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On the Validity of DWBA for Deuteron Stripping

I. The Mitra Three-Body Model

by

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ABSTRACT

Mitra's three-body, separable potential model of deuteron stripping is re-examined and new results are derived. It is shown that because a separable interaction is a one-dimensional projection operator, a DWBA amplitude can be formulated which is identical to the exact stripping amplitude. This result validates the general prescription of DWBA to retain an unpolarized or "unstretched" deuteron internal wave function, or equivalently, validates the replacement in the DWBA of a three-body deuteron wave function by a product of two-body wave functions, one the ground state and the other a center-of-mass wave function. The center-of-mass wave function is not that for elastic (deuteron) scattering, although it does obey an equation with a complex potential well which on the energy shell reduces to the complex potential well for elastic scattering. These results are in accord with the recent results of Johnson and co-workers who find that inclusion of continuum states via their adiabatic method as a means of improving the usual DWBA wave function leads to a one-body equation in which the optical potential is not that for elastic scattering. The work

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of Reiner and Jaffe is used to show that the elastic deuteron wave function used in the conventional DWBA, is a good approximation to the new DWBA center-of-mass wave function for the three-body model, thus justifying use of the conventional DWBA.

I. Introduction

Ever since the direct reaction concept was introduced by Butler[1], it has played an important role in nuclear physics, both as a tool for deducing spectroscopic information and as motive for developing theories of nuclear reactions[2]. At the present time, the standard method used in the theoretical analysis of those experiments believed to proceed via a direct reaction mechanism is the distorted wave Born approximation (DWBA). A vast number of calculations using the DWBA have been carried out, and the agreement between theory and experiment is usually good, sometimes surprisingly so. One reason for surprise is that the DWBA is an approximation scheme in which a non-calculable, many-body amplitude is reduced to manageable proportions through the use of approximations which, because of their many-body aspect, have never been quantitatively justified. For this reason, plus the fact that DWBA calculations use multi-parameter wave functions as imput, DWBA can possibly be regarded as a taxonomic device, sophisticated, but never-the-less more of a bookkeeping scheme than a true theoretical model for reactions. Its successes, from this point of view are too good. Criticism of this sort seems to go in and out of vogue, and attempts to understand or improve DWBA have often been made. There are two broad areas into which such attempts can be grouped. First is the set of calculations and/or simple physical arguments that have tried either to validate the assumptions of DWBA or explain them in terms of notions seemingly built into the method but generally not explicitly stated. Most previous three-body studies of direct reactions[3] fall into this category, as does the work of Johnson and co-workers[4], whose general prescription for carrying out DWBA calculations is quite closely related to the conclusions

of this paper. The other area includes those attempts to reformulate a direct reaction theory in such a way so as to eliminate the more "objectionable" features of DWBA. Probably the best known works in this area are those of Butler and co-workers[5] and of Pearson and co-workers[6] for (d,p) and (d,n) reactions. In these works, the notion of the deuteron optical model wave function[2] is replaced by other, supposedly more realistic and less objectionable concepts, with a resulting physical picture of the reaction which differs from that of DWBA. Objections to these new formulations have been raised[7,8] and the status of these works remains unsettled if not very doubtful.

On first glance, the use of a deuteron CM wave function in the DWBA description of deuteron stripping does seem unreasonable, since the three-body nature of the deuteron-nucleus wave function is apparently ignored. Improvements, even if they are not those already attempted, would thus seem to be needed. The work of Johnson and co-workers[4] is an important step in this direction since they partially include the effects of continuum deuteron states (breakup) in their description of the stripping process. However, as we show here, the partial inclusion of such effects occurs naturally in our modified DWBA for the separable-potential, three-body model. The modification allows us to validate, for this model, what we believe to be one of the basic assumptions of DWBA. The purpose of this paper and others that will follow on this topic will be to prove the preceding statement* and develop its consequences.

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*A brief outline of some of our results is given in Ref. 9.

Specifically, we shall show that in the three-body model just mentioned (Mitra's model[3]), the neglect of the "stretching" or distortion of the deuteron internal wave function can be completely justified. Equivalent to this is the statement that replacement, as in DWBA, of the three-body "deuteron" wave function by a product of two-body wave functions, one the deuteron ground state and the other a CM wave function (i.e., an optical model type of wave function), is valid. Proof of this follows from the demonstration that the exact and the reformulated DWBA amplitudes are identical. Furthermore, we shall see that the old and the new DWBA amplitudes are very similar. A similar identity between the reformulated DWBA and the direct reaction stripping amplitudes is found to hold in the many-body case as well, if Pauli principle exchange amplitudes are ignored and if the neutron-proton interaction is assumed to be of separable form, as we show in a subsequent article.

The remainder of this article is organized as follows. First we discuss direct (d,p) reactions and isolate the approximations leading to the DWBA amplitude used in calculations. Next we summarize the Mitra model, briefly rederiving some of his results and correcting others. We then prove the statements made above forthe model and investigate the properties of the new deuteron CM distorted wave function. Finally we discuss our results, in particular showing that conventional DWBA is a good approximation to the new DWBA, which itself is equal to the exact model stripping amplitude.

II. Review of DWBA for Stripping

The direct reaction model is a marriage of relatively simple nuclear structure properties such as the concept of the single particle state, and

the assumption that complicated nuclear degrees of freedom such as compound nuclear states are not excited in these reactions. For stripping, the semi-classical argument described by Sachs[10] probably provides the clearest insight into the model. The result of this argument is to predict that protons emerging from a (d,p) reaction* will be observed at unique angles determined by the relation $qR = \ell$, where $\underset{\sim}{q} = \underset{\sim}{k}_d - \underset{\sim}{k}_p$ is the momentum transferred to the nucleus by the captured neutron and $\underset{\sim}{k}_d$ and $\underset{\sim}{k}_p$ are the deuteron and proton wave vectors, R is the nuclear radius, and ℓ is the orbital angular momentum of the captured neutron. Indeed, the peak values of the proton angular distributions can be reasonably well predicted this way: as ℓ increases, so does the primary peak angle.

As wave vectors are emphasized in this argument, it was natural that some form of plane wave Born approximation, evaluated with the aid of special assumptions and parameters, was first used to fit data.[1,2] Poor agreement with absolute cross-sections magnitudes was obtained and only the small angle data was well fitted, but that was sufficient to yield values of ℓ, and thus the location of single particle states in nuclei.

Attempts to improve the model and in particular take account of the distortion of the motion of the deuteron and proton away from plane wave behavior due to the presence of the target and residual nuclei led to the present form of the DWBA amplitude[2]. Further improvements are still being undertaken, such as attempts to understand the role of the deuteron D-state[11], the role of excitation of initial and/or final nuclear

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* We take the (d,p) case as our prototype, though our remarks are intended to hold for the (d,n) case also.

states other than the target ground state and the residual state[12], etc. As mentioned above, improvements that replace DWBA by other models have also been undertaken. The basic question we ask and partially answer in this article is, why does the DWBA work so well? To see that this is a non-trivial question, we examine the standard set of assumptions that reduce the exact amplitude to DWBA form.* Pauli principle exchange effects are ignored, although we hope eventually to examine them in the context of a simple model.

The exact (d,p) matrix element T_{dp} is given by[2]

$$T_{dp} = (e^{i\underset{\sim}{k}_p \cdot \underset{\sim}{r}_p} \phi_f(\xi,n) | V_{np} + V_p | \Psi_{\underset{\sim}{k}_d}), \tag{1}$$

where $\phi_f(\xi,n)$ is the residual nuclear state describing the neutron n bound to the target nucleus collectively denoted ξ; V_{np} is the binding interaction between the neutron and the proton p in the deuteron; V_p is the interaction of the outgoing proton with the target nucleons ξ; $\underset{\sim}{r}_p$ is the coordinate of p measured from the CM of the target nucleus, which for simplicity, is assumed infinitely heavy; and $\Psi_{\underset{\sim}{k}_d}$ is the total scattering wave function generated by a deuteron of wave vector $\underset{\sim}{k}_d$ incident on the target state $\phi_i(\xi)$ and obeying outgoing wave boundary conditions in all open channels.

The Schrödinger equation which $\Psi_{\underset{\sim}{k}_d}$ satisfies is

$$(E-H)\Psi_{\underset{\sim}{k}_d} = 0$$

where H is the total Hamiltonian and E is the total energy, given by

$$E = \frac{\hbar^2 k_d^2}{4M} + \varepsilon_{io} + \varepsilon_d, \tag{2}$$

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* These have often been stated before: see e.g. Levin[13] or Johnson and Soper[4].

with M being the nucleon mass* and ε_{io} and ε_d being the binding energies of the target nucleus and the deuteron.

In terms of operators for n, p, and ξ, H may be written as

$$H = h_i + K_n + K_p + V_{pn} + V_p + V_n, \tag{3}$$

where K_n and K_p are kinetic energy operators for n and p, V_n is the interaction of n with the target nucleons ξ, and h_i is the internal Hamiltonian for the target nucleus. The sum $K_n + K_p$ may be re-expressed as the sum of relative and kinetic energy operators $K_{np} + K_{cm}$, in terms of which the deuteron bound state wave function ϕ_d obeys

$$(K_{np} + V_{np})\ \phi_d = \varepsilon_d\ \phi_d. \tag{4}$$

We assume the existence of a complete set of target states $\{\Phi_{i\alpha}(\xi)\}$, which obey

$$h_i\ \Phi_{i\alpha} = \varepsilon_{i\alpha}\ \Phi_{i\alpha};$$

$\alpha = 0$ denotes the ground state. Since continuum states are included in $\{\Phi_{i\alpha}\}$, we may use this set to expand $\Psi_{\underset{\sim}{k}d}$:

$$\Psi_{\underset{\sim}{k}d} = \sum_\alpha \Phi_{i\alpha}\ \psi_\alpha(n,\underset{\sim}{p}), \tag{5}$$

where** $\psi_\alpha \equiv (\Phi_{i\alpha}, \Psi_{\underset{\sim}{k}d})$. The ψ_α, when α corresponds to a bound state, describe all the events which can occur leaving the target in the state $\Phi_{i\alpha}$, and as such are three-body wave functions obeying coupled equations. For example, ψ_o will describe elastic scattering, stripping, and breakup.

