Atomic Energy of Canada Limited

OPTIMIZING ORGANIC-LIQUID-COOLED HEAVY-WATER NATURAL-URANIUM REACTOR DESIGN FOR SHUT-DOWN REFUELLING

DM-64

by

W.B. LEWIS

Chalk River, Ontario

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SYNOPSIS

Taking advantage of the expected properties of uranium monocarbide fuel in relatively massive rods, a specific (150 MWe) reactor design is suggested for shut-down refuelling. The suggested fuel consists of 7-rod bundles of 3 cm diam solid rods in about 300 channels 400 cm long. Such a design is also attractive for on-power refuelling.

Basic analysis shows that any practical and economical design for shut-down refuelling requires a combination of high fuel burn-up, high thermal-to-electrical conversion efficiency, moderate specific power rating and moderate total power.

To support the cost of the heavy water inventory, the total power (P eMW) must not be too low. At a specific power rating of the fuel (R) averaging 6 thermal MW/tonne U or less, the competitive neutron absorption by the fuel overrides the absorption in the organic coolant. A high burn-up (B) of about 10,000 MWD(th)/tonne natural U results. The high outlet temperature of the organic coolant, 400°C (752°F) will raise high pressure steam (up to 170 atmos (2500 psi)) yielding a net station efficiency of 35%. The reactivity drop over each operating period (t_f) can be made less than $Rt_f/45 = 6.6$ milli-k, for $t_f = 50$ days. Fuel management requiring each fuel element to occupy only two successive positions in the reactor limits re-positioning every 50 days to $2\,P.t_f/eB = P/35$ tonnes U or a fraction $2t_fR/B = 6\%$ of the total fuel.

The suggested specific design depends on some extrapolation from established reactor lattice and coolant channel measurements.

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1. Basis for Optimum Design

In the design of reactors different considerations have led to the adoption of on-power refuelling. For example, in the U.K. gas-cooled graphite reactors the structure is so massive that the rate of cooling must be slow. Moreover the natural uranium fuel operates at a relatively low specific power and the amount of fuel is large. Also the fuel burn-up is limited to about 3000 MWD/tonne U requiring either frequent changing or handling relatively large amounts of fuel at each shut-down. Consequently any shut-down period for refuelling would extend over several days or longer. On the other hand, for the CANDU heavy-water-moderated reactor, the specific power rating of the fuel is so high that the rate of decline of reactivity is relatively rapid. In consequence the frequency of shut-down would have to be high or a large reactivity reserve would be required at the start of each operating period. The provision of such a reactivity reserve would spoil the fuel economy by reducing the attainable burn-up. Moreover the high specific power magnifies the poison transient, due to xenon-135, that follows shut-down. Any shut-down for refuelling would therefore have to extend over about two days at least.

Provided the problems of thermal shock can be satisfactorily solved, it appears that by choosing a specific power rating lying between that of the U.K. graphite type and CANDU, and achieving long burn-up of the fuel, the disadvantages of shut-down refuelling would be minimized, and might even be outweighed by the simplification possible in the refuelling arrangements. Some additional selection of fuel programming is involved as discussed below.

The relevant simple relations derive from the following definitions:

t_d = fuel dwell time, the period that a given fuel element remains in one position in the reactor.

t_f = operating period between successive refuelling operations.

R = mean thermal power rating in thermal MW/tonne U.

B = mean burn-up in MWD/tonne U.

N = number of positions taken by a fuel element in the core.

P = total power in electrical MW.

e = thermal-to-electrical conversion efficiency.

Taking simple averages and neglecting even first-order corrections,

$$\text{Fuel dwell time} \quad t_d = B/NR \qquad (1)$$

Fraction of fuel to be re-positioned at each fuel change

$$= t_f/t_d = t_f NR/B \qquad (2)$$

The total tonnage of fuel = P/eR tonnes U. (3)

The tonnage to be re-positioned at each fuel change

$$= (P/eR)(t_f/t_d) = Pt_f N/eB \qquad (4)$$

It is proposed that it would be acceptable for a generating plant to be shut-down for 2 days every 8 weeks if the days for the shut-down can be selected in advance, with an allowed variation of a few weeks. It seems quite practicable, as will be shown, to design for this operating schedule a reactor using natural-uranium fuel, organic-liquid coolant, and heavy-water moderator.

It may be of interest to examine why such an operating schedule would be costly for the UK graphite and CANDU-type reactors. Selecting, for example, a reactor of the UK Trawsfynydd (1) plant, P = 250 eMW, R = 3.1 MW/tonne U, B = 3000 MWD/tonne U, e = 0.288. At such a low burn-up it is just possible for the fuel to occupy only one position in the reactor, so N = 1. The total tonnage of fuel = P/eR = 280 tonnes U. The reactor has 3720 fuel channels. The fraction of the fuel to be changed every 50 days would be $t_f NR/B$ = 50 x 3.1/3000 = 5.17% or 192 channels. The total tonnage to be replaced (Equation (4)) would be $Pt_f N/eB$ = 250 x 50/0.288 x 3000 = 15 tonnes. In view of the large thermal capacity of the reactor, the cooling and start-up time is long and such a major refuelling operation would be difficult in the short time allowed.

For CANDU, as already mentioned, the problem is different. The specific power rating of the fuel is high (2), R = 16.8 MW/tonne U, and consequently the reactivity decline over a 50-day period is appreciable. The exact calculation of reactivity change is somewhat elaborate for bidirectional fuelling, but an evaluation was made in DM-54 (3), Table A-3 for fuel characterized as Case VI of DR-39 (4). On the basis of this evaluation it is shown in Appendix 1 that the burn-up penalty would be about 850 MWD/tonne and a reactivity change of 18 mk over the 50-day period would have to be compensated by the control system.

Bidirectional fuelling is not, however, appropriate for shut-down refuelling at long intervals because almost all of the channels would require fuel movement. A comparable burn-up is offered by "axial-inversion," "two-position" fuelling. The fuel in a channel is divided at the mid-point and the two halves are interchanged after half the irradiation, preserving the axial direction. At the time of changeover the fuel may with advantage be transposed to a channel where the neutron flux is lower if the overall reactivity and flux pattern permits. Whether such a transposition is made or not, each fuel element takes up two, and only two, positions in the reactor. Hence such a system is characterized as "axial-inversion," "two-position" fuelling. The corresponding reactivity and burn-up penalties may be calculated as shown in Appendix 2 by a simple extension of the method developed in DM-42 (5). It is concluded that for 10,000 MWD/tonne burn-up for Case VI of DR-39, the reactivity drop over the operating period (t_f days) is given by

$$Rt_f/45 \text{ milli-k.} \qquad (5)$$

For CANDU and a 50-day period this would amount to 16.8 x 50/45 = 18.6 mk. The burn-up penalty would be 880 MWD/tonne U. The fraction of the fuel to be re-positioned at each fuel change by Equation (2) is $t_f NR/B$ = 100 R/9120 = 0.184 or 56 of the 306 channels in CANDU.

