UCRL-9692

THE GRAIN-DENSITY OF EMULSION TRACKS

Jack W. Patrick and Walter H. Barkas

Lawrence Radiation Laboratory
University of California
Berkeley, California

ABSTRACT

Measurements of the true grain density were made by several methods on the tracks of electrons, pions, K mesons, protons, Σ hyperons, and alpha particles. The curve of grain density versus velocity in K.5 emulsion was obtained. The results found by different objective methods and by different observers are in agreement. Owing to the finite density of silver-halide crystals in the emulsion, the grain density saturates. The nature of the saturation effect was studied. A decomposition of the grain density into primary and secondary components was made. Even at the minimum of grain density, some 25 percent of the grains are of secondary origin. Since only the primary grains are affected by the relativistic rise of the grain density, the interpretation of the plateau/minimum grain density ratio is affected. Special observations of the grain density in the relativistic region were made, taking precautions to avoid temperature, fading, and development-difference effects. A rise to the plateau of 18% in the primary grain density was found. This implies a mean excitation potential for AgBr of 442 e.v. Finally, indices that measure emulsion quality are suggested.

DISCLAIMER

THE GRAIN-DENSITY OF EMULSION TRACKS

Jack W. Patrick and Walter H. Barkas

Lawrence Radiation Laboratory
University of California
Berkeley, California

I. INTRODUCTION

The grain density in the emulsion-track of a charged particle is an indicator of its velocity. The information implicit in the track-structure, however, generally is only partly utilized. Some requirements for more rapid and accurate determination of the velocity of a particle from measurements on its track are (a) understanding of the connections between the various measures of the grain density and the true grain density; (b) knowledge of the measurements that will yield the optimum amount of information from a particular track segment; (c) knowledge of the connection between the true grain density and the particle velocity; (d) establishment of the relationship of the primary and secondary grain density in a track to each other, and to the particle charge and velocity (this is required especially for interpreting the ratio of plateau to minimum grain density); and (e) establishment of indices of merit for emulsions which define the density of information obtainable from a track and the velocity intervals in which the track can yield information regarding the particle velocity.

These questions recently were restudied theoretically, and considerable progress made.[1,2] In this paper we carry on the investigation, largely empirically, by analyzing the results of measurements at many velocities in some seven emulsion stacks, and in several additional plates. An essential preliminary is the operational definition of measurable and useful track quantities (ionization parameters).

A particle track is seen as a more or less continuous series of grain images. These are roughly circular in projection, but their centers, in general, are displaced around the particle trajectory, both vertically and horizontally. They may occult each other or be too close for resolution with the optical equipment employed. They vary in size.

Suppose the length projected on a plane perpendicular to the line of sight between the centers of two grain images is considered. This is a distance "a" when the grains can just be resolved into two objects. Then if "c" is the distance projected on the particle trajectory between the centers of two such grains, and the projected image of no other grain comes between them, a gap of length c-a is said to exist in the track. Since "a" varies for different pairs of grains, an expectation value, $\langle a \rangle = \alpha$ is defined which describes in one combined parameter the emulsion, the optical equipment, and the observer characteristics. The blob density, B, is defined as the linear density of gaps, or of clusters of unresolved grains in the track. A quantity $H = H(\ell)$ is the density of gaps exceeding the length ℓ. It is, of course, also equal to the density of clusters of grains in which are found no gaps exceeding ℓ. The blob density is, therefore, the special case of a cluster density in which $\ell = 0$, so that $B = H(0)$. If different values of ℓ, namely ℓ_1, ℓ_2, etc., are considered, several values of H, H_1, H_2, H_3, etc. are introduced.

The lacunarity, L, of the track is the linear fraction of it that consists of gaps. Thus

$$L = -\int_0^\infty \ell \frac{dH}{d\ell}\, d\ell. \qquad (1)$$

We define, but measure only indirectly, the primary grain density, g_p, the secondary grain density, g_s, and the true grain density, $g = g_s + g_p$. The primary grain density is the density of crystals that were penetrated by the particle in traversing the emulsion and subsequently developed into silver grains. The average number of silver-halide crystals penetrated per unit path is n. Then

$$n = \frac{3}{2} C \frac{\langle D^2 \rangle}{\langle D^3 \rangle} , \qquad (2)$$

where C is the silver-halide concentration, $\frac{\pi \langle D^2 \rangle}{4}$ is the mean projected area of a crystal, and $\frac{\pi \langle D^3 \rangle}{6}$ is the mean crystal volume. The maximum value of g_p, of course, is n. Secondary grains are those that develop along the path of the particle and are counted as part of the track, although they were not penetrated by the moving particle. Delta rays are projected from the path of the primary particle, and they render crystals developable that were not traversed by the primary particle. At velocities low enough for the delta-ray density to be high, however, the range of the delta rays is limited. In addition, there is a certain displacement, ρ, from the axis of a track that a grain cannot exceed and still be recognized as part of the track. The number, N, of grains per unit volume of emulsion is finite. The linear density of crystals, n_o, within a cylinder of radius ρ, therefore, is also limited, and n_o is a saturation value of the grain density that g does not exceed. As the charge on an ion increases, however, the density of delta rays produced by it and the effective value of ρ both increase also.

Photons produced by the ionizing particle in the transparent gelatin may be an additional source of secondary grains. No means for their identification, should they be present with an appreciable density, have been developed, however.

