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A PRACTICAL GUIDE TO THE OPTIMUM UTILIZATION OF COOLING CAPACITY OF BOIL-OFF GAS FROM CRYOGENIC APPARATUS

William Chamberlain and Harvey Maseck

April 29, 1963

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ABSTRACT

The loss rate of a liquid helium container is calculated by a method which includes the important effect of interception of heat conducted down the neck by boil-off gas flowing up the neck. The results indicated an optimum neck geometry for given flask design beyond which added neck length is not effective in reducing boil-off.

A radiation shield is introduced between the liquid helium flask and the liquid nitrogen shield. This intermediate shield is thermally tied to, and cooled by, the flask neck. Its effectiveness in reducing loss rate is demonstrated. It is further shown that when the intermediate shield is employed, large increases in the emissivity of the liquid helium flask have a negligible effect on loss rate.

A graph is presented (figure 4) which permits direct determination of loss rates with and without the intermediate shield for containers of known geometry and materials. The effect of instrument leads and supports either down the neck or through the vacuum can be included.

INTRODUCTION

Cryogenic liquid storage vessels and cryostats are currently in demand for both research and industrial use. As requirements for low loss rates and minimum weight and space grow more stringent, optimum design criteria becomes necessary. This paper serves to provide useful information in this area of need.

The units have been chosen on the basis of previous cryogenic engineering practice and convenience in application. In this regard, mass units throughout this paper appear as "liters", which refers to liters of liquid helium at 4.2°K and one atmosphere, or the equivalent mass of helium gas.

/ d

MASS FLOW RATE as a FUNCTION of TEMPERATURE and NECK GEOMETRY

A section of a typical heat neck for a cryogenic vessel is shown in Figure 1. Applying the first law of thermodynamics to the open system in steady flow designated by λ_a, and ignoring changes in velocity and elevation,

(1) $$Q\ net = H_{out} - H\ in$$

Where all energy quantities are on time rate basis. Neglecting radiation from the environment,

(2) $$Q\ net = W_c - W_{cL}$$

The temperature of the gas is assumed to be the same as that of the wall. (The validity of this assumption will be investigated later.) For a constant pressure process,

(3a) $$H_{out} - H_{in} = \dot{m} \int_{T_L}^{T} C_p\ (T)\ \ dT$$

where $\dot{m}$ is the time rate of mass flow. If C_p is assumed constant,

(3b) $$H_{out} - H_{in} = \dot{m}\ C_p\ (T - T_L)$$

Equation 3b is mathematically more convenient to handle.

Figure 7 presents C_p of helium at one atmosphere pressure as a function of temperature. The area under the curve represents the integral in equation 3a, whereas the smaller area under the straight line represents the quantity $C_p(T - T_L)$ in equation 3b. Figure 7 shows that the use of equation 3b is conservative with respect to this analysis because it has the effect of crediting the gas with a somewhat smaller capacity to intercept heat conducted down the tube wall than it actually possesses. The fractional error becomes smaller as the temperature interval is increased.

Equations 2 and 3b are used in equation 1 to obtain,

(4) $$W_c - W_{cL} = \dot{m}\ C_p\ (T - T_L)$$

For conduction down the tube wall

(5) $$W_c = K\ A_c\ (dT/dx)$$

where K is the thermalconductivity.

If K is a linear function of temperature,

(6) $$K = a\ T - b$$

Further, if K_L is defined as

(7) $$K_L \equiv a\,T_L - b$$

8) $$b = a\,T_L - K_L$$

(9) $$K = aT - (a\,T_L - K_L)$$

(10) $$K = a\,(T - T_L) + K_L$$

Combining equations 4, 5, and 10:

(11) $$W_c = W_{cL} + \dot{m}\,C_p\,(T - T_L)$$

(12) $$K\,A_c\,(dT/dx) = W_{cL} + \dot{m}\,C_p\,(T - T_L)$$

(13) $$a(T - T_L) + K_L \quad A_c(dT/dx) = W_{cL} + \dot{m}\,Cp\,(T - T_L)$$

Separating variables,

(14) $$\frac{dx}{A_c} = \left[\frac{K_L + a\,(T - T_L)}{W_{cL} + \dot{m}\,Cp\,(T - T_L)}\right] dT$$

Define

(15) $$(T - T_L) \equiv \tau, \quad dT = d\tau$$

Integrating equation 14,

(16) $$\frac{1}{A_c}\int_0^x dx = K_L\int_0^\tau \frac{d\tau}{W_{cL} + \dot{m}\,Cp\,\tau} + a\int_0^\tau \frac{\tau d\tau}{W_{cL} + \dot{m}\,Cp\,\tau}$$

(17) $$\frac{x}{A_c} = \left(\frac{K_L}{\dot{m}\,C_p}\right)\Big[\ln\,(W_{cL} + \dot{m}\,Cp\tau)\Big]_0^\tau + \left(\frac{a}{(\dot{m}\,C_p)^2}\right)\Big[W_{cL} + \dot{m}\,Cp\tau - W_{cL}\ln\,(W_{cL} + \dot{m}\,C_p\tau)\Big]_0^\tau$$

At $x = L$, $T = T_H$, and $\tau = \tau_H$; then equation 17 becomes,

(18) $$\frac{L}{A_c} = \left(\frac{1}{\dot{m}\,C_p}\right)\left\{a\,\tau_H + \left[K_L - W_{cL}\left(\frac{a}{\dot{m}\,C_p}\right)\right]\ln\left[\frac{W_{cL} + \dot{m}\,C_p\tau_H}{W_{cL}}\right]\right\}$$

Equation 18 appears in Reference 1 (p. 240, eq. 7.9)*. This equation may be written as

*Note that in Reference 1 the term $(1/\dot{m}\,C_p)$ is expressed as $(1/\dot{m})C_p$, a typographical error

(19)
$$\frac{\dot{m}\,(L/A_c)}{(a/C_p)} = \tau_H + \left[\frac{K_L}{a} - \frac{W_{cL}}{\dot{m}C_p}\right] \ln\left[1 + \frac{\dot{m}\,C_p\,\tau_H}{W_{cL}}\right]$$

Define

$$\theta \equiv \left[\frac{K_L}{a} - \frac{W_{cL}}{\dot{m}\,C_p}\right]$$

$$Z \equiv \frac{\dot{m}\,(L/A_c)}{a/C_p}$$

Then equation 19 may be written as,

(20)
$$Z \equiv \tau_H + \theta \ln\left[1 + \frac{\tau_H}{(K_L/a - \theta)}\right]$$

LIQUID HELIUM LOSS RATE AS A FUNCTION OF NECK GEOMETRY

A typical liquid nitrogen shielded helium dewar is shown in Figure 2. The features essential to this study are shown in Figure 3. Apply the first law of thermodynamics to the open system enclosed by the dashed line, designated λ_b:

(21) $$Q_{\lambda_b} - \dot{m}\,(u + pv)_{g,T_1} = \left(\frac{dU}{dt}\right)_{\lambda_b}$$

(22) $$Q_{\lambda_b} = W_{c_1} + W_{r\,31}$$

Where W_{c_1} is the heat conducted down the neck and W_{r31} is the heat radiated to the flask from temperature T_3.

(23) $$\left(\frac{dU}{dt}\right)_{\lambda_b} = -\,\dot{m}\,u_f\,,\ T_1$$

The subscripts f and g refer to the saturated liquid and vapor states respectively. Equation 21 can now be written as

(24) $$W_{c_1} + W_{r\,31} = \dot{m}\left[\,(u + pv)_g - u_f\,\right]_{T_1}$$

Define

(25) $$G \equiv u_{fg} + (pv)_g = h_{fg} - (pv)_f$$

Then

(26) $$\dot{m}\,G = W_{c_1} + W_{r\,31}$$

For the radiated heat,

(27) $$W_{r\,31} = \sigma\, E_{31}\, A_1\, (T_3^{\,4} - T_1^{\,4})$$

where σ is the Stephan-Boltzmann constant, E_{31} is the emissivity factor, and A_1 is the area of the liquid helium flask. Since both E_{31} and A_1 are quantities easily calculated if the geometry and materials of construction are known (which is usually the case), define the product

(28) $$\alpha\beta \equiv E_{31}\, A_1$$

The units of αB are area. This quantity may be regarded as an "equivalent black-body area" of the flask, because this area, at an emissivity factor of 1, would receive an amount of radiant heat from T_3 equivalent to that actually received by the flask.

