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FORMULATION OF THE COUPLED LATERAL-ROTATIONAL INTERACTIONS

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ABSTRACT

The overall objective of the work supported by this contract is to determine the extent to which the inertia of a large reactor structure will interact with the free-field seismic ground forces to alter the base motion of the structure from that of the free-field and hence to alter the structural forces caused by that base motion. In this report an analysis is made of the combined lateral and rocking motion of the foundation of an N-mass structure coupled to a two-dimensional elastic half-space the free-field motion of which simulates an earthquake. The problem model and the analytical technique are similar to the authors' earlier work in references (1) and (2) but this problem is considerably more complicated. The appropriate Lamb problem is solved in terms of integrals convergent in the Cauchy sense and three pairs of Volterra integral equations are derived - one pair neglecting base-mass effects and the other two pairs including them. The latter two pairs are programmed for numerical solution. Due to the fact that the Lamb solution is not sufficiently smooth to permit the required differentiations for obtaining those Volterra equations programmed, some integrals diverge and useful physical information in addition to that derived from the simpler models neglecting the rocking motion is not forthcoming. No attempt is made to program the equations which neglect base-mass effects. The integrals known to diverge do not appear in this formulation.

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NOMENCLATURE

a	Dilatation (P) wave velocity
A	Area of the structure base
b	Shear (S) wave velocity
c	Half the base width
E	Young's modulus
$F(t)$	Lateral force at the base of a structure
$f(t)$	Surface shear stress when $\|x\| < c$
h	$h = \frac{p}{a}$
I_o	Moment of inertia of base about an axis through the origin and perpendicular to it
k	$k = \frac{p}{b}$
M_k, M_k^1, N_k	Constants defined in equation (45)
m_o	Base mass
M_j	Effective mass of the jth mode
m_j	The jth structure mass
p	Transformed time variable
$u(x,y,t)$	Lateral displacement in the half-space (x-direction)
$u(t)$	Lateral displacement of the center of the base
$u_p(t)$	Free-field lateral displacement at the center of the base
$v(x,y,t)$	Vertical displacement in the half-space (y-direction)
V	Rayleigh wave velocity
$\bar{X}_{ij}$	Mode shape for the ith mass and jth mode

NOMENCLATURE (cont'd.)

β	Constant defined in equation (16)
γ	Constant defined in equation (29)
$\Theta(x,y,t)$	Half the out-of-plane component of the curl of the displacement vector. Measure the counterclockwise rotation.
$\Theta(t)$	Counterclockwise rotation at and about the origin
w_j	Natural circular frequency of the j^{th} mode
λ	Lame's constant $\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$
μ	Shear modulus $\mu = \frac{E}{2(1+\nu)}$
ν	Poisson's Ratio
ξ	Transformed x coordinate
ρ	Ground density
σ_x	Tensile stress in the x-direction
σ_y	Tensile stress in the y-direction
τ_{xy}	Shear stress
ϕ	Dilatation scalar potential
Φ	Counterclockwise rotation at the origin
ϕ_k	Spectrum integral (cf. equation (45))
ψ	Equivoluminal (shear) wave function
∇^2	Laplace operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

INTRODUCTION

The purpose of this report is to present the work done on the solution of the coupled rotational-lateral interaction problem associated with the motion of and forces on a structure laterally excited at the base by an earthquake. The structure subjected to this lateral motion at its base is modeled by considering its base rigidly displaced laterally and rigidly rotated about its center point. The structure model is lumped so the two variables of paramount importance become the lateral displacement of the center point of the base and the rotation at and about the center point. The earth is modeled by a plane elastic half-space. The models and the method of solution are similar to those employed by the authors to solve simpler problems of the same kind in references (1) and (2). The lateral displacement of the base center point and the rotation at and about it resulting from stress distributions on the plane elastic half-space which are implied from the assumption that the structure base moves rigidly laterally and rotates rigidly about the center point are determined by solving the appropriate Lamb problem. These solutions are found in terms integrals convergent in the Cauchy sense. The interaction equations exhibiting the relationship between the structure and the earth are formulated via normal mode theory both for the case in which the base mass is neglected and for the case in which it is not. A pair of coupled Volterra integral equations is developed for each of three cases. The latter two sets contain divergent integrals as confirmed analytically and also by the divergence of the corresponding digital computer programs. The three pairs of Volterra equations are presented in the Theory section, and the related computer programs for the latter two pairs containing base-mass effects are listed in the Appendix in the hope that they can jointly be modified by physically reasonable smoothing assumptions which will permit their solution.

THEORY

THE SOLUTION OF THE LAMB PROBLEM

On the boundary of a two-dimensional elastic half-space a time-dependent shearing stress uniform over a finite portion and zero elsewhere and a time-dependent linear skew symmetrical normal stress over the same finite portion and zero elsewhere are applied. For this boundary condition the resultant horizontal displacement of a base point symmetrically located on the boundary of the half-space together with the rotation at and about this base point is determined. If cartesian coordinates x and y are introduced so that the half-space is $y \geq 0$, the boundary-value problem consists of the Navier equations valid for $y > 0$ together with the prescribed stresses on the boundary $y = 0$ and the condition that all stresses vanish at $y = \infty$. (cf. fig. 1)

Specifically,

$$\left.\begin{aligned} \rho u_{tt} &= (\lambda+\mu)\,\theta_x + \mu\nabla^2 u \\ \rho v_{tt} &= (\lambda+\mu)\,\theta_y + \mu\nabla^2 v \end{aligned}\right\}\;, \quad y > 0 \qquad (1)$$

$$\left.\begin{aligned} \sigma_y &= \begin{cases} \frac{x}{c}\,\sigma(t) & , \ |x| < c \\ 0 & , \ |x| > c \end{cases} \\ \tau_{xy} &= \begin{cases} f(t) & , \ |x| < c \\ 0 & , \ |x| > c \end{cases} \end{aligned}\right\}, \quad y = 0 \qquad (2)$$

and $\sigma_x = \sigma_y = \tau_{xy} = 0 \quad , \ y = \infty$

where $f(t) = \sigma(t) = 0$ for $t \leq 0$. (The symbols used in the equations here and in the sequel are defined on the nomenclature page.) The quantities to be determined from equations (1) and (2) are u(0,0,t) and Θ(0,0,t) where $\Theta(x,y,t) = \frac{1}{2}[u_y(x,y,t) - v_x(x,y,t)]$. In order to uncouple equations (1), let $u = \phi_x+\psi_y$, $v = \phi_y-\psi_x$, and $\theta = \nabla^2\phi$. As is shown in references (1) and (2), substitution into equations (1), (2), and Hooke's Law yields the following boundary-value problem in $\phi(x,y,t)$ and $\psi(x,y,t)$.

$$\left.\begin{aligned}\phi_{tt} &= a^2\nabla^2\phi\\ \psi_{tt} &= b^2\nabla^2\psi\end{aligned}\right\} , \ y > 0 \qquad (3)$$

$$\left.\begin{aligned}\sigma_y &= \lambda\nabla^2\phi + 2\mu(\phi_{yy}-\psi_{xy})\\ \tau_{xy} &= \mu(2\phi_{xy}+\psi_{yy}-\psi_{xx})\end{aligned}\right\} , \ y = 0 \qquad (4)$$

Equations (3) and (4) are solved for ϕ and ψ by using the Laplace transformation with respect to t and the Fourier transformation with respect to x. Equations (5) define the notation and exhibit the invers Fourier transformation.

$$\begin{aligned}\bar{\phi}(x,y,p) &= L\{\phi(x,y,t)\} = \int_0^\infty \phi(x,y,t)\ e^{-pt}\ dt\\ \phi(x,y,t) &= L^{-1}\{\phi(x,y,p)\}\\ \bar{\bar{\phi}}(\xi,y,p) &= F\{\bar{\phi}(x,y,p)\} = \int_{-\infty}^{\infty} \bar{\phi}(x,y,p)\ e^{i\xi x}\ dx\\ \bar{\phi}(x,y,p) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{\bar{\phi}}(\xi,y,p)\ e^{-i\xi x}\ d\xi\end{aligned} \qquad (5)$$

Similar equations define the bar and two-bar notation relative to other variables. At t = 0, $\phi = \phi_t = \psi = \psi_t = 0$. As in references (1) and (2), the boundary-value problem for $\bar{\bar{\phi}}(\xi,y,p)$ and $\bar{\bar{\psi}}(\xi,y,p)$ is

$$\left.\begin{aligned}\bar{\bar{\phi}}_{yy} -(\xi^2+h^2)\bar{\bar{\phi}} &= 0\\ \bar{\bar{\psi}}_{yy} -(\xi^2+k^2)\bar{\bar{\psi}} &= 0\end{aligned}\right\} , \ y > 0 \qquad (6)$$

$$\left.\begin{aligned} \frac{2\bar{\sigma}i}{c\xi^2}(\sin c\xi - c\xi\cos c\xi) &= \lambda(\xi^2\bar{\bar{\phi}}+\bar{\bar{\phi}}_{yy}) + 2\mu(\bar{\bar{\phi}}_{yy}+i\xi\bar{\bar{\psi}}_y) \\ \frac{2\bar{f}\sin c\xi}{\xi} &= \mu(-2i\xi\bar{\bar{\phi}}_y+\bar{\bar{\psi}}_{yy}+\xi^2\bar{\bar{\psi}}) \end{aligned}\right\} , \ y = 0 \quad (7)$$

The solution of equation (6) which satisfies the condition at $y = \infty$ has the form

$$\begin{aligned} \bar{\bar{\phi}}(\xi,y,p) &= A(\xi,p)\, e^{-\sqrt{\xi^2+h^2}\; y} \\ \bar{\bar{\psi}}(\xi,y,p) &= B(\xi,p)\, e^{-\sqrt{\xi^2+k^2}\; y} \end{aligned} \qquad (8)$$

where the square roots are the principal-valued roots. Substitution of $\bar{\bar{\phi}}$ and $\bar{\bar{\psi}}$ from equations (8) with $y = 0$ into equations (7) yields two equations for $A(\xi,p)$ and $B(\xi,p)$. With A and B and thus $\bar{\bar{\phi}}$ and $\bar{\bar{\psi}}$ determined, $\bar{\bar{u}}$ can be computed from $\bar{\bar{u}} = -i\xi\bar{\bar{\phi}}+\bar{\bar{\psi}}_y$ and u is obtained from the Laplace and Fourier inversion. Finally Θ is obtained by noting that the defining relationships among u, v, ϕ, ψ, and Θ together with the second of equations (3) imply $\Theta = \frac{1}{2}\nabla^2\psi = \frac{1}{2b^2}\psi_{tt}$ and $\bar{\bar{\Theta}} = \frac{p^2}{2b^2}\bar{\bar{\psi}}$.

Since the boundary-value problem defined by equations (6) and (7) has already been solved in reference (1) for the case where $\sigma = 0$, it is convenient at this point to split the work into two parts (1) $\sigma = 0$ $f \neq 0$ and (2) $\sigma \neq 0$ $f = 0$, to denote the corresponding solutions with appropriate subscripts 1 or 2, and to superpose the results to determine the solutions $u(0,0,t)$ and $\Theta(0,0,t)$. Further since the origin is the only point of present interest it is convenient to write $u(0,0,t) = u(t)$ and $\Theta(0,0,t) = \Theta(t)$.

From reference (2) (p.5 equations (12) and (14))

$$u_1(t) = -\frac{b}{\mu}\int_0^t f(\tau)d\tau - \frac{b^2}{2\pi c\mu}\int_0^t\int_0^{t-\tau} f(\xi)\ \mathrm{Im}\ g_1(\frac{b\tau}{c})\ d\xi\ d\tau$$

$$\mathrm{Im}\ g_1(T) = \begin{cases} 0 & , \ 0 < T < \frac{1}{\sqrt{3}} \\ \dfrac{3T(1-T^2)\sqrt{T^2-1/3}}{2(T^2-\frac{1}{2})(T^2-\frac{3-\sqrt{3}}{4})(T^2-\frac{3+\sqrt{3}}{4})} & , \ \frac{1}{\sqrt{3}} < T < 1 \\ -\dfrac{3\sqrt{T^2-1}\ [(T^2-\frac{1}{2})^2 + T^2\sqrt{T^2-1/3}\ \sqrt{T^2-1}\ ]}{2(T^2-\frac{1}{2})(T^2-\frac{3-\sqrt{3}}{4})(T^2-\frac{3+\sqrt{3}}{4})} & , \ T > 1 \end{cases} \tag{9}$$

The integral in equation (9) is proper for $0 < t < \frac{c}{v}$. At $\tau = \frac{c}{v}$ $\mathrm{Im}\ g_1(\frac{b\tau}{c})$ has a singular point of order $(\tau - \frac{c}{v})^{-1}$. The displacement at $t = \frac{c}{v}$, however, is finite since the indeterminant form in the integrand, $\lim_{\tau\to\frac{c}{v}}\int_0^{\frac{c}{v}-\tau} f(\frac{c}{v}-\xi)d\xi(\tau-\frac{c}{v})^{-1}$, is finite. For $t > \frac{c}{v}$, because of the singular point at $t = \frac{c}{v}$, to be meaningful the integral in equation (9) must be considered as its Cauchy principal value. In which case $u_1(t)$ is a well defined finite value for all non negative t.

For problem (1) ($\sigma = 0$ and $f \neq 0$), the rotation solution evaluated on $y = 0$ is

$$\bar{\bar{\theta}}_1(\xi,0,p) = \frac{p^2}{2b^2}\ \psi_1(\xi,0,p) = \frac{p^2\bar{f}(p)}{2\mu b^2}\ \frac{\sin c\xi\ (\xi^2+\frac{p^2}{2b^2})}{\xi\ D(\xi,p)}$$

$$D(\xi,p) = (\xi^2+\frac{p^2}{2b^2})^2 - \xi^2\sqrt{\xi^2+\frac{p^2}{a^2}}\ \sqrt{\xi^2+\frac{p^2}{b^2}} \tag{10}$$

Inversion of the Fourier transform using the last of equations (5) with ϕ replaced by ψ_1 and with $x = 0$ and $\sin c\xi = \mathrm{Im}\ e^{ic\xi}$ yields

$$\bar{\theta}_1(0,0,p) = \frac{p^2\bar{f}(p)}{4\pi\mu b^2}\int_{-\infty}^{\infty}\frac{\xi^2+\frac{p^2}{2b^2}}{\xi\ D(\xi,p)}\ \mathrm{Im}\ e^{ic\xi}\ d\xi \tag{11}$$

which simplifies by halving the interval of integration and doubling the even integrand and by interchanging Im and the integration to

$$\bar{\Theta}_1(0,0,p) = \mathrm{Im}\left\{\frac{p^2\bar{f}(p)}{2\pi\mu b^2}\int_0^\infty \frac{\xi^2+\frac{p^2}{2b^2}}{\xi\, D(\xi,p)}\, e^{ic\xi}\, d\xi\right\} \tag{12}$$

As in the u(0,0,t) computation of reference (1) changing the dummy of integration by letting $ic\xi = -p\tau$ proves convenient.

$$\int_0^\infty \frac{\xi^2+\frac{p^2}{2b^2}}{\xi\, D(\xi,p)}\, e^{ic\xi}\, d\xi = \frac{b^3}{cp^2}\int_0^{-i\infty} g_2(T)\, e^{-p\tau}\, d\tau \tag{13}$$

where

$$T = \frac{b\tau}{c}$$

$$g_2(T) = \frac{3c(T^2-\frac{1}{2})\left[(T^2-\frac{1}{2})^2 - T^2\sqrt{1/3-T^2}\,\sqrt{1-T^2}\right]}{2b\tau(T^2-\frac{1}{4})(T^2-\frac{3-\sqrt{3}}{4})(T^2-\frac{3+\sqrt{3}}{4})} \tag{14}$$

and the integration in the complex τ-plane is down the negative imaginary axis. By using the fourth quadrant contour described in figure 2 and the same technique as in the $u_1(0,0,t)$ computation,

$$\int_0^{-i\infty} g_2(T)\, e^{-p\tau}\, d\tau = L\,\{g_2(\tfrac{bt}{c})\} + (\frac{\pi i}{2}\operatorname*{Res}_{\tau=0} + \pi i \operatorname*{Res}_{\tau=\frac{c}{V}})\; g_2(T)e^{-p\tau} \tag{15}$$

where

$$\operatorname*{Res}_{\tau=0}\; g_2(T)e^{-p\tau} = \frac{2c}{b}$$

$$\operatorname*{Res}_{\tau=\frac{c}{V}}\; g_2(T)e^{-p\tau} = \frac{\beta c}{b}\, e^{-\frac{c}{V}p} \tag{16}$$

$$\beta = \frac{3(X^2-\frac{1}{2})\left[(X^2-\frac{1}{2})^2 + X^2\sqrt{X^2-1/3}\,\sqrt{X^2-1}\right]}{4X^2(X^2-\frac{1}{4})(X^2-\frac{3-\sqrt{3}}{3})}$$

$$X = \frac{b}{v}$$

Substitution from equations (13) through (16) into equation (12) yields

$$\overline{\Theta}_1(0,0,p) = \mathrm{Im}\left\{\frac{b\ \overline{f}(p)}{2\pi\mu c}\ [L\{g_2(\tfrac{bt}{c})\} + \frac{\pi c}{b}\ i + \frac{\pi c\beta}{b}\ i\ e^{-\frac{c}{v}p}]\right\} \qquad (17)$$

Noting that the first factor following Im in equation (17) is real and that Im and L can be interchanged allows equation (17) to be simplified to

$$\overline{\Theta}_1(0,0,p) = \frac{b\ \overline{f}(p)}{2\pi\mu c}\ L\{\mathrm{Im}\ g_2(\tfrac{bt}{c})\} + \frac{\overline{f}(p)}{2\mu} + \frac{\beta\ \overline{f}(p)}{2\mu}\ e^{-\frac{c}{v}p} \qquad (18)$$

The first summand on the right side of equation (18) is in the form corresponding to a convolution in the t-space and the third summand is in the form corresponding to a shifting of the time scale in the t-space. This observation allows the inversion of the Laplace transformation. The solution in the notation $\Theta_1(0,0,t) = \Theta_1(t)$ is written in equation (19).

$$\Theta_1(t) = \frac{1}{2\mu}\ f(t) + \frac{\beta}{2\mu}\ f(t - \tfrac{c}{v}) + \frac{b}{2\pi\mu c}\int_0^t f(t-\tau)\,\mathrm{Im}\ g_2(\tfrac{b\tau}{c})\,d\tau \qquad (19)$$

where after simplification equation (14) yields

$$\mathrm{Im}\ g_2(T) = \begin{cases} \dfrac{3T(\frac{1}{2}-T^2)\ \sqrt{T^2-1/3}\ \sqrt{1-T^2}}{2T^2(T^2-\frac{1}{2})(T^2-\frac{3+\sqrt{3}}{3})(T^2-\frac{3-\sqrt{3}}{3}} & ,\ \frac{1}{\sqrt{3}} < T < 1 \\ \\ 0,\ 0 < T < \frac{1}{\sqrt{3}} \text{ and } T > 1 & \end{cases} \qquad (20)$$

and β is approximately .51. (β is given precisely from equations (16)). Therefore $\mathrm{Im}\ g_2(\frac{bt}{c})$ is zero for $0 < t < \frac{c}{a}$, negative for $\frac{c}{a} < t < \frac{c}{\sqrt{2}b}$, positive for $\frac{c}{\sqrt{2}b} < t < \frac{c}{b}$, and zero for $t > \frac{c}{b}$. The integral in equation (19) is a proper integral.

