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An Achromatic Translation System for Charged Particle Beams*

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ABSTRACT

An assembly of bending magnets and magnetic quadrupole lenses was found which causes the axis of a beam of charged particles of momenta $p \pm \Delta p$ to be shifted from its original axis to a parallel line displaced from its original axis by a distance d. A study of this transport system demonstrates that it is possible to choose the parameters of the system in such a way that the entire system has an acceptable effect on the transverse motion of the beam, and so that the system is achromatic to first order in $\Delta p/p$. A typical transport system is given, and results are presented in graphical form for a range of parameters encompassing the typical values.

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DISCLAIMER

I. INTRODUCTION

Beam transport systems generally consist of bending magnets and magnetic quadrupole lenses which bend, focus, and defocus a beam of charged particles so that it moves in a "satisfactory way" from one piece of apparatus to another.

The need arose at this laboratory for a beam transport system which would shift the axis of a charged particle beam from its original axis to some parallel line displaced from the original axis by a distance d. Transport systems of this type are referred to as beam translation systems.

In order to accommodate particles of momenta $p \pm \Delta p$, it is desirable that the net effect of the transport system be independent of $\Delta p/p$. Translation systems having this property are referred to as achromatic.

It is important to consider the effect of such transport systems on the transverse dimension of the beam. These considerations state simply that the transverse dimensions of the beam must not get unduly large at any point within the transport system. The importance of this consideration for a particular transport system depends on the size and collimation of the incident beam.

A general arrangement of beam transport elements was chosen which showed promise of yielding a satisfactory set of achromatic translation systems. A procedure for evaluating the performance was established, and a means was found for selecting suitable values for the numerous parameters describing the systems.

II. THE SYSTEMS

This study is restricted to beam transport systems having the general arrangement of elements shown in Fig. 1. The first element on the left is a bending magnet. The next five elements are magnetic quadrupole lenses and the last element is another bending magnet. The transport system is symmetric about the center quadrupole lens. The element spacings a, b, c, and the translation distance, d, are shown in the figure.

Both bending magnets bend axial particles of momentum p on a radius of curvature ρ, through an angle of magnitude α. The entrance and exit edge angles of the first element are β and zero respectively, where the angles are defined as shown in Fig. 1. The entrance and exit angles of the last element are zero and β respectively as required by the symmetry.

The strengths of the magnetic quadrupole lenses are given in terms of the lengths of the lenses and the gradients of the magnetic fields. All quadrupole lenses have the same length L. The magnetic field gradient in the second, third and fourth elements are S_2, S_3 and S_4 respectively. The gradients of the fifth and sixth elements are S_3 and S_2 respectively, again as a result of the symmetry. The second, fourth and sixth elements are oriented so as to be focusing for transverse motion in the plane of the translation. The third and fifth elements have a defocusing action on transverse motion in this plane.

III. TRAJECTORIES

It was felt that the arrangement of elements described above would yield a set of satisfactory achromatic translation systems. That this is reasonable can be advanced by considering the trajectories of a few particular test particles.

A. The Achromatic Condition

The trajectory of an axial particle of momentum p is shown as a solid line in Fig. 2. An axial particle of momentum $p + \Delta p$ will travel along some dashed line as shown in the same figure. The second and third elements form a quadrupole doublet lens combination which causes axial particles of momenta $p \pm \Delta p$ to intersect the axis near the point of symmetry in the center of the fourth element. The axial particle of momentum $p + \Delta p$ will continue relatively undisturbed through the weak field in the center of the fourth element, through the quadrupole doublet lens formed by the fifth and sixth elements, and finally through the second bending magnet. As a result of the symmetry about the center of the transport system, this off-momentum particle will rejoin the axis of the transport system on leaving the bending magnet. We see that axial particles of momenta $p \pm \Delta p$ all suffer a translation of a distance d, and a zero net change in angular direction independent of $\Delta p/p$ to first order. Hence we say the beam has undergone an achromatic translation.

B. The Transverse Motion

We now consider the transverse motion of the off-axis particles of momenta p. Let x be the coordinate normal to the axis of the transport system lying in the plane of the translation, which we will for simplicity take to be horizontal. Let y be the vertical coordinate (i.e., normal to the plane of the translation). Let x' and y' be dx/ds and dy/ds, where s is the arc length along the axis.

1. The Vertical Motion

The achromatic condition requires a quadrupole doublet (second and third elements) which has some net focusing in the horizontal plane. A net defocusing results from this doublet in the vertical plane. Vertical focusing is introduced by rotating the entrance face of the first magnet, and the exit face of the second magnet through an angle β from normal incidence. The angle β can be chosen so that a paraxial particle displaced initially a distance y above the plane of the translation is forced to pass through the plane at the point of symmetry and, as a result of the symmetry, to exit from the transport system as a paraxial ray displaced a distance y <u>below</u> the plane. A ray diagram of this motion is shown in Fig. 3. The transformation through the entire beam transport system for the (y, y') vector is, in matrix notation $\begin{vmatrix} -1 & +L_y \\ 0 & -1 \end{vmatrix}$. Were it not for the minus signs, the matrix would represent simply a free-space drift of the vector (y, y') for a distance L_y. L_y will be referred to as the "y" drift length.

2. The Horizontal Motion

Let us now consider the trajectory of a paraxial ray in the plane of the translation displaced horizontally a distance x from the axis on entrance to the transport system.

The focusing required by the achromatic condition causes an over-focused situation for the x motion. That is, the paraxial ray crosses the axis before arriving at the center of the transport system. The ray arrives at the plane of the fourth element with an x and an x' coordinate of the same algebraic sign. The role of the fourth element is to transform the coordinates of such particles from (x, x') to (x, - x'). This particle then continues through the transport system and,as a result of the symmetry, leaves the transport system as a paraxial ray displaced by a distance x from the axis. This motion is shown in Fig. 4. The transformation through the entire transport system for the (x, x') vector is, in matrix motion $\begin{vmatrix} 1 & L_x \\ 0 & 1 \end{vmatrix}$. This is simply the transformation for a drift of a distance L_x, where L_x is called the "x" drift length.

