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HIGH CURRENT GAS DISCHARGES

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ABSTRACT

In this review high current discharges are taken to be those discharges whose self magnetic fields play an important role in their behaviour. Such discharges have been studied mainly in connection with the quest for controlled thermonuclear reactions. Of the many possible configurations for the discharge tube and the electro-magnetic fields, the pinch discharges in straight or toroidal have received the most attention and accordingly, a major part of this review is devoted to these discharges. Other configurations are reviewed in a separate section. Following an historical and introductory review of the subject, a set of appropriate plasma equations is presented, and the common approximations and physical concepts associated with the theory are discussed. The properties of the pinch discharge are considered under the headings- discharge initiation and contraction, equilibrium, magnetohydrodynamic stability, particle heating and energy loss, and nuclear reactions- and in many cases it is necessary to subdivide the sections according to whether the rate of rise of current is high or low.

CONTENTS

1. INTRODUCTION.

1.1. The Class of Discharges to be Reviewed.

2.1. Introductory and Historical Survey.

2. THE PLASMA EQUATIONS AND SOME PHYSICAL CONCEPTS.

2.1. A Set of Plasma Equations.

2.2. The Magnetohydrodynamic Approximations.

2.3. The Collisionless Plasma Approximation.

2.4. Simple Physical Concepts.

3. DISCHARGE INITIATION AND CONTRACTION.

3.1. Ionization.

3.2. Discharge Contraction.

4. DISCHARGE EQUILIBRIUM.

4.1. Linear Discharges.

4.2. Toroidal Discharges.

5. MAGNETOHYDRODYNAMIC STABILITY.

5.1. Theory.

5.2. Experimental Observations on Stability.

5.3. Instability Waves.

6. PARTICLE HEATING AND ENERGY LOSS.

6.1. The Electron Temperature and Energy Loss.

6.2. The Positive Ion Temperature and Heating Mechanism.

7. NUCLEAR REACTIONS.

7.1. The Linear Pinch without an Axial Magnetic Field.

7.2. The Linear Pinch with an Axial Magnetic Field.

7.3. The Toroidal Pinch with Magnetic Field.

8. HIGH CURRENT DISCHARGES WITH OTHER GEOMETRICAL CONFIGURATIONS.

8.1. Azimuthal Discharges.

8.2. Annular Discharges ("Triax" and"Hard-core").

8.3. Rotating Plasmas.

8.4. Plasma"Guns".

REFERENCES

1. INTRODUCTION.

1.1. The Class of Discharges to be Reviewed.

Throughout this article the relative term "high current gas discharge" will be taken to mean a discharge in which the magnetic field produced by the current itself plays a major role in the mechanism of the discharge. This will be the case when the reaction between the self magnetic field and the current produce a force on the ionized gas comparable with the other forces acting on it, such as the gradient of pressure. The actual magnitude which a current must have to satisfy this condition will in general be higher the higher the gas density in the discharge, but as a rough guide it can be said that the currents referred to will be greater than about 10^3 amperes and the gas densities less than about 1 millimetre of mercury at N.T.P.

The impetus to study very high current discharges has come from the fact that they present a possible means of containing and heating a gas to very high temperatures and hence inducing controlled thermonuclear reactions. Most of the research in this field has been done with this ultimate goal in view. However, this paper is not a comprehensive review of thermonuclear research since it deals with only one of the two general methods of approach to produce such reactions. The systems dealt with here start with a cold gas which is ionized and heated by passing a high current through it. In the other class of devices a magnetic field configuration is created in a vacuum system by means of external coils, and a hot gas is generated by the injection of an ordered beam of energetic particles. Although the ultimate goal for this latter class of systems is the same, at their present state of development they bear little resemblence with gas discharges. Their physical properties and problems are closer to those for particle accelerators. It is therefore logical to omit such systems from the present review. For a general review of the various methods of approach to controlled thermonuclear power and the physics involved the reader is referred to the excellent accounts of Post (1956), Bishop (1958) and Simon (1959).

The most commonly adopted geometrical configurations for the study of high current discharges have been the straight cylindrical tube with end electrodes, and the torus without electrodes. Most of the sections in this article will deal with such discharge tubes. For many of the discharge properties the curvature of the toroidal tube is neglected and no distinction will be made between the two types of tubes. Since most of the discharge properties satisfy simple scaling laws (Bickerton and London, 1958) reference will seldom be made to the tube size in a particular experiment. High current discharges have been studied in other geometrical configurations; these will be reviewed in section 8.

1.2. Introductory and Historical Survey.

It has been well known to physicists and electrical engineers for many years that a group of parallel conductors carrying currents in the same direction mutually attract one another because of their magnetic fields. That a similar effect will also occur when a current is passed through an ionized gas was first considered theoretically by Bennett (1934), and later by Tonks (1939). Other early treatments were by Fetz (1949), Alfvén (1950), Schluter (1950), Blackman (1951), and Thonemann and Cowhig (1951). The parallel currents flowing in different parts of the gas attract one another and a cylindrical discharge (with current parallel to the axis) will contract in the radial direction towards its axis. This now well known "pinch effect" will continue until the attractive force is balanced by a radial pressure gradient. The force per unit volume on the ionized gas is $\underline{j} \wedge \underline{B}$, where $\underline{j}$ is the current density and $\underline{B}$ the magnetic field. In this simple case where $\underline{j}$ is parallel to the z-axis, and $\underline{B}$ is in the θ direction, if interaction with neutral gas can be neglected, the equilibrium is given by

$$j_z B_\theta = - \frac{dp}{dr} \tag{1}$$

This equation can be combined with Maxwell's equation for the current and integrated (see Pease, 1957) giving

$$N k \overline{T} = \frac{I^2}{2} + \pi b^2 p_b \tag{2}$$

where N is the number of particles (positive and negative) per unit length of the discharge tube, $\overline{T}$ their average temperature, p_b the pressure at the wall whose radius is b, and I the total current. Clearly the pinch effect will become important when $\frac{I^2}{2}$ is comparable with $N k \overline{T}$. At high currents the constriction of the discharge causes p_b to become small and

$$N k \overline{T} \simeq \frac{I^2}{2} \qquad (3)$$

This equation, often referred to as the Bennett relation, gives the magnitude of current required to "contain" a given number of particles at a given temperature so that the pressure at the wall will be small.

The first reported demonstration of any pinch effect in a gas discharge was by Fetz (1941), who showed that in a low pressure mercury arc of about 100 amperes the expected slight sharpening of the radial density distribution in the positive column did occur. Similar experiments were performed by Thonemann and Cowhig (1951), and Mamyrin (1953). The first person however, to study a current sufficiently high to be within the present criterion was Smith (1941) who induced currents of 400 amperes in a torus containing low pressure mercury vapour. The experiments suggested that the self magnetic field of the current was not causing complete containment of the ionized gas. (Smith, 1947). About the same time Steenbeck (1943), constructed a high current toroidal discharge device known as the "Wirbelrohr" in which current pulses up to 10,000 amperes were induced in an attempt to produce an electron accelerator. Steenbeck recognized that the self magnetic field within a high current discharge is ideally suited for the focusing and containment of energetic particles. He was also aware that since the Coulomb scattering cross section for charged particles decreases inversely as the square of the particle energy, some of the electrons in a low pressure discharge will be accelerated to high energies without making a collision. Although there was some evidence that a small number of X-rays were produced, it was not until about 13 years later (Gibson, 1957) that the electron accelerating properties of a toroidal discharge were confirmed. (These accelerated electrons are referred to as "runaway" electrons. See Harrison, 1958 and Dreicer, 1959.)

It was after the end of the second world war and the successful explosion of the atom bomb that several independent workers gave serious thought to the possibility of producing controlled thermonuclear reactions. To generate useful power from such reactions heavy hydrogen must be heated to temperatures of the order of 10^8 degrees K. If the gas contains n positive ions per cm^3, the time t in seconds for which it must be maintained at this temperature must be such that nt is greater than 10^{16} cm^{-3} sec for deuterium and 10^{14} for a deuterium-tritium mixture (Lawson, 1957). Since the temperatures required are more than 10^4 times higher than the highest temperatures which any material container could withstand without vaporizing, such a hot gas can be confined only by an electromagnetic field. This confinement is possible in principle since even well below this temperature hydrogen will be completely ionized into electrons and positive ions.

An electromagnetic field is required which will cause the gas in contact with the wall to have either a much lower density of a much lower temperature than the gas in the centre ofthe vessel, so that the particle bombardment of the walls involves only a small amount of heat. That is, there must be a large pressure gradient between the walls and the main body of the hot gas. From equation (1) it can be seen that the electromagnetic field of a high current discharge produces such a pressure gradient in the radial direction. There is, however, no pressure gradient parallel to the current and hence no containment in this direction. If the discharge is induced in a torus the need for containment in this direction is avoided.

Sir George Thompson, then at Imperial College, was among the first to realize the potentialities of the toroidal discharge (Ware, 1957 and Thonemann, 1958), and work started under his direction in 1947 on a discharge similar to Steenbeck's "Wirbelrohr" (Ware, 1951). Short current pulses of 2 x 10^4 amperes were achieved, and with the aid of high speed photography, Cousins and Ware (1951) demonstrated that a marked pinch effect was occurring. Because of the speed with which the high currents were generated (in a few microseconds) the constriction of the discharge was very rapid and the inertia of the collapsing gas lead to a radial oscillation of the discharge about its equilibrium configuration. This behaviour is shown in figure 1 which is a more recent streak photograph. Figure 2, which is a photograph taken with a glass toroidal discharge tube, illustrates the high degree of constriction which is momentarily achieved in such discharges. Other experimental work on the pinch effect was commenced by Thonemann at Oxford in 1948 (Thonemann and Cowhig, 1951) and in the U.S.A. by Tuck at Los Alamos in 1952 and Baker in Berkeley in 1952, (see Bishop, 1958) and in the U.S.S.R, by Artsimovich in Moscow about 1952 (see Kurchatov, 1956).

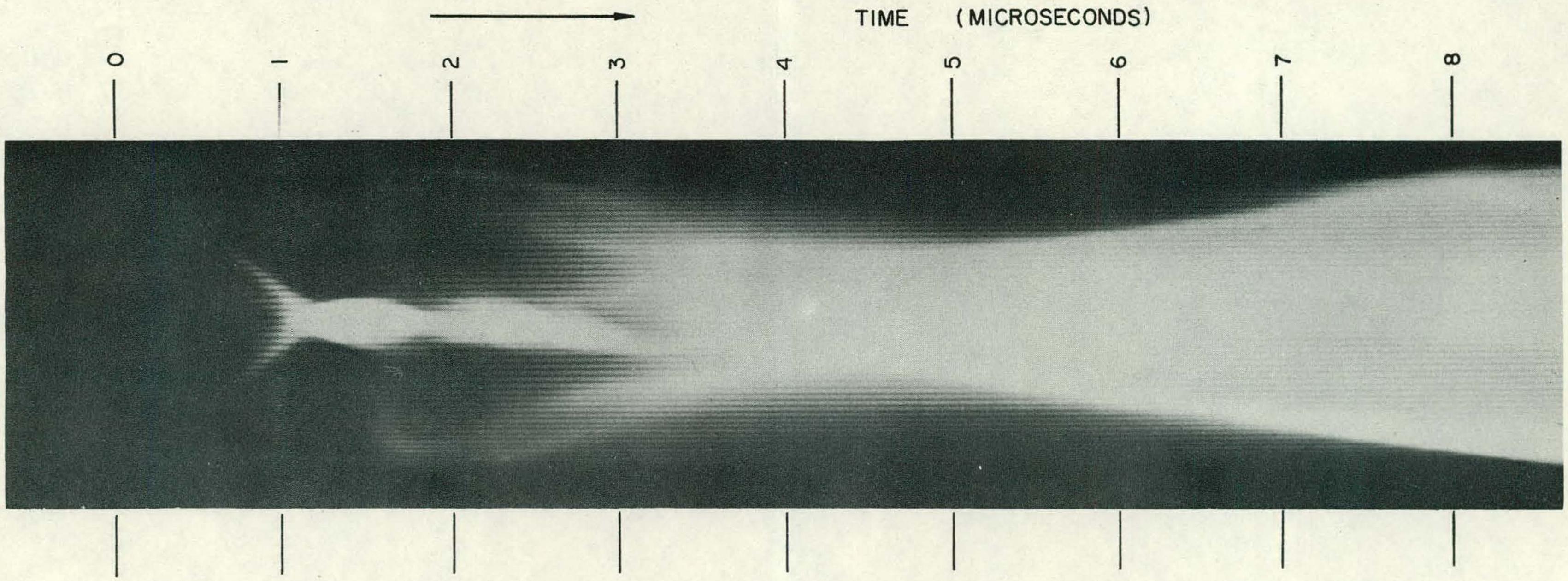


Figure 1 -- Streak photograph of a deuterium pinch discharge with 5% oxygen impurity taken by Gabriel (1959), showing the rapid contraction and "bouncing" of the discharge. The formation of a second current sheet can also be seen starting at 1.5 microseconds.

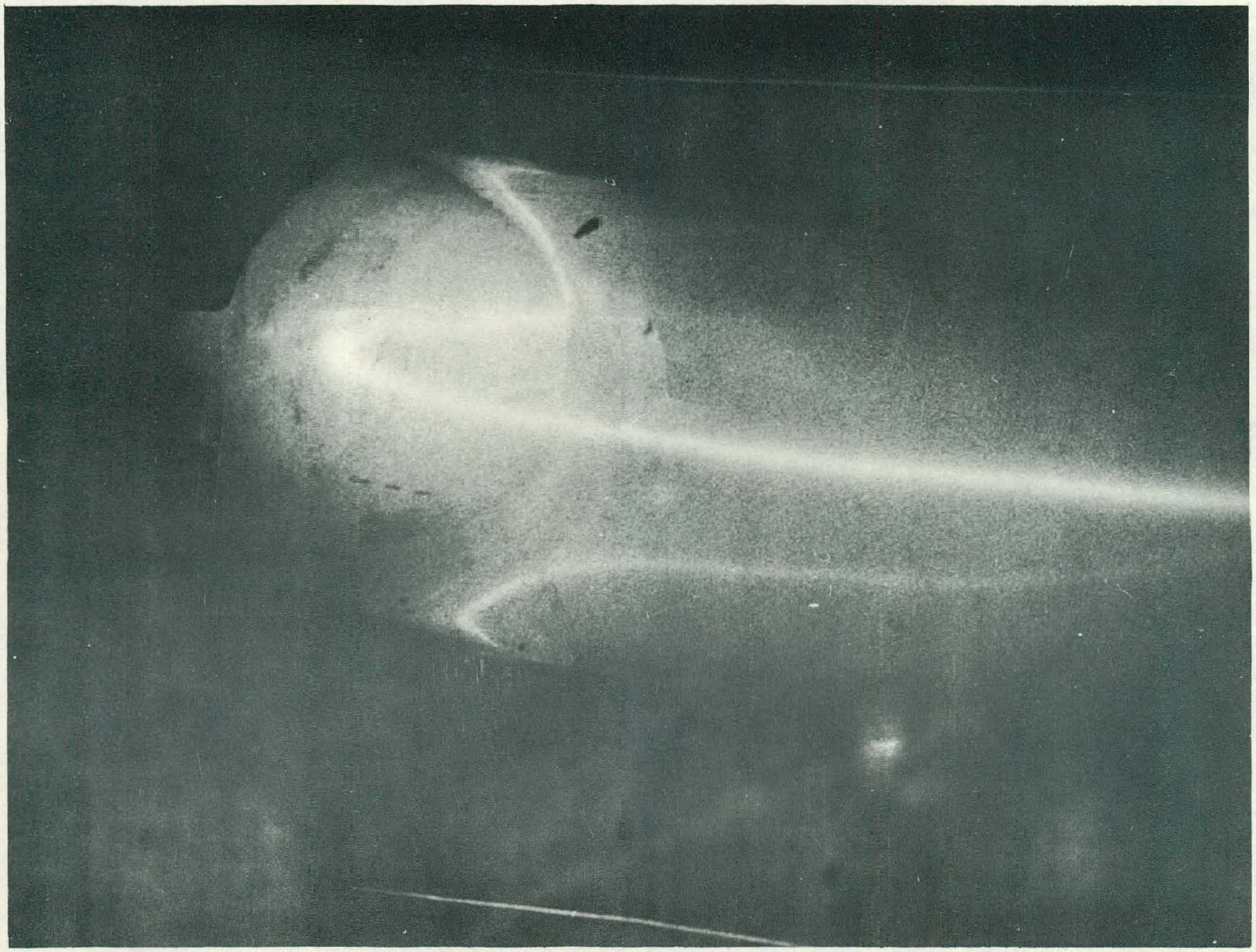


Figure 2 -- Toroidal Pinch discharge in Krypton. This photograph was taken by Burkhardt et al. (1957) using an open shutter. It shows the high degree of constriction momentarily achieved, at which time the discharge emits much more light.

At Princeton, U.S.A., a study was commenced in 1951 by Spitzer of a high current toroidal discharge with a large magnetic field applied parallel to the current so that little pinch effect occurred (see Bishop, 1958). Some early experimental work was carried out also by Bostick et al.,(1953). Subsequently many other laboratories in these and other countries have taken up the study of high current discharges. (See the Proceedings of the Second International Conference on Peaceful Uses of Atomic Energy, 1958, Volumes 31 and 32.) The work in the United Kingdom was made secret in 1950 and in the following year the research team at Oxford moved to the Atomic Energy Research Establishment at Harwell and the workers at Imperial College moved to the Associated Electrical Industries Research Laboratory, Aldermaston. The research in both the U.S.A. and U.S.S.R. was classified from its inception. As a result, despite the active research in progress there were very few publications on the subject until 1958 when the major powers declassified their work in this field.

In order to demonstrate just a small thermonuclear reaction a temperature of a few million degrees is required and must be maintained for a few microseconds or more, depending on the particle density. For a typical number of particles per unit discharge length of 10^{17} cm^{-1}, equation (4) shows that the current amplitude required is about 10^4 emu or 10^5 amperes. Such current pulses can be produced with quite modest condenser banks storing an energy of about 2×10^4 joules. The early work in this field was therefore aimed at producing such currents and inducing a small number of reactions.

Because of the short duration of the current pulse some of the experimental studies of such discharges can be conducted in straight tubes with electrodes. In the first few microseconds the heat conducted to the electrodes will be small and contamination by electrode vapours negligible. The simpler geometry of the cylinder compared with a torus greatly facilitates experimental work as well as the theory and analysis of the results.

Instability

The most important discovery made during the period of secrecy was that the magnetically constricted discharge is unstable. Almost any perturbation to the cylindrical symmetry or longitudinal uniformity of the discharge will cause the forces $j \wedge B$ and ∇p to change so that the resultant forces make the perturbation grow. Thus, for example, if the discharge remains symmetrical about the axis but becomes slightly more constricted at one or more points along its length B_Θ and j_z at these places both increase and the increase in the force $j_z B_\Theta$ causes the discharge to constrict still further. The excess plasma pressure causes an expansion of the discharge elsewhere. Again a kink in the discharge leads to an enhancement of $\underline{B}$ on the concave side of the kink and a reduction on the convex side, and the magnetic forces enlarge the kink.

