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MULTIGROUP CALCULATIONS OF EFFECTIVE NEUTRON FRACTION β_{eff}, PROMPT NEUTRON LIFETIME ℓ_p, AND RELATED KINETICS PARAMETERS FOR LARGE, FAST, PLUTONIUM-FUELED REACTORS

by

H. A. Sandmeier

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H. A. Sandmeier

Reactor Engineering Division

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ABSTRACT

Large, plutonium-fueled, fast reactors will have a different kinetic behavior than fast reactors fueled with U^{235}, due to the much smaller delayed-neutron fraction emitted in the fissioning of plutonium.

The fissioning of Pu^{240}, Pu^{241}, Pu^{242}, and especially U^{238}, which emit more delayed neutrons per fission than Pu^{239}, introduces a considerable increase in β_{eff} over that for a hypothetical pure Pu^{239} reactor.

The cases considered are 800-, 1500- and 2500-liter cores fueled with plutonium of different isotopic content, which lead to effective delayed-neutron fractions of approximately one-half the values of similar U^{235}-U^{238}-fueled fast reactors. The energy spectrum and the number of fissions weighted with their importance are recorded in order to reason physically on the obtained parameters.

INTRODUCTION

Due to the increased interest in large, plutonium-fueled, fast reactors, we obtained some important physics quantities related to the kinetics behavior and stability of such systems.

The prompt neutron lifetime ℓ_p and the effective delayed neutron fraction β_{eff} have been evaluated by using a machine program[1] which utilized as input the space and energy-dependent real and adjoint flux for multigroup solutions in spherical geometry. In order to be able to reason physically on the obtained parametric values, the total number of fissions for each Pu isotope, as well as for U^{238}, has been evaluated for all cores. Expressions proportional to the worth of prompt and delayed neutrons were obtained. All worth ratio functions are related to the worth of a prompt Pu^{239} neutron in the core. Finally, six β_{eff} values are obtained for each

delayed neutron precursor family. By using BUM,[2] the transfer function code, one gets the zero-power transfer function $ZP(j\omega)$. The cores used as representative examples were the ones reported by Yiftah and Okrent.[3] For the 1500-liter cores, the effect on the above parameters due to different Pu isotopes, i.e., Pu^{239}, Pu^{240}, Pu^{241}, and Pu^{242}, have been calculated, as well as the variations due to use of Pu metal, Pu oxide or Pu carbide fuels. The effect of core size was studied by considering 800-, 1500-, and 2500-liter, pure Pu^{239} metal, oxide, and carbide cores. The diluent in all cores, as well as the fertile material in the blanket, was U^{238}.

KINETICS PARAMETERS TO BE EVALUATED

The formula used to obtain β_{eff}, ℓ_p, and worth functions follows essentially the basic definitions given in the papers by Long et al.,[4] and by Meneghetti[5]:

$$\beta_{eff} = \frac{\sum_{\text{Fiss mat m}} \left\{ \beta^m \int \left(\sum_{i=1}^{16} (\nu\Sigma_f)_i^m \phi_i \right) \left(\sum_{j=1}^{16} \beta_{dj}^m \phi_j^* \right) dV \right\}}{\sum_{\text{Fiss mat m}} \left\{ \beta^m \int \left(\sum_{i=1}^{16} (\nu\Sigma_f)_i^m \phi_i \right) \left(\sum_{j=1}^{16} \beta_{dj}^m \phi_j^* \right) dV + (1-\beta^m) \int \left(\sum_{i=1}^{16} (\nu\Sigma_f)_i^m \phi_i \right) \left(\sum_{j=1}^{16} \chi_j^m \phi_j^* \right) dV \right\}} \qquad (1)$$

The first factor under the integral sign in the numerator of Eq. (1)

$$\sum_{i=1}^{16} (\nu\Sigma_f)_i^m \phi_i \qquad (2)$$

is proportional to the number of fission neutrons born in all groups. In order to get the number of delayed neutrons, we multiply the sum (2) by β^m, the delayed neutron fraction for material m, where

$$\beta^m = \left(\frac{n}{F}\right)^m \Big/ \bar{\nu}^m \quad . \qquad (3)$$

It is important to remember that β^m is obtained from two measured quantities, the total number of delayed neutrons per fission, $(n/F)^m$, for material m, and the total neutrons released per fission, $\bar{\nu}^m$, for material m. Both quantities are energy dependent, and it is advisable to state the reference for both values if numerical values for delayed neutron fractions are quoted.

The second factor under the integral sign in the numerator of Eq. (1),

$$\sum_{j=1}^{16} \beta_{dj}^m \phi_j^* \quad , \qquad (4)$$

where

$$\sum_{j=1}^{16} \beta_{dj}^{m} = 1 \quad ,$$

represents a feeding of the delayed neutrons with energies varying in the range from 200 to 500 kv into the proper energy group in the multigroup representation. Furthermore we have to take into account the importance ϕ_j^* of a neutron in this group.

In these calculations, both the real and adjoint flux are normalized, but since all quantities in this paper are ratios of ϕ and ϕ^*, we don't have to be concerned with this factor. The numerical value of Eq. (2) after integration over all regions and fissionable materials represents, therefore, a normalized number of fission neutrons. Finally, we integrate the numerator of Eq. (1) over all regions where fission takes place and sum over all fissionable materials.

In the denominator we sum up all the delayed and prompt neutrons born in all regions for all fissionable materials. The quantity χ_j^m separates the prompt fission neutrons born into the individual energy groups, and then we attach the proper importance to them expressed by ϕ_j^*, the adjoint flux.

For kinetics studies we must obtain the individual $\beta_{eff(n)}$ for the usual six families of precursors:

$$\beta_{eff(n)} = \frac{\sum_{Fiss.mat.m} \left\{ \beta_{(n)}^m \int \left(\sum_{i=1}^{16} (\nu\Sigma_f)_i^m \phi_i \right) \left(\sum_{j=1}^{16} \beta_{dj}^m \phi_j^* \right) dV \right\}}{\sum_{Fiss.mat.m} \left\{ \beta^m \int \left(\sum_{i=1}^{16} (\nu\Sigma_f)_i^m \phi_i \right) \left(\sum_{j=1}^{16} \beta_{dj}^m \phi_j^* \right) dV + (1-\beta^m) \int \left(\sum_{i=1}^{16} (\nu\Sigma_f)_i^m \phi_i \right) \left(\sum_{j=1}^{16} \chi_j^m \phi_j^* \right) dV \right\}} \qquad (5)$$

The only difference between Eq. (5) and Eq. (1) is that in the numerator we insert for the delayed neutron fraction $\beta_{(n)}^m$ the delayed neutron fraction for the precursor family n for each fissionable material m.