Equation 26 becomes

(29) $$\dot{m}G = W_{c1} + \sigma\alpha\beta\,(T_3^{\,4} - T_1^{\,4})$$

Solving for $(\dot{m}/\alpha b)$,

(30) $$(\dot{m}/\alpha\beta) = \frac{\sigma(T_3^{\,4} - T_1^{\,4})}{G - (W_{c1}/\dot{m})}$$

Equation 19, as applied to the neck of Figure 3 between T_1 and T_3, is seen to be of the form

(31) $$\dot{m}\,(L/A_c) = f_1\,(W_{c1}/\dot{m})$$

Similarly, equation 30 is of the form

(32) $$(\dot{m}/\alpha\beta) = f_2\,(W_{c1}/\dot{m})$$

Note also that

(33) $$\frac{1}{(L/A_c)\alpha\beta} = \frac{(\dot{m}/\alpha\beta)}{\dot{m}(L/A_c)} = \frac{f_2(W_{c1}/\dot{m})}{f_1(W_{c1}/\dot{m})}$$

or

(34) $$\frac{1}{(L/A_c)\alpha\beta} = f_3\,(W_{c1}/\dot{m})$$

The form of equations 32 and 34 indicate that, for various values of $(W_{c1}/\dot{m})$, corresponding values of both $\dot{m}/\alpha\beta$ and $\left[(L/A_c)\alpha\beta\right]^{-1}$ may be calculated. Thus $\dot{m}/\alpha\beta$ may be plotted as a function of $\left[(L/A_c)\alpha\beta\right]^{-1}$. Note that the value of $\left[(L/A_c)\alpha\beta\right]^{-1}$ may be calculated from knowledge of dewar geometry and emissivities of the construction materials. The corresponding value of $(\dot{m}/\alpha\beta)$ may then be read directly from the graph and converted to loss rate by simply multiplying by $\alpha\beta$.

This relationship is presented in Figure 4 as the line labelled "Dewar Without Shield"* for liquid helium dewars with stainless steel necks. Values of the constants used in evaluating equations 19 and 30 are listed in the appendix. The thermal conductivity of stainless steel from 4°K to 100°K is approximated by

(35) $$K\ \frac{\text{milliwatts}}{\text{inch}\ ^{\circ}\text{K}} = 3.0\,T\ ^{\circ}\text{K} - 6.0$$

*For this curve α is assigned the value of 1.

accurate to about 5%, as shown in Figure 5.

Particular notice should be taken of the shape of this curve on Figure 4, which exhibits a distinct "knee". Values of $\left[(L/A_c)\beta\right]^{-1}$ below about 12×10^{-4} $(\text{inches})^{-1}$ do not reduce the helium loss rate $\dot{m}/\beta$, which remains constant in this region at .0438 liters/day · inch^2. If further increase in the slenderness ratio, L/A_c, of the neck does not reduce the helium loss rate, then the conclusion may be drawn that conduction down the neck is essentially zero, and the loss rate is due entirely to radiant heat transfer to the flask. This conclusion may be verified by solving equation 30 for $\dot{m}/\beta$, with $W_{c_1} = 0$ and $\alpha = 1$:

(36) $$(\dot{m}/\beta) = \sigma\,(T_3^{\,4} - T_1^{\,4})\,/G$$

$$= \left(\frac{0.366}{10^{10}}\right)\left(\frac{\text{watts}}{\text{in}^2\,{}^{\circ}\text{K}^4}\right)\left(\frac{(.357/10^8)(\,{}^{\circ}\text{K}^{\,4})}{(0.0298)(\text{watt .days/liter})}\right) = 0.0438\left(\frac{\text{liters}}{\text{day .inch}^2}\right)$$

This result confirms the previous conclusion; therefore, the horizontal portion of the curve is duly labelled "Radiation Only", and the inclined portion is labelled "Radiation and Conduction". Due credit is given here to R.B. Scott, who, in Reference 2, p. 242, suggests that most dewar necks are made unnecessarily long and small in diameter. The knee of the curve under discussion substantiates his statement. It may be noted in passing that a long dewar neck should not be criticized solely on this basis. If it is intended that the dewar stand for long periods of time with a transfer tube in place, then a somewhat longer neck than indicated by the curve is justified. The operating condition may be found from the curve by adding the conduction area of the transfer tube to that of the neck.

The cases of "Radiation Only" and "Radiation and Conduction" have now been established. A curve representing "Conduction Only" would bracket all real cases on Figure 4. Proceeding toward this result, set $W_{r31} = 0$ in equation 26:

(37) $$(W_{c1}/\dot{m}) = G$$

This substitution in equation 19 yields

(38) $$\frac{\dot{m}\,(L/A_c)}{(a/C_p)} = \tau_H + \left[\frac{K_L}{a} - \left(\frac{G}{C_p}\right)\right] \ln\left[1 + \left(\frac{C_p\,\tau_H}{G}\right)\right]$$

Evaluation of equation 38 in terms of the constants (see appendix for numerical values) becomes

$$\dot{m}\ \left(\text{liters/day}\right) = 26.8\ \left(\text{liters/day .inch}\right)\left(\frac{A_c}{L}\right)\left(\text{inch}\right)$$

Both sides of this equation may be divided by β:

(40) $$\frac{\dot{m}}{\beta}\left(\frac{\text{liter}}{\text{day .inch}^2}\right) = 26.8\left(\frac{\text{liter}}{\text{day .inch}}\right)\frac{1}{(L/A_c)\beta}\left(\frac{1}{\text{inch}}\right)$$

For this case, if $(A_c/L) = 0$, then $\dot{m} = 0$, and equation 40 is therefore a straight line passing through the origin. This equation is plotted on Figure 4 and labelled "asymptote for Conduction Only".

LIQUID HELIUM LOSS RATE FOR A DEWAR WITH AN INTERMEDIATE GAS-COOLED RADIANT HEAT SHIELD

It is proposed that a metal radiation shield be interposed in the vacuum space between the liquid nitrogen shield and the liquid helium flask, as shown in Figure 6.

The open system designated λ_c is similar to λ_b. The first law of thermodynamics applied to λ_c yields an equation analogous to equation 29:

(41) $$\dot{m}G = W_{c_1} + W_{r21}$$

For the open system λd,

(42) $$Q_{\lambda d} - \dot{m}\,(u + pv)_{T_2} = \left(\frac{dU}{dt}\right)_{\lambda d}$$

(43) $$Q_{\lambda_d} = W_{c2} + W_{r\,3\,2}$$

(44) $$\left(\frac{dU}{dt}\right)_{\lambda d} = -\,\dot{m}\,u_f$$

Equations 43 and 44 employed in equation 42 give

(45) $$W_{c_2} + W_{r\,3\,2} = \dot{m}\,(h_{T_2} - u_f)$$

(46) $$h_{T_2} - h_{T_1} + h_{T_1} - u_f = h_{T_2} - h_{T_1} + u_{fg} + pv_g$$

Equation 25 defines G so that equation 46 becomes

(47) $$h_{T_2} - u_f = (h_{T_2} - h_{T_1}) + G$$

The gas undergoes a constant pressure process as it rises in the neck, so that, as previously justified, equation 45 becomes

(48) $$W_{c_2} + W_{\;32} = \dot{m}\left[C_p\,(T_2 - T_1) + G\right]$$

Define

(49) $$\beta \equiv E_{32}\,A_2$$

If

($$\alpha\beta = E_{31}\,A_1$$

(51) $$\alpha = (E_{31} A_1)/(E_{32} A_2)$$

If the emissivity of the liquid nitrogen surface is the same as that of the internal surface of the intermediate shield

(52) $$E_{31} = E_{21}$$

and

(53) $$\alpha = \frac{E_{31} A_1}{E_{32} A_2} = \frac{E_{21} A_1}{E_{32} A_2}$$

From Figure 6,

(54) $$W_{r\,21} = \sigma E_{21} A_1 (T_2^4 - T_1^4) = \sigma \alpha\beta (T_2^4 - T_1^4)$$

(55) $$W_{r\,32} = \sigma \beta (T_2^4 - T_1^4)$$

With these substitutions, equations 41 and 48 become

(56) $$\frac{W_{c1}}{\dot{m}\, C_p} = \frac{G}{C_p} - \left[\frac{\sigma\alpha\beta}{\dot{m}\, C_p}\right](T_2^{\;4} - T_1^{\;4})$$

(57) $$\frac{W_{c2}}{\dot{m}\, C_p} = \frac{G}{C_p} + (T_2 - T_1) - \left(\frac{\sigma\beta}{\dot{m}\, C_p}\right)(T_3^{\;4} - T_2^{\;4})$$

(58) $$\theta \equiv \frac{K_L}{a} - \frac{W_{cL}}{\dot{m}\, C_p}$$

(59) $$\frac{K_L}{a} = T_L - \frac{b}{a}$$

Therefore

(60) $$\theta = T_L - \frac{b}{a} - \frac{W_{cL}}{\dot{m}\, C_p}$$

Referring to Figure 6, and employing equations 56 and 57,

(61) $$\theta_{21} = T_1 - \frac{b}{a} - \left[\frac{G}{Cp} - \left(\frac{\sigma\alpha\beta}{\dot{m}C_p}\right)(T_2^4 - T_1^4)\right]$$

