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MAGNETIC FIELD MEASUREMENTS IN A CYLINDRICAL PINCH TUBE

by

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ABSTRACT

Magnetic field measurements in a cylindrical linear pinch tube with magnetic probes and a device that we named the Ḃ-loop are described. Tests were performed with a discharge current frequency of 20 kilocycles in nitrogen at an initial pressure of 0.1 mm Hg. Initial capacitor voltage was varied from 1000 to 5000 volts. Plots of the magnetic field distribution, the radial current distribution and the current density in the pinch tube are presented.

The experimental results indicate that the discharges are symmetric above 2000 volts. The symmetric discharges have the following characteristics. At the start of the discharge, current flows near the outer periphery of the pinch tube. A portion of the current concentrates in a thin shell which moves towards the axis at a high velocity. The remainder of the current flows between the rapidly advancing shell and the outer radius of the pinch tube. There is some evidence that after the formation of the first shell a second shell is formed and moves toward the axis.

It appears from our experiments that magnetic probes sometimes cause perturbations in the discharge which influence their output.

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SECTION I - INTRODUCTION

Magnetic field measurements have been made in a cylindrical linear pinch tube in order to obtain an understanding of the nature of discharges obtainable under certain parametric conditions of interest for our pinch engine development. Local azimuthal magnetic fields were measured at several radial positions with magnetic probes. The total azimuthal flux was measured with a device which we named the D-Loop. This device and some of its applications are described in Appendix A.

Our interpretations of magnetic probe signals are based on the assumption that the discharges are symmetric, that there is no axial variation in the magnetic field, that the magnetic field crossing the probe area is uniform over the area of the probe, and that the probes do not disturb the discharge. Interpretations of magnetic probe signals on this basis are discussed in References 1-3. Several arrangements of D-Loops and magnetic probes have been used to check the validity of these assumptions. The evidence which we have gathered indicates that the assumptions were applicable in most of the tests.

It was found that under the conditions of the experiment only a portion, about 50%, of the total current flows in a thin moving shell. The remainder flows between the advancing shell and the periphery of the pinch tube. There is some evidence that after a while a second shell forms and moves toward the axis.

Experiments were run at a number of different initial capacitor voltages. The results indicate that the symmetry of the discharges improves, and the velocity at which the current shell moves towards the axis increases with an increase in voltage. There are indications that the magnetic probes can cause discharge perturbations in their vicinity which influence their output. This effect caused some difficulty in the interpretation of data.

SECTION II - APPARATUS

The pinch tube and circuitry in which the magnetic field measurements were performed are described, in detail, in Reference 4. Here only basic features of the apparatus are discussed. The schematic diagram shown in Figure 1 represents the pinch tube circuit. Capacitor bank C, with a total capacitance of 515 μf, consists of 20 capacitors connected in parallel. The capacitor bank is charged from a high voltage power supply (not shown in sketch) to the desired voltage, V_o. The discharge is started by closing switch S (Jenings RH-4M-3121-1 Vacuum Switch). Current flows from the capacitor bank through the external circuit resistance R, the external inductance L, and the pinch tube with its resistance R_2 and inductance L_2. The pinch tube is cylindrical and has flat parallel aluminum electrodes 8 inches in diameter with a spacing of 2 inches between them. A cross-sectional view of the pinch tube with mounted D-Loop and magnetic probe is shown in Figure 2.

The D-Loop consists of a rectangular loop of insulated wire. The two ends of the loop are labeled F and F'. (Properties of the D-Loop are discussed in Appendix A.) The magnetic probe coils were wound on Teflon rods which were inserted into glass tubes 10 cm long and 0.4 cm O.D. as shown in Figure 2. Five turns of fine insulated wire were mounted on the rods with the coil areas in planes either parallel or perpendicular to the axis of the rod. The loops in planes perpendicular to the axis of the rod were circular and had a 0.178 cm diameter. The loops in planes parallel to the rod were rectangular (0.178 cm by 1 cm). The accuracy with which the probe coil areas are known determines the accuracy with which the induced magnetic field can be calculated. Our area dimensions are within 5% accurate.

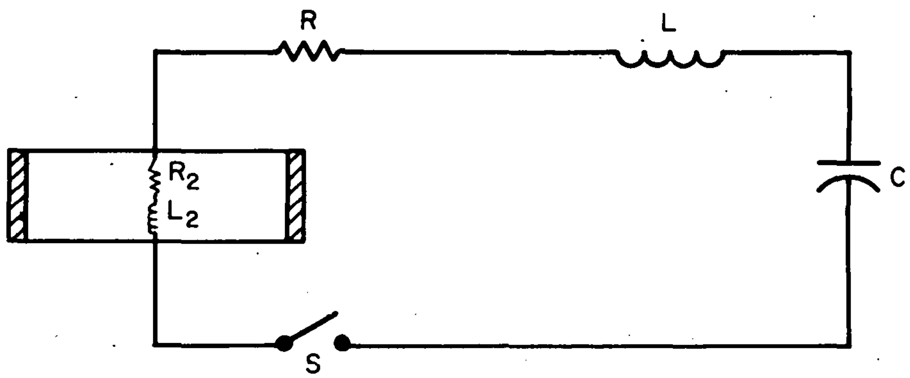


Figure 1

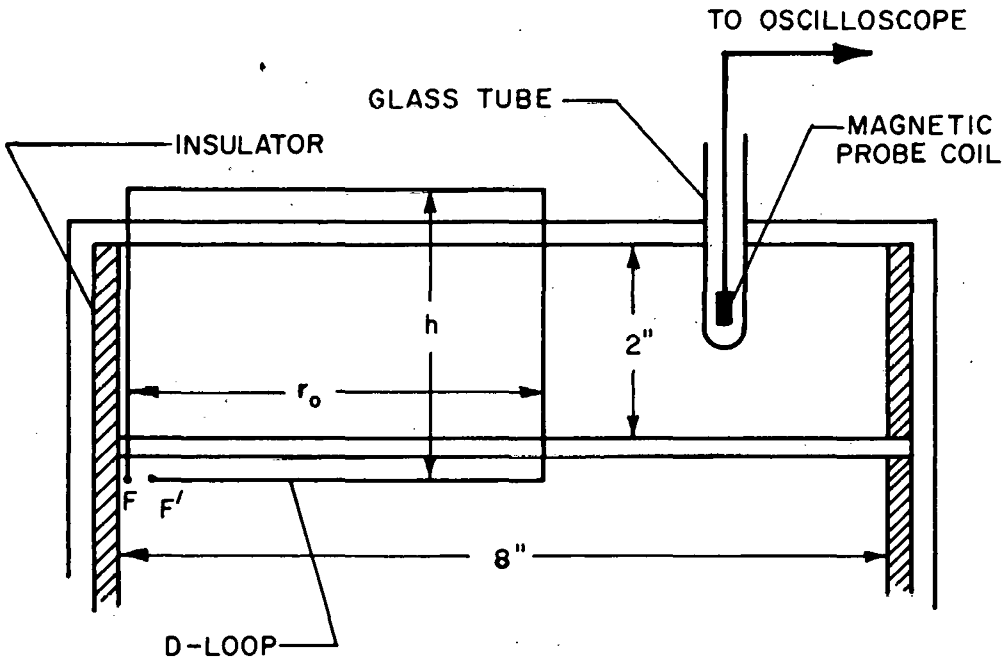


Figure 2

SECTION III - EXPERIMENTAL PROCEDURE

Experiments were conducted with several combinations of probes and D-Loops mounted in the pinch chamber. The experiments may be classified into two groups according to the information being sought:

(1) Magnetic Field Distribution Tests

(2) Symmetry Tests

Except for the distribution of probes and D-Loops the test procedure did not vary from experiment to experiment. In each case several magnetic probes or D-Loops would be inserted into the chamber. Their output signals were fed to oscilloscopes either directly or through RC integrating circuits. Integrating circuits with $R = 32,000$ ohms and $C = 10^{-8}$ farads were used. The total discharge current was measured with a specially calibrated Rogowski coil and displayed on an additional oscilloscope channel. Provisions were made to adjust the chamber pressure to the desired pre-discharge level, and to charge the capacitor bank to the desired initial voltage, V_o.