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* M will be the reduced mass if the target is not assumed to be infinitely heavy.

**We shall use parentheses for scalar products or matrix elements when the coordinate dependence of the wave functions is displayed, and the bra-ket notation for state vectors. In general we shall assume that, e.g. $\Phi_{i\alpha}(\xi) = \langle\xi|i\alpha\rangle$ or $F(\underset{\sim}{P}) = \langle\underset{\sim}{P}|F\rangle$, so that $|F\rangle$ is a state vector and $F(\underset{\sim}{P})$ is a wave function. Projections are similarly defined, and wave functions will not always be written with their arguments displayed.

We shall use the expansion (5) shortly.

With these preliminaries out of the way, we may now transform and reduce T_{dp} to desired form. The first step is to introduce the effect of an arbitrary potential U_p on the final state $\Phi_f(\xi,n)\exp[i(\underset{\sim}{k}_p\cdot\underset{\sim}{r}_p)]$. Let h_f be the internal Hamiltonian for the residual nucleus (including n). Then we may define a new state $\Xi_{\underset{\sim}{k}_p}^{(-)}$ obeying

$$(E - K_p - h_f - U_p)\,\Xi_{\underset{\sim}{k}_p}^{(-)} = 0,$$

generated from p incident on the final nucleus in state Φ_f and producing ingoing waves in all open channels at infinity. For example, if U_p is a one-body potential depending only on the coordinates of p, then

$$\Xi_{\underset{\sim}{k}_p}^{(-)} = \Phi_f \chi_{\underset{\sim}{k}_p}^{(-)}, \tag{6}$$

where $\chi_{\underset{\sim}{k}_p}^{(-)}$ obeys

$$(E - \varepsilon_f - K_p - U_p)\chi_{\underset{\sim}{k}_p}^{(-)} = 0$$

and ε_f is the binding energy of Φ_f.

In terms of $\Xi_{\underset{\sim}{k}_p}^{(-)}$ and U_p, T_{dp} may be written as

$$T_{dp} = (\Xi_{\underset{\sim}{k}_p}^{(-)} | V_{np} + (V_p - U_p) | \Psi_{\underset{\sim}{k}_d}), \tag{7}$$

and it is in this form that we begin introducing the approximations that lead to T_{dp}^{DW}, the standard DWBA amplitude.

The first and most important approximation consists of replacing the sum (5) by its first term:

$$\Psi_{\underset{\sim}{k}_d} \cong \Phi_{io}\psi_o \tag{8}$$

in the matrix element (7). This is clearly a many-body approximation which is unlikely, at least in the near future, to be validated. Nevertheless it is widely believed to embody the basic direct reaction character

of the process under consideration, and all theories of direct reactions employ it. We use it here without futher comment except to note that it may be extended to include several target states[12] in the event that they are strongly coupled through inelastic scattering.

The second approximation, and the one we are most concerned with, replaces ψ_o in (8) by the product $\phi_d \langle \phi_d | \psi_o \rangle \equiv \phi_d u_o$:

$$\psi_o \cong \phi_d u_o. \tag{9}$$

The coefficient u_o describes elastic scattering of the deuteron at the relevant energy ($\hbar^2 k_d^2/4M$) and with infinitely good energy resolution. In the notation of Feshbach[14] u_o is a wave function in a complex potential well, which when energy averaged leads to an optical model wave function. Use of this product wave function for ψ_o obviously replaces a three-body wave function by a two-body wave function, and leads to use of what has sometimes been described as the "barbarism" of a deuteron optical model wave function. Such use, and the ignoring of three-body effects in DWBA seem to have been the principle motives in the search for new direct reaction theories.[4,5,6] Clearly u_o is to ψ_o in (9) as ψ_o is to $\Psi_{\vec{k}_d}$ in (8), although its justification (and not its effectiveness) remains much more of a question, since we believe that in a direct stripping reaction the target nucleus is usually unaffected whereas we would strongly suspect that the internal wave function of a weakly bound particle such as the deuteron would be grossly distorted during the collision. However, we shall see that for the three-body model described in the next section, a form similar to (9) can be defined which is exact, thus leading to a redefinition of DWBA.

Let us examine the effect of these two approximations on T_{dp}. Equation (8) leads to

$$T_{dp} \cong (\Xi_{\underset{\sim}{k}_p}^{(-)} | V_{np} + (V_p - U_p) | \phi_{io} \psi_o), \tag{10}$$

while eq. (9), substituted into (10) yields

$$T_{dp} \cong T_{dp}^{ND} = (\Xi_{\underset{\sim}{k}_p}^{(-)} | V_{np} + (V_p - U_p) | \phi_{io} \phi_d \, u_o). \tag{11}$$

The superscript ND appears on the symbol for the matrix element in (11) to indicate that this is a "no distortion" approximation, i.e., that the exact deuteron internal (ground) state appears in (11), in contrast to (10) where ψ_o contains both ϕ_d and distorted internal states described as linear combinations of continuum deuteron states. We also note that for a model involving a structureless core, ϕ_{io} is unity and (10) is exact, as long as the other symbols are appropriately interpreted.

The two remaining approximations have been considered to be of lesser importance than (8) and (9) and to some extent represent assumptions introduced so as to help simplify calculations. The first of these deals with u_o. Despite the simplifications leading to eq. (11), u_o still cannot be calculated. Furthermore, experiments are not carried out with infinite energy resolution, so that an average over the energy range must be performed. Both of these problems are surmounted by replacing u_o in (11) by $\chi_{\underset{\sim}{k}_d}^{(+)}$, an optical model wave function describing deuteron elastic scattering in the usual way[2]. If (9) is valid, then the replacement

$$u_o \cong \chi_{\underset{\sim}{k}_d}^{(+)} \tag{12}$$

is very likely to be a reasonable approximation, particularly since u_o and $\chi_{\underset{\sim}{k}_d}^{(+)}$ can differ only near and in the nucleus where strong absorption effects should diminish the importance of a particular choice of wave function. Furthermore, from a practical point of view, a replacement such as (12) is dictated by the requirement that a calculable wave function is

needed in order to obtain a calculable amplitude. The central question here is whether, assuming use of u_o is valid, an optical potential, say U_d, can be constructed which will produce a satisfactory approximation to u_o. From the many very good fits to data, assuming (9) to be valid, it would seem that the appropriate U_d can be so constructed. The work of Johnson and Soper[4] modifies this conclusion but does not alter the fact that a suitable U_d can be determined; only the interpretation of U_d is changed, since assumption (12) is changed. They find that u_o in (11) should be replaced by a function which includes effects of deuteron breakup, and this in turn leads to a replacement of $\chi_{\underset{\sim}{k}_d}^{(+)}$ by a similar function. Hence their U_d is not an optical potential for elastic scattering alone. Our results confirm theirs, and in particular provide a basis for understanding why ϕ_d should be kept in (9), even though u_o is altered.

The final approximation concerns the difference $(V_p - U_p)$. Here again one faces a two-fold problem: production of a calculable $\Xi_{\underset{\sim}{k}_p}^{(-)}$ and minimization of $V_p - U_p$ to the point of ignoring it in the DWBA matrix element. The standard assumption has been that if U_p is an optical potential describing elastic scattering of protons of energy $\hbar^2 k_p^2/2M$, then $V_p - U_p$ may be neglected; $\Xi_{\underset{\sim}{k}_p}^{(-)}$ will then be given by a product such as (6), in which $\chi_{\underset{\sim}{k}_p}^{(-)}$ is indeed calculable. That an elastic scattering wave function must be present in $\Xi_{\underset{\sim}{k}_p}^{(-)}$ is evident, but that it is the only important term, or equivalently, that if U_p is an optical potential then $V_p - U_p \cong 0$, is not at all obvious. On the other hand it is a method that does work well; whether one should use elastic scattering from the target or the residual nucleus[15] seems an unimportant tertiary consideration, particularly as the optical potentials for these two different processes must be very similar. We therefore

assume that this approximation is also reasonable, and we use it in the form

$$\Xi_{\underset{\sim}{k}_p}^{(-)} \cong \Phi_f \chi_{\underset{\sim}{k}_p}^{(-)}, \qquad (13)$$

$$V_p - U_p \cong 0.$$

Two further remarks concerning (13) can be made in its support. First, (13) preserves Φ_f as the final nuclear state. If other states were to be mixed into $\Xi_{\underset{\sim}{k}_p}^{(-)}$ [excluding states strongly coupled in inelastic scattering] then there would be cases where other values of ℓ, the stripped neutron orbital angular momentum, would be present. The presence of such ℓ-values would tend to smear out the stripping patterns, at least near the first peak, that characterize these reactions as ℓ-meters. However, there seems to be no evidence for this, implying that only Φ_f is important in $\Xi_{\underset{\sim}{k}_p}^{(-)}$, which in turn supports the assumption made in (13). Secondly, one might try to define the DWBA amplitude as a particular matrix element of V_{np}, the states entering the matrix element being the ones that fit data best. We shall show later that as long as V_{np} is assumed separable, then any final state can be used and the equality between DWBA and "exact" matrix elements of the form of (10) with $V_p - U_p = 0$ will still hold. Unless it can be shown that use of a $\chi_{\underset{\sim}{k}_p}^{(-)}$ which is not the optical model wave function gives better fits to data than such a $\chi_{\underset{\sim}{k}_p}^{(-)}$, we will conclude that (13) is the best choice in that it leads to a DWBA which involves only a matrix element of V_{np}. There is no a priori reason to expect departures from this, and we therefore assume (13) to be valid.