(62) $$\theta_{32} = T_2 - \frac{b}{a} - \left[\frac{G}{Cp} + (T_2 - T_1) - \left(\frac{\sigma\beta}{\dot{m}\, Cp}\right)(T_3^{\;4} - T_2^{\;4})\right]$$

These may be reduced to

(63) $$\theta_{21} = \left[T_1 - \frac{b}{a} - \frac{G}{Cp}\right] + \left[\left(\frac{\sigma}{C_p}\right)(T_2^{\;4} - T_1^{\;4})\left(\frac{\alpha}{\dot{m}/\beta}\right)\right]$$

(64) $$\theta_{32} = \left[T_1 - \frac{b}{a} - \frac{G}{C_p}\right] + \left[\left(\frac{\sigma}{C_p}\right)\left(T_3^{\,4} - T_2^{\,4}\right)\left(\frac{1}{\dot{m}/\beta}\right)\right]$$

Equation 20 may be applied to Figure 6 as follows:

(65 $$Z_{21} = (T_2 - T_1) + \theta_{21} \ln \left[1 + \frac{(T_2 - T_1)}{T_1 - \frac{b}{a} - \theta_{21}} \right]$$

(66) $$Z_{32} = (T_3 - T_2) + \theta_{32} \ln \left[1 + \frac{(T_3 - T_2)}{T_2 - \frac{b}{a} - \theta_{32}} \right]$$

From the definition of Z

(67) $$Z = \dot{m} \frac{(L/A_c)}{(a/C_p)}$$

it can be seen that, for constant A_c, Z is proportional to L. Then

(68) $$\frac{L_{21}}{L_{21} + L_{32}} = \frac{Z_{21}}{Z_{21} + Z_{32}}$$

(69) $$L_T = L_{21} + L_{32}$$

(70) $$Z_T = Z_{21} + Z_{32}$$

Then

(71) $$\frac{L_{21}}{LT} = \frac{Z_{21}}{Z_T}$$

Equation 67 can be rearranged to become

(72) $$\frac{A_c}{L_T\beta} = \frac{(\dot{m}/\beta)}{(a/C_p)Z_T}$$

The functional dependence of equations 63, 64, 65, and 66 are as follows:

(73) $$Z_{21} = f\,(T_2, \dot{m}/\beta, \alpha)$$

(74) $$Z_{32} = f\,(T_2, \dot{m}/\beta)$$

Equations 70, 71, and 73 show that

(75) $$\frac{A_c}{L_T\beta} = f\,(T_2, \alpha, \dot{m}/\beta)$$

This relationship is plotted in figure 8 as a family of curves. Comparison of the curves for values of α from 1 to 4 shows that the corresponding values of the other parameters change very little. The important conclusion may be drawn that the flask may ice up by condensation of gas on the outer surface, and even though its emissivity is greatly increased, the loss rate of liquid from the flask will stay relatively constant. (This, of course, will be true only if the intermediate shield is present.)

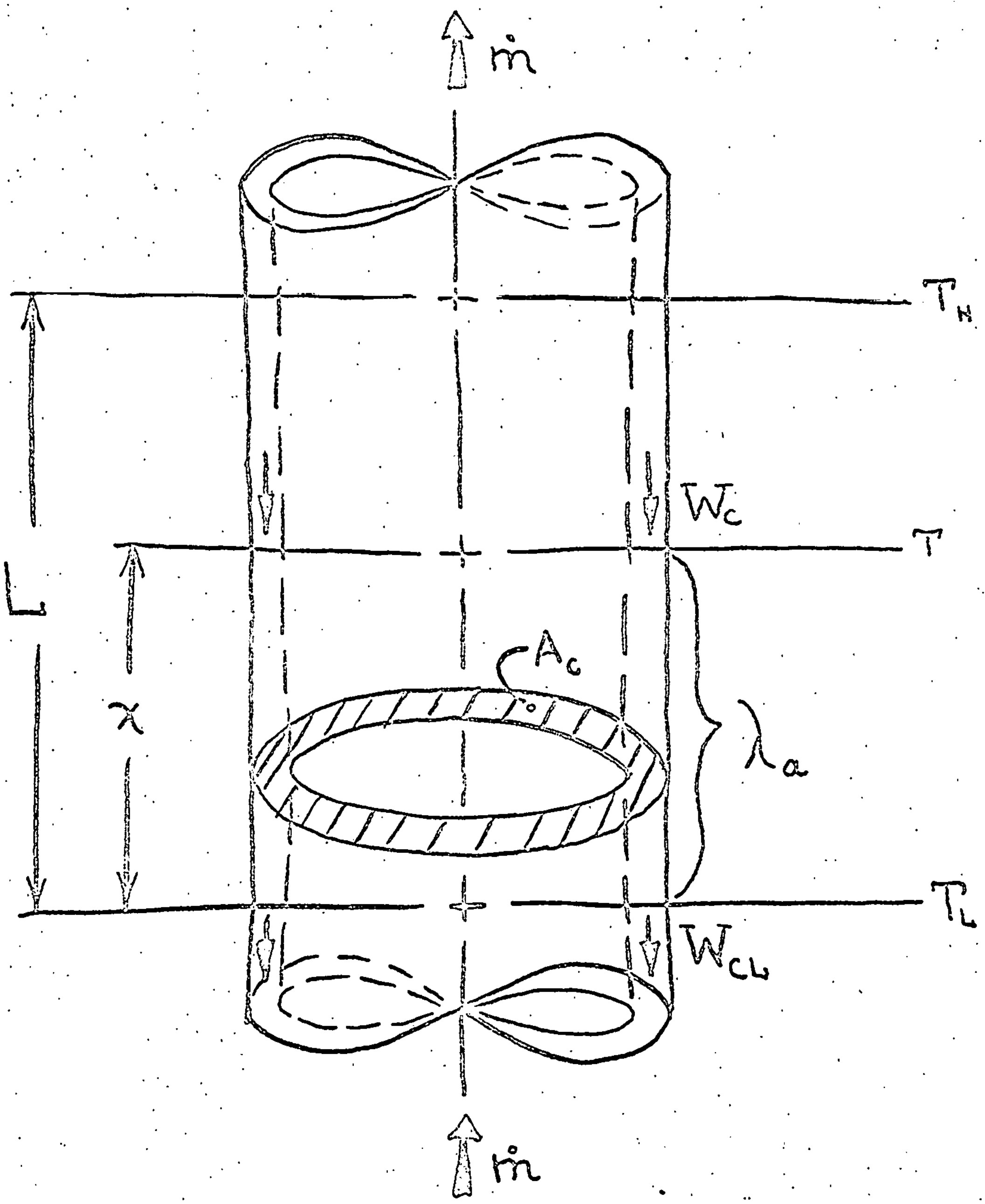
It may also be seen that the location of the shield connection to the neck is not critical.

Values are taken from Figure 8 at the 30% location to plot the line labelled "Shield Clamped to Neck at 30% from Bottom" on Figure 4. This curve clearly shows the advantage gained by the use of the shield in the region to the left of the knee in the uppermost curve.

