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A PROGRAM FOR CALCULATING HYPERGEOMETRIC PROBABILITY DISTRIBUTION TABLES ON THE IBM 704 EDPM

by

M. Katherine Weston

AUGUST 1961

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ABSTRACT

This report presents a SHARE coded program for tabulating the hypergeometric probability distribution for lot sizes less than or equal to 2000. Included are method of computation, instructions for using the program, a copy of the assembled program, and a sample of the computed data in format.

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A PROGRAM FOR CALCULATING HYPERGEOMETRIC PROBABILITY DISTRIBUTION TABLES ON THE IBM 704 EDPM

Introduction

A Sandia Corporation Technical Memorandum[1] gives descriptions of applications for hypergeometric probability distributions, and the method used for computing the tables. The program for tabulating these values has been modified to include lot sizes up to 2000. The same method of computation with the same nomenclature will be used in this report.

The following definitions are taken from page 5 of SCTM 215-59(51):[1]

N = Number of items in a lot

N* = Number of items in a sample taken from the lot

K* = Number of defective items in the lot

X = Number of defective items observed in the sample.

Then

$$\Pr\{\text{exactly X defectives in the sample}\} = PX^* = \frac{K^*!N^*!}{(K^* - X)!(N^* - X)!X!} \frac{(N - K^*)!(N - N^*)!}{N!(N - K^* - N^* + X)!},$$

[1] D. B. Owen, Sandia Corporation, "Tabulations of the Hypergeometric Probability Distribution for Lot Sizes Less Than or Equal to 50," SCTM 215-59(51), August 1959.

where X is an integer such that max [0, N* + K* - N] $\leq$ X $\leq$ min [N*, K*], and

$$\Pr X\{\text{defectives or less in the sample}\} = PX$$

$$= \sum_{i=\max[0,N^*+K^*-N]}^{X} \frac{K^*!N^*!}{(K^* - i)!(N^* - i)!i!} \frac{(N - K^*)!(N - N^*)!}{N!(N - K^* - N^* + i)!}$$

Since N* and K* may be interchanged in either of the probabilities PX and PX* without changing the value of the probabilities, it is necessary to tabulate only for K* $\leq$ N*. If N* < K*, it is necessary to enter the table with N* and K* interchanged, and the probabilities may be read directly from the table.

Program Description

Output

The program produces one line of output for each entry in the table. Fifty such lines appear on each page with a space after each set of ten lines unless a limit is met first. Appendix C gives a sample of this output.

Sense-switch settings that control output are as follows:

Sense switch	Position	
1	down	Tests K* as a limit.
1	up	Tests N as a limit.
2	down	Prints output on-line and off-line.
2	up	Prints output off-line only.
3	down	Deletes headings on each page.
3	up	Prints headings on each page.
4	down	Dumps program on Tape Unit 1 and halts.
4	up	Continues running until limit is met on last input.
5	down	Prints only when PX $\geq$ .000 0005 and prints PX = 1.000 000 only once in a set of data.
5	up	Prints a line for all values of PX.

Accuracy

Several accuracy checks on the output of this program have been made. One check was made by the Sandia Corporation Mathematical Research Department on the tables generated by the program for the publication Tables of the Hypergeometric Probability Distribution.[2]

This check was made by hand computing approximately every 500th entry in the table. Another check was made of the entries where the following conditions exist:

if N is even, $N^* = \frac{1}{2}N$, K^* is odd, and $X = \frac{1}{2}(K^* - 1)$, then $PX = \frac{1}{2}$.

Still another check made approximately every tenth page (50 answers per page) is indicated by the sum:

$$\sum_{X=\max[0,N^*+K^*-N]}^{K^*} PX = 1 + K^*\left(1 - \frac{N^*}{N}\right) \text{ for } N^* \geq K^*.$$

None of these checks indicated errors before the sixth decimal place.

Input

One input card is used for each set of data wanted. The card is punched in decimal and is read in by SAP routines: RDATA and RIC.

Any number of sets of data may be computed without interrupting the program by placing the input cards one behind the other in the card reader.