II. METHODS FOR MEASURING GRAIN DENSITIES

The true grain density, g, seldom can be measured merely by counting the developed grains. The error involved in such a measurement rises rapidly as the grain density increases. The density of silver-halide crystals in K.5 emulsion is about 100 per cubic micron. Even if just one per micron of particle path were rendered developable, only 37% would be resolvable as single objects under the best microscope, and the blob density would be about 0.6 per micron. As the grain density rises, it reaches a point where counting is hopeless and it becomes necessary to obtain estimates of the true grain density indirectly from the track features that remain measurable. The quantities defined in the introduction have been studied theoretically, and some are suitable for this purpose. Their definitions rather precisely prescribe the way in which measurements are to be taken, and only technical details may be varied. In practice, many of our measurements were made on a machine developed for the purpose. A description of an early model of the instrument has been given.[3] It provides a means for moving the plate parallel to the track at an adjustable velocity. The track is kept centered in the microscope field and in focus by an observer who holds a key depressed during the time that a fine reticle line perpendicular to the track crosses the track in a gap. He releases the key when the end of each gap is reached, and depresses it when each new gap first reaches it. The lengths of track and gap, the number of gaps, and the distribution of gaps in ten intervals of length are tabulated electronically. For reliable work the stage velocity must be decreased until no changes in results are produced by a further reduction in speed.

The connections between those measurements and the true grain density were established by theory.[1,2] We quote the pertinent results. The formulas

are written for unit length of track.

It was found that $H(\ell) = ge^{-g(\alpha+\ell)}$, where g is the expectation value of the grain density, and H is the expectation value of the cluster density. Good empirical evidence for the exponential gap length distribution was first put forward by O'Ceallaigh. The distribution of many hundreds of gaps is given by Menon and O'Ceallaigh.[4] The expectation value of the lacunarity is

$$L = e^{-g\alpha}$$

so that an estimate of g is obtained from

$$g_L = -\frac{\ell n\, L}{\alpha} \quad . \tag{3}$$

The parameter α must be measured in order to derive g from single measurements of B, H, or L. As g varies, it may be noted that $B\ (= ge^{-g\alpha})$ passes through a maximum at $g = 1/\alpha$, or $\alpha = (eB_{max})^{-1}$. Observation of the maximum value, B_{max}, of the blob density is simple, and is an operationally correct means for determining α. We have done this for each combination of emulsion, optical arrangement, and observer used in making these measurements.

The grain diameter, α, is perhaps even better obtained merely by observing B and L in tracks similar to those being measured. Then $-\frac{L}{B}\ \ell n\, L$ is an expression for α that takes account of the instrument and observer idiosyncrasies. The introduction of such an operationally defined parameter greatly improves the objectivity of grain density measurements. If each observer uses his own value of α, one expects and finds no systematic differences between observers. The quantity α here is <u>not</u> the symbol α used by Alexander and Johnston[5] in applying O'Ceallaigh's theory[6] of the track structure. Because in O'Ceallaigh's emulsion model the crystals are confined to lie with their centers on the particle trajectory, the crystal diameter

appears explicitly in their formulas. Our notation was selected to be in accord with that of Fowler and Perkins.[7]

The derivation of g from the measurements was carried out by a number of methods. The gap length coefficient method yields a value of g without requiring knowledge of α. As suggested by Fowler and Perkins,[7] one measures the value of H at two gap lengths, ℓ_1 and ℓ_2. Then

$$g = \frac{\ln \left[H(\ell_1)/H(\ell_2) \right]}{\ell_2 - \ell_1} \quad . \tag{4}$$

If only two values are to be measured, then one takes $\ell_1 = 0$ and $H(\ell_2) \approx g/5$. One can also calculate g from Eq. (3). Of course, g then contains any uncertainty that exists in α.

The most efficient means for utilizing the granularity information is the method of maximum likelihood, which combines the grain density estimate from the mean gap length with that from the mean blob length.[2] This method was not developed for application to grain density measurements at the time our work was carried out, however. To attain a given statistical accuracy, more work was required than would have been necessary by the improved procedure. On the other hand, the diversity of methods that we employed tested the theory more completely.

We have noted that because the density of silver-halide crystals is finite, a maximum grain density n_o will be observed in the track. The limiting lacunarity is $e^{-n_o\alpha}$ and the limiting gap density is $n_o e^{-n_o\alpha}$. Some conjectures regarding other emulsion defects have been made.[7,8] Although the possibility of their presence was mentioned in Reference 1, the effect of such defects in the emulsion was not readily treated. At very low particle velocities, nevertheless, a limiting empirical lacunarity L_o is found that does not vary with particle ionization. While we have no definite indication that

they are important, we may consider how they would be manifested in emulsion. A particle that produces delta rays copiously could presumably render developable all the crystals in a cylinder of one micron radius. Then in K.5 emulsion, the lacunarity would be less than e^{-150}, were there no defects in the emulsion. In such a track any gaps should be attributed to emulsion defects. Even in an emulsion free of defects, however, gaps occur in tracks when n_o is not large. We define the saturation gap distribution $H_o(\ell)$ as that gap distribution which does not change with increasing particle charge for a fixed particle velocity. Then, in principle, the saturation of g can be avoided if one defines ideal quantities $H'(\ell)$ and L' in terms of the observed $H(\ell)$ and L by

