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CLIP 1 - AN IBM-704 PROGRAM TO SOLVE THE P-3 EQUATIONS IN CYLINDRICAL GEOMETRY

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B. Anderson, J. Davis, E. Gelbard,
P. Jarvis, and J. Pearson

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A second order form of the cylindrical P-3 equations is obtained for the case of an isotropic source. The boundary conditions and numerical method are discussed. Input preparation and operating instructions are included.

CLIP 1 -- AN IBM-704 PROGRAM TO SOLVE THE P-3 EQUATIONS IN CYLINDRICAL GEOMETRY

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I. INTRODUCTION

CLIP is designed to solve the one velocity transport equation in one dimensional cylindrical geometry in a P-3 approximation. The P-3 equations are solved iteratively with the aid of standard finite differencing techniques. Anisotropic scattering is permitted, within the limitations of P-3, but the input source must be isotropic. Zero flux or zero gradient boundary conditions are available as options, and as a consequence of the method of solution, a P-1 solution can be obtained. CLIP is restricted to a maximum of 50 regions and 501 mesh points.

II. EQUATIONS AND METHOD OF SOLUTION

Fleck's (Ref 1) formulation of a P-3 approximation to the one-velocity transport equation in one-dimensional cylindrical geometry leads to a set of six linear first order differential equations, which may be written as

$$(\frac{d}{dr} + \frac{1}{r})\psi_3 + \Sigma_o\psi_o = S \tag{1}$$

$$\frac{d}{dr}\psi_o - \frac{d}{dr}\psi_1 + (\frac{d}{dr} + \frac{2}{r})\psi_2 + 3\Sigma_1\psi_3 = 0 \tag{2}$$

$$3(\frac{d}{dr} + \frac{1}{r})\psi_4 - (\frac{d}{dr} + \frac{1}{r})\psi_3 + 5\Sigma_2\psi_1 = 0 \tag{3}$$

$$2\frac{d}{dr}\psi_1 - \frac{1}{3}(\frac{d}{dr} + \frac{2}{r})\psi_2 + 7\Sigma_3\psi_4 = 0 \tag{4}$$

$$\frac{3}{2}\left(\frac{d}{dr}+\frac{3}{r}\right)\psi_5+3\left(\frac{d}{dr}-\frac{1}{r}\right)\psi_3-\frac{3}{2}\left(\frac{d}{dr}-\frac{1}{r}\right)\psi_4+5\Sigma_2\psi_2 = 0 \qquad (5)$$

$$5\left(\frac{d}{dr}-\frac{2}{r}\right)\psi_2+7\Sigma_3\psi_5 = 0 \qquad (6)$$

where, in terms of Fleck's notation (on the right-hand side)

$$\psi_o = \psi(0)$$

$$\psi_1 = \psi^{11}$$

$$\psi_2 = \psi^{11}+2\psi^{33}$$

$$\psi_3 = \psi^{3}$$

$$\psi_4 = \psi^{113}$$

$$\psi_5 = 3\psi^{113}+4\psi^{333}$$

and where S is the isotropic source, and $\Sigma_i = \Sigma_a + \Sigma_{so} - \Sigma_{si}$ $(i = 0,1,2,3)$. Here Σ_a is the absorption cross section, and Σ_{si} is the i-th Legendre component of the scattering cross section. ψ_o and ψ_3 are simply the scalar flux and the outward going radial current respectively.

The relations between the moments appearing in CLIP 1 and those appearing in BOUND* (Ref 2) are as follows:

$$\psi_o = \varphi_{o,o} \qquad \psi_3 = \varphi_{1,1}$$

$$\psi_1 = \varphi_{2,o} \qquad \psi_2 = \frac{1}{2}\varphi_{2,2}$$

$$\psi_4 = \frac{1}{3}\varphi_{3,1} \qquad \psi_5 = \frac{1}{6}\varphi_{3,3}$$

Now by means of a few simple substitutions, Eqs (1) through (6) can be written respectively as,

$$\left(\frac{d}{dr}+\frac{1}{r}\right)\psi_3+\Sigma_a\psi_o = S \qquad (7)$$

$$\frac{d\psi_o}{dr}+3\Sigma_1\psi_3 = \frac{d\psi_1}{dr}-\left(\frac{d}{dr}+\frac{2}{r}\right)\psi_2 \qquad (8)$$