DISCUSSION

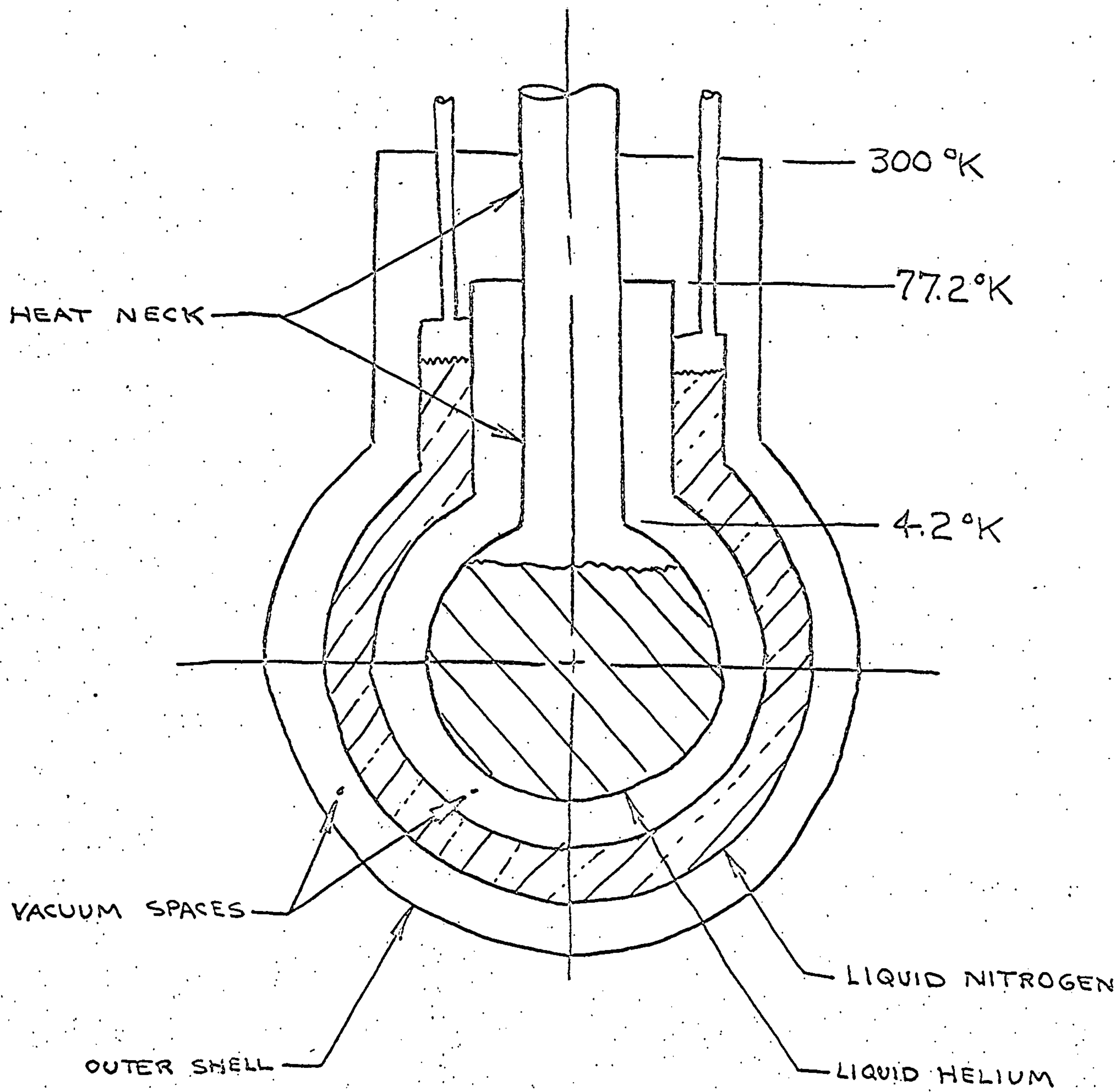
Figure 4 presents the results of the preceding analysis in a form directly applicable to design. The generality of the models employed permits application to be extended to other types of heat inputs to the flask. In particular all heat input crossing the vacuum space may be converted to equivalent black body area and added to the black body area of the flask. Heat entering the flask by metallic leads or supports in intimate contact with the boil-off gas may be converted to equivalent stainless steel neck area if they are so designed as to meet the requirements of the models herein employed. Thus, figure 4 will yield the operating characteristics of the typical research cryostat which may have leads or supports in the vacuum space and/or apparatus suspended in the neck.

The method of analysis presented here is general, and may be applied to containers for cryogenic liquids other than helium.