$$H'(\ell) = H(\ell) - (1 - L)\, H_o(\ell) \tag{5}$$

and

$$L' = -\int_0^\infty \ell \frac{dH'}{d\ell}\, d\ell \ . \tag{6}$$

III. OBSERVATIONAL PROCEDURE

Some dependence of the degree of development on depth in the emulsion is likely to exist. We believe we have eliminated this effect from consideration in these measurements. We used plates in which there was obviously little change of grain density with depth in the emulsion. All were developed by our method of immersion during the hot stage in developer of reduced concentration. In addition, we either calculated average values of g for tracks that were slightly inclined in the emulsion and sampled all strata, or we paired known and unknown tracks and observed grain-density ratios of tracks that were at the same depth in the emulsion.

The angle of inclination, δ, of the track was required to be small for the track to be used for a grain density measurement. If the correction for dip turned out not to be completely negligible, it was made according to the following general procedure: A universal relation $y = xe^{-x}$ exists between $y = H(\alpha+\ell)\sec\delta$ and $x = g(\alpha+\ell)\sec\delta$, where ℓ is the gap length in a projected image of the track, H is the average number of clusters in unit projected length of the track, and $(\pi/2) - \delta$ is the angle between the track in the unprocessed emulsion and the normal to the emulsion plane.[2]

In Table I is summarized the essential information on the emulsion stacks and plates used in this experiment. The bulk of the work was done with K.5 emulsion.

In the velocity interval between $\beta_o = 0$ and $\beta_o = 0.2$ in units of the light velocity, c, lacunarity measurements were made with 25 x 100 power magnification on the tracks of alpha particles, sigma hyperons, protons, K-mesons, and pions.

Each flat track was examined carefully to determine the identity of the particle. The ending was scrutinized to insure that the particle came to rest. A track was discarded if there was any reason to doubt its identity or the certainty of its coming to rest. The work was done by several trained observers who checked each other. (One of the satisfying aspects of this work was that when a value for α was found as prescribed, consistent values of g were found by all observers, and different methods of measurement also yielded the same result.)

For the velocity interval between $\beta_o = 0.2$ and $\beta_o = 0.5$, proton and K-meson tracks were observed under 25 x 100 magnification. The gap-length distribution was used in conjunction with the blob count so as to obtain the

TABLE I

Emulsion Designation	Emulsion Type	Size $(in.)^2$	Pellicle Thickness (μ)	Particles	$(\beta\gamma)_{max}$
A	K.5	6 x 9	600	K^-	0.870
				$\pi^+, \Sigma^\pm, p$	variable
B	G.5	6 x 9	600	K^-	2.33
BB	K.5	1 x 3	600	π^-	5.16
				e	1409.
ID	G.5	13 x 3	600	π^+	2.58
K-5	K.5	1 x 3	600	α	0.250
				p	variable
SD	G.5	1 x 3	200	p	6.61
				e	391.
Y	K.5	3 x 6	600	$\pi^\pm$	115.

gap length coefficient with the best precision. Data were collected using both a simple divided reticle in the microscope ocular and the special instrument mentioned above.

In the velocity interval $\beta_o > 0.7$, the method of blob counting also was used. If the available amount of track is not limited, the effort expended per unit of information is about the same for the two methods. Moreover, no special equipment is required for blob counting. The value of α appropriate to the observer, optics, and emulsion combination must be determined, however, in order that g be found from the relation $B = ge^{-g\alpha}$.

We used the tracks of protons, K-mesons, pions, and electrons. When the particle stopped in the emulsion, its tracks and terminal behavior were carefully studied to check its identity and to establish that it came to rest. Measurements began at the terminus and were carried up to any desired velocity by following the track.

In some cases we blob-counted tracks of particles that did not stop in the emulsion. Their velocities then usually were obtained by magnetic analysis. For example, the BB plates were exposed to a magnetically analyzed beam consisting of 70% pions and 30% electrons. The grain density identified the particles. A sample of about 80 tracks was blob counted. The distribution revealed two separated peaks.

A one-millimeter grid photographically printed on each pellicle was sufficiently accurate for the estimate of ranges exceeding about 3 centimeters, when allowance for the dip and large scattering angles was made. Short ranges were usually measured with a calibrated eyepiece reticle or on an automatic coordinate read-out microscope.[9]

If the range could not be measured directly, as in the case of particles that did not stop in the emulsion, then two procedures were available:

(1) The beam momentum was known and the corresponding range R_o when the particle entered the emulsion was calculated. The path length in emulsion R_A was subtracted, leaving the residual range $R = R_o - R_A$ at the point of measurement. From this range the velocity βc was determined. In the SD and BB stacks, the beam momentum was accurately known. The calculation of it at the point of grain density determination then was relatively simple. In the ID stack, protons that had been magnetically analyzed along with the pions were followed into the emulsion from three separate points on the beam edge of the pellicle. The beam momentum varied along the edge so these three points represented high, low, and medium momentum. The proton ranges were measured, and this gave the common momentum of protons and pions at entry in the stack.

For the BB stack the situation was slightly different. The identity of the particles could not be determined individually - only a statistical method was available, but the grain densities separated into two groups assumed to consist of π-mesons and electrons.