Putting all these approximations together leads to the result of interest, viz,

$$T_{dp} \cong T_{dp}^{DW} = (\chi_{k_p}^{(-)} \phi_f | V_{np} | \phi_{io} \phi_d \chi_{k_d}^{(+)}). \tag{14}$$

On examination of the approximations used to obtain (14), it is clear that (9) is the crucial one we must try to understand. Why should an approximation that ignores three-body effects in a situation where three-body effects are likely to be important lead to a result that agrees as well as it does with data? That is, why should we neglect components of ψ_o unless we can show they are negligible, and in particular, when we would expect them to be non-negligible? Certainly, as the deuteron approaches the target, it must be distorted away from ϕ_d, yet (11) and (14) fail to take this into account. Thus, on appearance, they should be poor approximations. The answer to these questions has been given by some workers, as we noted above,[5,6] through a reformulation of the theory. However, to some extent this is begging the question. One would still like to know why (14) does work, assuming that its overall agreement with experiment is not merely a massive coincidence. New formulations do not answer this fundamental question. It is our contention here that the answer lies in the fact that V_{np} may be very well approximated by a separable interaction. To see why this is the answer, we first examine the three-body model put forward by Mitra, and show that in this model, with $U_p = V_p$, a DWBA can be formulated which is identical to the exact amplitude. In a subsequent paper, we extend this result to the many-body case.

III. Resume of the Mitra Model.

The model studied by Mitra[3] consists of three spinless particles interacting via separable S-wave potentials. For the stripping reaction he

assumed that one of the three particles had infinite mass, corresponding to a structureless core or target, denoted A, and that the other two particles, denoted 1 and 2, were identical. We extend the model here by assuming that the masses of (the "nucleons") 1 and 2 are equal but that they have different interactions with the core. This assumption allows for inclusion of a knockout channel, so that any initial state can give rise to any one of four final states, as long as all channels are open.

We may thus write for the interaction V_i of particle i with A, that

$$\langle \underset{\sim}{P}_1 \underset{\sim}{P}_2 | V_i | \underset{\sim}{P}_1' \underset{\sim}{P}_2' \rangle = - \lambda^{(i)} \frac{\hbar^2}{2M} g_i(P_i) g_i(P_i') \delta(\underset{\sim}{P}_j - \underset{\sim}{P}_j'), \tag{15}$$

where $\underset{\sim}{P}_i$ is the momentum of particle i, M is the common "nucleon" mass, $\lambda^{(i)}$ is the strength of the interaction, and $g_i(P_i)$ is the form factor defining the separable interaction. In addition to these interactions, particles 1 and 2 also interact with each other via V_{12}:

$$\langle \underset{\sim}{P}_1 \underset{\sim}{P}_2 | V_{12} | \underset{\sim}{P}_1' \underset{\sim}{P}_2' \rangle = - \lambda \frac{\hbar^2}{2M} f(p) f(p') \delta(\underset{\sim}{P} - \underset{\sim}{P}'), \tag{16}$$

where*

$$2\underset{\sim}{p} = \underset{\sim}{P}_1 - \underset{\sim}{P}_2, \qquad \underset{\sim}{P} = \underset{\sim}{P}_1 + \underset{\sim}{P}_2, \tag{17}$$

λ is the strength of the interaction, and f is the form factor. Since V_{12} is an interparticle interaction its matrix elements are diagonal in the total momentum. Our notation is that used by Mitra[3].

The quantities λ and $\lambda^{(i)}$ are taken to be positive, so that each potential supports one bound state. The normalized bound state wave functions for V_i, denoted ϕ_i, are given by

$$\phi_i(q) = \frac{g_i(q)}{q^2 + \alpha_i^2}, \tag{18}$$

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*In operator form V_{12} may be written as $V_{12} = - |f\rangle(\hbar^2\lambda/2M)\langle f|$.

while that for the "deuteron", i.e. the bound state of V_{12}, denoted ϕ_d, is

$$\phi_d(q) = \frac{f(q)}{2(q^2 + \alpha_d^2)} . \tag{19}$$

Note that the assumption of normalized wave functions places a constraint on the form factors g_i and f. The binding energies are given by $\hbar^2\alpha_i^2/2M$ for ϕ_i and by $\hbar^2\alpha_d^2/M$ for ϕ_d, thus accounting for the factor of 2 in eq. (19). The values of α_i^2 and α_d^2 are related to $\lambda^{(i)}$ and λ via

$$\lambda^{(i)^{-1}} = \int d^3q \; g_i^2(q)/(q^2 + \alpha_i^2)^2 \tag{20}$$

and

$$\lambda^{-1} = \int d^3q f^2(q)/4(q^2 + \alpha_d^2)^2. \tag{21}$$

The three-particle Schrödinger equation for the state vector $|\Psi\rangle$ reads

$$(K-E)|\Psi\rangle = -(V_1 + V_2 + V_{12})|\Psi\rangle \tag{22}$$

where $K = K_1 + K_2$ is the kinetic energy operator. In a momentum space representation, eq. (22) takes the form

$$d(E)\Psi = [g_1(P_1)G_2(\underset{\sim}{P}_2) + g_2(P_2)G_2(\underset{\sim}{P}_1) + f(p)\, F(\underset{\sim}{P})], \tag{23}$$

where

$$d(E) = P_1^2 + P_2^2 - k_E^2 = \tfrac{1}{2}P^2 + 2p^2 - k_E^2,$$
$$E = \hbar^2 k_E^2/2M,$$

and the G_i and F are the spectator functions introduced by Mitra[3], although we have changed the subscripts on the G_i as compared with Mitra's definitions.

The spectator functions are formally defined by

$$G_i(\underset{\sim}{P}_i) = \lambda^{(j)} \int d^3q \; g_j(q)\Psi(\underset{\sim}{q},\underset{\sim}{P}_i), \; i \neq j, \tag{24}$$

and

$$F(\underset{\sim}{P}) = \lambda \int d^3p' f(p')\Psi(\underset{\sim}{p}',\underset{\sim}{P}); \tag{25}$$

they contain the physics of the problem to be solved and are discussed in

more detail below. We note here the extremely important result that each interaction is a one-dimensional projection operator[16] whose projection onto Ψ just yields the product of the form factor and the spectator functions. This property is the key to our proof of the equality of the exact and DWBA amplitudes.

Equations (24) and (25) represent coupled equations for the F and G_i. The form of these is made evident by solving (23) for Ψ, viz,

$$\Psi = d^{-1}[g_1 G_2 + q_2 G_1 + fF], \tag{26}$$

and substituting the result into the definitions of F and G_i. We find

$$[\lambda^{(j)-1} - h_j(P_i)]G_i(\underset{\sim}{P}_i) = \int d^3q\, B_{ij}(\underset{\sim}{P}_i, \underset{\sim}{q}; k_E^2) G_j(\underset{\sim}{q}) + \int d^3q\, B_i(\underset{\sim}{P}_i, \underset{\sim}{q}; k_E^2) F(\underset{\sim}{q}), \quad i \neq j,$$

and (27)

$$[\lambda^{-1} - \tfrac{1}{2} h(P)]F(\underset{\sim}{P}) = \sum_{i=1}^{2} \int d^3q B_i(\underset{\sim}{q}, \underset{\sim}{P}; k_E^2) G_j(\underset{\sim}{q}).$$

These are identical to Mitra's coupled equations[3] apart from a few notational changes. The kernals on the right hand sides of these equations are given by

$$B_{ij}(\underset{\sim}{P}_i, \underset{\sim}{q}; k_E^2) = \frac{g_i(P_i) g_j(q)}{P_i^2 + q^2 - k_E^2 - i\varepsilon}, \quad i \neq j, \tag{28}$$

and

$$B_i(\underset{\sim}{x}, \underset{\sim}{y}; k_E^2) = \frac{q_i(|\underset{\sim}{x} - \underset{\sim}{y}|) f(|\underset{\sim}{x} - \frac{1}{2}\underset{\sim}{y}|)}{2x^2 + y^2 - 2\underset{\sim}{x}\cdot\underset{\sim}{y} - k_E^2 - i\varepsilon}. \tag{29}$$

One can easily show that, on the energy shell, $[-(2M/\hbar^2)B_i(\underset{\sim}{k}_i, \underset{\sim}{k}_d; k_E^2)]$ is the Born approximation amplitude for stripping via the interaction V_{12}, while $[-(2M/\hbar^2)B_{ij}(\underset{\sim}{k}_i, \underset{\sim}{k}_d; k_E^2)]$ is the Born approximation amplitude for

knockout via the interaction V_i. In addition, the function h(P) is the same as the h(P) defined by Mitra[3],

$$h(P) = \int d^3q \, \frac{f^2(q)}{\frac{1}{4}P^2 + q^2 - \frac{1}{2}k_E^2 - i\varepsilon}$$

and $h_j(P_i)$ is a generalization of Mitra's[3] $h_1(P_1)$:

$$h_j(P_i) = \int d^3q \, \frac{g_j^2(q)}{P_i^2 + q^2 - k_E^2 - i\varepsilon}, \qquad i \neq j.$$

From the definitions of λ, $\lambda^{(j)}$ and the h's it follows, as shown by Mitra[3], that

$$[\lambda^{(j)-1} - h_j(P_i)] = (P_i^2 - k_i^2)\ell_j(P_i) \tag{30}$$

and

$$[\lambda^{-1} - \tfrac{1}{2}h(P)] = \tfrac{1}{2}(P^2 - k_d^2)\ell(P), \tag{31}$$

where the ℓ's are non-singular and reduce to unity on the energy shell since the bound states (18), (19) are normalized. They are defined by Mitra[3] and are trivial to work out.