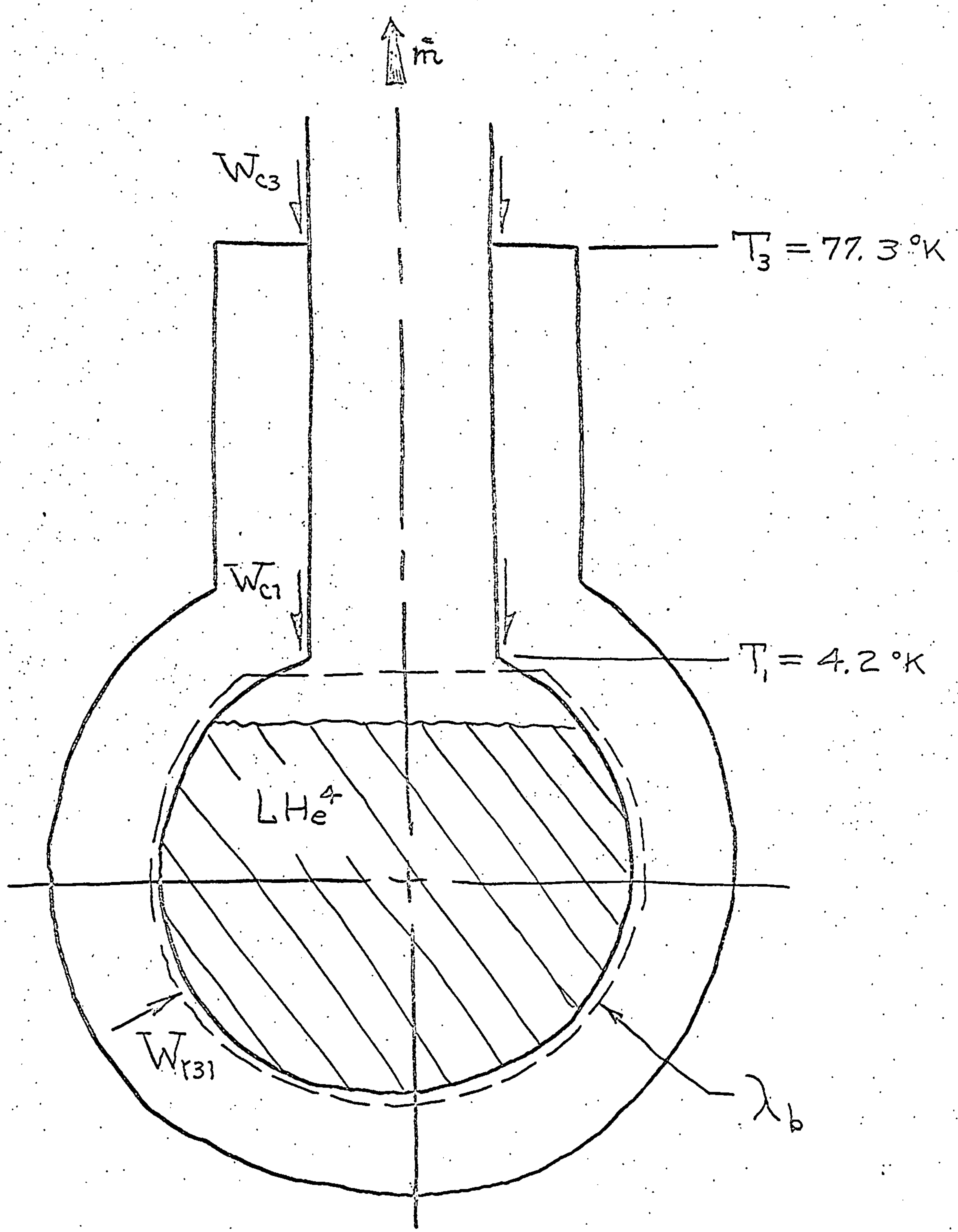
— FIGURE 1 —

SECTION OF A TYPICAL HEAT NECK



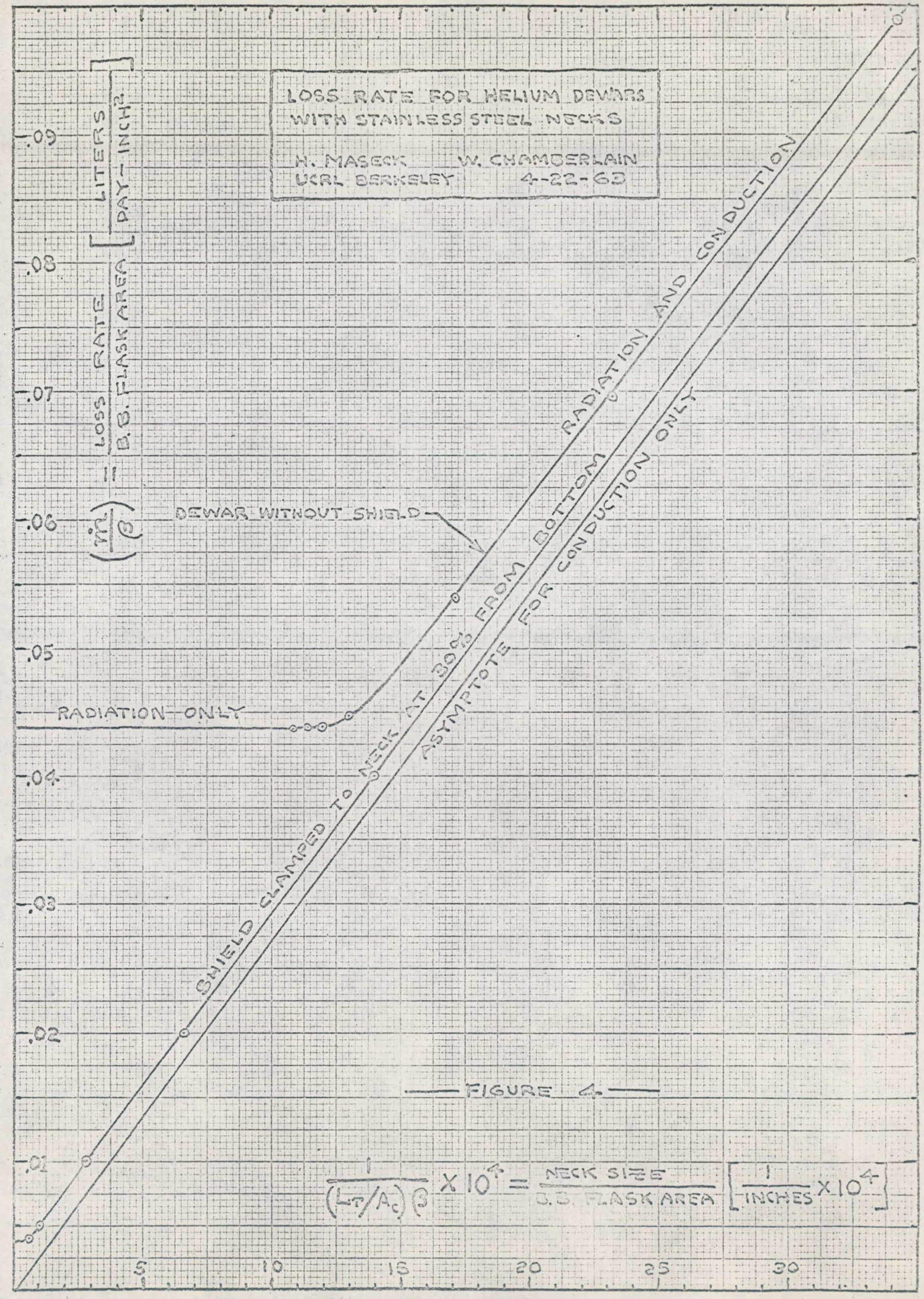
— FIGURE 2 —

A TYPICAL LIQUID HELIUM STORAGE DEWAR

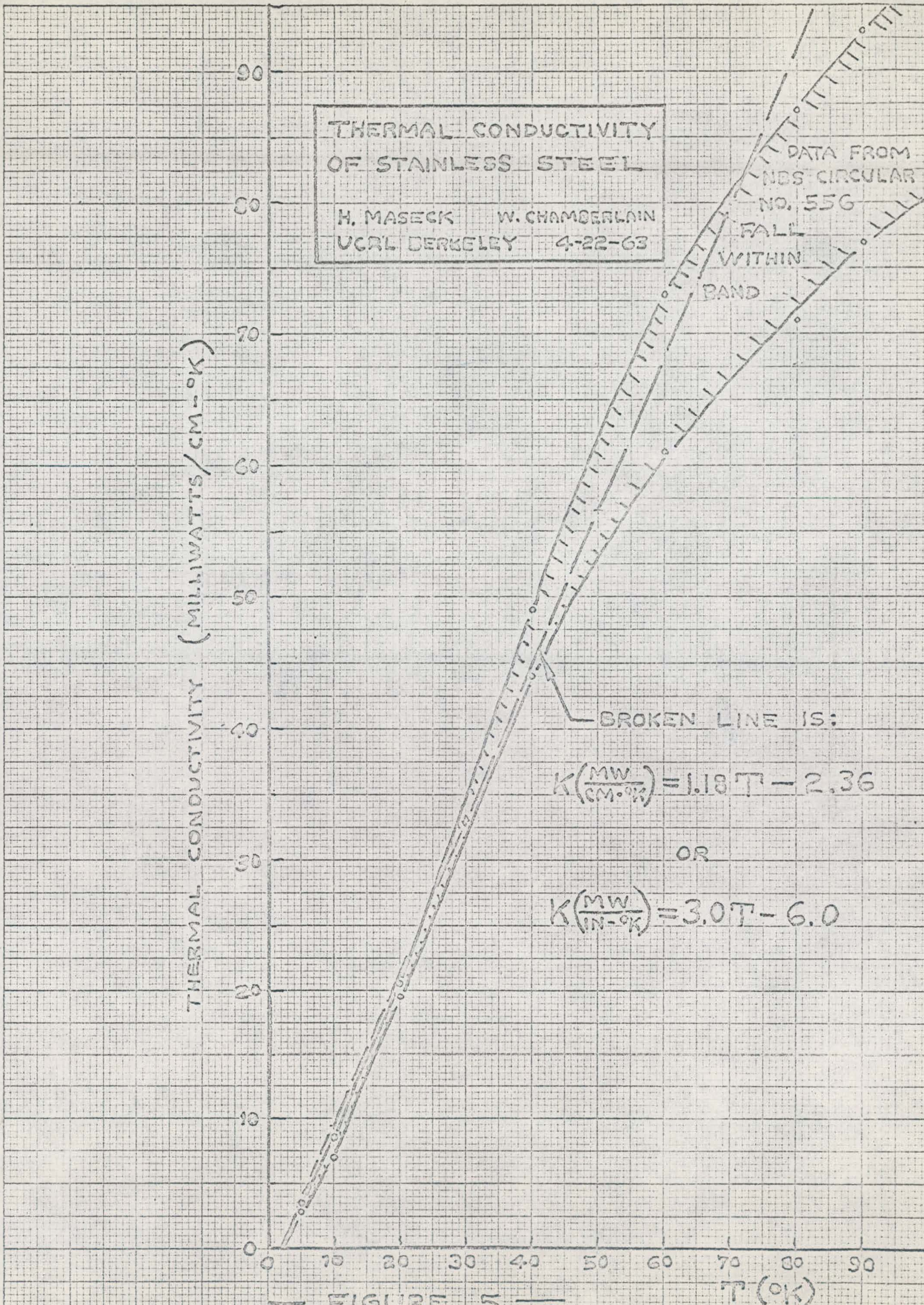


— FIGURE 3 —

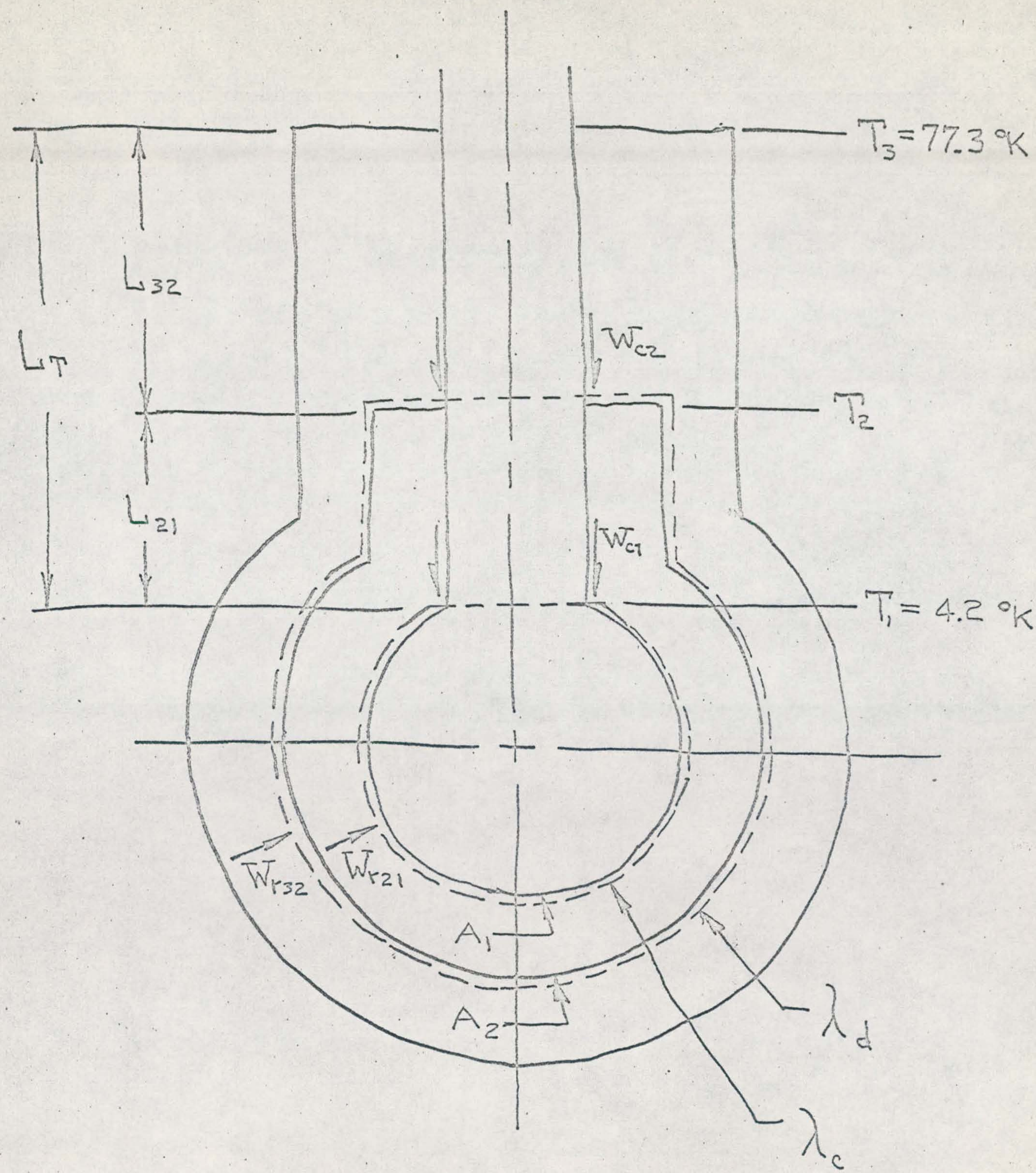
A TYPICAL LIQUID NITROGEN SHIELDED HELIUM DEWAR


LOSS RATE FOR HELIUM DEWARS
WITH STAINLESS STEEL NECKS
H. MASECK W. CHAMBERLAIN
UCRL BERKELEY 4-22-63
$\left(\frac{\dot{m}}{\beta}\right)$ = $\frac{\text{LOSS RATE}}{\text{B.B. FLASK AREA}}$ $\left[\frac{\text{LITERS}}{\text{DAY-INCH}^2}\right]$
.09
.08
.07
.06
.05
.04
.03
.02
.01
RADIATION AND CONDUCTION
