

Anisotropic Thermal Expansion of Single  
Crystals of Thallium, Yttrium, Beryllium, and Zinc  
at Low Temperatures\*

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ABSTRACT

The anisotropic thermal expansion of the hexagonal elements, thallium, yttrium, beryllium, and zinc, has been measured. The temperature range for the thallium, yttrium, and zinc measurements was 4.2-273.2°K, while for beryllium the range was 77-273.2°K. Dimensional changes were detected by an interferometric technique, and the movement of the interference fringes was detected with a photomultiplier tube. From the measured values of  $\Delta L/L_0$  the coefficients of thermal expansion and the  $c/a$  ratios of the metals were calculated as a function of temperature. A review of previous measurements of the thermal expansion of these metals is also included.

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## INTRODUCTION

Measurements of the anisotropic thermal expansion herein reported were originally undertaken to provide information for the dimensional correction of experimental data accumulated over an extensive temperature range. Since, however, thermal expansion data have in their own right an intrinsic value, an interferometric method of accurately determining both the linear dimensional changes and the thermal coefficients was utilized. Thallium, yttrium, and beryllium were chosen as test materials because measurements of their elastic parameters as a function of temperature were in progress. Zinc was also included in order to verify the existence of a negative coefficient of expansion at low temperature as reported by Gruneisen and Goens<sup>1</sup> for the direction perpendicular to the c-axis.

## EXPERIMENTAL TECHNIQUE

A diagram of the experimental apparatus is shown in Fig. 1. The basic element of the apparatus is a quartz tube dilatometer which is essentially the same as that employed by Henning.<sup>2</sup> The apparatus includes an interferometer as the detecting element to measure dimensional changes of the single crystalline specimens. Expansion or contraction of a test specimen operates through a

quartz push rod to move one optical flat of the interferometer. The second optical flat is fixed with respect to the base of the test specimen through the mounting head and a hollow tubular quartz envelope which supports the specimen and push rod. Dimensional changes in a test specimen thus produce a movement in an interference fringe pattern which can be detected with a photomultiplier tube.<sup>3,4</sup> A permanent record of the data may be made by coupling the output of the photomultiplier tube to a strip chart recorder with appropriate notation at periodic intervals on the chart of the specimen temperature. In the present instance specimen temperature was measured with a calibrated copper-constantan thermocouple; the techniques which were used to compensate for the inadequacy of this method of temperature measurement below 20°K are discussed in the results section.

For the work herein reported, the mounting head and interferometer were maintained at room temperature while the test specimen was cooled by a bath of liquified gas. Provision was made to allow evacuation of the thermal jacket around the lower portion of the apparatus in order to provide thermal isolation of the test specimen at any desired temperature. This thermal

isolation was not complete so that a slow downward drift toward the bath temperature could be followed, or alternatively heat could be applied through resistance windings so that a slow increase in temperature could be obtained. Thus dimensional changes could be observed both on heating and cooling. Helium exchange gas was introduced into the region between the quartz envelope and the push rod in order to provide for thermal exchange which would establish compensating thermal gradients along the envelope and along the push rod.

The apparatus was so designed that dimensional changes in the mounting head would cancel to a good approximation over the normal range of fluctuation of ambient temperature. This was accomplished by choosing compensating lengths for the brass, fused quartz, glass, and invar components. The mercury green line at  $5460.7 \overset{\circ}{\text{A}}$  was used in illuminating the interferometer so that the passage of one fringe indicated a dimensional change of  $2.7304 \times 10^{-5}$  cm. The data could be read to  $\pm 0.1$  fringe.

#### PREPARATION OF SINGLE CRYSTALLINE SPECIMENS

Thallium. Thallium was obtained from the American Smelting and Refining Company in the form of cast sticks with a specified purity of 99.95+%. This thallium was cast into cylindrical

forms about one inch in diameter and two inches in length. These cylinders were then vacuum annealed at 215°C for three to seven days to allow grain growth to occur. Several grains were obtained which were of sufficient size that suitable single crystalline test specimens could be cut from them. These thallium crystals, and similarly the crystals of zinc, yttrium, and beryllium, were aligned with a back reflection Laue camera<sup>5</sup> with a precision of  $\pm 0.5^\circ$ . Test specimens were cut from the oriented grains with an acid saw which was based on the design of Maddin and Asher.<sup>6</sup> The cutting mixture was one part 30% H<sub>2</sub>O<sub>2</sub> to three parts glacial acetic acid and the carrier was a silk thread. All test specimens were cut into rods with cross-section sufficiently small to fit inside the seven millimeter inner diameter of the quartz envelope. The thallium specimen whose longitudinal axis was parallel to the c-axis was 0.5730 inches long while the specimen with the long dimension perpendicular to the c-axis was 1.0000 inches long.

