

**Atomic Energy of Canada Limited**

**BUCKLINGS OF HEAVY WATER  
MODERATED LATTICES OF ZEEP RODS**

by

**F.N. McDONNELL and A. OKAZAKI**

Chalk River, Ontario

September 1971

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A B S T R A C T

The bucklings of heavy water moderated lattices of ZEEP rods (3.25 cm diameter natural U metal contained in aluminum tubes) were measured by the flux mapping method in the ZED-2 reactor. The isotopic composition of the moderator was 99.46 atom percent  $D_2O$ . The bucklings, corrected for the 0.34 atom percent difference in isotopic composition, are in excellent agreement with earlier measurements made in the ZEEP reactor.

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Laplaciens de réseaux de barres ZEEP  
modérés par eau lourde

par

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Résumé

Les laplaciens des réseaux de barres ZEEP modérés par eau lourde (barreaux d'uranium naturel métallique ayant 3,25 cm de diamètre et gainés dans des tubes d'aluminium) ont été mesurés par la méthode du cadrage du flux dans le réacteur ZED-2. La composition isotopique du modérateur était de 99,46% atomes de  $D_2O$ . Les laplaciens, corrigés pour la différence de 0,34% atomes dans la composition isotopique, sont en excellent accord avec les mesures précédentes effectuées dans le réacteur ZEEP.

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## BUCKLINGS OF HEAVY WATER MODERATED

### LATTICES OF ZEEP RODS

By F.N. McDonnell and A. Okazaki

#### 1. INTRODUCTION

The bucklings of heavy water moderated lattices of ZEEP rods, which are 3.25 cm diameter natural uranium metal contained in aluminum tubes, had been measured some time ago by flux mapping in the ZEEP reactor<sup>(1)</sup>. This report describes recent measurements of these bucklings in the ZED-2 reactor using standard ZED-2 techniques<sup>(2)</sup>. These are similar to those used in ZEEP but with some improvement in positioning of foils. At the time of the measurements in ZED-2 the heavy water contained 99.46 atom percent D<sub>2</sub>O. In the earlier experiment the moderator was 99.80 atom percent D<sub>2</sub>O. Since the buckling change for this 0.34 atom percent difference in isotope composition is large, a comparison of the ZED-2 and ZEEP results yields the dependence of the buckling on the D<sub>2</sub>O content of the moderator.

#### 2. EXPERIMENTAL MEASUREMENTS

##### 2.1 Lattice Arrangement

A cross-section of a ZEEP rod used in these experiments is shown in Figure 1. Each rod consists of natural uranium metal cylinders, 3.25 cm diameter and 15 cm long, stacked in a 2S aluminum alloy tube with 0.1 cm thick wall. The fuel length in the rod is 285 cm. The bottom of the fuel was 15 cm above the calandria floor.