(2) The second momentum determination procedure was used for electrons in the SD stack and for secondary pions in the Y stack. This was the method of multiple scattering.

The electrons were produced by the Lawrence Radiation Laboratory synchrotron. Their momentum at entry was found to be imperfectly defined although they had been magnetically separated. Electrons radiate so much energy while penetrating matter that it was considered wise to multiply-scatter every track at the same time that it was blob-counted. Then individual values of $p\beta_o$ and g were assigned to each track. The data from tracks found to lie in small intervals of $p\beta_o$ were eventually averaged.

The multiple scattering of the tracks was carried out using a Cooke, Troughton and Simms scattering microscope and a digitized Koristka MS-2 microscope.[10] The mean angle of scattering and the momentum were computed from the

data cards by an IBM 650 program.[10]

IV. GRAIN DENSITY VERSUS PARTICLE VELOCITY IN K.5 EMULSION

The grain density was obtained in each velocity interval by the methods described in Sections II and III. These data were then subjected to a statistical error evaluation appropriate to the method used.

In Fig. 1 the grain density measurements in K.5 emulsion are plotted as a function of β_o. The method of measurement is indicated by the character of the plotted point. There was considerable overlap of different methods in some velocity intervals, and in some intervals three methods were employed. No systematic difference in the results obtained by different methods was observed.

We notice that at low velocities the apparent grain density does not saturate at $g = n = 328/100$ microns, as the primary grain density must in this emulsion. The curve does tend to flatten, however, below $\beta_o = 0.07$. This portion of the curve is derived from lacunarity measurements near the termini on the tracks of singly charged particles. We are able to understand this behavior better by noticing the α-particle branch of the grain density curve. The same saturated grain density is reached by both singly and doubly charged particles. We interpret this to mean that in K.5 emulsion an irreducible track lacunarity L_o is present and is equal to 0.01-0.03, regardless of the rate of energy loss of the particle. Owing to this emulsion limitation, high rates of energy loss are not measurable in this emulsion by grain measurements.

On the other hand, when $\beta_o > 0.1$, the curves for charge 1 and charge 2 start to separate, and above $\beta_o = 0.15$, K.5 emulsion permits us to measure a difference in the rates of energy loss.

V. DECOMPOSITION OF THE GRAIN DENSITY INTO PRIMARY AND SECONDARY COMPONENTS

As mentioned above, the grain density in a sensitive emulsion at a low particle velocity may exceed n because the primary grain density saturates at

$g_p = n$, and, in addition, secondary grains are present.

We shall now calculate the density of such secondary grains produced by delta rays. The range-velocity relation for protons is

$$R \approx 3.6 \times 10^5 \beta^{10/3} \text{ microns.} \qquad (\beta < 0.3)$$

The range, R_e, of a low-velocity electron is obtained with satisfactory accuracy from the proton range merely by multiplying by the mass-ratio (1/1800):

$$R_e \approx 2 \times 10^2 \beta^{10/3} \quad . \tag{7}$$

The grain density at velocity β according to our measurements is given by:

$$g \approx 9.2 - 32 \beta \text{ per micron,} \qquad (\beta < 0.3)$$

for a singly-charged particle in K.5 emulsion. Then the number of grains, $G(\beta)$, in an electron track of initial velocity β can be found by integration.

$$G(\beta) = \int_0^\beta g \, dR_e = \frac{2000}{3} \int_0^\beta (9.2 - 32 \beta) \beta^{7/3} \, d\beta \tag{8}$$

$$= 0.18 w^{5/3} - 0.03 w^{13/6} \quad ,$$

where $w (= 256 \beta^2)$ is the electron energy in Kev. The number of delta rays in the energy interval w to $w + dw$ on the track of a particle with charge ze and velocity β_o is about $\frac{0.0255 z^2}{\beta_o^2} \frac{dw}{w^2}$ per micron. This formula breaks down at delta ray energies that are comparable to the electronic binding energies in the stopping material.

The number of grains g_s per micron produced by the delta rays along the path of a particle of charge ze and velocity β_o then is found by integrating over w.

$$g_s = \frac{A z^2}{\beta_o^2} \tag{9}$$

with $A = 0.68\ (w_m^{2/3} - w_o^{2/3}) - 1.1 \times 10^{-2}\ (w_m^{7/6} - w_o^{7/6})$.

In this expression, w_o represents the lowest average energy of delta rays contributing to the secondary grain density, and w_m is the maximum energy that a delta ray may have while its grains still are considered part of the track locus.

The delta rays of longest range are projected forward and tend to lie on the particle trajectory. Electrons are very much scattered, however, so that some will reach points off the particle path. The definition of w_m, therefore, must be made carefully. Obviously the judgement of the observer plays a part in its definition. Its value cannot be established without making reference to the technique of observation.

Two observers estimated at what residual range O^{16} tracks were so widened by delta rays that some of the grains would not have been considered part of the track locus had they occurred on the track of a singly-charged particle. Heavy-ion tracks were used so that a good density of delta rays would be present. This residual range was about 150 microns. Here the energy of the oxygen ion is $\approx$ 160 Mev.[12] The delta ray spectrum at this energy extends up to 22 Kev, and this value was adopted for w_m.

We believe that particularly in near-minimum tracks, where the grains are widely spaced, observers who are blob-counting accept grains as part of the track locus that extend 1.25 microns or more from the true trajectory of the particle.

