sqrt{2\ln(R/a)} = wR^2(1 + 0(\frac{a^2}{R^2})) \; . \tag{1}$$

In this case, the caustic is the locus of points where the particles with the largest amount of angular momentum ($\ell_{max} = \omega R^2$) are at their distance of closest approach to the galactic center. A calculation shows that, to leading order in an expansion in powers of a/R, the density distribution is given by:

$$d(a; \rho, z) = \frac{dM}{d\Omega dt} \frac{2}{v} \frac{1}{\sqrt{(r^2 - a^2)^2 + 4a^2z^2}} \tag{2}$$

where (ρ, z, θ) are cylindrical coordinates, $r = \sqrt{\rho^2 + z^2}$, $\frac{dM}{d\Omega dt}$ is the rate at which mass falls in per unit time and unit solid angle, and $v = v_{rot}\sqrt{2\ln(R/a)}$ is the velocity of particles near $r = a$. From Eq. (2), one readily finds the behaviour of the density near the caustic:

$$d(a; \rho, z) \simeq \frac{dM}{d\Omega dt} \frac{1}{v \, a \, \sigma} \tag{3}$$

where $\sigma = \sqrt{(\rho - a)^2 + z^2}$ is the distance to the ring.

For an isolated galaxy, there is a ring of radius a_1 due to particles falling through for the first time, a ring of radius a_2 due to particles falling through for the second time, and so on. Thus the quantities R, a, v, d and $\frac{dM}{d\Omega dt}$ carry an index $n = 1, 2, 3....$ One caustic ring is associated with each pair of velocity peaks [5,4] in the halo distribution. Eq. (2) implies $\frac{dM_n}{d\Omega dt}\frac{2}{v_n} = r^2 d_n(0; r)$. This relates the value of the prefactor $\frac{dM_n}{d\Omega dt}\frac{1}{v_n}$ for each infall to the density $d_n(0; r)$ the associated pair of velocity peaks contributes in the limit of zero angular momentum ($a = 0$). Estimates of $d_n(0, r)$ can be found in ref. [4] for the case of self-similar infall. Of course a caustic ring is perfectly sharp, with $d \sim \frac{1}{\sigma}$ for arbitrarily small σ, only in the limit where the velocity dispersion of the infalling CDM is zero. If the velocity dispersion is Δv_n, the caustic singularity spreads over a distance of order $\Delta a_n = R_n\frac{\Delta v_n}{v_n}$.

The infall is called self-similar [2,6] if it is time-independent after all distances are rescaled by the turn-around radius $R(t)$ at time t and all masses are rescaled by the mass $M(t)$

5

interior to $R(t)$. In the case of zero angular momentum and spherical symmetry, the infall is self-similar if the initial overdensity profile has the form $\frac{\delta M_i}{M_i} = (\frac{M_0}{M_i})^\epsilon$ where M_0 and ϵ are parameters [2]. ϵ must be in the range $0 \leq \epsilon \leq 1$. The rotation curve is flat if $0 \leq \epsilon \leq 2/3$ [2]. The infall model was generalized in ref. [4] to include the effect of angular momentum. It was found that self-similarity requires the angular momentum distribution $\ell(t)$ to have the time-dependence $\ell(t) = j\frac{R^2(t)}{t}$, where j is a dimensionless and time-independent distribution. A good agreement of the self-similar model with the properties of our own galaxy was found [4] for parameter values of order $\epsilon = 0.2$, $\bar{j} = 0.2$ and $h = 0.7$ where $\bar{j}$ is the average of the j distribution and h is the Hubble rate in units of 100 km/sec.Mpc . Using the model of ref. [4], the following values for the radii of the first five rings are obtained:

$$\{a_n : n = 1, 2...5\} \simeq (37, 18, 12, 9.5, 7.7) kpc \left(\frac{j_{max}}{0.25}\right) \left(\frac{0.7}{h}\right) \left(\frac{v_{rot}}{220\frac{km}{s}}\right) \tag{4}$$

where j_{max} is the maximum j value of the j distribution, and where the value $\epsilon = 0.2$ was used. For $\epsilon = 0.3$, $a_1 \simeq 31 kpc$ $\left(\frac{j_{max}}{0.25}\right) \left(\frac{0.7}{h}\right) \left(\frac{v_{rot}}{220 km/s}\right)$ but the ratios a_n/a_1 are almost the same as in the $\epsilon = 0.2$ case.

Luminous rings surrounding galaxies have been observed and such rings may be related to the caustics described here. However, luminous matter is presumably baryonic and the behaviour of baryons is more complicated than that of CDM particles. Infalling baryons do not necessarily behave in a collisionless manner and they may easily get stripped off the CDM flow. In contrast, the conclusion that isolated galaxies are surrounded by caustic rings of *dark* matter appears unavoidable if CDM exists. Because of this, we will limit ourselves in this paper to the observational implications of CDM rings which follow from their gravitational fields. CDM rings may nonetheless have a baryonic component. These baryons may have the same phase-space distribution as the CDM or they may have accreted onto the ring from neighboring space.