The various geometrical configurations which these magnetohydrodynamic instabilities can take are best described by Fourier components whose form, including their time variation, is the real part of

$$A = A'(r) \exp i(\omega t - m\Theta - kz) \tag{4}$$

where A represents the change in any of the discharge parameters such as density, pressure, magnetic field, etc., $A'(r)$ is a complex number and a function of radius, m is the number of wavelengths in the Θ-direction in the range 0 to 2π and $2\pi/k$ the wavelength parallel to the z axis. (Complex numbers are used as a convenient means of including the phase of the particular component.) The instabilities observed experimentally have been found in most cases to correspond to a single value of m, either 0 or 1, and to have a well defined fundamental for k. The $m = 0$ mode is often referred to as the "sausage" instability, and the $m = 1$ mode as the "wriggle" or "kink" instability. Examples of these two modes are shown in figure 3 and 4, figure 3 being a photograph with 0.2 microsecond exposure and figure 4 is a streak photograph. Figure 5 shows the corresponding instabilities induced in a falling column of mercury by passing through it a current of 300 amperes (Dattner et al, 1958). The ideal discharge configuration shown in figure 1 persists for only a microsecond or two after initiation before it degenerates to something like figures 4.

In the U.K., high currents approaching those needed for a small thermonuclear reaction were achieved as early as 1950 (see Allibone et al., 1958), but the hydrogen discharges were found to be highly polluted with impurities due to vaporization of the tube walls. The mechanism by which the heat

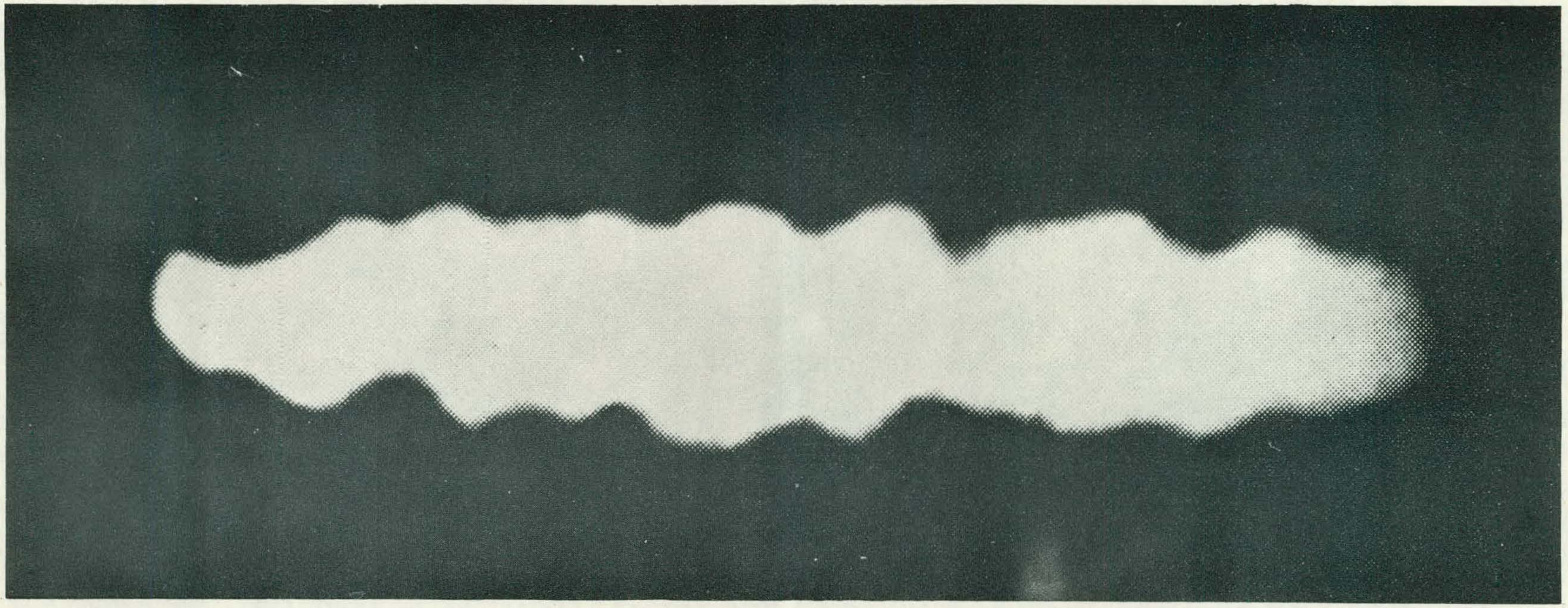


Figure 3 -- Example of an m = 0 or "sausage" instability in a linear pinch discharge in argon. This photograph was taken by Latham et al. (1959) using an image converter with 0.2 microsecond exposure time.

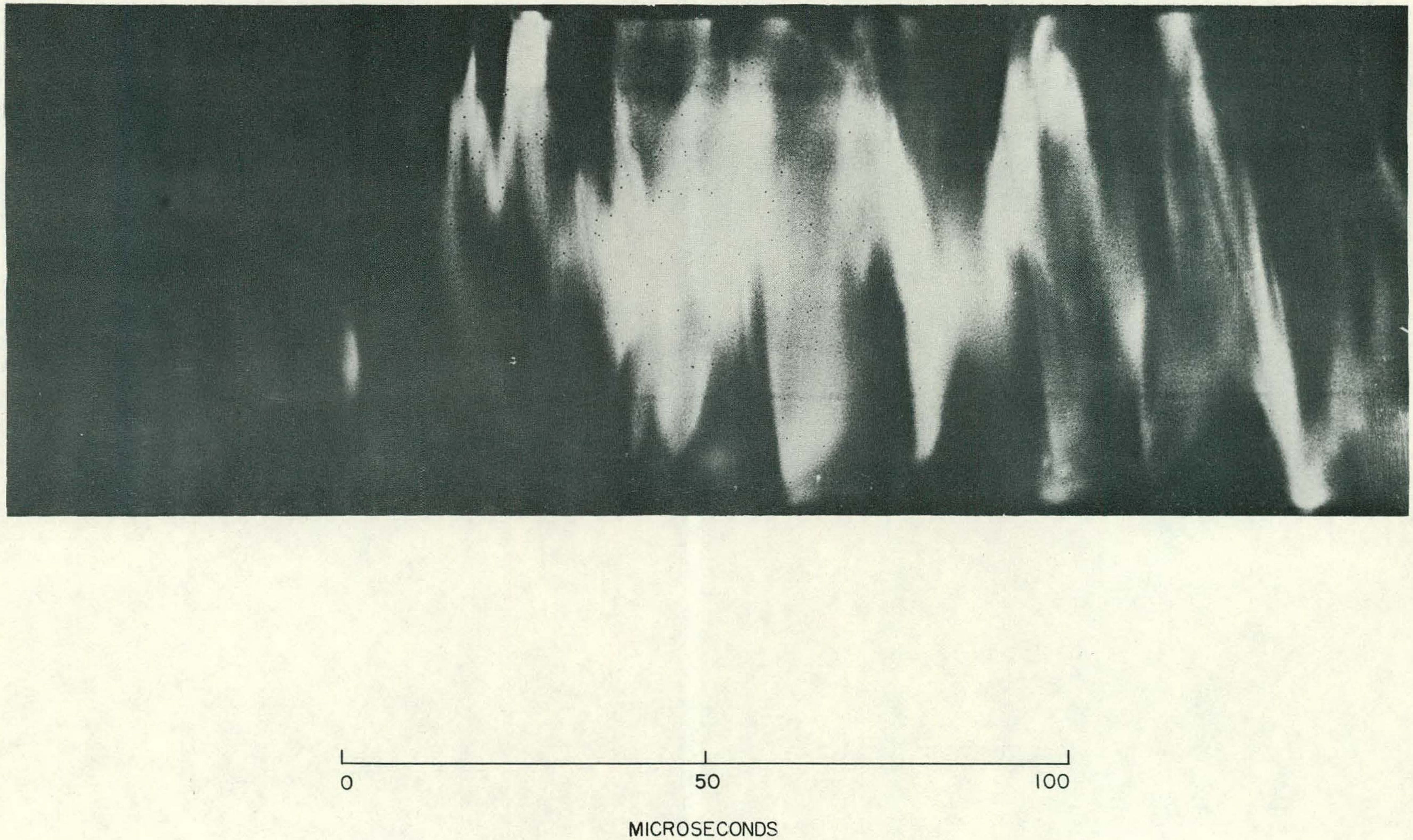
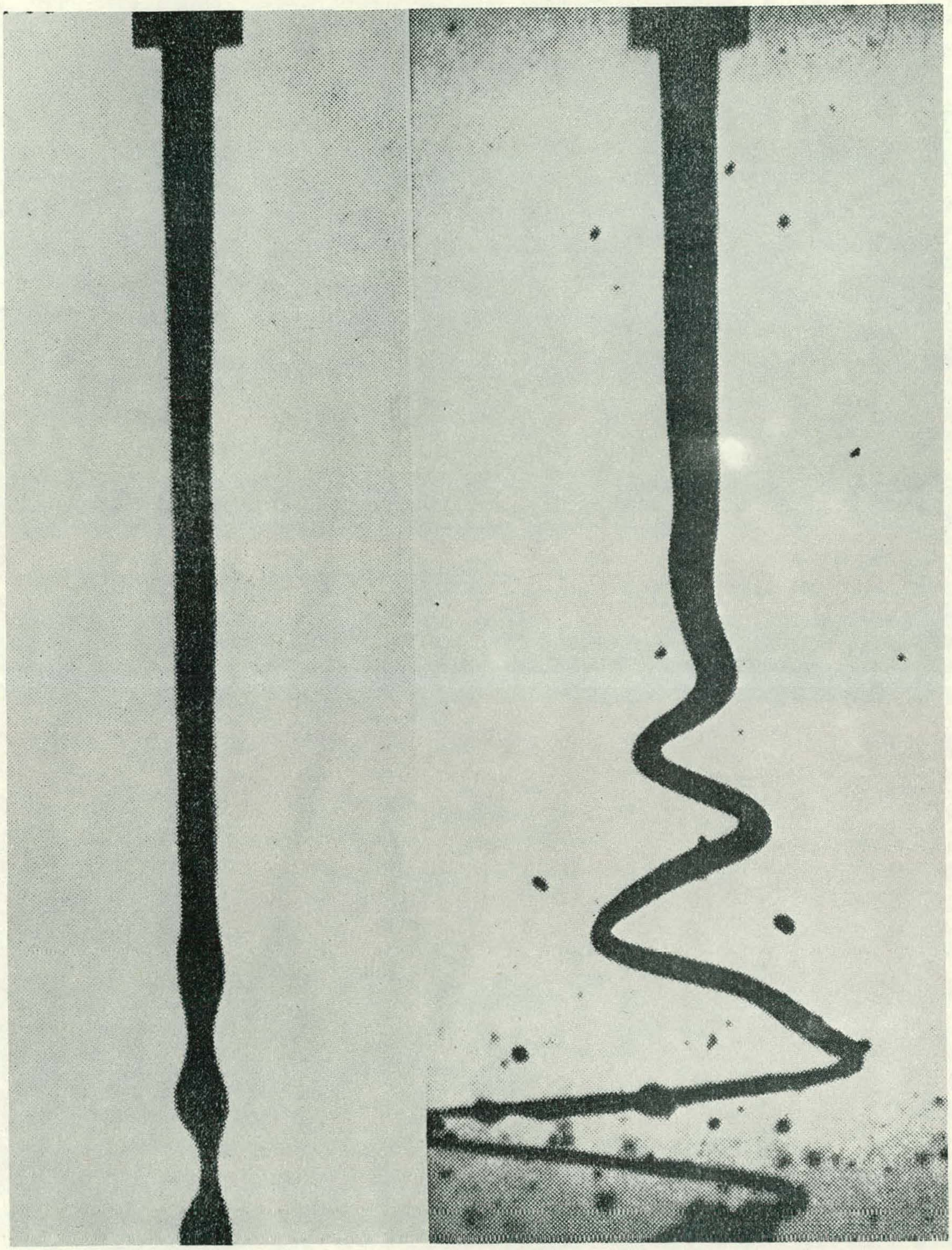


Figure 4 -- Streak photograph of an m = 1 or "wriggle" instability. This violent lashing of the discharge occurs at a later time than figures 1, 2 and 3.



Figures 5a & 5b -- Examples of "sausage" and "wriggle" instabilities in a column of mercury. Because the mercury is falling, the instability is further developed at the bottom. (Dattner et al., 1958). Figure (a) was taken was taken with no axial magnetic field and (b) with an axial magnetic field of 300 gauss.

generated in the gas was getting to the walls was almost certainly the discharge instability, but this was not recognized at the time. It was not until 1953 that the discharge instability was discovered by Carruthers and Davenport (1957) who were working on much lower current pulses in argon. With this heavier gas and lower current a fairly regular m = 1 instability was observed. These instabilities were discovered independently in Russia; the date is not given in the literature but was probably 1952, since means of stabilizing the discharge were studied theoretically in 1953 (Braginsky and Shafranov, 1958). A further independent discovery was in the work of Granovskii and Timofeeva (1955, 1956). In the U.S.A. it was predicted theoretically in the work of Kruskal and Schwarzschild (1954) before it was observed experimentally, a notable achievement.

The magnetohydrodynamic instabilities bring the plasma into contact with the tube wall and lead to extensive particle and heat losses and to wall vaporization. They have been the major obstacle to the achievement of high temperatures and probably still are. Many attempts were made to inhibit the instability with various types of applied magnetic field. In the U.K., where fairly low current discharges (10^3 to 10^4 amperes) with low rates of rise of current were used for instability studies, a simple magnetic field applied parallel to the discharge increases instability (Allibone et al., 1958). However, Bickerton (1957), working with higher currents (5×10^4 amperes) and a metal torus, found that a longitudinal field caused a marked decrease in the amplitude of the instability fluctuations. It was shortly afterwards that the theoretical work of Tayler (1957b) showed that a discharge would be stable if a longitudinal magnetic field exists within the constricted discharge with little or no longitudinal field outside, and if it is surrounded at not too great a distance by a metal wall. Corresponding developments occurred in the U.S.A. and U.S.S.R. culminating in the similar theoretical papers of Rosenbluth (1956) and Shafranov (1956). The equations of motion for an ionized gas in a magnetic field are very complex, and these theoretical advances were possible only when a very simplified theoretical model was taken. The model was a cylinder of perfectly conducting fluid with all currents flowing on the surface of the cylinder.

Discharges with configurations corresponding roughly to this theoretical model were produced in straight tubes (Bezbatchenko et al., 1956, Burkhardt, Lovberg and Phillips, 1958, Hagerman and Mather, 1958, Colgate, Ferguson and Furth, 1858), and in the toroidal discharge experiments "Zeta" (Thonemann et al., 1958),"Sceptre III" (Allen et al., 1958) and "Perhapsatron S3" (Honsaker et al., 1958). In the case of the discharges with rapid rise of current (rise times of a few microseconds), such as studied in the straight tubes and Perhapsatron S3, a distinct onset time is observed for instability. The stabilizing magnetic field was found to increase the onset time and to reduce the instability to a much smaller amplitude when it did occur. With the slow discharges no sharp onset is observed, but the stabilizing field caused a marked reduction in the instability amplitude. The substantial advance in stability in these experiments was accompanied by marked improvements in other discharge parameters including the electron and ion temperatures, impurity level and the nuclear reaction rate, particularly in the case of the toroidal experiments. Because of both the lower level of impurity and the higher electron temperature, the visible radiation from these discharges was reduced and normal high speed photography ceased to be able to detect the discharge channel when hydrogen was used. Time resolved spectroscopy (Allibone et al., 1958, Butt et al., 1958) revealed that there is a high temperature core to the discharge which moves very little and exhibits only small fluctuations . (See figure 6.) However, normal high speed photography has shown that outside this channel there is a considerable amount of colder plasma which emits light with fluctuating intensity (Allibone et al., loc.cit., Butt et al., loc. cit.). On rotating mirror streak photographs this light is sometimes seen to follow helical patterns and on other occasions bursts of light appear simultaneously at all points across the viewing slit, so that "bars" are seen on the streak photograph, as shown in figure 7. There is a close correlation between these light fluctuations and the magnetic field fluctuations (Ware, 1959b).

Simultaneously with this experimental work, the theoretical physicists were improving the simple model used in their analysis and were allowing for the fact that in practice the discharge currents are distributed over a sheath with finite radial thickness. Their results showed that unless the skin is very thin (Boon et al., 1958, Hubbard, 1958) or the magnetic field varies

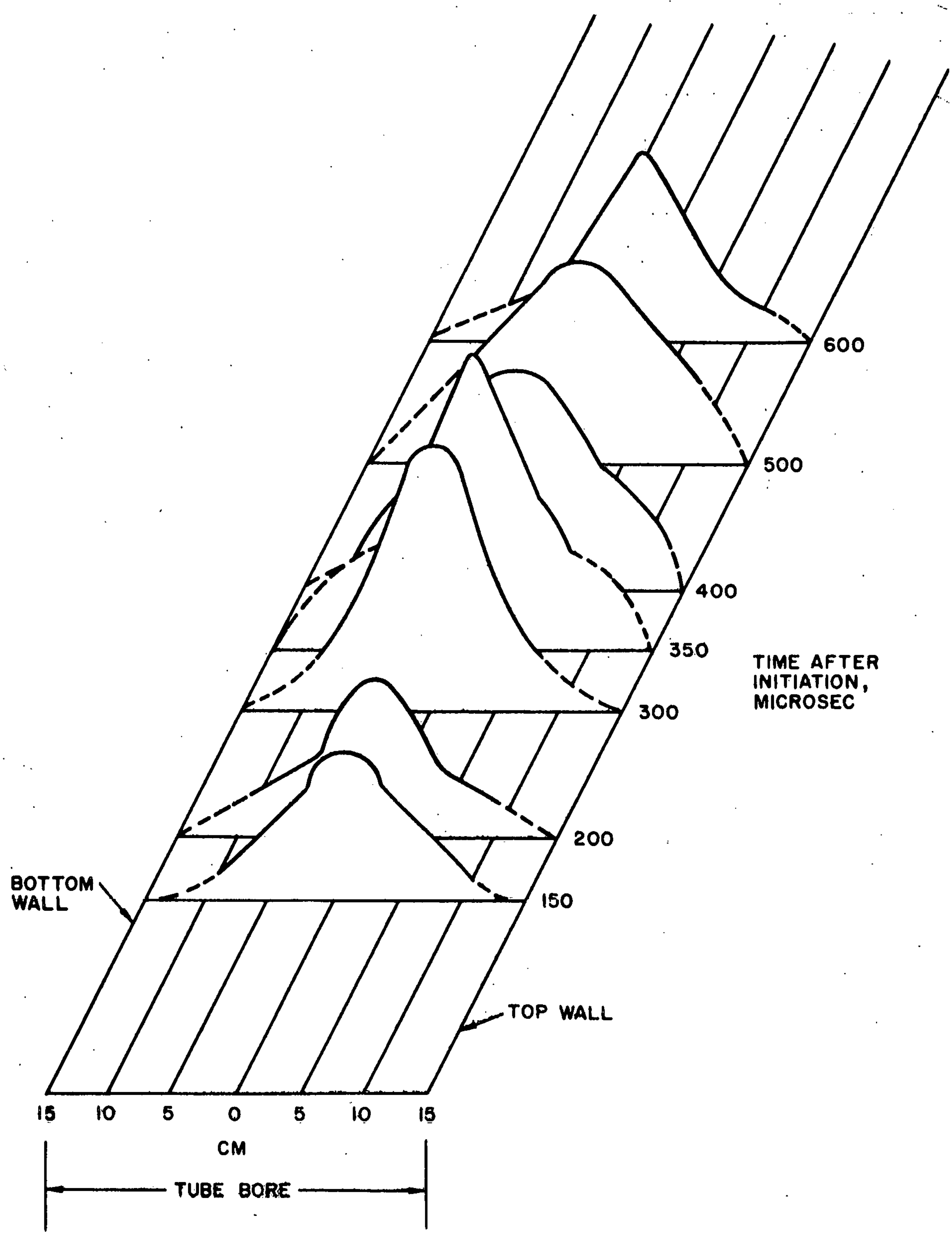


Figure 6 -- Spatial distribution of spectral line intensity from four times ionized oxygen at different times throughout the current pulse with Sceptre III.