PROMPT NEUTRON LIFETIME ℓ_p

The formula for the prompt neutron lifetime in multigroup notation is

$$\ell_p = \frac{\int \left(\sum_{i=1}^{16} \frac{\phi_i \, \phi_j^*}{v_i} \right) dV}{\sum_{Fiss.Mat.m} \int \left(\sum_{i=1}^{16} (\nu\Sigma_f)_i^m \, \phi_j \right) \left(\sum_{j=1}^{16} \chi_j^m \, \phi_j^* \right) dV} \qquad (6)$$

where v_i is the average speed of a neutron in a group:

$$\left(\frac{\overline{1}}{v_i}\right) = \frac{\int_{E_L}^{E_H} \frac{1}{v(E)}\ \phi\phi^*\ dE}{\int_{E_L}^{E_H} \phi\phi^*\ dE} \quad . \tag{7}$$

The above quantity was evaluated in two ways. First, it was assumed that both real and ajoint flux are the same (one-group perturbation theory); then the spectrum was assumed to be proportional to $1/E$, as one gets in a medium with pure scattering without absorption. Secondly, a 574-energy-group solution by Hummel[6] for an 800-liter plutonium (TYPE A) oxide core was used to evaluate average neutron speeds in the 16-group set of YOM.[7] The numerical variations on the prompt neutron lifetime by both methods were found to be insignificant.

ENERGY SPECTRUM FOR LARGE FAST REACTORS

As a representative illustration we show in Fig. 1 the spectrum of a 1500-liter Pu (TYPE C) metal, oxide, and carbide-fueled fast reactor. The adjoint flux for the oxide case is also shown.

WORTH FUNCTIONS

The evaluated worth functions for delayed and prompt neutrons are

$$W_d^m = \frac{\int \left(\sum_{i=1}^{16} (\nu\Sigma_f)_i^m\ \phi_i\right)\left(\sum_{j=1}^{16} \beta_{dj}^m\ \phi_j^*\right) dV}{\int \left(\sum_{i=1}^{16} (\nu\Sigma_f)_i^m\ \phi_i\right) dV} \tag{8}$$

and

$$W_p^m = \frac{\int \left(\sum_{i=1}^{16} (\nu\Sigma_f)_i^m\ \phi_i\right)\left(\sum_{j=1}^{16} \chi_j^m\ \phi_j^*\right) dV}{\int \left(\sum_{i=1}^{16} (\nu\Sigma_f)_i^m\ \phi_i\right) dV} \quad . \tag{9}$$

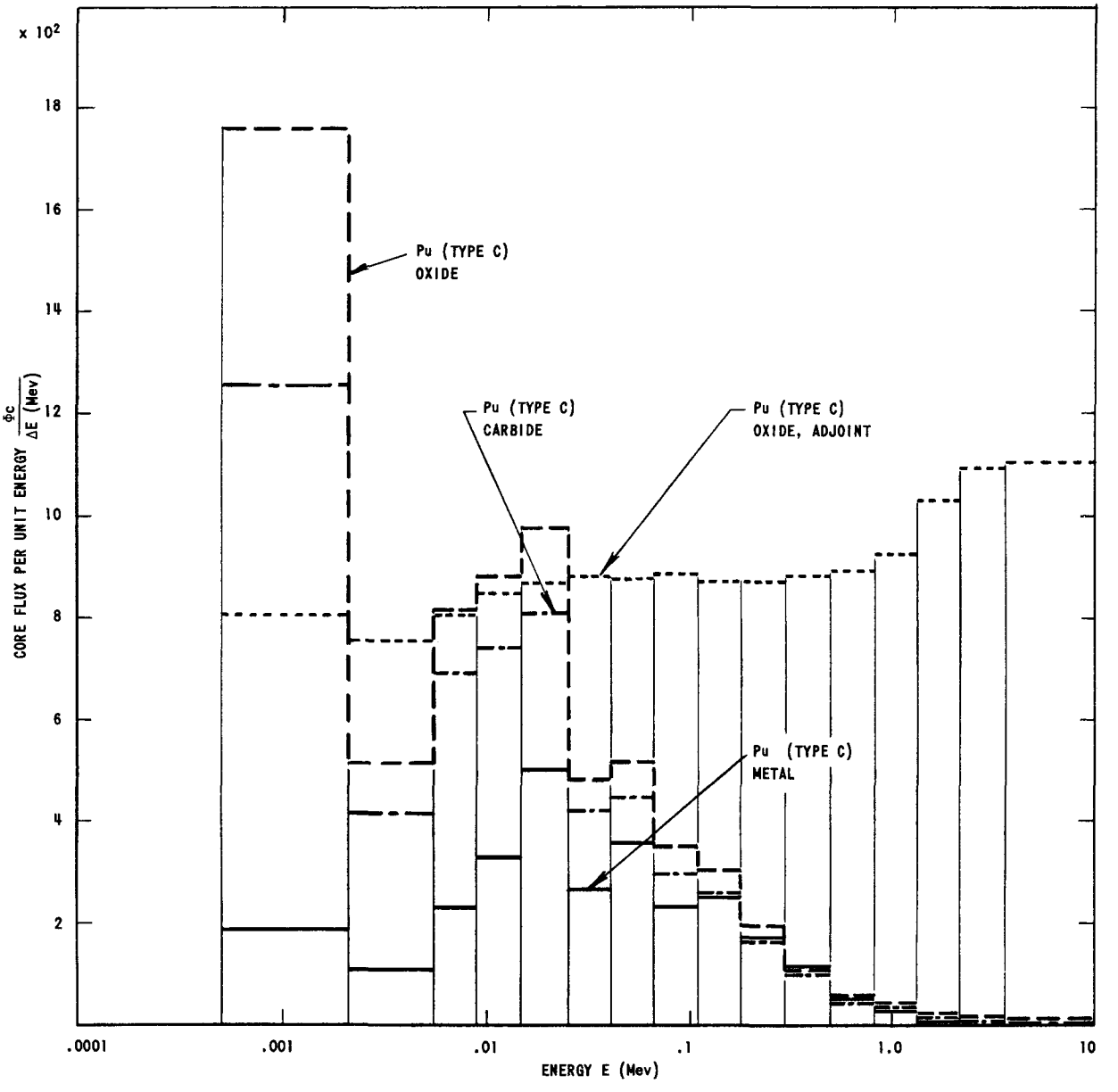

Fig. 1

COMPARISON OF SPECTRA CALCULATED FROM 1500-LITER METAL, OXIDE AND CARBIDE, PLUTONIUM (TYPE C) FUELED FAST REACTORS

FUNDAMENTAL DELAYED-NEUTRON DATA

The latest available delayed-neutron data for Pu^{239}, Pu^{240}, and Pu^{241} was taken from Cox,[8] for Pu^{242} from Smith,[9] and for U^{238} from ANL-5800.[10]

The delayed-neutron spectrum was assumed to be the same for all fissionable materials, i.e., χ_j^m is the same for all fissionable materials considered. Keepin[11] discusses the delayed-neutron energies for the 54-, 22-, 5.9-, 2.2-, 0.46-, and 0.13-sec half-life precursors of U^{235}. Stehney[12] suggests that it is reasonable to assume that the delayed-neutron spectrum for the Pu isotopes is the same as for U^{235}. This suggests that the same precursors are responsible for the delayed neutrons in either case, U^{235} or the Pu isotopes. For all Pu isotopes the delayed neutrons from the 55-sec half-life precursor have a mean energy of approximately 250 kv and are therefore fed into energy group 7 in YOM.[7] The five other groups with energies ranging from 300 to 500 kv are given off into YOM[7] energy group 6.

