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A Two-Dimensional Fast Solver for Arbitrary Vortex Distributions

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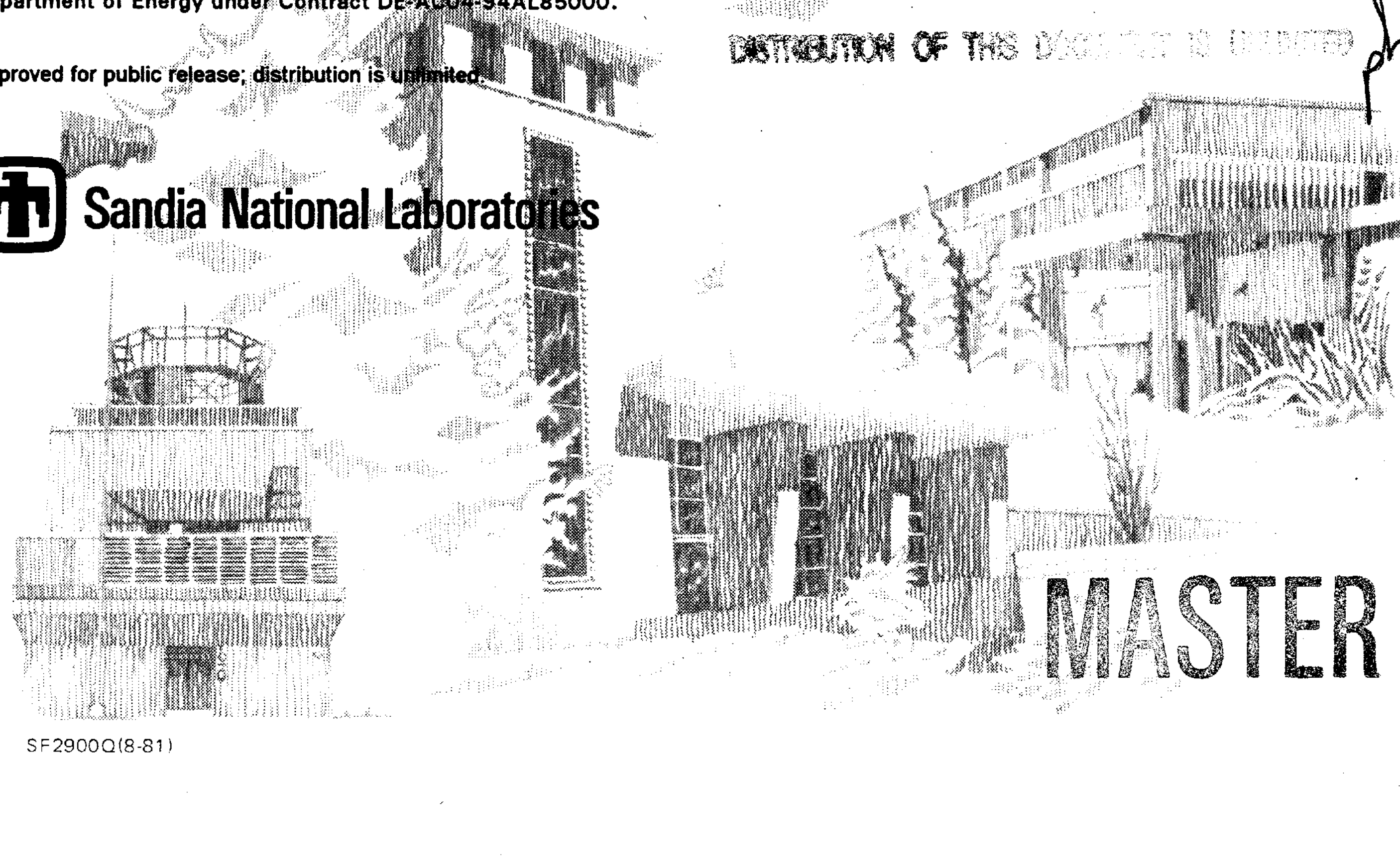

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A Two-Dimensional Fast Solver for Arbitrary Vortex Distributions

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Abstract

A method which is capable of an efficient calculation of the two-dimensional stream function and velocity field produced by a large system of vortices is presented in this report. This work is based on the adaptive scheme of Carrier, Greengard, and Rokhlin with the added feature that the evaluation or target points do not have to coincide with the location of the source or vortex positions. A simple algorithm based on numerical experiments has been developed to optimize the method for cases where the number of vortices N_V differs significantly from the number of target points N_T. The ability to specify separate source and target fields provides an efficient means for calculating boundary conditions, trajectories of passive scalar quantities, and stream-function plots, etc. Test cases have been run to benchmark the truncation errors and CPU time savings associated with the method. For six terms in the series expansions, non-dimensional truncation errors for the magnitudes of the complex potential and velocity fields are on the order of 10^{-5} and 10^{-3} respectively. We found that the CPU time scales as $\sqrt{N_V N_T}$ for N_V/N_T in the range of 0.1 to 10. For $\sqrt{N_V N_T}$ less than 200, there is virtually no CPU time savings while for $\sqrt{N_V N_T}$ roughly equal to 20,000, the fast solver obtains solutions in about 1% of the time required for the direct solution technique depending somewhat upon the configuration of the vortex field and the target field.

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Nomenclature

A_m	=	m^{th} multipole coefficient
A'_m	=	m^{th} shifted multipole coefficient
B_m	=	m^{th} local coefficient
B'_m	=	m^{th} shifted local coefficient
c	=	point
d	=	point
D_F	=	target domain
D_S	=	source domain
D_1	=	source domain
D_2	=	shifted source domain
D_3	=	target domain
D_4	=	shifted target domain
e	=	point
F_{pj}	=	complex potential at point p for a source at point j
$F_p(zp)$	=	complex potential at point zp
$F(z)$	=	complex potential
F_1	=	complex potential contribution from list 1 boxes
F_{24}	=	complex potential contribution from list 2 and 4 boxes
F_3	=	complex potential contribution from list 3 boxes
$G(\rho)$	=	core function for stream function
i	=	index, square root of -1
j	=	point, index
$K^{(m)}(z)$	=	m^{th} derivative of $\ln(z)$
l	=	box level
n	=	index
m	=	index
$Nmax$	=	maximum number of terms in multipole or local expansion
N_F	=	maximum number of target points in a box
N_S	=	maximum number of source points in a box

Nomenclature (continued)

N_T	=	total number of target points
N_V	=	total number of vortices
p	=	point, index
q	=	point
$q(\rho)$	=	core function for velocity
R_F	=	radius of target domain
R_S	=	radius of source domain
r	=	radius
S_l	=	box dimension at level l
$W(z)$	=	complex velocity
W_1	=	complex velocity contribution from list 1 boxes
W_{24}	=	complex velocity contribution from list 2 and 4 boxes
W_3	=	complex velocity contribution from list 3 boxes
x	=	horizontal coordinate
y	=	vertical coordinate
z	=	location in complex plane
zc	=	location of point c in complex plane
zd	=	location of point d in complex plane
ze	=	location of point e in complex plane
zj	=	location of point j in complex plane
zp	=	location of point p in complex plane
zq	=	location of point q in complex plane
ε	=	truncation error
Γ	=	circulation
Γ_j	=	circulation of j^{th} vortex
θ	=	angle
ρ	=	$\rho^2 \equiv \lvert z\rvert^2/\sigma^2$
σ	=	core radius
$\phi(z)$	=	velocity potential at z
$\psi(z)$	=	stream-function at z

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1 INTRODUCTION

1.1 Motivation for Work

Motivation for the present work results from a need to be able to perform fast yet accurate calculations of the two-dimensional flow field produced by large numbers of vortices. A vortex is simply a mathematical representation of a discretized region of vorticity. Vortices are often referred to in the literature as "vortex blobs." Such systems of vortices can be used, in turn, to model body surfaces, container boundaries, free-surfaces, plumes, jets, and wakes in unsteady flow fields. Simulations for most of these flows require that the flow structure evolve as a function of time, even for "steady flow" calculations. Large numbers N_V of vortices (1,000 to 100,000) are required for such simulations. Since the flow must be convected as part of the evolution process, the velocity perturbations from all vortices on all other vortices must be calculated at each time step. This requires an amount of work which is proportional to N_V^2 for each time step using direct calculation methods. The "Rokhlin-Greengard" method requires an amount of work which is proportional to $N_V \ln N_V$ which saves significant CPU time in large simulations. In the present work, we have added features that allow one to efficiently evaluate field variables at target points which do not have to coincide with the location of the source or vortex positions. The ability to specify separate source and target fields provides one with an efficient means for calculating boundary conditions, trajectories of passive scalar quantities, and field variable plots (stream function and velocity).

1.2 General Methodology

The general method comes from work developed by Rokhlin [1], Greengard and Rokhlin [2], and Carrier, Greengard and Rokhlin [3]. In the work of Carrier, Greengard and Rokhlin [3], it was shown that the asymptotic CPU time was of order $N_V \ln N_V$ for two-dimensional problems. A break-even point on the order of 100 particles was achieved for single precision accuracy for a uniform field in a square box. For 25,600 particles in the box, their method was roughly 100 times faster than the direct method. These fast methods take advantage of multipole and local series expansions which enables one to make calculations for interactions between groups of particles which are in well-separated spatial domains rather than having to consider interactions between every pair of particles. In the present work, the spatial domains in which vortices are located are obtained in an adaptive manner which is similar to the two-dimensional scheme used by Carrier, Greengard, and Rokhlin [3]. However, unlike their work, the locations of the source particles (vortices) need not be the same as the locations of the target (evaluation) points as shown in the work of Strickland and Baty [4].

As indicated, the present method takes advantage of source point (multipole) and field evaluation point (local) series expansions which enables one to make calculations for interactions between groups of vortices and field evaluation points which are in well-separated spatial domains rather than having to consider the interaction of every vortex on every evaluation point. For instance, consider the two domains D_S and D_F which are depicted in Figure 1. Domain D_S contains a set of vortices which are acting upon (producing perturbation velocities at) a set of evaluation points contained in domain D_F. In some cases, the set of evaluation points in domain D_F might also represent the centers of a second set of vortices. In other cases they might simply represent an arbitrary set of evaluation points.

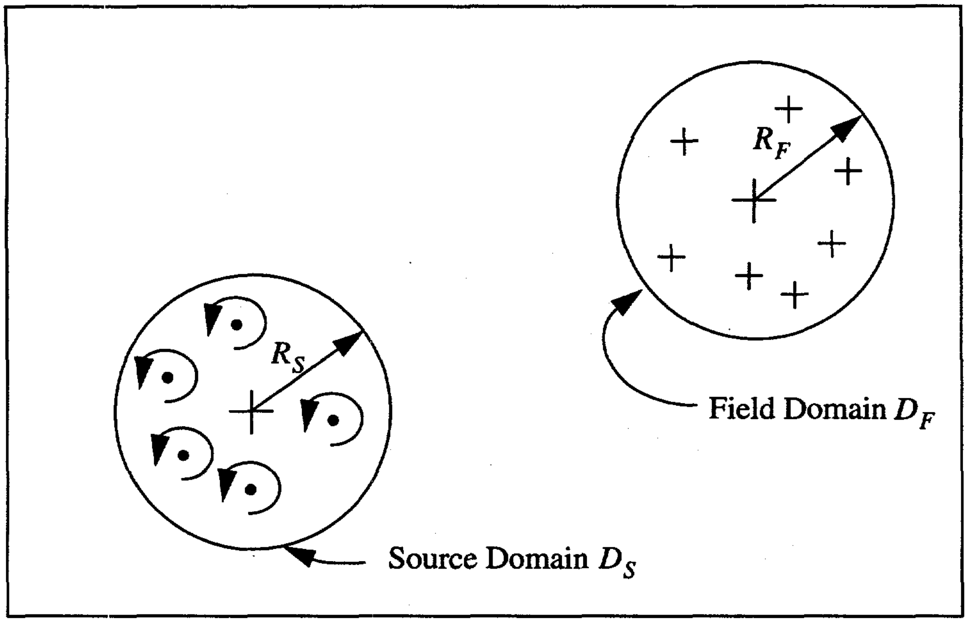


Figure 1. Source and Field Point Domains

In order to implement this method, a multipole series expansion for the complex potential about the center of the source domain D_S is first obtained. This expansion includes all of the vortices which are inside D_S. The series converges for field points which are outside of D_S. In order to be somewhat conservative, the expansion about the center of the source domain is used only for field points which are separated by at least R_S from the boundary of the source domain. Thus, for field points which are within a circle of radius $2R_S$ with respect to the source domain center, the influence of vortices in domain D_S on the field point must be computed in a more direct fashion. The center of the source domain expansion may be shifted without actual evaluation of the multipole series in order to allow one to form expansions efficiently for larger source domains which contain D_S.

A local or Taylor series expansion for the vector potential can also be obtained about the center of the field domain D_F. The series converges for field points which are inside D_F. The expansion about the center of the field domain is used only for source points which are separated by at least R_F from the boundary of the field domain. Thus, for source points that are within a circle of radius $2R_F$ with respect to the field domain center, the influence of any vortex in domain D_S on a field point must be computed in a more direct fashion. The center of the field point domain expansion may also be shifted without actual evaluation of the local series which allows one to form expansions for smaller field point domains contained in D_F.

In summary, the method allows one to calculate efficiently the influence of all of the vortices in D_S at all of the points in D_F when the two domains are "well separated" as defined above. There is additional efficiency inherent in the method if one exploits the possibility of moving the centers of expansion without actually having to evaluate the multipole or local series.