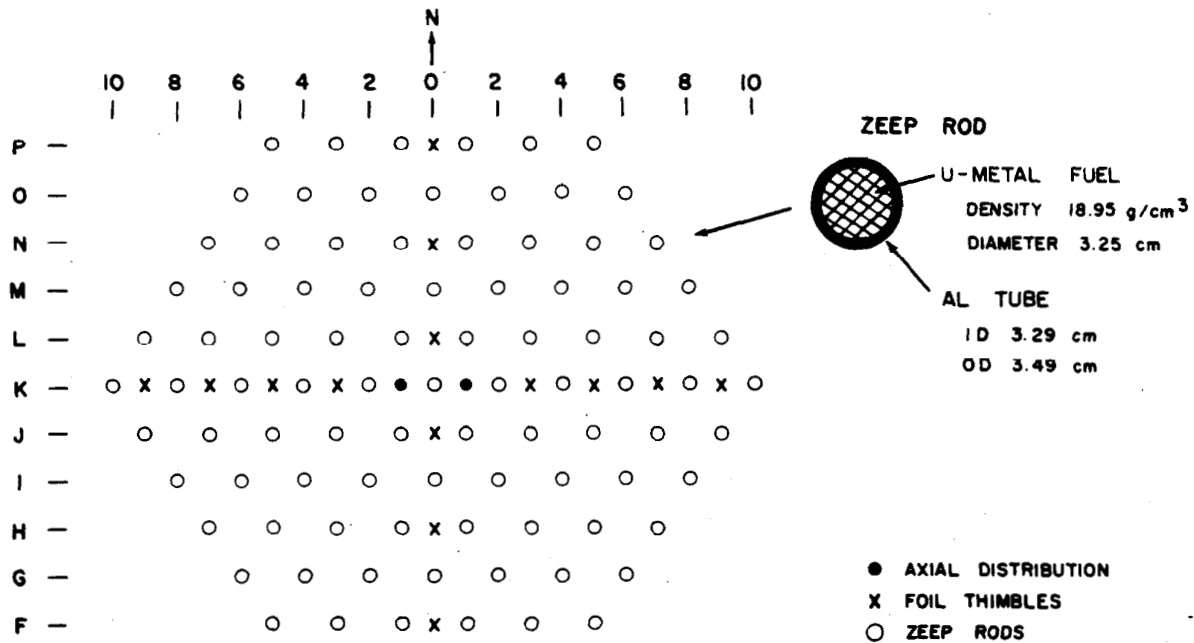


Fig. 1 Foil loading pattern and cross-section of ZEEP rod.

The buckling measurements were made in the cores containing 91 ZEEP rods at 5 triangular lattice pitches from 16 to 22 cm. The ZED-2 reactor tank<sup>(3)</sup> has a diameter of 336 cm so there was at least a 58 cm thick heavy water reflector around the cores. The graphite side and bottom reflectors are 60 cm and 90 cm thick respectively.

## 2.2 Buckling Measurement

Bucklings were determined from foil activation measurements at positions mid-way between fuel assemblies. Figure 1 shows the locations of the thimbles. For the radial distribution measurements, foils were placed at three elevations along a diameter of the lattice. The axial distribution was measured with foils placed at 10 cm intervals at two locations mid-way between the central rod and two opposite nearest neighbours.

Mn (11% Ni) alloy foils were used as neutron detectors. After irradiation the induced  $2.58 \text{ hr Mn}^{56}$  activity was measured with a TQOB electroscope. The experimental technique has been described previously in detail<sup>(2)</sup> and will not be repeated here.

The critical moderator height was measured with and without the foils in the reactor. While the height changes can be measured to  $\pm 0.01 \text{ cm}$ , the height above the calandria floor is estimated to be known to  $\pm 0.2 \text{ cm}$ .

### 3. RESULTS

#### 3.1 Foil Activity

The normalized Mn foil activities, corrected for decay and background, are given in Tables A-1 to A-5, Appendix A. The radius, elevation and position in the core (see Fig. 1) are given for each foil. All elevations are measured from the calandria floor.

#### 3.2 Determination of Buckling

Corrected axial distributions, well removed from the core boundaries, were fitted by the method of least squares to the function,

$$\phi(z) = b \cos\alpha(z-z_0) \quad (1)$$

where  $\phi(z)$  = the activity of the foil at elevation  $z$ ,

$$\alpha = \frac{\pi}{H} ,$$

and  $H$  = extrapolated height.

The bracketed activities in the tables of Appendix A were not included in the fit. The average of the two values of  $\alpha$  obtained from each irradiation is listed in the tables.

Radial distributions were fitted by the method of least squares to the following function<sup>(2)</sup>:

$$\phi(r) = A J_0(\lambda r) + C I_0(\beta r) \quad (2)$$

where

$$\beta^2 = \frac{1}{L^2} + \frac{1}{L_s^2} + 2\alpha^2 + \lambda^2 \quad (3)$$

$\phi(r)$  is the homogeneous two-group diffusion expression for the fundamental radial distribution of the thermal neutron flux.

