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FIELD-SCALE SIMULATION OF MATRIX-FRACTURE INTERACTIONS

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Introduction

Simulation of flow in fractured media continues to be among the most challenging problems faced in geothermal reservoir engineering. Because of a lack of information regarding specific matrix-fracture characteristics (e.g., fracture distribution, spacing, and aperture, and interfacial area for exchange of fluid), explicit representation of the reservoir is generally not feasible. Instead, a multiple (but usually dual) continua model is used. In multiple continua models, specific details of the reservoir are replaced with averaged properties (average fracture spacing, for example). Such averaging facilitates the simulation of fractured reservoirs; however, field-scale simulation remains numerically intensive. For example, it has been stated that 5-10 nested shells are required in the Multiple INteracting Continua (MINC; Pruess and Narasimhan, 1982) formulation in order to adequately resolve transient pressure and saturation gradients between the fracture and matrix domains (Zimmerman et al., 1992). While this results in a large amount of additional work (compared with a single porosity system of the same dimension), it should be noted that the MINC method *is* capable of resolving such transients, whereas most dual porosity simulators cannot.

Many of the numerical models used to simulate flow in fractured media invoke variations on the Warren and Root (1963) model of fractured reservoirs. The Warren and Root (W&R) model treats the fractured reservoir as dual continua, in which one continuum contains the fracture domain and the other contains the matrix domain. Interaction between the two is assumed to be linearly dependent on the pressure difference between the (numerical) grid block fracture pressure and average matrix pressure. This linear dependence has been referred to as the "pseudo-steady state assumption," and is known to be inaccurate at "small" times, especially in reservoirs with large fracture spacings and highly compressible fluids (see, for example, Najurieta, 1980).

This paper describes recent efforts at relaxing the assumptions inherent in the W&R formulation, through use of analytical solutions to the equations governing interporosity flow. For slightly compressible fluids, the governing equation is the well-known diffusion equation, for which analytical solutions are readily available (e.g., Crank, 1975). This work was recently presented (Shook, 1996) for the case of a single rock matrix surrounded by fractures. Those results will be presented, and the extension to field-scale simulation will be discussed.

Resolving the pressure gradient: Single block case

Mass conservation equations written for dual continua formulations contain a matrix-fracture interaction term, Q, describing the fluid flow between the two continua. This term Q can be derived from Darcy's law and the characteristics of the matrix-fracture interfacial area. Assuming single phase flow in a three-dimensional fracture network of spacing L, with all six sides of the rock matrix in contact with fractures, the interaction term can be written as (Shook, 1996):

$$Q = -\sum_{\#Blks}\sum_{faces} A\frac{k}{\mu}\vec{\nabla}P = -V_b\frac{6}{L}\frac{k}{\mu}\vec{\nabla}P \qquad (1)$$

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This expression is completely general, and only requires that the pressure gradient be correctly resolved.

Warren and Root (1963) and other workers in the field assumed that the pressure gradient could be approximated as the pressure difference between the two continua expressed over some characteristic distance. Using the fracture half-spacing as that distance, the assumption of pseudo-steady state results in the following expression for the pressure gradient.

$$\vec{\nabla} P = \frac{P_{fr} - P_{ma}}{L/2}$$

When this is used in Equation 1, the matrix-fracture interaction term becomes:

$$Q = - V_b \frac{12}{L^2} \frac{k}{\mu} (P_{fr} - \overline{P}_{ma})$$

where $12/L^2$ is known as the shape factor. Values used for the shape factor range from $12/L^2$ (Kazemi et al., 1976) to $60/L^2$ (Warren and Root, 1963).

It has long been known that this assumption of pseudo-steady flow is incorrect at small time. For example, Najurieta (1980) shows that the transition time to pseudo-steady state flow depends on, among other variables, rock and fluid compressibility and matrix dimensions. Zimmerman et al. (1992) show that the W&R-predicted response due to a step function change in pressure at the fracture face converges very slowly to the correct solution. Zimmerman et al. (1992) further state that the W&R-type equation (i.e., a constant shape factor) always predicts an incorrect time dependence on pressure at some time scale. In order to preserve the simplicity of the W&R formulation and correctly resolve pressure gradients, one must start with Eqn. 1 and remove the linear approximation for the pressure gradient.