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[2] D. B. Owen, Sandia Corporation and G. J. Lieberman, Stanford University, Tables of the Hypergeometric Probability Distribution, Stanford University Press, January 1961; gives the distribution for $N \leq 50$; N = 60(10)100; N = 1000, N^* = 500; N = 100 (100) 2000, N^* = 1/2 N and $K^* = N^* - 1$ and N^* and all values of X.

The format for the input card is:

Columns	
1 - 4	limit
5 - 8	N
9 - 12	N*
13 - 16	K*
17 - 20	X

Example: For starting with

N = 100; N* = 50; K* = 20; X = 0

and ending with

N = 100; N* = 99; K* = 99; X = 99

the input card would be

01000100005000200000

and Sense Switch 1 must be in up position.

For the above example if the last set of values wanted is

N = 100; N* = 50; K = 50; X = 50

the input card would be

00500100005000200000

and Sense Switch 1 must be down.

Or if the last set of values wanted is

N = 100; N* = 50; K* = 20; X = 20

the input card would be

00200100005000200000

and Sense Switch 1 must be down

Note: The initial value of X must always be zero to get the correct sum for PX.

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Operating Instructions

A. The program with the table of "Logarithms of Factorials" included is on Tape No. 139 for the Sandia Corporation IBM 704 Data Processing System. The operating procedure for using Tape No. 139 is:

1. Place Tape No. 139 on Tape Unit 1.
2. Place printer output tape on Tape Unit 2.
3. Set sense switches as desired.
4. Place input cards in card reader.
5. Press LOAD TAPE button on console. Program will continue until the limit on the last input card has been met, unless it is interrupted by Sense Switch 4. When the last limit has been met, OFF will be printed on the on-line printer.
6. If program is interrupted by Sense Switch 4, it may be continued before machine has been cleared by pressing the START button. To continue later:
   a. Place unread cards in card reader.
   b. Place dump tape (No. 139) on Tape Unit 1 and printer output tape on Unit 2.
   c. Load dump tape with LOAD TAPE button on console

B. To start program from program card deck and generate new program tape, the operating procedure is:

1. Place program tape on Tape Unit 1.
2. Place printer output tape on Tape Unit 2.
3. Place Tape No. 128 for the Sandia Corporation IBM 704 Data Processing System[3] on Tape Unit 3.

[3]D. B. Owen and C. M. Williams, Logarithms of Factorials from 1 to 2000, Sandia Corporation Monograph SCR-158, December 1959.

4. Set sense switches as desired.

5. Place program deck followed by input cards in card reader and press LOAD CARD button on the console.

6. At the end of running the program, tape on Unit 1 will contain the program and the table of "Logarithms of Factorials." Substituting this tape for Tape No. 139 in A above, the instructions in A may be followed for any subsequent computing.

Running Time

One page of data (50 lines) takes about 6 seconds of machine time when off-line printing is used.

