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SEAL-SHELL - A DIGITAL PROGRAM TO DETERMINE STRESSES AND DEFLECTIONS IN AN AXISYMMETRIC SHELL OF REVOLUTION

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SEAL-SHELL - A DIGITAL PROGRAM TO DETERMINE STRESSES AND DEFLECTIONS IN AN AXISYMMETRIC SHELL OF REVOLUTION

C. M. Friedrich

I. INTRODUCTION

SEAL-SHELL, a FORTRAN II program registered as code number M0077 at Bettis Atomic Power Laboratory, is written for the Philco 2000 computer with two tape units. The program is designed to determine loads, deflections, and stresses in a thin shell of revolution under axisymmetric end loads and pressure.

The shell is linear-elastic with variable thickness and with centerline shifts. The stress distribution developed by Reissner (Ref 1) is used in combination with elastic strain theory (Ref 2) and shell equilibrium equations (Ref 3) on short conical elements. These elements, which include flat plates and cylinders, are connected by C^tAC matrix transformation theory (Ref 4) to form the complete shell of revolution. Bending deflections, shear deflections, and pressure-squeezing deflections are included in the thin shell theory.

Comparison of SEAL-SHELL results and SET (Ref 5) results are shown in Fig. 12 for an elliptical head attached to a cylinder of different thickness. In SEAL-SHELL, the head was approximated by three circular arcs of 4, 3, and 3 elements, and the cylinder was divided into 10 elements, for a total of 20 elements or, equivalently, 21 locations along the centerline. Agreement between codes is good in the cylinder, but the three-arc approximation of the elliptical head in SEAL-SHELL produces spurious discontinuity stresses at the arc joints.

When the head is actually a combination of circular arcs, such as a torispherical head, the SEAL-SHELL program gives reasonable results. In Fig. 13 are shown SEAL-SHELL and photoelastic results for a complete pressure vessel model with a torispherical head and a flat bottom. In the sharp corner between the cylinder and the bottom plate, only three conical elements were used by SEAL-SHELL and the agreement is still satisfactory.

SEAL-SHELL-1 (M0077) is now in production status. Work has been started on an improved program which will include

1. a curved elliptical centerline on each element,
2. linearly varying thickness on each element,
3. thermal stresses,
4. thick shell theory,
5. axisymmetric loads along the shell as well as at the ends.

Three examples showing how SEAL-SHELL-1 input should be prepared are given in the Appendix.

II. THEORY

A. Loads Per Unit Length

Consider a conical section of length dx, width $Rd\theta$, and thickness H_e as shown in Fig. 1. Let the loads per unit edge length be as shown in Fig. 2, in agreement with the input notation on page 25.

Let ϕ = the cone angle such that $\begin{cases} \sin\phi = -dz/dx \\ \cos\phi = +dr/dx \end{cases}$

with ϕ independent of x.

Let

$$F_s = \begin{bmatrix} 1 \\ M_x \\ M_y \\ N_x \\ N_y \\ Q_x \end{bmatrix} = \text{force matrix of unit pressure and loads per unit edge.}$$

This matrix can be used to evaluate the local stresses:

$$F = \begin{bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{pp} \\ \tau_{xp} \end{bmatrix},$$ where

σ_{xx} = stress in x direction,

$\sigma_{\theta\theta}$ = stress in circumferential direction,

σ_{pp} = stress in normal direction to plate center surface,

τ_{xp} = shear stress by Q_x.

Let C = inward distance from the middle (negative if outward):

$$-H_e/2 \leq C \leq H_e/2.$$

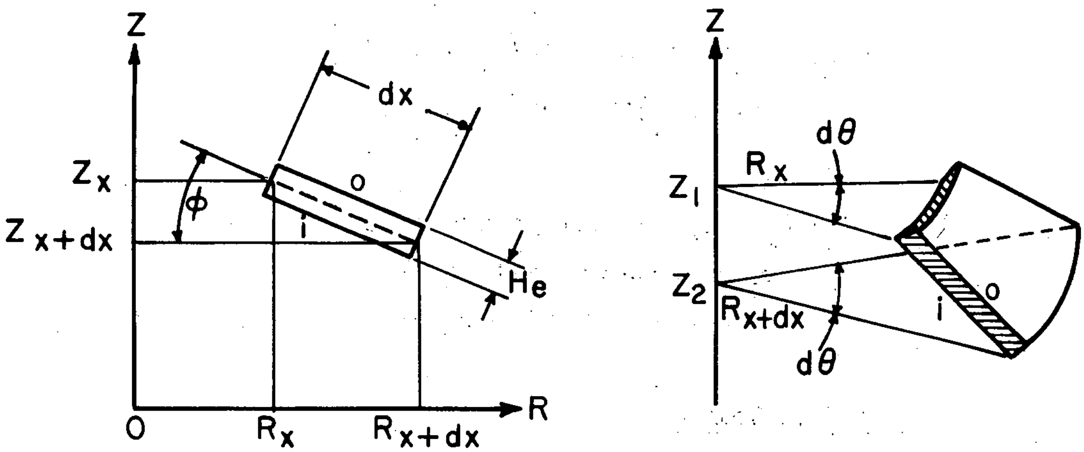


FIG. 1: CONE GEOMETRY

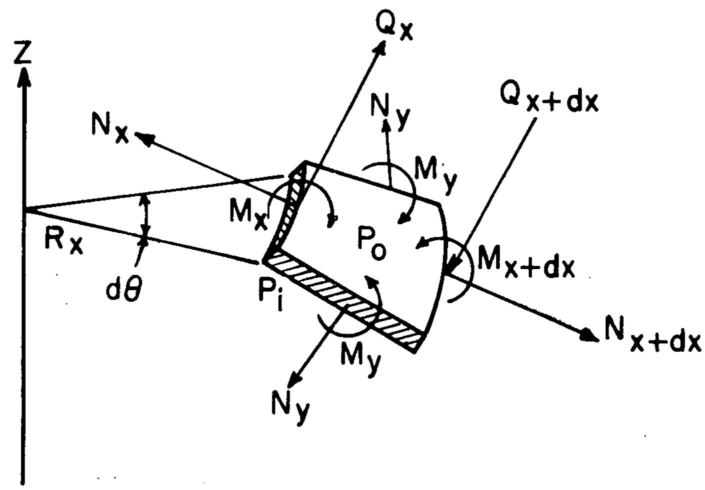


FIG. 2: LOADS PER UNIT EDGE LENGTH

Then[*]

$$\sigma_{xx} = (12C/H_e^3)M_x + (1/H_e)N_x \,,$$
$$\sigma_{\theta\theta} = (12C/H_e^3)M_y + (1/H_e)N_y \,,$$
$$\sigma_{pp} = -P_N + P_M(-1.5C/H_e + 2C^3/H_e^3) \,, \qquad (1)$$
$$\tau_{xp} = (1.5/H_e)(1 - 4C^2/H_e^2)Q_x \,,$$

where

$P_N = 0.5(P_i + P_o)$ = average pressure,
$P_M = P_i - P_o$ = pressure difference,
$\sigma_{pp} = -P_i$ at $C = +H/2$ and $-P_o$ at $C = -H_e/2$.

B. Strain Energy Per Unit Volume

The working deflections of F_ϵ are the strains[**]

$$\epsilon_x = \sigma_{xx}/E_m - \nu\sigma_{\theta\theta}/E_m - \nu\sigma_{pp}/E_m \,,$$
$$\epsilon_\theta = \sigma_{\theta\theta}/E_m - \nu\sigma_{xx}/E_m - \nu\sigma_{pp}/E_m \,,$$
$$\epsilon_p = \sigma_{pp}/E_m = \nu\sigma_{xx}/E_m - \nu\sigma_{\theta\theta}/E_m \,,$$
$$\gamma_{xp} = \tau_{xp}/E_{sm},$$

or in matrix form,

$$D_\epsilon = A_\epsilon \cdot F_\epsilon \qquad (2a)$$

where

$$D_\epsilon = \begin{bmatrix} \epsilon_x \\ \epsilon_\theta \\ \epsilon_p \\ \gamma_{xp} \end{bmatrix} \text{ and } A_\epsilon = \begin{bmatrix} 1/E_m & -\nu/E_m & -\nu/E_m & 0 \\ -\nu/E_m & 1/E_m & -\nu/E_m & 0 \\ -\nu/E_m & -\nu/E_m & 1/E_m & 0 \\ 0 & 0 & 0 & 1/E_{sm} \end{bmatrix} \qquad (2b)$$

The strain energy per unit volume is then

$$V_\epsilon = \frac{1}{2} F_\epsilon^t \cdot D_\epsilon = \frac{1}{2} F_\epsilon^t \, A_\epsilon \, F_\epsilon \,, \qquad (3)$$

where A_ϵ is called the flexibility matrix of the force system F_ϵ .

---

[*]Reference 1 - Reissner
[**]Reference 2 - Timoshenko

C. <u>Strain Energy of Loads Per Unit Length</u>

Equation (1) in matrix form is

$$F_\epsilon = C_\epsilon \cdot F_s \qquad (4a)$$

where C_ϵ = the connection matrix for F_ϵ in terms of F_s

$$= \left[\begin{array}{c|c|c|c|c|c} 0 & \frac{12C}{H_e^3} & 0 & \frac{1}{H_e} & 0 & 0 \\ \hline 0 & 0 & \frac{12C}{H_e^3} & 0 & \frac{1}{H_e} & 0 \\ \hline -P_N+P_M\left(\frac{-1.5C}{H_e} + \frac{2C^3}{H_e^3}\right) & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & \left(\frac{1.5}{H_e}\right)\left(1-\frac{4C^2}{H_e^2}\right) \end{array}\right] \qquad (4b)$$

The strain energy over the thickness H_e of the force system F_s is

$$V_s = \int_{-H_e/2}^{H_e/2} V_\epsilon \cdot dC ,$$

so that from Eq (3) and Eq (4),

$$V_s = \int_{-H_e/2}^{+H_e/2} \left[\frac{1}{2} (C_\epsilon F_s)^t A_\epsilon (C_\epsilon F_s)\right] dC ,$$

or

$$V_s = \frac{1}{2} F_s^t A_s F_s , \qquad (5a)$$

where

$$A_s = \int_{-H_e/2}^{+H_e/2} (C_\epsilon^t A_\epsilon C_\epsilon) dC \qquad (5b)$$

= the flexibility matrix of F_s.

By definition, the working deflection of F_s is D_s in the equation

$$V_s = \frac{1}{2} F_s^t D_s ,$$

so that from Eq (5a)

$$D_s = A_s F_s . \qquad (6a)$$

In engineering problems, the working deflection of the pressure is not used. Deleting the first row of D_s and of A_s gives

$$D_s' = A_s' F_s , \qquad (6b)$$

and evaluation of A_s' from Eqs (5b), (4b), and (2b) gives

$$A_s' = \begin{bmatrix} \frac{1.2\nu P_M}{E_m H_e} & \frac{12}{E_m H_e^3} & \frac{-12\nu}{E_m H_e^3} & 0 & 0 & 0 \\ \frac{1.2\nu P_M}{E_m H_e} & \frac{-12\nu}{E_m H_e^3} & \frac{12}{E_m H_e^3} & 0 & 0 & 0 \\ \frac{\nu P_N}{E_m} & 0 & 0 & \frac{1}{E_m H_e} & \frac{-\nu}{E_m H_e} & 0 \\ \frac{\nu P_N}{E_m} & 0 & 0 & \frac{-\nu}{E_m H_e} & \frac{1}{E_m H_e} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1.2}{E_{sm} H_e} \end{bmatrix} \qquad (6c)$$

D. Strain Energy of Unit Loads

The equilibrium equations for the force system F_s are*

$$\left.\begin{aligned} 0 &= \frac{dN_x}{dx} + \frac{\cos\phi}{R}(N_x - N_y), \\ 0 &= \frac{dQ_x}{dx} + \frac{\cos\phi}{R}Q_x + \frac{\sin\phi}{R}(N_y + H_e P_N) - P_M, \\ 0 &= \frac{dM_x}{dx} + \frac{\cos\phi}{R}(M_x - M_y) - Q_x . \end{aligned}\right\} \tag{7}$$

In order to separate out the pressure effects from the equilibrium equations, another unit force system F_u will be used. Let

$$\left.\begin{aligned} M_x &= \left(\frac{R^2}{6}\right) P_M + \left(\frac{1}{2\pi R}\right) F_{mx} \\ M_y &= \left(\frac{1}{2\pi}\right) F_{my} \\ N_x &= \left(\frac{R\sin\phi}{2} P_M - H_e P_N\right) + \frac{\cos\phi}{2\pi R} F_{rx} + \frac{\sin\phi}{2\pi R} F_{zx}, \\ N_y &= (R\sin\phi\, P_M - H_e P_N) + \left(\frac{1}{2\pi}\right) F_{ry}, \\ Q_x &= \left(\frac{R\cos\phi}{2} P_M\right) - \left(\frac{\sin\phi}{2\pi R}\right) F_{rx} + \left(\frac{\cos\phi}{2\pi R}\right) F_{zx} . \end{aligned}\right\} \tag{8a}$$

or in matrix form

$$F_s = C_s F_u \tag{8b}$$

---

*Reference 3, Timoshenko;
Also, for $Q_x = 0$ and $P_M = 0$, $N_y = N_x = -H_e P_N$.

where

$$F_u = \begin{bmatrix} 1 \\ F_{mx} \\ F_{rx} \\ F_{zx} \\ F_{my} \\ F_{ry} \end{bmatrix}$$

and C_s = connection matrix relating F_s and F_u

= (8a)

1	0	0	0	0	0
$\frac{R^2}{6} P_M$	$\frac{1}{2\pi R}$	0	0	0	0
0	0	0	0	$\frac{1}{2\pi}$	0
$\frac{R \sin\phi}{2} P_M - H_e P_N$	0	$\frac{\cos\phi}{2\pi R}$	$\frac{\sin\phi}{2\pi R}$	0	0
$R \sin\phi \, P_M - H_e P_N$	0	0	0	0	$\frac{1}{2\pi}$
$\frac{R \cos\phi}{2} P_M$	0	$\frac{-\sin\phi}{2\pi R}$	$\frac{\cos\phi}{2\pi R}$	0	0

Substitution of (8a) into (7) gives

$$\left.\begin{aligned} 0 &= \frac{dF_{rx}}{dx} - F_{ry} , \\ 0 &= \frac{dF_{zx}}{dx} , \\ 0 &= \frac{dF_{mx}}{dx} - (\cos\phi)(F_{ny} + F_{zx}) - (\sin\phi)F_{rx} ; \end{aligned}\right\} \qquad (9)$$

and these equilibrium equations are explicitly independent of pressure!