It is pertinent to note that the total tonnage of fuel to be re-positioned at each fuel change is given by $Pt_f N/eB$ (Equation (4)). Moreover if B exceeds 2500 MWD/tonne U, the loss of reactivity will in most cases require N to be 2 or more. Given the electric power P and operating period t_f, the tonnage to be moved can only be reduced in an economic reactor design by increasing the efficiency e and the burn-up B.

To summarize, the approach to design for shut-down refuelling is to aim at a high burn-up (B) and a high efficiency (e), and especially so for reactors of high power (P). The specific power rating of the fuel will have to be optimized. If it is too high, the reactivity drop over each operating period imposes a penalty on the control system and on the burn-up. Moreover, a high fuel rating implies a high neutron flux and consequently higher neutron wastage in the coolant and fuel cladding when the amount of these is set by heat transfer requirements. If the fuel rating is too low, the cost contribution from the fuel inventory is significant.

It has been indicated that fuel programming involving "axial inversion" and "two positions" is capable of yielding high burn-up. With such a system, however, it is not possible to achieve significant flux flattening in the axial direction. Usually in the design of reactors there is a thermal limit operating on the fuel. Either the central temperature is at a limit or the temperature of the cladding or the heat flux to the coolant. Under such circumstances the channel length for a given power output can be reduced only by flattening the flux distribution. Moreover reducing the total channel length is important for achieving an economical design for it reduces the size of the reactor, and the costly inventories of fuel and moderator. The reactor flux can be flattened both axially and radially without departing from "two-position" fuelling by dividing the fuel in a channel into three or more sections. Several reshuffling programs are available. For example, with three sections an inner channel may be paired always with an outer channel where higher reactivity has to be maintained for radial flux flattening. The centre fuel section is first fed into the outer channel, and at the fuel change is moved to become the centre section in the paired inner channel. At the same time the two outer sections may be interchanged to achieve axial inversion, and either replaced in the same channel or moved to the paired channel as required to yield the best overall flux pattern.

2. Approach to Specific Optimum Design

The specific reactor is required to yield 150 eMW and be suitable for refuelling in shut-down periods of 2 days after operating periods of 50 full power days. The objective is the lowest unit power cost for base-load operation at 80% load factor (7000 hr/yr) (i.e. utilization u = 0.8).

2.1 Heavy Water and Fuel Inventories

Provided the mass of D_2O is not less than 100 tonnes, a good lattice design results from a mass ratio of D_2O:U of 1.7. Selecting for trial R = 6 MW/tonne U and e = 0.35, then (by equation (3)) total U = P/eR = 150/6e = 71.4 tonnes.
$\therefore$ total D_2O = 122 tonnes.

The contribution from the heavy water inventory of D tonnes at $62/kg at an annual % charge of a_D is

$$\frac{62{,}000\ D}{P \times 8766u} \times \frac{a_D}{100} = \frac{62\ Da_D}{105{,}000} = \frac{62 \times 122}{105{,}000}\, a_D = 0.072\ a_D \text{ mill/kWh.}$$

Similarly the cost from the fuel inventory, adopting the practice of capitalizing 65% of a fuel charge (i.e. half a charge + 15% spares) in the reactor, and setting the price of fuel at $60/kg U is

$$0.65 \times \frac{P}{eR} \times \frac{1}{P.8766u} \times \frac{a_U}{100} = 0.0265\, a_U \,.$$

This shows that the trial value of R is probably satisfactory because in order to achieve the desired burn-up the D_2O inventory could not be less than 100 tonnes. An increase of R would not greatly reduce the D_2O inventory. Moreover R could not be twice as great if the desired burn-up is to be achieved. The extra inventory costs above the minimum for D = 100 tonnes and R = 12 MW/tonne U therefore do not exceed 0.0135 a_U + 0.013 a_D mill/kWh.

The values of a_U and a_D will be left to the reader's choice. For CANDU, a_U = 6.13, a_D = 5.43. The USAEC rules make a_U = (4 + extra on fabrication cost) and a_D = 12.5.

2.2 Size of Fuel Rods and Number of Channels

In order to achieve the highest burn-up, the fuel must be as dense as possible. For this reason, uranium carbide (UC) is the most promising fuel having a higher density than UO_2. Uranium metal is ruled out because it swells under irradiation and any practical design would have to leave an expansion space initially occupied by the coolant, which has a high neutron absorption. The design of uranium carbide fuel is not limited by the fuel temperature as it would be for UO_2 or U metal, but by heat transfer to the organic coolant. The fuel will accordingly be subdivided into rods to expose sufficient surface.

Let

W = maximum surface heat flux (watts/cm^2) averaged round the periphery of a rod

R_f = ratio of peak flux in the whole reactor averaged over the cross-section of a single rod to the mean flux in the fuel throughout the reactor.

H = length of fuel in reactor channel

N_B = number of rods per fuel bundle

0.95 = fraction of total power that passes from the fuel to the coolant. The remaining 5% is generated in the coolant tubes and moderator directly.

ρ = density of uranium in fuel in g U/cm^3.

Then

the total surface area required $= 0.95\ R_f P \times 10^6/We\ cm^2$ (6)

and since

the volume of the fuel $= P \times 10^6/eR\rho\ cm^3$ (7)

the radius r $= 2.(\pi r^2)/2\pi r$

$= 2 \times$ volume/surface $= 2W/0.95\ R_f R\rho$ (8)

the total length of fuel bundles

= volume/cross section

$= P \times 10^6/eR\rho N_B \pi r^2$

$= (0.95\ R_f)^2 R\rho P \times 10^6/4\pi N_B e W^2$ cm (9)

the total number of channels

= fuel length/reactor length

$= (0.95\ R_f)^2 R\rho P \times 10^6/4\pi N_B e W^2 H$ (10)

It is perhaps unexpected that for a constant N_B the minimum number of channels is proportional to the fuel rating, whereas the total amount of fuel is inversely proportional to the rating. This paradox serves as a pointer to the most significant feature to be optimized, the total fuel cross-section in a channel. The resonance escape factor and the flux depression in a fuel channel taken in relation to the fuel rating, R, determine the burn-up, and the flux depression has some effect on the ratio of peak-to-mean flux (R_f).

Before attempting to define the optimum it is of interest to evaluate some of the quantities for the specified reactor and trial value of the fuel power rating R = 6 MW/tonne U, assigning hoped-for values to W = 120 W/cm^2 (7) and R_f = 2.2. For uranium carbide, ρ = 12.85 g U/cm^3 corresponding to ρ_{UC} = 13.5 g/cm^3. By Equation (8), the rod radius r $= 2 \times 120/0.95 \times 2.2 \times 6 \times 12.85 = 1.49$ cm.

Assume N_B = 7 rods per fuel bundle and the reactor length H = 400 cm. Then the number of channels $(0.95 \times 2.2)^2$ $6 \times 12.85 \times 150 \times 10^6/4\pi \times 7 \times 0.35 \times (120)^2 \times 400 = 285$.

Since the total fuel is 71.4 tonnes U, the mass per channel = 250.5 kg U = 263 kg UC.