Yttrium. Yttrium metal produced at this laboratory<sup>7</sup> was arc melted under a helium atmosphere into rods about one inch in diameter and five inches long. Sections of these rods about two inches long were then placed in tantalum holders and vacuum

annealed for twenty-four hours at 1300°C. Again, grain growth was such that some of the grains were of adequate size to yield suitable test specimens. After these grains were aligned they were cut with a jeweler's saw. The ends of the samples were ground parallel with 600 grit polishing paper and subsequently etched to remove worked metal. The specimen whose major axis was parallel to the c-axis was 0.6742 inches long while a second specimen with the major axis oriented perpendicular to the c-axis was 0.5469 inches long.

Analyses made in the manner described by Banks et al.<sup>7</sup> indicated that the yttrium contained 212 ppm carbon, 148 ppm nitrogen, 112 ppm iron, 268 ppm titanium, 5000 ppm zirconium, 40 ppm calcium, <30 ppm magnesium, 2500 ppm oxygen, 125 ppm silicon, 260 ppm nickel, <10 ppm boron, and <1000 ppm of all rare earths combined. By difference the indicated purity is 99.0% yttrium.

Beryllium. Suitable beryllium crystals were cut with a jeweler's saw from a beryllium casting which contained several large grains. These crystals were polished and etched as in the case of yttrium. Qualitative spectrographic analysis showed weak lines corresponding to iron and aluminum and very weak lines from calcium,

chromium, copper, nickel, manganese, and magnesium. The purity of this beryllium is believed to be 99.5+%. The specimen with major axis parallel to the c-axis was 0.7700 inches long, and the specimen with major axis perpendicular to the c-axis was 0.9165 inches long.

Zinc. Zinc single crystals were grown in MgO crucibles by the Bridgman method.<sup>8</sup> Bunker Hill zinc with a purity of 99.99+% was used. Wilhelm and McCarley<sup>9</sup> report the following typical analysis for zinc of this grade: 10 ppm lead, 7 ppm copper, 3 ppm iron, 1 ppm cadmium, 0.3 ppm antimony, 0.1 ppm arsenic, and 99.9975% zinc. Test specimens with the desired orientations were cut from larger crystals with a jeweler's saw and were suitably polished and etched. The specimen with major axis parallel to the c-axis was 0.7900 inches long and the specimen with major axis perpendicular to the c-axis was 1.0315 inches long.

#### EXPERIMENTAL RESULTS

Several runs were made on each specimen from which values of  $\Delta L/L_0$  were determined at one to four degree temperature intervals with reference length  $L_0$  being the dimension at 0°C.

It was necessary to correct these values<sup>2</sup> for the dimensional change in the short length of the quartz envelope which concentrically surrounded the test specimen. Such a correction based upon the data of Scheel and Heaus<sup>10</sup> was applied. The composite data from a series of runs on a given test specimen were averaged and smoothed by what is essentially an iterative process.

Corrected values of  $\Delta L/L_0$  were tabulated as a function of temperature and values of the coefficient of thermal expansion,  $\alpha = (1/L_0)(dL/dT)$ , were computed for successive temperature increments (1-4°K) over the entire temperature range. Subsequent graphical integration of a plot of  $\alpha$  versus temperature yielded values of  $\Delta L/L_0$  which were in turn compared with the original tabulation. The process was repeated and  $\alpha$ -values were successively refined until the integrated values of  $\Delta L/L_0$  reproduced the original tabulated values.

Since temperature measurements in the range 4.2-20°K were inadequate, values of  $\alpha$  in this range were determined by extrapolating the  $\alpha$  versus temperature curve found at higher temperatures subject to the following three constraints. First, the extrapolated segment of the curve had to join smoothly with the higher temperature portion of the curve. Second, the

extrapolated segment was constrained to approach zero at 0°K. Third, graphical integration of the extrapolated segment was required to yield a  $\Delta L/L_0$  in agreement with the measured change between the liquid helium point and the lowest measured temperature. Because of these constraints the  $\alpha$ -values obtained from the extrapolated segment are considered to be comparable in reliability to the values determined at higher temperatures.

The equipment was tested on a specimen of OFHC-copper obtained from the American Brass Company. Data from three runs showed a reproducibility of  $\Delta L/L_0$  within 0.3% at all temperatures. The value of  $\Delta L/L_0$  found at 87.7°K was within 2.0% of the value reported by Nix and McNair.<sup>11</sup> The differences between the coefficients of expansion found between 20°K and 80°K and those reported by Rubin, Altman, and Johnson<sup>12</sup> and by Simmons and Balluffi<sup>13</sup> were at all temperatures less than  $0.2 \times 10^{-6} \text{ } ^\circ\text{K}^{-1}$ .

For thallium, yttrium, beryllium, and zinc, values of  $\Delta L/L_0$  both parallel and perpendicular to the c-axes and of  $\Delta V/V_0$  are given in Table 1, values of the expansion coefficients are given in Table 2, and a tabulation of relative c/a ratios is given in Table 3. A graphical representation of the expansion coefficients

as a function of temperature is shown in Fig. 2 and illustrates the pronounced variation both in magnitude and degree of anisotropy among the four materials.