As shown in Fig. 4, F_{mx} is a bending moment, F_{rx} is a radial force, and F_{zx} is an axial force. All three forces act over the entire circumference $2\pi R$ so that the strain energy of F_u is

$$V_u = 2\pi R V_s . \qquad (10a)$$

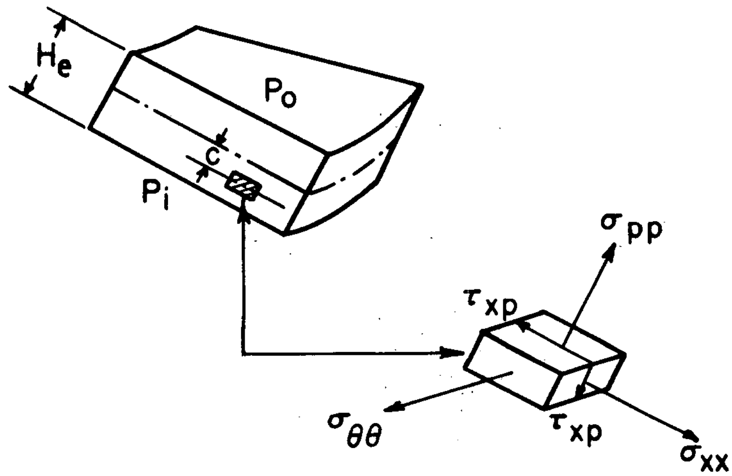


FIG. 3 : STRESSES

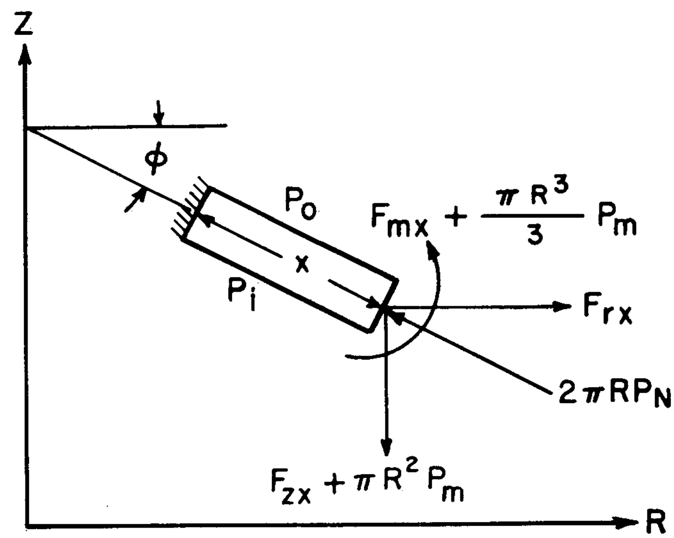


FIG. 4 : FORCE SYSTEM Fu

Substitution of F_s from Eq (8b) into V_s of Eq (5a) and then V_s into Eq (10a) gives

$$V_u = \frac{1}{2} F_u^t A_u F_u , \tag{10b}$$

where

$$A_u = (2\pi R)(C_s^t A_s C_s) . \tag{10c}$$

Then the working deflections of F_u are given by

$$D_u = A_u F_u . \tag{11}$$

E. Strain Energy of a Conical Element

The basic element used in the SEAL-SHELL program is the section of a cone as shown in Fig. 5.

Let
- e = subscript at the start of the element,
- $e+1$ = subscript at the end of the element,
- X_{12} = length of the element in the middle surface,
- H_{12} = uniform thickness of the element,
- U = fractional distance along the element; that is

$$X = X_e + UX_{12} ,$$
$$R = R_e + U(R_{e+1} - R_e) ,$$
$$Z = Z_e + U(Z_{e+1} - Z_e)$$

at any distance X from e towards e+1.

Let

$F_{ze} = F_{zx}$ at e, and $F_{ze+1} = F_{zx}$ at e+1,
$F_{re} = F_{rx}$ at e, $\quad F_{re+1} = F_{rx}$ at e+1,
$F_{me} = F_{mx}$ at e; $\quad F_{me+1} = F_{mx}$ at e+1

be the element loads at e and e+1.

As an approximation, F_{my} and F_{ry} are assumed to vary linearly from e to e+1. With this assumption, the following eight independent loads are found to act on the element in such a way that the equilibrium equations (9) are satisfied for $0 \leq U \leq 1$.

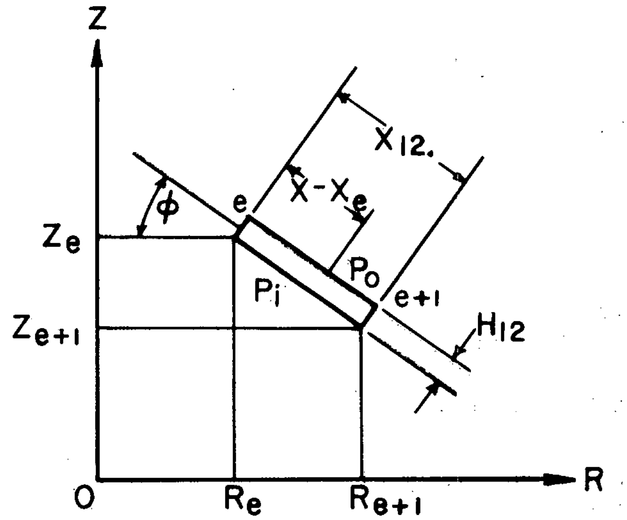


FIG. 5 : ELEMENT GEOMETRY

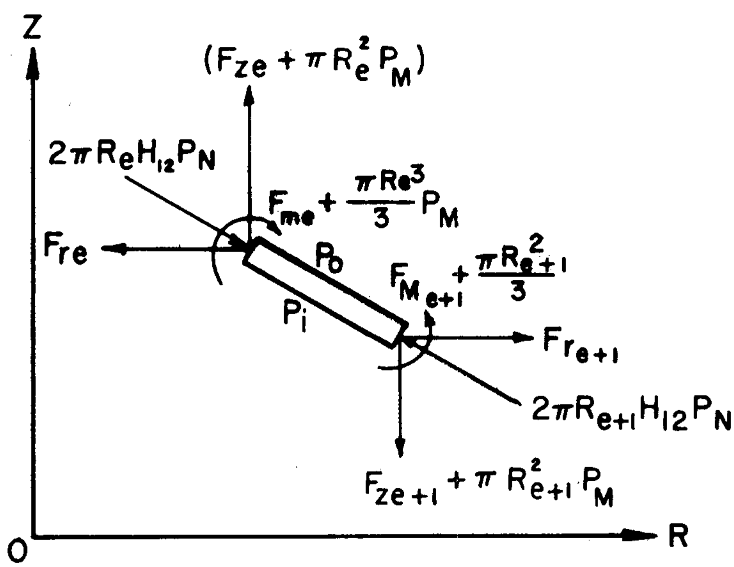


FIG. 6 : ELEMENT LOADS, Fe

1. Pressure (independent of Eq 9)

2. F_{re}, the radial load at e such that

$$F_{mx} = \left[(\sin\phi)(X_{12})(- U + 2U^2 - U^3)\right] F_{re}$$

$$F_{rx} = \left[1 - 4U + 3U^2\right] F_{re}$$

$$F_{zx} = 0$$

$$F_{my} = 0$$

$$F_{ry} = \left[(- 4 + 6U)/X_{12}\right] F_{re}$$

3. F_{re+1}, the radial load at e+1 such that

$$F_{mx} = \left[U^2 - U^3)X_{12} \sin\phi\right] F_{re+1}$$

$$F_{rx} = \left[- 2U + 3U^2\right] F_{re+1}$$

$$F_{zx} = 0$$

$$F_{my} = 0$$

$$F_{ry} = \left[(- 2 + 6U)/X_{12}\right] F_{re+1}$$

4. V_e, the axial load such that

$$F_{mx} = 0$$

$$F_{rx} = 0$$

$$F_{zx} = + F_{ze}$$

$$F_{my} = - F_{ze}$$

$$F_{ry} = 0$$

5. F_{me}, the moment at e such that

$$F_{mx} = \left[1 - U \cos^2\phi + (- 3U^2 + 2U^3) \sin^2\phi\right] F_{me}$$

$$F_{rx} = \left[(U - U^2)(6 \sin\phi)/X_{12}\right] F_{me}$$

$$F_{zx} = 0$$

$$F_{my} = \left[- (\cos\phi)/X_{12}\right] F_{me}$$

$$F_{ry} = \left[(1 - 2U)(6 \sin\phi)/X_{12}^2\right] F_{me}$$

6. F_{me+1}, the moment at e+1 such that

$$F_{mx} = \left[U \cos^2\phi + (3U^2 - 2U^3)\sin^2\phi \right] F_{me+1}$$

$$F_{rx} = \left[(-U + U^2)(6 \sin\phi)/X_{12} \right] F_{me+1}$$

$$F_{zx} = 0$$

$$F_{my} = \left[+ (\cos\phi)/X_{12} \right] F_{me+1}$$

$$F_{ry} = \left[(-1 + 2U)(6 \sin\phi)/X_{12}^2 \right] F_{me+1}$$

7. F_{mr}, a redundant force-moment distribution such that

$$F_{mx} = \left[(U - 3U^2 + 2U^3) \cos\phi \sin\phi \right] F_{mr}$$

$$F_{rx} = \left[(U - U^2)(6 \cos\phi)/X_{12} \right] F_{mr}$$

$$F_{zx} = 0$$

$$F_{my} = \left[(\sin\phi)/X_{12} \right] F_{mr}$$

$$F_{ry} = \left[(1 - 2U)(6 \cos\phi)/X_{12}^2 \right] F_{mr}$$

8. F_{mm}, a redundant moment distribution such that

$$F_{mx} = \left[(U - C^2)\cos\phi \right] F_{mm}$$

$$F_{rx} = 0$$

$$F_{zx} = 0$$

$$F_{my} = \left[(1 - 2U)/X_{12} \right] F_{mm}$$

$$F_{ry} = 0$$

In matrix form, the preceding equations are

$$F_u = C_u F_e , \qquad (12a)$$

where

$$F_e = \begin{bmatrix} 1 \\ F_{re} \\ F_{re+1} \\ V_e \\ F_{me} \\ F_{me+1} \\ F_{mr} \\ F_{mm} \end{bmatrix} ,$$

$$C_u = \left[\begin{array}{c|c|c|c|c|c|c|c}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & (-U+2U^2-U^3)X_{12}\sin\phi & (U^2-U^3)X_{12}\sin\phi & 0 & 1-U\cos^2\phi+(-3U^2+2U^3)\sin^2\phi & U\cos^2\phi+(3U^2-2U^3)\sin^2\phi & (U-3U^2+2U^3)\cos\phi\sin\phi & (U-U^2)\cos\phi \\
\hline
0 & 1-4U+3U^2 & -2U+3U^2 & 0 & \frac{(U-U^2)(6\sin\phi)}{X_{12}} & \frac{(-U+U^2)(6\sin\phi)}{X_{12}} & \frac{(U-U^2)(6\cos\phi)}{X_{12}} & 0 \\
\hline
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & -1 & \frac{-\cos\phi}{X_{12}} & \frac{\cos\phi}{X_{12}} & \frac{\sin\phi}{X_{12}} & \frac{1-2U}{X_{12}} \\
\hline
0 & \frac{-4+6U}{X_{12}} & \frac{-2+6U}{X_{12}} & 0 & \frac{(1-2U)(6\sin\phi)}{X_{12}^2} & \frac{(-1+2U)(6\sin\phi)}{X_{12}^2} & \frac{(1-2v)(6\cos\phi)}{X_{12}^2} & 0
\end{array}\right]$$