The fuel cross-section in a channel = $7\pi r^2$ = $48.8\ cm^2$.

In comment it may be noted that the flux depression may be unacceptably high and the resonance escape too low so that the optimum design might have more channels and rods of smaller radius. Possible departure from the 7-rod bundle seems, however, unlikely to give much benefit. A 9-rod configuration with a larger rod in the centre could achieve a more uniform heat flux through the sheathing but the optimum design in this respect would decrease the resonance escape and increase the flux depression. Internally-cooled rod designs, although reducing the neutron loss to the coolant, introduce problems of thermal insulation between the fuel and moderator.

Since the example is certainly not far from practical and acceptable, consideration will be restricted to the 7-rod bundle configuration.

2.3 Peak-to-Mean Flux Ratio R_f

Let us write $R_f = R_B R_R$ where R_B = peak-to-mean flux ratio for the cross-section of a fuel bundle or, more precisely, the ratio of the mean flux in an outer element to the mean flux for the bundle, and R_R = peak-to-mean flux ratio for position in the reactor. Assuming no axial flux flattening and a radial peak-to-mean ratio of 1/0.78 as for CANDU, $R_R = \pi/2 \times 0.78 = 2.014$. The calculation of the flux depression for a 7-rod bundle (R_B) is highly complex and is discussed further in approximation in Appendix 3. The geometry of the 7-element bundle tends, however, to keep the peak-to-mean ratio low, for if the ratio of the mean flux in the outer ring of 6 rods to that in the centre rod is R_I,

then $$R_B = \frac{R_I}{\frac{1}{7}(6R_I + 1)} = \frac{1}{1 - (1 - 1/R_I)/7}.$$ For example,

if $R_I = 1.5$, then $R_B = 1.05$, and if $R_I = 2$, then R_B = 1.077.

For this latter example, $R_f = R_R R_B = 2.014 \times 1.077 = 2.169$. This result is strictly applicable only if the sheath material is effectively a perfect thermal conductor so that the temperature around the periphery of an outer element, and the heat flux is constant. In practice, for such steep flux depression there would be some variation of heat flux around the rod that is further complicated although compensated by the coolant flow pattern and mixing swirl along the bundle.

2.4 Flux Depression, Resonance Escape and Burn-up

To find whether the flux depression is likely to destroy the advantage in competitive neutron absorption arising from the relatively massive fuel elements, a comparison is made with other design studies. Preliminary evaluations have been made for organic-cooled heavy-water-moderated reactors by Critoph, Pon and Primeau (8). The studies involve computations on the Datatron following a program similar to that originated and developed by Duret (9), based on the lattice recipes for reactors in use at Chalk River.

Such calculations have been extended by Primeau (10) to the massive rods here considered. Further studies will be needed to ensure that the extrapolation from experimentally studied lattices is justifiable but these will take time so the interim results are presented just as calculated.

Two postulated designs, Cases A and B, have 1.49 cm radius UC fuel rods. A seven-rod bundle is assumed and a square lattice pitch of 28 cm assigned. For Case A the spacing in the coolant between rods is 0.254 cm and for Case B, 0.127 cm. As derived in §2.2 above, for a reactor channel length of 400 cm and assigned flux flattening, a total of 285 channels is required for a 150 MWe plant. On a 28 cm pitch the core area is $285 \times (28)^2 = 223{,}440$ cm^2, corresponding to a radius of about 267 cm. Postulating a "reflector savings" of 60 cm on the radius from a 70 cm thick annular D_2O reflector makes the total D_2O volume $400 \times \pi \times (337)^2$ cm^3 = 142.7 m^3 less the space occupied by the fuel channels that is closely 10 m^3. This amount of D_2O, about 146 tonnes, is rather more than postulated in §2.1 above. The resulting geometrical buckling $B_G^2 = 1.158$ m^{-2} and the corresponding burn-up for unflattened flux is evaluated as 10,050 MWD/tonne U for Case A and 11,520 MWD/tonne U for Case B. The necessary flux flattening would reduce these by about 700 MWD/tonne U.

It therefore appears from this preliminary study that either Case A or Case B would give a satisfactory burn-up from natural uranium and further evaluation and optimizing would be worthwhile.

Further details of the preliminary analysis are given in the following table, together with comparable details for the fourth reference design (11) of CANDU computed by a similar program. Explanation of some details in the table is given in Appendix 4.

TABLE

			Case A	Case B	CANDU
Fuel spacing		cm	0.254	0.127	-
Fuel irradiation	ω	n/kb	2.10	2.40	2.20
Fast fission factor	ϵ		1.0489	1.05066	1.02666
Res. escape factor	p		0.8705	0.8727	0.8995
Mean neutron temp.					
in fuel	T_N	°C	355	340	151
in moderator	T_{NM}	°C	60	60	53.3
Buckling (material)	B_L^2	m^{-2}	1.08545	1.08558	1.07195
Thermal diffus. area	L^2	cm^2	163.26	170.48	150.1
Slowing-down area	L_s^2	cm^2	131.79	132.22	151.8
Reactivity (∞)	k_∞		1.0323	1.0331	1.0326
n yield th. fissions	$\bar{\eta}_0$		1.1818	1.1698	1.1766
Ratio of th. n density (moderator/fuel)			4.427	4.472	2.007
Initial conversion ratio			0.893	0.889	0.800
Burn-up (for ω and B_L^2) MWD/tonne U			10,390	11,865	9,082

Neutron Balance in bifa (barns per initial fissile atom)

		Case A		Case B		CANDU	
Yield	Y	1791.94		1794.16		1536.1	
$Y\epsilon$ + fast capture		1906.71		1917.69		1590.07	
Fast capture		27.16	267.14 (Fast capture + Resonance trap)	32.63	269.22 (Fast capture + Resonance trap)	13.72	169.7 (Fast capture + Resonance trap)
Resonance trap		239.98		236.60		156.0	
Fast leakage		26.51		26.67		25.25	
Thermal leakage		28.09		29.47		22.10	
Loss to sheathing		5.08		5.10		8.80	
Loss to inner coolant		18.47		15.02		0.23 (inner + outer coolant)	
Loss to outer coolant		14.42		6.79			
Loss to liner		4.17		3.86		Calandria Tube 13.55 (liner + thermal insulant)	
Loss to thermal insulant		1.36		1.27			
Loss to pressure tube		9.03		8.48		32.25	
Loss to moderator		20.76		21.22		13.44	
Total losses in reactor		73.29		61.73		68.27	
Total loss + leakage			127.89		117.88		115.62
Th. n capture in U-238			377.71		377.60		376.32
Th. n capture in remainder of fuel			1138.54		1156.18		929.18
Sum of loss, leak + capture			1911.27		1920.87		1590.82
Misbalance			4.56		3.19		0.75

3. Conclusion

The specific designs discussed may not be entirely practical. For Case B it remains to be established that

1) 120 W/cm^2 is a satisfactory design heat flux to the organic liquid collant.