DEWAR WITHOUT SHIELD
SHIELD CLAMPED TO NECK AT 30% FROM BOTTOM
ASYMPTOTE FOR CONDUCTION ONLY
RADIATION ONLY
$\frac{1}{(L_T/A_c)\beta} \times 10^4 = \frac{\text{NECK SIZE}}{\text{B.B. FLASK AREA}} \left[\frac{1}{\text{INCHES}} \times 10^4\right]$
5
10
15
20
25
30

— FIGURE 4 —

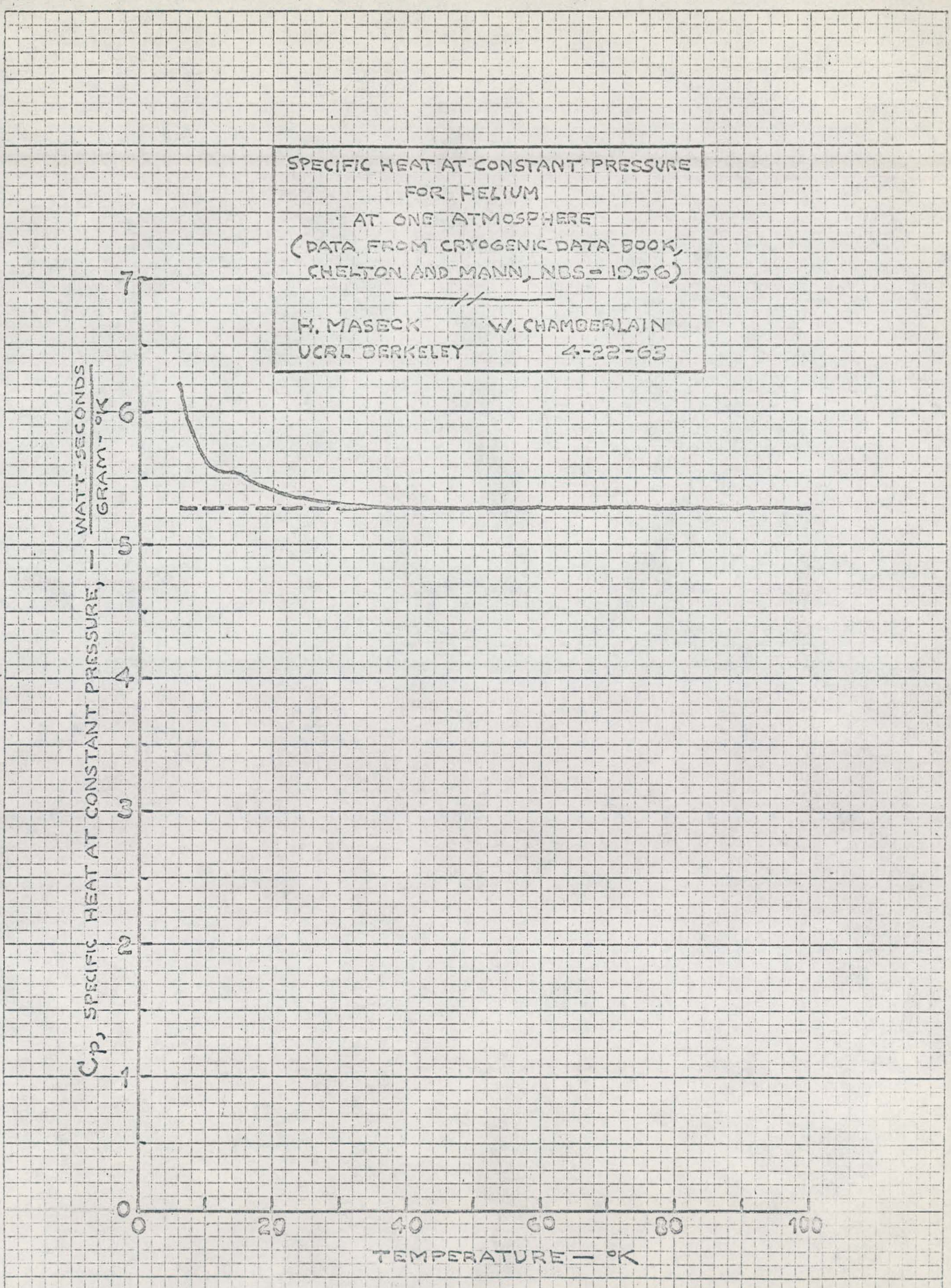

THERMAL CONDUCTIVITY
OF STAINLESS STEEL
H. MASECK W. CHAMBERLAIN
UCRL BERKELEY 4-22-63
DATA FROM
NBS CIRCULAR
NO. 556
FALL
WITHIN
BAND
BROKEN LINE IS:
$K\left(\frac{MW}{CM\cdot°K}\right) = 1.18T - 2.36$
OR
$K\left(\frac{MW}{IN\cdot°K}\right) = 3.0T - 6.0$
THERMAL CONDUCTIVITY (MILLIWATTS/CM-°K)
T (°K)
0
10
20
30
40
50
60
70
80
90