To record the transient data, up to three Tektronix No. 551 double-beam oscilloscopes were used simultaneously. It was necessary to ensure that each oscilloscope would start recording data at the same time. To accomplish this the Rogowski coil output was used as the trigger signal for each of the oscilloscopes which had been preset to trigger at the same signal level. To check whether the triggering was simultaneous, one common signal was recorded on all oscilloscopes. Small random delays could not be eliminated. These delays were probably caused by drifting in the trigger level and were of the order of 0.2 μsec.

During each set of experiments the initial capacitor voltage was the only variable circuit parameter. The gas used in all experiments was nitrogen at an initial pressure of 0.1 mm Hg. The discharge frequency was 20 kilocycles.

SECTION IV - MAGNETIC FIELD DISTRIBUTION TESTS

Figure 3 shows an electrode on which several glass tubes have been attached. (The tube near the center of the electrode was broken during a discharge at the beginning of the experiments.) A magnetic probe was inserted into each tube. In most tests the pickup coil of each probe was located halfway between the electrodes and was oriented to measure the azimuthal field. A few exploratory tests were made with probes oriented to measure radial and axial fields. The outermost probe was 0.5 inch from the edge of the electrode. Spacing between probes was 1 inch. We refer to the outermost probe as Probe 1. The other probes are numbered in sequence.

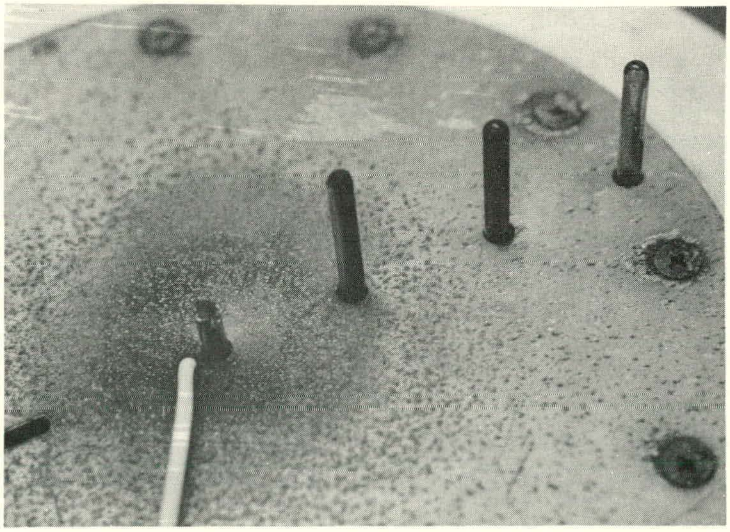


Figure 3

The wire visible in Figure 3 is part of the D-Loop which was mounted as indicated in Figure 2. The plane containing the D-Loop was set at an angle of 90° with the plane containing the probes.

Oscilloscope pictures obtained from some of the azimuthal field measurements are shown in Figure 4. The magnetic probe signals can be interpreted rather easily if the following assumptions concerning the discharge are made:

(1) The discharge is cylindrically symmetric.

(2) The magnetic field does not vary axially.

(3) The magnetic field crossing the probe at any instant is uniform over the area of the probe.

(4) Magnetic probes do not disturb the magnetic field distribution by their presence.

The relation between the magnetic field and the integrated probe voltage is then (See Reference 1 for example):

$$V_p = \frac{n\,A\,B}{RC} \tag{1}$$

where n = Number of turns

A = Cross-sectional area of loop

B = Magnetic field at probe location

RC = Integrating circuit constant

The relation between the probe voltage and the discharge current is

$$V_p = \frac{\mu_o\, I_r}{2\pi RC\, r} \tag{2}$$

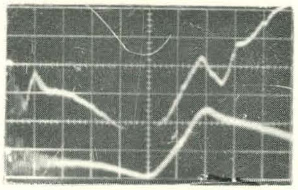
where I_r = Total current flowing between the electrodes inside the cylinder of radius r

r = Radius at which the probe is located

μ_o – Permeability of free space

It will be noted that in Figure 4 the signals from Probes 2 and 3 in all cases except the 1000 volt test are nearly zero for some initial time t. According to Equations 1 and 2 this means that up to time t, almost no current flows inside the cylinder of radius r, the probe radius, and there is no azimuthal magnetic field at the probes.

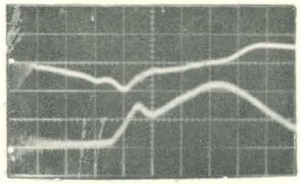
10 v/div



0.02 v/div

D-Loop

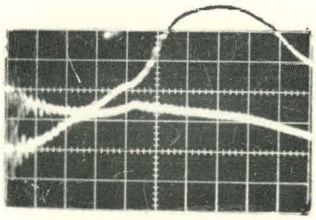
Probe 3



50 v/div

0.05 v/div

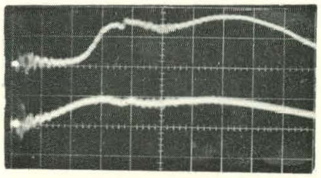
0.02 v/div



0.02 v/div

Probe 2

Probe 1



0.05 v/div

0.1 v/div

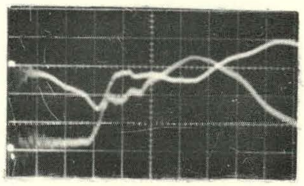
2 μsec/div

V_o = 1000 volts

2 μsec/div

V_o = 2000 volts

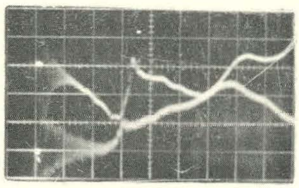
50 v/div



0.05 v/div

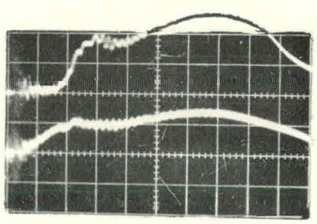
D-Loop

Probe 3



0.1 v/div

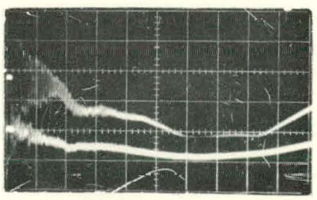
0.05 v/div



0.1 v/div

Probe 2

Probe 1



0.05 v/div

0.2 v/div

2 μsec/div

V_o = 3000 volts

2 μsec/div

V_o = 4000 volts

Figure 4

To be noted also is the fact that the rate of rise of the probe signal is relatively slow at Probe 1 and is increasingly rapid at positions 2 and 3. Also, as the voltage is increased the rate of rise of the signal increases and time t decreases. These results indicate that the current discharge starts at the periphery of the discharge tube, that the current has a well defined front, and that at least a portion of the current is concentrated in a thin shell which becomes better defined as the front moves inward.