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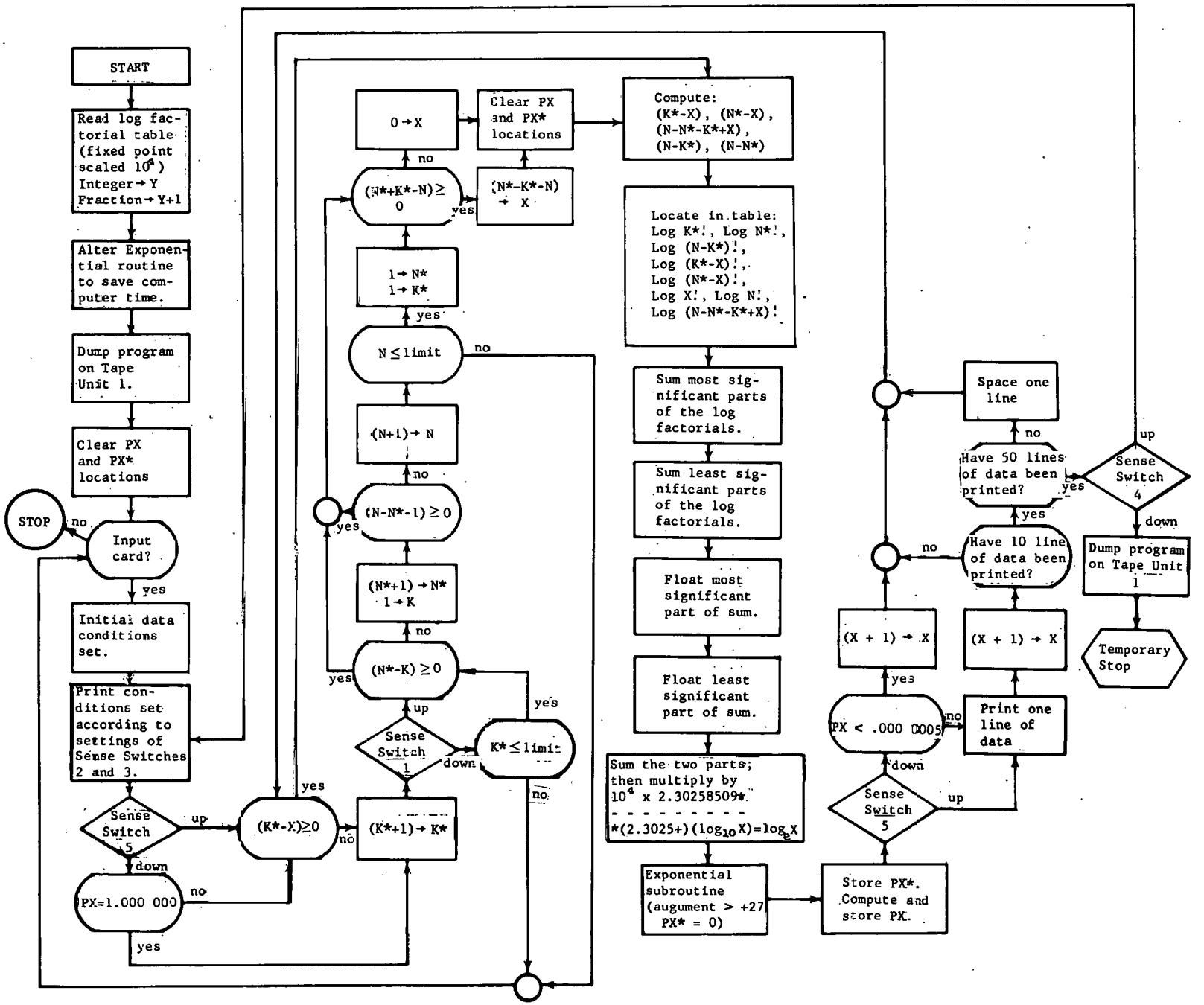
APPENDIX A