All numerical values used for Pu^{239}, Pu^{240}, Pu^{241}, Pu^{242}, and U^{238} are listed in Table I.

NUMERICAL EVALUATION FOR REPRESENTATIVE LARGE FAST REACTOR CORES

The cores investigated numerically are taken from Yiftah and Okrent.[3] For clarity of presentation we repeat here the pertinent data for the fuel and the plutonium compositions, as well as the geometrical dimensions of the reactors investigated.

Core Volume:

800, 1500, and 2500 liters
(Where necessary, some additional core volumes were used.)

Core Volume Fractions:

Fuel and Fertile Material	0.25
Structural Material	0.25
Coolant	0.5

Fuels:

	Density, g/cc
$Pu\text{-}U^{238}$	19.0
$PuO_2\text{-}UO_2$	8.4
PuC-UC	11.39

Plutonium Composition:

	Atom Per Cent			
TYPE	Pu^{239}	Pu^{240}	Pu^{241}	Pu^{242}
A	100.0	0	0	0
B	74.7	10.2	12.4	2.7
C	40.0	10.0	25.0	25.0

TABLE I

FUNDAMENTAL DELAYED-NEUTRON DATA

FISSIONABLE MATERIAL	$\lambda_{(\ )}$	$\beta_{(\ )}$	
Pu^{239}	(1) 0.0127690 (2) 0.0300846 (3) 0.1237760 (4) 0.3254225 (5) 1.1216010 (6) 2.6970817	(1) 0.00007241 (2) 0.00062759 (3) 0.00044483 (4) 0.00068621 (5) 0.00017931 (6) 0.00009310	β_{d6} = 0.96557700 β_{d7} = 0.03442440 β = 0.00210345 ν = 2.9 REF. (8)
Pu^{240}	(1) 0.0129415 (2) 0.0313076 (3) 0.1348540 (4) 0.3332452 (5) 1.3564570 (6) 4.0299400	(1) 0.00006666 (2) 0.00072121 (3) 0.00049090 (4) 0.00095454 (5) 0.00036061 (6) 0.00007273	β_{d6} = 0.97502430 β_{d7} = 0.02499800 β = 0.00266660 ν = 3.3 REF. (8)
Pu^{241}	(1) 0.0128000 (2) 0.0299000 (3) 0.1238000 (4) 0.3519000 (5) 1.6120000 (6) 4.6210000	(1) 0.00005168 (2) 0.00122483 (3) 0.00092282 (4) 0.00208050 (5) 0.00097315 (6) 0.00008389	β_{d6} = 0.99031600 β_{d7} = 0.00968305 β = 0.00533685 ν = 2.98 REF. (8)
Pu^{242}	(1) 0.0128360 (2) 0.0315060 (3) 0.1155250 (4) 0.3465750 (5) 1.3863000 (6) 3.4657500	(1) 0.00003030 (2) 0.00163636 (3) 0.00106060 (4) 0.00333330 (5) 0.00181818 (6) 0.00012121	β_{d6} = 0.96212100 β_{d7} = 0.03787800 β = 0.00799995 ν = 3.3 REF. (9)
U^{238}	(1) 0.0132000 (2) 0.0321000 (3) 0.1390000 (4) 0.3580000 (5) 1.4100000 (6) 4.0200000	(1) 0.00020400 (2) 0.00215100 (3) 0.00254300 (4) 0.00609200 (5) 0.00353300 (6) 0.00117800	β_{d6} = 0.98707000 β_{d7} = 0.01299000 β = 0.01570000 ν = 2.62 REF. (10)

Coolant:

Liquid Sodium 0.84 g/cc

Structural Material:	Blanket Thickness:
Stainless Steel	45 cm (uranium)

Blanket Volume Fractions:		Reflector Thickness:	
U^{238}	0.6	30 cm	
Na	0.2	Reflector Volume Fractions:	
Fe	0.2	Fe	0.6
		Na	0.4

The effect of different fuels was investigated by considering 1500-liter cores with different plutonium compositions: TYPE A, B, and C for all three fuels, i.e., Pu-U^{238}, PuO_2-UO_2 and PuC-UC in the core.

The effect of varying core volume was shown by considering 800-, 1500-, and 2500-liter cores with plutonium TYPE A; Pu-U^{238}, PuO_2-UO_2 and PuC-UC fuels.

The numerical values are shown in Tables II through VI.

ZERO-POWER TRANSFER FUNCTION ZP(jω)

For kinetics studies it is of interest to obtain the zero-power transfer function. Argonne code BUM[2] was used for this purpose. The data necessary to evaluate ZP(jω)[13] are the effective delayed neutron fractions, $\beta_{eff(1)} \cdots\cdots \beta_{eff(6)}$, and the neutron lifetime ℓ_p, listed in Tables II through VI.

The decay constants λ used for all transfer functions are the values for Pu^{239} listed in Table I. The effect of using different decay constants was investigated and found to be negligible.

The values for ZP(jω) in amplitude and phase are shown in Figs. II through IV. As a comparison we show a previously obtained zero-power transfer function ZP(jω) for EBR-I, Mark III, calculated by Okrent.[14]

TABLE II

EFFECTIVE DELAYED-NEUTRON FRACTION β_{eff} AND RELATED KINETICS DATA FOR 800-LITER, PLUTONIUM A METAL, OXIDE, AND CARBIDE FAST REACTOR CORES

$\beta_{eff}(\ )$	$W_d(\ )$ WORTH OF DELAYED NEUTRONS IN CORE (c) AND BLANKET (b) FOR FISSIONABLE MATERIALS	$W_p(\ )$ WORTH OF PROMPT NEUTRONS IN CORE (c) AND BLANKET (b) FOR FISSIONABLE MATERIALS	$\frac{FIS(\ )}{FIS(tot)}$	WORTH RATIOS

CORE: 800 LITER, PLUTONIUM A, METAL; $\ell_p = 1.66752 \times 10^{-7}$; $\beta_{eff} = 0.00390167$

(1) 0.00008187	$(c,U^{238}) = 0.962$	$(c,U^{238}) = 1.130$	$(c,tot) = 0.940$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.869$
(2) 0.00076924	$(c,Pu^{239}) = 0.939$	$(c,Pu^{239}) = 1.106$	$(b,tot) = 0.060$	
(3) 0.00070395			$(c,U^{238}) = 0.161$	$\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.849$
(4) 0.00142400			$(c,Pu^{239}) = 0.779$	
(5) 0.00067463				$\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.022$
(6) 0.00024798	$(b,U^{238}) = 0.288$	$(b,U^{238}) = 0.387$	$(b,U^{238}) = 0.060$	

CORE: 800 LITER, PLUTONIUM A, OXIDE; $\ell_p = 3.00751 \times 10^{-7}$; $\beta_{eff} = 0.00321875$