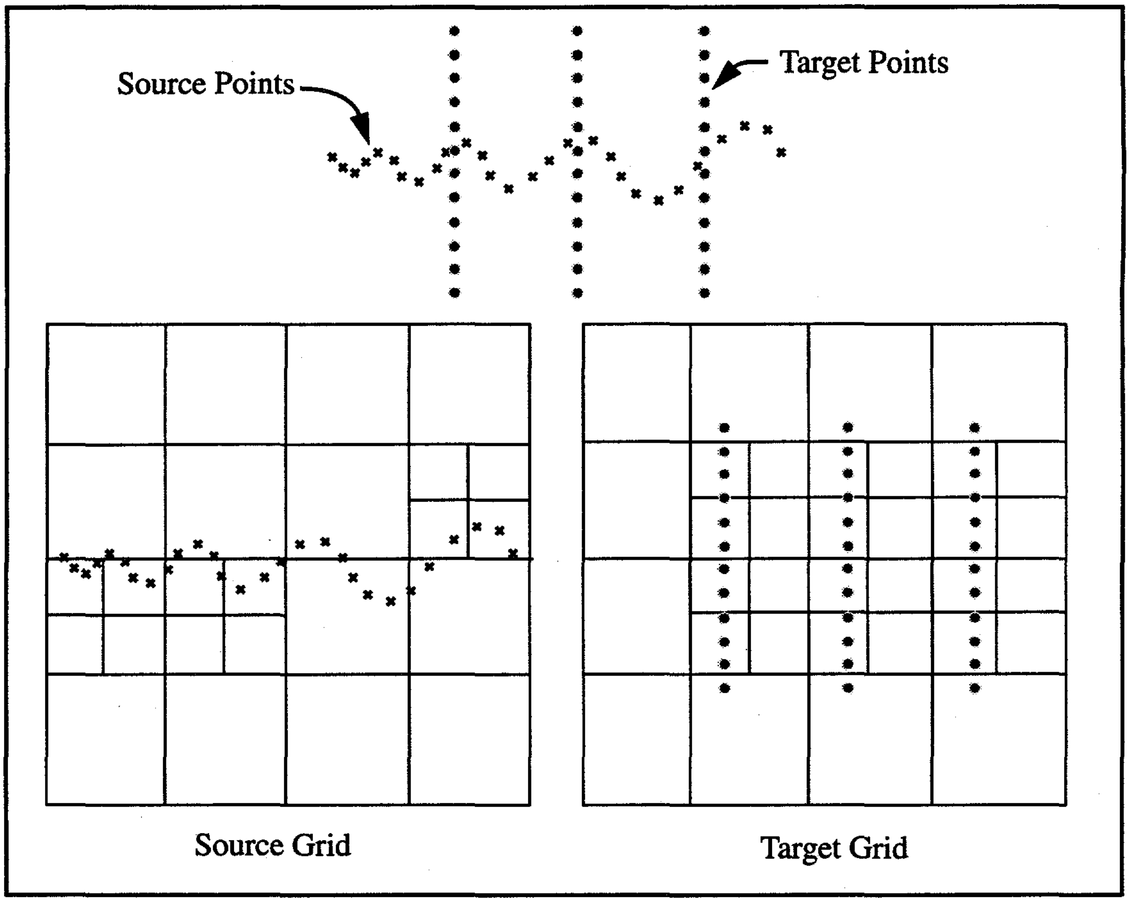


Figure 2. Adaptive Domain Mesh Scheme

As mentioned previously, the method used to divide the flow field into domains is an extension to that used by Carrier, Greengard, and Rokhlin [3]. The method requires that an adaptive domain mesh be generated that encloses all the source and target points in the flow. A square box is first constructed which encloses all of the sources in the flow as well as all of the target points. This box represents the zeroth level source box as well as the zeroth level target box. We first subdivide the source box into four boxes if it contains more than a specified number of vortices. Each of these boxes which contains more than the specified number of vortices is further subdivided. This process is continued until all of the boxes contain less than the specified number of vortices. The grid for the target points is generated in a similar manner. Figure 2 illustrates the generation of a source and target mesh. Each box size is associated with a box level with the original box level being equal to 0. At each level, empty boxes are ignored. The ability to ignore regions of the flow-field in which vortices or evaluation points are not present is a feature of the method which provides significant time savings. Boxes which can be subdivided are parent boxes and those which cannot are childless boxes.

A somewhat involved procedure is used to define the separation condition between a particular target box and each of the source boxes in the flow. This procedure determines the way in which vortices in a particular source box influence the complex potential at evaluation points in a particular target box. In general, each target box at each level has five possible relationships with each source box in the mesh. A formal description of these "box lists" will be given in a subsequent section. In general, the five lists produce the following types of restrictions on the use of the series expansions:

- Direct calculations must be made. Multipole and local series expansions cannot be used.
- Both multipole and local series expansions can be used.
- Multipole series expansions can be used, local series expansions cannot.
- Local series expansions can be used, multipole expansions cannot.
- Contributions from distant source boxes reside in the parent of the field box.

It should be pointed out, that the adaptive domain meshes which are generated here are not to be confused with the adaptive meshes which are generated for conventional CFD formulations. In the present case, the meshes represent a domain decomposition which is very simple when compared to meshes generated for conventional CFD formulations. Mesh generation for conventional CFD formulations is for the purpose of writing difference or element equations which satisfy the governing differential flow equations whereas the domain meshes generated here are for the purpose of grouping source or target points. The domain meshes in the present case do not have to conform to any flow boundaries, they extend only to regions where source or target points reside, and they are generated using a very simple algorithm. Generation of adaptive domain meshes is achieved using a very small amount of CPU time whereas adaptive mesh generation for conventional CFD problems can be quite CPU time and labor intensive.

2 SOURCE AND TARGET POINT EXPANSIONS

The basic solution method requires one to be able to write series expansions for source and field evaluation point domains. In addition, one must be able to translate the centers of the source and field point domains without actually having to evaluate the series expansions themselves. This section provides a description of the requisite expansions for the vector potential in a three-dimensional vortex field.

We begin by considering a single vortex located at the origin of the complex plane whose complex potential $F(z)$ is given by:

$$F(z) = -i\frac{\Gamma}{2\pi}\ln(z)\ . \tag{1}$$

Here Γ is the circulation strength and z is the position of the target point in the complex plane. Equation 1 represents a coreless vortex which is singular for $z = (0, 0)$. For the purposes of generating the multipole and local series expansions z will never be equal to zero and thus, this singularity will not be encountered. A core function will be added later to accommodate the direct calculations where z may approach zero. The contribution of this vortex to the complex velocity field $W(z)$ is given by:

$$W(z) = \frac{dF(z)}{dz} = -i\frac{\Gamma}{2\pi z}\ . \tag{2}$$

2.1 Multipole Expansion About the Source Domain Center

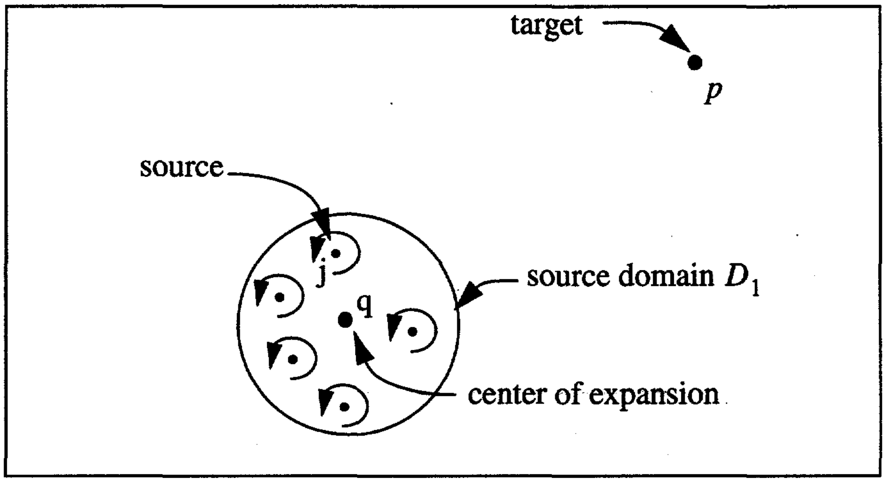


Figure 3. Multipole Expansion About Center of Source Domain

For a cluster of vortices as shown in Figure 3, a multipole expansion for the complex potential $F(z)$ can be obtained about the cluster center at zq. This expansion is valid for a target point at zp and will converge as long as zp is outside of the domain D_1 which encloses the cluster.

In order to obtain the multipole series, a Taylor series for the complex potential F_{pj} about the center of expansion due to an individual source at zj may be written as:

$$F_{pj}(zp - zj) = F_{pj}[(zp - zq) + (zq - zj)] \quad \text{or} \tag{3}$$

$$F_{pj}(zp - zj) = \sum_{m=0}^{Nmax} \frac{(zq - zj)^m}{m!} [F_{pj}^{(m)}(zp - zq)] + \varepsilon, \tag{4}$$

where $Nmax$ is the number of terms retained in the expansion, ε is the truncation error, and $F_{pj}^{(m)}(z) \equiv \frac{\partial^m F_{pj}}{\partial z^m}$. In the following expressions, it is understood that a truncation error ε exists although we will ignore it for notational simplicity.

The complex potential at zp with the center of expansion at zq due to the entire cluster of N_v sources in the source domain is:

$$F_p(zp) = \sum_{j=1}^{N_v} F_{pj}(zp - zj) \tag{5}$$

or

$$F_p(zp) = \sum_{j=1}^{N_v} \left\{ \sum_{m=0}^{Nmax} \frac{(zq - zj)^m}{m!} [F_{pj}^{(m)}(zp - zq)] \right\}. \tag{6}$$

Rearranging the order of summation in Equation 6 we can write:

$$F_p(zp) = \sum_{m=0}^{Nmax} A_m K^{(m)}(zp - zq) \quad , \tag{7}$$

where $K^{(m)}(z) \equiv \frac{\partial^m}{\partial z^m}[\ln(z)]$ which yields:

$$K^{(0)}(z) = \ln(z) \ , \tag{8}$$

$$K^{(m)}(z) = \frac{(m-1)!}{(-z)^m} \quad \text{for } m \geq 1 .$$

The coefficient A_m is given by:

$$A_m = -\frac{i}{2\pi} \sum_{j=1}^{N_v} \Gamma_j \frac{(zq - zj)^m}{m!} . \tag{9}$$

2.1 Translation of the Source Domain Center

It is advantageous to be able to move the center of the expansion given by Equation 3 such that the coefficients of the multipole expansion in domain D_1 can be added to the coefficients of other domains contained in a larger domain D_2. This situation is depicted in Figure 4.

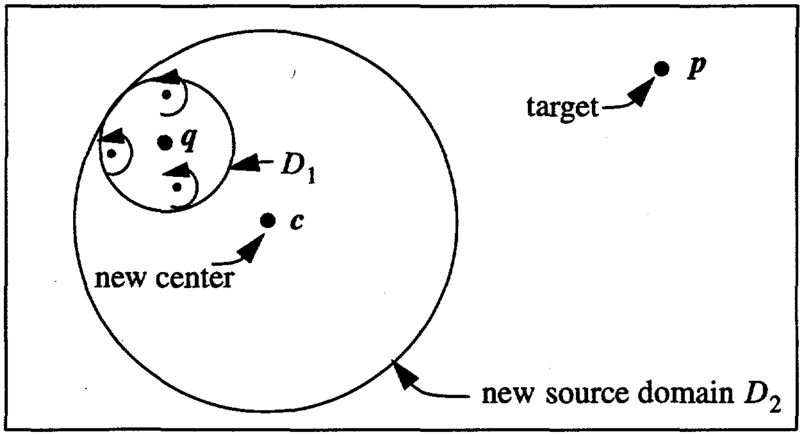


Figure 4. Translation of Source Domain Center

In order to shift the expansion to the point zc we first write:

$$F_p(zp) = \sum_{m=0}^{Nmax} A_m K^{(m)}[(zp - zc) + (zc - zq)] . \tag{10}$$

By expanding Equation 10 into a Taylor series we obtain:

$$F_p(zp) = \sum_m A_m \sum_p [K^{(m+p)}(zp - zc)] \frac{(zc - zq)^p}{p!} . \tag{11}$$

By moving A_m inside the second sum and substituting $n = m + p$, we obtain:

$$F_p(zp) = \sum_n \left\{ \left[\sum_p A_{n-p} \frac{(zc - zq)^p}{p!} \right] [K^{(n)}(zp - zc)] \right\} . \tag{12}$$

We now note that for Equation 12 to be well defined that $p \geq 0$, $n \geq 0$, $Nmax \geq (n - p) \geq 0$, $n \leq Nmax$, and $p \leq Nmax$. Therefore the limits on the sums are $0 \leq n \leq Nmax$ and $0 \leq p \leq n$. Therefore we may write the shifted multipole as:

$$F_p(zp) = \sum_{n=0}^{Nmax} A'_n K^{(n)}(zp - zc) , \tag{13}$$

where

$$A'_n = \sum_{p=0}^{n} A_{n-p} \frac{(zc - zq)^p}{p!} . \tag{14}$$

2.2 Local Expansion About the Target Domain Center

A local expansion will now be obtained about the center of a source point domain D_3 due to the influence of vortices in D_2. Referring to Figure 5, it can be seen that this expansion will be valid at field points inside D_3 if D_2 and D_3 are well separated in the sense that both multipole and local expansions may be used.

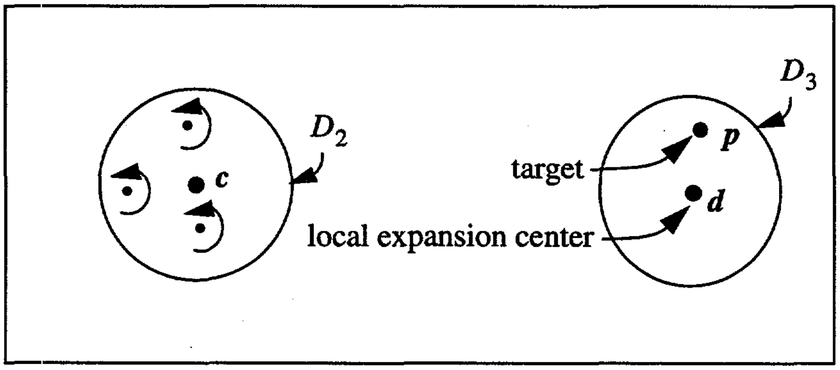


Figure 5. Local Expansion about Center of Target Domain

In order to shift the expansion to the point zd we first write Equation 13 as:

$$F_p(zp) = \sum_{m=0}^{Nmax} A'_m K^{(m)}[(zp - zd) + (zd - zc)]. \tag{15}$$

By expanding Equation 15 into a Taylor series we obtain:

$$F_p(zp) = \sum_m A'_m \sum_p [K^{(m+p)}(zd - zc)] \frac{(zp - zd)^p}{p!}. \tag{16}$$

Moving A'_m inside the second summation we can write:

$$F_p(zp) = \sum_p \left\{ \sum_m A'_m [K^{(m+p)}(zd - zc)] \right\} \frac{(zp - zd)^p}{p!}. \tag{17}$$

We now note that for Equation 17 to be well defined that $p \ge 0$, $m \ge 0$, $Nmax \ge (m+p) \ge 0$, $m \le Nmax$, and $p \le Nmax$. Therefore the limits on the sums are $0 \le p \le Nmax$ and $0 \le m \le Nmax - p$. We may therefore write the following with a slight change in index notation:

$$F_p(zp) = \sum_{n=0}^{Nmax} B_n \frac{(zp - zd)^n}{n!}, \tag{18}$$

where

$$B_n = \sum_{p=0}^{Nmax-n} A'_p [K^{(n+p)}(zd - zc)]. \tag{19}$$

2.3 Translation of the Target Domain Center

The last series expansion involves the translation of the center of the source domain. As Figure 6 depicts, the expansion for the domain D_3 can be used for any domain D_4 which is inside of D_3 by performing a simple translation of the expansion center.