Flow Chart

FLOW CHART



APPENDIX B

Printed Program

APPENDIX B

PRINTED PROGRAM

```
                                00003           ORG 3
                                00003 COMMON    BSS 100
                                00147 BDC       SYN 103
                                01021 DUMP0     SYN 529
                                01137 END       SYN 607
                                01165 EXP       SYN 629
                                01261 RDATA     SYN 689
                                01744 RIC       SYN 996
                                02231 RIT       SYN 1177
                                02300 WOP       SYN 1216
                                02532 WOT       SYN 1370
                                02567           ORG 1399
      02567 -0 53400 1 02572 START   LXD (4000),1
      02570  0 07400 4 03153         TSX READ,4
      02571  0 13104 1 13103                 TABLE+4000+2,1,TABLE+4000+2+1
      02572  0 07640 0 02570 (4000) HTR *-2,0,4000
      02573  2 00002 1 02570         TIX *-3,1,2
      02574  0 77200 0 00203         REW 3
      02575  0 50000 0 03214         CLA ALTEXP
      02576  0 60100 0 01252         STO EXP+53
      02577  0 77200 0 00201         REW 1
      02600  0 50000 0 02614         CLA DECRE1
      02601  0 07400 4 01021         TSX DUMP0,4
      02602  0 77200 0 00201 ST1     REW 1
      02603  0 60000 0 03151         STZ PX
      02604  0 60000 0 03152         STZ SMLPX
      02605  0 07400 4 03160         TSX READ1,4
      02606  0 03150 0 03144                 LIMIT,,X
      02607  1 00000 0 03137         TXI STOP,**,**
      02610  0 77200 0 00201 BEGIN   REW 1
      02611  0 53400 1 03226         LXA FIVE,1
      02612 -0 63400 1 03071         SXD A5,1
      02613  0 76000 0 00162         SWT 2
      02614  1 00001 0 02617 DECRE1  TXI ST2,0,1
      02615  0 07400 4 02300         TSX WOP,4
      02616  0 03110 0 03103                 IDENT,,IDENT+5
      02617  0 76000 0 00163 ST2     SWT 3
TD    02620  1 00000 0 02625         TXI ST3
      02621  0 76600 0 00202         WTD 2
      02622  0 07400 4 02532         TSX WOT,4
      02623  0 03112 0 03112                 RESTO,,RESTO
      02624  1 00000 0 02630 A10     TXI ST5,,**
      02625  0 76600 0 00202 ST3     WTD 2
      02626  0 07400 4 02532         TSX WOT,4
      02627  0 03110 0 03103                 IDENT,,IDENT+5
      02630  0 53400 1 03227 ST5     LXA TEN,1
      02631 -0 63400 1 02624 ST6     SXD A10,1
      02632  0 76000 0 00165 FIRST   SWT 5
TD    02633  1 00000 0 02642         TXI A2
```

