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TRANSIENT RESPONSE OF TRANSDUCERS
Definitions and Characteristics

BY

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ABSTRACT

SUMMARY - CONCLUSIONS

Discussion and reporting of transient response characteristics of transducers and measuring systems is often clouded by the confusing terminology associated with this particular branch of our modern technology. Meaningful communication between the designer, fabricator, evaluator, and user of such a system can result only if there is unified agreement concerning use of the terminology. It is thus intended that this document should provide clarification of definitions of many terms used in transducer instrumentation technology. An alphabetical listing of common terms and their definitions is presented. In addition, a discussion of the response of first- and second-order systems to step and ramp inputs is included.

It is shown that a single term does not adequately define the transient response of a transducer system. Response characteristics should be given in terms of well-defined parameters which are important to the measurement. If steady-state frequency response is of prime importance, the 3 dB bandwidth, phase characteristics, and amplitude peaking should be specified. If it is a transient response which is of importance, the risetime, delay time, overshoot, and settling time should be specified. The consistent use of well-defined terms will prevent many of the problems which have often been a part of the characterization of the transient response of transducer systems.

TABLE OF CONTENTS

TRANSIENT RESPONSE OF TRANSDUCERS

Definitions and Characteristics

LIST OF FIGURES

Page

TRANSIENT RESPONSE OF TRANSDUCERS

Definitions and Characteristics

I. INTRODUCTION

The "response time", "frequency response", or "speed of response" of a measurement system or component is often quoted in an ambiguous manner which leaves the reader uncertain as to the exact meaning of the parameter. Since many other terms are also used to describe the transient response characteristics of transducing systems, confusion is generally prevalent. Meaningful communication between the designer, fabricator, evaluator, and user of such a system can result only if there is uniform agreement on the terminology used to describe the system. It is thus intended that this document should provide definitions of the terminology used in describing the response and behavior of transducing systems. All phases of the system response, from application of the driving force until final steady-state outputs are attained, will be discussed. The transient response of a typical transducer can be divided into several regions including delay time, risetime, overshoot, and settling time. Each plays an important part in defining the overall response of the transducing system or component. Additional terminology must be used to define the above parameters and to further evaluate the system performance.

Accuracy, precision, range, span, and repeatability are terms which are also an integral part of the confusing transducer terminology. Unified definitions of each are necessary before meaningful communications in this field are possible. Thus, a list of pertinent terms and corresponding definitions has been prepared and is presented in following sections of this report. A discussion of transducer operating characteristics is also included to provide a working model of transducer response relationships.

II. TERMINOLOGY

Terms used to describe instrumentation components and systems are often used incorrectly. There is often misunderstanding concerning the definition of terms. For example, the terms "accuracy" and "precision" are often used interchangeably. Time constant, response time, and risetime are often incorrectly used to describe the same parameter. Terms which are commonly used in discussing the dynamic response of a transducer system have been separated into three categories. These include the general headings of A.) Response Time, B.) Accuracy, and C.) General System Behavior. Listings of the three groups of terms follow. Definitions of each of the terms are given in Section III of this report.

A. Response Time

Damping Time

Dead Time

Delay Time

Equivalent Time Constant (τ_e)

Frequency Response

Initial Delay Time

Lag Time

Response Time

Risetime (τ_r)

Settling Time

Time Constant (τ)

Transient Response

B. Accuracy

Accuracy

Deviation (d_i)

Drift

Error

Least Detectable Increment

Linearity

Noise

Precision

Random Errors

Repeatability

Sensitivity

Systematic Errors

Transient Error

Variance (σ^2)

C. General System Behavior

Amplitude Peaking (M)

Average Measurement

Bandwidth (BW)

Characteristic Equation

Critically Damped

Damped Natural Frequency (ω_d)

Damping Factor (ζ)

Differential Measurement

First-Order System

Hysteresis

Natural Frequency (ω_n)

Overdamped

Overshoot

Phase Shift

Range

Resonant Frequency (ω_r)

Ringing

Second-Order System

Span

Transfer Function

Underdamped

III. DEFINITIONS

Accuracy:

The accuracy of an instrument indicates the deviation of the reading from a known input. It is usually expressed as a number or quantity defining the error as the difference between the indicated and actual values. These can be expressed as a percent of the instrument range or as a percent of the actual value.

Amplitude Peaking (M):

The absolute value of the ratio of the maximum amplitude to the mean or average amplitude of an amplitude vs frequency plot. A second-order system response is presented in Figure 1.

$$M = \left| \frac{A_{max}}{A_{mean}} \right|$$

Average Measurement:

The arithmetic average (mean) of two or more simultaneous or sequential measurements of the parameter.

Bandwidth (BW):

The band of frequencies lying between the two frequencies where the system frequency response is $1/\sqrt{2}$ (3 dB down) or 70.7% of the peak system response. The bandwidth-system response relationship is shown in Figure 2. Bandwidth can also be obtained from the amplitude vs frequency plot (Bode diagram) presented in Figure 1. Bandwidth information is often presented as a normalized function of the damped and undamped natural frequencies of a system, i.e., the ratio of the 3 dB frequency to the natural frequency. Such a presentation is made in Figure 15 of Section IV of this report.

Characteristic Equation:

$am^2 + bm + c = 0$. The measuring system can be described by a second-order homogeneous differential equation of the form

$$a \frac{d^2x}{dt^2} + b \frac{dx}{dt} + cx = d. \qquad (1)$$

This can be expressed in terms of the damping factor ζ and natural frequency ω_n as follows:

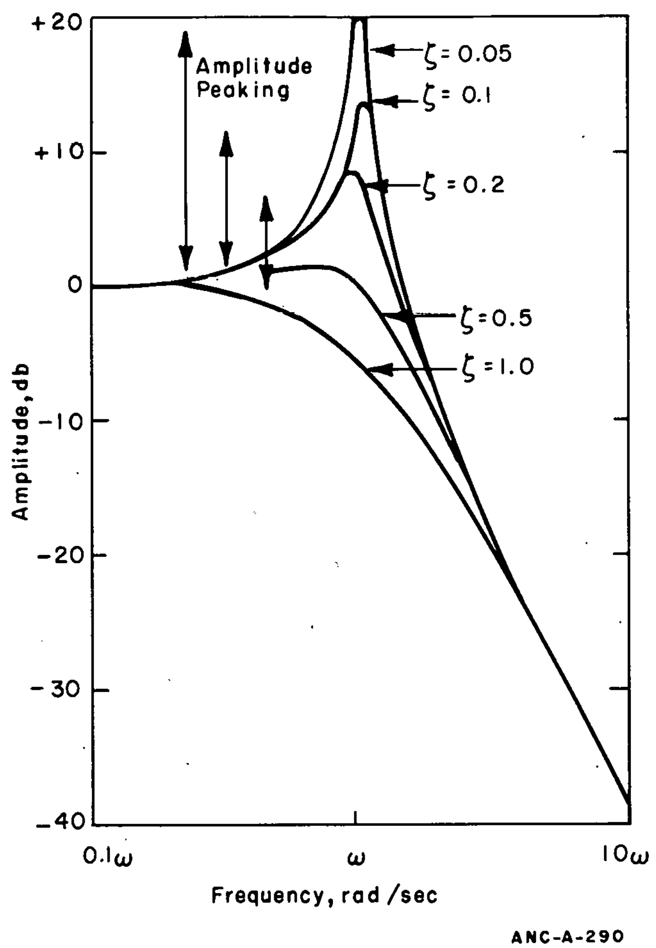

Figure 1. Amplitude versus Frequency Diagram for Second-Order Systems

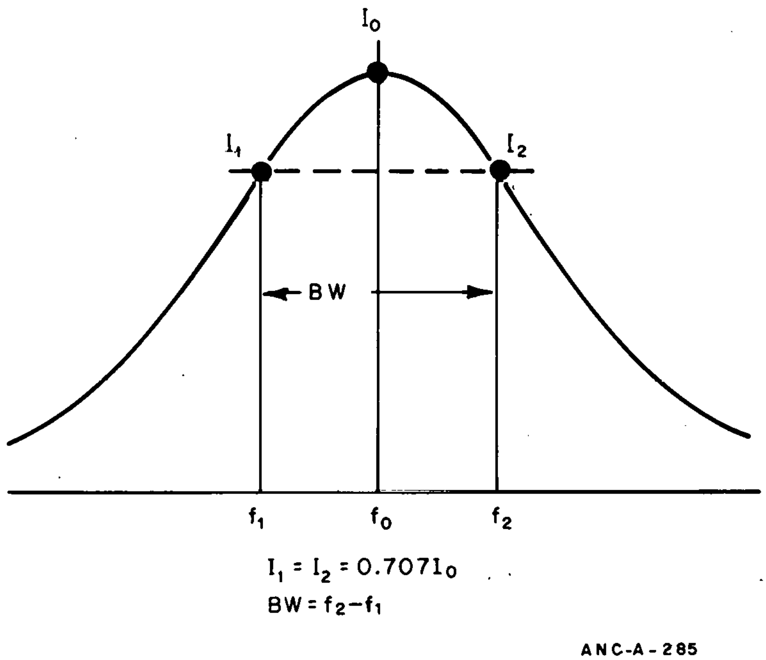

Figure 2. Bandwidth of a Resonant Circuit

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n \frac{dx}{dt} + \omega_n^2 + \omega_n^2 x = \frac{d}{a} \qquad (2)$$

where

$$\frac{c}{a} = \omega_n^2$$

$$\frac{b}{2\sqrt{ac}} = \zeta .$$

Solution of Equation (2) is of the form

$$x = Ae^{mt} . \qquad (3)$$

Substitution and factoring of Equation (1) yields the general form

$$e^{mt}(am^2 + bm + c) = 0 \qquad (4)$$

which must be satisfied if Equation (3) is a solution. Since e^{mt} can never be zero, it is necessary that

$$am^2 + bm + c = 0. \qquad (5)$$

This purely algebraic equation is known as the characteristic equation of Equation (1).