As noted above, the physics of any problem under consideration is contained in the spectator functions, specifically through the boundary conditions imposed on G_i and F. Three different cases can be distinguished depending on whether particle 1, particle 2, or the bound state of particles 1 and 2, which we shall now refer to as a "deuteron", is the projectile. Corresponding to these situations are three different forms for k_E^2:

$$\tfrac{1}{2}k_d^2 - 2\alpha_d^2 = k_E^2 = k_i^2 - \alpha_i^2, \qquad i = 1,2$$

where $\underset{\sim}{k}_d$ and $\underset{\sim}{k}_i$ are the wave vectors for a deuteron or for particle i incident. The boundary conditions are given in terms of these wave numbers.

For deuterons incident, we write

$$F(\underset{\sim}{P}) = \delta(\underset{\sim}{P}-\underset{\sim}{k}_d) - \frac{4M}{\hbar^2}\,[\ell(P)(P^2-k_d^{\,2}-i\varepsilon)]^{-1}\, T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d) \tag{32}$$

and

$$G_i(\underset{\sim}{P}_i) = -\frac{2M}{\hbar^2}\,[\ell_j(P_i)(P_i^{\,2} - k_i^{\,2} - i\varepsilon)]^{-1}\, T_{id}(\underset{\sim}{P}_i,\underset{\sim}{k}_d). \tag{33}$$

Our notation differs from that of Mitra[3] in that first we use the propagators of eqs. (30) and (31) since they enter into a) the defining equations (27), b) the derivation of the equation obeyed by $F(\underset{\sim}{P})$ in the Appendix, and c) allow us to identify the amplitudes directly with those of Lovelace[17,18]; second, we have used transition rather than scattering amplitudes; and third we have made explicit the dependence on wave vectors. Our model is slightly more general than Mitra's in that we have assumed $V_1 \neq V_2$, thus allowing for a knockout channel. The notation used here differs from that of our earlier work[9] in that we now use the full propagators rather than just the differences of the squares of the wave numbers; however we use the same symbols for the amplitudes, a trivial difference and one that cannot affect the on-shell interpretation of the amplitudes. On the energy shell ($P^2 = k_d^{\,2}$), $T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d)$ is the deuteron elastic scattering amplitude, and similarly, $T_{id}(\underset{\sim}{k}_i,\underset{\sim}{k}_d)$ is the deuteron stripping amplitude.

Alternatively, for particle i incident, we have[3]

$$G_i(\underset{\sim}{P}_i) = \delta(\underset{\sim}{P}_i-\underset{\sim}{k}_i) - \frac{2M}{\hbar^2}\,(P_1^{\,2}-k_i^{\,2}-i\varepsilon)^{-1} T_{ii}(\underset{\sim}{P}_i,\underset{\sim}{k}_i)/\ell_j(P_i) \tag{34}$$

and

$$F(\underset{\sim}{P}) = -\frac{4M}{\hbar^2}\,(P^2 - k_d^{\,2} - i\varepsilon)^{-1}\, T_{di}(\underset{\sim}{P},\underset{\sim}{k}_i)/\ell(P). \tag{35}$$

In these equations, T_{ii} is, for $P_i^{\,2} = k_i^{\,2}$, the elastic scattering amplitude and $T_{di}(\underset{\sim}{k}_d,\underset{\sim}{k}_i)$ is the pickup amplitude, which, by time reversal invariance, is equal to $T_{id}(\underset{\sim}{k}_i,\underset{\sim}{k}_d)$.

Substitution of these sets equations into (27) then leads to coupled equations[3] for the various amplitudes in G_i and F. By direct comparison with the results of the Faddeev and Lovelace equations method[17,18], it is straightforward to show that the amplitudes given in eqs. (32) - (36) obey the same integral equations as do the Lovelace amplitudes. Hence, our statements about the meaning of the various T's used above can be verified in detail. It is evident from the forms used for the spectator functions[3] that they represent the motion of particle i (G_i) or of the CM of the bound state of 1 and 2 (F), relative, respectively, to the bound state of j or to the core, A.

We shall not display here the sets of integral equations for the amplitudes, as they are practically the same as those derived by Mitra. Instead we turn to the question of the DWBA in this model. It is here that our results differ from those of Mitra. Apart from some questions concerning normalization, Mitra's major error in his derivation[3] of the DWBA arises from a notational confusion. In contrast to our eqs. (32) - (35), in which different amplitudes are differentiated by the use of different symbols, Mitra has, unintentially, used the same symbol, viz. b, for what we have denoted T_{id} and also for our T_{ii}, i.e., the stripping and nucleon elastic scattering amplitudes. Since his DWBA is defined as the matrix element of V_{12} taken between $\phi_d F$ and $\phi_j G_i$, but with his G_i containing T_{id} in place of T_{ii} (i.e. his amplitude b), he was led to an erroneous result concerning the validity of DWBA. Specifically, he concluded that DWBA (in the three-body model) should be valid when the deuteron elastic scattering amplitude is smaller than the stripping amplitude, or equivalently, when stripping cross sections are larger than elastic scattering cross sections. This is, of course, contrary

to what is observed experimentally, thus implying that the model is unrelated to everyday physics. Such a conclusion is, however, not implied by the works of Aaron and Shanley[3] and Reiner and Jaffe[3] using similar three-body models, since their DWBA angular distributions not only showed good stripping patterns but also were reasonably good approximations to the exact (d,p) cross sections. The resolution of this paradox consists in correcting Mitra's result and comparing the DWBA and the exact stripping amplitudes, which we now do.

IV. Distorted Wave Approximation.

We have discussed both DWBA in its general aspects and the main ideas of the three-body, separable potential model. We now examine the DWBA for the model.

In the model the exact stripping amplitude is $T_{id}(\underset{\sim}{k}_i, \underset{\sim}{k}_d)$, while in the general case, treated in section II, the exact amplitude is T_{dp} of eq. (7). In applying the DWBA approximations to the model, we noted that eq. (8), the direct reaction approximation, is exact, since the core, A, is structureless, implying that ϕ_{io} is a delta function. If we now use the correspondences $V_{np} \to V_{12}$, $V_p \to V_i$, and $\psi_o \to \Psi$, then T_{id} of the three-body model is identical to T_{dp} of eq. (10), where all quantities are to be evaluated on the energy shell. In other words

$$T_{id} = (\Xi_{\underset{\sim}{k}_i}^{(-)} | V_{12} + (V_i - U_i) | \Psi), \tag{36}$$

where $\Xi_{\underset{\sim}{k}_i}^{(-)}$ is the analog for the three-body model of $\Xi_{\underset{\sim}{k}_p}^{(-)}$ in the general case. For U_i a one-body operator, then by eq. (6), $\Xi_{\underset{\sim}{k}_i}^{(-)} = \phi_j \chi_{\underset{\sim}{k}_i}^{(-)}$. We shall assume this to be the case, and in particular will take $U_i = V_i$. Hence we have

$$T_{id} = (\chi_{\underset{\sim}{k}_i}^{(-)} \phi_j | V_{12} | \Psi), \tag{37}$$

which is the exact, three-body model amplitude evaluated on the energy shell.

Were we to follow the procedure of section II, we would introduce approximation (9), the traditional assumption, followed by approximation (12). However, this approach would then give us the tradiational form of DWBA amplitude, and no particular insight as to why it should work as well as it has, even though we have a simple model to deal with. In fact, we might find that it did not work so well at all, as indicated by the calculations of Shanley and Aaron[3], which we discuss in section VI. The reasons for their finding poor agreement between the exact three-body cross-section and the traditional DWBA cross-section are indicated in that section, and are not obviously connected with DWBA being a poor approximation - indeed, we believe it to be a good one - but their results do serve as a necessary reminder that DWBA may not always be applied straightforwardly.

The comments of the preceding paragraph suggest that a means other than that of section II to reduce T_{id} to simpler form might be useful to explore, if such could be found. One method is to try to improve DWBA, as in the work of Johnson and coworkers[4]. We comment on their procedure later, but do not follow it here. A different method was proposed by Mitra[3], who replaced steps (9) and (13). He used G_i in place of $\chi_{\underline{k}i}^{(-)}$ and $\phi_d F$ in place of $\phi_d u_o$ for Ψ (or ψ_o). In momentum space, G_i is given by eq. (34), ϕ_d by (19), and F is given by eq. (32). As we commented above, Mitra's DWBA result was incorrect because of a notational error. Had this error and a few trivial ones concerning normalization not been made, a result like eq. (41) below, which is our main result, could have been obtained. That is, one can show using $\phi_d F$ that a DWBA can be formulated in which the amplitude is identical to the exact one.

