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THE DESIGN AND ANALYSIS OF ELECTRONIC CIRCUITS BY DIGITAL COMPUTORS

by

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THE DESIGN AND ANALYSIS OF ELECTRONIC CIRCUITS BY DIGITAL COMPUTERS

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Dr. J. L. Wirth

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ABSTRACT

The role of the digital computer in the design and analysis of electronic circuits is discussed with special attention given to the problems created by steady-state and transient nuclear environments. Examples are presented which illustrate the usage, capability and limitations of several existing analysis programs. The current direction of research and programming efforts in the area of automatic circuit analysis programs is also considered.

ACKNOWLEDGMENT

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THE DESIGN AND ANALYSIS OF ELECTRONIC CIRCUITS BY DIGITAL COMPUTERS

Introduction

In order to efficiently apply computing machines to an area of science or technology, a well-defined theory must exist which provides a numerical algorithm for proceeding from a given set of initial data to the desired solution. Unfortunately, this prerequisite is not satisfied in the area of electronic circuit design because in all but the most trivial situation, there is no algorithm which leads from a specification sheet to the topology of a suitable circuit. Each designer has his own mental processes for subdividing a design problem into electronic subsystems which he can synthesize with circuits whose properties are known and which are part of his design repertoire. Once a topology has been selected, however, it is possible to use the general and mathematically rigorous theory of circuit analysis to investigate the properties of the circuit and determine the merit of the proposed design. As a result, most current applications of computers in the design and analysis of electronic circuits generally have been directed toward the analysis problem whereas the design problem has been treated through repeated analyses and optimizations of a predetermined circuit topology.

Whenever the subject of circuit analysis is discussed, advocates of the experimental approach immediately question the wisdom of using computer analysis techniques instead of the laboratory "bread-board" to verify or optimize designs. In view of the cost of computer time, the inaccuracy of component models and the difficulty in obtaining representative parameter values for these models, it must be admitted that this position is well founded. However, it should be realized that analysis by means of generalized computer programs does not compete with, but complements, the usual design procedures. The advantages of each method can be combined to increase the productivity of the design engineer and improve the performance and reliability of the final product. This paper is therefore intended to call attention to several existing circuit analysis programs and to indicate a number of ways in which these programs could be used to complement existing design procedures, especially in the design of radiation tolerant circuits.

Existing Circuit Analysis Programs

In order to show the usefulness of analysis programs, it is expedient to first discuss the input formats of some existing programs and thereby indicate their generality and ease of use. These discussions are by no means complete and should be considered only as introductions to the respective programs. These programs are, with one exception, documented and the reader is referred to these documents for additional detail.

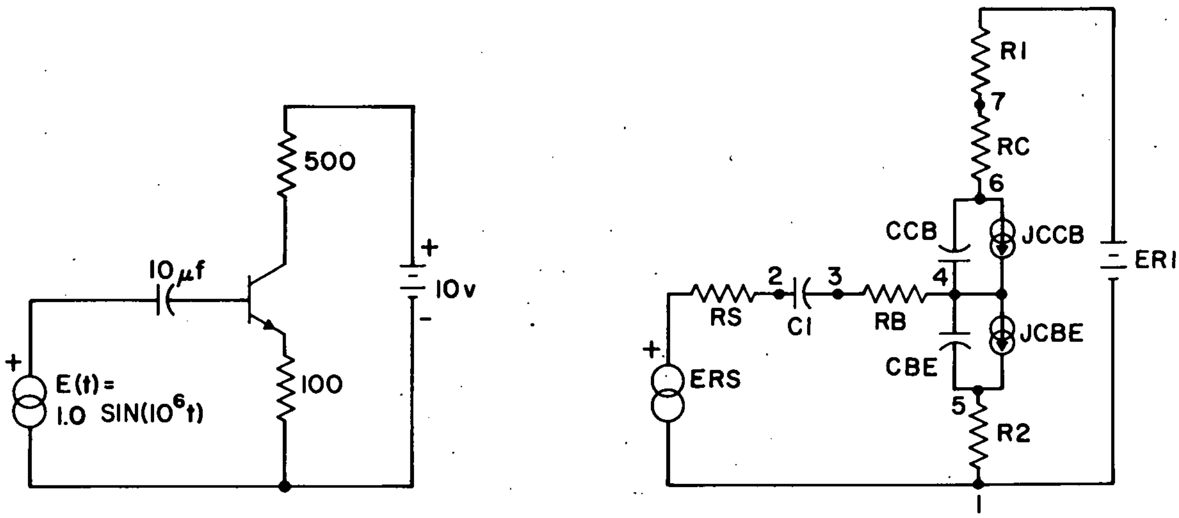
PREDICT

The PREDICT Analysis Program[1] was developed by the Radiation Effects Department of the International Business Machines Corporation, Owego, New York, to predict the effect of transient radiation upon electronic hardware. This program, which is written for the IBM 7090 computer, is predicated upon the assumption that every circuit can be modeled by a collection of resistive, capacitive and inductive components, voltage and current sources and mutually inductive components. The maximum circuit size is limited to 300 components and 2000 circuit data cards. Nonlinearities can be included by defining the resistance, capacitance, inductance and source coefficients as functions of the appropriate voltage or current. The form of these nonlinearities can be specified by equations in the Fortran format or by an array of tabular data.

An illustration of the steps required to analyze a simple amplifier using PREDICT is shown in Figure 1. First, the schematic diagram shown in Figure 1a is converted to an equivalent network by replacing each device by an appropriate equivalent circuit* as illustrated in Figure 1b. The PREDICT input format assumes that every voltage source is in series with and every current source is in parallel with a passive element. Therefore, the internal resistance of each driving source is normally included in the equivalent network. Next, every node (except those nodes between voltage sources and their respective series elements) is assigned a unique number and every passive component is assigned a unique alpha-numeric name with prefix, R, L or C to indicate a resistor, inductor, or capacitor. Names are assigned to current and voltage sources by preceding the name of their respective parallel or series element by a J or an E, respectively.