Critically Damped:

The characteristic equation of the system has two real, equal roots. The damping ratio $\zeta = 1$. The response of a critically damped second-order system can be seen in Figure 8 of Section IV.

Damped Natural Frequency (ω_d):

The frequency of vibration of real damped systems. The damped natural frequency of a second-order system can be expressed in terms of the undamped natural frequency ω_n and the damping factor ζ as follows:

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} .$$

Damping Factor (ζ):

A measure of the oscillatory behavior of the system. It has the following physical significance:

$$\zeta = \frac{\text{actual damping}}{\text{damping for critical response}} .$$

Since ζ influences the roots of the characteristic equation for a system, its value is a measure of the actual damping in the system. When,

$\zeta < 1$ the system is underdamped,

$\zeta = 1$ the system is critically damped,

$\zeta > 1$ the system is overdamped.

The damping factor of a second-roder system can be expressed in terms of the damped natural frequency ω_d and the undamped material frequency ω_n as follows:

$$\zeta = \sqrt{1 - \left(\frac{\omega_d}{\omega_n}\right)^2} .$$

Damping Time

Time interval measured from when the system output first reaches 90% of final value until the time after which it remains within a specified percentage of the final value. Damping time is shown schematically in the first-order response curve presented in Figure 6 of Section IV.

Dead Time:

The interval of time between the impression of the input on an element or system and the initial response to the input.

Delay Time:

Time interval between the application of a step function to a system input and the time at which the system output response reaches 10% of its final value. This is shown in the first-order response curve presented in Figure 6 of Section IV.

Deviation (d_1):

The difference between an individual measurement and the arithmetic mean or average of all measurements. Thus,

$$d_i = x_i - x_m$$

where

x_i represents an individual measurement,

x_m represents the arithmetic mean of the measurements.

The standard deviation or root-mean-square deviation is defined by

$$\sigma = \left[\frac{1}{n} \sum_{i=1}^{n} (x_i - x_m)^2\right]^{1/2} = \left[\frac{1}{n} \sum_{i=1}^{n} d_i^2\right]^{1/2} .$$

Differential Measurement:

The algebraic difference of two like parameters obtained by simultaneous measurement of these or, by direct measurement of the difference.

Drift:

A very low frequency component in output which is not produced by the input signal.

Equivalent Time Constant (τ_e):

The time required for an output quantity to change by an amount equal to 63.2% of the total change that it will experience in response to a step change in input. This is shown in the first-order response curve presented in Figure 6 of Section IV.

Error:

Difference between indicated and true value. Usually expressed as a percent of full scale or percent of reading.

First-Order System:

A system which has a behavior that can be described by a first-order differential equation. The transfer function for a first-order system is

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{\tau_e s + 1}$$

where

$E_o(s)$ is the system output,

$E_i(s)$ is the system input or driving function,

τ_e is the equivalent time constant of the system,

s is the Laplacian operator.

Frequency Response:

The frequency dependent ratio of system output to system input for sinusoidal signals. It is a complex function having both magnitude and phase. A frequency response plot in terms of amplitude (Bode diagram) is shown in Figure 1. Bandwidth specifications are often erroneously listed as frequency response requirements.

Hysteresis:

An instrument is said to exhibit hysteresis when there is a difference in the indicated value of a parameter as the measurement is approached from above or below the real value. That is, the output is dependent upon the sign of the rate-of-change of input, not upon the magnitude of the rate-of-change. Hysteresis may be the result of mechanical friction, magnetic effects, elastic deformation, or thermal effects. It is normally specified as a percent of the full range of the instrument.

Initial Delay Time:

See Dead Time.

Lag Time:

The time span between the actual occurrence of a given value of a parameter and when the instrumentation indicates this value. The lag time is shown on the transient error diagram in Figure 3.

Least Detectable Increment:

The minimum change in input which will produce a detectable change in output.

Linearity:

A measure of the deviation of the transfer function from a linear function. Linearity is usually expressed as the percent of full scale that the output deviates from a linear function. This concept is shown in Figure 4.

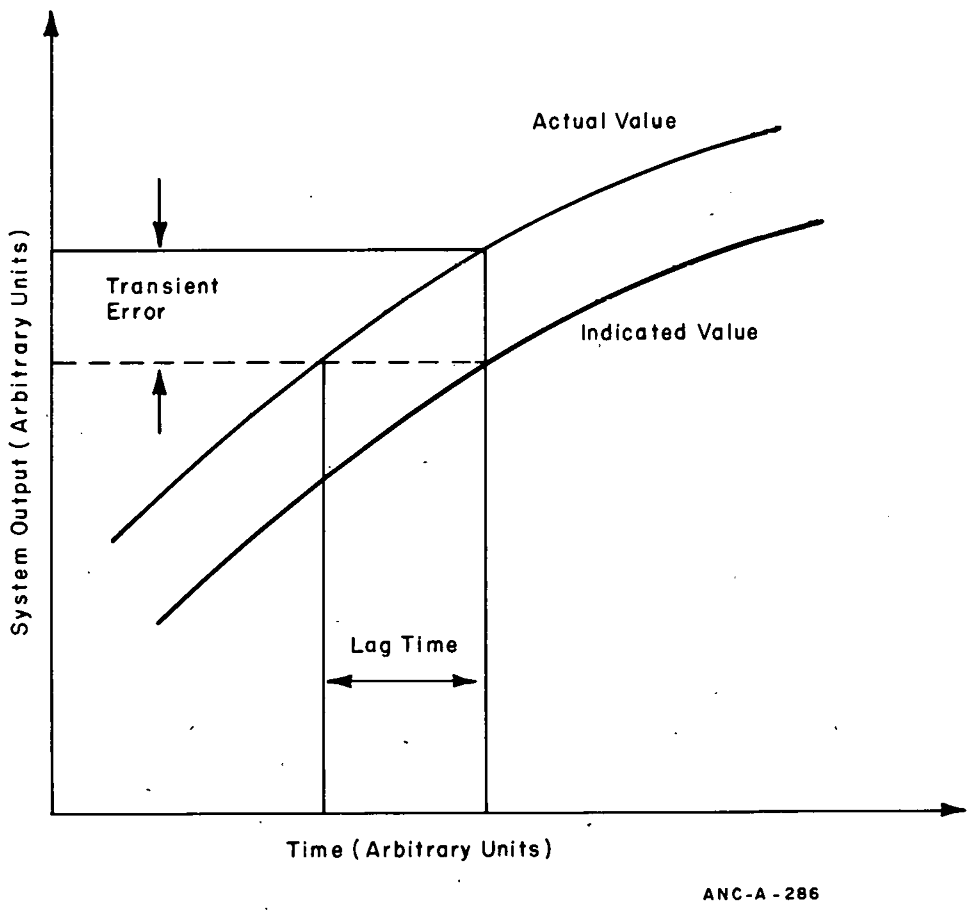

Figure 3. Transient Error in a Measuring Instrument

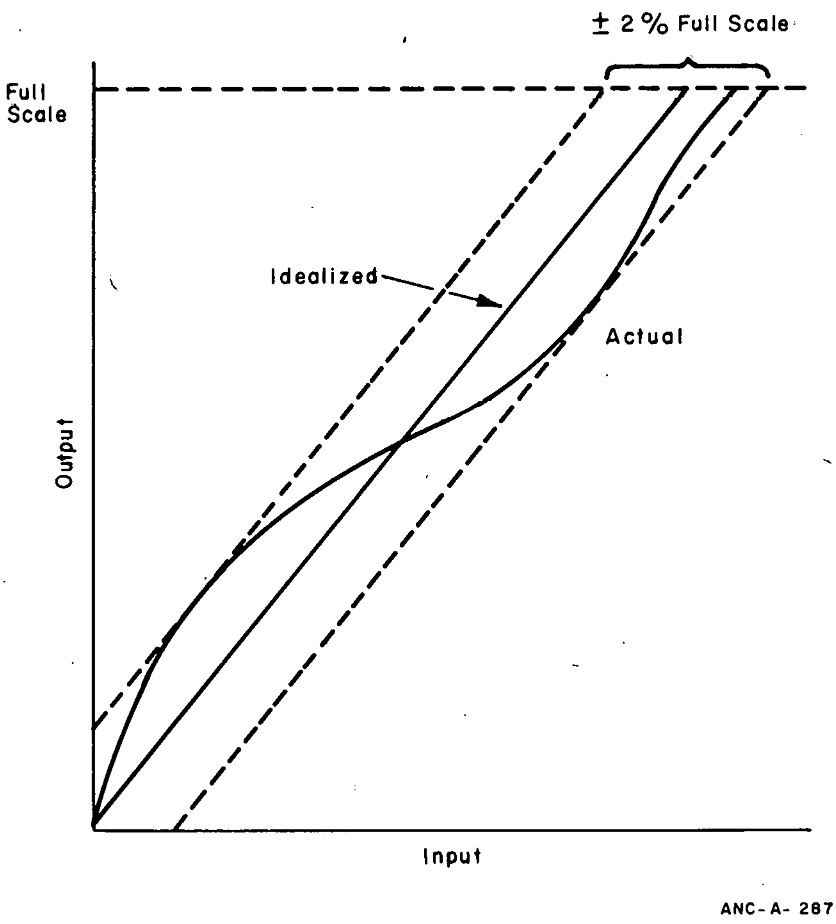

Figure 4. Departure of a Transfer Function from Linearity

Natural Frequency (ω_n):

The frequency of vibration or oscillation of an excited system which is free from the influence of external damping. Also called the undamped natural frequency. For a second-order system whose behavior is described by:

$$\alpha \frac{d^2x}{dt^2} + \beta \frac{dx}{dt} + \gamma x = f(x),$$

the natural frequency is

$$\omega_n = \sqrt{\frac{\gamma}{\alpha}}.$$

Noise:

Variation in system output when the system input is held constant. Noise levels may be expressed as peak-to-peak values or as a root-mean-square (rms) value.