The value of  $\lambda$  used in the calculation of  $\beta^2$  was obtained from a least squares fit of the radial distribution to the  $J_0(\lambda r)$  Bessel function.  $L^2$  and  $L_s^2$  were calculated with the LATREP<sup>(4)</sup> code and are listed in Table 1. The determination of the radial buckling by the above method is described in more detail in Reference (2). The  $\lambda$  values given in Tables A-1 and A-5 are derived from a fit to all radial activities (n), with the outermost point in each set omitted (n-1), and with the two outermost points in each set omitted (n-2).  $\lambda_{(n-1)}^2$  values are used in deriving the bucklings given in Table 2. The (n-1) point is approximately  $2\frac{1}{2}$  pitches from the equivalent core boundary  $R_c$ , where

$$R_c = 0.525 d\sqrt{N}$$

$$d = \text{lattice pitch}$$

$$\text{and } N = \text{number of rods (91)}$$

Table 1: LATREP Calculated  $L^2$  and  $L_S^2$  for  
ZEEP Rod Lattices\*

Triangular Pitch (cm)	$L^2$ (cm <sup>2</sup> )	$L_S^2$ (cm <sup>2</sup> )
16.0	142.9	111.9
17.3	168.0	111.1
18.9	201.5	110.4
19.68	221.4	110.1
22.0	277.7	109.5

\* Moderator conditions 25°C, 99.50 atom % D<sub>2</sub>O.

The buckling results of Hone et al.<sup>(1)</sup> were based on a radial fit to the one group  $J_0(\lambda r)$  distribution. For comparison, the results of the 5-point  $J_0(\lambda r)$  fit to the present data are also presented in Table 2.

The buckling results are summarized in Table 2. The quoted errors are based on the "goodness of fit" of the measured activities to the assumed distribution functions. The corrections for the foil and thimble loading were obtained from the critical heights measured with and without the loading, assuming that the radial buckling did not change.  $\Delta B_T^2$  is the LATREP calculated temperature correction to be applied to obtain the buckling at 25°C. The last column in Table 2 gives the bucklings for moderator conditions of 25°C and 99.46 atom percent D<sub>2</sub>O.

Table 2: Buckling of ZEEP Rod Lattices

Table	Pitch (cm)	Temp (°C)	Moderator Purity (atom %)	$\lambda^2$ * (m <sup>-2</sup> )	$\alpha^2$ (m <sup>-2</sup> )	$\alpha^2$ † (m <sup>-2</sup> )	$\Delta B_p^2$ (m <sup>-2</sup> )	$B^2$ (m <sup>-2</sup> ) 25°C, 99.46 a/o
A-1	16.00	21.3	99.46	4.080±0.012 (4.001±0.012)	3.834±0.024	3.884±0.024	-0.055	7.909±0.027
A-2	17.30	21.5	99.46	3.664±0.015 (3.601±0.015)	3.829±0.024	3.875±0.024	-0.050	7.489±0.028
A-3	18.90	20.9	99.46	3.206±0.005 (3.139±0.006)	3.729±0.023	3.741±0.023	-0.050	6.897±0.024
A-4	19.68	21.2	99.46	3.028±0.003 (2.989±0.004)	3.562±0.022	3.599±0.022	-0.050	6.577±0.023
A-5	22.00	20.8	99.46	2.560±0.011 (2.530±0.011)	3.208±0.021	3.247±0.021	-0.040	5.767±0.022

† Corrected for thimble and foil loading.

\* 5 point fit to  $J_0(\lambda r)$  distribution only in brackets.

To compare the present results with those obtained earlier, the bucklings have been corrected to 99.80 atom percent D<sub>2</sub>O, 25°C moderator. The moderator purity corrections,  $\Delta B_p^2$ , were calculated with LATREP. The results are summarized in Table 3 and plotted in Figure 2 as a function of lattice pitch. There is good agreement between the two sets of measurements. This shows that for this type of lattice the LATREP calculated corrections for the isotopic composition of the heavy water moderator are valid.