(1) 0.00007893	$(c,U^{238}) = 1.025$	$(c,U^{238}) = 1.109$	$(c,tot) = 0.916$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.945$
(2) 0.00072036	$(c,Pu^{239}) = 1.001$	$(c,Pu^{239}) = 1.085$	$(b,tot) = 0.084$	
(3) 0.00060705			$(c,U^{238}) = 0.071$	$\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.922$
(4) 0.00114192			$(c,Pu^{239}) = 0.844$	
(5) 0.00048226				$\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.022$
(6) 0.00018822	$(b,U^{238}) = 0.341$	$(b,U^{238}) = 0.434$	$(b,U^{238}) = 0.084$	

CORE: 800 LITER, PLUTONIUM A, CARBIDE; $\ell_p = 2.50021 \times 10^{-7}$; $\beta_{eff} = 0.00348409$

(1) 0.00008021	$(c,U^{238}) = 1.001$	$(c,U^{238}) = 1.114$	$(c,tot) = 0.924$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.918$
(2) 0.00074041	$(c,Pu^{239}) = 0.978$	$(c,Pu^{239}) = 1.091$	$(b,tot) = 0.076$	
(3) 0.00064502			$(c,U^{238}) = 0.103$	$\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.896$
(4) 0.00125111			$(c,Pu^{239}) = 0.821$	
(5) 0.00055609				$\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.021$
(6) 0.00021124	$(b,U^{238}) = 0.322$	$(b,U^{238}) = 0.420$	$(b,U^{238}) = 0.076$	

TABLE III

EFFECTIVE DELAYED-NEUTRON FRACTION β_{eff} AND RELATED KINETICS DATA FOR 1500-LITER, PLUTONIUM A METAL, OXIDE, AND CARBIDE FAST REACTOR CORES

$\beta_{eff}(\ )$	$W_d(\ )$ WORTH OF DELAYED NEUTRONS IN CORE (c) AND BLANKET (b) FOR FISSIONABLE MATERIALS	$W_p(\ )$ WORTH OF PROMPT NEUTRONS IN CORE (c) AND BLANKET (b) FOR FISSIONABLE MATERIALS	$\frac{FIS(\ )}{FIS(tot)}$	WORTH RATIOS
CORE: 1500 LITER, PLUTONIUM A, METAL; $\ell_p = 1.89851 \times 10^{-7}$; $\beta_{eff} = 0.00390760$				
(1) 0.00008131 (2) 0.00076528 (3) 0.00070345 (4) 0.00142813 (5) 0.00068010 (6) 0.00024934	$(c,U^{238}) = 0.973$ $(c,Pu^{239}) = 0.952$ $(b,U^{238}) = 0.258$	$(c,U^{238}) = 1.161$ $(c,Pu^{239}) = 1.139$ $(b,U^{238}) = 0.352$	$(c,tot) = 0.955$ $(b,tot) = 0.045$ $(c,U^{238}) = 0.174$ $(c,Pu^{239}) = 0.781$ $(b,U^{238}) = 0.045$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.854$ $\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.836$ $\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.020$
CORE: 1500 LITER, PLUTONIUM A, OXIDE; $\ell_p = 3.78915 \times 10^{-7}$; $\beta_{eff} = 0.00320268$				
(1) 0.00007830 (2) 0.00071507 (3) 0.00060362 (4) 0.00113731 (5) 0.00048167 (6) 0.00018771	$(c,U^{238}) = 1.040$ $(c,Pu^{239}) = 1.017$ $(b,U^{238}) = 0.309$	$(c,U^{238}) = 1.138$ $(c,Pu^{239}) = 1.116$ $(b,U^{238}) = 0.399$	$(c,tot) = 0.935$ $(b,tot) = 0.065$ $(c,U^{238}) = 0.082$ $(c,Pu^{239}) = 0.853$ $(c,U^{238}) = 0.065$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.932$ $\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.911$ $\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.020$
CORE: 1500 LITER, PLUTONIUM A, CARBIDE; $\ell_p = 2.99667 \times 10^{-7}$; $\beta_{eff} = 0.00347546$				
(1) 0.00007951 (2) 0.00073479 (3) 0.00064226 (4) 0.00124947 (5) 0.00055799 (6) 0.00021144	$(c,U^{238}) = 1.013$ $(c,Pu^{239}) = 0.991$ $(c,U^{238}) = 0.289$	$(c,U^{238}) = 1.144$ $(c,Pu^{239}) = 1.122$ $(b,U^{238}) = 0.385$	$(c,tot) = 0.942$ $(b,tot) = 0.058$ $(c,U^{238}) = 0.116$ $(c,Pu^{239}) = 0.826$ $(b,U^{238}) = 0.058$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.903$ $\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.883$ $\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.020$

TABLE IV

EFFECTIVE DELAYED-NEUTRON FRACTION β_{eff} AND RELATED KINETICS DATA FOR 1500-LITER, PLUTONIUM B METAL, OXIDE, AND CARBIDE FAST REACTOR CORES

$\beta_{eff}(\ )$	$W_d(\ )$ WORTH OF DELAYED NEUTRONS IN CORE (c) AND BLANKET (b) FOR FISSIONABLE MATERIALS	$W_p(\ )$ WORTH OF PROMPT NEUTRONS IN CORE (c) AND BLANKET (b) FOR FISSIONABLE MATERIALS	$\frac{FIS(\ )}{FIS(tot)}$	WORTH RATIOS

CORE: 1500 LITER, PLUTONIUM B, METAL; $\ell_p = 1.94110 \times 10^{-7}$; $\beta_{eff} = 0.00426902$

(1) 0.00007773	$(c,U^{238}) = 0.963$	$(c,U^{238}) = 1.161$	$(c,tot) = 0.955$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.846$
(2) 0.00083204	$(c,Pu^{239}) = 0.943$	$(c,Pu^{239}) = 1.139$	$(b,tot) = 0.045$	
(3) 0.00075384	$(c,Pu^{240}) = 0.957$	$(c,Pu^{240}) = 1.155$	$(c,U^{238}) = 0.174$	$\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.828$
(4) 0.00158773	$(c,Pu^{241}) = 0.941$	$(c,Pu^{241}) = 1.136$	$(c,Pu^{239}) = 0.625$	
(5) 0.00077303	$(c,Pu^{242}) = 0.956$	$(c,Pu^{242}) = 1.154$	$(c,Pu^{240}) = 0.018$	
(6) 0.00024465	$(b,U^{238}) = 0.257$	$(b,U^{238}) = 0.352$	$(c,Pu^{241}) = 0.133$	$\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.020$
			$(c,Pu^{242}) = 0.005$	
			$(b,U^{238}) = 0.045$	

CORE: 1500 LITER, PLUTONIUM B, OXIDE; $\ell_p = 3.88897 \times 10^{-7}$; $\beta_{eff} = 0.00369389$

