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HETEROGENEOUS REACTOR CALCULATION METHODS

Quarterly Progress Report No. 6
AEC Contract No. AT(30-1)-2375

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Abstract

This report includes the following results of heterogeneous reactor physics calculations: (1) A comparison of heterogeneous and homogeneous results for natural uranium rods in a loose-packed graphite lattice, (2) HERESY 2 calculations on the sensitivity of heterogeneous results to multiple resonance absorption and resonance fission parameters.

Detailed instructions on the use of the HERESY 1 code are given.

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Section 1. Introduction and Summary

In this Quarterly Report two new investigations relating to the further development of heterogeneous reactor calculation methods are presented. The first is a comparison between heterogeneous and homogeneous calculations for graphite lattices. The second investigation concerned the use of the HERESY 2 code which takes multiple resonance phenomena, including fission at resonance energies, into account. There is also presented a complete description of the HERESY 1 code, including operating instructions, input-output description, and a copy of the FORTRAN source program.

The comparison between homogeneous and heterogeneous calculations is given in Section 2 for loose-packed lattices of uranium metal rods in a graphite moderator. Lattices of various sizes were investigated, ranging from 5 by 5 to 12 by 12 rods. The reactivity and power pattern of these configurations were calculated using the HERESY 1 code. Two group, two region homogeneous calculations were also carried out. The results indicated that for lattices with one physical rod type the homogeneous and heterogeneous methods are quite comparable. Better agreement was obtained for large configurations than for small ones. Some interesting points of difference between the two sets of calculations appeared, however.

The HERESY 2 code has now been completed and debugged. This code differs from HERESY 1 primarily by including multiple resonance effects, both absorption resonances and fission resonances. A series of calculations to test the sensitivity of HERESY 2 results to the resonance parameters is reported in Section 3. It is found that for the low enrichment fuel elements considered, a variation of at least 0.5% in reactivity can be attributed to the choice of resonance energy in the case of absorption resonances. It is also found that even for such low enrichment elements the effects of fissions at resonance energy can be appreciable, several percent in reactivity. Since one wishes heterogeneous results to give reactivity values to a greater accuracy than one-half percent, one can conclude that accurate values of resonance data are required. The importance of interaction effects between resonances, i.e., the dependence of the slowing down density to a given resonance on depletion effects of higher energy resonances, is pointed out.

Section 4 is devoted to a detailed description of the HERESY 1 code for the IBM-704. Included are operating instructions, available options, input preparation and output format. Input and output for several sample problems are given.

A copy of the Fortran source program from which HERESY 1 can be compiled is given in Appendix 1. A table of symbol definitions is included.

A short Fortran source program for the calculation of age-diffusion kernel functions is given in Appendix 2.

Section 2. Comparison of Heterogeneous and Homogeneous Calculations

A series of calculations has been carried out to compare heterogeneous and homogeneous reactor physics results for uniform graphite lattices. Such a comparison is of interest in showing how small a uniform lattice must be in order for heterogeneous effects to be important. While Feinberg(1) has shown that for an infinite uniform lattice heterogeneous and homogeneous calculations become identical, it is important to know the extent of the discrepancy for a finite lattice in order to judge whether homogeneous calculations are justified.

Section 2.1 Homogeneous Calculation Methods

Two group, two region homogeneous calculations have been carried out on the TRG Alwac computer for comparison with heterogeneous calculations. A diffusion theory code has been written for cylindrical geometry in order to facilitate this comparison. The method of calculation is straightforward(2) except for the way in which the reactivity is introduced. The principal problem in the comparison is to choose two group constants in both core and reflector which are the homogeneous counterparts of the age used in the slowing down kernel of the heterogeneous calculation. (This is a variant of the customary problem of choosing appropriate multigroup constants.) This problem is discussed in detail in Section 2.2. The details of the two group calculation are presented here.

The fuel rods are taken to be unclad metallic natural uranium with a radius of 2 centimeters and of infinite axial length. These rods are identical with those used in previous heterogeneous calculations. The rod spacing is 20 cm, center-to-center, in the lattice. The core region in the homogeneous calculation is defined as follows: A square lattice of N rods in the heterogeneous calculation will correspond to a homogenized core of circular cross section and infinite height with a cylinder radius

$$R_o = \sqrt{\frac{Na^2}{\pi}} \qquad (2.1)$$

where a is the square lattice pitch. The graphite reflector is taken as extending from the edge of the core to infinity, corresponding to the infinite reflector used in the heterogeneous calculation.

In the core the equations for the fast and thermal groups are

$$-D_1 \nabla^2 \phi_1 + \sigma_1 \phi_1 = k_\infty \sigma_2 \phi_2 \tag{2.2}$$

$$-D_2 \nabla^2 \phi_2 + \sigma_2 \phi_2 = \sigma_1 \phi_1 \tag{2.3}$$

where ϕ_1 is the flux in fast group

ϕ_2 is the flux in thermal group

D_1, D_2 are the diffusion coefficients in the fast and thermal groups, respectively

σ_1 is the macroscopic removal cross-section in the fast group

σ_2 is the macroscopic absorption cross section in the thermal group

k_∞ is the multiplication factor in the core.

The equations in the reflector are of the same form except that k_∞ is zero in the reflector.

We assume that the configuration is fixed, e.g., the core radius is fixed and that the reactivity of the configuration is desired. The reactivity, k, of the configuration will be introduced into (2.2) in the same way it appears in the heterogeneous equations. Let η_m be the actual number of neutrons produced per absorption in the fuel, let f be the thermal utilization for an infinite lattice and let p be the resonance escape probability for an infinite lattice. Then for the actual fuel elements

$$k^{(m)} = \eta_m pf \tag{2.4}$$

where the superscript denotes the multiplication factor for the actual lattice. The fast fission factor is incorporated into η_m. We now define the reactivity, k as

$$k \equiv \frac{\eta_m}{\eta_c} \tag{2.5}$$

where η_c is the value of η_m that would make a specific configuration critical. In order for equation (2.2) to be valid in the steady state for the given core radius, k in that equation should be

$$k_\infty^{o} = \eta_c pf \tag{2.6}$$

Rewriting equation (2.2) using (2.4), (2.5), and (2.6) one obtains

$$-D_1\nabla^2\phi_1 + \sigma_1\phi_1 = \frac{k_\infty^{(m)}}{k} \sigma_2\phi_2 \tag{2.2'}$$

We shall henceforth use k_∞ to denote $k^{(m)}$.

The fast flux equation in standard texts(2)(3) is not usually written like this because the core radius is adjusted for criticality while the procedure indicated here is equivalent to adjusting the η of the fuel.

The criticality equation is a standard one aside from the point above. One eventually solves a determinantal equation of the form

$$\begin{vmatrix} X & Y & -Z_1 & 0 \\ S_1X & S_2Y & -S_3Z_1 & -Z_2 \\ D_{1c}X' & D_{1c}Y' & -D_{1r}Z_1' & 0 \\ S_1D_{2c}X' & S_2D_{2c}Y' & -S_3D_{2r}Z_1' & -D_{2r}Z_2' \end{vmatrix} = 0 \qquad (2.7)$$

where X, Y, X', Y', S_1 and S_2 are functions of k. The subscripts 1 and 2 refer to the fast and thermal groups, respectively, and c and r refer to core and reflector. We shall define

$$\mu^2(k) = \frac{1}{2}\left[-\left(\frac{1}{\tau_c} + \frac{1}{L_{2c}^2}\right) + \sqrt{\left(\frac{1}{\tau_c} + \frac{1}{L_{2c}^2}\right)^2 + \frac{4(k_\infty - k)}{k\tau_c L_{2c}^2}} \right] \qquad (2.8a)$$

$$-\gamma^2(k) = \frac{1}{2}\left[-\left(\frac{1}{\tau_c} + \frac{1}{L_{2c}^2}\right) - \sqrt{\left(\frac{1}{\tau_c} + \frac{1}{L_{2c}^2}\right)^2 + \frac{4(k_\infty - k)}{k\tau_c L_{2c}^2}} \right] \qquad (2.8b)$$

where $\tau_c \equiv \frac{D_{1c}}{\sigma_{1c}}$

$$L_{2c}^2 = \frac{D_{2c}}{\sigma_{2c}}$$

τ_c is the effective age in the core and L_{2c} is the homogeneous diffusion length. If R_o denotes the core radius one can express the symbols in the determinant in terms of Bessel functions.

$$X = J_o(\mu R_o) \tag{2.9a}$$

$$X' = -\mu J_1(\mu R_o) \tag{2.9b}$$

$$Y = I_o(\nu R_o) \tag{2.9c}$$

$$Y' = \nu I_1(\nu R_o) \tag{2.9d}$$

$$S_1 = \frac{D_{1c}}{\tau_c D_{2c}} \frac{1}{\frac{1}{L_{2c}^2} + \mu^2} \tag{2.10a}$$

$$S_2 = \frac{D_{1c}}{\tau_c D_{2c}} \frac{1}{\frac{1}{L_{2c}^2} - \nu^2} \tag{2.10b}$$

$$S_3 = \frac{1}{\tau_r} \frac{D_{1r}}{D_{2r}} \left(\frac{1}{\frac{1}{L_{2r}^2} - \frac{1}{\tau_r}} \right) \tag{2.10c}$$

The quantities Z_1 and Z_2 in the determinantal equation refer to the reflector. They are defined in terms of τ_r, the average age in the reflector, and L_{2r}, the thermal diffusion length in the reflector.

$$\tau_r = \frac{D_{1r}}{\sigma_{1r}}$$

$$L_{2r}^2 = \frac{D_{2r}}{\sigma_{2r}}$$

Z_1 and Z_2 are defined in terms of Bessel functions of imaginary argument as follows:

$$Z_1 = K_o \left(\frac{R_o}{\sqrt{\tau_r}}\right) \qquad (2.11a)$$

$$Z_1' = -\frac{1}{\sqrt{\tau_r}} K_1 \left(\frac{R_o}{\sqrt{\tau_r}}\right) \qquad (2.11b)$$

$$Z_2 = K_o\left(\frac{R_o}{L_{2r}}\right) \qquad (2.11c)$$

$$Z_2' = -\frac{1}{L_{2r}} K_1\left(\frac{R_o}{L_{2r}}\right) \qquad (2.11d)$$

The homogeneous code gives as its outputs the reactivity k and the individual terms of the matrix of (2.7) for the value of k. From this one obtains the eigenvector components (a_1, a_2, a_3, a_4) of this matrix and the thermal flux in the core is constructed from them

$$\phi_{2c}(r) = S_1 a_1 J_o(\mu r) + S_2 a_2 I_o(\nu r) \qquad (2.12)$$

Section 2.2 Two Group Parameters

The independent two group parameters needed for this calculation are as follows: k_∞, τ_c, τ_r, L_{2c}, L_{2r}, and the ratios D_{1c}/D_{2c} and D_{1r}/D_{2r}. These parameters must be obtained from the parameters used in the heterogeneous calculation for a valid comparison: τ, L, σ_{aM}, η, A, γ. The parameters τ, L and σ_{aM} refer to the pure moderator. From A and γ one can calculate p and f, respectively, by the methods discussed in previous reports(4) of this series. Actually the heterogeneous procedure has been to invert this order. From recipe-formulas for f and p in graphite lattices γ and A have been calculated for the rod type in an infinite one component lattice. This calculation has proceeded by the self-consistent method in which kernel functions corresponding to the given values of τ, L and σ_a have been used. It has been shown that this procedure tends to minimize errors in the calculated reactivity.

We therefore assume that values of the heterogeneous parameters τ, L, σ_{aM}, η, A, γ, f and p are available and ask for methods of calculating the homogeneous parameters from them. The following formulas have been used in our calculations to relate the two-group homogeneous parameters to the heterogeneous parameters:

$$k_\infty \cong k_\infty^{(m)} = \eta . p . f \qquad (2.12)$$

Thermal group parameters:

$$L_{2c}^2 = L^2(1-f) \tag{2.13a}$$

$$L_{2r} = L \tag{2.13b}$$

$$D_{2r} \equiv \frac{1}{3\sigma_{tr}} = \sigma_{aM} L^2 \tag{2.13c}$$

$$D_{2c} = D_{2r} \tag{2.13d}$$

Fast group parameters:

$$D_{1r} = \frac{1}{3(\bar{\sigma}_{tr})_r} \tag{2.14a}$$

where $(\bar{\sigma}_{tr})_r$ is the lethargy average in the fast group of the transport cross section in the moderator

$$D_{1c} = \frac{1}{3(\bar{\sigma}_{tr})_c} \quad \text{where} \tag{2.14b}$$

$$(\bar{\sigma}_{tr})_c = \frac{V_u}{V_u + V_M}(\bar{\sigma}_{tr})_u + \frac{V_M}{V_u + V_M}(\bar{\sigma}_{tr})_M \tag{2.14c}$$

where V_u and V_M are the volume of fuel and moderator, respectively, in the core, and $(\bar{\sigma}_{tr})_u$ and $(\bar{\sigma}_{tr})_M$ are, respectively, the lethargy average in the fast group of the transport cross section in the fuel and in the moderator.

The homogenization rules (2.13) and (2.14abc) can be derived directly from procedures for averaging $\overline{\sigma}_{tr}$, the transport cross section and $\overline{\sigma}_a$, the mean absorption cross section

$$\overline{\sigma}_a = \frac{\phi_u V_u \sigma_{au} + \phi_M V_M \sigma_{aM}}{\phi_u V_u + \phi_M V_M} \qquad (2.15)$$

where σ_{au} is the thermal absorption cross section in the fuel and where ϕ_u and ϕ_M are the mean flux values in the fuel and moderator, respectively. Now the thermal utilization, f, is defined as

$$f = \frac{\sigma_{au}\ \phi_u\ V_u}{\sigma_{au} V_u \phi_u + \sigma_{aM} V_M \phi_M} \qquad (2.16)$$

Equation (2.15) can be rewritten in terms of f

$$\overline{\sigma}_a = \frac{\sigma_{aM}}{1-f}\ \frac{\phi_M V_M}{\phi_u V_u + \phi_M V_M} \qquad (2.17)$$

The homogeneous mean transport cross section in the thermal group can be obtained by taking a similar space average

$$\overline{\sigma}_{tr} = \frac{\phi_u V_u \sigma_{tr,u} + \phi_M V_M \sigma_{tr,M}}{\phi_u V_u + \phi_M V_M} \qquad (2.18)$$

The thermal diffusion coefficient is then given by

$$D_{2c} = \frac{1}{3\overline{\sigma}_{tr}} = \frac{1}{3\sigma_{tr,M}}\ \frac{1 + \dfrac{\phi_u V_u}{\phi_M V_M}}{1 + \dfrac{\phi_u V_u \sigma_{tr,u}}{\phi_M V_M \sigma_{tr,M}}} \qquad (2.19)$$

The homogeneous thermal diffusion length is obtained from

$$L_{2c}^{2} = \frac{D_{2c}}{\bar{\sigma}_a} = L^{2}(1-f)\ \frac{\left[1 + \dfrac{\phi_u V_u}{\phi_M V_M}\right]^{2}}{1 + \dfrac{\phi_u V_u \sigma_{tr,u}}{\phi_M V_M \sigma_{tr,M}}} \tag{2.20}$$

For a loosely packed lattice $\phi_M V_M >> \phi_u V_u$ and therefore

$$L_{2c}^{2} \approx L^{2}(1-f)$$

$$D_{2c} \approx \frac{1}{3\sigma_{tr,M}}$$

Section 2.3 Multigroup Schemes

The homogeneous parameters given above do not uniquely define a multigroup calculation. For one thing the parameters τ_c and τ_r have not yet been given, since they depend on the choice of a multigroup scheme. The choice of multigroup scheme may be important since the reactivity and flux may depend considerably on the multigroup scheme that is used.