### 3.3 Extrapolation Lengths

The radial and axial extrapolation lengths are summarized in Table 4. The radial extrapolation length is

defined as

$$\begin{aligned} \delta R &= R - R_C \\ &= \frac{2.405}{\lambda} - R_C \end{aligned}$$

where  $R_C$ , effective core radius, is defined in Section 3.2. The axial extrapolation length ( $\delta H$ ) is defined as the difference between the extrapolated height ( $H$ ) and the length of fuel covered by the moderator, and is given by

$$\delta H = H - (H_C - 15)$$

where  $H_C$  is the critical height. Also listed in Table 4 are the upper and lower extrapolation lengths,  $\delta H_U$  and  $\delta H_L$ , and  $z_0$ , the elevation of the flux maximum. The extrapolation lengths are for the moderator conditions at the time of the experiment.

Table 3: Comparison of Buckling Measurements

Pitch (cm)	$B^2$ ( $m^{-2}$ ) 25°C, 99.46 a/o	$\Delta B_P^2$ ( $m^{-2}$ )	$B^2$ ( $m^{-2}$ ) 25°C 99.80 a/o	
			This Report	Reference (1)
16.00	7.909±0.027	0.214	8.123±0.027	-
17.30	7.489±0.028	0.274	7.763±0.028	7.961±0.35
18.90	6.897±0.024	0.346	7.243±0.024	7.278±0.14
19.68	6.577±0.023	0.373	6.950±0.023	7.072±0.046
22.00	5.767±0.022	0.448	6.215±0.022	6.212±0.084