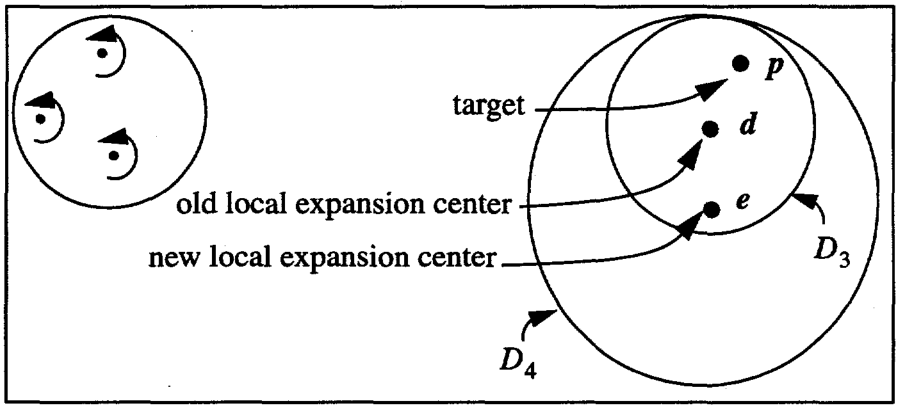


Figure 6. Local Expansion About Shifted Center of Target Domain

In order to shift the local expansion to the center ze we write Equation 18 as:

$$F_p(zp) = \sum_{n=0}^{Nmax} B_n \frac{[(zp-ze)+(ze-zd)]^n}{n!}. \tag{20}$$

The binomial expansion may be used to obtain:

$$[(zp-ze)+(ze-zd)]^n = \sum_p \frac{n!}{p!(n-p)!}(zp-ze)^{n-p}(ze-zd)^p. \tag{21}$$

Combining Equations 20 and 21 and letting $m = n-p$ we obtain:

$$F_p(zp) = \sum_m \left[\sum_p \frac{B_{m+p}}{p!}(ze-zd)^p\right]\frac{(zp-ze)^m}{m!}. \tag{22}$$

After obtaining the ranges on the sums and making a change in the index notation we obtain:

$$F_p(zp) = \sum_{n=0}^{Nmax} B'_n \frac{(zp-ze)^n}{n!}, \tag{23}$$

where

$$B'_n = \sum_{p=0}^{Nmax-n} B_{n+p} \frac{(ze-zd)^p}{p!}. \tag{24}$$

2.4 Local Series Expansion Due to Single vortices

As we shall see later, there will be a need to obtain a local series expansion about the point located at zd for a set of vortex sources located at zj as shown in Figure 7. In other words, in some situations, we will use the vortex centers in place of the center of the source domain D_2.

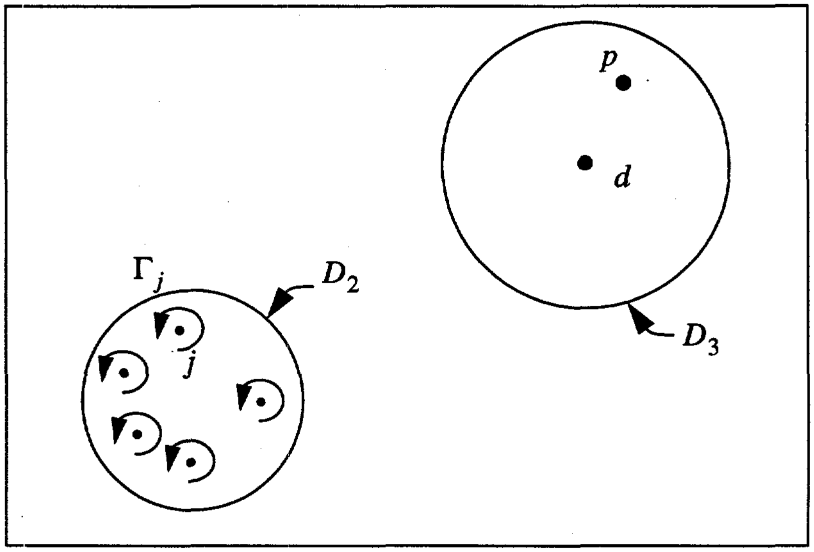


Figure 7. Local Expansion for Individual Sources

This occurs for situations when one cannot use the domain D_2 multipole expansion in domain D_3 since the two domains are not sufficiently well separated. This could be accomplished by formally applying the operations indicated in Equations 7 and 18 to each vortex in D_2 but would not be very efficient since only A_0 is non-zero in Equation 19 for this special case. Using this fact, one can show that the appropriate values for the coefficients B_n for use in Equation 18 which accounts for the m vortices in domain D_2 can be given by:

$$B_n = \sum_{j=1}^{m} -\frac{i}{2\pi}\Gamma_j K^{(n)}(zd - zj). \qquad (25)$$

3 METHOD OF SOLUTION

The present work is implemented in the computer code FAST2D developed at Sandia National Laboratories. This code requires that one specify a distribution of vortices by their positions (x, y), circulations Γ, and core radii σ and place such information in an input file. Positions of all target points are placed in a separate input file. The code also requires that one specify the domain of interest (the position and size of the square which encompasses all vortices and target points, the maximum number of vortices in a box N_S, the maximum number of target points in a box N_F and the order $Nmax$ of the series expansions to be used. FAST2D output consists of vortex input data and the values of the complex potential and complex velocity at all specified target points. The following describes the methodology used in the FAST2D code.

3.1 Adaptive Mesh

As mentioned in the introduction, the spatial domains in which vortices are located are obtained in an adaptive manner which is identical to the scheme used by Carrier, Greengard, and Rokhlin [3] for a field of charged particles. The spatial domains for the target points are obtained in an identical fashion. It should be noted that the only inherent commonality between the source and field meshes is that they both are derived from a common square box which contains all of the source and target points of interest.

3.1.1 Mesh Generation

The method requires that an adaptive mesh be generated that encloses all the N_V vortices and all of the N_T target points of interest in the flow. A square with dimensions S_0 is first constructed which encloses all of the chosen vortices and target points. If there happen to be vortices or target points outside of the chosen box or even on the box edges, they are discarded and an appropriate warning is issued. This box is subdivided into four equal boxes. If any of those four boxes contain more than a specified number of vortices N_S that particular box is subdivided into four more boxes. Vortices residing on mesh lines are associated with the box whose center has a more positive x or y value than the x or y value defining the mesh line. This process is continued until all of the boxes contain less than the specified number of vortices. The process is repeated for the field evaluation points. Each box size is associated with a box level l with the original box level being equal to 0. Therefore at a particular level l, the box dimension S_l is given by:

$$S_l = \frac{S_0}{2^l}. \tag{26}$$

At each level, empty boxes are ignored. The ability to ignore regions of the flow-field in which vortices or evaluation points are not present is a feature of the method which provides significant time savings. Boxes which can be subdivided are parent boxes and those which cannot are childless boxes. A list of vortices or field points residing in each parent or childless box at each level is maintained. Each vortex or field point is identified by a unique label (number).

The mesh generators in the FAST2D code allows a maximum level of $l = 10$. The box dimension S_{10} at that level will be $1/1024^{th}$ of the original box dimension. In some instances there will be more than the specified number of vortices or field points in some boxes at $l = 10$. In such cases, a warning is issued and the code proceeds leaving the extra vortices or field points in the boxes. It should be pointed out that the solution resolution is not limited by the maximum level l since interactions between vortices within the smallest box are still considered. It may also be possible that a maximum number of parent or childless boxes at a given level may be exceeded. This number is currently set at 1000 in the FAST2D code. If this number is exceeded a warning is issued along with a request to increase the value of N_S or N_F.

3.1.2 Box Lists

The purpose of defining a box list for each target point box at each level is to define the separation condition between the selected target box and all source boxes in the source box mesh. This in turn allows one to efficiently calculate the influence of vortices in a particular source box on the complex potential and velocity in the target box. In general, each target box at each level has five possible relationships with each source box. The five lists define restrictions on the use of the series expansions between domains defined by the source box and target box.

In order to formalize the box list definitions, let NBOXI refer to the target box for which a box list is being developed. As mentioned previously, parent boxes are boxes which can be subdivided into four smaller boxes, while childless boxes cannot. Thus, parent boxes contain more than N_S vortices or N_F target points. Empty boxes contain no vortices or target points, while childless boxes contain from 1 to N_S vortices or from 1 to N_F target points. A colleague box of NBOXI is a box which is adjacent to NBOXI and which has the same size (level) as NBOXI. There are at most eight colleague boxes associated with NBOXI. Definitions for the five box lists are as follows:

- Box List 1- In order for NBOXI to have any list 1 boxes, NBOXI must itself be childless. If NBOXI is a childless box, then list 1 boxes consist of all childless source boxes at all levels which are adjacent to NBOXI and all childless source boxes which are either contained within, congruent with, or which contain NBOXI. This list defines childless source boxes which are not sufficiently separated from NBOXI to allow any of the series expansions to be used. Direct calculations must be made for this list.

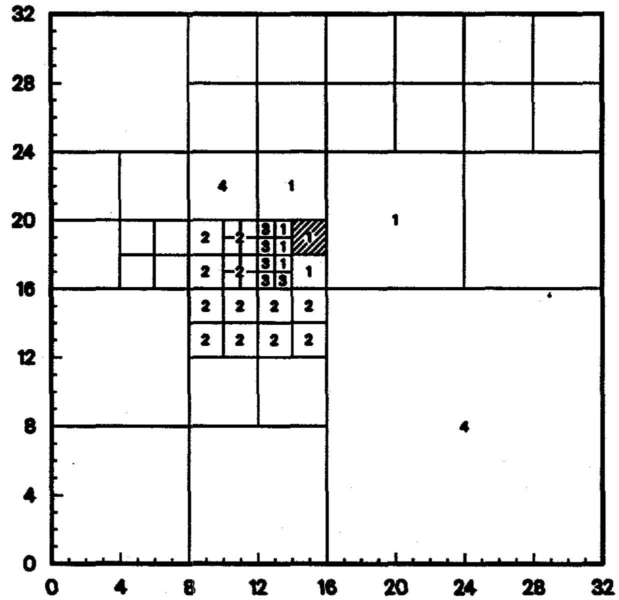
- Box List 2- List 2 boxes of NBOXI are source boxes which occupy the same positions that would be occupied by children of the colleagues of NBOXI's parent that are well separated from NBOXI. Note that the colleagues of NBOXI's parent do not actually have to exist or to have children. NBOXI and its list 2 source boxes can be either parent or childless boxes. NBOXI and its list 2 boxes will be the same size (level) and will be separated by at least the dimension of one of their sides. For this case both source domain (multipole) and target domain (local) series expansions can be used.

- Box List 3- In order for NBOXI to have any list 3 boxes, NBOXI must itself be childless. List 3 boxes can be either parent or childless source boxes. List 3 boxes occupy the positions that descendants of the colleagues of NBOX would occupy. Note that the colleagues of NBOX do not actually have to exist or to have descendants. The parent of the list 3 box must be adjacent to NBOXI but the list 3 box must not itself be adjacent to NBOXI. List 3 boxes will always be smaller than NBOXI. NBOXI will be separated from the list 3 box by one box which is the same size as the list 3 box. Source domain series expansions (multipole) can be used, target domain (local) expansions cannot.

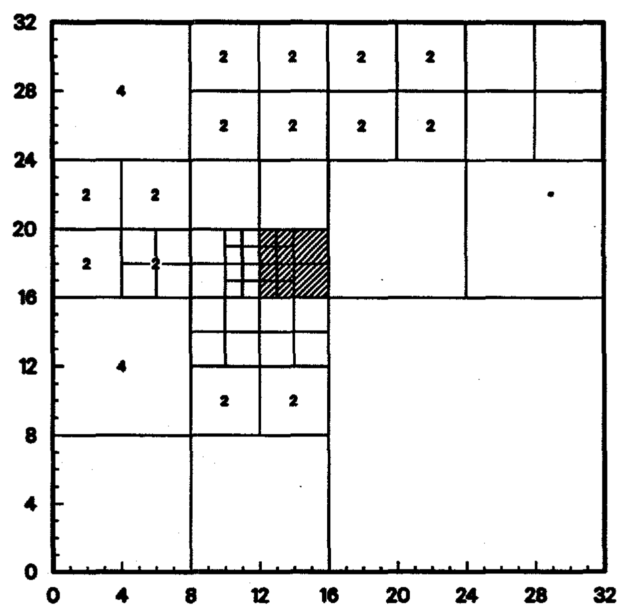
- Box List 4- NBOXI can be either a parent or childless box. List 4 boxes must be childless source boxes. NBOXI is in the position that a descendant of the colleagues of any list 4 box would occupy. Note that the colleagues of the list 4 box do not have to exist nor do they have to have descendants. The parent of NBOXI must be adjacent to the list 4 box but NBOXI must not itself be adjacent to the list 4 box. List 4 boxes will always be larger than NBOXI. NBOXI will be separated from the list 4 box by one box which is the same size as NBOXI. Target domain (local) series expansions can be used, source domain (multipole) expansions cannot.

- Box List 5- List 5 boxes consist of all source boxes that are well separated from NBOXI's parent. No calculations are necessary nor is any list ever made for these boxes. Contributions from these distant boxes reside in the parent of NBOXI.