By a multigroup scheme one means the relation between the average flux in a group and the flux at the upper and lower lethargy limits of the group. Thus for a two group calculation in which the fast group extends from U = 0 to U = U_{th} one must assume the relation

$$\frac{1}{U_{th}} \phi_f(r) = f_1 \phi(r,o) + f_2 \phi(r,U_{th}) \qquad (2.21)$$

where $\phi_f(r)$ is the total fast flux in the fast group of width U_{th} in lethargy, $\phi(r,o)$ is the flux per unit lethargy at the lower lethargy (upper energy) limit and $\phi(r,U_{th})$ is the flux per unit lethargy at the upper lethargy (lower energy) limit, just above thermal energy. The constants f_1 and f_2 characterize the multigroup scheme. The proper choice of f_1 and f_2 becomes more important as the lethargy width of the group increases because these parameters describe the lethargy variation of the flux through the group. Thus while for a many group scheme the proper choice of f_1 and f_2 may not be very important, for a two group scheme in which the lethargy variation of the flux through the fast group is considerable, the proper choice of f_1 and f_2 may be important. Hence, in comparing heterogeneous calculations with two group calculations one should examine the result for various sets of values of f_1 and f_2.

In order to do this it is convenient to derive the fast neutron equation of two group theory from Fermi age theory. One can write the age equation in the following form:

$$D_f \nabla^2 \phi = \xi \sigma_s \frac{\partial \phi}{\partial u} \tag{2.22}$$

within a lethargy interval containing no sources. If both sides of (2.22) are integrated from $U = 0_+$ (just below the source lethargy) to U_{th} one obtains the following equation, assuming cross sections are constant in energy

$$D_f \nabla^2 \phi_f = \xi \sigma_s \left[\phi(r, U_{th}) - \phi(r, 0_+) \right] \tag{2.23}$$

where ϕ_f is the fast flux, defined as

$$\phi_f(r) = \int_{0+}^{U_{th}} \phi(r, u)\, du \tag{2.24}$$

If one substitutes for $\phi(r, U_{th})$ in (2.23) from equation (2.21) one obtains

$$D_f \nabla^2 \phi_f = \frac{\xi \sigma_s}{f_2 U_{th}} \phi_f - \left(1 + \frac{f_1}{f_2}\right) S(r) \tag{2.25}$$

where $S(r) = \xi \sigma_s \phi(r, 0_+)$ is the source for the fast flux, which is given by

$$S(r) = k_\infty \sigma_a \phi_s \tag{2.26}$$

where ϕ_s is the thermal flux and σ_a is the thermal absorption cross section. For constant cross sections

$$\tau = \frac{DU_{th}}{\xi\sigma_s}$$

hence (2.25) can be rewritten as follows

$$\nabla^2\phi_f - \frac{1}{f_2\tau}\phi_f = -\left(1 + \frac{f_1}{f_2}\right)\frac{k_\infty}{L_{2c}^2}\phi_s \cdot \frac{D_s}{D_f} \qquad (2.27)$$

where D_s is the thermal diffusion coefficient. The thermal equation is given by

$$-D_s\nabla^2\phi_s + \sigma_a\phi_s = \xi\sigma_s\phi(r, U_{th}) \qquad (2.28)$$

$\phi(r, U_{th})$ can be eliminated from this equation by using (2.21) in the form

$$\phi(r, U_{th}) = \frac{1}{f_2}\frac{\phi_f}{U_{th}} - \frac{f_1}{f_2}\frac{k_\infty\sigma_a\phi_s}{\xi\sigma_s} .$$

Inserting this expression into (2.28) and collecting terms one obtains

$$-\nabla^2\phi_s + \left(1 + k_\infty\frac{f_1}{f_2}\right)\frac{\phi_s}{L_{2c}^2} = \frac{\phi_f}{f_2\tau} \cdot \frac{D_f}{D_s} \qquad (2.29)$$

Equations (2.27) and (2.29) are the two group equations corresponding to the f_1, f_2 scheme. The reactivity, k, is calculated from these equations by replacing k_∞ wherever it occurs by k_∞/k, according to our previous procedure. One can then define new parameters to make the set (2.27) and (2.29) formally equivalent to the previous two group equations (2.2) and (2.3) which correspond to the scheme $f_1 = 0$, $f_2 = 1$. We shall define

$$\overline{L}_c^{\,2} = \frac{L_{2c}^2}{1 + \frac{k_\infty}{k} \frac{f_1}{f_2}} \tag{2.30a}$$

$$\overline{L}_r^{\,2} = L_{2r}^2 \tag{2.30b}$$

$$\frac{1}{g} = \frac{1}{k} \cdot \frac{1 + f_1/f_2}{1 + \frac{k_\infty}{k} \frac{f_1}{f_2}} \tag{2.30c}$$

$$\tau_c = f_2 \tau \tag{2.30d}$$

$$\tau_r = f_2 \tau \tag{2.30c}$$

Equations (2.27) and (2.29) then take the following form in the core

$$- \nabla^2 \phi_f + \frac{\phi_f}{\tau_c} = \frac{k_\infty}{g} \frac{D_s}{D_f} \frac{\phi_s}{\overline{L}_c^2} \tag{2.31a}$$

$$- \nabla^2 \phi_s + \frac{\phi_s}{\overline{L}_c^2} = \frac{D_f}{D_s} \frac{\phi_f}{\tau_c} \tag{2.31b}$$

In solving equations (2.31) using the Alwac code one can take advantage of the fact that $\overline{L}_c^{\,2}$ is relatively insensitive to k. A guess is made for k and a value of $\overline{L}_c^2$ is computed. This value is used in the code to compute g. The actual reactivity is then computed from (2.30c)

$$k = g(1 + \frac{f_1}{f_2}) - k_\infty \frac{f_1}{f_2} .$$

If this value of k differs greatly from the previous guess, the calculation is iterated.

A number of f_1, f_2 schemes can be considered. The conventional scheme uses $f_2 = 1$, $f_1 = 0$, but this scheme is quite arbitrary. Various schemes can be generated by assuming a variation of $\phi(r,U)$ with U. For example, if one assumes a linear variation with U

$$\phi(r,U) = \phi(r,o) \left[1 - \frac{U}{U_{th}}\right] + \phi(r,U_{th}) \frac{U}{U_{th}} \qquad (2.32)$$

and substitutes this expression in (2.24) one obtains (2.21) with $f_1 = f_2 = \frac{1}{2}$.

Another approach is to use the age theory equation (2.22) subject to the condition that

$$\nabla^2\phi + \mu^2\phi = 0 \qquad (2.33)$$

which is valid in the central core region. Then substituting (2.33) into (2.22) and solving for $\phi(U)$, one obtains,

$$\phi(U) = Ce^{-a^2U} \qquad (2.34)$$

where C is a constant and $a^2 = \frac{D\mu^2}{\xi\sigma_s}$. Substituting this result into (2.24) and integrating we evaluate C in terms of ϕ_f, obtaining finally

$$\emptyset(U) = \left(\frac{\emptyset_f \quad a^2}{1-e^{-a^2 U_{th}}} \right) e^{-\ a^2 U} \qquad (2.35)$$

Comparing (2.35) with (2.21) one obtains

$$f_1 = \frac{1-e^{-\mu^2\tau}}{\mu^2\tau} \quad - f_2 e^{\mu^2\tau} \qquad (2.36)$$

One can choose f_2 arbitrarily as long as it leads to a non-negative value of f_1. Thus if $\mu^2\tau = \frac{1}{5}$, the following pairs of f values are given by (2.36)

$f_2 = 1$, $\quad f_1 = -0.3$ not allowable

$f_2 = .75$ $\quad f_1 = 1$

$f_2 = .5$ $\quad f_1 = .31$

$f_2 = .4$ $\quad f_1 = .49$

Section 2.4 Heterogeneous and Homogeneous Comparisons

Homogeneous calculations were performed on the standard 5 by 5, 9 by 9, and 11 by 11 natural uranium graphite lattices previously analyzed by HERESY 1 calculations.(5) Homogeneous calculations have also been carried out for the various Brookhaven natural uranium graphite configurations.(6) In these calculations the scheme $f_2 = 1$, $f_1 = 0$ was used. However, the sensitivity of the homogeneous calculations to the f_1, f_2 scheme was checked.

The results for the reactivity in the first set of calculations and a comparison with HERESY 1 heterogeneous results are given in Table 1. The results for the reactivity for the Brookhaven lattices and a comparison with both experiment and with HERESY 1 heterogeneous calculations are given in Table 2.

A comparison of the thermal flux shapes in the core as calculated by the homogeneous and heterogeneous methods is given for the 9 by 9 lattice in Figure 1 and for the 11 by 11 lattice in Figure 2.

The reactivity results in Tables 1 and 2 indicate that for the larger natural uranium graphite lattices (with 100 rods or more) the homogeneous and heterogeneous calculations agree to within 0.1%. However, for lattices with a smaller number of rods the homogeneous reactivities turn out higher. In the 5 by 5 lattice the homogeneous reactivity is higher by over 2%

Table 1

Homogeneous Results and Comparisons for First Set

No. Rods	k_{het}	k_{hom}	
25	.843	.862	5 x 5 lattice
81	.977	.982	9 x 9 lattice
121	1.006	1.009	11 x 11 lattice

Table 2

Homogeneous Results and Comparisons for Brookhaven Lattices

No. Rods	k_{exp}	k_{het}	k_{hom}
100	.902	.912	.913
144	.936	.943	.945
196	.961	.967	.966
256	.978	.982	.982

From figures 1 and 2 one concludes that the thermal flux shapes in the core calculated by homogeneous or heterogeneous methods are qualitatively similar. However, the homogeneous fluxes fall off somewhat more slowly from core center to edge than the heterogeneous results. In addition, the homogeneous fluxes show a characteristic upturn near the core edge which do not appear in the heterogeneous results. The scatter in the heterogeneous fluxes near the core edge is due to inclusion of rods along the diagonal of the square array as well as rods on the perimeter.

Alternate f_1, f_2 schemes were tested in the homogeneous calculations on the 9 by 9 lattice to determine the sensitivity of the reactivity to the f_1, f_2 scheme. The results are given in Table 3A. From this table one sees that for reasonable f_1, f_2 schemes the reactivities may vary through 0.5%. Similar calculations were done for cases in which the reflector was strongly poisoned. Here one finds an even greater variation of reactivity with the f_1, f_2 scheme.

It should be pointed out that there are two significant differences between heterogeneous calculations and multigroup calculations. One is the obvious difference in geometry. The other is the difference in the slowing down treatment. The sensitivity of k to the multigroup scheme for homogeneous calculations reflects the difference in the slowing down treatment.

Table 3A

Reactivity for 9 by 9 Lattice for Various f_1, f_2 Schemes

(Pure Graphite Reflector)

f_1/f_2	k
0	.982
1	.984
2.333	.987
9	.957

Table 3B

Reactivity for 9 by 9 Lattice for Various f_1, f_2 Schemes

(Poisoned Graphite Reflector)

f_1/f_2	k
0	.909
1	.896
2.33	.890
9	.875

When the number of groups is large and when reasonable schemes have been arrived at, this latter difference may not be as important as it is here.

We draw the following tentative conclusions from this comparison:

1. For uniform lattices the two group homogeneous calculation and the heterogeneous calculations are qualitatively similar, especially as the lattice becomes larger, e.g., for 9 by 9 lattices in graphite.
2. There are discrepancies in the flux pattern near the core edge.
3. The two group results show some sensitivity to the f_1, f_2 scheme.

Fig. 1. Ø(r) vs r for 9 x 9 lattice

het ⊡

hom ⊙

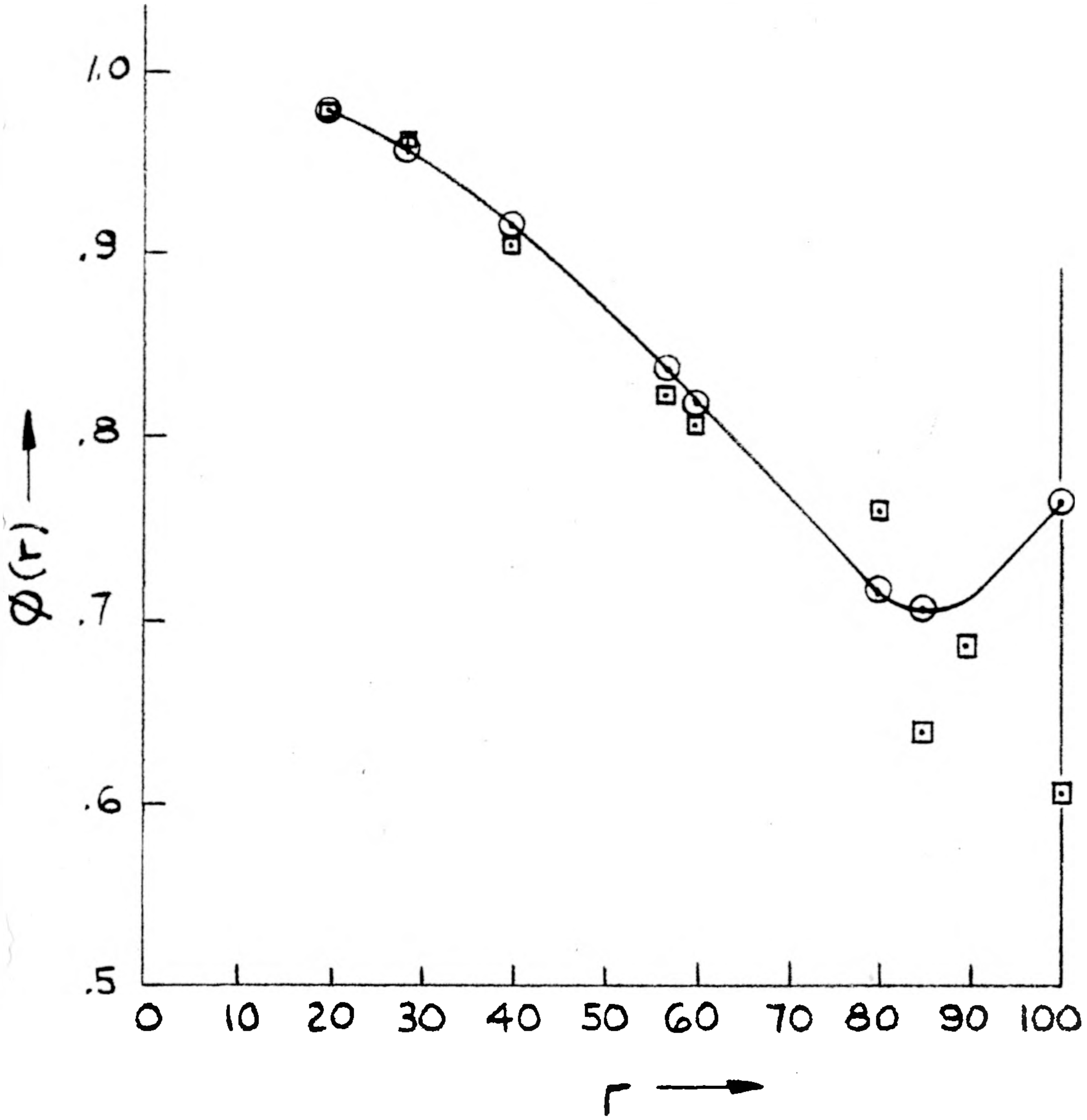
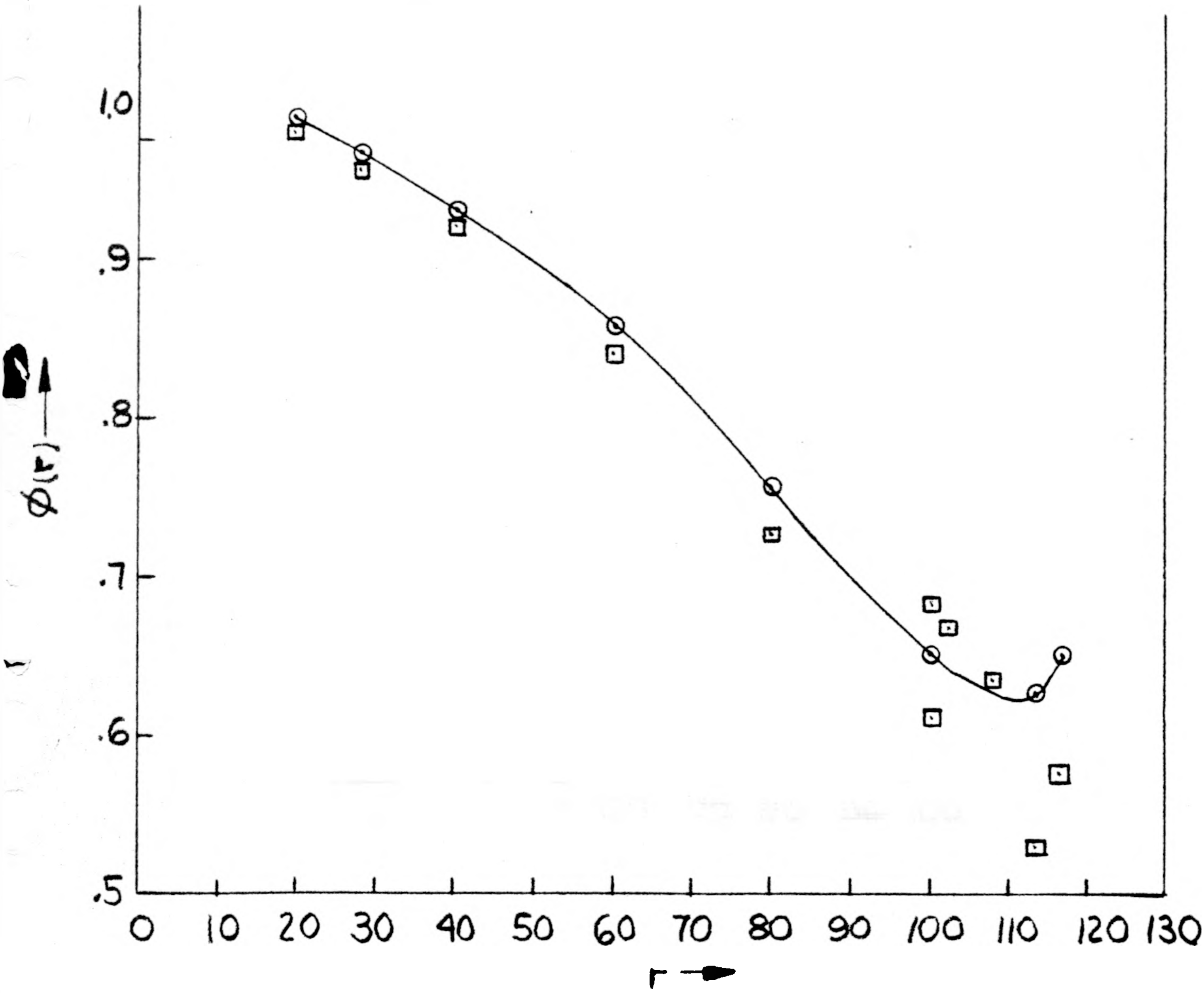