Rather than follow Mitra's approach[3] of substituting G_i for $\chi_{\underset{\sim}{k}_i}^{(-)}$ and F for u_o, we take a simpler and more general approach. We refer back to eqs. (22), (23), and (25), and note that in momentum space,

$$V_{12}\Psi = -\frac{\hbar^2}{2M} f(p)F(\underset{\sim}{P}). \tag{38}$$

Therefore on introducing a momentum space representation into T_{id} and substituting (38) into (37) we have

$$T_{id} = \frac{\hbar^2}{2M} \int d^3p d^3P \; \chi_{\underset{\sim}{k}_i}^{(-)}(\tfrac{1}{2}\underset{\sim}{P} + \underset{\sim}{p})\phi_i(\tfrac{1}{2}\underset{\sim}{P}-\underset{\sim}{p}) f(p)F(\underset{\sim}{P}). \tag{39}$$

Symbolically we write this as

$$T_{id} = -\frac{\hbar^2}{2M} \langle \chi_{\underset{\sim}{k}_i}\phi_i | fF\rangle, \tag{40}$$

which is an exact result.

We will prove below that eq. (40) is <u>identical</u> to a "no distortion" form of DWBA in which Ψ is replaced by $\phi_d F$. Therefore we propose a new "no distortion" DWBA assumption as a replacement for eq. (9) in this model:

$$\Psi(\underset{\sim}{p},\underset{\sim}{P}) \cong \Psi^{ND}(\underset{\sim}{p},\underset{\sim}{P}) \equiv \phi_d(p)F(\underset{\sim}{P}), \tag{9'}$$

an approximation which in (37) we now show is exact.

Substitution of (9') into (37) leads to the "no distortion" amplitude T_{id}^{ND}, given by

$$\begin{aligned} T_{id}^{ND} = \frac{\hbar^2}{2M} \int d^3p d^3p' d^3P d^3P' \; & \chi_{\underset{\sim}{k}_i}^{(-)}(\tfrac{1}{2}\underset{\sim}{P}+\underset{\sim}{p})\phi_i(\tfrac{1}{2}\underset{\sim}{P}-\underset{\sim}{p})\lambda f(p) \\ & x f(p')\delta(\underset{\sim}{P}-\underset{\sim}{P}')\phi_d(p')F(\underset{\sim}{P}') \\ = -\frac{\hbar^2}{2M} \int d^3p d^3P \chi_{\underset{\sim}{k}_i}^{(-)} & (\tfrac{1}{2}\underset{\sim}{P}+\underset{\sim}{p})\phi_i(\tfrac{1}{2}\underset{\sim}{P}-\underset{\sim}{p}) f(p)F(\underset{\sim}{P}) \\ & x\lambda \int d^3p' \frac{f(p')}{2(p'^2+\alpha_d^2)} . \end{aligned}$$

But from eq. (21), the p' integral cancels λ in the above equation. Hence

we find

$$T_{id}^{ND} = T_{id}, \tag{41}$$

or

$$\langle \chi_{\underset{\sim}{k}_i}^{(-)} \phi_i | V_{12} | \psi^{ND} \rangle = \langle \chi_{\underset{\sim}{k}_i}^{(-)} \phi_i | V_{12} | \psi \rangle,$$

which is the result we set out to establish and is the major conclusion of this paper.

We have thus shown that because of the projection operator nature of V_{12}, the no distortion approximation of (9') leads to a DWBA matrix element which is identical to the exact result T_{id}. The basic ingredient in obtaining this result is the equality

$$V_{12}\psi^{ND} = V_{12}\psi; \tag{42}$$

hence for any arbitrary final state, say Γ, we thus have

$$(\Gamma | V_{12} | \psi^{ND}) = (\Gamma | V_{12} | \psi). \tag{43}$$

We use this result in our subsequent discussion of the actual (many-body) nuclear case.

So far we have shown that a more or less conventional DWBA approximation analogous to (9), the standard one, can be formulated such that the no distortion DWBA matrix element is identical to the exact matrix element. Rephrasing this, we have shown that it is correct to keep only the deuteron ground state ϕ_d in the distorted wave ψ^{ND}. In addition, F is, by virtue of its argument $\underset{\sim}{P}$, a CM wave function. Hence we justify, in the three-body model, the basic DWBA approximation of replacing a three-body wave function (viz. ψ_o) by a product of two-body wave functions. We have therefore proved at least for the model, that a conventional DWBA type of approach to the description of deuteron stripping is valid, and that in principle, use of a CM deuteron wave function is quite proper. The new approaches to stripping can no longer be claimed to be necessary on the grounds that they are

replacing an incorrect or poorly understood method.

Clearly, the most important aspect of this present development of DWBA is an understanding of the function $F(\underset{\sim}{P})$. We discuss some of its properties in the next section.

V. Properties of the Spectator Function F.

To understand F, we go back to eq. (26). For incident energies below the three-body-breakup threshold, F uniquely yields on the energy shell, only the elastic amplitude: the stripping amplitudes are contained in the G_i. Above threshold, however, F contributes to the breakup amplitude as clearly shown in the discussion of Watson and Nuttall[18] following the arguments of Lovelace[17]. Off the energy shell F yields both elastic and breakup amplitudes, although only a portion of the latter since the G_i contribute as well. Because of these points, it is clear that F must obey an equation in which the one-body potential is not that for elastic scattering, but is altered due to the off-shell behavior of F. We shall derive this potential later in this section.

Let us first compare $F(\underset{\sim}{P})$ with $u_o(\underset{\sim}{P})$, the elastic scattering wave function in momentum space. From section II we know that $u_o = \langle\phi_d|\psi_o\rangle$ and in the three-body model we are considering $u_o(\underset{\sim}{P}) = (\phi_d(\underset{\sim}{p})|\Psi(\underset{\sim}{p},\underset{\sim}{P}))$. Clearly $u_o(\underset{\sim}{P})$ must have the form

$$u_o(\underset{\sim}{P}) = \delta(\underset{\sim}{k}_d-\underset{\sim}{P}) - \frac{4M}{\hbar^2}\,(P^2-k_d^2-i\varepsilon)^{-1}T_{d,el}(\underset{\sim}{P},\underset{\sim}{k}_d) \tag{44}$$

where $T_{d,el}(\underset{\sim}{P},\underset{\sim}{k}_d)$ is the half-off-shell deuteron elastic scattering amplitude. It is straightforward to show that

$$T_{d,el}(\underset{\sim}{P},\underset{\sim}{k}_d) = \langle\underset{\sim}{P},\phi_d|V_1+V_2|\Psi\rangle, \tag{45}$$

where Ψ is the total (three-body model) scattering wave function generated by a deuteron of momentum $\underset{\sim}{k}_d$ incident on the core A. Of course, we can also formulate $T_{d,el}$ in terms of a complex potential well description or as a matrix element of a T-operator, etc., but all these forms are equivalent.

The function u_o must have this form since it is the coefficient of ϕ_d in ψ_o. That is, we implicitly assume an expansion of ψ_o in states of $h_{12} = K_{12}+V_{12}$; the coefficients of the continuum terms will only contribute to breakup or stripping and not to elastic scattering. Let us now re-examine eq. (26) in this light. The sum of the three terms in $\Psi(=\psi_o$ in the model) does not correspond to an expansion of Ψ in the states of h_{12}; each term can therefore contribute to breakup as well to the elastic scattering or stripping processes we associate with the forms of F or G_i given by eqs. (32) and (33).

Let us now try to isolate ϕ_d in eq. (26). To do so, we rewrite the denominator d^{-1} as

$$d^{-1} \equiv [\tfrac{1}{2}(p^2-k_d^2) + 2(p^2+\alpha_d^2)]^{-1}$$

$$= [2(p^2+\alpha_d^2)]^{-1} - \frac{\frac{1}{2}(p^2-k_d^2)}{2(p^2+\alpha_d^2)}[\tfrac{1}{2}(p^2-k_d^2)+2(p^2+\alpha_d^2)]^{-1}. \qquad (46)$$

Substituting eq. (46) into eq. (26), and rearranging, we find

$$\Psi(\underset{\sim}{p},\underset{\sim}{P}) = \phi_d(\underset{\sim}{p})F(\underset{\sim}{P})+d^{-1}[-\tfrac{1}{2}(p^2-k_d^2)\phi_d(\underset{\sim}{p})F(\underset{\sim}{P})+g_1G_2+g_2G_1]. \qquad (47)$$

It is evident that $F(\underset{\sim}{P})$ is not equal to $u_o(\underset{\sim}{P})$, the deuteron elastic scattering wave function. In fact, u_o is not simply identifiable in terms of F since the expression $F - \frac{1}{2}(P^2-k_d^2)d^{-1}F$, which from (47) would be the natural choice for u_o is <u>not</u> a function of P alone: it also depends on p. This follows from the p-dependence of d. Thus, even though the factor $(-\frac{1}{2}(p^2-k_d^2)F)$ itself leads to a simple expression, namely

$$-\frac{1}{2}(P^2-k_d^2)F = \frac{2M}{\hbar^2}\,T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d)$$

[which follows from eq. (32)], the product $-\frac{1}{2}(P^2-k_d^2)d^{-1}\phi_d(\underset{\sim}{p})F(\underset{\sim}{P})$ is still a three-body wave function, and not a product of two-body wave functions. This, of course, reflects the way that breakup occurs in the three-body model, namely through scattering of a quasi-bound pair followed by decay of the pair. Since a range of momenta are now allowed, f(p) and $F(\underset{\sim}{P})$ must be linked via d^{-1}, or equivalently, the correction term to $\phi_d F$ above must have the form $d^{-1}\phi_d(\underset{\sim}{p})T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d)$, representing off-shell scattering followed by breakup; the latter statement simply emphasizes the fact that $d^{-1}\phi_d(\underset{\sim}{p})$ is not the deuteron ground state.