From the oscilloscope data it is possible to calculate an average current front velocity between two probes. Thus, for example, the average front velocity, U_{23}, between Probes 2 and 3 can be calculated from the relation

$$U_{23} = \frac{D}{t_3 - t_2} \qquad (3)$$

where t_2 and t_3 are the times of arrival of the current front at Probes 2 and 3, and D is the distance between the probes. From the relation

$$U_{23}\tau_3 - \ell' \approx \ell_3 \qquad (4)$$

where τ_3 = the rise time of the signal at Probe 3

ℓ' = radial dimension of probe coil (0.178 cm)

We can estimate a current shell thickness ℓ_3. In Table I velocities and shell thickness calculated from Equations 3 and 4 are tabulated versus initial voltage and maximum discharge current.

TABLE I

Initial Voltage	I_0	t_2	t_3	τ_3	U_{23}	ℓ_3	γ_3
Volts	Amperes	μsec	μsec	μsec	cm/μsec	cm	
2000	128,000	4	7.2	2	0.78	1.4	0.43
3000	192,000	3.2	5.8	1.6	0.96	1.4	0.40
4000	256,000	3	5.0	1	1.25	1.1	0.40

It will be noted that the front velocity increases with an increase in initial capacitor voltage.

The 1000 volt test results do not indicate a sharp inward moving current front. Probably this discharge was asymmetric and diffuse.

From Equations 1 and 2 and the probe signals we can calculate the magnetic field, B, at each probe location, r, and the total discharge current, I_r, inside a cylinder of radius r coaxial with the axis of the electrodes. From the relation

$$J_Z = \frac{1}{2\pi r}\left(\frac{\partial I_r}{\partial r}\right)_r = \frac{1}{\mu_o r}\frac{\partial rB}{\partial r} \tag{5}$$

we can calculate the axial current density, J_Z. In Figures 5, 6 and 7, respectively, we have plotted B, I_r and J_Z versus radius at 2, 4, 6, 7, 8 and 10 μsec after the beginning of the discharge. Data was taken from the 3000 volt test and the values of $\left(\frac{\partial I_r}{\partial r}\right)_r$ were obtained from the slope of Figure 6. The implosion time of the first pinch was less than 8 μsec in this experiment.

On the plots, the positions r_o, r_1, r_2, r_3 indicate the outer radius of the pinch tube and the radii of Probes 1, 2, and 3, respectively. The plotted values at r_o were calculated from the total current flow measured by the Rogowski coil. This is justified because the output of a probe at the outer periphery of the discharge should always be proportional to the total discharge current. Examination of Figure 6 points out a few features about the discharge. Figure 6, as already noted, is a plot of the current, I_r, flowing inside a cylinder of radius r, versus r. The total discharge current I_{r_o} is the current at r_o. It is interesting to examine how the ratio $\frac{I_r}{I_{r_o}} = \gamma$ varies as a function of time at a given probe location. At Probe 3, γ is small, about 0.01, for the first 6 μsec. Between 6 and 7 μsec, it suddenly increases to about 0.4. Thereafter γ does not exceed 0.41. At r_2 we again see a jump in γ between 4 and 6 μsec, however it is always less than 0.63. These results indicate that although there is a fairly sharp imploding