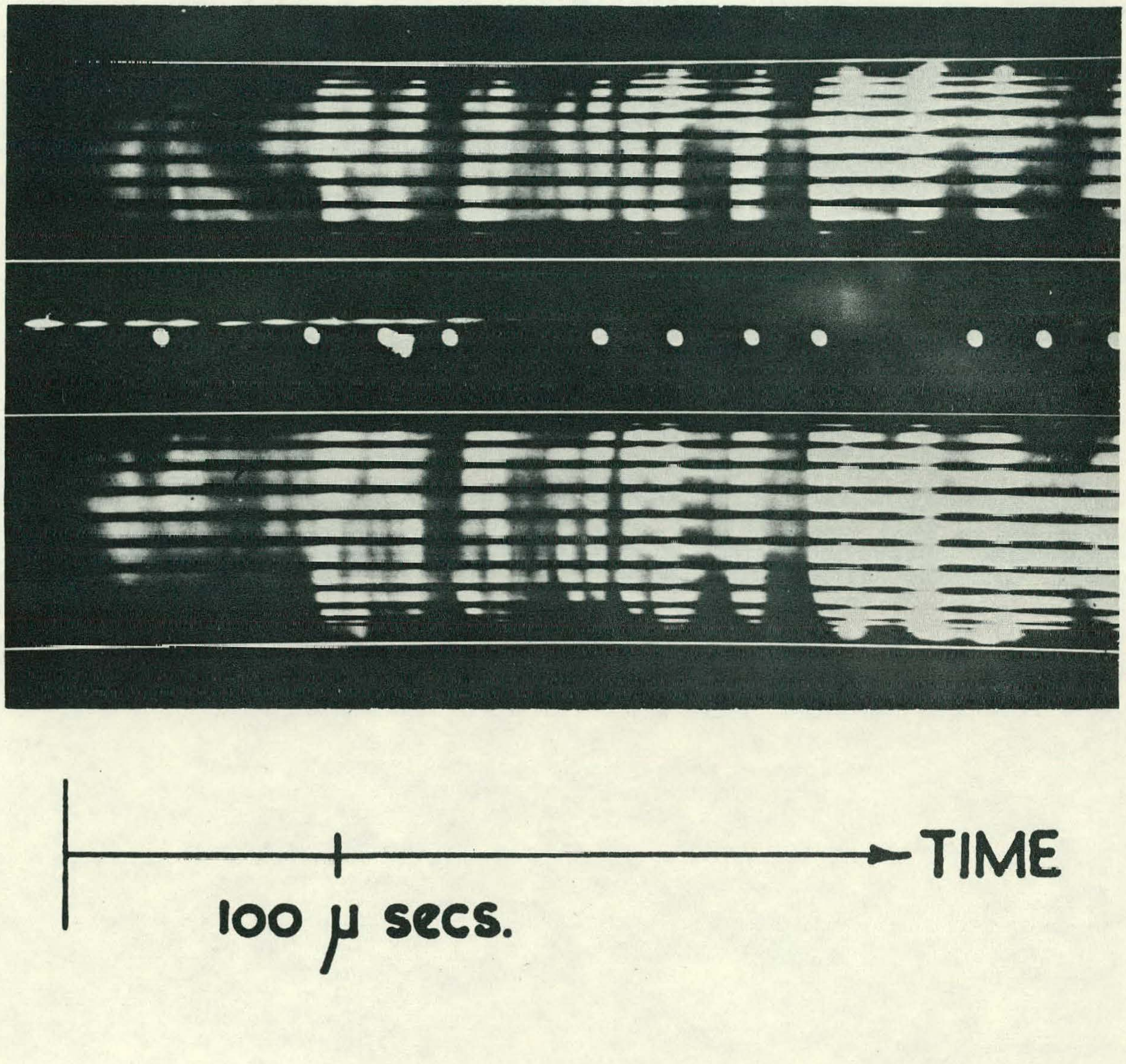


Figure 7 -- Streak photographs of a hydrogen discharge in Sceptre III, showing the so-called "bars". (The two streak pictures are different views of the same discharge.)

with radius according to fairly stringent conditions, (Rosenbluth, 1958, Suydam, 1958) the discharge will be unstable within the current sheet. The magnetic field configurations in all the above experiments did not satisfy these new conditions; hence these layer instabilities should occur and they present the simplest explanation of the small amplitude fluctuations which are still observed. More recent experimental results favour this assumption (Birdsall et al., 1959a, Tuck, 1959, Ware, 1959b and 1959c).

Nuclear Reactions

Deuterium fusion reactions were detected in 1952 in the linear discharges studied in the U.S.S.R (Kurchatov, 1956) and in 1955 in similar discharges in the U.S.A. (Anderson et al., 1957). In these experiments with no axial magnetic field, current pulses with amplitude about 10^5 amperes were generated in a few microseconds so that very rapid constriction to a small diameter occurred, followed by several oscillations about the equilibrium diameter. The nuclear reactions — detected by the neutrons emitted — occurred at the second and sometimes subsequent times of maximum constriction. Since the rapid collapse generates a shock wave, considerable heating of the positive ions was expected and there were hopes that the fusion reactions were thermonuclear, that is, induced by a high temperature. Subsequently the energy of neutrons emitted parallel to the current was found to be slightly in excess of the energy of those emitted in the opposite direction. This non-isotropy indicated a centre of mass velocity for the reacting deuterons of about 10^8 cm sec^{-1} parallel to the current. This result is most easily explained by assuming that a small proportion of the deuterons in the discharge are accelerated by some extraneous mechanism to high energies, and that some of these particles react with relatively stationary deuterons. The most likely cause of the particle acceleration is the $m = 0$ instability, (Anderson et al., 1957) which causes large inductive voltages at the points of maximum constriction.

The number of neutrons generated in these experiments was found to be greatly reduced by the addition of a small percentage of an impurity gas or a small axial magnetic field. In other experiments with very low external curcuit inductance, the nuclear reactions were much less sensitive to both impurity and axial magnetic field (Allen, K.W., et al, 1957 and Curran et al., 1958). However, neutron energy measurements again showed a moving centre of mass for the reacting particles.

In the experiments with stabilizing magnetic fields neutron emission was observed in both the toroidal experiments (Thonemann et al., 1958, Allen et al., 1958, Honsaker et al., 1958) and in the straight discharge tubes (Hagerman and Mather, 1958, Burkhardt and Lovberg, 1958a, Anderson, Baker, Colgate et al., 1958). Since these reactions were occurring in discharges which were much more stable than previously, and where the positive ion temperature from spectroscopic measurements were of the right order of magnitude there was again considerable hope that thermonuclear reactions had been achieved. Once again, however, the neutron emission was found to be anisotropic. The cause of the particle acceleration associated with these reactions has yet to be satisfactorily explained.

One of the other types of high current discharge studied in the past few years has been a cylindrical discharge in which a very rapidly rising axial magnetic field induces a high current in the θ-direction. (Post, 1958, Colgate and Wright, 1958, Elmore et al., 1958, Kolb, 1958 and Osovets et al, 1958). The field is often increased in magnitude at each end of the cylinder to provide magnetic mirrors to reduce end losses. The detailed mechanism of this type of discharge (see section 8) is somewhat different from the pinch discharge but an analogous constriction of the plasma occurs. At the time of the Second International Conference on the Peaceful Uses of Atomic Energy (Geneva, September, 1958) this was the only discharge for which a possible thermonuclear reaction was claimed (Elmore et al, 1958). Since then these discharges have been studied in considerable detail and although the heating of the deuterons may well be due to an instability (Kolb, Dobbie and Griem, 1959) there is now very strong evidence that the reactions are truly thermonuclear (Ribe, 1959). These discharges are also unique for another reason; they are the only discharges in which electrons have been heated to sufficiently high temperatures with sufficiently high densities for their bremsstrahlung radiation to have a detectable intensity at soft X-ray wavelengths (Griem et al, 1959 and Boyer et al., 1959). (Bremsstrahlung photons are those generated by free-free collisions between electrons and positive ions.) It is not surprising that the study of this type of discharge has been taken up by many other laboratories. (See the Proceedings of the Fourth International Conference on Ionization Phenomena in Gases, Uppsala, 1959.)

Energy Loss

A characteristic feature of the high current discharge is the very large rate at which energy is generated in the discharge due to Ohmic heating. Instantaneous powers as high as 900 megawatts have been quoted and since this power is being dissipated in an amount of gas weighing less than 10^{-3} grams, there is clearly no problem at present in delivering heat to the discharge. In many experiments the amount of energy introduced by the time peak current has been reached has been enough to heat all the particles in the gas to several tens of millions of degrees. Even in the presence of a stabilizing magnetic field and in the toroidal discharge experiments where there are no end losses, the amount of thermal energy retained in the gas is at least an order of magnitude less than this figure (Colgate, Ferguson and Furth, 1958, Connor et al., 1958, Allen and Liley, 1959). Thus, energy is being lost from the discharge at roughly the same rate as it is delivered. The search for the mechanism of this heat loss has been the main problem associated with these discharges since early in 1958.

The first workers to systematically study the possible heat loss processes were Colgate, Ferguson and Furth (1958). In their "Gamma" toroidal pinch experiment, during the first eighth of a cycle (about 5 microseconds) the main heat loss was found to be due to energetic electrons (energy $\sim$ 1.5 kev) striking the tube walls. After the first quarter of a cycle the amount of ultraviolet radiation due to impurity ions in the discharge became large and was the main energy loss mechanism throughout the rest of the current pulse. It was inferred that electrons with lower energies were being lost during the second eighth of a cycle, since the measuring technique could not detect electrons with energy less than 1 kev.

These results suggest that the fundamental problem is electron loss from the discharge. Since the electron bombardment will cause the quartz discharge tube used in this experiment to vaporize, the impurity radiation later in the pulse is probably a secondary effect. More recent results of other workers are in general agreement with these early conclusions. (See section 6.1.) It has been proposed by Ware (1959c) that the hydromagnetic instability provides the mechanism by which the electrons traverse the confining magnetic fields.

2. THE PLASMA EQUATIONS AND SOME PHYSICAL CONCEPTS.

2.1. A Set of Plasma Equations.

The mechanics of an ionized gas, even when it is completely ionized hydrogen, is a very complicated subject. This is because such a gas is a mixture of two types of particles (electrons and positive ions) and because it has many characteristic frequencies, such as the electron and positive cyclotron frequencies, the electron plasma frequency and the electron-electron, electron-ion and ion-ion collision frequencies. Last, and not least, is the fact that the equations of motion are non-linear. The only comprehensive and exact equations are the Boltzmann equations for the velocity distribution functions of the electrons and positive ions. If $f_i(\underline{w},\underline{r},t)dw_x\, dw_y\, dw_z\, dx\, dy\, dz$ is the number of ions in the volume element dx dy dz at time t whose velocity components w_x, w_y, w_z are in the ranges dw_x, dw_y, dw_z, the Boltzmann equation for f_i is

$$\frac{\partial f_i}{\partial t} = - \underline{w} \cdot \nabla f_i - \frac{e}{M}\left(\underline{E} + \frac{1}{c}\,\underline{w} \wedge \underline{B}\right) \cdot \nabla_w f_i + \left(\frac{\partial f_i}{\partial t}\right)_{coll} \qquad (5)$$

(See Chapman and Cowling, 1939.). The first term on the right hand side is the rate at which ions leave the volume element dx dy dz due to their velocity, the second term is the rate at which ions leave the corresponding volume element $dw_x\, dw_y\, dw_z$ in velocity space due to their acceleration $e(E + \underline{w} \wedge \underline{B}/c)/M$ and the third term is the change in f_i due to particle collisions. There is a corresponding equation for the electron velocity distribution function f_e.

In order to solve these equations and obtain transport equations relating such macroscopic parameters as pressure, mass velocity and acceleration, current density, etc., it is necessary to make several simplifying assumptions, and even then the solutions are obtained only by a series of successive approximations. The order of approximation achieved has generally been dictated by the mathematical difficulty of proceeding further. Since a particular set of approximations will not have general validity, the transport equations obtained will apply for only a limited range of discharge conditions.

Here a set of transport equations are presented which are obtained from the Boltzmann equations assuming that the distribution functions are only slightly perturbed from Maxwellian distributions. In the case of phenomena which occur at frequencies low compared with the particle collision frequencies the equations will be a good first order approximation. In particular they will probably be valid for most of the hydromagnetic instabilities observed so far, since instability frequencies have generally been less than the collision frequencies. The plasma equations have been taken from Chapman and Cowling (1939) and Liley (1959), but they are similar to those which can be obtained from Landshoff (1948) or Spitzer (1956). The equations are for completely ionized hydrogen or deuterium. The reader is referred to the above references for the changes which must be made when a gas with higher ionic charge is considered. For completeness the continuity equation and Maxwell's equations for the electromagnetic fields are given. Since we are dealing with low frequencies the displacement current has been ignored. Lastly, radiation has been neglected.

Plasma Momentum Balance.

$$\rho \frac{dV}{dt} = \frac{1}{c} \underline{j} \wedge \underline{B} - \nabla p \tag{6}$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \underline{V} . \nabla$$

Electric Conduction Equation.

$$\underline{E} + \frac{1}{c} \underline{V} \wedge \underline{B} - \frac{1}{nec} \underline{j} \wedge \underline{B} + \frac{1}{ne} \nabla p_e = \eta \underline{j} - \alpha \underline{q}_e \tag{7}$$

Electron Heat Flux Equation.

$$\frac{5 p_e k}{2m} \nabla T_e + \frac{e}{mc} \underline{q}_e \wedge \underline{B} = - \beta \underline{q}_e - \gamma \underline{j} \tag{8}$$

Thermal Energy Equations.

$$\frac{\partial T_e}{\partial t} + \underline{v}_e \cdot \nabla T_e + \frac{2}{3kn} \left\{ p_e \nabla \cdot \underline{v}_e + \nabla \cdot \underline{q}_e \right\} = \frac{2}{3kn} \underline{j} \cdot \left| \underline{E} + \frac{1}{c} \underline{V} \wedge \underline{B} \right| + \frac{1}{\tau_{ei}} \left| T_i - T_e \right| \tag{9}$$

$$\frac{\partial T_i}{\partial t} + \underline{V}_i \cdot \nabla T_i + \frac{2}{3kn}\left\{ p_i \nabla \cdot \underline{V}_i + \nabla \cdot \underline{q}_i \right\} = \frac{1}{\tau_{ei}} \left| T_e - T_i \right| \quad (10)$$

Continuity Equation

$$\frac{\partial n}{\partial t} = \nabla \cdot (n \underline{V}_i) = \nabla \cdot (n v_e) \quad (11)$$

Maxwell's Equations

$$\nabla \wedge \underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t} \quad (12)$$

$$\nabla \wedge \underline{B} = 4\pi \underline{j}/c \quad (13)$$

$$\nabla \cdot \underline{B} = 0 \quad (14)$$

where Gaussian units have been used and where

ρ = $n(M + m)$ = plasma density
M = ion mass
m = electron mass
n = number of electrons per unit volume $\triangleq$ number of positive ions per unit volume.
$\underline{V}$ = mass velocity = $(M \underline{V}_i + m \underline{v}_e)/(M + m)$
$\underline{V}_i$ = mean ion velocity
$\underline{v}_e$ = mean electron velocity
$\underline{j}$ = current density
p = $p_i + p_e$ = total plasma pressure
p_i = positive ion pressure
p_e = electron pressure
$\underline{E}$ = electric field
$\underline{B}$ = magnetic field
c = velocity of light
e = proton charge

η = the resistivity perpendicular to the magnetic field when no Hall current flows $= 1.43 \times 10^{-9} \ \ln \Lambda / T_e^{3/2}$ esu
($= 1.29 \times 10^{4} \ \ln \Lambda / T_e^{3/2}$ ohm cm)

$$\Lambda = \frac{3}{2e^3} \left(\frac{k^3 T_e^{\,3}}{\pi n} \right)^{1/2}$$

$\alpha = 3 \eta e / 5kT_e$

$\underline{q}_e$ = heat flux due to electrons, referred to the mean electron velocity

$\underline{q}_i$ = heat flux due to positive ions, referred to the mean ion velocity

k = Boltzmann's constant

T_e = electron temperature

T_i = ion temperature

$\beta = 1.9 \eta n \, e^2/m$

$\gamma = 1.5 \eta e \, n \, kT_e/m$

$$\tau_{ei} = \frac{3m \, Mk^{3/2}}{8(2\pi)^{1/2} \, ne^4 \, \ln \Lambda} \left(\frac{T_e}{m} + \frac{T_i}{M} \right)^{3/2}$$

Equation (6) is the well known equation of motion for the plasma and shows that the mass acceleration is given by the difference in the two forces on the right hand side. The electric conduction equation (sometimes call the generalized Ohm's law) is essentially the equation of motion for the electrons with the acceleration term neglected because of the small electron mass. The first three terms constitute the effective electric field acting on the moving electrons; the second term is the contribution due to the mean plasma velocity and the third term (the Hall emf) allows for the difference between the mean electron velocity and the plasma velocity. The fourth term is the emf due to the electron pressure gradient. The terms on the right hand side are proportional to the mean force acting on the electrons due to collisions with the positive ions, one term being proportional to the electron current and one to the electron heat flux.

In equation 9 the first two terms constitute the total rate of change with time of the temperature of a particular group of electrons. The first term in the brackets on the left hand side gives the change of T_e due to compression or expansion of the electron gas, and the second term in the brackets is the change due to the divergence of the electron heat flux. The two terms on the right hand side represent respectively the heating of the electrons due to the Ohmic heating of the current and due to energy transfer from the positive ions by collisions. Equation (10) is the corresponding equation for T_i, except that the Ohmic heating of the ions is small and has been neglected.

