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Abstract. The potential of e^+/e^- storage rings for producing short bunches is reviewed. We begin with a review of existing storage rings and proceed to discuss possible future developments. The effects which limit the production of short bunches in a ring are also discussed. Finally, the emission of coherent synchrotron radiation is examined.

SHORT BUNCHES IN AN e^+/e^- STORAGE RING

The electron bunch length in a storage ring can be considered in two states: equilibrium and non-equilibrium. Both scenarios are of interest for either steady state or pulsed phenomena.

Equilibrium or Steady-State Operation

According to the theory of e^+/e^- storage rings, the equilibrium electron distribution is Gaussian with a bunch length at low currents, σ_{L0}, given as[1]

$$\sigma_{L0} \propto \left(\frac{\alpha_1 E^3}{f_{RF} V_{RF}} \right)^{1/2}, \tag{1}$$

where α_1 is the momentum compaction, E is the electron energy, f_{RF} is the RF frequency, and V_{RF} is the RF cavity peak voltage. In Tables 1 and 2 we list the equilibrium electron bunch lengths for many existing and planned e^+/e^- storage rings. It can be seen that in the present state of the art a short bunch is one with $\sigma_{L0} \approx 3\text{-}5$ mm, apart from a few exceptions which we will now address.

From Equation (1) it can be seen that to produce short electron bunches one can use the following techniques in isolation or in combination:

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- Large RF voltage,[2]
- High frequency RF system,
- Low energy,
- Small momentum compaction.[3-5]

The best candidates for using a large RF voltage in combination with a 'low' energy are the high physics machines such as PEP and LEP or a high energy light source such as the ESRF.[2,6] From Table 1 it can be seen that the HEP machines are capable of producing bunch lengths of a few millimeters for the parameters given. At the present time the benchmark for the shortest bunch actually measured in a ring is at the ESRF ring, which was recently operated at 1 GeV and produced bunches with $\sigma_{L0} \approx 0.35$ mm (1 picosecond).[6]

In the realm of low energy rings there are two proposed compact rings which also incorporate a very high frequency RF system with $f_{RF} \approx 3$ GHz to shorten the bunch. The proposed Coherent Synchrotron Radiation Experiment (CSRE) at the NSLS is based on an existing 150 MeV ring with an 8.5 meter circumference in which a superconducting 3 GHz cavity with a 1.5 MV RF gap voltage would produce 1 picosecond electron bunches.[7] Similarly, Yamada has proposed a single dipole weak focusing ring operating at 50 MeV with a 2.5 GHz normal conducting RF cavity to produce 1 picosecond bunches.[8]

TABLE 1. Bunch lengths in existing and proposed damping rings and colliders.

Ring	E [GeV]	σ_{L0} [mm]
DAΦNE	0.51	30
SLC DR	1.15	5.4
CLIC DR	1.8	2.1
NLC DR	2	4.1
τ-CHARM	2.5	7
SLAC B	9	10
PEP	6	2.7
PEP	8	4.1
LEP INJ	20	3
LEP	45	20

TABLE 2. Bunch lengths in existing and proposed light sources.

Ring	E [GeV]	σ_{L0} [mm]	V_{RF} [MV]	f_{RF} [GHz]
NSLS CSRE	0.15	0.33	1.5	2.856
ALS	1.5	3.5	1.5	0.5
NSLS XRAY	0.75	6.8	1	0.05
NSLS XRAY	2.6	49	1	0.05
ESRF	1	0.35	8	0.35
ESRF	6	5.7	8	0.35

Low Momentum Compaction

The phase slip factor in a ring, η, is defined in terms of the revolution time, T, and the fractional energy deviation, δ, as follows:

$$\eta \equiv \frac{1}{\delta}\frac{T(\delta)-T(0)}{T(0)}. \tag{2}$$

The phase slip factor can be written as a Taylor series expansion in δ,

$$\eta \equiv \alpha_1 + \alpha_2\delta + \cdots, \tag{3}$$

where the lead term is called the momentum compaction and is defined as

$$\alpha_1 \equiv \frac{1}{L}\oint ds \frac{\eta_0(s)}{\rho(s)} - \frac{1}{\beta^2\gamma^2}, \tag{4}$$

and where $\eta_0(s)$ is the dispersion, ρ is the dipole bending radius, and L is the ring circumference. For e^+/e^- rings the second term in Equation (4) is usually negligible.