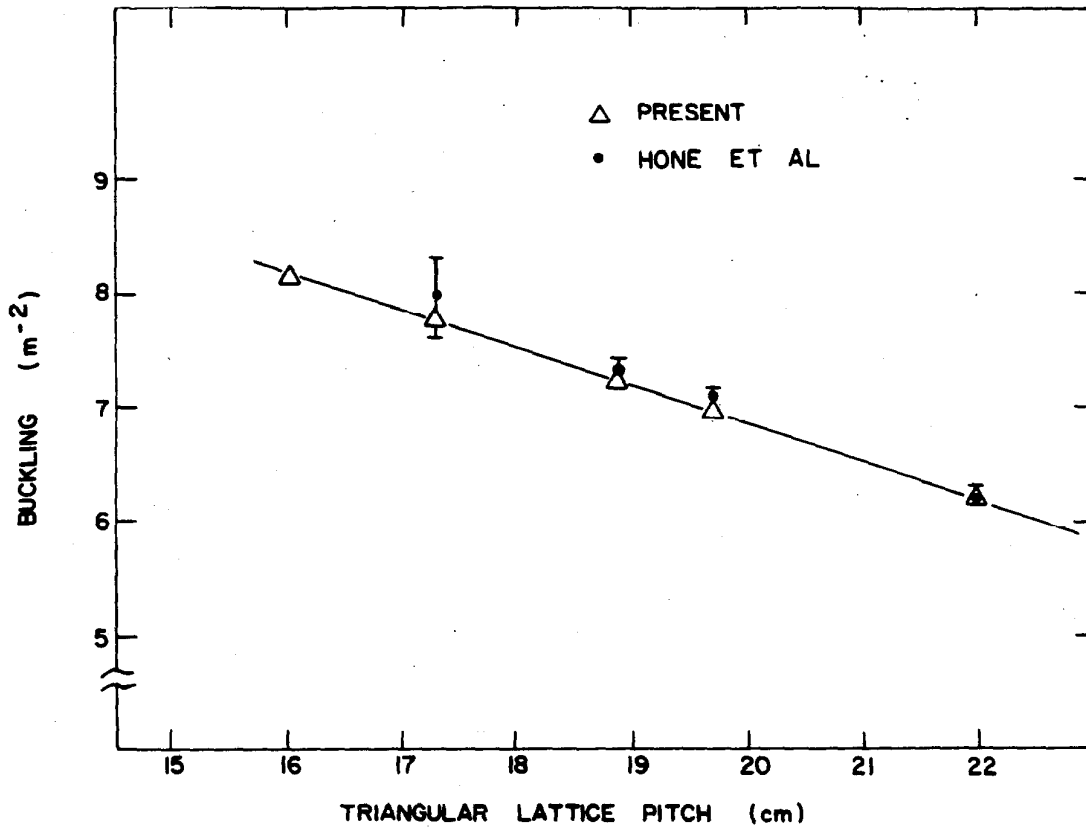


Fig. 2 Buckling of ZEEP rods in D<sub>2</sub>O

Table 4: Summary of Extrapolation Lengths\*

Table	Pitch (cm)	R (cm)	$\delta R$ (cm)	H (cm)	H <sub>c</sub> (cm)	Z <sub>o</sub> (cm)	$\delta H$ (cm)	$\delta H_u$ (cm)	$\delta H_l$ (cm)
A-1	16.0	119.04±0.20	38.90±0.20	160.45±0.80	141.24±0.2	66.15±0.11	34.21±0.82	5.14±0.61	29.07±0.57
A-2	17.3	125.63±0.20	38.98±0.20	160.54±0.75	141.19±0.2	66.10±0.11	34.35±0.77	5.18±0.58	29.17±0.54
A-3	18.9	134.30±0.10	39.64±0.10	162.68±0.80	144.63±0.2	68.18±0.11	33.05±0.82	4.89±0.61	28.16±0.57
A-4	19.68	138.19±0.20	39.62±0.20	166.44±0.90	147.35±0.2	69.53±0.13	34.09±0.92	5.40±0.67	28.69±0.64
A-5	22.0	150.30±0.20	40.11±0.20	175.39±0.70	158.73±0.2	75.01±0.10	31.66±0.73	3.98±0.55	27.68±0.50

\* Values for moderator conditions at time of experiment.

#### 4. SUMMARY

The bucklings of heavy water moderated lattices of ZEEP rods were determined by flux mapping and were compared with earlier measurements made about fifteen years ago in the ZEEP reactor. The present results were corrected for the significant difference in D<sub>2</sub>O isotopic composition of the moderator in the two experiments using LATREP calculations. There is good agreement with the earlier results.

#### 5. ACKNOWLEDGMENTS

We wish to thank the ZED-2 staff for assistance in the careful performance of this experiment. D.H. Walker and P.D.J. Ferrigan supervised the ZED-2 operation and were assisted by E. Pleau and D.J. Roberts. D.A. Kettner looked after the foil counting, and Mrs. A. Brum ran the counting rate reduction programs and made the drawings.

#### 6. REFERENCES

1. D.W. Hone et al.; "Natural-Uranium Heavy-Water Lattices, Experiment and Theory", Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, 12, 351 (1958), (AECL-622).
2. See for example, K.J. Serdula, "Lattice Measurements with 28-element Natural UO<sub>2</sub> Fuel Assemblies Part 1: Bucklings for a Range of Spacings with Three Coolants", AECL-2606\*, (1966).

3. G.A. Beer and D.W. Hone; "Lattice Measurements with 7-element  $\text{UO}_2$  Clusters in ZED-2", AECL-1505\*, (1962).
4. G.J. Phillips and J. Griffiths; "LATREP Users Manual", AECL-3857\*, (1971).

\* Atomic Energy of Canada Limited Publication

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Table A-2: Buckling Data - 17.30 cm Lattice

November 30, 1970		Manganese Foils						Temp: 21.5°C		Moderator Purity: 99.46 atom%				h <sub>c</sub> = 141.19 cm			
Direction	W	S	W	S	W	W	S	W	E	N	E	E	N	E	N	E	
Thimble	K9	P0	K7	H0	K5	K3	J0	K1	K1	L0	K3	K5	N0	K7	P0	K9	
Radius cm	77.85	74.91	60.55	44.95	43.25	25.95	14.98	8.65	8.65	14.98	25.95	43.25	44.95	60.55	74.91	77.85	
Elevation cm																	
135									(0.1845)	(0.1960)							
125									(0.3858)	(0.3976)							
115									0.5656	0.5763							
105									0.7195	0.7256							
95									0.8423	0.8514							
85									0.9319	0.9389							
75	0.5381	0.5787	0.6885	0.8232	0.8357	0.9287	0.9734	0.9832	0.9897	0.9774	0.9329	0.8318	0.8214	0.6875	0.5810	0.5336	
65	0.5435	0.5876	0.6956	0.8311	0.8467	0.9395	0.9911	1.0010	1.000	0.9888	0.9458	0.8444	0.8352	0.6984	0.5899	0.5410	
55	0.5306	0.5698	0.6768	0.8119	0.8246	0.9143	0.9596	0.9747	0.9745	0.9649	0.9191	0.8233	0.8150	0.6791	0.5160	0.5243	
45									0.9097	0.9072							
35									0.8116	0.8106							
25									(0.7006)	(0.6972)							
15									(0.6066)	(0.6049)							