In order to demonstrate how the set of box lists brings information into the field box NBOXI, consider the example given in Figures 8 and 9. The field box NBOXI is shown as the cross-hatched box in Figure 8 a). Assume that the indicated mesh has been generated for a set of vortices and that there are no empty boxes. It should be pointed out that although the target box NBOXI is congruent with one of the source boxes in this example, this need not be the case in general. Since there are vortices in every source box, one would expect every box to contribute to the complex potential and velocity in NBOXI. From Figure 8 a), it can be seen that there are contributions from NBOXI's own box list and that this list contains box lists 1-4. As indicated previously, box list 5 does not contribute directly and is thus not even indicated in Figures 8 and 9. In Figure 8 b), there are several list 2 and 4 boxes that contribute to NBOXI's parent. Since this is a parent box, it does not have any list 1 or 3 boxes associated with it. The information from NBOXI's parent is fed into NBOXI by translation of the target domain center of the parent of NBOXI to the center of NBOXI. Information from NBOXI's grandparent's list boxes (see Figure 9 c)) were previously fed into NBOXI's parent in the same fashion. As indicated in Figure 9 d), NBOXI's great grandparent has a null box list and therefore does not contribute in this case. It can be noted that every box in the mesh has contributed to NBOXI. In general, it can be seen that the box list concept allows information to be passed down through successive levels in an orderly and efficient fashion.

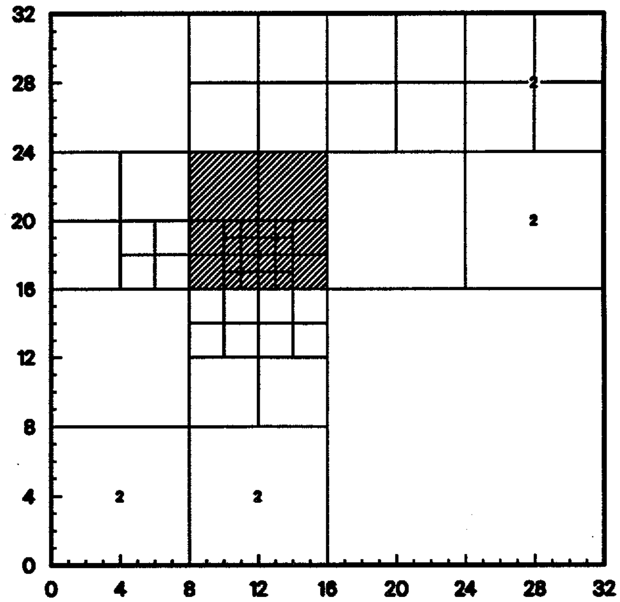


(a) Contribution from NBOXI's own box list.

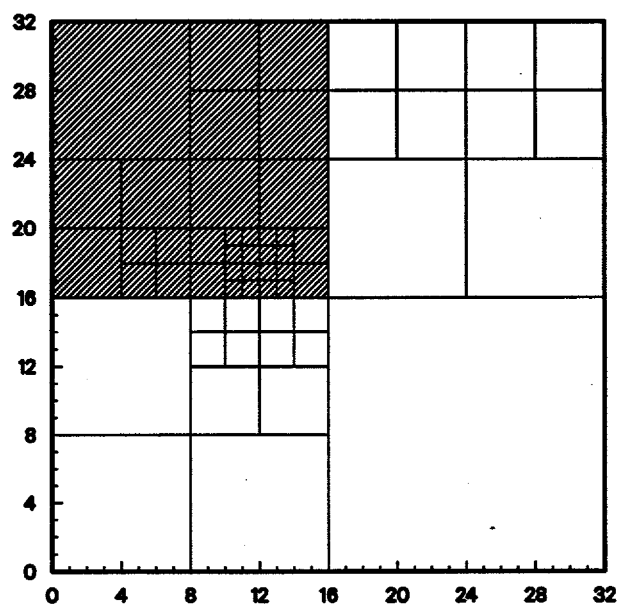


(b) Contribution from NBOXI's parent's box list.

Figure 8. Box List Example



(c) Contribution from NBOXI's grandparent's box list.



(d) Contribution from NBOXI's great grandparent's box list.

Figure 9. Box List Example (continued)

3.2 Generation of Series Coefficients

The series expansions are used only in cases where the source list box associated with a target box NBOXI is a list 2, list 3, or list 4 box. Information from the multipole coefficients (the A coefficients) is used in cases where NBOXI has list 2 or list 3 boxes, while information from the local coefficients (the B coefficients) is used in cases where NBOXI has list 2 or 4 boxes. A set of A coefficients is obtained for each source box (may be either a childless or parent box) whereas a set of B coefficients is obtained for each target box (childless or parent).

3.2.1 Multipole Coefficients

The A_n or A'_n coefficients defined in Equations 9 and 14 are used to calculate the B_n coefficient when the source box is a list 2 box. Also, for list 3 boxes, the A_n coefficients are used directly in the calculation of field variables. The A_n coefficients are calculated by first using Equation 9 to generate values of A_n for all childless source boxes. The centers of expansion for the smallest (highest level) source child boxes are then shifted to their parent's centers using Equation 14. The contributions from the child boxes to the parent box are added together producing a set of A_n coefficients for the parent box. This process is repeated for decreasing levels (larger boxes) until the A_n coefficients are calculated for all parent and childless source boxes at all levels. This process may be thought of as the process by which information concerning source domain expansions is promulgated from smaller domains into larger domains.

3.2.2 Local Coefficients

The B_n coefficients are only used for list 2 and list 4 boxes. For list 2 boxes, the B_n coefficients can be calculated from Equation 19. For list 4 boxes, the contribution from each vortex must be considered separately since the source domain is not well separated from the center of the field box NBOXI. For this case Equation 25 is used. Therefore, the B_n coefficients are calculated by first using Equation 19 for list 2 boxes and Equation 25 for list 4 boxes to generate values of B_n for all boxes. Next, the centers of expansion for the largest (lowest level) parent boxes which have list 2 or list 4 boxes associated with them are shifted to their children's centers using Equation 23. The contribution from the parent box to the child box is added together with the list 2 and list 4 contributions associated with the child box itself. This produces a set of B_n coefficients for the child box. This process is repeated for increasing levels (smaller boxes) until the B_n coefficients are calculated for all parent and childless boxes at all levels. This process may be thought of as the process by which information concerning field domain expansions is promulgated from larger domains into smaller domains.

3.3 Calculation of Field Variables

The field variables $F(z)$ and $W(z)$ at a point p are calculated according to the type of list box containing the source information. For list 1 boxes, the field variable at point p must be calculated by considering the influence of each individual vortex in the source box on the point p in the target box. For list 2 and 4 source boxes, a local series about the center of the field box is used. This series uses the B coefficients. For list 3 source boxes, a series containing the A coefficients is used.

3.3.1 Complex Potential

The complex potential at a point p in the target box for list 1 source boxes must be calculated directly. Contributions from each source in each list 1 box are added together. In order to insure that the complex potential is bounded as z approaches zero, a finite core is introduced to regularize the expression:

$$F(z, \sigma) = \phi(z, \sigma) + i\psi(z, \sigma). \tag{27}$$

Here, σ is the "core radius" associated with the vortex. A more explicit description of the functions $\phi(z, \sigma)$ and $\psi(z, \sigma)$ will be given in a subsequent section.

The contributions from <u>all</u> list 2 and 4 source boxes at a point p in a given target box are obtained by simply using the local series expansion given by Equation 18. Each target box has its own set of B coefficients which contain all of the necessary information from its list 2 and 4 source boxes as well as from list 2 and 4 boxes associated with its parent, grandparent, etc.

The contributions to the complex potential from list 3 source boxes at a point p in a given target box are obtained by using the multipole expansion of Equation 7. This requires that at a point p that contributions from the multipole expansions from all list 3 source boxes be added together.

In summary, the contribution to the complex potential at a point p from list 1 and list 3 source boxes is obtained by adding the contributions from all list 1 and list 3 boxes together. The contribution from all list 2 and 4 boxes is inherent in the B coefficients which are associated with the field box itself. This contribution is added to the contributions from all list 1 and list 3 source boxes. This may be illustrated by the following equation:

$$F = \sum_{list1} F_1 + \sum_{list3} F_3 + F_{24}, \tag{28}$$

where the subscripts (1,2,3,4) indicate the source box list number.

3.3.2 Complex Velocity

The complex velocity at a point p is calculated using box list information in a manner similar to that for the complex potential. For example, the contribution to the complex velocity at a point p from list 1 and list 3 source boxes is obtained by adding the contributions from all list 1 and list 3 boxes together. The contribution from all list 2 and 4 boxes is inherent in the B coefficients which are associated with the target box itself. This contribution is added to the contributions from all list 1 and list 3 source boxes yielding the following equation:

$$W = \sum_{list1} W_1 + \sum_{list3} W_3 + W_{24}, \tag{29}$$

where the subscripts (1,2,3,4) indicate the source box list number.

Using the relationship between the complex velocity and the complex potential given by Equation 2 along with Equation 7, we see that the appropriate multipole expansion for the complex velocity is given by:

$$W_p(zp) = \sum_{m=0}^{Nmax} A_m K^{(m+1)}(zp - zq)\,. \tag{30}$$

From Equation 2 and 18 the appropriate local expansion for the complex velocity is:

$$W_p(zp) = \sum_{n=1}^{Nmax} B_n \frac{(zp - zd)^{n-1}}{(n-1)!}\,. \tag{31}$$

3.3.3 Core Functions

In order to regularize the interactions in list 1 boxes, a core function must be introduced. A comprehensive discussion of the suitability of core functions is beyond the scope of the present work but an excellent presentation of such matters is given by Winckelmans and Leonard [5]. It should be noted that the influence of the core region does not extend outside of list 1 boxes associated with a particular target box. Therefore one should get results which are within the expected precision when compared with direct calculations so long as the support of the core function is less than the list 1 source box size. This is not an especially onerous condition since typically, there are several vortices in each source box whose supports overlap by something on the order of the distance between the vortices.

In the present work, we chose to use the high-order algebraic smoothing of Winckelmans and Leonard [5]. The complex velocity is then expressed by:

$$W_p(z) = -\frac{i\Gamma}{2\pi z} q(\rho), \tag{32}$$

where

$$q(\rho) = \frac{\rho^2(\rho^2 + 2)}{(\rho^2 + 1)^2}\,. \tag{33}$$

Here $\rho^2 \equiv \frac{|z|^2}{\sigma^2}$ where σ is the core radius. For $\rho \geq 5.5$ a value of $q(\rho) = 1.0$ may be used resulting in less than a 0.1% difference from Equation 33 in the calculated value of $W_p(z)$.

The stream function which is consistent with this high order algebraic smoothing is given by:

$$\psi = -\frac{\Gamma}{2\pi}[G(\rho) + \ln(\sigma)]\ , \tag{34}$$

where

$$G(\rho) = \frac{1}{2}\left[\ln(\rho^2 + 1) - \frac{1}{\rho^2 + 1}\right]\,. \tag{35}$$

It can be noted that for $\rho \geq 3.5$, $G(\rho) \approx \ln(\rho)$ and $\psi = -\frac{\Gamma}{2\pi}\ln(r)$. The error in ψ is approximately 0.1% at $\rho = 3.5$.

A difficulty arises in representing the complex potential in the core region since the velocity potential is not defined in rotational portions of the flow field. Since we are only interested in the stream-function we will simply set the velocity potential equal to:

$$\phi = \frac{\Gamma}{2\pi}q(\rho)\theta , \tag{36}$$

where θ is an azimuthal angle about the vorticity vector. Equation 36 will yield valid results for areas outside of the viscous regions where $q(\rho)$ approaches unity. We note that the complex potential is not analytic in the core region.

4 BENCHMARK TESTS

4.1 General Description of Benchmark Tests

The important dependent variables in the bench mark tests are the CPU run times and the truncation errors associated with the calculation of field variables. The independent variables are the number of vortex sources and target points in the field, the distribution of the sources and target points in the field, the number of terms used in the multipole and local series expansions, and the maximum number of source and target points allowed in any childless box in the source and target meshes. In this work, we are especially interested in how to set the maximum number of source and target points allowed in any childless box in the source and target meshes in order to minimize CPU time, irrespective of the configuration of source and target points. In view of the large number of parametric variations which might ultimately be useful, we have run a somewhat limited set of test cases to examine truncation error and CPU run times.

Four general configurations of vortex and target point placements have been studied as indicated in Figure 10. In the first set of cases, the source and target points are uniformly placed. The total number of source points may be different from the total number of target points which holds true for any of the configurations studied. This is labeled as the uniform-uniform (U-U) case. For the second set of cases, all of the source and target points are arranged on a spiral. This case is designated as the spiral-spiral (S-S) case. For the third set of cases, the sources are on the spiral while the targets are uniformly spaced (S-U). Finally, for the fourth set, the sources are uniformly spaced while the targets are on the spiral (U-S).

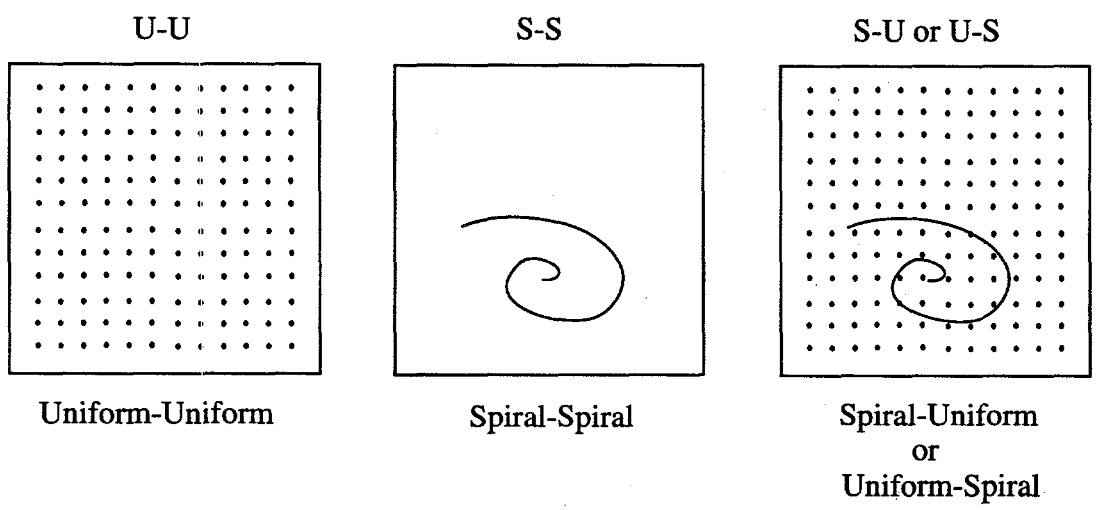


Figure 10. Benchmark Test Cases

Before presenting the results for the various benchmark cases, a metric for the truncation error must be defined. One such measure of the error incurred by truncation of the multipole and local series after the N^{th} order terms can be obtained for the stream-function and the magnitude of the velocity vector field by means of the following equations:

$$\varepsilon_\psi = \left[\frac{\sum_{i=1}^{N_V} (|\psi|_i - |\psi|_{ei})^2}{\sum_{i=1}^{N_V} |\psi|_{ei}^2}\right]^{\frac{1}{2}},$$

$$\varepsilon_U = \left[\frac{\sum_{i=1}^{N_V} (|U|_i - |U|_{ei})^2}{\sum_{i=1}^{N_V} |U|_{ei}^2}\right]^{\frac{1}{2}}. \tag{37}$$

Here, $|\psi|_i$ and $|U|_i$ are the magnitudes of the stream function and velocity vector at point i as calculated from the truncated series. The quantities $|\psi|_{ei}$ and $|U|_{ei}$ are the magnitudes of the stream function and velocity vector at point i as calculated exactly using the direct method. As indicated, these error calculations are obtained from summations over all of the N_V vortices in the flow. Assuming that single precision accuracy means that any number stored in the computer will be accurate through 6 significant digits, then single precision round off errors will produce values of ε_ψ and ε_U up to about 1×10^{-6}. Therefore, errors calculated from Equations 37 which are equal to or less than 1×10^{-6} imply that the machine round off errors are possibly as large as the truncation errors. For such cases, one should not continue to add terms to the series expansions for added accuracy unless the machine precision is also increased.