For problem (2) ($\sigma \neq 0$ and $f = 0$) equations (6) and (7) have the solution which for $y = 0$ can be written

$$\bar{\bar{\phi}}_2(\xi,0,p) = \frac{(\xi^2+\frac{p^2}{2b^2})\,\bar{\sigma}(p)(\sin c\xi - c\xi\cos c\xi)i}{c\;\xi^2\mu\; D(\xi,p)}$$

$$\bar{\bar{\psi}}_2(\xi,0,p) = \frac{\xi\sqrt{\xi^2+\frac{p^2}{a^2}}\;\bar{\sigma}(p)\;(\sin c\xi - c\xi\cos c\xi)}{c\;\xi^2\mu\; D(\xi,p)} \tag{21}$$

where $D(\xi,p)$ is defined by equations (10) and where all square roots are the principal-valued roots. Substituting $\bar{\bar{\phi}}_2$ and $\bar{\bar{\psi}}_2$ from equations (21) into $\bar{\bar{u}}_2 = -i\xi\,\bar{\bar{\phi}}_2 + \bar{\bar{\psi}}_{2y}$ and simplifying yields equations (22) for $\bar{\bar{u}}_2$.

$$\bar{\bar{u}}_2(\xi,0,p) = \frac{\sigma(p)(\sin c\xi - c\xi\cos c\xi)(\xi^2+\frac{p^2}{2b^2} - \sqrt{\xi^2+\frac{p^2}{a^2}}\sqrt{\xi^2+\frac{p^2}{b^2}}\,)}{\mu\; c\;\xi\; D(\xi,p)} \tag{22}$$

From use of the Fourier inversion integral (the last of equations (5) with ϕ replaced by u)with $x = 0$ and the fact that $\bar{\bar{u}}$ is an even function of ξ,

$$\bar{u}_2(0,0,p) = \frac{\bar{\sigma}(p)}{\pi\mu}\;(I_1 - I_2) \tag{23}$$

where

$$I_1 = \int_0^\infty \frac{\sin c\xi}{c\xi}\;\frac{\xi^2+\frac{p^2}{2b^2} - \sqrt{\xi^2+\frac{p^2}{a^2}}\;\sqrt{\xi^2+\frac{p^2}{b^2}}}{D(\xi,p)}\;d\xi$$

$$I_2 = \int_0^\infty \cos c\xi\;\frac{\xi^2+\frac{p^2}{2b^2} - \sqrt{\xi^2+\frac{p^2}{a^2}}\;\sqrt{\xi^2+\frac{p^2}{b^2}}}{D(\xi,p)}\;d\xi \tag{24}$$

By equating $\sin c\xi$ to $\operatorname{Im} e^{ic\xi}$ and $\cos c\xi$ to $\operatorname{Re} e^{ic\xi}$, by noting all other factors other than the exponential in the integrands of I_1 and I_2 are real, and by interchanging Im and Re with the integration, I_1 and I_2 can be rewritten.

$$I_1 = \text{Im}\{\int_0^\infty \frac{\xi^2+\frac{p^2}{2b^2}-\sqrt{\xi^2+\frac{p^2}{a^2}}\sqrt{\xi^2+\frac{p^2}{b^2}}}{c\xi\, D(\xi,p)}\, e^{ic\xi}\, d\xi\ \}$$

$$I_2 = \text{Re}\ \{\int_0^\infty \frac{\xi^2+\frac{p^2}{2b^2}-\sqrt{\xi^2+\frac{p^2}{a^2}}\sqrt{\xi^2+\frac{p^2}{b^2}}}{D(\xi,p)}\, e^{ic\xi}\, d\xi\ \} \tag{25}$$

By putting $ic\xi = -p\tau$ in equations (25) I_1 and I_2 can be rewritten in a convenient form.

$$I_1 = \frac{b^3}{c^2p^2}\ \text{Im}\ \{\int_0^{-i\infty} g_3(\frac{b\tau}{c})\ e^{-p\tau}\ d\tau\}$$

$$I_2 = -\frac{b^3}{c^2p}\ \text{Im}\ \{\int_0^{-i\infty} \tau\ g_3(\frac{b\tau}{c})\ e^{-p\tau}\ d\tau\} \tag{26}$$

where

$$g_3(T) = \frac{3[T^2-\frac{1}{2}-\sqrt{T^2-1/3}\,\sqrt{T^2-1}\,][(T^2-\frac{1}{2})^2+T^2\sqrt{T^2-1/3}\,\sqrt{T^2-1}\ ]}{2T(T^2-\frac{1}{4})(T^2-\frac{3-\sqrt{3}}{4})(T^2-\frac{3+\sqrt{3}}{4})} \tag{27}$$

and the integration in the complex τ-plane is down the negative imaginary axis. By using the fourth quadrant contour described in figure 2, the integrals down the imaginary axis can be written as a Laplace transformation plus residue terms.

$$I_1 = \frac{b^3}{c^2p^2}\ \text{Im}\ \{L\{g_3(\frac{bt}{c})\} + (\frac{\pi i}{2}\underset{\tau=0}{\text{Res}} + \pi i \underset{\tau=\frac{c}{v}}{\text{Res}})\, g_3(\frac{b\tau}{c})e^{-p\tau}\ \}$$

$$I_2 = -\frac{b^3}{c^2p}\ \text{Im}\ \{\ L\{t\ g_3(\frac{bt}{c})\} + \pi i \underset{\tau=\frac{c}{v}}{\text{Res}}\ \tau\ g_3(\frac{b\tau}{c})e^{-p\tau}\ \} \tag{28}$$

where

$$\operatorname*{Res}_{\tau=0} g_3(\tfrac{b\tau}{c})\ e^{-p\tau} = \frac{2(3-2\sqrt{3})}{3}\ \frac{c}{b}$$

$$\operatorname*{Res}_{\tau=\frac{c}{v}} g\ (\tfrac{b\tau}{c})\ e^{-p\tau} = \gamma\ \frac{c}{b}\ e^{-\frac{c}{v}p} \qquad (29)$$

$$\operatorname*{Res}_{\tau=\frac{c}{v}} \tau\ g_3(\tfrac{b\tau}{c})\ e^{-p\tau} = \gamma\ \frac{c^2}{bv}\ e^{-\frac{c}{v}p}$$

$$\gamma = \frac{3[x^2-\tfrac{1}{2} - \sqrt{x^2-1/3}\ \sqrt{x^2-1}\,][(x^2-\tfrac{1}{2})^2 + x^2\ \sqrt{x^2-1/3}\ \sqrt{x^2-1}\,]}{4\ x^2\ (x^2-\tfrac{1}{2})(x^2-\ \frac{3-\sqrt{3}}{4}\ )}$$

$$x = \frac{b}{v}$$

Substitution from equations (28) and (29) into equation (23) and the interchange of Im and L gives

$$\bar{u}_2(0,0,p) = \frac{b^3}{\pi\mu c^2}\left\langle \frac{\bar{\sigma}(p)}{p^2}\left[L\{\text{Im } g_3(\tfrac{bt}{c})\} + \frac{\pi(3-2\sqrt{3})}{3}\ \frac{c}{b} + \right.\right.$$
$$\left.\left. + \pi\ \gamma\ \frac{c}{b}\ e^{-\frac{c}{v}p}\right] + \frac{\bar{\sigma}(p)}{p}\left[L\{\text{Im } t\ g_3(\tfrac{bt}{c})\} + \pi\ \gamma\ \frac{c^2}{bv}\ e^{-\frac{c}{v}p}\right]\right\rangle \qquad (30)$$

Application of the operational formula $L\{\int_0^t\int_0^\tau \sigma(\eta)d\eta\ d\tau\} = \frac{\sigma(p)}{p^2}$ and the Convolution Theorem yields the final solution, $u_2(0,0,t)$, of the original boundary-value problem stated by equations (1) and (2) with $f = 0$. This solution in the notation $u_2(t) = u_2(0,0,t)$ is written in equation (31).

$$u_2(t) = \frac{b^2}{\pi\mu c}\ \ \{ - \frac{\pi(2\sqrt{3}\ -3}{3}\ G\ (t) + \pi\ \gamma\ G\ (t-\ \tfrac{c}{v}) +$$
$$+ \frac{\pi\gamma c}{v}\ \dot{G}\ (t-\ \tfrac{c}{v}) + \frac{b}{c}\int_0^t G\ (t-\tau)\ \text{Im } g_3(\tfrac{b\tau}{c})\ d\tau\ + \qquad (31)$$
$$+ \frac{b}{c}\int_0^t \dot{G}\ (t-\tau)\ \tau\ \text{Im } g_3(\tfrac{b\tau}{c})\ d\tau\ \}$$

where

$$G(t) = \begin{cases} 0 \ , & t \leq 0 \\ \int_0^t \int_0^\tau \sigma(\eta)\, d\eta \, d\tau \ , & t > 0 \end{cases} \tag{32}$$

$$\text{Im } g_3(T) = \frac{1}{2T^2} \text{ Im } g_2(T)$$

Therefore Im $g_3(\frac{bt}{c})$ is zero for $0 < t < \frac{c}{a}$, negative for $\frac{c}{a} < t < \frac{c}{\sqrt{2}\, b}$, positive for $\frac{c}{\sqrt{2}\, b} < t < \frac{c}{b}$, and zero for $t > \frac{c}{b}$. The integrals in equation (31) are proper integrals.

For problem (2) ($\sigma \neq 0$ and $f = 0$), the rotation solution evaluated on $y = 0$ is

$$\bar{\bar{\Theta}}_2(\xi,0,p) = \frac{p^2}{2b^2} \bar{\bar{\psi}}_2(\xi,0,p) = \frac{p^2\, \bar{\sigma}(p) \sqrt{\xi^2 + \frac{p^2}{a^2}}}{2\mu b^2 c\xi \ D(\xi,p)} \left[\text{Im } e^{ic\xi} - c\xi \text{ Re } e^{ic\xi}\right] \tag{33}$$

Inversion of the Fourier tranformation using the last of equation (5) with ϕ replaced by ψ, with $x = 0$ and with Re and Im interchanged with the integration yields

$$\bar{\Theta}_2(0,0,p) = \frac{p^2 \bar{\sigma}(p)}{2\pi\mu b^2} (I_3 - I_4) \tag{34}$$

where

$$\begin{aligned} I_3 &= \text{Im } \left\{ \int_0^\infty \frac{\sqrt{\xi^2 + \frac{p^2}{a^2}}}{c\xi \ D(\xi,p)} \ e^{ic\xi} \ d\xi \right\} \\ I_4 &= \text{Re } \left\{ \int_0^\infty \frac{\sqrt{\xi^2 + \frac{p^2}{a^2}}}{D(\xi,p)} \ e^{ic\xi} \ d\xi \right\} \end{aligned} \tag{35}$$

By putting $ic\xi = -p\tau$ in equations (35), I_3 and I_4 can be rewritten in a convenient form.

$$I_3 = \frac{b^4}{c^2p^3} \text{ Im } \{\int_0^{-i\infty} g_4(\frac{b\tau}{c}) \; e^{-p\tau} \; d\tau\}$$

$$I_4 = -\frac{b^4}{c^2p^2} \text{ Im } \{\int_0^{-i\infty} \tau \; g_4(\frac{b\tau}{c}) \; e^{-p\tau} \; d\tau\} \tag{36}$$

where

$$g_4(T) = -\frac{3\sqrt{1/3 - T^2} \; [(\tfrac{1}{2}-T^2)^2 - T^2\sqrt{1/3-T^2}\sqrt{1-T^2}\;]}{2\;T(T^2-\tfrac{1}{4})(T^2 - \frac{3-\sqrt{3}}{4})(T^2 - \frac{3+\sqrt{3}}{4})} \tag{37}$$

and the integration in the complex τ-plane is down the negative imaginary axis. By using the fourth quadrant contour described in figure 2 and the technique illustrated earlier, the integrals down the imaginary axis can be written as a Laplace transformation plus residue terms.

$$I_3 = \frac{b^4}{c^2p^3} \text{ Im } \{L\{g_4(\frac{bt}{c}) + (\frac{\pi i}{2} \operatorname*{Res}_{\tau=0} + \pi i \operatorname*{Res}_{\tau=\frac{c}{v}}) \; g_4(\frac{b\tau}{c}) \; e^{-p\tau}\;\}$$

$$I_4 = -\frac{b^4}{c^2p^2} \text{ Im } \{L\{t \; g_4(\frac{bt}{c})\} + \pi i \operatorname*{Res}_{\tau=\frac{c}{v}} \; \tau \; g_4(\frac{b\tau}{c}) \; e^{-p\tau}\;\} \tag{38}$$

where

$$\operatorname*{Res}_{\tau=0} g_4(\frac{b\tau}{c}) \; e^{-p\tau} = \frac{4c}{\sqrt{3}\;b} \tag{39}$$

and the other two residues are imaginary and therefore do not contribute to the value of I_3 and I_4. Equations (38) simplify to

$$
\begin{aligned}
I_3 &= \frac{b^4}{c^2 p^3}\, L\{\,\mathrm{Im}\; g_4(\tfrac{bt}{c})\} + \frac{2\pi b^3}{\sqrt{3}\; c p^3} \\
I_4 &= -\frac{b^4}{c^2 p^2}\, L\{t\; \mathrm{Im}\; g_4(\tfrac{bt}{c})\}
\end{aligned}
\tag{40}
$$

and substitution into equation (34) gives

$$
\begin{aligned}
\overline{\Theta}_2(0,0,p) = {} & \frac{b^2}{2\pi\mu c^2}\, \frac{\overline{\sigma}(p)}{p}\, L\{\mathrm{Im}\; g_4(\tfrac{bt}{c})\} + \\
& + \frac{b}{\sqrt{3}\;\mu c}\, \frac{\overline{\sigma}(p)}{p} + \frac{b^2}{2\pi\mu c^2}\, \overline{\sigma}(p)\;\; L\{t\; \mathrm{Im}\; g_4(\tfrac{bt}{c})\}
\end{aligned}
\tag{41}
$$

Application of the operational formula $L\{\int_0^t \sigma(\eta)\,d\eta\} = \frac{\overline{\sigma}(p)}{p}$ and the Convolution Theorem yields the solution, $\Theta_2(0,0,t)$. This solution in the notation $\Theta_2(t) = \Theta_2(0,0,t)$ is written in equation (42)

$$
\begin{aligned}
\Theta_2(t) = {} & \frac{b}{\sqrt{3}\;\mu c}\, \dot{G}\,(t) + \frac{b^2}{2\pi\mu c^2} \Bigg[\int_0^t \dot{G}\,(t-\tau)\; \mathrm{Im}\; g_4(\tfrac{b\tau}{c})\, d\tau\; + \\
& + \int_0^t \sigma(t-\tau)\;\tau\; \mathrm{Im}\; g_4(\tfrac{b\tau}{c})\, d\tau \Bigg]
\end{aligned}
\tag{42}
$$

where

$$
\dot{G}\,(t) = \int_0^t \sigma(\eta)\, d\eta
$$

$$
\mathrm{Im}\; g_4(T) = \begin{cases}
0\;, & 0 < T < \frac{1}{\sqrt{3}} \\[2ex]
-\dfrac{3\sqrt{T^2-1/3}\;\;(\frac{1}{2}-T^2)^2}{2T(T^2-\frac{1}{4})(T^2-\frac{3-\sqrt{3}}{4})(T^2-\frac{3+\sqrt{3}}{4})}\;, & \frac{1}{\sqrt{3}} < T < 1 \\[2ex]
-\dfrac{3\sqrt{T^2-1/3}\;[(\frac{1}{2}-T^2)^2 + T^2\sqrt{T^2-1/3}\;\;\sqrt{T^2-1}\;]}{2T(T^2-\frac{1}{4})(T^2-\frac{3-\sqrt{3}}{4})(T^2-\frac{3+\sqrt{3}}{4})}\;, & T > 1
\end{cases}
\tag{43}
$$

Therefore Im $g_4(\frac{bt}{c})$ is zero for $0 < t < \frac{c}{a}$, positive for $\frac{c}{a} < t < \frac{c}{b}$, positive for $\frac{c}{b} < t < \frac{c}{v}$, and negative for $t > \frac{c}{v}$. The integrals in equation (42) are proper integrals for $0 < t < \frac{c}{v}$. At $\tau = \frac{c}{v}$ Im $g_4(\frac{b\tau}{c})$ has a singular point of order $(\tau - \frac{c}{v})^{-1}$. The rotation at $t = \frac{c}{v}$, however, is finite since the indeterminant forms in the integrand, $\lim_{\tau\to\frac{c}{v}} \dot{G}\ (\frac{c}{v} - \tau)(\tau - \frac{c}{v})^{-1}$ and $\lim_{\tau\to\frac{c}{v}} \sigma(\frac{c}{v} - \tau)(\tau - \frac{c}{v})^{-1}$, are finite. For $t > \frac{c}{v}$, because of the singular point at $\tau = \frac{c}{v}$, to be meaningful the integrals in equation (42) must be considered as their Cauchy principal values. In which case $\Theta_2(t)$ is a well defined finite value for all nonnegative t.

THE INERTIAL HORIZONTAL FORCE AND MOMENT OF THE STRUCTURE

The multistoried structure is modeled as a lumped system with masses m_i located at points distances l_i above the base point of the structure on the surface and with a base mass m_o. The horizontal force exerted at the base point is F(t) and the moment exerted about the base point is M(t). (cf.fig.3) From normal mode theory the following equations have been derived. (5)

$$\begin{aligned} \sum_{k=1}^{n} M_k \ddot{\zeta}_k(t) - \sum_{k=1}^{n} M_k^1 \ddot{\phi}_k(t) &= F(t) \\ \sum_{k=1}^{n} M_k^1 \ddot{\zeta}_k(t) - \sum_{k=1}^{n} N_k \ddot{\phi}_k(t) &= M(t) \end{aligned} \tag{44}$$

where

$$\begin{aligned} M_k &= \frac{\left(\sum_{k=1}^{n} m_i x_{ik}\right)^2}{\sum_{i=1}^{n} m_i x_{ik}^2} \\ M_k^1 &= \frac{\left(\sum_{i=1}^{n} m_i x_{ik}\right)\left(\sum_{i=1}^{n} m_i l_i x_{ik}\right)}{\sum_{i=1}^{n} m_i x_{ik}^2} \\ N_k &= \frac{\left(\sum_{i=1}^{n} m_i l_i x_{ik}\right)^2}{\sum_{i=1}^{n} m_i x_{ik}^2} \; . \end{aligned} \tag{45}$$

x_{ik} the i^{th} component of k^{th} eigenvector of $(m_j a_{ij})$

$A = (a_{ij})$ the flexibility matrix

$$\ddot{\zeta}_k = w_k \int_o^t \ddot{U}(\tau) \sin w_k (t-\tau)\, d\tau$$

$$\ddot{\phi}_k = w_k \int_o^t \ddot{\Phi}(\tau) \sin w_k (t-\tau)\, d\tau$$

w_k^2 the reciprocal of the k^{th} eigenvalue of $(m_j\ a_{ij})$

$U(t)$ is the displacement of the base point

$\Phi(t)$ is the rotation of the base point ($\Phi > 0$ counterclockwise)

In this work $F(t)$ is equated to the sum of the x-components of the force due to the prescribed boundary stresses over the entire boundary given by equations (1) and (2). Likewise $M(t)$ is equated to the sum of the moments about the origin due to the same boundary stresses over the entire boundary. In computing $F(t)$ the second set of stresses make no contribution and in computing $M(t)$ the first set of stresses make no contribution. The integration from minus infinity to plus infinity simplifies in both cases to an integration from $-c$ to c.

$$F(t) = A\, f(t)$$

$$M(t) = \frac{cA}{3}\, \sigma(t) \qquad (46)$$

where A is the area underneath the structure.