IV. THE APPROACH

The system is described in Section II in terms of twelve variables; a, b, c, d, p, L, S_2, S_3, S_4, α, β, and ρ. The variable c is simply $d/(2 \sin \alpha)$.

It is of interest to group the variables in such a way that they are readily applicable to transport systems producing a translation of distance d, for particles of momentum $p \pm \Delta p$. The variables, other than d and p, are grouped as follows: a/c, b/c, L/d, G_2, G_3/G_2, G_4,

α, β, ρ/d, where the G's are unitless parameters proportional to the quadrupole field gradients S. G_n is defined as $S_n d^2/p$ where S_n has the units of gauss/unit length, and p is expressed as magnetic rigidity in units of gauss x unit length. All nine quantities above are unitless, and hence require no scaling between transport systems of different d's and p's.

Assuming that d and p are given, we now seek a way to determine suitable values for the nine unitless quantities required to complete the description of the transport system. Application of the three conditions presented in Section III, serves to make the three quantities G_2, β, G_4 dependent on the remaining six quantities. Given these six quantities, a procedure described in Section VIII facilitates the solution for the values of G_2, β, and G_4.

We began to probe the nature of these transport systems by choosing $\alpha = 20^\circ$, $\rho/d = 0.5467$, $L/d = 0.1366$, and $b/c = 0.25$. A range of values was chosen for a/c and G_3/G_2. Specification of the latter two parameters completes the required information for a particular achromatic translation system.

V. BEHAVIOR OF L_y

Examination of the resulting beam translation systems confirms the fact that the net result on the y motion of the entire system is simply a drift-like transformation* with a drift length of L_y. However, the

*See Section III.

magnitude of L_y for some of the systems was quite a surprise. The values of L_y ranged from values which were unacceptably large by most standards down to values of zero in some cases. These data are presented in Fig. 5. Each dot represents a particular transport system where the resulting L_y is plotted against the ratio G_3/G_2. Systems having the same value of a/c are joined by a line and the value of a/c is indicated.

The x motion may likewise be described as a drift-like transformation with a drift length L_x. L_x is always small, and is apparently insensitive to the particular parameters of the transport system.

The behavior of L_y gave rise to a fourth condition on the choice of parameters. The decision was made to choose the ratio G_3/G_2 so as to make L_y small, and in the subsequent runs, G_3/G_2 was chosen to correspond to the first root of L_y as shown in Fig. 5, for every set of values chosen for the remaining variables.

VI. RESULTS

There remain but five choices to be made now to complete the description of the transport system. The choices have been reduced to the deflection angle α, and the radius of curvature ρ/d of the bending magnet, the quadrupole spacings a/c and b/c, and the quadrupole length L/d.

This study was made in connection with a search for a beam translation system for a particular purpose. Values were chosen for the five quantities above which were suitable for this particular purpose, namely, $\alpha = 20^\circ$, $\rho/d = 0.5467$, $a/c = 0.50$, $b/c = 0.25$, and $L/d = 0.1366$.

In order to reveal the effect of each of the quantities α, ρ/d, a/c and b/c, they were varied about their design value and the results are presented in graphical form in Figs. 6-9. It was felt that the effect of the parameter L/d on the performance of the translation system was small and no attempt was made to study the system while varying this parameter.

Figure 6 is a plot of the parameters G_2, G_4 and β as a function of α for α between 15° and 30°, where the parameters ρ/d, a/c, b/c, and L/d have their design value. The solution of G_3/G_2 proved to be 0.75 for all values of α. The vertical dashed line marks the design value of α.

Figure 7 is a plot of the parameters G_2, G_4 and β as a function of ρ/d, where the parameters α, a/c, b/c and L/d have their design value. Here again, G_3/G_2 is 0.75 for all values of ρ/d.

Figure 8 is a plot of the parameters G_2, G_3, G_4 and β as a function of a/c, where the parameters α, ρ/d, b/c and L/d have their design value. The solution for G_3/G_2 is a function of a/c as shown by the two curves.

Figure 9 is a plot of the parameters G_2, G_3, G_4 and β as a function of b/c, where the parameters α, ρ/d, a/c and L/d have their design value. Here the solution for G_3/G_2 is a function of b/c.

The particle beams exhibit their maximum width in the x plane at the central quadrupole lens. The ratio of x_{max} to the width of the incident beam, x_i, is shown in Figures 6-9.

Table I presents the actual parameters for a beam translation system, along with values for the corresponding unitless quantities defined in this paper. The translation system described on the left-hand side of Table I was designed to translate a beam of 200 Mev protons by a translation distance of 12 feet.

TABLE I

Actual Translation System	Unitless Representation
Proton Beam	
Energy - 200 Mev	
d = 12 ft. = 365.8 cm.	d
p = 2.1496×10^6 gauss cm.	p
a = 267.5 cm.	a/c = 0.50
b = 133.75 cm.	b/c = 0.25
c = 535.0 cm.	
L = 50.0 cm.	L/d = 0.1366
S_2 = 465 gauss/cm.	G_2 = 28.95
S_3 = 348 gauss/cm.	G_3/G_2 = 0.75
S_4 = 420 gauss/cm.	G_4 = 26.14
$\alpha = 20^\circ$	$\alpha = 20^\circ$
$\beta = 28.75^\circ$	$\beta = 28.75^\circ$
ρ = 200 cm.	ρ/d = 0.5467

VII. DISCUSSION

The beam translation systems described here have a number of desirable features. They are achromatic to first order in $\Delta p/p$. The transformations describing the transverse motions are essentially plus and minus the identity transformation. The systems are made up of standard beam transport elements and all systems are symmetric about their center element.

An achromatic beam translation system[1] was described in the literature in 1954, which is in fact a special case of the beam translation systems described in the present paper. The system described in the earlier paper had no elements corresponding to the third and fifth elements of the present system. The earlier system, therefore, is the special case of the present system where G_3 or the ratio G_3/G_2 is zero.

We have found the third and fifth element essential for the reduction of the magnitude of the drift length L_y. A reference to Fig. 5 supports this finding.

VIII. DIGITAL COMPUTATION PROGRAM

The large number of variables involved, and the nature of the four conditions imposed on the solution, make it difficult to proceed analytically. We decided to seek our understanding of these transport systems with the aid of a digital computation program.