By using analytical solutions to the diffusion equation, Shook (1996) obtained a semi-analytical expression that correctly describes the pressure gradient over all time scales. The exact solution to the problem is given as (assuming spherical coordinates; Crank, 1975, p 91):

$$\frac{\partial P}{\partial r} = (P_{fr} - P_{ma}) \frac{\pi^2}{3a} \frac{\sum_{n=1}^{\infty} \exp\left(-\frac{n^2\pi^2 Dt}{a^2}\right)}{\sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n^2\pi^2 Dt}{a^2}\right)} \qquad (2)$$

where r is the spatial coordinate in the matrix, a is the characteristic matrix half length (in spherical or cylindrical coordinates, the radius; in linear coordinates, it is fracture half spacing), t is time, and D is the diffusivity of the matrix (fluid + rock):

$$D = \frac{k}{\varphi \mu c_t}$$

Equation 2 contains two infinite series; however, these series converge relatively rapidly, and may typically be truncated after only a few terms. The number of terms required to obtain an accurate solution is:

$$N_{Terms} = \frac{L}{\sqrt{4Dt}}$$

Thus, the infinite series (which are intractable in a numerical model) are approximated by a finite-limit DO loop, and the pressure gradient for interposity flow is easily obtained.

The new method was validated by comparing solutions against fine grid simulations in 1-, 2-, and 3-D, as described in Shook (1996). Here, we show only the validation results for the 3-D case. For comparison purposes, W&R results are also shown. A schematic is given for the test problem in Figure 1. The grid employed for the fine grid simulation was 21 x 21 x 21. Both the new formulation and the W&R simulation used two grid blocks; one for the fracture domain and one for the matrix. Aside from the differences in numerical grid, properties were identical between the fine grid and dual porosity simulations.

The test case is an example of matrix mass depletion. From a uniform initial condition of 1000 kPa, fracture pressure was dropped to 100 kPa at t=0, and was held constant throughout the simulation. Because of the pressure difference between the matrix and fracture, flow occurs from matrix to fracture. Interporosity mass flow rates are shown in Figure 1. Excellent agreement is observed between the fine grid simulation and the new formulation, except at very early times. Further analysis of the fine grid simulation indicates that this grid was insufficiently fine to capture the correct pressure gradient at early time (i.e., the grid blocks are too large). Comparisons between the new formulation and analytical results indicates that the new method is extremely accurate over all time scales. That is to be expected, since this formulation is based on a truncated version of the analytical solutions. In contrast, the W&R simulation exhibits significant error over all time scales.

Generalization for Field-Scale
The formulation described above works well for a single rock matrix surrounded by fractures which are subjected to a single change in pressure. It appears that one could readily generalize the formulation to account for multiple matrix blocks, so long as the change in pressure were restricted to a single step function change. However, a more realistic situation is one in which there are an arbitrary number of changes in fracture pressure occurring on a field scale (i.e., an arbitrary number of numerical grid blocks). As discussed below, that problem is substantially more difficult to solve using an extension of the above formulation.

