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Title: **PROBABILITY OF SURVIVAL FROM MULTIPLE WEAPON ATTACKS**

Author(s): Gregory H. Canavan, DDP

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PROBABILITY OF SURVIVAL FROM MULTIPLE WEAPON ATTACKS

Gregory H. Canavan

Exact and approximate~~ies~~ PROBABILITIES of missiles surviving attacks by multiple weapons are derived and discussed. Both are readily computable; the exponential approximation is also suited to analytic optimizations.

This note discusses the survival probabilities for multiple weapons missile attacks. It derives analytic and approximate expressions for average survival probabilities. Both are readily computable. The exponential approximation is sufficiently useful for rough calculations and is well suited to analytic optimizations

Model. The probability of survival of a missile silo against the attack of a weapon with a kill probability p is q = 1 - p. The probability of survival against two independent weapons is q^2; the probability of survival against weapons is q^3; etc.

In an attack of N weapons on M missiles, the number of weapons per silo will vary from silo to silo. There is an average of n = N/M weapons per silo, but the number of weapons on each silo must be an integer. If the integer part of n is i = Int (N/M) and the remainder is r = n - i, the most efficient allocation of weapons is to place i + 1 weapons on a fraction r of the silos and i weapons on the remaining 1 - r, which gives an average probability of survival

$$Q = rq^{i+1} + (1-r)q^i, \qquad (1)$$

which is shown in Fig. 1 as a function of the number of weapons N of kill probability p = 0.6 and 0.8 for M = 40 silos. For each value of p, the curves fall linearly for N up to 40, at which each reaches the single coverage values of Q = 1 - p, which are 0.4 and 0.2, respectively. They again fall linearly with different slopes for N up to 80, at which each reaches the single coverage values of $Q = (1 - p)^2$, which are 0.16 and 0.04, respectively. These values of p and the N/M of 1/4 to 2 roughly bound the attacks of interest.

The approximate probability of survival can be bounded by treating the number of weapons per silo, r, to be a fraction, which gives an approximate average probability of survival

$$Q' = q^r , \qquad (2)$$

which is shown and compared with Q in Fig. 2. The piecewise linear curves are those from Fig. 1. The smooth curves are from Eq. (2). As expected, the differences between them are small for large N/M, where the error introduced by treating r as continuous should be small. The approximation is exact at N = 40 and 80, i.e., r = 1, and 2. The error is greatest for N small and p large, where it is about 10-20%. There i = 0, r = N/M, and

$$Q(r < 1) = (N/M)(1 - p) + (1 - N/M)q^0 = 1 - (N/M)\,p, \qquad (3)$$

while

$$Q' = (1 - p)^{N/M}, \qquad (4)$$

which only agrees with Q to first order. Between the integer values of r, for which it is exact, Q' always lies below Q. so Q' always underestimates the survival probability by a modest extent.

Summary and conclusions. This note discusses the survival probabilities for multiple weapons missile attacks. It derives analytic and approximate expressions for average survival probabilities. Both are readily computable. The exponential approximation is sufficiently useful for rough calculations and is well suited to analytic optimizations.