In order to analyze the amplifier shown in Figure 1a, the information contained in Figure 1c is punched on data cards. The beginning of the problem is signified by a card containing the word START followed by one or more title cards which serve to identify the problem. All passive elements are entered after the word BRANCHES by listing the element name, its respective node pair and the coefficient value. In the case of nonlinear elements, the coefficient is defined by either an equation or a tabular array as illustrated by the fourth entry under BRANCHES in Figure 1c. Sources are encoded as the source name, the node number toward which the assumed positive direction of the source is oriented and the source value or equation. The name of the voltage or current associated with a particular element is derived by prefixing the element name with a V or I, respectively. Initial conditions and the desired output are then specified uniquely by these variable names as illustrated in Figure 1c under INITIAL CONDITIONS and OUTPUT. Nonlinearities specified in equation form and the data tables are included after a FUNCTIONS card. The equations, which are written in the usual Fortran format, may contain

*Diode and transistor models are presented in Appendix I.



(a) (b)

```
START
    SAMPLE PROBLEM - SIMPLE AMPLIFIER     10-21-64

BRANCHES
    RS,    1 - 2 = 0.050
    C1,    2 - 3 =10.0E6
    RB,    3 - 4 = 0.120
    CBE,   4 - 5 = EQUATION 1
    R2,    5 - 1 = 0.100
    CCB,   6 - 4 = EQUATION 2
    RC,    7 - 6 = 0.060
    R1,    1 - 7 = 0.500

SOURCES
    ERS,  2     = EQUATION 3
    JCBE, 5     = EQUATION 4
    JCCB, 4     = EQUATION 5
    ER1,  7     = 10.0

INITIAL CONDITIONS
    VC1         = 0.68
    VCBE        = 0.58
    VCCB        = 8.76

OUTPUT
    ERS
    VR1
    VR2

FUNCTIONS
    EQUATION  1 = 5.0 + 10.6/SQRTF(VCBE + 1.0) + 3.0E-9*EXPF(39.0*VCBE)
    EQUATION  2 = 5.0 +  5.6/SQRTF(VCCB + 1.0)
    EQUATION  3 = SINF (1.0E6*TIME)
    EQUATION  4 = 1.52E-10*(EXPF(39.0*VCBE) - 1.0)
    EQUATION  5 = 0.982*1.52E-10*(EXPF(39.0*VCBE) - 1.0)

STOPTIME
    3000..0001  1.0E-6  0.01  10.0  1.0   6.0  1.0E-3  1.0E-9
```

(c)

Figure 1 Predict Example and Data Format exponentiation, multiplication, division, addition and subtraction operations as well as square root, sine, cosine, exponential, arctangent, hyperbolic tangent and natural logarithm functions. These equations may be functions of time, tables, voltage or current. The STOPTIME card signifies the conclusion of the circuit specification. This card is followed immediately by a card containing the real problem time at which the solution is to be terminated, the allowable numerical integration errors, maximum integration step size, the maximum computer running time, etc. This is the only card in the PREDICT data which requires a fixed field format.

After reading the STOPTIME data card, PREDICT formulates a mathematical model of the circuit and numerically solves for the transient response of the desired variables. This solution is printed in tabular form and, if plots are requested, in a form required for the Calcomp plotter. Continue, Plot and Message operating modes are also provided in order that a solution may be continued past a previous termination point, additional data may be plotted or special operating instructions may be issued to the computer operator.

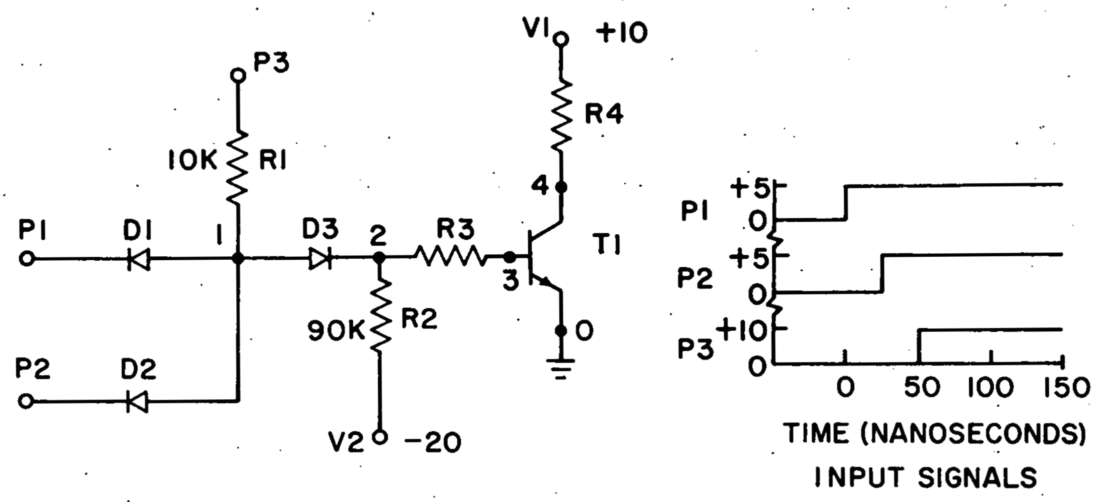
NET-1

The NET-1 Analysis Program[2] was originally written for and developed on the MANIAC II computer. However, the recent demand for circuit analysis programs has prompted a translation of this code to the IBM 7090/7094 language. This translation should be completed and made available to the public in the near future.

The input format for NET-1 is illustrated in Figure 2 where a typical schematic diagram and the necessary NET-1 input data are shown. As indicated in Figure 2, the format for resistors, capacitors and inductors is, except for punctuation and naming conventions, identical to the PREDICT format. However, in contrast to the PREDICT code, the coefficients of these components must be constants. Therefore, time varying coefficients, which are useful in modeling such things as radiation-induced conductivity modulation, cannot be included in a NET-1 analysis.