What we have seen is that F contains breakup parts as well as elastic scattering portions, with all amplitudes being half-off-energy shell. To see how much more F yields, we can compare $T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d)$ with $T_{d,el}(\underset{\sim}{P},\underset{\sim}{k}_d)$. This latter term is, from eq. (44), just

$$T_{d,el}(\underset{\sim}{P},\underset{\sim}{k}_d) = \int d^3p'\ \phi_d^*(\underset{\sim}{p}')\ \langle \underset{\sim}{P},\underset{\sim}{p}'|V_1+V_2|\Psi\rangle,$$

where $\phi_d^*(\underset{\sim}{p}') = \langle\phi_d|\underset{\sim}{p}'\rangle$. In terms of f(p') this is

$$T_{d,el}(\underset{\sim}{P},\underset{\sim}{k}_d) = \int d^3p'\ \frac{f(p')}{2(p'^2+\alpha_d^2)}\ \langle \underset{\sim}{P},\underset{\sim}{p}'|V_1+V_2|\Psi\rangle. \tag{48}$$

The only off-energy-shell quantity in (48) is the momentum $\underset{\sim}{P}$.

The corresponding expression for $T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d)$ can be derived by projecting $|\Psi\rangle$ onto $\langle f|$ and using the integral equation formulation for $|\Psi\rangle$ as in the Appendix. Equivalently, we can proceed simply by referring to Lovelace's analysis[17,18]. The amplitude he defines as $X_{\beta\alpha}(\underset{\sim}{p}_\beta,\underset{\sim}{p}_\alpha)$ is equal to our $T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d)$ when β and α each correspond to the deuteron channel (incident and emergent) and we take $\underset{\sim}{P} = \underset{\sim}{p}_\beta$, $\underset{\sim}{k}_d = \underset{\sim}{p}_\alpha$. For the case of elastic scattering,

it is straightforward to show, by either approach noted above, that*

$$T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d) = -\frac{\hbar^2}{2m}\,\langle \underset{\sim}{P},f|G_o(V_1+V_2)|\Psi\rangle \tag{49}$$

where $G_o = (E^+-K)^{-1}$ is the free particle Green's function and $[-G_o|f,P>]$ is the final state, which on the energy shell ($P^2=k_d^2$) is $|\phi_d,\underset{\sim}{P}\rangle$. In momentum space of course $G_o = -D^{-1}(E)$. Putting (49) in a form similar to (48) yields

$$T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d) = \int d^3p' \,\frac{f(p')}{\frac{1}{2}(P^2-k_d^2) + 2(p'^2+\alpha_d^2)}\,\langle \underset{\sim}{P},\underset{\sim}{p}'|V_1+V_2|\Psi\rangle\ . \tag{50}$$

The difference between (48) and (50) is in the denominators, and cannot be put in such a way that T_{dd} is a simple multiplicative function of $\underset{\sim}{P}$ times $T_{d,el}$. Clearly, $T_{d,el}$ and T_{dd} are equal on the energy shell. Off the energy shell we may express T_{dd} as a sum of $T_{d,el}$ plus a correction term by using eq. (46) in (50) to give

$$T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d) = T_{d,el}(\underset{\sim}{P},\underset{\sim}{k}_d)+\frac{\hbar^2}{4M}\,(P^2-k_d^2)\langle \underset{\sim}{P},\phi_d|G_o(V_1+V_2)|\Psi\rangle. \tag{51}$$

The correction term obviously takes on a simple form.

Were $T_{d,el}$ the only term on the right hand side of (51), then $F(\underset{\sim}{P})$ would be equal to u_o. It would obey a one-body equation containing Feshbach's complex potential well[14], which would give rise to the exact elastic scattering amplitude $T_{d,el}$ for infinitely good energy resolution. However, the correction term implies that $F \neq u_o$. Hence the complex potential well which yields T_{dd} is therefore not the same as the one which produces elastic scattering, as noted in a somewhat similar context by Johnson and Soper[4].

The form of the complex potential well which yields T_{dd} can be obtained

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*The correction term $Z_{\beta\alpha}$ arising in the general equation for $X_{\beta\alpha}$ vanishes for elastic scattering[17,18].

from T_{dd} if we can express $(V_1+V_2)|\Psi\rangle$ in terms of $V_{12}|\Psi\rangle$, since this latter term, in momentum space, is proportional to $f(p)F(\underset{\sim}{P})$. We start by noting that $(-\hbar^2/2M)\langle \underset{\sim}{P}, f| = \langle \underset{\sim}{P}, \phi_d|V_{12}$, and therefore that (49) can be written as

$$T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d) = \langle \underset{\sim}{P}, \phi_d|V_{12}G_o(V_1+V_2)|\Psi\rangle. \tag{52}$$

Next we observe that Ψ of eq. (26) in operator form is

$$|\Psi\rangle - G_oV_{12}|\Psi\rangle = G_o(V_1+V_2)|\Psi\rangle. \tag{53}$$

Multiplying both sides of (53) from the left by (V_1+V_2), rearranging, and inverting $(E^+-K-V_1-V_2)$ then gives

$$G_o(V_1+V_2)|\Psi\rangle = (E^+-K-V_1-V_2)^{-1}(V_1+V_2)G_oV_{12}|\Psi\rangle.$$

On substituting this into (52) we see that T_{dd} becomes

$$T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d) = \langle \underset{\sim}{P}, \phi_d|V_{12}(E^+-K-V_1-V_2)^{-1}(V_1+V_2)G_oV_{12}|\Psi\rangle.$$

Finally, writing $V_{12}|\Psi\rangle = -\frac{\hbar^2}{2M}|fF\rangle$, we have

$$T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d) = -\frac{\hbar^2}{2M}\langle \underset{\sim}{P}, \phi_d|V_{12}(E^+-K-V_1-V_2)^{-1}(V_1+V_2)G_o|fF\rangle. \tag{54}$$

The complex potential well operator, denoted $\mathcal{V}$, is, from (54), just

$$\mathcal{V} = -\frac{\hbar^2}{2M}\langle \phi_d|V_{12}(E^+-K-V_1-V_2)^{-1}(V_1+V_2)G_o|f\rangle \tag{55}$$

and is a non-local operator acting on $|F\rangle$. It follows from eq. (32) that $F(\underset{\sim}{P})$ obeys

$$[\tfrac{1}{2}(P^2-k_d^2)\ell(P)]F(\underset{\sim}{P}) = -\frac{2M}{\hbar^2}T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d)$$

and from (54) and (55) this is equal to

$$\frac{\hbar^2}{4M}[(P^2-k_d^2)\ell(\underset{\sim}{P})]F(\underset{\sim}{P}) + \langle \underset{\sim}{P}|\mathcal{V}|F\rangle = 0, \tag{56}$$

a form which is perhaps more transparent for establishing that $\mathcal{V}$ is indeed the complex potential well which produces T_{dd}.

In the definition (55) of $\mathcal{V}$ we have deliberately retained the factor

$\langle\phi_d|V_{12}$ to emphasize the fact that $F(\underset{\sim}{P})$ obeys an equation in which the potential is given by the matrix element of an operator taken between $|f\rangle$ and $\langle\phi_d|$. Clearly we can just as easily put $(-\hbar^2/2M)|f\rangle$ equal to $V_{12}|\phi_d\rangle$ and rewrite $\mathcal{V}$ as

$$\mathcal{V} = \langle\phi_d|V_{12}(E^+-K-V_1-V_2)^{-1}(V_1+V_2)G_oV_{12}|\phi_d\rangle, \tag{57}$$

thus emphasizing even more strongly the fact that we are taking a deuteron ground state matrix element to obtain $\mathcal{V}$. Nevertheless $\mathcal{V}$ is not the complex potential well for elastic scattering. We can, however, express $\mathcal{V}$ as an "elastic" portion plus a correction, which will evidently contain a factor $(P^2-k_d^2)$, as indicated by eq. (51).

It follows from (51) that $T_{d,el}(\underset{\sim}{P},\underset{\sim}{k}_d)$ must be of the form

$$T_{d,el}(\underset{\sim}{P},\underset{\sim}{k}_d) = \langle\underset{\sim}{P}|\mathcal{V}_{el}|F\rangle,$$

where $\mathcal{V}_{el}$ is the complex potential well producing the elastic scattering <u>when it acts on F;</u> it is given by

$$\mathcal{V}_{el} = \langle\phi_d|(V_1+V_2)[1+(E^+-K-V_1-V_2)^{-1}(V_1+V_2)]G_oV_{12}|\phi_d\rangle.$$

From eq. (51) we can write the correction to $\mathcal{V}_{el}$, denoted $\mathcal{V}_R$, as

$$\mathcal{V}_R = \langle\phi_d|(K+V_{12}-E)(E^+-K-V_1-V_2)^{-1}(V_1+V_2)G_oV_{12}|\phi_d\rangle,$$

where $\langle\underset{\sim}{P},\phi_d|(K+V_{12}-E) = \frac{\hbar^2}{4M}(P^2-k_d^2)\langle\underset{\sim}{P},\phi_d|$.

Hence,

$$\mathcal{V} = \mathcal{V}_{el} + \langle\phi_d|(K+V_{12}-E)(E^+-K-V_1-V_2)^{-1}(V_1+V_2)G_oV_{12}|\phi_d\rangle \equiv \mathcal{V}_{el}+\mathcal{V}_R \tag{58}$$

and thus we have

$$\frac{h^2}{4M}[(P^2-k_d^2)\,\mathcal{L}(P)]F(\underset{\sim}{P}) + \langle\underset{\sim}{P}|\mathcal{V}_{el}|F\rangle + \langle\underset{\sim}{P}|\mathcal{V}_R|F\rangle = 0. \tag{59}$$

To the extent that the $\langle\underset{\sim}{P}|\mathcal{V}_R|F\rangle$ term of (59) is small, $F(\underset{\sim}{P}) \cong u_o(\underset{\sim}{P})$. However, unless $\mathcal{V}_R = 0$, it is clear that $F \neq u_o$, even though F does obey a complex equation.