$\lambda(m^{-1})$		Thimble		
		K1W	K1E	Average
$\lambda(n)$	1.8865 ± 0.0032	$\alpha(m^{-1})$ 1.9589 ± 0.008	1.9550 ± 0.009	1.9569 ± 0.006
$\lambda(n-1)$	1.9142 ± 0.0040	$\alpha_{corr}(m^{-1})$ 1.9685 ± 0.006		
$\lambda(n-2)$	1.8871 ± 0.0075			

Table A-3: Buckling Data - 18.90 cm Lattice

December 7, 1970		Manganese Foils										Temp: 20.9°C		Moderator Purity: 99.46 atom%		h <sub>c</sub> = 144.63 cm				
Direction	W	S	W	S	W	S	W	S	W	S	W	S	W	S	W	S	W	S	E	
Thimble	K9	F0	K7	H0	K5	K3	J0	K1	K1	L0	K3	K5	NO	K7	P0	K9				
Radius cm	85.05	81.84	66.15	49.10	47.25	28.35	16.37	9.45	9.45	16.37	28.35	47.25	49.10	56.15	81.84	85.05				
Elevation																				
cm																				
135																				
125																				
115																				
105																				
95																				
85																				
75																				
65																				
55																				
45																				
35																				
25																				
15																				
	(0.2409)	(0.4140)	0.5775	0.7127	0.8203	0.8954	0.9369	0.9408	0.9802	0.9701	0.8980	0.7497	(0.6407)	(0.5437)						
	(0.2530)	(0.4431)	0.6138	0.7539	0.8704	0.9491	0.9789	0.9843	1.000	0.9789	0.8980	0.7497	(0.6815)	(0.5786)						
	0.5155	0.5523	0.6783	0.8179	0.8300	0.9369	0.9789	0.9843	1.000	0.9789	0.8980	0.7497	(0.6815)	(0.5786)						
	0.5188	0.5580	0.6839	0.8246	0.8378	0.9408	0.9843	1.000	0.9874	0.9424	0.8374	0.8281	0.6825	0.5596	0.5123					
	0.5029	0.5378	0.6613	0.7969	0.8102	0.9802	0.9519	0.9701	0.9672	0.9532	0.9105	0.8099	0.8007	0.6606	0.5407	0.4973				
	0.9965	0.9779	0.9379	0.8314	0.8217	0.6789	0.5554	0.5108	0.9965	0.9779	0.9379	0.8314	0.8217	0.6789	0.5554	0.5108				
	0.9965	0.9779	0.9379	0.8314	0.8217	0.6789	0.5554	0.5108	0.9965	0.9779	0.9379	0.8314	0.8217	0.6789	0.5554	0.5108				

$\lambda(n)$  1.7742 ± 0.0021  
 $\lambda(n-1)$  1.7907 ± 0.0013  
 $\lambda(n-2)$  1.7720 ± 0.0037  
 $\alpha(m^{-1})$  1.9392 ± 0.008  
 $\alpha_{corr}(m^{-1})$  1.9342 ± 0.006  
 $\alpha(m^{-1})$  1.9233 ± 0.009  
 $\alpha_{corr}(m^{-1})$  1.9312 ± 0.006