The general solution for matrix pressure as a function of time and space (again

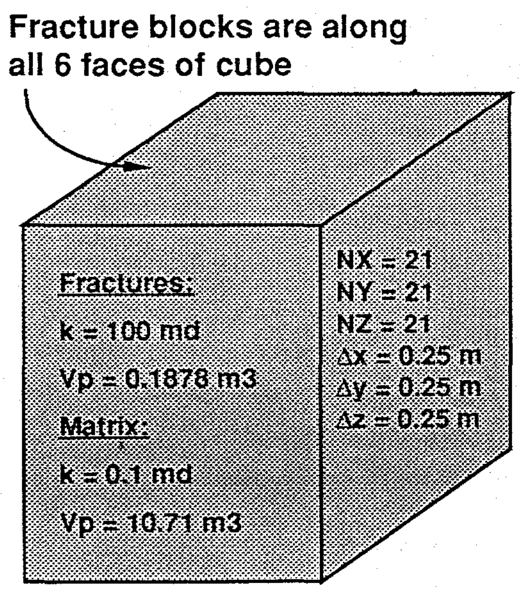

Figure 1. 3-D fracture validation problem

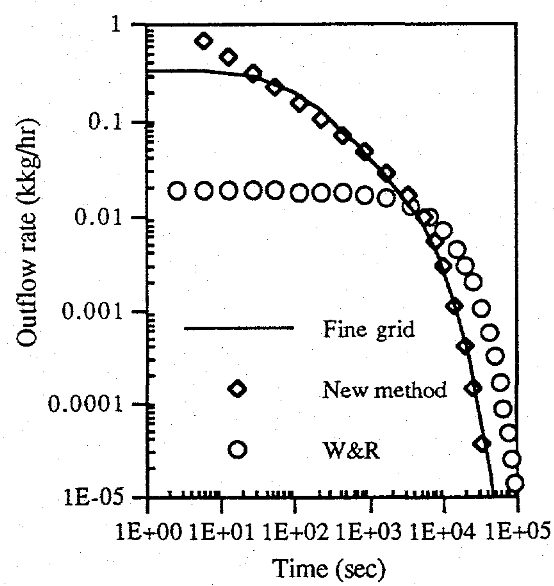

Figure 2. 3-D validation results from Shook, 1996.

assuming spherical coordinates) is (Carslaw and Jaeger, 1959, p 233):

$$P(r,t) = \frac{2}{ar} \sum_{n=1}^{\infty} \exp\left(\frac{-n^2\pi^2 Dt}{a^2}\right) \sin\left(\frac{n\pi r}{a}\right) x$$

$$\left\{ \int_0^a r'P(r')\sin\left(\frac{n\pi r'}{a}\right)dr' - n\pi D(-1)^n \int_0^t \exp\left(\frac{n^2\pi^2 D\tau}{a^2}\right) P(\tau)d\tau \right\} \quad (3)$$

The first integral above accounts for the initial condition, and the second for changes in fracture pressure, both of which are damped through time by the exponential decay term (term 1).

One may take either of two approaches in solving Eqn 3. Typically, the initial condition is treated as a constant, and all subsequent variations in pressure are captured in the second integral. While this is mathematically tractable, it appears to be numerically difficult. The first integral is trivially solved, but solution of the second integral requires that all previous changes in fracture pressure be stored. Furthermore, since the previous changes in fracture pressure are damped in time, it is not a matter of "updating" the effects of previous changes in pressure, but rather a recalculation at each time step. While it is true that "old" changes in pressure decay with time (and therefore could be omitted from consideration), an *a priori* means of evaluating how many such changes can be omitted does not appear to exist.

A second means of evaluating the general solution, and one that is currently being investigated, is to update the initial condition at the end of every time step, and consider only the current change in fracture pressure. That is, identify a parametric function that accurately describes the pressure distribution in the matrix, and evaluate the second integral only over the current time step. In this approach, constraint equations are used to identify the unknowns in the expression P(r,t). The idea of a parametric expression for pressure has been used by several researchers (e.g., Vinsome and Westerveld, 1980; Pruess and Wu, 1993). Our current approach is simpler in that we are attempting to describe the matrix pressure explicitly; therefore, no iteration on the unknowns in the expression are required. Constraint equations in this case include fracture pressure, pressure gradients at the matrix-fracture interface and matrix center, and average matrix pressure - all known from the converged solutions to the governing equations from the last time step. This solution has not yet been proven out, but remains the current topic of research on this project.

Summary and Future Work

A new method for semi-analytically resolving the pressure gradient in fractured reservoirs was presented and validated for a single step change in fracture pressure. This new method, while generalizable to field-scale problems, is likely to be restricted to cases in which a single change in fracture pressure occurs. In order to develop the approach for field-scale work, one must go back to the general solution for matrix pressure, and either, 1) treat the initial condition as a constant and store all previous variations in fracture pressure, or 2) update the initial condition at each time step, and treat only the current variation in fracture pressure as a perturbation acting on the system. From our preliminary studies, the second option appears to be more readily implemented. Current efforts are focused on the identification of a parametric expression for pressure that is both simple enough for use, and accurate in describing the initial pressure distribution at each time step.

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Nomenclature

English

a characteristic diffusion length in rock matrix (fracture half-spacing in 1-D, effective matrix radii in 2-D and 3-D) [=] m

A Cross sectional area [=] m^2

c_t Total compressibility (rock + fluid) [=] kPa^{-1}

D diffusivity (k/jmc) [=] m^2/s

k permeability [=] m^2

L Characteristic rock matrix length (fracture spacing) [=] m

P Pressure [=] kPa

P_{ma} Average matrix pressure [=] kPa

q Volumetric flux [=] m/s

t Time [=] s

Q Matrix/fracture Source/sink term [=] m^3/s

V_b. Grid block bulk volume [=] m^3

Greek

φ Porosity [=] vol. pore space / bulk volume

μ Viscosity [=] mPa-s

Subscripts

fr fracture

ma matrix

I initial (condition)

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