A program entitled BEAM TRANSPORT[2] was prepared for an IBM 704 computer, which could simulate the action of an arbitrary collection of beam transport elements on a beam of charged particles.

The transformations which simulate the action of the elements of the transport system include only the linear terms and are conveniently recorded in the form of (3 x 3) matrices which operate on the column vectors $\begin{pmatrix} x \\ x' \\ \Delta p/p \end{pmatrix}$ and $\begin{pmatrix} y \\ y' \\ \Delta p/p \end{pmatrix}$.

An excellent review of the considerations leading to these transformations appears in an article by S. Penner.[3]

Tracing particle rays through a particular beam transport system with the aid of this program is so rapid (of the order of one second), that it is practical to solve for G_2, β, G_4, and G_3/G_2 in an iterative way.

The nature of the beam entering the first element of the transport system was specified as a set of particles each having x, x', y, y', and $\Delta p/p$ coordinates. This set of particles included the three particles pertinent to the application of the four conditions described earlier.

A run would be initiated by specification of the five quantities α, ρ/d, a/c, b/c and L/d, along with guesses at the values of G_2, β, G_4, and G_3/G_2. The initial set of particles would then be traced through the transport system.

An "adjust" subroutine was prepared for the computer program, which applied the four conditions described earlier to the results of the ray tracing, and which made incremental changes to the values of G_2, β, G_4, and G_3/G_2 in such a direction as to satisfy more closely the imposed conditions. After an average of 100 iterations (i.e., 100 seconds), the values of G_2, β, G_4 and G_3/G_2 would represent appropriate solutions for these parameters.

IX. ACKNOWLEDGMENT

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REFERENCES

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2. D. A. Swenson, Midwestern Universities Research Association report MURA-645 (1962, unpublished).
3. S. Penner, Rev. Sci. Instr. 32, 150 (1961).

FIGURES CAPTIONS

Fig. 1. General arrangement of elements.

Fig. 2. Ray diagram showing as a dashed line the trajectory of an axial particle of momentum $p + \Delta p$. The F and D refer to the action of the quadrupole lenses on transverse motion in the plane of the translation.

Fig. 3. Ray diagram showing as a dashed line the y motion for a paraxial particle of momentum p. The y coordinate is the coordinate normal to the plane of the translation. Here F and D refer to the action of the quadrupole lenses on y.

Fig. 4. Ray diagram showing as a dashed line the x motion for a paraxial particle of momentum p. The x coordinate is measured normal to the axis of the transport system in the plane of the translation. Here F and D refer to the action of the quadrupole lenses on x.

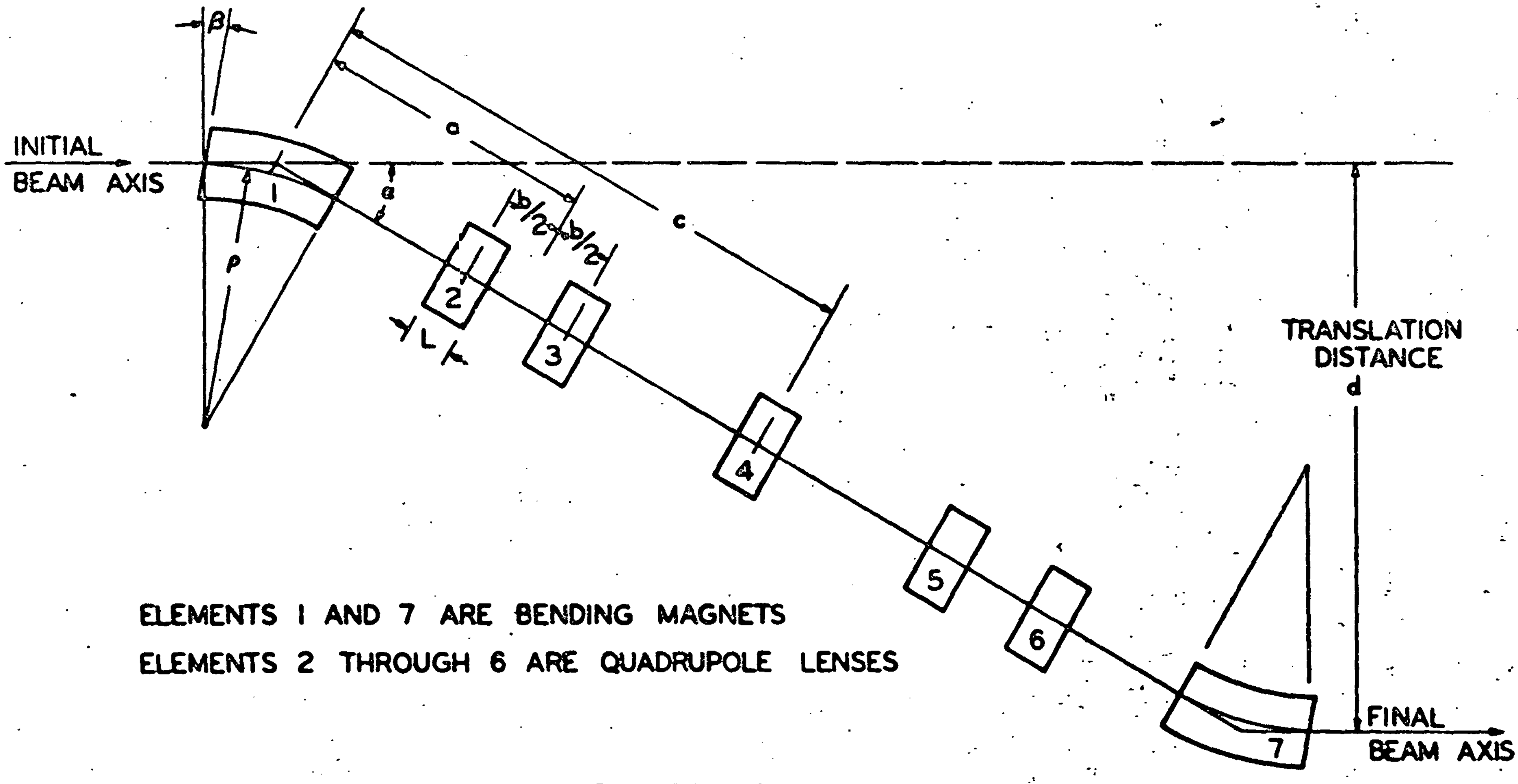
Fig. 5. The y drift length versus the ratio G_3/G_2 for three different values of a/c.