Fig. 2. $\phi(r)$ vs r for 11 by 11 lattice

het ⊡

hom ⊙



Section 3. HERESY 2 Resonance Sensitivity Calculations

A number of schematic calculations have been performed with the **HERESY 2** code to determine the sensitivity of reactivity and power pattern in a uranium graphite lattice to the characteristics of resonance absorption.

One set of calculations studied the effect of the neutron energy at which the resonance absorption takes place. These calculations (which we designate A) indicate an appreciable dependence of the reactivity on the location of the resonance in energy.

A second set of calculations (which we designate B) studied the effect of resonance fissions on reactivity. These calculations indicated that the effect is considerable, between one and two percent.

A third set of calculations (designated C) studied the effect of the neutron energy at which the resonance fissions take place. These calculations indicated that the effect of varying the energy at which the fission resonance is found is relatively small.

The calculations will be described in detail. However, no wide generalizations can be inferred from them. Only a narrow range of uranium graphite lattices was studied, characterized by low enrichment and large moderator to fuel volume ratios. The rod parameters used were schematic rather than realistic. The calculations did not take account of interference effects between resonances. Nevertheless the results are of conceptual interest because they indicate that heterogeneous reactor calculations are sensitive to these details of the resonance structure.

The motivation for these calculations is the following: If one expects a high degree of accuracy from heterogeneous calculations one must take into account any physical effect which will effect reactivity by a fraction of a percent.

One such effect is the energy of the resonance. The higher the resonance energy the less important the absorbed neutron would be in causing fissions since the likelihood of its leakage is greater. As fuel to moderator volume ratio increases the probability of resonance absorption increases, and the more important one would expect the effect of resonance energy to be.

Another important effect is that of resonance fission. This may change the reactivity by a few percent in thermal or epithermal reactors, depending on the fuel enrichment and on the fuel-moderator volume ratio. The importance of the competition between resonance fission and thermal fission is that at thermal a larger fraction of the neutrons may be captured parasitically (in fuel, cladding and moderator) than at resonance energies, and therefore resonance fission may contribute more to reactivity. On the other hand η is generally lower at resonance energies. The resonance fission factor β gives the ratio

$$\beta = \frac{\text{reactivity with fission resonances included}}{\text{reactivity with fission resonances omitted}} .$$

An important effect has been omitted from HERESY 2 calculations to date. This is interaction between resonances or interference effects between resonances. The resonances at higher energies deplete the slowing down density at the lower resonance energies. When resonance absorption is appreciable and when resonance fission is an important contributor to reactivity, this effect may alter the reactivity by several percent, and may also alter the power distribution. Interference effects between resonances can be properly taken into account in the slowing down kernels but the calculation is quite onerous. A formalism for this calculation has been given in detail in a previous report in this series.

Calculation Set A

The effect of the resonance energies was studied by dividing one lumped resonance at $\tau_r = 262.5\ \text{cm}^2$, $A = 51\ \text{cm}^2$ into four equal absorption resonances at different energies.

$\tau_1 = 87.5\ \text{cm}^2$	$A_1 = 12.75$
$\tau_2 = 175\ \text{cm}^2$	$A_2 = 12.75$
$\tau_3 = 262.5\text{cm}^2$	$A_3 = 12.75$
$\tau_4 = 325\ \text{cm}^2$	$A_4 = 12.75$

The lattices studied were the 5 by 5, 7 by 7 and 9 by 9 lattices previously discussed. The reactivity results are given in Table 4A. The power distribution is substantially unaltered.

Table 4A

Effect of Energy of Resonance Absorption on Reactivity

5 by 5 Lattice

	One Resonance	Four Resonances	$\frac{\Delta k}{k}$
k	.8428	.8394	-.0040
η	1.34	1.34	
p	.9117	.9058	
f	.6899	.6915	

7 by 7 Lattice

	One Resonance	Four Resonances	$\frac{\Delta k}{k}$
k	.9279	.9255	-.0026
η	1.34	1.34	
p	.8998	.8957	
f	.7696	.7711	

9 by 9 Lattice

	One Resonance	Four Resonances	$\frac{\Delta k}{k}$
k	.9772	.9736	-.0037
η	1.34	1.34	
p	.8927	.8897	
f	.8167	.8167	

Calculation Set B

The effect of resonance fission on reactivity has been studied by comparing two situations, in one of which the rods had a single lumped absorption resonance at $\tau_r = 262.5\ \text{cm}^2$, $A = 51\ \text{cm}^2$ and in the other situation the rods had this lumped absorption resonance plus a lumped fission resonance at $\tau_f = 275\ \text{cm}^2$, $A_f = 25.5\ \text{cm}^2$. The rods were assumed to have 1.3% enrichment with $\eta = 1.605$, $\gamma = .178$. Thus the situation without resonance fission is identical to some previous calculations using HERESY 1. The fission η was also taken to be 1.605.

The lattices studied were the 5 by 5, 7 by 7, and 9 by 9 lattices previously discussed. The reactivity results are given in Table 4B. Note that the effect of resonance fission is to decrease p and to decrease f, while increasing k. The decrease in p is due to the additional resonance absorption by the fission resonances in U^{235}. The decrease in f is due to the diminution of thermal neutrons in the core (where fuel absorption predominates) while thermal neutron flux in the reflector (where only moderator absorption takes place) is not diminished. β values of about 1.015 are obtained, indicating that resonance fission increases reactivity by about 1.5%. It should be pointed out that when these β values are used in a five factor formula

$$k_\infty = \eta\ \epsilon\ p\ f\ \beta$$

the p and f values are those for the situation <u>without</u> inclusion of fission resonances.

Calculation Set C

The effect of the energy of the fission resonances has been studied by comparing situations in which the fission resonance A value quoted above is divided among two and four fission resonances at different energies. This calculation was done for the lattices enumerated above. The maximum variation in reactivity was about 0.2%.

Table 4B

Effect of Resonance Fission on Reactivity

5 by 5 Lattice β = 1.016

	Without Resonance Fission	With Resonance Fission
k	1.041	1.058
p	.9123	.8682
f	.7077	.7052

7 by 7 Lattice β = 1.014

k	1.138	1.155
p	.9001	.8503
f	.7878	.7829

9 by 9 Lattice β = 1.013

k	1.192	1.208
p	.8937	.8397
f	.8263	.8276

Section 4. Instructions for using HERESY 1

HERESY 1 is written for the IBM 704 with a 32 K memory. A description of the mathematical and physical basis of the code is given in a previous report,(5) which gives a detailed account of the computation scheme. The code was written using the Fortran II interpretive compiler. The Fortran II source program is given in Appendix 1 of this report. In this section we present the operating instructions and the input preparation.

Section 4.1 HERESY 1 Operating Instructions

This section explains the details of the following sequence of operations, which constitute the actual execution of the problem runs.

1. Select and ready tapes
2. Set Sense Switches
3. Place program deck in card reader, 9-edge in, face down
4. Place input deck in card reader, 9-edge in, face down
5. Set to Automatic the automatic-manual switch
6. Depress CLEAR resetting magnetic core to zeros
7. Press LOAD CARDS
8. Any number of problems may be run in succession since the program automatically returns to the initial instruction (read series 1000 cards)
9. Read on-line output
10. Save tape 3 and print off-line output.

If HERESY 1 is on a magnetic tape, replace steps 3 and 7 with 3' and 7'.

3'. Ready the program tape on tape logical unit 1
7'. Press LOAD TAPE.

Item 1. Ready Tapes

Logical Tape No.	When Necessary to put in Ready Condition	Purpose	Save or Temporary Tape	Mode Written in
1	When HERESY Code is on tape	Read Machine Language HERESY Code	Save	Binary
	When changing this tape	Change Tape	Save	Binary
	When making a copy of HERESY on logical Tape 1 from cards	Make a tape copy of code	Save	Binary
3	All problems	Output	Temporary	Hollerith or BCD

Item 2. Set Sense Switches

At start of problem switches 1, 2, 3, and 5 are normally in the UP position

Switch	When Used	Up	Down
1	Changing the HERESY Tape or producing another copy of HERESY Tape	(Normal Position)	Writes on any logical tape 1 the contents of core memory before input is read in.*
2	After matrix inversion failure the number of performed iterations is equal to N_1 but $\underline{L}_i \underline{L}_i^{-1} \neq I$ to S_f significant figures, where $E_1 = 10^{-S_f}$. Machine stops at Pause 2	Retain E_1 convergence criterion	$E_1 \times 10 \rightarrow E_1$ There is a reduction of one significant figure after each Pause 2
3	Matrix $\underline{L}$ inversion failure	Prints $\underline{L}$ once, and E_1, IT, $\underline{L}.\underline{L}^{-1}$ afterwards	adds $\underline{L}^{-1}$ to the inversion failure printout
4	Infinite lattice computation	Finite lattice	Infinite lattice calls for JMAX, radius $\Delta x, \Delta y$, x_1, y_1 as input from cards
5	After eigenvalue determination failure the number of performed iterations = the limit N_2, but V2(I),I=1,2,...,JMAX does not equal V1(I),I=1,2,..., JMAX to S_F significant figures, where $E_2 = 10^{-S_F}$	E_2 convergence condition retained	$E_2 \times 10 \rightarrow E_2$ There is a reduction of one signficant figure after each Pause 5
6	Not presently used	-	-

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* The tape is actually written after kernel functions are read in and hence saves the current kernel functions permanently. Transfer control to location $151)_8$ to avoid reading new kernel functions.

Item 3. Error Stops

The following table explains the 5 error conditions detected by HERESY 1.

If the machine stops, note the contents of the Storage Register, which will contain the instruction

$$(042\ 000\ 000\ \underline{aaa})_{octal}.$$

The last 3 octal digits aaa are listed in numerical order below.

Identification of pause: aaa	Explanation of Pause	Correction Procedure or Procedures
002	EL(I,J) matrix inversion failure. The number of iterations performed is equal to the limit N1 but $L.L^{-1} \neq$ to I to S_F significant figures where $E_1 = 10^{-S_F}$.	Depress sense switch 2, if necessary, to reduce the number of required significant figures by 1. Press Start button. When Start is pressed, two changes automatically occur: (1) N1 + 3 → N1 (2) the machine prints EL(I,J) once, and E1,IT,RLT(I,J) each iteration. See Item 2, "Set Sense Switches", and Symbol Table Appendix 1(B).

Identification of pause: aaa	Explanation of Pause	Correction Procedure or Procedures
005	Determination of eigenvector and eigenvalue of the matrix RLG(I,J) has failed. The number of iterations performed is equal to the limit N2, but V2(I) I=1,2,...JMAX does not equal V_1(I), I=1,2,...,JMAX, to S_F significant figures, where $E_2 = 10^{-S_F}$	Depress Sense Switch 5, if necessary, to reduce the number of significant figures required by 1. Press Start button. When the start button is pressed, two changes automatically occur: (1) N2 + 3 → N2 (2) the machine prints out IT2, the number of performed iterations; FK2, the eigenvalue; V2(I) I=1,2,...,JMAX- the current eigenvector VRAT(I) I=1,2,..., JMAX, the current eigenvector normalized so that its maximum component is 1.
006	Input of GAMMA(I) I=1,2,...,JMAX table has a zero value. Inverse Gamma matrix can not be computed	Check GAMMA(I) I=1,2,...,JMAX table. Replace entries construed as zero with correct values. Load the card reader with first card (1001 series) of current problem. Depress Start button. Machine will rerun this problem.

Identification of pause: aaa	Explanation of Pause	Correction Procedure or Procedures
778	distance (D) between the representative receiver rod and the present neutron donor rod is too large for correct interpolation, i.e. $D \geq H(IJMAX-2)$	H, the change in argument of the Kernel Function Table must be increased and/or geometric data must be changed in input. Pressing Start will ignore this rod-to-rod* contribution.
780	distance (D) between the representative receiver rod and the present donor rod is too small for correct interpolation, i.e. $D < 2H$ This pause will also be reached if 2 different rods are found to occupy the same spacial position.	H, the change in argument of the kernel function table must be decreased so that the minimum non-zero distance $D \geq 2H$ and/or geometric data must be changed in input. Do not continue without performing* the above changes.

Note that at the end of a normally completed HERESY 1 run the machine asks for a new problem input deck.

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* Transfer control to location $64)_8$ to read in kernel functions and continue.

Item 4. HERESY 1 Decks

To run HERESY 1 problems one requires the following:

1. HERESY 1 program (260 cards)*
2. Kernel function deck (61 cards)
3. Problem decks consisting of
    title card
    input

   for each problem.

Thus having the HERESY 1 program deck one needs two kinds of data

1) kernel function deck
2) problem decks.

These are punched on Hollerith (BCD) cards unlike the program deck which is on binary cards.

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*If HERESY is on tape logical unit 1, the program deck is not inserted.

Section 4.2 Kernel Function Deck Preparation

Kernel function tables must be loaded before the data of the first problem to be run by means of a kernel function card deck.

This deck consists of the following data:

1) The number of entries IJMAX in a kernel function table (usually is 60, which is the maximum number of entries)
2) The change in argument distance Δh
3) The $F(r)$ table
4) The $F^{res}(r)$ table
5) The $f(r)$ table
6) The $g^{res}(r)$ table.