THE COUPLING OF THE LAMB PROBLEM WITH THE STRUCTURE

In equations (44) let

$$\begin{aligned} U(t) &= u_1(t) + u_2(t) + U_p(t) \\ \Phi(t) &= \Theta_1(t) + \Theta_2(t) + \Phi_p(t) \\ F(t) &= A\, f(t) \\ M(t) &= \frac{cA}{3}\, \sigma(t) \end{aligned} \tag{47}$$

where $u_1(t)$ is given by equation (9); $u_2(t)$ is given by equation (31); $\Theta_1(t)$ is given by equation (19); $\Theta_2(t)$ is given by equation (42); and $U_p(t)$ and $\Phi_p(t)$ are particular solutions for the horizontal displacement and rotation at the origin corresponding to the earthquake input into the system.

$$\begin{aligned} u_1(t) &= -\frac{b}{\mu} \int_0^t f(\tau)\, d\tau - \frac{b^2}{2\pi c\mu} \int_0^t \int_0^{t-\tau} f(\xi)\, \mathrm{Im}\, g_1\!\left(\frac{b\tau}{c}\right) d\xi\, d\tau \\ u_2(t) &= \frac{b^2}{\pi\mu c} \Big\{ -\frac{\pi(2\sqrt{3}-3)}{3}\, G(t) + \pi\gamma\, G\!\left(t-\frac{c}{v}\right) + \\ &\quad + \frac{\pi\gamma c}{v}\, \dot{G}\!\left(t-\frac{c}{v}\right) + \frac{b}{c} \int_0^t G(t-\tau)\, \mathrm{Im}\, g_3\!\left(\frac{b\tau}{c}\right) d\tau \\ &\quad + \frac{b}{c} \int_0^t \dot{G}(t-\tau)\, \tau\, \mathrm{Im}\, g_3\!\left(\frac{b\tau}{c}\right) d\tau \Big\} \end{aligned} \tag{48}$$

$$\Theta_1(t) = \frac{1}{2\mu} f(t) + \frac{\beta}{2\mu} f\!\left(t-\frac{c}{v}\right) + \frac{b^3}{\pi\mu c^3} \int_0^t f(t-\tau)\, \tau^2\, \mathrm{Im}\, g_3\!\left(\frac{b\tau}{c}\right) d\tau$$

$$\begin{aligned} \Theta_2(t) &= \frac{b}{\sqrt{3}\,\mu c}\, \dot{G}(t) + \frac{b^2}{2\pi\mu c^2} \Big[\int_0^t \dot{G}(t-\tau)\, \mathrm{Im}\, g_4\!\left(\frac{b\tau}{c}\right) d\tau + \\ &\quad + \int_0^t \sigma(t-\tau)\, \tau\, \mathrm{Im}\, g_4\!\left(\frac{b\tau}{c}\right) d\tau \Big] \end{aligned}$$

The sum $u_1(t) + u_2(t)$ represents the horizontal displacement of the base of a structure which exerts on the surface of the half-space the superposed shear and normal stress distributions given by equations (2). $U_p(t)$ represents the horizontal displacement of the base of a structure due to the excitation of an earthquake. Analytically, $U_p(t)$ is the evaluation at the origin of the x-component of the solution of the initial-value boundary-value problem consisting of the Navier equations (1) valid for $y > 0$, the boundary conditions implied by $\sigma_x = \sigma_y = \tau_{xy} = 0$ on $y = 0$ and by boundedness at infinity, and arbitrary initial conditions. $\Phi_p(t)$ is the evaluation at the origin of half the component perpendicular the xy plane of the curl of the vector solution of this initial-value boundary-value problem.

After substitution of $F(t) = A\, f(t)$ and $M(t) = \frac{cA}{3}\,\sigma(t)$ equations (44) are solved for $f(t)$ and $\sigma(t)$ and the result substituted into equations (47) solved for $u_1(t) + u_2(t)$ and $\Theta_1(t) + \Theta_2(t)$ respectively. These equations for $u_1(t) + u_2(t) = U(t) - U_p(t)$ and $\Theta_1(t) + \Theta_2(t) = \Phi(t) - \Phi_p(t)$ are differentiated twice to give equations (49).

$$\begin{aligned}\ddot{U}(t) = &- \frac{b}{\mu}\,\dot{f}(t) - \frac{b^2}{2\mu c\pi}\int_0^t \dot{f}(t-\tau)\ \mathrm{Im}\ g_1\!\left(\frac{b\tau}{c}\right)d\tau - \left(\frac{2\sqrt{3}-3}{3}\right)\frac{b^2}{\mu c}\,\sigma(t)\\ &+ \frac{\gamma b^2}{\mu c}\,\sigma\!\left(t-\frac{c}{v}\right) + \frac{\gamma b^2}{\mu v}\,\dot{\sigma}\!\left(t-\frac{c}{v}\right) + \frac{b^3}{\pi\mu c^2}\int_0^t \sigma(t-\tau)\ \mathrm{Im}\ g_3\!\left(\frac{b\tau}{c}\right)d\tau\\ &+ \frac{b^3}{\pi\mu c^2}\int_0^t \dot{\sigma}(t-\tau)\ \tau\ \mathrm{Im}\ g_3\!\left(\frac{b\tau}{c}\right)d\tau + \ddot{U}_p(t)\end{aligned}$$

$$\ddot{\Phi}(t) = \frac{1}{2\mu}\,\ddot{f}(t) + \frac{\beta}{2\mu}\,\ddot{f}\!\left(t-\frac{c}{v}\right) + \frac{b^3}{\pi\mu c^3}\int_0^t \ddot{f}(t-\tau)\ \tau^2\ \mathrm{Im}\ g_3\!\left(\frac{b\tau}{c}\right)d\tau$$

$$+\frac{b}{\sqrt{3}\;\mu c}\,\dot{\sigma}(t) + \frac{b^2}{2\pi\mu c^2}\int_0^t \dot{\sigma}(t-\tau)\ \mathrm{Im}\ g_4(\frac{b\tau}{c})\ d\tau \qquad (49)$$

$$+\ \frac{b^2}{2\pi\mu c^2}\int_0^t \ddot{\sigma}(t-\tau)\ \tau\ \mathrm{Im}\ g_4(\frac{b\tau}{c})\ d\tau + \ddot{\Phi}_p(t)$$

where

$$f(t) = \Sigma\,\frac{M_k\ w_k}{A}\int_0^t \ddot{U}(\tau)\ \sin\ w_k(t-\tau)\ d\tau - \Sigma\,\frac{M_k^1\ w_k}{A}\int_0^t \ddot{\Phi}(\tau)\ \sin\ w_k(t-\tau)\ d\tau$$

$$\dot{f}(t) = \Sigma\,\frac{M_k\ w_k^2}{A}\int_0^t \ddot{U}(\tau)\ \cos\ w_k(t-\tau)\ d\tau - \Sigma\,\frac{M_k^1\ w_k^2}{A}\int_0^t \ddot{\Phi}(\tau)\ \cos\ w_k(t-\tau)\ d\tau$$

$$\ddot{f}(t) = -\ \Sigma\,\frac{M_k\ w_k^3}{A}\int_0^t \ddot{U}(\tau)\ \sin\ w_k(t-\tau)\ d\tau + (\Sigma\,\frac{M_k\ w_k^2}{A})\ \ddot{U}(t) \qquad (50)$$

$$+\ \Sigma\,\frac{M_k^1\ w_k^3}{A}\int_0^t \ddot{\Phi}(\tau)\ \sin\ w_k(t-\tau)\ d\tau - (\Sigma\,\frac{M_k^1\ w_k^2}{A})\ \ddot{\Phi}(t)$$

and $\sigma(t)$, $\dot{\sigma}(t)$, & $\ddot{\sigma}(t)$ are like $f(t)$, $\dot{f}(t)$, & $\ddot{f}(t)$ except with M_k replaced by $\frac{3M_k^1}{c}$ and M_k^1 replaced by $\frac{3N_k}{c}$.

The $\ddot{U}$ equation is obtained by substitution of $\dot{f}$, σ, & $\dot{\sigma}$ into the first of equations (49)

$$\ddot{U}(t) = -\ \frac{b}{\mu A}\ \Sigma\ M_k\ w_k^2\int_0^t \ddot{U}(\tau)\ \cos\ w_k(t-\tau)\ d\tau$$

$$-\ \frac{(2\sqrt{3}\ -3)b^2}{\mu A c^2}\ \Sigma\ M_k^1\ w_k\int_0^t \ddot{U}(\tau)\ \sin\ w_k(t-\tau)\ d\tau$$

$$+\ \frac{3\gamma b^2}{\mu A c^2}\ \Sigma\ M_k^1\ w_k\int_0^{t-\frac{c}{v}} \ddot{U}(\tau)\ \sin\ w_k\ (t-\frac{c}{v}-\tau)\ d\tau$$

$$+ \frac{3\gamma b^2}{\mu A c v} \Sigma M_k^1 w_k^2 \int_0^{t-\frac{c}{v}} \ddot{U}(\tau) \cos w_k(t-\frac{c}{v}-\tau)\, d\tau$$

$$- \frac{b^2}{2\pi\mu A c} \Sigma M_k w_k^2 \int_0^t \int_0^{t-\tau} \ddot{U}(\xi) \cos w_k(t-\tau-\xi)\, d\xi \ \mathrm{Im}\ g_1(\frac{b\tau}{c})\, d\tau$$

$$+ \frac{3b^3}{\pi\mu A c^3} \Sigma M_k^1 w_k \int_0^t \int_0^{t-\tau} \ddot{U}(\xi) \sin w_k(t-\tau-\xi)\, d\xi \ \mathrm{Im}\ g_3(\frac{b\tau}{c})\, d\tau$$

$$+ \frac{3b^3}{\pi\mu A c^3} \Sigma M_k^1 w_k^2 \int_0^t \int_0^{t-\tau} \ddot{U}(\tau) \cos w_k(t-\tau-\xi)\, d\xi \ \tau \ \mathrm{Im}\ g_3(\frac{b\tau}{c})\, d\tau$$

$$+ \frac{b}{\mu A} \Sigma M_k^1 w_k^2 \int_0^t \ddot{\Phi}(\tau) \cos w_k(t-\tau) d\tau \qquad (51)$$

$$+ \frac{(2\sqrt{3}-3)b^2}{\mu A c^2} \Sigma N_k w_k \int_0^t \ddot{\Phi}(\tau) \sin w_k(t-\tau)\, d\tau$$

$$- \frac{3\gamma b^2}{\mu A c^2} \Sigma N_k w_k \int_0^{t-\frac{c}{v}} \ddot{\Phi}(\tau) \sin w_k(t-\frac{c}{v}-\tau)\, d\tau$$

$$- \frac{3\gamma b^2}{\mu A c v} \Sigma N_k w_k^2 \int_0^{t-\frac{c}{v}} \ddot{\Phi}(\tau) \cos w_k(t-\frac{c}{v}-\tau)\, d\tau$$

$$+ \frac{b^2}{2\pi\mu A c} \Sigma M_k^1 w_k^2 \int_0^t \int_0^{t-\tau} \ddot{\Phi}(\xi) \cos w_k(t-\tau-\xi)\, d\xi \ \ \mathrm{Im}\ g_1(\frac{b\tau}{c})\, d\tau$$

$$- \frac{3b^3}{\pi\mu A c^3} \Sigma N_k \cdot w_k \int_0^t \int_0^{t-\tau} \ddot{\Phi}(\xi) \sin w_k(t-\tau-\xi)\, d\xi \ \mathrm{Im}\ g_3(\frac{b\tau}{c})\, d\tau$$

$$- \frac{3b^3}{\pi\mu A c^3} \Sigma N_k w_k^2 \int_0^t \int_0^{t-\tau} \ddot{\Phi}(\xi) \cos w_k(t-\tau-\xi)\, d\xi \ \tau \ \mathrm{Im}\ g_3(\frac{b\tau}{c})\, d\tau$$

$$+ \ddot{U}_p(t)$$

The $\ddot{\Phi}$ equation is obtained by substitution of $\ddot{f}$, $\dot{\sigma}$, & $\ddot{\sigma}$ into the second of equations (49).

$$\begin{aligned}
&[1 + \frac{1}{2\mu A} \quad (\Sigma\, M_k^1\, w_k^2)]\ \ddot{\Phi}(t) + \frac{\beta}{2\mu A} (\Sigma\, M_k^1\, w_k^2)\ \ddot{\Phi}(t- \frac{c}{v}) \\
&= \frac{1}{2\mu A} (\Sigma\, M_k\, w_k^2)\ \ddot{U}(t) + \frac{\beta}{2\mu A} (\Sigma\, M_k\, w_k^2)\ \ddot{U}(t- \frac{c}{v}) \\
&- \frac{1}{2\mu A}\ \Sigma\, M_k\, w_k^3 \int_0^t \ddot{U}(\tau) \sin w_k(t-\tau)\, d\tau \\
&- \frac{\beta}{2\mu A}\ \Sigma\, M_k\, w_k^3 \int_0^{t-\frac{c}{v}} \ddot{U}(\tau) \sin w_k\ (t- \frac{c}{v} -\tau)\, d\tau \\
&+ \frac{\sqrt{3}\, b}{\mu A c^2}\ \Sigma\, M_k^1\, w_k^2 \int_0^t \ddot{U}(\tau) \cos w_k(t-\tau)\, d\tau \\
&+ \frac{b^3}{\pi\mu A c^3}\ \Sigma\, M_k\, w_k^2 \int_0^t \ddot{U}(t-\tau)\tau^2\ \mathrm{Im}\ g_3(\frac{b\tau}{c})\, d\tau \\
&+ \frac{3b^2}{2\pi\mu A c^3}\ \Sigma\, M_k^1\, w_k^2 \int_0^t \ddot{U}(t-\tau)\ \tau\ \mathrm{Im}\ g_4(\frac{b\tau}{c})\, d\tau \qquad (52) \\
&- \frac{b^3}{\pi\mu A c^3}\ \Sigma\, M_k\, w_k^3 \int_0^t\int_0^{t-\tau} \ddot{U}(\xi) \sin w_k(t-\tau-\xi)\, d\xi\ \tau^2\ \mathrm{Im}\ g_3(\frac{b\tau}{c})\, d\tau \\
&+ \frac{3b^2}{2\pi\mu A c^3}\ \Sigma\, M_k^1\, w_k^2 \int_0^t\int_0^{t-\tau} \ddot{U}(\xi) \cos w_k(t-\tau-\xi)\, d\xi\ \ \mathrm{Im}\ g_4(\frac{b\tau}{c})\, d\tau \\
&- \frac{3b^2}{2\pi\mu A c^3}\ \Sigma\, M_k^1\, w_k^3 \int_0^t\int_0^{t-\tau} \ddot{U}(\xi) \sin w_k(t-\tau-\xi)\, d\xi\ \tau\ \mathrm{Im}\ g_4(\frac{b\tau}{c})\, d\tau \\
&+ \frac{1}{2\mu A}\ \Sigma\, M_k^1\, w_k^3 \int_0^t \ddot{\Phi}(\tau) \sin w_k(t-\tau)\, d\tau \\
&+ \frac{\beta}{2\mu A}\ \Sigma\, M_k^1\, w_k^3 \int_0^{t-\frac{c}{v}} \ddot{\Phi}(\tau) \sin w_k(t- \frac{c}{v} -\tau)\, d\tau
\end{aligned}$$

$$- \frac{\sqrt{3}\, b}{\mu A c^2} \; \Sigma \, N_k \, w_k^2 \int_0^t \ddot{\Phi}(\tau) \cos w_k(t-\tau) \, d\tau$$

$$- \frac{b^3}{\pi \mu A c^3} \; \Sigma \, M_k^1 \, w_k^2 \int_0^t \ddot{\Phi}(t-\tau) \, \tau^2 \, \mathrm{Im} \, g_3(\frac{b\tau}{c}) \, d\tau$$

$$- \frac{3b^2}{2\pi \mu A c^3} \; \Sigma \, N_k \, w_k^2 \int_0^t \ddot{\Phi}(t-\tau) \, \tau \, \mathrm{Im} \, g_4(\frac{b\tau}{c}) \, d\tau$$

$$+ \frac{b^3}{\pi \mu A c^3} \; \Sigma \, M_k^1 \, w_k^3 \int_0^t \int_0^{t-\tau} \ddot{\Phi}(\xi) \sin w_k(t-\tau-\xi) \, d\xi \; \tau^2 \, \mathrm{Im} \, g_3(\frac{b\tau}{c}) \, d\tau$$

$$- \frac{3b^2}{2\pi \mu A c^3} \; \Sigma \, N_k \, w_k^2 \int_0^t \int_0^{t-\tau} \ddot{\Phi}(\xi) \cos w_k(t-\tau-\xi) \, d\xi \; \mathrm{Im} \, g_4(\frac{b\tau}{c}) \, d\tau$$

$$+ \frac{3b^2}{2\pi \mu A c^3} \; \Sigma \, N_k \, w_k^3 \int_0^t \int_0^{t-\tau} \ddot{\Phi}(\xi) \sin w_k(t-\tau-\xi) \, d\xi \; \tau \, \mathrm{Im} \, g_4(\frac{b\tau}{c}) \, d\tau$$

$$+ \ddot{\Phi}_p(t)$$

If the heavy base is taken into account, equations (44) are modified to

$$\sum_{k=1}^{n} M_k \, \ddot{\zeta}_k(t) - \sum_{k=1}^{n} M_k^1 \, \ddot{\phi}_k(t) + m_o \, \ddot{U}(t) = F(t)$$

$$\sum_{k=1}^{n} M_k^1 \, \ddot{\zeta}_k(t) - \sum_{k=1}^{n} N_k \, \ddot{\phi}_k(t) - I_o \, \ddot{\Phi}(t) = M(t) \qquad (53)$$

Substitution from equations (48) and manipulation similar to that illustrated in obtaining equations (51) and (52) yields the following pair of coupled Volterra equations for $\ddot{U}(t)$ and $\ddot{\Phi}(t)$ in the case where base-mass effects are included.