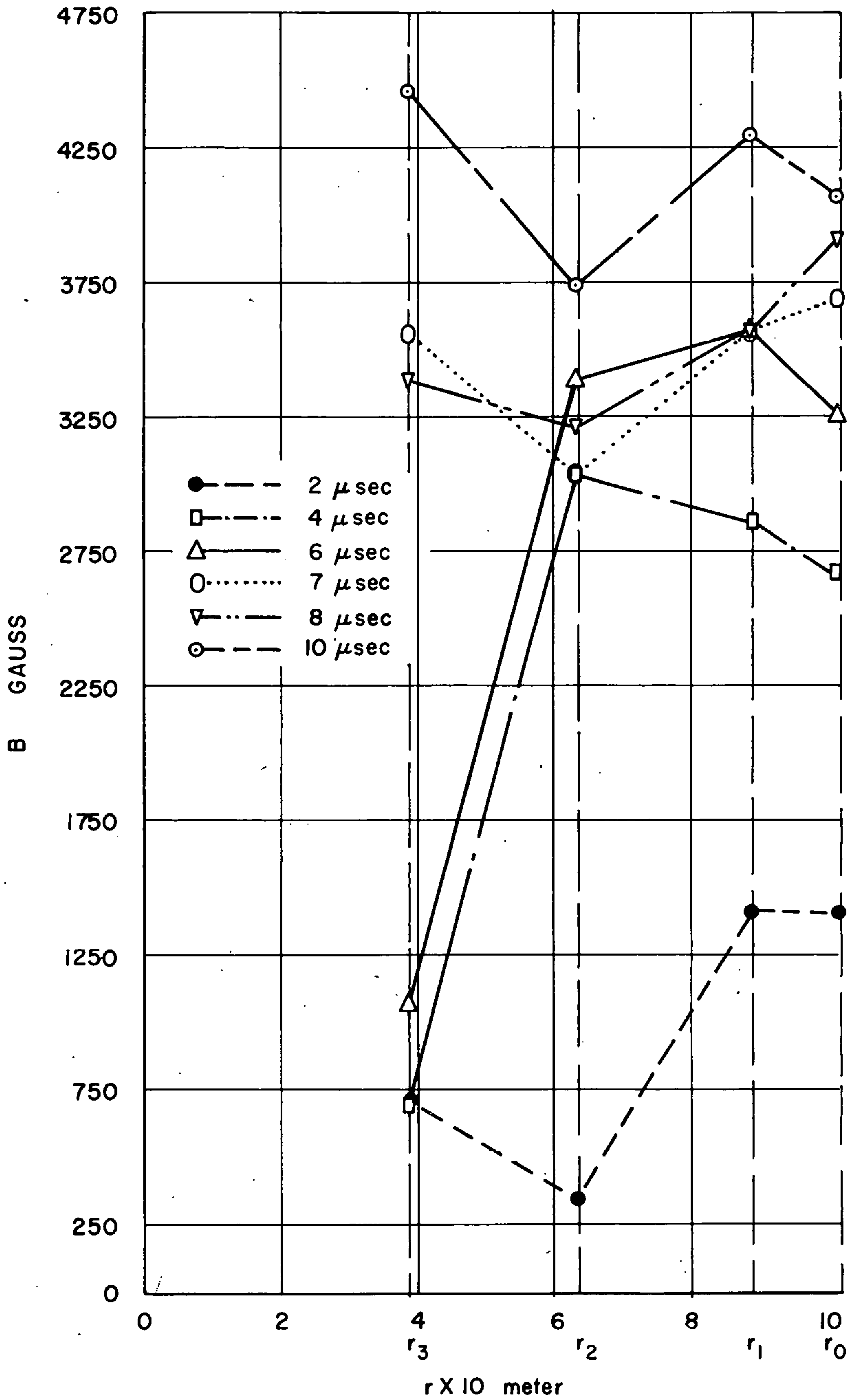
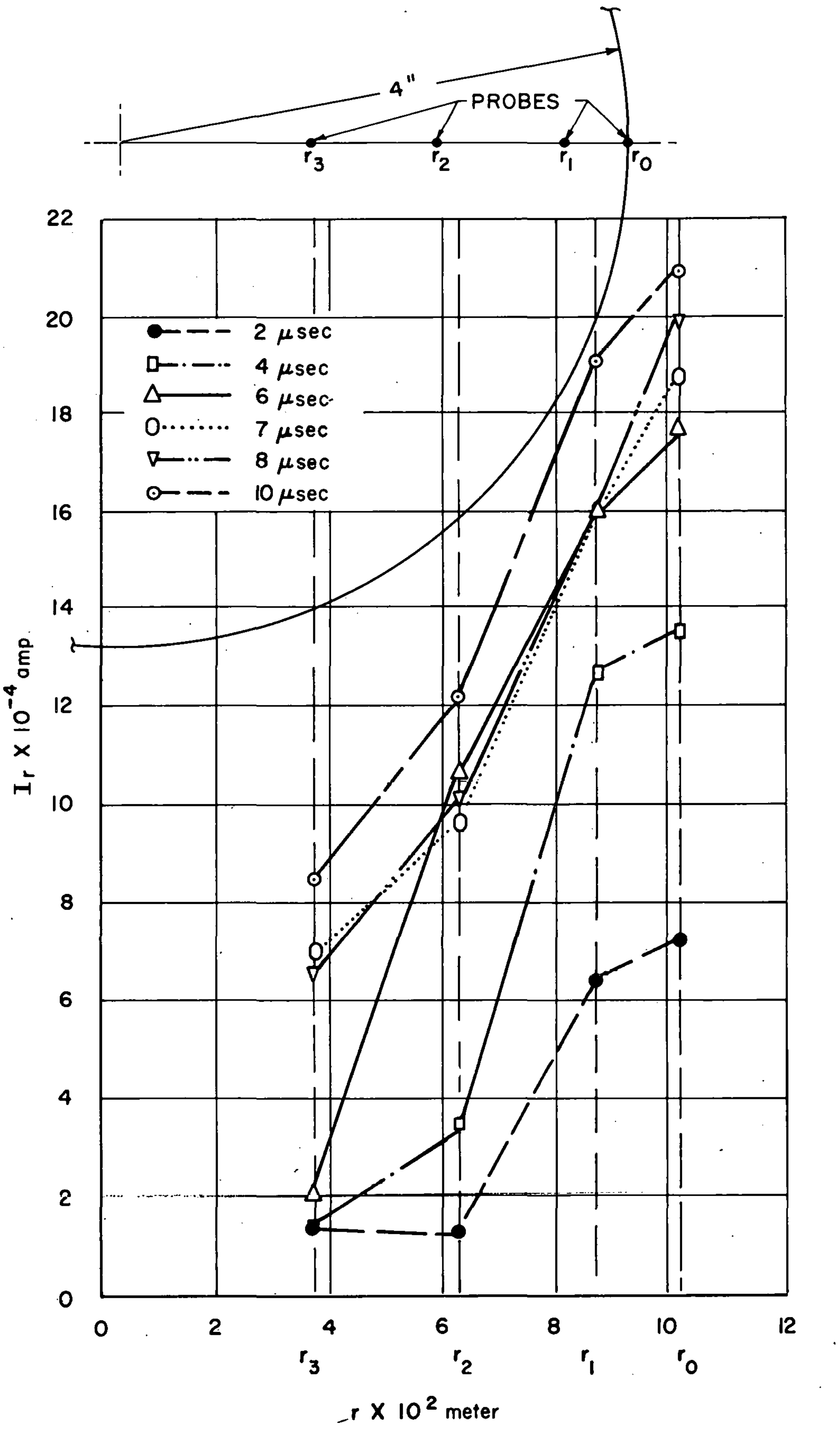
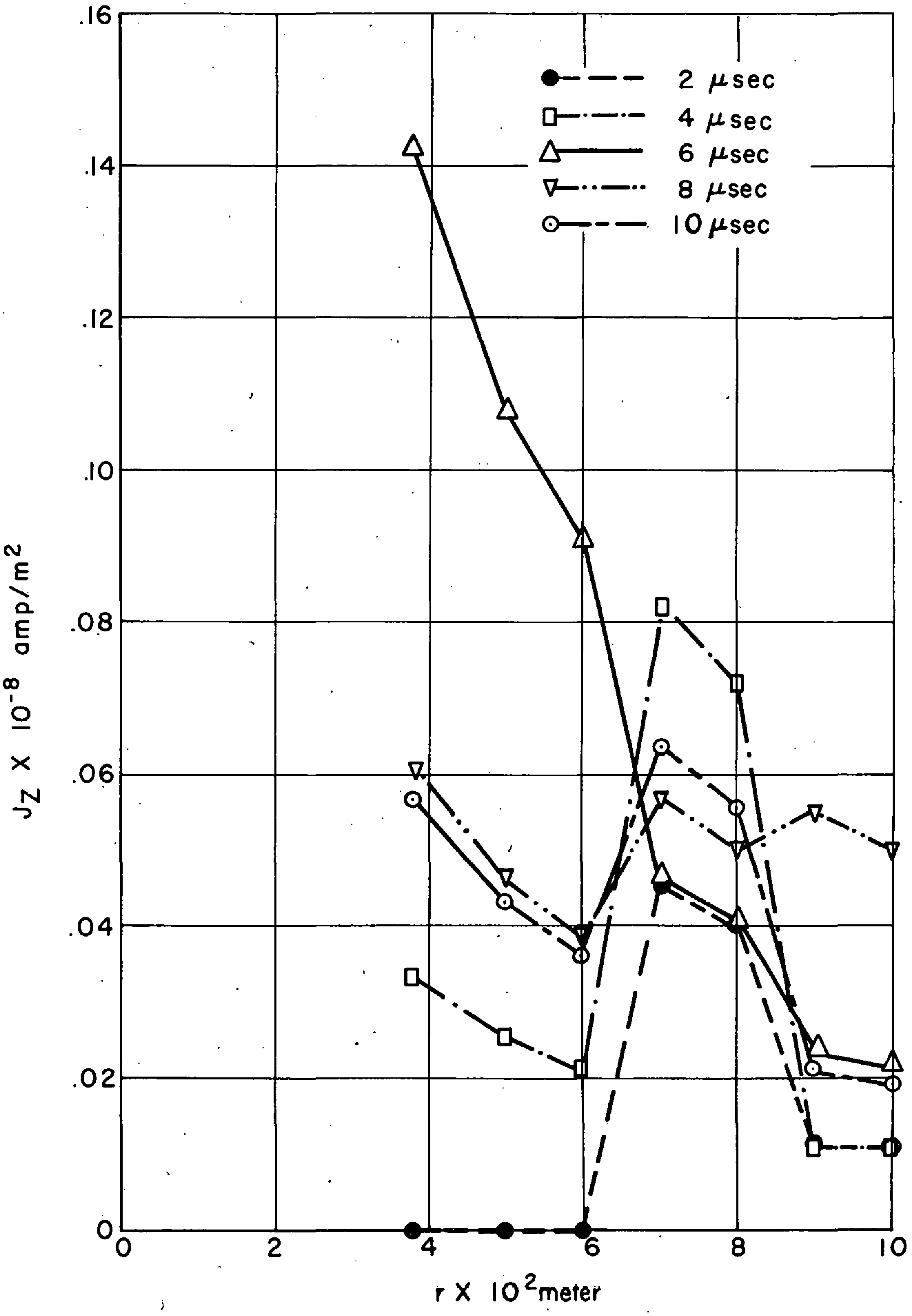

4750
4250
3750
3250
2750
2250
1750
1250
750
250
0
B GAUSS
2 μ sec
4 μ sec
6 μ sec
7 μ sec
8 μ sec
10 μ sec
0
2
4
6
8
10
r_3
r_2
r_1
r_0
r X 10 meter