Table A-4: Buckling Data - 19.68 cm Lattice

November 25, 1970		Manganese Foils						Temp: 21.2°C		Moderator Purity: 99.46 atom%				h <sub>C</sub> = 147.35 cm			
Direction	W	S	W	S	W	W	S	W	E	N	E	E	N	E	N	E	
Thimble	K9	F0	K7	H0	K5	K3	J0	K1	K1	L0	K3	K5	N0	K7	P0	K9	
Radius cm	88.56	85.22	68.88	51.13	49.20	29.52	17.04	9.84	9.84	17.04	29.52	49.20	51.13	68.88	85.22	88.56	
Elevation cm																	
135								(0.3017)	(0.3100)								
125								(0.4848)	(0.4915)								
115								0.6478	0.6531								
105								0.7853	0.7880								
95								0.8910	0.8911								
85								0.9629	0.9645								
75	0.5067	0.5438	0.6753	0.8188	0.8341	0.9388	0.9868	0.9994	1.000	0.9873	0.9428	0.8335	0.8188	0.6771	0.5514	0.5051	
65	0.5078	0.5462	0.6771	0.8223	0.8325	0.9414	0.9873	0.9981	1.0029	0.9871	0.9463	0.8313	0.8223	0.6772	0.5507	0.5035	
55	0.4893	0.5267	0.6526	0.7903	0.8045	0.9062	0.9486	0.9681	0.9660	0.9528	0.9129	0.8026	0.7903	0.6545	0.5330	0.4851	
45								0.8972	0.8934								
35								0.7926	0.7919								
25								(0.6768)	(0.6723)								
15								(0.5679)	(0.5624)								

$\lambda(m^{-1})$		Thimble		
		K1W	K1E	Average
$\lambda(n)$	1.7231 ± 0.0019	$\alpha(m^{-1})$ 1.8846 ± 0.008	1.8904 ± 0.009	1.8875 ± 0.006
$\lambda(n-1)$	1.7402 ± 0.0010			
$\lambda(n-2)$	1.7269 ± 0.0014	$\alpha_{corr}(m^{-1})$ 1.8971 ± 0.006		

Table A-5: Buckling Data - 22.0 cm Lattice

November 24, 1970		Manganese Foils						Temp: 20.8°C			Moderator Purity: 99.46 atom%				h <sub>c</sub> = 158.73 cm		
Direction	W	S	W	S	W	W	S	W	E	N	E	E	N	E	N	E	
Thimble	K9	P0	K7	H0	K5	K3	J0	K1	K1	L0	K3	K5	N0	K7	P0	K9	
Radius cm	99.0	95.26	77.0	57.16	55.0	33.0	19.05	11.0	11.0	19.05	33.0	55.0	57.16	77.0	95.26	99.0	
Elevation cm																	
155								(0.1140)	(0.12352)								
145								(0.3020)	(0.3113)								
135								(0.4721)	(0.4812)								
125								0.6251	0.6331								
115								0.7577	0.7636								
105								0.8620	0.8691								
95								0.9419	0.9435								
85	0.4759	0.5065	0.6559	0.8003	0.8144	0.9289	0.9740	0.9883	0.9899	0.9760	0.9279	0.8113	0.7998	0.6558	0.5107	0.4734	
75	0.4821	0.5161	0.6644	0.8149	0.8262	0.9422	0.9879	1.0047	1.000	0.9906	0.9422	0.8239	0.8147	0.6565	0.5184	0.4815	
65	0.4756	0.5075	0.6516	0.8018	0.8160	0.9243	0.9690	0.9866	0.9866	0.9749	0.9275	0.8117	0.8006	0.6515	0.5103	0.4720	
55								0.9413	0.9285								
45								0.8618	0.8574								
35								0.7565	0.7529								
25								(0.6396)	(0.6325)								
15								(0.5219)	(0.5157)								

$\lambda$ (m <sup>-1</sup> )	Thimble		
	K1W	K1E	Average
$\lambda(n)$ 1.5957 ± 0.0022	$\alpha$ (m <sup>-1</sup> ) 1.7960 ± 0.007	1.7874 ± 0.007	1.7917 ± 0.005
$\lambda(n-1)$ 1.6000 ± 0.0033			
$\lambda(n-2)$ 1.5897 ± 0.0063	$\alpha_{corr}$ (m <sup>-1</sup> ) 1.8012 ± 0.005		