2) UC fuel is mechanically satisfactory in thick rods 1.49 cm radius at a rating of 6 MW/tonne U to burn-up of 12,000 MWD/tonne. UO_2 in an even slightly larger size is satisfactory in NRX (12).

3) the resonance escape factor is sufficiently high.

4) 0.127 cm (0.050") spacing between rods in the organic coolant is sufficient.

5) the flux depression in the fuel bundle is not excessive after the required burn-up.

If departures from any of these points are necessary, it may be noted that

1) The rod diameter may be reduced if

   (a) The mean fuel power rating is raised above 6 MW/tonne U. This change would reduce the burn-up and increase the rate of decline of reactivity. The calculations of reactivity change in Appendix 2 are based on Case VI of DR-39 for which the neutron temperature $T = 227^oC$ and $p = 0.9$. For a lower p the rate of decline of reactivity with irradiation at 10,000 MWD/tonne would be lowered. This effect would be aided by the accompanying increase in fast fission and probably only slightly offset by the increase in T and flux depression in the fuel.

   or

   (b) The number or length of channels is increased.

2) The heat flux to the coolant may be reduced if the number of channels is increased or the flux flattening is improved.

3) The spacing between rods may be increased. The burn-up penalty may be assessed by considering the difference between Cases A and B.

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References

1. J.W. Ashley, "Trawsfyndd Nuclear Power Station. I - Engineering Design," Nuclear Power 4, No. 43, 90, 1959.

2. W.B. Lewis, "Competitive Nuclear Power for Canada," Nucleonics 18, No. 10, p. 57, 1960.

3. W.B. Lewis, "Initial Fuelling of Power Reactors," AECL-696, DM-54, 1958.

4. W.B. Lewis, "Low Cost Fuelling without Recycling," AECL-382, DR-39, 1956.

5. W.B. Lewis, "Fuelling System for Natural Uranium Reactor with Long Rods for High Burn-up without Recycling," AECL-436, DM-42, 1957.

6. See, e.g. -

   "Progress in Carbide Fuels" by various authors, USAEC TID-7589, 1960.

   A. Strasser, "Uranium Carbide as Fuel (including Data Sheet)," Nuclear Engineering 5, No. 51, 353, 1960.

   J.A.L. Robertson, "Uranium Carbide - A Preliminary Assessment of its Use as Fuel for Organic-Cooled Reactors, Part II - Properties of UC as Fuel," CEI-123 (AECL Internal Report - Not for Publication), 1961.

7. W.R. Martini, "Status of Heat Transfer Development for Organic Reactors," Proceedings of the Organic Cooled Reactor Forum, October 6 to 7, 1960, NAA-SR-5688, p. 45, USAEC Office of Technical Information, Washington 25, D.C.

8. AECL Chalk River, unpublished.

9. M.F. Duret and R. Mariott, "A Computer Program for Reactor Studies," AECL-911, CRRP-873, 1959.

   M.F. Duret, "Comments on the Optimization Program for CANDU," RPI-67 (AECL Internal Report - Not for Publication), 1960.

   D.L. Booth, "An Interim Method for Lattice Calculations in CANDU Type Reactors," RPI-79 (AECL Internal Report - Not for Publication), 1961.

10. AECL unpublished.

11. W.J. Henderson and D.G. Hurst, "The Effect of Flux Flattening on the Economics of Heavy Water Moderated Reactors," AECL-949, CRRP-872, Appendix 6, 1961.

12. M.B. Watson and J.D. Craigie, "Assembling and Testing Experimental UO_2 Fuel Rods for the NRX Reactor," FD-53 (AECL working document - Not for Publication), 1961.

13. C.H. Westcott, "Effective Cross Section Values for Well-Moderated Thermal Reactor Spectra," AECL-1101, CRRP-960 (3rd edition corrected), 1960.

14. W.H. Walker, "Yields and Effective Cross Sections of Fission Products and Pseudo Fission Products," AECL-1054, CRRP-913, 1960.

15. G.C. Hanna, "A Survey of Published Values of ν in the Fast Fission of U-238," AECL-1056, CRP-936, TNCC(CAN)23, 1960.

16. M.R. Fleishman and H. Soodak, "Methods and Cross Sections for Calculating the Fast Effect," Nuc. Sci. and Eng. 7, 217, 1960.

17. R. Sher and J. Leroy, J. Nuc. Energy, 12, No. 3, 101, 1960.

18. W.B. Lewis, "Calculations on the Long Irradiation of Uranium - Revising Appendix A, DM-52," DM-62, to be published.

APPENDIX 1

Burn-up Penalty with Bidirectional Slug Fuelling

It was shown in DM-54, Table A-3, for fuel characterized by Case VI of DR-39 taken to an irradiation of 2.5 n/kb or about 10,000 MWD/tonne U, that the reactivity loss for 1/16 of the burn-up would be 17.6 bifa (136.22 - 118.66 bifa). Converting to milli-k by the ratio $(1 + B_g^2M^2)/Y\epsilon p$ in the DR-39 notation for $1 + B_g^2M^2 = 1.05$ and $Y\epsilon p = 1390$ bifa, this loss amounts to 13.3 mk. The calculation assumed the neutron flux pattern was a sinusoidal axial distribution and uniform radially.

For a burn-up of 10,000 MWD/tonne U in CANDU the mean duration of irradiation is 10,000/16.8 = 595 days. A period of 50 days corresponds to 8.4% of the total and the reactivity change would be 13.3 x 8.4/6.25 = 17.9 mk.

For Case VI it was shown in DR-39, Fig. 2 and Table A.II.3, that, at 10,000 MWD/tonne U, $\sigma_{e.av.}$ the average "excess bifa" falls by 143.3 - 91.9 = 51.4 bifa per 1.634 - 1.118 = 0.516 MWD/g U-235 = 3670 MWD/tonne U. The burn-up penalty is therefore 3670/51.4 = 71.4 MWD/tonne U per bifa. The penalty for the 50-day operating period is ½ x 17.9 = 8.95 mk = 11.86 bifa = 847 MWD/tonne U.

APPENDIX 2

Reactivity and Burn-up Penalty for Shut-down Refuelling using "Axial-inversion," "Two-position" Fuel Changes

It was shown in DM-42 (pp. 5 and 6) that the effect of irradiation of a single rod in the reactor results in a change of reactivity given by

$$\delta k_{rod} = \frac{1 + B_g{}^2 M^2}{Y \epsilon p} \cdot \frac{2F^2 \delta\sigma_{av}}{N_t \overline{F^2}}$$

where Y = yield cross-section in bifa

ϵ = fast fission factor

p = resonance escape factor

$B_g{}^2$ = geometrical buckling

M^2 = migration area

F = flux factor for the rod position

= $\phi_{max.rod}/\phi_{max.reactor}$

$\overline{F}^2$ = average F^2 over all rods

N_t = total number of rods

$\delta\sigma_{av}$ = $\frac{2}{\ell}\int_0^{\ell/2} \delta\sigma_e(\phi/\phi_{max})^2 dx$

$\delta\sigma_e$ = change due to irradiation of "excess bifa" σ_e

ϕ = neutron flux at given position along rod

= $\phi_{max}\sin(\pi x/\ell)$

dx = element of length along rod.