Fig. 6. The effect of varying α about its design value of 20° on several parameters. The solution for G_3/G_2 is 0.75 for all values of α.

Fig. 7. The effect of varying ρ/d about its design value 0.5467 on several parameters. The solution for G_3/G_2 is 0.75 for all values of ρ/d.

Fig. 8. The effect of varying a/c about its design value of 0.50 on several parameters.

Fig. 9. The effect of varying b/c about its design value of 0.25 on several parameters.



ACHROMATIC TRANSLATION SYSTEM

Fig 1

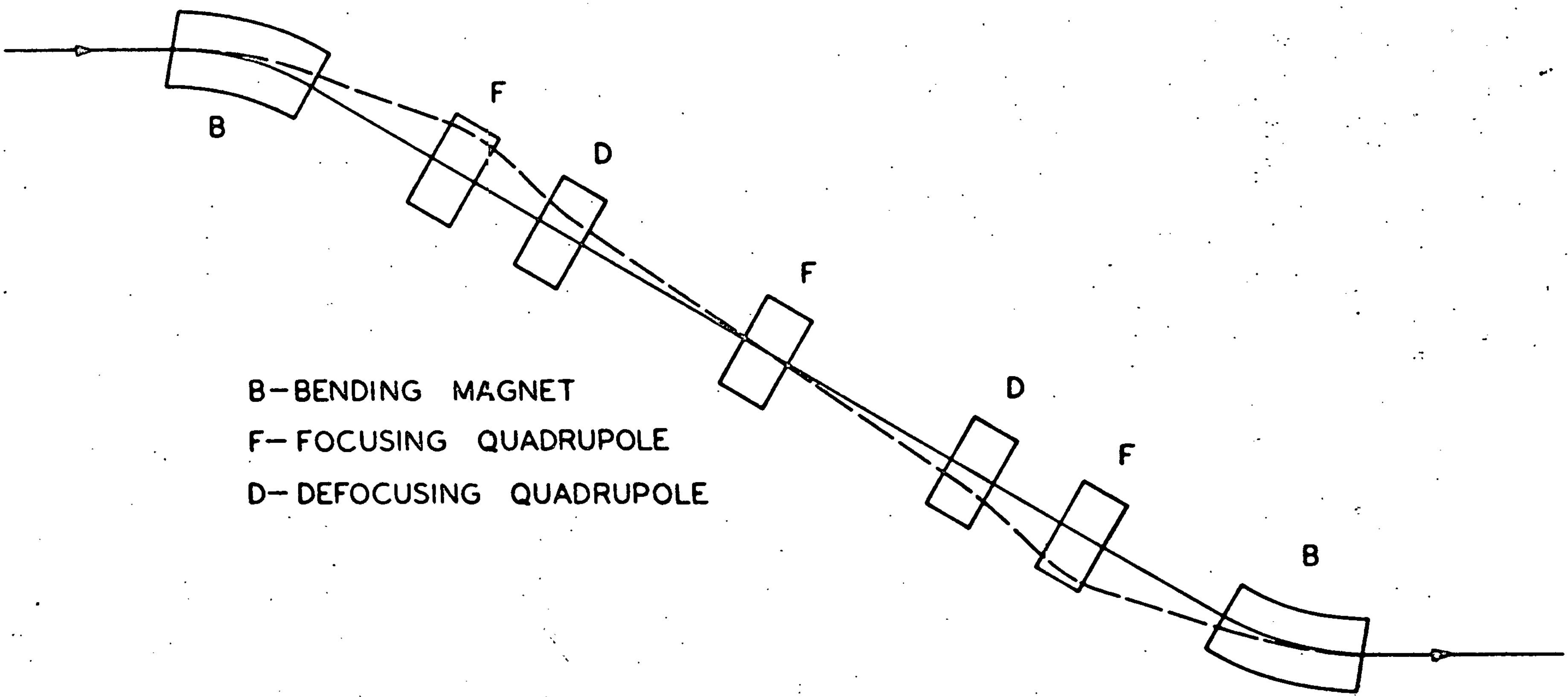

