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Plume Rise

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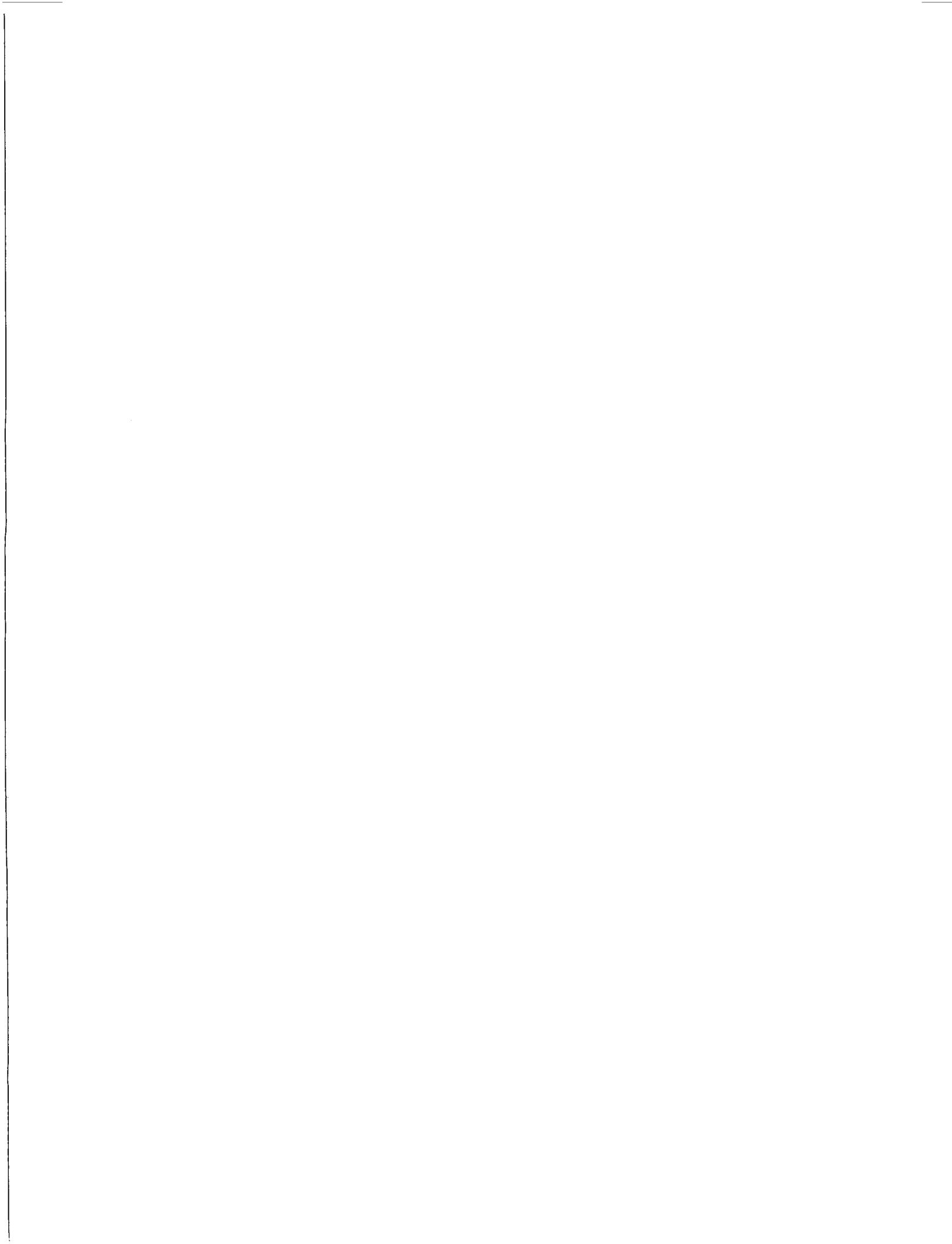
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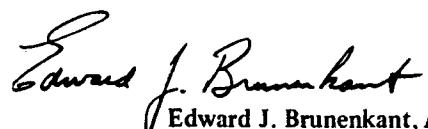
FOREWORD

Scientists and technologists have been concerned in recent years about the "explosion" of original literature engendered by the staggering volume of research and development being undertaken throughout the world. It has proved all but impossible for scientific workers to keep up with current progress even in quite narrow fields of interest. Automated retrieval systems for identifying original literature pertinent to the interests of individuals are being developed. These systems are only a partial solution, however, because the original literature is too large, too diverse, too uneven in quality, to fully satisfy by itself the information needs of scientists.

In this situation of vastly expanding knowledge, there is increasing recognition of the valuable role that can be played by critical reviews of the literature and of the results of research in specialized fields of scientific interest. Mr. Briggs's study, the third published in the AEC Critical Review Series, is an excellent example of this genre.

This review is also significant as a further step in the unceasing effort of the AEC to assure that nuclear plants operate safely. *Plume Rise* is a much needed addition in a field in which a meteorologist must choose from over 30 different plume-rise formulas to predict how effluents from nuclear plants are dispersed into the atmosphere. Mr. Briggs presents and compares all alternatives, simplifies and combines results whenever possible, and makes clear and practical recommendations.

The Atomic Energy Commission welcomes any comments about this volume, about the AEC Critical Review Series in general, and about other subject areas that might beneficially be covered in this Series.



Edward J. Brunenkant
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Division of Technical Information
U. S. Atomic Energy Commission

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SYNOPSIS

The mechanism of plume rise and dispersion is described in qualitative terms with emphasis on possible aerodynamic, meteorological, and topographical effects. Plume-rise observations and formulas in the literature are reviewed, and a relatively simple theoretical model is developed and compared with other models. All available data are used to test the formulas for a number of idealized cases.

The inverse wind-speed relation, $\Delta h \propto u^{-1}$, is shown to be generally valid for the rise of a hot plume at a fixed distance downwind in near-neutral conditions. Nine formulas of this type are compared with data from sixteen different sources, and the best agreement is obtained from the "2/3 law," $\Delta h = 1.6F^{4/5}u^{-1}x^{2/5}$, modified by the assumption that a ceiling height is reached at a distance of ten stack heights downwind. The term F is proportional to the heat emission. In uniform stratification buoyant plumes are seen to follow the 2/3 law until a ceiling height of 2.9 (F/us)^{1/5} is approached, where s is proportional to the potential temperature gradient. In calm conditions the formula $\Delta h = 5.0F^{4/5}s^{-3/5}$ is in excellent agreement with a wide range of data.

Formulas of a similar type are recommended for nonbuoyant plumes on the basis of much more limited data.

1 INTRODUCTION

The calculation of plume rise is often a vital consideration in predicting dispersion of harmful effluents into the atmosphere, yet such a calculation is not straightforward. The engineer or meteorologist must choose from more than thirty different plume-rise formulas, and a casual search through the literature for help in choosing is likely to be confusing. The purpose of this survey is to present an overall view of the pertinent literature and to simplify and combine results whenever possible, with the objective of setting down clear, practical recommendations.

The importance of stack height and buoyancy in reducing ground concentrations of effluents has been recognized for at least 50 years.¹ In a 1936 paper Bosanquet and Pearson² showed that under certain conditions the maximum ground concentration depends on the inverse square of stack height, and experience soon confirmed this relationship.³ Later the stack height in this formula came to be replaced by the "effective stack height," which was defined as the sum of the actual stack height and the rise of the plume above the stack. Since smoke plumes from large sources of heat often rise several stack heights above the top of the stack even in moderately high winds, plume rise can reduce the highest ground concentration by an order of magnitude or more.

In spite of the importance of plume rise in predicting dispersion, there is much controversy about how it should be calculated. A recent symposium on plume behavior,⁴ held in 1966, summarizes the current state of affairs. Lucas expressed a desire for better agreement between empirical results and stated flatly, "There are too many theoretical formulae and they contradict one another!" Spurr lamented, "The argument for and against different plume rise formulae can be discussed clinically by

physicists and theoreticians, but the engineer who has to apply the formulae is obliged to make a choice." He then compared five recent formulas for a specific example and concluded that the results varied by a factor of 4 in the calculated maximum ground concentration. Even worse examples were given in the same symposium.

There are reasons for the lack of agreement. Different techniques for measuring plume height and wind speed can account for some of the disparity in the data, but the differences in the results are due primarily to the different concepts of what constitutes effective stack height. A recent paper by Slawson and Csanady states:

With an ostrich-like philosophy, the effective stack height is often defined to be the point where the plume is just lost sight of. It is then not very surprising to find that the observed thermal rise of the plume depends, for example, on a power of the heat flux ranging from $\frac{1}{4}$ to 1.0, influenced by a number of factors including, presumably, the observer's eyesight.†

It was natural for early plume-rise observers to assume that a smoke plume leveled off in all conditions and that the plume was near the height of leveling off when it was inclined only slightly above the horizontal; subsequent observations suggest otherwise. The early formula of Holland,⁶ sometimes called the Oak Ridge formula, was based on photographic data that followed the plumes only 600 ft downwind,⁷ yet recent data of the Tennessee Valley Authority (TVA) show plumes still rising at 1 and even 2 miles downwind. Over this distance even a small inclination above the horizontal becomes important. The plume height normally of greatest concern is that above the point of maximum ground concentration, and it seems logical to define this as the effective stack height, as suggested by Lucas.⁴ A major difficulty with this definition is that none of the present observations goes that far downwind. In practice we must choose formulas for plume rise on the basis of agreement with data on hand and, at the same time, be aware of the limitations of the data.

General plume behavior, which is discussed briefly in the next chapter, has been described in greater detail in other publications. The textbook by Sutton⁸ first reviewed all aspects of diffusion, including plume rise. Pasquill⁹ surveyed the subject in considerably more detail and on the basis of more data than was previously available. The first edition of *Meteorology and Atomic Energy*¹⁰ adequately covered the qualitative aspects of plume rise and diffusion, but the new edition¹¹ is quantitatively more up-to-date. An excellent survey by Strom¹² reviewed all aspects of plume behavior, including the potential for modeling dispersion. Smith briefly reviewed the main qualitative considerations in plume rise and diffusion¹³ and more recently discussed the practical aspects of dispersion from tall stacks.¹⁴ The practical experience of TVA has been described by Thomas,¹⁵ by Gartrell,¹⁶ and by Thomas, Carpenter, and Gartrell.¹⁷ The British experience with diffusion from large power plants and their tall-stack policy has been analyzed by Stone and Clark.¹⁸

Several attempts have been made at setting down definite procedures for calculating diffusion, including the plume rise. The first, primarily concerned with dust

†Ref. 5, page 311.

deposition, was by Bosanquet, Carey, and Halton.¹⁹ Hawkins and Nonhebel⁷ published a procedure based on a revised formulation for plume rise by Bosanquet.²⁰ More recently, Nonhebel²¹ gave detailed recommendations on stack heights, primarily for small plants, based on the Bosanquet plume-rise formula and the Sutton diffusion formula.^{8,9} Many of these recommendations were adopted in the British *Memorandum on Chimney Heights*,²² which has been summarized by Nonhebel.²³ Scorer and Barrett²⁴ outlined a simple procedure applicable to long-term averages. A CONCAWE† publication^{25,26} presented a method for determining stack height for a plant built on flat, open terrain with a limited range of gas emissions; this method included a formula for plume rise based on regression analysis of data. The American Society of Mechanical Engineers (ASME)²⁷ has prepared a diffusion manual with another formula for plume rise. The implications of this formula and the CONCAWE formula are discussed in Ref. 28. Further discussions of plume-rise questions can be found in Refs. 29 to 33.

†CONCAWE (Conservation of Clean Air and Water, Western Europe), a foundation established by the Oil Companies' International Study Group for the Conservation of Clean Air and Water.

2 BEHAVIOR OF SMOKE PLUMES

Plume dispersion is most easily described by discussing separately three aspects of plume behavior: (1) aerodynamic effects due to the presence of the stack, buildings, and topographical features; (2) rise relative to the mean motion of the air due to the buoyancy and initial vertical momentum of the plume; and (3) diffusion due to turbulence in the air. In reality all three effects can occur simultaneously, but in the present state of the art they are treated separately and are generally assumed not to interact. This is probably not too unrealistic an assumption. We know that undesirable aerodynamic effects can be avoided with good chimney design. Clearly the rise of a plume is impeded by mixing with the air, but there is not much agreement on how important a role atmospheric turbulence plays. It is known that a rising plume spreads outward from its center line faster than a passive plume, but this increased diffusion rate usually results in an only negligible decrease of ground concentrations.

The following sections discuss the three aspects of plume diffusion. Symbols and frequently used meteorological terms are defined in Appendixes B and C.

DOWNWASH AND AERODYNAMIC EFFECTS

Downwash of the plume into the low-pressure region in the wake of a stack can occur if the efflux velocity is too low. If the stack is too low, the plume can be caught in the wake of associated buildings, where it will bring high concentrations of effluent

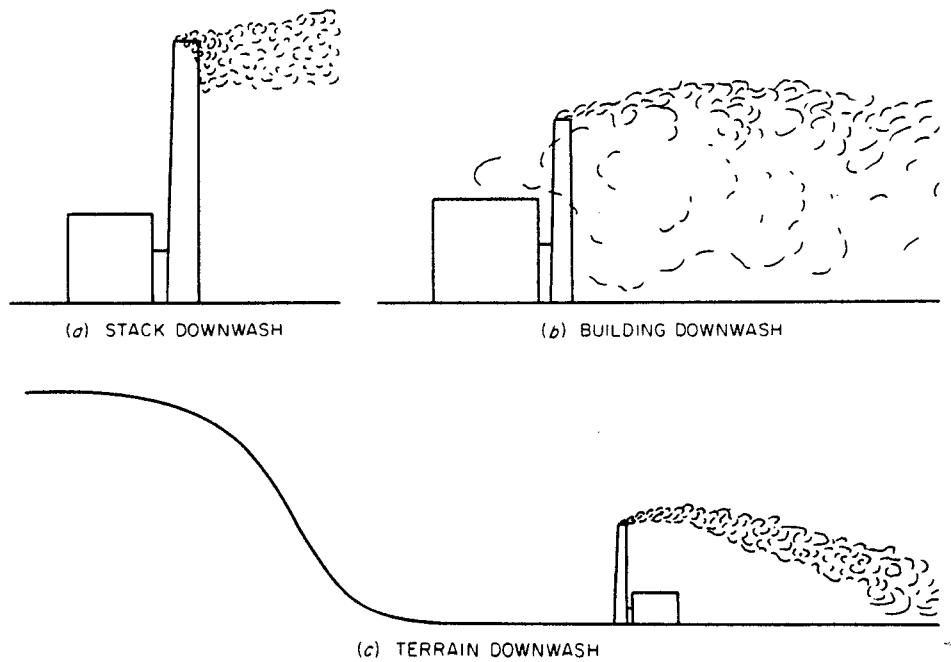


Fig. 2.1 Undesirable aerodynamic effects.

to the ground and even inside the buildings. A similar effect can occur in the wake of a terrain feature. These three effects are illustrated in Fig. 2.1.

The wind-tunnel studies of Sherlock and Stalker³⁴ indicate that downwash is slight as long as $w_0 > 1.5u$, where w_0 is the efflux velocity of gases discharging from the stack and u is the average wind speed at the top of the stack. These results are consistent with elementary theoretical considerations: when $w_0 > 1.8u$, the upward momentum of the stack gases should overcome the downward pressure gradient produced by the wind blowing around the stack on the basis of the pressure distribution around an infinite circular cylinder in a crosswind given by Goldstein;³⁵ when $w_0 < 0.8u$, the smoke can be sucked into the lower pressure region across the entire back of the chimney. If the plume is very buoyant, i.e., if the efflux Froude number, Fr , is 1.0 or less, the buoyancy forces are sufficient to counteract some of the adverse pressure forces, and the preceding criterion for w_0 could be relaxed. This factor probably abates downwash at the Tallawarra plant, cited in Table 5.1, where

$$Fr = \frac{w_0^2}{g(\Delta T/T)D} = 0.5$$

Experiments are still needed to determine quantitatively the effect of the efflux Froude number on the abatement of downwash, unfortunately, the experiments of

Sherlock and Stalker involved only high values of Fr , and thus buoyancy was not a significant factor.

Nonhebel²¹ recommends that w_0 be at least 20 to 25 ft/sec for small plants (heat emission less than 10^6 cal/sec) and that w_0 be in the neighborhood of 50 to 60 ft/sec for a large plant (e.g., with a heat emission greater than 10^7 cal/sec). Larger efflux velocities are not necessary since such high winds occur very rarely; in fact, much higher velocities may be detrimental to the rise of a buoyant plume because they are accompanied by more rapid entrainment of ambient air into the plume. Scorer³⁶ reports that, when efflux velocity must be low, placing a horizontal disk that is about one stack diameter in breadth about the rim of the chimney top will prevent downwash.

One of the most enduring rules of thumb for stack design was a recommendation³⁷ made in 1932 that stacks be built at least 2.5 times the height of surrounding buildings, as illustrated in Fig. 2.2. If such a stack is designed with sufficient efflux velocity to avoid downwash, the plume is normally carried above the region of downflow in the wake of the building. If the stack height or efflux velocity is slightly

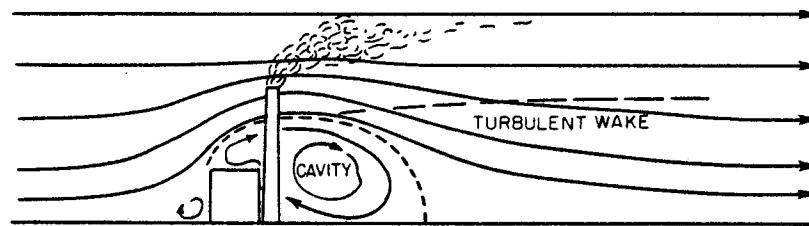


Fig. 2.2 Flow past a typical power plant.

lower, in high winds the plume will get caught in the downflow and be efficiently mixed to the ground by the increased turbulence in the wake of the building. If the stack is less than twice the building height, at least part of the plume is likely to be caught in the cavity of air circulating in the lee of the building; this can bring high concentrations of effluent to the ground near the building and even into the building. The streamlines in Fig. 2.2 also illustrate the advantage of constructing a chimney on the side of the building facing the prevailing wind, where the air is still rising.

Still, this is only a rough rule, because the air-flow pattern around a building depends on the particular shape of the building and on the wind direction. Details on these effects are given by Halitsky.³⁸ Also, for sources with very small emissions, the rule for stacks 2.5 times higher than nearby buildings may be impractical. Lucas³⁹ suggested a correction factor for smaller stacks, and this has been incorporated into the British *Memorandum on Chimney Heights*.²² The correction factor is also reported by Ireland⁴⁰ and Nonhebel.²³ The behavior of effluents from very short stacks has been discussed by Barry,⁴¹ Culkowski,⁴² and Davies and Moore.⁴³ For such sources plume rise is probably negligible.

It is much more difficult to give any rules about the effect of terrain features, partly because of the great variety of possibilities. Fortunately the general effect of terrain and buildings on a plume can be fairly well modeled in a wind tunnel, such as the one at New York University or at the D.S.I.R. (National Physics Laboratory, England). Stümke^{44,45} gives a method for correcting effective stack height for a simple step in the terrain, but only streamline flow is considered.

A curious aerodynamic effect sometimes observed is bifurcation, in which the plume splits into two plumes near the source. This is discussed by Scorer,³⁶ and a good photograph of the phenomenon appears in Ref. 46. Bifurcation arises from the double-vortex nature of a plume in a crosswind, but it is not clear under what conditions the two vortices can separate. However, bifurcation is rare and appears to occur only in light winds.

Sciven⁴⁷ discusses the breakdown of plumes into puffs due to turbulent fluctuations in the atmosphere. Scorer⁴⁶ discusses the breakdown into puffs of buoyant plumes with low exit velocity and includes a photograph. The process appears to be associated with a low efflux Froude number, but a similar phenomenon could be initiated through an organ-pipe effect, e.g., if the vortex-shedding frequency of the stack corresponds to a harmonic mode of the column of gas inside the stack.

PLUME RISE

Although quantitative aspects of plume rise are the concern of the bulk of this report, only the qualitative behavior is discussed in this section. More detailed discussions can be found in a paper by Batchelor⁴⁸ and a book by Scorer.⁴⁶ It is assumed that the plume is not affected by the adverse aerodynamic effects discussed in the previous section since these effects can be effectively prevented.

The gases are turbulent as they leave the stack, and this turbulence causes mixing with the ambient air; further mechanical turbulence is then generated because of the velocity shear between the stack gases and the air. This mixing, called entrainment, has a critical effect on plume rise since both the upward momentum of the plume and its buoyancy are greatly diluted by this process. The initial vertical velocity of the plume is soon greatly reduced, and in a crosswind the plume acquires horizontal momentum from the entrained air and soon bends over.

Once the plume bends over, it moves horizontally at nearly the mean wind speed of the air it has entrained; however, the plume continues to rise relative to the ambient air, and the resulting vertical velocity shear continues to produce turbulence and entrainment. Measurements of the mean velocity distribution in a cross section of a bent-over plume show the plume to be a double vortex, as shown in Fig. 2.3. Naturally the greatest vertical velocity and buoyancy occur near the center of the plume, where the least mixing takes place. As the gases encounter ambient air above the plume, vigorous mixing occurs all across the top of the plume. This mixing causes the plume diameter to grow approximately linearly with height as it rises.

If the plume is hot or is of lower mean molecular weight than air, it is less dense than air and is therefore buoyant. If the heat is not lost and the atmosphere is well mixed, the total buoyant force in a given segment of the moving plume remains constant. This causes the total vertical momentum of that segment to increase at a constant rate, although its vertical velocity may decrease owing to dilution of the momentum through entrainment.



Fig. 2.3 Cross section of mean velocity distribution in a bent-over plume.