$$
\begin{aligned}
\ddot{U}(t) = & - \frac{3b\gamma I_o}{vcm_o}\,\ddot{\Phi}(t-\tfrac{c}{v}) - \frac{3b^2 I_o}{\pi c^3 m_o}\int_o^t \ddot{\Phi}(t-\tau)\,\tau\,\mathrm{Im}\,g_3(\tfrac{b\tau}{c})\,d\tau \\
& - \frac{3b\gamma I_o}{c^2 m_o}\int_o^{t-\frac{c}{v}} \ddot{\Phi}(\tau)\,d\tau + \frac{\sqrt{3}\,(2-\sqrt{3}\,)\,bI_o}{c^2 m_o}\int_o^t \ddot{\Phi}(\tau)\,d\tau \\
& + \sum_{k=1}^{n} \frac{M_k^1\,w_k}{m_o}\int_o^t \ddot{\Phi}(\tau)\,\sin w_k(t-\tau)\,d\tau \\
& - \frac{3b\gamma}{cvm_o}\sum_{k=1}^{n} N_k\,w_k\int_o^{t-\frac{c}{v}} \ddot{\Phi}(\tau)\,\sin w_k(t-\tfrac{c}{v}-\tau)\,d\tau \\
& + \frac{b}{2\pi cm_o}\sum_{k=1}^{n} M_k^1\,w_k\int_o^t\int_o^{t-\tau} \ddot{\Phi}(\xi)\,\sin w_k(t-\tau-\xi)\,d\xi\;\mathrm{Im}\,g_1(\tfrac{b\tau}{c})\,d\tau \\
& + \frac{\sqrt{3}\,(2-\sqrt{3}\,)\,b}{c^2 m_o}\sum_{k=1}^{n} N_k\,w_k\int_o^t\int_o^{\tau} \ddot{\Phi}(\xi)\,\sin w_k(\tau-\xi)\,d\xi d\tau \\
& - \frac{3b\gamma}{c^2 m_o}\sum_{k=1}^{n} N_k\,w_k\int_o^{t-\frac{c}{v}}\int_o^{\tau} \ddot{\Phi}(\xi)\,\sin w_k(\tau-\xi)\,d\xi d\tau \qquad (54) \\
& - \frac{3b^2 I_o}{\pi c^3 m_o}\int_o^t\int_o^{t-\tau} \ddot{\Phi}(\xi)\,d\xi\;\mathrm{Im}\,g_3(\tfrac{b\tau}{c})\,d\tau \\
& - \frac{3b^2}{\pi c^3 m_o}\sum_{k=1}^{n} N_k\,w_k\int_o^t\int_o^{t-\tau} \ddot{\Phi}(\xi)\,\sin w_k(t-\tau-\xi)\,d\xi\;\tau\;\mathrm{Im}\,g_3(\tfrac{b\tau}{c})\,d\tau \\
& - \frac{3b^2}{\pi c^3 m_o}\sum_{k=1}^{n} N_k\,w_k\int_o^t\int_o^{t-\tau}\int_o^{\xi} \ddot{\Phi}(\eta)\,\sin w_k(\xi-\eta)\,d\eta d\xi\;\mathrm{Im}\,g_3(\tfrac{b\tau}{c})\,d\tau \\
& - \frac{\mu A}{bm_o}\int_o^t \ddot{U}(\tau)\,d\tau - \sum_{k=1}^{n} \frac{M_k\,w_k}{m_o}\int_o^t \ddot{U}(\tau)\,\sin w_k(t-\tau)\,d\tau
\end{aligned}
$$

$$- \frac{b}{2\pi c} \int_0^t \ddot{U}(t-\tau)\ \mathrm{Im}\ g_1(\tfrac{b\tau}{c})\ d\tau + \frac{3b\gamma}{cvm_o} \sum_{k=1}^{n} M_k^1\ w_k \int_0^{t-\frac{c}{v}} \ddot{U}(\tau)\ \sin w_k(t-\tfrac{c}{v}-\tau)\ d\tau$$

$$- \frac{b}{2\pi c m_o} \sum_{k=1}^{n} M_k\ w_k \int_0^t \int_0^{t-\tau} \ddot{U}(\xi)\ \sin w_k(t-\tau-\xi)\ d\xi\ \mathrm{Im}\ g_1(\tfrac{b\tau}{c})\ d\tau$$

$$- \frac{\sqrt{3}\,(2-\sqrt{3}\,)}{c^2 m_o} \sum_{k=1}^{n} M_k^1\ w_k \int_0^t \int_0^\tau \ddot{U}(\xi)\ \sin w_k(\tau-\xi)\ d\xi d\tau$$

$$+ \frac{3b\gamma}{c^2 m_o} \sum_{k=1}^{n} M_k^1\ w_k \int_0^{t-\frac{c}{v}} \int_0^\tau \ddot{U}(\xi)\ \sin w_k(\tau-\xi)\ d\xi d\tau$$

$$+ \frac{3b^2}{\pi c^3 m_o} \sum_{k=1}^{n} M_k^1\ w_k \int_0^t \int_0^{t-\tau} \ddot{U}(\xi)\ \sin w_k(t-\tau-\xi)\ d\xi\ \tau\ \mathrm{Im}\ g_3(\tfrac{b\tau}{c})\ d\tau$$

$$+ \frac{3b^2}{\pi c^3 m_o} \sum_{k=1}^{n} M_k^1\ w_k \int_0^t \int_0^{t-\tau} \int_0^\xi \ddot{U}(\eta)\ \sin w_k(\xi-\eta)\ d\eta d\xi\ \mathrm{Im}\ g_3(\tfrac{b\tau}{c})\ d\tau$$

$$+ \frac{\mu A}{b m_o}\ \dot{U}_p(t)$$

$$\ddot{\Phi}(t) = -\frac{\mu c^2 A}{\sqrt{3}\ b I_o} \int_0^t \ddot{\Phi}(\xi)\ d\xi + \frac{c^2}{2\sqrt{3}\ b I_o} \sum_{k=1}^{n} M_k\ w_k^2 \int_0^t \ddot{U}(\tau)\ \cos w_k(t-\tau)\ d\tau$$

$$- \frac{c^2}{2\sqrt{3}\ b I_o} \sum_{k=1}^{n} M_k^1\ w_k^2 \int_0^t \ddot{\Phi}(\tau)\ \cos w_k(t-\tau)\ d\tau + \frac{m_o\ c^2}{2\sqrt{3}\ b I_o}\ \dddot{U}(t)$$

$$+ \frac{c^2\beta}{2\sqrt{3}\ b I_o} \sum_{k=1}^{n} M_k\ w_k^2 \int_0^{t-\frac{c}{v}} \ddot{U}(\tau)\ \cos w_k(t-\tfrac{c}{v}-\tau)\ d\tau \qquad (55)$$

$$- \frac{c^2\beta}{2\sqrt{3}\ b I_o} \sum_{k=1}^{n} M_k^1\ w_k^2 \int_0^{t-\frac{c}{v}} \ddot{\Phi}(\tau)\ \cos w_k(t-\tfrac{c}{v}-\tau)\ d\tau + \frac{c^2\beta m_o}{2\sqrt{3}\ b I_o}\ \dddot{U}(t-\tfrac{c}{v})$$

$$+ \frac{c}{2\sqrt{3}\,\pi I_o} \sum_{k=1}^{n} M_k w_k^2 \int_0^t \int_0^{t-\tau} \ddot{U}(\xi) \cos w_k(t-\tau-\xi)\, d\xi \,\mathrm{Im}\, g_2\left(\frac{b\tau}{c}\right) d\tau$$

$$- \frac{c}{2\sqrt{3}\,\pi I_o} \sum_{k=1}^{n} M_k^1 w_k^2 \int_0^t \int_0^{t-\tau} \ddot{\Phi}(\xi) \cos w_k(t-\tau-\xi)\, d\xi \,\mathrm{Im}\, g_2\left(\frac{b\tau}{c}\right) d\tau$$

$$+ \frac{m_o c}{2\pi\sqrt{3}\, I_o} \int_0^t \dddot{U}(t-\tau) \,\mathrm{Im}\, g_2\left(\frac{b\tau}{c}\right) d\tau$$

$$+ \frac{1}{I_o} \sum_{k=1}^{n} M_k^1 w_k \int_0^t \ddot{U}(\tau) \sin w_k(t-\tau)\, d\tau$$

$$- \frac{1}{I_o} \sum_{k=1}^{n} N_k w_k \int_0^t \ddot{\Phi}(\tau) \sin w_k(t-\tau)\, d\tau$$

$$+ \frac{\sqrt{3}\, b}{2\pi c I_o} \sum_{k=1}^{n} M_k^1 w_k \int_0^t \int_0^{t-\tau} \ddot{U}(\xi) \sin w_k(t-\tau-\xi)\, d\xi \,\mathrm{Im}\, g_4\left(\frac{b\tau}{c}\right) d\tau$$

$$- \frac{\sqrt{3}\, b}{2\pi c I_o} \sum_{k=1}^{n} N_k w_k \int_0^t \int_0^{t-\tau} \ddot{\Phi}(\xi) \sin w_k(t-\tau-\xi)\, d\xi \,\mathrm{Im}\, g_4\left(\frac{b\tau}{c}\right) d\tau$$

$$- \frac{\sqrt{3}\, b}{2\pi c} \int_0^t \ddot{\Phi}(t-\tau) \,\mathrm{Im}\, g_4\left(\frac{b\tau}{c}\right) d\tau$$

$$+ \frac{\sqrt{3}\, b}{2\pi c I_o} \sum_{k=1}^{n} M_k^1 w_k^2 \int_0^t \int_0^{t-\tau} \ddot{U}(\xi) \cos w_k(t-\tau-\xi)\, d\xi \; \tau \,\mathrm{Im}\, g_4\left(\frac{b\tau}{c}\right) d\tau$$

$$- \frac{\sqrt{3}\, b}{2\pi c I_o} \sum_{k=1}^{n} N_k w_k^2 \int_0^t \int_0^{t-\tau} \ddot{\Phi}(\xi) \cos w_k(t-\tau-\xi)\, d\xi \; \tau \,\mathrm{Im}\, g_4\left(\frac{b\tau}{c}\right) d\tau$$

$$- \frac{\sqrt{3}\, b}{2\pi c} \int_0^t \dddot{\Phi}(t-\tau)\, \tau \,\mathrm{Im}\, g_4\left(\frac{b\tau}{c}\right) d\tau + \frac{\mu c^2 A}{\sqrt{3}\, b\, I_o} \int_0^t \Phi_p(\tau)\, d\tau$$

For prescribed $\ddot{U}_p(t)$ and $\ddot{\Phi}_p(t)$ equations (54) and (55) are to be solved numerically for $\ddot{U}(t)$ and $\ddot{\Phi}(t)$. The output $\ddot{U}(t)$ and $\ddot{\Phi}(t)$ could then be compared with the input $\ddot{U}_p(t)$ and $\ddot{\Phi}_p(t)$. Comparisons also could be made in the output and input spectra.

Unfortunately, although the integrals in the $u_1(t)$, $u_2(t)$, $\Theta_1(t)$ and $\Theta_2(t)$ are meaningful when singular integrals are considered in the Cauchy sense, the differentiations necessary to obtain equations (54) and (55) render some of the integrals therein divergent. Hence equation (54) and (55) do not provide a meaningful set of equations for $\ddot{U}(t)$ and $\ddot{\Phi}(t)$. These computer programs are included in the Appendix B. They could prove useful in subsequent calculations if reasonable smoothing conditions could be determined. A smoother revision of equation (55) which does not contain any third derivatives can be obtained by integrating that equation. The equation resulting is written following as equation (56). Equations (54) and (56) have been programmed for numerical solution but as in the case of equations (54) and (55), the integrals still diverge. This program is also listed in the Appendix for possible use in jointly modifying the equations and the program.

$$\dot{\Phi}(t) = -\frac{\mu c^2 A}{\sqrt{3}\, bI_o}\int_o^t\int_o^\tau \ddot{\Phi}(\xi)\, d\xi d\tau + \frac{c^2}{2\sqrt{3}\, bI_o}\sum_{k=1}^{n} M_k w_k\int_o^t \ddot{U}(\tau)\sin(t-\tau)\, d\tau$$

$$- \frac{c^2}{2\sqrt{3}\, bI_o}\sum_{k=1}^{n} M_k^1 w_k\int_o^t \ddot{\Phi}(\tau)\sin w_k(t-\tau)\, d\tau + \frac{m_o c^2}{2\sqrt{3}\, bI_o}\ddot{U}(t)$$

$$+ \frac{c^2\beta}{2\sqrt{3}\, bI_o}\sum_{k=1}^{n} M_k w_k\int_o^{t-\frac{c}{v}} \ddot{U}(\tau)\sin w_k\left(t-\frac{c}{v}-\tau\right) d\tau$$

$$\begin{aligned}
&- \frac{c^2\beta}{2\sqrt{3}\, bI_o} \sum_{k=1}^{n} M_k^1 w_k \int_o^{t-\frac{c}{v}} \ddot{\Phi}(\tau) \sin w_k(t-\tfrac{c}{v}-\tau)\, d\tau + \frac{c^2\beta m_o}{2\sqrt{3}\, bI_o}\, \ddot{U}(t-\tfrac{c}{v}) \\
&+ \frac{c}{2\sqrt{3}\,\pi I_o} \sum_{k=1}^{n} M_k w_k \int_o^{t}\int_o^{t-\tau} \ddot{U}(\xi) \sin w_k(t-\tau-\xi)\, d\xi \,\operatorname{Im} g_2(\tfrac{b\tau}{c})\, d\tau \\
&- \frac{c}{2\sqrt{3}\,\pi I_o} \sum_{k=1}^{n} M_k^1 w_k \int_o^{t}\int_o^{t-\tau} \ddot{\Phi}(\xi) \sin (t-\tau-\xi)\, d\xi \,\operatorname{Im} g_2(\tfrac{b\tau}{c})\, d\tau \\
&+ \frac{m_o c}{2\pi\sqrt{3}\, I_o} \int_o^{t} \ddot{U}(t-\tau) \,\operatorname{Im} g_2(\tfrac{b\tau}{c})\, d\tau \\
&+ \frac{1}{I_o} \sum_{k=1}^{n} M_k^1 w_k \int_o^{t}\int_o^{\tau} \ddot{U}(\xi) \sin w_k(\tau-\xi)\, d\xi\, d\tau \\
&- \frac{1}{I_o} \sum_{k=1}^{n} N_k w_k \int_o^{t}\int_o^{\tau} \ddot{\Phi}(\xi) \sin w_k(\tau-\xi)\, d\xi\, d\tau \\
&+ \frac{\sqrt{3}\, b}{2\pi\, cI_o} \sum_{k=1}^{n} M_k^1 w_k \int_o^{t}\int_o^{t-\tau}\int_o^{\xi} \ddot{U}(\eta) \sin w_k(\xi-\eta)\, d\eta\, d\xi \,\operatorname{Im} g_4(\tfrac{b\tau}{c})\, d\tau \\
&- \frac{\sqrt{3}\, b}{2\pi\, cI_o} \sum_{k=1}^{n} N_k w_k \int_o^{t}\int_o^{t-\tau}\int_o^{\xi} \ddot{\Phi}(\eta) \sin w_k(\xi-\eta)\, d\eta\, d\xi \,\operatorname{Im} g_4(\tfrac{b\tau}{c})\, d\tau \\
&- \frac{\sqrt{3}\, b}{2\pi c} \int_o^{t}\int_o^{t-\tau} \ddot{\Phi}(\xi)\, d\xi \,\operatorname{Im} g_4(\tfrac{b\tau}{c})\, d\tau \\
&+ \frac{\sqrt{3}\, b}{2\pi\, cI_o} \sum_{k=1}^{n} M_k^1 w_k \int_o^{t}\int_o^{t-\tau} \ddot{U}(\xi) \sin w_k(t-\tau-\xi)\, d\xi\, \tau \,\operatorname{Im} g_4(\tfrac{b\tau}{c})\, d\tau
\end{aligned} \tag{56}$$

$$- \frac{\sqrt{3}\ b}{2\pi\ cI_o} \sum_{k=1}^{n} N_k\, w_k \int_o^t \int_o^{t-\tau} \ddot{\Phi}(\xi)\ \sin w_k(t-\tau-\xi)\, d\xi\ \tau\ \mathrm{Im}\ g_4(\frac{b\tau}{c})\ d\tau$$

$$- \frac{\sqrt{3}\ b}{2\pi c} \int_o^t \ddot{\Phi}(t-\tau)\ \tau\ \mathrm{Im}\ g_4(\frac{b\tau}{c})\ d\tau$$

$$+ \frac{\mu c^2 A}{\sqrt{3}\ bI_o}\ \Phi_p(t)$$

CONCLUSIONS

By the same technique and a similar model as was utilized successfully in references (1) and (2), the solution of an extended Lamb Problem formulated in equations (1) and (2) for the horizontal displacement at the origin due to the earthquake excitation has been obtained. In addition the rotation at and about origin for this model has been determined. These results are summarized in equations (48) in terms of integrals convergent in the Cauchy sense. Three sets of complicated coupled Volterra equations (equations (51) and (52) for no-base-mass case and for the base-mass included case equations (54) and (55) and equations (54) and (56)) are obtained. Equations (54) and (55) and equations (54) and (56) are programmed for numerical computations which would permit comparisons of the input and output as well as of the input and output spectra. Each of these sets programmed, however, contains divergent integrals and thus yield no results as written. The divergent computer programs for solving each of these sets of Volterra equations are listed in the Appendix B for the purpose of expediting later solution via possible joint modification of the equations and the corresponding programs.

The source of the difficulty has been that the solution of this Lamb Problem, unlike the simpler ones of references (1) and (2), is not sufficiently smooth to allow the differentiations necessary to obtain the Volterra equations in those cases programmed. (Equations (51) and (52) have not been programmed and integrals known to diverge do not appear in them.) It could be that some physically reasonable smoothing is possible. In which case the Volterra equations and digital computer programs tabulated herein might be useful. Or it could be that the model chosen is inadequate for this refinement of the simpler problems for which it yielded the results of references (1) and (2).

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*This document can be obtained through the United States Department of Commerce, Washington, D.C.