Then the strain energy of the element is

$$V_e = \int_0^{X_{12}} V_u dx ,$$

so that from Eq (12a) and Eq (10b),

$$V_e = \frac{1}{2} F_e^t D_e , \qquad (13a)$$

where

$$D_e = A_e F_e \ . \tag{13b}$$

and

$$A_e = \int_0^1 (C_u^t A_u C_u)\ X_{12}\ dU \ . \tag{13c}$$

As usual, D_e gives the working deflections of F_e. Since the redundant loads F_{mr} and F_{mm} should do no work, their deflections should be zero. Let

$$F_{we} = \begin{bmatrix} 1 \\ F_{re} \\ F_{re+1} \\ F_{ze} \\ F_{me} \\ F_{me+1} \end{bmatrix} = \text{working loads}$$

and

$$F_{oe} = \begin{bmatrix} F_{mr} \\ F_{mm} \end{bmatrix} = \text{nonworking loads,}$$

so that

$$F_e = \begin{bmatrix} F_{we} \\ F_{oe} \end{bmatrix} \ . \tag{14a}$$

Partitioning Eq (13b) to correspond to Eq (14a) gives

$$\begin{bmatrix} D_{we} \\ D_{oe} \end{bmatrix} = \begin{bmatrix} A_{wwe} & A_{woe} \\ A_{owe} & A_{ooe} \end{bmatrix} \times \begin{bmatrix} F_{we} \\ F_{ow} \end{bmatrix} ,$$

and setting $D_{oe} = 0$ gives

$$F_{oe} = C_{re} F_{we} \quad \text{where} \quad C_{re} = -\ (A_{ooe})^{-1}(A_{owe}) , \tag{14b}$$

so that

$$\underline{D_{we} = A_{we} F_{we}} , \tag{15a}$$

where

$$A_{we} = A_{wwe} + A_{woe} \cdot C_{re} \ . \tag{15b}$$

Equation (15a) is the basic building block used in assembling a shell of revolution from a set of conical elements.

F. Connecting Elements

Suppose n = e-1 elements have been connected with the resulting flexibility equations

$$V_n = \frac{1}{2} F_{wn}^t D_{wn} , \tag{16a}$$

where

$$D_{wn} = A_{wn} F_{wn} , \tag{16b}$$

and

$$F_{wn} = \begin{bmatrix} 1 \text{ (Pressure)} \\ F_{z1} \\ F_{r1} \\ F_{m1} \\ F_{re} \\ F_{me} \end{bmatrix} . \tag{16c}$$

It is desired to attach element e with flexibility equation (15a). Let

$$F_{n+1} = \begin{bmatrix} 1 \\ F_{z1} \\ F_{r1} \\ F_{m1} \\ F_{re+1} \\ F_{me+1} \\ F_{re} \\ F_{me} \end{bmatrix} = \text{matrix of all loads on n and e.} \tag{17}$$

Then

$$F_{wn} = F_{n+1}$$

and

$$F_e = C_e F_{n+1} , \tag{18}$$

where

$$C_{wn} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$C_e = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

The total strain energy becomes

$$V_{n+1} = V_n + V_e = \frac{1}{2} (C_{wn} F_{n+1})^t A_{wn} (C_{wn} F_{n+1}) + \frac{1}{2} (C_e F_{n+1})^t A_e (C_e F_{n+1})$$

or

$$V_{n+1} = \frac{1}{2} F_{n+1}^t D_{n+1} , \tag{19a}$$

where

$$D_{n+1} = A_{n+1} F_{n+1} , \tag{19b}$$

and

$$A_{n+1} = C_{wn}^t A_{wn} C_{wn} + C_e^t A_e C_e . \tag{19c}$$

Once n and e are joined, the loads F_{re} and F_{me} are redundant and have zero net deflections. Let

$$F_{wn+1} = \begin{bmatrix} 1 \\ F_{z1} \\ F_{r1} \\ F_{m1} \\ F_{re+1} \\ F_{me+1} \end{bmatrix} = \text{working loads on n+1}, \tag{20a}$$

and

$$F_{on+1} = \begin{bmatrix} F_{re} \\ F_{me} \end{bmatrix} = \text{nonworking loads between n and e,} \qquad (20b)$$

so that

$$F_{n+1} = \begin{bmatrix} F_{wn+1} \\ F_{on+1} \end{bmatrix} . \qquad (20c)$$

Partitioning Eq (19b) to correspond with Eq (20c) gives

$$\begin{bmatrix} D_{wn+1} \\ D_{on+1} \end{bmatrix} = \begin{bmatrix} A_{wwn+1} & A_{won+1} \\ A_{own+1} & A_{oon+1} \end{bmatrix} \cdot \begin{bmatrix} F_{wn+1} \\ F_{on+1} \end{bmatrix}$$

and setting $D_{on+1} = 0$ gives

$$F_{on+1} = C_{rr} F_{wn+1} \quad \text{where} \quad C_{rn} = - (A_{oon+1})^{-1} (A_{own+1}) , \qquad (21)$$

so that

$$\underline{D_{wn+1} = A_{wn+1} F_{wn+1}} , \qquad (22a)$$

where

$$A_{wn+1} = A_{wwn+1} + A_{won+1} C_{rn} . \qquad (22b)$$

Thus there are now n = e elements connected and Eq (22a) is equivalent to Eq (16b) for n = e-1 elements.

G. Shifting Centerlines

In the SEAL-SHELL program, elements of zero thickness are introduced to allow a centerline shift between flexible elements. Suppose element e is such a shift, as shown in Fig. 7.

By use of the equilibrium equations (9) or from inspection of Fig. 7, the force systems at e and e+1 are equipollent if

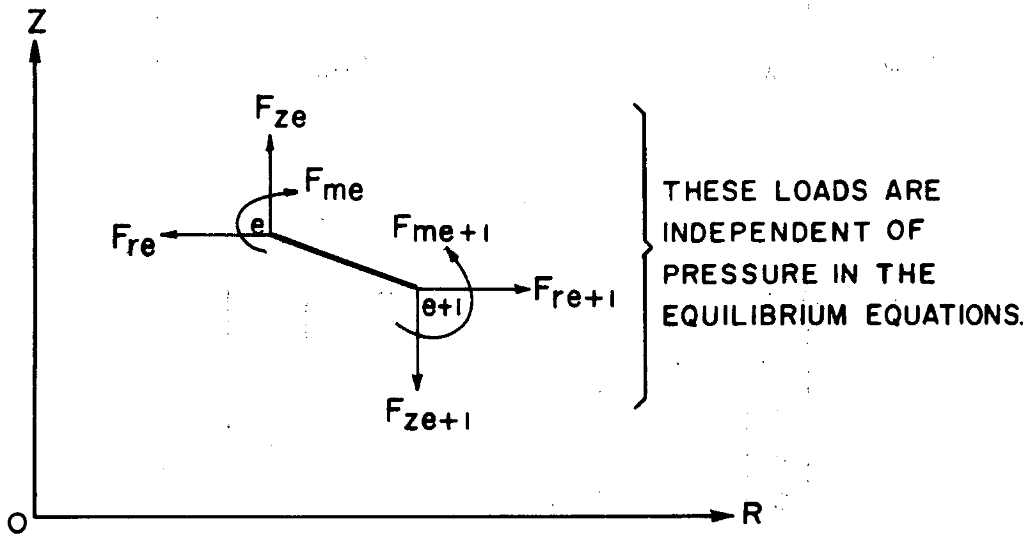


FIG. 7 : CENTERLINE SHIFT FROM e TO e+1

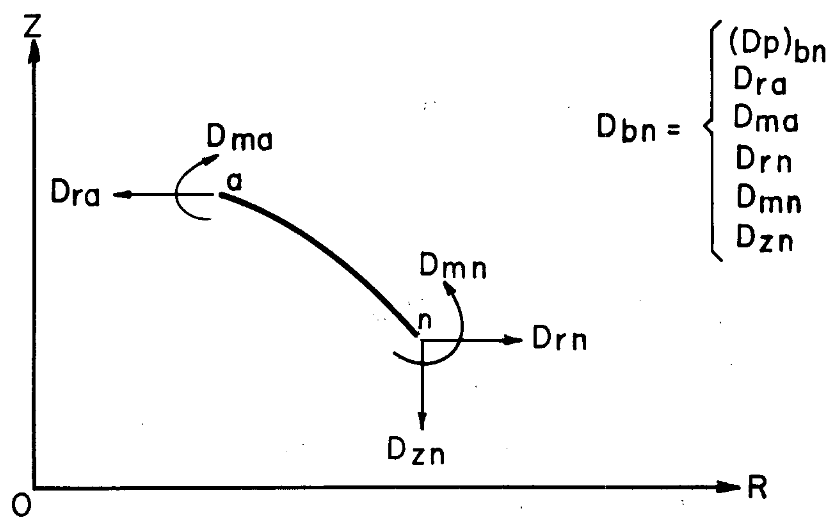


FIG. 8 : END DEFLECTION

$$\left.\begin{aligned} F_{re} &= F_{re+1} , \\ F_{ze} &= F_{ze+1} , \\ F_{me} &= F_{me+1} - (r_{e+1}-r_e)F_{ze+1} + (z_e-z_{e+1})F_{re+1} . \end{aligned}\right\} \quad (23a)$$

Thus

$$F_{we} = C_{wn}F_{wn+1} , \quad (23b)$$

where

$$C_{wn} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & z_e-z_{e-1} & 0 & 0 & r_e-r_{e-1} & 1 \end{bmatrix} , \quad (23c)$$

and

$$V_{n+1} = \frac{1}{2} F^t_{wn+1} D_{wn+1} , \quad (24a)$$

where

$$D_{wn+1} = A_{wn+1} F_{wn+1} , \quad (24b)$$

and

$$A_{wn+1} = C^t_{wn} A_{wn} C_{wn} . \quad (24c)$$

H. End Loads of Shell

Let

$$\left\{\begin{aligned} F_{ra} &= (F_{re} - P_N 2\pi R_e H_e \cos \phi)_{e=1} \\ F_{ma} &= (F_{me} + P_M \pi R_o^3/3)_{e=1} \\ F_{rn} &= (F_{re+1} - P_N 2\pi R_{e+1} H_e \cos \phi)_{e=N_e} \\ F_{mn} &= (F_{me+1} + P_M \pi R^3_{e+1}/3)_{e=N_e} \\ F_{zn} &= (F_{ze+1} - P_N 2\pi R_{e+1} H_e \sin \phi)_{e=N_e} \end{aligned}\right.$$

be the loads on the boundaries a and n of the shell of revolution. (In the SEAL-SHELL program, these loads as input are given per unit circumference $2\pi R$.) In matrix form,

$$F_{bn} = C_{bn} F_{wn_e} , \tag{25a}$$

where

$$F_{bn} = 1 \text{ (Pressure)} , \quad F_{wne} = F_{wn} \text{ at } n=n_e, \text{ and} \tag{25b}$$

$C_{an} =$

1	0	0	0	0	0
$-P_N(2\pi R_e H_e \cos\phi)_1$	0	1	0	0	0
$+P_M(\pi R_e{}^3/3)_1$	0	0	1	0	0
$-P_N(2\pi R_{e+1} H_e \cos\phi)N_e$	0	0	0	1	0
$+P_M(\pi R^3_{e+1}/3)N_e$	0	0	0	0	1
$-P_N(2\pi R_{e+1} \sin\phi)N_e$	1	0	0	0	0

Then the total strain energy in the shell is

$$V_{bn} = \frac{1}{2} F^t_{bn} D_{bn} , \tag{26a}$$

where

$$D_{bn} = A_{bn} F_{bn} , \tag{26b}$$

and

$$A_{bn} = C^t_{bn} (A_{bn})N_e C_{bn} . \tag{26c}$$

D_{nb} gives the deflections of F_{bn}, as shown in Fig. 8.

Any consistent combination of the variables in D_{bn} and F_{bn} may be used as input to the SEAL-SHELL program.