To shorten a bunch one can reduce $\alpha_1 \Rightarrow 0$ by manipulating the ring optics according to any of the following methods:

- Generate a negative value of η in a bending magnet with a positive ρ. This is the method used for most synchrotron light source lattices.
- Introduce bending magnets with a negative ρ in a region of the lattice with a positive η. This method had been proposed for the UCLA Φ factory.[9]
- An as yet untried method would be to take advantage of the fact that wiggler magnets naturally have a negative momentum compaction. One could introduce a sufficient number of wigglers in a dispersionless region to reduce the overall value of $\alpha_1 \Rightarrow 0$.

Numerous experimental programs have been undertaken to study the reduction of the bunch length as $\alpha_1 \Rightarrow 0$, the results of which are reviewed in detail elsewhere in this volume.[10] Here it suffices to note that the bunch lengths have been reduced to several millimeters but the continued reduction is limited by the necessity to control the α_2 term in the phase slip factor.

As a matter of historical record it should be noted that in 1966 Ken Robinson suggested a 30 MeV ring with a momentum compaction $\alpha = 1.5\times10^{-7}$, incorporating a 23 GHz RF system to produce $\sigma_{L0} \approx 0.1$ μm long bunches.[11] The ring was never constructed and the details are sketchy, but the paper does incorporate all the above techniques for producing short bunches.

Longitudinal Potential Well Distortion

If the electron beam in the ring interacts with an impedance, the simple Gaussian result given in Equation (1) for the longitudinal equilibrium bunch distribution is modified by potential well distortion. It is unlikely that longitudinal potential well distortion alone is a sufficient technique to get from long bunches to short bunches. However it is important to determine if a bunch grows or shrinks due to potential well distortion as this can affect the microwave instability threshold and growth rate.

Bane gives an expression for the longitudinal equilibrium bunch distribution, I(t), for an electron beam interacting with a wakefield,[12]

$$I(t) = K\exp\left[-\frac{t^2}{2\sigma_{L0}^2} - \frac{1}{\dot{V}_{RF}\sigma_{L0}^2}\int_{-\infty}^{\infty} S(t')I(t-t')dt'\right], \tag{5}$$

where K is a normalization constant, t is the time relative to the synchronous particle, $\dot{V}_{RF}$ is the slope of the RF voltage at the synchronous phase and the wakefield is characterized by S(t), the response to a step function input. Note that S(t) is the indefinite integral of the delta function wakefield $W_\delta(t)$.

Introducing the impedance, $Z_L(\omega)$, as the Fourier transform of $W_\delta(t)$, the above can be rewritten as

$$I(t) = K\exp\left[-\frac{t^2}{2\sigma_{L0}^2} - \frac{i}{\dot{V}_{RF}\sigma_{L0}^2}\int_{-\infty}^{\infty} \frac{I(\omega)Z_L(\omega)}{\omega} e^{-i\omega t} d\omega\right]. \tag{6}$$

To determine the exact equilibrium bunch distribution one must solve this implicit equation for a given impedance. However, one can get a qualitative idea of the effects of an impedance by Taylor expanding $e^{-i\omega t}$ in Equation (6) and taking one iteration of the equation starting from the Gaussian distribution, $I_0(\omega)$. It can be seen that the resistive part of the impedance leads to a shift of the centroid of the bunch proportional to

$$-\frac{1}{\dot{V}_{RF}}\int_{-\infty}^{\infty} \mathrm{Re}\, Z_L(\omega) I_0(\omega) d\omega . \tag{7}$$

This simply corresponds to a shift in the synchronous phase as the electron moves higher on the RF waveform to compensate for the energy loss. The direction of motion is determined by the sign of $\dot{V}_{RF}$, which in turn depends on the sign of the momentum compaction since $\alpha\dot{V}_{RF} > 0$ for stable synchrotron oscillations.

