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Small Amplitude Electro-Acoustic Plasma Oscillations

ROBERT A. WEIR
A.F. WICKERSHAM, JR.

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SMALL AMPLITUDE ELECTRO-ACOUSTIC PLASMA OSCILLATIONS

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Approved for publication. . . . F. E. Butterfield
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SYLVANIA ELECTRIC PRODUCTS INC.

SMALL AMPLITUDE ELECTRO-ACOUSTIC PLASMA OSCILLATIONS

Robert A. Weir
A. F. Wickersham, Jr.

Electrostatic-acoustic disturbances will propagate with but slight attenuation through an ionized medium, even if the medium includes high concentrations of neutral particles. From such observations we conclude that there must exist a propagation mode in which most of the kinetic energy resides in the electrons, rather than the ions. In the following we obtain a theory, comprising both an ionic and an electronic mode, by deriving a set of differential equations which describe small amplitude, longitudinal, coherent oscillations in a plasma. In addition we briefly compare such theory with recent observations.

For simplicity of presentation we consider first a medium containing only electrons $(-e, m_2, \rho_2)$ and one specie of singly charged ions $(+e, m_1, \rho_1)$. The linear equation of motion for the ion "fluid" at a point in the plasma is

$$(1) \qquad -\vec{\nabla}P - \frac{e}{m_1}\rho_1(\vec{\nabla}\phi_1 + \vec{\nabla}\phi_2) = \rho_1\frac{\partial\vec{V}_1}{\partial t} \quad ,$$

where m, ρ, and P are mass, density, and pressure, and where ϕ_1 and ϕ_2 are electrostatic potentials derived from ions and electrons at points other than the one under consideration. To avoid self-forces we take the divergence of (1) to obtain

$$(2) \qquad \vec{\nabla}\cdot\left(\rho_1\frac{\partial\vec{V}_1}{\partial t}\right) + \left.\frac{\partial P}{\partial\rho_1}\right|_0 \nabla^2\rho_1 = -\frac{e}{m_1}\vec{\nabla}\rho_1\cdot(\vec{\nabla}\phi_1 + \vec{\nabla}\phi_2)$$
$$- \frac{e}{m_1}\rho_1(\nabla^2\phi_1 + \nabla^2\phi_2) \ ,$$

where we have assumed $\partial P/\partial\rho_1$ is not a function of position for small variations in pressure. The last term on the right-hand side vanishes since the sources of potential are elsewhere, and the remaining term may be written as

$$- \frac{e}{2m_1}\left[\nabla^2(\rho_1\phi_1) - \phi_1\nabla^2\rho_1 - \rho_1\nabla^2\phi_1 + \nabla^2(\rho_1\phi_2) - \phi_2\nabla^2\rho_1 - \rho_1\nabla^2\phi_2\right] .$$

Here the third and last terms vanish and after re-arrangement our equation becomes

$$(3) \qquad \vec{\nabla}\cdot\left(\rho_1\frac{\partial\vec{V}_1}{\partial t}\right) + \left(\nabla^2\rho_1\right)\left(\frac{\partial P}{\partial\rho_1}\Big|_0 - \frac{e\phi_1}{2m_1} - \frac{e\phi_2}{2m_1}\right) = - \frac{e}{2m_1}\left[\nabla^2(\rho_1\phi_1) + \nabla^2(\rho_1\phi_2)\right] .$$

We neglect the electrostatic energy of coherent motion, $e\phi/2m$, compared to the heat energy per particle, $\partial P/\partial\rho_1$, and we use Poisson's Equation to replace $\nabla^2\rho_1$ by $-\frac{m}{4\pi e}\nabla^4\phi_1$; thus, equation (3) is reduced to

$$(4) \qquad \vec{\nabla}\cdot\left(\rho_1\frac{\partial\vec{V}_1}{\partial t}\right) - \frac{m_1}{4\pi e}\frac{\partial P}{\partial\rho_1}\Bigg|_0 \nabla^4\phi_1 = - \frac{e}{2m_1}\left[\nabla^2(\rho_1\phi_1) + \nabla^2(\rho_1\phi_2)\right] .$$

From Poisson's Equation and the equation of continuity it follows that

$$\nabla^2\frac{\partial^2\phi_1}{\partial t^2} = \frac{4\pi e}{m_1}\vec{\nabla}\cdot\left(\vec{V}_1\frac{\partial\rho_1}{\partial t} + \rho_1\frac{\partial\vec{V}_1}{\partial t}\right) , \text{ and}$$

we shall neglect the first term on the right-hand side. Thus we can substitute the last expression for the first term in (4), and our equation becomes

$$(5) \qquad \nabla^2 \left\{ \frac{\partial^2 \phi_1}{\partial t^2} - \left.\frac{\partial P}{\partial \rho_1}\right|_0 \nabla^2 \phi_1 + \frac{2\pi \rho_1 e^2}{m_1^2} (\phi_1 + \phi_2) \right\} = 0 \quad \text{or,}$$

$$(6a) \qquad \frac{\partial^2 \phi_1}{\partial t^2} - \left.\frac{\partial P}{\partial \rho_1}\right|_0 \nabla^2 \phi_1 + \frac{\omega^2_{p1}}{2} (\phi_1 + \phi_2) = 0 \quad \text{, where}$$

ω_{p1} is the ion "plasma frequency". A similar equation attains for the electrons:

$$(6b) \qquad \frac{\partial^2 \phi_2}{\partial t^2} - \left.\frac{\partial P}{\partial \rho_2}\right|_0 \nabla^2 \phi_2 + \frac{\omega^2_{p2}}{2} (\phi_1 + \phi_2) = 0 \quad \text{, where}$$

ω_{p2} is the (electron)plasma frequency.