At some point downwind of the stack, the turbulence and vertical temperature gradient of the atmosphere begin to affect plume rise significantly. If the atmosphere is well mixed because of vigorous turbulent mixing, it is said to be neutral or adiabatic. In such an atmosphere the temperature decreases at the rate of 5.4°F per 1000 ft. This rate of decrease, which is called the adiabatic lapse rate (Γ), is the rate at which air lifted adiabatically cools owing to expansion as the ambient atmospheric pressure decreases. If the temperature lapse of the atmosphere is less than the adiabatic lapse rate, the air is said to be stable or stably stratified. Air lifted adiabatically in such an environment becomes cooler than the surrounding air and thus tends to sink back. If the temperature actually increases with height, the air is quite stable. Such a layer of air is called an inversion. If the temperature lapse of the atmosphere is greater than the adiabatic lapse rate, the air is said to be unstable or unstably stratified. Air lifted adiabatically in such an environment becomes warmer than the surrounding air, and thus all vertical motions tend to amplify.

The potential temperature, θ , is defined as the temperature that a sample of air would acquire if it were compressed adiabatically to some standard pressure (usually 1000 millibars). The potential temperature is a convenient measure of atmospheric stability since

$$\frac{\delta\theta}{\delta z} = \frac{\delta T}{\delta z} + \Gamma \quad (2.1)$$

where $\Gamma = 5.4^{\circ}\text{F}/1000 \text{ ft} = 9.8^{\circ}\text{C}/\text{km}$. Thus the potential temperature gradient is positive for stable air, zero for neutral air, and negative for unstable air.

If the ambient air is stable, i.e., if $\delta\theta/\delta z > 0$, the buoyancy of the plume decays as it rises since the plume entrains air from below and carries it upward into regions of warmer ambient air. If the air is stable throughout the layer of plume rise, the plume eventually becomes negatively buoyant and settles back to a height where it has zero buoyancy relative to the ambient air. The plume may maintain this height for a distance of 20 miles or more from the source. In stable air atmospheric turbulence is suppressed and has little effect on plume rise.

If the atmosphere is neutral, i.e., if $\delta\theta/\delta z = 0$, the buoyancy of the plume remains constant in a given segment of the plume provided the buoyancy is a conservative property. This assumes no significant radiation or absorption of heat by the plume or loss of heavy particles. Since a neutral atmosphere usually comes about through vigorous mechanical mixing, a neutral atmosphere is normally turbulent. Atmospheric turbulence then increases the rate of entrainment; i.e., it helps dilute the buoyancy and vertical momentum of the plume through mixing.

If the atmosphere is unstable, i.e., if $\delta\theta/\delta z < 0$, the buoyancy of the plume grows as it rises. Increased entrainment due to convective turbulence may counteract this somewhat, but the net effect on plume rise is not well known. The few usable data for unstable situations seem to indicate slightly higher plume rise than in comparable neutral situations. On warm, unstable afternoons with light wind, plumes from large sources rise thousands of feet and even initiate cumulus clouds.

Measurements are made difficult by fluctuations in plume rise induced by unsteady atmospheric conditions. On very unstable days there are large vertical velocity fluctuations due to convective eddies that may cause a plume to loop, as shown in Fig. 2.5d. Figure 2.4 illustrates the large variations in plume rise at a fixed distance downwind during unstable conditions. On neutral, windy days the plume trajectory at any one moment appears more regular, but there still may be large fluctuations in plume rise due to lulls and peaks in the horizontal wind speed. Since the wind is responsible for the horizontal stretching of plume buoyancy and momentum, the wind strongly affects plume rise. In stable conditions there is very little turbulence, and plume rise is also less sensitive to wind-speed fluctuations. This can be seen in Fig. 2.4. In this case the plume leveled off in stable air, and its rise increased in a smooth fashion as the air became less and less stable owing to insolation at the ground.

One might ask whether plume rise is affected by the addition of latent heat that would occur if any water vapor in the stack gases were to condense. This is an important question because there may be as much latent heat as there is sensible heat present in a plume from a conventional power plant. It is true that some water vapor may condense as the plume entrains cooler air, but calculations show that in most conditions the plume quickly entrains enough air to cause the water to evaporate again. Exceptions occur on very cold days, when the air has very little capacity for water vapor, and in layers of air nearly saturated with water vapor, as when the plume rises through fog. Observations by Serpolay⁴⁹ indicate that on days when cumulus clouds are present condensation of water from entrained air may increase the buoyancy of the plume and enhance its ability to penetrate elevated stable layers.

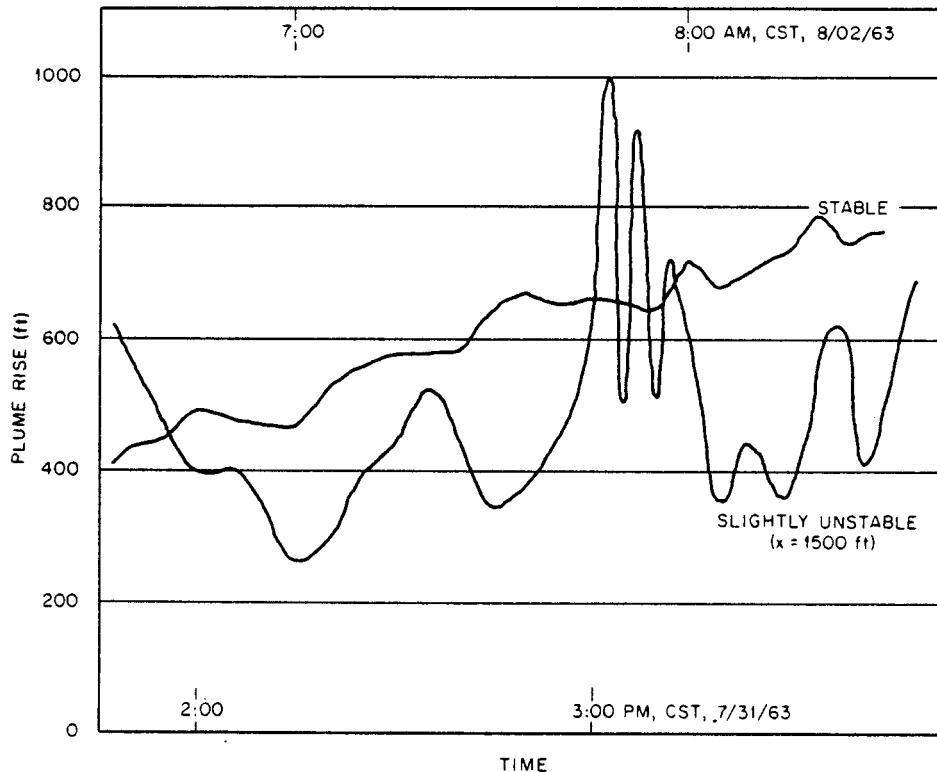


Fig. 2.4 Fluctuations of plume rise with time (Gallatin Plant, Tennessee Valley Authority).

Ordinarily only the sensible heat of the plume should be used in calculations.

One might also ask whether thermal radiation can significantly alter the heat content of a plume, i.e., its buoyancy. Not much is known about the radiative properties of smoke plumes, but crude calculations show that radiation is potentially important only for very opaque plumes some thousands of feet downwind and should have little effect on clean plumes from modern power plants or on plumes from air-cooled reactors. Plumes from TVA plants have been observed to maintain a constant height for 20 miles downwind in the early morning; thus there appears to be negligible heat loss due to radiation.

DIFFUSION

Detailed diffusion calculations are beyond the scope of this review, but the main types of diffusion situations should be discussed with regard to plume rise. On a clear

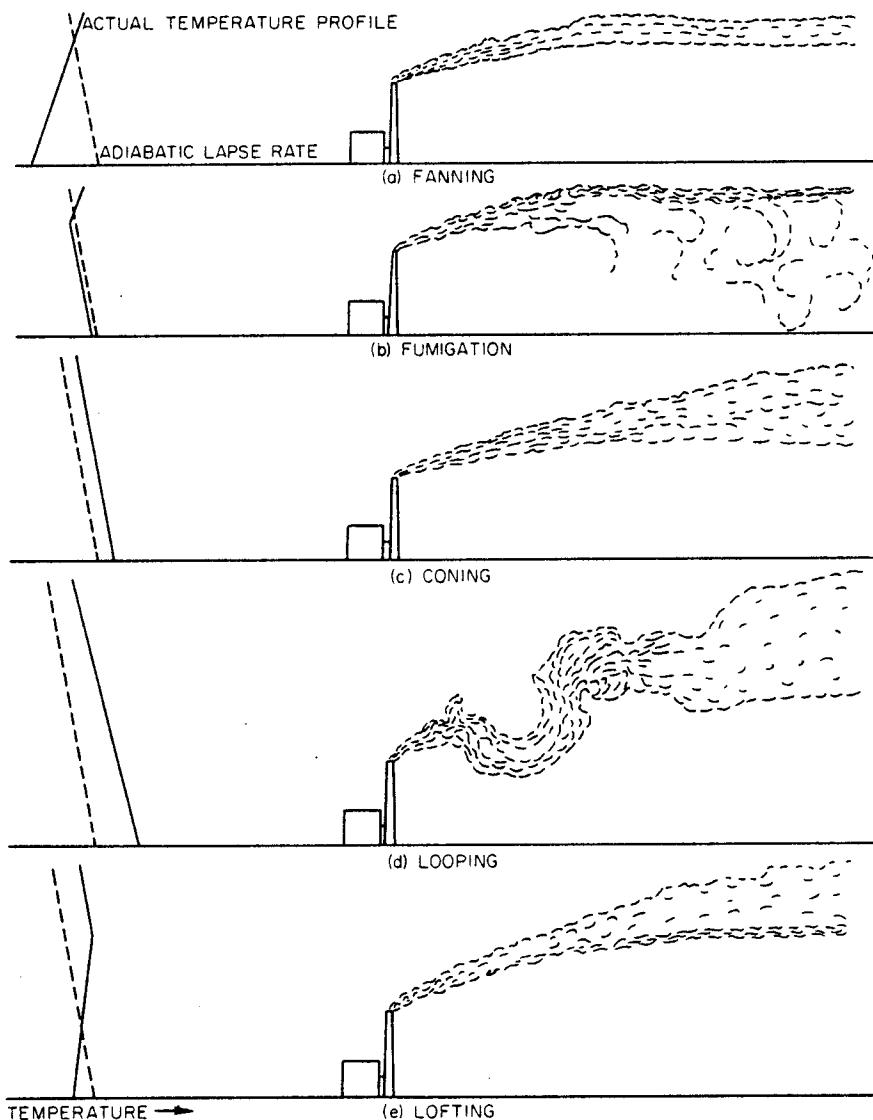


Fig. 2.5 Effect of temperature profile on plume rise and diffusion.

night the ground radiates heat, most of which passes out into space. In this process the air near the ground is cooled, and an inversion is formed. The stable layer may be several thousand feet deep; so most plumes rising through it lose all their buoyancy and level off. This behavior is called fanning and is pictured in Fig. 2.5a. When the sun comes up, convective eddies develop and penetrate higher and higher as the ground warms up. As the eddies reach the height at which the plume has leveled off, they rapidly mix the smoke toward the ground while the inversion aloft prevents upward diffusion. This phenomenon, called fumigation, can bring heavy concentrations of effluent to the ground (Fig. 2.5b). Just after an inversion has been broken down by convective eddies or in cloudy, windy conditions, the atmosphere is well mixed and nearly neutral. Then the plume rises and diffuses in a smooth fashion known as coning (Fig. 2.5c). As the heating of the ground intensifies, large convective eddies may develop and twist and fragment the plume in a looping manner (Fig. 2.5d). Diffusion is then more rapid than in a neutral atmosphere. The convection dies out as the sun gets lower, and an inversion again starts to build from the ground up. This ground inversion is weak enough at first that the plume can penetrate it, and the plume diffuses upward but is prevented by the stability below from diffusing downward. This lofting period (Fig. 2.5e) is the most ideal time to release harmful effluents since they are then least likely to reach ground.

The meteorological conditions that should be considered in stack design depend on the size of the source, the climatology of the region, and the topography. In reasonably flat terrain, high wind with neutral stratification usually causes the highest ground concentrations since there is the least plume rise in these conditions. The mean concentration of the effluent in the plume is reasonably well described by a Gaussian distribution, for which the maximum ground concentration is given by

$$\chi = \frac{\sigma_z}{\sigma_y} \frac{2Q}{\pi eu h^2} = 0.164 \frac{Q}{uh^2} \quad (2.2)$$

where Q is the rate at which pollutant is emitted, u is the mean wind speed at the source height, and h is the effective stack height (defined as the sum of the actual stack height, h_s , and the plume rise, Δh); σ_z/σ_y is the ratio of the vertical dispersion to the horizontal dispersion and is equal to about 0.7 in a neutral atmosphere for an averaging period of 30 min.²⁵ Variation with distance has been neglected in deriving Eq. 2.2. This equation is valid only when the atmosphere is neutral from the ground up to at least twice the effective stack height. Inversions may exist below this height even in windy conditions. A diffusion model for this case is given by Smith and Singer.⁵⁰ If the plume reaches the height of the inversion and penetrates it, as can be predicted by Eq. 4.30,† none of the effluent reaches the ground. If the plume does not penetrate, the inversion acts as an invisible ceiling and prevents upward diffusion.

A good measure of the efficiency of the diffusion process on a given occasion is

†See "Basic Theory Simplified" in Chapter 4.

Q/χ , the effective ventilation, which has the dimensions of volumetric flow rate (ft^3/sec). For the case just described,

$$\frac{Q}{\chi} = 6.1u h^2 = 6.1u(h_s + \Delta h)^2 \quad (2.3)$$

Naturally the effective ventilation is large for extremely high wind speeds, but it is also large at low values of u because of very high plume rise. It is at some intermediate wind speed that Q/χ attains a minimum, i.e., χ attains a maximum; this wind speed is called the critical wind speed. If the dependence of Δh on u is known, Eq. 2.3 can be differentiated and set equal to zero to find the critical wind speed. The result can be substituted into the plume-rise equation and into Eq. 2.2 to find the highest expected ground concentration for the neutral, windy case, x_{\max} .

There is evidence that fumigation during calm conditions may lead to the highest ground-level concentrations at large power plants. This type of fumigation can occur near the center of large slow-moving high-pressure areas in so-called "stagnation" conditions. Such high-pressure systems usually originate as outbreaks of cold, relatively dense air, and, as these air masses slow down, they spread out much in the manner of cake batter poured into a pan. Since the air underneath the upper surface of these air masses is appreciably colder than the air above it, a subsidence inversion forms and presents a formidable barrier to upward mixing; such an inversion normally occurs 1500 to 4000 ft above the ground.⁵¹ In combination with a near-zero wind speed, a subsidence inversion severely limits atmospheric ventilation, and the little ventilation that occurs is due to convective mixing from the ground up to the inversion.

Fortunately such circumstances are rare except in certain geographical areas. The southeastern United States, one such region, averages 5 to 15 stagnation days a year with the higher figure occurring in the Carolinas and Georgia.⁵¹ Nevertheless, there is only one outstanding case of fumigation during stagnation in all the years of monitoring SO_2 around TVA power plants. In this instance ground concentration near an isolated plant was 50% higher than the maximum observed in windy, neutral conditions, and this condition continued for most of one afternoon. The wind speed was 0 to 1 mph, and the effective ventilation, as defined above, was $1.5 \times 10^8 \text{ cu ft/sec}$ ($4.3 \times 10^6 \text{ m}^3/\text{sec}$). This value is adequate for a small plant but too small for a large plant. There is not much hope of improving the effective ventilation in this rare condition, for a stack would have to be thousands of feet high to ensure that the plume could penetrate a subsidence inversion. The only way to reduce ground concentrations in this case seems to be to reduce the emission of pollutants; accordingly, TVA stockpiles low-sulfur coal for use when the Weather Bureau predicts stagnation conditions.

Similar conditions occur under marine inversions, such as are found along the Pacific coast of the United States. The inversions there are sometimes less than 1000 ft above the ground,⁵¹ and plumes from high stacks can often penetrate them. Such penetration can be predicted by equations presented in later chapters.

Fumigation associated with inversion breakdown may be serious when topography is prominent. If the plume does not rise out of a deep valley during the period of the nighttime inversion, the pollutant will mix fairly uniformly down to the ground during fumigation; therefore concentration is given by

$$x = \frac{Q}{uhW} \quad (2.4)$$

where u is the average velocity of the along-valley drainage flow at night, h is the effective stack height at night, and W is the average width of the valley up to height h .

An elevated plateau can also be subjected to intensified fumigation if during an inversion the plume rises slightly higher than the plateau and drifts over it. This has occurred at a plant on the Tennessee River where the river cuts a 1000-ft-deep gorge through the Cumberland Plateau.¹⁷ Careful consideration should be given to this possibility at such a site. Topographic effects are discussed by Hewson, Bierly, and Gill.⁵²

3 OBSERVATIONS OF PLUME RISE

Dozens of plume-rise observations have been made, and each is unique in terms of type of source and technique of measurement. Observations have been made in the atmosphere, in wind tunnels, in towing channels, and in tanks. Brief descriptions of these experiments are given in this chapter.

MODELING STUDIES

Plume rise is a phenomenon of turbulent fluid mechanics and, as such, can be modeled; i.e., it can be simulated on some scale other than the prototype. There are obvious advantages to modeling plume rise. For example, the model plume can be measured much less expensively than the real plume since it is not necessary to probe high above the ground, and the variables can be controlled at will. The main difficulty is in ensuring that the behavior of the model plume essentially duplicates that of a real smoke plume. The most obvious requirements are that all lengths be scaled down by the same factor and that the wind speed and efflux velocity be scaled down by identical factors. For exact similarity the Reynolds number has to be the same in model and in prototype. The Reynolds number is defined by

$$Re = \frac{vl}{\nu} \quad (3.1)$$

where v is a characteristic velocity, l is a characteristic length, and ν is the kinematic viscosity of air or the fluid in which the model is measured. Exact similarity is seldom

possible in modeling since Re is of the order of 10^6 for a real plume. Fortunately fully turbulent flow is not very dependent on Reynolds number so long as it is sufficiently high. In most experiments Re is at least 10^3 on the basis of efflux velocity and stack diameter, but the adequacy of this value is not certain.

For buoyant plumes the Froude number must be the same in model and in prototype. Since we are unable to scale down gravity, which is a prerequisite for the existence of buoyancy, the basic requirement is that

$$\left(\frac{v^2}{l}\right)_{\text{model}} = \left(\frac{v^2}{l}\right)_{\text{prototype}} \quad (3.2)$$

provided the temperature or density ratios are kept unchanged.

Numerous measurements have been made on the simple circular jet.^{53,54} Schmidt⁵⁵ first investigated the heated plume with zero wind. Yih⁵⁶ studied the transition from laminar to turbulent flow in a heated plume. Later, Rouse, Yih, and Humphreys⁵⁷ studied the detailed distribution of vertical velocity and temperature in a fully turbulent hot plume from a gas flame near the floor of an airtight, high-ceilinged room. They measured temperature with a thermocouple and velocity with a $1\frac{1}{4}$ -in. vane on jeweled bearings. The important result of all these investigations is that both jets and hot plumes are cone shaped in calm, unstratified air. The half-cone angle is smaller for the heated plume than for a jet, and the decreases of temperature and velocity with distance above the source are consistent with heat and momentum conservation principles. Also, the cross-sectional distributions of vertical velocity and temperature excess are approximately Gaussian except close to the source. The characteristic radius describing the temperature distribution in a heated plume is 16% greater than that for the velocity distribution.

Several modeling studies have been made on heated plumes rising through a stable environment. Morton, Taylor, and Turner⁵⁸ confirmed predictions by using measured releases of dyed methylated spirits in a 1-m-deep tank of stratified salt solution. Crawford and Leonard⁵⁹ ran a similar experiment with a small electric heater to generate a plume on the floor of an ice rink. The invisible plume was observed with the Schlieren technique, and convection thermocouples were used to measure the temperature profile of the air above the ice. Their results are, in fact, in good agreement with those of Ref. 58, although they miscalculated the constant in the equation of Ref. 58 by a factor of $2\frac{1}{4}$. Vadot⁶⁰ conducted experiments with an inverted plume of heavier fluid in a tank of salt solution. His inversions were quite sharp in contrast to the smooth density gradients used in the preceding studies.

A number of wind-tunnel investigations of jets in a crosswind have been made. The early study of Rupp and his associates⁶¹ has been used as the basis for a momentum contribution to plume rise by several investigators. Callaghan and Ruggeri⁶² measured the temperature profile of heated jets in experiments in which the efflux velocities were of the order of the speed of sound. Keffer and Baines⁶³ measured rise for only four stack diameters downwind and obtained some velocity and turbulence intensity measurements within the jets. Halitsky⁶⁴ and Patrick⁶⁵ summarized the work of

previous investigators. In addition, Patrick presented new measurements to about 20 stack diameters downstream, including detailed profiles of velocity and concentration of a tracer (nitrous oxide).