---

* BOUND is an IBM-704 program in which the analytic solution for the moments is evaluated at discrete points.

$$3\left(\frac{d}{dr} + \frac{1}{r}\right)\psi_4 + 5\Sigma_2\psi_1 = S - \Sigma_a\psi_o \tag{9}$$

$$\frac{5}{3}\frac{d\psi_1}{dr} + 7\Sigma_3\psi_4 = -\frac{1}{3}\frac{d\psi_o}{dr} - \Sigma_1\psi_3 \tag{10}$$

$$\frac{3}{4}\left(\frac{d}{dr} + \frac{3}{r}\right)\psi_5 + \frac{5}{2}\Sigma_2\psi_2 = S - \Sigma_a\psi_o + \frac{3}{4}\left(3\frac{d}{dr} + \frac{1}{r}\right)\psi_4 - 3\frac{d\psi_3}{dr} + \frac{5}{2}\Sigma_2\psi_1 \tag{11}$$

$$\frac{5}{4}\left(\frac{d}{dr} - \frac{2}{r}\right)\psi_2 + \frac{7}{4}\Sigma_3\psi_5 = 0 \tag{12}$$

If now the values of ψ_3, ψ_4, and ψ_5 as determined by Eqs (8), (10), and (12) are inserted into Eqs (7), (9), and (11) then the following second order equations result.

$$-D_i\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)\psi_i + \Sigma_{Ti}\psi_i = S_i \quad (i = 0,1,2) \tag{13}$$

where, $D_o = \dfrac{1}{3\Sigma_1}$ $\qquad$ $\Sigma_{To} = \Sigma_o$

$D_1 = \dfrac{5}{7\Sigma_3}$ $\qquad$ $\Sigma_{T1} = 5\Sigma_2$

$D_2 = \dfrac{3}{4}D_1$ $\qquad$ $\Sigma_{T2} = \dfrac{5}{2}\Sigma_2 + \dfrac{1}{r^2}\left(\dfrac{16}{5}D_1 + 6D_o\right)$

$$S_o = S - D_o\left[\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)\psi_1 - \left(\frac{d^2}{dr^2} + \frac{3}{r}\frac{d}{dr}\right)\psi_2\right]$$

$$S_1 = \left(1 + \frac{D_1}{5D_o}\right)S - \frac{D_1}{5D_o}S_o - \Sigma_a\psi_o$$

$$S_2 = -\frac{5}{4}S + \frac{5}{4}\Sigma_a\psi_o - \frac{3}{2}\frac{D_o}{r}\frac{d\psi_o}{dr} - \frac{\Sigma_1}{4}\psi_1 + \left(\frac{3}{5}D_1 + 3D_o\right)\frac{1}{r}\frac{d\psi_1}{dr} - \left(\frac{D_1}{10} + 3D_o\right)\frac{1}{r}\frac{d\psi_2}{dr}$$

Equations (13) are solved iteratively in CLIP 1 according to the following procedure: Assume that ψ_1^k and ψ_2^k are available where the superscript k denotes the k-th iterate solution. S_o^k is constructed from ψ_1^k and ψ_2^k and a solution obtained for ψ_o which we denote as ψ_o^k. Then S_1^{k+1} is formed from S_o^k and ψ_o^k, and ψ_1^{k+1} is determined.

Next S_2^{k+1} is constructed from ψ_o^k, ψ_1^{k+1}, ψ_2^k and a new solution is obtained for ψ_2 which in our notation is ψ_2^{k+1}. Now ψ_1^{k+1} and ψ_2^{k+1} are available and the process can be repeated. The iterative process is initiated in CLIP by taking $\psi_1^1 = \psi_2^1 = 0$; hence ψ_o^1 is the P-1 solution. Thus subsequent iterates make corrections to the P-1 solution. If the P-1 solution were not very different from the P-3 solution, one might expect that the process would converge in a few iterations, and conversely, if the P-1 solution were very different from the P-3 then the process might require a larger number of iterations. In practice, the number of iterations required to satisfy a specific convergence criteria on CLIP ranges from 8-12 as compared to 4-8 for an equivalent problem on FLIP (Ref 3).