From the range-energy relation, w_o must be around 2 Kev for a crystal radius of about 0.1μ (K.5 emulsion). With w_m as high as 22 Kev, the value of A then is insensitive to w_o. This evaluation of A fails when $2\ mc^2\beta_o^2$ falls below w_m. Then the nominal upper limit of the delta ray spectrum does not reach the limit w_m set by the observer. If the above formulas were exact,

A would fall to zero and remain zero for all β_o such that $2mc^2\beta_o^2 < w_o$. However, in a sensitive emulsion the observed grain density does not fall to n at very low particle velocities. Actually the cross section for producing delta rays of any energy less than that of the particle itself never completely vanishes. As mentioned above, our simple formula for the delta ray density was derived with the supposition that the atomic electron velocities are small compared to the particle velocity. A much more elaborate study would be necessary to treat correctly the terminal portion of the track.

We find $A = 3.9$ per 100 microns, and adopt the relation $g_s = 3.9\ z^2/\beta_o^2$ per 100 microns for all high velocities.

The difference $g - g_s$ is presumably the primary grain density. It is almost surely rather complicated in its dependence on the rate of energy loss of the primary particle, as discussed in Ref. 1. Long ago, however, a formula was derived[11] with simplifying assumptions that may approximate the true situation. The assumptions were merely that an increment in grain density is to be attributed to the product of three factors: the increment in the effective rate of energy loss, the remaining density of grains not already rendered developable, and a parameter measuring the emulsion sensitivity. The relationship found was

$$g_p = n\,(1 - e^{-\lambda I'}). \qquad (10)$$

Here λ measures the emulsion sensitivity, and I' is the <u>effective</u> rate of energy loss.

In Fig. 2 we have plotted $-\ln(1 - g_p/n)$ vs. I', where I' is taken to be the restricted rate of energy loss[12] with a cutoff at 2 Kev. We can assign a sensitivity, λ, of 0.048 gm/Mev-cm^2 to this K.5 emulsion if we approximate the data by a straight line through the origin.

VI. GRAIN DENSITY IN THE RELATIVISTIC REGION

In virtue of the simple behavior of both the primary and secondary grain densities when the track is highly unsaturated and $\beta_o \approx 1$, we can write

$$g \approx n \lambda z^2 i' + A z^2/\beta_o^2 \qquad (11)$$

where $z^2 i' = I'$. The second term is new, and we are certain that it is not negligible. In K.5 emulsion our crude estimate of A implies that 25 percent of the grain density at the minimum is of secondary origin. The relativistic rise of I' does not affect the secondary grain density; on the contrary, g_s falls some 6 percent between the minimum and the plateau. For this reason, the minimum of grain density may be non-existent or at least less pronounced in tracks of multiply-charged particles.[1] In such tracks the grain density will already be tending to saturate at the ionization minimum.

In the relativistic region where small differences are important, special precautions were taken in our observations. One was careful control of the temperature of the emulsion at the time of exposure. Another was minimizing the time lapse between different exposures. All tracks to be compared were produced in the emulsion at the same temperature and within a time interval of a few hours at most. They were produced in the same emulsion pellicle and were developed simultaneously. Different kinds of particles near the minimum generally were caused to traverse the emulsion at 90° to each other so as to be readily identified.

We always measured ratios of grain densities near the minimum and have normalized all grain densities, g, to unity at the minimum. The results are shown in Fig. 3.

The primary grain density rises from the minimum to plateau by a factor of 1.18. The theoretical curve of g_p, assuming that it is proportional to the restricted rate of energy loss in AgBr, is also graphed in Fig. 3. The

curve is drawn for a mean excitation potential of AgBr equal to 442 e.v., and a delta ray cut-off of 2 Kev. While we believe these results to be reliable, we do not consider them to be final and decisive for the question of the behavior of the relativistic rise in grain density. In order for this rise to be used quantitatively to measure trans-minimum particle velocities, much more very painstaking work will be required.

VI. INDICES OF MERIT FOR EMULSION

The information density in a track is limited by the maximum gap density, B_{max}. In general, the higher this quantity the more information the track can yield. Of course, such optical resolution then must be employed that B_{max} does not change with the optical resolving power. It is directly related to the developed grain diameter, α ($= 1/eB_{max}$).

The sensitivity of the emulsion is usually measured by the grain density, g_{min}, produced at the minimum of ionization. It is necessary to determine this at some preselected fog level. It may perhaps be arbitrarily established at one background grain per 1000 cubic microns. If one states the quantity (g_p/n) at the minimum, he gives a more absolute measure of the sensitivity for the halide concentration is then eliminated from the measurement. For our K.5 emulsion this ratio was 0.037.

The saturation lacunarity, L_o, limits the amount of information that is obtainable from tracks of slowly moving particles. As the ionization increases, eventually a point is reached at which the gap-structure of the track measures the quality of the emulsion rather than the particle velocity. We have mentioned the limitation caused by the finite density of grains. If, in addition, the emulsion contains a population of totally insensitive grains, the gap density will be increased. Transparent or soluble occlusions will have the same effect.

The quantities B_{max}, g_{min}, and L_o are all operationally defined and can be used to describe emulsion quality. It should be noticed that g_{min} can be given for an emulsion of any sensitivity if, as is reasonable to assume, g varies linearly with I' in the very unsaturated region. Alternatively λ can be quoted. Both B_{max} and L_o, of course, must be measured with such an optical resolution that it does not affect the measurement.