The first entry in each table is always for the distance $r= 0$. The n-th entry is for the distance $(n-1)$ h. There is a maximum of 60 entries allowable for each table so that the maximum distance is $r_{max} = 59$ h. A sample kernel function card preparation sheet is found in Figure 9, Appendix 2. Appendix 2 completely describes a kernel function program that will compute a 1, 2, or 3 Gaussian fit of age-diffusion kernels.

Second and Successive Cards

The remaining cards of kernel function input data are in floating point notation. Each card contains 4 fifteen column wide fields, defined by columns 1-15, 16-30, 31-45, and 46-60. The cards beyond the first card carry successively without any gaps within a table, or between tables, the remainder of the F(r) table for r = 3H, 4H, ..., (IJMAX-1)H; followed immediately by the $F^{res}(r)$ for r = 0,H,2H,...,(IJMAX-1)H; f(r) for r = 0,H,2H, ...,(IJMAX-1)H; $g^{res}(r)$ for I = 0,H,2H,...,(IJMAX-1)H, tables. There are IJMAX + 1 cards for kernel function input.

Note that $5 \leq \text{IJMAX} \leq 60$.

Section 4.3 Problem Deck Input

Input to HERESY 1 is organized into five series of cards:

Series	General Description
1001	Titles
200N	Geometrical and control data
300N	Convergence & composition data
400N	Geometry data
500N	Number of rods/rod type for each rod type

It is the purpose of this section to explain in careful detail the preparation of input cards within these five categories. At the end of the section is a concise table of input instruction for HERESY 1.

Note that on all input cards in HERESY 1, columns 77-80 inclusive are used to carry the card series number. This number is used only for identification purposes. As an example the second card of the 400N (Geometry Input) series is shown below:.

Column number	01				72	73	74	75	76	77	78	79	80
Contents in the column			(numerical data)			T	R	G	P	4	0	0	2

Identification of cards within series.

Columns 73-76 inclusive may be used for further identification.

Note that all fields begin in the first column unless otherwise stated.

Series 1001

This series consists of 1 card carrying 71 characters of alpha-numeric information. Column 1 must be left blank. Columns 2 to 72 inclusive may contain any capital alphabetic letters, all decimal digits, and the special symbols (blank)

.   ,   +   -   *   (   )   /   & and $

This card carries a title for identification purposes in output. Arrange the 1001 card so that a) the number of the problem being identified has its last digit in approximately column 23, and b) the last figure of the date is in approximately column 32. Space from 33 to 72 is available for description.

There is 1 card in this series, identified in columns 77-80 as 1001. THE FORMAT STATEMENT IS (72H).

Series 200N

All numbers in the 200N series of input cards are in integer notation. Each card is divided into 14 successive fields of five columns width and within each field the integers are punched so that the units place occupies the extreme right hand position*.

This series consists of JMAX, the number of different rod types, followed by a table KMAX(I), I=1,2,...,JMAX entered

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*Caution:in integer notation,blanks are regarded as zeroes.

successively. KMAX(I) is the number of individual rods in rod type I. This is followed by N1, the maximum number of iterations permitted within the matrix inversion routines; and lastly by N2, the maximum number of iterations permitted within the eigenvalue determination routine.

Hence the card 2001 has JMAX in columns 1-5, KMAX (Rod Type 1) in columns 6-10, KMAX (Rod Type 2) in columns 11-15,etc. The remainder of the KMAX table follows successively through the remaining 5 column fields, continuing if necessary onto columns 1-5 of the next card in this series. The maximum iteration counts N1, and N2 follow without gap in the next 2 integer fields.

There are $N_{1000} = \left\lceil \frac{JMAX + 3}{14} \right\rceil^{*}$ cards in this series.

The FORMAT STATEMENT is 14I5

Series 300N

This series consists entirely of numbers in floating point notation. Each card contains 4 fifteen column wide fields, defined by columns 1-15, 16-30, 31-45, and 36-60. Convergence criteria E1, E2, and E3 are loaded onto the first 3 fields of the first card 3001. E1 determines the number of significant figures S_f obtained within the matrix inversion routines, where $E = 10^{-S_f}$. E2 determines the number of significant figures obtained in the eigenvector determination routine. E3 determines the

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* $\lceil x \rceil$ is the smallest integer $\geqq$ x.

First Card of Kernel Function Input to HERESY 1

a. Columns 1-5(inclusive) hold the number of entries for each of the four kernel tables, IJMAX

b. Columns 6-20 give the change in distance argument Δh in floating point form, H

c. Fields (21-35), (36-50), (51-65) carry the first entries of the F(r) table.

Footnote:

a. The number of entries is in integer data form (without a decimal point) with the least significant figure in column 5. Non leading blanks are interpreted as zeros.
b. and c. These are in floating point number form (with decimal points) with the exponent taking the last 4 figures to the extreme right in the respective fields.

Numbers for floating point (E) conversion need not have 4 columns used for the exponent field. The start of an exponent field must be marked by an E, or if that is omitted by a + or - (not a blank).

Identical permissible exponential fields are

E+03, E 03, E03, E+3, E3, +03, +3

In this code all numbers are in integer form (no decimal point) or in floating point form.

Useful Notes:

1. + signs may be omitted
2. The last digit in the exponent field (E±ii) must be in the last column provided for the number
3. Exact zeroes may be entered by all blanks

number of significant figures obtained in an iteration to system criticality. The last entry on card 3001 is the Gamma for rod type 1. The remaining cards carry without gap the remainder of the Gamma(I) table for I = 2,3,.... JMAX followed successively by Eta(I) for I = 1, 2, ...JMAX; $V_o(I)$ for I = 1,2,...JMAX; W(I) for I = 1, 2,... JMAX; A(I) for I = 1, 2, ... JMAX; and SMFO(I), for I = 1, 2, ... JMAX. The number of cards in this series, $(N_{3000}) = \left[\frac{3 + 6JMAX}{4}\right]^*$. The FORMAT is 4E15.4

400N Series

The 400N series consists of position data for rods within the reactor. Two options exist within HERESY 1 for obtaining coordinate data as defined by setting sense switch 4. If sense switch 4 is left in the up position, option U is chosen. If sense switch 4 is placed in the down position, option D is taken.

Arbitrary Configurations of Rods, Option U, Sense Switch 4 up

In this option all numbers are in floating point notation where each number occupies a 12 column wide field. Arbitrary configurations can be read directly from input cards, where each coordinate is multiplied by a number (DIMEN) to give correct lattice spacing. This latter value may be used to expand or contract standardized geometric systems carried on labelled decks for this code where the minimum rod-center to rod-center distance on the standardized deck is equal to one.

The U4001 card always contains only this single floating point number, DIMEN, within columns 1-12.

The following cards contain 5 twelve column wide fields, defined by columns 1-12, 13-24, 25-36, 37-48, and 49-60. The first cards in this format contain the x-coordinates without gap for all the rods in rod type 1. Beginning on a new card, the y-coordinates for rods of rod type 1 are then read in without gap in exactly the same individual rod order as chosen for the x-coordinate data. Note that the sequence of individual rods within the rod type is completely arbitrary, however, once a sequence is chosen for the x-coordinates, it must be followed for the y-coordinates. This form of read-in is followed successively for x,y coordinates for each rod type 1, 2, ...,JMAX, the first x-coordinate of each rod type starting a new card and the first y-coordinate of each rod type beginning a new card. The number of cards in this series, N_{4000} ,$= 1 + 2\sum_{I=1}^{JMAX}\left[\frac{KMAX(I)}{5}\right]^{*}$ numbered in columns 77-80 as 4001, 4002, ... 400N. The Format is (5E12.5).

Infinite Lattices, Option D, Sense Switch 4 Down

This option is used to compute the reactivities of infinite complex lattices consisting of 2 or more rectangular interstitial lattices of infinite extent. The rod compositions and/or lattice pitches may vary among the individual rectangular arrays.

The integer JMAX is the first input quantity on card D4001 and is entered in the integer field defined by columns 1-5. This is followed by the floating point number RADIUS in columns 6-20. RADIUS is the radius of a circle located at the origin such that any rods outside the periphery of this circle will give negligible flux contributions to representative receiver rods located nearest to our coordinate origin.

The second and following cards of the 400N series option Down, consist entirely of numbers in floating point notation. Each card contains 4 fifteen column wide fields, defined by columns 1-15, 16-30, 31-45, and 46-60. Card D4002 contains the necessary geometry data for the infinite array of rod type I=1: Δx, Δy, x_1, y_1 for rod type 1 are punched successively, in floating point notation, within the 4 fields of this card. Δx for rod type 1 is the pitch in the x direction for rods composing the infinite rectangular array 1. Similarly Δy is the pitch in the y direction for rods composing the infinite rectangular array 1. x_1 is the x coordinate of the rod of type 1 located closest to the coordinate origin. y_1 is the y coordinate of this same rod. This input form is followed successively for each rod type $I = 2, 3, \ldots, J_{Max}$, the geometry input for each rod type taking up a single card. There are JMAX + 1 cards in the 400N series

D option. The FORMATS are (I5, E15.5) for the first card followed by (4E15.5) for the remaining JMAX cards.

Series 500N

This series consists entirely of numbers in floating point notation. Each card contains 4 fifteen column wide fields, defined again by columns 1-15, 16-30, 31-45, and 46-60. The 500N series cards contain a table, FKMAX(I), 1, 2, ..., JMAX entered successively where FKMAX(I) is the number of individual rods in rod type I.

For the case of infinite lattices (option 4000D), the exact theoretical ratio of the number of individual rods in one distinct rod type to the number of individual rods in any other distinct rod type will generally not be maintained because of peculiarities of the choice of a circle to limit the infinite arrays. These theoretical ratios are required in the data processing sections at the end of HERESY 1. The 500N series of input allows the proper ratios to be entered into the machine. This floating point input table may legitimately be normalized by dividing each entry of the FKMAX(I) input table by the smallest number in the table. The FKMAX(I) table is reduced then to the set of theoretical ratios required by the code.

Table for HERESY 1 Problem Deck Input

Information presented within a box is read in successively without gaps, the order of input following the vertically downward direction in the table. If in any line the notation Q(I), I = 1, 2, ..., J_{MAX} is used, this indicates that all the Q's are entered as input before proceeding to the next line below. The topmost input quantity in any box is always entered on a new card. Unless otherwise stated, the first field on any card begins with column 1. A --------separates 2 possible options. (e.g. see 400N Series)

Series Number	Input	Description of Input	Format Statement	Number of Cards in Series
1001	Title Information	Column 1 must be blank any punchable character in columns 2-72	(72H)	1
200N	JMAX	Number of rod types	(14I5)	$\left[\frac{\text{JMAX} + 3}{14}\right]^*$
	KMAX(I), I=1,2,...,JMAX	Number of individual rods of type(I) entered successively according to rod type		
	N1	Maximum No. iterations for matrix inversion		
	N2	Maximum No. iterations for eigenvalue determination		
300N	E_1;	Determines number of significant figures S_F for matrix inversion $E_1 = 10^{-S_F}$	(4E15.5)	$\left[\frac{3 + 6(\text{JMAX})}{4}\right]^*$
	E_2;	Determines S_F for eigenvalue determination		
	E_3;	Determines S_F for iteration of reactivity to criticality		
	GAMMA(I) I=1,2,...,JMAX ;	Thermal neutron rod absorption parameter		
	ETA(I) I=1,2,...,JMAX ;	Number of fission neutrons emitted per neutron absorbed in rod of type I		

Series Number	Input	Description of Input	Format Statement	Number of Cards in Series
	VO(I) I = 1,2,...,JMAX	Initial guess to eigenvector		
	w(I) I = 1,2,...,JMAX;	Power ratios		
	A(I) I = 1,2,...,JMAX;	Resonance absorption Parameter for Rod Type I		
	SMFO(I) I = 1,2,...,JMAX	Self thermal Diffusion kernel		
400N	DIMEN	Lattice Spacing Factor	1E12.5	
Arbitrary input of x,y data	x coordinates for all the rods of Type 1	x coordinates of rod centers	5E12.5	$1 + 2\sum_{I=1}^{JMAX}\left[\frac{KMAX(I)}{5}\right]^{*}$
Option U (Sense Switch 4 up)	followed by a separate card or cards for the y coordinates of the corresponding rods of Type 1.	y coordinates of rod centers. For rod type I, there are a KMAX(I) number of x coordinates read in without gap. The same number of y coordinates follows starting on a new card		
	This form is followed successively for each rod type→2,3,... JMAX			

Series Number	Input	Description of Input	Format Statement	Number of Cards in Series
400N	JMAX;	Number of rod types	(15,E15.5)	
Infinite Lattices	Radius ;	Radius of circular area of contributing rods		
Option D	$\Delta x(1)$	x pitch for the rods of the infinite rectangular lattice composing rod type 1.		
(Sense Switch 4 Down)	$\Delta y(1)$	y pitch for type 1 rods.	(4E15.5)	1 + JMAX
	x_1	x coordinate for the rod of type 1 closest to the origin		
	y_1	y coordinate for rod of type 1 closest to the origin		
	This form is followed successively for each rod type→2,3,...JMAX			
500N	FKMAX(I) I=1,2,...,JMAX	Number of individual rods of Type I, entered successively according to rod type	(4E15.5)	$\left[\frac{JMAX}{4}\right]^{*}$

Section 4.4 Examples

Let us consider the preparation of input for the following problems:

Problem I

A finite 3 x 3 lattice (20 cm pitch) of natural uranium rods in a graphite moderator. The lattice is shown in Fig. 5. There are 3 rod types and all rods are of the same composition and radius (R_o = 2 cm). The rod properties are given below.

$f_o(2\text{ cm}) = .4824$ for age diffusion kernels with $L^2 = 2800;\ \sigma_a = 4 \times 10^{-4}$.

$\eta = 1.34$

$\gamma = .255\text{ cm}^{-1}$

$A = 51\text{ cm}^2$

From Fig. 5 we note that rod type 1 only includes one rod whereas rod types 2 and 3 each include 4 individual rods. We will take as our convergence criteria:

$N_1 = 10$

$N_2 = 40$

$E_1 = 10^{-6}$

$E_2 = 10^{-5}$

$E_3 = 10^{-5}$

As our eigenvector guess take V_o = 1 for all 3 rod types. For simplicity we take the power conversion factor W = 1 for all rod types.

The output for this problem (using age diffusion kernels with $L^2 = 2800$; $\Sigma_a = 4 \times 10^{-4}$; $\tau = 350$; $\tau_r = 262.5$) is found in Fig. 7 of this section.

Problem II

Two superimposed infinite square arrays as shown in Fig. 6. In this problem we will take rods of type 1 and type 2 to be of different compositions. We note there are four times as many rods of type 1 as of type 2. The rod properties for the two rod types are indicated below:

Rod Type 1	Rod Type 2
$f_o = .4824$	$f_o = .4824$
$\eta = 1.34$	$\eta = 1.605$
$\gamma = .255 \text{ cm}^{-1}$	$\gamma = .178 \text{ cm}^{-1}$
$A = 51 \text{ cm}^2$	$A = 51.0 \text{ cm}^2$

We will take our radius cutoff as 280 cm. That is, we know that rods located outside of this radius will give negligible flux contributions to representative receiver rods located nearest to our coordinate origin. We will take convergence criteria to be the same as the last problem. As our eigenvector guess we take $V_o(1) = 1$; $V_o(2) = 1$. The power factor is taken as 1. The output for this sample problem is shown in Fig. 8 of this section.