In some cases, such as when a large mass velocity gradient exists, the isotropic pressure p must be replaced by a pressure tensor so as to include the effect of viscosity. In other cases, such as in discharge models which assume a plasma edge, it will be necessary to allow for a net volume or surface electric charge in the plasma. The appropriate terms in the plasma equations are given by Liley (loc. cit.). The fourth Maxwell equation (for $\nabla \cdot \underline{E}$) will also be required.

It should be noted that the electron heat-flux equation does not include the positive ion contribution to heat conduction. In the absence of a magnetic field the contribution to the thermal conductivity due to protons in ionized hydrogen is about 1/15 of the electron component (Chapman, 1954). Since, however, in the presence of a magnetic field the electron heat conduction is reduced by the factor $1/(1 + \omega_e^2 \tau_e^2)$ where ω_e is the electron cyclotron frequency and τ_e the mean electron collision frequency, quite modest magnetic fields will reduce the electron component below that for the ions. The positive ion heat flux equation is given by Braginsky (1957a).

2.2. <u>The Magnetohydrodynamic Approximations.</u>

Although many simplifying assumptions have already been made in deriving the above equations, their solutions for a particular problem would still be a very formidable mathematical task. Progress has been made possible only by making further assumptions. Most notable of these assumptions are:

(a) the neglect of the Hall emf. $(\underline{j} \wedge B/ne)$ and the electron pressure gradient emf. $(\nabla p_e/ne)$ from the electric conduction equation, and

(b) the neglect of heat conduction from the thermal energy equation.

With these further approximations the set of equations become independent of the fine structure of the plasma and they apply equally to a continuous conducting fluid; they are known as the magnetohydrodynamic equations.

A further approximation which is frequently made is to neglect the two terms in the electric conduction equation which are proportional to the collision frequency, namely $\eta \underline{j}$ and $\alpha\, \underline{q}_e$. Together with the approximations (a) above, equation (7) reduces to the simple form associated with a perfect metallic conductor, namely

$$\underline{E} + \frac{1}{c}\underline{V} \wedge \underline{B} = 0 \tag{15}$$

2.3. The Collisionless Plasma Approximation.

For effects which occur rapidly compared with the particle collision frequency, the assumption of approximately Maxwellian velocity distributions will not be valid. This will often be the case as higher temperatures are achieved since the collision frequency is proportional to $T^{-3/2}$. When the motions of the plasma are rapid compared with the collision frequency there will be little coupling between the particle velocities parallel to the magnetic field and those perpendicular to the field. The distribution function will cease to be isotropic and the component of pressure parallel to the field $p_{\parallel}$ may be appreciably different from the perpendicular components $p_{\perp}$. The problem of solving Boltzmann's equations for these conditions has been tackled by several workers (Chew, Goldberger and Low, 1956, Watson, 1956, Chandrasekhar, Kaufman and Watson, 1957, Rosenbluth and Rostoker, 1958, Kruskal and Oberman, 1958, and Sagdeyev, et al., 1958). Although the procedures differ in other respects, they have all assumed that the magnetic field is large and obtained solutions for the transport equations by using a series expansion in terms of the parameter ρ_1/L or ω/ω_i, where ρ_1 is the particle Larmor radius, L is the characteristic length over which quantities change in the discharge, $\omega/2\pi$ is the frequency with which quantities change and $\omega_i/2\pi$ is the ion cyclotron frequency. The transport equations obtained in this way have been used mainly in the theory of discharge instability. At the degree of approximation to which the solutions have been taken the results obtained have not been greatly different from those obtained with the magnetohydrodynamic equations. One reason for the similarity in the results is that to first order in the expansion parameter the electric conduction equation is identical with the perfect conductivity approximation (equation (15)).

2.4. Simple Physical Concepts.

In order to assist the theoretical physicist in choosing the most suitable approximations, and to assist the experimenter in understanding his results it has been necessary to conceive simple models to replace the complex plasma. The perfectly conducting compressible cylinder is an example. In order to visualize the physical processes involved, several physical concepts have been of great assistance. Most notable among these are the concepts of "magnetic pressure", and the "freezing in" of magnetic lines of force; these are summarized below. For fuller descriptions of these and other plasma processes the reader is referred to the works of Spitzer (1956) and Cowling (1957).

Magnetic Pressure.

(a) Magnetic fields with no curvature.

When the magnetic lines of force are straight it follows directly from equation (13) that the force $\underline{j} \wedge \underline{B}$ is equal to $-\nabla(B^2/8\pi)$. In this case therefore the quantity $B^2/8\pi$ acts as a scalar pressure, and for equilibrium, equation (5) can be integrated to give

$$p + \frac{B^2}{8\pi} = \text{constant} \tag{16}$$

(b) Magnetic fields with curvature.

In this case the concept of a simple scalar pressure breaks down. Taking as an example the pinch effect where there is cylindrical symmetry and the only component of $\underline{j}$ is j_z, the force is

$$-j_z B_\theta = -\frac{\partial}{\partial r}\left(\frac{B_\theta^{\,2}}{8\pi}\right) - \frac{B_\theta^{\,2}}{4\pi r} \tag{17}$$

The extra term arises because magnetic tubes of force behave as if they have a tension along their length of $B^2/8\pi$ per unit area in addition to the pressure in the lateral direction. In the special case where a pinch discharge has a current sheath thin compared with the radius, equation (17) can be integrated over the sheath thickness δ giving

$$\int_{r}^{r+\delta} - j_z B_\Theta \, dr \simeq - B_{\Theta e}^{\;2} \Big/ 8\pi \tag{18}$$

where $B_{\Theta e}$ is the field just outside the sheath and where the field inside the sheath is assumed zero. In this special case the magnetic field can once again be regarded as a pressure $B_{\Theta e}^{\;2} \Big/ 8\pi$ pushing inwards on the discharge.

"Freezing in" of Magnetic Lines of Force.

When the perfect conductivity approximation is made, a simple relationship can be obtained by taking the curl of equation (15), and substituting for $\nabla \wedge \underline{E}$ and $\nabla \bullet \underline{V}$ from equations (12) and (11), (Cowling, 1957, page 7) namely

$$\frac{d}{dt} \left(\frac{B}{n}\right) = \left(\frac{B}{n}\right) \bullet \nabla \underline{V} \tag{19}$$

Thus for motion of the plasma for which the component of $\nabla \underline{V}$ parallel to $\underline{B}$ is zero, $\underline{B}/n$ is a constant of the mass motion and the lines of force can be regarded as frozen to the plasma and moving with the mass velocity.

If, however, the more exact conduction equation (7) is used, a substantially different result is obtained. Assuming that the collision frequency is low enough to make the two terms on the right hand side of (7) negligible the relation corresponding to equation (19) is (Schluter, 1956)

$$\left(\frac{d}{dt}\right)_e \left(\frac{B}{n}\right) = \frac{B}{n} \bullet \nabla \underline{v}_e - \frac{1}{n^3 e} \nabla n \wedge \nabla p_e \tag{20}$$

where $$\left(\frac{d}{dt}\right)_e = \frac{\partial}{\partial r} + \underline{v}_e \cdot \nabla$$

In this case, if $\nabla n \wedge \nabla p_e$ is small, for motion such that the component of $\nabla \underline{v}_e$ parallel to $\underline{B}$ is zero the magnetic field is frozen to the electrons and the lines of force move with the mean electron velocity. If $\nabla n \wedge \nabla p_e$ is not zero there is a generation of magnetic field within the plasma.

When the finite conductivity of the plasma is taken into account the lines of force are found to diffuse out of a region in which they are frozen. The time constant for the exponential decay of the field from a region of dimension L is $4\pi L^2 c^2/\eta$ (Spitzer, 1956, page 38).

3. DISCHARGE INITIATION AND CONTRACTION.

3.1. Ionization.

The initial formation of high current discharges is particularly complex due to the many processes taking place simultaneously with different time constants. The main processes can be listed as follows:

1. Ionization.
2. Radiation.
3. Heating of (a) the electrons and (b) the ions.
4. Increase in current.
5. Penetration of the Magnetic Field (or current) into the discharge.
6. Contraction of Discharge (and formation of a shock wave).

Because of the number and complexity of these processes and the coupling between them it is not surprising that a comprehensive theory has not been undertaken. In certain cases it has been possible to neglect several of the processes, and even then a substantial problem for an electronic computor remains.

With a few exceptions very little work has been done on the ionization processes. One notable exception has been the computation of the ionization and heating of a very low electric field (0.1 volt cm^{-1}) and low pressure (10^{-4}mm Hg) discharge in helium (Berger et al., 1958). The electric field was assumed constant across the discharge and with respect to time, and contraction of the discharge was assumed inhibited by a large axial magnetic field, so that processes (5) and (6) were eliminated. The currents, temperatures and spectral line intensities were obtained throughout the first and second stages of ionization. Kogan (1958) has developed a method of estimating the electron temperature during the ionization and contraction stage of the linear discharge with high rate of rise of current; the electron temperature during this phase is kept down by the ionization process and the values found were in the range 2.5 to 4 ev depending on the initial hydrogen pressure

(Borzunov, Kogan and Orlinsky, 1958). Corresponding calculations for the slow toroidal pinch discharge have been made by McWhirter (1959) and measurements by McWhirter et al., (1959).

Interest has been shown recently in the ionization process in connection with chosing the optimum electric field for most rapid ionization. That the electric fields can be too high as well as too low for rapid ionization is shown clearly in the breakdown curves obtained by Etievant et al., (1959) for a toroidal discharge. The growth of ionization at high electric fields has been studied theoretically by Gerjuoy and Stuart, (1960).

3.2 Discharge Contraction.

(a) High Rate of Rise of Current, No Axial Magnetic Field.

High current discharges have been studied with rates of rise of current (dI/dt) varying over the wide range 10^7 to 10^{12} amperes sec^{-1}. It is not surprising that different initial behavior has been observed. In the case of high rates of rise of current, where the high applied electric field causes a high degree of ionization in a time short compared with the discharge contraction time, and where the rapid rise of current limits the current carrying region to a thin annular sheath on the outside of the discharge (the skin effect), a fairly well defined type of discharge contraction occurs. The high current density in the sheath rapidly ionizes most of the gas in this region and the $\underline{j} \wedge \underline{B}$ force gives this plasma an inward acceleration. Since the force operates only on the outer layer, a high pressure is built up on the inside of the contracting sheath.

The sheath will accelerate rapidly to a terminal velocity close to the value cE_z/B_θ where E_z and B_θ are the electric and magnetic fields at the edge of the discharge. This is the velocity at which the back emf IdL/dt balances the electric field at the edge of the discharge. (L is the inductance per unit length of the discharge.) If this velocity is comparable with or exceeds the speed of sound for the gas in the central region of the discharge the high pressure will propogate inwards as a shock wave, the shock front having a velocity somewhat higher than the current carrying sheath. (See figure 8.) The outer current carrying edge can be regarded as the contact front between the magnetic pressure and the gas. The shock front is "reflected" from the axis of the discharge and then travels radially outwards.

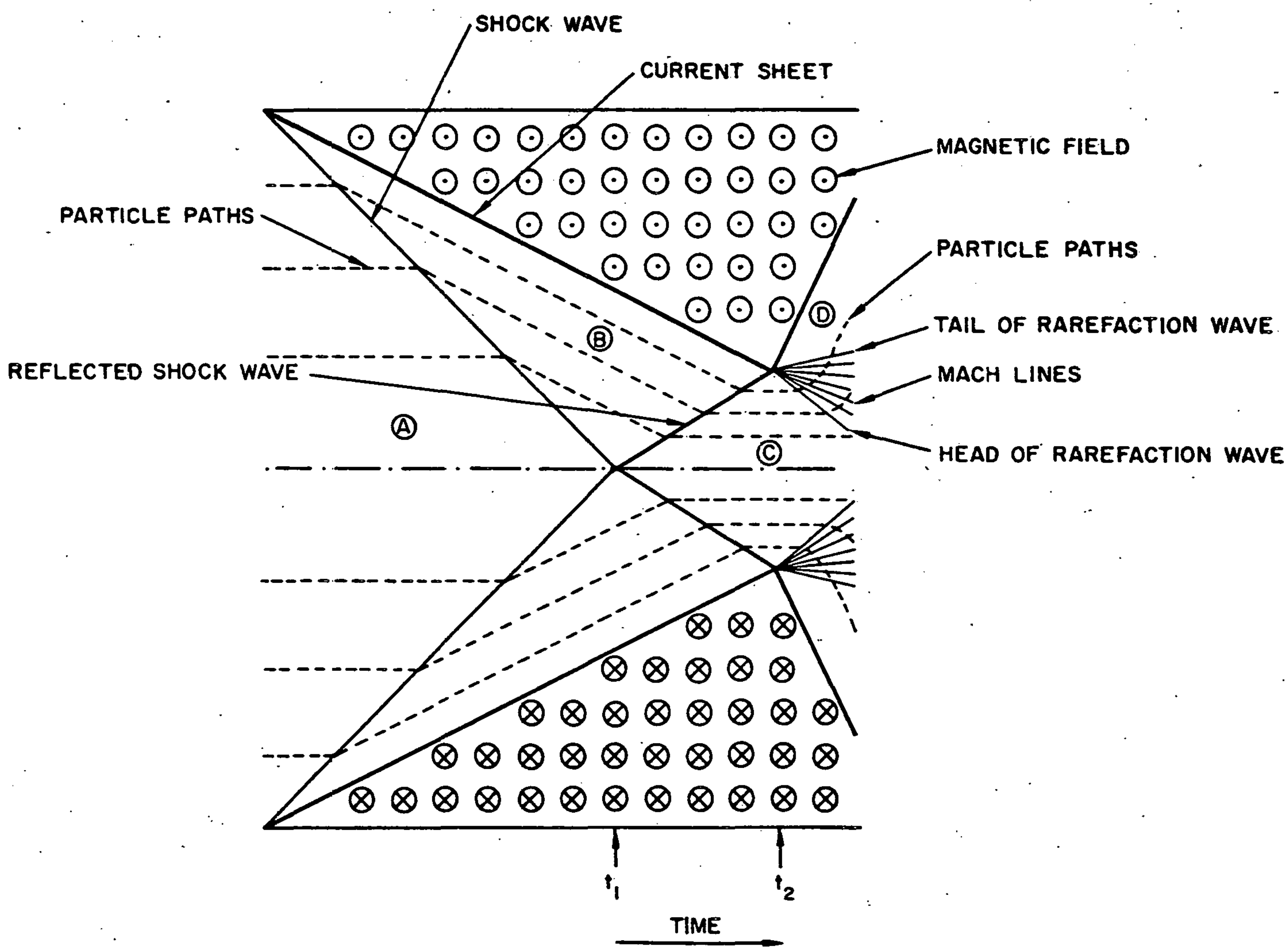


Figure 8 -- Diagrammatic representation of the pinch discharge with high rate of rise of current, showing the contracting discharge edge and the shock wave preceding it. Acceleration has been ignored. Taken from J. E. Allen (1957).

When it meets the incoming edge of the discharge it causes the edge to change its radial velocity from a negative to a positive value. The discharge edge moves outwards until the $\underline{j} \wedge \underline{B}$ force dissipates the inertia of the shock wave. The discharge then contracts again and a series of these "bounces" occur. As many as six bounces have been observed (Siegbahn and Ohlin, 1958), before instability caused the discharge to break up. In spectroscopic studies Lukyanov and Sinitsyn (1956, 1958 and 1959) have measured densities up to thirty times the initial gas density and ion energies corresponding to 10^6 to 2×10^6 degrees K.

The shock wave nature of the contraction can be seen in streak photographs of such discharges (for example, Cousins and Ware, 1951, Allen and Reynolds, 1957) and has been analyzed by J.E. Allen (1957), from whom figure 8 is taken, and by Jukes (1958) who allowed for the cylindrical nature of the shock wave. If the contact and shock fronts are assumed to have roughly equal velocities, a simple "snow plough" model can be adopted and the equation of motion for the sheath can be solved simultaneously with the current equation (Rosenbluth, et al., 1954). If the applied electric field (E_o) is assumed constant throughout the contraction, although the equations have to be integrated numberically, a simple formula is obtained for the time to maximum constriction (t_p) namely

$$t_p = b(4\pi \rho_o/c^2 E_o^{\,2})^{1/4} \tag{21}$$

where b is the tube radius and ρ_o the initial gas density. The results show reasonable agreement with experiment (Anderson et al.,1957, and Sundstrom and Svennerstedt, 1959). A more elaborate model analyzed by Braginsky and Migdal (1958) who allowed for finite electrical conductivity and the compression and heating of the gas, showed similar reasonable agreement with experiment (Andrianov et al., 1958). A further theoretical treatment is given by Kulikovskii (1957), and the trapping of the neutral gas in the collapsing discharge has been studied by Phillips (1959).

In the case where the contraction of the discharge sheath is slow compared with the speed of sound for the gas in the centre of the tube, sound waves will equalize the pressure throughout the volume within the discharge sheath and the gas will compress approximately adiabatically. No shock wave will occur but there will still be bouncing due to the inertia of the contracting discharge. An analysis appropriate to this type of contraction was done by Leontovich and Osovets (1956). Since the time taken for the discharge to contract is proportion to the fourth root of the discharge mass and the square root of the current it is not very sensitive to the type of model taken for the discharge.

The rapid contractions and expansions of the discharge cause large changes in the discharge inductance and because the external electric circuit has appreciable impedance the emf I dL/dt causes large fluctuations in the voltage across the discharge tube (Artsimovich, Andrianov, Bazilevskaia et al., 1956, Anderson et al., 1957). These fluctuations enable the times of maximum constriction to be accurately determined.

Several experiments indicate that some neutral gas is left behind by the contracting discharge or is being evaporated from the tube walls. Magnetic probe measurements have shown that although the current density is largest in the luminous constricted discharge channel there is some current density throughout the whole tube. The current density in the outer regions is often negative, (Artsimovich et al., loc. cit. and Burkhardt and Lovberg, 1958a) and in some cases a further positive current sheath appears on the outside of the negative current region and moves towards the centre of the tube (Artsimovich et al., loc. cit., Komelkov et al., 1958, Bodin, Peacock and Reynolds, 1959). The existence of this second current sheath has been observed photographically with discharges in impure deuterium (Gabriel, 1959, see figure 1)and in gases other than deuterium (Borzunov, Orlinsky and Osovets, 1959 and Bodin, Peacock and Reynolds, loc.cit.). It has also been observed by Latham (1959) who found that it no longer occurred when the quartz tube was shielded on the inside by a series of overlapping copper cylinders. This last result is strong evidence that the gas is coming from the tube wall, since copper will undergo a much smaller temperature rise than quartz for a given heat pulse (Craston et al., 1958). When a pyrex glass discharge tube is used the initial discharge stays close to the tube wall for a longer time, (Anderson et al., 1958) and at very large dI/dt a similar effect may be occurring with quartz tubes since the contraction times are longer than predicted by theory (Curran et al., 1958).