VII. ACKNOWLEDGMENTS

We thank the Bevatron and cyclotron staffs for their assistance in the exposures. The Visual Measurements' technicians who have helped in various phases of the experiment include: Miss Ernestine Beleal, Mr. Alan Betz, Mr. Robert Blinkenberg, Mrs. Marilyn Mollin, Mr. Walter Nelson, Mr. Thomas Thompson, Mrs. Penny Vedder, Mr. Sheldon Welles, Mrs. Hester Yee, and Mr. Gordon Younce. Physicists to whom we are indebted include especially Mr. Donald Delise, who initiated some of the early phases of the experiment, and Professor John N. Dyer, who provided helpful suggestions during the data reduction and also made available his IBM 650 program for the reduction of multiple scattering measurements.

This work was performed under the auspices of the U. S. Atomic Energy Commission.

REFERENCES

1. Walter H. Barkas, "The Theory of Emulsion Track Structure," Third International Conference on Nuclear Photography, Moscow, 1960. Also, Walter H. Barkas, Lawrence Radiation Laboratory Report UCRL 9181.

2. Walter H. Barkas, Lawrence Radiation Laboratory Report UCRL 9638.

3. Norris N. Nickols, Lawrence Radiation Laboratory Report UCRL 8692 (thesis) (1959).

4. M. G. K. Menon and C. O'Ceallaigh, Proc. Roy. Soc. A221, 292 (1954).

5. G. Alexander and R. H. W. Johnston, Nuovo Cimento series 10, 5, 363 (1957).

6. C. O'Ceallaigh, CERN Report B.S. 11 (1954).

7. P. H. Fowler and D. H. Perkins, Phil. Mag. 46, 587 (1955).

8. J. P. Baillard, "Contribution a la methodoligie des traces de particles emulsions ionographiques a grain fin," (thesis) Strasbourg (1958).

9. Conrad J. Mason, Lawrence Radiation Laboratory Report UCRL 9297 (thesis) (1960).

10. John N. Dyer, Lawrence Radiation Laboratory Report UCRL 9450 (thesis) (1960).

11. Walter H. Barkas, Colloque sur la senibilite des cristaux et des emulsions photographiques, Paris (1951).

12. R. M. Sternheimer, Phys. Rev. 91, 256 (1953) and 103, 511 (1956). For our purpose it was convenient to interpolate the density effect calculated for different values of the mean excitation potential in these two articles.

FIGURE CAPTIONS

Figure 1

The observed grain density of singly and doubly-charged particles in Ilford K.5 emulsion versus velocity β_o. Measurements by different methods were made in overlapping regions of β_o. The legend is: * minimum in this K.5 emulsion, o tracks of Σ hyperons (lacunarity method), Δ tracks of K mesons, protons and π mesons (lacunarity method), $\square$ tracks of the same particles using the gap length coefficient method, ∇ tracks of K mesons, π mesons and electrons (blob count method), and $\bullet$ tracks of α particles (lacunarity method). Typical errors are indicated.

Figure 2

The quantity $-\ln\left(1 - \frac{g_p}{n}\right)$ as a function of the restricted rate of energy loss i'_2. The cutoff energy for i' was taken to be 2 Kev, hence the subscript. Typical errors are shown.

Figure 3

The ratio of primary grain density to that at the minimum plotted versus $(1 - \beta_o)^{1/2}$ in the relativistic region. The solid curve is the restricted energy-loss rate normalized to unity at the minimum. The legend is: O measured grain density ratio in K.5 emulsion, ∇ measured grain density ratio in G.5 emulsion. The errors shown are statistical standard deviations.