APPENDIX A

FIGURES

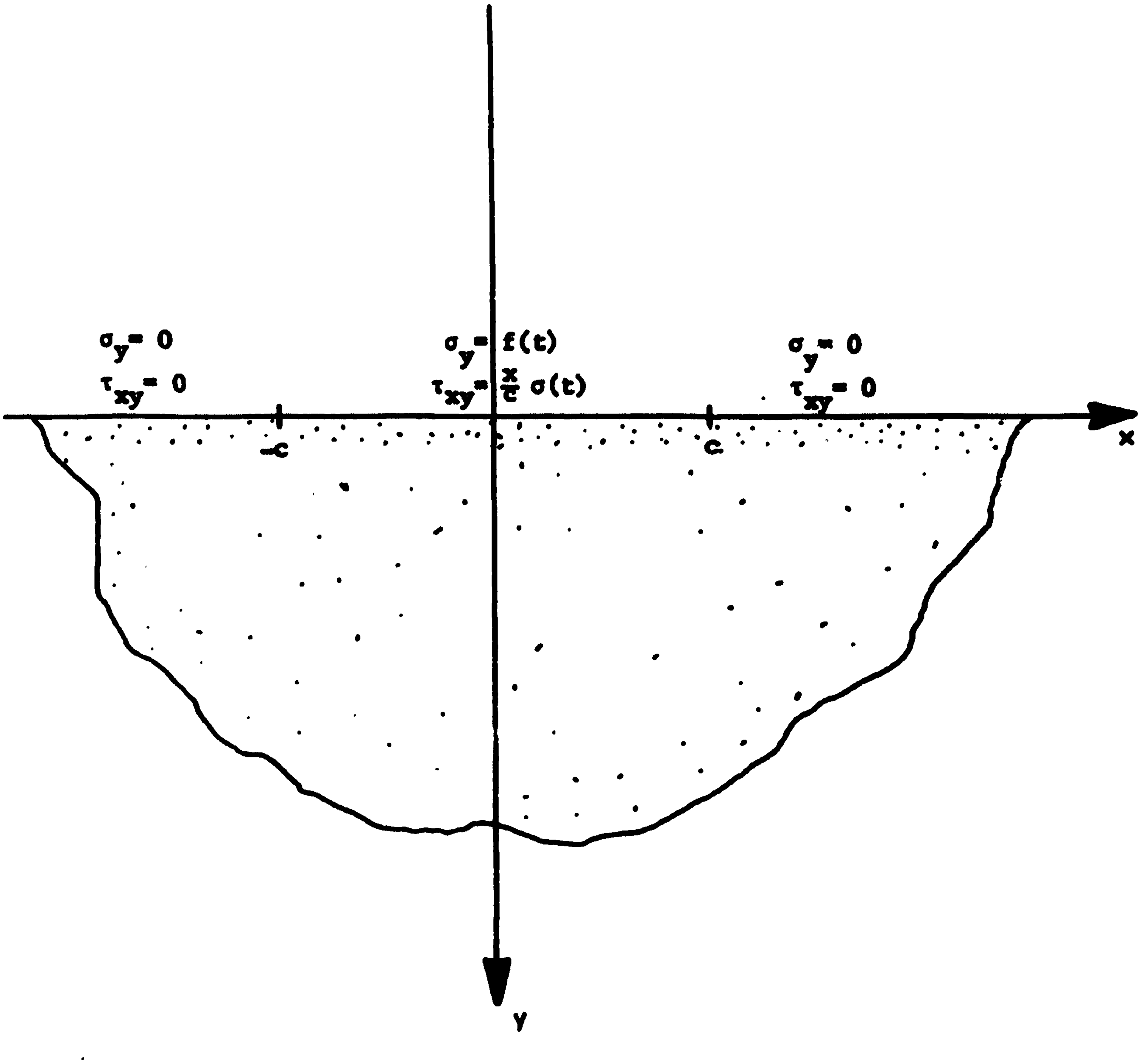

Figure 1. The coordinate system used in the solution of the Lamb Problem stated in equations (1) and (2).

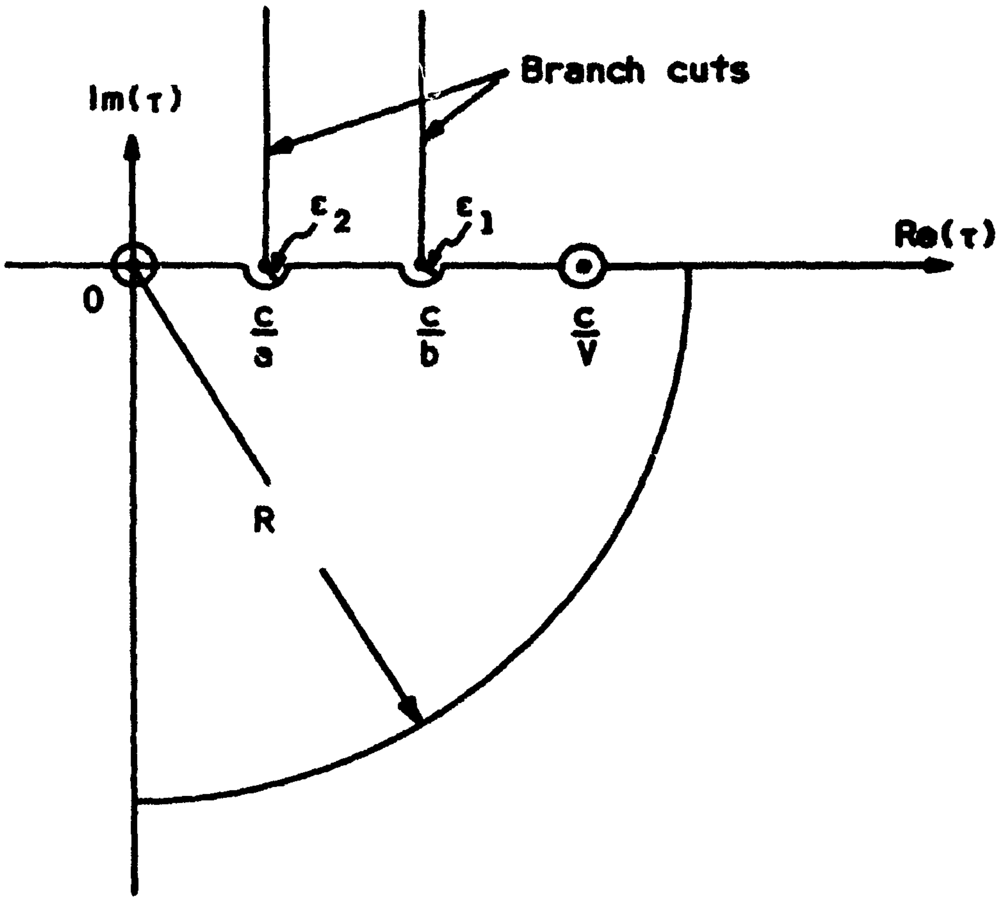

Figure 2. The fourth quadrant contour used in evaluating the inverse Laplace transformations.

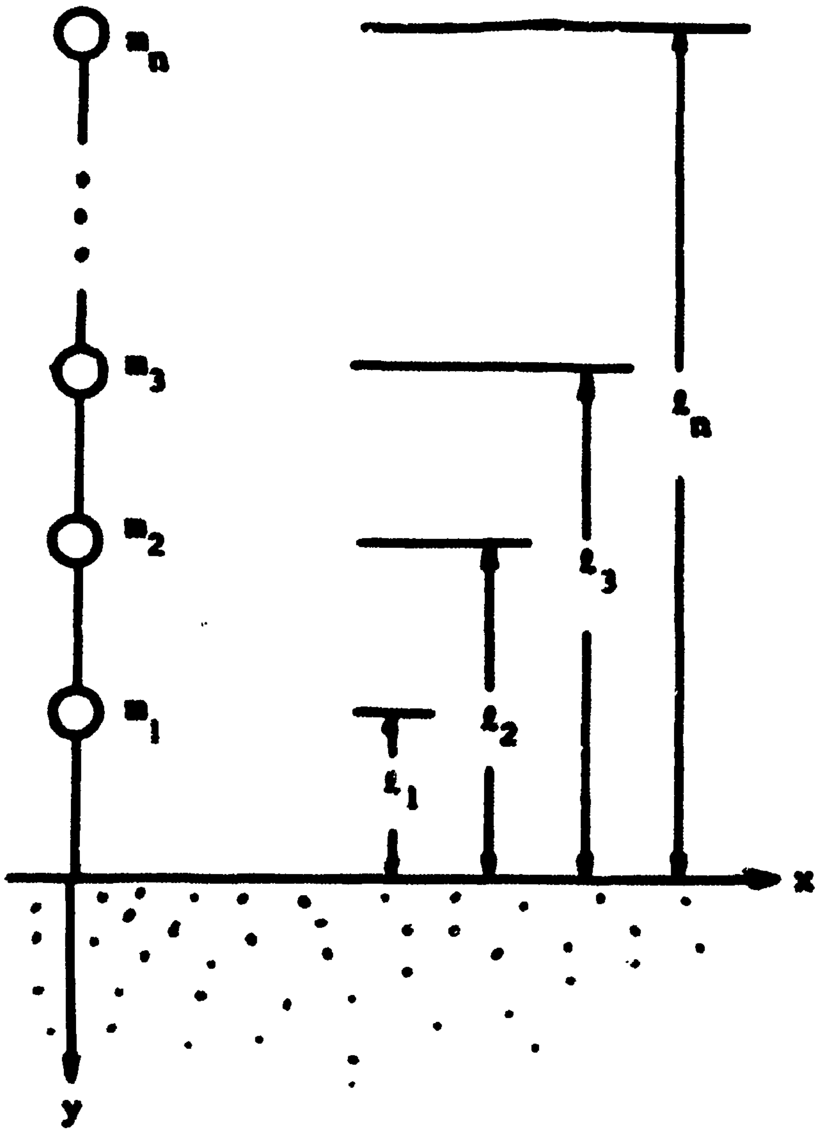

Figure 3. The lumped-mass structure model used in the normal mode theory.

APPENDIX B

PROGRAM LISTINGS

Two programs are developed in attempting to solve the coupled lateral-rotation interaction problem. However, as pointed out in the main body of this report neither of these programs converged. Each is listed in order to show the progress to date made in the solution of this problem.

Program 1 is an attempt to use numerical iteration to solve the first formulation of the coupled interaction problem with base inertia presented in equations (54) and (55). The second program, Program 2, is a listing using the second formulation of the interaction problem with base inertia (equations (54) and (56)). The main program controls the input data except $\ddot{u}_p(t)$ and $\ddot{\phi}_p(t)$, the summations of terms, the iteration procedure, the convergence criterion and the program output. Each type of integral on the right hand side of the equations is solved in a subroutine. In the development of these subroutines, known functions of $\ddot{u}(t)$ or $\ddot{\phi}(t)$ are used as input. Thus, analytical solutions could be compared to numerical values. Two investigators verified each subroutine. The digitized input values, $\ddot{u}_p(t)$ and $\ddot{\phi}_p(t)$, are read from cards using function subroutines.

Input quantities are defined in the comment cards listed in each program. Because the programs do not converge, detailed input formats are not presented.