Overdamped:

The characteristic equation of the system has two real, unequal roots. The damping ratio $\zeta > 1$. Overdamped response in a second-order system is shown in Figure 8 of Section IV.

Overshoot:

The difference between the peak and steady-state or equilibrium system outputs for a step change in input. In terms of the damping factor ζ, the percent overshoot is expressed as:

$$\%\ \text{overshoot} = 100 \exp - \frac{\zeta\pi}{\sqrt{1-\zeta^2}}.$$

Overshoot in the output of a second-order system is shown in Figure 7 of Section IV.

Phase Shift:

The frequency dependent difference between the phase of an output sinusoid and the phase of the corresponding input sinusoid. Defined for sinusoidal signals only. The effect of the damping factor ζ on phase shift over a range of input frequencies is shown in Figure 5.

Precision:

The ability to reproduce an output for a constant input with a given accuracy.

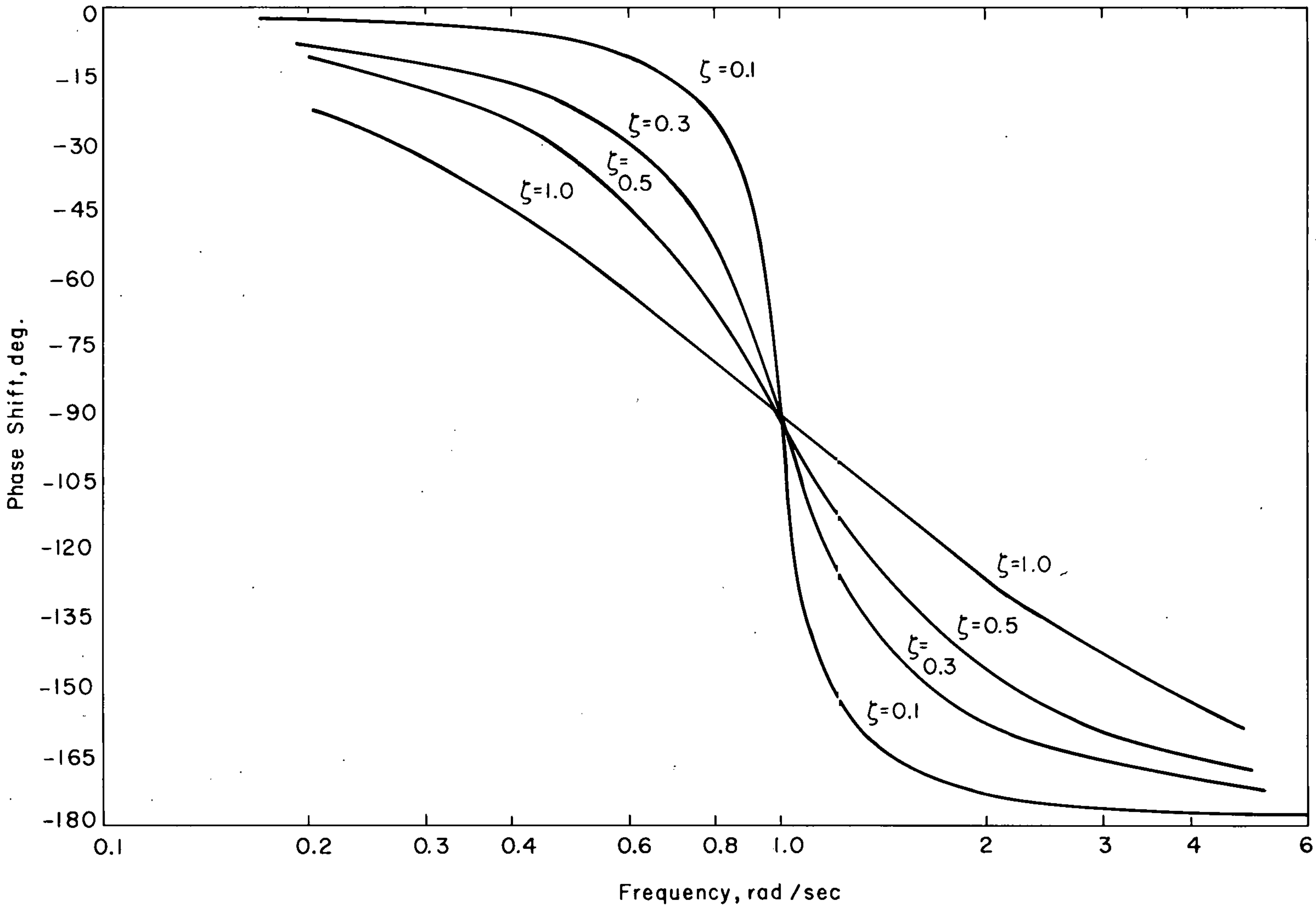

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Figure 5. Effect of the damping Factor on Phase Shift for a Second-Order System

Random Errors:

Errors resulting from random fluctuations in the instruments, various influences of friction, etc.

Range:

The difference between the maximum and minimum parameter values to which a system may be applied.

Repeatability:

See Precision.

Resonant Frequency (ω_r):

The frequency at which the maximum gain (ratio of output to input) occurs. In terms of the damping factor ζ, the natural frequency ω_n, the resonant frequency for a second-order system is:

$$\omega_r = \omega_n \sqrt{1-2\zeta^2} \ .$$

Response Time:

The interval of time from the initiation of a step change in input until the output reaches 90% of the final value. The response time is shown schematically in Figure 6 of Section IV.

Ringing:

Oscillatory output resulting from a step input. As in a greatly under-damped system. (See Figure 8 of Section IV, $\zeta = 0.1$.)

Risetime (τ_r):

The length of time between the 10% and 90% values on the transient response curve for a step input. The risetime is indicated in Figure 6 of Section IV.

Second-Order System:

A system which has a behavior that can be described by a second-order differential equation. The transfer function for a second-order system is

$$\frac{E_o(S)}{E_i(S)} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

where

$E_o(S)$ is the system output

$E_i(S)$ is the system input or driving function

ω_n is the undamped natural frequency

ζ is the system damping factor

S is the Laplacian operator.

It should be noted that $S^2 + 2\zeta\omega_n S + \omega_n^2 = 0$ is the characteristic equation for this system.

Sensitivity:

The ratio of the output change to a specific change in the measured parameter.

Settling Time (τ_Δ):

Sum of the delay time, risetime, and damping time. Time required for the output to reach and stay within a specified percentage of the steady-state value. The time to settle within 5% is indicated in Figure 6 of Section IV.

Span:

That portion of the instrument range for which the instrument is calibrated to perform a definite function.

Systematic Errors:

Fixed errors which cause repeated measurements to be in error by roughly the same amount. Usually inherent in the system.

Time Constant (τ):

Equal to the equivalent time constant τ_e for transducer systems. $\tau = RC$ for a simple circuit containing resistance R and capacitance C.

Transfer Function:

The relationship between the system input and its output. This is usually expressed in the following form:

$$E_o(S) = f[E_i(S)],$$ where S is a Laplacian operator.

Transient Error:

The difference between the system error at any time and the steady-state system error for a specified input. The transient error is shown schematically in Figure 3.

Transient Response:

The response or output history of a system for a step input.

Underdamped:

The roots of the characteristic equation are complex conjugate numbers. The damping factor $\zeta < 1$. The system tends to display an oscillatory behavior in response to a step input. Underdamped response in a second-order system can be seen in Figure 8 of Section IV.

Variance (σ^2):

The square of standard or root-mean-square deviation.

IV. TRANSDUCER SYSTEM CHARACTERISTICS - TRANSIENT AND STEADY-STATE BEHAVIOR

As previously mentioned, there is much confusion about the terminology applied to transient response characteristics. To help clarify the definitions given in the preceding section, a discussion of some of these terms, as applied to first and second-order systems, is presented below. Parameters used to describe the transient response of a transducer system are presented in a normalized form as functions of the system damping factor. Thus, a common basis of comparison is obtained.

A. First-Order Systems

1. Step Input

Consider a first-order system having no initial delay. The transfer function for such a system is

$$\frac{E_o(S)}{E_i(S)} = \frac{1}{\tau S+1} \qquad (1)$$

where

$E_o(S)$ is the system output

$E_i(S)$ is the system input or driving function

τ is the time constant of the system

S is the Laplacian operator.

The response of this system to a step input such that for time $t < 0$, $E_i(S) = 0$ and for $t > 0$, $E_i(S) = A$ is

$$E_o(t) = A \left(1-e^{\frac{-t}{\tau}} \right) . \qquad (2)$$

The response of such a system to a step input is shown graphically in Figure 6. As can be seen from Figure 6, the risetime τ_r is about 2.2 times the time constant τ. Thus, for a first-order system:

$$\tau_r = 2.2\ \tau . \qquad (3)$$

2. Ramp Input

Because physical phenomena occur at a finite rate, a step input analysis will not always be appropriate. Thus, consider the response of a first-order system to a ramp input. For this case the response can be expressed as

$$E_o(t) = M \left[t+\tau \left(e^{\frac{-t}{\tau}} - 1 \right) \right] \qquad (4)$$

where M is the rate of change of the input or driving function. The response of a first-order system to a ramp input is shown in Figure 7. As can be seen, the dynamic error soon attains a fixed value. This results in a decreasing error percentage as time progresses ($t >> \tau$). Referring again to Figure 7, it can be seen that the maximum error occurs after several time constants have passed. The magnitude of the error is the difference between the ideal and actual outputs of the system. Since this value divided by time interval τ is equal to the slope of the input ramp, the maximum error can be expressed as follows,

$$\text{Maximum error} = \tau M. \qquad (5)$$

Where, τ is the time constant of the system and M is the rate of change of the ramp input.