GENERAL PURPOSE CODING FORM

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PROGRAM: PROBLEM INPUT DECK FOR HERESY I

INPUT FOR 3 X 3 LATTICE-OPTION U

```
  GRAPHITE NAT UR  26    4/19/60  FINITE 9 ROD SQUARE LATTICE           GRU11001
    3    1    4    4   10   40                                          GRU12001
        1.0E-06        1.0E-05        1.0E-05       .255E 00            GRU13001
       .255E 00       .255E 00       1.34E 00       1.34E 00            GRU13002
       1.34E 00        1.0E 00        1.0E 00        1.0E 00            GRU13003
        1.0E 00        1.0E 00        1.0E 00        5.1E 01            GRU13004
        5.1E 01        5.1E 01      4.824E-01      4.824E-01            GRU13005
      4.824E-01                                                         GRU13006
      2.E 01                                                            GRU14001
       0E 00                                                            GRU14002
       0E 00                                                            GRU14003
    -1.0E 00      1.0E 00    0.0E 00     0.0E 00                        GRU14004
     0.0E 00      0.0E 00    1.0E 00    -1.0E 00                        GRU14005
     1.0E 00      1.0E 00   -1.0E 00    -1.0E 00                        GRU14006
     1.0E 00     -1.0E 00    1.0E 00    -1.0E 00                        GRU14007
        1.0E 00        4.0E 00        4.0E 00                           GRU15001
```

. 4

GENERAL PURPOSE CODING FORM

TRG, Incorporated

PROGRAMMER:

PAGE 1 OF 1

PROGRAM: PROBLEM INPUT DECK FOR HERESY 1

INPUT FOR TWO SUPERIMPOSED INFINITE LATTICES - OPTION D

```
        COMPLEX INF 33   4/19/60   2 SQUARE ARRAYS  20CM,40CM               1001
    2    4    1   10   40                                                   2001
        1.0E-06        1.0E-05        1.0E-05       2.55E-01                3001
       1.78E-01       1.34E 00      1.605E 00        1.0E 00                3002
        1.0E 00        1.0E 00        1.0E 00        5.1E 01                3003
        5.1E 01       4.824E-1      4.824E-01                               3004
    2        2.8E 02                                                        4001
        2.0E 01        2.0E 01        0.0E 00        0.0E 00                4002
        4.0E 01        4.0E 01       -1.0E 01       -1.0E 00                4003
        4.0E 00        1.0E 00                                              5001
```

Fig. 5. A 3 x 3 square lattice of Pitch 20 cm

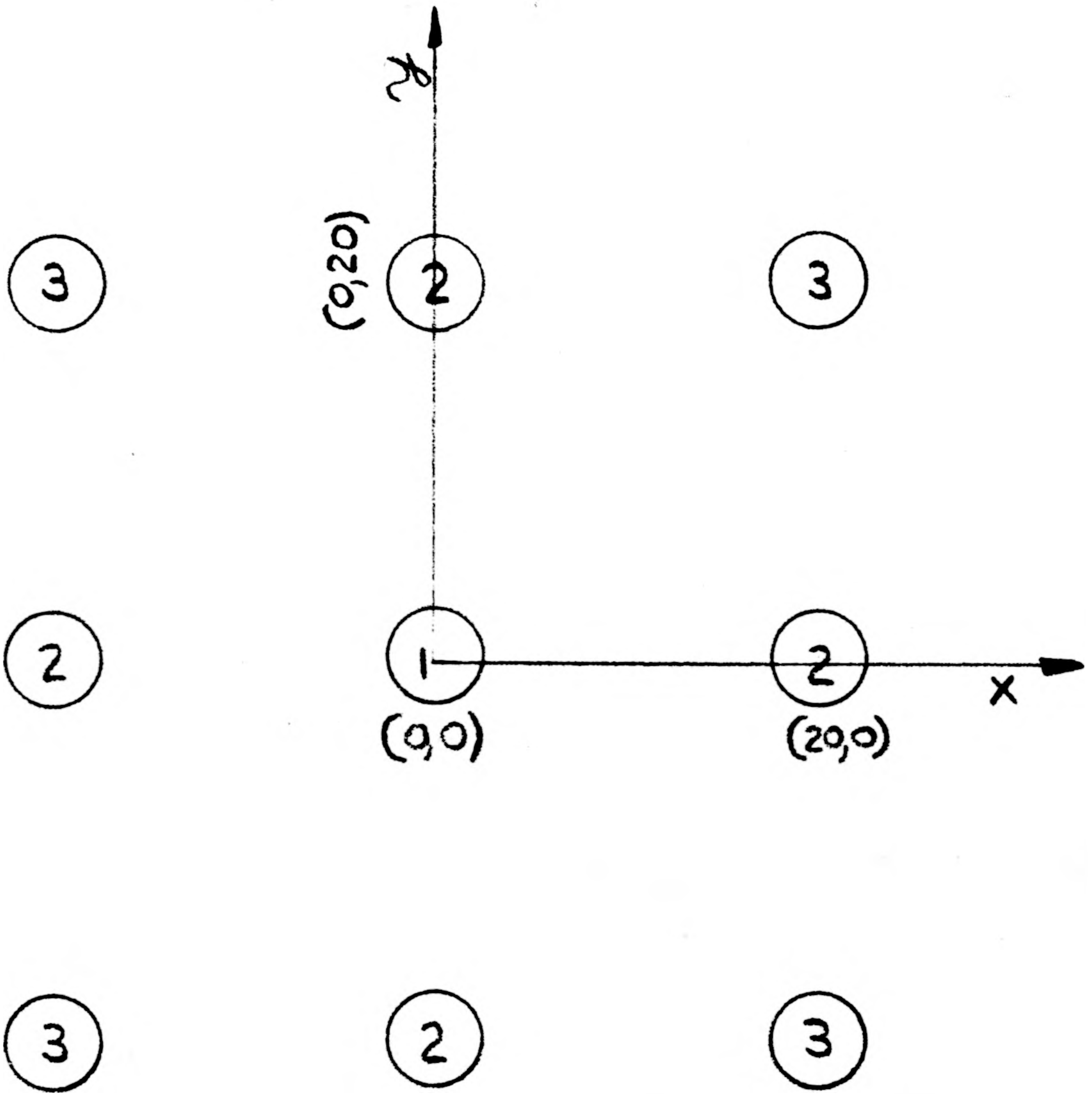
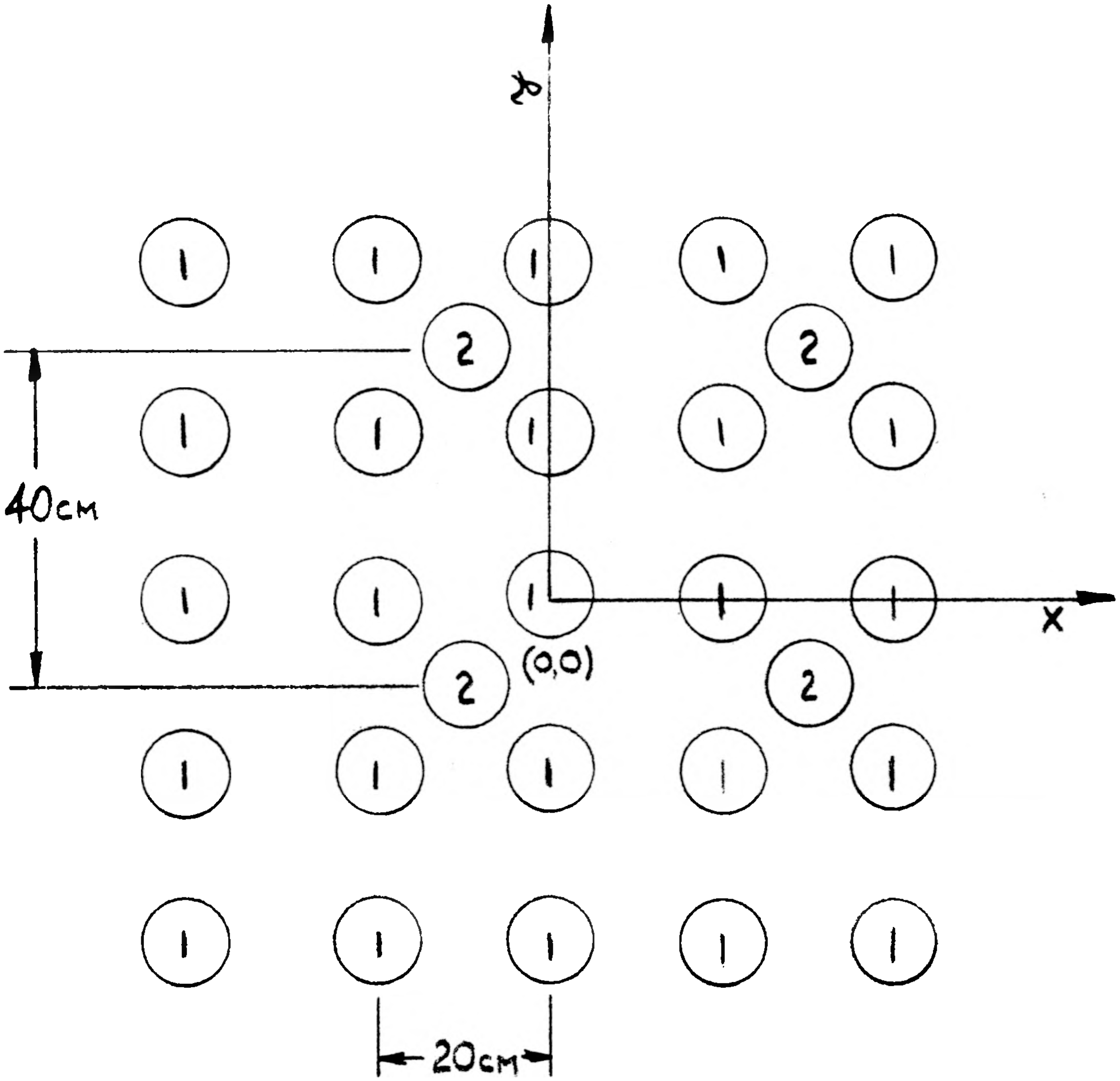


Fig. 6. An Infinite Interstitial Lattice



PROBLEM I:

Fig. 7. HERESY 1 output for 3 x 3 rod lattice of natural uranium rods in a graphite reactor

```
HERESY                 Problem No.   Date                  Description

Graphite-Nat.Ur.          26       4/19/60  Finite 9 rod square lattice
Type No. or rods        Rod SMF             Gamma
   1             1    0.48240E-00    0.25500E-00
   2             4    0.48240E-00    0.25500E-00
   3             4    0.48240E-00    0.25500E-00
K2=   0.6635451E  00          ETA            A           W           ABSRB         POWER
                   1    0.1340E  01   0.5100E  02   0.1000E 01  0.9318E 00   0.9318E 00
                   2    0.1340E  01   0.5100E  02   0.1000E 01  0.9771E 00   0.9771E 00
                   3    0.1340E  01   0.5100E  02   0.1000E 01  0.1000E 01   0.1000E 01
Average Rod Absorption =        0.982243E 00
Average ETA =    0.1340E 01     Resonance Escape Probability =  0.9348E 00
Thermal Utilization Factor  =  0.5297E 00
```

PROBLEM II:

Fig. 8. HERESY 1 output for complex infinite array of (1) natural uranium (2) enriched uranium rods in a graphite moderator

```
HERESY                  Problem No.   Date                  Description

Complex Inf                 33       4/19/60  2 Square Arrays 20CM,40CM
Type No. of rods          Rod SMF             Gamma
   1           611    0.48240E-00     0.25500E-00
   2           147    0.48240E-00     0.17800E-00
K2=  0.1119853E  01           ETA            A           W            ABSRB         POWER
                  1     0.1340E  01   0.5100E  02   0.1000E  01  0.8694E 00   0.8694E 00
                  2     0.1605E  01   0.5100E  02   0.1000E  01  0.1000E 01   0.1000E 01
Average Rod Absorption =       0.895534E  00
Average ETA =    0.1399E  01   Resonance Escape Probability = 0.8406E  00
Thermal Utilization Factor =  0.9522E 00
```