B
F
D
F
D
F
B
B-BENDING MAGNET
F-FOCUSING QUADRUPOLE
D-DEFOCUSING QUADRUPOLE

Fig 2

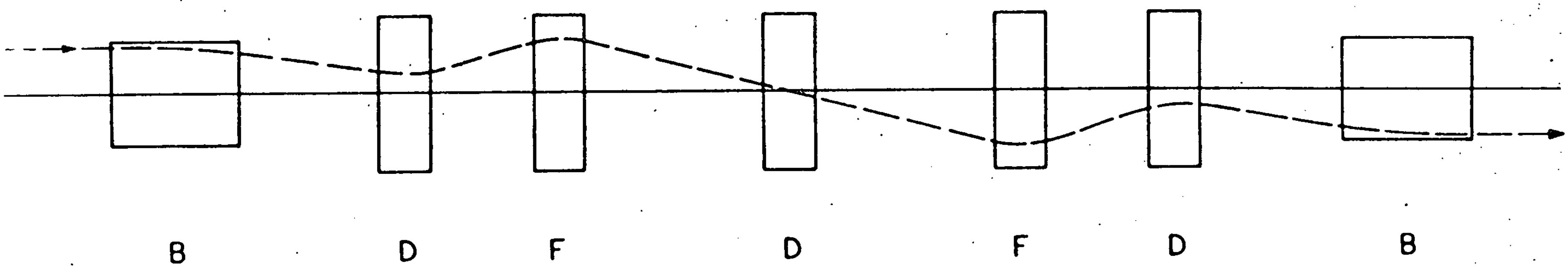


Fig 3

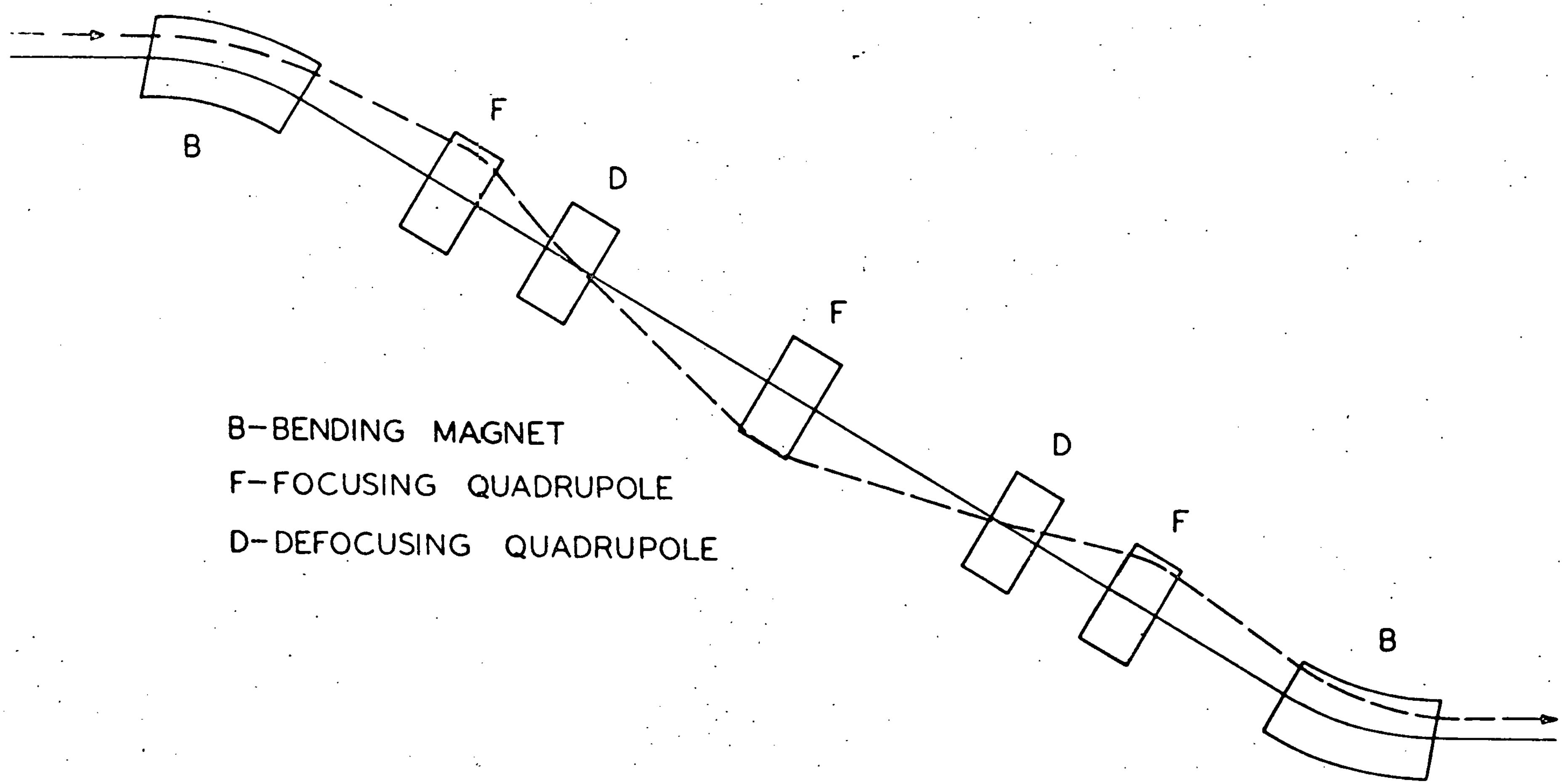
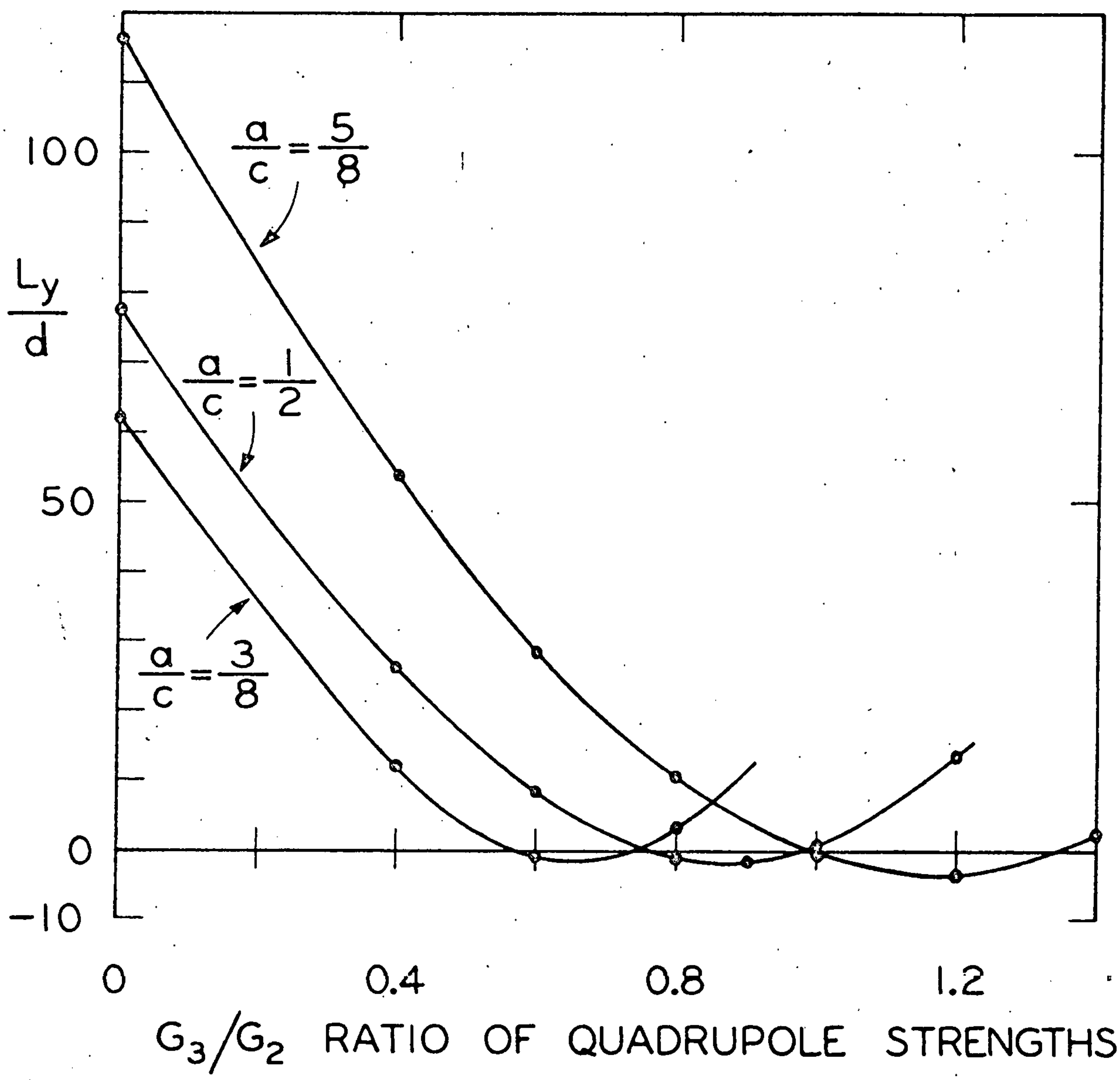
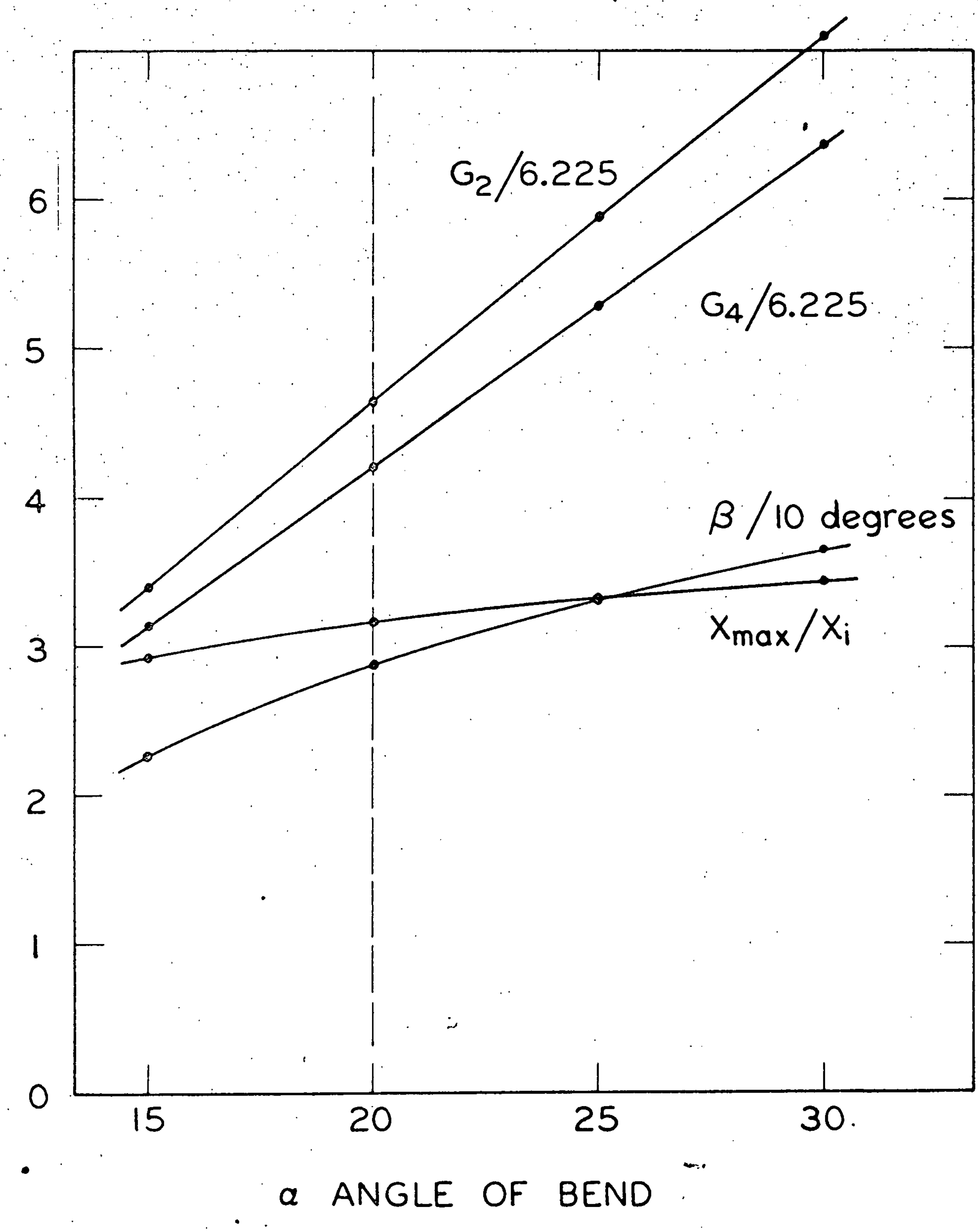