— FIGURE 5 —

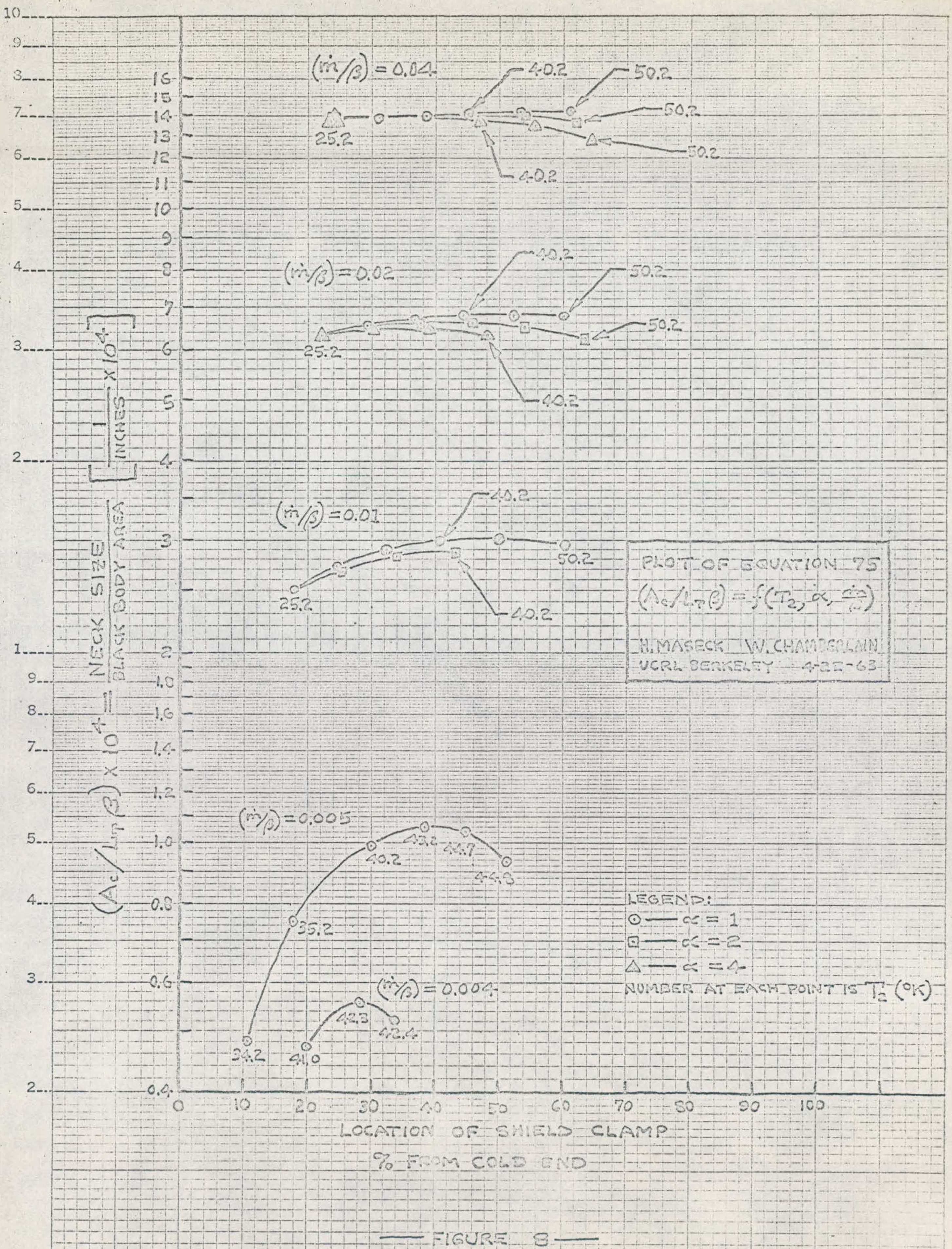


— FIGURE 6 —

LIQUID HELIUM DEWAR WITH GAS-COOLED SHIELD


SPECIFIC HEAT AT CONSTANT PRESSURE
FOR HELIUM
AT ONE ATMOSPHERE
(DATA FROM CRYOGENIC DATA BOOK,
CHELTON AND MANN, NBS-1956)
H. MASECK W. CHAMBERLAIN
UCRL BERKELEY 4-22-63
C_P, SPECIFIC HEAT AT CONSTANT PRESSURE, — WATT-SECONDS / GRAM-°K
0
1
2
3
4
5
6
7
0
20
40
60
80
100
TEMPERATURE — °K

— FIGURE 7 —


$(\dot{m}/\beta) = 0.04$
$(\dot{m}/\beta) = 0.02$
$(\dot{m}/\beta) = 0.01$
$(\dot{m}/\beta) = 0.005$
$(\dot{m}/\beta) = 0.004$
PLOT OF EQUATION 75
$(A_c/L_T\beta) = f(T_2, \alpha, \frac{\dot{m}}{\beta})$
H. MASECK W. CHAMBERLAIN
UCRL BERKELEY 4-22-63
LEGEND:
$\odot$ — $\alpha = 1$
$\boxdot$ — $\alpha = 2$
$\triangle$ — $\alpha = 4$
NUMBER AT EACH POINT IS T_2 (°K)
$(A_c/L_T\beta) \times 10^4 = \frac{\text{NECK SIZE}}{\text{BLACK BODY AREA}} \left[\frac{1}{\text{INCHES}} \times 10^4\right]$
LOCATION OF SHIELD CLAMP
% FROM COLD END
SEMI-LOGARITHMIC 359-61G
KEUFFEL & ESSER CO.
2 CYCLES X 70 DIVISIONS

— FIGURE 8 —

REFERENCES

1. Chelton, D. B., and Mann, D. B., Cryogenic Data Book, NBS, Cryogenic Engineering Laboratory, Boulder, Colorado, 1956

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APPENDIX

SYMBOL DEFINITIONS

Symbol	Definition	Value	Units
C_p	Specific heat at constant pressure for helium	5.23	watt-sec/gm °K
a	See equation 6	3	mw/in $(°K)^2$
b	See equation 6	6	mw/in °K
T_1		4.2	°K
T_3		77.3	°K
G	See equation 25	29.8	mw-days/liter
σ	Stephan Boltzmann const.	36.8×10^{-9}	mw/in^2 °K^4
$\dot{m}$	Mass flow rate		liters/day*
W_c	Heat flow by conduction		milliwatts
W_r	Heat flow by radiation		milliwatts
T_2	Temperature of shield		°K
τ	Temperature difference		°K
K	Thermal conductivity of stainless steel		$\frac{\text{mw}}{\text{in-°K}}$
θ	See equation 20		°K
Z	See equation 20		°K
λ	Designates systems		
L	Length of neck		inches
A_c	Conduction area		inches^2
A_1, A_2	Radiation area		inches^2
β	Equivalent radiation area		inches^2
α	See equation 53		
E	Effective emissivity		

* These liters are liters of liquid helium at 4.2°K, or the equivalent mass of He gas.

CALCULATIONS FOR DEWAR WITHOUT SHIELD

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EQ. 19, 31				EQ 30		EQ 33	EQ 20	
ASSUMED VALUES OF $W_{CL}/\dot{m} C_p \tau_H$	$\ln\left[1+\frac{1}{①}\right]$	$\left[\frac{K_H}{a\tau_H} - \frac{W_{CL}}{\dot{m} C_p \tau_H}\right] = .0302 - ①$	$\frac{Z}{\tau_H} = 1 + ② \times ③$	$\frac{G - W_{CL}/\dot{m}}{C_p \tau_H} = .0302 - ④$	$\frac{\dot{m}}{\beta} = \left[\frac{\sigma(T_3^4 - T_1^4)}{C_p \tau_H}\right]\frac{1}{⑤} = .002355 \div ⑤$	$\frac{A_0}{L_1 \beta} = \frac{⑥}{④\left(\frac{a}{C_p}\right)\tau_H} = \left[\frac{⑥}{④\, 28.8}\right]$	$Z = \frac{⑦}{⑥}$	
.000001	13.8	.0302	1.416	.0538	.0438	.001073		
.00001	11.5	.0302	1.347	.0538	.0438	.001130		
.0001	9.2	.0301	1.277	.0537	.0439	.001194		
.001	6.9	.0292	1.201	.0529	.0446	.001290		
.01	4.61	.0202	1.093	.0438	.0538	.001710		
.1	2.40	-.0698	.8325	-.0462	-.0510	-.00213		NOT SOLUTION TO REAL PROB
1	0.693	-.9698	.329	-.99[illegible]	[illegible]	-.0[illegible]263		
.02	3.93	.0102	1.0402	.0338	.0695	.00232	.0339	
.03	3.53	.0002	1.0007	.0238	.0 88	.00343	.0348	
.05	3.04	-.0198	.940	.0038	.620	.0229	.0370	
.0526	2.99	-.0224	.933	.0012	1.96	.0730	.0373	
.0535	2.98	-.0233	.9306	.0003	7.85	.293	.0374	
①	②	③	④	⑤	⑥	⑦	⑧	

CALCULATIONS FOR DEWAR WITH SHIELD

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1	2	3	4	5	6
T_2 (ASSUMED VALUES)	$(0.1 T_2)^4$	$(0.1T_2)^4 - (0.1T_1)^4 = [(2) - 0.03]$	$(0.1T_3)^4 - (0.1T_2)^4 = [3550 - (2)]$	$T_2 - T_1 = T_2 - 4.2$	$T_3 - T_2 = 77.2 - T_2$
°K	°K^4	°K^4	°K^4	°K	°K
				(1) − 4.2	77.2 − (1)
25.2	40.3	40.3	3510	21	52
30.2	84.0	84.0	3460	26	47
35.2	155.0	155.0	3395	31	42
40.2	261.0	261.	3289	36	37
45.2	420.	420	3130	41	[illegible]
50.2	640.	640.	2910	46	27
33.2	121.5	121.5	3430	29	44
34.2	136.8	136.8	3413	30	43
37.7	202	2[illegible]2	3348	33.5	39.5
41.2	288	288	3262	37	36
39.5	243	243	3307	35.3	37.7
43.2	348	348	3202	39	34
44.2	382.	382	3168	40	[illegible]
44.7	399	399	3151	40.5	32.5
44.8	403	403	3147	40.6	32.4
41.0	282	2[illegible]2	3267	36.8	36.2
42.0	312	312	3238	37.8	35.2
42.3	320	320.	3230	38.1	34.9
[illegible]	3[illegible]	323	3227	38.2.	34.8

CALCULATIONS FOR DEWAR WITH SHIELD
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CASE (A) LET $\alpha = 1.0$
CASE (B) LET $\alpha = 2.0 \longrightarrow Z_{21B} = Z_{21A}$ FOR $(\frac{\dot{m}}{\beta})(\frac{1}{2}) \longrightarrow \theta_{21B} = \theta_{21A}$ FOR $(\frac{\dot{m}}{\beta})(\frac{1}{2})$
CASE (C) LET $\alpha = 4.0 \longrightarrow Z_{21C} = Z_{21A}$ FOR $(\frac{\dot{m}}{\beta})(\frac{1}{4}) \longrightarrow \theta_{21C} = \theta_{21A}$ FOR $(\frac{\dot{m}}{\beta})(\frac{1}{4})$