With these last few results we have achieved our purpose of characterizing F and deriving the one-body equation it obeys. We shall not pursue in greater detail here the properties of F, since our main purpose has been to discuss the formal aspects of the distorted wave approximation $\Psi \cong \Psi^{ND}$ as used in the stripping matrix element. Both F and u_o are calculable in the three-body model, and we postpone to the future a detailed comparison between them, as well as an investigation ofthe terms $\mathcal{V}_{el}$ and $\mathcal{V}_R$, although we do note here that $\mathcal{V}_R$ can be put into a form simple enough to allow us to attempt an analytic study of it. However, we need not do any further calculation to observe that in the model, F and u_o have very similar effects in the DWBA matrix element, even though they may differ in their functional forms. This statement follows from the numerical work of Reiner and Jaffe[3] which we discuss in the next section.

VI. Discussion

1. Summary

Our results for the three-body model may be summarized as follows. Using the projection operator nature of a separable potential, we have reformulated the DWBA in such a way that the DWBA amplitude for stripping is identical to the exact stripping amplitude. This justifies, in the model, the basic DWBA approximation of replacing a three-body wave function by a product of two-body wave functions in the matrix element.

The new DWBA wave function, denoted Ψ^{ND}, is given by

$$\Psi^{ND} = \phi_d F,$$

where ϕ_d is the deuteron ground state and

$$F = \lambda \langle f|\Psi\rangle$$

is the spectator (CM) wave function, with the two-body interaction having the form

$$V_{12} = -|f\rangle(\hbar^2\lambda/2M)\langle f|$$

We have shown that $F(\underset{\sim}{P})$ obeys a one-body equation which contains a complex potential well that yields the exact elastic scattering amplitude on the energy shell and a more complicated amplitude off the energy shell. This potential well is the ground state matrix element of an operator that we have shown is a sum of $\mathcal{V}_{el}$, which gives rise to the elastic scattering amplitude (on and off shell), and a term $\mathcal{V}_R$ which vanishes on the energy shell. If we were unable to calculate $\langle \chi^{(-)}_{\underset{\sim}{k}_i} \phi_j | V_i - U_i | \Psi \rangle$ we could define the distorted wave approximation as that one for which the second term in (36) was a minimum, since use of $\Psi \cong \Psi^{ND}$ in the first term is still exact [see eq. (43)]. In this sense, DWBA is a variational approach; one may infer for the many-body nuclear case, on the basis of good agreement with experiment, that the choice of V_i to be the optical potential producing the elastic amplitude is the optimum choice. We shall refer to this point again in the following article.

The analysis of section III was concerned with a model in which A, the target or core, was assumed infinitely heavy. The general results of eqs. (41)-(43) however are independent of this assumption, and depend only on the separable character of V_{12}. No changes result if A is assumed to recoil, although in that situation the equations obeyed by the G_i and F will be slightly different. Such effects, while they change the kinematics, cannot alter the basic physics inherent in the separable form for V_{12}.

2. Reformulation of DWBA

In terms of this, we can now reformulate the approximations of section II which lead to the DWBA amplitude. The first approximation, (8), is exact for the model, and in general is one which we retain unquestioned, as in the succeeding article. The second approximation, (9), is to be discarded in favor of

$$\Psi \cong = \Psi^{ND} \doteq \phi_d F, \tag{9'}$$

where the three-body wave function is just ψ_o of section II. Use of (9') in the model stripping matrix element gives an exact result, and thus use of (9) rather than (9') becomes a question of how well u_o approximates F. We discuss this point shortly. The fact that replacement of Ψ by Ψ^{ND} leads to an equality thus justifies retention of an unpolarized or unstretched deuteron ground state wave function in the DWBA matrix element, at least for the model. Alternatively, this justifies the use of a deuteron CM wave function in DWBA, thus allowing us to regard it as "natural" and not as a barbarism.

The third approximation, (12), is now to be replaced by

$$F \cong \chi_d^{(+)} \tag{12'}$$

where $\chi_d^{(+)}$ is an optical model wave function yielding the energy-averaged scattering amplitude arising from F. The function $\chi_d^{(+)}$ would obey an equation with an optical model potential which would not yield the elastic amplitude, just as with the improved DWBA wave function of Johnson and Soper[4].

The final approximation, (13), is not relevant to our work so far

since we have chosen $U_i = V_i$ in the three-body model. We shall discuss this approximation further in the next paper in the series.

3. Remarks on other calculations

In light of these remarks, it is of interest to examine the results of other calculations dealing with this topic, particularly those of Reiner and Jaffe[3] and of Shanley and Aaron[3], each of whom has performed numerical calculations using three-body models similar to Mitra's. Each of these calculations is interesting in its own way: the former for its ability to relate u_o to F; the latter for a comparison of T^{ND} with a DWBA amplitude using the standard DWBA assumption noted above.

3a. Meaning of the Reiner-Jaffe results.

The calculations of Reiner and Jaffe[3] employ different potentials V_1 and V_2, the difference being an attempt to include coulomb effects on their proton (say particle 1) by altering the form factor (our g_1) compared to the form factor occurring in g_2. They compute the exact stripping amplitude, our T_{id}, and compare it with a particular DWBA amplitude T^{DW}_{RJ} calculated as follows. Their final state wave function $\chi^{(-)}_{k_f}$ is computed by setting $U_1 = V_1$, as in section II. Hence, their $\chi^{(-)}_{k_f}$ gives rise to real phase shifts. However, their initial distorted wave corresponds to our u_o rather than F, just as is called for in the standard DWBA prescription (approximation (9)). Hence, since use of Ψ^{ND} is exact as we have explained above, their comparison of T^{DW}_{RJ} with T_{id} is actually a comparison of T^{DW}_{RJ} with T^{ND}, or equivalently compares the

effects of using u_o and F in the stripping matrix element. This is precisely the comparison we need in order to verify if standard DWBA, as represented by (9), i.e. by use of u_o, is a good approximation to "exact" DWBA, as represented by use of (9').

We note first that the change in form factors (V_1 compared to V_2) produces the desired effect of decreasing the (d,p) cross-sections compared to the (d,n) cross-sections. Hence, they are employing a realistic three-body model. More important than this, however, is their finding that replacement of F by u_o (use of T_{RJ}^{DW} rather than T^{ND}) is a valid approximation: the angular distributions from T_{RJ}^{DW} agree quite well with those from $T^{ND} = T_{id}$ except for forward angles ($\lesssim 20°$) and then only at the lower of the two energies investigated. Specifically, the differences in magnitude are roughly of the order of 20% at a "deuteron" energy of 6.7 MeV (DWBA < exact for (d,p) and DWBA > exact for (d,n)) and are approximately zero at 11.2 MeV. This implies that the difference between u_o and F (at least in their effect in the matrix element) decreases as the energy increases, a point we are investigating.

The conclusion to be drawn from this, at least for the three-body model, is not that $\Psi \cong \phi_d u_o$ is a good approximation, but that $u_o \cong F$ is a good approximation, since we already know that use of $\Psi \cong \Psi^{ND} = \phi_d F$ is exact in the matrix element. We shall refer to this conclusion in the next paper where we treat the many-body case of actual nuclear reactions.

3b. Remarks on Johnson's Adiabatic Approximation

Another very interesting connection is with the work of Johnson and co-workers[4]. Their improved CM wave function is conceptually very much

like our F, although they use optical potentials to calculate their wave function, so it would probably be nearer quantitatively to $\chi_d^{(+)}$ of (12') than to F. However, we shall refer here to F rather than to $\chi_d^{(+)}$. Both their improved wave function and F obey one-body equations that do not give rise to an elastic scattering amplitude, and both contain effects from continuum or break-up deuteron states. Use of their improved wave function has led to improved fits to data in all cases but one, and it would seem likely that their method is in general the correct one to follow for more accurate results. The connection between their method of including continuum effects and the inclusion of such effects in F, i.e., the relation between the two wave functions remains to be established. We conjecture that they are similar, and will use the three-body model as a means to compare the two and so verify or disprove this conjecture. It is interesting to note that the exact role played by ψ^{ND} in the separable-potential, three-body model was recognized by Johnson and Soper[4] and used to help formulate their adiabatic method[19].

3c. Interpretation of the Shanley-Aaron Calculations.

These authors compared their exact (d,p) computations with a standard DWBA cross-section. The calculations were done for the case in which only the interactions V_1 and V_{12} are non-zero: setting V_2=0 means that the knock-out channel is absent and that only nucleon 1 can be stripped from the deuteron. To get optical model wave functions to use in the DWBA matrix element (eq. (14)) they matched the elastic proton and deuteron cross sections as determined from their three-body model with those generated from a Saxon-Wood type optical potential. Excellent agreement was found for the elastic

angular distributions at the relevant energies, although the agreement was not so good when considered as a function of energy,the deuteron elastic scattering parameters being fitted better. Their fits of the DWBA cross-sections to the exact stripping cross-section was, however, not very good. At best, the peak cross-section was underestimated by a factor of about 1.7, while the first minimum was closer to the origin by at least 10^{o}. In addition, the secondary peak was also underestimated and a secondary dip or diffraction minimum was predicted by DWBA although it was not present in the exact result.