III. INPUT FOR SEAL-SHELL

A. Deck Arrangement

An input deck for one problem consists of the accounting card, problem cards 1000 through 8000, and a blank card

Accounting Card
Card 1000

Card 8000
Blank Card

If a set of problems is to be run successively, the arrangement consists of the accounting card, sets of problem cards 1000 through 8000, and one blank card:

Accounting Card
First Problem: Card 1000

First Problem: Card 8000

Last Problem: Card 1000

Last Problem: Card 8000
Blank Card

An explanation of the input table follows.

TABLE I: INPUT ARRANGEMENT

ACCOUNTING

1-8	9		11-15		17-21		25-32	33-37	38-72
NUMBER	$		M0077		USER		CHARGE	PRODN	

1000 TO 4000

1-6	7-12	13-18	19-24	25-30	31-36	37-42	43-48	49-54	55-60	61-66	67-72
M0077	NAME			PHONE			DATE				1000
OPENING COMMENTS											2000
N_e	P_i		P_o		E_m		E_{sm}		P_{nu}		3000
K_{an}	G_{ra}		G_{ma}		G_{rn}		G_{mn}		G_{zn}		4000

5000, 5001, ETC.

1-6	7-12	13-18	19-24	25-30	31-36	37-42	43-48	49-54	55-60	61-66	67-72
	R_a		Z_a								5000
I_e	R_e		Z_e		H_e		ANGLE				5001
I_e	R_e		Z_e		H_e		ANGLE				ETC.

6000, 6001, ETC

1-6	7-12	13-18	19-24	25-30	31-36	37-42	43-48	49-54	55-60	61-66	67-72
I_e	K_{ffd}	I_e	K_{ffd}	I_e	K_{ffd}	I_e	K_{ffd}	I_e	K_{ffd}		6000
I_e	K_{ffd}	I_e	K_{ffd}	I_e	K_{ffd}	I_e	K_{ffd}	I_e	K_{ffd}		6001
I_e	K_{ffd}	I_e	K_{ffd}	I_e	K_{ffd}	I_e	K_{ffd}	I_e	K_{ffd}		ETC.

7000, 7001, ETC.

1-6	7-12	13-18	19-24	25-30	31-36	37-42	43-48	49-54	55-60	61-66	67-72
I_e	K_{ss}	I_e	K_{ss}	I_e	K_{ss}	I_e	K_{ss}	I_e	K_{ss}		7000
I_e	K_{ss}	I_e	K_{ss}	I_e	K_{ss}	I_e	K_{ss}	I_e	K_{ss}		7001
I_e	K_{ss}	I_e	K_{ss}	I_e	K_{ss}	I_e	K_{ss}	I_e	K_{ss}		ETC.

8000, AND BLANK CARD (NO COMMENTS IN COLUMNS 73-80 OF BLANK CARD)

1-66	67-72
CLOSING COMMENTS	8000

B. <u>Format of Words</u>

nX = n blank columns.

nA = n columns of alphabet letters, digits, blanks, or any combination.

In = n columns for an integer number, which is placed to the right. For example, using the format I6:

300 =				3	0	0
7 =						7
0 =						0

E12.4 = 12 columns of an exponential number in the form

$$(\pm\ a.bcde) \times 10^{(\pm\ fgh)} =$$

±	a	.	b	c	d	e		±	f	g	h
		number					blank or E	exponent			

where the decimal point is in the third column and the exponential sign is in the ninth column. For example,

- 3 =	-	3	.						+			0
0.06080 =		6	.	0	8				-			2
$6x10^{12}$ =		6	.						+		1	2
0 =		0	.						+			0

Columns 2, 3, 9, and 12 should not be blank; and column 8 should be blank or contain the letter E.

C. Input Cards and Notations

Accounting Card

At Bettis Atomic Power Laboratory, each problem deck for the Philco 2000 has an accounting card of this form:

Columns	1-8	9	10	11-15	16	17-21	22-80
Formats	I8	A1	X1	A5	X1	A5	X59
Words	Number	$	Blank	Code	Blank	User	Blank

The number in columns 1-8 is assigned to each user. A dollar sign is in column 9. For SEAL-SHELL, the code number in columns 11-15 is M0077. The user's code name is in columns 17-21.

Card 1000

Card 1000 is the title card for a problem, and has this form:

Columns	1	2-6	7-24	25-42	43-60	61-66	67-72
Formats	X1	A5	A18	A18	A18	X6	I6
Words	Blank	M0077	Name	Phone	Date	Blank	1000

This card should contain the code number, user's name, phone number and extension, date, and card number.

In all cards 1000 through 8000, columns 67-72 are reserved for the card number, and columns 73-80 may be used by the user for comments or sequencing if he so desires.

Card 2000

Card 2000 is for comments on the particular problem:

Columns	1-66	67-72
Formats	A66	I6
Words	Comments	2000

Card 3000

Card 3000 contains the basic parameters of the problem.

Consider the intersection of the shell of revolution in a meridian plane. Let

a = the starting point of the centerline of intersection. Either end may be chosen as such.

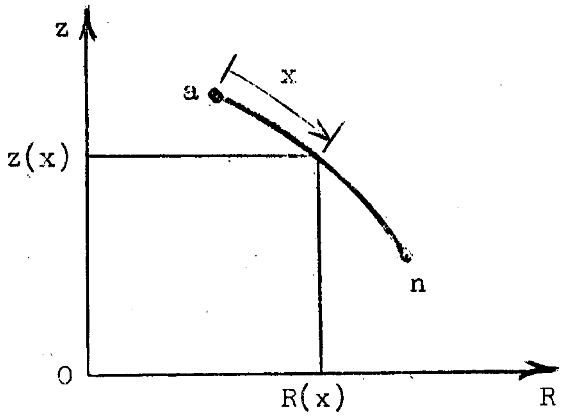
n = the ending point of the centerline.

x = a distance along the centerline from the end a, in.

R = R(x) = the radius of the centerline at x from the axis of revolution, in. $0 \leq R \leq 1000$ in.

Z = Z(x) = the elevation of the centerline at x on the axis of revolution, in. $0 \leq Z \leq 1000$ in.

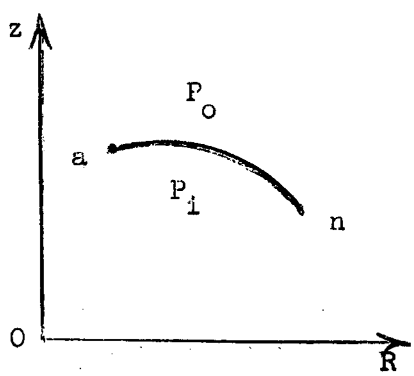
Construct the R-Z axes from an origin 0 so that the +R axis is 90° cw* from the +Z axis.

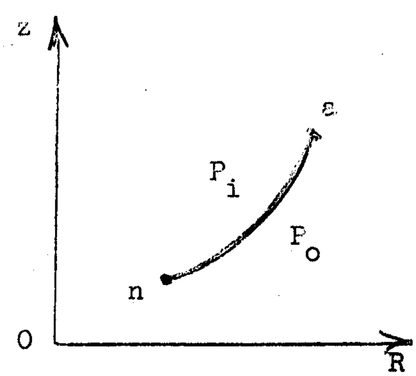


When an observer moves along the centerline from a towards n, the region to his right in the R-Z plane is defined as the inside and the region to his left is the outside. Let

P_i = the inside pressure, psi.

P_o = the outside pressure, psi.





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* ccw = counterclockwise

cw = clockwise

Let

E_m = the tensile modulus of elasticity, psi. In the code, $1 \leq E_m$.

E_{sm} = the shear modulus of elasticity, psi. $1 \leq E_{sm}$.

P_{nu} = Poisson's ratio. $0 \leq P_{nu} \leq 0.4999$.

N_e = the number of elements used in flexibility calculations = 2 to 100. The elements are arranged in sequence from a to n along the centerline of the shell in the R-Z plane.

The preceding six parameters are placed as follows on card 3000:

Columns	I-6	7-18	19-30	31-42	43-54	55-66	67-72
Formats	I6	E12.4	E12.4	E12.4	E12.4	E12.4	I6
Words	N_e	P_i	P_o	E_m	E_{sm}	P_{nu}	3000

Card 4000

Card 4000 specifies the types of boundary conditions. At ends a and n, there can be three loads applied per unit circumference and for each load, there is an associated displacement.

Let

F_{za} = force per unit circumference of a, parallel to the Z axis, lbf/in. F_{za} is positive if it is in the same direction as the positive Z axis (+↑) and negative if in the opposite direction (-↓).

D_{za} = axial deflection of F_{za}, in. (+↑ and -↓)

F_{ra} = force per unit circumference of a, parallel to the R axis, lbf/in. F_{ra} is positive if it is in the direction opposite to the +R axis (←+) and negative if in the same direction (-→).

D_{ra} = inward radial deflection of F_{ra}, in. (←+ and -→)

F_{ma} = bending moment per unit circumference of a, in.-lbf/in. F_{ma} is positive if cw (↻+) and negative if ccw (↺-).

D_{ma} = rotation of F_{ma}, radians. (↻+ and ↺-)

F_{zn} = force per unit circumference of n, parallel to the Z axis, lbf/in. F_{zn} is positive if opposite to the +Z axis (+↓) and negative if in the same direction (-↑).

D_{zn} = axial deflection of F_{zn}, in. (+↓ and -↑)

F_{rn} = force per unit circumference of n, parallel to the R axis, lbf/in. F_{rn} is positive if it is in the same direction as the +R axis ($\xrightarrow{+}$) and negative if opposite ($\xleftarrow{-}$).

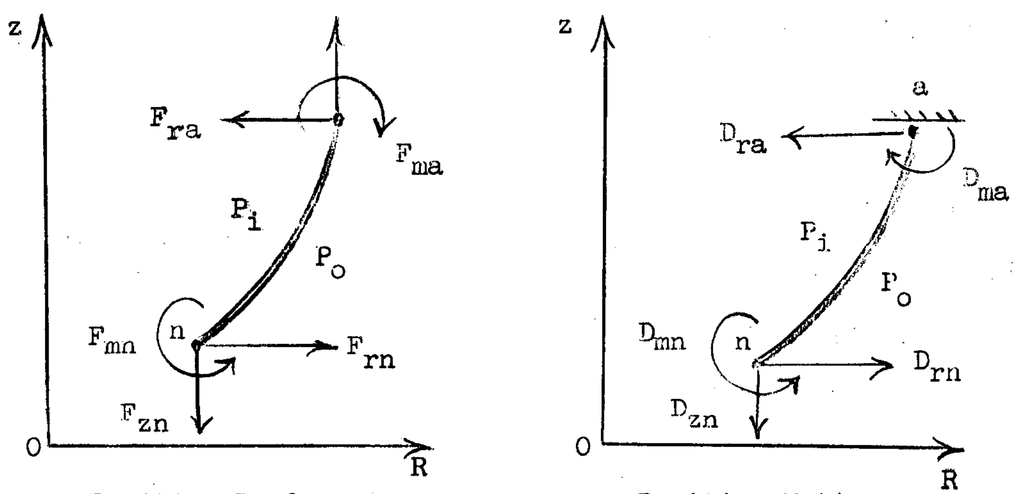
D_{rn} = radial deflection of F_{rn}, in. ($\xrightarrow{+}$ and $\xleftarrow{-}$)

F_{mn} = bending moment per unit circumference of n, in.-lbf/in. F_{mm} is positive if ccw (↺+) and negative if cw (↻−).

D_{mn} = rotation of F_{mn}, radians. (↺+ and ↻−)

If P_i, P_o, and F_{zn} are given, then F_{za} is found from a vertical equilibrium equation and may not be specified independently. Thus a can be considered as a vertical anchor point which, however, can move radially and rotate, with $D_{za} \equiv 0$.

Equilibrium Reaction



Positive Loads   Positive Motions

In the code, these loads are multiplied by circumference and used that way.

Let

K_{ra} = input option for D_{ra} or F_{ra};
= 0 if D_{ra} is given, or 1 if F_{ra} is given.

G_{ra} = given value of D_{ra} or F_{ra}.

K_{ma} = input option for D_{ma} or F_{ma};
= 0 if D_{ma} is given, or 1 if F_{ma} is given.

G_{ma} = given value of D_{ma} or F_{ma}.

K_{rn} = input option for D_{rn} or F_{rn};
= 0 if D_{rn} is given, or 1 if F_{rn} is given.

G_{rn} = given value of D_{rn} or F_{rn}.

K_{mn} = input option for D_{mn} or F_{mn};
= 0 if D_{mn} is given, or 1 if F_{mn} is given.

G_{mn} = given value of D_{mn} or F_{mn}.

K_{zn} = input option for D_{zn} or F_{zn};
= 0 if D_{zn} is given, or 1 if F_{zn} is given.

G_{zn} = given value of D_{zn} or F_{zn}.