At the conclusion of each iterative solution for the scalar flux (with the exception of the first) the following pointwise ratios are computed,

$$\lambda^k(n) = \frac{\psi_o^{k-1}(n)}{\psi_o^k(n)}$$

The iterative process is terminated on the k-th iterate when

$$\left|[\lambda^k(n)]_{max} - 1\right| \leq \epsilon$$

and

$$\left|[\lambda^k(n)]_{min} - 1\right| \leq \epsilon$$

where ϵ is an input parameter. In general, the proper choice for ϵ can be made only after gaining some experience with CLIP. In problems which have run so far, satisfactory results have been achieved by choosing ϵ equal to 0.00005.

III. BOUNDARY CONDITIONS

A. Origin

At the origin, the directional flux should have conical symmetry about the axis of the cylinder. This requirement, if explicitly imposed, would force ψ_2, ψ_3, ψ_4, and ψ_5 to be equal to zero, and the solution at the origin would be overdetermined. To avoid this difficulty in CLIP, the first derivative with respect to r of ψ_o, ψ_1, and ψ_2 are set equal to zero at the origin, and in addition the $1/r^2$ term in Σ_{T2} is taken to be a very large but finite number. As a result of the presence of the $1/r^2$ term and of the boundary condition which is imposed, ψ_2, ψ_3, ψ_4, and ψ_5 become zero or very small in magnitude at the origin.

B. Interface

The internal boundary conditions which must be satisfied if the moments are to be continuous (and also compatible with the iterative process) are

$$\left[D_o \frac{d\psi_o^k}{dr} - D_o \frac{d\psi_1^k}{dr} + D_o\left(\frac{d}{dr} + \frac{2}{r}\right)\psi_2^k\right]_+ = \left[\qquad\right]_- \tag{14}$$

$$\left[D_1 \frac{d\psi_1^{k+1}}{dr} + \frac{D_1}{5}\frac{d\psi_1^k}{dr} - \frac{D_1}{5}\left(\frac{d}{dr} + \frac{2}{r}\right)\psi_2^k\right]_+ = \left[\qquad\right]_- \tag{15}$$

$$\left[D_2 \frac{d\psi_2^{k+1}}{dr} - \frac{2D_2\psi_2^{k+1}}{r}\right]_+ = \left[\qquad\right]_- \tag{16}$$

The symbols (-) and (+) indicate the evaluation of the bracketed expression at the interface with quantities characteristic of the left- and right-hand medmium respectively. This convention will be adopted consistently throughout the remainder of the report.

C. Cell

Strictly speaking, the concept of a cell as a repeating section of an infinite lattice is, from a physical standpoint, a meaningless one in cylindrical geometry. One usually associates with a cell the property that there is no net flow of neutrons into or out of the cell, but aside from this there is no physical consideration which will select a unique set of boundary conditions. In CLIP, cell boundary conditions are taken to be

$$\frac{d\psi_o}{dr} = 0, \quad \frac{d\psi_1}{dr} = 0 \text{ and } \left(\frac{d}{dr} + \frac{2}{r}\right)\psi_2 = 0 . \tag{17}$$

This particular choice guarantees that the neutron current is zero, and forces the gradient of the scalar flux to be zero. Conditions (17) differ from the conventional boundary conditions which are obtained by imposing reflecting symmetry upon the directional flux. See Refs 4 and 5 for a discussion of cell boundary conditions in cylindrical geometry.

D. Zero Flux

In addition to cell problems CLIP will also solve reactor problems. The boundary condition which will be taken to correspond to this latter type of problem will be denoted as "Zero Flux".

It is not possible, in the general case, to terminate a reactor calculation at some finite radius by imposing boundary conditions which will be exact in incorporating the effect of the environment on the reactor. In practice however, a reflector of sufficient size is usually present so that the calculation is relatively insensitive to the choice of boundary conditions. In diffusion theory, the scalar flux is chosen to be equal to zero. In CLIP it was found convenient to set ψ_o, ψ_1, and ψ_2 equal to zero. Thus the zero flux condition in CLIP may be considered as an extension of the zero flux condition which is employed in conventional diffusion codes such as WANDA.