(1) 0.00007436	$(c,U^{238}) = 1.033$	$(c,U^{238}) = 1.138$	$(c,tot) = 0.934$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.926$
(2) 0.00080521	$(c,Pu^{239}) = 1.011$	$(c,Pu^{239}) = 1.116$	$(b,tot) = 0.066$	
(3) 0.00067342	$(c,Pu^{240}) = 1.026$	$(c,Pu^{240}) = 1.131$	$(c,U^{238}) = 0.082$	$\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.906$
(4) 0.00135141	$(c,Pu^{241}) = 1.010$	$(c,Pu^{241}) = 1.114$	$(c,Pu^{239}) = 0.676$	
(5) 0.00060539	$(c,Pu^{242}) = 1.026$	$(c,Pu^{242}) = 1.131$	$(c,Pu^{240}) = 0.017$	
(6) 0.00018410	$(b,U^{238}) = 0.308$	$(b,U^{238}) = 0.400$	$(c,Pu^{241}) = 0.155$	$\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.020$
			$(c,Pu^{242}) = 0.004$	
			$(b,U^{238}) = 0.066$	

CORE: 1500 LITER, PLUTONIUM B, CARBIDE; $\ell_p = 3.07530 \times 10^{-7}$; $\beta_{eff} = 0.00382487$

(1) 0.00007574	$(c,U^{238}) = 1.006$	$(c,U^{238}) = 1.144$	$(c,tot) = 0.942$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.896$
(2) 0.00081750	$(c,Pu^{239}) = 0.985$	$(c,Pu^{239}) = 1.122$	$(b,tot) = 0.058$	
(3) 0.00070596	$(c,Pu^{240}) = 0.999$	$(c,Pu^{240}) = 1.137$	$(c,U^{238}) = 0.115$	$\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.877$
(4) 0.00144617	$(c,Pu^{241}) = 0.984$	$(c,Pu^{241}) = 1.121$	$(c,Pu^{239}) = 0.656$	
(5) 0.00067184	$(c,Pu^{242}) = 0.998$	$(c,Pu^{242}) = 1.137$	$(c,Pu^{240}) = 0.017$	
(6) 0.00020766	$(b,U^{238}) = 0.289$	$(b,U^{238}) = 0.385$	$(c,U^{238}) = 0.148$	$\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.020$
			$(c,Pu^{242}) = 0.005$	
			$(b,U^{238}) = 0.058$	

TABLE V

EFFECTIVE DELAYED-NEUTRON FRACTION β_{eff} AND RELATED KINETICS DATA FOR 1500-LITER, PLUTONIUM C METAL, OXIDE, AND CARBIDE FAST REACTOR CORES

$\beta_{eff}(\ )$	$W_d(\ )$ WORTH OF DELAYED NEUTRONS IN CORE (c) AND BLANKET (b) FOR FISSIONABLE MATERIALS	$W_p(\ )$ WORTH OF PROMPT NEUTRONS IN CORE (c) AND BLANKET (b) FOR FISSIONABLE MATERIALS	$\frac{FIS(\ )}{FIS(tot)}$	WORTH RATIOS

CORE: 1500 LITER, PLUTONIUM C, METAL; $\ell_p = 1.96703 \times 10^{-7}$; $\beta_{eff} = 0.00486485$

$\beta_{eff}(\ )$	$W_d(\ )$	$W_p(\ )$	$\frac{FIS(\ )}{FIS(tot)}$	WORTH RATIOS
(1) 0.00007039	$(c,U^{238}) = 0.937$	$(c,U^{238}) = 1.162$	$(c,tot) = 0.954$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.823$
(2) 0.00094009	$(c,Pu^{239}) = 0.918$	$(c,Pu^{239}) = 1.139$	$(b,tot) = 0.046$	
(3) 0.00082769	$(c,Pu^{240}) = 0.931$	$(c\ Pu^{240}) = 1.155$	$(c,U^{238}) = 0.173$	$\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.806$
(4) 0.00185693	$(c,Pu^{241}) = 0.916$	$(c,Pu^{241}) = 1.136$	$(c,Pu^{239}) = 0.395$	
(5) 0.00093462	$(c,Pu^{242}) = 0.931$	$(c,Pu^{242}) = 1.155$	$(c,Pu^{240}) = 0.020$	$\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.020$
(6) 0.00023514	$(b,U^{238}) = 0.252$	$(b,U^{238}) = 0.351$	$(c,Pu^{241}) = 0.316$	
			$(c,Pu^{242}) = 0.050$	
			$(b,U^{238}) = 0.046$	

CORE: 1500 LITER, PLUTONIUM C, OXIDE; $\ell_p = 3.95275 \times 10^{-7}$; $\beta_{eff} = 0.00447424$

$\beta_{eff}(\ )$	$W_d(\ )$	$W_p(\ )$	$\frac{FIS(\ )}{FIS(tot)}$	WORTH RATIOS
(1) 0.00006663	$(c,U^{238}) = 1.013$	$(c,U^{238}) = 1.139$	$(c,tot) = 0.934$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.908$
(2) 0.00094628	$(c,Pu^{239}) = 0.991$	$(c,Pu^{239}) = 1.116$	$(b,tot) = 0.066$	
(3) 0.00077465	$(c,Pu^{240}) = 1.006$	$(c,Pu^{240}) = 1.132$	$(c,U^{238}) = 0.079$	$\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.888$
(4) 0.00169858	$(c,Pu^{241}) = 0.990$	$(c,Pu^{241}) = 1.114$	$(c,Pu^{239}) = 0.423$	
(5) 0.00081130	$(c,Pu^{242}) = 1.006$	$(c,Pu^{242}) = 1.131$	$(c,Pu^{240}) = 0.019$	$\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.021$
(6) 0.00017678	$(b,U^{238}) = 0.304$	$(b,U^{238}) = 0.398$	$(c,Pu^{241}) = 0.364$	
			$(c,Pu^{242}) = 0.047$	
			$(b,U^{238}) = 0.066$	

CORE: 1500 LITER, PLUTONIUM C, CARBIDE; $\ell_p = 3.12706 \times 10^{-7}$; $\beta_{eff} = 0.00464446$

$\beta_{eff}(\ )$	$W_d(\ )$	$W_p(\ )$	$\frac{FIS(\ )}{FIS(tot)}$	WORTH RATIOS
(1) 0.00006822	$(c,U^{238}) = 0.985$	$(c,U^{238}) = 1.145$	$(c,tot) = 0.941$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.877$
(2) 0.00094771	$(c,Pu^{239}) = 0.964$	$(c,Pu^{239}) = 1.122$	$(b,tot) = 0.059$	
(3) 0.00079831	$(c,Pu^{240}) = 0.978$	$(c,Pu^{240}) = 1.137$	$(c,U^{238}) = 0.113$	$\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.859$
(4) 0.00176745	$(c,Pu^{241}) = 0.963$	$(c,Pu^{241}) = 1.121$	$(c,Pu^{239}) = 0.411$	
(5) 0.00086293	$(c,Pu^{242}) = 0.977$	$(c,Pu^{242}) = 1.137$	$(c,Pu^{240}) = 0.020$	$\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.020$
(6) 0.00019983	$(b,U^{238}) = 0.285$	$(b,U^{238}) = 0.383$	$(c,Pu^{241}) = 0.349$	
			$(c,Pu^{242}) = 0.048$	
			$(b,U^{238}) = 0.059$	