The five K's are combined into one number K_{an}. For example, if

$$\left\{\begin{array}{l} K_{ra} = 0 \\ K_{ma} = 1 \\ K_{rn} = 1 \\ K_{mn} = 0 \\ K_{zn} = 0 \end{array}\right\}, \quad \text{then } K_{an} = 01100 \text{ or } 1100$$

Card 4000 has this form:

Columns	1-6	7-18	19-30	31-42	43-54	55-66	67-72
Formats	I6	E12.4	E12.4	E12.4	E12.4	E12.4	I6
Words	K_{an}	G_{ra}	G_{ma}	G_{rn}	G_{mn}	G_{zn}	4000

Card 5000

Card 5000 contains the radius and elevation at a:

Columns	1-6	7-18	19-30	31-42	43-54	55-66	67-72
Formats	I6	E12.4	E12.4	E12.4	E12.4	E12.4	I6
Words	Blank	R_a	Z_a	Blank	Blank	Blank	5000

Cards 5001, 5002, etc.

Instead of giving the radius, elevation, and thickness at the end of each element I_e for I_e = 1 to N_e, where N_e can be 100, the user can specify regions of data, as follows:

Let

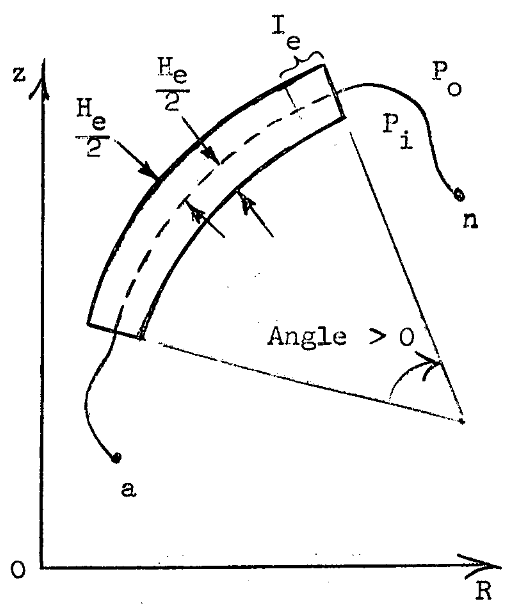
I_e = the element number at the end of a region.

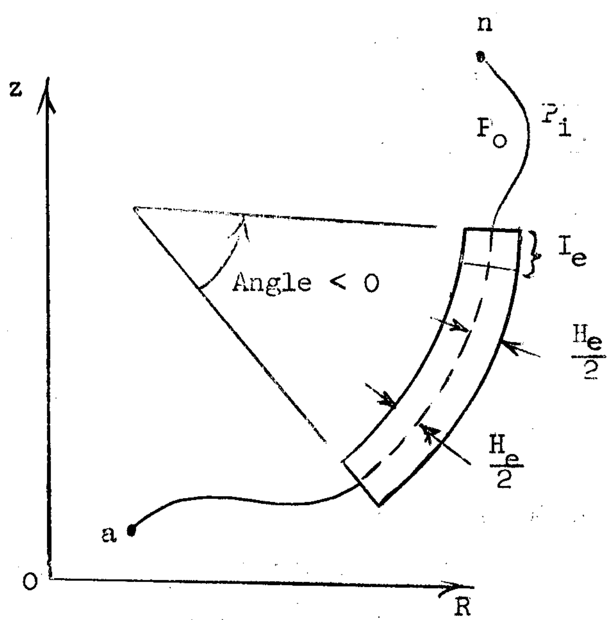
R_e = the radius at the end of the region.

Z_e = the elevation at the end of the region.

H_e = the thickness of the elements in the region, in. $0 \leq H_e \leq 100$ in.

Angle = the angle through which the region turns toward the inside of the shell, degrees. This angle is negative if the region turns towards the outside and is 0 if the centerline is straight. The regions must be chosen so that $-90 \leq \text{Angle} \leq 90$.

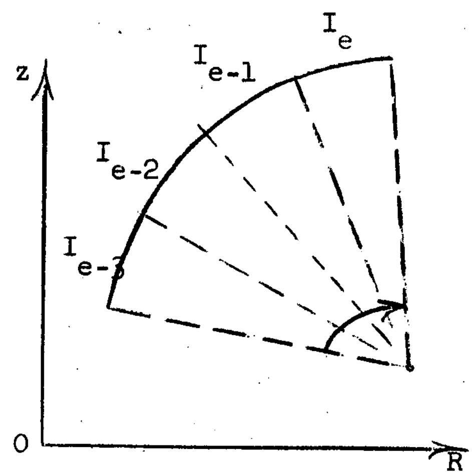




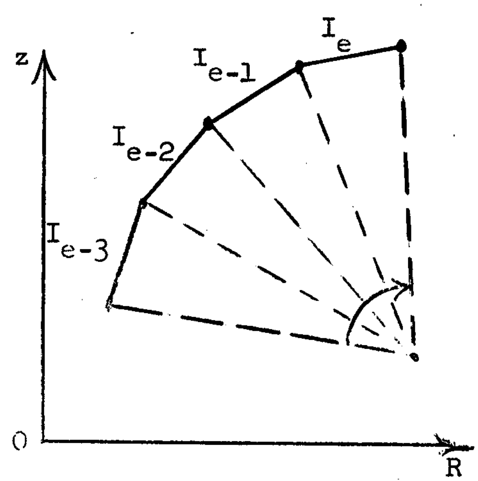
Each region has uniform thickness, uniform curvature, and at least one element. The cards are numbered for the regions in succession: 5001, 5002, etc. The element number I_e for each region is higher than for the preceding region, and the regions continue until $I_e = N_e$. The card for region I has the form:

Columns	1-6	7-18	19-30	31-42	53-54	55-66	67-72
Formats	I6	E12.4	E12.4	E12.4	E12.4	E12.4	I6
Words	I_e	R_e	Z_e	H_e	Angle	Blank	5000+I

In the computer program, each region is broken into equal length elements along the arc of the centerline. For example, if a region has four elements, the region is broken into elements as shown:



Then each element is approximated as a conical surface of zero curvature, so that the centerline becomes kinked into straight segments:

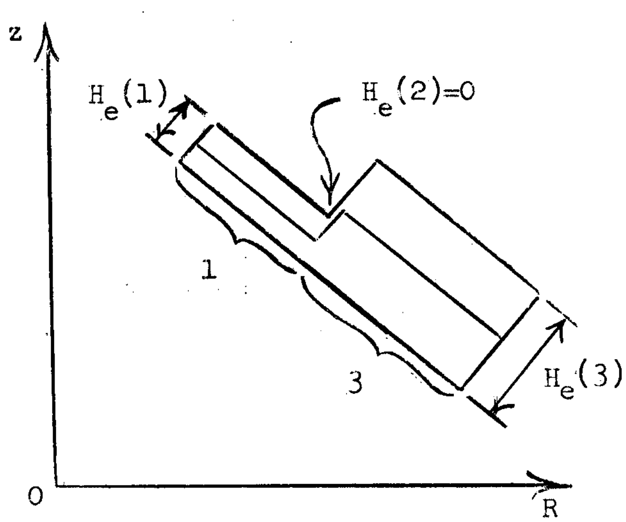


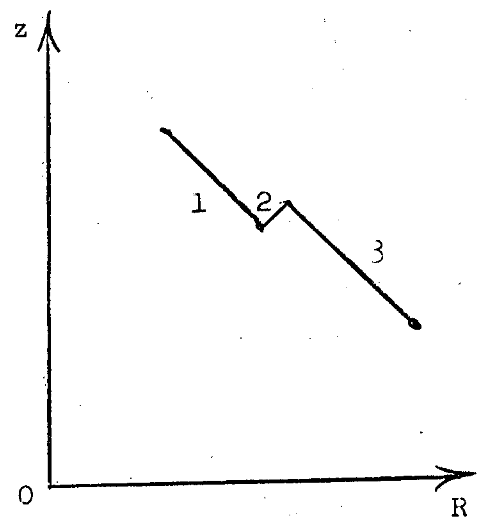
The end points of each element are still on the original curved centerline of the region, but the element interiors come inside the curve as chords.

Note: for accuracy in the flexibility calculations, the element length along the centerline should not be much less than the thickness H_e or be as large as the quantity

$$\sqrt[4]{\left(\frac{R_e}{\sin \phi}\right)^2 \cdot \frac{H_e^{\,2}}{3(1-\nu^2)}} \quad .$$

The thickness of each element is assumed to be bisected by the straight centerline through the element. If two adjacent flexible elements have centerlines which do not meet at the interface, the shift in centerlines is indicated by introducing a region of zero thickness. For example, in the sketch, element 2 is a centerline shift between elements 1 and 3:





A region of zero thickness should contain only one element.

<u>Cards 6000, 6001, etc.</u>

Regions of output are specified for bending moments, tensile forces, shear forces, and deflections. Consider an element I_e.

Let

M_x = bending moment per unit circumference, in.lb/in..

N_x = tensile force per unit circumference, lb/in.

Q_x = shear force per unit circumference, lb/in.

M_y = bending moment per unit centerline length, in. lb/in.

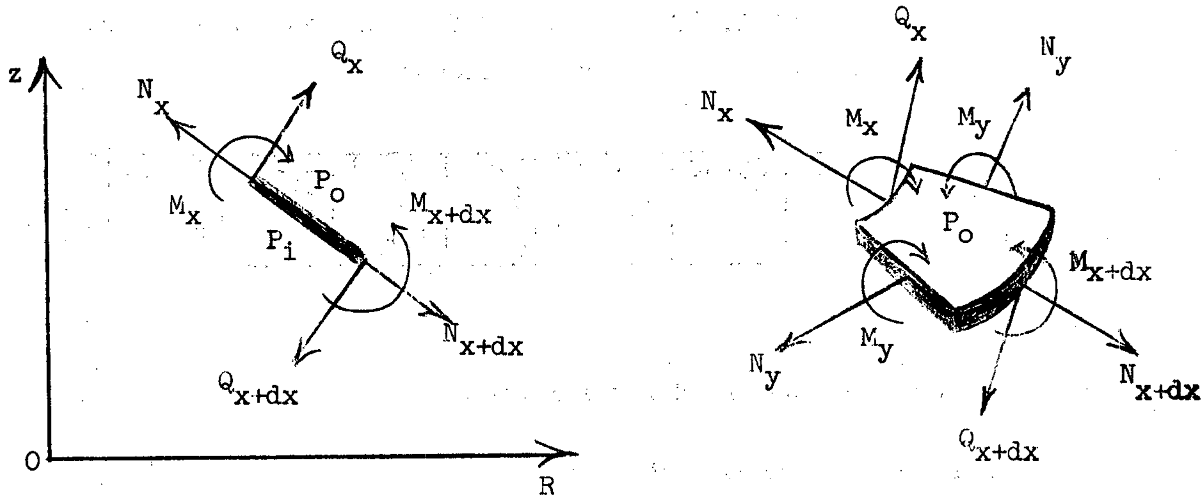
N_y = tensile force per unit centerline length, in. lb/in.

D_m = rotation, radians (↶+).

D_r = radial deflection, in. ($\overset{+}{\rightarrow}$).

D_z = axial deflection, in. (+↓).

Positive sign conventions are shown in the following sketches:



Let

K_{fo} = load option of printing M_x, N_x, Q_x, M_y, and N_y at the start of an element I_e;
= 1 if yes, 0 if no.

K_{f1} = load option of printing M_x, N_x, Q_x, M_y, and N_y at the end of the element I_e;
= 1 if yes, 0 if no.

K_{d1} = load option of printing D_m, D_r, and D_z at the end of the element I_e;
= 1 if yes, 0 if no.

K_{ffd} = a combination of K_{fo}, K_{f1}, and K_{d1}. For example, if

$$\left\{\begin{array}{l} K_{fo} = 0 \\ K_{f1} = 1 \\ K_{d1} = 1 \end{array}\right\}, \text{ then } K_{ffd} = 011 \text{ or } 11.$$

In cards 6000, 6001, etc., regions and K's are given in pairs, five pairs per line, until $I_e = N_e$. Each region is identified by the last element I_e in that region, as in the 5000 series of cards. For example, with $N_e = 50$, the sequence

$$\left\{10,\ 100,\ 32,\ 0,\ 50,\ 1\right\}$$

indicates that the first 10 regions have M_{xo}, N_{xo}, Q_{xo}, M_{yo}, and N_{yo} printed out; regions 11 to 32 have nothing printed out; regions 33 to 50 have D_{ml}, D_{rl}, and D_{zl} printed in the output to the problem.

The cards are numbered 6000, 6001, etc. If five or less pairs reach $I_e = N_e$, only one card is needed. The card follows:

Columns	1-6	7-12	13-18	19-24	25-30	31-36	37-42	43-48	49-54	55-60	61-66	67-72
Formats	I6	I6	I6	I6	I6	I6	I6	I6	I6	I6	I6	I6
Words	I_e	K_{ffd}	I_e	I_{ffd}	I_e	K_{ffd}	I_e	I_{ffd}	I_e	K_{ffd}	Blank	6000+I

Cards 7000, 7001, etc.