IV. NUMERICAL SOLUTION

In the interior of a region, the set of Eqs (13) is approximated at a mesh point, n, as,

$$- D_1\left(\frac{\psi^k_{i,n+1} - 2\psi^k_{i,n} + \psi^k_{i,n-1}}{h^2}\right) - D_1\left(\frac{\psi^k_{i,n+1} - \psi^k_{i,n-1}}{2hr_n}\right) + \Sigma_{Ti,n}\psi^k_{i,n} = S^k_{i,n} \qquad (18)$$

Here $\psi^k_{i,n-1}$, $\psi^k_{i,n}$, $\psi^k_{i,n+1}$ are the i-th moments at the n-1, n, n+1 mesh points respectively, belonging to the k-th iterate, and h is the interval between mesh points. The $S^k_{i,n}$ are

$$S^k_{o,n} = S_n - D_o\left[\frac{\psi^k_{1,n+1} - 2\psi^k_{1,n} + \psi^k_{1,n-1}}{h^2} + \frac{\psi^k_{1,n+1} - \psi^k_{1,n-1}}{2hr_n}\right.$$
$$\left. -\left(\frac{\psi^k_{2,n+1} - 2\psi^k_{2,n} + \psi^k_{2,n-1}}{h^2}\right) - \frac{3}{2}\left(\frac{\psi^k_{2,n+1} - \psi^k_{2,n-1}}{hr_n}\right)\right]$$

$$S^k_{1,n} = \left(1 + \frac{D_1}{5D_o}\right)S_n - \frac{D_1}{5D_o}S^{k-1}_{o,n} - \Sigma_a\psi^{k-1}_{o,n}$$

$$S^k_{2,n} = -\frac{5}{4}S_n - \frac{\Sigma_1}{4}\psi^k_{1,n} + \left(\frac{3}{5}D_1 + 3D_o\right)\left(\frac{\psi^k_{1,n+1} - \psi^k_{1,n-1}}{2hr_n}\right)$$
$$- \frac{3}{2}\frac{D_o}{r_n}\left(\psi^{k-1}_{o,n+1} - \psi^{k-1}_{o,n-1}\right) + \frac{5}{4}\Sigma_a\psi^{k-1}_{o,n} - \left(\frac{D_1}{10} + 3D_o\right)\left(\frac{\psi^{k-1}_{2,n+1} - \psi^{k-1}_{2,n-1}}{2hr_n}\right)$$

For notational simplicity, the i and k indices will henceforth be suppressed. When there is a possibility for ambiguity they will be explicitly displayed.

Equation (18) may be rewritten as

$$\psi_{n+1} = k_n\psi_n - \ell_n\psi_{n-1} - m_n \tag{19}$$

and by utilizing Stark's matrix inversion technique (Ref 6), the three point formula can be converted to the two point formula

$$\psi_n = \frac{\psi_{n+1} + \beta_n}{\alpha_{n+1}} \tag{20}$$

where
$$\alpha_{n+1} = k_n - \frac{\ell_n}{\alpha_n}, \quad \beta_n = \frac{\ell_n\beta_{n-1}}{\alpha_n} + m_n \tag{21}$$

and
$$k_n = \frac{\frac{2D}{h^2} + \Sigma_{Tn}}{g_n}, \quad \ell_n = \frac{\frac{D}{h^2}(1 - \frac{h}{2r_n})}{g_n} \tag{22}$$

$$m_n = \frac{S_n}{g_n}$$

with
$$g_n = \frac{D}{h^2}(1 + \frac{h}{2r_n}) \tag{23}$$

In order to reduce roundoff errors, the delta scheme, as used in WANDA (Ref 6), has been incorporated into CLIP. This consists in introducing the variables

$$\delta_n = \alpha_n - 1, \quad \text{and} \quad p_n = k_n - \ell_n - 1 = \frac{\Sigma_{Tn}}{g_n}. \tag{24}$$

An examination of the recursion relations will show that if the mesh width is very small, and if α is very close to one, then

$$\alpha_{n+1} \approx 2 + \frac{h^2\Sigma_{Tn}}{D} - \alpha_n .$$

From this it is clear that the full numerical significance of Σ_{Tn} does not enter into the calculation of α. On the other hand, in the same situation

$$\delta_{n+1} \approx \delta_n + \frac{h^2\Sigma_{Tn}}{D} .$$

Now δ is on the order of h^2, and thus a greater degree of numerical significance of Σ_{Tn} enters into the recursion relation. If the small mesh width persists for a good number of mesh intervals, the error will be accumulative. It has been found that the delta scheme reduces this type of error.