		EQ 63	EQ 64	EQUATION 65							
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
T_2	$\dot{m}/\beta$	θ_{21}	θ_{32}	$[\,]_{21}$	$[\,]_{32}$			Z_{21A}	Z_{32}	Z_{21B}	Z_{21C}
		$\frac{4.85(3)}{10^5(12)}-1.72$	$\frac{4.85(4)}{10^5(12)}-1.72$	$\frac{(5)}{2.2-(13)}$	$\frac{(6)}{75.2-(14)}$	(13) ln[1+(15)]	(14) ln[1+(16)]	(5)+(17)	(6)+(18)		
	.005	-1.33	32.3	5.9[illegible]	-5.72	-2.5[illegible]	—	17.4	—	—	—
25.2	.01	-1.52	15.3	5.64	6.59	-2.37	31.0	18.1	83.0	18.4	—
	.02	-1.62	6.8	5.44	3.17	-3.0[illegible]	9.7	18.0	61.7	18.1	18.4
	.04	-1.67	2.53	[illegible]	2.51	[illegible]	3.18	17.7	55.2	17.9	18.1
	.005	-.90	31.9	[illegible]	-12.[illegible]	-2.[illegible]	(—)	24.0	—	—	—
30.2	.01	-1.31	15.1	7.[illegible]	3.59	-2.[illegible]	23.0	23.2	70.0	24.0	—
	.02	-1.52	5.67	6.[illegible]	2.1[illegible]	-3.16	7.73	22.[illegible]	54.7	23.2	24.0
	.04	-1.62	2.47	6.8[illegible]	1.[illegible]	-3.32	2.57	[illegible]	[illegible]6	22.8	23.2
	.005	-.17	31.2	13.1	21.0	-.419	96.3	[illegible]	[illegible]	—	—
35.2	.01	-.97	14.73	9.7[illegible]	2.[illegible]	-2.[illegible]	17.4	2[illegible]	[illegible]	30.6	—
	.02	-1.34	6.51	8.75	1.57	~~-3.[illegible]~~	6.14	28.0	[illegible]	28.7	30.6
	.04	-1.53	2.40	8.31	1.3[illegible]	-3.41	2.06	27.6	44.1	2[illegible]	28.7
	.005	+.81	30.2	25.9	4.62	+2.[illegible]7	52.2	38.7	87.2		—
40.2	.01	-.45	14.3	13.6	1.5[illegible]	-1.2[illegible]	13.4	34.8	50.4	38.7	
	.02	-1.09	6.26	10.9	1.16	-2.7[illegible]	4.92	33.3	41.8	34.8	38.7
	.04	-1.40	2.27	10.0	1.03	-3.[illegible]	1.61	32.7	[illegible]	33.3	[illegible]
	.005	+2.36	28.7	-256	2.21	(—)	33.4	—	65.4		
45.2	.01	+.32	13.5	+21.8	1.08	+1.[illegible]	9.89	42.0	41.7	—	[illegible]
	.02	-.70	5.87	14.1	.858	-1.10	3.63	37.1	[illegible]	-	—
	.04	-1.21	2.08	12.0	.778	-3.10	1.20	37.7	[illegible]		42.0
	.005	4.48	2[illegible]	-20.2	1.24	---	21.4	—	[illegible]	—	—
50.2	.01	1.38	12.4	+56	.753	+[illegible]	6.95	51.6	3[illegible]	—	---
	.02	-.17	5.33	17.9	.627	-5.12	2.60	45.5	27.6	51.6	—
	.04	-.95	1.80	14.5	1.582	-2.60	.825	43.4	27.8	45.5	51.6

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						$\times 10^{-4}$					
23	24	25	26	27	28	29	30	31	32	33	34
$(Z_{TT})_A$	$(Z_{TT})_B$	$(Z_T)_C$	$\left(\frac{L_{21}}{L_{TT}}\right)_A$	$\left(\frac{L_{21}}{L_T}\right)_B$	$\left(\frac{L_{21}}{L_T}\right)_C$	$\left(\frac{A_c}{L_T\beta}\right)_A$	$\left(\frac{A_c}{L_T\beta}\right)_B$	$\left(\frac{A_c}{L_T\beta}\right)_C$			
20 + 19	20 + 21	20 + 22	19 / 23	21 / 24	22 / 25	$\frac{12}{.395\ 23}$	$\frac{12}{.395\ 24}$	$\frac{12}{.395\ 25}$			
—	—	—	—	—	—	—	—	—			
101.1	101.4	—	.179	.182	—	2.51	2.50	—			
77.7	77.8	80.1	.225	.227	.230	6.36	6.35	6.32			
73.1	73.1	73.3	.245	.245	.247	13.86	13.85	13.82			
—	—	—	—	—	—	—	—	—			
93.2	94.0	—	.249	.256	—	2.72	2.70	—			
77.5	77.9	79.1	.294	.298	.305	6.54	6.50	6.45			
72.3	72.4	72.7	.314	.315	.319	14.0	14.0	13.92			
168.9	—	—	.181	—	—	.75	—	—			
88.1	90.[illegible]	—	.326	.341	—	2.88	2.82	—			
76.1	76.8	77.7	.363	.374	.38[illegible]	~~6.[illegible]~~	6.60	6.44			
71.7	72.1	72.[illegible]	.3[illegible]	.389	.394	14.12	14.06	13.92			
127.9	—	—	.302	—	—	.99	—	—			
85.2	87.1	—	.408	.434	—	2.[illegible]	2.85	—			
75.1	76.6	80.5	.444	.454	.481	6.75	6.62	6.29			
71.3	7[illegible]	73.4	.456	.464	.475	14.20	14.10	13.80			
—	—	—	—	—	—	—	—	—			
8[illegible]	—	—	.502	—		3.02	—	—			
74.7	7[illegible]	—	.524	.542	—	6.78	6.53	—			
71.1	[illegible]	75.2	.534	.542	.5[illegible]	14.25	14.01	13.48			
—	—	—	—	—	—	—	—	—			
75.5	—	—	.604	—	—	2.[illegible]	—	—			
75.1	81.2	—	.605	.636	—	6.75	6.24	—			
71.2	73.3	77.4	.609	.621	.651	14.12	13.82	12.85			

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		EQ 63	EQ 64	EQUATION 65							
11	12	13	14	15	16	17	18	19	20	21	22
T_2	$\dot{m}/\beta$	θ_{21A}	θ_{32}	$[\ ]_{21}$	$[\ ]_{32}$			Z_{21A}	Z_{32}		
		$\frac{4.85(3)}{10^5(12)}-1.72$	$\frac{4.85(4)}{10^5(12)}-1.72$	$\frac{(5)}{2.2-(13)}$	$\frac{(6)}{75.2-(6)-(14)}$	$(13)\ln[1+(15)]$	$(14)\ln[1+(16)]$	$(5)+(17)$	$(6)+(18)$		
34.2	.005	-.90	31.3	11.56	+477	-1.91	193	27.0	236		
42.33											
42.36											
[illegible]											
42.4											
41.0	[illegible]	1.70	38.0	73.5	36.2	7.33	137.	44.1	173.2		
42.0	[illegible]	[illegible]	37.6	270.	14.7	11.50	103.	49.3	138.2		
42.3	[illegible]	2.[illegible]5	37.5	761.	12.5	14.30	97.5	[illegible].4	132.4		
42.[illegible]	.004	[illegible]	37.4	38200	11.[illegible]	23.2	94.8	61.4	127.6		
42.[illegible]	.0[illegible]4	2.199	37.4	382000	11.6	28.3	94.8	66.5	127.6		
43.2	.005	+1.66	29.3	72.2	2.[illegible]	7.11	38.6	46.1	73.6		
44.2	.005	+[illegible]	29.0	182.0	2.5[illegible]	11.3	36.9	50.[illegible]	6[illegible].4		
44.70	[illegible]	+2.[illegible]	28.8	810.0	+2.33	44.4	34.7	54.9	67.2		
44.80	.005	+2.195	2[illegible]	8100.	+2.31	19.8	34.5	60.4	66.9		
44.8[illegible]	.005	+2.199	2[illegible]	40[illegible]00.	+2.31	23.3	34.5	63.[illegible]	66.9		
44.[illegible]	.005	+2.1999	28.8	406000	+2.[illegible]	28.4	34.5	69.0	66.9		
41.0	.0035	-[illegible]	43.6	0	-7.[illegible]	[illegible]					
39.5	.0050	-[illegible]0	50.3	110	-3.31	0	—)	35.3			
37.7	.0035	-[illegible]20	65.0	0	-1.35	0	—)	33.5			

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23	24	25	26	27	28	29	30	31	32	11	12
z_{TA}		L_{21}/L_{TA}				$\left(\frac{A_c}{L_T \beta_A}\right) \times 10^{-4}$				T_2	$\dot{m}/\beta$
(20)+(19)		(19)/(23)				(12)/(.595 (23))					
265			.1015			.478				34.2	.005
217.3			.203			.466				41.0	.004
187.5			.263			.541				42.0	.004
184.8			.294			.549				42.3	.004
191.0			.322			.551				42.4	.004
196.1			.339			.517				42.[illegible]	.004
117.7			.385			1.0[illegible]				43.2	.005
119.7			.420			1.058				44.2	.005
122.1			.449			~~1.052~~				44.7	.005
127.3			.474			.995				44.3	.005
130.8			.489			.968				44.8	.005
135.9			.508			.931				44.8	.005

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