For approximate working write

$$1 + B_g{}^2 M^2 = 1.05;$$

$$Y \epsilon p = 1390 \text{ bifa};$$

and neglect the radial flux variation so that

$$\delta k_{reactor} = \frac{2}{1.325} \delta\sigma_{av} \text{ milli-k for } \delta\sigma_{av} \text{ in bifa.}$$

For the chosen trial case

$$R = 6 \text{ MW/tonne U}$$

$$\therefore \quad \phi = 1.8 \times 10^{13} \text{ n/cm}^2\text{/sec.}$$

Hence in 50 days the mean irradiation is 7.78×10^{19} n/cm^2. More precisely, 32 equal irradiations of 7.8125×10^{19} n/cm^2 will amount to 2.5 n/kb or about 10,000 MWD/tonne of natural uranium.

At each fuel change when equilibrium has been reached for axial inversion fuelling, 1/32 of all the rods would be replaced with fresh rods and 1/32 of all the rods would be axially inverted. Each rod is therefore changed after 16 steps and there will be rods in the reactor at each of the 16 step irradiations.

$\delta\sigma_{av}$ may be read from Fig. 3 of DM-42 reproduced as Fig. A2-1 for 16 steps of 7.8125×10^{19} n/cm^2, giving a total mean irradiation of 1.25 n/kb. A similar curve could then be used for irradiations after axial inversion. The calculation of $\delta\sigma_{av}$ for the whole reactor may, however, be abbreviated by noting that at the start of each irradiation period there will be rods at stages 0 to 31 and at the end of each period rods at stages 1 to 32. The net change (in the approximation used) is only 1/16 of the differences between the initial and final $\delta\sigma_{av}$ for the two situations before and after axial inversion. The calculation of these values by a five-point approximation is given in Table A2-1. From this we derive

TABLE A2-1

			First Position		Second Position			
			Finish		Start		Finish	
x	ϕ/ϕ_{max}	$(\phi/\phi_{max})^2$	I	$\delta\sigma_e(\phi/\phi_{max})^2$	I	$\delta\sigma(\phi/\phi_{max})^2$	I	$\delta\sigma_e(\phi/\phi_{max})^2$
0	0	0	0	0	1.9635	0	1.9635	0
$\ell/8$	0.382683	0.146446	0.7514	- 1.7746	1.8140	- 18.512	2.5654	- 30.7188
$\ell/4$	0.707107	0.5000	1.3884	- 39.5827	1.3884	- 39.5827	2.7768	- 116.6130
$3\ell/8$	0.923880	0.853554	1.8140	-107.8952	0.7514	- 10.3433	2.5654	- 179.0432
$\ell/2$	1.0000	1.0000	1.9635	-142.951	0	0	1.9635	- 142.9511
5 point sum (1+4+2+4+1)				-660.795		-194.586		-1215.225
$\delta\sigma_{av}$ = Approx. mean = Sum/12				- 55.066		- 16.216		- 101.269

Final mean irradiation = 2.5 n/kb $\therefore$ I_{max} at change = 2.5 x $\pi/4$ = 1.9635 n/kb

$$\delta\sigma_e = 75(1 - e^{4I}) - 111\ I$$

I n/kb	$\delta\sigma_e$ bifa
0.7514	- 12.1179
1.3884	- 79.1654
1.8140	-126.4070
1.9635	-142.9511
2.5654	-209.7620
2.7768	-233.2259

TABLE A2-2

	Before Inversion		After Inversion	
	Mean Irradiation n/kb	$\delta\sigma_{av}$ bifa	Mean Irradiation n/kb	$\delta\sigma_{av}$ bifa
	0	0	1.25	- 16.22
	1.25	-55.07	2.5	-101.27
Difference		-55.07		- 85.05
Difference ÷ 16		- 3.442		- 5.316

Average - 4.379 bifa

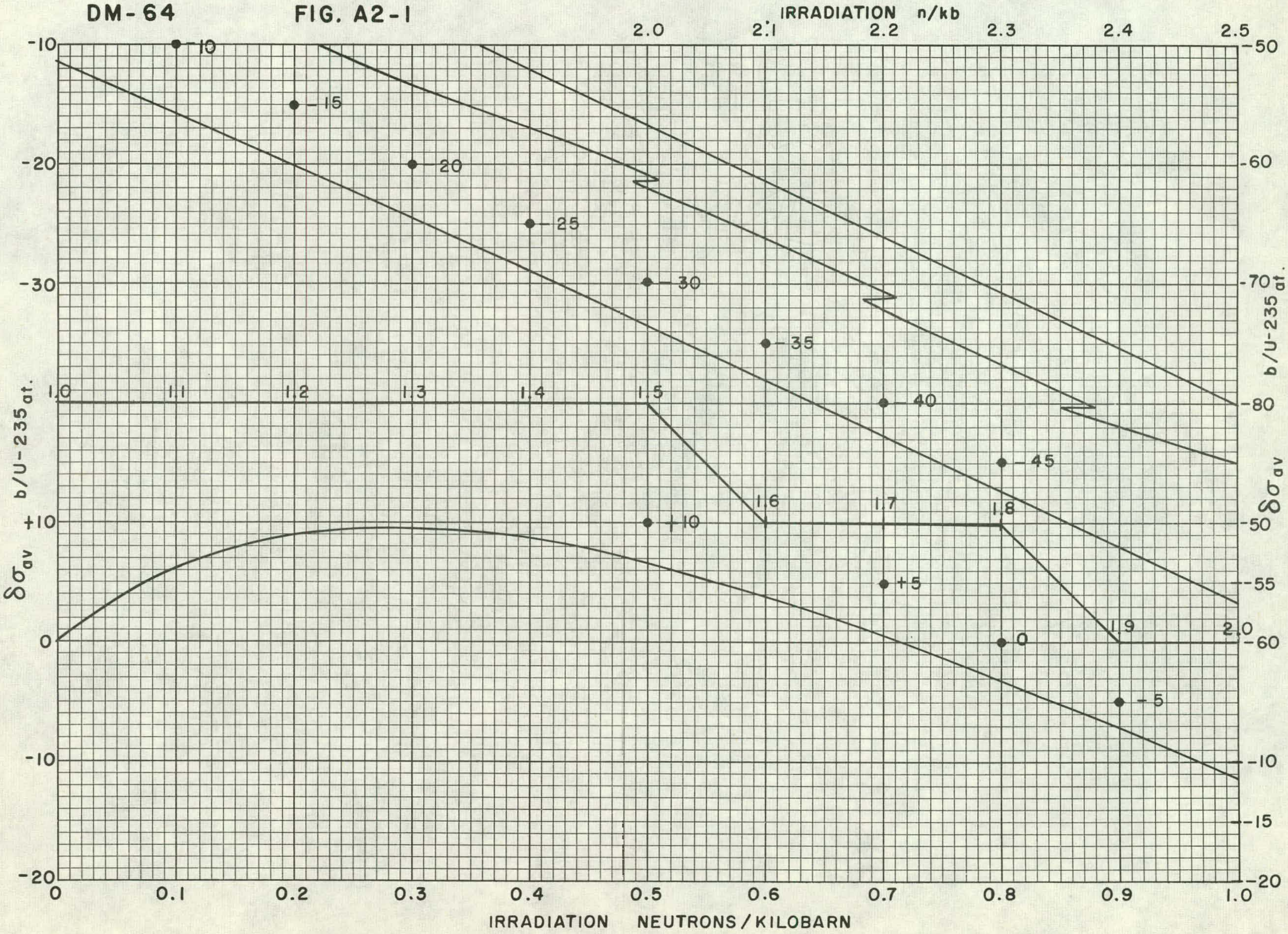
$$\text{Hence } \delta k = \frac{-2}{1.325} \times 4.379 = -6.61 \text{ mk.}$$