B
F
D
F
D
F
B
B-BENDING MAGNET
F-FOCUSING QUADRUPOLE
D-DEFOCUSING QUADRUPOLE

Fig 4



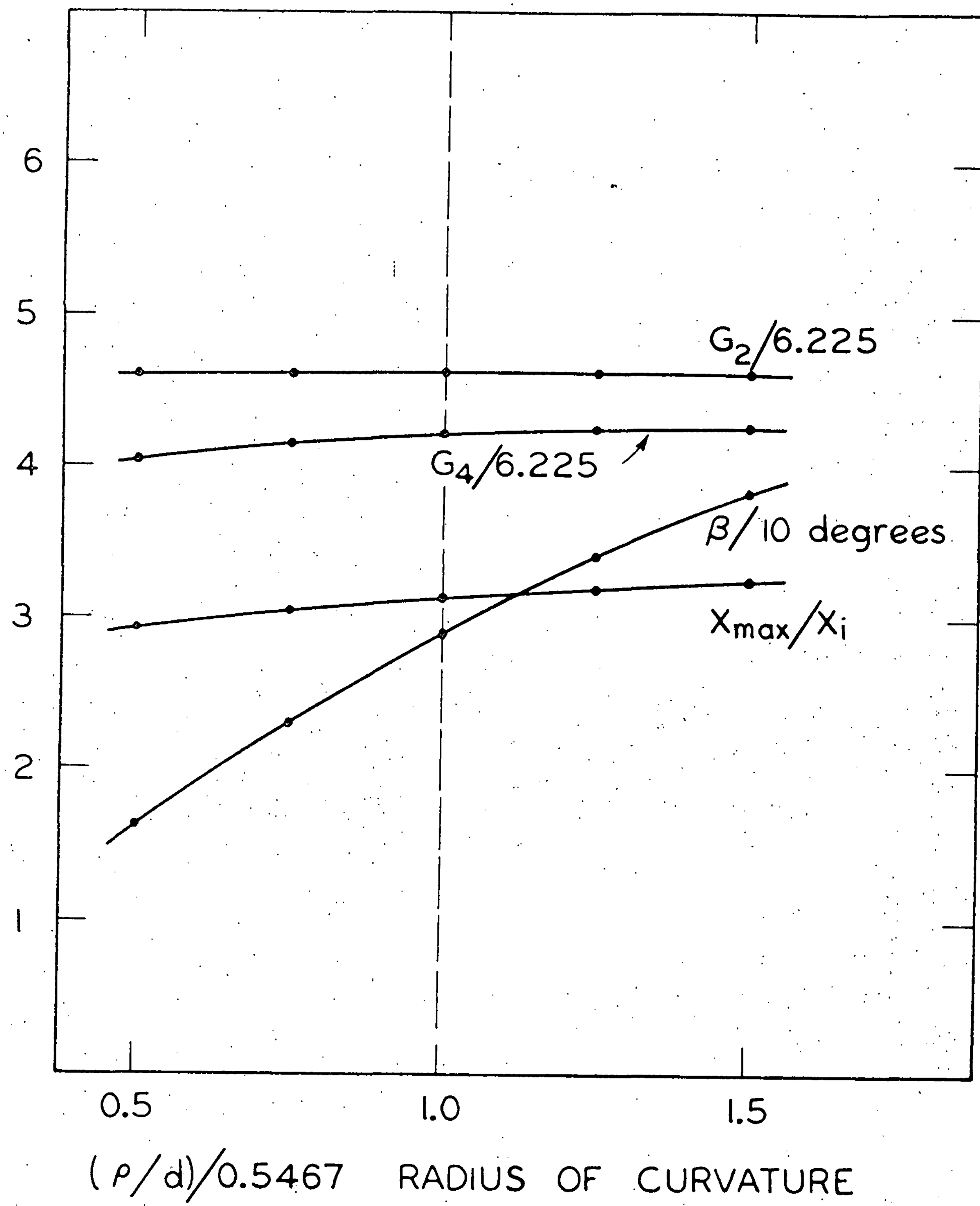
100
50
0
−10
$\frac{L_y}{d}$
$\frac{a}{c}=\frac{5}{8}$
$\frac{a}{c}=\frac{1}{2}$
$\frac{a}{c}=\frac{3}{8}$
0
0.4
0.8
1.2
G_3/G_2 RATIO OF QUADRUPOLE STRENGTHS

Fig 5



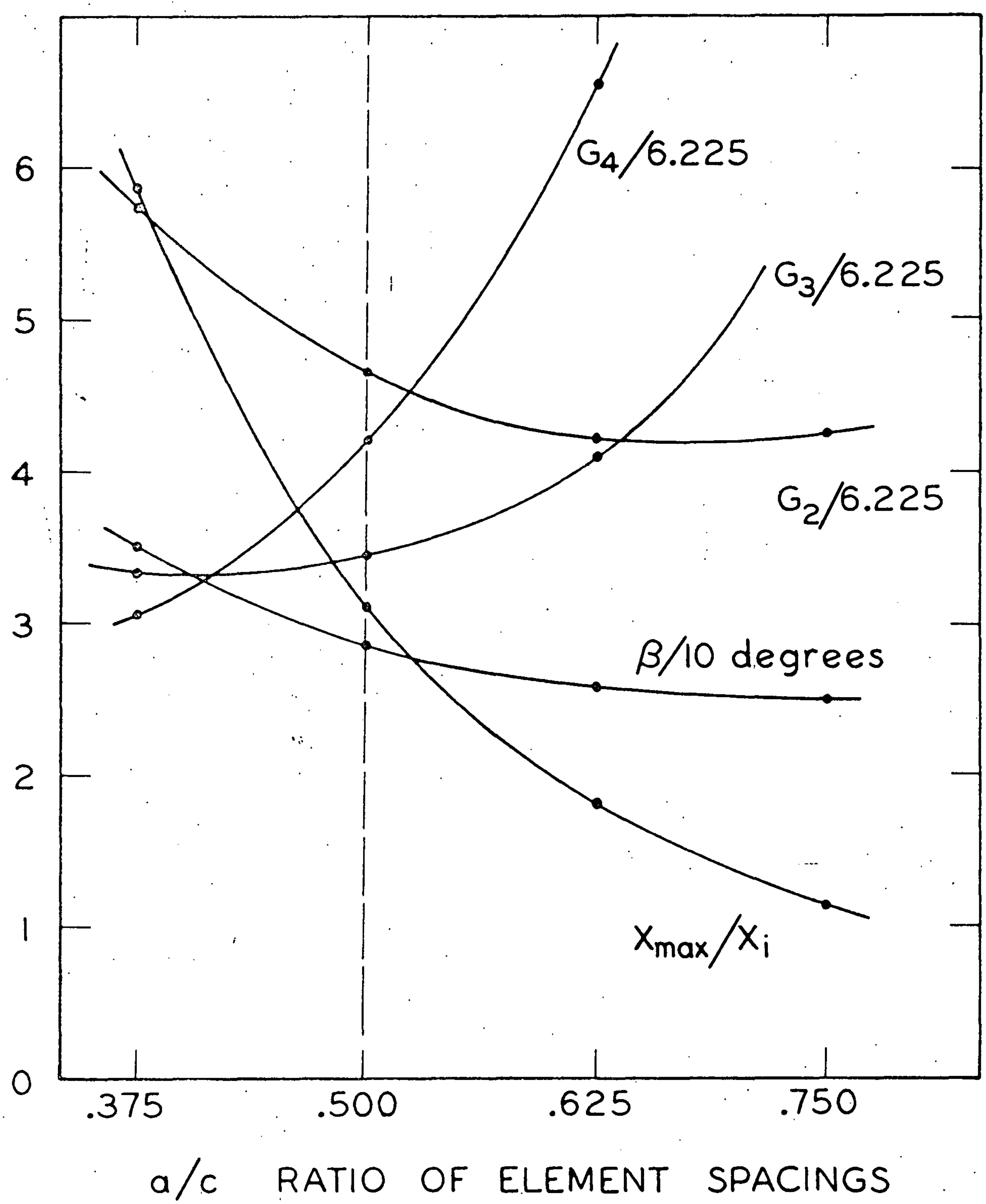
G2/6.225
G4/6.225
β /10 degrees
Xmax/Xi
6
5
4
3
2
1
0
15
20
25
30
α ANGLE OF BEND