The reactive part of the impedance leads to a change in the beam width,

$$\frac{1}{\sigma_{L0}^2} \Rightarrow \frac{1}{\sigma_{L0}^2} + \frac{1}{\dot{V}_{RF}\sigma_0^2}\int_{-\infty}^{\infty} \omega\, \mathrm{Im}\, Z_L(\omega) I_0(\omega) d\omega . \tag{8}$$

For example, for a pure inductor, $Z_L(\omega) = -i\omega L$, so the bunch starts to widen but the centroid does not shift. For a capacitor the bunch length would start to shrink. A complete solution of these examples can be found in reference 12. The contribution of the synchrotron radiation impedance (wakefield) to potential well distortion is examined in reference 13.

Negative Momentum Compaction

As seen from Equation (8), for a given reactive impedance one can switch from growth to shrinking of the bunch due to potential well distortion, and vice versa, by varying the sign of the momentum compaction. SUPERACO has been operated with a negative α but there it was observed that the bunch still widened, just not as rapidly as in the positive α case.[10]

Changing the sign of the momentum compaction has several other interesting consequences.[14] When considering the Robinson instability or the longitudinal symmetric coupled bunch instability, the presence of a high-Q impedance on an upper sideband of a rotation line causes the instability for $\alpha > 0$. For $\alpha < 0$, impedances straddling the lower sidebands will lead to an instability. Also the head-tail instability growth rate depends on the sign of the momentum compaction. For $\alpha < 0$ one must have a negative chromaticity to stabilize the head-tail instability. This implies one could construct a ring without sextupoles if the ring could tolerate the tune spread caused by the uncorrected chromaticity. A proof of principle experiment was performed on the NSLS VUV ring, where currents in excess of 100 ma in a single bunch and 400 ma in seven bunches have been stored with $\alpha < 0$ and no sextupoles.[15]

It has also been suggested that a negative momentum compaction could delay the onset of the microwave instability in electron rings.[16]

Non-Equilibrium or Pulsed Operation

The standard method for producing short bunches in a linac-driven bunch compressor is to impart an energy deviation correlated with the position along the length of the bunch and then to make the bunch traverse a magnetic lens arrangement where the path length depends on the energy of the particles. Hofmann has suggested that this method could be used in the PEP storage ring to produce bunch lengths on the order of 1.5-5 ps.[2] A compressor of this type could also be installed in a bypass section of a storage ring.

Zholents and Zolotorev have suggested that stochastic cooling techniques, which are routinely used in proton rings, could be extended to optical frequencies for use in electron rings. Using stochastic cooling in conjunction with bunch compressors, these authors propose to produce 30 μm electron bunches in a ring.[17]

In lieu of shortening the entire bunch it is also possible to impart an intra-bunch microstructure as would take place if there were a free-electron laser operating in the ring. The FEL interaction creates microbunches on the scale of the radiation wavelength which could be as short as 250 nanometers in a storage ring. Additional information can be obtained from the discussion of storage ring FELs given elsewhere in these proceedings.[18]

COLLECTIVE EFFECTS

Beam Lifetime and Intra-beam Scattering

Storing a large current in a short bunch leads to a high density of electrons in the bunch. This increases the rate of Coulomb scattering of the particles in the bunch, which can either increase the dimensions of the bunch (so-called intra-beam scattering) or the particles can be knocked out of the bunch altogether resulting in a reduced Touschek lifetime.[19] Since the Coulomb scattering cross section increases rapidly as the energy of the electrons is reduced, these problems are most restrictive for low energy rings, E < 500 MeV.

For some applications it might be possible to trade off an increased transverse beam dimension, due either to intra-beam scattering or to 'heating' the beam with a noise source on a stripline, in order to preserve a desired bunch length. In the end, the only surefire way to reduce the effects of intra-beam scattering is to decrease the number of particles in the bunch. To combat a short Touschek lifetime one could use a full energy injector running in top-off mode. Each specific application will dictate what beam sizes, lifetimes and stored currents are acceptable.

Microwave Instability

Perhaps the greatest impediment to high current, short electron bunches is the interaction of the beam with impedances in the ring, which can give rise to instabilities. Paramount among these is the so-called microwave instability whose peak current threshold, I_p^{th}, is estimated as a function of the ring broad band impedance, Z_L, and energy spread, σ_ε, by the following guideline:[20]

$$I_p^{th}[A] \cong \frac{2\pi\alpha_1\sigma_\varepsilon^2 E[eV]}{|Z_L / n|[\Omega]}. \quad (9)$$

This expression itself is simple enough, the complication lies in specifying the proper broad band impedance.