The relations applicable in the interior of a medium now become

$$\psi_n = \frac{\psi_{n+1} + \beta_n}{(1 + \delta_{n+1})} \tag{25}$$

where $$\delta_{n+1} = p_n + \frac{\ell_n \delta_n}{(1 + \delta_n)}, \quad \beta_n = \frac{\ell_n \beta_{n-1}}{1 + \delta_n} + m_n \tag{26}$$

Equations (25) through (26) are valid in the interior of a medium. We seek relations of this form which are valid at the origin, at the interface between two media, and at the external boundary.

At the origin (in view of Section III-A) the set of Eqs (13) can be written

$$- 2D_i \frac{d^2\psi_i^k}{dr^2} + \Sigma_{Ti}\psi_i^k = S_i^k \tag{27}$$

which in finite difference form become

$$- \frac{4D_i}{h^2}(\psi_{i,1}^k - \psi_{i,o}^k) + \Sigma_{Ti,o}\psi_{i,o}^k = S_{i,o}^k \tag{28}$$

where $$S_{o,o}^k = S_o - \frac{4D_o}{h^2}[(\psi_{1,1}^k - \psi_{1,o}^k) - 2(\psi_{2,1}^k - \psi_{2,o}^k)]$$

$$S_{1,o}^k = (1 + \frac{D_1}{5D_o})S_o - \frac{D_1}{5D_o} S_{o,o}^{k-1} - \Sigma_a \psi_{o,o}^{k-1}$$

$$S_{2,o}^k = - \frac{5}{4} S_o + \frac{5}{4} \Sigma_a \psi_{o,o}^{k-1} - \frac{3D_o}{h^2}(\psi_{o,1}^{k-1} - \psi_{o,o}^{k-1}) - \frac{\Sigma_1}{4} \psi_{1,o}^k$$

$$+ (\frac{3}{5} D_1 + 3D_o)\frac{2}{h^2}(\psi_{1,1}^k - \psi_{1,o}^k) - (\frac{D_1}{10} + 3D_o)\frac{2}{h^2}(\psi_{2,1}^{k-1} - \psi_{2,o}^{k-1}) .$$

It follows then that $$\delta_{i,1} = \frac{\Sigma_{Ti,o} h^2}{4D_i} \quad \text{and} \quad \beta_{i,o}^k = \frac{h^2 S_{i,o}^k}{4D_i} . \tag{29}$$

Now at an interface point, which will be denoted by the subscript I, the fictitious moments ψ^*_{I+1} and ψ^*_{I-1} are introduced which satisfy the following equations:

$$\psi^*_{I+1} = [k_I]_- \psi_I - [\ell_I]_- \psi_{I-1} - [m^*_I]_- \tag{30}$$

$$\psi_{I+1} = [k_I]_+ \psi_I - [\ell_I]_+ \psi^*_{I-1} - [m^*_I]_+ . \tag{31}$$

Equation 30 (31) defines a continuation of the solution in the medium to the left (right) of the interface into the right (left) as if the right (left) hand medium and right (left) hand mesh width did not differ from the left (right).

It can be easily shown that

$$\psi_I = \frac{\psi^*_{I+1} + \beta^*_I}{1 + \delta^*_{I+1}}, \quad \psi^*_{I-1} = \frac{\psi_I + \beta^*_{I-1}}{1 + \delta^*_I} \tag{32}$$

where

$$\delta^*_{I+1} = [p_I]^- + \frac{[\ell_I]^- \delta_I}{1 + \delta_I}, \quad \beta^*_I = \frac{[\ell_I]^+ \beta_{I-1}}{1 + \delta_I} + [m^*_I]^+ \tag{33}$$

$$\delta_{I+1} = [p_I]^+ + \frac{[\ell_I]^+ \delta^*_I}{1 + \delta^*_I}, \quad \beta_I = \frac{[\ell_I]^- \beta^*_{I-1}}{1 + \delta^*_I} + [m^*_I]^- . \tag{34}$$