This change is almost directly proportional to the operating period (t_f) and the fuel power rating (R). For this example, R = 6, t_f = 50 days, so we may write

$$\delta k = -Rt_f/45.4 \text{ mk.}$$

The burn-up penalty is almost the same as for the bi-directional fuelling case discussed in Appendix 1 and may be taken as 94.6 MWD/tonne per mk applied to the net mk wastage that is ½ the decline between fuel changes. For the chosen example the penalty is 94.6 x 3.305 = 313 MWD/tonne U.

DM-64 FIG. A2-1


IRRADIATION n/kb
2.0
2.1
2.2
2.3
2.4
2.5
-10
-20
-30
+10
0
-10
-20
-50
-60
-70
-80
-50
-55
-60
-10
-15
-20
b/U-235 at.
$\delta\sigma_{av}$
-10
-15
-20
-25
-30
-35
-40
-45
+10
+5
0
-5
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
IRRADIATION NEUTRONS / KILOBARN

APPENDIX 3

Flux Depression in 7-rod Bundle

To estimate the flux pattern an experimental measurement for a 7-rod UO_2 bundle of 2.385 cm diam pellets of density 10.4 g/cm^2 will be taken as the basis. Measurements on such a bundle were made by R.E. Green (AECL-PR-RRD-17, March 1959) who concluded that the ratio of mean flux in an outer rod to that in the centre rod was 1.260 ± 0.005.

To extend this the bundle will be assumed homogenized in a cylinder characterized by a radius r, that need not be directly specified. The flux distribution in this cylinder will be assumed to take the Bessel function form $\phi = \phi_o I_o(k\sqrt{\rho}\,r)$ where ρ = density of uranium in the material of a rod, k is a constant, and ϕ is the flux at radius r from the centre of the bundle. ϕ_o is the flux at the centre (r = 0) and ϕ_1 is the flux at the effective boundary of the homogenized cylinder (where $r = r_1$). From this flux distribution the area mean flux ϕ_m within radius r is derived $\phi_m = \frac{2}{r^2}\int_o^r \phi r.dr$.

The mean flux in the centre rod (ϕ_7) will be identified as the area mean flux over a central circle of area = 1/7 of the homogenized cylinder. The mean flux in an outer rod ϕ_2 is then given by $\phi_7 + 6\,\phi_2 = 7\phi_m$. From which is derived ϕ_2/ϕ_7, the ratio measured by R.E. Green. Explicitly, $\phi_2/\phi_7 = (7\,\phi_m/\phi_7 - 1)/6$

and $\phi_7 = \frac{2}{r^2}\int_o^{r_1/\sqrt{7}} \phi r.dr = (\phi_m)_{r=r/\sqrt{7}}$

TABLE A3-1

I_o Bessel Function Flux Ratios

$k^2\rho r^2$	$k\sqrt{\rho}.r$	ϕ/ϕ_o $I_o(k\sqrt{\rho}.r)$	ϕ_m/ϕ_o	ϕ/ϕ_m	ϕ_2/ϕ_7 R_I	ϕ_2/ϕ_m R_B
0		1.0000	1.0000	1.0000	1.0000	1.0000
0.4	0.63246	1.103	1.052	1.049	1.049	1.008
0.8	0.89443	1.210	1.104	1.097	1.100	1.013
1.2	1.0954	1.323	1.158	1.143	1.154	1.019
1.6	1.2649	1.443	1.214	1.188	1.208	1.025
2.0	1.4142	1.567	1.272	1.232	1.265	1.031
2.4	1.5492	1.698	1.332	1.274	1.323	1.036
2.8	1.6733	1.834	1.394	1.315	1.380	1.041
3.2	1.7889	1.976	1.458	1.355	1.441	1.046
3.6	1.8974	2.124	1.523	1.394	1.503	1.050
4.0	2.00	2.280	1.591	1.433	1.565	1.054
4.4	2.0976	2.442	1.661	1.470	1.628	1.058
4.8	2.1909	2.612	1.733	1.507	1.690	1.062
5.2	2.2804	2.790	1.808	1.544	1.756	1.066
5.6	2.3664	2.976	1.884	1.579	1.825	1.069

Figure A3-1 shows ϕ_1/ϕ_m, ϕ_m/ϕ_o and ϕ_2/ϕ_7 plotted (from Table A3-1) against $k\sqrt{\rho}.r$. Green's value, $\phi_2/\phi_7 = 1.260$, may be seen from this figure to correspond to $k\sqrt{\rho}.r = 1.40$. Writing $r = k_1 a$ where a = radius of a single rod and k_1 = const. since $\rho = 10.4 \times 238/270 = 9.167$ g U/cm^3 and $a = 1.1925$ cm; $kk_1 = 1.40/1.1925\sqrt{9.167} = 0.3877$.

For the example of §2.2 of the main text, $a = 1.49$ cm and $\rho = 12.85$ g U/cm^3 so $k\sqrt{\rho}.r = kk_1 a\sqrt{\rho} = 0.391 \times 1.49\sqrt{12.85} = 2.07$. Reading from Fig. A3-1 we find

(Mean flux in outer rod)/(Mean flux in centre rod) $= \phi_2/\phi_7 = 1.61.$

$$\frac{\text{Flux at periphery of homogenized cylinder}}{\text{Mean flux in fuel}} = \phi_1/\phi_m = 1.46.$$

It should be noted that simplifying assumptions and approximations have been involved in reaching these results. The ratios introduced in §2.3 of the text, namely R_I and R_B may be identified with ϕ_2/ϕ_7 and ϕ_2/ϕ_m respectively.

It may also be noted that the flux depression calculated by Primeau (10) for Cases A and B of §2.4 is considerably greater. Further experimental work and calculations are required to resolve the discrepancy.