Since the standard approximations were used to derive their DWBA it is no surprise that Shanley and Aaron failed to get good agreement between their T_{id} and T_{id}^{DW}. Two points are of interest here. First, an elastic deuteron wave function was used in their DWBA, whereas our work quite clearly shows that a different optical model wave function such as $\chi_d^{(+)}$ should have been used. Second, they used an elastic scattering "proton" wave function. But since $V_2 = 0$, they should in fact have used a plane wave for the proton final state. Or equivalently, since a final state potential V_2 was used to generate a $\chi_{k_2}^{(-)}$, then the second matrix element in eq. (36) should have been computed as well. One can only conclude that their obtaining of a DWBA angular distribution with the same general features of the exact one demonstrates that use of the neutron-proton interaction in the matrix element plus the proper angular momentum coupling are the main ingredients in producing a cross-section with the proper shape.

3d. Comment on the Model of Baz', Demin and Kuz'min

Baz', Demin, and Kuz'min have investigated the properties of a three-body model in which the interaction V_{12} is taken to be a point interaction, while

V_1 and V_2 are assumed to be one-body potential wells[20]. Numerical calculations of deuteron elastic scattering and stripping were carried out. In particular, exact and DWBA results were compared and it was found that DWBA was a poor approximation at an energy of 2.2 MeV (the only energy for which calculations are reported). The authors state that the reason for poor agreement is that the penetrability of the centrifugal barrier by the deuteron for S-, P-, and D-waves is large in this problem, and in such cases approximate methods are inapplicable. Only for higher partial waves is the penetrability small and the exact and approximate partial wave amplitudes in close agreement.

These results can be understood on the basis of the foregoing work. First, we note that the model of point interaction is one that involves a limiting process: a delta function potential as such at r=0 does not have a bound state. If we keep the limiting process in mind, but write the interaction as a delta function, then it is straightforward to show that the interaction is separable in the sense used in this article. For example, if we are dealing with $V(\underset{\sim}{r}) = \lambda\delta(\underset{\sim}{r})$, then what is really meant is that

$$\langle \underset{\sim}{r}' | V | \underset{\sim}{r} \rangle = V(\underset{\sim}{r})\ \delta(\underset{\sim}{r}-\underset{\sim}{r}') = \lambda\delta(\underset{\sim}{r})\delta(\underset{\sim}{r}-\underset{\sim}{r}'),$$

which is equivalent to

$$\langle \underset{\sim}{r}' | V | \underset{\sim}{r} \rangle = \lambda\delta(\underset{\sim}{r})\delta(\underset{\sim}{r}'),$$

a separable form. A similar result holds for $V(\underset{\sim}{r}) = r^n\ \delta(r)$, where n need not be integral.

Given this, it is obvious that an exact DWBA can be formulated for the model of Baz' et al. The finding that standard DWBA (i.e.: use of u_o) is not a good approximation at low energies is not entirely inconsistent with the lower energy calculation of Reiner and Jaffe[3]; rather, we simply seem to have an example where replacement of (the appropriate) F by u_o is inadequate. Whether this is a result due mainly to the choice of interaction, the very low value of

of the energy, or both, we cannot say. Another possibility, that replacement of the separable forms for V_1 and V_2 by local potentials also has influenced the result, will be investigated in the future.

The result of Baz' et al should not be confused with a possible inapplicability of the zero-range approximation as used in DWBA[2]. This is an approximation applying to the ranges of values of the neutron and proton spatial coordinates that enter into a spatial coordinate evaluation of the usual stripping matrix element, and implies that the relevant integrals can be accurately evaluated if these two coordinates are taken to be the same. This is not necessarily inconsistent with use of (any form of) a separable interaction; the form factor occurring in the interaction need only be short-ranged enough to permit a zero-range approximation to be used.

4. Other Processes

So far we have considered the case of (d,p) reactions in this model. The equality of the DWBA (as we have formulated it) and the exact amplitudes follows from the projection operator nature of the separable interaction. It is clear that each of the interactions acting between the particles produces the same effect and thus we can also prove an equality between DWBA and exact amplitudes for the knock out process as well, and of course for elastic scattering too, although this latter point is of less interest for the many-body problem since elastic scattering is conventionally discussed in terms of the optical model[2]. The (exact) DWBA in this model for knock out involves a matrix element of V_i (i incident) and not V_{12} as one would expect on the basis of calculations done for actual (p,n) or even (p,p') reactions. The significance of this result is currently being investigated; we hope to report our findings on this point in the future.

5. Divergence of DWBA Series

The main feature of the present model is the fact that the DWBA and exact stripping amplitudes are identical if DWBA is properly defined. It thus allows us to see what the ordinary or standard DWBA is trying to approximate, at least within the context of the model. However another interesting feature is that by using F in the initial state distorted wave, we produce a DWBA series which converges, in contrast to a) the generally expected behavior of the Born series[20], and b) the likelihood that the DWBA series will also diverge in general[22]. We intend to return to this point elsewhere.

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Appendix

We derive here equation (32) obeyed by $F(\underset{\sim}{P})$. We could start with eq. (53) and project it onto $\langle f|$, but it is simpler to begin with the Lippmann-Schwinger equation

$$|\Psi\rangle = |\underset{\sim}{k}_d\rangle|\phi_d\rangle + (E^+-K-V_{12})^{-1}(V_1+V_2)|\Psi\rangle, \tag{A1}$$

where $E^+ = E + i\varepsilon$, $\varepsilon>0$, and $K = K_{np}+K_{cm}$. Projecting both sides of (A1) onto $\lambda\langle f|$ then gives

$$\lambda\langle f|\Psi\rangle \equiv |F\rangle = \lambda|\underset{\sim}{k}_d\rangle\langle f|\phi_d\rangle + \lambda\langle f|(E^+-K-V_{12})^{-1} \times (V_1+V_2)|\Psi\rangle. \tag{A2}$$

By definition [eq. (21)], $\lambda\langle f|\phi_d\rangle = 1$, so the first term on the right hand side of (A2) is just $|\underset{\sim}{k}_d\rangle$. We note that (A1) suffers from the usual problems of an integral equation for more than two particles, but that (A2) does not since it is a one-particle equation.

The key to solving (A2) is the separable nature of V_{12}. In detail, the propagator is

$$(E^+-K-V_{12})^{-1} = (E^+ - K + \frac{\hbar^2\lambda}{2M}|f\rangle\langle f|)^{-1},$$

which is easily expressed in terms of $(E^+-K)^{-1}$. Some straightforward algebra leads to

$$(E^+-K-V_{12})^{-1} = (E^+-K)^{-1}[1 + V_{12}(E^+-K)^{-1}/(1+\frac{\hbar^2\lambda}{2M}\langle f|(E^+-K)^{-1}|f\rangle)] \tag{A3}$$

and on substitution into (A2) we have

$$|F\rangle = |\underset{\sim}{k}_d\rangle + \lambda\langle f|(E^+-K)^{-1}(V_1+V_2)|\Psi\rangle + \lambda\langle f|(E^+-K)^{-1}V_{12}(E^+-K)^{-1}(V_1+V_2) \times\{1/[1 + \hbar^2\lambda\langle f|(E^+-K)^{-1}|f\rangle/2M]\}|\Psi\rangle. \tag{A4}$$

Equation (A4) can be put in more familiar form by examining its momentum representative:

$$F(\underset{\sim}{P}) \equiv \langle \underset{\sim}{P}|F\rangle = \delta(\underset{\sim}{P}-\underset{\sim}{k}_d) + \lambda\langle \underset{\sim}{P},f|(E^+ - \frac{\hbar^2P^2}{4M} - K_{np})^{-1}(V_1+V_2)$$
$$\times |\psi\rangle + \lambda\langle \underset{\sim}{P},f|(E^+ - \frac{\hbar^2P^2}{4M} - K_{np})^{-1} V_{12}(E^+ - \frac{\hbar^2P^2}{4M} - K_{np})^{-1}$$
$$\times\{(V_1+V_2)/[1 + \hbar^2\lambda\langle f|\ E^+ - \frac{\hbar^2P^2}{4M} - K_{np}|f\rangle/2M]\}|\psi\rangle. \qquad (A5)$$

If the separable form of V_{12} is used in the last matrix element of (A5), the result can be combined with the preceding term to give

$$F(\underset{\sim}{P}) = \delta(\underset{\sim}{P}-\underset{\sim}{k}_d) + [\lambda^{-1} - \tfrac{1}{2}\, h(P)]^{-1}\langle \underset{\sim}{P},f|G_o(V_1+V_2)|\psi\rangle, \qquad (A6)$$

where h(P) is the function defined by Mitra[3] and also above eq. (30) in the main text, and where we use the operator form $(E^+-K)^{-1} = G_o$ for the factor $E^+ - K_{np} - \hbar^2P^2/4M$ appearing in the matrix element.

On now transforming the propagator via eq. (31), we may put (A6) into the form

$$F(\underset{\sim}{P}) = \delta(\underset{\sim}{P}-\underset{\sim}{k}_d) - [\ell(P)(P^2-k_d^{\,2}-i\varepsilon)]^{-1}\,\frac{4M}{\hbar^2}\,T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d), \qquad (A7)$$

where

$$T_{dd}(\underset{\sim}{P},\underset{\sim}{k}_d) = -\frac{\hbar^2}{2M}\langle \underset{\sim}{P},f|G_o(V_1+V_2)|\psi\rangle. \qquad (A8)$$

It is clear by inspection that (A7) is identical to eq. (32), and that (A8) is identical to (49), as claimed in the text. Furthermore, use of eqs. (15) in (A6) leads to the last equation in (27); similar manipulations provide the other members of the coupled equations.

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