Notice that the $[m^*_I]^{\pm}$ contain derivatives which are to be evaluated by the use of the fictitious moments. The manner in which this is accomplished will be indicated later in this section

The boundary crossing conditions are given by Eqs (14-16), Sec. III-B. The general form is

$$\left[\frac{Dd\psi}{dr} - \frac{Dc\psi}{r} - s\right]_+ = \left[\qquad\right]_- . \tag{35}$$

The finite difference approximation to Eq (35) is then written as

$$\left[D\left(\frac{\psi_{I+1} - \psi^*_{I-1}}{2h}\right) - \frac{Dc\psi_I}{r_I} - s^*_I\right]_+ = \left[D\left(\frac{\psi^*_{I+1} - \psi_{I-1}}{2h}\right) - \frac{Dc\psi_I}{r_I} - s^*_I\right]_- . \tag{36}$$

If now the fictitious moments in Eq (36) are replaced by their values as determined by Eqs (30) and (31) and if ψ_{I-1} is replaced by $\frac{\psi_I + \beta_{I-1}}{1 + \delta_I}$, then the desired relation at the interface can be obtained. It is

$$\psi_I = \frac{\psi_{I+1} + \beta_I}{1 + \delta_{I+1}} \tag{37}$$

where
$$\delta_{I+1} = p_I + \frac{\ell_I \delta_I}{1 + \delta_I}, \quad \beta_I = \frac{\ell_I \beta_{I-1}}{1 + \delta_I} + m_I \tag{38}$$

with
$$p_I = \frac{[P_I]^-[\ell_I]^+ + t[P_I]^+}{t(1 + [\ell_I]^+)}, \quad \ell_I = \frac{[\ell_I]^+(1 + [\ell_I]^-)}{t(1 + [\ell_I]^+)}$$

$$m_I = ([M_I]^- + t\left[\frac{M_I}{\ell_I}\right]^+) \frac{[\ell_I]^+}{t([\ell_I]^+ + 1)} \tag{39}$$

and
$$[P_I]^{\pm} = [p_I]^{\pm} \mp \left[\frac{cD(2r_I \mp h)}{hr_I^2}\right]^{\pm}$$

$$[M_I]^{\pm} = [m_I^*]^{\pm} \mp \left[\frac{(2r \mp h)Ds_I^*}{hr_I}\right]^{\pm}$$

$$t = \frac{[\frac{D}{h}]^+}{[\frac{D}{h}]^-}. \tag{40}$$

It now remains to determine δ's and β's such that the external boundary conditions are satisfied. We distinguish the outer boundary mesh point with the subscript B.

The zero flux condition is trivial and results simply in

$$\psi_{B-1} = \frac{\beta_{B-1}}{1 + \delta_B}. \tag{41}$$

The ccll conditions, which are given by Eqs (17), can be written as

$$\left(\frac{d\psi}{dr}\right)_B + \left(\frac{c'\psi}{r}\right)_B = 0, \tag{42}$$

and in finite difference form become

$$\frac{\psi_{B+1}^* - \psi_{B-1}}{2h} + \frac{c'\psi_B}{r_B} = 0 \tag{43}$$

where
$$\psi_{B+1}^* = k_B\psi_B - \ell_B\psi_{B-1} - m_B^*.$$

Now if

$$\psi_B = \frac{\psi^*_{B+1} + \beta^*_B}{1 + \delta^*_{B+1}} \tag{44}$$

then

$$\delta^*_{B+1} = p_B + \frac{\lambda_B \delta_B}{1 + \delta_B}, \quad \beta^*_B = \frac{\lambda_B \beta_{B-1}}{1 + \delta_B} + m^*_B . \tag{45}$$

With the aid of the preceding relations it can be easily shown that

$$\psi_B = \left(\beta^*_B + \frac{\beta_{B-1}}{1 + \delta_B}\right) \Big/ \left(\delta^*_{B+1} + \frac{\delta_B}{1 + \delta_B} + \frac{c'}{r_B}\right) . \tag{46}$$