TABLE VI

EFFECTIVE DELAYED-NEUTRON FRACTION β_{eff} AND RELATED KINETICS DATA FOR 2500-LITER, PLUTONIUM A METAL, OXIDE, AND CARBIDE FAST REACTOR CORES

β_{eff}( )	W_d( ) WORTH OF DELAYED NEUTRONS IN CORE (c) AND BLANKET (b) FOR FISSIONABLE MATERIALS	W_p( ) WORTH OF PROMPT NEUTRONS IN CORE (c) AND BLANKET (b) FOR FISSIONABLE MATERIALS	$\frac{FIS(\ )}{FIS(tot)}$	WORTH RATIOS
CORE: 2500 LITER, PLUTONIUM A, METAL; $\ell_p = 2.06918 \times 10^{-7}$; $\beta_{eff} = 0.00390745$				
(1) 0.00008086 (2) 0.00076188 (3) 0.00070239 (4) 0.00142937 (5) 0.00068300 (6) 0.00024997	$(c,U^{238}) = 0.986$ $(c,Pu^{239}) = 0.967$ $(b,U^{238}) = 0.233$	$(c,U^{238}) = 1.190$ $(c,Pu^{239}) = 1.169$ $(b,U^{238}) = 0.323$	$(c,tot) = 0.965$ $(b,tot) = 0.035$ $(c,U^{238}) = 0.183$ $(c,Pu^{239}) = 0.782$ $(b,U^{238}) = 0.035$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.843$ $\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.827$ $\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.018$
CORE: 2500 LITER, PLUTONIUM A, OXIDE, $\ell_p = 4.43106 \times 10^{-7}$; $\beta_{eff} = 0.00319037$				
(1) 0.00007777 (2) 0.00071051 (3) 0.00060062 (4) 0.00113320 (5) 0.00048105 (6) 0.00018723	$(c,U^{238}) = 1.054$ $(c,Pu^{239}) = 1.033$ $(b,U^{238}) = 0.281$	$(c,U^{238}) = 1.166$ $(c,Pu^{239}) = 1.145$ $(b,U^{238}) = 0.368$	$(c,tot) = 0.948$ $(b,tot) = 0.052$ $(c,U^{238}) = 0.090$ $(c,Pu^{239}) = 0.858$ $(b,U^{238}) = 0.052$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.921$ $\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.903$ $\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.019$
CORE: 2500 LITER, PLUTONIUM A, CARBIDE; $\ell_p = 3.37748 \times 10^{-7}$; $\beta_{eff} = 0.00346535$				
(1) 0.00007892 (2) 0.00072995 (3) 0.0063957 (4) 0.00124686 (5) 0.00055871 (6) 0.00021134	$(c,U^{238}) = 1.025$ $(c,Pu^{239}) = 1.006$ $(b,U^{238}) = 0.263$	$(c,U^{238}) = 1.172$ $(c,Pu^{239}) = 1.152$ $(b,U^{238}) = 0.353$	$(c,tot) = 0.954$ $(b,tot) = 0.046$ $(c,U^{238}) = 0.124$ $(c,Pu^{239}) = 0.830$ $(b,U^{238}) = 0.046$	$\left[\frac{W_d(c,U^{238})}{W_p(c,Pu^{239})}\right] = 0.890$ $\left[\frac{W_d(c,Pu^{239})}{W_p(c,Pu^{239})}\right] = 0.873$ $\left[\frac{W_p(c,U^{238})}{W_p(c,Pu^{239})}\right] = 1.018$

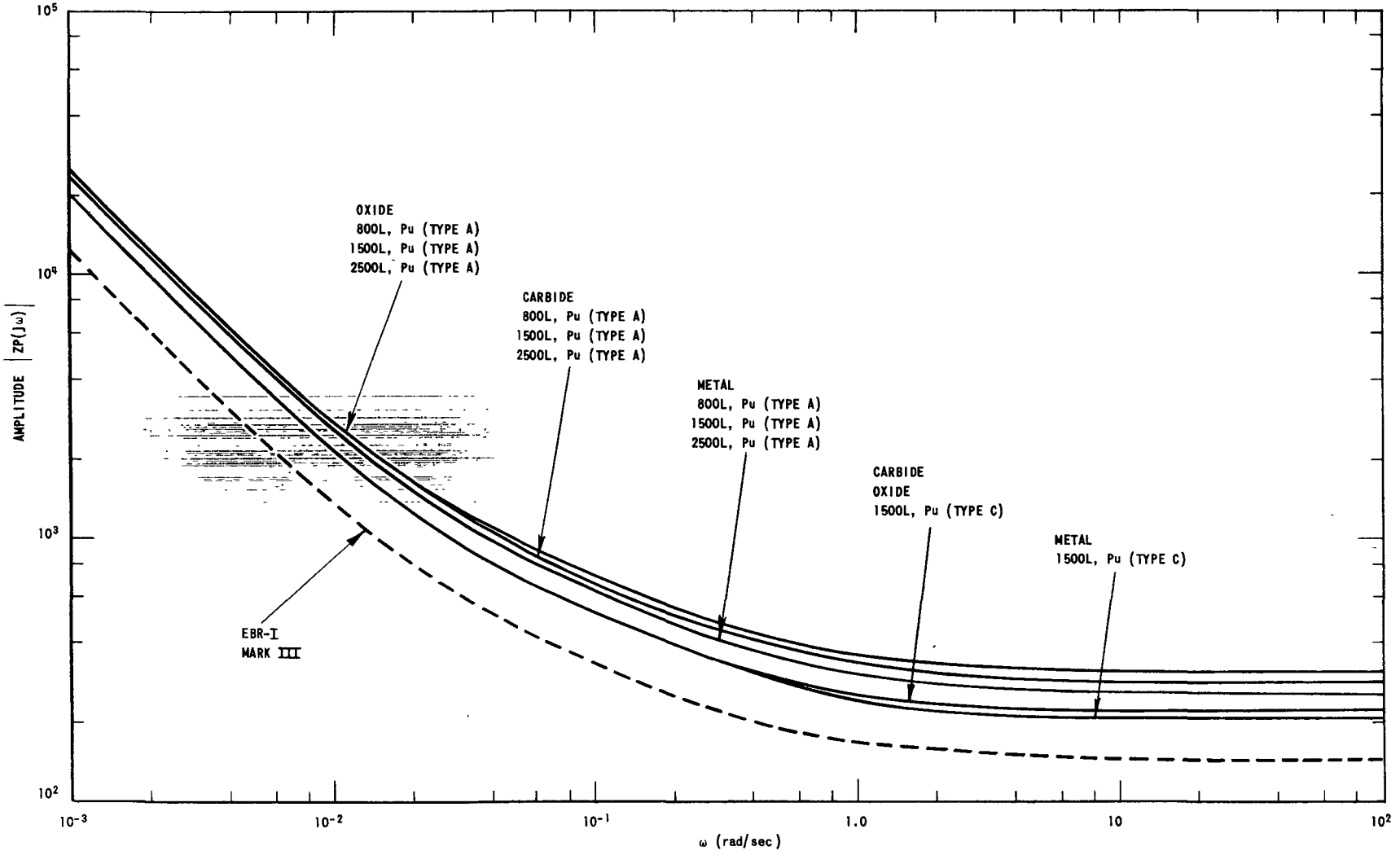

FIG. II

ZERO POWER TRANSFER FUNCTION AMPLITUDE $|ZP(j\omega)|$ FOR LARGE PLUTONIUM (TYPE A AND C) OXIDE, CARBIDE AND METAL FUELED FAST REACTORS