The nonlinear behavior of diodes and transistors is included in an analysis by using preprogrammed diode and Ebers-Moll transistor models. These models are specified by simply writing T (transistor) or D (diode) followed by the appropriate node numbers and device code. The NET-1 program then queries a library tape to locate the appropriate model coefficients. While this feature makes the program very easy to use, it restricts the versatility of the code by forcing a built-in model upon the user and not permitting the use of models which are more appropriate for a particular problem.

In addition to obtaining the transient response of a circuit, the NET-1 program also performs other tasks frequently required in the design of circuits. During a transient analysis, NET-1 automatically prints the state of all diodes and transistors and the time at which any state changes occur. The power dissipation of each semiconductor device is also computed and, if maximum ratings are exceeded, recorded as part of the output data. The power dissipation of each semiconductor device is also computed and, if maximum ratings are exceeded, the maximum dissipation is recorded as part of the output data. NET-1 can also perform steady-state



(a)

```
"SAMPLE PROBLEM ** AND GATE INVERTER"

RI   P3     I      10.0
R2   V2     2      90.0
R3   2      3      0.500
R4   VI     4      3.3
DI   I      PI     IN279
D2   I      P2     IN279
D3   I      2      IN279
TI   0   3  4      2N709
VI   +10.0
V2   -20.0
PI   PULSE   0.0   5.0   10.0
P2   PULSE   0.0   5.0   25.0
P3   PULSE   0.0  10.0   50.0
RESOLUTION 1.
INTERRUPT  500.
END
```

(b)

Figure 2 NET-1 Example and Data Format

analyses to determine whether or not a power supply failure or an improper power supply turn-on sequence will jeopardize any of the diodes or transistors in the circuit.

CIRCUS

CIRCUS is being developed by the Radiation Effects Unit, The Boeing Company Aero-Space Division, for the purpose of simulating the effect of transient radiation upon systems and circuits. This program is currently in the final development stages and documentation on formats and computer requirements is not available at this time. The program is written in Fortran and has been run on IBM 7094 and Univac 1107 computers.

MISSAP

The MIchigan State System Analysis Program[3] is being developed at Michigan State University under the sponsorship of International Business Machines Corporation. Although this program has not been released to the general public, it is of particular interest because it represents one of the first attempts to automatically analyze a system composed of both distributed (transmission lines) and lumped parameter components. In addition to formulating a mathematical model of the lumped parameter part of the system, MISSAP generates difference equation approximations to the partial differential equations which describe transmission lines. These difference equations and the equations characterizing the lumped parameter part of the system are then solved numerically and the solutions are tabulated or plotted. Fourier transforms of the transient solutions are also tabulated and plotted upon request.

The input format for MISSAP is similar to NET-1 although a fixed field format is required and no provision is made for a transistor or diode library. In addition, voltmeter and ammeter components are used to specify the desired output. Special data cards are also needed to indicate the placement of transmission lines in the circuit.

Except for the pre-programmed transistor and diode models, all components considered in a MISSAP analysis must be linear with constant coefficients. The diode and transistor models are of the Ebers-Moll form but do not include the nonlinear depletion and diffusion capacitance terms. These capacitance effects can be approximated by including linear capacitance between the external terminals of the device.

ECAP

The original version of the Electronic Circuit Analysis Program,[4] ECAP, was programmed for the IBM 1620 computer. This programming was done in the Fortran language and can therefore be easily converted to other machines. ECAP is capable of performing ac, dc and transient analyses of circuits containing up to 20 nodes and 60 R, L or C components. Nonlinear elements are admitted in the analysis by automatically changing component coefficients whenever a specified component current passes through zero. This program also contains provisions for automatically determining sensitivity coefficients (the rate of change of a voltage or current with respect to some circuit parameter), worst-case solutions, quiescent initial conditions and the standard deviation of node voltages.

Although ECAP contains several features which are not found in the programs discussed previously, the manner in which ECAP treats nonlinear elements makes it inconvenient to obtain accurate dc or transient solutions of circuits containing semiconductor devices. In order to approximate exponential functions, which are encountered in describing the junction capacitance as well as the dc characteristics of many semiconductor devices, a large number of piece-wise linear segments must be used and the number of extraneous circuit elements required to approximate the behavior of a single device becomes excessive.

Applications of Analysis Programs

It is evident from the above discussion that the modern circuit analysis program is easy to use and yet contains sufficient generality to determine the transient and/or quiescent solution for a large class of electronic circuits. Because of these properties, the utility of such programs is bounded only by the imagination of the analyst and the economic considerations associated with a particular problem.

Circuit Design

In the general field of circuit design there are many applications where these programs can be used to advantage. The comparison of several tentative topologies, for example, can be done economically on a computer because it saves the expense and delay associated with constructing the actual circuits. In addition to identifying inferior topologies, these preliminary analyses may reveal design oversights which could result in excessive power dissipation and damage to expensive components. The automatic analysis program is also a useful tool in "debugging" preliminary models of a circuit design. For example, the hypothesis that a certain anomalous oscillation or transient response is caused by parasitic capacitance or inductance can be quickly tested by analysis.

Many of the problems associated with finalizing a design are also expedited by analysis programs. For example, the need for safeguarding a circuit against overloads produced by adverse input or output conditions, power supply failures or power supply turn-on or turn-off sequence can be determined by analysis. Circuit analysis programs are also useful for performing worst-case analyses to obtain an estimate of the electrical reject rate which might be encountered in production. Other production problems might also be anticipated by calculating the effects of such things as the stray wiring capacitance associated with a new package geometry or the substitution of a new component type.

Contract Monitoring

Contract monitoring organizations can use circuit analysis programs to evaluate a final design and thereby be assured of a quality product. Moreover, the design can be evaluated at intermediate stages of evolution in order to demonstrate the ultimate feasibility of the design and to avoid needless "deadend" projects and loss of time. These factors are particularly important when designs are required to operate in abnormal environments, such as nuclear radiation.

Radiation Effects

One of the most important uses of analysis programs is in the simulation of environment or test conditions not readily achieved in the laboratory. An excellent example of such a situation is the simulation of the transient and permanent effects of nuclear radiation.