Broad Band Impedance in a Storage Ring

Heifets has recently reviewed the various models characterizing the broad band impedance in an electron storage ring.[21] He gives the following expression for the broad band impedance, which displays the key elements encountered in the vacuum chamber and RF cavities:

Inductance from Bellows, Slots, Etc.
Low Frequency Resistance
Resistive Wall

$$Z_L(\omega) = -i\omega L + R + (1-i)\sqrt{\omega}B + \frac{(1+i)Z_C}{\sqrt{\omega}} + Z_0\frac{\Gamma(2/3)}{3^{1/3}}\left[\frac{\sqrt{3}}{2} + i\frac{1}{2}\right]\left(\frac{\omega}{\omega_0}\right)^{1/3}. \quad (10)$$

Pillbox at High Frequency
Synchrotron Radiation

The last term in the above equation has been added to Heifets' expression to account for the impedance due to the free-space synchrotron radiation.[22-24] Which term dominates the broad band impedance depends on the specifics of any given storage ring. While it may be possible to dream away the first four terms in the above if one imagines a completely smooth, resistance-free chamber and an inverse free electron laser as the RF system, the last term remains, as the particles must circulate in the ring and thereby give rise to synchrotron radiation. In what follows we focus our attention on the impedance due to synchrotron radiation.

Synchrotron Radiation Impedance

The synchrotron radiation impedance given in Equation (10) is for an electron circulating in free space and is accurate for $n \ll 3\gamma^3/2$. The free space impedance must be modified to account for the presence of the storage ring vacuum chamber. The simplest model is to consider the chamber as a pair of conducting plates on the top and bottom of the electron beam. The complete expression for the impedance for an electron beam circulating midway between a pair of infinite conducting plates can be found in the references.[25-27] Here it suffices to examine an approximate expression derived by Warnock for the low n behavior of $\mathrm{Re}Z_n/n$,[27]

$$\frac{\mathrm{Re}\,Z_n}{n} \approx 2Z_0\left(\frac{\pi\rho}{hn}\right)^2 \exp\left[-\frac{2}{3n^2}\left(\frac{\pi\rho}{h}\right)^3\right], \qquad 0 < n \le \sqrt{\frac{2}{3}\left(\frac{\pi\rho}{h}\right)^3}, \quad (11)$$

where h is the full gap between the plates. According to Equation (11) the impedance is modified for values of n on the order $(\pi\rho/h)^{3/2}$. For $n < \pi(\rho/h)^{3/2}$, $\mathrm{Re}Z_n$ is vanishingly small. For larger values of n the parallel plate impedance matches up to the free space value. Using Equation (11), Warnock shows that the maximum value of $\mathrm{Re}Z_n/n$ is

$$\frac{\mathrm{Re}\,Z_n}{n} \approx 132\frac{h}{\rho} \quad \text{for} \quad \hat{n} = \sqrt{\frac{2}{3}\left(\frac{\pi\rho}{h}\right)^3}, \quad (12)$$

which compares with a result derived earlier by Faltens and Laslett.[26] Unfortunately, there is no simple expression for ImZ_n/n, one must resort to numerical techniques to explore the effects of the plates on ImZ_n/n.

As an example of the behavior of the impedance, in Figure 1 we plot the exact numerical result for ReZ_n/n and ImZ_n/n versus n for the free-space case and for the parallel plates case with $\rho/h = 10$. It can be seen that with the plates in place, ReZ_n/n is indeed strongly suppressed for low values of n, it reaches a maximum near $\hat{n}$, and soon thereafter it matches up with the free-space result. ImZ_n/n is similarly suppressed; for low n it flips sign and then matches up with the free- space result. Therefore the plates will suppress the low frequency coherent synchrotron radiation, which is proportional ReZ_n, and they will reduce $|Z_L / n|$. As such the plates should increase the threshold current for the onset of the microwave instability. In addition, since the plates reverse the sign of ImZ_n for some portion of the spectrum, this could affect the bunch length due to potential well distortion.