The explanation for a negative current density occurring while the applied electric field is still positive may be a positive radial pressure gradient caused by gas evolved from the tube wall. When the applied electric field reverses and the total discharge current is decaying, the negative currents in the outer regions of the tube are observed to become much larger while the positive current in the core decreases only slowly and persists well into the second half cycle (Burkhardt and Lovberg, 1958a). This latter behavior is the same skin effect phenomenon which occurs in any thick electrical conductor. It has been analyzed by Haines (1959).

In several experiments (Komelkov, 1959, Bodin, Peacock and Reynolds, 1959, Gabriel, 1959) multiple filaments have been observed in the contracting discharge sheath.

(b) <u>High Rate of Rise of Current with an Axial Magnetic Field</u>.

The presence of an axial magnetic field has two main effects on the contraction of the discharge. Firstly, since the current sheath is a good conductor its radial motion will cause currents in the θ-direction on the inner edge of the sheath so as to maintain a constant axial magnetic flux within the cylindrical sheath. The magnetic field increases as the discharge constricts so that $\pi a^2 B_z$ is a constant, where a is the radius of the sheath. This increase in B_z within the discharge causes a back emf which will either make the current in the coil producing the initial B_z fall close to zero, or induce a negative current in the metal shield if such a shield is present which is continuous in the θ-direction. In either case the magnetic field outside the discharge falls to a low value. Therefore, as far as the axial magnetic field is concerned there is an excess pressure within the discharge sheath of approximately $B_z^2/8\pi$, and the discharge will not constrict and achieve this state until the pinch effect pressure $B_{\theta e}^2/8\pi$ is equal to $B_z^2/8\pi$.

Secondly, if the gas in the central part of the discharge is sufficiently ionized to be a reasonably good electrical conductor, the presence of the magnetic field increases the speed of sound in this region to $\sqrt{C_s^2 + \frac{B_z^2}{8\pi\rho}}$ where C_s is the speed of sound in the absence of B_z. In addition the rate of contraction is much slower because the pinch force $j_z B_\theta$ is still counteracted to a large extent by the outward force $j_\theta B_z$. As a result in most experiments the contraction velocity has been small compared with the sound speed and the contraction has been of the adiabatic type referred to in (a) above with no shock wave.

Detailed studies using magnetic probes have been made of this type of discharge contraction with straight discharge tubes (Burkhardt, Lovberg and Phillips, 1958, Burkhardt and Lovberg, 1958b, Colgate, Ferguson and Furth, 1958, and Brower et al., 1958) and in toroidal tubes (Conner et al., 1958). In most cases with high rate of rise of current a skin current is formed, as in the absence of B_z, and there is sufficient inertia in the contraction for some bouncing of the discharge to occur after maximum contraction (Kerst, 1958, Sundstrom and Svennerstedt, 1959). The reduced contraction means less compressional heating of the discharge and Ohmic heating becomes an important contribution. Since the currents are confined to a sheath the Ohmic heating causes the plasma pressure to build up mainly in this annular region.

In experiments with high magnetic fields in straight tubes (Kerst, 1958), in spite of the high rate of rise of current, the discharge current was observed to be approximately uniformly distributed across the tube even as early as 0.3 microseconds after initiation. This may have been due to distortion of the axial magnetic field near the electrodes since copper electrodes were used and the axial magnetic field was applied as a short pulse.

(c) Low Rate of Rise of Current, No Axial Magnetic Field.

When discharges are produced with low electric fields so that the rate of increase of current is slow compared with the skin depth penetration time, the initiation of the discharge is markedly different to that described above. Instead of the discharge forming in a sheath near the tube wall the high speed photographs show the discharge growing from the start as a constricted channel along the tube axis (Hemmings et al., 1957). This channel grows in brightness with time and often expansion rather than contraction is observed. (Allibone et al., 1958).

A possible explanation is as follows. Since the discharge remains a poor conductor for a considerable time during the slow ionization of the gas, the electric field will penetrate to the centre of the tube, and initially, ionization will occur fairly uniformly over the tube cross section. When the self magnetic field becomes large enough to affect the electron motion it will inhibit further ionization in the outer regions of the discharge since this is where the magnetic field is largest. In the centre where the magnetic field is small the ionization can proceed unimpeded. Further, the growth of the central region will be enhanced by the pinch effect drawing the changed particles from the outer regions into the central core. The expansion of the channel may be due to Ohmic heating increasing the plasma pressure faster than the growth of the magnetic pressure.

(d) Low Rate of Rise of Current with Axial Magnetic Field.

At fairly low currents (10^3 - 10^4 amperes in tubes 10 cm - 30 cm diameter) in the presence of an axial magnetic field the discharge forms over progressively more of the tube as the applied magnetic field is increases. (Hemmings et al., 1957, Allibone et al., 1958 and Dolgov-Saveliev, 1958). With high fields the discharge fills most of the tube throughout the pulse. A notable feature of these discharges is the appearance of multiple channels on the streak photographs at the very beginning of the luminosity and continuing throughout the pulse in some cases(Hemmings et al., loc cit.).

At higher currents ($\sim 10^5$ amperes)in the toroidal tubes Zeta and Sceptre III the discharge forms over the whole tube but as the current builds up the streak photographs show a fairly sharp edged channel constricting to about half the tube diameter (Thonemann et al., 1958 and Allen et al., 1958). This constriction occurs before B_θ becomes equal to the applied magnetic field B_{zo}. The intermittent light observed outside the main channel (Ware, 1959b) and magnetic probe measurements (Allen and Liley, 1959) indicate a considerable amount of plasma left behind by the constricting discharge. During or shortly after constriction the channel becomes invisible to normal high speed photography but its continued existence has been demonstrated using more sensitive detection in the ultraviolet spectrum (Allibone et al., 1958), as shown in figure 6. By measuring the current density and axial field at the centre of the discharge with magnetic probes, Lees and Rusbridge (1959) have inferred that the constriction ceases when the discharge is about half the tube diameter, and that when the parameter $2I/bB_{zo}$ exceeds about 1.4, negative B_z is generated in the outer parts of the discharge.

High currents with low dI/dt in straight tubes, with very high pulsed magnetic fields (Golovin et al., 1958), showed discharges forming at the centre of the tube and then expanding as the current increased. The width of the channel at peak current was smaller the higher the magnetic field. Here again large copper electrodes were used and the magnetic field was pulsed, so that the constriction may have been caused by the non-uniformity of the magnetic field near the electrodes.

4.1. Linear Discharges.

Strictly speaking, complete equilibrium for a discharge, with all the parameters remaining constant in time, can never be achieved. There must at least be some heat generation due to particle collisions. However, a steady state can be considered in which electric fields (or fusion reactions) provide the heat energy and thermal conduction or radiation dissipate the heat. For discharges configurations which involve magnetic fields frozen into the plasma, the discharge is not even in a steady state. However, a quasi-steady state can be assumed for times short compared with the time taken by the magnetic field to diffuse out of the plasma.

The prime condition for equilibrium is that the gas has no acceleration so that from equation (6)

$$\frac{1}{c}\,\underline{j}\wedge\underline{B} = \nabla nk(T_e + T_p) \quad . \tag{22}$$

Even for a straight discharge with assumed cylindrical symmetry this equation together with Maxwell's equation (13) for $\underline{j}$ has four too many unknowns. Ideally the heat flux and energy transfer equations must be solved simultaneously. Such detailed calculations have not been carried out. Not only are the equations complex but the temperature predicted by such solutions would greatly exceed the observed temperatures and would not be very relevant to present experimental results. The practice has been to make simplifying assumptions about the particle temperatures.

In the case where the self field B_θ is the only component of $\underline{B}$ present, equation (22) has a simple integral giving the mean temperature (or mean pressure) in the discharge, (see section 1, equation (2)) but this solution gives no information about the discharge configuration. In the original treatment by Bennett (1934) a plasma column was considered in which the electrons were assumed to have an axial drift velocity constant with respect to radius. With constant temperatures throughout the discharge this leads to the simple density distribution

$$n = n_o/(1 + A\,r^2)^2 \tag{23}$$

where A is a constant. Essentially, similar distributions were obtained by Tonks (1939), Thonemann and Cowhig (1951) and Blackman (1951). The effect of bremsstrahlung radiation on the equilibrium conditions for a highly constricted discharge was considered by Braginsky (1957b) and Pease (1957), who showed that the current in such a discharge in the absence of B_z could not exceed about 2×10^6 amperes.

When an axial magnetic field is also present and is assumed frozen into the plasma there is an infinite number of quasi-steady state solutions depending on the relative magnitudes of B_Θ and B_z and their gradients. The simplest configuration occurs when the discharge is initiated rapidly in the presence of a uniform B_z. In this case B_z is approximately constant within the discharge channel and falls rapidly at the edge to a low value outside. B_Θ is zero inside and rises rapidly to a large value outside where it falls off with radius as $1/r$. The width of the transition layer depends on the conductivity of the plasma and the speed with which the discharge is generated. When this sheath thickness is small compared with the radius, equation (22) can be integrated over the sheath in a similar manner to equation (16), to give

$$B_{\Theta e}^2/8\pi + B_{ze}^2/8\pi = B_{zi}^2/8\pi + p \tag{24}$$

where the subscripts "e" and "i" refer to fields on the external and internal surfaces of the sheath, and where both $B_{\Theta i}$ and the external plasma pressure are assumed zero.

In the case with applied B_z where a true steady state has been reached, (i.e. $\partial \underline{B}/\partial t$ truly zero) current will still flow in the Θ-direction due to the anisotropic conductivity of an ionized gas in the presence of a magnetic field. This was first pointed out by Braginsky and Shafranov, (see Bezbatchenko et al., 1956). The equilibrium solution for the case where the radial electron temperature gradient is assumed zero, has been obtained by Bickerton (1958), and a similar result has been given by Longmire (1958).

4.2. Toroidal Discharges.

The change to toroidal geometry introduces two main effects on discharge equilibrium. Firstly, since the B_Θ magnetic field is larger on the concave side of the curved discharge than on the convex side, this component of field exerts a net outward force on the discharge tending to increase the torus diameter. A further component of magnetic field is required to give equilibrium. When trapped fields are not considered this must be a field vertical to the plane of the torus and of the required sign to exert an inward force on the toroidal current (Shafranov, 1957 and 1959 and Biermann et al., 1957). Experimentally if the discharge tube possesses a metal sheath the movement of the discharge towards the outer wall induces eddy currents which generate such a transverse field. The equilibrium position for the centre of the discharge is displaced outwards from the centre of the tubing an amount depending on the ratio of the torus and tube diameters. When a toroidal magnetic field parallel to the discharge is trapped in the plasma, this will reduce the tendency for the discharge to expand but in general some externally applied field (e.g. due to eddy currents) is needed for equilibrium.

The other effect is connected with the toroidal magnetic field and the fact that its magnitude must be inversly proportional to R, where R is the radius of curvature of the lines of force. When this field component is not zero, equation (22) requires that $\nabla \cdot \underline{j}_{\perp} \neq 0$, where $\underline{j}_{\perp}$ is the component of $\underline{j}$ perpendicular to $\underline{B}$ (Spitzer, 1958). $\nabla \cdot \underline{j}_{\perp}$ will have a different sign above and below the equatorial plane of the torus. Since equilibrium requires that $\nabla \cdot \underline{j}$ be zero, this will be possible only if currents can flow along the lines of force between regions of opposite sign for $\nabla \cdot \underline{j}_{\perp}$. When there is a B_{θ} magnetic field present, as with the pinch discharge, the lines of force do pass from top to bottom of the torus and equilibrium is possible. When only j_{θ} currents, and hence only B_z, are present, the lines of magnetic force must be given a rotational transform by some other means. The conditions for this equilibrium , and in particular the limitation on the plasma pressure, have been considered by Spitzer (1958) and Kruskal and Kulsrud (1958).

Some of the equilibrium configurations which are possible in a torus when trapped fields are considered have been derived by Laing, Roberts and Whipple (1959), who have looked for configurations close to those observed in the Zeta experiment (Butt et al., 1958).

5. MAGNETOHYDRODYNAMIC STABILITY.

5.1. Theory.

Many theoretical studies have been made on the stability of magnetically confined plasmas and in particular of the pinch discharge. In most cases the starting point has been the transport equations obtained by making the magnetohydrodynamic approximations referred to in section 2. Other treatments based on approximate solutions of the Boltzmann equation ignoring collisions (see section 2.3) have been made by Chandrasekhar, Kaufmann and Watson, 1958, Rosenbluth and Longmire, 1957, Kruskal and Oberman, 1958 and Sagdeyev et al., 1958, but the results are in close agreement with magnetohydrodynamic solutions. One reason for this is the fact that to the degree of approximation to which the solutions have been taken the electric conduction equation is identical with the perfect conductivity approximation (equation 15). Also it has been shown by Rosenbluth and Rostoker (1958) that to the same degree of approximation, if the magnitude of $\underline{B}$ does not vary along a line of force, as is the case in pinch discharge, the plasma momentum balance equation is the same as in the magnetohydrodynamic approximation. This is valid for marginally stable configurations, which are the ones studied in deciding stability criteria. The few treatments which have used a higher order approximation for the electric conduction equation are considered in section 5.3 below.

The conditions for stability are obtained by studying the behavior of small perturbations to the discharge. Terms which are of second order or higher in the small increments to the discharge parameters are neglected so that a set of linear equations are obtained. Any arbitrary displacement can be regarded as a sum of Fourier components with time variation of the form $\exp(i\omega t)$, so that the increments in the various discharge parameters can be represented by expression of the type given in equation (4). Because of the linearity of the equations each Fourier component (or normal mode) can be treated independently. One procedure is therefore to substitute expressions (4) into the plasma equations. Since all the terms have the same t, θ and z variation the exponential terms cancel, and a series of linear simultaneous differential equations remain which must be solved for the radial dependence of the amplitudes $A'(r)$. Finally by applying the boundary condition a dispersion relation results giving ω as a function of k.

In general ω can be real, imaginary or complex, but when the magneto-hydrodynamic approximations are made, including the perfect conductivity approximation, a property of this particular set of equations is that ω^2 is always real. In the case when $\omega^2 > 0$ the solutions represent oscillations or waves with constant amplitude and the discharge is stable. (There are no dissipative effects in this set of equations so the oscillations are not damped.) When $\omega^2 < 0$ then ω can have a negative imaginary value and the perturbations will grow with time. The conditions for stability are therefore obtained by studying the dispersion relations in the neighborhood of the marginal stability where $\omega = 0$.

An alternative treatment which avoids some of the complexity of the normal mode analysis and which yields stability conditions but no dispersion relation, is to evaluate the change in the electromagnetic and thermal energy of the system which results from an arbitrary displacement of the plasma. If the change in this "potential energy" is positive for all displacements of the plasma, the configuration will be stable. This energy principle was first applied to a plasma by Lundquist (1951) and has been developed by Bernstein, Frieman et al. (1958) and Hain, Lust and Schluter (1957). The potential energy (δW) is obtained in the form of a volume integral which is a function of the discharge configuration and the displacement ($\underline{\xi}$) undergone by the plasma, where $\underline{\xi}$ is initially an arbitrary function of position. The usual procedure is to find the displacement which minimises δW. This will be the most unstable displacement and if δW is positive the discharge will be stable. For a given instability mode (m,k) provided

$$mB_\theta + krB_z \neq 0 \tag{25}$$

or at most only vanishes at discrete radii, the energy principle shows that the most unstable displacements are those for which $\nabla \underline{\xi} = 0$, i.e., displacements involving no compression or expansion of the plasma. Most of the theoretical treatments of stability have studied such discharges. The exceptional case is considered under (c) below.

In neither approach are large amplitude displacements considered, since they lead to intractable non-linear equations. Only on very simplified models of the discharge have such effects been considered (Roberts and Tayler, 1957, Rosenbluth and Longmire, 1957 and Tayler, 1959c).

The following is a summary of the results of linear stability theory:

(a) Perfectly Conducting Compressible Cylinder with Infinitely Thin Current Sheath.

This simplified model, which gives the discharge a sharp edge and which assumes it to be a perfect electrical conductor, was introduced originally by Kruskal and Schwarzschild (1954) and uas used in much of the early stability theory. With it Shafranov (1956), Tayler (1957b) and Kruskal and Tuck (1958) showed that a magnetic field trapped in the discharge has a stabilizing effect but that modes with long wavelength in the z-direction were still unstable. Complete stability is obtained when this discharge model is surrounded by a tube with conducting walls, as was shown by Shafranov (1956), Rosenbluth (1956) and Tayler (1957b). Figure 9 (taken from Tayler) shows the conditions for stability. With increasing discharge compression the allowable values for the external B_z field and for the β value ($\beta = p/[B_{\Theta e}{}^2/8\pi]$) decrease. For a compression greater than 25 ($b/a > 5$) there is no stability.

(b) Perfectly Conducting Cylinder with Finite Current Sheath.