Transient Radiation Effects -- When a transistor or diode is exposed to a pulse of ionizing radiation, such as gamma or X-rays, hole-electron pairs are generated throughout the device. Some of the carriers generated near a junction will traverse the junction and produce transient variations in the terminal voltage or current.[5,6] These effects can be modeled by including the appropriate current sources in the device models as discussed in Appendix I.

To illustrate how one might use an automatic analysis program to design a radiation-hardened circuit, consider the problem of comparing the radiation sensitivity of the two amplifiers shown in Figure 3. Both of these amplifiers have a gain of approximately ten, an input resistance of approximately one hundred kilohms and a low output impedance. The sensitivity of each circuit to transient pulses of ionizing radiation is determined by using the circuit analysis programs discussed previously and the models presented in Appendix I. The results of these analyses, shown by the solid lines in Figure 4, indicate that for identical exposures circuit A has a peak response one-third as large as the response of circuit B. Circuit A is therefore preferred for those systems with an amplitude failure threshold. However, if a systems failure is determined by pulse duration, circuit B is preferred. An experimental evaluation of the two circuits is shown by the dotted lines of Figure 4.

The radiation sensitivity of a circuit is, of course, dependent upon the choice of components as well as circuit topology. For purposes of illustration, the proposed amplifier designs were also evaluated using 2N336 transistors instead of the 2N1051 code specified in Figure 3. These analyses, shown in Figure 5, indicate that although the relative behavior of the circuits is independent of device type the absolute sensitivity of the circuit is approximately doubled by using the 2N336 transistor. These predictions are experimentally confirmed by the measured data shown in Figure 5.

A study of the discrepancies between the predicted and measured response indicates two likely sources of error. First, the stray wiring capacitance, particularly that between the collector and base leads of the transistors, was not included in the analyses and, as a result, the predicted waveforms show a much faster decay than the measured waveforms. Thus, even though circuit analysis programs greatly expedite the analysis of circuits, the analyst must still exercise considerable judgment in order to include those circuit parameters which are important and exclude those which are superfluous. Secondly, errors in the measurement of transistor parameters and inaccuracies in the transistor models also contribute to the discrepancies