The effect of buoyancy on plume rise near the stack was studied by Bryant and Cowdrey^{66,67} in low-speed wind in a tunnel. Vadot⁶⁰ made a study of buoyancy effects in a towing channel with both stratified and unstratified fluids. This study was unusual in that the ambient fluid was at rest and the effect of crosswind was incorporated by towing the source at a constant speed down the channel. This is a valid experimental technique since motion is only relative. However, Vadot's source was a downward-directed stream of dense fluid. There is some question whether a bent-over plume from such a source behaves as a mirror image of a bent-over plume from an upward-directed stream of light fluid. Subtle changes in the entrainment mechanism could take place owing to centrifugal forces acting on the more dense fluid inside the plume. The recent treatment by Hoult, Fay, and Forney⁶⁸ of past modeling experiments tends to confirm this. The bent-over portion of a hot plume behaves much like a line thermal, which was modeled for both dense and light plumes by Richards,⁶⁹ who found that the width of the thermals increased linearly with vertical displacement from their virtual origins, just as had been observed for jets and plumes that were not bent over. The line thermal was also modeled numerically by Lilly.⁷⁰ Lilly did not have enough grid points to reach the shape-preserving stage found in laboratory thermals, but, as larger computers are developed, numerical modeling should be quite feasible. Extensive experiments made recently by Fan⁷¹ in a modeling channel included plume rise both into a uniform crossflow and into a calm stream with a constant density gradient. In the latter case most of the plumes were inclined; i.e., the stacks were not vertical. Although the buoyancy of these plumes was varied, they were momentum dominated for the most part. The behavior of plumes with negative buoyancy in a crosswind was modeled by Bodurtha.⁷²

ATMOSPHERIC STUDIES

The first full-scale plume-rise data were given in an appendix to the Bosanquet, Carey, and Halton paper¹⁹ of 1950. The center lines of plumes from four chimneys were traced from visual observation onto a Perspex screen. The observations were carried only as far as 800 ft downwind of the stacks, where apparently the visibility of the plumes was lost. These observations also appear in a paper by Priestley.⁷³ Holland⁶ published some of the details of the observations that he used in deriving the Oak Ridge formula, but the distance of observation was not mentioned. According to Hawkins and Nonhebel⁷ the plume heights were measured at only two or three stack heights downwind and were obtained from photographs. Holland found only a small correlation between plume rise and the temperature gradient, which was measured near the ground. However, the plume is affected only by the temperature gradient of the air through which it is rising, and the gradient near the ground is not a good

measure of the gradient higher up. Stewart, Gale, and Crooks^{74,75} published a survey of plume rise and diffusion parameters at the Harwell pile. Vertical surveys of the invisible plume were conducted by mounting up to ten Geiger counter units on the cable of a mobile barrage balloon. The stack was a steady, known source of radioactive argon (⁴¹Ar), and the Geiger units were arranged to measure the disintegration of beta particles, which have a maximum range of only 3 m in air. Again, the temperature gradient was measured well below plume level except for a few runs that were made in neutral conditions. Most of the wind-speed measurements were also made at a height well below the plume height. Since wind speed generally increases with height, the reported wind speeds are probably too low for such runs.

Ball⁷⁶ made measurements on very small plumes from lard-pail-type oil burners. The heights were estimated at 30 and 60 ft downwind by visual comparison with 10-ft poles and were averaged over 2 or 3 min. There was some tendency for the burning rate to increase with wind speed. Moses and Strom⁷⁷ ran experiments on a source with about the same heat emission, but here the effluent was fed into an 111-ft experimental stack with a blower. Plume-rise data at 30 and 60 m downwind were obtained photographically and averaged over 4 min. Wind speed was interpolated at plume level from measurements from a nearly 150-ft meteorological tower. The temperature gradient was measured between the 144- and 5-ft levels of the tower. This provided only a fair measure of the actual gradient at plume level since the gradient above 111 ft may be quite different from that near the ground. In only 2 of the 36 runs, the plume appeared to level off owing to stable conditions. These data tend to be dominated by momentum rise.

Danovich and Zeyger⁷⁸ published some plume-rise data obtained from photographs. However, the effective rise was assumed to occur when the plumes were still inclined at 10 to 15° above horizontal, and plumes have been observed to rise many times the height at this point. Some interesting data were obtained from exhaust plumes of rocket motors by van Vleck and Boone,⁷⁹ including some runs with complete temperature profiles furnished. The sources ranged up to 1000 Mw, which is about ten times the heat-emission rate of a large power plant stack. However, they were not true continuous sources since burning times varied from 3 to 60 sec.

Extensive plume photography was carried out at two moderate-size power plants in Germany by Rauch.⁸⁰ Plume center lines were determined for 385 runs at Duisburg and for 43 runs at Darmstadt. Each determination consisted of two or three time exposures of about 1 min each, together with five instantaneous pictures taken at set time intervals. The horizontal speed of the plume was calculated by following irregular features of the plume from one negative to the next. This method should provide a good measure of the wind speed experienced by the plume. In most of the photographs, the plume center line could not be determined for a distance downwind of more than 1000 ft, although a few could be determined out to 3000 ft. The accuracy of the temperature-gradient measurements was such that only general stability classifications could be made. In practice no measurements in very unstable conditions were made because of looping, and no measurements in stable conditions were made far enough downwind to show the plume leveling off. In fact, not one of

the 428 plume center lines leveled off. It would therefore seem that Rauch's extrapolation of these center lines to a final rise is rather speculative.

Much more extensive observations, consisting of about 70 experiments on more than 30 smoke stacks in Sweden, were recently made by Bringfelt,⁸¹ and some of the preliminary data have been reported by Högström.⁸² Each experiment consisted in taking about one photograph a minute for 30 to 60 min. The center lines were measured up to 9000 ft downwind, and wind speed and temperature gradient were measured at the plume level.

Some observations of plume rise at a small plant were reported by Sakuraba and his associates.⁸³ The best fit to the data was given by $\Delta h \propto u^{-\frac{3}{2}}$, but downwash was likely at the higher wind speeds since the wind speeds exceeded the efflux velocity. The temperature gradient and distance downwind were not given. More observations were carried out by the Central Research Institute of Electric Power Industry, Japan,⁸⁴ in which temperature and wind profiles were measured, as well as the vertical profile of SO_2 concentration at 1 km downwind.

Several groups have shown continuing interest in plume-rise measurements. The Meteorology Group at Brookhaven National Laboratory has conducted several programs by burning rocket fuel on the ground near their well-instrumented 420-ft meteorological tower. Limited data⁸⁵ were published in 1964 from tests in which there was some difficulty in obtaining a constant rate of heat release. This problem has been overcome, and more detailed data are available.⁸⁶

Csanady published plume-rise observations⁸⁷ from the Tallawarra power station in New South Wales in 1961. Plume rise was measured photographically, and wind speed was determined from displacement of plume features in a succession of photographs. Csanady has been conducting a continuing program of plume-rise and dust-deposition research at the University of Waterloo in Ontario since 1963. More-elaborate photographic measurements of plume rise made at the Lakeview Generating Station were published by Slawson and Csanady.^{5,88} Tank, wind-tunnel, and small-scale outdoor studies are now in progress.^{89,90}

The Central Electricity Research Laboratory in England has been conducting plume-rise studies for some time. In 1963 they published results from the Earley and Castle Donington power stations.⁹¹ The measurements were unique in that the plumes were traced a long distance downwind by injecting balloons into the base of the chimney.⁹² The balloons were observed to stay within the plumes when the plumes were purposely made visible, but there may have been systematic errors due to the relative inertia and buoyancy of the balloons. Although some of the balloons continued to rise beyond 2 miles downwind, the reported rises were in the range 3600 to 6000 ft downwind. The motion of the balloons provided a convenient measure of wind speed. More recently measurements were made by Hamilton⁹³ at the Northfleet Power Station by using lidar to detect the plume. Lidar is an optical radar that uses a pulsed ruby laser. It measures the range and concentration of light-reflecting particles and can detect smoke plumes even when they become invisible to the eye.^{94,95} Some searchlight determinations of the height of the Tilbury plant plume are also given in Refs. 93 and 96.

The Tennessee Valley Authority has also conducted plume-rise measurements over many years. The plume-rise and dispersion results^{9,9,8} published in 1964 were based on helicopter probes of SO₂ concentrations in the plume. The helicopter also measured the temperature gradient up to the top of the plume. Plumes in inversions were observed to become level and maintain a nearly constant elevation as far as 9 miles downwind. Much more detailed studies at six TVA plants have recently been completed.⁹⁹ Heat emissions ranged from 20 to 100 Mw per stack with one to nine stacks operating. Complete temperature profiles were obtained by helicopter, and wind profiles were obtained from pibal releases about twice an hour. Such intermittent sampling of wind speed does not provide a good average value, however, and may account for some of the scatter in the results. After several different techniques were tried, with good agreement among them, infrared photography was used to detect the plume center line. Complete plume trajectories as far as 2 miles downwind were obtained from the photographs.

There have been a few atmospheric studies concerned particularly with plume rise in stable air. Vehrenberg, Ambrosio, and Romie¹⁰⁰ conducted tests on the Mojave Desert, where very steep surface inversions occur in the early morning. The heat sources were shallow depressions, 2.5, 5, 10, and 20 ft in diameter, containing ignited diesel oil. Temperature profiles were measured with a thermocouple attached to a balloon, and the dense black plumes were easily photographed. Davies¹⁰¹ reported a 10,000-ft-high plume rise from an oil fire at a refinery in Long Beach, Calif. The heat release was estimated to be of the order of 10,000 Mw;¹⁰² i.e., about 100 times the heat emission from a large power plant stack. Observations of plume rise into multiple inversions over New York City were presented recently by Simon and Proudfit.¹⁰³ The plumes were located with a fast-response SO₂ analyzer borne by helicopter, and temperature profiles were also obtained by helicopter.

4 FORMULAS FOR CALCULATING PLUME RISE

There are over 30 plume-rise formulas in the literature, and new ones appear at the rate of about 2 a year. All require empirical determination of one or more constants, and some formulas are totally empirical. Yet the rises predicted by various formulas may differ by a factor greater than 10. This comes about because the type of analysis and the selection and weighting of data differ greatly among various investigators.

Emphasis is given here on how the formulas were derived and on the main features of each. Complicated formulations are omitted since readers may check the original references. For convenience all symbols are defined in Appendix B.

EMPIRICAL FORMULAS

Formulas for Buoyant Plumes

Of the purely empirical plume-rise formulas, the first to be widely used was that suggested by Holland⁶ on the basis of photographs taken at three steam plants in the vicinity of Oak Ridge, Tenn. The observed scatter was large, but the rise appeared to be roughly proportional to the reciprocal of wind speed. Holland used the wind-tunnel results of Rupp and his associates⁶¹ for the momentum-induced part of the rise and, by assuming a linear combination of momentum and buoyancy rises, found the best fit to the data with

$$\Delta h = 1.5 \left(\frac{w_0}{u} \right) D + 4.4 \times 10^{-4} \left[\frac{\text{ft-ft/sec}}{\text{cal/sec}} \right] \frac{Q_H}{u} \quad (4.1)$$

The dimensions of constants are given in brackets. Thomas^{1,5} found that a buoyancy term twice as large as that in Eq. 4.1 gave a better fit to observations at the TVA Johnsonville plant, and Stümke^{1,04} recommended a rise nearly three times that given by Eq. 4.1 on the basis of comparisons with many sets of observations.

Another early empirical formula was suggested by Davidson^{1,05} in 1949 on the basis of Bryant's^{6,6} wind-tunnel data:

$$\Delta h = \left(\frac{w_0}{u} \right)^{1.4} D \left(1 + \frac{\Delta T}{T_s} \right) \quad (4.2)$$

Equation 4.2, although a dimensionally homogeneous formula, is physically oversimplified in that the buoyancy term ($\Delta T/T_s$) does not take into consideration the total heat emission or the effect of gravity, without which buoyancy does not exist. The main weakness of Eq. 4.2 is that it is based on data obtained at only seven stack diameters downwind and often greatly underestimates observed rises.

Berlyand, Genikhovich, and Onikul^{1,06} suggested

$$\Delta h = 1.9 \left(\frac{w_0}{u} \right) D + 5.0 \frac{F}{u^3} \quad (4.3)$$

where F is a quantity that is proportional to the rate of buoyancy emission from the stack. This formula is dimensionally consistent, but few details are given about the observations on which it is based. The constant in the buoyancy term, 5.0, is curiously almost two orders of magnitude smaller than the constants recommended by Csanady,^{8,7} by Briggs,^{11,107} and by the new ASME manual.^{2,7}

On the basis of data from four stacks, namely, the Harwell stack,^{74,75} Moses and Strom's experimental stack,⁷⁷ and the two stacks reported by Rauch,⁸⁰ Stümke^{1,08} derived the formula

$$\Delta h = 1.5 \left(\frac{w_0}{u} \right) D + 118 \left[\frac{\text{ft}^{\frac{1}{2}}}{\text{sec}} \right] D^{\frac{1}{3}} \left(1 + \frac{\Delta T}{T_s} \right)^{\frac{1}{4}} u^{-1} \quad (4.4)$$

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The argument for omitting emission velocity from the buoyancy term is not clear. The constants and exponents for the various terms resulted from applying the method of least squares to the observed and calculated rises.

Lucas, Moore, and Spurr^{9,1} fitted observed plume rises at two of their plants with

$$\Delta h = 258 \left[\frac{\text{ft-ft/sec}}{(\text{cal/sec})^{\frac{1}{4}}} \right] \frac{Q^{\frac{1}{4}}}{u} \quad (4.5)$$

The heat emissions varied from 4 to 67 Mw, and the plumes were traced to about a mile downwind by releasing balloons in the stacks (see "Atmospheric Studies" in Chap. 3). The formula is based on a simplification of Priestley's theoretical plume-rise model.^{7,3} The best values for the constant in Eq. 4.5 differed by 25% at the two

plants, and further variations have been observed at other plants.⁹³ Lucas¹⁰⁹ noted some correlation with stack height and suggested a modification of Eq. 4.5:

$$\Delta h = (134 + 0.3 [ft^{-1}] h_s) \left[\frac{ft \cdot ft/sec}{(cal/sec)^{1/4}} \right] \frac{Q_H^{1/4}}{u} \quad (4.6)$$

Recently a CONCAWE working group^{25,26} developed a regression formula based on the assumption that plume rise depends mainly on some power of heat emission and some power of wind speed. The least-squares fit to the logarithms of the calculated-to-observed plume-rise ratios was

$$\Delta h = 1.40 \left[\frac{ft \cdot (ft/sec)^{1/4}}{(cal/sec)^{1/4}} \right] \frac{Q_H^{1/4}}{u^{1/4}} \quad (4.7)$$

Data from eight stacks were used, but over 75% of the runs came from Rauch's⁸⁰ observations at Duisburg, i.e., from just one stack. Most of these data fall into a small range of Q_H and of u , and therefore it is difficult to establish any power-law relation with confidence.

Even more recently Moses and Carson¹¹⁰ developed a formula of the same type as Eq. 4.7 with data for ten different stacks, but again the Duisburg observations were heavily weighted. A momentum term of the type that appears in the formulas of Holland,⁶ Berlyand and his associates,¹⁰⁶ and Stümke¹⁰⁸ was included, but the optimum value of the constant turned out to be very small. The least-squares fit was given by

$$\Delta h = 1.81 \left[\frac{ft \cdot ft/sec}{(cal/sec)^{1/4}} \right] \frac{Q_H^{1/4}}{u} \quad (4.8)$$

Actually, changing the exponent of Q_H to $1/3$ or $3/4$ increased the standard error very little. This insensitivity is due partly to the small range of Q_H into which the bulk of the data fell. Another shortcoming of this analysis, as well as of the analysis by Stümke, is that absolute values of the error in predicted rises were employed. This tended to weight the analysis in favor of situations with high plume rise; cases with high wind speed counted very little since both the predicted and the observed rises, and hence their differences, were small. Relative or percentage error, such as used by CONCAWE by means of logarithms, results in more even weighting of the data.

Formulas for Jets

One of the first empirical relations for the rise of pure jets was given by Rupp et al.⁶¹ This relation was determined from photographs of a plume in a wind tunnel. The investigators found the height of the jet center line at

$$\Delta h = 1.5 \left(\frac{w_0}{u} \right) D \quad (4.9)$$

the point at which the plume became substantially horizontal, i.e., when its inclination was only 5 to 8°.

Subsequent investigators have all given empirical relations for the jet center line as a function of downwind distance. The results are summarized in Table 4.1 for the case in which the density of the jet is the same as that of air. A theoretical formula to be given later in the chapter is included for comparison.

Table 4.1
COMPARISON OF EMPIRICAL RESULTS FOR JET
CENTER LINES AS A FUNCTION OF DOWNDOWN DISTANCE

Investigator	Range of $R = (w_0/u)$	Maximum x/D	$\Delta h/D$	$\Delta h/D$ at 5.7° Inclination
Eq. 4.33			$1.44 R^{0.67} (x/D)^{0.33}$	$3.2 R^{1.00}$
Rupp et al. ⁶¹	2 to 31	47		$>1.5 R^{1.00}$
Callaghan and Ruggeri ⁶²		81	$1.91 R^{0.61} (x/D)^{0.30}$	$4.0 R^{0.87}$
Gordier (by Patrick ⁶⁵)			$1.31 R^{0.74} (x/D)^{0.37}$	$3.3 R^{1.17}$
Shandorov (by Abramovich ¹¹¹)	2 to 22		$0.84 R^{0.78} (x/D)^{0.39}$	$1.8 R^{1.28}$
Patrick ⁶⁵				
Concentration	6 to 45	22	$1.00 R^{0.85} (x/D)^{0.34}$	$1.9 R^{1.29}$
Velocity	8 to 54	34	$1.00 R^{0.85} (x/D)^{0.38}$	$2.3 R^{1.37}$

The early Callaghan and Ruggeri⁶² experiments involved heated, supersonic jets in a very narrow wind tunnel; so application of their results to free, subsonic jets is questionable. Since the penetration was determined as the highest point at which the temperature was 1°F above the free-stream temperature, the rises given represent the very top of the plume and are noticeably higher than in other experiments. The Gordier formula was obtained from total-head traverses in a water tunnel as reported by Patrick.⁶⁵ The formula attributed to Shandorov by Abramovich¹¹¹ was based on experiments that included various angles of discharge and density ratios. The Patrick⁶⁵ formulas were based both on the height or maximum concentration of nitrous oxide tracer and on the height of maximum velocity as determined by a pitot tube.

THEORETICAL FORMULAS

There are many theoretical approaches to the problem of plume rise, and some of them are quite complex. To reproduce them all here would be tedious and of little help to most readers. Instead, the various theories are compared with a relatively simple basic plume-rise theory based on assumptions common to most of the theories.

It will be shown later that this basic theory in its simplest form gives good agreement with observations.

Basic Theory

In most plume-rise theories, buoyancy is assumed to be conserved; i.e., the motion is considered to be adiabatic. This means that the potential temperature of each element of gas remains constant. It is also assumed that pressure forces are small and have little net effect on the motion, that they merely redistribute some of the momentum within the plume. Molecular viscosity is also negligible because the plume Reynolds number is very high, and local density changes are neglected. These assumptions lead to three conservation equations:

$$\nabla \cdot \rho_p \vec{v}_p = 0 \quad (\text{continuity of mass}) \quad (4.10)$$

$$\frac{d\theta_p}{dt} = 0 \quad (\text{buoyancy}) \quad (4.11)$$

$$\frac{d\vec{v}_p}{dt} = \frac{g}{T} \theta' \vec{k} \quad (\text{momentum}) \quad (4.12)$$

where \vec{v}_p = the local velocity of the gas in the plume

ρ_p = the local gas density

θ_p = the local potential temperature

$\theta' = \theta_p - \theta$ = the departure of the potential temperature from the temperature of the environment at the same height

\vec{k} = the unit vector in the vertical direction (buoyancy acts vertically)

Equations 4.10, 4.11, and 4.12 can be transformed to describe the mean motion of a plume by integrating them over some plane that intersects the plume. It is most convenient to integrate over a horizontal plane because then the mean ambient values of potential temperature (θ), density (ρ), and velocity (\vec{v}_e) can be considered constant over the plane of integration and are assumed to be functions of height only. Furthermore, if \vec{v}_e is assumed to be horizontal, the vertical component of \vec{v}_p , denoted by w' , is due entirely to the presence of the plume. Thus w' is a convenient variable with which to identify the plume.

A further simplification results from assuming that the vertical velocity and the buoyancy are everywhere proportional to each other in a horizontal section of the plume since it is then unnecessary to assume any specific distribution of either. This assumption is approximately true for measured cross sections of vertical plumes.^{5,7} Admittedly it does not hold near the height of final rise in a stable atmosphere, because buoyancy decays more rapidly than vertical velocity in such a situation.

A steady state is assumed. To obtain Eq. 4.13, we combine Eq. 4.10 times θ' with Eq. 4.11 times ρ_p and integrate the resulting equation over a horizontal plane,

assuming that the vertical velocity and the buoyance are everywhere proportional to each other. Similarly, to obtain Eq. 4.14, we combine Eq. 4.10 times \vec{v}_p with Eq. 4.12 times ρ_p and integrate the resulting equation over the same horizontal plane. The plane of integration must completely intersect the plume so that $\theta' \neq 0$ around the perimeter of the plane. The resulting equations for the net buoyancy flux and momentum flux in a plume are

$$\frac{dF_z}{dz} = -sV \quad (4.13)$$

$$\frac{d(\vec{v}V)}{dz} = \frac{F_z}{w} \vec{k} + \vec{v}_e \frac{dV}{dz} + \text{drag} \quad (4.14)$$

where

$$V = \frac{\iint \rho_p w' dx dy}{\pi \rho} \quad (\text{vertical volume flux}) \quad (4.15)$$

$$s = \frac{g}{T} \frac{\partial \theta}{\partial z} \quad (\text{stability parameter}) \quad (4.16)$$

$$F_z = \frac{\iint (g/T) \theta' \rho_p w' dx dy}{\pi \rho} \quad (\text{buoyancy flux}) \quad (4.17)$$

$$\vec{v}V = \frac{\iint \vec{v}_p \rho_p w' dx dy}{\pi \rho} \quad (\text{momentum flux}) \quad (4.18)$$

The vertical volume flux of the plume, as defined in Eq. 4.15, is the total vertical mass flux divided by $\pi \rho$, where ρ is the environmental density. The stability parameter, s , can be interpreted as the restoring acceleration per unit vertical displacement for adiabatic motion in a stratified atmosphere (either stable or unstable); in an unstable atmosphere, s is negative; F_z is the vertical flux of the buoyant force divided by $\pi \rho$; \vec{v} is an average plume velocity at a given height, as defined by the total velocity field at that height weighted by the normalized vertical mass flux; w is the vertical component of \vec{v} and is the velocity of plume rise at any given height. The drag term in Eq. 4.14 is not written out since it will be dropped later, but it can be interpreted as the net horizontal advection of momentum deficit across the boundary of the plane of integration.