Regions of output are specified for stresses.

Let

S_{xx} = normal stress produced by M_x and N_x, psi.

S_{yy} = normal stress produced by M_y and N_y, psi.

S_{zz} = the third normal stress, psi. At the inside surface, $S_{zz} = -P_i$; and at the outside surface, $S_{zz} = -P_o$.

S_{xz} = shear stress produced by Q_x, psi.

S_{max} = twice the maximum shear stress by Mohr's Circle calculation, psi.

K_{io} = option for printing out the stresses on the inside surface at the start of element I_e, 0 or 1.

K_{co} = option for printing out the stresses on the centerline at the start of element I_e, 0 or 1.

K_{oo} = option for printing out the stresses on the outside surface at the start of element I_e, 0 or 1.

$K_{so} = 4 \times K_{io} + 2 \times K_{co} + K_{oo}$. For example, if

$\left\{ \begin{array}{l} K_{io} = 1 \\ K_{co} = 0 \\ K_{oo} = 1 \end{array} \right\}$, then $K_{so} = 4 \times 1 + 2 \times 0 + 1 = 5$ and the stresses on the inside and outside surfaces are printed in the code output.

K_{il} = option for printing out the stresses on the inside surface at the end of element I_e, 0 or 1.

K_{cl} = option for printing out the stresses on the centerline at the end of element I_e, 0 or 1.

K_{ol} = option for printing out the stresses on the outside surface at the end of element I_e, 0 or 1.

$K_{sl} = 4 \times K_{il} + 2 \times K_{cl} + K_{ol}$.

$K_{ss} = 10 \times K_{so} + K_{sl}$ = stress option of element I_e. For example, $K_{ss} = 12$ indicates that the outside surface stresses at the start of the element and the centerline stresses at the end of the element are to be printed in the code output.

In cards 7000, 7001, etc., regions and K's are given in pairs, five pairs per line, until $I_e = N_e$. Each region is identified by the last element I_e in the region. The card form is as follows:

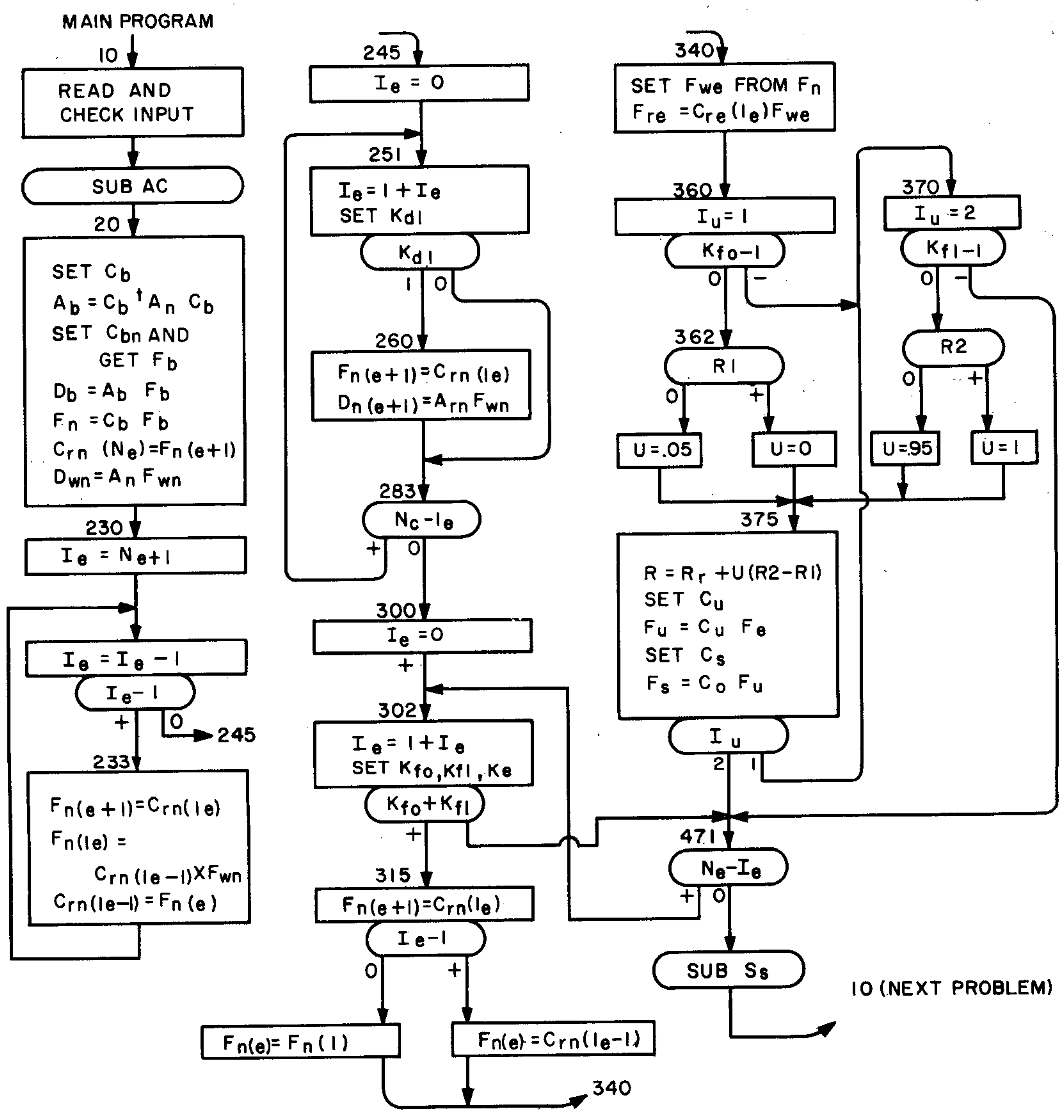
Columns	1-6	7-12	13-18	19-24	25-30	31-36	37-42	43-48	49-54	55-60	61-66	67-72
Formats	I6	I6	I6	I6	I6	I6	I6	I6	I6	I6	I6	I6
Words	I_e	K_{ss}	I_e	K_{ss}	I_e	K_{ss}	I_e	K_{ss}	I_e	K_{ss}	Blank	7000+I

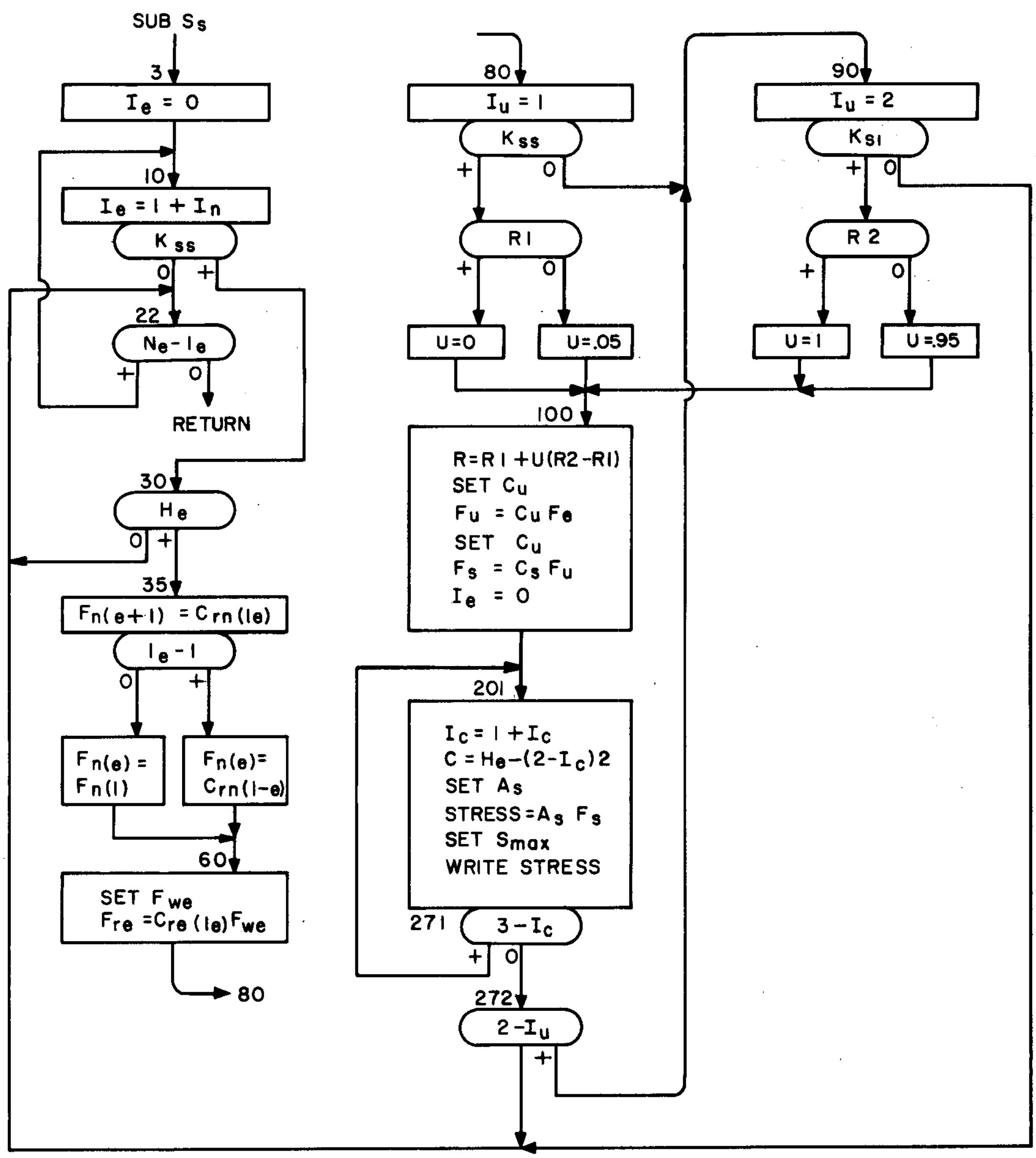
Card 8000

Card 8000 is the last card of each problem:

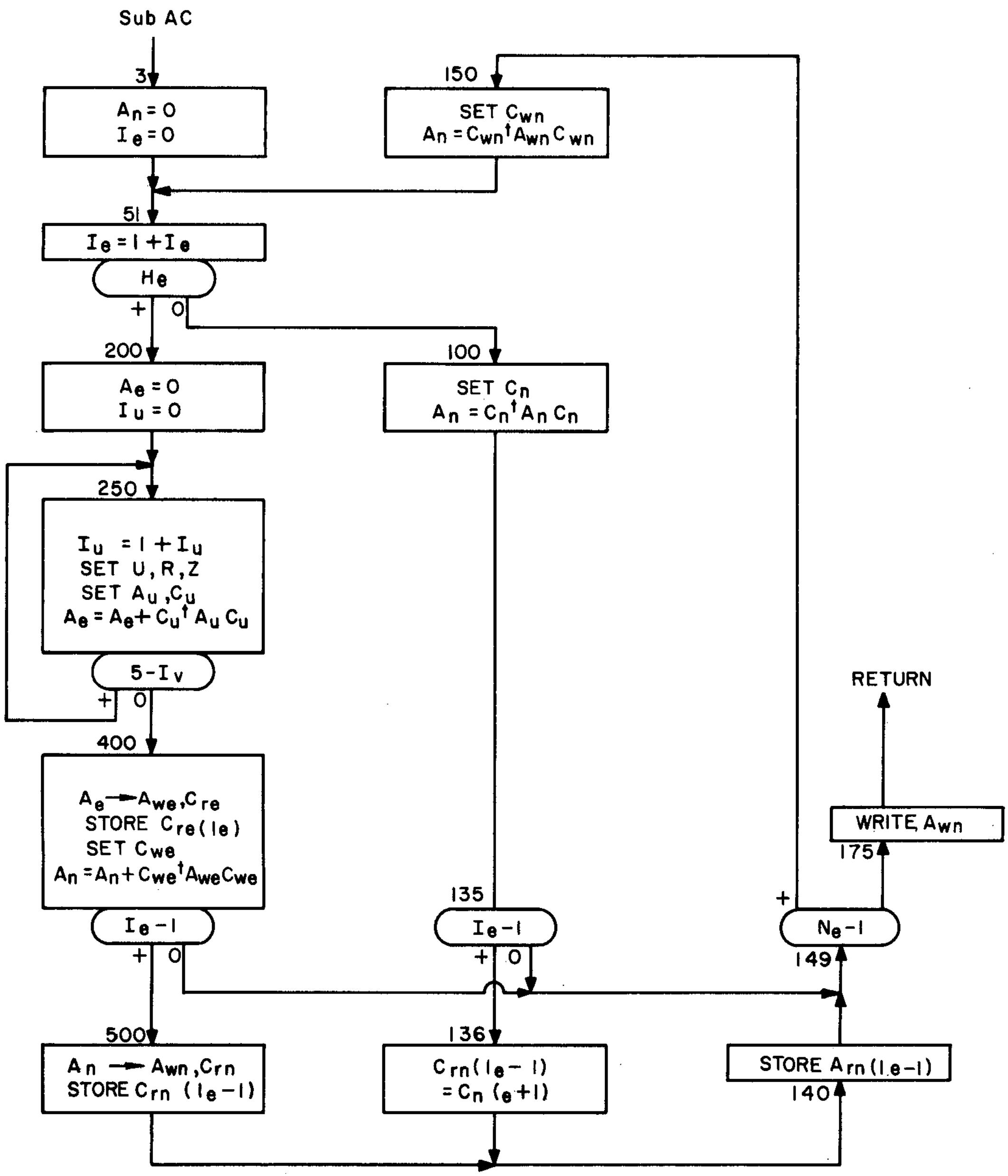
Columns	1-66	67-72
Formats	A66	I6
Words	Comments	8000