APPENDIX 1(A) HERESY 1 FORTRAN II SOURCE PROGRAM LISTING

(See Appendix 1(B) for Symbol Table)

```
C        Main Program HETEROGENEOUS REACTOR SYSTEM CODE

         Dimension BGF(60),BGFR(60),SMF(60),SMFO(50),SMGR(60)
         Dimension KMAX(50),GAMMA(50),ETA(50),VO(50),W(50),A(50)
         Dimension Z(50,50),T(50,50),TR(50,50)SR(50,50),
        1EL(50,50),RL(50,50),RLX(50,50),RLT(50,50),EM1(50,50),
        2RLG(50,50),G(50,50),EM5(50,50),  ASR(50,50)
         Dimension HGMIN(50),GAMIN(50),HA(50),FKMAX(50),HETA(50),
        2V1(50), V2(50),VRAT(50),HGMIG(50),HGMIZ(50)
         Dimension X(6500),Y(6500)
         Dimension          HASR(50),HASRE(50)
         Common JMAX,IJMAX,H,N1,N2,E1,E2,E3,IT,IT2,IT3,I,J,HETAI,HI,REESPR
        2,AVETA,HGMIGI,QUE,HGMIZI,THUT,NI,IT3IND,NN,
        3 BGF,BGFR,SMF,SMFO,SMGR,KMAX,GAMMA,ETA,VO,W,A,Z,T,TR,SR,EL,RL,
        4RLX,RLT,EM1,RLG,G,EM5,  ASR,HGMIN,GAMIN,HA,FKMAX,HETA,V1,V2,VRAT,
        5HGMIG,HGMIZ,X,Y
         Equivalence (T,RLG),(X,EL),(X(2501),RLX,EM5,G),(X(5001),RL),(Y(
        11),RL(1501)),(Y(1001),RLT,EM1), (Y(3501), ASR),(Y(6001),V1),
        2(Y(6051),V2),(Y(6101),HGMIN),(Y(6151),GAMIN),(Y(6201),HA),
        3(Y(6251),FKMAX),(Y(6301),HETA),(Y(6351),VRAT),(Y(6401),HGMIG)
        4,(Y(6451),HGMIZ)
         Dimension ABSRB(50), Power (50)
         Read 3, IJMAX,H,(BGF(I),I=1,IJMAX),(BGFR(1),I=1,IJMAX),
        X(SMF(I),I=1,IJMAX),
        X(SMGR(I),I=1,IJMAX)
    3      Format (15,(4E15.6))
         If (SENSE SWITCH 1) 1,2
    1    Y=SOSF(2)
    2    Read 9
    9     Format(72H
        1                                      )
         Read 5,JMAX,(KMAX(I),I=1,JMAX),N1,N2
    5     Format(1415)
         Read 4,E1,E2,E3,(GAMMA(I),I=1,JMAX),
        X(ETA(I),I=1,JMAX),(VO(I),I=1,JMAX),(W(I),I=1,JMAX),
        X(A(I),I=1,JMAX)     ,(  SMFO(I),I=1, JMAX)
    4     Format (4E15.5)
         Call EFM
C        Obtain X,Y from single index input or generating routine
         If (SENSE SWITCH 4)998,997
  998    Call GETXYC
         Go to 999
  997    Call GET XY
C        Compute Z,T,TR,SR
  999    Call MATELM
         Call MXADDG (Z,GAMMA,EL)
         IT=0
```

```
      Assign 213 to NI
       DO 87 I = 1, JMAX
       DO 87 J = 1, JMAX
      RLX(I,J)=EL(I,J)
       IF(I-J)89,88,89
 88   RL(I,J)=1.0
      Go to 87
 89   RL(I,J)=0.0
 87   Continue
      LL=XLOCF(RLX(1,1)     )
      LRLO=XLOCF(RL(1,1)      )
      DET=MATEQF(LL,LRLO,JMAX,JMAX,50,50)
 41   Call MXMPMX(EL,RL,RLT)
      DO 22 I=1,JMAX
      DO 22 J=1,JMAX
      IF(I-J)24,23,24
 23   IF(ABSF(RLT(I,J)-1.0)-E1)22,22,26
 24   IF(ABSF(RLT(I,J))-E1)22,22,26
 22   Continue
      GO to 42
 26   IT=IT+1
       IF(IT-N1)27,27,28
 28   Assign 210 to NI
      Pause 2
      Print 317,IT,E1
317    Format(12H1 Iteration I3,10H  EPSILON=E15.8,9H L Matrix)
      DO 319I=1,JMAX
319   Print 320,(EL(I,J),J=1,JMAX)
320    Format(1H08E14.7/(1H 8E14.7))
       N1=N1+3
      IF (SENSE SWITCH 2) 211,210
211   E1=E1*10.0
 27       GO TO NI, (210,213)
210   Call PRTINV(IT,E1)
213   DO 32I=1,JMAX
      DO 32J=1,JMAX
      RLX(I,J)=RL(I,J)
      IF(I-J)33,34,33
 34   EM1(I,J)=2.0-EM1(I,J)
      GO TO 32
 33   EM1(I,J)=-EM1(I,J)
 32   Continue
      Call MXMPMX(RLX,EM1,RL)
      GO TO 41
 42   Call DGMPMX(A,SR,ASR)
```

```
      Call MXMPMX(TR, ASR,EM5)
      Call MXSBMX(T,EM5,T)
      Call MXMPDG(T,ETA,G)
      Call MXMPMX(RL,G,RLG)
      NN=2
      DO 51 I=1,JMAX
  51  V1(I)=V0(I)
      IT2=0
      Call ABMXVC(V1,A1,IMAX1)
      FK2=1.0
  60  Call MXMPCV(RLG,V1,V2)
      Call ABMXVC(V2,A2,IMAX2)
      IF (V1(IMAX2))599,56,599
 599  FK2=V2(IMAX2)/V1(IMAX2)
          GO TO (100,101),NN
C     Print Out of Successive Eigenvectors
 100  DO 102 I=1,JMAX
 102  VRAT(I)=V2(1)/A2
      Print 103,112,FK2,(V2(1),VRAT(I),I=1,JMAX)
 103  Format(15H1 Iteration No. I4,29H Eigenvector   V2/A2    E14.5/
     X(E36.4,E12.4))
 101  DO 500 I=1,JMAX
      IF (ABSF(V1(I)-V2(I)/FK2)-E2*A1)500,500,56
 500  Continue
      GO TO 55
  56  IT2=IT2+1
      IF(IT2-N2)57,57,58
  58  Call PRTEGV
  57  DO 59 I=1,JMAX
  59  V1(I)=V2(I)

      A1=A2
      IMAX1=IMAX2
      FK1=FK2
      GO to 60
  55  IF(ABSF(FK2-1.0)-E3)501,501,502
 502  Call ITERAT
         GO TO (501,42),IT3IND
 501  DO 63 I=1,JMAX
      ABSRB(I)=V2(I)/A2
  63  Power(I)=ABSRB(I)*W(I)
      Write Output Tape 3,112
 112  Format(55H1 HERESY    Problem No.   Date          Description)
      Write Output Tape 3,9
      Write Output Tape 3,111,(I,KMAX(1),SMFO  (I),GAMMA(I),I=1,JMAX)
 111  Format(41H  Type No. of Rods    Rod SMF      GAMMA  /(I6,I12,2E12.
      X5)
```

```
          Write Output Tape 3,110,FK2,(I,ETA(I),A(I),W(I),
         XABSRB(I), Power(I),I=1,JMAX)
 110      Format(5H   K2=E12.4,61H       ETA           A            W
         1 ABSRB        Power/(I20,5E12.4))
 600     Read 4,(FKMAX(I),I=1,JMAX)
         Call RVMPDG(FKMAX,ETA,HETA)
         Call RVMPCV(FKMAX,ABSRB
        1                      ,HI)
         Call RVMPCV(HETA,ABSRB
        1                     ,HETAI)
         AVETA=HETAI/HI
         Call RVMPDG(FKMAX,A,HA)
         Call RVMPMX(HA,SR,HASR)
         Call RVMPDG(HASR,ETA,HASRE)
         Call RVMPCV(HASRE,ABSRB
        1  ,HASREI)
         REESPR=1.0-HASREI/HETAI
         DO 603 I=1,JMAX
         IF (GAMMA(I))603,604,603
 603     GAMIN(I)=1.o/GAMMA(I)
         Call RVMPDG(FKMAX,GAMIN,HGMIN)
         Call RVMPMX(HGMIN,G,HGMIG)
         Call RVMPCV(HGMIG,ABSRB
        1                      ,HGMIGI)
         QUE=HGMIGI/(HETAI-HASREI)
         Call RVMPMX(HGMIN,Z,HGMIZ)
         Call RVMPCV(HGMIZ,ABSRB
        1                      ,HGMIZI)
         THUT=HI*QUE/(HI+HGMIZI)
          FKMASB = 0.0
         SMFKMX = 0.0
         DO 1002 I = 1, JMAX
         SMFKMX=SMFKMX+KMAX(I)
 1002    FKMASB=FKMAX(I)*ABSRB(I)+FKMASB
         FKMASB=FKMASB/SMFKMX
         Write Output Tape 3,1010,FKMASB
 1010    Format(26H Average Rod Absorption = E15.6)
         Write Output Tape 3,610,AVETA,REESPR,THUT
  610    Format(15H Average ETA = E12.4,32H  Resonance Escape Probability =
         1E12.4,/
         X30H Thermal Utilization Factor =E12.4)
  601    GO TO 2
  604    Pause 6
         GO TO 2
         End (0,1,0,0,0)
```

```
       subroutine ABMXVC (V,A,IMAX)

C      SR 1.   Purpose:  Finds the largest element A, at position
C      IMAX, of a given vector V.

       Dimension, Common, Equivalence Statements

       Dimension V(50)

       IMAX = 1
       A = ABSF(V(1))
       Do 54 I = 2, JMAX
       If (A- ABSF(V(I))) 53,54,54
 53    A = ABSF (V(I))
       IMAX = I
 54    Continue
       Return
       End(0,1,0,0,0)
```

```
       subroutine DGMPMX (XS,YS,ZS)

C      SR 2.  Purpose:  Computes product matrix ZS of a diagonal matrix XS
C      multiplied by a matrix YS.

       Dimension, Common, Equivalence Statements

       Dimension ZS(50,50), XS(50), YS(50,50)

       DO 1  I = 1,JMAX
       DO 1  J = 1,JMAX
 1     ZS(I,J) = XS(I)*  YS(I,J)
       Return
       End (0,1,0,0,0)
```

```
       subroutine EFM

C      SR 3.  Purpose:  Prevents underflow error stops by replacing
C      floating point numbers less than  10^-38 emerging from the com-
C      putation by an exact zero.

       Standard New York University Mathematical Institute Subroutine
```

```
      subroutine GETXY
C     SR 4.  Purpose:  Obtains the coordinate positions of rod centers,
C     grouped by rod type, from input.

      Dimension, Common, Equivalence Statements
      Read 1, DIMEN
      KMN = 1
      DO 3 J=1, JMAX
      N = KMAX(J)
      KMNN = KMN + N-1
      Read 1, (X(K), K=KMN,KMNN)
      Read 1, (Y(K), K=KMN,KMNN)
      DO 118 K=KMN,KMNN
      X(K)= X(K) *DIMEN
118   Y(K)= Y(K) *DIMEN
  3   KMN = KMN + N
  1   Format (5E12.4)
      Return
      End(0,1,0,0,0)                 ---------------
      subroutine GETXYC
C     SR 5.  Purpose:  Computes a circle of contributing rods of as many
C     as 50 interstitial infinite rectangular arrays.  Primitive rect-
C     angles have their sides parallel to the coordinate axes.

      Dimension, Common, Equivalence Statements

100   Read 1,JMAX,Radius
  1   Format(I5,E15.5)
      IX=1
      IT3=2
      RSQ=Radius**2
      DO 2 J=1,JMAX
      Read 3,WDX,WDY,WX1,WY1
  3   Format(4E15.5)
      KMAX(J)=0
      NMAX=Radius/WDX
      X(IX)=0.0
      DO 4 N=1,NMAX
      LMAX=(SQRTF(RSQ-X(IX)**2))/WDY
      Y(IX)=0.0
      KMAX(J)=KMAX(J)+1
      DO 5 L=1,LMAX
      IXL = IX+ L
      Y(IXL)  = Y(IXL  -1)   + WDY
      X(IX L)=X(IX)
```

```
  5   KMAX(J)=KMAX(J)+1
      IU=IX+LMAX+1
      X(IU)=X(IX)+WDX
  4   IX=IU
      KNQ2=0
      IX=IX-1
      IQ1=IX-KMAX(J)
      KMAXJ = KMAX(J)
      DO 6 L=1,KMAX J
      IQ1L    =  IQ1 + L
      IF(X(IQ1 L))777,6,7
  7   KNQ2=KNQ2+1
      IXL = IX + KNQ2
      Y(IX L)=Y(IQ1 L)
      X(IX L)=-X(IQ1 L)
  6   Continue
      IX=IX+KNQ2
      KN34Q=0
      K12Q=KNQ2+KMAX(J)
      IQ12=IX-K12Q
      DO10 L=1,K12Q
      IQ12L =  IQ12 + L
      IF(Y(IQ12 L))777,10,12
 12   KN34Q=KN34Q+1
      IXL = IX + KN34Q
      Y(IX L)=-Y(IQ12 L)
      X(IX L)=X(IQ12 L)
 10   Continue
      IX=IX+KN34Q+1
      KMAX(J)=KN34Q+K12Q
      KMXDX=KMAX(J)
      ISHIFT=IX-KMAX(J)-1
      DO 2 L=1,KMXDX
      ISHIFL  = ISHIFT + L
      Y(ISHIF  L)=Y(ISHIF  L)+WY1
  2   X(ISHIF  L)=X(ISHIF  L)+WX1
      Return
777   Pause 777
      Go to 100
      End(0,1,0,0,0)
```

```
        subroutine ITERAT

C       SR 6.  Purpose:  Provides a calling sequence after the comparison
C       of the current reactivity to 1.000 .  If k is sufficiently far
C       from unity, input parameters will be recursively modified by a
C       scheme to be developed.

        Dimension, Common, Equivalence Statements

        IT3IND = 1
        Return
        End(0,1,0,0,0)
```

```
        subroutine MATELM

C       SR 7.  Purpose:  Computes and stores elements of the matrices
C       T(I,J), TR(I,J), SR(I,J), and Z(I,J).

        Dimension, Common, Equivalence Statements

        FIJMAX = IJMAX
        HIJM2 = H*(FIJMAX-2.0)
        KMN=1
        DO 5 J=1,MAX
        N=KMAX(J)
        KR=1
        KMNN = KMN + N - 1
        DO 1 I=1,JMAX
        T(I,J)=0.0
        TR(I,J)=0.0
        SR(I,J)=0.0
        Z(I,J)=0.0
        DO 6 K=KMN,KMNN
        IF(KR-K)4,3,4
 3      T(I,J)=T(I,J)+BGF(1)
        TR(I,J)=TR(I,J)+BGFR(1)
        SR(I,J)=SR(I,J)+SMGR(1)
        Z(I,J)=Z(I,J)+SMFO(J)
        Go to 6
 4      D=SQRTF((X(KR)-X(K))**2+(Y(KR)-Y(K))**2)
        IF (D-HIJM2) 7,8,8
 8      Pause 778
        Go to 6
 7      IF (D-H*2) 780, 9,9
 780    Pause 780
        Go to 6
 9      M=D/H + 1.0
```

```
      U=(MODF(D,H))/H
C     BESSELS INTERPOLATION FORMULA
      P=U*(U-1.0)/4.0
      Q = 1.0 - U - P
      R=U-P
      T(I,J) = T(I,J) + P*(BGF(M-1)+BGF(M+2))+Q*BGF(M)+R*BGF(M+1)
      TR(I,J) = TR(I,J)+P*(BGFR(M-1) +BGFR(M+2))+Q*BGFR(M)+R*BGFR(M+1)
      SR(I,J) = SR(I,J)+P*(SMGR(M-1)+SMGR(M+2))+Q*SMGR(M)+R*SMGR(M+1)
      Z(I,J) = Z(I,J)+P*SMF(M-1)+SMF(M+2))+Q*SMF(M)+R*SMF(M+1)
 6    Continue
 1    KR = KR+KMAX(I)
 5    KMN = KMN+N
      Return
      End(0,1,0,0,0)
```

```
      subroutine MATEQ

C     SR 8.  Purpose:  As used, computes the inverse matrix L^-1, from
C     the matrix L.

      Standard New York University Mathematical Institute Subroutine
```

```
      subroutine MXADDG(XS,YS,ZS)

C     SR 9.  Purpose:  Computes the sum matrix ZS, from the matrix XS
C     plus the diagonal matrix YS.

      Dimension, Common, Equivalence Statements

      Dimension XS(50,50), YS(50), ZS(50,50)
      DO 10  J=1,JMAX
      DO 10  I=1,JMAX
      IF(I-J)  11,12,11
 11   ZS(I,J) =  XS(I,J)
      GO to 10
 12   ZS(I,J) = XS(I,J)  +  YS(I)
 10   Continue
      Return
      End(0,1,0,0,0)
```

```
      subroutine MXMPCV(XS,YS,ZS)

C     SR 10.  Purpose:  Computes the product vector ZS, from the matrix
C     XS, and the column vector YS.

      Dimension, Common, Equivalence Statements

      Dimension XS(50,50), YS(50), ZS(50)
      DO 1 I = 1, JMAX
      ZS(I)  =  0.0
      DO 1 J = 1, JMAX
 1    ZS(I)  =   ZS(I)   +   XS(I,J)  *  YS(J)
      Return
      End(0,1,0,0,0)
```

```
      subroutine  MXMPDG(XS,YS,ZS)

C     SR 11.  Purpose:  Computes the product matrix ZS, from the matrix
C     XS, and the diagonal matrix YS.

      Dimension, Common, Equivalence Statements

      Dimension XS(50,50),  YS(50),  ZS(50,50)
      DO 1 I = 1, JMAX
      DO 1 J = 1, JMAX
 1    ZS(I,J)     = XS(I,J) * YS(J)
      Return
      End (0,1,0,0,0)
```

```
      subroutine MXMPMX (XS,YS,ZS)

C     SR 12.  Purpose:  Computes the product matrix ZS, from the matrix
C     XS, and the matrix YS.

      Dimension, Common, Equivalence Statements

      Dimension  XS(50,50), YS(50,50) ZS(50,50)
      DO 1 I = 1, JMAX
      DO 1 J = 1, JMAX
      ZS(I,J)  = 0.0
      DO 1 K = 1, JMAX
 1    ZS(I,J) = ZS(I,J) + XS(I,K)*YS(K,J)
      Return
      End(0,1,0,0,0)
```

```
        subroutine MXSBMX(XS,YS,ZS)

C       SR 13.  Purpose:  Computes the matrix ZS, equal to the matrix XS
C       minus the matrix YS.

        Dimension, Common, Equivalence Statements

        Dimension, XS(50,50), YS(50,50), ZS(50,50)
        DO 1 J = 1, JMAX
        DO 1 I = 1, JMAX
1       ZS(I,J) = XS(I,J) - YS(I,J)
        Return
        End(0,1,0,0,0)
```

```
        subroutine PRTEGV

C       SR 14.  Purpose:  Provides a calling sequence for additional eigen-
C       value determination failure routine. (e.g. alternate eigenvalue
C       subroutine)

        Dimension, Common, Equivalence Statements

        NN = 1
        N2 = N2 + 3
        PAUSE 5
        Return
        End(0,1,0,0,0)
```

```
        subroutine PRTINV

C       SR 15.  Purpose:  This error routine prints on the on-line printer
C       three successive sets of the test matrix L L^{-1}, and in addition if
C       Sense Switch 3 is depressed this routine prints the corresponding
C       inverse matrix L^{-1}.  The inverse matrix is denoted RL in the code.

        Dimension, Common, Equivalence Statements

        IF (SENSE SWITCH 3) 230, 231
 230    PRINT 217, IT, E1
        DO 221 I = 1, JMAX
 221    PRINT  218, (RL(I,J), J = 1, JMAX )
 218    FORMAT (1H08E14.6/(1H 8E14.6))
 217    FORMAT(12H1 ITERATION I3,11H  EPSILON=E14.6,17H L INVERSE MATRIX)
 231    PRINT 237, IT , E1
 237    FORMAT(12H1 ITERATION I3,11H  EPSILON = E14.6,25H L TIMES L INVERSE
        1 MATRIX)
        DO  241  I = 1, JMAX
 241    PRINT 218,(RLT(I,J), J = 1, JMAX)
        Return
        End(0,1,0,0,0)
```

```
        subroutine RVMPCV (XS,YS,ZS)

C       SR 16.  Purpose:  Computes the scalar ZS, from the row vector XS,
C       multiplied by the column vector YS.

        Dimension, Common, Equivalence Statements

        Dimension  XS(50),YS(50)
        ZS  =  0.0
        DO  1  I = 1, JMAX
 1      ZS  =  ZS + XS(I) * YS(I)
        Return
        End(0,1,0,0,0)
```

```
      subroutine RVMPDG(XS,YS,ZS)

C     SR 17. Purpose:  Computes the product row vector ZS, from the row
C     vector XS, multiplied by the diagonal matrix YS.  This same routine
C     is used to multiply a diagonal matrix by a column vector.

      Dimension, Common, Equivalence Statements

      Dimension XS(50), YS(50), ZS(50)
      DO 1  I = 1, JMAX
 1    ZS(I)  = XS(I) * YS(I)
      Return
      End(0,1,0,0,0)
```

```
      subroutine RVMPMX(XS,YS,ZS)

C     SR 18.  Purpose:  Computes the product row vector XS, from the row
C     vector XS, multiplied by the matrix YS.

      Dimension, Common, Equivalence Statements

      Dimension  XS(50),  YS(50,50),  ZS(50)
      DO 1 J = 1, JMAX
      ZS(J)  =  0.0
      DO 1 I = 1, JMAX
 1    ZS(J)  = ZS(J) + XS(I) * YS(I,J)
      Return
      End(0,1,0,0,0)
```

```
      subroutine SOS

C     SR 19.  Purpose:  As modified by TRG, this subroutine produces
C     a binary tape of the HERESY I program.  This tape is normally used
C     to load the HERESY I machine program into the 704.

      Standard New York University Mathematical Institute Subroutine
```

subroutine XLOC

C SR 20. Purpose: Subroutine No. 8, MATEQ, requires the machine
C addresses of the matrices L, and L^{-1} . This routine No. 20 obtains
C these addresses.