#### DISCUSSION

Thallium. Eight previous investigations of the thermal expansion of thallium have been reported. Of these only the work of Erfling<sup>14</sup> was done on a single crystal, and his measurements were made for a direction oriented  $47^\circ$  from the c-axis over the temperature range,  $57^\circ\text{K}$  to room temperature. Values of the expansion coefficient for the  $47^\circ$  direction were computed from the expansion coefficients for the parallel and perpendicular directions which were determined in the present investigation. These values agree with the values reported by Erfling within 6% or less at all temperatures.

The measurements of Schulze,<sup>15</sup> Fizeau,<sup>16</sup> Smith,<sup>17</sup> and Swenson<sup>18</sup> were made on polycrystalline specimens and the volume coefficients of expansion obtained from their data may be used as a basis of comparison. The value of the volume coefficient of expansion at room temperature obtained in the present investigation lies between the values reported by Schulze and by Fizeau with the difference in both cases being of the order of 1%.

The value reported by Smith is about 5% lower, while the value reported by Swenson is nearly 20% lower. It seems probable that Swenson's low value may have been due to preferred orientation in his sample which is a possibility he himself noted.

The remaining three investigations by Shinoda,<sup>19</sup> by Schneider and Heymer,<sup>20</sup> and by Meyerhoff and Nitteberg<sup>21</sup> were made by means of x-ray techniques. In all three cases the volume coefficients of thermal expansion for the room temperature range are within 10% of the present value. However, the linear coefficients for the c and a directions differ widely both among themselves and from the present values. Only the values obtained by Meyerhoff and Nitteberg,  $\alpha_{\perp c} = 27.5 \times 10^{-6} \text{ }^{\circ}\text{K}^{-1}$  and  $\alpha_{\parallel c} = 40.0 \times 10^{-6} \text{ }^{\circ}\text{K}^{-1}$ , are close to the present values. It is believed that these discrepancies in values obtained from x-ray measurements arise from the fact that thallium tends to form as a coarse grained material. Thus observation of both HKO and OOL reflections in the back-reflection region is unlikely, and as a result the two expansion coefficients must be resolved from measurement of the shifts with temperature of two HKL reflections. The simultaneous equations which must be solved are such that small errors in measurement of d-spacings may easily reflect a disproportionately large error in the linear coefficients of expansion. However,

the errors in the resolved linear coefficients are of opposite sign so that a reasonable value for the volume coefficient of expansion may still be attained.

Yttrium. Spedding, Hanak, and Daane<sup>22</sup> have measured the coefficients of thermal expansion of yttrium above room temperature by an x-ray method. Born<sup>23</sup> has measured the linear coefficient of thermal expansion below room temperature for a polycrystalline specimen by a macroscopic method. The value of  $\alpha_v$  found in the present investigation at 0° C is within 1% of the value found by Born. The agreement between the present value of  $\alpha_{||c}$  at room temperature and the value reported by Spedding et al. is also within 1%; however, the discrepancy in the case of  $\alpha_{\perp c}$  is nearly 25% which represents a difference of  $\sim 1.5 \times 10^{-6} \text{ } ^\circ\text{K}^{-1}$ . The coefficients of Spedding et al were obtained from a least squares fit of the experimental data by a computer, and since room temperature represents the lower temperature limit of their data, the lower temperature points may have been weighted disproportionately. It should be noted that extrapolation of the present data to higher temperature shows the two sets of data to be in quite good agreement at elevated temperatures.

Beryllium. Thermal expansion measurements by x-ray methods have been made by Gordon,<sup>24</sup> Owen and Richards,<sup>25</sup> and Kossolapov and Trapeznikov<sup>26</sup> with the work of Gordon covering the widest temperature range with the best precision. Macroscopic measurement of the coefficients of thermal expansion have been made by Hidnert and Sweeney<sup>27</sup> and by Losana<sup>28</sup> on polycrystalline specimens and by Erfling<sup>14</sup> and by Treco<sup>29</sup> on single crystalline specimens. The present values of the coefficients of thermal expansion at 0°C are in best agreement with the values of Erfling and with extrapolated values of Gordon. In the case of Erfling's values, the agreement is within 9% for the volume coefficient and for both linear coefficients, while the extrapolated values of Gordon are in even better agreement with the exact degree of mismatch being subject to the bias of the extrapolator.

Zinc. Among a large number of determinations<sup>1,30-39</sup> of the thermal expansion of zinc, the values obtained by Owen and Yates<sup>30</sup> and by Grüneisen and Göens<sup>1</sup> are believed to be the most reliable. Owen and Yates measured the anisotropic thermal expansion by an x-ray technique from room temperature to the melting point. Grüneisen and Göens made macroscopic measure-

ments on single crystalline specimens over the range from 20°K to room temperature. The results of the present investigation are in good agreement with the results of the previous investigations and in particular the reported negative values for  $\alpha_{\perp c}$  at low temperatures are confirmed. In the present instance  $\alpha_{\perp c}$  was found to become negative below 71°K and to pass through a minimum at 48°K; corresponding temperatures reported by Grunelsen and Goens are 80°K and 45°K which in view of the small dimensional changes at these temperatures must be considered excellent agreement.

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Table 1. Tabulation of dimensional change as a function of temperature.

Table 2. Coefficients of thermal expansion.