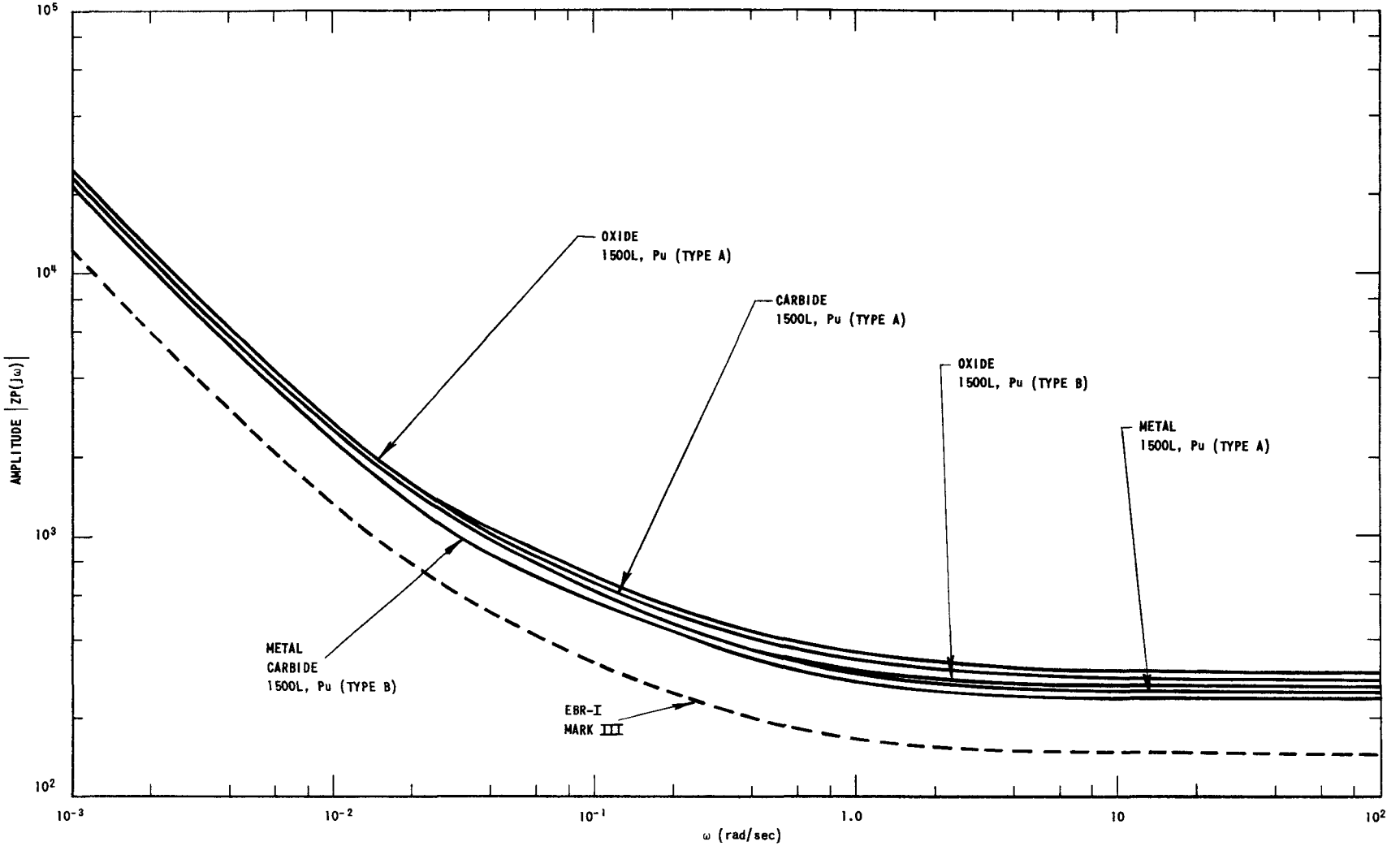

FIG. III

ZERO POWER TRANSFER FUNCTION AMPLITUDE $|ZP(j\omega)|$ FOR LARGE PLUTONIUM (TYPE A AND B) OXIDE, CARBIDE AND METAL FUELED FAST REACTORS