Figure 5



4"
PROBES
r_3
r_2
r_1
r_0
2 μsec
4 μsec
6 μsec
7 μsec
8 μsec
10 μsec
$I_r \times 10^{-4}$ amp.
0
2
4
6
8
10
12
14
16
18
20
22
$r \times 10^2$ meter

Figure 6



.16
.14
.12
.10
.08
.06
.04
.02
0
0
2
4
6
8
10
2 μsec
4 μsec
6 μsec
8 μsec
10 μsec
$J_Z \times 10^{-8}$ amp/m^2
r X 10^2meter

Figure 7

current front, some current flows through the entire region between the outer pinch tube radius and the instantaneous front position. Note that for a pinch in which the entire current flows in a thin imploding sheet, the value of γ would be constant and equal to 1 for r greater than the radius which the current shell has passed and would be equal to 0 for r smaller than the shell radius. In Table I, values of γ when the shell passes probe position 3 are listed. These values were calculated by taking the ratio of the current calculated at time $(t_3 + \tau_3)$ from Equation 2 to the total measured discharge current at the same time.

The D-Loop signals in Figure 4 were not integrated. The general expression relating the voltage induced in the unintegrated D-Loop to the azimuthal magnetic field B is

$$V_D = -\frac{d\phi}{dt} = -\frac{d}{dt}\int_S B \cdot dS \tag{5}$$

where ϕ = magnetic flux crossing D-Loop

S = surface enclosed by D-Loop

B = magnetic field normal to S.

In Appendix A it is shown that for an ideal pinch

$$-\frac{d\phi}{dt} = -\left[\frac{\mu_o h}{2\pi}\left(\ln\frac{r_o}{r_p}\right)\frac{dI}{dt} - \frac{\mu_o h}{2\pi}\,\frac{I}{r_p}\,\frac{dr_p}{dt}\right] \tag{6}$$

where

r_o = width of D-Loop (See Figure 2)

h = height of D-Loop (See Figure 2)

r_p = instantaneous radius of cylindrical current shell

I = total discharge current.

An ideal pinch is defined as one in which the current flows in a thin cylindrical shell symmetric with respect to the axis. The shell forms at the periphery of the discharge tube and is pushed inward by its own magnetic field. Equation 6 can give us a qualitative understanding of the D-Loop signals. The first term in Equation 6 gives the rate of change of flux through the instantaneous fraction of the D-Loop area where $|B| > 0$. The change in flux resulting from this term comes from the rate of change of current. This term may also be thought of as the instantaneous pinch tube inductance $\frac{\mu_o h}{2\pi} \ln \frac{r_o}{r_p}$ multiplied by $\frac{dI}{dt}$. The second term gives the rate of change of flux through the D-Loop to the change in the D-Loop area through which $|B| > 0$. The quantity $\frac{h dr_p}{dt}$ is the rate of increase of this area. The quantity $\frac{\mu_o I}{2\pi r}$ is the instantaneous value of the magnetic field just outside the cylindrical shell. For the conditions of our experiments the first term in Equation 6 is much smaller than the second, after the first μsec, so that we can write

$$V_D \approx \frac{\mu_o h}{2\pi} \frac{I}{r_p} \frac{dr_p}{dt} . \tag{7}$$

Note that the D-Loop signals in Figure 4 at first have negative slopes which after some time change direction rather suddenly. The change in direction in each case occurs a little after the current front passes Probe 3. It should be noted that the rate at which the D-Loop output changes increases with an increase in initial capacitor bank voltage. The explanation suggested by Equation 7 is that the reversal in slope is caused by the decrease in the current front velocity as the front approaches the axis of the discharge tube.

Interpretation of the oscilloscope signals after the current front reaches its position of closest approach to the pinch tube axis is more difficult. Therefore, the explanations which follow should be considered tentative. It has already been mentioned that when the current front is slowed down near the pinch tube axis the slopes of the D-Loop signals change direction. The signals increase and eventually become positive. Sometime after the signals have become positive a second reversal in slope occurs. It should also be noted that the probe outputs after the passage of the current shell are rather plateau-like or actually decreasing for a time and only a little later do they start rising again. This behavior suggests the following tentative explanations: When the current shell approaches the pinch axis it is first slowed down and then pushed out again by the high pressure created. A second current shell starts moving towards the pinch tube axis sometime after the first one. The slowing down of the first shell causes the D-Loop signal to increase and the reversal of its motion causes a positive D-Loop signal. The inward motion of the second shell causes a negative D-Loop signal.

In the 4000 volt test the effect of the second shell motion is not large when the first shell is slowing down, and hence we have a very sharp rise in D-Loop output. The output becomes positive due to the bouncing back of the first shell. A little while later it decreases again due to the inward motion of the second shell. In the 2000 and 3000 volt tests the implosion times for the first shell are longer. The second shell is already moving in at an appreciable rate by the time the first one is slowed down. As a result the rate of increase of the D-Loop signal is smaller than in the 4000 volt case. Eventually the second shell approaches center and is bounced back producing the positive D-Loop outputs.

It is interesting to compare the predicted D-Loop outputs on the basis of Equation 7 with actual signals. If we assume that current flows in a thin shell moving at the front velocity we can calculate the D-Loop output at each probe location, r. This calculation can be performed in two ways. In one case we assume that the entire discharge current is concentrated in the shell. In the second case we calculate the current in the shell from the probe signal at time $(t + \tau)$ and Equation 2. We assume that the probe in question is at the center of the current shell at time $(t + \tau/2)$ and we determine the total current at time $(t + \tau)$ from the Rogowski coil measurements. The front velocity is obtained from Equation 3. Results of such calculations at Probe 3, together with the measured D-Loop signal at $(t + \tau/2)$, are presented in Table II.

TABLE II

Initial Capacitor Voltage	$t_3 + (\tau/2)_3$	I_{r_o}	I_S	V_E	V_T	V_S
Volts	Sec	Amperes	Amperes	Volts	Volts	Volts
2000	8.2	110,000	47,500	50	100	56
3000	6.6	185,000	67,000	70	210	90
4000	5.5	190,000	70,000	90	300	110

In the above table V_E is the measured D-Loop signal in volts. V_T is the D-Loop signal calculated using Equation 7 and the total current I_{r_o}. V_S is the output calculated using the shell current, I_S.

It will be noted that the measured values are considerably smaller than the values calculated on the basis of total current. Values calculated on the basis of the shell current are in surprisingly good agreement with the measured D-Loop outputs. These results are interpreted as further indication that the shell current is moving inward at a high front velocity and that at least before the first shell starts slowing down, the D-Loop output is essentially determined by the current in this shell. The current between the shell and the outer pinch tube radius, r_o, on the other hand, diffuses inward slowly during this time and has little effect on the D-Loop output.

A few tests have been performed with magnetic probes oriented to measure the radial and the axial fields. These measurements indicate that the axial and radial fields are considerably smaller than the azimuthal field at our test conditions. The signals obtained were not negligible, however, and a systematic measurement of these fields would be interesting.

SECTION V - SYMMETRY TESTS

In the preceding section magnetic probe outputs were interpreted on the basis of several assumptions concerning the discharges. Tests made to check the assumption that the discharges are axially symmetric are discussed in this section. The probe arrangement used is shown in Figure 8. Note the difference between this arrangement and the previous one by comparing Figure 3 with Figure 8. Probe coils were oriented to measure azimuthal fields and were of the same size as the ones already described. The radial position of the probes and the number assigned to each position are shown in Figure 9. Probes 1, 3, 5 and 6 are located on the same circle and are called Group A probes. Probes 2 and 4 are on a second, smaller, circle and are called Group B probes.