Table 3. Variations of the ratio,  $(c/a)_T / (c/a)_{273.20K}$ , with temperature.

Figure 1. Diagram of experimental apparatus.

Figure 2. Temperature variation of the linear coefficients of expansion.

Table 1. Tabulation of dimensional change as a function of temperature.

TEMP. °K	BERYLLIUM			YTTRIUM			ZINC			THALLIUM			TEMP °K
	$\frac{\Delta L}{L_0}$ X10 <sup>4</sup> LC	$\frac{\Delta L}{L_0}$ X10 <sup>4</sup> HC	$\frac{\Delta V}{V_0}$ X10 <sup>4</sup>	$\frac{\Delta L}{L_0}$ X10 <sup>4</sup> LC	$\frac{\Delta L}{L_0}$ X10 <sup>4</sup> HC	$\frac{\Delta V}{V_0}$ X10 <sup>4</sup>	$\frac{\Delta L}{L_0}$ X10 <sup>4</sup> LC	$\frac{\Delta L}{L_0}$ X10 <sup>4</sup> HC	$\frac{\Delta V}{V_0}$ X10 <sup>4</sup>	$\frac{\Delta L}{L_0}$ X10 <sup>4</sup> LC	$\frac{\Delta L}{L_0}$ X10 <sup>4</sup> HC	$\frac{\Delta V}{V_0}$ X10 <sup>4</sup>	
273.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	273.2
270	0.344	0.246	0.934	0.145	0.615	0.905	0.364	1.939	2.667	0.837	1.163	2.837	270
265	0.868	0.622	2.358	0.371	1.575	2.317	0.924	4.971	6.819	2.141	2.972	7.254	265
260	1.376	0.988	3.740	0.595	2.534	3.724	1.491	8.005	10.987	3.441	4.773	11.655	260
255	1.868	1.345	5.081	0.819	3.492	5.130	2.049	11.041	15.139	4.737	6.566	16.040	255
250	2.344	1.692	6.380	1.042	4.448	6.532	2.603	14.080	19.286	6.029	8.350	20.408	250
245	2.804	2.029	7.637	1.265	5.402	7.932	3.153	17.121	23.427	7.317	10.126	24.760	245
240	3.247	2.356	8.850	1.486	6.353	9.325	3.698	20.165	27.561	8.600	11.893	29.093	240
235	3.674	2.673	10.021	1.707	7.301	10.715	4.238	23.212	31.688	9.880	13.651	33.411	235
230	4.085	2.979	11.149	1.927	8.247	12.101	4.773	26.262	35.808	11.155	15.401	37.711	230
225	4.480	3.275	12.235	2.146	9.190	13.482	5.302	29.314	39.918	12.427	17.142	41.996	225
220	4.861	3.560	13.282	2.364	10.130	14.858	5.825	32.369	44.019	13.694	18.874	46.266	220
215	5.227	3.834	14.288	2.581	11.068	16.230	6.341	35.427	48.109	14.957	20.597	50.511	215
210	5.578	4.097	15.253	2.797	11.999	17.593	6.850	38.487	52.187	16.216	22.311	54.743	210
205	5.914	4.349	16.177	3.012	12.928	18.952	7.352	41.550	56.254	17.470	24.015	58.955	205
200	6.236	4.590	17.062	3.226	13.852	20.304	7.846	44.615	60.307	18.720	25.710	63.150	200
195	6.543	4.819	17.905	3.439	14.771	21.649	8.331	47.683	64.345	19.966	27.395	67.327	195
190	6.835	5.036	18.706	3.651	15.685	22.987	8.807	50.754	68.368	21.208	29.071	71.487	190
185	7.113	5.240	19.466	3.861	16.593	24.315	9.273	53.828	72.374	22.446	30.737	75.629	185
180	7.376	5.432	20.184	4.069	17.495	25.633	9.729	56.904	76.362	23.679	32.393	79.751	180
175	7.624	5.611	20.859	4.275	18.389	26.939	10.174	59.983	80.331	24.908	34.039	83.855	175
170	7.858	5.777	21.493	4.480	19.275	28.235	10.608	63.065	84.281	26.133	35.675	87.941	170
165	8.077	5.929	22.083	4.683	20.153	29.519	11.030	66.150	88.210	27.354	37.300	92.008	165
160	8.281	6.066	22.628	4.885	21.022	30.792	11.439	69.237	92.115	28.570	38.915	96.055	160
155	8.471	6.190	23.132	5.084	21.882	32.050	11.834	72.327	95.995	29.782	40.520	100.084	155
150	8.646	6.300	23.592	5.282	22.733	33.297	12.241	75.420	99.902	30.990	42.113	104.093	150
145	8.806	6.397	24.009	5.477	23.573	34.527	12.580	78.515	103.675	32.194	43.695	108.083	145
140	8.952	6.482	24.386	5.669	24.402	35.740	12.930	81.613	107.473	33.394	45.265	112.053	140
135	9.085	6.555	24.725	5.857	25.219	36.933	13.263	84.714	111.240	34.589	46.823	116.001	135
130	9.204	6.617	25.025	6.042	26.023	38.107	13.579	87.818	114.976	35.780	48.369	119.929	130
125	9.310	6.669	25.289	6.223	26.813	39.259	13.878	90.924	118.680	36.966	49.902	123.834	125
120	9.403	6.712	25.518	6.400	27.588	40.388	14.157	94.031	122.345	38.148	51.423	127.719	120
115	9.482	6.746	25.710	6.572	28.347	41.491	14.415	97.137	125.967	39.326	52.930	131.582	115
110	9.548	6.773	25.869	6.739	29.089	42.567	14.651	100.242	129.544	40.500	54.423	135.423	110
105	9.602	6.793	25.997	6.900	29.812	43.612	14.864	103.339	133.067	41.669	55.902	139.240	105
100	9.647	6.807	26.101	7.054	30.514	44.622	15.051	106.424	136.526	42.833	57.366	143.032	100
95	9.684	6.815	26.183	7.199	31.194	45.592	15.209	109.495	139.913	43.994	58.814	146.802	95
90	9.714	6.818	26.246	7.335	31.849	46.519	15.337	112.547	143.221	45.150	60.247	150.547	90
85	9.738	6.819	26.295	7.461	32.478	47.400	15.435	115.578	146.448	46.301	61.662	154.264	85
80	9.757	6.819	26.333	7.576	33.076	48.228	15.499	118.583	149.581	47.448	63.059	157.955	80
75				7.681	33.639	49.001	15.528	121.552	152.608	48.589	64.435	161.613	75
70				7.774	34.167	49.715	15.520	124.477	155.517	49.722	65.791	165.235	70
65				7.857	34.657	50.371	15.480	127.347	158.307	50.845	67.123	168.813	65
60				7.929	35.108	50.966	15.415	130.147	160.977	51.957	68.429	172.343	60
55				7.990	35.516	51.496	15.333	132.852	163.518	53.056	69.706	175.818	55
50				8.041	35.878	51.960	15.242	135.477	165.921	54.138	70.951	179.227	50
45				8.083	36.188	52.354	15.148	137.847	168.143	55.198	72.161	182.557	45
40				8.116	36.449	52.681	15.056	140.002	170.114	56.229	73.331	185.789	40
35				8.141	36.659	52.941	14.971	141.922	171.864	57.217	74.454	188.888	35
30				8.160	36.813	53.133	14.899	143.597	173.395	58.055	75.521	191.631	30
25				8.173	36.918	53.264	14.842	145.017	174.701	58.714	76.518	193.946	25
20				8.181	36.982	53.344	14.801	146.162	175.784	59.104	77.425	195.633	20
15				8.186	37.007	53.379	14.775	147.052	176.602	59.312	78.203	196.827	15
10				8.189	37.019	53.397	14.761	147.602	177.124	59.436	78.787	197.659	10
5				8.190	37.023	53.403	14.755	147.812	177.322	59.500	79.053	198.053	5
0				8.190	37.024	53.404	14.754	147.847	177.355	59.517	79.123	198.157	0