The numerical solution is thus achieved as follows. At any point in the iterative cycle one always has available

$$\psi_{j,n}, \quad \psi^*_{j,I+1}, \quad \psi^*_{j,I-1}, \quad \psi_{j,B}, \quad \psi^*_{j,B+1}$$

from which $m_{i,n}$, $m_{i,I}$, and $m_{i,B}$ can be constructed. One then calculates the δ's and the β's associated with the internal and external points, and proceeds to calculate the i-th moment by means of Eqs (46), (or 41), (37), (20). The solution for each moment i, for each iteration is obtained in an identical fashion. The process is terminated when the convergence criteria has been satisfied, or when a specified number of iterations have been completed.

Since the S_i contain derivatives of the moments, loss of numerical significance can result if these derivatives are evaluated straightforwardly by standard differencing techniques in situations where the moments vary little from one mesh point to another. To avoid this, a refinement is introduced into the calculation of these derivatives. At a point n, these derivatives would ordinarily be evaluated by terms of the form

$$\psi_{n+1} - \psi_{n-1} \quad \text{and} \quad \psi_{n+1} - 2\psi_n + \psi_{n-1} . \tag{48}$$

Now if

$$\gamma_n = \psi_{n+1} - \psi_n \tag{49}$$

then expressions (48) can be rewritten as

$$\gamma_n + \gamma_{n-1} \quad \text{and} \quad \gamma_n - \gamma_{n-1} \tag{50}$$

respectively.

Rather than evaluate γ_n by means of Eq (49), we introduce the δ's and β's such that

$$\gamma_n = \frac{\psi_{n+1}\delta_{n+1} - \beta_n}{1 + \delta_{n+1}} \tag{51}$$

An inspection of (51) will show that γ_n is being computed to a greater degree of significance than a computation of γ_n by (49) when either h is small or δ is small.

Thus at points interior to a medium, γ_n is computed by means of Eq (51). Similarly,

$$\gamma_I^* = \psi_{I+1}^* - \psi_I = \delta_{I+1}^*\psi_I - \beta_I^*$$

$$\gamma_{I-1}^* = \psi_I - \psi_{I-1}^* = \frac{\delta_I^*\psi_I - \beta_{I-1}^*}{1 + \delta_I^*}$$

$$\gamma_B^* = \psi_{B+1}^* - \psi_B = \delta_{B+1}^*\psi_B - \beta_B^* .$$

V. PREPARATION OF INPUT

A. General Description

Input to the CLIP 1 program consists of a title card, control information, a description of the geometry, pointwise or regionwise sources, and cross sections for each region. Each type of data is identified by a series number ranging from 10000 to 60000.

B. Description of Series

1. Title Card

Columns 1-67 are available for problem identification
Columns 68-72 must contain CLIP 1.

2. Card 10000

a. Number of regions ($\leq$ 50).
b. Number of points ($\leq$ 501), with the origin counted as point number one.
c. Right-hand boundary condition (0 = zero flux, 1 = zero gradient).
d. Type of input source (0 = regionwise, 1 = pointwise).
e. Maximum number of iterations the problem is to run. Use a zero if the problem is to run until it converges.
f. Number of fuel regions. Use a zero if there are no fuel regions specified. The purpose for specifying fuel regions is to obtain an edit of:

$$\sum_{\text{fuel regions}} \int \psi_o r dr \quad \text{and} \quad \sum_{\text{fuel regions}} \Sigma_a \int \psi_o \, r dr .$$

If fuel regions are not specified this edit will be omitted.

g. Convergence criterion (a floating point number).

3. Card(s) 20000

This series contains, in order, the region numbers of the fuel regions.

4. Card(s) 30000

This series contains, in order, the interface point numbers. The last number must be the outer boundary, and all numbers must be odd.

5. Card(s) 40000

This series contains the mesh spacing for each region.

6. Card(s) 50000

This series contains the regionwise sources. If regionwise sources are not to be specified, this series must not be used.

7. Card(s) 51000

This series contains the pointwise sources. At an interface, list only the source which is dependent on the region to the left. If pointwise sources are not specified, do not use this series.

8. Card(s) 52000

This series contains those interface sources which are dependent on the region to the right of the interface.