If the discharges are symmetric, the outputs of all Group A probes should be the same; similarly, all Group B probe outputs should be the same, but should be different from the Group A signals. Furthermore, if the pinch starts at r_o we should obtain signals from Group A before the signals from Group B.

Our symmetry experiments yielded data which were not always completely reproducible. In most tests above 2000 volts, at least two probes in either Group A or Group B gave similar output signals. Signals from the third or the fourth probe were sometimes similar to the first two, but more often they were not. Similar outputs were often obtained from different combinations of probes. Whenever similar outputs from Group A and Group B probes were obtained, the signal appeared first at the probes of Group A as would be expected. Tests at 1000 volts produced dissimilar outputs at all probes.

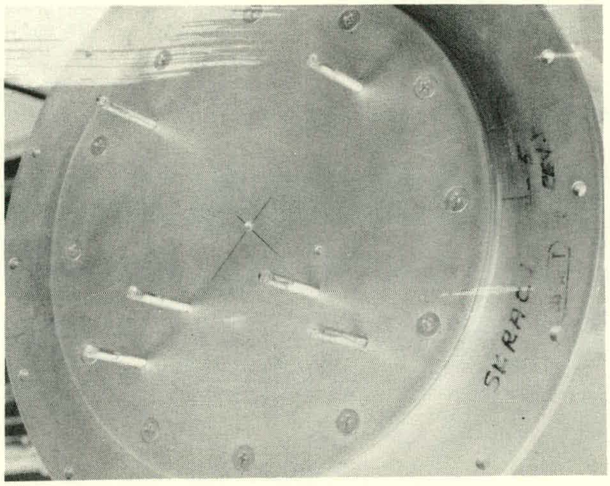


Figure 8

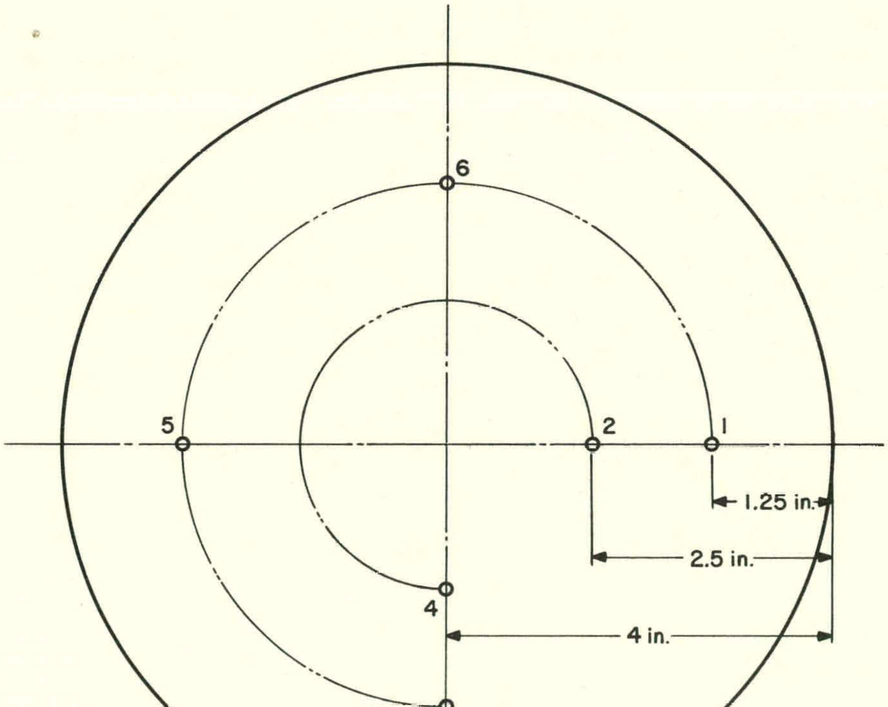
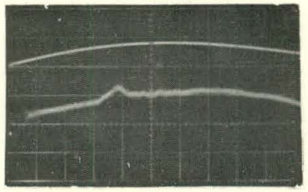
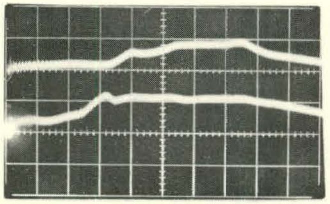