PROGRAM 1

```
//RS2       JOB NODUMP,1177       RJS
// EXEC FORTRAN(BCD)
C     MAIN PROGRAM
      DIMENSION F(4,1000),OMM(3),CCC(1,1000),SSS(1,1000),SSD(2,3),SS(2,3
     1),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2,1000),
     2DS3(2,3),ST(2,4,3),UD(2,2)  ,W(3),WL(3),WM(3),XL(3),XB(3,3),
     3WN(3),C(22),SL(19)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND
C     II  II=0 - MODAL INPUT
C     COM DIM F,CCC,SSS,U,F  SUB-DSDLI,FTAU  SUBDS2IL,CI,SI  SUB-DSDIL,V
C     SUB-TPLIG,ETA
C         II=1 - STRUCTURE INPUT
C     IB  IB=LESS THAN ZERO OR ZERO  - NO ROCKING INPUT
C         ABS(IB)=2 TERM OUTPUT
C         IB=1 OR 2 - ROCKING INPUT  - VELOCITY UNITS
C     NM=NUMBER OF MODES
C     W(I)=EFFECTIVE MASS LB-SEC*SEC/FT
C     WL(I)=MODAL EFFECTIVE FIRST MOMENT
C     WN(I)=MODAL EFFECTIVE SECOND MOMENT
C     WIO=BASE MOMENT OF INERTIA  (LB*SEC*SEC*FT*FT)/FT
C     WO =BASE MASS LB-SEC*SCE/FT
C     B=SHEAR WAVE VELOCITY  FT/SEC
C     G=SHEAR MODULUS  LB/FT/FT
C     AL=BUILDING WIDTH  FT
C     T,DT-SEC
C     AA BASE AREA FT*FT
C     UP(T)=LATERAL FREE FIELD ACCELERATION
C     UR(T)=FREE FIELD ROTATION (VELOCITY)
C     INPUT
      IPU=7
      IR=5
      IP=6
  100 READ(IR,1)II,IB,NM,TMAX,DT,B,G,AL,AA,ERR
      IF(NM)101,100,103
    1 FORMAT(1X3I3,7E10.3)
  103 READ(IR,2)WC,WIO
      READ(IR,2) (OMM(I),I=1,NM)
    2 FORMAT (8E10.3)
      WRITE(IP,5)
    5 FORMAT(1X9H II IB NM,10X4HTMAX,12X2HDT,13X1HB,13X1HG,
     112X2HAL,12X2HAA,11X3HERR)
      WRITE(IP,6)II,IB,NM,TMAX,DT,B,G,AL,AA,ERR
    6 FORMAT(1X3I3,7E14.4)
      WRITE(IP,3)WC,WIO
    3 FORMAT(13X2HMO,12X2HIO/(1X2E14.2))
      WRITE(IP,7)(OMM(I),I=1,NM)
    7 FORMAT(7X8HOMEGA(I)/(1X8F14.3))
      IF(II)20,20,21
   20 READ(IR,2) (W(I),I=1,NM)
      READ(IR,2) (WL(I),I=1,NM)
      READ(IR,2) (WN(I),I=1,NM)
      GO TO 24
   21 READ(IR,2) (WM(I),I=1,NM)
```

```
      READ(IR,2)(XL(I),I=1,NM)
      DO 22 I=1,NM
   22 READ(IR,2) (XB(J,I),J=1,NM)
      DO 51 I=1,NM
      C1=0.0
      C2=0.0
      C3=0.0
      DO 23 J=1,NM
      C1=C1 WM(J)*XB(J,I)
      C2=C2 WM(J)*XB(J,I)*XB(J,I)
   23 C3=C3 WM(J)*XB(J,I)*XL(J)
      W(I)=C1*C1/C2
      WL(I)=C1*C3/C2
   51 WN(I)=C3*C3/C2
      WRITE(IP,11) (WM(I),I=1,NM)
      WRITE(IP,12) (XL(I),I=1,NM)
      WRITE(IP,13)
      DO 25 I=1,NM
   25 WRITE(IP,15) (XB(J,I),J=1,NM)
   24 CONTINUE
      WRITE(IP,14)
      WRITE(IP,8) (W(I),I=1,NM)
      WRITE(IP,9) (WL(I),I=1,NM)
      WRITE(IP,10) (WN(I),I=1,NM)
   10 FORMAT(11X4HN(J)/(1X8F14.3))
    9 FORMAT(10X5HML(J)/(1X8F14.3))
    8 FORMAT(11X4HM(J)/(1X8F14.3))
   14 FORMAT(1X16HMODAL PROPERTIES)
   11 FORMAT(1X20HSTRUCTURE PROPERTIES//1X4HW(I)/1X8F14.3)
   12 FORMAT(1X4HL(I)/1X8F14.3)
   13 FORMAT(1X6HX(I,J))
   15 FORMAT(1X8F14.3)
   16 FORMAT(1X6HOUTPUT//(14X1HT,10X4HU(T),10X4HR(T),9X5HUP(T),9X5HRP(T)
     1))
   17 FORMAT(15X18HTERMS OF EQUATIONS)
   18 FORMAT(1X7F14.4)
   19 FORMAT(15X6E14.5)
   50 FORMAT(8F10.5)
  104 FORMAT (9H DIVERGED)
C     OUTPUT
      WRITE(IP,16)
       IA=IABS(IB)
      IF(IA-2)65,26,26
   26 WRITE(IP,17)
      UG2=0.0
      ERA=ERR/100.
      ERS=ERR/100000.
      UG22=0.0
C     CONSTANTS FOR FIRST EQUATION
   65 T1=G*AA/B/WO
      T2=1.0/WO
      PI=3.14159
      T4=B/2.0/PI/AL/WO
      T6=T4*WO
      C1=0.464102
```

```
      T7=B*C1/AL/AL/WO
      T9=T7*WIO
      GA=0.211325
      T10=3.0*B*GA/AL/AL/WO
      T12=T10*WIO
      VR=B*0.9194
      T13=3.0*B*GA/VR/WO/AL
      T15=T13*WIO
      T16=3.0*B*B/PI/AL/AL/AL/WO
      T18=T16*WIO
C     CONSTANTS FOR SECOND EQUATION
      R3=1.73205
      BET=0.5
      ND=AL/0.9194/B/DT 0.5
      W1=G*AL*AL*AA/R3/B/WIO
      W2=AL*AL/2.0/R3/B/WIO
      W4=W2*WO
      W5=W2*BET
      W7=W5*WO
      W8=AL/2./R3/PI/WIO
      W10=W8*WO
      W11=1.0/WIO
      W13=3.0*B/2.0/PI/R3/WIO/AL
      W15=W13*WIO
      W19=W1
C     MODAL CONSTANTS
      T=0.0
      NP=TMAX/DT
      NP1=NP+1
      DO 28 I=1,NM
      DO 28 J=1,NP1
      CCC(I,J)=COS(OMM(I)*T)
      SSS(I,J)=SIN(OMM(I)*T)
   28 T=T DT
      U(1,1)=0.0
      U(2,1)=0.0
      T=0.0
      UD(2,1)=0.0
      UD(2,2)=0.0
      CALL FG124
      IF(IB)29,29,30
   30 UD(2,2)=UR(T)
   29 UD(1,2)=UP(T)
      UG2D=0.0
      UG2DD=0.0
      T=0.0
      WRITE(IP,18)T,U(1,1   ),U(2,1   ),UD(1,2),UD(2,2)
      T=DT
      DO 41 IT=2,NP1
      IT1=IT-1
      IF(IB)31,31,32
   32 UD(2,1)=UD(2,2)
      UD(2,2)=UR(T)
   31 UD(1,1)=UD(1,2)
      UD(1,2)=UP(T)
```

```
      U(1,IT)=U(1,IT1)
      U(2,IT)=U(2,IT1)
      CALL TPLIG(T)
      CALL DS2IL
      CALL UFSIG
      CALL SSIGL
      CALL DSPIL
      CALL DSDLI
      DO 35 ITT=1,25
      CALL SIGL1
      DO 33 I=1,22
   33 C(I)=0.0
      C(1)=-T1*SUJ(1)
      C(6)=-T6*USF(1,1)
      C(9)=-T9*SUJ(2)
C     C(9)=-T9*UG2
      C(9)=+T9*UG2
C     C(12)=+T12*SUD
      C(12)=-T12*SUD
      C(15)=0.0
      C(18)=T18*DSF(1)*(-1.0)
C     C(18)=T18*DSF(1)
      IND=IT-ND
      IF(IND)53,53,52
   52 C(15)=T15*U(2,IND)*(-1.0)
C  52 C(15)=T15*U(2,IND)
C  53 C(21)=T18*USF(3,2)
   53 C(21)=T18*USF(3,2)*(-1.0)
      C(22)=T1*SUP(1)
      DO 34 I=1,NM
      C(2)=C(2)-SS(1,I)*T2*W(I)*OMM(I)
C     C(3)=C(3)-SS(2,I)*T2*WL(I)*OMM(I)
      C(3)=C(3)+SS(2,I)*T2*WL(I)*OMM(I)
      C(4)=C(4)-T4*W(I)*OMM(I)*DS2(1,1,I)
C     C(5)=C(5)-T4*WL(I)*OMM(I)*DS2(2,1,I)
      C(5)=C(5)+T4*WL(I)*OMM(I)*DS2(2,1,I)
      C(7)=C(7)-T7*WL(I)*DS3(1,I)*OMM(I)
C     C(8)=C(8)-T7*WN(I)*DS3(2,I)*OMM(I)
      C(8)=C(8)+T7*WN(I)*DS3(2,I)*OMM(I)
      C(10)=C(10)+T10*WL(I)*DS1(1,I)*OMM(I)
C     C(11)=C(11)+T10*WN(I)*DS1(2,I)*OMM(I)
      C(11)=C(11)-T10*WN(I)*DS1(2,I)*OMM(I)
      C(13)=C(13)+T13*WL(I)*SSD(1,I)*OMM(I)
C     C(14)=C(14)+T13*WN(I)*SSD(2,I)*OMM(I)
      C(14)=C(14)-T13*WN(I)*SSD(2,I)*OMM(I)
      C(16)=C(16)+T16*WL(I)*ST(1,3,I)*OMM(I)
C     C(17)=C(17)+T16*WN(I)*ST(2,3,I)*OMM(I)
      C(17)=C(17)-T16*WN(I)*ST(2,3,I)*OMM(I)
      C(19)=C(19)+T16*WL(I)*DS2(1,3,I)*OMM(I)
   34 C(20)=C(20)-T16*WN(I)*DS2(2,3,I)*OMM(I)
C  34 C(20)=C(20)+T16*WN(I)*DS2(2,3,I)*OMM(I)
      UG1=0.0
      DO 44 I=1,22
   44 UG1=UG1 C(I)
      DO 36 I=1,19
```

```
   36 SL(I)=0.0
      SL(4)=+W4*U(1,IT)
C     SL(4)=-W4*U(1,IT)
      SL(15)=-W15*DSF(2)
      SL(1)=W1*DU2*(-1.0)
      SL(7)=0.0
      IF(IND)54,54,55
   55 SL(7)=+U(1,IND)*W7
C  55 SL(7)=-U(1,IND)*W7
   54 SL(10)=+W10*USF(2,1)
C  54 SL(10)=-W10*USF(2,1)
      SL(18)=-W15*USF(4,2)
      SL(19)=+W19*SUP(2)
C     SL(19)=-W19*SUP(2)
      DO 37 I=1,NM
      SL(2)=SL(2)+W2*W(I)*SS(1,I)*OMM(I)
C     SL(2)=SL(2)-W2*W(I)*SS(1,I)*OMM(I)
      SL(3)=SL(3)-W2*WL(I)*SS(2,I)*OMM(I)
C     SL(5)=SL(5)-W5*W(I)*SSD(1,I)*OMM(I)
      SL(5)=SL(5)+W5*W(I)*SSD(1,I)*OMM(I)
      SL(6)=SL(6)-W5*WL(I)*SSD(2,I)*OMM(I)
C     SL(8)=SL(8)-W8*W(I)*DS2(1,2,I)*OMM(I)
      SL(8)=SL(8)+W8*W(I)*DS2(1,2,I)*OMM(I)
      SL(9)=SL(9)-W8*WL(I)*DS2(2,2,I)*OMM(I)
      SL(11)=SL(11)+W11*WL(I)*DS3(1,I)*OMM(I)
C     SL(11)=SL(11)-W11*WL(I)*DS3(1,I)*OMM(I)
      SL(12)=SL(12)-W11*WN(I)*DS3(2,I)*OMM(I)
C     SL(13)=SL(13)-W13*WL(I)*ST(1,4,I)*OMM(I)
      SL(13)=SL(13)+W13*WL(I)*ST(1,4,I)*OMM(I)
      SL(14)=SL(14)-W13*WN(I)*ST(2,4,I)*OMM(I)
C     SL(16)=SL(16)-W13*WL(I)*DS2(1,4,I)*OMM(I)
      SL(16)=SL(16)+W13*WL(I)*DS2(1,4,I)*OMM(I)
   37 SL(17)=SL(17)-W13*WN(I)*DS2(2,4,I)*OMM(I)
      UG22=UG2
      UG2=0.0
      DO 48 I=1,19
   48 UG2=UG2 SL(I)
      UG3=(3.0*UG2-4.0*UG2D UG2DD)/DT/2.0
      UG3=(UG2-UG2D)/DT
      IF(ITT-3)75,75,79
   79 DIFU=ABS(UG1-U(1,IT))
      DIFS=ABS(UG3-U(2,IT))
      DIFV= ABS(UG2-UG22)
      IF(DIFV-ERS)76,76,75
C     ERROR CRITERION
   76 IF(DIFU-ERR)38,75,75
   38 IF(DIFS-ERA)40,75,75
   75 U(1,IT)=UG1
      U(2,IT)=UG3
   35 CONTINUE
      GO TO 1033
   40 UG2DD=UG2D
      UG2D=UG2
      CALL DSPIL
      CALL DS2IL
```

```
      CALL TPLIG(T)
      CALL DSDLI
      CALL SIGL1
      CALL SSIGL
      CALL UFSIG
      U(1,IT)=UG1
      U(2,IT)=UG3
      WRITE(IP,18)T,U(1,IT ),U(2,IT ),UD(1,2),UD(2,2)
      IF(IA-2)41,42,42
   42 WRITE(IP,904) ITT,UG1,UG2,UG3
      WRITE(IP,19) (C(I),I=1,22)
      WRITE(IP,19) (SL(I),I=1,19) ,SUJ(2)
   41 T=T+DT
  904 FORMAT(20X15,3E14.5)
      WRITE(IPU,50)(U(1,I),I=1,NP1)
      WRITE(IPU,50)(U(2,I),I=1,NP1)
      GO TO 100
 1033 WRITE (IP,104)
      WRITE(IP,904) ITT,UG1,UG2,UG3
      WRITE(IP,19) (C(I),I=1,22)
      WRITE(IP,19) (SL(I),I=1,19) ,SUJ(2)
  101 CALL EXIT
      END
      SUBROUTINE DSDLI
C     INTEGRATION   OF U(XI)*SIN(W*(TAU-XI)) WITH TIME DELAY
      DIMENSION F(4,1000),OMM(3),CCC(1,1000),SSS(1,1000),SSD(2,3),SS(2,3
     1),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2,1000),
     2DS3(2,3),ST(2,4,3),UD(2,2)   ,DSS(2,3),D33(2,3)
     2 ,DUS(2,3),DUSS(2,3),DUC(2,3),DUCC(2,3),FTAU(2,1,1000)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND
      IF(T-DT)12,12,11
   12 X=0.C
      NI=1
      DO 14 I=1,NM
      DO 14 J=1,2
      FTAU(J,I,1)=0.0
      DS3(J,I)=0.0
      DS1(J,I)=0.0
      DUC(J,I)=0.0
      DUS(J,I)=0.0
      DUSS(J,I)=0.0
   14 DUCC(J,I)=0.0
   11 IF(X-T)10,13,13
   10 X=X+DT
      NI=NI+1
      NI1=NI-1
      DO 15 J=1,2
      DO 15 I=1,NM
      DUSS(J,I)=DUS(J,I)
      DSS(J,I)=DS1(J,I)
      D33(J,I)=DS3(J,I)
   15 DUCC(J,I)=DUC(J,I)
   13 DO 16 J=1,2
```

```
      DO 16 I=1,NM
      DUC(J,I)=DUCC(J,I)+DT/2.*(U(J,NI)*SSS(I,NI)+U(J,NI1)*SSS(I,NI1))
      DUS(J,I)=DUSS(J,I)+DT/2.0*(U(J,NI)*CCC(I,NI)+U(J,NI1)*CCC(I,NI1))
      FTAU(J,I,NI)=SSS(I,NI)*DUS(J,I)-CCC(I,NI)*DUC(J,I)
   16 DS3(J,I)=D33(J,I)+DT/2.0*(FTAU(J,I,NI)+FTAU(J,I,NI1))
      IF(NI-ND)17,17,18
   18 DO 20 J=1,2
      DO 20 I=1,NM
      N=NI-ND+1
      N1=N-1
      DS1(J,I)=DSS(J,I) DT/2.0*(FTAU(J,I,N) FTAU(J,I,N1))
   20 CONTINUE
   17 RETURN
      END
      SUBROUTINE UFSIG
C     UFS(F(4),UD OR US)
C     INTEGRATION OF U(J,T-TAU)*F(K,T)
      DIMENSION F(4,1000),OMM(3),CCC(1,1000),SSS(1,1000),SSD(2,3),SS(2,3
     1),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2,1000),
     2DS3(2,3),ST(2,4,3),UD(2,2)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND
      IF(T-DT)12,12,11
   12 X=0.0
      NI=1
   11 IF(X-T)10,13,13
10    X=X+DT
      NI=NI+1
      NI1=NI-1
   13 N=NI
      DO 15 I=1,2
      DO 15 J=1,4
   15 USF(J,I)=0.0
      TT=0.0
      DO 20 IT=1,NI1
      IT1=IT+1
      NN=N-1
      TT1=TT+DT
      DO 21 J=1,2
   21 USF(J,1)=DT/2.0*(U(1,N)*F(J,IT)+U(1,NN)*F(J,IT1))+USF(J,1)
      DO 22 J=3,4
   22 USF(J,2)=USF(J,2)+DT/2.0*(U(2,N)*F(J,IT)*TT U(2,NN)*F(J,IT1)*TT1)
      TT=TT+DT
   20 N=N-1
C     USF(F(4), UD OR US)
      RETURN
      END
      SUBROUTINE FG124
C     CALCULATION OF LAMB FUNCTIONS
      DIMENSION F(4,1000),OMM(3),CCC(1,1000),SSS(1,1000),SSD(2,3),SS(2,3
     1),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2,1000),
     2DS3(2,3),ST(2,4,3),UD(2,2)
        COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
```

```
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND
      YRT=1.0/SQRT(3.0)
      Y=0.0
      NPI=NP+1
      DO 21 I=1,NPI
      X=B*Y/AL
      X2=X*X
      X4=X2*X2
      CK1=(3.0+SQRT(3.0))/2.0
      CK2=(3.0-SQRT(3.0))/2.0
      CK3=CK1/2.0
      CK4=CK2 /2.0
      R1=X2-1.0
      RTD=X2-1.0/3.0
      RP5=X2-0.5
      R14=X2-0.25
      SR14P=X2-CK3
      SR14M=X2-CK4
      IF(X-YRT)20,20,11
   11 DOM4=2.0*X*R14*SR14P*SR14M
      IF(X-1.0)12,12,13
   20 DO 22 J=1,4
   22 F(J,I)=0.0
      GO TO 21
   12 DOM1=RP5**4-X4*RTD*R1
      SR11=SQRT(1.0-X2)
      SRTD=SQRT(RTD)
      SQTD=SRTD
      SQ14=SQRT(R14)
      F(1,I)= X*R1*SRTD/DOM1
      F(2,I)=-3.*RP5*X2*SRTD*SR11/DOM4
      F(3,I)=-3.0*RP5*SRTD*SR11/2.0/DOM4
      F(4,I)=-3.0*SQTD*RP5*RP5/DOM4
      GO TO 21
   13 SR1=SQRT(X2-1.0)
      SRTD=SQRT(RTD)
      SQTD=SRTD
      SQ14=SQRT(R14)
      F(1,I)=SR1/(X*(RP5*RP5-X2*SRTD*SR1))
      F(2,I)=0.0
      F(3,I)=0.0
      F(4,I)=-3.0*SRTD*(RP5*RP5+X2*SRTD*SR1)/ DOM4
   21 Y=Y+DT
      RETURN
      END
      SUBROUTINE TPLIG (X)
C     TRIPLE  INTEGTATION
      DIMENSION F(4,1000),OMM(3),CCC(1,1000),SSS(1,1000),SSD(2,3),SS(2,3
     1),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2,1000),
     2DS3(2,3),ST(2,4,3),UD(2,2),
     3FFP(2,3),ETA(2,1,1000),FF(2,3),C(2,3),S(2,3 ),SXS(2,3),SXC(2,3)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND
      IF(X-DT)12,12,11
```

```
   12 DO 13 I=1,NM
      DO 13 J=1,2
      SXS(J,I)=0.0
      SXC(J,I)=0.0
      ETA(J,I,1)=0.0
      C(J,I)=0.0
      S(J,I)=0.0
      FFP(J,I)=0.0
   13 FF(J,I)=0.0
      Y=0.0
      NI=1
   11 IF(Y-X)10,14,14
   10 Y=Y+DT
      NI=NI+1
      NI1=NI-1
      DO 19 J=1,2
      DO 19 I=1,NM
      SXC(J,I)=C(J,I)
      SXS(J,I)=S(J,I)
   19 FFP(J,I)=FF(J,I)
C     XI INTEGRATION
   14 DO 15 J=1,2
      DO 15 I=1,NM
      C(J,I)=SXC(J,I)+DT/2.0*(U(J,NI)*CCC(I,NI)+U(J,NI1)*CCC(I,NI1))
   15 S(J,I)=SXS(J,I)+DT/2.0*(U(J,NI)*SSS(I,NI)+U(J,NI1)*SSS(I,NI1))
C     ETA INTEGRATION
      DO 16 J=1,2
      DO 16 I=1,NM
      FF(J,I)=SSS(I,NI)*C(J,I)-CCC(I,NI)*S(J,I)
   16 ETA(J,I,NI)=ETA(J,I,NI1)+DT/2.0*(FF(J,I)+FFP(J,I))
C     TAU INTEGRATION
      DO 18 L=1,NM
      DO 18 J=1,2
      DO 18 K=3,4
      N=NI
      ST(J,K,L)=0.0
      DO 18 I=1,NI1
      N1=N-1
      NN1=I+1
      ST(J,K,L)=ST(J,K,L)+DT/2.0*(F(K,NN1)*ETA(J,L,N1)+F(K,I)*ETA(J,L,N)
     1)
   18 N=N-1
C     ST(UD OR US,F(4),MODE)
      RETURN
      END
      SUBROUTINE SIGL1
C     SIMPLE AND DOUBLE INTEGRATION ROUTINE
      DIMENSION F(4,1000),OMM(3),CCC(1,1000),SSS(1,1000),SSD(2,3),SS(2,3
     1),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2,1000),
     2DS3(2,3),ST(2,4,3),UD(2,2)
     3      ,YU(2),YP(2)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND
       IF(T-DT)12,12,11
```

```
   12 X=0.0
      NI=1
      DO 16 J=1,2
      SUP(J)=0.0
   16 SUJ(J)=0.0
      SUD=0.0
      SUJJ=0.0
      DU2=0.0
   11 IF(X-T)10,13,13
   10 NI=NI+1
      NI1=NI-1
      X=X+DT
      DU=DU2
      SUJJ=SUJ(2)
      SU=SUD
      DO 17 J=1,2
      YU(J)=SUJ(J)
   17 YP(J)=SUP(J)
   13 DO 14 J=1,2
      SUJ(J)=YU(J)+DT/2.0*(U(J,NI)+U(J,NI1))
   14 SUP(J)=YP(J)+DT/2.*(UD(J,1)+UD(J,2))
      DU2=DU DT/2.0*(SUJ(2)+SUJJ )
      IF(NI-ND)19,18,19
   19 N=NI-ND+1
      N1=N-1
      SUD=SU+DT/2.0*(U(2,N)+U(2,N1))
   18 RETURN
      END
      SUBROUTINE DS2IL
C     DOUBLE INTEGRATION ROUTINE - MODIFIED
      DIMENSION F(4,1000),OMM(3),CCC(1,1000),SSS(1,1000),SSD(2,3),SS(2,3
     1),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2,1000),
     2DS3(2,3),ST(2,4,3),UD(2,2)
     3,CI(2,1,1000),SI(2,1,1000)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND
      IF(T-DT)12,12,11
   12 X=0.0
      DO 14 I=1,NM
      DO 14 J=1,2
      CI(J,I,1)=0.0
   14 SI(J,I,1)=0.0
      NI=1
   11 IF(X-T) 10,13,13
   10 X=X+DT
      NI=NI+1
      NI1=NI-1
   13 DO 15 L=1,NM
      DO 15 J=1,2
      CI(J,L,NI)=CI(J,L,NI1)+( U(J,NI1)*CCC(L,NI1)+U(J,NI)*CCC(L,NI))*DT
     1/2.0
   15 SI(J,L,NI)=SI(J,L,NI1)+( U(J,NI1)*SSS(L,NI1)+U(J,NI)*SSS(L,NI))*DT
     1/2.0
      DO 21 L=1,NM
```

```
      DO 21 J=1,2
      DO 21 K=1,4
      C1=0.0
      C3=0.0
      C2=0.0
      C4=0.0
      N=NI
      XT=0.0
      DO 20 I=1,NI1
      N1=N-1
      NN1=I+1
      IF(K-3)17,18,18
   18 CC=F(K,I)*CCC(L,I)*XT
      RY=F(K,I)*SSS(L,I)*XT
      XT1=XT+DT
      SS1=F(K,NN1)*SSS(L,NN1)*XT1
      CC1=F(K,NN1)*CCC(L,NN1)*XT1
      GO TO 19
   17 RY=F(K,I)*SSS(L,I)
      CC=F(K,I)*CCC(L,I)
      CC1=F(K,NN1)*CCC(L,NN1)
      SS1=F(K,NN1)*SSS(L,NN1)
   19 C1=(CC*CI(J,L,N)+CC1*CI(J,L,N1))*DT/2.0+C1
      C2=(CC*SI(J,L,N)+CC1*SI(J,L,N1))*DT/2.0+C2
      C3=(RY*CI(J,L,N)+SS1*CI(J,L,N1))*DT/2.0+C3
      C4=(RY*SI(J,L,N)+SS1*SI(J,L,N1))*DT/2.0+C4
      XT=XT+DT
   20 N=N-1
      CC=CCC(L,NI)
      RY=SSS(L,NI)
   21 DS2(J,K,L)=RY*C1-CC*C2-CC*C3-RY*C4
C     DS2(UD OR US,F(4),MODE)
  100 RETURN
      END
      SUBROUTINE DSPIL
C     DOUBLE INTEGRATION OF U(2,T)*F(3 OR 4,TAU)
      DIMENSION F(4,1000),OMM(3),CCC(1,1000),SSS(1,1000),SSD(2,3),SS(2,3
     1),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2,1000),
     2DS3(2,3),ST(2,4,3),UD(2,2),V(1000)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND
      IF(T-DT)12,12,11
   12 V(1)=0.0
      X=0.0
      NI=1
   11 IF(X-T)10,13,13
   10 NI=NI+1
      NI1=NI-1
      X=X+DT
   13 V(NI)=V(NI1)+DT/2.0*(U(2,NI)+U(2,NI1))
      DSF(1)=0.0
      DSF(2)=0.0
      DO 15 K=3,4
      N=NI
```

```
      DO 15 I=1,NI1
      N1=N-1
      I1=I+1
      K1=K-2
      DSF(K1)=DSF(K1)+DT/2.0*(V(N)*F(K,I)+V(N1)*F(K,I1))
   15 N=N-1
C     D(F(3,T) OR F(4,T))   FOR U(2,T)
      RETURN
      END
      SUBROUTINE SSIGL
C     SINGLE INTEGRATION ROUTINE- MODIFIED   - TIME DELAY INCLUDED
      DIMENSION F(4,1000),OMM(3),CCC(1,1000),SSS(1,1000),SSD(2,3),SS(2,3
     1),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2,1000),
     2DS3(2,3),ST(2,4,3),UD(2,2)
     4, DIS(2,3),DIC(2,3),C(2,3),S(2,3) ,CD(2,3),SD(2,3),DDC(2,3),DDS(2,
     53)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND
       IF(T-DT)12,12,11
   12 DO 15 I=1,NM
      DO 15 J=1,2
      DIS(J,I)=0.0
      DIC(J,I)=0.0
      SSD(J,I)=0.0
      DDS(J,I)=0.0
      DDC(J,I)=0.0
      CD(J,I)=0.0
      SD(J,I)=0.0
      C(J,I)=0.0
   15 S(J,I)=0.0
      X=0.0
      NI=1
   11 IF(X-T)10,13,13
   10 X=X+DT
      NI=NI+1
      NI1=NI-1
      DO 16 J=1,2
      DO 16 I=1,NM
      DDC(J,I)=DDC(J,I)+CD(J,I)
      DDS(J,I)=DDS(J,I)+SD(J,I)
      DIC(J,I)=DIC(J,I)+C(J,I)
   16 DIS(J,I)=DIS(J,I)+S(J,I)
   13 DO 17 J=1,2
      DO 17 I=1,NM
      C(J,I)=(U(J,NI1)*CCC(I,NI1)+U(J,NI)*CCC(I,NI))*DT/2.0
      S(J,I)=(U(J,NI1)*SSS(I,NI1)+U(J,NI)*SSS(I,NI))*DT/2.0
   17 SS(J,I)=SSS(I,NI)*(C(J,I)+DIC(J,I))-CCC(I,NI)*(S(J,I)+DIS(J,I))
      IF(NI-ND)18,18,19
   19 N=NI-ND 1
      N1=N-1
      DO 20 J=1,2
      DO 20 I=1,NM
      CD(J,I)=(U(J,N1)*CCC(I,N1)+U(J,N)*CCC(I,N))*DT/2.0
      SD(J,I)=(U(J,N1)*SSS(I,N1)+U(J,N)*SSS(I,N))*DT/2.0
```

```
 20 SSD(J,I)=SSS(I,N )*(CD(J,I)+DDC(J,I))-CCC(I,N)*(SD(J,I)+DDS(J,I))
 18 RETURN
    END
    FUNCTION UP(X)
    DIMENSION F(4,1000),OMM(3),CCC(1,1000),SSS(1,1000),SSD(2,3),SS(2,3
   1),
   1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2,1000),
   2DS3(2,3),ST(2,4,3),UD(2,2)
     COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
   1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND
    IF(X-0.8)10,10,11
 10 UP=X/8.0*SIN(31.4159*X)*5.0
    GO TO 100
 11 IF(X-2.)12,12,13
 12 UP=(2.0-X)/14.0*SIN(31.4159*X)*5.0
    GO TO 100
 13 UP=0.0
100 RETURN
    END
    FUNCTION UR(X)
    DIMENSION F(4,1000),OMM(3),CCC(1,1000),SSS(1,1000),SSD(2,3),SS(2,3
   1),
   1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2,1000),
   2DS3(2,3),ST(2,4,3),UD(2,2)
     COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
   1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND
    IF(X-0.8)10,10,11
 10 UR=X/8.0*SIN(31.4159*X)*5.0
    GO TO 100
 11 IF(X-2.)12,12,13
 12 UR=(2.0-X)/14.0*SIN(31.4159*X)*5.0
    GO TO 100
 13 UR=0.0
100 RETURN
    END
/*
// EXEC LNKEDT
/*
// EXEC
```