Using this more realistic model Boon et al. (1958) estimated that for a compression factor of 4, instability would occur for current sheaths greater than 15% of the discharge radius. However, even within thin current sheaths there will always be a radius at which a particular instability mode (m,k) is parallel to the helical magnetic field lines so that

$$mB_\Theta + krB_z = 0 \qquad (26)$$

Rosenbluth (1958) and Suydam(1958) showed that because of this singularity, instability will occur unless stringent conditions are satisfied by the magnetic field configuration. By solving the equations for marginal stability Rosenbluth showed that only certain discharge configurations which have a negative B_z external to the discharge will be stable. The required variation of the field components through the current layer are not those which would arise from the internal and external fields diffusing into one another. However, provided the negative field is present and the degree of constriction limited, the instability growth rate in this last case will be very small (Drummond and Rosenbluth, 1960). Suydam found a necessary but not sufficient condition

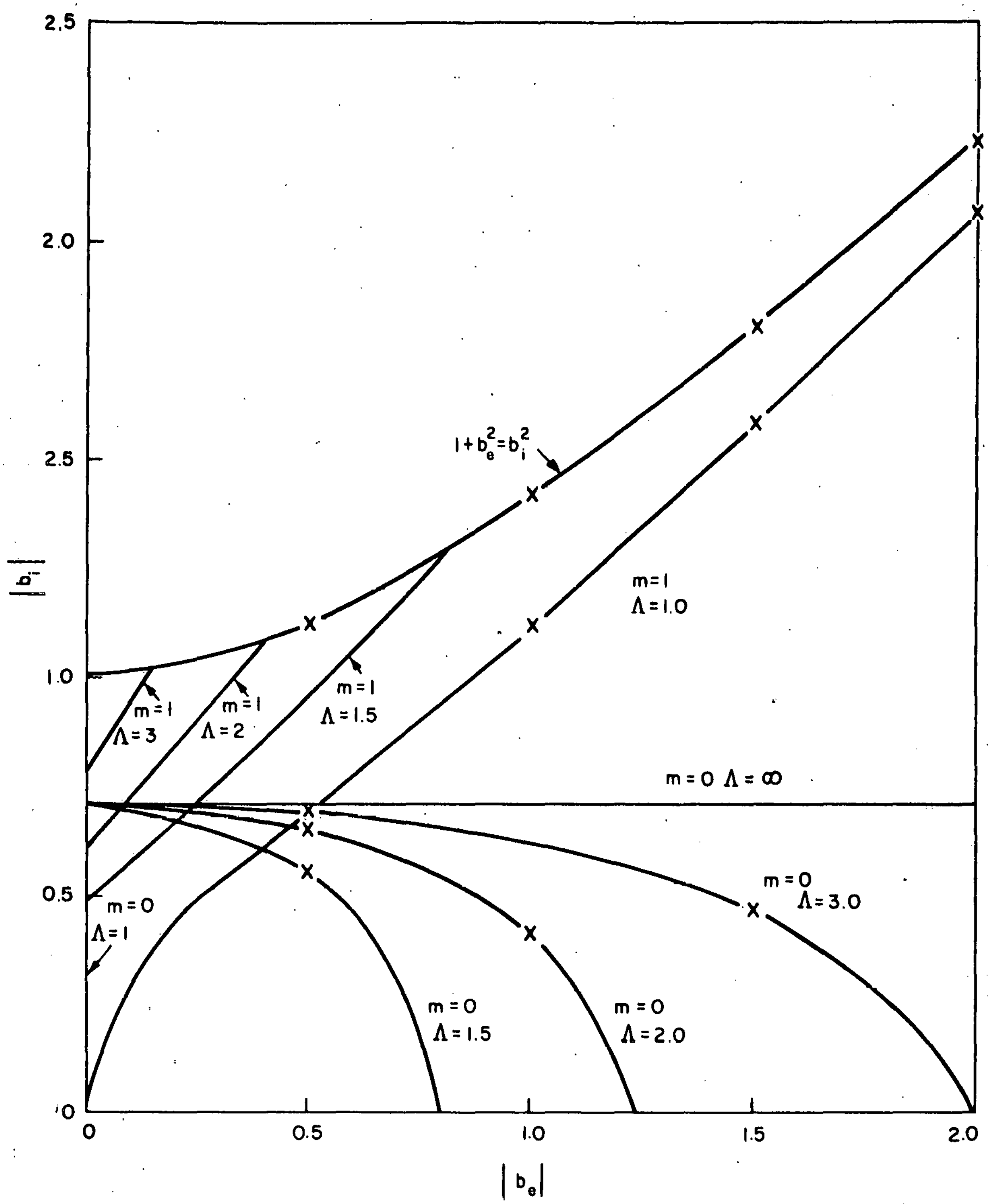


Figure 9 -- Stability diagram for a perfectly conducting cylinder of radius a with surface currents and surrounded by a conducting wall at radius b. Stability occurs for a given $\Lambda(=b/a)$ for points above both the m = 0 and m = 1 curves with this Λ and below the curve $1 + b_e^2 = b_i^2$ where $b_e = B_{ze}/B_{\theta e}$ and $b_i = B_{zi}/B_{\theta e}$. The quantity $\beta = 1 + b_e^2 - b_i^2$ (Tayler, 1957b).

for stability to be

$$\frac{r}{4}\left(\frac{1}{\mu}\ \frac{d\mu}{dr}\right)^2 \geqslant -\ \frac{8\pi}{{B_z}^2}\ \frac{dp}{dr} \tag{27}$$

where $\mu = B_\theta/rB_z$, so that $2\pi/\mu$ is the pitch of the lines of force. The quantity in the brackets is a measure of the shear in the lines of force as one passes from one radius to another. For stability therefore the magnetic field must have sufficient shear.

These instabilities will be localized in a thin layer at the radius where (26) is satisfied but it is unknown how they will affect the remainder of the discharge when their amplitude grows. Since they arise because of a singularity at a discrete radius, whereas the charged particles perform helical paths and experience the fields over a finite band of radii, Hubbard (1958) made an approximate calculation to estimate the effect of the finite Larmor radii of the particles, since this is neglected in the magnetohydrodynamic approximation. His results suggest that stability may still be achieved with discharges which have thin current layers and which satisfy the stability conditions for the infinitely thin layer model.

(c) Perfectly Conducting Discharge with Constant Pitch Magnetic Field.

If $\partial(B_\theta/rB_z)/\partial r$ is zero over the whole discharge so that the helical lines of force have a constant pitch, there will be a set of m and k which will satisfy equation (26) at all radii. These instabilities are parallel to the magnetic field. In this case a helical line of force with the plasma frozen to it, will move outwards (or inwards) at all points along its length and will accurately match the line of force it has displaced. In some cases little or no change in the magnitude of the magnetic field occurs; the energy for the instability coming from the expansion of the hot plasma. Such instabilities have been studied by Johnson et al. (1958) who refer to them as "interchange instabilities", by Kadomtsev and Braginsky (1958), and by Kadomtsev (1959) who uses the equally appropriate name "convective instabilities".

The most unstable perturbations in the case of a convective instability satisfy the condition

$$\nabla \cdot \underline{\xi} = 2B_\theta^{\,2}\ \xi_r \Big/ r(B_\theta^{\,2} + B_z^{\,2} + \gamma p) \tag{28}$$

as shown by Tayler (1959) where γ is the ratio of the specific heats and is equal to 5/3. The condition for stability is then

$$\frac{dp}{dr} > -\ 2\gamma p B_\theta^{\,2} \Big/ r(B_\theta^{\,2} + B_z^{\,2} + \gamma p) \tag{29}$$

Thus the pressure gradient must be less negative than the negative quantitiy on the right hand side. Close to marginal stability these convective modes produce little change in the magnetic field (Ware, 1960b). However, it can be shown that these convective instabilities are not the most unstable modes for a constant pitch discharge. Other modes which are not parallel to $\underline{B}$ and which satisfy $\nabla \cdot \underline{\xi} = 0$, require a positive pressure gradient for stability (Ware, loc. cit.).

(d) Discharges with $B_z \gg B_\theta$.

In the case of infinitely thin skin currents with no external conducting tube, and where both within the discharge and external to it B_z is large compared with B_θ, the discharge is stable against all modes except m = 1. With m = 1 the discharge is unstable only for wavelengths greater than $2\pi a B_z/B_{\theta e}$ (Tayler, 1957b and Kruskal and Tuck, 1958). a is the discharge radius.

This result is important in the case of a torus, since round the closed path of the torus there must be an integral number of instability wavelengths. The torus circumference (L_T) is an upper limit to the possible wavelength and for m = 1 the discharge will be stable if $L_T < 2\pi a B_z/B_\theta$ (Kruskal, Johnson et al., 1958, Tayler, 1957b, and Braginsky and Shafranov, 1958).

In the case of a torus which has a rotational transform angle ι due to some cause other than B_θ the condition for stability becomes (Kruskal, Johnson et al., 1958)

$$L \leqslant B_z\ (\iota + 2\pi n)\ r\ /\ B_\theta \tag{30}$$

where h is the integer (positive or negative) which gives the right hand side its smallest positive value. In this case unless ι is an integral multiple of π the criterion depends on the relative signs of B_θ and B_z. The values of B_θ are those which reduce the transform angle to 0 or increase it to 2π. When distributed currents are considered (Johnson et al., 1958) the same condition is obtained for stability against m = 1 modes. However, in this case higher m modes can be unstable and the corresponding stability conditions have been obtained.

(e) Discharge Models with Dissipative Effects.

Very little has been done on the affect of particle collisions on stability because of the increased complexity of the plasma equations. Viscosity can be expected to decrease the growth rates for short wavelength instabilities and this has been shown for one case by Tayler (1957a). The inclusion of finite conductivity is particularly difficult (Tayler, 1959a).

(f) Stable Configurations.

Using the energy principle Laing (1958) has constructed discharge configurations which were obviously stable by requiring that all the terms in the integral for δW be positive. The unusual features of these configurations are (a) a positive pressure gradient in the centre of the discharge and (b) negative axial current in the outer regions so that the total axial current is zero. Such current configurations were observed by Burkhardt and Lovberg (1958a) at the current zeros in their oscillating discharge and the fluctuations at these times were small. Johnson et al. (1958) have shown that stable configurations can be produced in high B_z discharges with helical windings.

(g) Toroidal Discharges.

Several workers (Suydam et al., 1960 and Rotenberg et al., 1960) are studying the difficult problem of analysing the instability of a toroidal discharge, but no conclusive results have yet been reached. In the case of small plasma pressure Johnson (1960) has shown that the curvature does not affect the wriggle instability.

5.2. Experimental Observations on Stability.

(a) Discharges with Trapped Axial Magnetic Field.

For those discharges which are generated rapidly so that a skin effect occurs and the B_z field is trapped in the plasma, there is a qualitative agreement between theory and experiment. Thus if only a small B_z is used so that the discharge compresses to a small diameter a large amplitude instability is observed (Tuck, 1958 and Colgate, Ferguson and Furth, 1958) involving lateral movements of the discharge (Burkhardt and Lovberg, 1958b). With larger fields however these large amplitude fluctuations cease and only a small flutter is observed on the magnetic probe signals (Burkhardt and Lovberg, 1958a, Honsaker et al., 1958 and Colgate, Ferguson and Furth, 1958). If a thicker insulating tube is used between the discharge and the metal tube so that a larger B_z field is left outside the discharge, Colgate, Ferguson and Furth (loc. cit.) observed that the large amplitude instability occurred with less discharge compression. These results are in qualitative agreement with the conditions for gross stability referred to in 5.1(a) above.

It is an obvious inference that the remaining small amplitude fluctuations are the layer instabilities referred to in 5.1(b), since in these experiments the current sheaths were thicker than those required for complete stability, even on the basis of Hubbard's calculations. However, a study of these fluctuations (Burkhardt and Lovberg, 1958b) has yielded little information about their structure and experimental proof that they are the layer instabilities is lacking. Typical mean frequencies for these fluctuations are in the range 10^6 - 10^7 cycles per second.

Attempts were made by Colgate, Ferguson and Furth (1958) and by Burkhardt and Lovberg (see Tuck, 1959) to stabilize the discharge completely by programming the B_z field so that B_z became negative outside the discharge but were unsuccessful. However, there is some evidence (Birdsall et al., 1959b and Aitken et al., 1959) that the fluctuations observed in straight discharge tubes (as used in these experiments) are generated at the electrodes. An interesting result obtained by Burkhardt and Lovberg (Tuck, loc. cit.) was that, as the magnetic shear in the discharge was increased by increasing the negative B_z, the pressure gradient also rose so as to satisfy approximately the equality in the Suydam criterion (equation 27). Figure 10a shows a radial plot of B_θ , B_z and p from the results, and figure 10b shows the two terms in the Suydam criterion. The pressure in the discharge is apparently increasing until the pressure gradient is sufficient for instability.

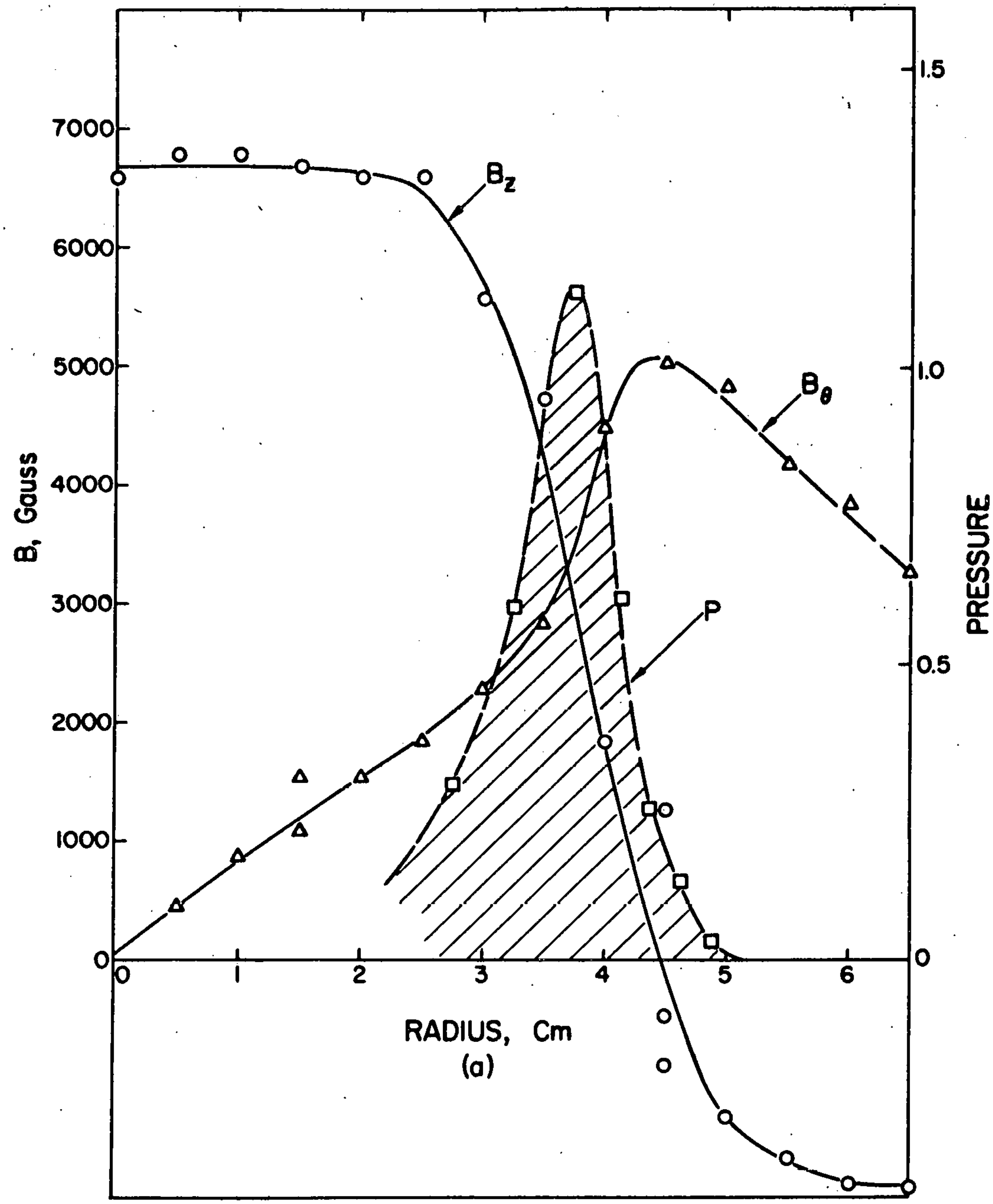


Figure 10a -- Radial distribution of measured B_θ and B_z, and the deduced plasma pressure, in the linear pinch discharge experiment Columbus S-4 (Tuck, 1959).

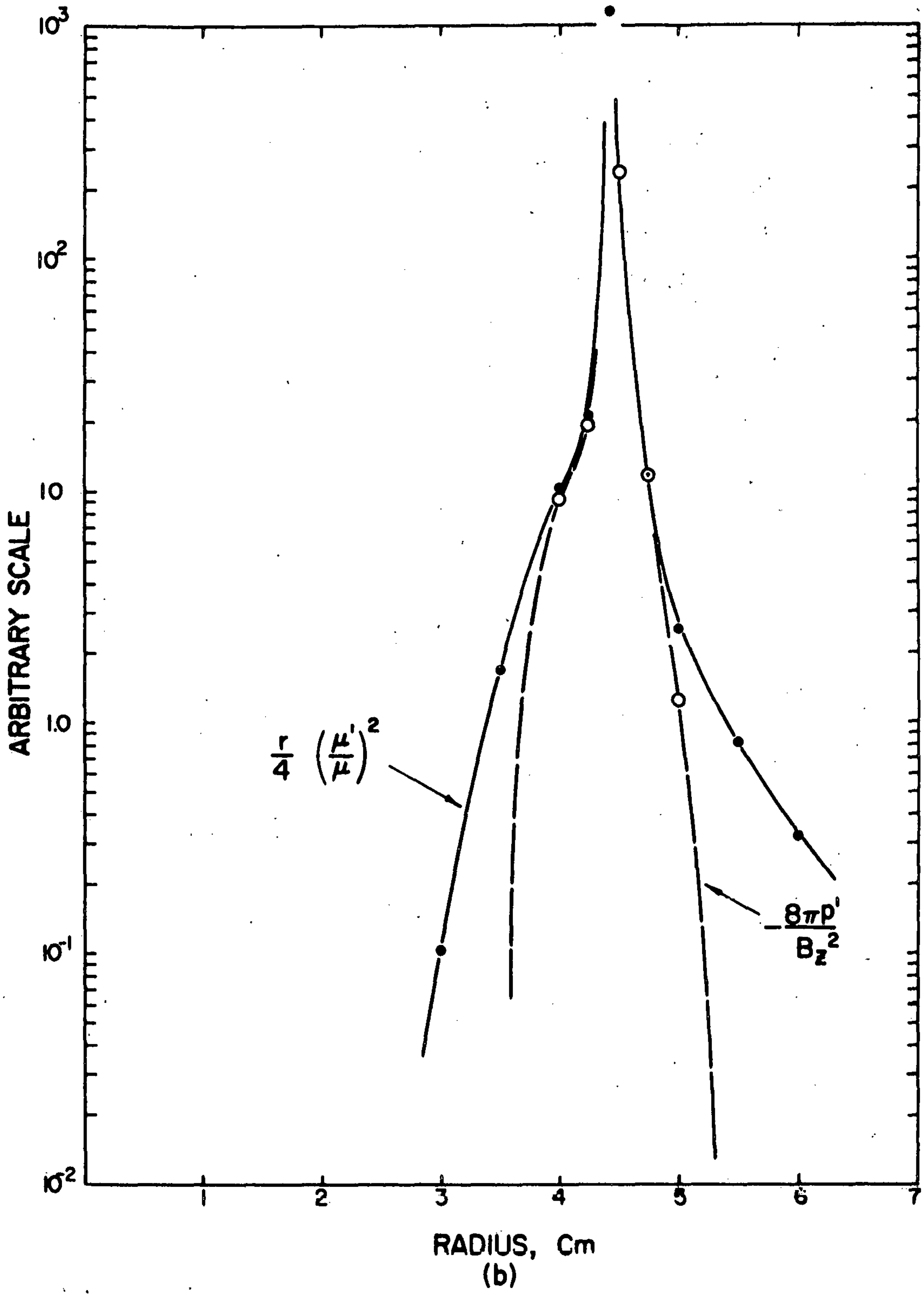


Figure 10b -- Plots of the experimental values of the two terms in the Suydam stability criterion vs. radius for the conditions shown in Figures 10a (Tuck, 1959).

(b) Discharges with Approximately Constant Pitch Magnetic Fields.

In discharges with applied B_z and a low rate of rise of current no skin effect is observed and the magnetic field is observed to have approximately the same pitch at all radii (Ware, 1959c). With low B_z these discharges also exhibit large fluctuations (Butt et al., 1958), and again with higher B_z only small amplitude fluctuations remain (Butt et al., loc. cit., Golovin et al., 1958, Allen and Liley, 1959). The frequency of these fluctuations has generally been in the range 10^4 to 10^5 cycles per second.