Figure 9



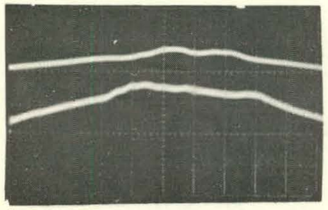
Current
8×10^4
amp/div

Probe 6
0.2 v/div



Probe 4
0.5 v/div

Probe 3
0.2 v

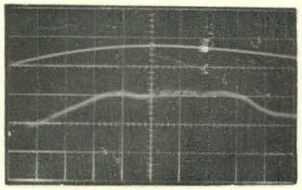


Probe 2
0.5 v/div

Probe 1
0.2 v/div

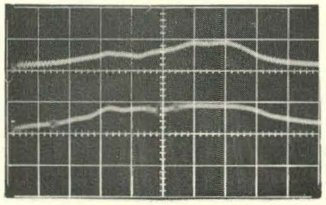
2 μsec/div

V_o = 1000 volts



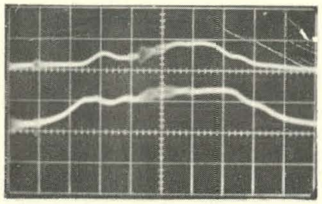
Current
1.6×10^5
amp/div

Probe 6
0.05 v/div



Probe 4
0.1 v/div

Probe 3
0.05 v/div

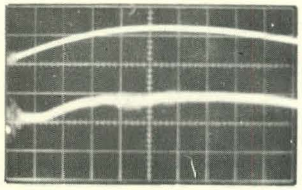


Probe 2
0.1 v/div

Probe 1
0.05 v/div

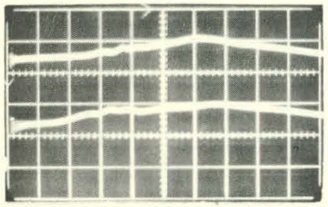
2 μsec/div

V_o = 2000 volts



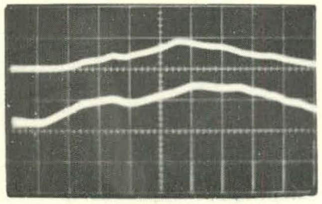
Current
1.6×10^5
amp/div

Probe 6
0.1 v/div



Probe 4
0.2 v/div

Probe 3
0.1 v/div



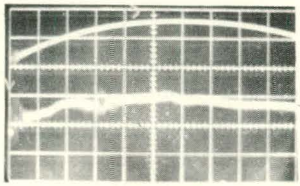
Probe 2
0.2 v/div

Probe 1
0.1 v/div

2 μsec/div

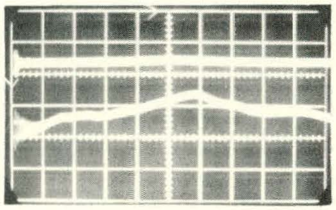
V_o = 3000 volts

Figure 10(a). Probe Outputs for Pinch Discharges



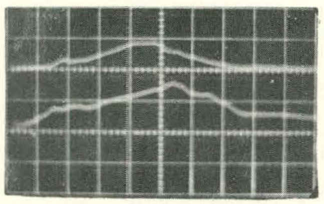
Current
1.6×10^5
amp/div

Probe 6
0.1 v/div



Probe 4
0.2 v/div

Probe 3
0.1 v/div

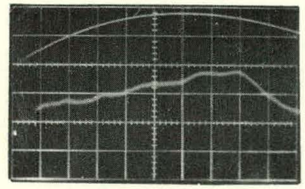


Probe 2
0.2 v/div

Probe 1
0.1 v/div

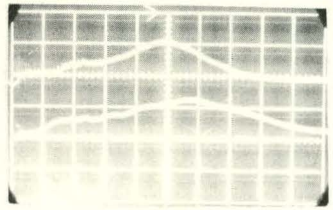
2 μsec/div

V_o = 4000 volts



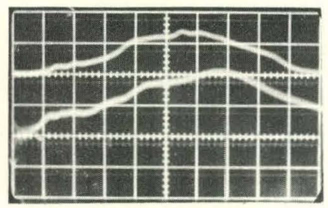
Current
1.6×10^5
amp/div

Probe 6
0.1 v/div



Probe 4
0.2 v/div

Probe 3
0.1 v/div



Probe 2
0.2 v/div

Probe 1
0.1 v/div

2 μsec/div

V_o = 5000 volts

Figure 10(b). Probe Outputs for Pinch Discharges

The occurance of similar outputs at two or more probes of Group A or Group B is an indication of symmetry. This conclusion was reached because it seems unlikely that similar outputs would be obtained at two or more probes so many times from asymmetric discharges. The random dissimilarities of some of the signals are attributed to perturbation of the discharge in the vicinity of the probes. Such perturbation can, for example, be induced by a small current flow on the probe surface.

In Figure 10 some oscilloscope pictures obtained in these experiments are shown. Since symmetry is not expected after the first pinch, the pictures should be compared only during the first implosion time. In the 5000 volt test the implosion time is about 5 μsec. It will be noted that the outputs of the comparable probes are nearly the same during this time. Furthermore, as expected, the signals of Group A probes start rising before the signals of Group B probes. The 4000 volt test shows similar results, except for the signal from Probe 4 which was lost due to a faulty connection at the oscilloscope. The implosion time in this case was 7 μsec. In the 3000 volt pictures only the outputs of Probes 1 and 6 are similar (implosion time 8 μsec). Probes 3 and 4 start giving out signals from the start of the discharge. This is believed to be due to a current flow along the surface of these probes. Results similar to the 3000 volt test were also obtained frequently at 2000, 4000 and 5000 volts.

In the typical 1000 volt tests the output from each probe was different. It is believed that this indicates an asymmetric discharge.

We have also made symmetry tests using D-Loops. Two D-Loops were mounted as indicated in Figure 2 with their loop areas at an angle of 180° to each other. The integrated D-Loop outputs were fed to the oscilloscope. Figure 11 is an oscilloscope picture of the D-Loop signals at 5000 volts. The fact that the two outputs are alike is an indication of symmetry.

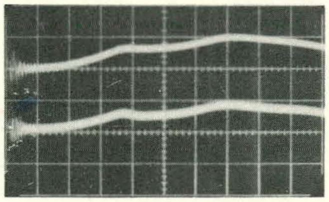


Figure 11

SECTION VI - DISCUSSION

We have interpreted our magnetic probe data on the basis of Equations 1 and 2 and have reached certain conclusions regarding the nature of the pinch discharges. Our conclusions can only be considered tentative unless it can be shown that the assumptions on which Equations 1 and 2 are based were valid under our experimental conditions.

We have extensively tested the assumption of axial symmetry of the discharges, and have concluded that the discharges were symmetrical when the initial capacitor voltage was above 2000 volts. The other assumptions on which Equations 1 and 2 are based have been checked by using a procedure outlined in Appendix A. The approach involves the calculation of the D-Loop output from a magnetic field map obtained from magnetic probe readings using Equation 1. The comparison of the calculated D-Loop output with the measured output constitutes a check of the magnetic field map and hence also of the assumptions on which the map was calculated. A D-Loop output was calculated from a magnetic field map obtained from the 4000 volt magnetic field distribution test. We have compared this to the measured integrated D-Loop signal obtained under the same test conditions and have found good agreement between the calculated and the measured signals. The results of these checks encourage us to believe that the discharges and measuring technique satisfied the assumptions in Equations 1 and 2.

We believe however that additional checks of the above assumptions can and should be made. The axial variation of the magnetic field can be checked by mounting magnetic probe coils at several axial positions and comparing their outputs during a discharge. An overall check of the assumptions can be obtained from a probe located at the edge of the pinch tube. The validity of the assumptions may be tested by comparing the output signal with a calculated output signal obtained by using the measured value of the total discharge current in Equation 2. In addition, it appears worthwhile to repeat the D-Loop check on the basis of a magnetic field map obtained from probe measurements at a greater number of points.

In the tests reported here the only variable was the initial capacitor bank voltage. We believe that the effect on the discharge of other parameters such as capacitance, discharge frequency, nature of gas, and initial pressure would constitute an interesting study and we expect that this will be undertaken at some other time.

SECTION VII - CONCLUSIONS

We can make the following conclusions regarding the nature of the discharges studied:

(1) The symmetry of the discharge improves as the initial capacitor bank voltage is increased.

(2) The discharges with an initial capacitor bank voltage above 2000 volts have the following characteristics:

- The discharges are symmetric.
- The main discharge current starts flowing near the periphery of the pinch tube.
- The region of current flow spreads toward the axis of the pinch tube with a well defined front velocity.
- The front velocity increases with an increase in initial capacitor voltage (the only parameter that was varied).
- A portion of the total current (less than 50%) flows in a thin shell moving towards the axis with the front. The estimated shell thickness is about one-tenth of the pinch tube radius.
- The remainder of the current flows in the region between the current shell and the outer pinch tube radius. This current diffuses inward slowly during the first few μsec of the discharge.

REFERENCES

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2. L. Aronowitz, "A Note on the Use of Magnetic Probes to Measure the Field in the Magnetic Piston," Republic Aviation Corporation PPL Note PPL-TN-60-9, May 27, 1960.

3. L. Aronowitz and T. Donner, "Calibration of a Magnetic Probe," Republic Aviation Corporation PPL Note PPL-TN-60-17, November 17, 1960.

4. M. J. Minneman, "An Experimental Plasma Propulsion System," PPL Report No. 115, Republic Aviation Corporation, 1959.

APPENDIX A

APPENDIX A

1. THE D-LOOP

A typical D-Loop,* mounted to measure the azimuthal flux, is shown schematically in Figure A-1.