Because these systems are represented by linear differential equations, the solutions can be superposed. Thus, combinations of ramp and step inputs can be used in the analysis of a problem. This greatly simplifies the analysis of the dynamic response of first-order systems.

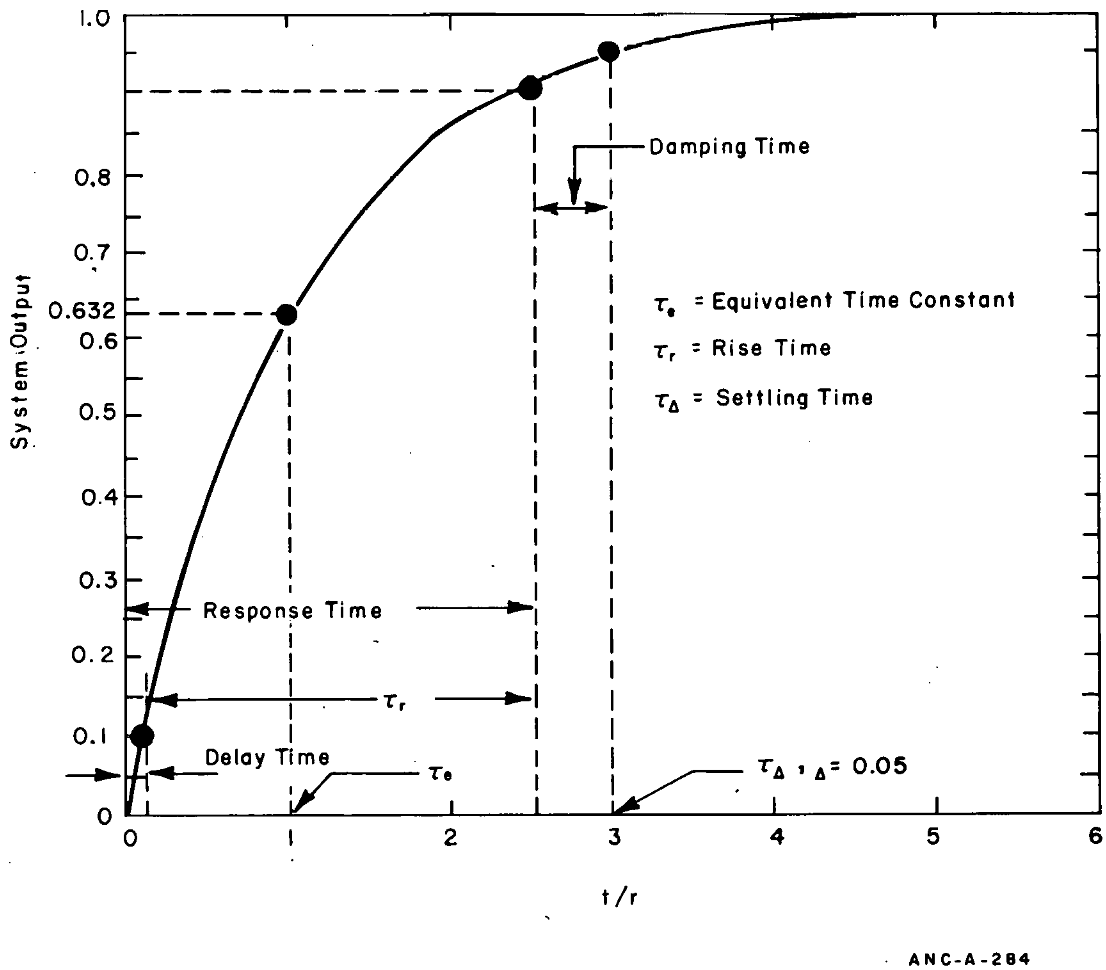

Figure 6. Response of a First-Order System to a Step Input

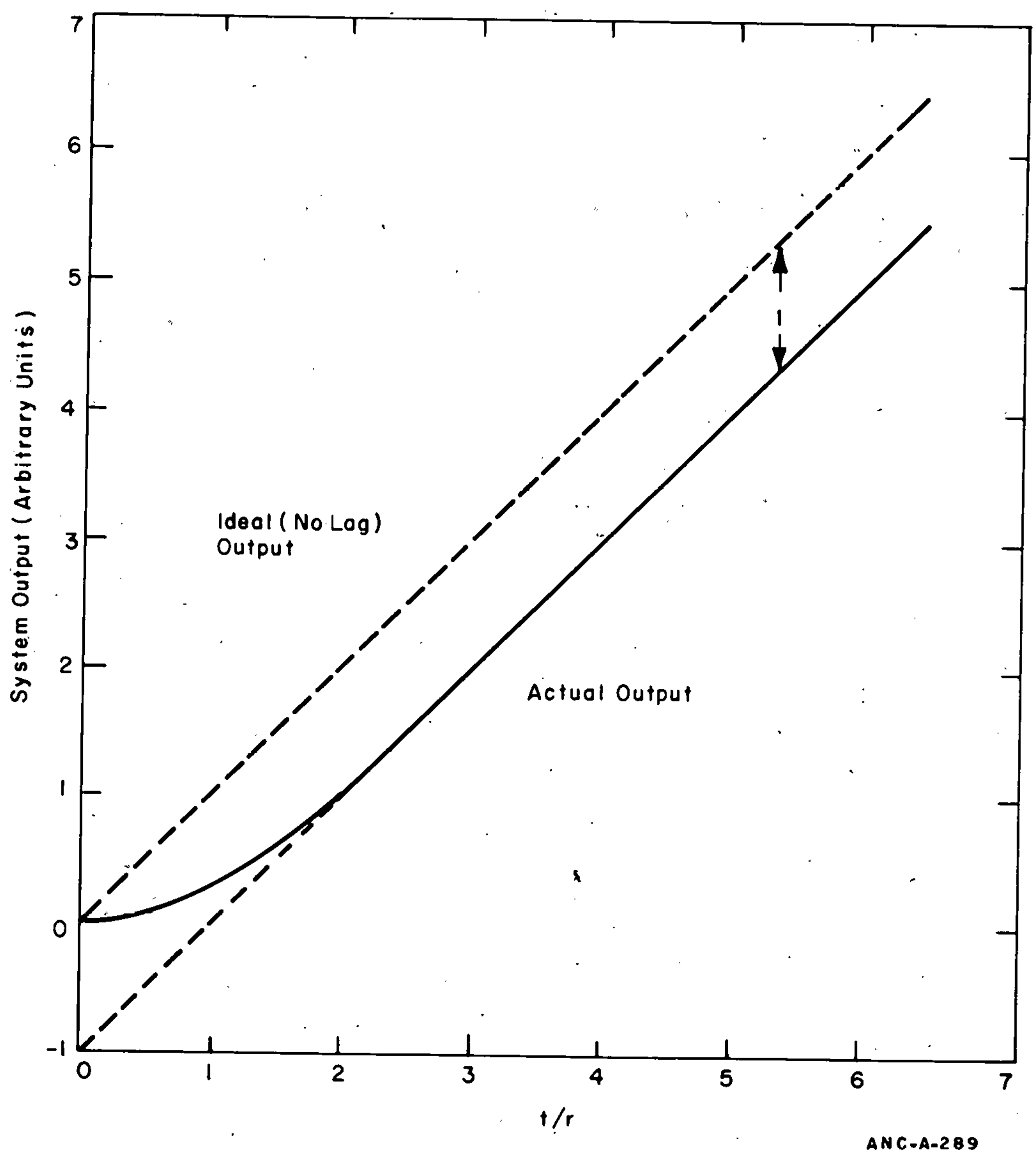

Figure 7. Response of a First-Order System to a Ramp Input

3. Steady-State Frequency Response

The bandwidth of the first-order system is determined from the two frequencies at which the system response is reduced by 3 dB (70.7% of peak response). The 3 dB frequency is given by

$$f_{3dB} = \frac{\omega_{3db}}{2\pi} = \frac{0.35}{\tau_r} = \frac{0.16}{\tau} . \qquad (6)$$

B. Second-Order Systems

1. Step Input

Although some transducer systems can be adequately described by the mathematics of a first-order system, the real effects of friction, inertia, and finite mass distributions usually place transducers in the realm of second-order systems. This in effect requires a more precise mathematical description of the system parameters.

The general transfer function for the second-order system is:

$$\frac{E_o(S)}{E_i(S)} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2} \qquad (7)$$

where

$E_o(S)$ is the system output

$E_i(S)$ is the system input

$\omega_n = 2\pi f_n$, the undamped natural frequency

ζ is the damping factor

S is a Laplacian operator.

The transducer response described by Equation (7) can be obtained by solving for the roots of its characteristic equation,

$$S^2 + 2\zeta\omega_n S + \omega_n^2 = 0 . \qquad (8)$$

Thus the roots are

$$S_{1,2} = \omega_n \left(- \zeta \pm \sqrt{\zeta^2-1}\right) \qquad (9)$$

From this solution three distinct cases develop which are dependent upon the value of the damping factor ζ.

For:

$\zeta < 1$, the roots are conjugate complex numbers and the system is underdamped.