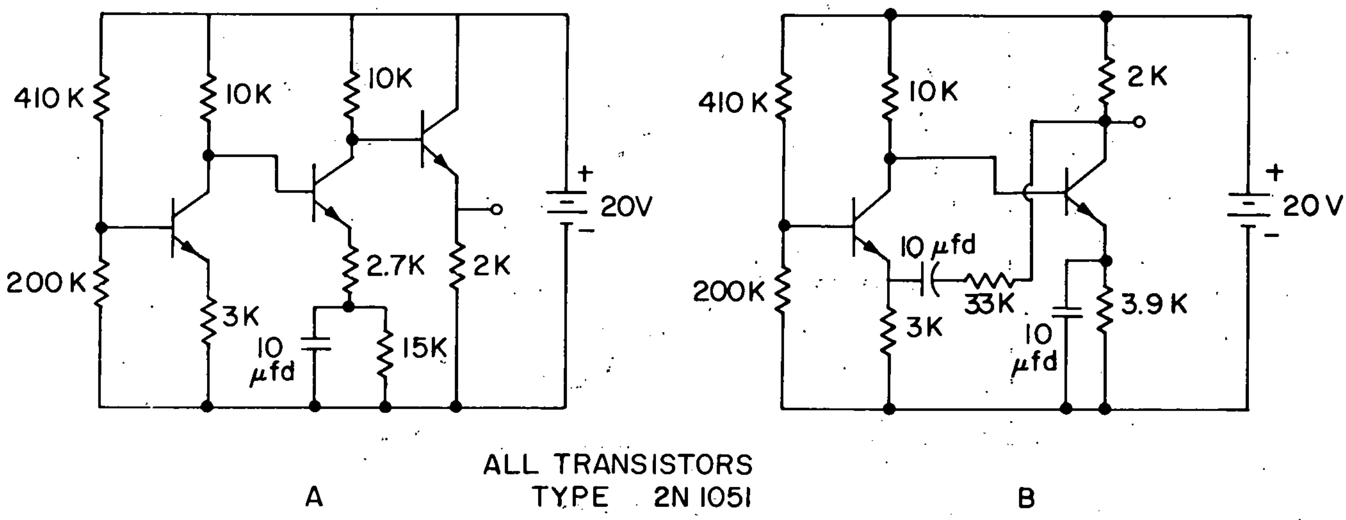


Figure 3 Typical Amplifier Designs

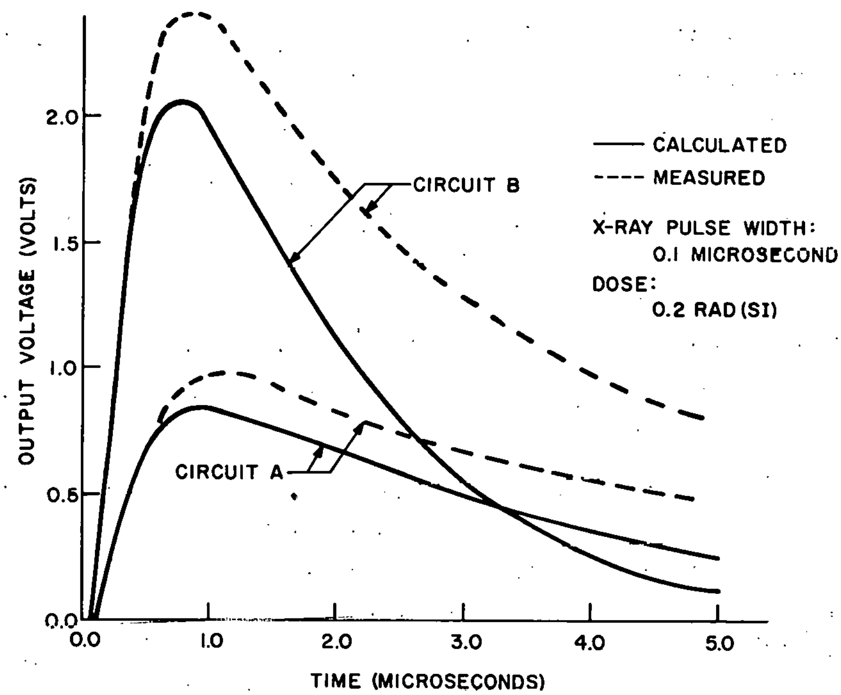


Figure 4 Photoresponse of Amplifiers Using 2N1051 Transistors

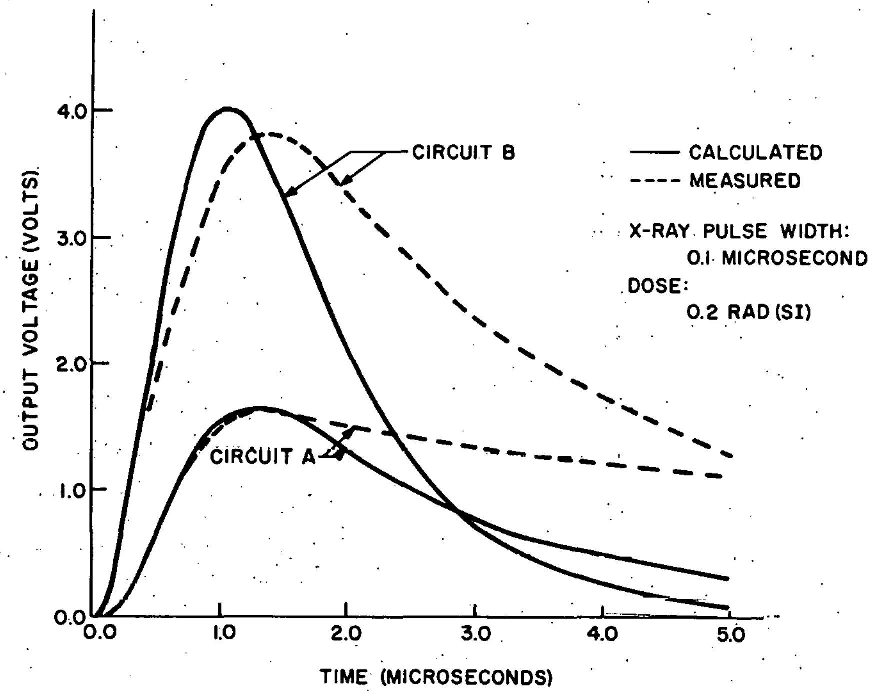


Figure 5 Photoresponse of Amplifiers Using 2N336 Transistors

between the predicted and measured response but, in this case, to a lesser degree than stray capacitance effects. It should be emphasized, however, that even with these errors and omissions, the predicted data yields an accurate comparison between the proposed designs and therefore provides a means for determining the better design.

The above examples also illustrate the economic considerations involved in the use of analysis programs. If the device parameters are known, the input data required for each of the above analyses can be prepared by an engineer in about one hour. The analyses shown in Figure 4 were performed with PREDICT (on an IBM 7090 computer) and required approximately four minutes per solution. The data presented in Figure 5, which were computed with CIRCUS (on a Univac 1107 computer), required about one minute per solution. Conclusions concerning the relative computing efficiency of PREDICT and CIRCUS should not be drawn from these data because the two programs used different transistor models and were run on different computers.

Permanent Radiation Effects -- When semiconductor material is exposed to a flux of high energy particles, such as neutrons, some of the incident particles interact with the atoms of the crystalline lattice and displace these atoms from their normal positions. This process damages the lattice structure and therefore produces permanent changes in the properties of semiconductor devices.[7,8,9]

Permanent damage effects can be included in an analysis by simply making the model coefficients a function of the incident particle flux. The common base current gain, for example, is shown as a function of the integrated neutron flux in Figure 6. If these data and similar curves characterizing the other model parameters are used in the analysis, a circuit can be evaluated at any arbitrary neutron flux and, through repeated analyses, the failure threshold of the circuit (i.e., the minimum neutron flux at which the circuit fails to meet specifications) can be determined.

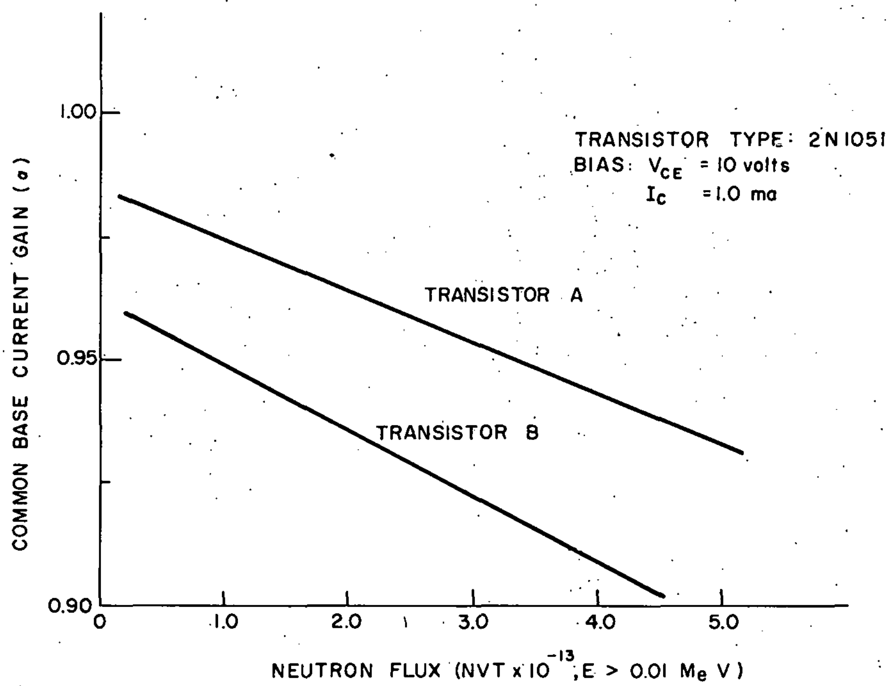


Figure 6 Common Base Current Gain Versus Neutron Flux

The transient and permanent damage mechanisms have been considered independently in the above discussions. However, situations exist where both effects occur simultaneously and must therefore be modeled simultaneously. This can be done by expressing each coefficient of the device model as a function of the partial integral of the particle flux rate, i.e.,

$$K_i(t) = K_{io}\left(\int_0^t n(\tau)d\tau\right)$$

where $K_{io}(\Phi)$ is the measured variation of the coefficient K_i with integrated flux Φ and $n(t)$ is the flux rate. The ionizing effect of the incident radiation can be simulated by including the appropriate photocurrent sources in the models as discussed previously.