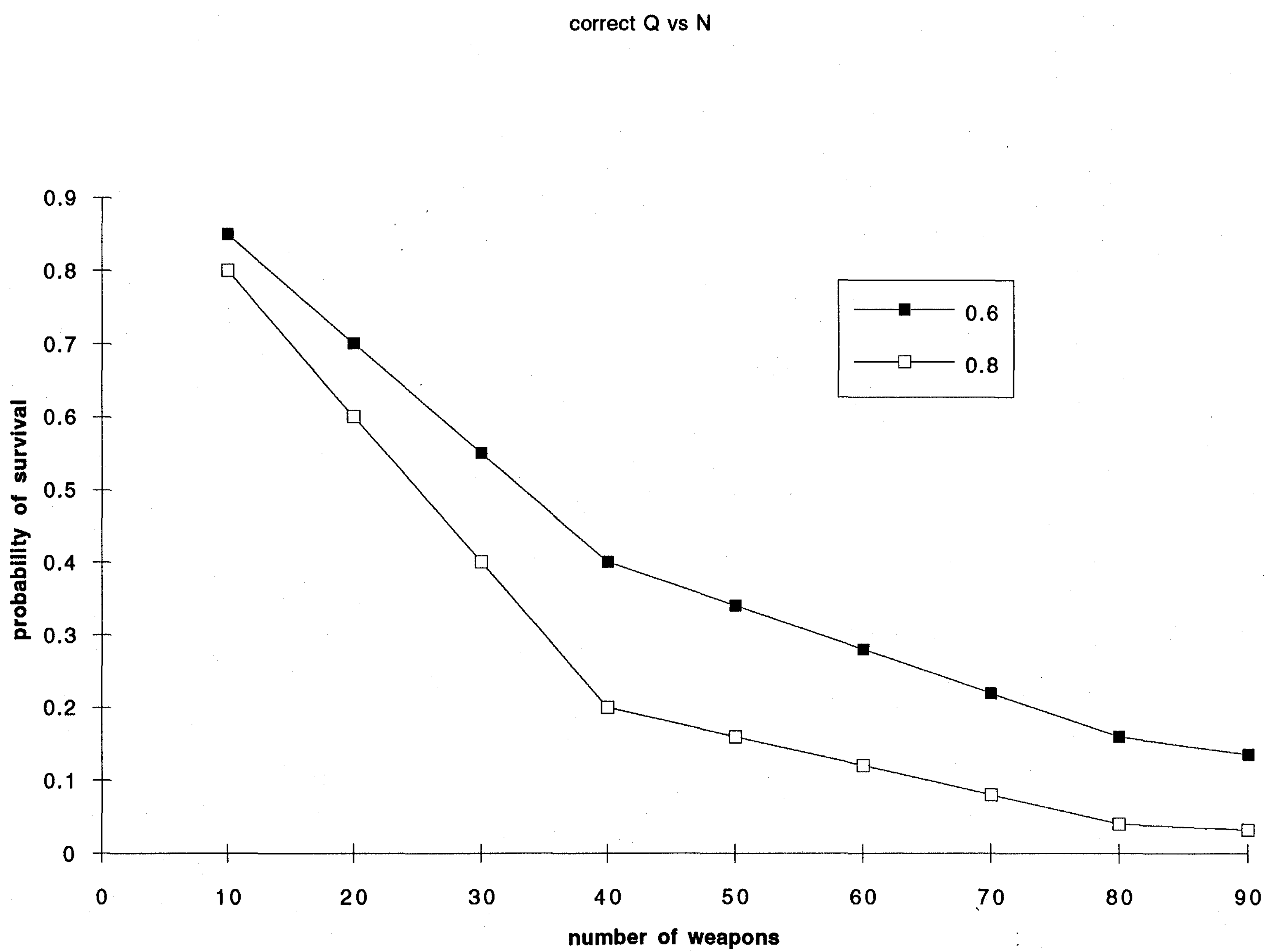

Fig. 1. Average probability of survival vs number os weapons for 40 silos.

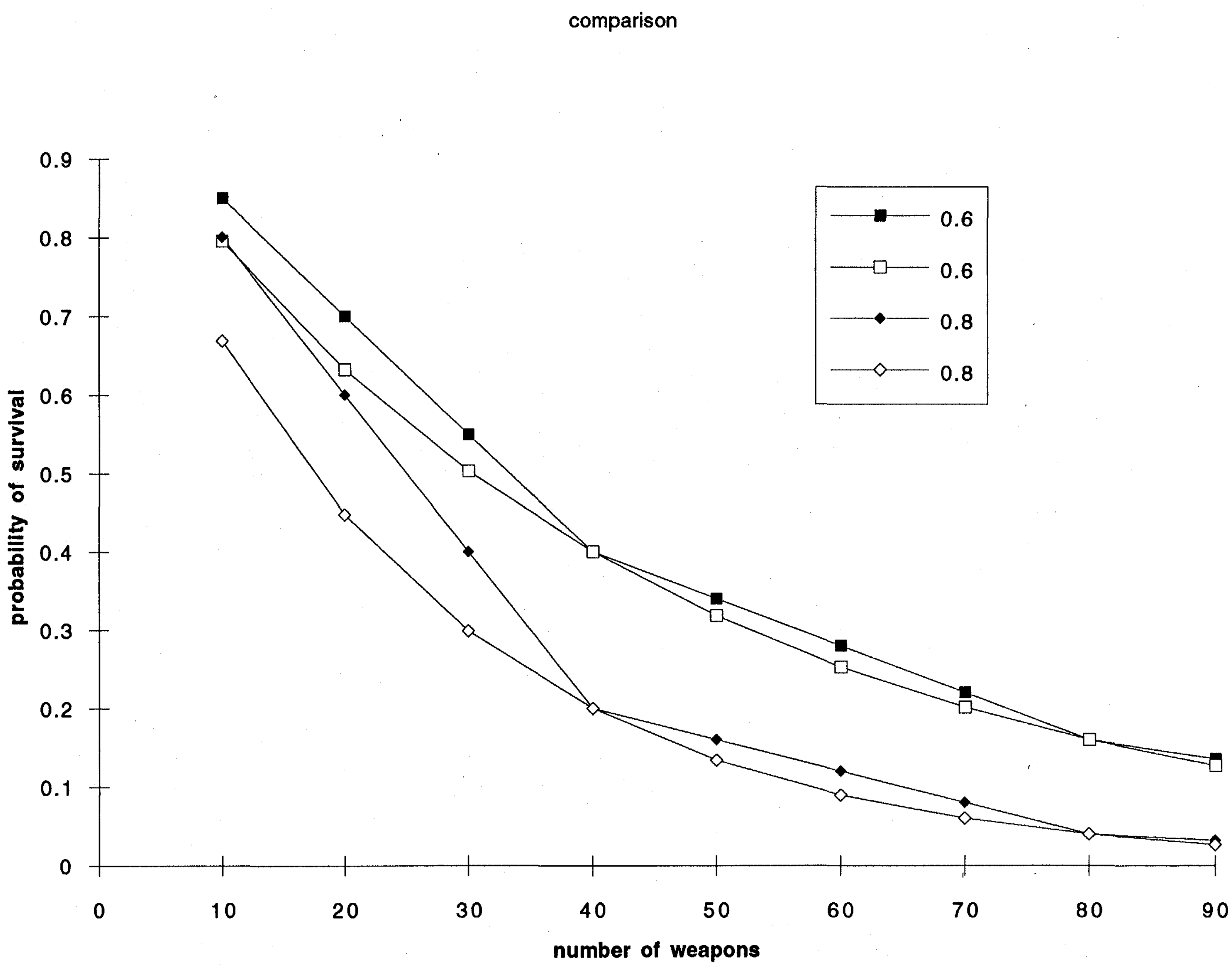

Fig. 2. Exact (upper) and approximate (lower) probabilities of survival of 40 silos vs various numbers of weapons.