Standard New York University Mathematical Institute Subroutine.

APPENDIX 1(B) SYMBOL TABLE FOR HERESY 1

Notation: The Fortran Symbols of Input Quantities are Underlined in this Table

Fortran Listing Form of Symbol	Equation Form of Symbol	Definition	Units	No. Spaces Saved	Location of Highest Space Saved
A(I)	A_i	Resonance Absorption Parameter for Rod Type (I)	cm^2	50	40150
ABSRB(I)	i	Eigenvector V2(I) normalized so that largest component V2(IMAX2) = 1.0	-	50	2174
ASR(I,J)	$\underline{\underline{AS}}^r$	Product of diagonal matrix A(I) post multiplied by matrix SR(I,J)		2500	47136
A1	-	Value of largest element of eigenvector V1(IMAX1)	-	1	2020
A2	-	Value of largest element of eigenvector V2(IMAX2)	-	1	2021
AVETA	$\bar{\eta}$	Average Eta for reactor	neutrons emitted per absorption	1	41226
BGF(I)	$F(d)$	Slowing down diffusion kernel from source energies	cm^{-1}	60	41216
BGFR(I)	$F^r(d)$	Slowing down diffusion kernel from resonance energies	cm^{-1}	60	41122
D	d	Distance from present donor to present receiver rod	cm	1	17244
DIMEN	-	Lattice spacing factor	-	1	-

SYMBOL TABLE (continued)

Fortran Listing Form of Symbol	Equation Form of Symbol	Definition	Units	No. Spaces Saved	Location of Highest Space Saved
DET	-	Dummy Variable required by Subroutine No.8 MATEQ	-	1	—
EL(I,J)	L_{ij}	Sum matrix of the matrix Z(I,J) + Diagonal matrix gamma(I): $L=Z+\gamma$		2500	72556
EM1(I,J)	-	Working matrix within matrix-inversion routine		2500	54042
EM5(I,J)	-	Working storage of $T^r AS^r$ matrix		2500	65652
ETA(I)	η_i	No. fission neutrons emitted per neutron absorbed in rod of type I		50	40376
E1	-	E1 determines the number of significant figures, S_F, obtained for each element of L^{-1}_{ij}. $E_1 = 10^{-S_F}$	-	1	41241
E2	-	E2 determines the number of significant figures, S_F obtained in each element of the eigenvector, V2, $E_2 = 10^{-S_F}$	-	1	41240
E3	-	E_3 can be used to determine the number of significant figures to which the computed reactivity K_2 equals unity	-	1	41237

SYMBOL TABLE (continued)

Fortran Listing Form of Symbol	Equation Form of Symbol	Definition	Units	No. Spaces Saved	Location of Highest Space Saved
FIJMAX	-	IJMAX in floating point form	-	1	-
FKMASB	-	Average rod absorption	-	1	2025
FKMAX(I)	H_i	Number of rods in Type I	-	50	41640
FK1	k_1	Eigenvalue corresponding to the eigenvector $V_1(I)$	-	1	2023
FK2	k_2	Eigenvalue or reactivity of system; corresponding to eigenvector V2(I)	-	1	2024
G(I,J)	G_{ij}	$\underline{\underline{G}} = \underline{(T-T^r AS^r)\eta}$	-	2500	65652
GAMIN(I)	γ_{ij}^{-1}	Gamma inverse matrix	cm^2	50	42004
GAMMA(I)	γ_{ij}	Thermal neutron rod absorption parameter; diagonal matrix	cm^{-2}	50	40460
HA(I)	$H_i A_{ii}$	Product of vector FKMAX(I), i.e. number of rods in each type multiplied by the diagonal matrix A(I) or A_{ii}		50	41722
HASR(I)	HAS^r	Product of vector FKMAX(I) by matrix A(I) by matrix SR(I,J)		50	2340
HASRE(I)	$HAS^r\eta$	Product of vector FKMAX(I) by matrix A(I) by matrix SR(I,J) by matrix Eta(I)		50	2256
H	Δh	Change in argument for kernel function tables	cm	1	41244

SYMBOL TABLE (continued)

Fortran Listing Form of Symbol	Equation Form of Symbol	Definition	Units	No. Spaces Saved	Location of Highest Space Saved
HASREI	$HAS^r\eta i$	Product of vector FKMAX(I) by matrix A(I) by matrix SR(I,J) by matrix Eta(I) by normalized eigenvector ABSRB(I)	-	1	2026
HETA	$H\eta$	Product of vector FKMAX(I) by diagonal matrix Eta(I) or $H_i\eta_{ii}$		50	41556
HETAI	$H\eta i$	Product of vector FKMAX(I) by matrix Eta(I) by vector ABSRB(I)		1	41231
HGMIG	$H\gamma^{-1}G$	Product of vector FKMAX(I) by matrix GAMIN(I) by matrix G(I,J)		50	41412
HGMIGI	$H\gamma^{-1}Gi$	Product of vector FKMAX(I) by matrix GAMIN(I) by matrix G(I,J) by column vector ABSRB(I)		1	41225
HGMIN	$H\gamma^{-1}$	Product of row vector FKMAX(I) by the diagonal matrix GAMIN(I)		50	42066
HGMIZ	$H\gamma^{-1}Z$	Product of row vector FKMAX(I) by the diagonal matrix GAMIN(I) by the matrix Z(I,J)		50	41330
HGMIZI	$H\gamma^{-1}Zi$	Product of row vector FKMAX(I) by diagonal matrix GAMIN(I) by the matrix Z(I,J) by vector ABSRB(I)		1	41223

SYMBOL TABLE (continued)

Fortran Listing Form of Symbol	Equation Form of Symbol	Definition	Units	No. Spaces Saved	Location of Highest Space Saved
HI	$(\underline{H})\underline{i}$	Product of row vector FKMAX(I) by column vector ABSRB(I)	-	1	41230
HIJM2	-	H(IJMAX-2.0) :maximum inter-rod distance	-	1	-
I	-	General Index	-	1	41233
IJMAX	-	Maximum number of entries in each kernel function table	-	1	41245
IMAX1	-	Index of largest element of the vector V1(I)	-	1	2027
IMAX2	-	Index of the largest element of the eigenvector V2(I)	-	1	2030
IQ1	-	Index of rod no. in infinite array calculations IQ1 = IX-KMAX(J)	-	1	-
IQ1L	-	Index	-	1	-
IT	-	Current number of iterations completed through the matrix inversion routines	-	1	41236
IU	-	Index giving number of rod whose position is to be computed in the infinite array calculations	-	1	-
IX	-	Index of current pair of coordinates being computed for compute-infinite-lattice option	-	1	-
IXL	-	Index	-	1	-
ISHIFT	-	Index	-	1	-
ISHIFL	-	Index	-	1	-

SYMBOL TABLE (continued)

Fortran Listing Form of Symbol	Equation Form of Symbol	Definition	Units	No. Spaces Saved	Location of Highest Space Saved
IT2	-	Current number of iterations completed through the eigenvalue determination routines	-	1	41235
IT3	-	Saved to count number of iterations completed through iteration towards criticality, i.e. $k_{IT3} \to 1 \pm E3$	-	1	41234
IT3IND	-	Space saved in common for use in iteration towards criticality routine	-	1	41220
J	j	General Index	-	1	41232
JMAX	J_{MAX}	Number of different rod types	-	1	41246
KMAX(I)	H_i	Table of (No. of rods per type), in integer notation	-	50	40542
KMXDX	-	Index	-	1	-
KR	-	Index of receiver rod, used in MATELM	-	1	-
KNQ2	-	Index of rod No. in infinite array calculations in Quadrant II	-	1	-
KN34Q	-	Index of rod No. in infinite array calculations in Quadrant III and IV	-	1	-
K12Q	-	Index	-	1	-
KMNN	-	Upper Limit Index of Donor rod within rod type used in MATELM	-	1	-

SYMBOL TABLE (continued)

Fortran Listing Form of Symbol	Equation Form of Symbol	Definition	Units	No. Spaces Saved	Location of Highest Space Saved
KMN	-	Lower limit index of donor rod within rod type used in MATELM	-	1	-
K	-	Index	-	1	-
L	-	Index giving No. of the individual rod within a given rod type; used in infinite array calculations	-	1	-
LL	-	Contains the address of RLX(1,1)	-	1	2017
LRLO	-	Contains the address of RL(1,1)	-	1	2016
LMAX	-	$(RSQ-X(IX)^2)/WDY$	cm	1	
M	-	Parameter within MATELM: $M=D/H + 1.0$	-	1	-
NI	-	Indicator: If NI = 210_{10}, inversion of matrix $\underline{\underline{L}}$ failed. Enter disaster routines. If NI = 213_{10}, proceed through normal iteration coding	-	1	41221
NMAX	-	Radius/WDX in compute infinite array	-	1	-

SYMBOL TABLE (continued)

Fortran Listing Form of Symbol	Equation Form of Symbol	Definition	Units	No. Spaces Saved	Location of Highest Space Saved
NN	-	Indicator: If NN = 1, eigenvalue determination failed. Enter disaster routines. If NN=2, proceed through normal iteration form.	-	1	41217
N1	-	Maximum number of iterations permitted through the matrix inversion routines before disaster routine is automatically entered	-	1	41243
N2	-	Maximum number of iterations permitted through the eigenvalue determination routine before eigenvalue disaster routine is automatically entered	-	1	41242
P		Parameter used within MATELM subroutine: P=U(U-1)/4	-	1	-
POWER(I)	(iW)	Rod absorption multiplied by power factor W	Power per absorption	50	2112
QUE	Q	Parameter used within data processing routine within main program: $Q = (\underline{H}\ \underline{\underline{\gamma}}^{-1}\ \underline{\underline{G}}\ \underline{i})/(H\eta i-H\ A\ S^{r}\eta\ i)$	-	1	41224
R	-	Parameter within MATELM R=U-P		1	-
RADIUS	-	Radius of circle of contributing rods in infinite arrays	cm	1	-
REESPR	p	Resonance Escape Probability	-	1	41227

SYMBOL TABLE (continued)

Fortran Listing Form of Symbol	Equation Form of Symbol	Definition	Units	No. Spaces Saved	Location of Highest Space Saved
RSQ	-	Radius square of circle of contributing rods in infinite arrays	cm^2	1	-
RL(I,J)	L^{-1}_{ij}	(1) Identity matrix, then (2) Inverse EL(I,J) matrix	-	2500	60746
RLG(I,J)	$\underline{\underline{L}}^{-1}_{ij}$	Product of RL(I,J) matrix, i.e. $\underline{\underline{L}}^{-1}$, post multiplied by the G(I,J) matrix	-	2500	77462
RLT(I,J)	$\underline{\underline{L.L}}^{-1}$	Product of matrix $\underline{\underline{L}}$, i.e. EL(I,J) post multiplied by the current $\underline{\underline{L}}^{-1}$, i.e. RL(I,J) matrix. This product is compared to $\underline{\underline{I}}$.	-	2500	54042
RLX(I,J)		Working storage for matrix $\underline{\underline{L}}$, i.e. EL(I,J)	-	2500	65652
SMF(I)	f(d)	Thermal diffusion kernel table	cm^{-1}	60	41026
SMFKMX	$\sum_{j=1}^{Jmax} H_j$	Total number of individual rods within the reactor (sum of components of KMAX(I))	-	1	2015
SMFO(I)	$f(r_I)$	Self thermal diffusion kernel	cm^{-1}	50	40732
SMGR(I)	$g^r(d)$	Slowing down kernels to resonance energy	cm^{-2}	60	40636

SYMBOL TABLE (continued)

Fortran Listing Form of Symbol	Equation Form of Symbol	Definition	Units	No. Spaces Saved	Location of Highest Space Saved
SR(I,J)	S^r_{ij}	Matrix whose elements are defined by $S^r_{ij}=\Sigma g^r(i,\alpha;j,\beta)$ where α = representative receiver rod of type i(α=1) and β= all donor rods of type j $g^r(d) \equiv$ SMGR(I)	-	2500	26256
T(I,J)	T_{ij}	Matrix whose elements are defined by $T_{ij} = \sum_{\beta} F(i,\alpha;j,\beta)$ where α = representative receiver rod of type i(α=1), and β= all donor rods of type j $F(d) \equiv$ BGF(I)	-	2500	77462
THUT	f	Thermal utilization factor		1	41222
TR(I,J)	T^r_{ij}	Matrix whose elements are defined by $T^r_{ij} = \sum_{\beta} F^r(i,\alpha;j.\beta)$ where α = representative receiver rod of type i(α=1) and β = all donor rods of type j $F^r(d) \equiv$ BGFR(I)		2500	33162
U	-	Parameter used within subroutine MATELM $U = (D-\lfloor D/H \rfloor H)/H$ where $\lfloor x \rfloor$ = integral part of x	-	1	-

SYMBOL TABLE (continued)

Fortran Listing Form of Symbol	Equation Form of Symbol	Definition	Units	No. Spaces Saved	Location of Highest Space Saved
VO(I)	V_{o_i}	Initial guess to eigenvector	-	50	40314
V1(I)	V_{1_i}	Eigenvector at next to last performed iteration	-	50	42232
V2(I)	V_{2_i}	Eigenvector	-	50	42150
VRAT(I)	-	Eigenvector V2(I) normalized by dividing each component by V2(IMAX2)	-	50	40232
W	W(I)	Power conversion factor	-		
WDX	Δx	Distance from rod to rod on primative rectangle of infinite array of rod type I in direction of x axis	cm	1	-
WDY	Δy	Distance from rod to rod on primative rectangle of infinite array of rod type I in direction of y axis	cm	1	-
WX1	x_1	x coordinate for the rod of Type I closest to the origin	cm	1	-
WY1	y_1	y coordinate for the chosen rod of Type I closest to the origin	cm	1	-
X(I)	x_i	x coordinates, grouped by rod type, of all rods within the reactor	cm	6500	72556
XS	-	Dummy arguments for subroutines	-	0	-
Y(I)	y_i	y coordinates, grouped in chosen order by rod type, of all rods within the reactor	cm	6500	56012
YS	-	Dummy arguments for subroutines	-	0	-

SYMBOL TABLE (continued)

Fortran Listing Form of Symbol	Equation Form of Symbol	Definition	Units	No. Spaces Saved	Location of Highest Space Saved
Z(I,J)	Z_{ij}	Matrix whose elements are defined by $Z_{ij} = \sum_{\beta} f(i,\alpha;j,\beta)$ where α= representative rod of type i(α=1) β=all donor rods of type j f(d) = SMF(I) Note that where i=j and $\alpha = \beta$, $f(r_I)$(i.e.SMFO(I)) replaces f(d).	—	2500	40060
ZS	-	Dummy argument for output of subroutines	-	0	-

Appendix 2. Kernel Function Computation

This appendix presents a typical program to compute kernel tables required by HERESY 1.