4.2 Results From U-U case

In order to investigate the relationship between accuracy, CPU time, and optimal values for the maximum number of sources and target points in a box, a series of runs were made in which 10,202 vortices each with unit strength were used. A direct calculation required 356 seconds on a Sun Sparc 20 workstation. In this series, the maximum number of sources N_S and target points N_F in a box were set equal to each other and varied between 10 and 60. The number of terms used in the series expansions N_{max} was varied between 4 and 20. Results from these runs are tabulated in Appendix A.1 and plotted in Figures 11 and 12.

From Figure 11 it can be seen that the error in the stream function ε_ψ is on the order of 1×10^{-6} when the number of terms in the series expansion $N_{max} \geq 6$. The error in the velocity ε_U for $N_{max} \geq 6$ is less than 1×10^{-4} which is probably acceptable for most engineering calculations. The error appears to be insensitive to the values of N_S and N_F over the range of 10 to 60. From Figure 12 it can be seen that a range of values for N_S and N_F between 20 and 60 will yield good results with regard to optimal CPU times.

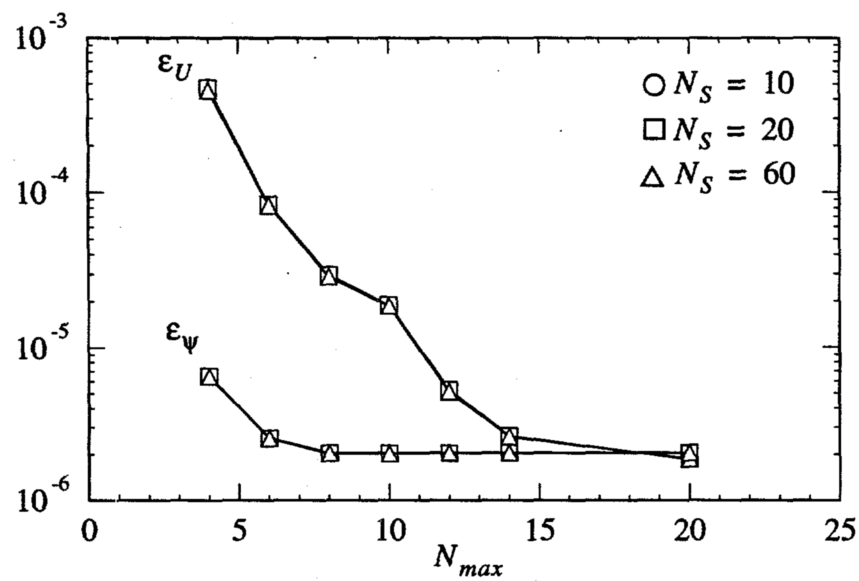


Figure 11. Error Plots for Uniform Distribution of 10,201 Vortices

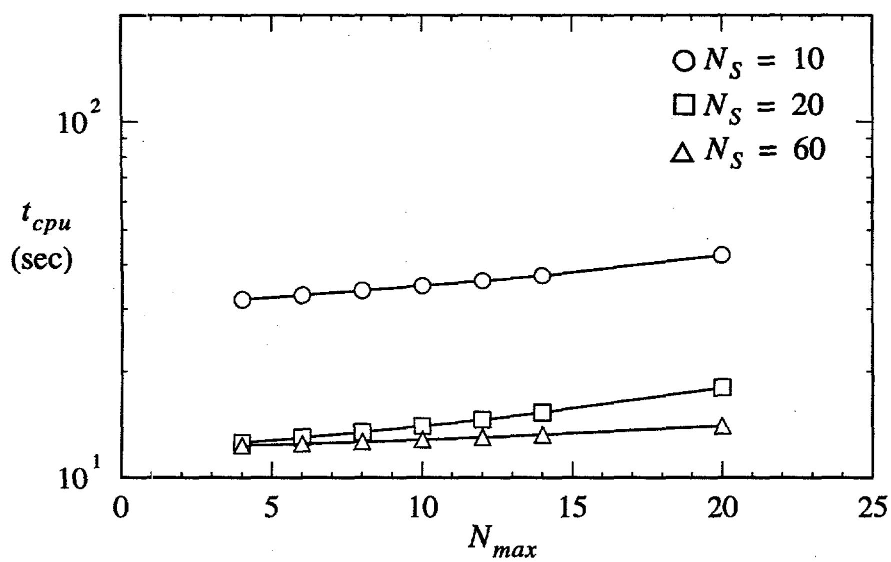


Figure 12. CPU Times for Uniform Distribution of 10,201 Vortices

In order to examine the effect of varying N_S, N_F, N_V, and N_T on CPU time, 150 cases were run with $N_{max} = 6$. A set of six contour plots are given in Figure 13. The contours represent lines of constant CPU time which are normalized by dividing actual CPU time by the minimum CPU time associated with a given plot. These plots suggest that in order to make optimal choices for N_S and N_F an algorithm such as the following might be used:

$$N_S = 30max(1, \sqrt{N_V/N_T}) \qquad (38)$$

$$N_F = 30max(1, \sqrt{N_T/N_V}) \quad .$$

The symbol ⊠ in the following contour plots represents application of Equation 38. It is also worth noting that if one lets $N_S = N_F = 30$, the CPU time will be within about 150% of the optimal value for the cases shown in Figure 13. Tabular data are given in Appendix A.5.

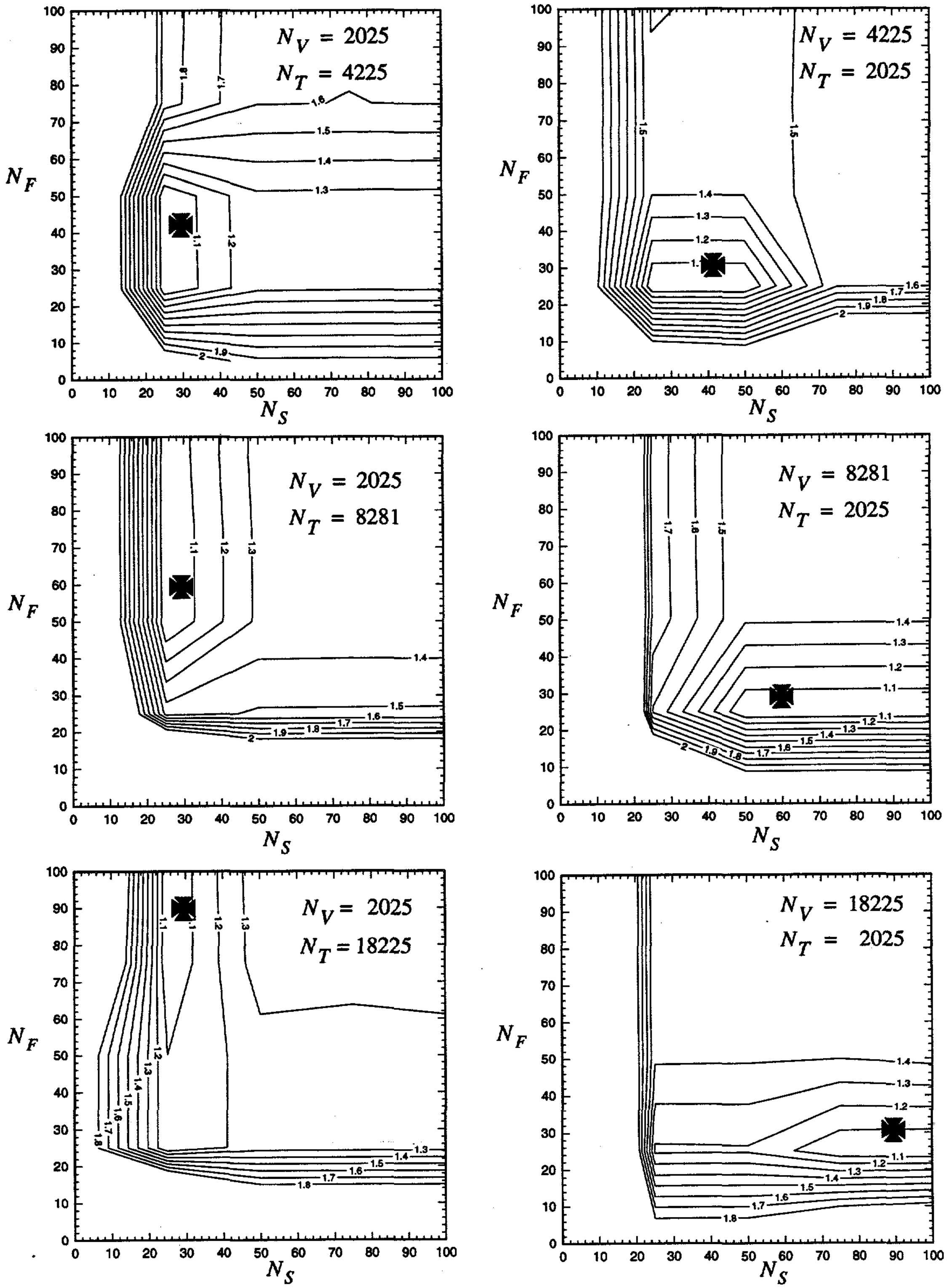


Figure 13. Optimum N_S and N_F for U-U Case (N_{max} = 6)

We now examine the CPU times and errors associated with U-U cases when $N_{max} = 6$ and N_S and N_F are governed by Equation 38. A set of runs were made with $N_V = N_T$ where the number of vortices and target points varied from about 100 to 30,000. In addition, four cases were run where $N_V/N_T = (1/9, 1/2, 2, 9)$. All of these cases are tabulated in Appendix A.1 and plotted in Figures 14 and 15. In order to plot the results for the four cases where $N_V \neq N_T$, a value of $\sqrt{N_V N_T}$ was used on the abscissa.

From Figure 14 and the data in Appendix A.1 it can be shown that the CPU times for the direct calculations are proportional to the square of N_V or the square of N_T when $N_V = N_T$. When the number of source and target points are not equal, the CPU time for the direct calculation is proportional to $N_V N_T$. The CPU times for the fast solver are well represented by the following equation in the indicated range of $N_V N_T$:

$$t_{cpu} = 1.7\times10^{-4}(N_V N_T)^{0.60}. \tag{39}$$

It is satisfying to see that the data for different numbers source and target points collapses onto the same CPU time curves.

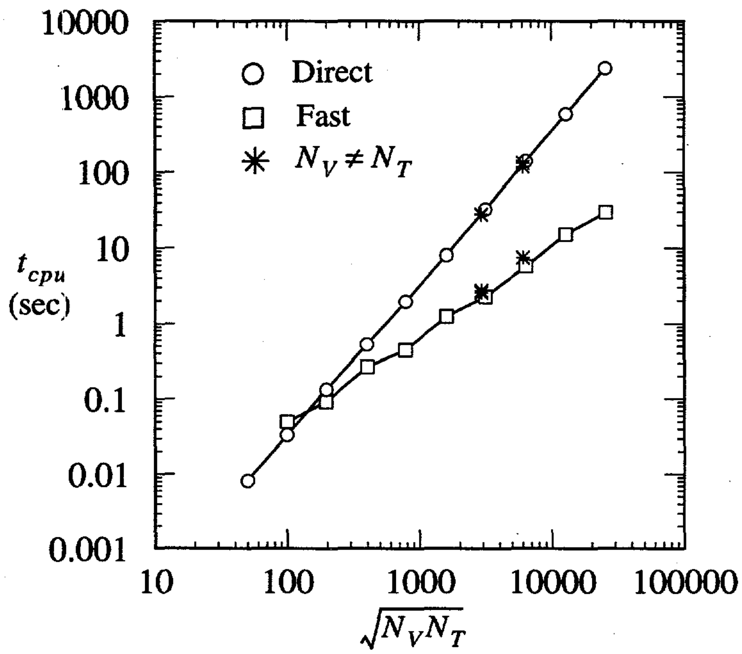


Figure 14. CPU Times for U-U Case (Direct versus Fast)

From Figure 15 we see that the error in the stream function is on the order of 10^{-6} while the error in the velocity magnitude is on the order of 10^{-4} except for $\sqrt{N_V N_T} = 100$. For this case the solution is essentially a direct solution and the error should be very small. It is again interesting to note that the error for the cases where $N_V \neq N_T$ collapses onto the same curves where $N_V = N_T$.