Limitations

One of the principle shortcomings of existing programs is that an excessive amount of computer time is required to solve for the transient response of circuits whose solutions contain extremely small time constants. While this difficulty can be alleviated somewhat by the choice of component models, it still represents a serious limitation; one which can only be removed by a continued search for more stable numerical solution methods. A second basic limitations imposed by existing programs is associated with the solution of nonlinear algebraic equations. With the exception of MISSAP, none of the above programs provide the proper numerical methods and convergence criteria to simultaneously solve nonlinear algebraic and differential equations. Therefore, junction capacitance must be included in every diode and transistor in order to eliminate all nonlinear algebraic equations. While this process is theoretically acceptable, it is usually expensive because small time constants are introduced in the solution and the number of integration steps required for the desired solution may be excessive.

Apart from the fundamental limitations imposed by numerical solution techniques, the general philosophy and organization of the existing programs have imposed other limitations. The PREDICT program, for example, does not adequately treat the problem of coupling between elements and, as a result, small signal H-parameter models of the transistor cannot be used when conditions permit. The NET-1, CIRCUS and MISSAP programs do not admit nonlinear capacitive or inductive components. Although the ECAP program does admit a limited form of nonlinear elements, this form is not general enough to accurately characterize many devices. Moreover, none of the above programs accept generalized mathematical models of multi-terminal components; a feature which is very important in areas such as radiation effects studies where a large fraction of the immediate problem centers around the determination of adequate component models.

The problems associated with obtaining realistic parameter values and component models must also be recognized. Although there is no problem for resistors or capacitors taken from a clearly labeled stock cabinet, transistor and diode specification sheets do not normally contain values for such parameters as junction diffusion capacitance or radiation-induced photocurrent. Furthermore, the measurement[10] of device parameters is not a simple matter, particularly for high frequency devices.

Conclusion

The material presented in this paper is primarily intended to review several existing circuit analysis programs and to indicate a few of the many ways in which these programs can be used to complement design procedures. Although these programs are not as general as might be desired, many of the limitations inherent in the present versions are being systematically eliminated through the continued development and evolution of new coding and numerical techniques.

In addition to the normal process of evolution, programming developments in other scientific areas could also influence future analysis programs. Because of the work being done in pattern recognition, it is not unrealistic to think of submitting the network topology or parameter data in the form of schematic diagrams or plotted curves. Even with the present technology, it is possible to have a computer automatically construct a schematic diagram, a parts list and a cost estimate from the input data required by the analysis programs discussed above.

The recent theoretical interest in the formulation of state models of electro-mechanical systems indicates that future analysis programs could be oriented toward generalized mathematical models of multi-terminal components without regard for the particular technology associated with the problem in question. In this way, one program or, more appropriately, a system of subprograms, could (1) perform detailed analyses of small subsystems, (2) obtain simplified mathematical representations of these subsystems and (3) use these simplified representations in the analyses of the entire electro-mechanical system.

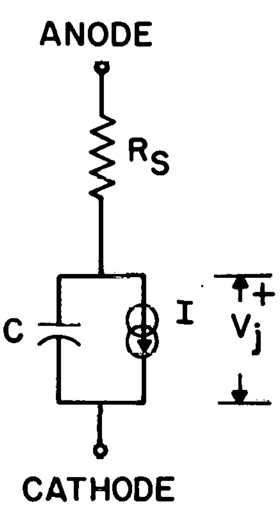
APPENDIX I

Diode and Transistor Models

Because of the importance of component models in the analysis of electronic systems, models of the diode and transistor are reviewed in this appendix and discussed from the viewpoint of their applicability to circuit analysis programs. The effects of transient radiation are also considered.

Diode Model

The most widely used model for predicting the transient and dc behavior of diodes is shown in Figure I-1. This model assumes that the diode can be modeled by a bulk resistance, R_s, in series with the parallel combination of a current source, I, and a capacitor, C. The magnitude of C is made up of three components: (1) a stray capacitance,* C_s, introduced by the diode package, (2) a transition region component, C_t, which varies as the reciprocal of the junction depletion region width, and (3) a diffusion component, C_d, which varies as the exponential of the junction voltage and, therefore, linearly with the ideal diode current I(t).



$$I = I_S(\exp(\theta V_j) - 1)$$

$$\theta = q/MkT$$

$$C = C_S + C_t + C_d$$

$$C_t = C_{to}/(V_Z - V_j)^N$$

$$C_d = q(I + I_S)/2\pi MkTF$$

Figure I-1 Diode Model

*Normally, the stray capacitance is included between the external terminals of the device, and as a result, extremely short time constants are introduced in the solution. These time constants can be eliminated with little or no sacrifice in model accuracy by including the stray capacitance with the junction capacitance.

Although this model provides a reasonable representation of most junction diode, it should be used with caution for diodes with significant conductivity modulation. Furthermore, since this model is derived by making a first order approximation to the solution of the continuity equation, it should be used cautiously when time variations are fast compared to the normal recovery time of the diode.