A detailed study of these fluctuations (Allen, Ware and Williams, 1960) has shown that they have small amplitude in a central core of the discharge where the pitch of the lines of force is fairly accurately constant, and that over the outer regions of the discharge there is an $m = 1$ instability with a larger regular amplitude. (See section 5.3 for further details.) Since the magnetic fluctuations are small in the core only interchange instabilities can be occurring in this region with appreciable amplitude (Ware, 1960b). In fact, the observed pressure gradients (Allen and Liley, 1959) are in reasonable agreement with the marginal stability condition for such modes (equation 27). This result is at variance with theory, since, in such a discharge, modes which are not parallel to the magnetic field should still be unstable and these would produce large magnetic fluctuations.

High speed photography of these discharges always show large fluctuation in light intensity (Allibone et al., 1958). The fluctuations sometimes appear as bars of light perpendicular to the time axis on streak photographs and sometimes are helical in shape. A close correlation has been observed between these light fluctuations and the magnetic fluctuations (Ware, 1959b). (See also section 5.3.)

(c) Discharges with $B_z \gg B_\theta$.

This type of discharge has been studied extensively at Princeton University and it is one of the few cases where close quantitative agreement has been found between stability theory and experiment (Kruskal, Johnson et al., 1958). With increasing voltages applied to the discharge the current is found to increase rapidly with voltage until a critical current is reached, after which it increases very slowly with voltage. (See figure 11.) Above the

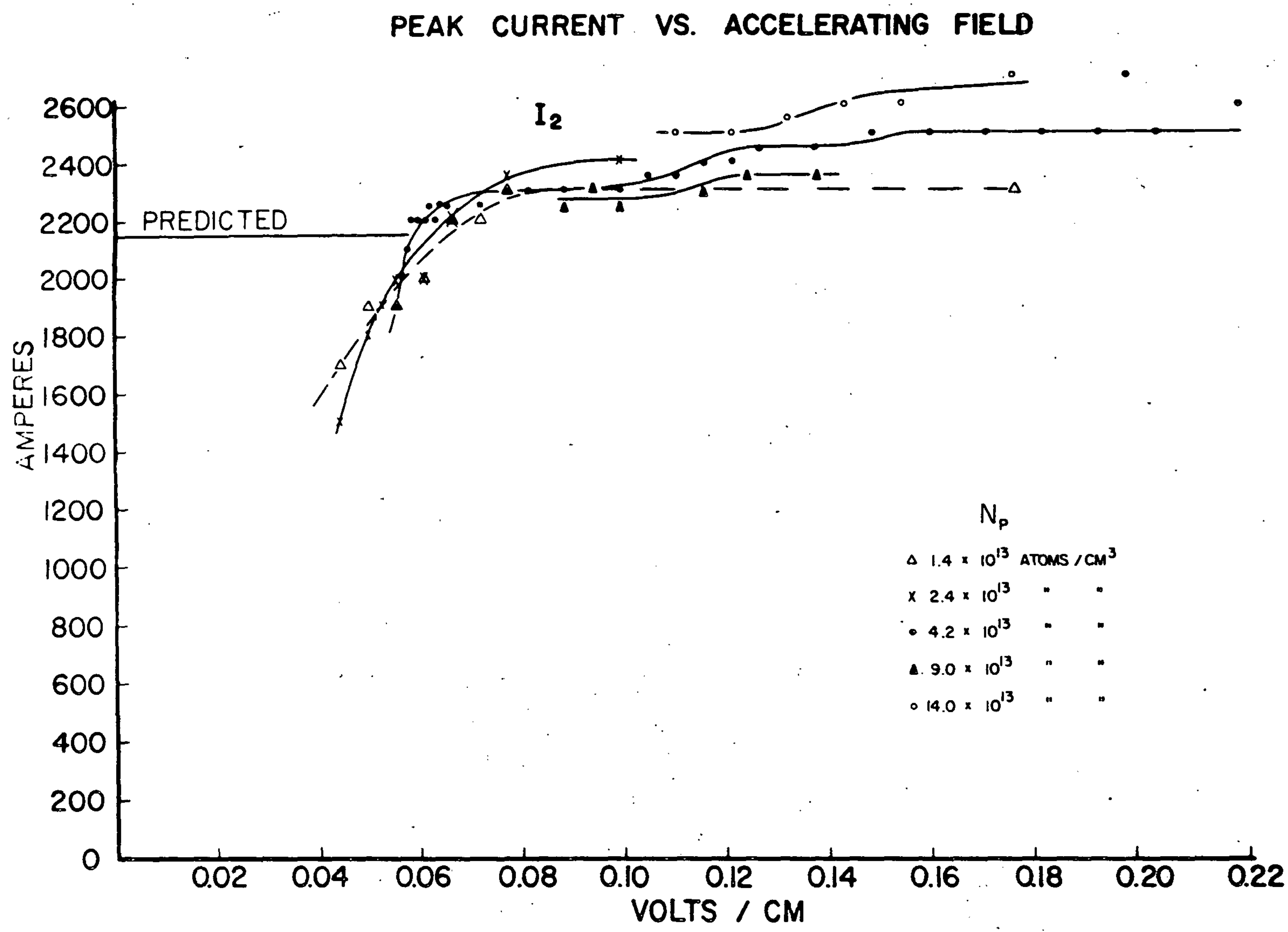


Figure 11 -- Peak current as a function of applied electric field at various pressures of helium in the B-1 Stellarator. The Kruskal limit predicted for the m = 1 instability is shown by the horizontal line (Kruskal et al., 1958).

critical current fluctuations are observed and large amounts of impurity appear. The magnitude of the critical current is different when the sign of the current is changed, and the values of these two critical currents are in good agreement with the two values of B_θ for the limits of stability given by equation (30).

At that time instability modes with higher values of m, which were expected because of the distributed discharge current, were not observed. These modes have since been detected by Bernstein et al.(1959).

5.3. Instability Waves.

As mentioned above, in most stability theory the approximation of perfect conductivity (equation 15) has been used. The solutions for ω^2 are then always real. An instability configuration will remain fixed in position and merely grow in amplitude with no oscillations or wave motion occurring. Experimentally, however, oscillations are always observed in connection with instabilities. The motion associated with these oscillations could be a mass motion of the discharge or a wave motion. Of the many observations on instabilities, until recently only two experiments involved measurements capable of answering this question (T.K. Allen, 1957 and Baker et al., 1958). Both indicated wave motion. The discharges studied were fairly low current and the instabilities were m = 1 modes with observed wavelengths roughly equal to the tube diameter. The wave velocities in the z-direction were in the range 10^5 to 10^6 cm sec^{-1}, being in the opposite direction to the gas current. Similar velocities have been observed with a growing m = 1 instability. (See Allibone et al., 1958.) Further properties of these waves have recently been studied by Lees et al.(1960).

It has been pointed out by Ware (1960a) that if the Hall emf ($\underline{j} \wedge \underline{B}/ne$) is retained in the electric conduction equation (see equation (7)) such wave motion is expected. In this case the magnetic lines of force are frozen to the mean electron velocity (see section 2.5) and the instability perturbations to the magnetic field are carried along by the electrons. For the positive ions it is a wave motion. Ware estimated that the wave velocity should be approximately the mean electron velocity perpendicular to the unperturbed field, namely

$$V_w = \nabla p \wedge \underline{B} / neB^2 \qquad (31)$$

That such a wave velocity is obtained when the full set of plasma equations are solved with the Hall emf retained has been shown independently for the special case of an $m = 0$ mode in a discharge with $B_z = 0$ in the work of Kadomtsev (1959) and Ridakov and Sagdeev (1959). This wave velocity agrees in sign and order of magnitude with the observed wave velocity and has the correct dependence on magnetic field (Ware, loc. cit.).

A search for such waves in the high current toroidal discharge experiments Sceptres III and IV has revealed two such instability waves (Allen, Ware and Williams, 1960). In the central core of the discharge where dp/dr is negative the fluctuations travel in the opposite direction to the discharge current; in the outer regions where dp/dr is positive, the $m = 1$ instability observed in this region moves in the same direction as the current. These results agree in sign and order of magnitude with equation (31). (The possibility that two instabilities might occur simultaneously in different regions of the same discharge was suggested by Kadomtsev (1959).) Since in earlier work it has been shown that the "bars" on the streak photographs exhibit similar velocities (Allibone et al., 1959) and that there is a close correlation between "bars" and the magnetic fluctuations (Ware, 1959b), the evidence is strong that the "bars" are identical with the magnetohydrodynamic instabilities in the outer regions of the discharge.

6. PARTICLE HEATING AND ENERGY LOSS.

6.1. The Electron Temperature and Energy Loss.

A characteristic feature of high current discharges is the large amount of energy dissipated in the discharge; instantaneous powers as high as 900 megawatts have been quoted (Conner et al., 1958). Equally characteristic is the fact that although most of this energy is given to the electrons the electron temperature remains low. Electron temperature measurements from the plasma conductivity (Artsimovich, 1958, Colgate, Ferguson and Furth, 1958, Tuck, 1958, Allen and Liley, 1959) and from spectroscopic observations (Kaufman and Williams, 1958 and 1960) have yielded values less than 5×10^5 degrees K, whereas the energy input was sufficient to produce temperatures between one and two orders of magnitude greater. The electrons must be losing energy at roughly the same high rate as they receive it.

The electrons can lose energy at these rates either by exciting radiation from the bound electrons of impurity ions, if a sufficient concentration of impurity ions is present, or by the electrons transporting the energy to the walls. Calculations of the rates of radiation expected for given impurity concentration and electron temperatures have been made by Knorr (1958) assuming that the discharge pulse time is long compared with the ionization times for the multi-ionized ions concerned. These results have been applied by Kogan (1959). In connection with particle transport of energy, since the above temperature and energy measurements are for toroidal discharges, any such transport must be across the magnetic field. The energy flux due to normal diffusion and heat conduction across the magnetic field as given by equations (7) and (8) is much smaller than the observed energy loss, and hence if the electrons are transporting energy at this rate some extra mechanism must be involved to enable the particles to move across the magnetic field.

The first experiments to determine which of these two processes was the main energy loss were made by Colgate, Ferguson and Furth (1958) with their ceramic toroidal discharge tube "Gamma pinch". They found that for the first eighth of a cycle of the current (about 5 microseconds) the main energy loss to the walls was by electrons with energies of about 1.5 kev. These electrons were detected by the soft X-rays they induced. After about 10 microseconds the ultraviolet light increased greatly in amplitude and became the main loss from then on. During the second eighth of a cycle neither the loss of energetic electrons nor the radiation was sufficient to explain the energy loss. However, since the lower limit for soft X-ray detection was 1 kev, lower energy electrons could have been escaping undetected during this period.

In the toroidal experiment Perhapsatron S4, which had a quartz torus, ultraviolet radiation became intense after about 7 microseconds, and when integrated for the duration of the discharge was the main energy loss (Tuck, 1959). With Perhapsatron S5, by improving the vacuum conditions, the onset of this radiation has been delayed to 40 microseconds (Karr, 1960). Few X-rays were detected during this period but low energy electrons were not studied. In experiments in a straight porcelain tube Kirillov (1959) found the ultraviolet radiation due to impurities to be the main energy loss when integrated over the current pulse.

In the large metal torus Zeta, however, the radiation was found to account for only 10% of the energy loss throughout the discharge pulse (Gibson et al., 1960).Soft X-ray emission from the tube walls was intense but only at the end of the current pulse when a rapid fall of current occurred. A large density of slow electrons was detected near the tube walls and was consistent with these electrons carrying most of the energy to the walls. In the smaller metal torus Sceptre III the energy loss by electrons with energy greater than 2 kev was found to be small, but large currents to the walls of lower energy electrons were detected (Ware, 1959b).

These various results can be reconciled if it is assumed that the fundamental loss process is an electron particle loss. In the case of ceramic tubes this leads to wall vaporization and contamination of the discharge, so that radiation then becomes the main energy loss mechanism. In the metal tubes the particle loss will cause a much smaller rise in wall temperature and hense less contamination.

It has been suggested by Ware (1960c) that the instability waves (see section 5.3) provide a mechanism by which electrons can escape across the magnetic field. In a magnetohydrodynamic instability there are regions where both electrons and ions drift radially outwards. Since these regions are carried along with approximately the same velocity as the equilibrium mean electron velocity, the unfortunate electrons in such a region will remain in it and will continue to move outwards until they are lost to the walls. They will be replaced by cold electrons (from secondary emissions, arc spots or ionization of neutral gas emitted from the walls,) which move into the discharge in those regions of the instability where the net movement is inwards. The positive ions, however, undergo an oscillating radial drift since the instability configuration passes over them, and their losses will be much less. Such a preferential electron loss is indicated by experiment since there is considerable evidence that the positive ion temperature exceeds the electron temperature.

Other instabilities which have been suggested as explanations of the electron particle loss, are ion wave instabilities (Bernstein et al., 1960) and electron plasma oscillations excited by runaway electrons (see for example Tuck, 1958, Buneman, 1958 and Ellis et al., 1959). The ion wave instabilities will occur when the electron drift velocity becomes comparable with the thermal velocities in the plasma. (The exact critical drift velocity depends on the assumptions made about the electron and ion temperatures.)

These waves will produce space charge fields and enhanced diffusion in the same manner as the instability waves referred to above, but there is as yet no evidence that they occur experimentally.

Runaway electrons will excite electron plasma oscillations if $dn'/dv > 0$ where $n'dv$ is the number of runaway electrons in the velocity range v to $v + dv$. As indicated above, in most of the toroidal pinch experiments high energy electrons are not very numerous throughout the main part of the current pulse, an exception being the first eighth of a cycle in the Gamma pinch torus. In Zeta only at the end of the pulse does the plasma density fall low enough for an appreciable number of runaway electrons to occur (Gibson et al., 1960). In the stellerator experiments runaway electrons may be more important (Bernstein, Chen et al., 1958) and there is indirect evidence that they play a part in the loss of plasma from the discharge (Ellis et al., 1959).

6.2. The Positive Ion Temperature and Heating Mechanism.

In the toroidal discharges Zeta and Sceptre III the spectral lines of highly ionized impurities show marked Doppler broadening (Thonemann et al., 1958 and Allen et al., 1958). The shape of the line profiles, which are integrals for the long exposure times of many complete discharge pulses, corresponds to a Maxwellian velocity distribution (Allibone et al., 1958) and is independent of the direction in which the discharge is viewed (Hughes and Kaufman, 1959). These results suggest that the broadening is due to thermal motion of the ions, the temperatures involved being in the range 10^6 to 7×10^6 degrees K. However, a turbulent mass motion of the plasma could give a Maxwellian velocity distribution when averaged over the long exposure times. Evidence against this has been obtained by Hughes, Kaufman and Williams (1960) who have compared the broadening of ions of different mass. Thus FVI and BIV, which have atomic masses 19 and 10.8 and which emit light from the same central region of the discharge, show different velocities but the same temperature.

If thermal motion is the cause of the broadening, since the energy exchange time between these impurity ions and deuterons is short compared with the current pulse, the deuteron temperature should be of the same order of magnitude. This is provided the rate of heating or cooling of either the deuterons or the impurities is not excessively large. The apparent high ion temperature is one of the paradoxes of the slow toroidal discharge, since it was expected that the electron temperature would always exceed the ion temperature because most of the Ohmic heat goes to the electrons.

A study of the Ohmic heating of impure deuterium has been made by Ware and Wesson (1960). In the case where a cooling mechanism keeps the electron temperature low the presence of highly ionized impurity ions leads to a larger fraction of the Ohmic heat being given directly to the deuterons. With a 14% concentration of OV the deuterons receive 7% of the heat when their temperature is 2×10^6 degrees K. This rate of heating is sufficient to explain the observed ion temperatures, but an attempt at experimental verification proved inconclusive.

Other heating mechanisms which have been suggested are the instability waves (Ware, 1959c) and magnetohydrodynamic waves (Reagan, 1959, 1960).

7. NUCLEAR REACTIONS.

The two nuclear reactions which can occur between deuterons and which occur with approximately equal probability are

(1) $D + D \longrightarrow (He^3 + 0.8 \text{ Mev}) + (n + 2.45 \text{ Mev})$

(2) $D + D \longrightarrow (T + 1.0 \text{ Mev}) + (p + 3.0 \text{ Mev})$

The cross sections for these reactions increases rapidly with energy up to about 100 kev (Post, 1956) and since the population of energetic particles increases exponentially with temperature the reaction rate is a particularly rapidly varying function of temperature (Post, 1956 and Thompson, 1957).

These reactions can be studied by detecting any one of the four reaction products, the neutrons being the easiest to detect since they penetrate the tube walls. The reaction products are emitted isotropically with respect to the centre of mass of the reacting particles with the energies indicated in the above equations. Since in a hot gas the centre of mass of pairs of colliding particles will have a Maxwellian velocity distribution, the observed energy spectrum of a particular type of reaction product will show a broadening about the appropriate energy indicated above, but no shift.

If, however, reactions are caused by a directed beam of deuterons colliding with comparatively stationary deuterons, the particles emitted parallel to the beam will have a higher energy in the laboratory frame of reference, and those emitted in the reverse direction a lower energy than the above value. The measured difference in energy for the two directions can be used to calculate the mean centre of mass velocity of the reacting particles. This measurement is therefore one of the simple but not conclusive tests applied to determine if the reactions are thermonuclear in origin. Until very high temperatures and copious reactions occur, very stringent tests are needed before the origin of observed reactions can be ascertained conclusively (see Tuck, 1958).

7.1. The Linear Pinch without an Axial Magnetic Field.

Only in straight pinch discharges have neutrons been detected when no stabilizing magnetic field was used. (The rates of rise of current have generally been lower with toroidal discharges.) The reactions occur in bursts of short duration usually at the second time, but sometimes at the first or subsequent times of maximum constriction. Up to 3×10^9 neutrons per discharge pulse have been observed (Andrianov et al., 1958). The yield increases with current but at the highest currents shows a saturation and sometimes a decrease (Artsimovich, Andrianov, Dobrokhotov et al., 1956 and Anderson et al., 1957). The yield is quenched by small percentages of impurity and greatly reduced by small applied axial magnetic fields. A study of the energy of the neutrons has invariably shown a large asymmetry indicating mean centre of mass velocities in the range 7×10^7 to 1.6×10^8 cm sec^{-1} parallel to the gas current. If the reactions are due to accelerated particles colliding with stationary ones, the corresponding energies of the former would be 20 - 100 kev (Anderson et al., loc. cit. and Andrianov et al., loc. cit.).

The most plausible explanation of these reactions is the theory of Anderson et al., loc. cit.) which is based on the large inductive voltages associated with the necking off of the discharge at various points along its length due to the m = 0 instability. Among the evidence in favour of this theory are the large voltage pulses which are sometimes detected simultaneously with the neutron pulse (Anderson et al., loc, cit., Siegbahn and Ohlin, 1958). The presence of high voltages in the discharge is indicated also by the generation of high energy electrons with energies up to 300 kev (Podgornyi et al., 1958), and recently the accelerated deuterons have been detected (Hartman et al., 1960).