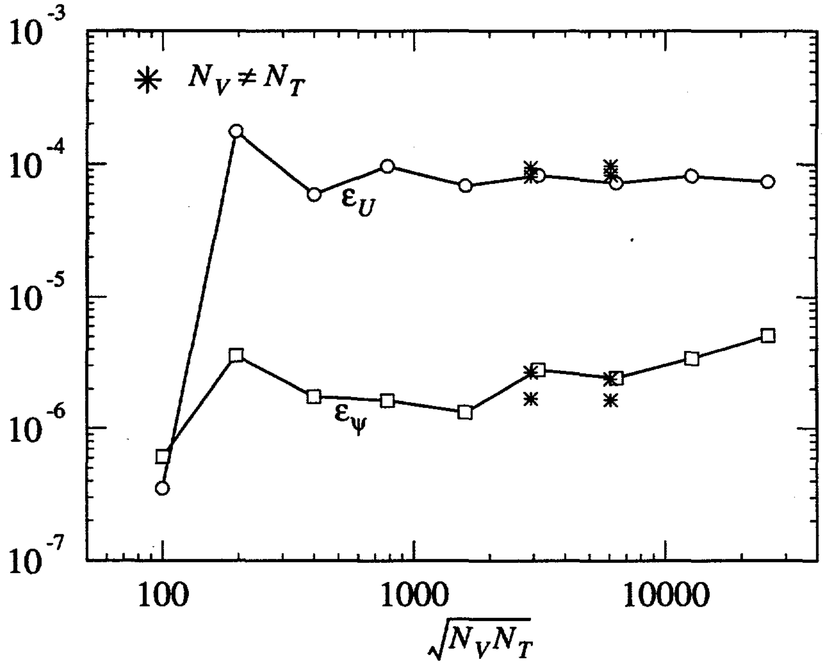


Figure 15. Errors Using Fast Solver for U-U Case

4.3 Results From S-S Case

In this case, all of the source and target points are placed on a spiral curve. Equations for points x_p and y_p on the spiral curve are given by:

$$x_p = x_c + S_r \cos\theta,$$

$$y_p = y_c + S_r \sin\theta, \tag{40}$$

$$S_r = \frac{2\theta}{\pi}\left(\sin^2\theta + \frac{1}{4}\cos^2\theta\right)^{-0.5},$$

where $x_c = 16.00$ and $y_c = 10.67$. The angle θ is measured counter clockwise about the point (x_c, y_c) from an axis parallel to the x axis. Vortices of unit strength were placed on the spiral at equal intervals of θ where θ ranges from 0 to 2.8π.

In order to examine the effect of varying N_S, N_F, N_V, and N_T on CPU time, 150 cases were run with $N_{max} = 6$. Tabular data are given in Appendix A.5. A set of six contour plots are given in Figure 16. The contours represent lines of constant CPU time which are normalized by dividing actual CPU time by the minimum CPU time associated with a given plot. It can be noted that Equation 38 (indicated by the symbol ⊞) provides values for N_S and N_F which yield very close to optimum values of CPU time.

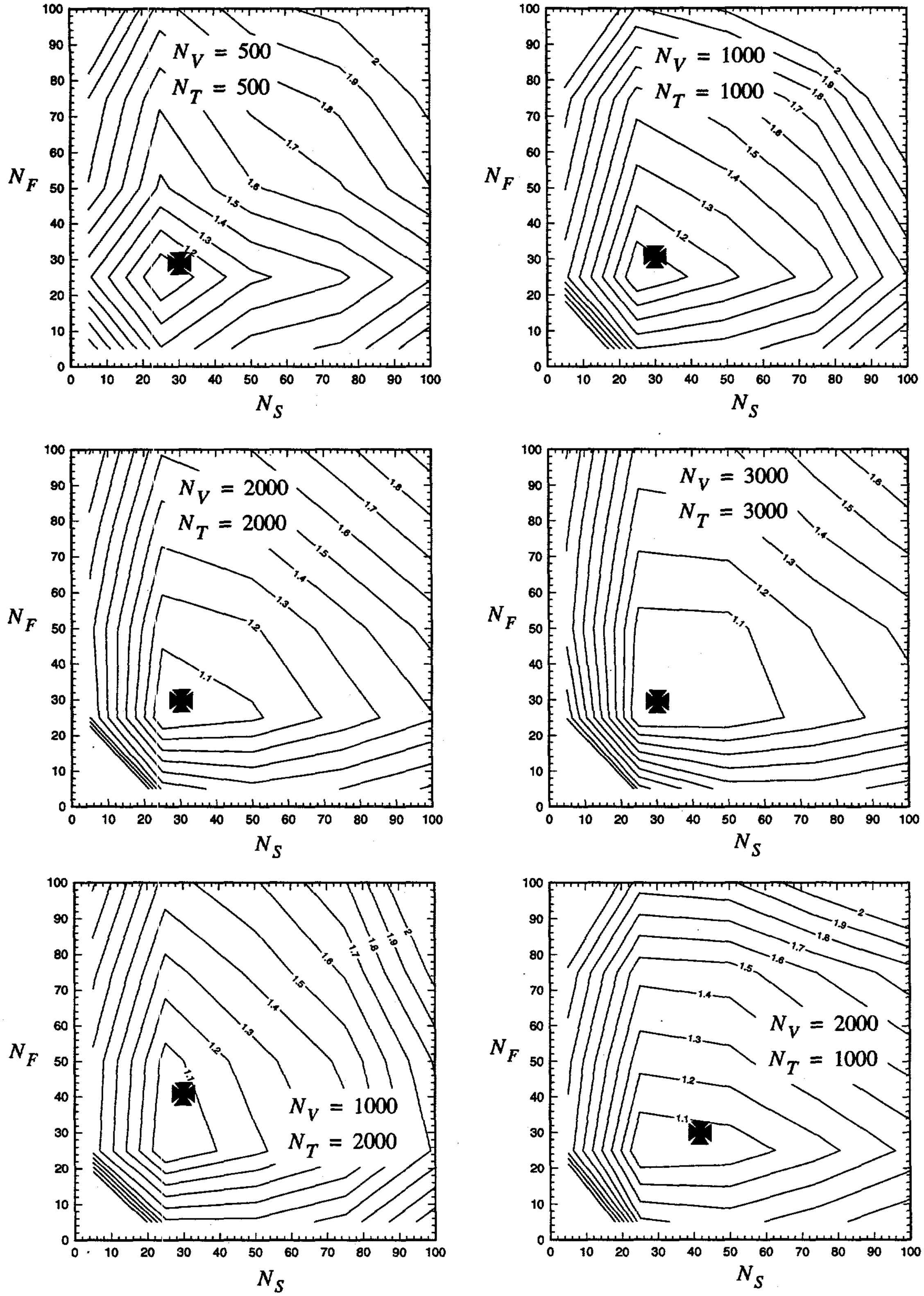


Figure 16. Optimum N_S and N_F for S-S Case ($N_{max} = 6$)

We now examine the CPU times and errors associated with S-S cases when $N_{max} = 6$ and N_S and N_F are governed by Equation 38. A set of runs were made with $N_V = N_T$ where the number of vortices and target points varied from about 50 to 6400. In addition, four cases were run where $N_V/N_T = (1/4, 1/2, 2, 4)$. All of these cases are tabulated in Appendix A.2 and plotted in Figures 17 and 18.

From Figure 17 and the data in Appendix A.2 it can be shown that the CPU times for the direct calculations are proportional to $N_V N_T$. The CPU times for the fast solver are reasonably well represented by the following equation:

$$t_{cpu} = 2.9\times10^{-4}(N_V N_T)^{0.55}. \qquad (41)$$

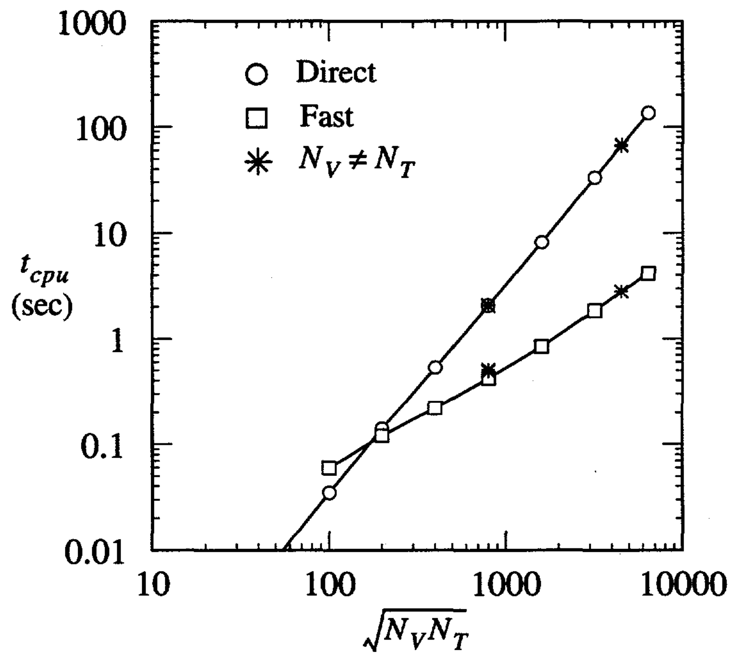


Figure 17. CPU Times for S-S Case (Direct versus Fast)

From Figure 18, one can see that the error in the stream function is on the order of 10^{-5} while the error in the velocity magnitude is on the order of 10^{-4} to 10^{-3}. Errors in the S-S configuration tend to be larger than those associated with the U-U case. In addition, the errors in the velocity magnitude vary depending on whether or not the maximum number of vortices in a box N_S is equal to the maximum number of target points in a box N_F. Moreover, the error does not drop into the 10^{-6} range for 100 vortices as it does for the U-U case.

Part of this behavior may be attributed to the very small mesh box sizes which are generated in resolving the S-S configuration. Even for 100 source or target points placed on the spiral, a number of relatively small boxes are generated. It may also be noted that most of the source and target boxes tend to be well separated which means that a major portion of the computation is achieved using multipole and local expansions as opposed to direct calculations.

There appears to be a gradual increase in the velocity error with increasing numbers of source and target points. This might possibly be due to the size of the core radii which was used in the computations. The core radii was set equal to 0.001 times the box size (32.0) which is on the order of 0.001 times the total arc length along the spiral curve. Source and or target boxes of this size are generated near the center of the spiral when something on the order of a few thousand sources or targets are used. This means that the support of the core will extend outside of the mesh boxes resulting in a loss of accuracy. It is not clear why the same trend is not seen in the errors associated with the stream function.

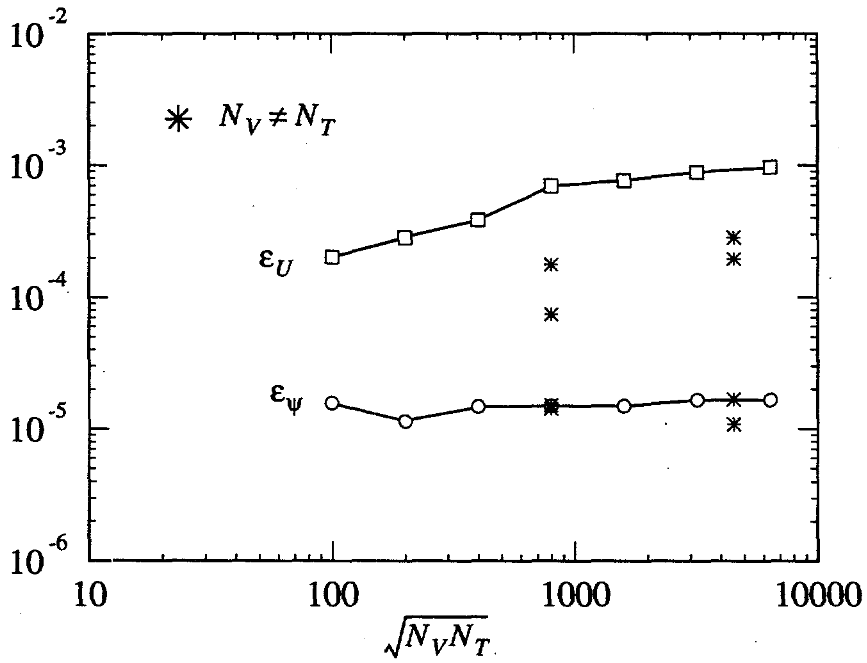


Figure 18. Errors Using Fast Solver for S-S Case

4.4 Results From S-U and U-S Cases

In order to examine the effect of varying N_S, N_F, N_V, and N_T on CPU time, 150 S-U cases (sources on spiral, targets uniformly spaced) and 100 U-S cases (sources uniformly spaced, targets on spiral) were run with $N_{max} = 6$. Tabular data are given in Appendix A.5. A set of six contour plots are given in Figure 19 for the S-U case and four contour plots are given in Figure 20 for the U-S case. As a reminder, the contours represent lines of constant CPU time which are normalized by dividing actual CPU time by the minimum CPU time associated with a given plot. Again, it can be noted that Equation 38 (indicated by the symbol ⊞) provides values for N_S and N_F which yield very close to optimum values of CPU time. The worst case appears to be the S-U case for $N_V = N_T = 3025$ where use of Equation 38 would lead to an increase of about 30% above the minimum CPU.