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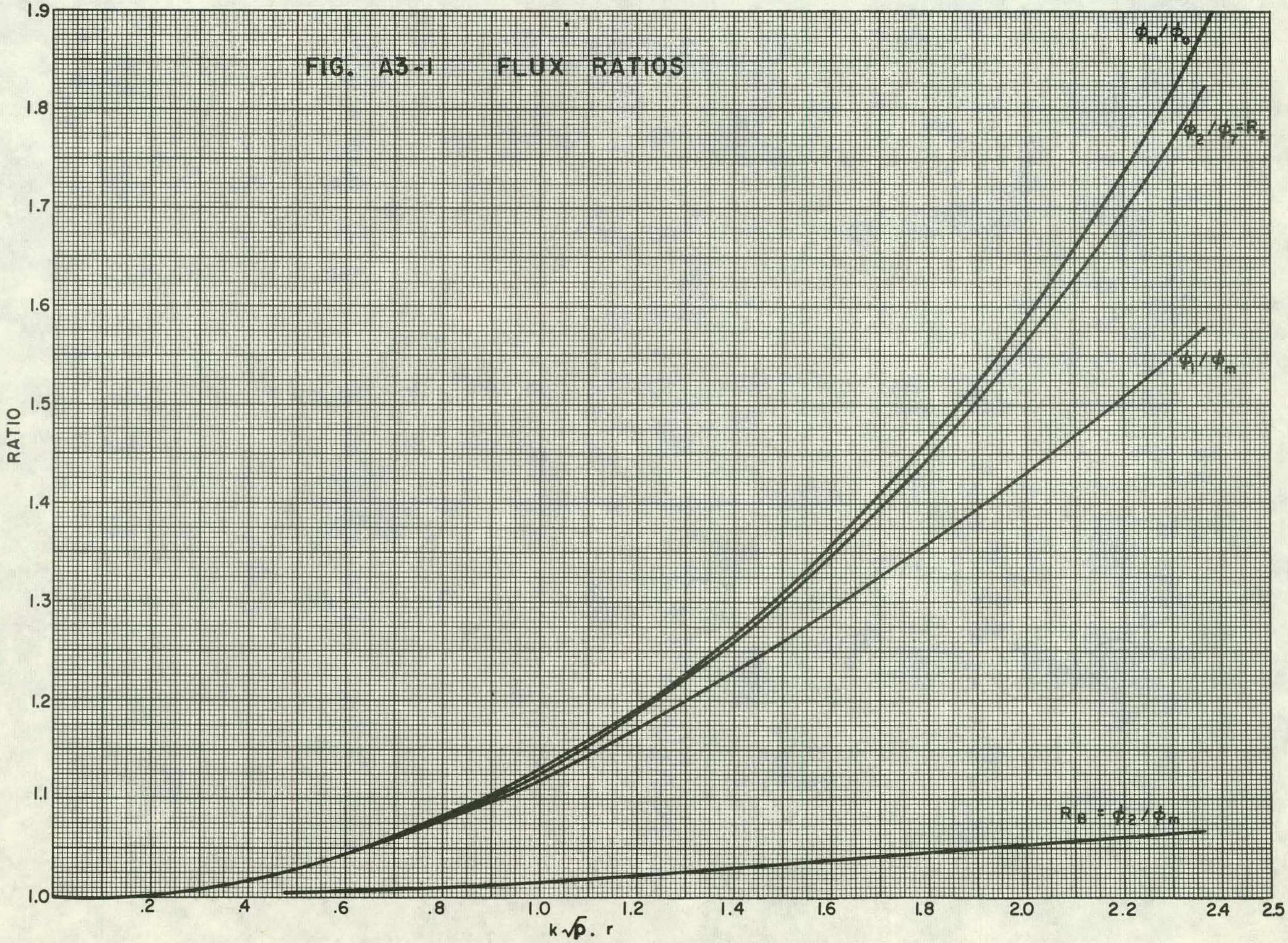

FIG. A3-1 FLUX RATIOS
RATIO
$k\sqrt{p} \cdot r$
ϕ_m/ϕ_o
$\phi_2/\phi_7 = R_s$
ϕ_1/ϕ_m
$R_B = \phi_2/\phi_m$
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.5

APPENDIX 4

Basis for Table in §2.4

The reactivity balance given by the four-factor diffusion equation

$$\eta \, \epsilon \, p \, f = (1 + B_L{}^2 L_s{}^2)(1 + B_L{}^2 L^2) \qquad \text{---(A4.1)}$$

is solved for the buckling ($B_L{}^2$) by a revised version of the computer program set by M.F. Duret in CRRP-873 (9). Irradiation of fuel initially natural uranium changes η, f and L^2, while ϵ, p and $L_s{}^2$ are regarded as constant. Fuel properties are averaged over irradiation to an upper limit ω(n/kb), the varying parameters η, f, L^2 and $B_L{}^2$ are derived from the mean fuel properties and lattice dimensions using lattice recipes based on Hone et al. (1958). The recipes are adjusted by choice of $\xi \, \Sigma_s$ for D_2O and E_R, the effective energy of the resonance absorption to fit experimental measurements of buckling and neutron density fine structure, while using the world best values of nuclear cross-sections and fission yields as reviewed by C.H. Westcott in CRRP-960 (3rd Edition corrected) (13) 1960 and W.H. Walker, CRRP-913 (1960) (14). In addition fast fission, capture and scattering constants for U-238 in the form of UO_2 or U-metal (the latter used for UC) are assigned from recent reviews of the literature by G.C. Hanna (1960) (15) and others (16) (17) as follows:

Components in Neutron Balance

Yield cross section in bifa $Y = 137.8\,\hat{\sigma}_8\,\overline{B}/D$ ---(A4.2)

Resonance Trap " $= Y\epsilon(1 - p)/(1 + B_L^2 L_s^2)$ ---(A4.3)

Fast Plutonium Production (F.P.P.)
i.e. by fast neutron capture in U-238

$$= R'\left\{\frac{C_A}{F_A} + \frac{C_B}{F_A}\,\frac{f_B}{f_A}\left[\frac{1 + R'\left(\frac{I_A}{F_A f_B} + \nu_8\right)}{1 + R'\left(\left[1 + \frac{I_A}{F_A} + \frac{C_A}{F_A} - \frac{C_B}{F_A}\right]/f_A - \nu_8\right)}\right]\right\} \quad \text{(A4.4)}$$

Fast neutron factor $\epsilon = 1 + R'(\nu_8 - 1) - (\text{F.P.P.})$ ---(A4.5)

Fast neutron leakage $Y\epsilon\, B_L^2 L_s^2/(1 + B_L^2 L_s^2)$ ---(A4.6)

Neutrons becoming thermal

$$Y\epsilon p/(1 + B_L^2 L_s^2) \quad \text{---(A4.7)}$$

Thermal neutron leakage

$$Y\epsilon p\, B_L^2 L^2/(1 + B_L^2 L_s^2)(1 + B_L^2 L^2) \quad \text{---(A4.8)}$$

Neutrons absorbed in fuel

$$Y\epsilon p f'/(1 + B_L^2 L_s^2)(1 + B_L^2 L^2) \quad \text{---(A4.9)}$$

$$= Y/\overline{\eta} \quad \text{---(A4.10)}$$

Neutrons absorbed in other material X

$$= Y\epsilon p f_x/(1 + B_L^2 L_s^2)(1 + B_L^2 L^2) \quad \text{---(A4.11)}$$

Definitions

1. Non-thermal plutonium production factor

$$D = [\epsilon(1 - p)/(1 + B_G^2 L_s^2)] + (\text{F.P.P.})$$

2. Ratio of non-thermal to thermal Pu production

$$\overline{B} = YD/137.8\,\hat{\sigma}_8$$

3. Geometrical buckling B_G^2

4. Lattice Buckling B_L^2

Note that except when $B_G^2 = B_L^2$ there is a slight discrepancy between the numbers assigned to the resonance plutonium production in D (and $\overline{B}$) and in the neutron balance components.