Fig 6



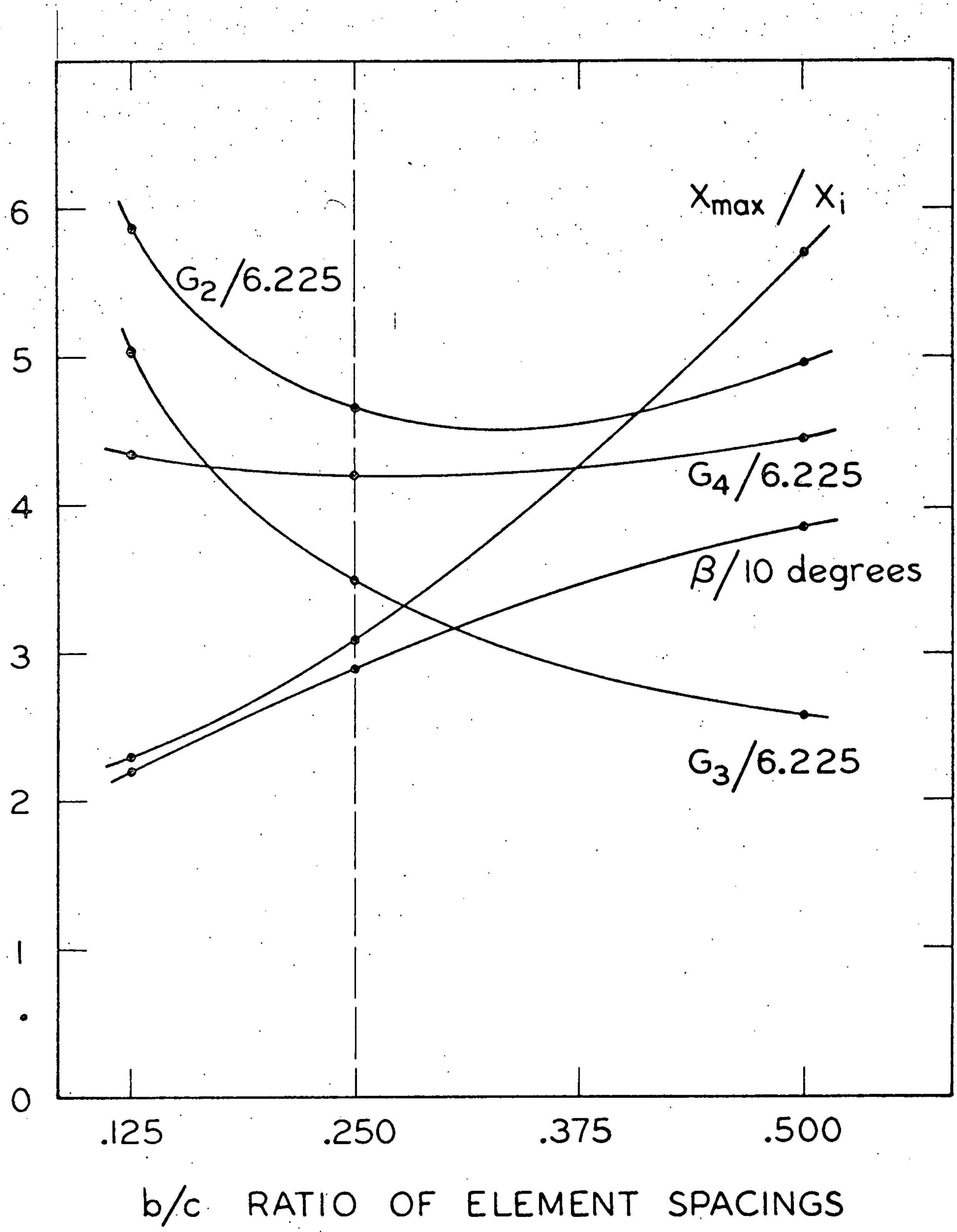
6
5
4
3
2
1
0.5
1.0
1.5
G2/6.225
G4/6.225
β/10 degrees
Xmax/Xi
(ρ/d)/0.5467 RADIUS OF CURVATURE

Fig 7



0
1
2
3
4
5
6
.375
.500
.625
.750
a/c RATIO OF ELEMENT SPACINGS
G4/6.225
G3/6.225
G2/6.225
β/10 degrees
Xmax/Xi

Fig 8



Xmax / Xi
G2/6.225
G4/6.225
β/10 degrees
G3/6.225
0
1
2
3
4
5
6
.125
.250
.375
.500
b/c RATIO OF ELEMENT SPACINGS

Fig 9