Transient Radiation Model -- When a diode is exposed to ionizing radiation, the hole-electron pairs created within an average of one diffusion length on either side of the junction traverse the junction and produce a transient photocurrent. This effect can be taken into account by including an additional term, $I_{pp}(t)$, in the ideal diode current expression as follows:

$$I = I_s\ (\exp(\theta V_j) - 1) - I_{pp}(t)\ . \qquad (I.1)$$

The negative sign preceding the photocurrent term indicates that this current is oriented in the reverse direction, i.e., from the cathode to the anode. Although $I_{pp}(t)$ is actually a function of the junction voltage, diode current and radiation waveform, reasonable results can generally be obtained by assuming a fixed photocurrent waveform and scaling the magnitude of $I_{pp}(t)$ linearly with dose. The photocurrent waveform can be determined experimentally by measuring the radiation-induced leakage current of the reverse-biased diode or it can be calculated by using the geometric and material properties of the device.[5]

The accuracy of the radiation model can be improved by using difference equations to obtain approximate solutions for the carrier density distributions in the device during the radiation exposure. In this way, electric field effects, storage effects and conductivity modulation of the bulk material can also be included in the model. A physical interpretation of a difference equation approximation is presented by Linvill.[9]

General Discussion -- The diode model given in Figure I-1 is well suited to automatic analysis programs since it provides a reasonably accurate model of the device and at the same time is amenable to solution by existing numerical techniques. Practical situations exist, however, for which this model should not be used because of the excessive computer time required to obtain solutions. If, for example, a very fast diode (recovery time approximately one nanosecond) is incorporated in a circuit which has a relatively slow response, integration steps of the order of one nanosecond may be required to maintain numerical stability and one million integration steps may be required to produce one millisecond of real solution time. This problem can be reduced by using a slower diode since the circuit operation clearly does not depend upon the diode recovery characteristics. This can be done artificially in the analysis by arbitrarily increasing the diode capacitance as much as possible without affecting the solution. A second approach is to delete the junction capacitance and characterize the diode by equations with the following form:

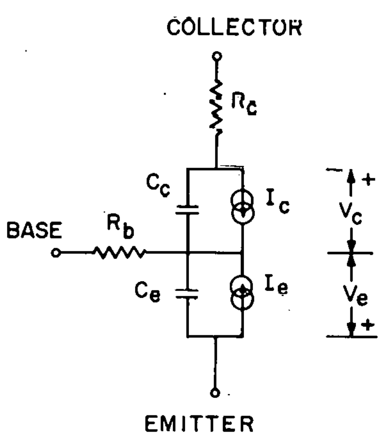
$$I = I_s \ (\exp(\theta(V - R_s I)) - 1) + I_{pp}(t) \ . \qquad (I.2)$$

Unfortunately, this leads to nonlinear algebraic equations which are beyond the scope of most automatic analysis programs and is therefore not generally acceptable.

Transistor Models

Both the Ebers-Moll[12] and Charge Control[13] transistor models have been used extensively to describe the transient and dc characteristics of junction transistors. Although these models differ considerably in concept and schematic representation, it can be shown that the commonly used forms presented below are mathematically identical. Therefore, only one of these models is considered in detail.

Ebers-Moll Model -- The schematic representation of the Ebers-Moll model, shown in Figure I-2, consists of two resistors, R_b and R_c, two junction capacitors, C_c and C_e, and two current sources, I_c and I_e. As in the case of the diode model presented earlier, the capacitance values are comprised of stray, depletion and diffusion components. The detailed form of the two capacitance values and current sources can be expressed by analytical approximations to the measured characteristics of the device, as indicated in Figure I-2, or by interpolation between actual data points.



$$I_c = \left[I_{cs} \ (\exp(\theta_1 V_c) - 1) - a_n I_{es} \ (\exp(\theta_2 V_e) - 1) \right] / (1 - a_n a_i) + I_{pp}(t),$$

$$I_e = \left[I_{es} \ (\exp(\theta_2 V_e) - 1) - a_i I_{cs} (\exp(\theta_1 V_c) - 1) \right] / (1 - a_n a_i),$$

$$\theta_1 = q/M_c kT, \quad \theta_2 = q/M_e kT,$$

$$C_c = C_{cs} + C_{ct} + C_{cd}$$

$$C_{ct} = C_{cto}/(V_{zc} - V_c)^{N_c}$$

$$C_{cd} = q \ I_{cs} \exp(\theta_1 V)/2\pi M_c kTFI$$

$$C_e = C_{es} + C_{et} + C_{ed}$$

$$C_{et} = C_{eto}/(V_{ze} - V_e)^{N_e}$$

$$C_{ed} = q \ I_{es} \exp(\theta_2 V)/2\pi M_e kTF_n$$

Figure I-2 Ebers-Moll Transistor Model

Transient Radiation Model -- When a transistor is exposed to a pulse of ionizing radiation, holes and electrons generated near the junctions drift and diffuse across the junctions and produce photo-enhanced leakage currents similar to those discussed in the diode case. However, in contrast to the diode, these primary photocurrents are amplified by the gain of the transistor and produce a secondary collector current which can be many times as large as the primary photocurrent.

As in the case of the diode, the effect of transient radiation can be incorporated in the model by including the primary photocurrents associated with both transistor junctions. As a practical matter, however, the emitter contribution is usually insignificant compared to the collector component because the diffusion length of the emitter is usually very short compared to that of the collector. Consequently, the term $I_{pp}(t)$ is included only in $I_c(t)$ as shown in Figure I-2.

In much of the analysis work reported to date, the photocurrent $I_{pp}(t)$ has been obtained by linearly scaling an experimentally determined waveform.* While this approach is sufficiently accurate for many purposes, conditions may arise where photocurrents obtained in this manner are not meaningful.[5] The accuracy of the radiation model can be improved by obtaining $I_{pp}(t)$, minority carrier storage and bulk resistance from approximate solutions for the carrier densities in the collector and base regions. These solutions can be determined by approximating the continuity equation by the appropriate difference equations as discussed in the diode case.

General Discussion -- The Ebers-Moll model has been used extensively by the NET-1 analysis program and, based upon this experience and a recent paper by Wilfinger, et. al.,[12] appears to be a very useful form for modeling many devices. Mathematically, this model is characterized by nonlinear differential and linear algebraic equations and, therefore, does not present any solution difficulties which are beyond the scope of most analysis programs. Practical situations do arise, however, which require excessive computer time to obtain solutions. As in the diode case, the analysis of a basically slow amplifier containing fast transistors is costly because the maximum solution step, in a sense, is determined by the "fastest" component. More subtle difficulties arise when the collector terminal is connected to the emitter through a low impedance element such as a capacitor or a small collector resistor. Under these conditions, the maximum integration step size is controlled by the time constant of the collector circuit $(R_c C_c)$ and is typically 10^{-10} seconds.