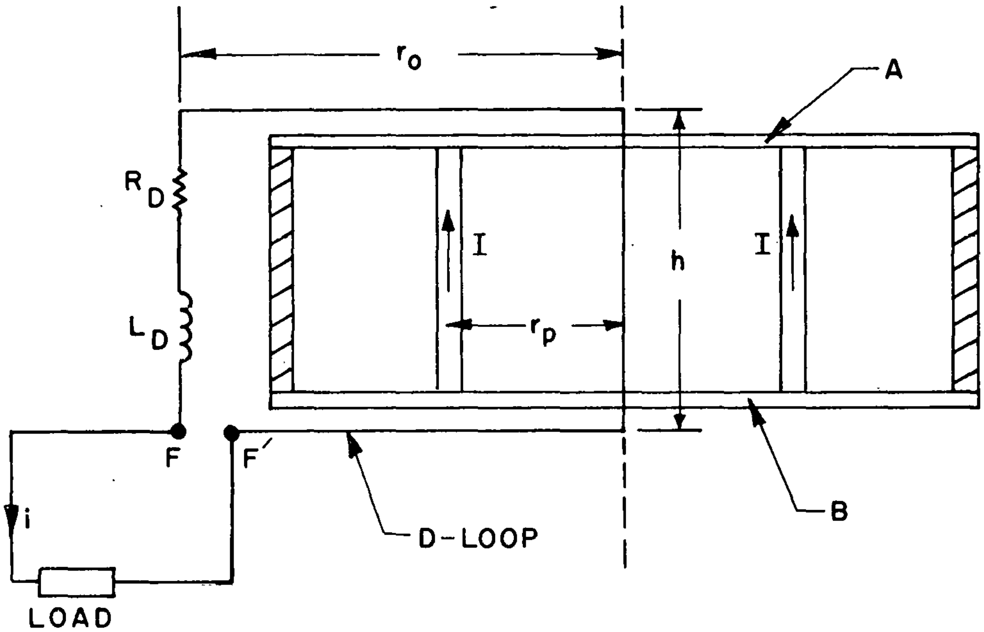

Figure A-1

A and B are flat circular electrodes contained in a cylindrical linear pinch tube. The D-Loop is an insulated wire placed along the axis of the tube and closed into a rectangular loop which encircles the electrodes. The outer edge of the loop is

* T. Donner, "The D-Loop and Some of its Applications in Pinch Discharge Diagnostics," PPL-TN-61-19, Republic Aviation Corporation, July 15, 1961.

at a radius r_o from the axis. R_D and L_D are the loop resistance and inductance. Leads from the D-Loop to the oscilloscope or other voltage measuring instruments are shielded from magnetic pickup.

The voltage output of the D-Loop is proportional to the rate of change of flux through the loop area, hr_o.

$$V = -\frac{d\phi}{dt} = -\frac{d}{dt}\int_S B \cdot dS \qquad \text{(A-1)}$$

where ϕ = total flux crossing the loop surface, hr_o.
S = surface area enclosed by D-Loop
B = magnetic field strength perpendicular to loop.

The voltage, V_D, appearing across the D-Loop terminals F and F′ when connected to a load is

$$V_D = -\frac{d\phi}{dt} + R_D i + L_D \frac{di}{dt} \qquad \text{(A-2)}$$

where R_D = resistance of D-Loop circuit
L_D = inductance of the D-Loop
i = current flowing through the D-Loop

The D-Loop output may be integrated by using an RC integrating circuit. If this is done, the D-Loop current is negligible and the voltage output from the integrator is approximately

$$\left(V_D\right)_i = -K\phi \qquad \text{(A-3)}$$

where K is the integrating circuit constant, $\frac{1}{RC}$.

Since $d\phi/dt$ in the D-Loop is related to the rate of change of current distribution in the discharge, relations between the D-Loop voltage and current distribution may be written. As an illustration, a simple situation will be considered.

In Figure A-1 an idealized linear pinch discharge is shown schematically. The discharge current, I, flows in a thin cylindrical sheet that is symmetric with respect to the axis. The sheet originally forms at the periphery and is pushed inward by its own magnetic field.

For this case the azimuthal field as a function of distance from the axis and current is

$$B = \frac{\mu_o I}{2\pi r}, \quad r > r_p$$

$$= 0, \quad r < r_p$$

where I = the total discharge current

r = radius where the magnetic field is equal to B

r_p = instantaneous radius of the current sheet.

Therefore,

$$\phi = \int_S B \cdot dS = h \int_{r_p}^{r_o} B \cdot dr_p$$

$$= \frac{\mu_o h I}{2\pi} \ln \frac{r_o}{r_p}$$

and

$$\frac{d\phi}{dt} = \frac{\mu_o h}{2\pi} \left[\left(\ln \frac{r_o}{r_p} \right) \frac{dI}{dt} - \frac{I}{r_p} \frac{dr_p}{dt} \right]$$

Substituting the above into Equations 2 and 3, we obtain, respectively, for $r_o \geq r_p$

$$V_D = - \frac{\mu_o h}{2\pi} \left[\left(\ln \frac{r_o}{r_p} \right) \frac{dI}{dt} - \frac{I}{r_p} \frac{dr_p}{dt} \right] + R_D i + L_D \frac{di}{dt} \qquad \text{(A-4)}$$

and $$\left(V_D\right)_i = -\frac{K\mu_o h}{2\pi} I \ln \frac{r_o}{r_p} \tag{A-5}$$

For $r_o < r_p$, both V_D and $\left(V_D\right)_i$ are equal to zero. Note that the terms $R_D i$ and $L_D \frac{di}{dt}$ in Equation A-4 can be neglected when the load resistance is sufficiently large.

2. THE D-LOOP AS A CHECK OF MAGNETIC PROBE MEASUREMENTS

As in the case of the D-Loop, the output of a magnetic probe is proportional to the rate of change of flux normal to the probe area. Therefore Equations A-1 through A-3 are applicable to the magnetic probe if R_D and L_D are interpreted as the probe resistance and inductance. Because of the small size of the probes it is usually assumed that the magnetic field at any given time is constant over the probe area. It is also assumed that the voltage drop in the probe is negligible. With these assumptions the voltage induced in the probe becomes

$$V_p = n\,A\,\frac{dB}{dt}$$

and if the output is integrated

$$\left(V_p\right)_i = K'\,n\,A\,B \tag{A-6}$$

where n = number of turns in probe coil

A = coil area

B = magnetic field at probe location (perpendicular to the plane of the probe coil)

K′ = integrating circuit constant = $\frac{1}{R'C'}$

By inserting probes at various positions in the discharge and measuring the voltage, the magnetic field in the pinch tube can be mapped as a function of time and space. The reliability of such a map is usually questioned because the probes, by their presence in the discharge, can cause local perturbations in the current distribution. If such perturbations are induced they may have a strong effect on the probe output. The unperturbed field distribution could then be quite different from that indicated by the probes.

The D-Loop offers a possibility of checking whether the probes are yielding the desired unperturbed field map. It is possible to calculate the D-Loop output from a series of magnetic probe measurements. The calculated output may then be compared to the measured D-Loop output. If the two agree we have fairly good evidence that the magnetic probe measurements were not strongly distorted by induced local perturbations. The assumption implied here is of course that the D-Loop itself is unaffected by perturbations it may cause. This assumption seems reasonable because the only disturbance in the discharge caused by the D-Loop is due to the insulated wire through the axis of the pinch tube. The effect of this on the magnetic field crossing the rather large D-Loop area should be small. Furthermore, the D-Loop may be present while the magnetic probe measurements are made so that any disturbance caused by the D-Loop would not affect the comparison.

The D-Loop output may be obtained from magnetic probe readings in the following manner. The integrated voltage output of the D-Loop in Figure A-1 is

$$\left(V_D \right)_i = - K\phi = - K \int_o^h \int_o^{r_o} B\,dr\,dh \qquad \text{(A-7)}$$

We can take magnetic probe readings on the surface enclosed by the D-Loop. From these and Equation 6 we can obtain the magnetic field at the probe location. To obtain the integral of Equation 8 we plot these results and integrate graphically. In practice it is often found that the axial variation of the magnetic field in pinch discharges is very small. If this is the case, to calculate $\left(V_D \right)_i$, magnetic field measurements are necessary only along one radius of the pinch tube.

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