PROGRAM FLOW DIAGRAM





SUB S_s
3
$I_e = 0$
10
$I_e = 1 + I_n$
K_{ss}
22
$N_e - I_e$
RETURN
30
H_e
35
$F_{n(e+1)} = C_{rn}(I_e)$
$I_e - 1$
$F_{n(e)} = F_{n(1)}$
$F_{n(e)} = C_{rn}(1-e)$
60
SET F_{we}
$F_{re} = C_{re}(I_e)F_{we}$
80
80
$I_u = 1$
K_{ss}
R1
U=0
U=.05
100
R=R1+U(R2-R1)
SET C_u
$F_u = C_u F_e$
SET C_u
$F_s = C_s F_u$
$I_e = 0$
201
$I_c = 1 + I_c$
$C = H_e - (2 - I_c)2$
SET A_s
STRESS=$A_s F_s$
SET S_{max}
WRITE STRESS
271
$3 - I_c$
272
$2 - I_u$
90
$I_u = 2$
K_{s1}
R 2
U=1
U=.95



Sub AC
3
$A_n = 0$
$I_e = 0$
150
SET C_{wn}
$A_n = C_{wn}^{\dagger} A_{wn} C_{wn}$
51
$I_e = 1 + I_e$
He
+
0
200
$A_e = 0$
$I_u = 0$
100
SET C_n
$A_n = C_n^{\dagger} A_n C_n$
250
$I_u = 1 + I_u$
SET U, R, Z
SET A_u, C_u
$A_e = A_e + C_u^{\dagger} A_u C_u$
$5 - I_v$
+
0
RETURN
400
$A_e \rightarrow A_{we}, C_{re}$
STORE $C_{re}(I_e)$
SET C_{we}
$A_n = A_n + C_{we}^{\dagger} A_{we} C_{we}$
WRITE, A_{wn}
175
135
$I_e - 1$
$I_e - 1$
+
$N_e - 1$
+
0
+
0
149
500
$A_n \rightarrow A_{wn}, C_{rn}$
STORE $C_{rn}(I_e - 1)$
136
$C_{rn}(I_e - 1)$
$= C_n(e + 1)$
STORE $A_{rn}(I_e - 1)$
140

1. Listing of Input

The output of M0077 first lists the input, checking for errors in data limits. A complete listing of R, Z, and H is made for all elements N_e.

2. Matrices in General

All load, deflection and stress matrices have 7 columns.

Column 1 gives the effect of pressure alone.
Column 2 gives the effect of GRA alone.
Column 3 gives the effect of GMA alone.
Column 4 gives the effect of GRN alone.
Column 5 gives the effect of GMN alone.
Column 6 gives the effect of GZN alone.
Column 7 gives the combined effect of the first 6 columns.

3. Matrices in Particular

a. Boundary Force Matrix, F_{bn}

The boundary force matrix is F_{bn} in Eq (25b). The rows printed in the output are F_{ra}, F_{ma}, F_{rn}, F_{mn}, F_{zn}. These loads are over the entire circumference $2\pi R$, not per unit length.

b. Boundary Flexibility Matrix, A'_{bn}

c. Boundary Deflection Matrix, D_{bn}

D_{bn} is the working deflection matrix corresponding to F_{bn}.
$D_{bn} = A_{nb} F_{bn}$.

d. End Force Matrix, F_{wn}

The end force matrix is F_{wn} in Eq (16c) for $I_e = N_e + 1$. The rows are F_{z1}, F_{r1}, F_{m1}, F_{re}, F_{me}.

e. End Force Flexibility Matrix, A'_{wn}

f. End Deflection Matrix, D_{wn}

D_{wn} is the working deflection matrix corresponding to F_{wn}.
$D_{wn} = A_{wn} F_{wn}$.

g. Inbetween Deflection Matrix, D_{wn}

The inbetween deflection matrix is the matrix D_{wn} for an inbetween $I_e = 1$ to N_e. The rows are the deflections for F_{ze+1}, F_{re+1} and F_{me+1}.

h. Local Load Matrix, F_s

The local load matrix is F_s in Eq (8b). The rows are M_x, M_y, N_x, N_y, and Q_x at position U on element I_e.

i. Local Stress Matrix

The local stress matrix at position U on element I_e has five rows: σ_{xx}, $\sigma_{\theta\theta}$, σ_{pp}, τ_{xp}, and σ_{max}. σ_{max} is twice the maximum shear stress as determined by the following formula:

$$\sigma_{ave} = 0.5\,(\sigma_{ss} + \sigma_{pp})$$

$$\sigma_{rad} = \sqrt{(\sigma_{xx} - \sigma_{ave})^2 + (\tau_{xp})^2}$$

$$\sigma_1 = |2\sigma_{rad}|$$

$$\sigma_2 = |\sigma_{ave} + \sigma_{rad} - \sigma_{\theta\theta}|$$

$$\sigma_3 = |\sigma_{ave} - \sigma_{rad} - \sigma_{\theta\theta}|$$

$$\sigma_{max} = \text{Max of } (\sigma_1, \sigma_2, \sigma_3)$$

REFERENCES

1. E. Reissner, "The Effect of Transverse Shear Deformation on the Bending of Elastic Plates," Journal of Applied Mechanics, 12, 1945, pp A68-77.

2. S. Tomoshenko, Theory of Elasticity, McGraw-Hill, New York, 1934.

3. S. Timoshenko, Theory of Plates and Shells, McGraw-Hill, New York, 1940.

4. C. M. Friedrich, "Deflections and Load Distributions in Linear Elastic Structures--CTAC and MODE Codes," WAPD-TM-182, January 1960.

5. G. G. Bilodeau, J. B. Callaghan, and H. Kraus, "The SET Codes--IBM-704 Codes for the Calculation of the Stresses in a Pressure Vessel with an Ellipsoidal Head," WAPD-TM-174, June 1959.

APPENDIX OF SAMPLE PROBLEMS

A. Plate

Consider a pressurized circular plate with a hole as shown in Fig. 9. The outside edge, a, is simply supported and the inside edge, N, is free.

The input of Table II is based on the following requirements:

1. use four flexible elements,
2. determine deflection and slope at the end of each element,
3. determine stresses at all three surfaces at all locations.

B. Seal Ring

Consider a pressurized seal ring as shown in Fig. 10. End a is built-in and n deflects .010 in. vertically without rotation.

The input of Table II is based on the following requirements:

1. use 20 flexible elements at 6^o intervals,
2. determine inside and outside surface stresses at all locations.

C. Hemisphere and Cylinder

Table II also gives the input for determining stresses at the joint between a hemisphere and cylinder as shown in Fig. 11. Fifteen intervals in the hemisphere and 20 intervals in the cylinder are used.

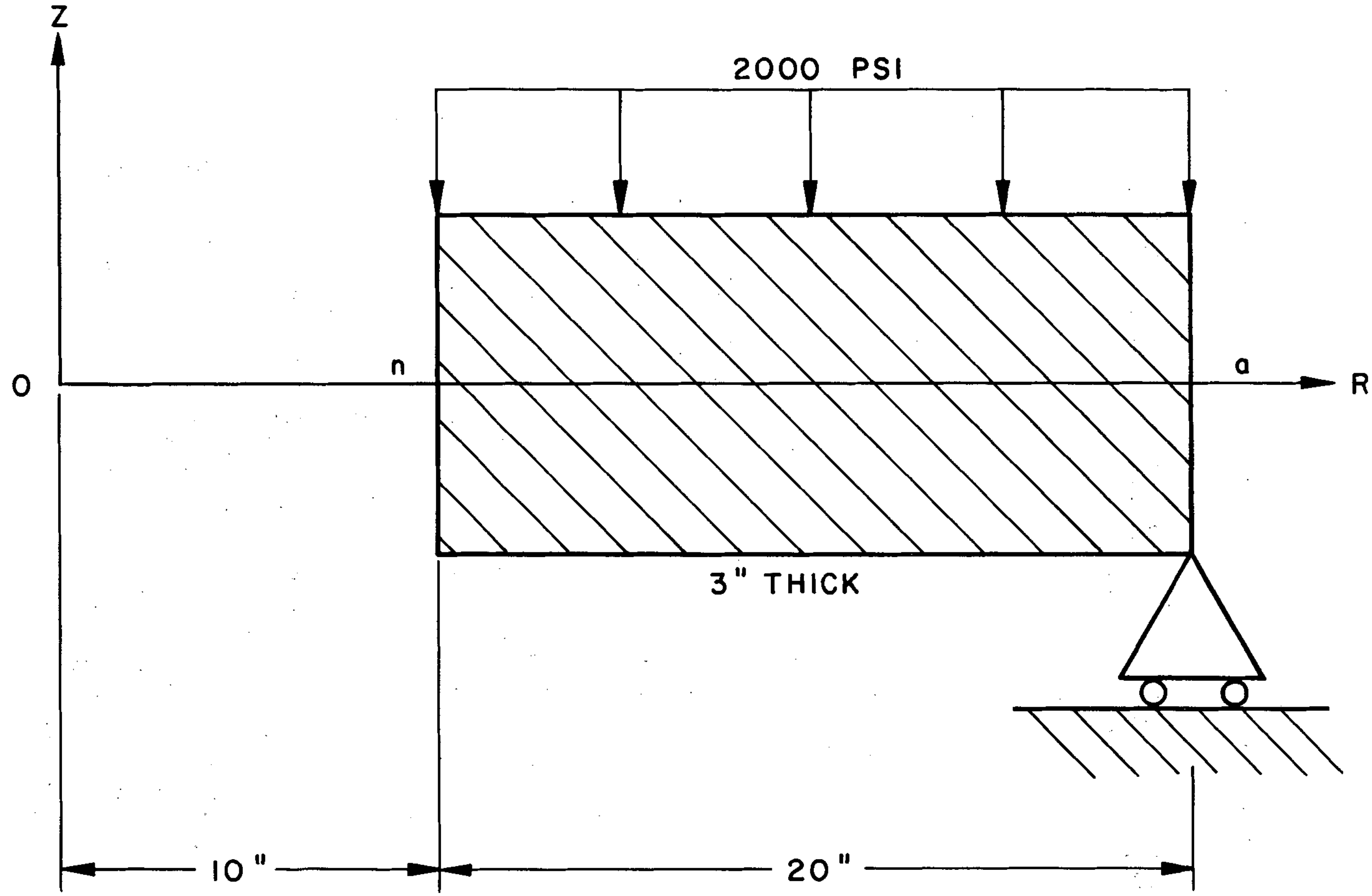
In all three problems, $E_m = 3 \times 10^7$ psi,

$E_{sm} = 1.15 \times 10^7$ psi,

$P_{nu} = 0.3$.