The initial conditions are

$$\vec{v} = w_0 \vec{k} \quad (4.19a)$$

$$\vec{v}V = \frac{\rho_0}{\rho} w_0^2 r_0^2 \vec{k} \equiv F_m \vec{k} \quad (4.19b)$$

and

$$F_z = \left(1 - \frac{\rho_0}{\rho}\right) g w_0 r_0^2 \equiv F \quad (4.19c)$$

For a hot source

$$\begin{aligned} F &= \frac{g Q_H}{\pi c_p \rho T} \\ &\doteq 4.3 \times 10^{-3} \left[\frac{\text{ft}^4/\text{sec}^3}{\text{cal/sec}} \right] Q_H \\ &\doteq 3.7 \times 10^{-5} \left[\frac{\text{m}^4/\text{sec}^3}{\text{cal/sec}} \right] Q_H \end{aligned} \quad (4.20)$$

where c_p is the specific heat of air at constant pressure.

Equations 4.13 and 4.14 can be solved for the mean motion of a plume through any atmosphere, including one with stability varying with height and wind shear. However, the equations cannot be solved until some specific assumption is made about the growth of volume flux with the height (dV/dz). This assumption, called an entrainment assumption, is necessary to describe the bulk effect of turbulence in diffusing momentum and buoyancy in a plume.

Basic Theory Simplified

It is desirable to reduce the basic theory to the simplest form that works. To be more specific, we would like to derive from the basic theory simple formulas that agree with data. To do this, we must make the simplest workable entrainment and drag-force assumptions, assume simple approximations for the atmosphere, treat the stack as a point source, and treat the plume as being either nearly vertical or nearly horizontal, i.e., ignore the complicated bending-over stage.

When the wind speed is sufficiently low, a plume rises almost vertically, and the drag force and mechanically produced atmospheric turbulence are negligible. The turbulence that causes entrainment of ambient air is generated within the plume by the shear between the vertical plume motion and the almost stationary environment. The simplest workable entrainment hypothesis for this case is that the entrainment velocity, or the average rate at which outside air enters the plume surface, is proportional to the characteristic vertical velocity (w) at any given height. This assumption, based on dimensional analysis, will be called the Taylor entrainment assumption after the author¹¹² who suggested it in 1945. If $(V/w)^{1/4}$ is defined as a characteristic plume radius, the rate at which the volume flux grows in a given increment of height is then $2\pi(V/w)^{1/4} \alpha w$, where α is called the entrainment constant

and is dimensionless. The complete set of equations governing the vertical plume are then

$$\frac{dF_z}{dz} = -sV$$

which was given as Eq. 4.13,

$$\frac{d(wV)}{dz} = F_z \quad (4.21)$$

and

$$\frac{dV}{dz} = 2\alpha(wV)^{1/4} \quad (4.22)$$

This set of equations is equivalent to the relations given by Taylor¹¹² in 1945 and further developed in 1956 in a classic paper⁵⁸ by Morton, Taylor, and Turner, who found that a value of 0.093 for the entrainment constant gave the best fit to observed profiles of heated plumes. Briggs¹¹³ found that $\alpha = 0.075$ gives the best predictions of the height of the top of stratified plumes in stable air, based on the height at which the buoyancy flux decays to zero. The latter value is used here. The direct empirical determination of entrainment in jets by Ricou and Spalding¹¹⁴ yields a comparable value of 0.080.

The case of a bent-over plume, in which the vertical velocity of the plume is much smaller than the horizontal velocity, is simpler. Both the total plume velocity and its horizontal component are then very close to the ambient wind speed, u , which is assumed constant; wind shear is neglected. It is more reasonable in this case to integrate Eqs. 4.10 to 4.12 over a vertical plane intersecting the plume since a vertical plane is more nearly perpendicular to the plume axis. When this is done, the resulting equations are identical to Eqs. 4.13 and 4.21, provided that s is constant over the plane of integration, that F_z , V , and wV are defined as fluxes of plume quantities through a vertical plane, and that the drag term is zero. Measurements by Richards⁶⁹ of the mean streamlines near horizontal thermals suggest that the drag term is zero provided the chosen plane of integration is large enough. This is also intuitively evident since one would not expect a vertically rising plume to leave a very extensive wake underneath it.

In the initial stage of rise of a bent-over plume, the self-induced turbulence dominates the mixing process, and the Taylor entrainment hypothesis can be used again. The main difference from a vertical plume is that in this case the velocity shear is nearly perpendicular to the plume axis, rather than parallel to it. This apparently results in more efficient turbulent mixing since the entrainment constant for a bent-over plume is about 5 times as large as that for a vertical plume. With a characteristic plume radius defined as $(V/u)^{1/4}$, the rate at which the volume flux grows

in a given increment of axial distance is $2\pi(V/u)^{1/2} \gamma w$, where γ is the entrainment constant for a bent-over plume. If this is transformed to vertical coordinates, the plume rise is governed by Eqs. 4.13, 4.21, and

$$\frac{dV}{dz} = 2\gamma(uV)^{1/2} \quad (4.23)$$

which is comparable to Eq. 4.22. Since u is a constant, Eq. 4.23 can readily be integrated. For a point source this yields a characteristic radius equal to γz . The relation is confirmed by modeling experiments of Richards^{6,9} and by photographs of full-scale plumes made by TVA^{9,9} (see Fig. 4.1). On the basis of these photographic plume diameters, $\gamma \approx 0.5$.

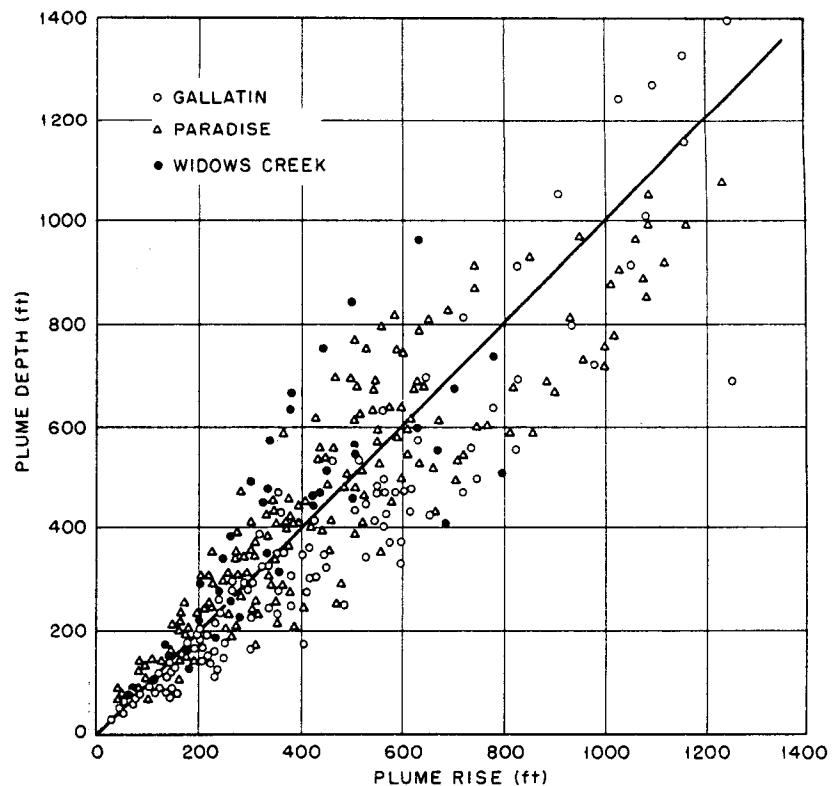


Fig. 4.1 Photographic plume depth (top to bottom) vs. plume rise (center line) at TVA plants.

Atmospheric turbulence is small in a stable environment and can be neglected, in which case Eq. 4.23 is valid up to the point where a bent-over plume reaches its maximum rise. However, in a neutral or unstable atmosphere, turbulence is vigorous enough to eventually dominate the entrainment process. This occurs some distance

downwind of the stack when the vertical velocity of the plume becomes small compared with ambient turbulent velocities. The simplest measure of the effective intensity of atmospheric turbulence is the eddy energy dissipation, ϵ , because it adequately describes the part of the turbulence spectrum that is most effective at diffusing the plume relative to its axis, i.e., the inertial subrange. The characteristic radius of the plume, $(V/u)^{1/4}$, determines the range of eddy sizes that most efficiently diffuse the plume. If these two terms are adequate enough to characterize entrainment, the effective entrainment velocity must be given by $\beta\epsilon^{1/4}(V/u)^{1/4}$, where β is a dimensionless constant; the exponents of the terms result from dimensional considerations. Since the entrainment velocity in the initial stage of plume rise is γw , for the simplest model of a bent-over plume an abrupt transition to an entrainment velocity of $\beta\epsilon^{1/4}(V/u)^{1/4}$ is assumed to occur when $\gamma w = \beta\epsilon^{1/4}(V/u)^{1/4}$.

The solution for the bending-over stage of a plume in a crosswind is less certain because both shear parallel to the plume axis and shear perpendicular to the axis are present. Both mechanisms operate at once to cause turbulent entrainment. Drag force could contribute to the bending over of the plume since there could be an extensive wake downwind of the plume in this case, but the drag force will have to be neglected at present owing to insufficient knowledge. In the early stage of bending over, the vertical-plume model is applicable except that there is a perpendicular shear velocity nearly equal to u . If the two contributions to entrainment can be summed in the manner of vectors, the resultant entrainment velocity becomes $(\alpha^2 w^2 + \gamma^2 u^2)^{1/4}$, and the plume center line is given by Eqs. 4.13, 4.21, and

$$\frac{dV}{dz} = 2 \left(\frac{V}{w} \right)^{1/4} (\alpha^2 w^2 + \gamma^2 u^2)^{1/4} \quad (4.24)$$

Before applying models of the vertical plume and bent-over plume to specific cases, some approximations about the source can be made. Usually it is reasonable to assume that either the initial vertical momentum or the buoyancy dominates the rise. In the former case the plume is called a jet, and we set F equal to 0. Unheated plumes composed mostly of air are in this category. Most hot plumes are dominated by buoyancy, and we can neglect the initial vertical momentum flux, F_m . At a sufficient distance from the stack, e.g., beyond 20 stack diameters downwind, we can neglect the finite size of the source and treat the stack merely as a point source of momentum flux or buoyancy flux.

Some of the approximations that come out of the simplified theory are given in Eqs. 4.25 to 4.34. Vertical plumes are indicated by the term "calm" and bent-over plumes by "wind." For rise into stable air in which s is constant, we have

$$\Delta h = 5.0 F^{1/4} s^{-1/4} \quad (\text{buoyant, calm}) \quad (4.25)$$

$$\Delta h = 2.4 \left(\frac{F}{u s} \right)^{1/4} \quad (\text{buoyant, wind}) \quad (4.26)$$

$$\Delta h = 4 \dagger \left(\frac{F_m}{s} \right)^{1/4} \quad (\text{jet, calm}) \quad (4.27)$$

$$\Delta h = 1.5 \left(\frac{F_m}{u} \right)^{1/4} s^{-1/4} \quad (\text{jet, wind}) \quad (4.28)$$

In the calm case, Eq. 4.25 gives the height at which the buoyancy goes to zero. In the windy cases for a bent-over plume, the equations are integrated to the point where $w = 0$, and the plume is assumed to fall back to the level at which the buoyancy is zero with no further mixing. More details are given by Briggs.¹¹³ The plume will penetrate a ground-based inversion or stable layer if the preceding formulas predict a rise higher than the top of the stable air. If the air is neutrally stratified above this level, a buoyant plume will continue to rise since it still has some buoyancy. A jet will fall back and level off near the top of the stable air because it acquires negative buoyancy as it rises.

The model predicts penetration of a sharp, elevated inversion of height z_i through which the temperature increases by ΔT_i if

$$z_i \leq 7.3 F^{0.4} b_i^{-0.6} \quad (\text{buoyant, calm}) \quad (4.29)$$

$$z_i \leq 2.0 \left(\frac{F}{u b_i} \right)^{1/4} \quad (\text{buoyant, wind}) \quad (4.30)$$

$$z_i \leq 1.6 \dagger \left(\frac{F_m}{b_i} \right)^{1/4} \quad (\text{jet, calm}) \quad (4.31)$$

where $b_i = g \Delta T_i / T$. The buoyant plume is assumed to penetrate if its characteristic temperature excess, given by $(T/g)F_z/V$, exceeds ΔT_i at the height of the inversion.

For the first stage of rise, the bent-over model predicts plume center lines given by

$$\Delta h = 1.8 F^{1/4} u^{-1} x^{3/4} \quad (\text{buoyant, wind}) \quad (4.32)$$

$$\Delta h = 2.3 F_m^{1/4} u^{-3/4} x^{1/4} \quad (\text{jet, wind}) \quad (4.33)$$

For the general case where s is positive and constant, Eqs. 4.13 and 4.21 can be combined with the transform $dz = (w/u) dx$ to give

$$\frac{d^2(wV)}{dx^2} = - \frac{s}{u^2} (wV)$$

This is the equation of a simple harmonic oscillator. Since V always increases, the plume center line behaves like a damped harmonic oscillator (the author has observed

[†]Empirical; numerical value difficult to determine from present model.

such behavior at a plant west of Toronto in the early morning). Since $V \simeq u\gamma^2 z^2$, the preceding expression can be integrated and satisfies the initial conditions when

$$[(\gamma^2/3)us^{1/2}] \Delta h^3 = F_m \sin(xs^{1/2}/u) + Fs^{-1/2} [1 - \cos(xs^{1/2}/u)]$$

This equation is valid only up to the point of maximum rise because beyond this point a negative entrainment velocity would be implied. According to this equation a jet ($F = 0$) reaches its maximum height at $x = (\pi/2) us^{-1/2}$ and a buoyant plume ($F_m = 0$) reaches its maximum height at $x = \pi us^{-1/2}$. At much smaller distances the plume center line is approximated by

$$\Delta h = 2.3 F_m^{1/2} u^{-1/2} x^{1/2} \left(1 + \frac{Fx}{2F_m u}\right)^{1/2}$$

From this equation it is seen that the ratio $Fx/F_m u$ is a general criterion of whether a bent-over plume is dominated by buoyancy or by momentum at a given distance downwind. It, in fact, represents the ratio of buoyancy-induced vertical momentum to initial vertical momentum.

For the buoyant bent-over plume in neutral conditions, the first stage of rise is given by Eq. 4.32 up to the distance at which atmospheric turbulence dominates the entrainment. The complete plume center line is given by Eq. 4.32 when $x < x^*$ and by

$$\Delta h = 1.8 F^{1/2} u^{-1} x^{3/2} \left[\frac{2}{5} + \frac{16}{25} \frac{x}{x^*} + \frac{11}{5} \left(\frac{x}{x^*} \right)^2 \right] \left(1 + \frac{4}{5} \frac{x}{x^*} \right)^{-2} \quad (4.34)$$

when $x > x^*$, where x^* is the distance at which atmospheric turbulence begins to dominate entrainment. This distance is given by

$$x^* = 0.43 F^{2/3} (\beta^{-3} u \epsilon^{-1})^{1/3}$$

Results from puff and cluster diffusion data and from measurements of eddy energy dissipation rates, given in Appendix A, show that $\beta = 1$ is acceptable as a somewhat conservative approximation. In the surface layer of the atmosphere defined by constant stress, e.g., the lowest 50 ft or so, it is well established¹¹⁵ that $\epsilon = u^{*3}/0.4\bar{z}$, where \bar{z} is the height above the ground and u^* is the friction velocity. If we approximate \bar{z} by $\bar{z} = \Delta h$, the final plume rise given by Eq. 4.34 is $\Delta h = 4.5 F/u u^{*2}$; since $u \propto u^*$ and changes only gradually with height in the neutral surface layer, this result is similar to those of earlier theories^{36,46,107} that predict $\Delta h \propto F/u^3$. Unfortunately, this clear relation between ϵ and u^* breaks down at heights more typical of smoke plumes. In Appendix A, data from 50 to 4000 ft above the ground give more support to the empirical relation

$$\epsilon \simeq 0.73 \left[\frac{\text{ft}^2}{\text{sec}^2} \right] \frac{u}{\bar{z}}$$

up to $\bar{z} \approx 1000$ ft, then becoming constant with height. If we conservatively approximate \bar{z} with the stack height, the resulting estimate for x^* becomes

$$x^* = 0.52 \left[\frac{\sec^{\frac{1}{3}}}{\text{ft}^{\frac{1}{3}}} \right] F^{\frac{2}{3}} h_s^{\frac{3}{5}} \quad (h_s < 1000 \text{ ft})$$

$$x^* = 33 \left[\frac{\sec^{\frac{1}{3}}}{\text{ft}^{\frac{1}{3}}} \right] F^{\frac{2}{3}} \quad (h_s > 1000 \text{ ft}) \quad (4.35)$$

Other Theories

There is such a variety of plume-rise theories in the literature that only the briefest discussion of each must suffice. One can only be amazed, and perhaps perplexed, at the number of different approaches to the solution of this fascinating fluid-dynamics problem. The theories will be discussed chronologically, first for the calm case and then for the crosswind case.

The first theoretical treatment was of a jet in calm surroundings and was given by Tollmien¹¹⁶ in 1926. Rather than making an entrainment assumption, he used the Prandtl mixing-length hypothesis to derive a specific velocity-profile law that agrees quite well with data. A similar approach was used for heated plumes in calm air by Schimdt⁵⁵ in 1941. Rouse, Yih, and Humphreys⁵⁷ treated the same problem by assuming eddy viscosity diffusion of the buoyancy and momentum by a process analogous to molecular diffusion. They determined experimentally that the mean temperature and velocity profiles are approximately Gaussian with the characteristic plume radius growing linearly with height. Yih⁵⁶ also considered the case of a laminar plume, which does not apply to full-scale plumes.

Batchelor⁴⁸ considered the same problem in 1954 by dimensional analysis. He included the case of a stratified environment and found power-law expressions for the mean plume velocity and temperature as functions of height in an unstable atmosphere whose potential temperature gradient is also approximated by a power law. The first theoretical model for a vertical plume rising through any type of stratification was given by Priestley and Ball¹¹⁷ in 1955. Their equations are similar to the preceding equations for the vertical plume except that the entrainment assumption, Eq. 4.22, is replaced by an energy equation involving an assumption about the magnitude and distribution of the turbulent stress. Vehrencamp, Ambrosio, and Romie¹⁰⁰ were the first to apply the results from an entrainment model to final rise in stable air by using the Taylor entrainment assumption. A general model involving this assumption and complete with experimental verification was put forth by Morton, Taylor, and Turner⁵⁸ in 1956. This model is called the M,T,&T model in the discussion that follows. The M,T,&T model is virtually identical to the vertical-plume model presented in the section "Basic Theory Simplified" of this chapter and differs from the Priestley and Ball¹¹⁷ model mainly by predicting a wider half-cone angle for jets than for buoyant plumes. This is actually observed in the laboratory. Both the M,T,&T model and the Priestley and Ball model predict a linear increase of radius with height in the

unstratified case and give similar results for the final plume height but disagree somewhat on the values of the numerical constants. Estoque¹¹⁸ further compares these two theories.

Morton¹¹⁹ extended the numerical integrations of the M,T,&T model to the case of a buoyant plume with nonnegligible initial momentum and concluded that increasing the efflux velocity can actually lessen rise in stable conditions because of increased entrainment near the stack level. In another paper,¹²⁰ he extended the theory to include augmented buoyancy due to the condensation of moisture of the entrained air. Hino^{121,122} made further calculations with the M,T,&T model, including the effects of a finite source radius. Turner¹²³ coupled the M,T,&T model with a vortex ring model to predict the speed of rise for a starting plume in neutral surroundings. Okubo¹²⁴ expanded the M,T,&T model to the case of a plume rising through a salinity gradient in water.

A generalized theory for steady-state convective flow incorporating several of these solutions was given by Vasil'chenko.¹²⁵ Recently Telford¹²⁶ proposed another type of entrainment assumption in which the entrainment velocity is proportional to the magnitude of turbulent fluctuations in the plume as calculated from a turbulent kinetic energy equation. Telford's results are similar to those of the M,T,&T model for a buoyant plume, except near the stack, but his model predicts too-rapid growth for a jet. This happens because the model is, in effect, based on the assumption that the scale of the energy-containing turbulent eddies is proportional to the plume radius, but this is not true for a jet, because most of the turbulent energy is generated while the jet radius is relatively small. Morton¹²⁷ has further criticized Telford's model in a recent note.

Lee¹²⁸ developed a model for a turbulent swirling plume. He used the Prandtl mixing-length hypothesis. Still another problem was explored by Fan,⁷¹ who extended the M,T,&T theory to the case of nonvertical emissions and tested the result in a modeling tank with linear density stratification.

One of the earliest theories for a bent-over buoyant plume was given by Bryant⁶⁶ in 1949. A drag-force assumption was included, and the entrainment assumption was in the form of a fairly complicated hypothesis about how the plume radius grows with distance from the source along its center line. Eventually the radius in this model becomes proportional to $x^{\frac{1}{3}}$, which is too small a growth rate compared with subsequent observations.

In 1950 Bosanquet, Carey, and Halton¹⁹ published a well-known theory that was later revised by Bosanquet.²⁰ The entrainment assumptions were similar to those made in the simplified theory here except that the same entrainment constant was applied to both the vertical and the bent-over stages of plume development, i.e., $\gamma = \alpha$. In addition, a contribution to the entrainment velocity due to environmental turbulence was assumed that was proportional to the wind speed. This assumption eventually led to a linear growth of plume radius with distance downwind and resulted in a final height for a bent-over jet and rise proportional to $\log x$ for a buoyant plume. The theory tends to underestimate rise at large distances downwind (see Fig. 5.3 in the next chapter).

About the same time, Sutton¹²⁹ developed a simple theory for a buoyant plume in a crosswind which was based on Schmidt's⁵⁵ result for a vertical plume, i.e., $w \propto (F/z)^{1/4}$. Sutton replaced z in this relation with the distance along the plume center line and took the horizontal speed of the plume to be equal to u . The expression is dimensionally correct and, at large distances, approaches the form given by Eq. 4.32.