5. Subscripts A and B denote the two energy groups into which fission neutrons are divided. Group A has the higher energy and alone contributes to fast fission. The fractions in groups A and B are f_A and f_B so that $f_B = 1 - f_A$. C, F and I are the probabilities that a given fast neutron is captured, causes fission or is inelastically scattered out of the energy group on its next collision. Since these quantities appear only as ratios they have been substituted by the ratios of the corresponding cross sections $\sigma_C(A)$, $\sigma_C(B)$, $\sigma_F(A)$, $\sigma_I(A)$.

6. ν_8 is the average yield of neutrons for fast fission in U-238.

7. $R' = \gamma/\nu_{th}$ in the notation of CRRP-873 (9),

$= R_I/\nu_{th}$ in the notation of DM-62 (18)

where γ or R_I is the ratio of fast to thermal fissions and ν_{th} is the mean neutron yield for thermal fission. R' and ϵ are related by equation A4.5 and both assumed constant and independent of fuel irradiation, hence R' is assigned the value obtained by extrapolation from experiments with natural uranium lattices $= R_I/\nu_5$. It follows that γ or R_I is a basic parameter that determines ϵ, R' and the fast capture.

8. The thermal utilization in the fuel, f', and the various other materials, f_x, are calculated by the program, but two modifications have been introduced, one an error and the other an attempt to make a correction for the effect of moderation by the coolant within the fuel denoted by subscript "o".

The error is that the macroscopic fuel thermal cross-section $\hat{\Sigma}_f^a$ has been calculated as 138.8/137.8 of its true value. As long as $\hat{\Sigma}_f^a$ is not used otherwise the result of this from the program is that the f_x are low by about 1 in 2000 and (1 - f) correspondingly high.

The correction for moderation by the coolant is introduced by multiplying f by a factor 1.002 in Case B and 1.0036 in Case A; this causes the misbalance recorded in the Table in §2.4. The f_x should be correspondingly reduced in the same ratio but this would have only a small effect towards restoring the balance.

The misbalance is, however, to be considered as insignificant, if the attempted correction is justified.

Assigned Values (in addition to those given in the table in §2.4)

All Cases

	ν_5	2.4498
	ν_8	2.8415
	ν_9	2.885
	ν_1	3.060
	η_1	2.22303
For UC	$\sigma_c(A)$	0.54 b
For UO_2	$\sigma_c(A)$	0.99 b
For UC and UO_2	$\sigma_c(B)$	0.137 b
For UC and UO_2	$\sigma_F(A)$	0.549 b
For U metal	$\sigma_I(A)$	2.07 b
For UO_2	$\sigma_I(A)$	2.555 b
	f_A	0.561; $f_B = 1 - f_A = 0.439$
	$\xi\overline{\Sigma_s}$	0.178234 cm^{-1} at 20°C

Cases A and B

	$B_G{}^2$	0.0001158 cm^{-2}
Effective Res. Energy	E_R	25 eV
	ρ_{UC}	13.5 g/cm^3

CANDU

	ρ_{UO_2}	10.4 g/cm^3

		Case A	Case B	CANDU
Epithermal factor	r	0.0839000	0.0839000	0.0600
Fast fission ratio	$\gamma = R_I$	0.088690626	0.091583494	0.0476
	$\hat{\sigma}_8$	2.74100 b	2.7401824 b	2.73093 b
	$\hat{\sigma}_5$	645.07 b	645.6198 b	655.216 b
	$\hat{\sigma}_{f5}$	534.49 b	535.17455 b	547.457 b
	η_5	2.0299	2.030716	2.04690
	$\hat{\sigma}_9$	1939.3 b	1906.5233 b	1467.814 b
	$\hat{\sigma}_{f9}$	1248.97 b	1230.1963 b	981.769 b
	η_9	1.85802	1.86158	1.92967
	$\hat{\sigma}_1$	1919.23 b	1902.1155 b	1666.077 b
	$\hat{\sigma}_{f1}$	1394.28 b	1381.8718 b	1210.372 b
Fast Inelastic σ (fuel)	$\sigma_I(A)$	2.58 b*	2.47 b*	2.07 b⌿

* i.e. for U-238 + organic

⌿ This value from Fleishman and Soodak (1960) (16) was used rather than 2.555 b used by Hone et al. (1958) but is not necessarily preferable.

9. It should be possible to derive Y, etc. in bifa from the relation

$$\hat{\Sigma}^a_f = \rho \times \frac{6.025 \times 10^{23}}{250.1185} \times \frac{1}{138.8} \times \frac{\Sigma(N\sigma)}{\Sigma N_5} \times 10^{-24}$$

$$\frac{cm^2}{cm^3} \quad \frac{g(UC)}{cm^3} \times \frac{\text{Atom U}}{g(UC)} \times \frac{\text{Initial At U-235}}{\text{Atom U}} \times \frac{\text{Barn}}{\text{Atom U-235}} \times \frac{cm^2}{\text{barn}}$$

$$= \rho \times \frac{.002408858}{138.8} \times \text{fuel absorption in bifa.}$$

For $\rho = 13.5$

$$\text{fuel absorption (bifa)} = 4268.20\ \hat{\Sigma}^a_f\ (cm^{-1})$$

This relation was not used because of the error in $\hat{\Sigma}^a_f$ and the slight neutron misbalance.

10. The epithermal component, Westcott's "r" (13) of the neutron spectrum that is important in setting the neutron cross-sections is derived from the approximate relation

$$r = \frac{\bar{\eta}\ \epsilon p\ \hat{\Sigma}_f^a\ V_F \sqrt{\dfrac{\pi T_o}{4T_N}}}{(1 + B_G{}^2 L_s{}^2) V_{cell} (\xi \Sigma_s)}$$

The relative volume of fuel V_F/V_{cell} is the same as used in $\hat{\Sigma}_f^a$.

V_{cell} for Cases A and B = $(28\ cm)^2$ = 784 cm^3 per cm length.

T_o is the Westcott (13) standard temperature 293.6°K.

11. Miscellaneous Assumptions:

11.1 D_2O is 99.8% D_2O and 0.2% H_2O.

11.2 Organic coolant density is 0.832 g/cm^3.

11.3 Cases A and B. V_F = 48.82 cm^3 per cm of channel.

11.4 Initial fission product (high cross-section, i.e. Xe-135 and Sm) poison 39.088 bifa.

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