$\zeta = 1$, the roots are real and equal, the system is critically damped.

$\zeta > 1$, the roots are real and unequal and the system is said to be overdamped.

Second-order system solutions corresponding to these three cases are as follows.

For a step input in which $E_i(S) = 0$ for $t < 0$ and $E_i(S) = A$ for $t > 0$, the solution for the underdamped case ($\zeta < 1$) is:

$$E_o(t) = A\left[1 - \frac{1}{\sqrt{1-\zeta^2}} \exp(-\zeta\omega_n t)\sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \phi\right)\right] \tag{10}$$

where $$\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\,. \tag{11}$$

The system response to a step input when critically damped ($\zeta = 1$) is described by:

$$E_o(t) = A\,[1-(1+\omega_n t)\exp(-\omega_n t)]\,. \tag{12}$$

The overdamped system ($\zeta > 1$) will respond to a step input A in the following manner:

$$E_o(t) = A\left[1- \frac{1}{2\sqrt{\zeta^2-1}}\left(\frac{\exp(-\zeta+\sqrt{\zeta^2-1}\omega_n t}{\zeta-\sqrt{\zeta^2-1}} - \frac{\exp(-\zeta-\sqrt{\zeta^2-1}\omega_n t}{\zeta+\sqrt{\zeta^2-1}}\right)\right]\,. \tag{13}$$

The response of a second-order system to a step input is shown for various values of the damping factor ζ in Figure 8.

The risetime τ_r of a second-order system in response to a step input can be approximated by the following expression:

$$\tau_r \cong \frac{4.4}{\omega_n}\sqrt{\frac{\zeta^4 + 0.46\,\zeta^2 + 0.0889}{\zeta^2 + 1.655}} \tag{14}$$

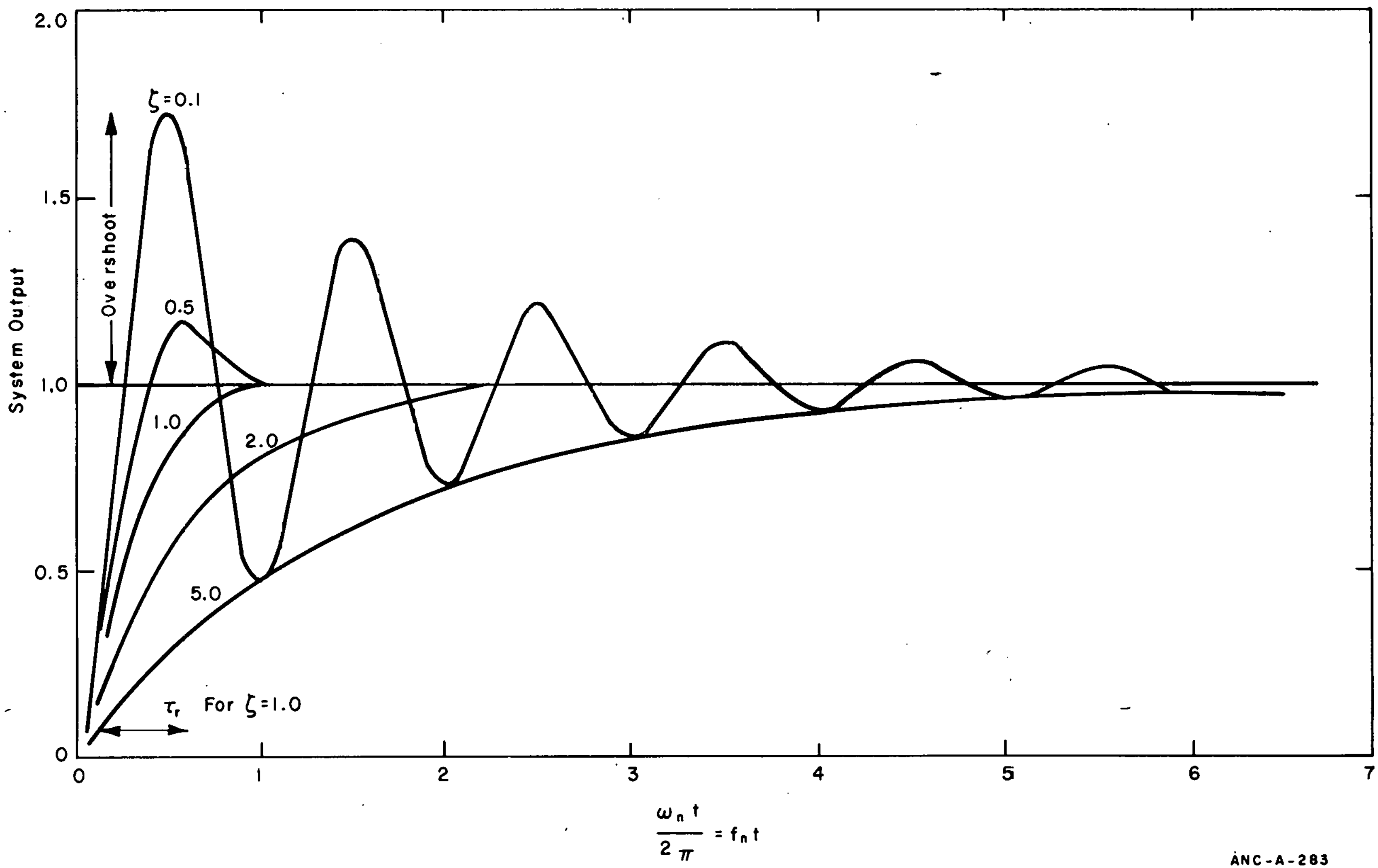

Figure 8. Response of a Second-Order System to a Step Input

The risetime of such a system is shown as a function of the damping factor in Figure 9. It will be noted that two curves are shown in this figure. One curve represents the risetime which has been normalized to the undamped natural frequency ω_n, while the other is normalized to the damped natural frequency ω_d by using the relationship $\omega_d = \omega_n \sqrt{1-\zeta^2}$. The risetime of a typical second-order response to a step input is also indicated in Figure 8.

The equivalent time contant τ_e (time constant) is calculated from second-order response curves such as shown in Figure 8. It is the time required for the output to reach 63.2% of the step input and, as can be seen, is also a strong function of the damping factor. Equivalent time constants, normalized to both the damped and undamped natural frequencies are shown in Figure 10.

Just as the time constant can be determined from the system response curves, so too can the settling time τ_Δ. Since the settling time is defined as the time required for the output to settle to within a specified percentage of the steady-state value, one time is obtained for each specified percentage. Part of this family of settling time curves are shown as functions of the damping factor in Figure 11. Again, the settling time has been normalized to the natural frequency ω_n.

Also apparent in the response of a second-order system to a step input is overshoot and ringing. As shown in Figure 8, these also are controlled by the system damping factor. The percent overshoot for a second-order system can be calculated from the following expression.

$$\text{Percent Overshoot} = 100 \exp\left[\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right] \tag{15}$$

The overshoot as a function of the damping factor is shown in Figure 12.

2. Ramp Input

Solutions to the characteristic equation [Eq. (8)] when the input increases linearly with time (ramp) are given by the following. For an underdamped system where $\zeta < 1$:

$$E_o(t) = \frac{M}{\omega_n}\left[\frac{1}{\sqrt{1-\zeta^2}} \exp\left(-\zeta\omega_n t\right) \sin\left(\sqrt{1-\zeta^2}\ \omega_n t+\phi\right. - 2\zeta+\omega_n t\right] \tag{16}$$

$$\text{where} \quad \phi = \tan^{-1} \frac{\zeta\sqrt{1-\zeta^2}}{\zeta^2-1/2} \quad \text{and} \tag{17}$$

M is the rate-of-change of the input.