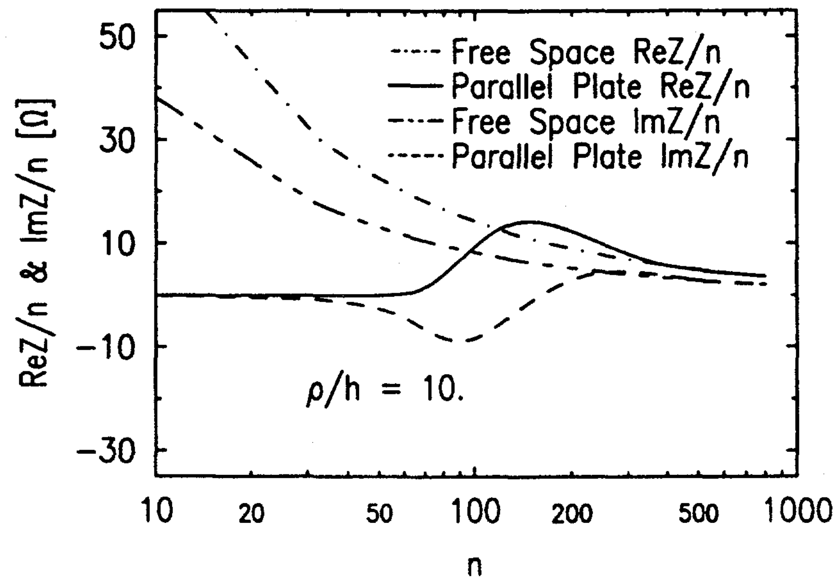

Figure 1: ReZ/n and ImZ/n for Free Space and Parallel Plates with ρ/h = 10.

COHERENT SYNCHROTRON RADIATION

The power radiated as coherent synchrotron radiation is given by[25-27]

$$P_{coh} = 2\sum_{n=1}^{\infty} I_n^2 \cdot \mathrm{Re}\, Z_n . \qquad (13)$$

Of particular interest is the case of Gaussian electron bunches, for which I_n is given by

$$I_n = \frac{Nec\beta}{2\pi\rho}\exp\left[-\frac{1}{2}\left(\frac{n\sigma}{\rho}\right)^2\right]. \tag{14}$$

Equation (14) implies that to have significant coherent synchrotron radiation at a wavelength λ requires the electron bunch length to satisfy the following constraint:

$$\pi\sigma \le \lambda. \tag{15}$$

According to Equation (11), to have ReZ_n be unaffected by the vacuum chamber shielding requires that

$$\lambda \le 2h\cdot\left(\frac{h}{\rho}\right)^{1/2}. \tag{16}$$

Combining the results in Equations (15) and (16) indicates that coherent synchrotron radiation is permitted in the range of wavelengths given by

$$\pi\sigma \le \lambda \le 2h\cdot\left(\frac{h}{\rho}\right)^{1/2}. \tag{17}$$

Hence, if you want to generate coherent synchrotron radiation in a storage ring, design an experiment with short bunches (σ) in a small ring (ρ) with a large vacuum chamber aperture (h). For example in reference 7, $\sigma = 0.33$ mm, $\rho = 603.7$ mm and $h = 38$ mm so one can expect coherent synchrotron radiation in the range 1 mm $\le \lambda \le$ 19 mm.

In a large storage ring (large ρ), the right-hand side of Equation (17) can be smaller than the left-hand side; then the coherent synchrotron radiation is suppressed. However the short bunches would still produce short pulses of incoherent x-rays.

CONCLUDING REMARKS

At present short bunches in e^+/e^- rings are about $\sigma \approx$ 3-5 mm (10 ps) long. Experimental work has begun to generate bunches that are an order of magnitude smaller, for example ESRF has produced 1 ps bunches by operating the ring at a reduced energy of 1 GeV. There are numerous impediments to obtaining large currents in short bunches, such as intra-beam scattering, short Touschek lifetimes, potential well distortion and the microwave instability. Additional experimental and theoretical work is needed to determine the ultimate limit of short bunches in e^+/e^- storage rings. Although coherent synchrotron radiation has not yet been observed in a ring, it should be possible to generate it with short bunches in a small ring with a large vacuum chamber aperture. In a large ring, where the coherent synchrotron radiation is likely to be suppressed, the short electron bunches will produce short pulses of incoherent synchrotron radiation extending into the x-ray region of the spectrum.

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