Table 2. Coefficients of thermal expansion.

TEMP °K	BERYLLIUM			YTTRIUM			ZINC			THALLIUM			TEMP °K
	$\alpha_{LC}$ $\times 10^6$ °K <sup>-1</sup>	$\alpha_{HC}$ $\times 10^6$ °K <sup>-1</sup>	$\alpha_V$ $\times 10^6$ °K <sup>-1</sup>	$\alpha_{LC}$ $\times 10^6$ °K <sup>-1</sup>	$\alpha_{HC}$ $\times 10^6$ °K <sup>-1</sup>	$\alpha_V$ $\times 10^6$ °K <sup>-1</sup>	$\alpha_{LC}$ $\times 10^6$ °K <sup>-1</sup>	$\alpha_{HC}$ $\times 10^6$ °K <sup>-1</sup>	$\alpha_V$ $\times 10^6$ °K <sup>-1</sup>	$\alpha_{LC}$ $\times 10^6$ °K <sup>-1</sup>	$\alpha_{HC}$ $\times 10^6$ °K <sup>-1</sup>	$\alpha_V$ $\times 10^6$ °K <sup>-1</sup>	
270	10.64	7.68	28.96	4.52	19.22	28.26	11.36	60.60	83.32	26.14	36.33	88.61	273.2-270
265	10.48	7.52	28.48	4.51	19.20	28.22	11.30	60.64	83.24	26.08	36.19	88.35	270 -265
260	10.16	7.32	27.64	4.49	19.18	28.16	11.24	60.68	83.16	26.00	36.02	88.02	265 -260
260 -255	9.84	7.14	26.82	4.48	19.16	28.12	11.16	60.72	83.04	25.92	35.86	87.70	260 -255
255 -250	9.52	6.94	25.98	4.46	19.12	28.04	11.08	60.78	82.94	25.84	35.68	87.36	255 -250
250 -245	9.20	6.74	25.14	4.45	19.08	27.98	11.00	60.82	82.82	25.76	35.51	87.03	250 -245
245 -240	8.86	6.54	24.26	4.43	19.02	27.88	10.90	60.88	82.68	25.67	35.34	86.68	245 -240
240 -235	8.54	6.34	23.42	4.42	18.96	27.80	10.80	60.94	82.54	25.59	35.17	86.35	240 -235
235 -230	8.22	6.12	22.56	4.40	18.92	27.72	10.70	61.00	82.40	25.51	34.99	86.01	235 -230
230 -225	7.90	5.92	21.72	4.38	18.86	27.62	10.58	61.04	82.20	25.43	34.82	85.68	230 -225
225 -220	7.62	5.70	20.94	4.36	18.80	27.52	10.46	61.10	82.02	25.34	34.64	85.32	225 -220
220 -215	7.32	5.48	20.12	4.34	18.72	27.40	10.32	61.16	81.80	25.26	34.46	84.98	220 -215
215 -210	7.02	5.26	19.30	4.32	18.66	27.30	10.18	61.20	81.56	25.18	34.28	84.64	215 -210
210 -205	6.72	5.04	18.48	4.30	18.58	27.18	10.04	61.26	81.34	25.09	34.09	84.27	210 -205
205 -200	6.44	4.82	17.70	4.28	18.48	27.04	9.88	61.30	81.06	25.00	33.90	83.90	205 -200
200 -195	6.14	4.58	16.86	4.26	18.38	26.90	9.70	61.36	80.76	24.92	33.70	83.54	200 -195
195 -190	5.84	4.34	16.02	4.23	18.28	26.74	9.52	61.42	80.46	24.84	33.51	83.19	195 -190
190 -185	5.56	4.08	15.20	4.20	18.16	26.56	9.32	61.48	80.12	24.75	33.32	82.82	190 -185
185 -180	5.26	3.84	14.36	4.16	18.04	26.36	9.12	61.52	79.76	24.67	33.12	82.46	185 -180
180 -175	4.96	3.58	13.50	4.13	17.88	26.14	8.90	61.58	79.38	24.58	32.92	82.08	180 -175
175 -170	4.68	3.32	12.68	4.10	17.72	25.92	8.68	61.64	79.00	24.50	32.72	81.72	175 -170
170 -165	4.38	3.04	11.80	4.06	17.56	25.68	8.44	61.70	78.58	24.41	32.51	81.33	170 -165