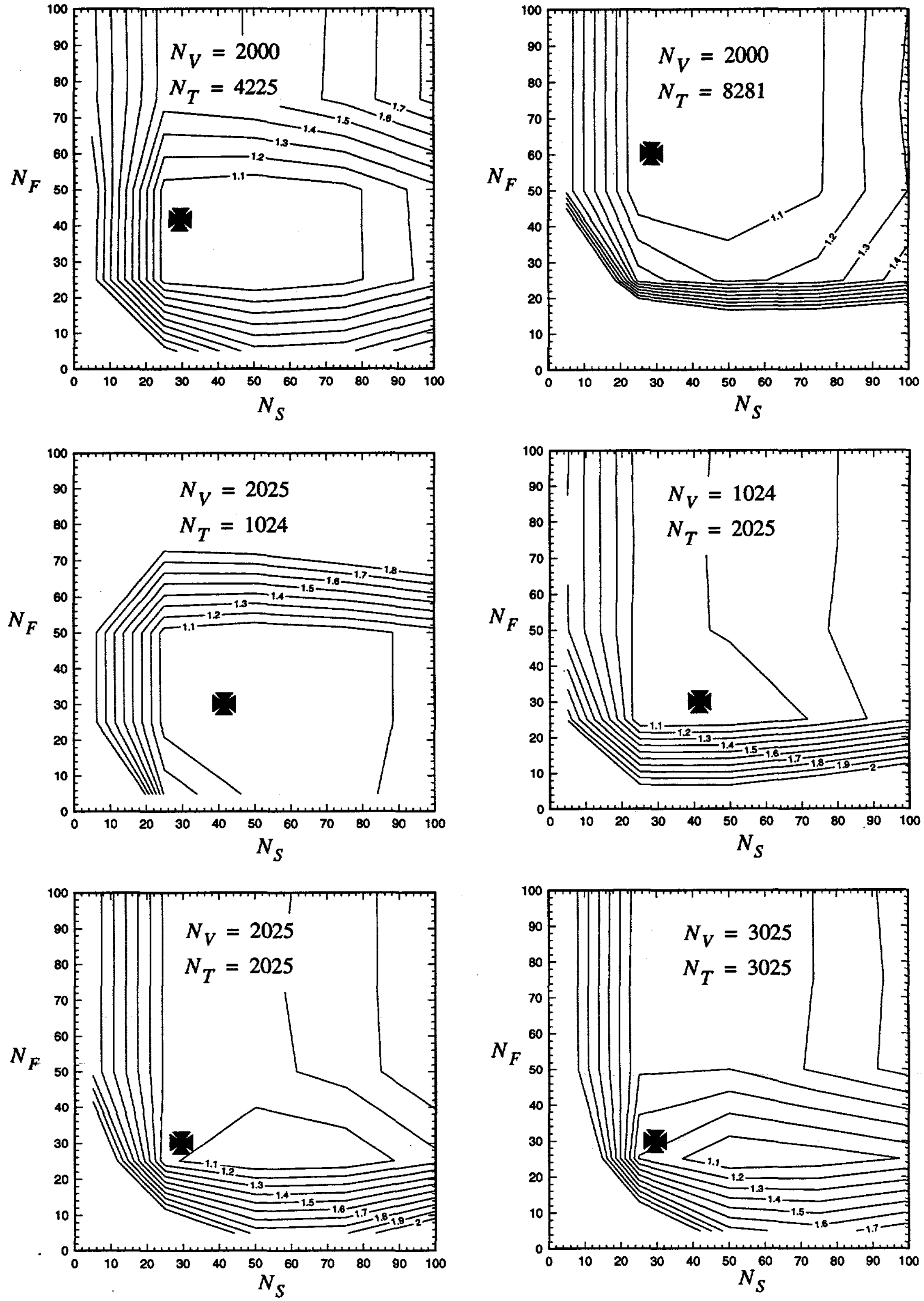


Figure 19. Optimum N_S and N_F for S-U Case ($N_{max} = 6$)

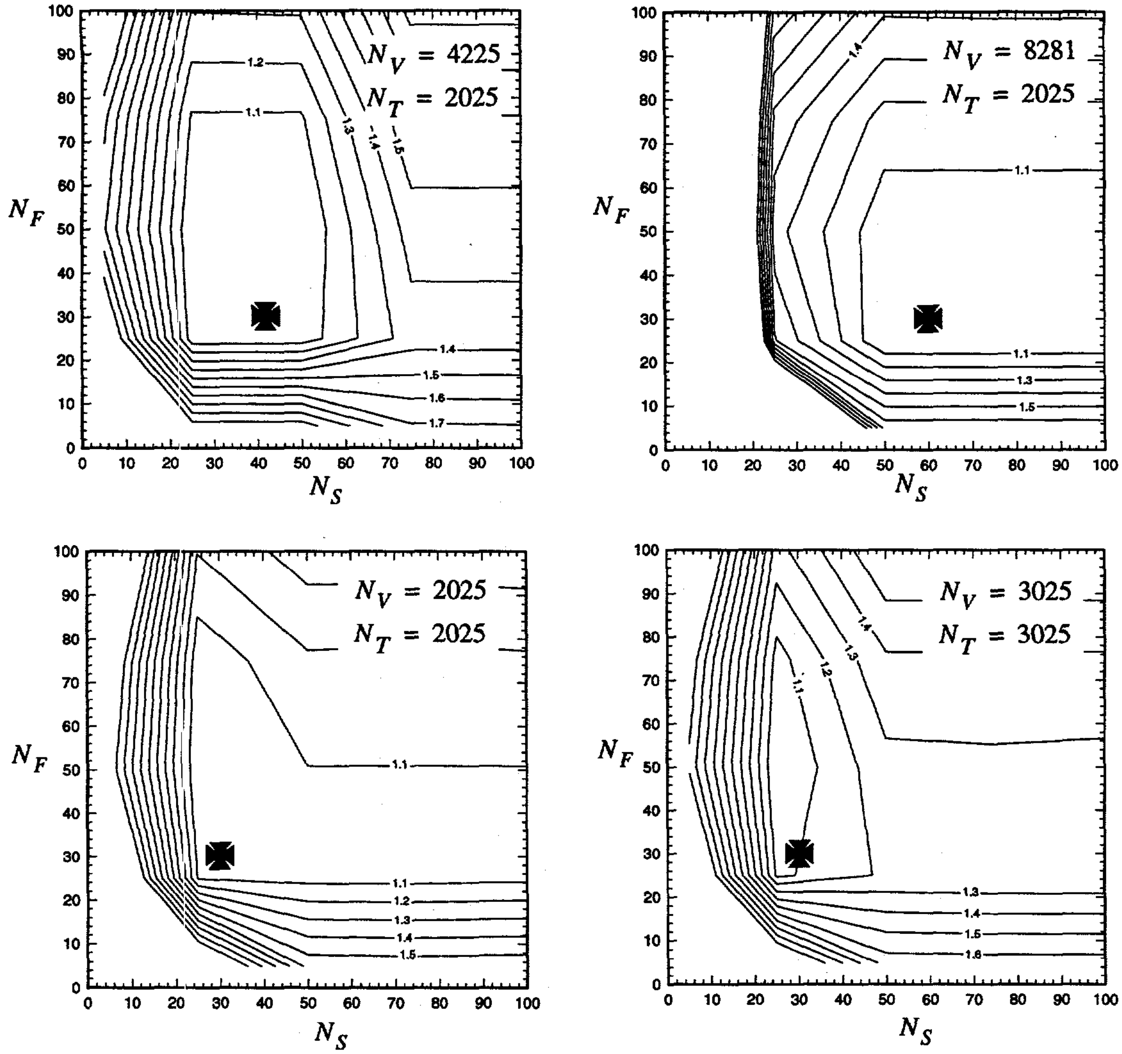


Figure 20. Optimum N_S and N_F for U-S Case ($N_{max} = 6$)

We now examine the CPU times and errors associated with the S-U and U-S cases when $N_{max} = 6$ and N_S and N_F are governed by Equation 38. A set of runs were made with $N_V = N_T$ where the number of vortices and target points varied from about 50 to 7,000. In addition, four cases were run where $N_V/N_T \approx (1/4, 1/2, 2, 4)$. All of these cases are tabulated in Appendices A.3 and A.4 and plotted in Figures 21 through 24.

From Figure 21 and 22 it can be seen that the CPU times for the direct calculations are proportional to $N_V N_T$. We note that the CPU times for the fast solver are essentially the same for both the S-U and U-S cases and are well represented by the following equation in the indicated range of $N_V N_T$:

$$t_{cpu} = 5.8\times10^{-4}(N_V N_T)^{0.50}. \tag{42}$$

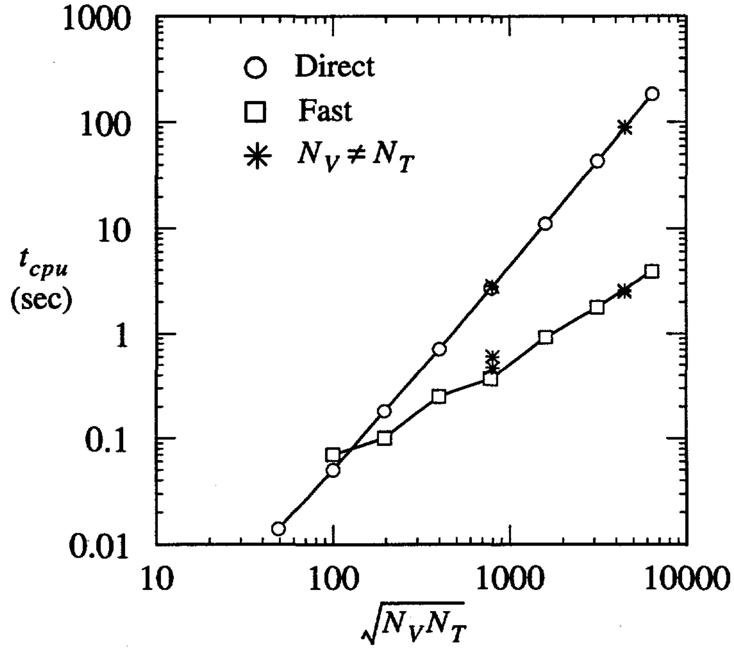


Figure 21. CPU Times for S-U Case (Direct versus Fast)

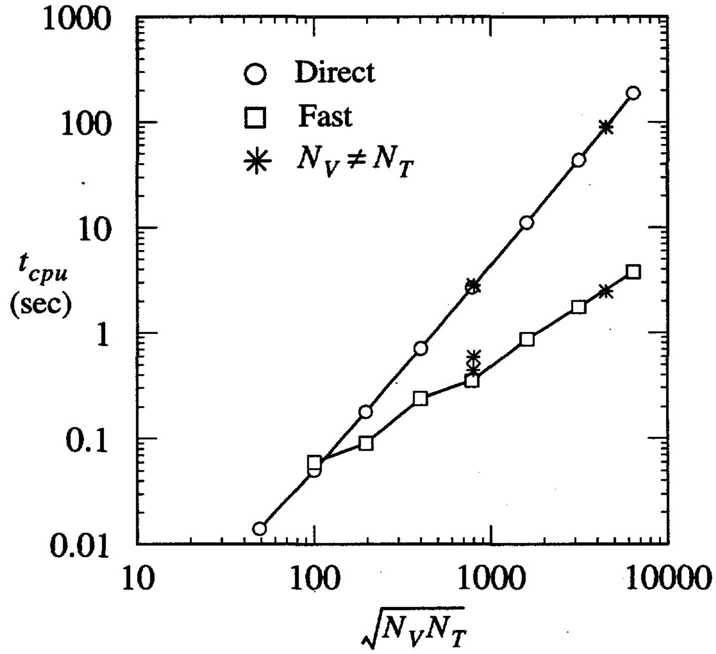


Figure 22. CPU Times for U-S Case (Direct versus Fast)

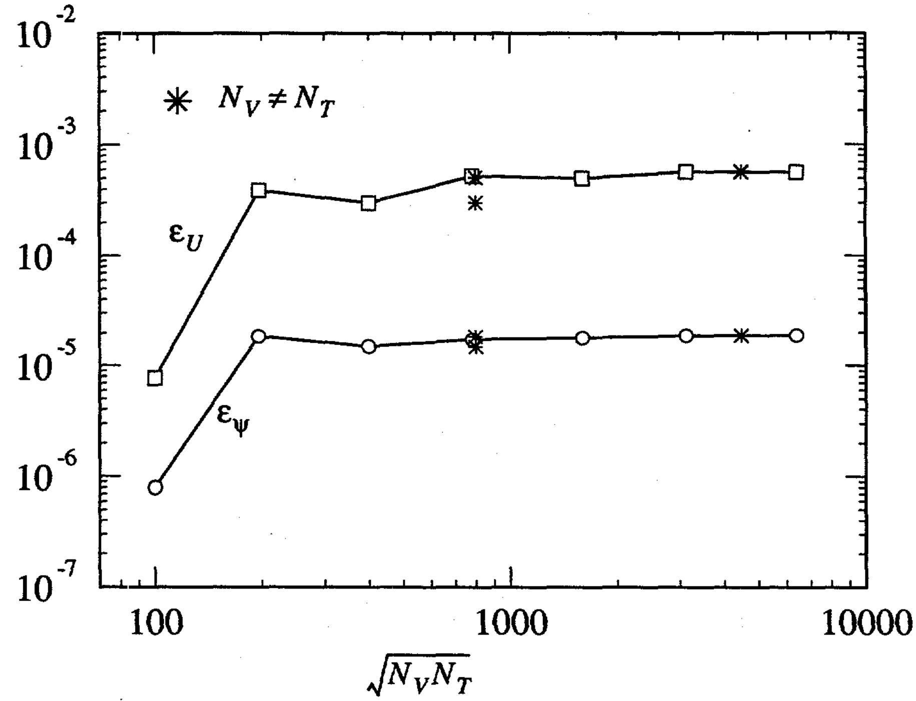


Figure 23. Errors Using Fast Solver for S-U Case

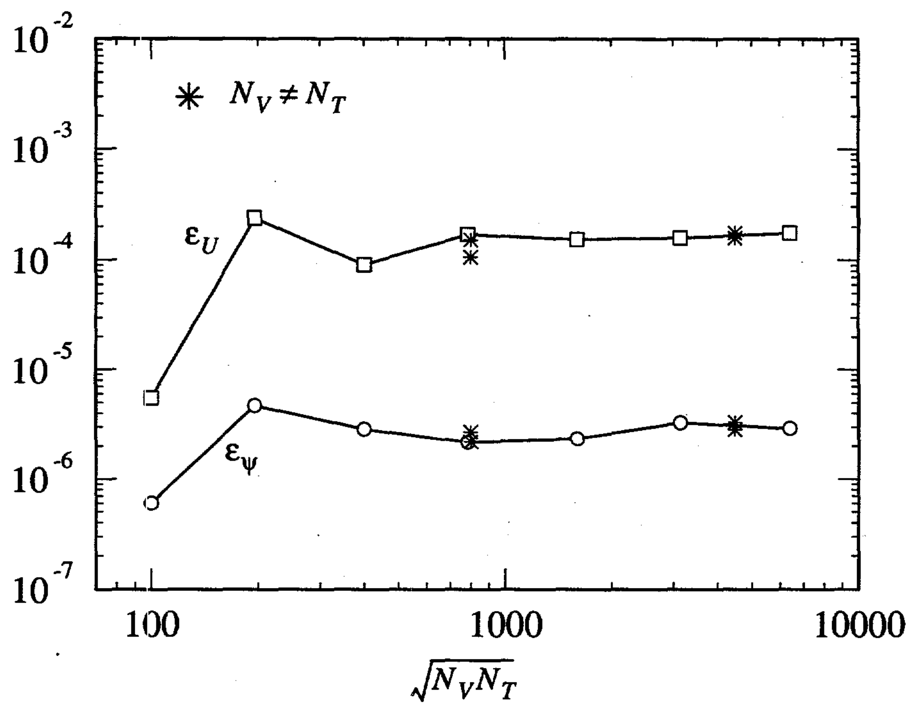


Figure 24. Errors Using Fast Solver for U-S Case

5 SUMMARY

The work accomplished can be summarized as follows:

- A Fortran computer code "FAST2D" has been written to generate solutions for the stream function and velocity vector fields for an arbitrary two-dimensional distribution of vortices and an arbitrary two-dimensional distribution of evaluation points. Multipole and local series expansions written in terms of complex variables were used to make calculations for the influence of groups of vortices on well-separated groups of evaluation points.