In the experiments of Curran et al. (1958), when the external circuit inductance was made very small so that dI/dt was very large, smaller neutron yields were detected. The yields, however, were much less sensitive to both impurity and an applied axial magnetic field than in the experiments described above. These results suggest that a different mechanism is inducing the nuclear reactions in this case, although similar energy shifts have been detected in the neutron energies (Bodin, Fitch and Peacock, 1960). Other workers (Hagerman and Mather, 1958, Siegbahn and Ohlin, 1958) subsequently found that after an initial rapid drop in neutron yield with increasing magnetic field, a small yield remained which was relatively insensitive to magnetic field and impurity. Colgate (1958a) has suggested that the two reaction inducing mechanisms are the m = 0 and m = 1 instabilities, the m = 0 case being the mechanism sensitive to magnetic field and impurities. He suggested that in the experiments of Curran et al. (loc. cit.) the low circuit inductance leads to a broad pinch boundary, (because the electric field near the walls stays high for a longer time,) and that this favors the m = 1 instability.

7.2. The Linear Pinch with an Axial Magnetic Field.

As mentioned above nuclear reactions continue to occur in the presence of an axial magnetic field but at a lower rate. The reaction rate varies with discharge conditions in a manner markedly different to the case where $B_z = 0$. Thus, the neutron yield shows only a slow decrease with increase in B_z and is relatively insensitive to impurity. In addition the neutron burst lasts for a much longer time (Hagerman and Mather, 1958) and is often centred at the current maximum (Burkhardt and Lovberg, 1958a). It is not correlated with the "bouncing" of the discharge.

Mather and Williams (1958) have found that the neutron energy shift is much smaller than in the $B_z = 0$ case, and their results show that the reactions cannot be produced by a simple axial acceleration mechanism since the necessary deuteron current would exceed the observed discharge current. No satisfactory mechanism has yet been proposed to explain the nuclear reactions.

In experiments in which a pulsed B_z field was applied simultaneously with the gas current, Colgate, Ferguson, Furth and Wright (1958) detected a short neutron burst occurring simultaneously with the onset of an m = 1 instability.

No detailed mechanism has yet been proposed whereby the m = 1 instability can cause particle acceleration.

7.3. The Toroidal Pinch with Magnetic Field.

The properties of the neutron yields obtained with toroidal discharges are similar to those for straight tubes with applied magnetic field. The one major exception is the variation with magnetic field. No reactions have been observed with zero magnetic field. A small field is needed before a detectable yield is obtained and beyond this point the yield increases with magnetic field (Colgate, Ferguson and Furth, 1958, Allibone et al., 1958). In most cases the yield passes through a maximum and decreases at high fields (Harding et al., 1958, Connor et al., 1958 and Ware, 1959b). The emission period extends throughout most of the current pulse - for as long as 2 milliseconds in the case of Zeta (Thonemann et al., 1958) - and sometimes into the second half cycle (Allibone et al., loc. cit.). The reactions occur in the central core of the discharge (Harding et al., loc. cit., Conner et al., loc. cit.). Although there is evidence for high temperatures in these discharges (see section 6.2) the observed reaction rates exceed the expected thermonuclear reaction rates in many cases (Thonemann et al., loc. cit., Allibone et al., loc. cit.).

Energy shifts similar to those for the linear pinch with applied field have been detected for the neutrons (Rose et al., 1958 and Connor et al., loc. cit.) and for the protons (Jones et al., 1958). The mean velocity for the centre of mass of the reacting particles is about 5×10^7 cm/sec. Evidence has also been obtained indicating the mean centre of mass velocity has a θ component of about 2×10^7 cm/sec (Hunt, 1960). Such a component is expected since high energy deuterons confined in the discharge should move approximately along the helical lines of force. That the energy shift is not due to a simple mass motion of a thermonuclear-reacting gas has been shown by the Doppler shift measurements of Hughes and Kaufman (1959). However, if a small fraction of the deuterons have a high temperature and are moving through the remaining deuterons with a velocity of the order 10^7 cm/sec this would give the observed energy shift (Herdan and Hughes, 1960). Hunt (1960) has analyzed the observed proton spectra making the assumption that all thermal motion of the deuterons can be neglected and that the reactions are caused by accelerated deuterons colliding with stationary deuterons. The results indicate a wide spread of deuteron energies the population increasing with decreasing deuteron energy.

8. HIGH CURRENT DISCHARGES WITH OTHER GEOMETRICAL CONFIGURATIONS.

High current discharges can be produced with an infinite variety of shapes. Not only can the shape of the discharge tube be varied, but also the geometry of the electric and magnetic fields. Lastly the ingenious experimenter has the fourth dimension at his disposal since both of the fields can be "programmed", i.e., varied in time. In many cases both theory and experiment are tending towards more complex systems in order to achieve a stable discharge. Here only the more widely studied discharge configurations will be reviewed.

8.1. Azimuthal Discharges.

As mentioned in the introduction, discharges with currents induced in cylindrical discharge tubes in the Θ-direction are now being widely studied. The standard practice is to use a low inductance coil wound on the tube and fed from a low inductance condenser bank. The high current in the coil is allowed to ring. The rapidly rising axial magnetic field induces a Θ current in the gas. Generally, only a poor degree of ionization is achieved in the first half cycle although a contracting shock wave and "bounces" are sometimes observed (Bodin, Green et al., 1959b). This is due to the axial magnetic field limiting the energy which one electron can gain between collision to $2\ mc^2E^2/B^2$. By the end of the first half cycle there is a sufficient plasma density so that space charge forces can counteract the limiting effect of the magnetic field. In the second half cycle, therefore, large gas currents are induced at the periphery of the discharge, and as in the case of the pinch discharge with high dI/dt, a contracting shock wave is generated. This is shown by A in figure 12 which is taken from Kolb (1958). B is the boundary between the plasma and the magnetic field.

As in the pinch discharge a series of bounces then occur (C D E F G in figure 12). If a uniform coil is used a damped train of many oscillations is observed (Bodin et al., 1959b) but the plasma is rapidly lost longitudinally along the magnetic lines of force. To avoid this most workers have designed their coils to give the magnetic field a larger magnitude at each end so that magnetic mirrors are formed. (For the principle of a magnetic mirror see Post, 1958.) These mirrors greatly reduce the end losses, and in addition the radial components of magnetic field at these points exert a longitudinal force on the Θ currents and cause longitudinal shock waves which travel towards

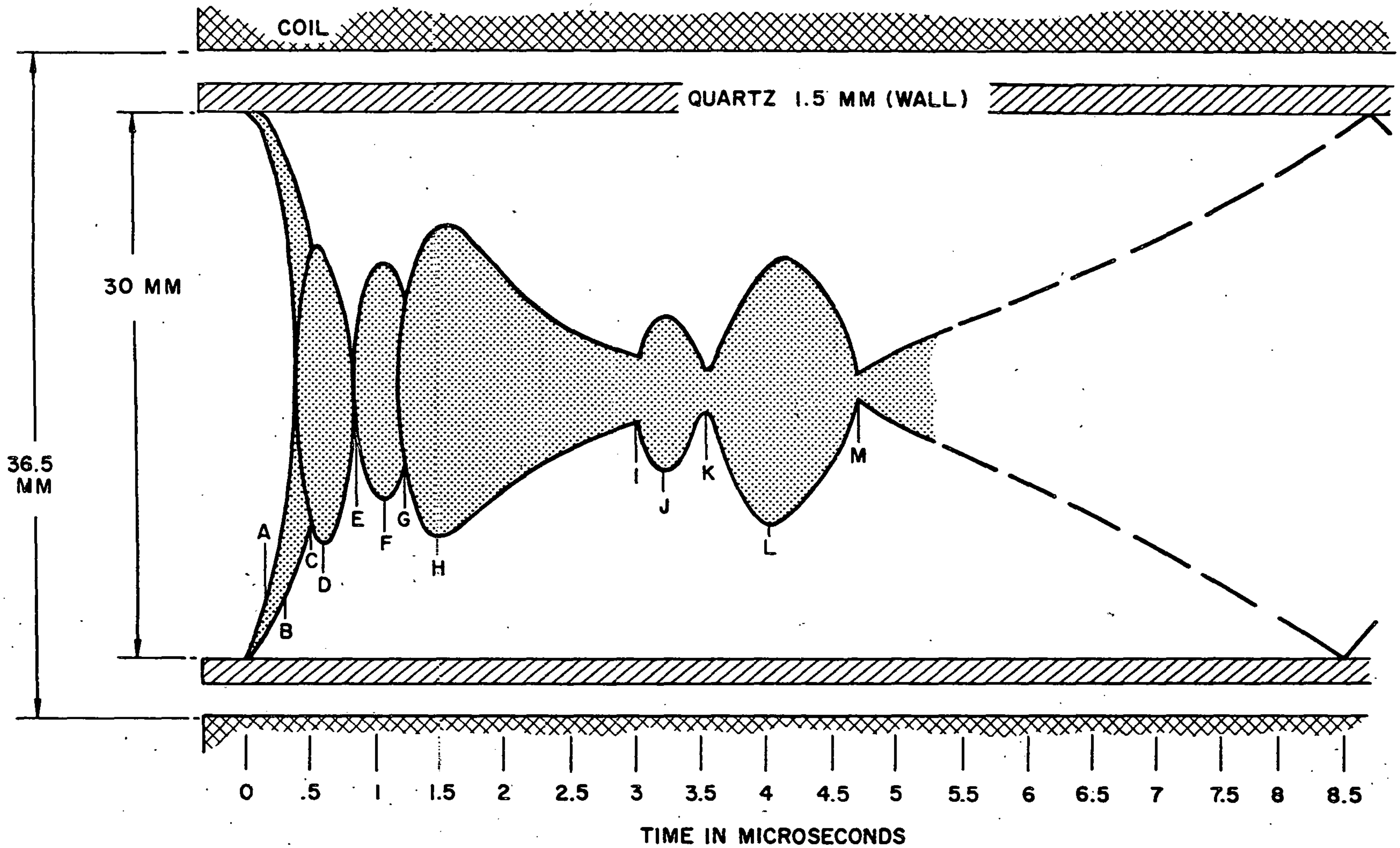


Figure 12 -- Diagrammatic representation of the streak photograph for the second half cycle of an azimuthal discharge with magnetic mirror ends (Kolb, 1958).

the centre of the coil. These waves cause an expansion of the plasma cylinder to about half the tube diameter (H in figure 12). Subsequently the rising magnetic field compresses the discharge again to a small diameter. Then as indicated at I J K L M, Kolb has observed expansions and contractions of the plasma cylinder suggesting some form of discharge oscillation. In some cases these oscillations have the appearance of a rotating instability (Kolb, Griem and Faust, 1959 and Bodin et al., 1959a). In Kolb's experiment the visible part of these oscillations on the streak photographs does not expand to the walls. Other workers, however, have observed instabilities which do appear to extend to the walls (Alidieres et al., 1959, Bodin et al., 1959a and Kvartzhava, 1959). In all cases the instability occurs in the second half cycle.

Another important feature associated with the second half cycle is the magnetic field within the plasma. The field produced by the coil in the second half cycle will be defined as a positive field. Magnetic probe measurements (Kolb, Dobbie and Griem, 1959 and Fay et al., 1959) have shown that some of the negative flux which penetrated the plasma in the first half cycle remains trapped in the centre of the discharge during part of the second half cycle, and then the field abruptly changes to a positive value.

Still further features associated with this interesting second half cycle are the emission of neutrons and of soft X-rays from the plasma. In experiments with small or unity magnetic mirror ratios the neutron emission occurs in a short pulse (Kolb, Griem and Faust, 1959, Fay et al., 1959 and Bodin et al., 1959a) and in some cases there is correlation with the sudden change of trapped magnetic field. When strong magnetic mirrors are used, however, much larger neutron pulses of longer duration are observed (Elmore et al., 1958, Tuck, 1959 and Kolb, Griem and Faust, 1959). The Scylla experiment has yielded most of the latter results and in this experiment very extensive studies have been made of the reaction products. The neutrons show no energy shift (Elmore et al., 1958), and accurate measurements of the protons and tritons have revealed a Maxwellian distribution for the centre of mass velocities of the reacting pairs of deuterons (Ribe, 1959 and Nagle et al., 1959). This is strong evidence that the deuteron motion is close to thermal motion in this experiment. The indicated temperature is about 1.3×10^7 degrees K.

The soft X-ray emission (Griem et al., 1959 and Boyer et al., 1959) has a spectrum consistent with bremsstrahlung radiation corresponding to an electron temperature of about 2 X 10^6 degrees K (Ribe, 1959).

The heating mechanism causing the high ion temperature has yet to be determined. The shock wave and subsequent adiabatic compression will make substantial contributions, but the experimental results suggest that the instability is playing a major part. Thus, the neutron production can be advanced to the first half cycle provided the gas is adequately pre-ionized and provided an initial magnetic field is applied to the gas in the opposite direction to that generated by the coil in the first half cycle (Kolb, Dobbie and Greim, 1959 and Quinn et al., 1960). It has been pointed out by Furth (see Kolb, Dobbie and Greim loc. cit.) that the presence of the trapped reverse field leads to an unstable configuration and has suggested the rapid dissipation of the reverse field as the cause of the heating. An attractive feature is that the instability changes the discharge to a more stable configuration and the particles heated by the instability will be contained.

8.2. Annular Discharges, ("Triax" and "Hardcore").

At the Lawrence Radiation Laboratory some interesting annular discharges have been developed. These discharges are generated in the annular space between two concentric insulating cylinders of different diameter. The discharge is thus a hollow cylinder. By means of a copper conductor within the inner insulating tube a negative B_Θ magnetic field is produced within the discharge which keeps the plasma from touching the inner tube.

In the "Triax" experiment (Anderson, Baker, Ise et al., 1958) the current in the gas is larger than in the inner copper conductor and, hence, a positive B_Θ is generated on the outer surface of the discharge and this keeps the plasma from the outer wall. No axial field is used. The two B_Θ fields compress the plasma to a thin sheet and some "bouncing" is observed. Such a system should be more stable than a normal pinch discharge without an axial magnetic field but not completely stable. Experimentally the discharge is stable for a longer time but an instability sets in which is accompanied by an increase in gas resistance and nuclear reactions. It was possible to measure the neutron energies with only limited accuracy but the results showed that any energy shift must be less than that which would be caused by 3 kev accelerated deuterons reacting with stationary deuterons (B. Pyle, private communciation).

As pointed out by Furth (see Kolb, Dobbie and Griem, 1959) the magnetohydrodynamic configuration in this discharge, in which two parallel magnetic fields of opposite sign are separated by a thin sheet of plasma, has many similarities with the second half cycle of the azimuthal discharge. The mechanisms for deuteron heating or acceleration may be the same.

In the "Hardcore" experiment (Birdsall et al., 1959a)the current in the gas is approximately equal to the current in the inner conductor, so that there is little or no B_θ outside the discharge. The plasma is compressed from the outside by a B_z magnetic field. This configuration is very attractive since the plasma is on the outside of the curved lines of force. Many authors have shown that such a configuration should be stable (see for example Berkowitz et al., 1958). Experimentally some of the discharges with this general configuration are free from magnetic field fluctuations whereas others are not (Birdsall et al., loc. cit.). The unstable discharges are those with large axial current densities. However, there is evidence that these instabilities form initially at the cathode and propogate along the tube (Birdsall, et. al., 1959b and Aitken et al., 1959). Similar discharges with the B_z field missing, and which are therefore not in equilibrium, were studied by Anderson et al., 1958).

Because of the electrode effects on stability and because even with a stable discharge large heat losses will occur to the electrodes, a toroidal version of this type of discharge has been proposed by Colgate and Furth (1958). The results obtained with this experiment which is at present under construction (Birdsall et al., 1959b) should prove very interesting.

8.3. Rotating Plasmas.

Since a plasma is an electrical conductor many of the electromagnetic processes which can be applied to a piece of copper can also be applied to a plasma. In particular a discharge can be made to rotate in a manner analogous with the Faraday disc or with the rotor of a homopolar motor. The rotation is generated in an annular discharge tube by creating a radial current density j_r between coaxial electrodes in the presence of an axial magnetic field B_z. The force $j_r B_z$ causes the angular acceleration and magnetic pressure balances the centrifugal force. Such rotating plasmas with circumferential velocities up to 6×10^6 cm sec^{-1} were generated in the Homopolar I experiment (Anderson, Baker, Bratenahl et al., 1958). Because of the energy of the rotating plasma, the discharge acts like a condenser containing a medium of very high dielectric constant of 10^6 to 10^7 (Anderson et al., 1959).

The centrifugal force can be used to aid the confinement of a plasma in experiments aimed at thermonuclear reactions. In the experiment "Ixion" the rotating plasma is produced in a magnetic mirror field (Boyer et al., 1958) and in Homopolar III curved lines of force are used so that the centrifugal force will prevent the loss of particles along the magnetic field (Colgate, 1958b). Instabilities have been observed (Baker et al., 1959), and short curcuiting of the radial electric field by breakdown at the surface of the end insulators is suspected (Baker et al., loc. cit., and Baker and Hammel, 1960).

8.4. Plasma"Guns".

By suitably designing the leads to a high current discharge the magnetic field on one side of the discharge can be made to exceed greatly the field on the other side. This will cause an acceleration of the plasma in the transverse direction. If there are no material obstacles the plasma will often break loose from the electrodes and be ejected with a high velocity (Bostick, 1956, Kolb, 1957, Scott et al., 1958 and Finkelstein et al., 1958). The ejected volume of plasma often contains circulating currents and trapped magnetic fields, and has been termed a "plasmoid" by Bostick (loc. cit.). Growing interest has recently been shown in such plasma guns because of their possible application to propulsion in outer space (Von Engel, 1959) and to the injection of energetic plasmas into confining magnetic fields in thermonuclear research (Tuck, 1959).

More efficient acceleration can be achieved by using a pair of parallel rails as the electrodes. In this way the moving plasma stays in contact with the electrodes for a longer time and since a current continues to flow in the plasma, there is a longer period of acceleration and higher velocities are achieved for a given energy input. Such "rail guns" have been studied experimentally by Artsimovich et al., (1957), Bostick (1958) and theoretically by Morozov (1957). Similar propulsion was achieved by Marshall (1958) using a moving ring discharge which was achieved by a travelling magnetic wave. More recently rail guns with coaxial electrodes have been studied (Dattner, 1959 and Tuck, 1959). Overall efficiencies from 30 to 40 % have been achieved in the conversion of stored energy to plasma energy(Tuck, loc. cit., Marshall, 1960, Bostick et al., 1960), involving several litres of plasma with density 10^{16} cm^{-3} and velocities up to 1.5×10^{7} cm sec^{-1}. The highest plasma velocities (4×10^{7} cm/ sec) have been obtained in the axial flow from a conical pinch discharge (Hartman et al., 1960).

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