179 015

```
          02634 -0 50000 0 00056          CAL COMMON+43
          02635  0 77100 0 00006          ARS 6
          02636  0 76000 0 00001          LBT
TD        02637  1 00000 0 02642          TXI A2
          02640  0 60000 0 00056          STZ COMMON+43
TD        02641  1 00000 0 02645          TXI A1
          02642  0 50000 0 03147 A2       CLA K
          02643  0 40200 0 03150          SUB X
          02644  0 12000 0 02713          TPL FOURTH
          02645  0 50000 0 03147  A1       CLA K
          02646  0 40000 0 03225          ADD ONE
          02647  0 60100 0 03147          STO K
          02650  0 76000 0 00161          SWT 1
          02651  1 00000 0 02655          TXI SECOND,0,**
          02652  0 50000 0 03144          CLA LIMIT
          02653  0 40200 0 03147          SUB K
          02654 -0 12000 0 02602          TMI ST1
          02655  0 50000 0 03146 SECOND   CLA SMALLN
          02656  0 40200 0 03147          SUB K
          02657  0 12000 0 02702          TPL THIRD
          02660  0 50000 0 03146          CLA SMALLN
          02661  0 40000 0 03225          ADD ONE
          02662  0 60100 0 03146          STO SMALLN
          02663  0 50000 0 03225          CLA ONE
          02664  0 60100 0 03147          STO K
          02665  0 50000 0 03145          CLA N
          02666  0 40200 0 03146          SUB SMALLN
          02667  0 40200 0 03225          SUB ONE
          02670  0 12000 0 02702          TPL THIRD
          02671  0 50000 0 03145          CLA N
          02672  0 40000 0 03225          ADD ONE
          02673  0 60100 0 03145          STO N
          02674  0 50000 0 03144          CLA LIMIT
          02675  0 40200 0 03145          SUB N
          02676 -0 12000 0 02602          TMI ST1
          02677  0 50000 0 03225          CLA ONE
          02700  0 60100 0 03146          STO SMALLN
          02701  0 60100 0 03147          STO K
          02702  0 50000 0 03146 THIRD    CLA SMALLN
          02703  0 40000 0 03147          ADD K
          02704  0 40200 0 03145          SUB N
          02705  0 12000 0 02710          TPL *+3
          02706  0 60000 0 03150          STZ X
          02707  0 02000 0 02711          TRA *+2
          02710  0 60100 0 03150          STO X
          02711  0 60000 0 03151          STZ PX
          02712  0 60000 0 03152          STZ SMLPX
          02713  0 50000 0 03147 FOURTH   CLA K
          02714  0 40200 0 03150          SUB X
          02715  0 60100 0 03207          STO K.X
          02716  0 50000 0 03146          CLA SMALLN
          02717  0 40200 0 03150          SUB X
          02720  0 60100 0 03210          STO (N).X
          02721  0 50000 0 03150          CLA X
          02722  0 40000 0 03145          ADD N
          02723  0 40200 0 03146          SUB SMALLN
          02724  0 40200 0 03147          SUB K
          02725  0 60100 0 03212          STO N(N)KX
          02726  0 50000 0 03145          CLA N
          02727  0 40200 0 03147          SUB K
          02730  0 60100 0 03206          STO N.K
```

```
02731  0 50000 0 03145          CLA N
02732  0 40200 0 03146          SUB SMALLN
02733  0 60100 0 03211          STO N.(N)
02734  0 53400 1 03230          LXA EIGHTN,1
02735  0 50000 0 03147          CLA K
02736  0 07400 4 03125          TSX LOOKUP,4
02737  0 50000 0 03146          CLA SMALLN
02740  0 07400 4 03125          TSX LOOKUP,4
02741  0 50000 0 03206          CLA N.K
02742  0 07400 4 03125          TSX LOOKUP,4
02743  0 50000 0 03211          CLA N.(N)
02744  0 07400 4 03125          TSX LOOKUP,4
02745  0 50000 0 03207          CLA K.X
02746  0 07400 4 03125          TSX LOOKUP,4
02747  0 50000 0 03210          CLA (N).X
02750  0 07400 4 03125          TSX LOOKUP,4
02751  0 50000 0 03150          CLA X
02752  0 07400 4 03125          TSX LOOKUP,4
02753  0 50000 0 03145          CLA N
02754  0 07400 4 03125          TSX LOOKUP,4
02755  0 50000 0 03212          CLA N(N)KX
02756  0 07400 4 03125          TSX LOOKUP,4
02757  0 50000 0 03164 FIFTH    CLA KFACT
02760  0 40000 0 03166          ADD KFACT+2
02761  0 40000 0 03170          ADD KFACT+4
02762  0 40000 0 03172          ADD KFACT+6
02763  0 40200 0 03174          SUB KFACT+8
02764  0 40200 0 03176          SUB KFACT+10
02765  0 40200 0 03200          SUB KFACT+12
02766  0 40200 0 03202          SUB KFACT+14
02767  0 40200 0 03204          SUB KFACT+16
02770  0 60100 0 03215          STO TEMP
02771  0 50000 0 03165          CLA KFACT+1
02772  0 40000 0 03167          ADD KFACT+3
02773  0 40000 0 03171          ADD KFACT+5
02774  0 40000 0 03173          ADD KFACT+7
02775  0 40200 0 03175          SUB KFACT+9
02776  0 40200 0 03177          SUB KFACT+11
02777  0 40200 0 03201          SUB KFACT+13
03000  0 40200 0 03203          SUB KFACT+15
03001  0 40200 0 03205          SUB KFACT+17
03002  0 60100 0 03216          STO TEMP+1
03003  0 50000 0 03215          CLA TEMP
03004 -0 50100 0 03222          ORA OCT233
03005  0 30000 0 03222          FAD OCT233
03006  0 60100 0 03215          STO TEMP
03007  0 50000 0 03216          CLA TEMP+1
03010  0 76500 0 00013          LRS 35-24
03011  0 22000 0 03223          DVH 10E8
03012 -0 60000 0 03220          STQ TEMP+3
03013  0 50000 0 03220          CLA TEMP+3
03014 -0 50100 0 03224          ORA OCT203
03015  0 30000 0 03224          FAD OCT203
03016  0 30000 0 03215          FAD TEMP
03017  0 76500 0 00043          LRS 35
03020  0 26000 0 03221          FMP CONVRT
03021  0 07400 4 01165          TSX EXP,4
03022  1 00000 0 03114          TXI BIG,0,**
03023  0 60100 0 03152          STO SMLPX