PROGRAM 2

```
//RS4TAEC  JOB .NODUMP,1177        R SCAVUZZO
// EXEC FORTRAN(BCD)
C      MAIN PROGRAM
       DIMENSION F(4, 900),OMM(3),CCC(1, 900),SSS(1, 900),SSD(2,3),SS(2,3
      1), SC(2,3),UV(1000),US(1000),XCD(2,3),DYX(2,3),
      1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2, 900),
      2DS3(2,3),ST(2,4,3),UD(2,2) ,W(3),WL(3),WM(3),XL(3),XB(3,3),
      3WN(3),C(22),SL(19)
        COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
      1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND,SC,UV,US,YFX,XCD,DYX
C      II  II=0 - MODAL INPUT
C      COM DIM F,CCC,SSS,U,F  SUB-DSDLI,FTAU  SUBDS2IL,CI,SI  SUB-DSDIL,V
C      SUB-TPLIG,ETA
C          II=1 - STRUCTURE INPUT
C      IB  IB=LESS THAN ZERO OR ZERO  - NO ROCKING INPUT
C          ABS(IB)=2 TERM OUTPUT
C          IB=1 OR 2 - ROCKING INPUT  - VELOCITY UNITS
C      NM=NUMBER OF MODES
C      W(I)=EFFECTIVE MASS LB-SEC*SEC/FT
C      WL(I)=MODAL EFFECTIVE FIRST MOMENT
C      WN(I)=MODAL EFFECTIVE SECOND MOMENT
C      WIO=BASE MOMENT OF INERTIA  (LB*SEC*SEC*FT*FT)/FT
C      WO =BASE MASS LB-SEC*SCE/FT
C      B=SHEAR WAVE VELOCITY  FT/SEC
C      G=SHEAR MODULUS  LB/FT/FT
C      AL=BUILDING WIDTH  FT
C      T,DT-SEC
C      AA BASE AREA FT*FT
C      UP(T)=LATERAL FREE FIELD ACCELERATION
C      UR(T)=FREE FIELD ROTATION (VELOCITY)
C      INPUT
       IPU=7
       IR=5
       IP=6
       REAL*8 UG1,UG12,UG13,UG14,UG15,UG2,UG22,UG23,UG24,UG25
   100 READ(IR,1)II,IB,NM,TMAX,DT,B,G,AL,AA,ERR
       IF(NM)101,100,103
     1 FORMAT(1X3I3,7E10.3)
   103 READ(IR,2)WO,WIO
       READ(IR,2) (OMM(I),I=1,NM)
     2 FORMAT (8E10.3)
       WRITE(IP,5)
     5 FORMAT(1X9H II IB NM,10X4HTMAX,12X2HDT,13X1HB,13X1HG,
      112X2HAL,12X2HAA,11X3HERR)
       WRITE(IP,6)II,IB,NM,TMAX,DT,B,G,AL,AA,ERR
     6 FORMAT(1X3I3,7E14.4)
       WRITE(IP,3)WO,WIO
     3 FORMAT(13X2HMO,12X2HIO/(1X2E14.2))
       WRITE(IP,7)(OMM(I),I=1,NM)
     7 FORMAT(7X8HOMEGA(I)/(1X8F14.3))
       IF(II)20,20,21
    20 READ(IR,2) (W(I),I=1,NM)
       READ(IR,2) (WL(I),I=1,NM)
       READ(IR,2) (WN(I),I=1,NM)
       GO TO 24
```

```
   21 READ(IR,2) (WM(I),I=1,NM)
      READ(IR,2)(XL(I),I=1,NM)
      DO 22 I=1,NM
   22 READ(IR,2) (XB(J,I),J=1,NM)
      DO 51 I=1,NM
      C1=0.0
      C2=0.0
      C3=0.0
      DO 23 J=1,NM
      C1=C1+WM(J)*XB(J,I)
      C2=C2+WM(J)*XB(J,I)*XB(J,I)
   23 C3=C3+WM(J)*XB(J,I)*XL(J)
      W(I)=C1*C1/C2
      WL(I)=C1*C3/C2
   51 WN(I)=C3*C3/C2
      WRITE(IP,11) (WM(I),I=1,NM)
      WRITE(IP,12) (XL(I),I=1,NM)
      WRITE(IP,13)
      DO 25 I=1,NM
   25 WRITE(IP,15) (XB(J,I),J=1,NM)
   24 CONTINUE
      WRITE(IP,14)
      WRITE(IP,8) (W(I),I=1,NM)
      WRITE(IP,9) (WL(I),I=1,NM)
      WRITE(IP,10) (WN(I),I=1,NM)
   10 FORMAT(11X4HN(J)/(1X8F14.3))
    9 FORMAT(10X5HML(J)/(1X8F14.3))
    8 FORMAT(11X4HM(J)/(1X8F14.3))
   14 FORMAT(1X16HMODAL PROPERTIES)
   11 FORMAT(1X20HSTRUCTURE PROPERTIES//1X4HW(I)/1X8F14.3)
   12 FORMAT(1X4HL(I)/1X8F14.3)
   13 FORMAT(1X6HX(I,J))
   15 FORMAT(1X8F14.3)
   16 FORMAT(1X6HOUTPUT//(14X1HT,10X4HU(T),10X4HR(T),9X5HUP(T),9X5HRP(T)
     1))
   17 FORMAT(15X18HTERMS OF EQUATIONS)
   18 FORMAT(1X7F14.4)
   19 FORMAT(15X6E14.5)
   50 FORMAT(8F10.5)
  104 FORMAT (9H DIVERGED)
C     OUTPUT
      WRITE(IP,16)
       IA=IABS(IB)
      IF(IA-2)65,26,26
   26 WRITE(IP,17)
      UG2=0.0
      ERA=ERR/100.
      ERS=ERR/100000.
      UG22=0.0
C     CONSTANTS FOR FIRST EQUATION
   65 T1=G*AA/B/WO
      T2=1.0/WO
      PI=3.14159
      T4=B/2.0/PI/AL/WO
      T6=T4*WO
```

```
      C1=0.464102
      T7=B*C1/AL/AL/WO
      T9=T7*WIO
      GA=0.211325
      T10=3.0*B*GA/AL/AL/WO
      T12=T10*WIO
      VR=B*0.9194
      T13=3.0*B*GA/VR/WO/AL
      T15=T13*WIO
      T16=3.0*B*B/PI/AL/AL/AL/WO
      T18=T16*WIO
C     CONSTANTS FOR SECOND EQUATION
      R3=1.73205
      BET=0.5
      ND=AL/0.9194/B/DT 0.5
      W1=G*AL*AL*AA/R3/B/WIO
      W2=AL*AL/2.0/R3/B/WIO
      W4=W2*WO
      W5=W2*BET
      W7=W5*WO
      W8=AL/2./R3/PI/WIO
      W10=W8*WO
      W11=1.0/WIO
      W13=3.0*B/2.0/PI/R3/WIO/AL
      W15=W13*WIO
      W19=W1
C     MODAL CONSTANTS
      T=0.0
      NP=TMAX/DT
      NP1=NP+1
      WRITE(IP,88)T1,T2,T4,T6,T7,T9,T10,T12
      WRITE(IP,88)T13,T15,T16,T18,C1,GA,VR
      WRITE(IP,88)W1,W2,W4,W5,W7,W8,W10,W11
      WRITE(IP,88)W13,W15,W19
   88 FORMAT(1X8E14.4)
      DO 28 I=1,NM
      DO 28 J=1,NP1
      CCC(I,J)=COS(OMM(I)*T)
      SSS(I,J)=SIN(OMM(I)*T)
   28 T=T DT
      U(1,1)=0.0
      U(2,1)=0.0
      T=0.0
      UD(2,1)=0.0
      UD(2,2)=0.0
      CALL FG124
      DO 108 J=1,NP1
  108 WRITE(IP,15)(F(I,J),I=1,4)
      IF(IB)29,29,30
   30 UD(2,2)=UR(T)
   29 UD(1,2)=UP(T)
      UV(1)=0.0
      US(1)=0.0
      T=0.0
      WRITE(IP,18)T,U(1,1  ),U(2,1  ),UD(1,2),UD(2,2)
```

```
      T=DT
      UGG1=0.0
      UG12=0.0
      UG13=0.0
      UG14=0.0
      UG15=0.0
      UG25=0.0
      UG24=0.0
      UG23=0.0
      UG22=0.0
      DO 41 IT=2,NP1
      IT1=IT-1
      IF(IB)31,31,32
   32 UD(2,1)=UD(2,2)
      UD(2,2)=UR(T)
   31 UD(1,1)=UD(1,2)
      UD(1,2)=UP(T)
      U(1,IT)=U(1,IT1)
      U(2,IT)=U(2,IT1)
      UV(IT)=UV(IT1)
      US(IT)=US(IT1)
      CALL TPLIG(T)
      CALL DS2IL
      CALL UFSIG
      CALL SSIGL
      CALL DSPIL
      CALL DSDLI
      DO 35 ITT=1,25
      CALL SIGL1
      DO 33 I=1,22
   33 C(I)=0.0
      C(1)=-T1*SUJ(1)
      C(6)=-T6*USF(1,1)
      C(9)=-T9*SUJ(2)
      C(12)=-T12*SUD
      C(15)=0.0
      C(18)=T18*DSF(1)*(-1.0)
      IND=IT-ND
      IF(IND)53,53,52
   52 C(15)=T15*U(2,IND)*(-1.0)
   53 C(21)=T18*USF(3,2)*(-1.0)
      C(22)=T1*SUP(1)
      DO 34 I=1,NM
      C(2)=C(2)-SS(1,I)*T2*W(I)*OMM(I)
      C(3)=C(3)+SS(2,I)*T2*WL(I)*OMM(I)
      C(4)=C(4)-T4*W(I)*OMM(I)*DS2(1,1,I)
      C(5)=C(5)+T4*WL(I)*OMM(I)*DS2(2,1,I)
      C(7)=C(7)-T7*WL(I)*DS3(1,I)*OMM(I)
      C(8)=C(8)+T7*WN(I)*DS3(2,I)*OMM(I)
      C(10)=C(10)+T10*WL(I)*DS1(1,I)*OMM(I)
      C(11)=C(11)-T10*WN(I)*DS1(2,I)*OMM(I)
      C(13)=C(13)+T13*WL(I)*SSD(1,I)*OMM(I)
      C(14)=C(14)-T13*WN(I)*SSD(2,I)*OMM(I)
      C(16)=C(16)+T16*WL(I)*ST(1,3,I)*OMM(I)
      C(17)=C(17)-T16*WN(I)*ST(2,3,I)*OMM(I)
```

```
      C(19)=C(19)+T16*WL(I)*DS2(1,3,I)*OMM(I)
   34 C(20)=C(20)-T16*WN(I)*DS2(2,3,I)*OMM(I)
      UG1=0.0
      DO 44 I=1,22
   44 UG1=UG1+C(I)
      DO 36 I=1,19
   36 SL(I)=0.0
      SL(4)= W4*UV(IT)
      SL(15)=-W15*USF(4,2)
      SL(1)=W1*SUJ(2)*(-1.0)
      SL(7)=0.0
      IF(IND)54,54,55
   55 SL(7)=W7*UV(IND)
   54 SL(10)= W10*USF(2,1)
      SL(18)=-W15*YFX
      SL(19)= W19*SUP(2)
      DO 37 I=1,NM
      SL(2)=SL(2)+W2*W(I)*SC(1,I)*OMM(I)*OMM(I)
      SL(3)=SL(3)-W2*WL(I)*SC(2,I)*OMM(I)*OMM(I)
      SL(5)=SL(5)+W5*W(I)*XCD(1,I)*OMM(I)*OMM(I)
      SL(6)=SL(6)-W5*WL(I)*XCD(2,I)*OMM(I)*OMM(I)
      SL(8)=SL(8)+W8*W(I)*DS2(1,2,I)*OMM(I)*OMM(I)
      SL(9)=SL(9)-W8*WL(I)*DS2(2,2,I)*OMM(I)*OMM(I)
      SL(11)=SL(11)+W11*WL(I)* SS(1,I)*OMM(I)
      SL(12)=SL(12)-W11*WN(I)* SS(2,I)*OMM(I)
      SL(13)=SL(13)+W13*WL(I)*DYX(1,I)*OMM(I)
      SL(14)=SL(14)-W13*WN(I)*DYX(2,I)*OMM(I)
      SL(16)=SL(16)+W13*WL(I)*DS2(1,4,I)*OMM(I)*OMM(I)
   37 SL(17)=SL(17)-W13*WN(I)*DS2(2,4,I)*OMM(I)*OMM(I)
      UG2=0.0
      DO 48 I=1,19
   48 UG2=UG2+SL(I)
      UV(IT)=1.0/70.0/DT*(26.0*UG15-27.0*UG14-40.0*UG13-13.0*UG12+54.0*
     1UG1)
      US(IT)=1.0/70.0/DT*(26.0*UG25-27.0*UG24-40.0*UG23-13.0*UG22+54.0*
     1UG2)
  892 IF(ITT-3)75,75,79
C     ERROR CRITERION
   79 DIFU=DABS(UG1-U(1,IT))
      DIFS=DABS(UG2-U(2,IT))
   76 IF(DIFU-ERR)38,75,75
   38 IF(DIFS-ERA)40,75,75
   75 U(1,IT)=UG1
      U(2,IT)=UG2
   35 CONTINUE
      GO TO 1033
   40 CALL DSPIL
      CALL DS2IL
      CALL DSDLI
      CALL SIGL1
      CALL TPLIG(T)
      CALL SSIGL
      CALL UFSIG
  203 U(1,IT)=UG1
      U(2,IT)=UG2
```

```
      UG15=UG14
      UG14=UG13
      UG13=UG12
      UG12=UG1
      UG25=UG24
      UG24=UG23
      UG23=UG22
      UG22=UG2
  204 WRITE(IP,18)T,U(1,IT ),U(2,IT ),UD(1,2),UD(2,2)
      IF(IA-2)41,42,42
   42 WRITE(IP,904) ITT,UG1,UG2,UV(IT),US(IT)
      WRITE(IP,19) (C(I),I=1,22)
      WRITE(IP,19) (SL(I),I=1,19) ,SUJ(2)
   41 T=T+DT
  904 FORMAT(20XI5,4E14.5)
      WRITE(IPU,50)(U(1,I),I=1,NP1)
      WRITE(IPU,50)(U(2,I),I=1,NP1)
      GO TO 100
 1033 WRITE (IP,104)
      WRITE(IP,904) ITT,UG1,UG2
      WRITE(IP,19) (C(I),I=1,22)
      WRITE(IP,19) (SL(I),I=1,19) ,SUJ(2)
  101 CALL EXIT
      END
      SUBROUTINE TPLIG (X)
C     TRIPLE  INTEGTATION
      DIMENSION F(4, 900),OMM(3),CCC(1, 900),SSS(1, 900),SSD(2,3),SS(2,3
     1), SC(2,3),UV(1000),US(1000),XCD(2,3),DYX(2,3),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2, 900),
     2DS3(2,3),ST(2,4,3),UD(2,2),
     3FFP(2,3),ETA(2,1, 900),FF(2,3),C(2,3),S(2,3 ),SXS(2,3),SXC(2,3)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND,SC,UV,US,YFX,XCD,DYX
      IF(X-DT)12,12,11
   12 DO 13 I=1,NM
      DO 13 J=1,2
      SXS(J,I)=0.0
      SXC(J,I)=0.0
      ETA(J,I,1)=0.0
      C(J,I)=0.0
      S(J,I)=0.0
      FFP(J,I)=0.0
   13 FF(J,I)=0.0
      Y=0.0
      NI=1
   11 IF(Y-X)10,14,14
   10 Y=Y+DT
      NI=NI+1
      NI1=NI-1
      DO 19 J=1,2
      DO 19 I=1,NM
      SXC(J,I)=C(J,I)
      SXS(J,I)=S(J,I)
   19 FFP(J,I)=FF(J,I)
C     XI INTEGRATION
```

```
   14 DO 15 J=1,2
      DO 15 I=1,NM
      C(J,I)=SXC(J,I)+DT/2.0*(U(J,NI)*CCC(I,NI)+U(J,NI1)*CCC(I,NI1))
   15 S(J,I)=SXS(J,I)+DT/2.0*(U(J,NI)*SSS(I,NI)+U(J,NI1)*SSS(I,NI1))
C     ETA INTEGRATION
      DO 16 J=1,2
      DO 16 I=1,NM
      FF(J,I)=SSS(I,NI)*C(J,I)-CCC(I,NI)*S(J,I)
   16 ETA(J,I,NI)=ETA(J,I,NI1)+DT/2.0*(FF(J,I)+FFP(J,I))
C     TAU INTEGRATION
      DO 18 L=1,NM
      DO 18 J=1,2
      DO 18 K=3,4
      N=NI
      ST(J,K,L)=0.0
      DO 18 I=1,NI1
      N1=N-1
      NN1=I+1
      ST(J,K,L)=ST(J,K,L)+DT/2.0*(F(K,NN1)*ETA(J,L,N1)+F(K,I)*ETA(J,L,N)
     1)
   18 N=N-1
C     ST(UD OR US,F(4),MODE)
      RETURN
      END
      SUBROUTINE DSDLI
C     INTEGRATION   OF U(XI)*SIN(W*(TAU-XI)) WITH TIME DELAY
      DIMENSION F(4, 900),OMM(3),CCC(1, 900),SSS(1, 900),SSD(2,3),SS(2,3
     1), SC(2,3),UV(1000),US(1000),XCD(2,3),DYX(2,3),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2, 900),
     2DS3(2,3),ST(2,4,3),UD(2,2)   ,DSS(2,3),D33(2,3)