165 -160	4.08	2.74	10.90	4.03	17.38	25.44	8.18	61.74	78.10	24.33	32.30	80.96	165 -160
160 -155	3.80	2.48	10.08	3.99	17.20	25.18	7.90	61.80	77.60	24.24	32.09	80.57	160 -155
155 -150	3.50	2.20	9.20	3.95	17.02	24.92	7.60	61.86	77.06	24.16	31.86	80.18	155 -150
150 -145	3.20	1.94	8.34	3.90	16.80	24.60	7.32	61.90	76.54	24.08	31.64	79.80	150 -145
145 -140	2.92	1.70	7.54	3.84	16.58	24.26	7.00	61.96	75.96	23.99	31.40	79.38	145 -140
140 -135	2.66	1.46	6.78	3.77	16.34	23.88	6.66	62.02	75.34	23.90	31.16	78.96	140 -135
135 -130	2.38	1.24	6.00	3.70	16.08	23.48	6.32	62.08	74.72	23.82	30.92	78.56	135 -130
130 -125	2.12	1.04	5.28	3.62	15.80	23.04	5.98	62.12	74.08	23.73	30.67	78.13	130 -125
125 -120	1.86	0.86	4.58	3.54	15.50	22.58	5.58	62.14	73.30	23.64	30.41	77.69	125 -120
120 -115	1.58	0.68	3.84	3.44	15.18	22.06	5.16	62.12	72.44	23.56	30.14	77.26	120 -115
115 -110	1.32	0.54	3.18	3.34	14.84	21.52	4.72	62.10	71.54	23.47	29.86	76.80	115 -110
110 -105	1.08	0.40	2.56	3.22	14.46	20.90	4.26	61.94	70.46	23.38	29.58	76.34	110 -105
105 -100	0.90	0.28	2.08	3.08	14.04	20.20	3.74	61.70	69.18	23.29	29.28	75.86	105 -100
100 - 95	0.72	0.16	1.60	2.90	13.60	19.40	3.16	61.42	67.74	23.21	28.97	75.39	100 - 95
95 - 90	0.60	0.06	1.26	2.72	13.10	18.54	2.56	61.04	66.16	23.12	28.65	74.89	95 - 90
90 - 85	0.48	0.02	0.98	2.52	12.58	17.62	1.96	60.62	64.54	23.03	28.31	74.37	90 - 85
85 - 80	0.38	<0.01	0.76	2.30	11.96	16.58	1.28	60.10	62.66	22.94	27.94	73.82	85 - 80
80 - 75				2.09	11.26	15.44	0.58	59.38	60.54	22.81	27.53	73.15	80 - 75
75 - 70				1.87	10.56	14.30	-0.16	58.50	58.18	22.66	27.11	72.43	75 - 70
70 - 65				1.65	9.80	13.10	-0.80	57.40	55.80	22.46	26.63	71.55	70 - 65
65 - 60				1.44	9.02	11.90	-1.30	56.00	53.40	22.25	26.12	70.62	65 - 60
60 - 55				1.22	8.16	10.60	-1.64	54.10	50.82	21.98	25.54	69.50	60 - 55
55 - 50				1.02	7.24	9.28	-1.82	51.70	48.06	21.64	24.90	68.18	55 - 50
50 - 45				0.84	6.20	7.88	-1.88	48.20	44.44	21.19	24.20	66.58	50 - 45
45 - 40				0.66	5.22	6.54	-1.84	44.10	40.42	20.62	23.40	64.64	45 - 40
40 - 35				0.51	4.20	5.22	-1.70	38.40	35.00	19.77	22.47	62.01	40 - 35
35 - 30				0.37	3.08	3.82	-1.44	33.50	30.62	16.75	21.33	54.83	35 - 30
30 - 25				0.26	2.10	2.62	-1.14	28.40	26.12	13.18	19.95	46.31	30 - 25
25 - 20				0.17	1.26	1.62	-0.82	23.30	21.66	7.80	18.13	33.73	25 - 20
20 - 15				0.10	0.50	0.70	-0.52	17.40	16.36	4.16	15.57	23.89	20 - 15
15 - 10				0.05	0.24	0.34	-0.28	11.00	10.44	2.48	11.67	16.63	15 - 10
10 - 5				0.02	0.08	0.12	-0.12	4.20	3.96	1.28	5.32	7.88	10 - 5
5 - 0				<0.01	0.02	0.03	-0.02	0.70	0.66	0.34	1.40	2.08	5 - 0