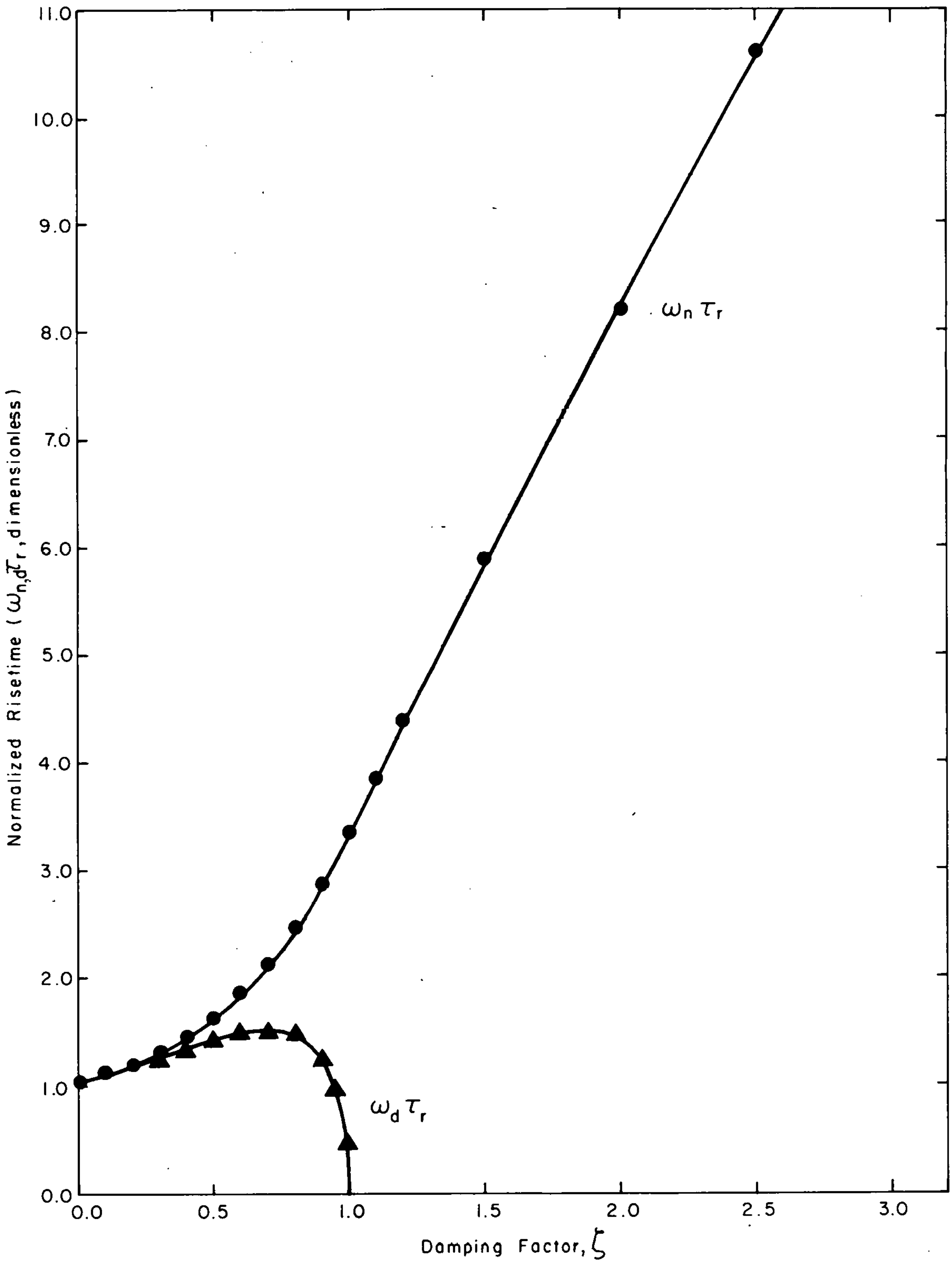

Figure 9. Normalized Risetime versus Damping Factor for a Second-Order System Having a Step Input

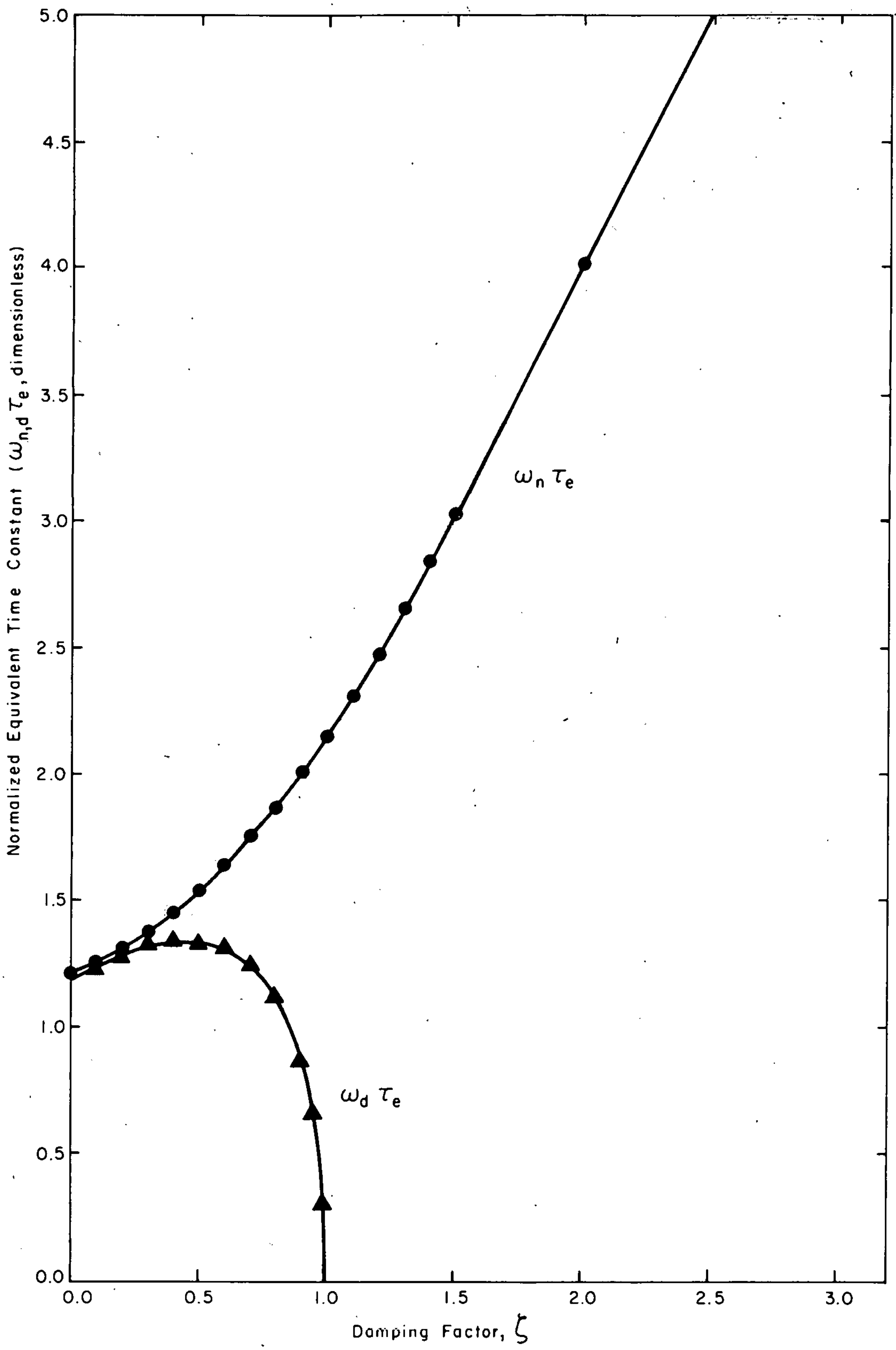

Figure 10. Normalized Equivalent Time Constant versus Damping Factor for a Second-Order System Having a Step Input

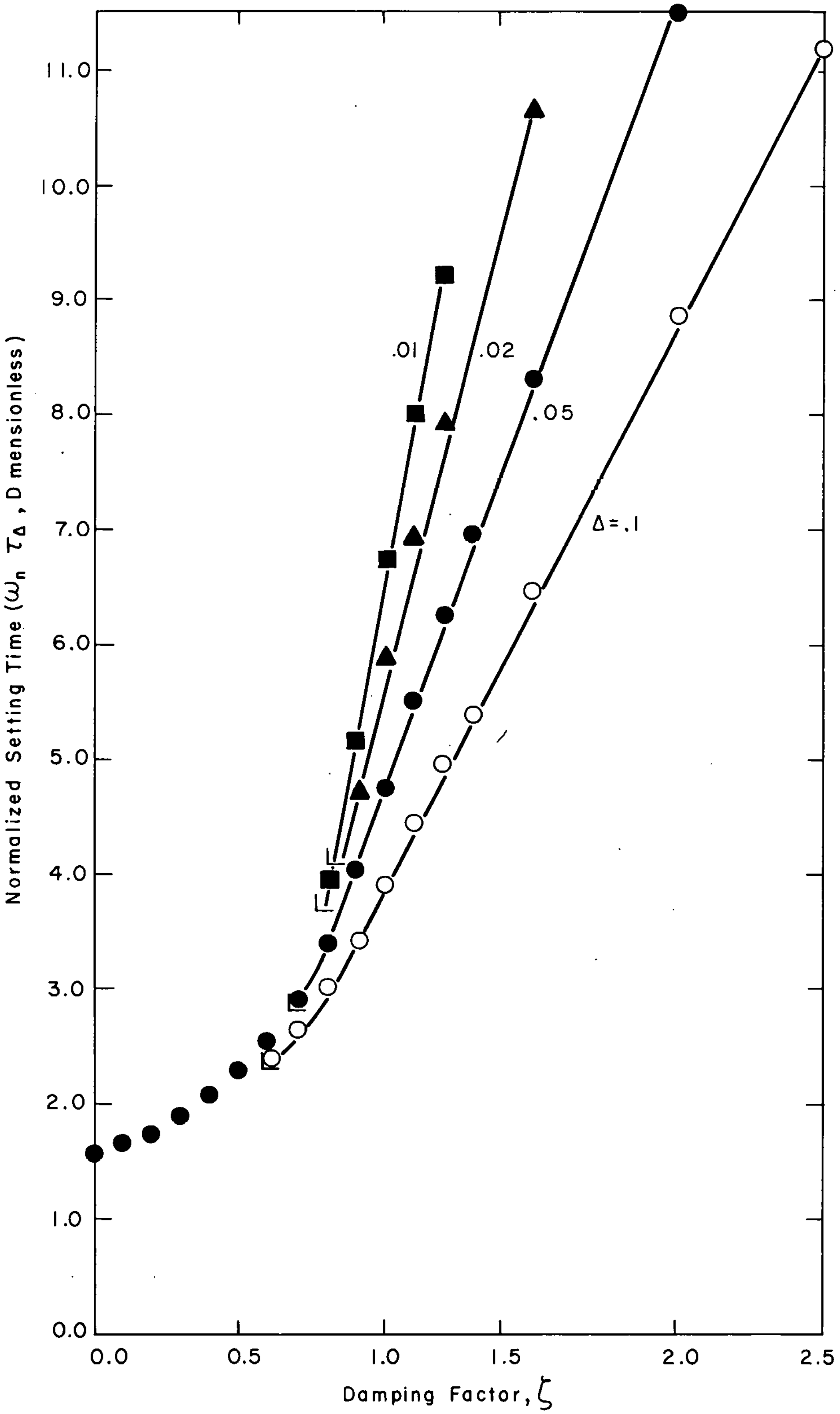

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Figure 11. Normalized Settling Time versus Damping Factor for a Second-Order System Having a Step Input. Settling to within Δ of the Final Value.