     2 ,DUS(2,3),DUSS(2,3),DUC(2,3),DUCC(2,3),FTAU(2,1, 900)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND,SC,UV,US,YFX,XCD,DYX
      IF(T-DT)12,12,11
   12 X=0.0
      NI=1
      DO 14 I=1,NM
      DO 14 J=1,2
      FTAU(J,I,1)=0.0
      DS3(J,I)=0.0
      DS1(J,I)=0.0
      DUC(J,I)=0.0
      DUS(J,I)=0.0
      DUSS(J,I)=0.0
   14 DUCC(J,I)=0.0
   11 IF(X-T)10,13,13
   10 X=X+DT
      NI=NI+1
      NI1=NI-1
      DO 15 J=1,2
      DO 15 I=1,NM
      DUSS(J,I)=DUS(J,I)
      DSS(J,I)=DS1(J,I)
      D33(J,I)=DS3(J,I)
   15 DUCC(J,I)=DUC(J,I)
```

```
   13 DO 16 J=1,2
      DO 16 I=1,NM
      DUC(J,I)=DUCC(J,I)+DT/2.*(U(J,NI)*SSS(I,NI)+U(J,NI1)*SSS(I,NI1))
      DUS(J,I)=DUSS(J,I)+DT/2.0*(U(J,NI)*CCC(I,NI)+U(J,NI1)*CCC(I,NI1))
      FTAU(J,I,NI)=SSS(I,NI)*DUS(J,I)-CCC(I,NI)*DUC(J,I)
   16 DS3(J,I)=D33(J,I)+DT/2.0*(FTAU(J,I,NI)+FTAU(J,I,NI1))
      IF(NI-ND)17,17,18
   18 DO 20 J=1,2
      DO 20 I=1,NM
      N=NI-ND+1
      N1=N-1
      DS1(J,I)=DSS(J,I)+DT/2.0*(FTAU(J,I,N)+FTAU(J,I,N1))
   20 CONTINUE
   17 RETURN
      END
      SUBROUTINE UFSIG
C     UFS(F(4),UD OR US)
C     INTEGRATION OF U(J,T-TAU)*F(K,T)
      DIMENSION F(4, 900),OMM(3),CCC(1, 900),SSS(1, 900),SSD(2,3),SS(2,3
     1), SC(2,3),UV(1000),US(1000),XCD(2,3),DYX(2,3),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2, 900),
     2DS3(2,3),ST(2,4,3),UD(2,2)
        COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND,SC,UV,US,YFX,XCD,DYX
      IF(T-DT)12,12,11
   12 X=0.0
      NI=1
   11 IF(X-T)10,13,13
10    X=X+DT
      NI=NI+1
      NI1=NI-1
   13 N=NI
      DO 15 I=1,2
      DO 15 J=1,4
   15 USF(J,I)=0.0
      YFX=0.0
      TT=0.0
      DO 20 IT=1,NI1
      IT1=IT+1
      NN=N-1
      TT1=TT+DT
   21 USF(1,1)=DT/2.0*(U(1,N)*F(1,IT)+U(1,NN)*F(1,IT1))+USF(1,1)
      USF(2,1)=DT/2.0*(UV(N)*F(2,IT)+UV(NN)*F(2,IT1))+USF(2,1)
      USF(3,2)=DT/2.0*(U(2,N)*F(3,IT)*TT U(2,NN)*F(3,IT1)*TT1)+USF(3,2)
      USF(4,2)=USF(4,2)+DT/2.0*(U(2,N)*F(4,IT) +  U(2,NN)*F(4,IT1))
      YFX=YFX+DT/2.0*(US(N)*F(4,IT)*TT+US(NN)*F(4,IT1)*TT1)
      TT=TT+DT
   20 N=N-1
C     USF(F(4), UD OR US)
      RETURN
      END
      SUBROUTINE FG124
C     CALCULATION OF LAMB FUNCTIONS
      DIMENSION F(4, 900),OMM(3),CCC(1, 900),SSS(1, 900),SSD(2,3),SS(2,3
     1), SC(2,3),UV(1000),US(1000),XCD(2,3),DYX(2,3),
```

```
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2, 900),
     2DS3(2,3),ST(2,4,3),UD(2,2)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND,SC,UV,US,YFX,XCD,DYX
      YRT=1.0/SQRT(3.0)
      Y=0.0
      NPI=NP+1
      DO 21 I=1,NPI
      X=B*Y/AL
      X2=X*X
      X4=X2*X2
      CK1=(3.0+SQRT(3.0))/2.0
      CK2=(3.0-SQRT(3.0))/2.0
      CK3=CK1/2.0
      CK4=CK2 /2.0
      R1=X2-1.0
      RTD=X2-1.0/3.0
      RP5=X2-0.5
      R14=X2-0.25
      SR14P=X2-CK3
      SR14M=X2-CK4
      IF(X-YRT)20,20,11
   11 DOM4=2.0*X*R14*SR14P*SR14M
      IF(X-1.0)12,12,13
   20 DO 22 J=1,4
   22 F(J,I)=0.0
      GO TO 21
   12 DOM1=RP5**4-X4*RTD*R1
      SR11=SQRT(1.0-X2)
      SRTD=SQRT(RTD)
      SQTD=SRTD
      SQ14=SQRT(R14)
      F(1,I)= X*R1*SRTD/DOM1
      F(2,I)=-3.*RP5*X2*SRTD*SR11/DOM4
      F(3,I)=-3.0*RP5*SRTD*SR11/2.0/DOM4
      F(4,I)=-3.0*SQTD*RP5*RP5/DOM4
      GO TO 21
   13 SR1=SQRT(X2-1.0)
      SRTD=SQRT(RTD)
      SQTD=SRTD
      SQ14=SQRT(R14)
      F(1,I)=SR1/(X*(RP5*RP5-X2*SRTD*SR1))
      F(2,I)=0.0
      F(3,I)=0.0
      F(4,I)=-3.0*SRTD*(RP5*RP5+X2*SRTD*SR1)/ DOM4
      IF(ABS(F(1,I))-24.)30,30,31
   31 F(1,I)=24.0*F(1,I)/ABS(F(1,I))
   30 IF(ABS(F(4,I))-24.)32,32,33
   33 F(4,I)=24.0*F(4,I)/ABS(F(4,I))
   32 CONTINUE
   21 Y=Y DT
      RETURN
      END
      SUBROUTINE SIGL1
C     SIMPLE AND DOUBLE INTEGRATION ROUTINE
```

```
   DIMENSION F(4, 900),OMM(3),CCC(1, 900),SSS(1, 900),SSD(2,3),SS(2,3
  1), SC(2,3),UV(1000),US(1000),XCD(2,3),DYX(2,3),
  1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2, 900),
  2DS3(2,3),ST(2,4,3),UD(2,2)
  3     ,YU(2),YP(2)
    COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
  1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND,SC,UV,US,YFX,XCD,DYX
   IF(T-DT)12,12,11
12 X=0.0
   NI=1
   DO 16 J=1,2
   SUP(J)=0.0
16 SUJ(J)=0.0
   SUD=0.0
   SUJJ=0.0
   DU2=0.0
11 IF(X-T)10,13,13
10 NI=NI+1
   NI1=NI-1
   X=X+DT
   DU=DU2
   SUJJ=SUJ(2)
   SU=SUD
   DO 17 J=1,2
   YU(J)=SUJ(J)
17 YP(J)=SUP(J)
13 DO 14 J=1,2
   SUJ(J)=YU(J)+DT/2.0*(U(J,NI)+U(J,NI1))
14 SUP(J)=YP(J)+DT/2.*(UD(J,1)+UD(J,2))
   IF(NI-ND)18,18,19
19 N=NI-ND+1
   N1=N-1
   SUD=SU+DT/2.0*(U(2,N)+U(2,N1))
18 RETURN
   END
   SUBROUTINE DS2IL
   DOUBLE INTEGRATION ROUTINE - MODIFIED
   DIMENSION F(4, 900),OMM(3),CCC(1, 900),SSS(1, 900),SSD(2,3),SS(2,3
  1), SC(2,3),UV(1000),US(1000),XCD(2,3),DYX(2,3),
  1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2, 900),
  2DS3(2,3),ST(2,4,3),UD(2,2)
  3,CI(2,1, 900),SI(2,1, 900)
    COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
  1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND,SC,UV,US,YFX,XCD,DYX
   IF(T-DT)12,12,11
12 X=0.0
   DO 14 I=1,NM
   DO 14 J=1,2
   CI(J,I,1)=0.0
14 SI(J,I,1)=0.0
   NI=1
11 IF(X-T) 10,13,13
10 X=X+DT
   NI=NI+1
   NI1=NI-1
```

```
13 DO 15 L=1,NM
   DO 15 J=1,2
   CI(J,L,NI)=CI(J,L,NI1)+( U(J,NI1)*CCC(L,NI1)+U(J,NI)*CCC(L,NI))*DT
  1/2.0
15 SI(J,L,NI)=SI(J,L,NI1)+( U(J,NI1)*SSS(L,NI1)+U(J,NI)*SSS(L,NI))*DT
  1/2.0
   DO 21 L=1,NM
   DO 21 J=1,2
   DO 21 K=1,4
   C1=0.0
   C3=0.0
   C2=0.0
   C4=0.0
   N=NI
   XT=0.0
   DO 20 I=1,NI1
   N1=N-1
   NN1=I 1
   IF(K-3)17,18,18
18 CC=F(K,I)*CCC(L,I)*XT
   RY=F(K,I)*SSS(L,I)*XT
   XT1=XT DT
   SS1=F(K,NN1)*SSS(L,NN1)*XT1
   CC1=F(K,NN1)*CCC(L,NN1)*XT1
   GO TO 19
17 RY=F(K,I)*SSS(L,I)
   CC=F(K,I)*CCC(L,I)
   CC1=F(K,NN1)*CCC(L,NN1)
   SS1=F(K,NN1)*SSS(L,NN1)
19 C1=(CC*CI(J,L,N)+CC1*CI(J,L,N1))*DT/2.0+C1
   C2=(CC*SI(J,L,N)+CC1*SI(J,L,N1))*DT/2.0+C2
   C3=(RY*CI(J,L,N)+SS1*CI(J,L,N1))*DT/2.0+C3
   C4=(RY*SI(J,L,N)+SS1*SI(J,L,N1))*DT/2.0+C4
   XT=XT DT
20 N=N-1
   CC=CCC(L,NI)
   RY=SSS(L,NI)
   IF(K-2)30,31,32
32 IF(K-4)30,31,21
30 DS2(J,K,L)=RY*C1-CC*C2-CC*C3-RY*C4
   GO TO 21
31 DS2(J,K,L)=CC*C1+RY*C2+RY*C3-CC*C4
21 CONTINUE
   DO 23 L=1,NM
   DO 23 J=1,2
   C1=0.0
   C2=0.0
   C3=0.0
   C4=0.0
   N=NI
   XT=0.0
   DO 24 I=1,NI1
   N1=N-1
   NN1=I+1
   RY=F(4,I)*SSS(L,I)
```

```
      CC=F(4,I)*CCC(L,I)
      CC1=F(4,NN1)*CCC(L,NN1)
      SS1=F(4,NN1)*SSS(L,NN1)
      C1=(CC*CI(J,L,N)+CC1*CI(J,L,N1))*DT/2.0+C1
      C2=(CC*SI(J,L,N)+CC1*SI(J,L,N1))*DT/2.0+C2
      C3=(RY*CI(J,L,N)+SS1*CI(J,L,N1))*DT/2.0+C3
      C4=(RY*SI(J,L,N)+SS1*SI(J,L,N1))*DT/2.0+C4
      XT=XT+DT
   24 N=N-1
      CC=CCC(L,NI)
      RY=SSS(L,NI)
   23 DYX(J,L)   =RY*C1-CC*C2-CC*C3-RY*C4
C     DS2(UD OR US,F(4),MODE)
  100 RETURN
      END
      SUBROUTINE DSPIL
C     DOUBLE INTEGRATION OF U(2,T)*F(3 OR 4,TAU)
      DIMENSION F(4, 900),OMM(3),CCC(1, 900),SSS(1, 900),SSD(2,3),SS(2,3
     1), SC(2,3),UV(1000),US(1000),XCD(2,3),DYX(2,3),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2, 900),
     2DS3(2,3),ST(2,4,3),UD(2,2),V( 900)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND,SC,UV,US,YFX,XCD,DYX
      IF(T-DT)12,12,11
   12 V(1)=0.0
      X=0.0
      NI=1
   11 IF(X-T)10,13,13
   10 NI=NI+1
      NI1=NI-1
      X=X+DT
   13 V(NI)=V(NI1)+DT/2.0*(U(2,NI)+U(2,NI1))
      DSF(1)=0.0
      DSF(2)=0.0
      DO 15 K=3,4
      N=NI
      DO 15 I=1,NI1
      N1=N-1
      I1=I+1
      K1=K-2
      DSF(K1)=DSF(K1)+DT/2.0*(V(N)*F(K,I)+V(N1)*F(K,I1))
   15 N=N-1
C     D(F(3,T) OR F(4,T))   FOR U(2,T)
      RETURN
      END
      SUBROUTINE SSIGL
C     SINGLE INTEGRATION ROUTINE- MODIFIED   - TIME DELAY INCLUDED
      DIMENSION F(4, 900),OMM(3),CCC(1, 900),SSS(1, 900),SSD(2,3),SS(2,3
     1), SC(2,3),UV(1000),US(1000),XCD(2,3),DYX(2,3),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2, 900),
     2DS3(2,3),ST(2,4,3),UD(2,2)
     4, DIS(2,3),DIC(2,3),C(2,3),S(2,3) ,CD(2,3),SD(2,3),DDC(2,3),DDS(2,
     53)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND,SC,UV,US,YFX,XCD,DYX
```

```
      IF(T-DT)12,12,11
 12   DO 15 I=1,NM
      DO 15 J=1,2
      DIS(J,I)=0.0
      DIC(J,I)=0.0
      XCD(J,I)=0.0
      SSD(J,I)=0.0
      DDS(J,I)=0.0
      DDC(J,I)=0.0
      CD(J,I)=0.0
      SD(J,I)=0.0
      C(J,I)=0.0
 15   S(J,I)=0.0
      X=0.0
      NI=1
 11   IF(X-T)10,13,13
 10   X=X+DT
      NI=NI+1
      NI1=NI-1
      DO 16 J=1,2
      DO 16 I=1,NM
      DDC(J,I)=DDC(J,I)+CD(J,I)
      DDS(J,I)=DDS(J,I)+SD(J,I)
      DIC(J,I)=DIC(J,I)+C(J,I)
 16   DIS(J,I)=DIS(J,I)+S(J,I)
 13   DO 17 J=1,2
      DO 17 I=1,NM
      C(J,I)=(U(J,NI1)*CCC(I,NI1)+U(J,NI)*CCC(I,NI))*DT/2.0
      S(J,I)=(U(J,NI1)*SSS(I,NI1)+U(J,NI)*SSS(I,NI))*DT/2.0
      SC(J,I)=CCC(I,NI)*(C(J,I)+DIC(J,I))+SSS(I,NI)*(S(J,I)+DIS(J,I))
 17   SS(J,I)=SSS(I,NI)*(C(J,I)+DIC(J,I))-CCC(I,NI)*(S(J,I)+DIS(J,I))
      IF(NI-ND)18,18,19
 19   N=NI-ND+1
      N1=N-1
      DO 20 J=1,2
      DO 20 I=1,NM
      CD(J,I)=(U(J,N1)*CCC(I,N1)+U(J,N)*CCC(I,N))*DT/2.0
      SD(J,I)=(U(J,N1)*SSS(I,N1)+U(J,N)*SSS(I,N))*DT/2.0
      XCD(J,I)=CCC(I,N)*(CD(J,I)+DDC(J,I))+SSS(I,N)*(SD(J,I)+DDS(J,I))
 20   SSD(J,I)=SSS(I,N )*(CD(J,I)+DDC(J,I))-CCC(I,N)*(SD(J,I)+DDS(J,I))
 18   RETURN
      END
      FUNCTION UP(X)
      DIMENSION F(4, 900),OMM(3),CCC(1, 900),SSS(1, 900),SSD(2,3),SS(2,3
     1), SC(2,3),UV(1000),US(1000),XCD(2,3),DYX(2,3),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2, 900),
     2DS3(2,3),ST(2,4,3),UD(2,2)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND,SC,UV,US,YFX,XCD,DYX
       IF(X-0.8)10,10,11
 10   UP=X/8.0*SIN(31.4159*X)*5.0
      GO TO 100
 11   IF(X-2.)12,12,13
 12   UP=(2.0-X)/14.0*SIN(31.4159*X)*5.0
      GO TO 100
```

```
   13 UP=0.0
  100 RETURN
      END
      FUNCTION UR(X)
      DIMENSION F(4, 900),OMM(3),CCC(1, 900),SSS(1, 900),SSD(2,3),SS(2,3
     1), SC(2,3),UV(1000),US(1000),XCD(2,3),DYX(2,3),
     1DSF(2),SUJ(2),SUP(2),DS1(2,3),DS2(2,4,3),USF(4,2),U(2, 900),
     2DS3(2,3),ST(2,4,3),UD(2,2)
       COMMON F,OMM,CCC,SSS,SSD,SS,DSF,SUJ,SUP,SUD,DS1,DS2,USF,U ,
     1DS3,ST,UD,T,DT,DU2,  B,AL,NP,NM,ND,SC,UV,US,YFX,XCD,DYX
      IF(X-0.8)10,10,11
   10 UR=X/8.0*SIN(31.4159*X)*5.0
      GO TO 100
   11 IF(X-2.)12,12,13
   12 UR=(2.0-X)/14.0*SIN(31.4159*X)*5.0
      GO TO 100
   13 UR=0.0
  100 RETURN
      END
/*
// EXEC LNKEDT
/*
// EXEC
```