Table 3. Variations of the ratio,  $(c/a)_T / (c/a)_{273.2^\circ\text{K}}$ , with temperature.

T <sup>°K</sup>	Thallium	Yttrium	Beryllium	Zinc
273.2	1.0000000	1.0000000	1.0000000	1.0000000
260	.9998667	.9998061	1.0000388	.9993485
240	.9996704	.9995132	1.0000891	.9983527
220	.9994812	.9992232	1.0001302	.9973441
200	.9992735	.9989371	1.0001647	.9963202
180	.9991265	.9986569	1.0001925	.9952779
160	.9989625	.9983855	1.0002207	.9942136
140	.9988089	.9981256	1.0002472	.9931228
120	.9986674	.9978798	1.0002694	.9920013
100	.9985404	.9976523	1.0002843	.9908489
80	.9984314	.9974481	1.0002941	.9896756
60	.9983441	.9972799		.9885091
40	.9982801	.9971644		.9874866
20	.9981570	.9971175		.9868424
0	.9980276	.9971142		.9866710

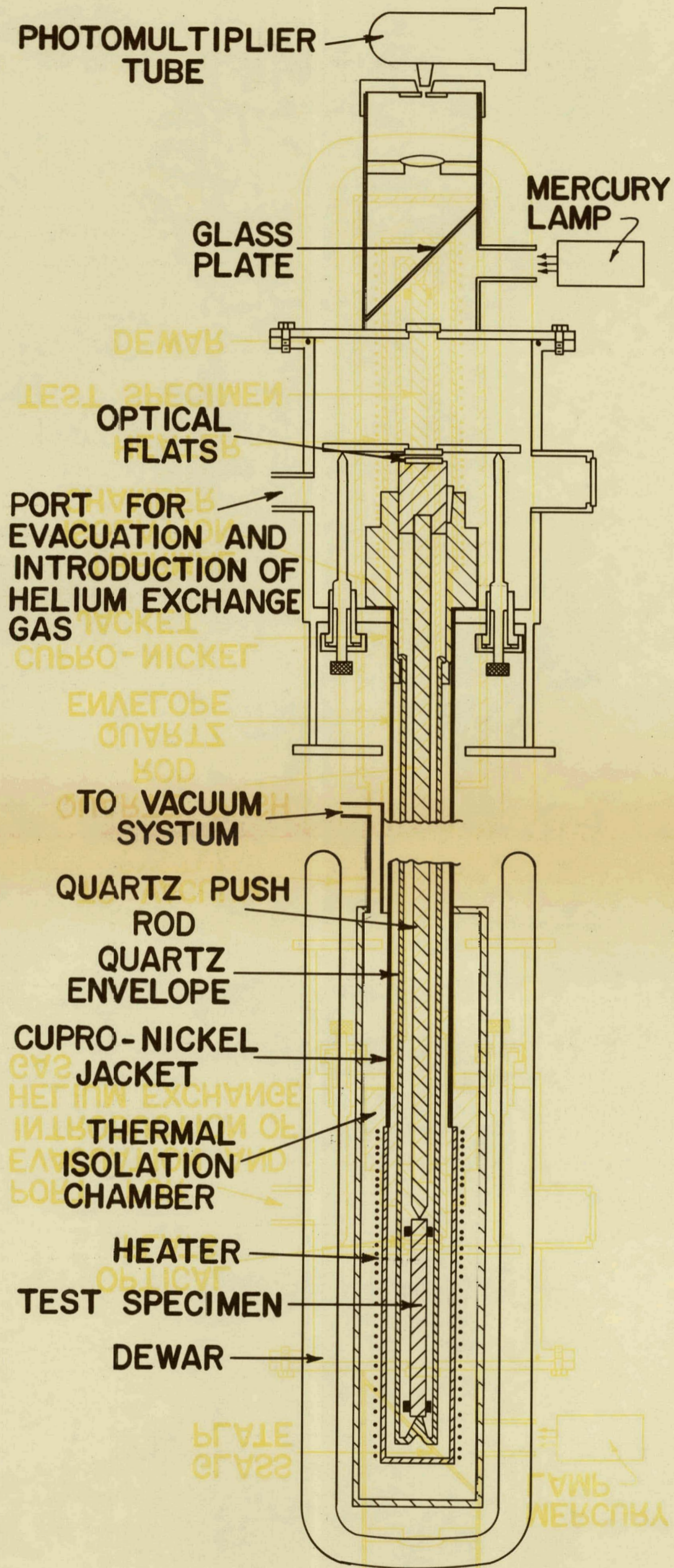


Fig. 1. Diagram of experimental apparatus.

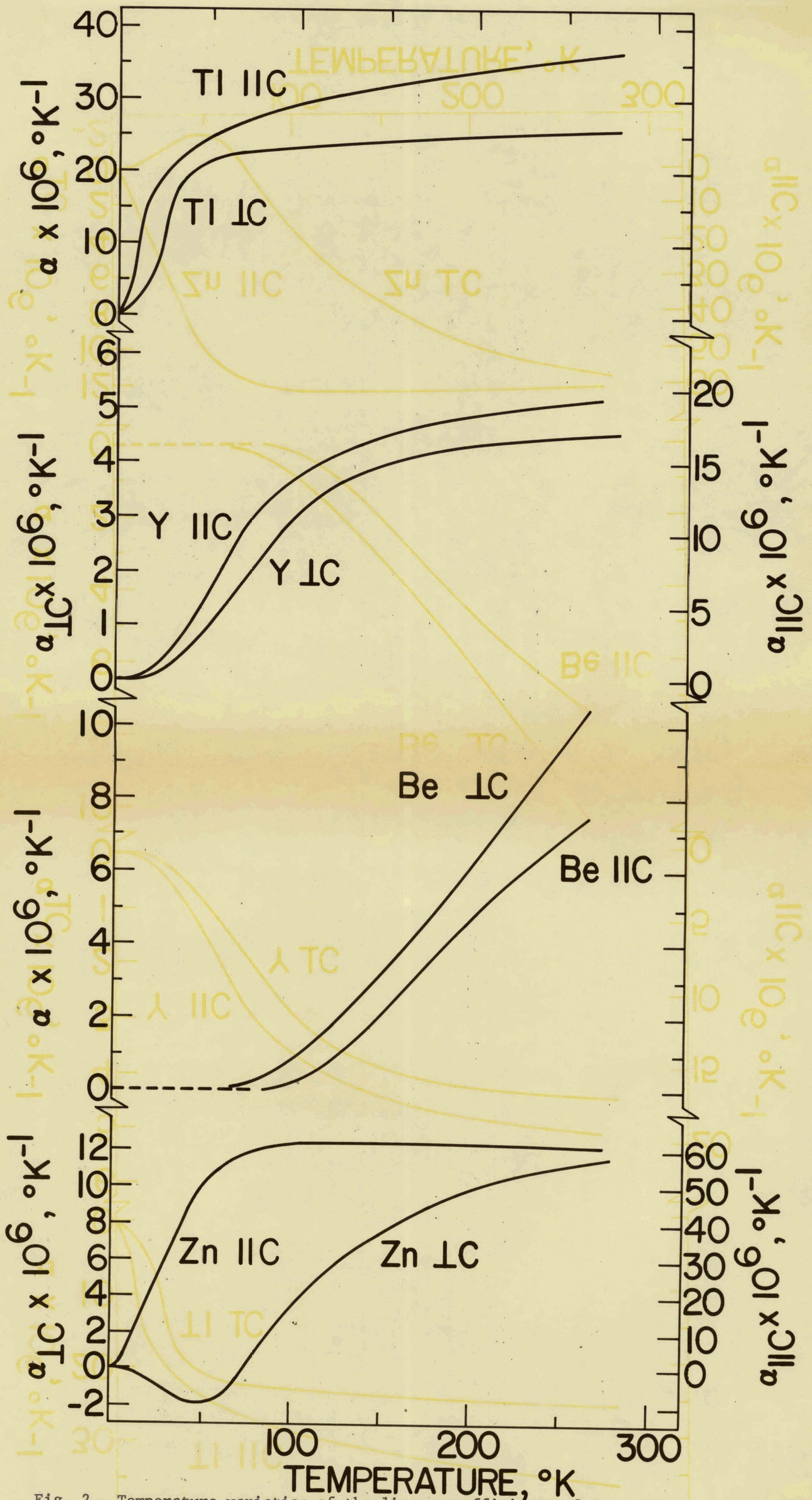


Fig. 2. Temperature variation of the linear coefficients of expansion.