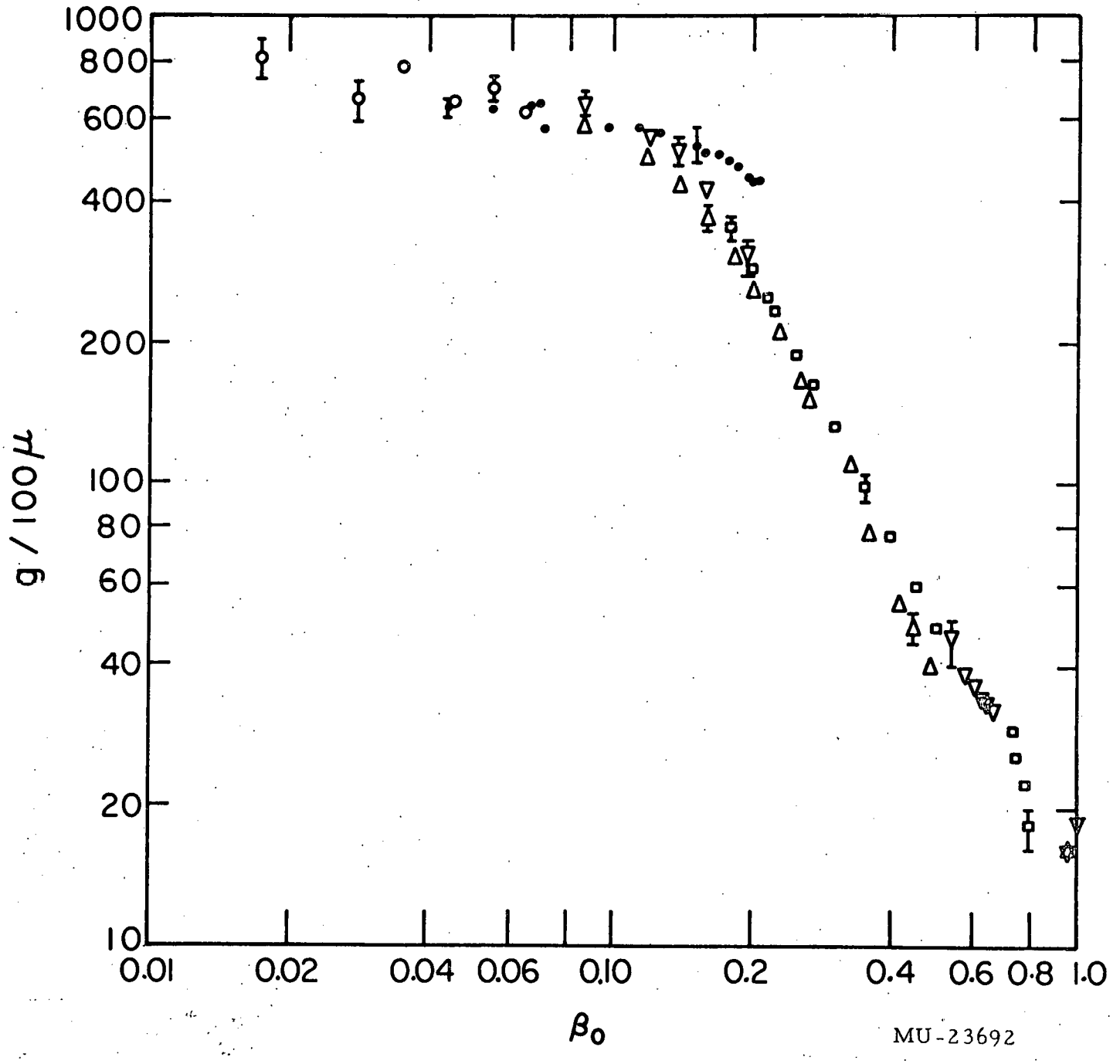
1000
800
600
400
200
100
80
60
40
20
10
g / 100 μ
0.01
0.02
0.04
0.06
0.10
0.2
0.4
0.6
0.8
1.0
β0
MU-23692

Fig. 1

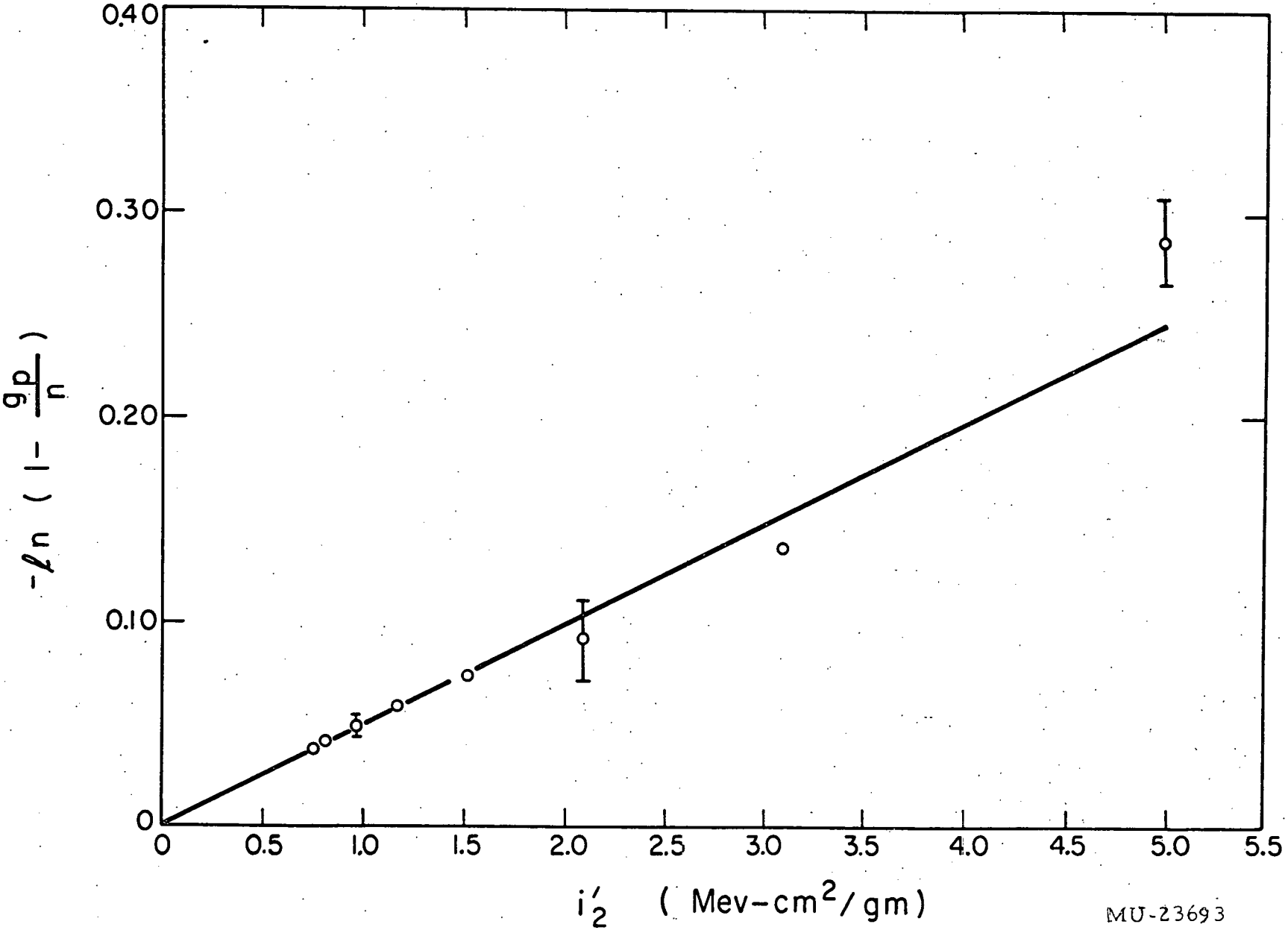
0.40
0.30
0.20
0.10
0
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
$-\ell n\left(1-\frac{g_p}{n}\right)$
i_2' (Mev-cm^2/gm)
MU-23693