03024  0 30000 0 03151          FAD PX
03025  0 60100 0 03151          STO PX
```

```
        03026  0 76000 0 00165          SWT 5
 TD     03027  1 00000 0 03040          TXI PRNT
        03030  0 34000 0 03213          CAS RND
 TD     03031  1 00000 0 03040          TXI PRNT
 TD     03032  1 00000 0 03040          TXI PRNT
        03033  0 60000 0 00056          STZ COMMON+43
        03034  0 50000 0 03150          CLA X
        03035  0 40000 0 03225          ADD ONE
        03036  0 60100 0 03150          STO X
 TD     03037  1 00000 0 02642          TXI A2
        03040  0 76000 0 00162 PRNT     SWT 2
 TD     03041  1 00000 0 03044          TXI *+3
        03042  0 07400 4 03073          TSX FORM,4
        03043  0 03152 0 03145              N,,SMLPX
        03044  0 07400 4 03077          TSX FORM1,4
        03045  0 03152 0 03145              N,,SMLPX
        03046  0 50000 0 03150          CLA X
        03047  0 40000 0 03225          ADD ONE
        03050  0 60100 0 03150          STO X
        03051 -0 53400 1 02624 SPACE    LXD A10,1
        03052  2 00001 1 02631          TIX ST6,1,1
        03053 -0 53400 1 03071          LXD A5,1
        03054  2 00001 1 03065          TIX SPACE1,1,1
        03055  0 76000 0 00164          SWT 4
 TD     03056  1 00000 0 02611          TXI BEGIN+1
        03057  0 77200 0 00201          REW 1
        03060  0 50000 0 02614          CLA DECRE1
        03061  0 07400 4 01021          TSX DUMP0,4
        03062  0 10000 0 02610          TZE BEGIN
        03063  0 77200 0 00201          REW 1
        03064  0 00000 0 02611          HTR BEGIN+1
        03065  0 76600 0 00202 SPACE1   WTD 2
        03066  0 07400 4 02532          TSX WOT,4
        03067  0 03072 0 03072              SP1,,SP1
        03070 -0 63400 1 03071          SXD A5,1
        03071  1 00000 0 02630 A5       TXI ST5,,**
        03072  606060606060     SP1     BCD 1
        03073  0 02000 0 00147 FORM     TRA BDC
        03074  0 02000 0 02300          TRA WOP
        03075  043105730226             BCD 24I5,2F10.6
        03076  010033066060
        03077  0 02000 0 00147 FORM1    TRA BDC
        03100  1 00002 0 02532          TXI WOT,,2
        03101  043105730226             BCD 24I5,2F10.6
        03102  010033066060
        03103  016060604560     IDENT   BCD 71   N   N*   K*    X    PX     PX*
        03104  606045546060
        03105  604254606060
        03106  606760606060
        03107  476760606060
        03110  606047675460
        03111  606060606060
        03112  016060606060     RESTO   BCD 21
        03113  606060606060
        03114  0 12000 0 03117 BIG      TPL BGPL
 T      03115 -0 75400 0 00000          PXD
        03116  0 02000 4 00002          TRA 2,4
        03117  0 07400 4 02300 BGPL     TSX WOP,4
        03120  0 03124 0 03122              B,,B+2
 A      03121  0 00000 0 00000          HTR
        03122  606025674760     B       BCD 3  EXP TOO BIG
```

```
03123  634646602231
03124  276060606060
03125  0 76700 0 00001 LOOKUP ALS 1
03126  0 40200 0 03225        SUB ONE
03127  0 76000 0 00006        COM
03130  0 73400 2 00000        PAX ,2
03131  0 50000 2 03241        CLA TABLE,2
03132  0 60100 1 03206        STO KFACT+18,1
03133  0 50000 2 03242        CLA TABLE+1,2
03134  0 60100 1 03207        STO KFACT+19,1
03135  1 77776 1 03136        TXI *+1,1,-2
03136  0 02000 4 00001        TRA 1,4
03137  0 50000 0 02614 STOP   CLA DECRE1
03140  0 07400 4 01021        TSX DUMP0,4
03141  0 10000 0 02602        TZE ST1
03142  0 77200 0 00201        REW 1
TD     03143  1 00000 0 01137        TXI END
                      03144 LIMIT  BSS 1
                      03145 N      BSS 1
                      03146 SMALLN BSS 1
                      03147 K      BSS 1
                      03150 X      BSS 1
                      03151 PX     BSS 1
                      03152 SMLPX  BSS 1
03153  0 02000 0 01261 READ   TRA RDATA
03154  1 00003 0 02231        TXI RIT,,3
03155  046273311073           BCD 34S,I8,I8
03156  311060606060
03157  606060606060
03160  0 02000 0 01261 READ1  TRA RDATA
03161  0 02000 0 01744        TRA RIC
03162  053104606060           BCD 25I4
03163  606060606060
                      03164 KFACT  BSS 2
                      03166 FACTN  BSS 2
                      03170 NKFACT BSS 2
                      03172 NNFACT BSS 2
                      03174 KXFACT BSS 2
                      03176 XNFACT BSS 2
                      03200 XFACT  BSS 2
                      03202 NFACT  BSS 2
                      03204 NNKXFT BSS 2
                      03206 N.K    BSS 1
                      03207 K.X    BSS 1
                      03210 (N).X  BSS 1
                      03211 N.(N)  BSS 1
                      03212 N(N)KX BSS 1
03213 +154414336750    RND    DEC .0000005
03214 +205660000000    ALTEXP DEC 27.
                      03215 TEMP   BSS 4
03221 +164742706141    CONVRT DEC .0002302585092994046
03222 +233000000000    OCT233 OCT 233000000000
03223 +000575360400    10E8   DEC 100000000
03224 +203000000000    OCT203 OCT 203000000000
03225 +000000000001    ONE    DEC 1
03226 +000000000005    FIVE   DEC 5
03227 +000000000012    TEN    DEC 10
03230 +000000000022    EIGHTN DEC 18
03231  0 76100 0 00000        NOP
03232  0 76100 0 00000        NOP
03233  0 76100 0 00000        NOP
```

```
03234   0 76100 0 00000          NOP
03235   0 76100 0 00000          NOP
03236   0 76100 0 00000          NOP
03237   0 76100 0 00000          NOP
03240   0 76100 0 00000          NOP
                      03241 TABLE  BSS 4002
                      02567        END START
```