We consider now a one-dimensional example and take $\left.\frac{\partial P}{\partial \rho}\right|_0 = \gamma \frac{KT}{m}$, where γ is the ratio of specific heats, K is Boltzmann's constant, and T is temperature. The assumption of sinusoidal solutions, $A \exp i(kx - \omega t)$, to the set of equations (6ab) leads to

$$(7a) \qquad A_1 \left[\omega^2_{p1} (h^2 k^2 + 1/2) - \omega^2 \right] + A_2 \frac{\omega^2_{p1}}{2} = 0 \; ,$$

$$(7b) \qquad A_1 \frac{\omega^2_{p2}}{2} + A_2 \left[\omega^2_{p2} (h^2 k^2 + 1/2) - \omega^2 \right] = 0 \; ,$$

where A_1 and A_2 are the amplitudes of the ionic and electronic components of the disturbance and where $h^2 = \gamma KT/4\pi ne^2$ defines the Debye shielding length. The corresponding secular equation is

(8) $$\omega^4 - \omega^2 (h^2 k^2 + 1/2) (\omega^2 p_1 + \omega^2 p_2) + \omega^2 p_1 \omega^2 p_2 (h^2 k^2 + 1/2)^2 - \frac{\omega^2 p_1 \omega^2 p_2}{4} = 0$$

and its solution gives the dispersion equation,

(9) $$\omega^2 = (h^2 k^2 + 1/2)\left(\frac{\omega^2 p_1 + \omega^2 p_2}{2}\right)\left(1 \pm \sqrt{1 - \frac{4 m_1 m_2 h^2 k^2 (h^2 k^2 + 1)}{(m_1 + m_2)^2 (h^2 k^2 + 1/2)^2}}\right)$$

The positive sign of the radical in (9) gives relations for the electronic mode, analagous to the optic branch of propagation in periodic structures, and the negative sign gives the ionic mode, analagous to the acoustic branch. We shall denote the two modes by + and - subscripts. For all values of the wave number, k, it is permissible to expand the radical in (9). Thus from the dispersion equation the following relations can be derived.

IONIC MODE

(10) $$\omega_-^2 = (\omega^2 p_1 + \omega^2 p_2) \frac{m_1 m_2}{(m_1 + m_2)^2} h^2 k^2 \left(\frac{h^2 k^2 + 1}{h^2 k^2 + 1/2}\right)$$

(11) $$U_-^2 = \left(\frac{\omega_-}{k}\right)^2 = \left(\frac{\gamma K T}{m_1 + m_2}\right)\left(\frac{h^2 k^2 + 1}{h^2 k^2 + 1/2}\right)$$

(12) $$U_g = \frac{d\omega_-}{dk} = \frac{\omega_-}{k} \left[\frac{h^2 k^2 (h^2 k^2 + 1) + 1/2}{(h^2 k^2 + 1)(h^2 k^2 + 1/2)}\right] .$$

ELECTRONIC MODE

(13) $\omega_+^2 = (\omega^2_{p_1} + \omega^2_{p_2})\ (h^2 k^2 + 1/2)$

(14) $U_+^2 = \left(\frac{\gamma KT}{m_{re}}\right)\left(1 + \frac{1}{2h^2 k^2}\right), \quad m_{re} = \frac{m_1 m_2}{m_1 + m_2}$

(15) $U_g = \left(\frac{\gamma KT}{m_{re}}\right)^{1/2} \left(1 + \frac{1}{2h^2 k^2}\right)^{-1/2}$.

To compare equations (13) - (15) with observations we first differentiate (15) to obtain

(16) $\frac{\Delta U_g}{U_g} = \frac{\Delta k}{4k\ (h^2 k^2 + 1/2)}$.

It can be shown that $\Delta k \approx \frac{\omega_g}{U_g}$, where ω_g is the group or lower beat frequency. The quantity $\left(\frac{\Delta U_g}{U_g}\right)$ has been determined experimentally to be 0.0962, and from measurements of group velocity we have determined that $h^2k^2 \approx m_2/m_1$. Combining our results we have

$$\omega_g = 2\sqrt{2}\left(\frac{\Delta U_g}{U_g}\right)\sqrt{\left(\frac{m_2}{m_1}\right)\left(\frac{4\pi n e^2}{m_1}\right)}$$ or numerically,

with $m_1 = 16$ amu,

(17) $f_g = \frac{\omega_g}{2\pi} = 0.0835\sqrt{n}$.

For values of the charged particle density, n, from 10^3 to $2 \cdot 10^5$ cm^{-3} equation (17) gives frequencies from 2.6 to 37.4 cps. Modulation frequencies in the range 2 ±1 to 28 ± 10 cps have been observed repeatedly by J. E. Don Carlos and C. R. McClung at these laboratories. The electronic mode frequency or corresponding group frequency of a plasma oscillation would modulate radio frequency energy also propagating through the medium. Such modulations could account for sudden frequency shifts observed in long range radio propagation.[1]

In the foregoing we have assumed equal numbers of electrons and ions and equal temperatures of the electronic and ionic components. The dynamical equations (6ab) are independent of charge sign and can be extended to include several ionic species; also, they can be extended readily to include attenuation effects owing to the presence of neutral particles.

---

[1] L.C. Edwards and G.D. Thome, J. Geophys. Res. 67, 6, 2573 (1962); R. C. Fenwick and O.G. Villard, Jr., J. Geophys. Res. 65, 10, 3249 (1960).

(i + 6)