Table of Contents

1. Age Diffusion Kernel Function Program

In this appendix, the age-diffusion kernel computation subroutine is given. These kernels describe the slowing down and diffusion of neutrons after they have been emitted isotropically from infinite line source rods. The slowing down kernel function g(d) is taken as a linear superposition of three gaussian kernels. In this way we can compute not only the standard age diffusion kernels,[7] where two of the weighting factors(B_i's) are set equal to 0, but also the slowing down-diffusion kernels for those cases where an empirical slowing down distribution can be fitted by two or three gaussians.

2. Kernel Function Equations

In the equations below, d is the perpendicular distance out into the moderator from any given rod. Note that $f(r_I)$ is not computed in this subroutine since the values of $f(r_I)$ are entered by a separate input table within HERESY 1. However for completeness we include the equation for $f(r_I)$ below.

$$f(d) = \frac{1}{2\,L^2\sigma_a} \quad K_o\left(\frac{d}{L}\right)$$

$$f(r_I) = \frac{1}{2\,L^2\sigma_a} \quad K_o\left(\frac{r_I}{L}\right)$$ where r_I = radius of the fuel rod of the I^{th} type

$$g^r(d) = \sum_{i=1}^{3} \frac{B_i}{4\,\tau_i{}^r}\, e^{-\frac{d^2}{4\tau_i{}^r}}$$

$$F(d) = \sum_{i=1}^{3} B_i \frac{e^{\tau_i/L^2}}{2\pi L^2 \sigma_a} \left[K_o\left(\frac{d}{L}\right) + \frac{\tau_i}{2L^2} e^{-d^2/4\tau_i} - 1/2\left(1 + \frac{d^2}{4L^2}\right) E_1\left(\frac{d^2}{4\tau_i}\right) \right].$$

To calculate $F^r(d)$ replace τ_i by $\tau_i - \tau_i^r$ in the equation for $F(d)$.

Here

$f(d)$ = thermal diffusion kernel

$g^r(d)$ = empirical or age theory slowing down kernels to resonance energy

$F(d)$ = slowing down-diffusion kernel from fast energies

$F^r(d)$ = slowing down-diffusion kernel from resonance energies

L = diffusion length of thermal neutrons in the moderator

σ_a = macroscopic thermal absorption cross section

τ = neutron age to thermal energies

τ_r = neutron age to resonance energy

B_i = linear weighting coefficients which are usually obtained in an empirical fit to the slowing down density.

3. Operating Instructions for KF Code

1. Set sense switches to up position
2. Place Kernel Function program deck in the card reader, 9-edge in, face down.

3. Place input deck in hopper of card reader - 3 cards for each distinct set of kernel function tables
4. Set to AUTOMATIC the automatic-manual switch
5. Depress the CLEAR button resetting magnetic cores to zeroes
6. Press LOAD CARDS button
7. Any number of separate kernel functions may be computed in succession since the program automatically returns to the initial instruction (read KF 1 card)
8. The card punch will produce kernel functions in correct order suitable for immediate use with HERESY 1.

4.(a) Preparation of the Three Input Cards

Input consists of one integer followed by 12 floating point numbers, on three cards.

Columns on Card KF1	1 - 5	6-20	21-35	36-50	51-65
Contents of Columns	IJMAX$(5 \leqq \text{IJMAX} \leqq 60)$	Δh	L^2	σa	B_1

Columns on Card KF2	1 - 15	16-30	31-45	46-60
Contents of Columns	B_2	B_3	τ_1	τ_2

Columns on Card KF3	1 - 15	16-30	31-45	46-60
Contents of Columns	τ_3	τ_1^r	τ_2^r	τ_2^r

For a discussion of integer and floating point notation see Section 4.2 of this report.

4.(b) Below is the input preparation sheet for the computation of kernels with

$$\begin{aligned} \text{IJMAX} &= 10, \quad \Delta h = 5.0 \\ L^2 &= 2800 \\ \sigma_a &= 4 \times 10^{-4} \\ B_1 &= 1, \; B_2 = 0, \; B_3 = 0; \\ \tau_1 &= 350, \quad \tau_2 = 0, \quad \tau_3 = 0; \\ \tau_1^r &= 262.5 \quad \tau_2^r = 0, \quad \tau_3^r = 0. \end{aligned}$$

See Figure 9 (next page).

5. Output

The first card of output consists of one integer in columns (1 - 5), followed by four floating point numbers, in columns (6-20), (21-35), (36-50) and (51-65). Each succeeding card of output contains four floating point numbers in columns (1-15), (16-30), (31-45), and (46-60). On the first card is the integer IJMAX, which is the number of entries in the kernel function tables; Δh, the change in argument of the table; and the first three entries of the F(d) table. The remainder of the F(d) table follows on the next cards, followed without gap by the $F^r(d)$, f(d), and $g^r(d)$ tables. Hence there are 4(IJMAX) + 1 floating point numbers punched out and the total number of cards is (IJMAX + 1).

A sample output sheet for the above kernel function input is also included in Figure 9.

G. 3

TRG, Incorporated

GENERAL PURPOSE CODING FORM

PROGRAMMER:

PAGE 1 OF 1

PROGRAM: AGE DIFFUSION KERNEL FUNCTIONS

INPUT PREPARATION SHEET FOR THE COMPUTATION OF KERNELS

```
   10           5.E0        2800.E0          4.E-4           1.E0       AGEDFKF1
             E0             E0         350.E0             E0            AGEDFKF2
             E0        262.5E0             E0             E0            AGEDFKF3
```

OUTPUT OF AGE DIFFUSION KERNEL FUNCTION PROGRAM (THIS CAN BE INPUT FOR HERESY 1)

```
   10          .5E01     .131026-00    .1300423-00     .127159-00       STDKF1
     .122554-00     .116500-00     .109331-00     .101406-00            STDKF2
     .930786-01     .846661-01     .764337-01     .214068-00            STDKF3
     .209554-00     .196991-00     .178887-00     .158279-00            STDKF4
     .137754-00     .118945-00     .102536-00     .885706-01            STDKF5
     .767832-01     .000000-00     .352886-00     .256807-00            STDKF6
     .202466-00     .165472-00     .138098-00     .116856-00            STDKF7
     .998629-01     .859767-01     .744523-01     .303150-03            STDKF8
     .296020-03     .275613-03     .244680-03     .207117-03            STDKF9
     .167168-03     .128650-03     .944026-04     .660509-04            STDKF10
     .440649-04                                                         STDKF11
```

6(a). Symbol Table

I. Input Symbols

Listing Form	Equation Form	Significance
B(I)	B_i	Linear weighting coefficients which are usually obtained in an empirical fit to the slowing down density. Note that $\sum_{i=1}^{3} B_i$ is unity.
EL SQRE	L^2	Square of the diffusion length of thermal neutrons in the moderator
H	Δh	The change in argument for the kernel tables. Argument values are 0, H, 2H, 3H...MH... (IJMAX-1)H
IJMAX		The number of entries in each of the four kernel function tables
SIGMA A	σ_a	Macroscopic moderator thermal absorption cross section
TAU(I)	τ_i	Neutron age to thermal energies
TAUR(I)	τ_i^r	Neutron age to resonance energies

II. Output Symbols (listed in order of computation)

Listing Form	Equation Form	Significance
BGF(J)	$F(d)$	Slowing down diffusion kernel from fast energies.
BGFR(J)	$F^r(d)$	Slowing down diffusion kernel from resonance energies.
SMF(J)	$f(d)$	Thermal diffusion kernel
SMGR(J)	$g^r(d)$	Empirical or age theory slowing down kernels to resonance energy

III. Computation Symbols

Listing Form	Equation Form	Significance
ARG	d/L	
COEF	$\frac{1}{2\pi L^2 \sigma a}$	
D	d	Distance, equivalent to nH, n=0,1,2,...,(IJMAX-1) for which table values are being computed
D2BY4	$d^2/4$	
IFPATH		Control word for code
FIZARG		Current argument for obtaining I_o Bessel Function
FIZC0N (I)		Coefficients of 6th degree[8] polynomial to compute Bessel function I_o
FIZER	I_o(current argument)	Bessel function I_o(argument)
FJM1		The value of the control index J minus one, (J-1)
FKZARG	$\frac{d^2}{4L^2}$ or $\frac{2L}{d}$	
FKZCN1 (I)		Coefficients of 6th degree[8] polynomial to compute $K_o(x)$ where $x \leq 2$
FKZCN2 (I)		Coefficients of 6th degree[8] polynomial to compute $K_o(x)$ where $x > 2$.
FKZER	K_o(current argument)	Bessel function
FL	L	Square root of ELSQRE
NN		Number of non-zero weighting factors Bi. $1 \leq NN \leq 3$.

Listing Form	Equation Form	Significance
WBGF		Working or temporary F(d) or $F^r(d)$
WTAU(I)		Working or temporary τ_i or τ_i^r

IV. Function Routines used in the Kernel Function Computation

Function	Yields
EXPF (x)	e^x
EXPIF(x)	exponential integral of x
LOGF (x)	ln x
SQRTF (x)	$\sqrt{x}$

6(b). FORTRAN Listing

Kernel Function Program

```
C         Purpose:  Computes line source Age Diffusion Kernel Functions

          Dimension, Common, Equivalence Statements for HERESY 1

          Dimension B(3), TAU(3), TAUR(3), WTAU(3)
          Dimension FIZCON(7), FKZ CN1(7), FKZCN2(7)
          FIZCON(1)=.0045813   (See Reference No. 8)
          FIZCON(2)=.0360768
          FIZCON(3)=.2659732
          FIZCON(4)=1.2067492
          FIZCON(5)=3.0899424
          FIZCON(6)=3.5156229
          FIZCON(7)=1.00000
          FKZCN1(1)=.00000740
          FKZCN1(2)=.00010750
          FKZCN1(3)=.00262698
          FKZCN1(4)=.03488590
          FKZCN1(5)=.23069756
          FKZCN1(6)=.42278420
          FKZCN1(7)=-.57721566
          FKZCN2(1)=.00053208
          FKZCN2(2)=-.00251540
          FKZCN2(3)=.00587872
          FKZCN2(4)=-.01062446
          FKZCN2(5)=.02189568
          FKZCN2(6)=-.07832358
          FKZCN2(7)=1.25331414
```

```
1          READ 2, IJMAX, H,
           S ELSQRE,  SIGMAA, (B(I), I = 1,3), (TAU(I) I = 1,3 ), (T1AUR(I),
           I = 1,3)
           CALL EFM
           FL = SQRTF(ELSQRE)
           COEF = 1.0/(6.283184 * ELSQRE*SIGMAA)
           IF (ABSF(B(2))) 777, 16, 15
15         IF (ABSF(B(333))) 777, 18, 14
16         NN = 1
           GO to 17
18         NN = 2
           GO to 17
14         NN = 3
17         DO 105 J = 1, IJMAX
           FJM1 * H
           ARG = D/FL
           IF (ARG) 777, 6, 5
6          IFPATH = 3
           GO to 13
5          IFPATH = 1
           FKZER = 0.0
           IF (ARG-2.)) 7,7,8
2          Format(15,(4E15.6))
7          FIZARG = (ARG/3.75)**2
           FIZER = 0.0
           DO 9  I = 1,7
9          FIZER = FIZER*FIZARG + FIZCON(I)
10         FKZARG = (ARG/2.0)**2
           DO 11  I = 1,7
11         FKZER = FKZER* FKZARG + FKZCN1(I)
           FKZER   FKZER - LOGF(.5*ARG)*FIZER
           GO TO I3
8          FKZARG = 2.0/ ARG
           DO 12 I = 1,7
12         FKZER = FKZER*FKZARG + FKZCN2(I)
           FKZER = FKZER/(SQRTG(ARG)*EXPF(ARG))
13         SMGR(J) = 0.0
           D2BY 4 = D**2/4.0
           DO 100 I = 1, NM
100        SMGR(J) = SMGR(J) + B(I)*EXPF(-D2BY4/TAUR(I))/(TAUR(I)*12.566370
           16)
           DO 99 I = 1, NN
99         WTAU(I) = TAU(I)
101        WBGF = 0.0
           GO TO (405,405,401,401), IFPATH
401        DO 402 I = 1, NN
402        WBGF = WBGF + (B(I)/2.0)*COEF*EXPF(WTAU(I)/ELSQRE)*(-EXPIF(-WTAU
           1(I)/ELSQRE))
           GO TO (777,777,403,109) , IFPATH
```

```
 403   BGF(J) = WBGF
       IFPATH = 4
       GO TO 409
 405   DO 102 I = 1, NN
 102   WBGF = WBGF + COEF*EXPF(WTAU(I)/ELSQRE)*B(I)*(FKZER+(WTAU(I)/2.0
       1 *ELSQRE))*EXPF(-D2BY4/WTAU(I))+(.5+D2BY4/(2.0*ELSQRE))EXPIF(-D2BY
       24/WTAU(I)))
       GO TO (103,109,777,777),IFPATH
 103   BGF(J) = WBGF
       IFPATH = 2
 409   DO 107 I = 1,NN
 107   WTAU(I) = TAU(I) - TAUR(I)
       GO TO (777,101,777,101), IFPATH
 777   PAUSE 777
C      This pause 777 denotes a machine error.
       GO TO 1
 109   BGFR(J) = WBGF
       SMF(J) = COEF *FKZER
 105   Continue
       IF (SENSE SWITCH 6) 209, 210
 210   PUNCH615, IJMAX,H,(BGF(I),I = 1, IJMAX), (BGFR(I),I = 1,IJMAX),
       (SMF(I), I=1, IJMAX),
       (SMGR(I),I=1, IJMAX)
C      Kernel functions are on cards in correct
C      order suitable for immediate use with HERESY 1.
       GO TO 1
 209   SENSE LIGHT 4
       End (0,1,0,0,0)
```

References

(1) Feinberg, S. M., "Heterogeneous Methods for Calculating Reactors" 1955 Geneva Conference Proceedings, Vol.5, page 490 et seq

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(3) Weinberg and Wigner, "The Physical Theory of Neutron Chain Reactors", University of Chicago Press (1958)

(4) Klahr, C. N., Mendelsohn, L. B., "Heterogeneous Reactor Calculation Methods", Quarterly Report No. 4, NYO-2676, March 31, 1960

(5) Klahr, C. N., Mendelsohn, L. B., Heitner, J., "Heterogeneous Reactor Calculation Methods", Quarterly No. 3, NYO-2675, Dec. 31, 1959

(6) Klahr, C. N., Mendelsohn, L. B., "Heterogeneous Reactor Calculation Methods", Quarterly Report No. 5, NYO-2677, June 30, 1960

(7) Reference (1), page 495

(8) Allen, E. E., "Polynomial Approximations to Some Modified Bessel Functions", Mathematical Tables & Other Aids to Computation; Vol. X Nos. 53-56, 1956, (pp. 162-163).