Charge-Control Model -- In its simplest form, the charge-control model proposed by Beaufoy and Sparks[13] provides a conceptually and mathematically simple tool for predicting the approximate behavior of circuits containing transistors operated in the active region. However, when a more detailed and accurate analysis is desired, the basic charge-control model must be augmented

*This waveform is determined by measuring the radiation-induced leakage current of a reverse-biased collector-base junction.

to include the effects of junction capacitance, saturated and inverted operation, etc. These changes lead to the more complicated model[14] shown in Figure I-3.

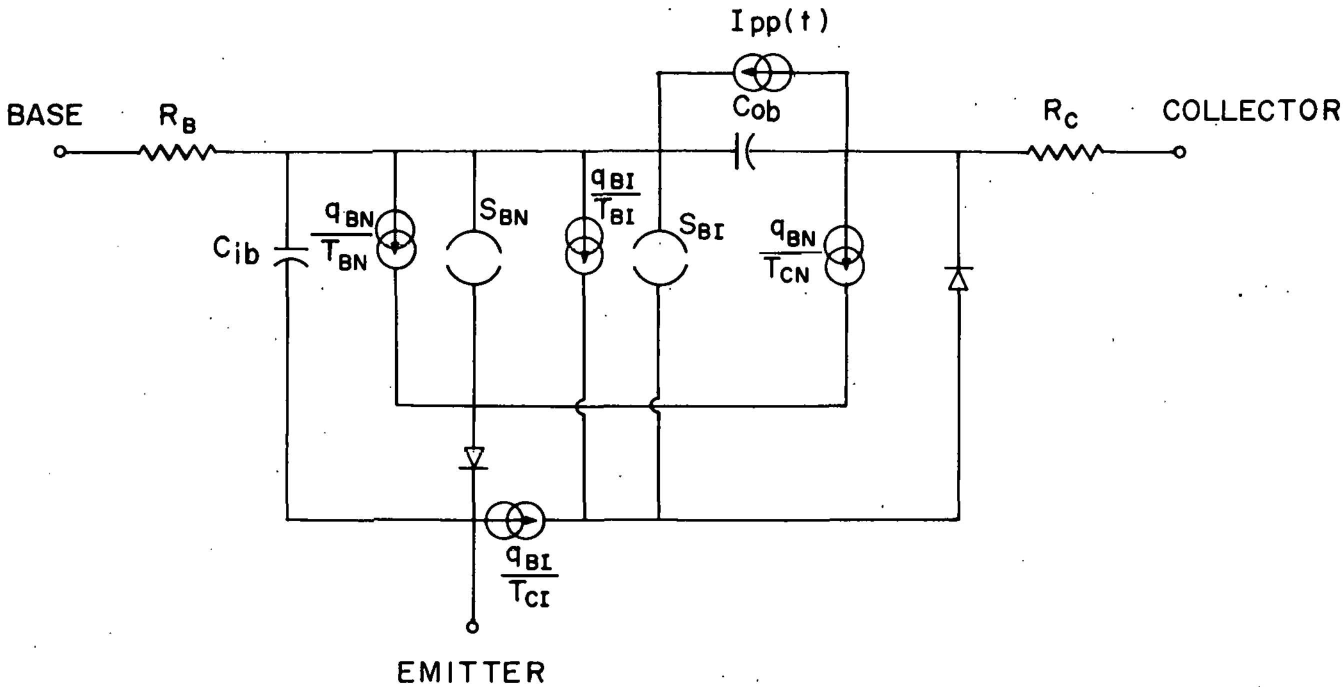


Figure I-3 Charge-control Model

From the equations describing the Charge-Control and Ebers-Moll models, it can be shown that although these models appear radically different from the standpoint of schematic representation, they are identical in a mathematical sense. Therefore, one can readily convert between Charge-Control and Ebers-Moll parameters and thereby use the existing tabulation[14] of Charge-Control parameters in the NET-1, PREDICT and MISSAP analysis programs.

Definitions of Symbols

α_n, α_i	Normal and inverted common base current gains
θ, θ_1, θ_2	Exponential coefficient for diode, collector and emitter junctions ($volts^{-1}$)
C	Total diode capacitance (farads)
C_c, C_{ob}	Total collector-base capacitance (farads)
C_d, C_{cd}, C_{ed}	Diffusion capacitance of diode, collector and emitter junctions (farads)
C_s, C_{cs}, C_{es}	Stray capacitance of diode, collector and emitter junction (farads)
C_e, C_{ib}	Total base-emitter capacitance (farads)
C_t, C_{ct}, C_{et}	Transition capacitance of diode collector and emitter junctions (farads)
F, F_i, F_n	Proportionality constants ($time^{-1}$)
I_s, I_{cs}, I_{es}	Saturation Current of the diode, collector and emitter junctions (amperes)
$I_{pp}(t)$	Primary photocurrent (amperes)
k	Boltzmann constant (ergs/degree Kelvin)
M, M_c, M_e	Emission constants of diode, collector and emitter junctions
n(t)	Neutron flux rate (nvt)
N, N_c, N_e	Grading constant of diode, collector and emitter junctions
q	Electronic charge (coulombs)
q_{BN}, q_{BI}	Normal and inverted base charge (coulombs)
R_s, R_b, R_c	Series resistance of diode, base and collector (ohms)
S_{BN}, S_{BI}	Normal and inverted base store
T	Temperature (degrees Kelvin)
T_{BN}, T_{BI}	Normal and inverted base time constant (seconds)

Definitions of Symbols (cont)

T_{CN}, T_{CI}	Normal and inverted collector time constant (seconds)
V_j, V_c, V_e	Diode, collector and emitter junction voltage (volts)
V_z, V_{zc}, V_{ze}	Contact potential of diode, collector and emitter junction (volts)

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