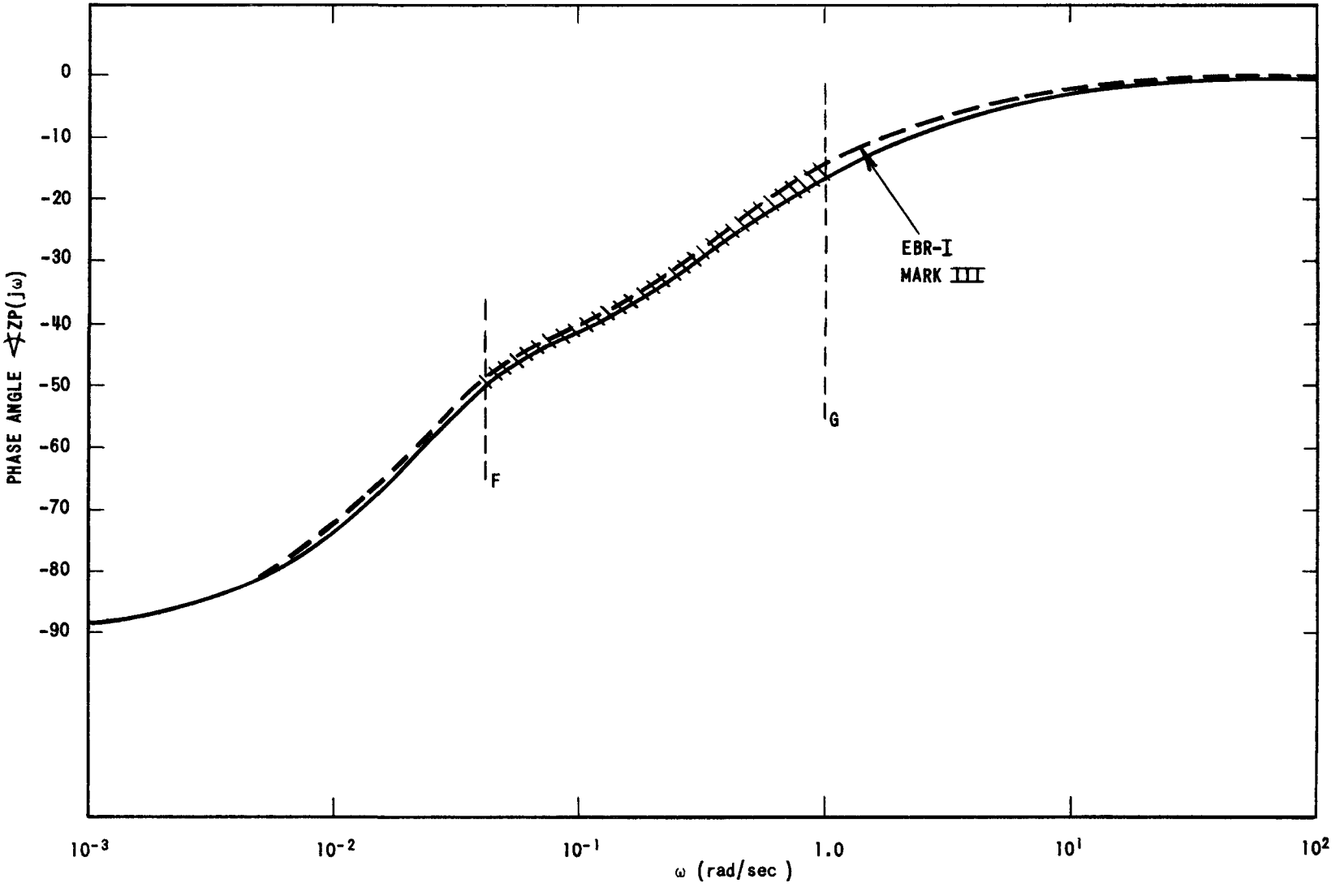

FIG. IV

ZERO POWER TRANSFER FUNCTION PHASE ANGLE ∢ZP(jω) FOR LARGE PLUTONIUM (TYPE A,B AND C) OXIDE, CARBIDE AND METAL FUELED FAST REACTORS. (800L, 1500L AND 2500L CORES)

CONCLUSIONS

Tables I through VI are arranged in such a way as to be largely selfexplanatory. One important quantity for stability studies is the value $1/\beta_{eff}$, as it determines the amplification in the plateau region of the zero-power transfer function. Some numbers for representative systems are

	β_{eff}	$1/\beta_{eff}$
EBR-I, Mark III (U^{235}-U^{238}) System	0.00683	146
Typical Large Fast (Pu-U^{238}) System	0.0035	286

We note that the value $1/\beta_{eff}$ is doubled up by going from the EBR-I, Mark III (U^{235}-U^{238}) system to a typical large fast (Pu-U^{238}) system.

Let us assume two reactors with equal heat transfer characteristics, sizes, and other pertinent data for the feedback function, i.e., $PK(j\omega)$[13] (power coefficient). Furthermore, we assume that the U^{235}-U^{238} system has a small bump in the load power transfer function $LP(j\omega)$, as shown in Fig. V. By replacing the U^{235}-U^{238} neutronics system with a Pu-U^{238} system, we would get a significant increase in the tendency towards resonance instability due to the doubling up of the amplification. This is, in fact, equivalent to a doubling up of the power level in the U^{235}-U^{238} system.

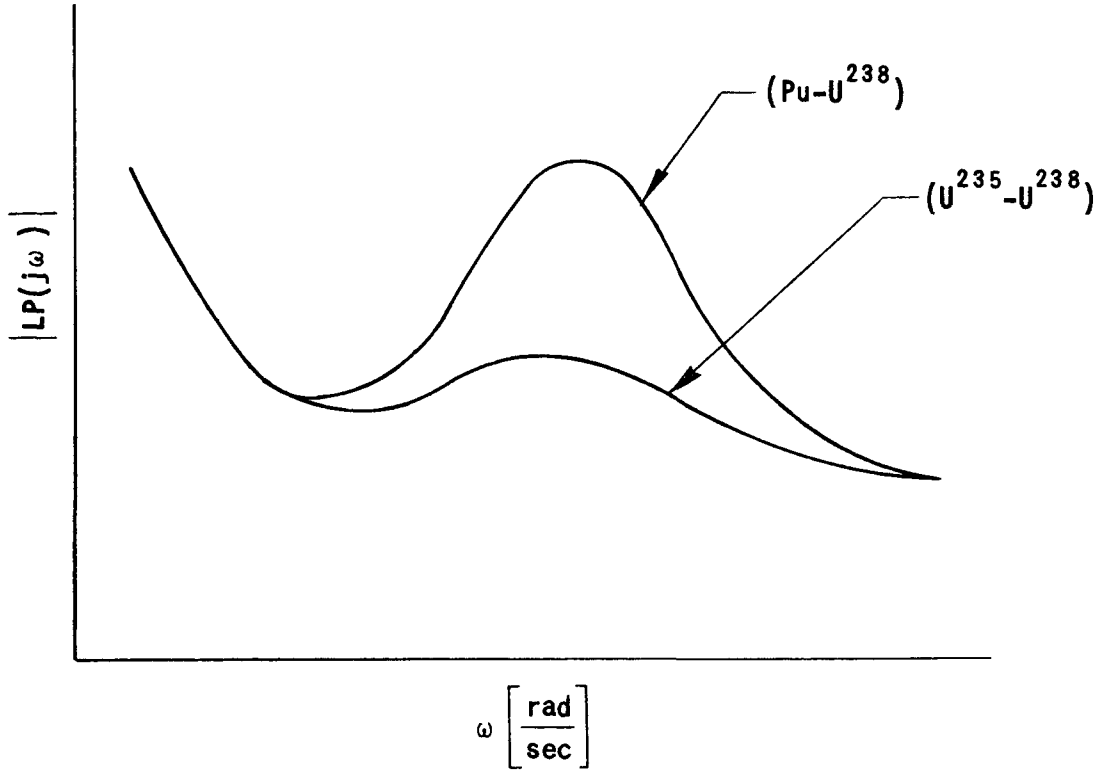

FIG. V

LOAD POWER TRANSFER FUNCTION FOR FAST PLUTONIUM AND U^{235} REACTOR

The effect of raising power level in a linear system is studied in detail in Ref. 13. Bethe[15] pointed out that it is relatively easy to correct such resonances if their mechanism is visualized during the design stages of the reactor. The core considered here is, of course, quite hypothetical, but it is well to remember that the neutronics characteristics of a plutonium system will introduce additional amplification because the delayed-neutron fractions of Pu^{239} are three times smaller than the delayed-neutron fractions of U^{235}.[13] A considerable increase of β_{eff} is introduced in the Pu cases due to Pu^{240}, Pu^{241}, Pu^{242}, and U^{238}. Especially U^{238} contributes considerably, since the U^{238} fissions in the 1500-liter Pu (TYPE C) oxide core shown in Table V, for example, are close to 20% of the fissions of Pu^{239} in the core.

In the last column of Table V one notes for the above case that the worth of a delayed U^{238} neutron in the core is slightly higher than the worth of a delayed Pu^{239} neutron in the core.

From Figs. II and III one could single out fuel compositions and core sizes which would be worse than others with respect to introducing amplification, but it is felt that generalized conclusions should not be drawn at this point. Figure IV shows that the phase for these cores considered is not effected very much, as indicated by the variation between F and G.

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