9. Card(s) 60000

This series contains the cross sections for each region. There will be one card for each region containing Σ_a, Σ_{so}, Σ_{s1}, Σ_{s2}, Σ_{s3}.

10. Format

a. Card numbers must start in column one.
b. Each piece of data is separated from the next by a comma. The last number on a card must not be followed by a comma.
c. The first blank column on a card indicates the end of data for that card.
d. Columns 1-72 are available for data.
e. If it is necessary to continue the data within a series on additional cards, these cards must be numbered consecutively, i.e. 60000, 60001, etc.
f. The following parameters must be integers: all control parameters appearing on card 10000 except for the convergence criterion, interface numbers, and fuel region numbers. These numbers must not contain a decimal point.
g. All other data must be in floating-point form. There are several ways of expressing a floating point number. The value 0.001234 may be written as 0.001234, or 0.1234E-2.

VI. DESCRIPTION OF OUTPUT

The following information will be edited for each problem:

1. A listing is made of the data deck.

2. The calculated group parameters, D_i and Σ_{Ti} are printed.

3. Values for λ_{max} and λ_{min} will be printed after each iteration except the first.

4. The point number, radius, and fluxes will be printed after convergence has taken place or after the problem has been stopped after a given number of iterations.

5. The width of each region is listed.

6. The following integrals calculated over each region by Simpson's Rule, will be printed:

   a. $\int \psi_o r dr$

   b. $\int S r dr$

   c. $\Sigma_a \int \psi_o r dr$

7. Balance checks are printed. A comparison of the following values should give some estimate of the adequacy of the mesh spacing.

   a. $$r\psi_3 \Big|_b^B = \int_b^B (S - \Sigma_a \psi_o) r dr$$

   where

   $$r_n \psi_{3,n} = D_o \left\{ r_n \left[\frac{\psi_{1,n+1} - \psi_{1,n-1}}{2h} - \frac{\psi_{2,n+1} - \psi_{2,n-1}}{2h} - \frac{\psi_{o,n+1} - \psi_{o,n-1}}{2h} \right] - 2\psi_{2,n} \right\}$$

   b. $$r\psi_4 \Big|_b^B = \frac{1}{3} \int (S - \Sigma_a \psi_o - \Sigma_1 \psi_1) r dr$$

   where

   $$r_n \psi_{4,n} = -\frac{D_1}{5} \left\{ r_n \left(\frac{\psi_{1,n+1} - \psi_{1,n-1}}{h} \right) - \frac{r_n}{3} \left(\frac{\psi_{2,n+1} - \psi_{2,n-1}}{2h} \right) - \frac{2}{3} \psi_{2,n} \right\}$$

   c. $$r^3 \psi_5 \Big|_b^B = \frac{4}{3} \int_b^B r^3 \left[S_2 - \left(\frac{D_1}{5} + 6D_o \right) \left(\frac{\psi_2}{r^2} \right) - \frac{5}{2} (\Sigma_a + \Sigma_s - \Sigma_{s2}) \psi_2 \right] dr$$

   where

   $$r_n^3 \psi_{5,n} = -\frac{4}{3} D_2 r_n^3 \left[\frac{\psi_{2,n+1} - \psi_{2,n-1}}{2h} - \frac{2}{r_n} \psi_{2,n} \right] .$$

VII. OPERATING INSTRUCTIONS

A. Printer Board: AEROJET

B. Tapes:

1. Logical number 1 - BCD input tape. This tape is not used if the input data is on cards.

2. Logical number 5 - output tape.

C. Sense Switches:

1. Number 1 - down for BCD card input, normal for BCD tape input.
2. Number 5 - normal except when the operator desires an on-line monitor of the convergence. With this sense switch depressed, which may be done at any time during the running of the problem, values of λ_{max} and λ_{min} are printed on-line after each iteration. Each of these parameters must be approaching unity if the problem is to converge.

D. Console: CLEAR, LOAD CARDS

E. Card Reader: START

Any number of consecutive problems may be run either from cards or BCD tape.

One blank card must be inserted after each problem.

F. Transfer of Control:

If desired, control may be transferred to the beginning of the program by manually executing a transfer to 140_8.

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