Priestley⁷³ adapted his and Ball's vertical-plume model to the bent-over case. The average radius of a horizontal section was assumed to grow linearly with height, and the entrainment constant was modified by a factor proportional to $u^{1/4}$. Thus the equations of rise were identical to those for a vertical plume except for the entrainment constant modification. Priestley coupled this first-phase theory with a second phase in which atmospheric turbulence dominates the mixing. This latter phase is complicated and yields some unrealistic results, as was mentioned by Csanady.⁸⁷ The first phase leads to an asymptotic formula identical to Eq. 4.32 times a factor proportional to $(F/x)^{-1/12}$; namely,

$$\Delta h = 2.7 \left[\left(\frac{\text{ft}}{\text{sec}} \right)^{1/4} \right] F^{1/4} u^{-1} x^{1/4} \quad (4.36)$$

Lucas, Moore, and Spurr⁹¹ were able to simplify Priestley's theory considerably. For the first stage of rise, they obtained a plume rise 15% greater than that given by Eq. 4.36, and, for the atmospheric-turbulence-dominated stage, they obtained

$$\Delta h = 258 \left[\frac{\text{ft-ft/sec}}{(\text{cal/sec})^{1/4}} \right] \frac{Q_H^{1/4}}{u} \left[1 - \left(0.6 + 0.2 \frac{x - x_1}{1000 \text{ ft}} \right) \exp \left(-\frac{x - x_1}{1000 \text{ ft}} \right) \right] \quad (4.37)$$

where x_1 is the distance of transition to the second stage. It was estimated that $x_1 = 660$ ft, in contrast to the transition distance x^* given by Eq. 4.35, which depends on both the source strength and the height in the atmosphere.

Scorer^{36,130} introduced a simple plume-rise model for which he assumed that the plume radius grows linearly with height (see Fig. 4.1). The constant governing the growth rate depends on whether the plume is nearly vertical or bent over and also on whether it is dominated by momentum or by buoyancy in a given stage. Scorer considered all the separate possibilities and then matched them at the bend-over point to get a complete set of formulas for rise in neutral conditions. The predictions for transitional rise, the plume center line before final height is reached, are similar to those given by Eqs. 4.32 and 4.33. In addition, he postulated that the active rise terminates when the vertical velocity of the plume reduces to the level of atmospheric turbulence velocities, which he took to be some fraction of the wind speed. This led to the prediction that $\Delta h \propto F/u^3$ for a very buoyant plume. This type of formula has been given by many authors, but the leveling off of the plume in neutral conditions has not yet actually been observed. Furthermore, it now appears that atmospheric turbulence velocities are less strongly related to wind speed at typical plume heights.¹³¹

A great variety of work has been done in the last 6 years. Lilly⁷⁰ constructed a numerical model of the two-dimensional vortex pair seen in a vertical cross section of a bent-over buoyant plume. Keffer and Baines⁶³ presented a model for the bending-over stage of a jet with an entrainment assumption similar to the one in the bending-over model given in this review except that only the horizontal shear was included. Danovich and Zeyger⁷⁸ developed a theory along the lines of the Priestley theory for the first stage but with the second-stage dynamics determined by the diffusion of buoyancy by atmospheric turbulence. The type of diffusion assumed was essentially the same as that observed for total diffusion of gases in a passive plume. However, total diffusion includes the meandering of the plume axis caused by shifts in wind direction, whereas the action of buoyancy on the plume is affected only by the diffusion of buoyancy relative to the plume axis. Only relative diffusion should be used. The same criticism applies to a theory developed by Schmidt,¹³² which is based on the assumption that the spread of material equals that given by the total diffusion of a passive plume. There is also the criticism that the diffusion of a rising plume, especially in its early stages, is not the same as for a passive plume, because the rising plume generates its own turbulence in addition to the ambient turbulence. These problems were also pointed out by Moore.¹³³

Equations 4.25 through 4.28 and Eqs. 4.32 and 4.33 were proposed by Briggs¹⁰⁷ on the basis of rather elementary dimensional analysis as an extension of Batchelor's⁴⁸ and Scorer's⁴⁶ approaches. Briggs¹¹³ recently considered in some detail the penetration of inversions by plumes of all types by using a model based on the simplified theory given here. Gifford¹³⁴ extended this type of model to the case of a bent-over plume whose total buoyancy flux increases linearly with time as it moves away from the source, again using the Taylor entrainment assumption. Modeling experiments of Turner¹³⁵ with thermals of increasing buoyancy support this assumption.

A model by Csanady¹³⁶ for the bent-over buoyant plume included the effect of eddy-energy dissipation and of inertial subrange turbulence in the relative diffusion of plume buoyancy. In a later paper by Slawson and Csanady,⁵ a three-stage model was proposed. In the first stage, self-generated turbulence dominates, and the governing equations are in fact the same as those given in the bent-over plume model here. The second stage is dominated by inertial subrange atmospheric turbulence, and, in the third stage, the plume is supposed to be large enough for the eddy diffusivity to be essentially constant, as is the case for molecular diffusion. This model yields a radius proportional to $x^{1/4}$ and a constant rate of rise in the final stage rather than any limiting height of rise.

Very recently a model along the lines of the basic theory presented here was developed by Hoult, Fay, and Forney,¹³⁷ in which entrainment velocity depends on the longitudinal and transverse shear velocities. This theory is more elaborate than the simplified theory presented here, in that γ may be a function of w_0/u and the Froude number at the stack but does not take into account the effect of atmospheric turbulence.

5 COMPARISONS OF CALCULATED AND OBSERVED PLUME BEHAVIOR

NEUTRAL CONDITIONS

Buoyant Plumes in Neutral Conditions

Some previous comparisons of plume-rise formulas with data for the case of hot bent-over plumes in near-neutral conditions were reviewed by Moses, Strom, and Carson¹³⁸ and are only summarized here. Moses and Strom¹³⁹ compared a number of formulas with data from their experimental stack. However, there was much scatter in the data, and only the absolute differences between observed and calculated values were used in the analysis, rather than their ratios. The results of the comparisons were rather inconclusive. Rauch⁸⁰ made a brief comparison of the Holland⁶ formula and that of Lucas, Moore, and Spurr⁹¹ with his own data and found the latter formula, multiplied by a factor of 0.35, to be a better fit. Stümke¹⁰⁴ made more extensive comparisons between 8 different formulas and the data of Bosanquet, Carey, and Halton,¹⁹ of Stewart, Gale, and Crooks,⁷⁵ and of Rauch.⁸⁰ By computing the ratios of calculated to observed rises, Stümke concluded that the Holland formula, multiplied by a factor of 2.92, works best.

Since these comparisons were made, a number of new formulas have appeared, including those by Stümke,¹⁰⁸ Moses and Carson,¹¹⁰ CONCAWE,^{25,26} the modified Lucas formula,¹⁰⁹ and Eq. 4.34, published for the first time here. In addition, more data are now available, especially the data from three Central Electricity plants and six TVA plants; therefore comparisons can now be made over a much wider range of conditions.

First, a simple wind-speed relation would be convenient since this would allow some reduction of a large amount of data that covers a wide range of wind speeds, source strengths, and measuring distances. Many formulas, both empirical and theoretical, suggest that plume rise is inversely proportional to wind speed, at least at a fixed point downwind. In Fig. 5.1, data from a large number of sources tend to confirm this. In each graph the plume rise at one or more fixed distances is plotted against wind speed on logarithmic coordinates so that $\Delta h \propto u^{-1}$ is represented by a straight line with a slope of -1 ; such lines are indicated for reference.

For most of the sources, $\Delta h \propto u^{-1}$ is the best elementary relation. It would be difficult to make a case for $\Delta h \propto u^{-\frac{3}{4}}$, as appears in the CONCAWE^{25,26} formula. A better fit would result only for the Duisburg data, upon which the CONCAWE formula is very largely based. A few of the sources, in particular Shawnee and Widows Creek, show a greater decrease of Δh with increasing u , which probably indicates some form of downwash at higher wind speeds. However, the Davidson-Bryant¹⁰⁵ prediction that Δh is proportional to $u^{-1.4}$ would not fit most of the data.

With the inverse wind-speed law reasonably well established for neutral conditions, we can now average the product of plume rise and wind speed for all wind speeds to greatly reduce the volume of data. Such a presentation was first employed by Holland.⁶ In Fig. 5.2, $u \Delta h$ is plotted as a function of x for all available data sources. The average heat efflux per stack, in units of 10^6 cal/sec, is given in parentheses following each identification code, along with the number of stacks if more than one. The key to the code is given in Table 5.1. In system A at Harwell (HA), wind measurements were at a height of 27 m, whereas in system B (HB) the measurements were at 152 m, which is much closer to the height of the plume. A considerable amount of data is presented in Fig. 5.2. A general criterion was that each point plotted should represent at least three periods of 30 to 120 min duration each and that each period should be represented by at least five samples of plume rise or some equivalent amount of data.

The outstanding feature of Fig. 5.2 is that all the plume center lines continued to rise as far as measurements were made; there is no evidence of leveling off. In general, the plume center lines approximate a $2/3$ slope, as predicted by the "2/3 law" in Eq. 4.32. This means that the final rise has not definitely been measured in neutral conditions, and therefore we will have to find some other way of defining effective stack height.

The same data as in Fig. 5.2, along with the data of Ball,⁷⁶ are plotted in Fig. 5.3. Both the rise and the distance downwind are made nondimensional by means of the length $L = F/u^3$. The result is a somewhat entangled family of curves that lie between 1.0 and 3.0 times $F^{\frac{1}{3}}u^{-1}x^{\frac{2}{3}}$. Rise for a buoyant plume according to the Bosanquet theory²⁰ and the asymptotic plume rises according to Csanady⁸⁷ in 1961 and to Briggs¹⁰⁷ in 1965 are shown. They all underestimate rise at large values of x/L .

The Bosanquet²⁰ formulation underestimates plume rise when $x/L > 10^3$. The CONCAWE^{25,26} relation that Δh is proportional to $u^{-\frac{3}{4}}$ and the Davidson-Bryant¹⁰⁵ relation that Δh is proportional to $u^{-1.4}$ are not valid for most data sources. Formulas of the type $\Delta h \propto L = F/u^3$ are difficult to test because they apply only to final rise in

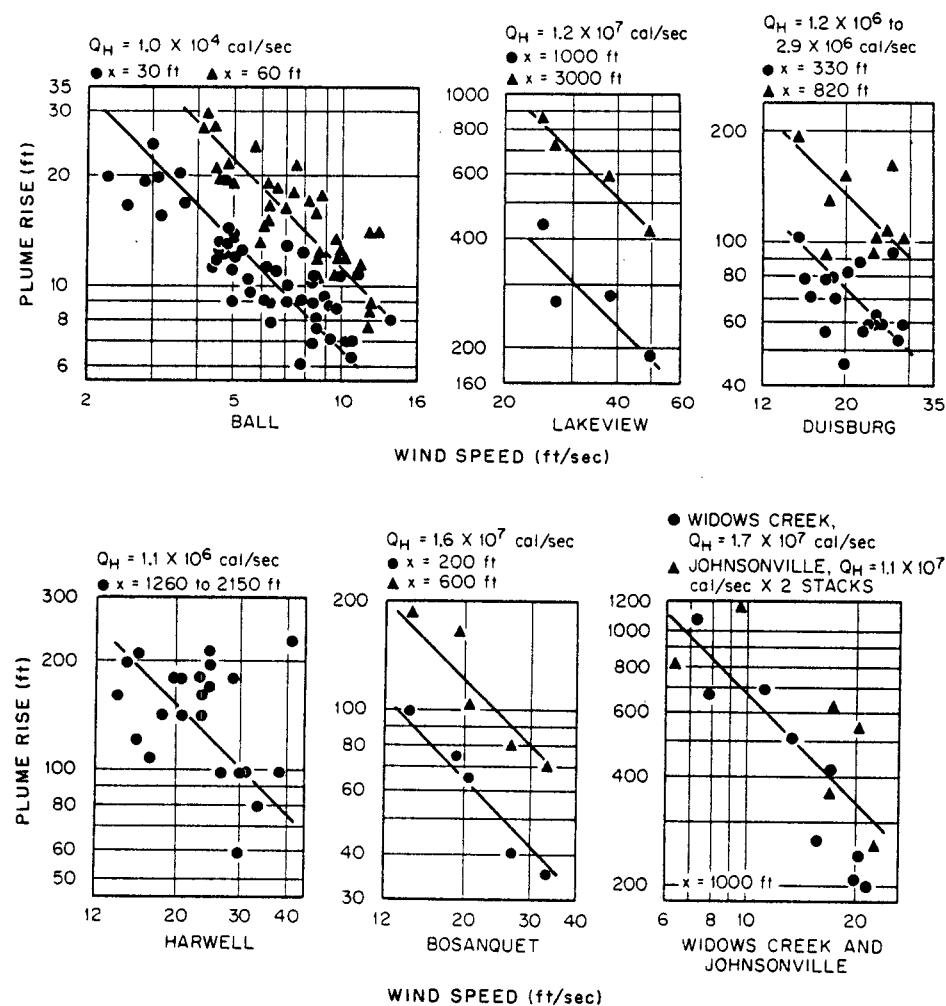
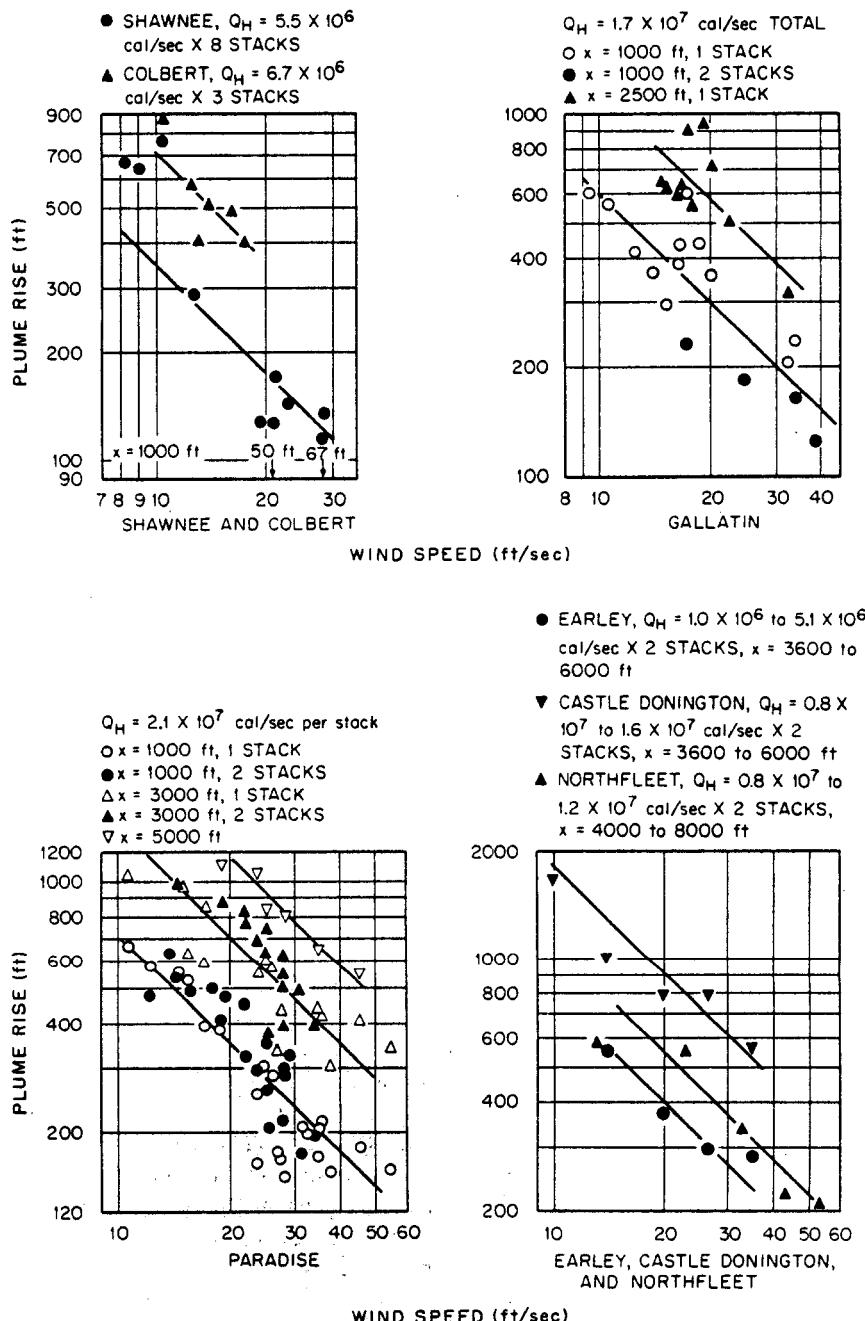


Fig. 5.1 Plume rise vs. wind speed in near-neutral conditions.

NEUTRAL CONDITIONS

41



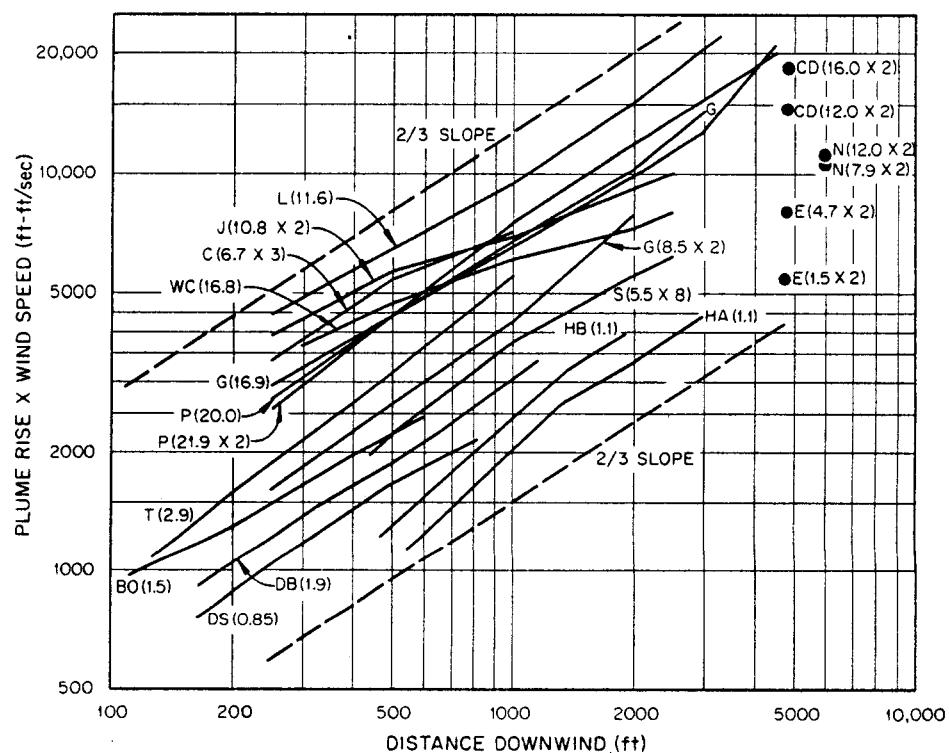


Fig. 5.2 Plume rise times wind speed vs. downwind distance in near-neutral conditions. The average heat efflux per stack, in units of 10^6 cal/sec, and the number of stacks, if more than one, are given in parentheses. See Table 5.1 for identification of sources and for additional data.

neutral conditions, which has not yet been clearly observed. Therefore only relations of the type $\Delta h \propto u^{-1}$ have been chosen for the comparison shown in Table 5.1. Data are given for the plume rises at the maximum distance downwind for which there was sufficient information to meet the data criterion set up for Fig. 5.2. The ratio of calculated to observed plume rises times wind speed was computed for each source and each formula, and the results were analyzed on a one-source one-vote basis. The exceptions to this rule were plants that were run both with one stack and with two stacks emitting (Paradise and Gallatin) and plants at which there were substantial amounts of data for different rates of heat emission (Earley, Castle Donington, Northfleet). The median value of the ratio was also computed for each plume-rise formula, along with the average percentage deviation from the median. The same computation was repeated for a selected set of data that excluded the following data sources: Ball, source very small; Harwell A, wind speed measured much below plume and obviously lower than that measured with system B; Bosanquet, no stack heights indicated and length of runs uncertain; Darmstadt, low efflux velocity and insufficient

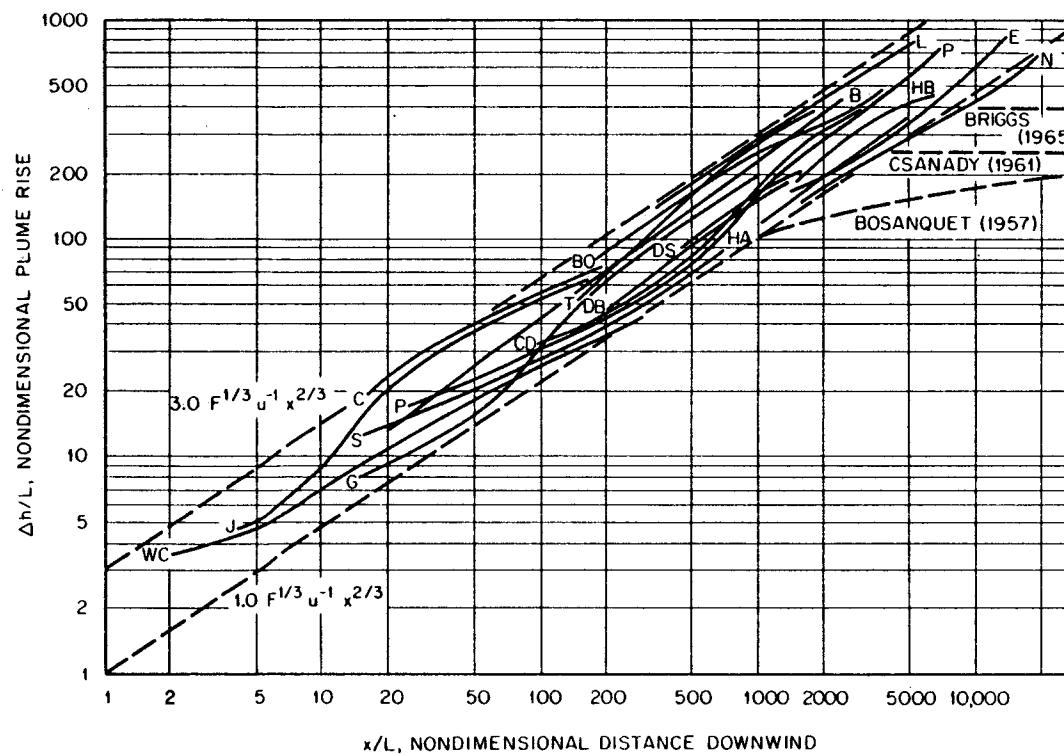


Fig. 5.3 Nondimensional plume rise vs. nondimensional distance downwind in near-neutral conditions. Rise for a buoyant plume according to Bosanquet²⁰ and the asymptotic plume rises according to Csanady⁸⁷ and Briggs¹⁰⁷ are shown. See Table 5.1 for identification of other curves and for additional data.