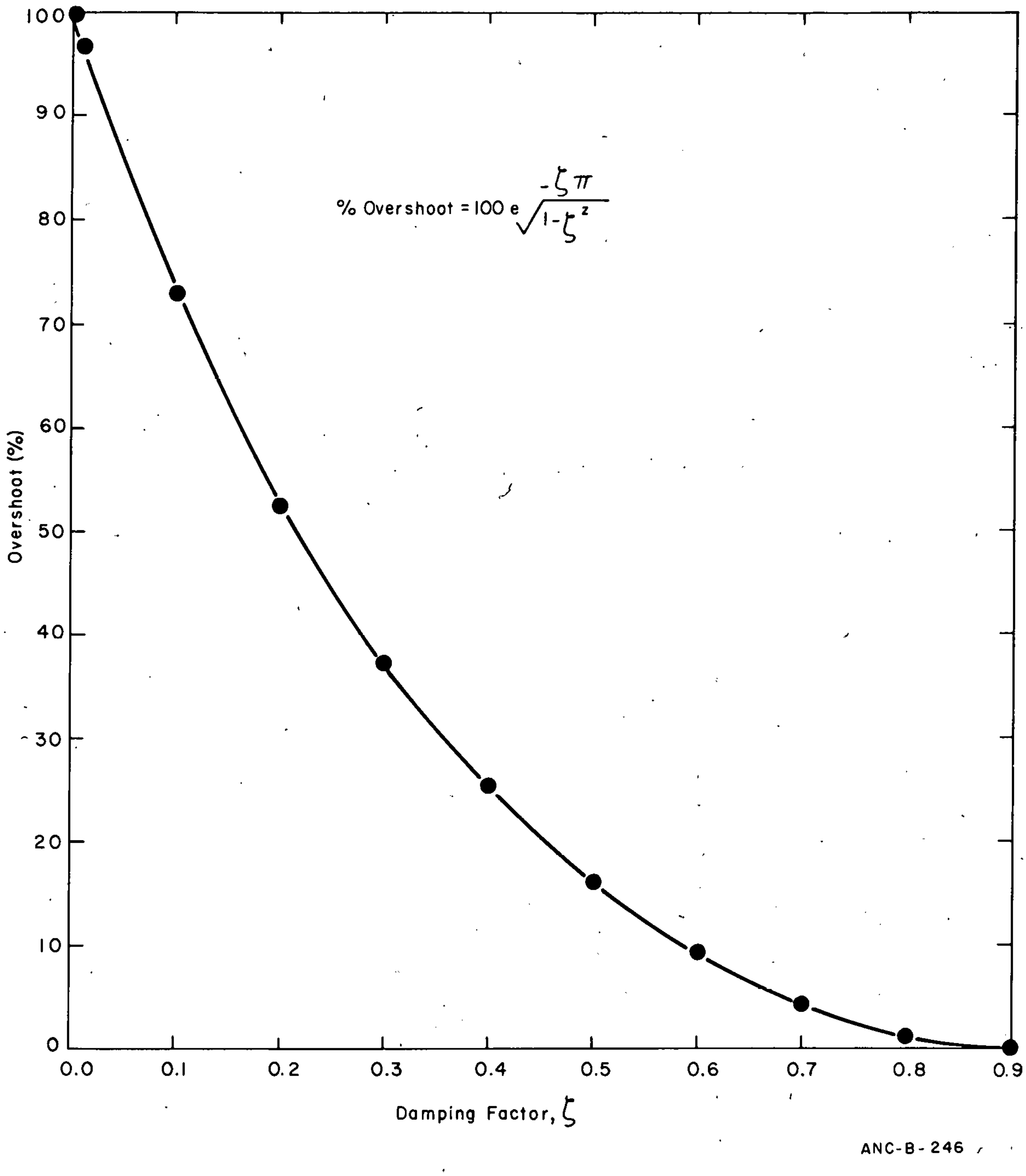

Figure 12. Overshoot versus Damping Factor for a Second-Order System Having a Step Input

When the system is critically damped and $\zeta = 1$:

$$E_o(t) = \frac{M}{\omega_n}[(2+\omega_n t)\exp(-\omega_n t) + \omega_n t - 2]. \tag{18}$$

The overdamped case in which $\zeta > 1$ is described by the following equation:

$$E_o(t) = \frac{M}{\omega_n}\left\langle \frac{\nu\sqrt{\nu}}{\nu-1}\left[\exp\left(-\frac{\omega_n t}{\sqrt{\nu}}\right) - \frac{1}{\nu^2}\exp(\sqrt{\nu}\,\omega_n t)\right] - \frac{\nu+1}{\sqrt{\nu}} + \omega_n t\right\rangle \tag{19}$$

where $$\nu = \frac{\zeta + \sqrt{\zeta^2-1}}{\zeta - \sqrt{\zeta^2-1}} . \tag{20}$$

The response of a second-order system to a ramp input is shown for several values of the damping factor ζ in Figure 13.

The maximum error for a given rate of change of input M is given by:

$$\text{Maximum Error} = \frac{2\zeta M}{\omega_n} . \tag{21}$$

This will define the response parameter ζ/ω_n and if the damping factor ζ is selected to comply with optimum design criteria, the risetime, time constant, etc., can be determined in a manner similar to that used for the step input.

3. Steady-State Frequency Response

The amplitude peaking for a second-order system can also be expressed as a function of the damping factor. The peaking factor M, which represents the ratio of the maximum to mean signal amplitudes, is expressed as a function of the damping factor as follows:

$$M = \left|\frac{A_{max}}{A_{mean}}\right| = \frac{1}{2\zeta\sqrt{1-\zeta^2}} . \tag{22}$$

This relationship is shown graphically in Figure 14. For convenience the peaking is shown both in terms of the peaking factor M, and in decibels.

The 3 dB bandwidth of a second-order system can also be obtained from a knowledge of the system damping factor. For