Fig. 2

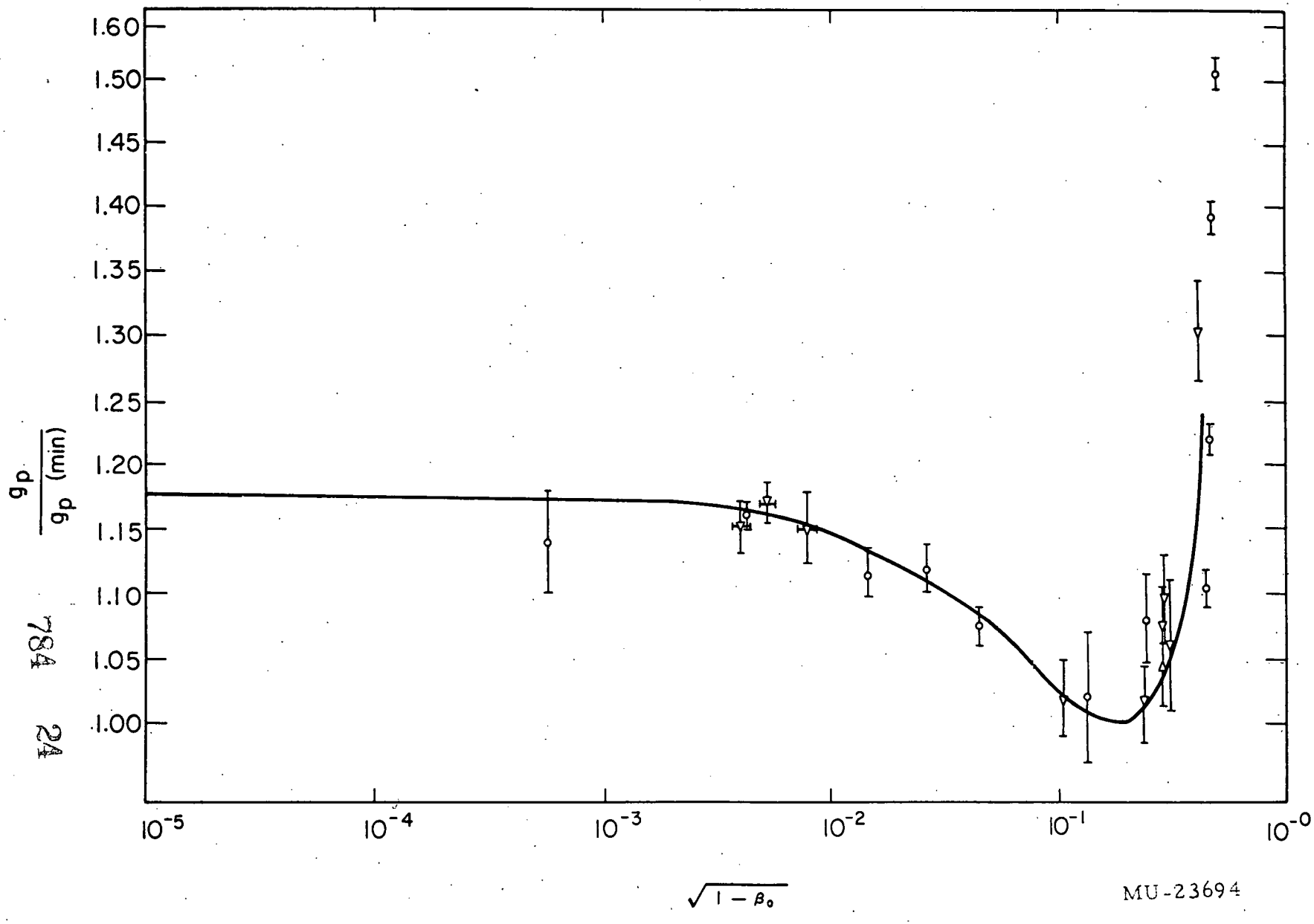
1.60
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
$\frac{g_p}{g_p\,(\min)}$
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{-0}
$\sqrt{1-\beta_0}$
MU-23694

Fig. 3