Table 5.1

COMPARISON OF CALCULATED VALUES WITH OBSERVATIONS FOR NEUTRAL CONDITIONS

Code	Source	Reference	Number of stacks	h_s , ft	D, ft	w_0 , ft/sec	Range of u , ft/sec	Q_H /stack, 10^6 cal/sec	$x^*,†$, ft	x, ft	$u \Delta h$, ft-ft/sec
B	Ball‡	76					2 to 14	0.0096	14§	60	112
HA	Harwell A‡	75	1	200	11.3	32.6	14 to 30	1.10	370	2950	4,430
HB	Harwell B	75	1	200	11.3	32.6	17 to 38	1.10	370	1900	3,980
BO	Bosanquet‡	73	1		6.5	31.9	14 to 33	1.54	485¶	600	2,450
DS	Darmstadt‡	80	1	246	7.5	15.7	16 to 25	0.855	380	820	2,150
DB	Duisburg	80	1	410	11.5	28.0	15 to 29	1.88	705	1150	3,400
T	Tallawarra‡	87	1	288	20.5	12.0	20 to 23	2.93	680	1000	5,500
L	Lakeview‡	88	1	493	19.5	65.0	25 to 49	11.6	1630	3250	22,100
CEGB plants											
E	Earley	91	2	250	12.0	18.3	14 to 35	1.54	485	4800	5,580
E	Earley	91	2	250	12.0	56.0	14 to 35	4.72	760	4800	8,150
CD	Castle Donington	91	2	425	23.0	40.9	10 to 26	11.95	1510	4800	14,800
CD	Castle Donington	91	2	425	23.0	54.7	10 to 35	16.0	1700	4800	18,600
N	Northfleet‡	93	2	492	19.7	46.3	13 to 52	7.9	1400	5900	10,900
N	Northfleet‡	93	2	492	19.7	70.0	13 to 52	11.95	1660	5900	11,150
TVA plants											
S	Shawnee‡	99	8	250	14.0	48.7	8 to 29	5.45	805	2500	6,210
C	Colbert‡		3	300	16.5	42.9	10 to 17	6.74	975	1000	7,200
J	Johnsonville		2	400	14.0	94.8	6 to 22	10.8	1400	2500	10,100
WC	Widows Creek‡		1	500	20.8	71.5	8 to 21	16.8	1910	2500	8,000
G	Gallatin		1	500	25.0	52.4	7 to 34	16.9	1920	3000	14,250
G	Gallatin		2	500	25.0	23.7	5 to 39	8.55	1460	2000	7,850
P	Paradise		1	600	26.0	51.3	6 to 55	20.2	2300	4500	21,200
P	Paradise		2	600	26.0	57.2	12 to 34	21.9	2380	4500	20,000

†Calculated from Eq. 4.35.

‡Not included in selected data.

§ Height chosen for computing $x^* = 20$ ft.¶ Height chosen for computing $x^* = 250$ ft.

Table 5.1 (Continued)

Ratio of calculated to observed values of $u \Delta h$										
Code	Source	Reference	Moses and Carson ¹¹⁰	Stümke ¹⁰⁸	Holland ⁶	Priestley ^{73,87} (first phase)	Lucas, Moore, and Spurr ⁹¹	Lucas ¹⁰⁹	Eq. 4.32 ("2/3 law")	Eq. 4.34
B	Ball	76	1.59		0.04	1.31	1.51	0.78	0.86	0.72
HA	Harwell A	75	0.43	0.74	0.23	2.00	1.70	1.27	1.40	0.95
HB	Harwell B	75	0.48	0.83	0.25	1.60	1.59	1.19	1.17	0.93
BO	Bosanquet	73	0.92	0.75	0.44	1.19	1.38	1.12	0.98	0.98
DS	Darmstadt	80	0.78	1.04	0.25	1.49	1.69	1.36	1.13	1.09
DB	Duisburg	80	0.74	1.12	0.38	1.47	1.62	1.60	1.17	1.16
T	Tallawarra	87	0.57	1.53	0.30	0.91	1.02	0.87	0.76	0.75
L	Lakeview	88	0.28	0.41	0.31	0.78	0.62	0.68	0.66	0.64
CEGB plants										
E	Earley	91	0.40	0.72	0.18	2.49	1.59	1.30	1.73	1.05
E	Earley	91	0.48	0.57	0.37	2.26	1.44	1.16	1.71	1.25
CD	Castle Donington	91	0.43	0.74	0.44	1.57	1.01	1.01	1.29	1.17
CD	Castle Donington	91	0.39	0.62	0.47	1.34	0.86	0.86	1.13	1.05
N	Northfleet	93	0.47	0.79	0.44	2.24	1.24	1.35	1.75	1.46
N	Northfleet	93	0.56	0.84	0.65	2.43	1.35	1.47	1.96	1.73
TVA plants										
S	Shawnee	99	0.68	0.90	0.54	1.88	1.69	1.36	1.53	1.40
C	Colbert		0.66	0.96	0.55	0.86	0.96	0.82	0.77	0.77
J	Johnsonville		0.59	0.66	0.66	1.37	1.24	1.21	1.19	1.17
WC	Widows Creek		0.94	1.32	1.18	1.94	1.76	1.92	1.73	1.72
G	Gallatin		0.53	0.90	0.64	1.35	1.05	1.15	1.10	1.09
G	Gallatin		0.68	1.53	0.58	1.41	1.37	1.50	1.21	1.20
P	Paradise		0.39	0.65	0.51	1.19	0.80	0.96	1.03	1.00
P	Paradise		0.42	0.70	0.58	1.28	0.85	1.03	1.11	1.09
Median for all data										
Median for selected data										
			$0.54 \pm 34\%$	$0.79 \pm 27\%$	$0.44 \pm 37\%$	$1.44 \pm 26\%$	$1.36 \pm 21\%$	$1.18 \pm 20\%$	$1.17 \pm 23\%$	$1.09 \pm 19\%$
			$0.48 \pm 19\%$	$0.72 \pm 24\%$	$0.47 \pm 26\%$	$1.41 \pm 18\%$	$1.24 \pm 22\%$	$1.16 \pm 14\%$	$1.17 \pm 12\%$	$1.09 \pm 7\%$

data; Tallawara and Lakeview, much higher rise than comparable sources in Fig. 5.2, possibly due to lakeshore effect; Widows Creek, downwash, possibly due to a 1000-ft plateau nearby, shown in Figs. 5.1 and 5.2; Northfleet, terrain downwash reported by Hamilton⁹³ and rise much lower than at Castle Donington at same emission; Colbert and Shawnee, many stacks. The results in Table 5.1 help justify the exclusion of these data, since with the selected data the average deviation from the median is considerably reduced for seven of the eight formulas.

The first three formulas tested in Table 5.1 are completely empirical and do not allow for the effect of distance of measurement on plume rise as the remaining five formulas do; consequently, these three formulas give poorer agreement with data. The Holland⁶ formula (Eq. 4.1) in particular shows a high percentage of scatter. The formula of Stümke¹⁰⁸ (Eq. 4.4) is perhaps slightly preferable to that of Moses and Carson¹¹⁰ (Eq. 4.8), although the latter shows less scatter in comparison with the selected data. All three of these formulas underestimate plume rise, but this shortcoming can be corrected by multiplying the formulas by a constant that optimizes the agreement.

The next three formulas are based on the Priestley⁷³ theory. The first is the asymptotic formula for the first-phase theory⁸⁷ (Eq. 4.36), which predicts a rise proportional to $x^{3/4}$. Even though this is a transitional-rise formula, which does not apply to a leveling off stage of plume rise, it shows less scatter compared with observations than the three empirical final-rise formulas. The next formula (Eq. 4.37), by Lucas, Moore, and Spurr,⁹¹ includes both a transitional- and a final-rise stage and gives a little better agreement with data. When Eq. 4.37 is multiplied by the empirical stack-height factor suggested by Lucas,¹⁰⁹ i.e., $0.52 + 0.00116 h_s$, the agreement is considerably better. However, one should be cautious about applying this formula to plants with heat emission less than 10 Mw, because it predicts continued plume rise to almost 1 km downwind regardless of source size. For instance, for the very small source used by Ball,⁷⁶ the predicted final rise is 12 times the rise measured at 60 ft downwind; it seems unlikely that such a weakly buoyant plume so close to the ground, where turbulence is stronger, will continue to rise over such a long distance.

The last two formulas are based on the simplified theory given in the section, "Basic Theory Simplified" in Chapter 4. The "2/3 law" (Eq. 4.32), another transitional-rise formula, agrees about as well with these data as the Lucas¹⁰⁹ formula just discussed. Equation 4.34, which includes both a transitional-rise and a final-rise stage, gives both improved numerical agreement and much less percentage of scatter. Clearly it is the best of the eight formulas tested in Table 5.1 and is the one recommended for buoyant plumes in neutral conditions (for optimized fit it should be divided by 1.09).

Eq. 4.34 should not be applied beyond $x = 5x^*$, because so few data go beyond this distance. In some cases the maximum ground concentration occurs closer to the source than this, and Eq. 4.34 applied at the distance of the maximum gives the best measure of effective stack height. (Beyond this distance plumes diffuse upward, and the interaction of diffusion with plume rise cannot be neglected.) One conservative approach is to set $x = 10 h_s$, which is about the minimum distance downwind at which maximum ground concentration occurs. For the fossil-fuel plants of the Central

Electricity Generating Board (CEGB) and TVA in Table 5.1, at full load this distance turns out to be in the range $2.5 < (x/x^*) < 3.3$. At $x/x^* = 3.3$, Eq. 4.34 gives a plume rise only 10% lower than Eq. 4.32, but at twice this distance the plume rise is increased by only 27%. This suggests a rule of thumb that Eq. 4.34 can be approximated by Eq. 4.32, the "2/3 law," up to a distance of 10 stack heights, beyond which further plume rise is neglected, i.e.,

$$\begin{aligned}\Delta h &= 1.8 F^{\frac{1}{3}} u^{-1} x^{\frac{2}{3}} & (x < 10 h_s) \\ \Delta h &= 1.8 F^{\frac{1}{3}} u^{-1} (10 h_s)^{\frac{2}{3}} & (x > 10 h_s)\end{aligned}\quad (5.1)$$

For other sources a conservative approximation to Eq. 4.34 is to use Eq. 4.32 up to a distance of $x = 3x^*$ and then to consider the rise at this distance to be the final rise. Surprisingly, Eq. 5.1 compares even better with the data in Table 5.1 than the recommended Eq. 4.34. Excluding Ball's data, which were for a ground source, the median ratio of calculated to observed plume rises is about 1.13, and the average deviations are $\pm 17\%$ for all data and $\pm 4\%$ for the selected data. Because of the nature of the approximation used in Eq. 5.1 and the scarcity of data beyond $x = 5x^*$, Eq. 5.1 is recommended as an alternative to Eq. 4.34 only for fossil-fuel plants with a heat emission of at least 20 Mw at full load.

For multiple stacks the data show little or no enhancement of plume rise over that from comparable single stacks in neutral conditions. Observations at the Paradise Steam Plant were about equally split between one-stack operation and two-stack operation with about the same heat emission from the second stack. In Fig. 5.1 the plume rises in these two conditions can be seen to be virtually indistinguishable. However, the same figure shows a clear loss in plume rise at Gallatin for the cases in which the same heat emission was split between two stacks. In Table 5.1 average plume rises for plants with two stacks are somewhat less than those for plants with one stack, at least in comparison with Eq. 4.34. Colbert, with three stacks, seems to have an enhanced rise, but Shawnee, with eight or nine stacks operating, has a lower rise than would be expected for a single stack. This may be due to downwash, as noted in the discussion of Fig. 5.1. In summary, the observations do not clearly support any additional allowance for plume rise when more than one stack is operating. It is beneficial to combine as much of the effluent as possible into one stack to get the maximum heat emission and the maximum thermal plume rise. This has been the trend for large power plants both in England and in the United States.

Few data are available to evaluate plume rise in unstable conditions. Slawson⁸⁸ found a just slightly higher average rise in unstable than in neutral conditions, as well as more scatter, as might be expected owing to convective turbulence. The same general features are evident in the TVA data. The buoyancy flux of the plume increases as it rises in unstable air, but there is also increased atmospheric turbulence; it is not clear which influence has the greater effect on the plume. However, because of lack of empirical evidence, it is possible only to recommend for unstable conditions the same formulas that apply in neutral conditions, specifically Eq. 5.2.

Jets in Neutral Conditions

Most data for jets in a crosswind do not extend very far downwind; so in Fig. 5.4 they are compared with the bending-over plume model in "Basic Theory Simplified," Chapter 4; $\Delta h/D$ is plotted as a function of $R = w_0/u$ for two different distances

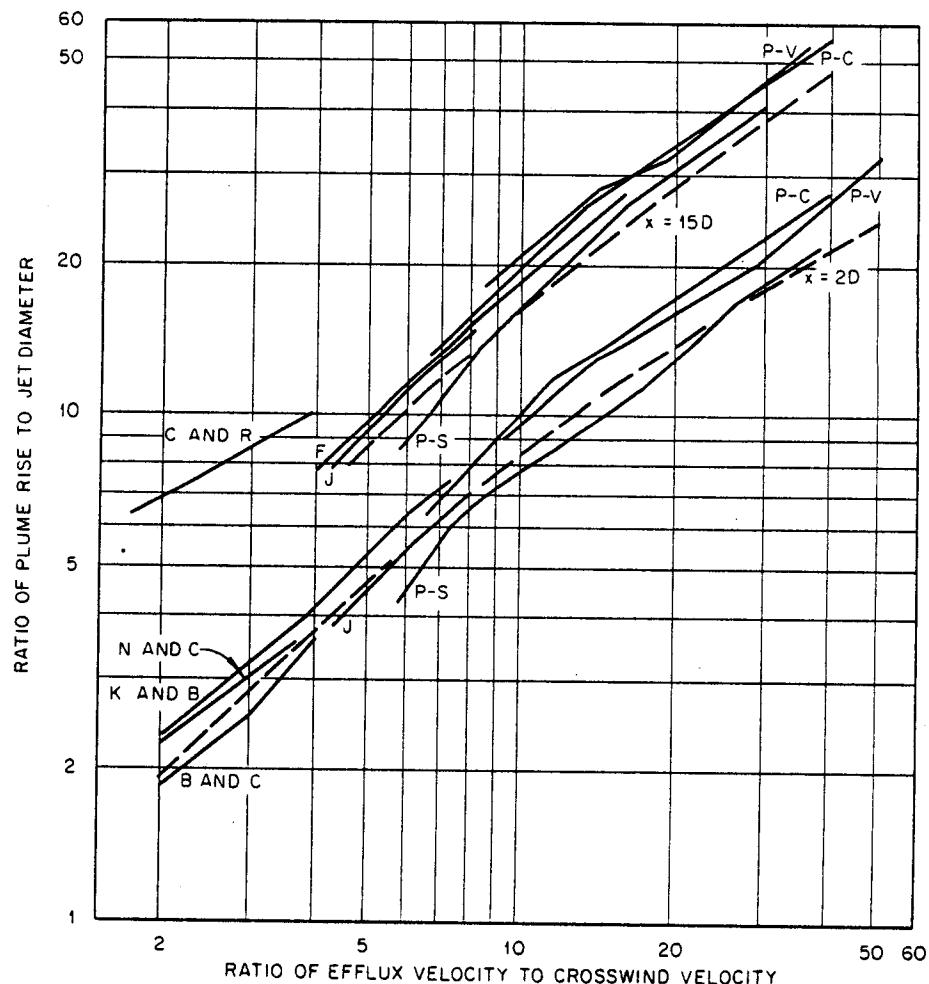


Fig. 5.4 Plume rise of jets in crosswind compared with values for bending-over plume model.

B and C, Bryant and Cowdry⁶⁷

C and R, Callaghan and
Ruggeri⁶²

F, Fan⁷¹

J, Jordinson⁶⁵

K and B, Keffer and
Baines⁶³

N and C, Norster and Chapman⁶⁵

P-C, concentration profiles,
Patrick⁶⁵

P-S, Schlieren photographs,
Patrick⁶⁵

P-V, velocity profiles,
Patrick⁶⁵

downwind, $x = 2D$ and $x = 15D$. The two families of curves group together rather well, considering the variety of experiments and measurement techniques, which include the photographic center lines by Bryant and Cowdry^{6,7} (B and C), the temperature survey by Norster and Chapman^{6,5} (N and C), the velocity survey by Keffer and Baines^{6,3} (K and B), the total pressure measurements by Jordinson^{6,5} (J), the top of the temperature profile measured by Callaghan and Ruggeri^{6,2} (C and R), the photographic measurements by Fan^{7,1} (F), and the three different sets of measurements made by Patrick,^{6,5} i.e., concentration profiles (P-C), velocity profiles (P-V), and Schlieren photographs (P-S). The data are fit rather well by the dashed line that represents the formula given by the bending-over plume model (Eqs. 4.14 and 4.24); the resultant formula is probably not of practical value since it applies only near the source and, being unwieldy, is not written out. This is just a test of the entrainment assumption. Only the Callaghan and Ruggeri data do not fit the pattern. A number of reasons are possible, one being that the jet velocities were near supersonic and another being that this jet was more nearly horizontal, the distance downwind being about twice the rise. The main reason this curve is higher is probably that it represents the top of the jet rather than the center line.

A comparison of values from Eq. 4.33 with the few sets of data that go as far as 100 or 200 stack diameters downwind is shown in Fig. 5.5. Equation 4.33 does fairly

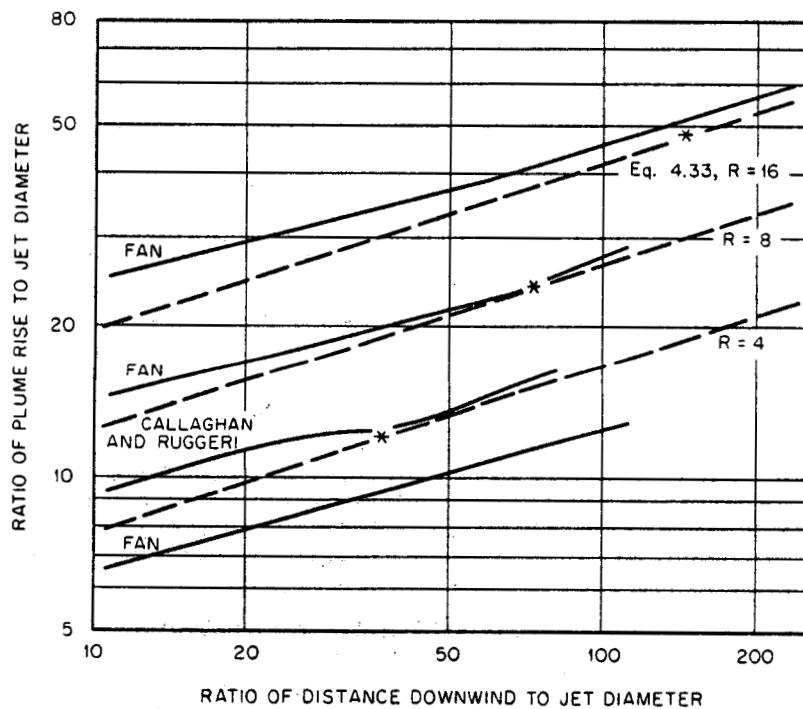


Fig. 5.5 Plume rise of jets in crosswind compared with values from Eq. 4.33. ($R = w_0/u$; asterisks denote $\Delta h/D = 3.0 R$.)

well even when the plume is more vertical than horizontal ($\Delta h > x$) and works quite well when the plume is more horizontal. The exception is that it overestimates the rise measured by Fan at the lower value of $R = w_0/u$, specifically at $R = 4$. This lends some credence to the suggestion made by Hoult, Fay, and Forney^{6,8} that the entrainment constant γ may be a function of R although the particular function that they suggest works poorly in the present model. It should be noted that Fan's plumes were partially buoyant, but these effects are minimized by rejecting data for which $F_x/F_m u$, the ratio of buoyancy-induced momentum flux to initial momentum flux, is greater than 0.5.