The Newtonian gravitational force per unit mass due to the density distribution of Eq. (3) is

$$\vec{F}_n(\vec{\sigma}) = -\frac{2\pi G \ C_n}{a_n}\hat{\sigma} \tag{5}$$

for small σ, where $C_n \equiv \frac{dM_n}{d\Omega dt} \frac{2}{v_n} = r^2 \, d_n(0; r)$. Hence there is a discontinuity in the rotation velocity

$$\frac{\Delta_n v_{rot}}{v_{rot}} = \frac{1}{2} \frac{d_n(0; a_n)}{d(a_n)} \quad , \tag{6}$$

directly across the nth caustic, in the ideal case where this caustic is perfectly sharp and lies in the galactic plane. In Eq. (6), $d(r) \equiv \frac{v_{rot}^2}{4\pi G r^2}$ is the total density at r in the limit of spherical symmetry and perfectly flat rotation curves. The ratios $f_n \equiv \frac{1}{2} \frac{d_n(0;r)}{d(r)}$ were found to be of order $(12, 5.3, 3.3, 2.4, 1.9, ...)10^{-2}$ in the self-similar infall model with $\epsilon = 0.2$. (For $r << R$ and small n, the f_n are nearly r-independent.) There are of course a number of effects that will smooth out sudden variations in the measured rotation velocities. One effect is that the CDM ring may be some distance away from the galactic plane where the rotation velocities are measured. Secondly, the measured rotation velocities are spatial averages over some distance across the galactic plane. Thirdly, the CDM ring may be fuzzy because of velocity dispersion as mentioned earlier. Thus the discontinuities of the ideal case are smoothed out into bumps. The bumps occur in the measured rotation curve near the location of CDM rings if the latter happen to be close to the galactic plane.

Galactic rotation curves often do have bumps. Of special interest here are those bumps which occur at radii larger than the disc radius because they cannot readily be attributed to inhomogeneities in the luminous matter distribution. Consider the rotation curve of NGC 3198 [7], one of the best measured and often cited as providing compelling evidence for the existence of dark halos. It appears to have bumps near 28, 13.5 and 9 kpc, assuming $h = 0.75$. Although the statistical significance of these bumps is not great, let's assume for the moment that they are real effects. Note then that their existence is inconsistent with the assumption that the dark halo is a perfect isothermal sphere. On the other hand, the radii at which the bumps occur are in close agreement with the ratios:

$$\{a_n/a_{n+1} : n = 1, 2, 3, 4\} = (0.48, \ 0.68, \ 0.77, \ 0.81) \tag{7}$$

predicted by Eq. (4) assuming that the bumps are caused by the gravitational fields of the first three caustic rings of NGC 3198. Since $v_{rot} = 150$ km/sec, we find that $j_{max} = 0.3$

in this case if $\epsilon = 0.2$. The uncertainty in h drops out. A fit of the infall model to our own galaxy produced $\bar{j} = 0.2$ for $\epsilon = 0.2$. If the turnaround sphere is taken to be rigidly rotating, one has $j_{max} = \frac{4}{\pi}\bar{j}$. Thus the values of $\bar{j}$ for our own halo and that of NGC 3198 are found to be similar.

The ratios of caustic ring radii given in Eq.(7) are characteristic of the t-dependence in the angular momentum distribution $l(\hat{r}, t) = j(\hat{r})R(t)^2/t$. The main reason for using this ansatz in ref. [4] was that it produces exact self-similarity in the time evolution of the halo. However, there is a broader justification for a time-dependence close to the one of the ansatz. Consider particles which at some early initial time t_i are at a distance r_i from the center of a large overdensity which will grow into a galactic halo. These particles have magnitude of angular momentum with respect to the overdensity's center: $l(\vec{r}_i) = r_i v_{i\perp}(\vec{r}_i)$ where $v_{i\perp}$ is the magnitude of the component of the initial velocity $\vec{v}_i(\vec{r}_i)$ transverse to $\vec{r}_i$. Because $\vec{v}(\vec{0}) = 0$, it is reasonable to assume $v_{i\perp}(\vec{r}_i)$ is proportional to r_i when comparing values of r_i which are of the same order of magnitude. In that case, $l(\vec{r}_i) \sim r_i^2 \sim M_i^{\frac{2}{3}} \sim R(r_i)^2/t(r_i)^{\frac{4}{3}}$ where M_i is the mass interior at time t_i to the sphere of radius r_i, $R(r_i)$ and $t(r_i)$ are the turn-around radius and turn-around time of particles initially at radius r_i, and where we used $GM_i = \pi^2 R(r_i)^3/8t(r_i)^2$. Except for the relatively slowly-varying factor of $1/t^{\frac{1}{3}}$, this is the time dependence which leads to self-similarity. Exact self-similarity corresponds to the case $v_{i\perp}(r_i) \sim r_i^{1+3\epsilon/2}$, or $l(r_i) \sim r_i^{2+3\epsilon/2}$. Using the methods of ref. [4], the ratios of caustic ring radii are found to be of order $\{a_{n+1}/a_n : n = 1, 2, 3, 4\} = (0.43, 0.64, 0.74, 0.78)$ when $l(r_i) \sim r_i^2$. Comparison with Eq. (7) provides an estimate of the model-dependence of the ratios of caustic ring radii and suggests that the values given in Eq.(7) are fairly robust. At any rate, since the gravitational field around a large isolated overdensity is approximately spherically symmetric and hence angular momentum about the center of the overdensity is approximately conserved, the study of bumps in rotation curves may inform us about the peculiar velocities associated with primordial density perturbations and thus constrain theories on the origin of these perturbations.