APPENDIX C

Sample Output

APPENDIX C

Sample Output

N	N*	K*	X	PX	PX*
1000	500	59	12	0.000001	0.000001
1000	500	59	13	0.000005	0.000004
1000	500	59	14	0.000020	0.000014
1000	500	59	15	0.000065	0.000046
1000	500	59	16	0.000199	0.000133
1000	500	59	17	0.000555	0.000356
1000	500	59	18	0.001429	0.000874
1000	500	59	19	0.003406	0.001977
1000	500	59	20	0.007532	0.004126
1000	500	59	21	0.015493	0.007961
1000	500	59	22	0.029719	0.014226
1000	500	59	23	0.053295	0.023576
1000	500	59	24	0.089571	0.036276
1000	500	59	25	0.141447	0.051876
1000	500	59	26	0.210448	0.069000
1000	500	59	27	0.295863	0.085415
1000	500	59	28	0.394313	0.098450
1000	500	59	29	0.500000	0.105687
1000	500	59	30	0.605687	0.105687
1000	500	59	31	0.704137	0.098450
1000	500	59	32	0.789552	0.085415
1000	500	59	33	0.858553	0.069000
1000	500	59	34	0.910429	0.051876
1000	500	59	35	0.946705	0.036276
1000	500	59	36	0.970281	0.023576
1000	500	59	37	0.984507	0.014226
1000	500	59	38	0.992468	0.007961
1000	500	59	39	0.996594	0.004126
1000	500	59	40	0.998571	0.001977
1000	500	59	41	0.999445	0.000874
1000	500	59	42	0.999801	0.000356
1000	500	59	43	0.999934	0.000133
1000	500	59	44	0.999980	0.000046
1000	500	59	45	0.999994	0.000014
1000	500	59	46	0.999998	0.000004
1000	500	59	47	1.000000	0.000001