As for the final rise of a jet, again it appears that none has been measured, but the asterisks in Fig. 5.5 at $\Delta h/D = 3.0 R$ (see Table 4.1) indicate a reasonable value for maximum observed rise; i.e.,

$$\Delta h = 3 \frac{w_0}{u} D \quad (5.2)$$

This is twice the value given by Eq. 4.9, the often-cited formula of Rupp and his associates.^{6,1}

STABLE CONDITIONS

Penetration of Elevated Inversions

A hot plume will penetrate an inversion and continue to rise if at that elevation the plume is warmer than the air above the inversion, i.e., if its temperature excess exceeds ΔT_i . A jet, on the other hand, must have enough momentum to force its way through an inversion, and then it must eventually subside back to the level of the inversion since it is cooler than the air above. For the case of no wind, the simplified vertical model with boundary conditions implies that penetration ability is a function of b_i , z_i , F_m , and F . Then conventional dimensional analysis predicts penetration when

$$z_i b_i^{0.6} F^{-0.4} \leq f(F_m b_i^{0.8} F^{-1.2}) \quad (5.3)$$

where f is a function to be determined empirically. Relevant empirical data, collected in a modeling experiment of Vadot,^{6,0} are plotted in Fig. 5.6 to confirm Eq. 5.3. The two data points on the left side of Fig. 5.6 provide some support for Eq. 4.29 for penetration by a buoyant plume. As F_m is increased, the penetration ability actually decreases because a jet expands faster than a buoyant plume and therefore any heat content is diluted more rapidly. For very large values of the nondimensional momentum flux, the height of possible penetration becomes proportional to $(F_m/b_i)^{1/4}$. Three data points on the right side of Fig. 5.6 suggest that the constant of

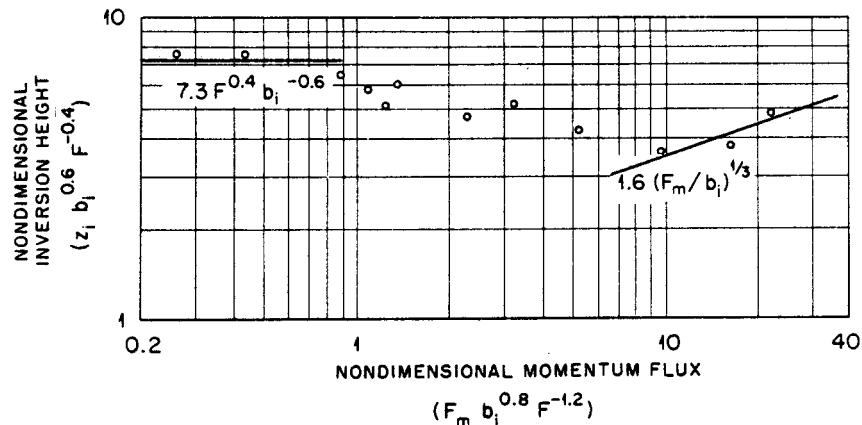


Fig. 5.6 Maximum nondimensional inversion height for penetration by plume vs. nondimensional momentum flux (based on data from Vadot⁶⁰).

proportionality is roughly 1.6, as given in Eq. 4.31. As a simple, conservative criterion for a vertical plume, Vadot's experiments suggest penetration when

$$z_i \leq 4F^{0.4} b_i^{-0.6} \quad (5.4)$$

A bent-over buoyant plume rising through neutrally stratified air should penetrate an inversion at height z_i if, as expressed by Eq. 4.30,

$$z_i \leq 2.0 \left(\frac{F}{ub_i} \right)^{1/4}$$

This equation (Eq. 4.30) was derived from the simplified bent-over plume model, which gives a characteristic temperature excess of the plume of

$$\theta' = 4.0 \frac{T}{g} \frac{F_z}{uz^2} \quad (5.5)$$

for a plume rising through neutral air. Eq. 5.5 is easier to apply to cases where there are two or more inversions separated by neutral stratification. Initially $F_z = F$, and θ' decreases with the inverse square of the height above the source until the plume reaches the first inversion. As the plume rises through the inversion, its potential temperature is unaffected, but the potential temperature of the ambient air increases by ΔT_i ; thus θ' is reduced by ΔT_i . If θ' remains positive, the plume is buoyant and continues to rise with θ' proportional to z^{-2} until it reaches the height of the next inversion. The same procedure is repeated until the plume reaches an inversion it cannot penetrate, i.e., until $\theta' < \Delta T_i$.

The results obtained by applying this procedure to the data of Simon and Proudfit¹⁰³ from the Ravenswood plume in New York City, which include plume penetrations of multiple inversions, are shown in Table 5.2, along with the temperature excesses of the plume relative to the air above the inversion as calculated by subtracting ΔT_i from Eq. 5.5 applied at the top of the inversion. It can be seen that every one of the eight nonpenetrations is predicted by a negative calculated θ' . In one case penetration is questionable because the plume center line ascended only 10 m higher than the inversion; so the lower part of the plume was undoubtedly below the inversion. Only one of the five penetrations was not predicted, and that was with a negative θ' of only 0.2°C , near the limits of the accuracy of temperature measurements. The procedure given in the discussion following Eq. 5.5 appears to be a good predictor but, perhaps, just slightly conservative.

Rise Through Uniform Temperature Gradient

Also of particular interest is the case in which the plume rises through air with a fairly uniform temperature gradient. In this case we can approximate s as a constant. For the calm case the simple vertical model predicts that the buoyancy of a hot plume decays to zero according to Eq. 4.25. This formula was derived by M,T,&T⁵⁸ from virtually the same model, and a similar formula was derived by Priestley and Ball.¹¹⁷ The ability of Eq. 4.25 to predict the final height of the tops of plumes is shown in Fig. 5.7. Data are plotted from the modeling experiment in stratified salt solution by M,T,&T⁵⁸ from the modeling experiment in air near the floor of an ice rink of Crawford and Leonard,⁵⁹ from the experiments of Vehrencamp, Ambrosio, and Romie¹⁰⁰ on the Mojave Desert, and from the observation by Davies^{101,102} of the plume from a large oil fire. Equation 4.25 correctly approximates the top of the massive smoke plume that billowed out of the Surtsey volcano in 1963.¹⁴⁰ The rate of thermal emission was estimated to be of the order of 100,000 Mw,¹⁴¹ or about a thousand times greater than the heat emission from a large stack. For the average lapse rate observed in the troposphere ($6.5^\circ\text{C}/\text{km}$), Eq. 4.25 gives a rise of 5 km, or about 16,000 ft; the observed cloud top ranged from 3 to 8 km.

As the nondimensional momentum flux is increased, Morton's¹¹⁹ numerical solution indicates lessened plume rise, just as inversion penetration ability was seen to decline in Fig. 5.6. There are no data to show this, but three experiments with vertical plumes by Fan⁷¹ indicate gradual enhancement of rise over that given by Eq. 4.25 when $F_m s^{1/4}/F > 1.8$. Dimensional analysis of the vertical model indicates that

$$\Delta h = CF_m^{1/4} s^{-1/4} \quad (5.6)$$

for a pure jet, where C is a constant. The values of C that correctly describe Fan's plumes, which were momentum dominated but not pure jets, are 4.53, 4.43, and 4.18. A value of $C = 4$ is suggested as an approximation, as in Eq. 4.27.

Table 5.2
INVERSION PENETRATION AT THE RAVENSWOOD PLANT†

Date	Time	\bar{Q}_H , 10^7 cal/sec	\bar{u} , m/sec	Plume height, m	Inversion height, m		ΔT_i , $^{\circ}$ C	θ , $^{\circ}$ C	Calculated Penetration
					Bottom	Top			
May 25	1825	1.97	9.0	295	145	180	0.2	15	Yes
					325	475	0.7	-0.5	No
July 20	0552-0559	0.98	10.5	350	255	275	0.3	0.05	Yes
	0617-0820	1.11	7.3	360	365	395	2.0	-2.0	No
July 21	0600-0724	1.13	4.3	360	410	450	0.6	-0.45	No
	0828	1.64	2.7	510	240	280	0.6	1.7	Yes
					360	410	0.4	0.0	Yes
September 8	0648-0930	1.66	7.5	410	360	400	0.8	-0.6	?
	1000-1020	1.77	5.4	560	620	650	0.4	-0.3	No
September 9	0640-0705	1.20	9.6	350	360	400	2.1	-2.0	No
	0747-0850	1.54	9.1	370	260	300	0.7	-0.2	Yes
					370	410	1.6	-1.6	No
	0930-1000	2.13	9.6	390	420	530	1.8	-1.7	No

† Stack height, 155 m.

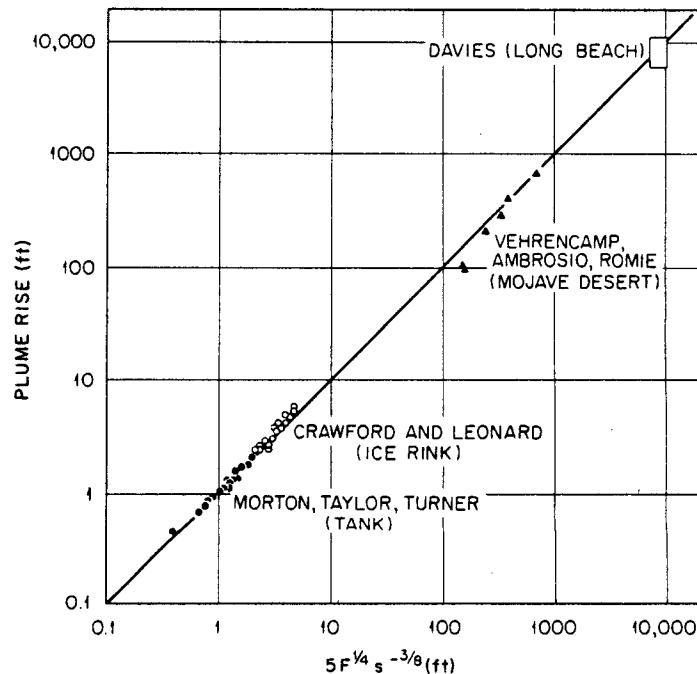


Fig. 5.7 Rise of buoyant plumes in calm, stable air.

For the case of a bent-over plume rising through stable air with constant s , the quasi-horizontal model can be applied both to a buoyant plume and to a jet to yield Eqs. 4.26 and 4.28, respectively. There are no data to test Eq. 4.28, but Eq. 4.26 and several other formulas can be compared with data from buoyant plumes released in stable air. These data include nine runs made at Brookhaven⁸⁶ with 15-sec ignitions of rocket fuel, six runs by TVA⁹⁹ with large single stacks, and seven runs by Van Vleck and Boone⁷⁹ with 60-sec firings of horizontal rocket motors. Admittedly the plumes were not continuous in two of these experiments, and the plume rises were defined somewhat differently in each case. In each case the ratios of the calculated to observed rises were computed. The resulting median values of this ratio and mean deviation from the median are

Holland ⁶	$0.44 \pm 131\%$
Priestley ⁷³	$0.42 \pm 43\%$
Bosanquet ²⁰	$1.22 \pm 26\%$
Briggs, Eq. 4.26	$0.82 \pm 13\%$

Holland⁶ suggested that Eq. 4.1 be reduced by 20% to predict rise in stable conditions, but this may be seen to work poorly. The Priestley⁷³ and Bosanquet²⁰ theoretical formulations are both complex; so they were simplified to the case for a buoyant

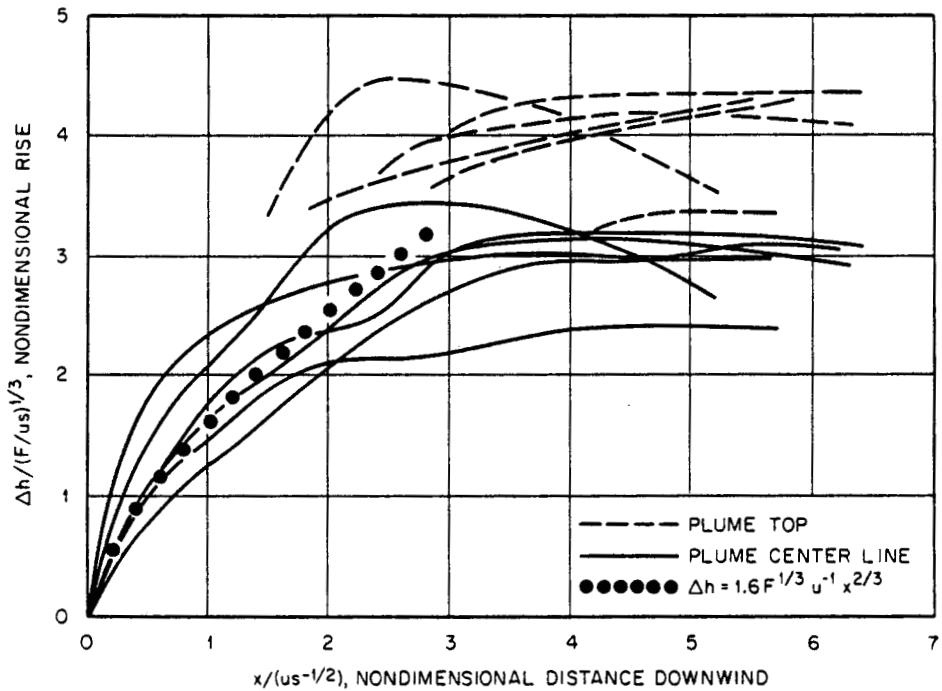


Fig. 5.8 Rise of buoyant plumes in stable air in crosswind at the TVA Paradise and Gallatin plants.

point source. Clearly Eq. 4.26 gives the most consistent agreement, and on the average it slightly underestimates rise. A constant of $2.4/0.82 = 2.9$ works best, i.e.,

$$\Delta h = 2.9 \left(\frac{F}{us} \right)^{1/3} \quad (5.7)$$

A further test of the simplified theory for bent-over plumes is shown in Fig. 5.8 for six periods of TVA data, which include the complete trajectories of the plume center lines and plume tops in stable air. The center lines follow the "2/3 law" in the first stage of rise with a fairly typical amount of scatter and reach a maximum in the neighborhood of $x = \pi us^{-1/2}$ as is predicted by theory. There is less scatter in the final-rise stage, where four of the six trajectories almost coincide. The actual final heights range from 450 to 1500 ft. The plume tops level out at

$$\Delta h = 4.0 \left(\frac{F}{us} \right)^{1/3} \quad (5.8)$$

When two or three stacks were operating at the TVA plants, there was some evidence of enhanced final rise in stable conditions. The maximum enhancement that

could be expected according to Eq. 5.7 would be 26 and 44% for two and three stacks, respectively, if the total heat emission could simply be lumped together in computing F . The averaged observed enhancement relative to Eq. 5.7 was +20% with two stacks operating and +30% with three stacks operating except that when the wind was blowing along the line of three stacks at Colbert the enhancement was +40%. Enhancement also depends on stack spacing since the plumes can hardly be expected to interact with each other if they are too far apart, especially if the wind is perpendicular to the line of stacks. In the preceding cases the stacks were spaced less than $0.9(F/\text{us})^{1/4}$, or about one-fourth of the plume rise apart.

6 CONCLUSIONS AND RECOMMENDATIONS

There is no lack of plume-rise formulas in the literature, and selection is complicated by the fact that no one formula applies to all conditions. For a given situation many different predictions emerge, as is shown in Table 5.1. The variety of theoretical predictions follows from the great variety of assumptions used in the models; the disagreement among empirical formulas is due to the different weighting of data used in their formulations and to variability among the data. Another factor is the frequent disregard of the dependence of plume rise on distance downwind of the stack. In the formulas recommended in the following paragraphs, all symbols are given in Appendix B, and the constants in the formulas are optimized for the best fit to data covered by this survey. Readjustment of the constants in previously cited equations is indicated by primes on the equation numbers.

An important result of this study is that buoyant plumes are found to follow the "2/3 law" for transitional rise for a considerable distance downwind when there is a wind, regardless of stratification; i.e.,

$$\Delta h = 1.6 F^{\frac{1}{3}} u^{-1} x^{\frac{2}{3}} \quad (4.32')$$

The bulk of plume-rise data are fit by this formula.

In neutral stratification Eq. 4.32' is valid up to the distance $x/x^* = 1$, beyond which the plume center line is the most accurately described by

$$\Delta h = 1.6 F^{\frac{1}{3}} u^{-1} x^{* \frac{2}{3}} \left[\frac{2}{5} + \frac{16}{25} \frac{x}{x^*} + \frac{11}{5} \left(\frac{x}{x^*} \right)^2 \right] \left(1 + \frac{4}{5} \frac{x}{x^*} \right)^{-2} \quad (4.34')$$

where

$$x^* = 0.52 \left[\frac{\sec^{\frac{1}{3}}}{ft^{\frac{2}{3}}} \right] F^{\frac{2}{3}} h_s^{\frac{1}{3}} \quad (h_s < 1000 \text{ ft}) \quad (4.35)$$

$$x^* = 33 \left[\frac{\sec^{\frac{1}{3}}}{ft^{\frac{2}{3}}} \right] F^{\frac{2}{3}} \quad (h_s > 1000 \text{ ft})$$

Equation 4.35 is the best approximation of x^* at present for sources 50 ft or more above the ground; for ground sources an estimated plume height can be used in place of h_s . Equation 4.34' applies to any distance such that $x/x^* > 1$, but owing to lack of data at great distances downwind $x/x^* = 5$ is suggested as the maximum distance at which it be applied at present. Even though Eq. 4.34' is the best of the dozen or so formulas considered, the average plume rise at a given plant may deviate from the value given by Eq. 4.34' by $\pm 10\%$ if the site is flat and uniform and by $\pm 40\%$ if a substantial terrain step or a large body of water is nearby. Furthermore, normal variations in the intensity of turbulence at plume heights at a typical site cause x^* to vary by about $\pm 20\%$ on the average, with corresponding variations in Δh . For fossil-fuel plants with a heat emission of 20 Mw or more, a good working approximation to Eq. 4.34' is given by

$$\Delta h = 1.6 F^{\frac{1}{3}} u^{-1} x^{\frac{2}{3}} \quad (x < 10 h_s) \quad (5.1')$$

$$\Delta h = 1.6 F^{\frac{1}{3}} u^{-1} (10 h_s)^{\frac{2}{3}} \quad (x > 10 h_s)$$

For other sources, a conservative approximation to Eq. 4.34' is to use Eq. 4.32' up to a distance of $x = 3x^*$, then to consider the rise at this distance to be the final rise.

Equations 4.34' and 5.1' are also recommended for the mean rise in unstable conditions although larger fluctuations about the mean should be expected (see Fig. 2.4).

In stable stratification Eq. 4.32' holds approximately to a distance $x = 2.4u_s^{-\frac{1}{3}}$, beyond which the plume levels off at about

$$\Delta h = 2.9 \left(\frac{F}{u_s} \right)^{\frac{1}{3}} \quad (5.7)$$

as illustrated in Fig. 5.8. The top of the stratified plume is about 38% higher than that predicted by Eq. 5.7, which describes the plume center line. Although no significant increase in transitional rise is found when more than one stack is operating, some enhancement of the final rise in stable conditions is observed provided the stacks are close enough. If the wind is so light that the plume rises vertically, the final rise is given accurately by

$$\Delta h = 5.0 F^{\frac{1}{3}} s^{-\frac{1}{3}} \quad (4.25)$$

In computing s for Eqs. 4.25 and 5.7, an average potential temperature gradient is calculated for the stable layer or for the layer expected to be traversed by the plume.

A buoyant plume will penetrate a ground inversion if both Eq. 5.7 and Eq. 4.25 give a height higher than the top of the inversion. The plume will penetrate an elevated inversion if the top of the inversion lies below both Eq. 5.4 and Eq. 4.30, i.e.,

$$z_i \leq 4F^{0.4} b_i^{0.6} \quad (\text{calm}) \quad (5.4)$$

$$z_i \leq 2.0 \left(\frac{F}{u b_i} \right)^{1/4} \quad (\text{wind}) \quad (4.30)$$

All the preceding formulas apply to buoyant plumes, which include most plumes from industrial sources, and they are fairly well confirmed by observations. Because of a relative lack of data, it is more difficult to make firm recommendations of formulas for jets. It appears that in neutral, windy conditions the jet center line is given by

$$\Delta h/D = 1.44 \left(\frac{w_0}{u} \right)^{1/4} \left(\frac{x}{D} \right)^{1/4} \quad (4.33)$$

at least up to the point that

$$\Delta h = 3 \frac{w_0}{u} D \quad (5.2)$$

as long as $w_0/u \geq 4$. It can be only tentatively stated that in windless conditions the jet rises to

$$\Delta h = 4 \left(\frac{F_m}{s} \right)^{1/4} \quad (5.6)$$

where 4 is used as the value of C . This is on the basis of only three experiments. If there is some wind and the air is stable, the minimum expected theoretical rise is

$$\Delta h = 1.5 \left(\frac{F_m}{u} \right)^{1/4} s^{-1/4} \quad (4.28)$$

Unfortunately there are no published data for this case, and it would be presumptuous to recommend any formula without testing it. However, since Eq. 4.28 is based on the same model, we should not use Eq. 5.6 or Eq. 5.2 if it gives a higher rise than Eq. 4.28 does. The most conservative of the three formulas is the one that best applies to a given situation. The same can be said of Eqs. 4.34', 5.7, and 4.25 for a buoyant plume.

Obviously more experiments are needed to complete our basic understanding of plume rise. In particular they are needed for jets at large distances downwind for all

stability conditions and for buoyant plumes at distances greater than ten stack heights downwind in neutral conditions. Once the fundamental results are complete, it will be worthwhile to study in detail the effect of the finite source diameter, the bending-over stage of plume rise, the effect of wind shear and arbitrary temperature profiles, the interaction of plumes from more than one stack, and the interaction of plume-rise dynamics with diffusion processes.

APPENDIX: **A** EFFECT OF ATMOSPHERIC TURBULENCE ON PLUME RISE

As discussed in "Basic Theory Simplified," in Chapter 4, entrainment of ambient air into the plume by atmospheric turbulence is due mostly to eddies in the inertial subrange; so, for a bent-over plume or a puff in a neutral atmosphere, the entrainment velocity, or velocity of growth, is given by

$$dr/dt = \beta \epsilon^{1/4} r^{1/4} \quad (A.1)$$

where β is a dimensionless entrainment constant, ϵ is the eddy energy dissipation rate, and r is a characteristic radius defined as $(V/u)^{1/4}$ for a bent-over plume. To apply this entrainment assumption, some simple method of estimating ϵ at plume heights is needed, and β must be determined.