Finally, let's remark that if the caustic rings lie in the galactic plane, the halo density is

enhanced there compared to the case of spherical symmetry. If, for the sake of *an example*, we use the parameter values $\epsilon = 0.2$, $j_{max} = 0.254$ and $h = 0.7$ to model our own galactic halo, then its caustic rings are at the radii: 38, 18, 12.5, 9.7, 7.9 kpc ... . If all rings lie in the galactic plane, we at 8.5 kpc would be between the 4th and 5th ring. The 4th ring would have passed by us approximately 650 million years ago assuming that our distance to the galactic center remained constant. The 5th ring would pass by us approximately 370 million years from now. If the angular momentum distribution is the one $\frac{dn}{dj} = \frac{j\theta(j_{max}-j)}{j_{max}(j_{max}^2-j^2)}$ characterizing a turn-around sphere which is initially rigidly rotating, the contributions of the first eight incoming peaks to the local halo density would be approximately (0.8, 1.7, 3.1, 8.6, 13.9, 4.3, 2.7, 1.9) $10^{-26}\frac{gr}{cm^3}$ whereas the averages [4] over all locations at the same distance from the galactic center as us are approximately (0.8, 1.4, 2.0, 3.0, 3.4, 2.1, 1.6, 1.3) $10^{-26}\frac{gr}{cm^3}$. The 4th and 5th velocity peaks are considerably enhanced in this example because of our proximity to the corresponding rings. If all the caustic rings lie in the galactic plane, the local halo density would be boosted from approximately 0.5 $10^{-24}\frac{gr}{cm^3}$, which is the usual estimate in the case of spherical symmetry, to approximately 0.75 $10^{-24}\frac{gr}{cm^3}$. Such an enhancement is consistent with a recent estimate of the local density based on a flattened model of our galactic halo [8]. These results are relevant to the axion [9] and WIMP [10] dark matter searches which in fact provided the original impetus for this work. It is conceivable that these experiments will measure some day the contributions of the velocity peaks to the local density and thereby provide us with detailed information about the structure of our galactic halo.

ACKNOWLEDGMENTS

9

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a)
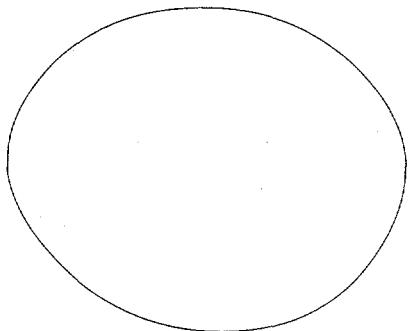
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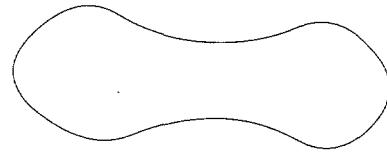
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d)

e)
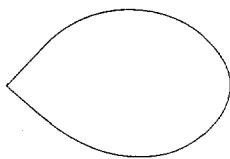
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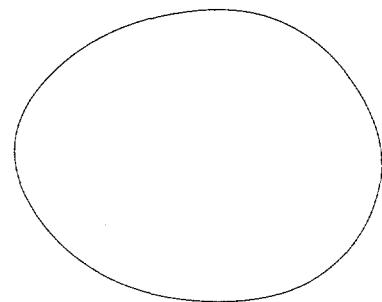
FIG. 1. Infall of all particles on a given initial turnaround sphere. The sphere crosses itself between frames b) and c). The cusps in frames d) and e) are at the intersection of the ring caustic with the plane of the figure.