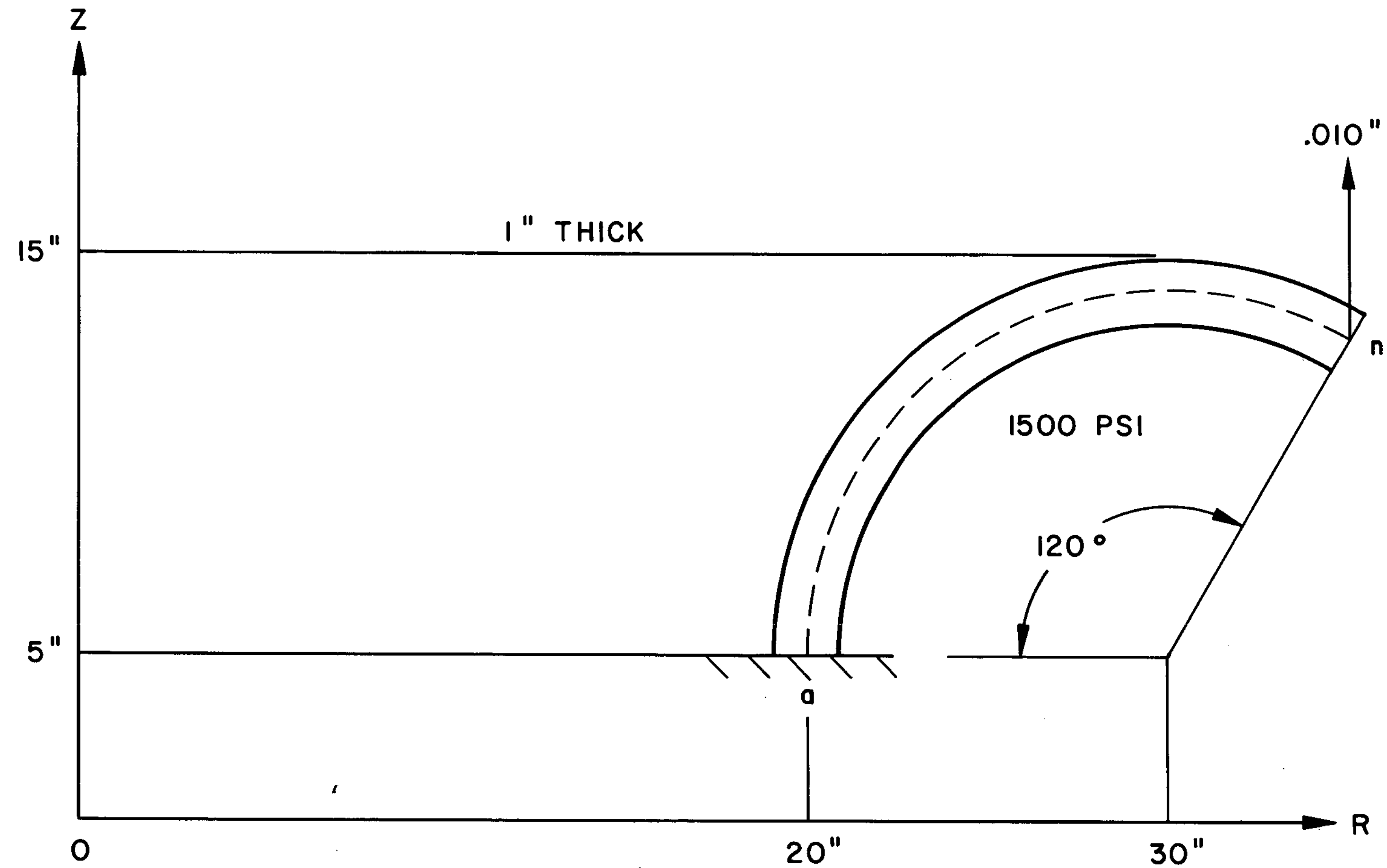
TABLE II

```
14001205$ M0077 FRIRH        48408666PRODN
 M0077 FRIEDRICH             EXT. 381           APRIL 17, 1961               1000      1
 CIRCULAR PLATE INPUT                                                        2000      1
     4 2.       + 3 0.       + 0 3.      + 7 1.15    + 7 3.        - 1  3000      1
 11111 0.       + 0 0.       + 0 0.      + 0 0.      + 0 0.        + 0  4000      1
       3.       + 1 0.       + 0                                        5000      1
     4 1.       + 1 0.       + 0 3.      + 0                            5001      1
     4   001                                                            6000      1
     3    70      4      77                                             7000      1
 END OF INPUT                                                           8000      1
 M0077 FRIEDRICH             EXT. 381           APRIL 17, 1961               1000      2
 SEAL RING INPUT                                                        2000      2
    20 1.5      + 3 0.       + 0 3.      + 7 1.15    + 7 3.        - 1  3000      2
 00000 0.       + 0 0.       + 0 0.      + 0 0.      + 0 1.        - 2  4000      2
       2.       + 1 5.       + 0                                        5000      2
    15 3.       + 1 1.5      + 1 1.      + 0 9.      + 1                5001      2
    20 3.5      + 1 1.366    + 1 1.      + 0 3.      + 1                5002      2
    20                                                                  6000      2
    19      50   20     55                                              7000      2
 END OF INPUT                                                           8000      2
 M0077 FRIEDRICH             EXT. 381           APRIL 17, 1961               1000      3
 HEMISPHERE AND CYLINDER INPUT                                          2000      3
    35 1.75     + 3.0.       + 0 3.      + 7 1.15    + 7 3.        - 1  3000      3
 00000 0.       + 0 0.       + 0 0.      + 0 0.      + 0 0.        + 0  4000      3
       0.       + 0 3.       + 1                                        5000      3
    15 1.       + 1 2.       + 1 5.      - 1 9.      + 1                5001      3
    35 1.       + 0 0.001    + 0 1.      + 0                            5002      3
    35                                                                  6000      3
    14      0   15     05     16     50     35      0                   7000      3
 END OF INPUT                                                           8000      3
```



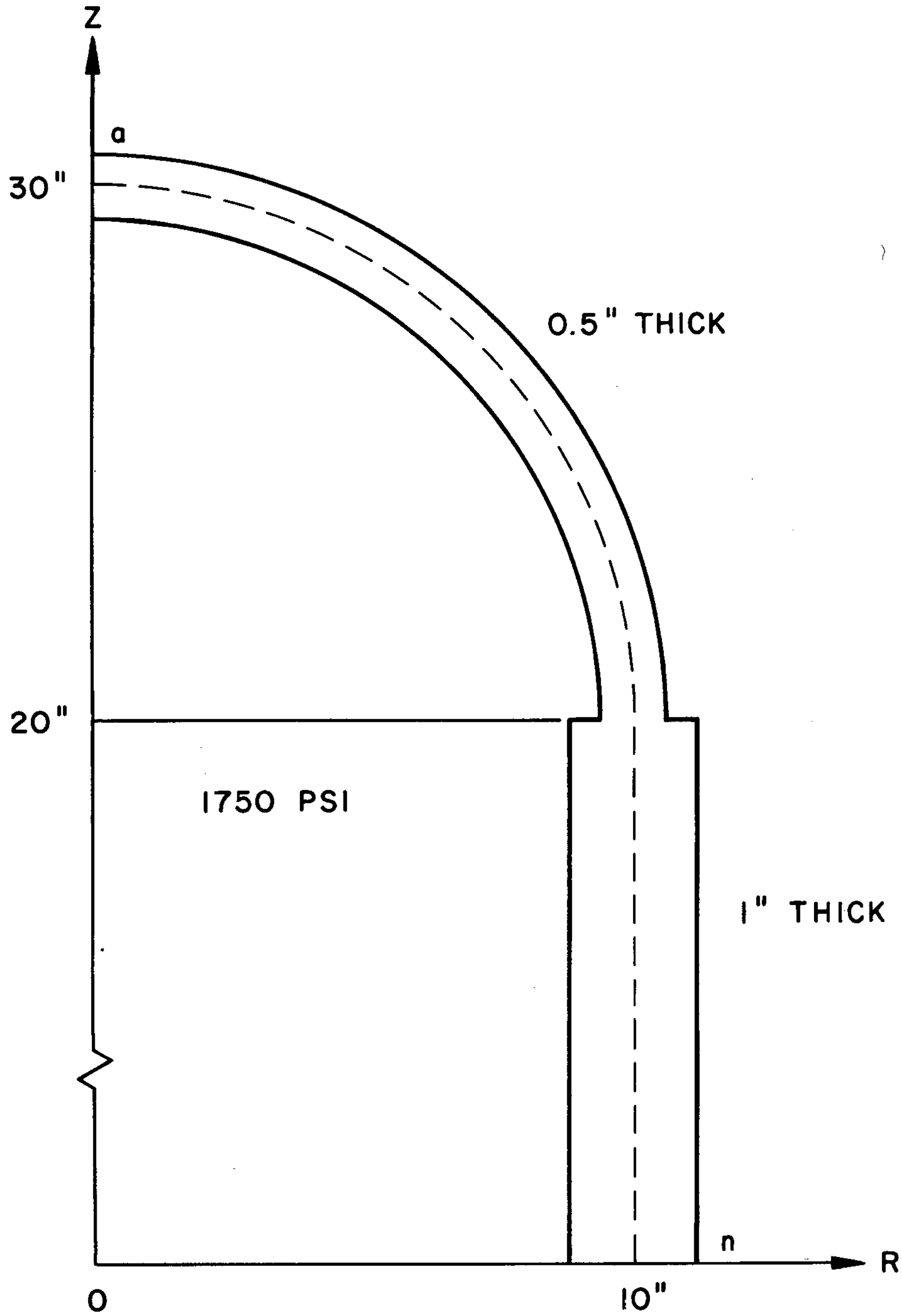
CIRCULAR PLATE

FIG. 9



SEAL RING

FIG.10



HEMISPHERE AND CYLINDER

FIG. 11

FIG. 13: SEAL-SHELL ANALYSIS

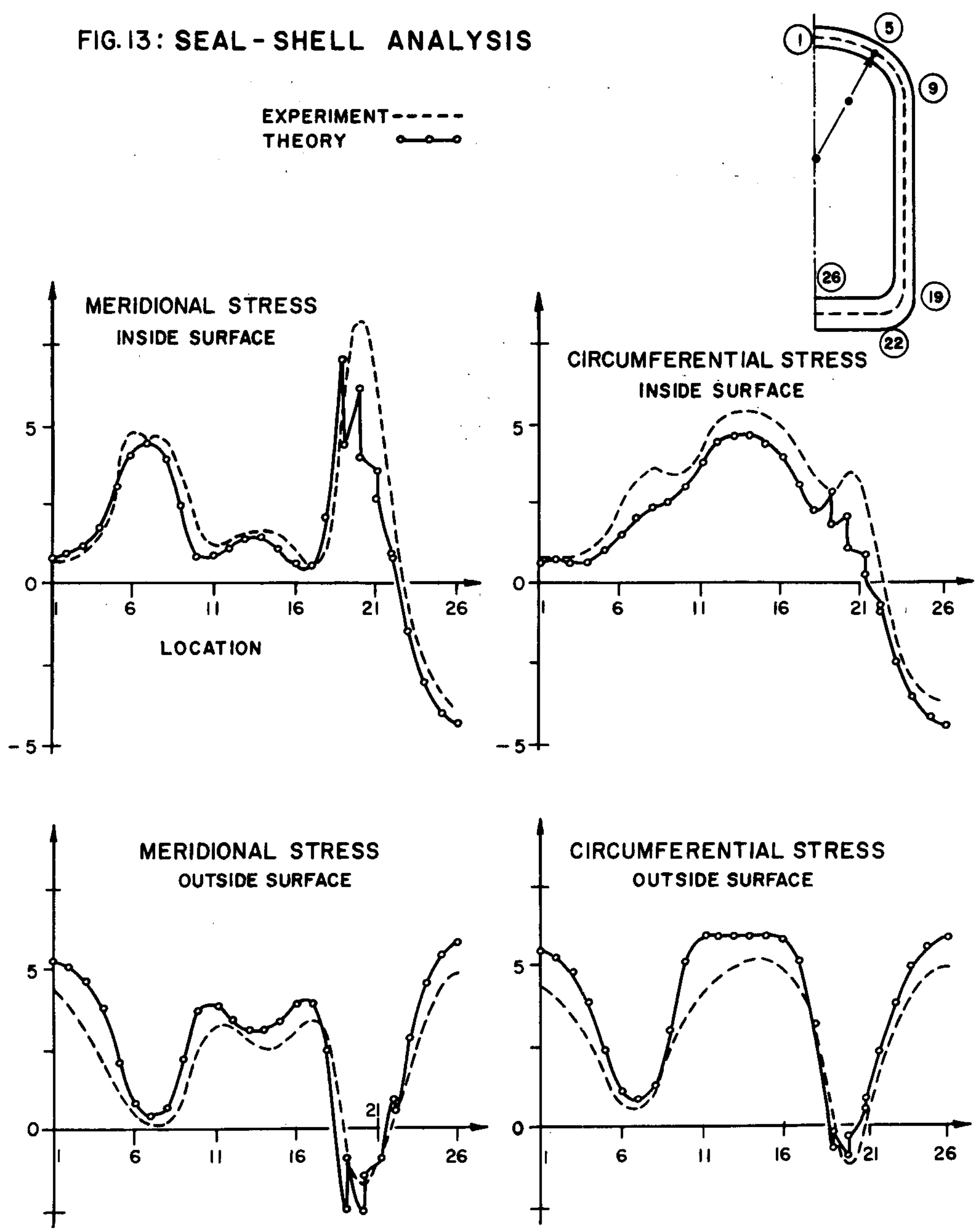


FIG. 12: SEAL-SHELL ANALYSIS