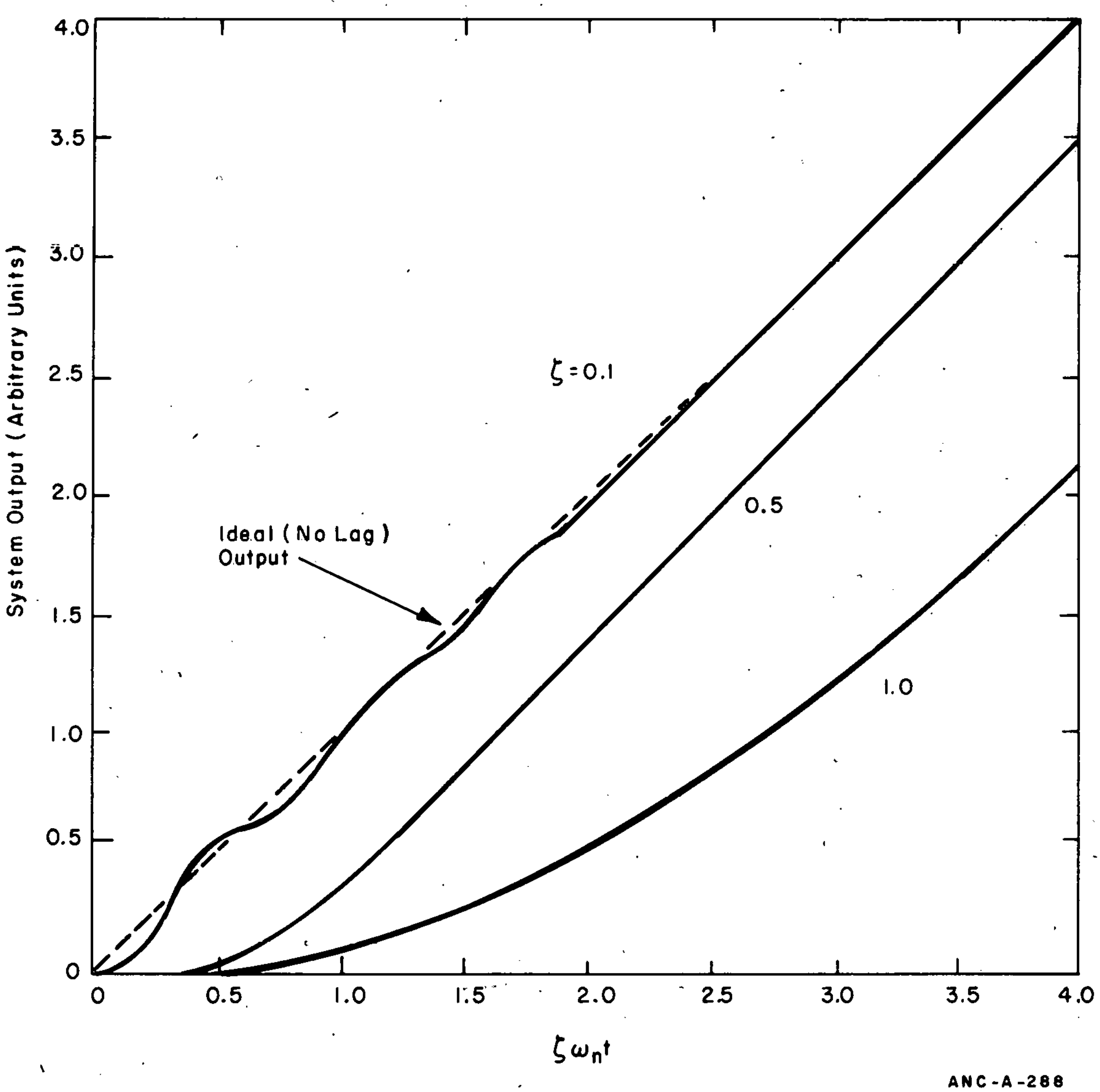

Figure 13. Response of a Second-Order System to a Ramp Input

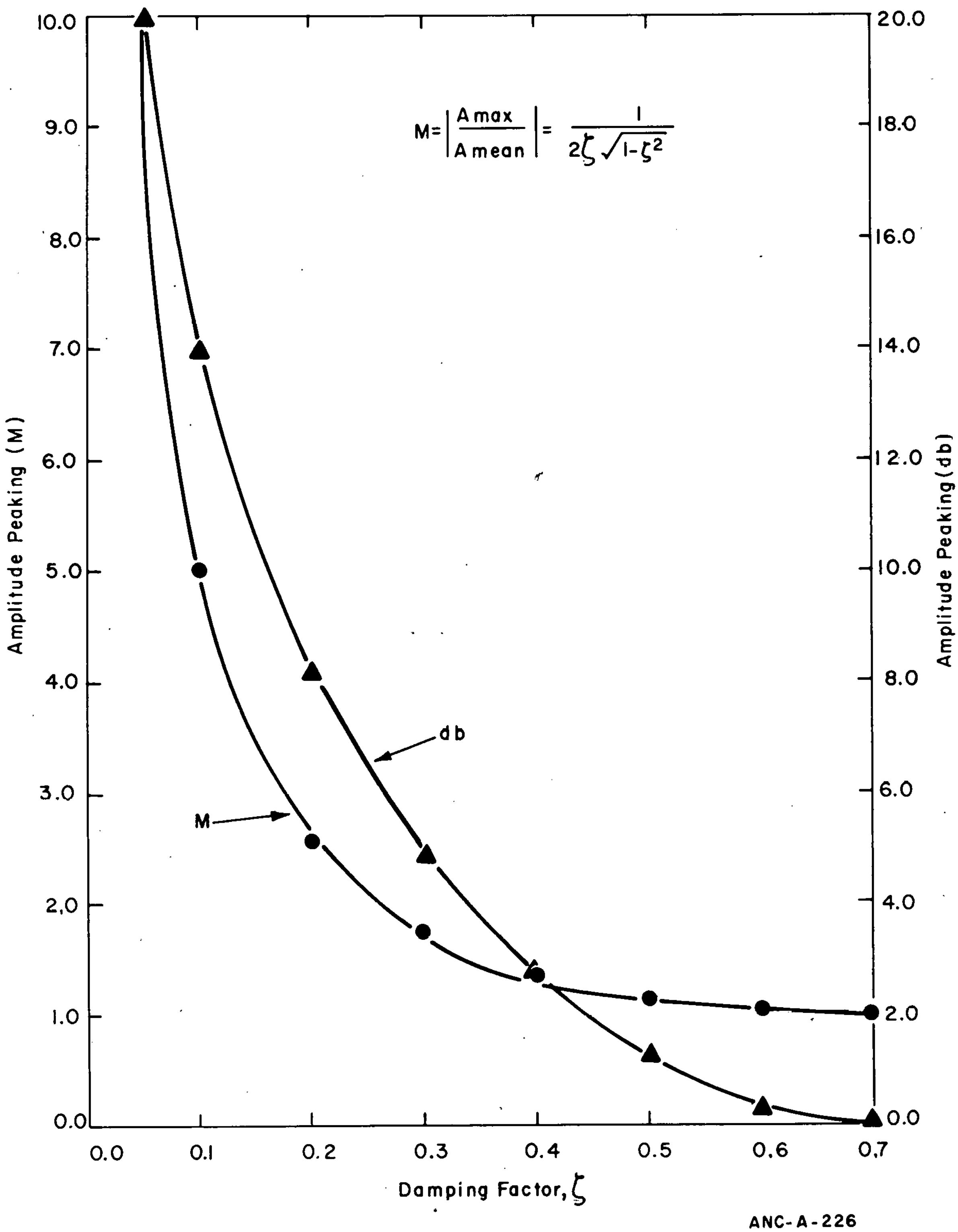

Figure 14. Amplitude Peaking versus Damping Factor for a Second-Order System Having a Step Input

convenience the bandwidth is normalized to either the damped or undamped natural frequencies of the system. The bandwidth normalized to the resonant frequency is expressed as follows:

$$\frac{\omega_{3dB}}{\omega_n} = \left(1 - 2\zeta^2 + \sqrt{2-4\zeta^2+4\zeta^4}\right)^{1/2} . \qquad (23)$$

This relationship, and one in which ω_{3dB} is normalized to ω_d, is shown in Figure 15.

As revealed by the preceding figures, the damping factor ζ exerts a strong influence on the response of a system. Thus, care should be taken to insure that the damping factor is such that the system response does not adversely effect the measurement.

More complex systems are composed of combinations of ramp and step inputs and their analysis becomes increasingly cumbersome. However, the system response can sometimes be approximated by considering the lowest frequency term. If some initial delay is included as with acoustic, electrical, or pneumatic transmission lines, then this must be considered separately as an additional delay in the transient response.

V. CONCLUSIONS AND RECOMMENDATIONS

A single term such as "response time" does not adequately define the transient response behavior of a transducer system. Thus, such terms should not be used to specify transducer response requirements. Response characteristics should be given in terms of well-defined parameters which are important to the measurement.

If steady-state frequency response is of prime importance, the 3 dB bandwidth, phase characteristics, and amplitude peaking should be specified. If it is a transient response which is of importance, then the risetime, delay time, overshoot, and settling time should be specified. Often the maximum allowable error for a given rate-of-change of input is a parameter which must be specified. The use of well-defined terms, such as those given in Section III of this report, will help to prevent some of the problems which have occurred in the preparation of specifications for transient response systems.

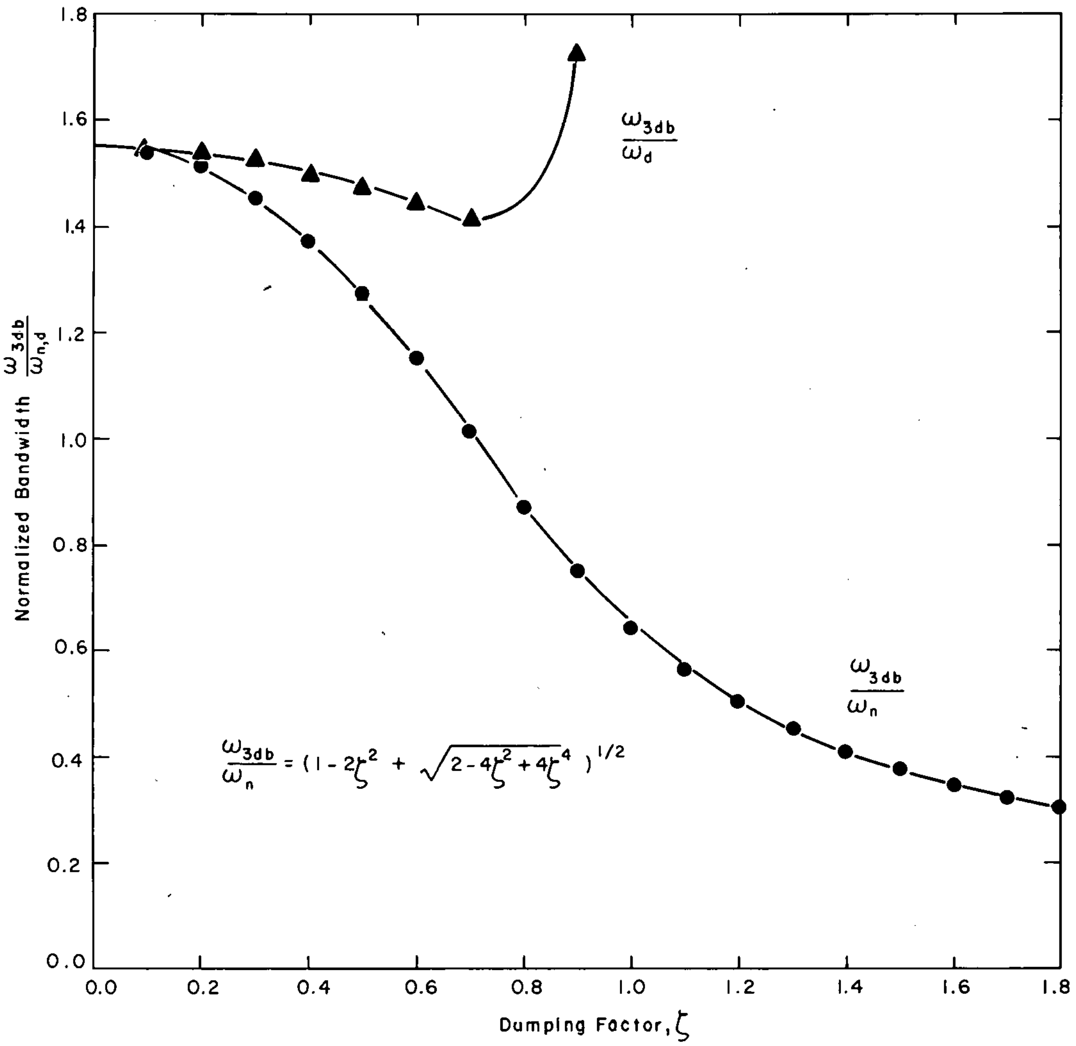

Figure 15. Normalized Bandwidth (3 db) versus Damping Factor for a Second-Order System Having a Step Input.

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