Ideally ϵ would be related in some simple way to wind speed (u) and height above the ground (z). In the neutral surface layer, e.g., the lowest 50 ft or so, such a relation is well described by the expression¹¹⁵ $\epsilon = u^{*3}/0.4z$, where u^* is the friction velocity and is proportional to the wind speed at some fixed height. Unfortunately, at typical plume heights no such simple relation is found to exist. The turbulence becomes more intermittent and is affected more by departures from neutral stability and by terrain irregularities over a wide area. Still, enough data exist to estimate mean values of ϵ along with the amount of variability that should be expected.

Recent estimates of ϵ were made by Hanna,¹⁴² who used vertical-velocity spectra measured in a variety of experiments, and by Pasquill,¹⁴³ who used high-frequency standard deviations of wind inclination measured with a lightweight vane mounted on captive balloons at Cardington, England. Hanna used data from towers at Round Hill,

Mass.,¹⁴⁴ and Cedar Hill, Tex.,¹⁴⁵ from aircraft measurements made over a great variety of terrain by the Boeing Company,¹⁴⁶ and from several low-level installations (below 50 ft). These values of ϵ are used in Table A.1 to test the relation $\epsilon^{\frac{1}{3}} \propto u_m^{\frac{1}{3}}$ by computing the median value of $\epsilon^{\frac{1}{3}} u^{-m}$ and the average deviation from the median value for $m = 0, \frac{1}{3}, \frac{2}{3}$, and 1 at each height of each experiment. Because ϵ is sensitive to atmospheric stability, only runs in which $-1.0 < R_i < 0.15$ were used from the Round Hill and Cedar Hill data, where R_i is the local Richardson number; the Boeing runs during very stable conditions and Pasquill's measurements above inversions were omitted. Also omitted were the few runs made during very low wind speeds, i.e., less than 2 m/sec.

Table A.1 shows that the excellence of the fit is rather insensitive to increasing the value of m , especially at Round Hill and Cedar Hill. The best overall fit is with $m = \frac{1}{3}$; the average percentage deviation from the median is lowest with $m = \frac{1}{3}$ for four of the eight sets of data and, on the average, is only 9% greater than the minimum value of percentage deviation (indicated by \dagger in Table A.1). This is fortunate because the expression for x^* , the distance at which atmospheric turbulence begins to dominate entrainment, turns out to be independent of wind speed when $\epsilon^{\frac{1}{3}} \propto u^{\frac{1}{3}}$ (see Eq. 4.35 and the preceding discussion in Chap. 4). It is therefore very desirable to adopt this approximation, keeping in mind the scatter about the median values shown in the table.

It is evident in Table A.1 that $\epsilon^{\frac{1}{3}}/u^{\frac{1}{3}}$ decreases with height. With a power law relation of $\epsilon^{\frac{1}{3}}/u^{\frac{1}{3}} \propto \bar{z}^{-n}$, the optimum value of n depends on which data are used. The best least-squares fit to $\log \epsilon^{\frac{1}{3}}/u^{\frac{1}{3}} = \text{constant} - n \log \bar{z}$ is $n = 0.29$ for all the data but $n = 0.37$ if the Pasquill data at 4000 ft are omitted. At Round Hill $n = 0.31$ between 50 ft and 300 ft, and at Cedar Hill $n = 0.39$ between 150 ft and 450 ft, but in Pasquill's data n is only 0.15 between 1000 ft and 4000 ft. These values are roughly consistent with the following three published conclusions: (1) Hanna^{142,147} confirmed the relation $\epsilon^{\frac{1}{3}} = 1.5 \sigma_w \lambda_m^{-\frac{1}{3}}$ for a wide variety of data, where σ_w is the variance of vertical velocity and λ_m is the wavelength of maximum specific energy in the vertical-velocity spectra; (2) data compiled in a note by Moore¹³¹ indicate almost no dependence of σ_w on height from about 100 to 4000 ft except for very high wind speeds ($u > 10$ m/sec); (3) Busch and Panofsky¹⁴⁸ conclude that $\lambda_m \propto \bar{z}$ near the ground and reaches a maximum or a constant value somewhere above $\bar{z} = 200$ m. The simplest expression consistent with all of the preceding evidence is $\epsilon^{\frac{1}{3}}/u^{\frac{1}{3}} \propto \bar{z}^{-\frac{1}{3}}$ up to a height of the order of 1000 ft and then becomes constant with height. In the last column of Table A.1, an expression of this type is compared with the data. The best estimate of energy dissipation appears to be

$$\begin{aligned} \epsilon^{\frac{1}{3}} &= 0.9 [\text{ft}^{\frac{1}{3}}/\text{sec}^{\frac{1}{3}}] u^{\frac{1}{3}} \bar{z}^{-\frac{1}{3}} & (\bar{z} < 1000 \text{ ft}) \\ \epsilon^{\frac{1}{3}} &= 0.09 [\text{ft}^{\frac{1}{3}}/\text{sec}^{\frac{1}{3}}] u^{\frac{1}{3}} & (\bar{z} > 1000 \text{ ft}) \end{aligned} \quad (\text{A.2})$$

There remains the problem of how to determine the value of the dimensionless constant β , particularly when no observations of plume, puff, or cluster growth include

Table A.1
ENERGY DISSIPATION VS. WIND SPEED AND HEIGHT

Source	Height, ft	Number of runs	$\epsilon^{1/4}$, ft $^{1/4}$ /sec	$\epsilon^{1/4}/u^{1/4}$, ft $^{1/4}$ /sec $^{1/4}$	$\epsilon^{1/4}/u^{1/4}$, sec $^{-1/4}$	$\epsilon^{1/4}/u$, ft $^{-1/4}$	$(\epsilon z/u)^{1/4}$, ft $^{1/4}$ /sec $^{1/4}$
Round Hill	50	8	$0.636 \pm 17\%$	$0.266 \pm 14\%$ †	$0.103 \pm 16\%$	$0.042 \pm 18\%$	0.98
Round Hill	150	11	$0.495 \pm 11\%$	$0.177 \pm 10\%$	$0.063 \pm 8\%$ †	$0.022 \pm 10\%$	0.94
Cedar Hill	150	9	$0.457 \pm 20\%$	$0.159 \pm 18\%$	$0.057 \pm 16\%$ †	$0.020 \pm 19\%$	0.84
Round Hill	300	4	$0.470 \pm 11\%$	$0.151 \pm 7\%$ †	$0.049 \pm 7\%$ †	$0.017 \pm 7\%$ †	1.01
Cedar Hill	450	6	$0.331 \pm 9\%$ †	$0.104 \pm 9\%$ †	$0.034 \pm 11\%$	$0.010 \pm 17\%$	0.80
Boeing	750	22	$0.256 \pm 20\%$ †	$0.083 \pm 24\%$	$0.028 \pm 34\%$	$0.009 \pm 53\%$	0.75
Pasquill	1000	31	$0.269 \pm 38\%$ †	$0.097 \pm 44\%$	$0.042 \pm 46\%$	$0.018 \pm 53\%$	0.97
Pasquill	4000	10	$0.172 \pm 49\%$	$0.079 \pm 42\%$ †	$0.030 \pm 47\%$	$0.011 \pm 59\%$	0.79†

†Minimum value of percentage deviation.

‡ $\bar{z} = 1000$ ft.

simultaneous, independent measurements of ϵ . The approach used in this review is to assume the validity of Eq. A.2 at the time and place of diffusion experiments and to compare the results with Eq. A.1.

Frenkiel and Katz¹⁴⁹ used two motion-picture cameras to photograph smoke puffs released above an island in the Chesapeake Bay. The puffs were produced by small detonations of gunpowder from an apparatus on the cable of a tethered balloon. The radii of the puffs were calculated from their visible areas at 1-sec intervals. The values of $\beta\epsilon^{1/4}$ shown in Table A.2 were calculated from the first 2 sec of puff growth by using Eq. A.1 as a finite difference equation, i.e., by setting $dr/dt = \Delta r/\Delta t$. Smith and Hay¹⁵⁰ published some data from several experiments on the expansion of clusters of particles. In their short-range experiments, Lycopodium spores were released at a height of 2 m and were collected on adhesive cylinders lined up perpendicular to the wind at 100 m downwind, yielding a lateral standard deviation of particle distribution (σ_y). In their medium-range experiments, fluorescent particles were released from an airplane at heights of 1500 to 2500 ft several miles upwind of a sampling apparatus mounted on the cable of a captive balloon, yielding a vertical standard deviation of particle distribution (σ_z). The values of $\beta\epsilon^{1/4}$ shown in Table A.2 for the Smith and Hay experiments were calculated from the integral of Eq. A.1 for a point source, namely,

$$\frac{3}{2} r^{1/4} = \beta\epsilon^{1/4} t = \beta\epsilon^{1/4} \frac{x}{u}$$

Interpreting the effective radius of a rising plume in terms of σ_y or σ_z is difficult, but in this case it was assumed that $\sigma_z = \sigma_y$ and that $r = 2^{1/4} \sigma_y$, as is true in the "top hat" model equivalent to a Gaussian plume in the Morton, Taylor, and Turner⁵⁸ theory.

The last column of Table A.2 shows the value of $\beta\epsilon^{1/4}$ inferred from the diffusion data divided by the value of $\epsilon^{1/4}$ calculated from Eq. A.2. The values of β inferred from this calculation range from 0.62 to 0.82, a remarkably small range considering the

Table A.2
GROWTH RATE OF PUFFS AND PARTICLE CLUSTERS

Source	Number of runs	\bar{z} , ft	\bar{u} , ft/sec	$\beta\epsilon^{1/2}$, ft $^{3/2}$ /sec	$\beta\epsilon^{1/2}/\epsilon^{1/2}$ calculated
Smith and Hay					
Runs 1-5	5	$14 = \sigma_y$	18	$0.60 \pm 7\%$	0.62
Runs 7-10	4	$13 = \sigma_y$	30	$0.96 \pm 18\%$	0.82
Frenkiel and Katz					
$\bar{z} = 15$ to 22 m	6	58	19	$0.40 \pm 7\%$	0.64
$\bar{z} = 39$ to 61 m	7	164	52	$0.48 \pm 23\%$	0.78
Smith and Hay (May 7, 1959)	4	2500	16	$0.17 \pm 17\%$	0.74

indirectness of this approach and the wide range of variables involved. Note that the short-range experiments of Smith and Hay were probably carried out within the surface layer, where Eq. A.2 is not actually valid; nevertheless, the error in estimating ϵ is not large for moderate wind speeds at these heights. Table A.2 suggests that $\beta \approx 0.7$, but, considering the small number of data and the indirectness of this analysis, the more conservative value of $\beta = 1.0$ is recommended.

It should be cautioned that the characteristic plume radius, r , that appears in Eq. A.1 is not necessarily the same as the visible radius or other measures of size of a passive puff or plume, and so the evaluation of β made in Appendix A is not directly applicable to diffusion problems other than plume rise.

B APPENDIX: NOMENCLATURE

Dimensions of each term are given in brackets: l = length, t = time, τ = temperature, m = mass.

b_i	Inversion parameter = $g \Delta T_i / T [l/t^2]$
C_D	Drag coefficient [dimensionless]
D	Internal stack diameter [l]
F	Buoyancy flux parameter [l^4/t^3] ; see Eqs. 4.19c and 4.20
F_m	Momentum flux parameter [l^4/t^2] ; see Eq. 4.19b
F_z	Vertical flux of buoyant force in plume divided by $\pi \rho$ [l^4/t^3] ; see Eq. 4.17
Fr	Froude number = $w_0^2 / [g(\Delta T/T)D]$ [dimensionless]
g	Gravitational acceleration [l/t^2]
h	Effective stack height = $h_s + \Delta h [l]$
h_s	Stack height [l]
Δh	Plume rise above top of stack [l]
\vec{k}	Unit vector in the vertical direction [dimensionless]
L	Characteristic length for buoyant plume in crosswind = $F/u^3 [l]$
Q	Emission rate of a gaseous effluent [m/t]
Q_H	Heat emission due to efflux of stack gases [m^2/t^3]
R	Ratio of efflux velocity to average windspeed = w_0/u [dimensionless]
r	Characteristic radius of plume or puff, defined as $(V/u)^{1/4}$ for a bent-over plume [l]
r_0	Internal stack radius [l]

s	Restoring acceleration per unit vertical displacement for adiabatic motion in atmosphere [t^{-2}]; see Eq. 4.16
T	Average absolute temperature of ambient air [τ]
T_s	Average absolute temperature of gases emitted from stack [τ]
ΔT	Temperature excess of stack gases = $T_s - T$ [τ]
ΔT_i	Temperature difference between top and bottom of an elevated inversion [τ]
$\partial T / \partial z$	Vertical temperature gradient of atmosphere [τ/l]
t	Time [t]
u	Average wind speed at stack level [l/t]
u^*	Friction velocity in neutral surface layer [l/t]; see Ref. 115
V	Vertical volume flux of plume divided by π [l^3/t]; see Eq. 4.15
\hat{v}	Average velocity of plume gases [l/t]; see Eq. 4.18
\hat{v}'	Velocity excess of plume gases = $\hat{v}_p - \hat{v}_e$ [l/t]
\hat{v}_e	Average velocity of ambient air [l/t]
\hat{v}_p	Average local velocity of gases in plume [l/t]
w	Vertical component of $\hat{v} = \vec{k} \cdot \hat{v}$ [l/t]
w'	Vertical component of $\hat{v}_p = \vec{k} \cdot \hat{v}_p$ [l/t]
w_0	Efflux speed of gases from stack [l/t]
x	Horizontal distance downwind of stack [l]
x^*	Distance at which atmospheric turbulence begins to dominate entrainment [l]; see Eq. 4.34.
y	Horizontal distance crosswind of stack [l]
z	Vertical distance above stack [l]
\bar{z}	Height above the ground [l]
z_i	Height of penetrable elevated inversion above stack [l]
α	Entrainment constant for vertical plume [dimensionless]; see Eq. 4.22
β	Entrainment constant for mixing by atmospheric turbulence [dimensionless]; see Eq. A.2
Γ	Adiabatic lapse rate of atmosphere = $5.4^\circ\text{F}/1000 \text{ ft}$ [τ/l]
γ	Entrainment constant for bent-over plume [dimensionless]; see Eq. 4.23
ϵ	Eddy energy dissipation rate for atmospheric turbulence [l^2/t^3]; see Ref. 115
θ	Average potential temperature of ambient air [τ]
θ'	Potential temperature excess of plume gases = $\theta_p - \theta$ [τ]
θ_p	Average potential temperature of gases in plume [τ]
$\partial \theta / \partial z$	Vertical potential temperature gradient of atmosphere [τ/l]; see Eq. 2.1
ρ	Average density of ambient air [m/l^3]
ρ_0	Density of gases emitted from stack [m/l^3]
ρ_p	Average density of gases in plume [m/l^3]
σ_z/σ_y	Ratio of vertical dispersion to horizontal dispersion [dimensionless]
x	Concentration of a gaseous effluent [m/l^3]

C APPENDIX: GLOSSARY OF TERMS

Adiabatic lapse rate The rate at which air lifted adiabatically cools owing to the drop of pressure with increasing height, $5.4^{\circ}\text{F}/1000 \text{ ft}$ in the earth's atmosphere.

Advection The transport of a fluid property by the mean velocity field of the fluid.

Buoyant plume A plume initially of lower density than the ambient fluid after the pressure is adiabatically brought to equilibrium. Usually, the term "buoyant plume" refers to a plume in which the effect of the initial momentum is small, and the term "forced plume" refers to a plume with buoyancy in which the effect of the initial momentum is also important.

Convection Mixing motions in a fluid arising from the conversion of potential energy of hydrostatic instability into kinetic energy. It is more precise to term this motion "free convection" to distinguish it from "forced convection," which arises from external forces.

Critical wind speed In the context of this critical review, the wind speed at the height of an elevated plume for which the maximum ground concentration is highest in neutral conditions.

Diffusion The mixing of a fluid property by turbulent and molecular motions within the fluid.

Downwash The downward motion of part or all of a plume due to the lower pressure in the wake of the stack or building or due to a downward step of the terrain.

Effective stack height Variously defined. The three most common definitions are: (1) the height at which a plume levels off, which has been observed only in stable conditions; (2) the height of a plume above the point of maximum ground concentration; (3) the virtual height of plume origin based on the diffusion pattern

at large distances downwind of the stack. Definition 1 is the easiest to apply in stable conditions; definition 2 is the most practical in neutral and unstable conditions; definition 3 is comprehensive but difficult to apply.

Efflux velocity The mean speed of exiting stack gases.

Entrainment The dilution of plume properties due to mixing with the ambient fluid.

Final rise The total plume rise after leveling off, if this occurs, especially as opposed to the term "transitional rise."

Froude number The ratio of pressure forces to buoyant forces. The efflux Froude number of a stack may be defined as $w_0^2/[g(\Delta T/T)D]$.

Fumigation The downward diffusion of pollutants due to convective mixing underneath an inversion that prevents upward diffusion.

Inversion A layer of air in which temperature increases with height. Such a layer is also stable.

Jet A nonbuoyant plume.

Lapse rate The rate at which temperature drops with increasing altitude; the negative of the vertical temperature gradient.

Neutral In hydrostatic equilibrium. A neutral atmosphere is characterized by an adiabatic lapse rate, i.e., by potential temperature constant with height.

Plume rise The rise of a plume center line or center of mass above its point of origin due to initial vertical momentum or buoyancy, or both.

Potential temperature The temperature that a gas would obtain if it were adiabatically compressed to some standard pressure, usually 1000 mb in meteorological literature.

Stable Possessing hydrostatic stability. A stable atmosphere has a positive potential temperature gradient.

Stratification The variation of potential temperature with height. Usually the term "stratified fluid" refers to a fluid possessing hydrostatic stability, as does the atmosphere when the potential temperature gradient is positive.

Temperature gradient In meteorology, usually the vertical gradient of mean temperature.

Transitional rise The rise of a plume under the influence of the mean wind and the properties of the plume itself; i.e., the rise before atmospheric turbulence or stratification has a significant effect.

Turbulence Three-dimensional diffusive motions in a fluid on a macroscopic scale. According to Lumley and Panofsky,¹¹⁵ turbulence is also rotational, dissipative, nonlinear, and stochastic.

Unstable Possessing hydrostatic instability. An unstable atmosphere has a negative potential temperature gradient.

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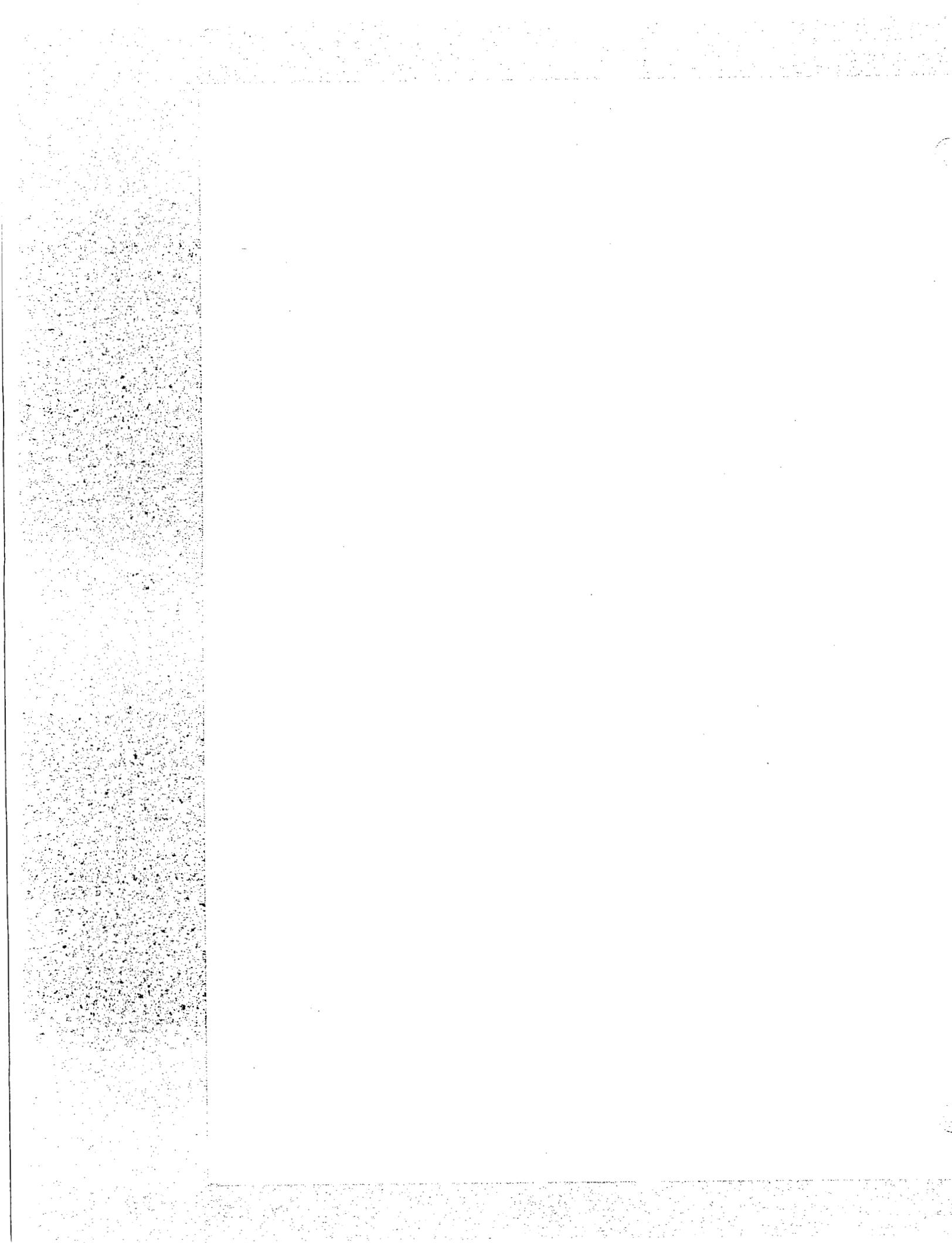
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