- The basic Greengard-Rokhlin adaptive domain decomposition algorithms were used which dismiss regions of the global domain in which no vortices or evaluation points are present. We have extended their method such that non-congruent vortex (source) and evaluation point (target) fields may be handled.

- A simple dynamic algorithm has been developed to better optimize values of N_S and N_F which are the maximum number of vortices in a box and the maximum number of evaluation points in a box respectively. This optimization is based on numerical experiments and is especially useful for situations in which there is a large difference in the total number of source points N_V and target points N_T. With this algorithm in place, the CPU time scales as $\sqrt{N_V N_T}$.

- Preliminary benchmark tests were run on four basic configurations of the vortex and evaluation point field. For multipole and local series expansions of order $N = 6$ the error associated with the stream function was on the order of 10^{-5} whereas the error associated with the velocity was on the order of 10^{-3} or better. No improvement in CPU time, relative to that obtained using direct calculation of all interactions, was obtained for less than about 200 vortices. For 25,600 vortices, the CPU time using the fast solver was on the order of 1% of that required by the direct solution technique.

6 REFERENCES

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APPENDIX

A.1 Data for U-U Configuration

Table 1: U-U Configuration

N_V	N_T	N_S	N_F	N_{max}	t_{cpu} (sec)	ε_U	ε_ψ
10201	10201	10201	10201	1	356.00	0	0
10201	10201	10	10	4	31.76	4.73e-04	6.49e-06
10201	10201	20	20	4	12.55	4.68e-04	6.48e-06
10201	10201	30	30	4	12.55	4.68e-04	6.48e-06
10201	10201	60	60	4	12.34	4.53e-04	6.45e-06
10201	10201	10	10	6	32.71	8.38e-05	2.55e-06
10201	10201	20	20	6	13.00	8.35e-05	2.54e-06
10201	10201	30	30	6	12.96	8.35e-05	2.54e-06
10201	10201	60	60	6	12.49	8.27e-05	2.55e-06
10201	10201	10	10	8	33.74	2.96e-05	2.05e-06
10201	10201	20	20	8	13.47	2.95e-05	2.04e-06
10201	10201	30	30	8	13.50	2.94e-05	2.03e-06
10201	10201	60	60	8	12.66	2.88e-05	2.03e-06
10201	10201	10	10	10	34.80	1.88e-05	2.04e-06
10201	10201	20	20	10	14.01	1.88e-05	2.03e-06
10201	10201	30	30	10	14.00	1.88e-05	2.03e-06
10201	10201	60	60	10	12.84	1.86e-05	2.02e-06
10201	10201	10	10	12	35.99	5.26e-06	2.04e-06
10201	10201	20	20	12	14.62	5.26e-06	2.03e-06
10201	10201	30	30	12	14.63	5.28e-06	2.03e-06
10201	10201	60	60	12	13.03	5.26e-06	2.04e-06
10201	10201	10	10	14	37.22	2.63e-06	2.04e-06
10201	10201	20	20	14	15.31	2.63e-06	2.04e-06
10201	10201	30	30	14	15.33	2.63e-06	2.04e-06

Table 1: U-U Configuration

N_V	N_T	N_S	N_F	N_{max}	t_{cpu} (sec)	ε_U	ε_ψ
10201	10201	60	60	14	13.25	2.61e-06	2.04e-06
10201	10201	10	10	20	42.59	1.85e-06	2.04e-06
10201	10201	20	20	20	18.02	1.85e-06	2.04e-06
10201	10201	30	30	20	18.02	1.85e-06	2.03e-06
10201	10201	60	60	20	14.05	1.86e-06	2.04e-06
100	100	30	30	6	0.05	3.54e-07	6.13e-07
196	196	30	30	6	0.09	1.76e-04	3.58e-06
400	400	30	30	6	0.27	5.90e-05	1.74e-06
784	784	30	30	6	0.44	9.70e-05	1.63e-06
1600	1600	30	30	6	1.28	6.89e-05	1.33e-06
3136	3136	30	30	6	2.76	8.30e-05	2.77e-06
6400	6400	30	30	6	5.76	7.25e-05	2.41e-06
12769	12769	30	30	6	15.09	8.19e-05	3.41e-06
25600	25600	30	30	6	29.62	7.43e-05	5.12e-06
100	100	100	100	1	0.04	0	0
196	196	196	196	1	0.13	0	0
400	400	400	400	1	0.52	0	0
784	784	784	784	1	1.95	0	0
1600	1600	1600	1600	1	8.22	0	0
3136	3136	3136	3136	1	32.20	0	0
6400	6400	6400	6400	1	141.31	0	0
12769	12769	12769	12769	1	586.83	0	0
25600	25600	25600	25600	1	2363.31	0	0
2025	4225	2025	4225	1	27.40	0	0
4225	2025	4225	2025	1	27.89	0	0
2025	18225	2025	18225	1	118.08	0	0
18225	2025	18225	2025	1	131.86	0	0

Table 1: U-U Configuration

N_V	N_T	N_S	N_F	N_{max}	t_{cpu} (sec)	ε_U	ε_ψ
2025	4225	30	43	6	2.56	9.44e-05	1.68e-06
4225	2025	43	30	6	2.72	8.11e-05	2.64e-06
2025	18225	30	90	6	7.52	9.71e-05	1.65e-06
18225	2025	90	30	6	7.51	8.35e-05	2.37e-06

A.2 Data for S-S Configuration

Table 2: S-S Configuration

N_V	N_T	N_S	N_F	N_{max}	t_{cpu} (sec)	ε_U	ε_ψ
50	50	50	50	1	.0083	0	0
100	100	100	100	1	.035	0	0
200	200	200	200	1	.14	0	0
400	400	400	400	1	.53	0	0
800	800	800	800	1	2.06	0	0
1600	1600	1600	1600	1	8.15	0	0
3200	3200	3200	3200	1	33.19	0	0
6400	6400	6400	6400	1	135.41	0	0
50	50	30	30	6	.02	2.14e-07	1.49e-07
100	100	30	30	6	.06	2.02e-04	1.56e-05
200	200	30	30	6	.12	2.83e-04	1.14e-05
400	400	30	30	6	.22	3.84e-04	1.47e-05
800	800	30	30	6	.42	6.96e-04	1.49e-05
1600	1600	30	30	6	.84	7.64e-04	1.49e-05
3200	3200	30	30	6	1.85	8.84e-04	1.65e-05
6400	6400	30	30	6	4.15	9.61e-04	1.65e-05
1600	400	1600	400	1	2.05	0	0
400	1600	400	1600	1	2.09	0	0
6400	3200	6400	3200	1	67.75	0	0
3200	6400	3200	6400	1	66.16	0	0
1600	400	60	30	6	.49	1.77e-04	1.42e-05
400	1600	30	60	6	.51	7.40e-05	1.51e-05
6400	3200	43	30	6	2.79	2.83e-04	1.67e-05
3200	6400	30	43	6	2.80	1.95e-04	1.08e-05

A.3 Data for S-U Configuration

Table 3: S-U Configuration

N_V	N_T	N_S	N_F	N_{max}	t_{cpu} (sec)	ε_U	ε_ψ
49	49	49	49	1	.02	0	0
100	100	100	100	1	.05	0	0
196	196	196	196	1	.18	0	0
400	400	400	400	1	.71	0	0
784	784	784	784	1	2.69	0	0
1600	1600	1600	1600	1	11.15	0	0
3136	3136	3136	3136	1	43.35	0	0
6400	6400	6400	6400	1	184.04	0	0
49	49	30	30	6	.03	3.35e-07	6.90e-07
100	100	30	30	6	.07	7.71e-06	7.96e-07
196	196	30	30	6	.10	3.90e-04	1.85e-05
400	400	30	30	6	.25	2.99e-04	1.48e-05
784	784	30	30	6	.37	5.22e-04	1.71e-05
1600	1600	30	30	6	.93	4.97e-04	1.78e-05
3136	3136	30	30	6	1.78	5.67e-04	1.86e-05
6400	6400	30	30	6	3.91	5.63e-04	1.87e-05
1600	400	1600	400	1	2.80	0	0
400	1600	400	1600	1	2.83	0	0
6400	3136	6400	3136	1	90.21	0	0
3136	6400	3136	6400	1	88.82	0	0
1600	400	60	30	6	.60	2.99e-04	1.47e-05
400	1600	30	60	6	.47	5.05e-04	1.82e-05
6400	3136	43	30	6	2.49	5.71e-04	1.85e-05
3136	6400	30	43	6	2.58	5.61e-04	1.87e-05

A.4 Data for U-S Configuration

Table 4: U-S Configuration

N_V	N_T	N_S	N_F	N_{max}	t_{cpu} (sec)	ε_U	ε_ψ
49	49	49	49	1	.02	0	0
100	100	100	100	1	.05	0	0
196	196	196	196	1	.18	0	0
400	400	400	400	1	.71	0	0
784	784	784	784	1	2.69	0	0
1600	1600	1600	1600	1	11.17	0	0
3136	3136	3136	3136	1	43.51	0	0
6400	6400	6400	6400	1	187.07	0	0
49	49	30	30	6	.03	1.46e-07	0
100	100	30	30	6	.06	5.51e-06	6.01e-07
196	196	30	30	6	.09	2.37e-04	4.70e-06
400	400	30	30	6	.24	9.05e-05	2.86e-06
784	784	30	30	6	.35	1.70e-04	2.17e-06
1600	1600	30	30	6	.87	1.54e-04	2.34e-06
3136	3136	30	30	6	1.75	1.59e-04	3.28e-06
6400	6400	30	30	6	3.77	1.77e-04	2.92e-06
1600	400	1600	400	1	2.80	0	0
400	1600	400	1600	1	2.83	0	0
6400	3136	6400	3136	1	90.16	0	0
3136	6400	3136	6400	1	88.72	0	0
1600	400	60	30	6	.44	1.52e-04	2.19e-06
400	1600	30	60	6	.59	1.06e-04	2.70e-06
6400	3136	43	30	6	2.46	1.76e-04	2.84e-06
3136	6400	30	43	6	2.47	1.59e-04	3.30e-06

A.5 Detail Timing Data for U-U, S-S, S-U, and U-S Configurations.

The data in this appendix contains a set of timing data for different configurations of source and target points. These data were generated on a Sun Sparc 20 workstation which has a slower CPU than the one used in the previous appendices. All of the data used from this series of runs for Figures 13, 16, 19, and 20 were non-dimensionalized. The following definitions apply:

Ns = maximum number of sources per box.
Nf = maximum number of field points per box.
t1 = time in sec to generate source mesh.
t2 = time in sec to generate field mesh.
t3 = time in sec to generate multipole coefficients.
t4 = time in sec to generate local coefficients.
t5 = time in sec to calculate field L1.
t6 = time in sec to calculate field L3.
t7 = time in sec to calculate field L24.
t8 = total time in sec.

Figures 13, 16, 19, and 20 contain non-dimensionalized contour plots of t8 versus Ns and Nf.

Uniform-Uniform Distribution (3)
uniform = 2025 (source points), uniform = 4225 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	6.000E-02	1.000E-01	7.700E-01	7.470E+00	4.600E+00	8.000E-02	2.600E-01	1.334E+01
5.000E+00	2.500E+01	6.000E-02	7.000E-02	7.600E-01	6.600E-01	1.440E+00	3.860E+00	2.400E-01	7.090E+00
5.000E+00	5.000E+01	6.000E-02	7.000E-02	7.600E-01	6.600E-01	1.440E+00	3.860E+00	2.400E-01	7.090E+00
5.000E+00	7.500E+01	6.000E-02	5.000E-02	7.800E-01	1.000E-01	1.290E+00	6.470E+00	2.300E-01	8.980E+00
5.000E+00	1.000E+02	6.000E-02	5.000E-02	7.600E-01	9.000E-02	1.290E+00	6.480E+00	2.300E-01	8.960E+00
2.500E+01	5.000E+00	3.000E-02	1.000E-01	2.000E-01	3.490E+00	1.600E+00	5.000E-02	2.400E-01	5.710E+00
2.500E+01	2.500E+01	3.000E-02	7.000E-02	1.900E-01	7.300E-01	1.360E+00	1.000E-02	2.200E-01	2.620E+00
2.500E+01	5.000E+01	3.000E-02	6.000E-02	1.900E-01	7.300E-01	1.360E+00	1.000E-02	2.300E-01	2.630E+00
2.500E+01	7.500E+01	3.000E-02	5.000E-02	1.900E-01	1.000E-01	2.080E+00	2.170E+00	2.200E-01	4.840E+00
2.500E+01	1.000E+02	4.000E-02	5.000E-02	1.900E-01	9.000E-02	2.080E+00	2.190E+00	2.200E-01	4.860E+00
5.000E+01	5.000E+00	3.000E-02	1.000E-01	1.100E-01	3.160E+00	1.360E+00	5.000E-02	2.300E-01	5.040E+00
5.000E+01	2.500E+01	3.000E-02	7.000E-02	1.100E-01	1.130E+00	1.790E+00	1.000E-02	2.200E-01	3.350E+00
5.000E+01	5.000E+01	3.000E-02	7.000E-02	1.100E-01	1.130E+00	1.790E+00	1.000E-02	2.300E-01	3.360E+00
5.000E+01	7.500E+01	3.000E-02	5.000E-02	1.100E-01	1.000E-01	3.690E+00	0.000E+00	2.200E-01	4.200E+00
5.000E+01	1.000E+01	3.000E-02	5.000E-02	1.100E-01	1.000E-01	3.710E+00	0.000E+00	2.200E-01	4.210E+00
7.500E+01	5.000E+00	3.000E-02	1.000E-01	1.100E-01	3.160E+00	1.360E+00	5.000E-02	2.300E-01	5.040E+00
7.500E+01	2.500E+01	3.000E-02	7.000E-02	1.100E-01	1.120E+00	1.790E+00	1.000E-02	2.300E-01	3.360E+00
7.500E+01	5.000E+01	3.000E-02	7.000E-02	1.100E-01	1.130E+00	1.790E+00	1.000E-02	2.200E-01	3.350E+00
7.500E+01	7.500E+01	3.000E-02	5.000E-02	1.100E-01	1.000E-01	3.690E+00	0.000E+00	2.200E-01	4.190E+00
7.500E+01	1.000E+02	3.000E-02	5.000E-02	1.100E-01	9.000E-02	3.710E+00	0.000E+00	2.200E-01	4.210E+00
1.000E+02	5.000E+00	3.000E-02	1.000E-01	1.100E-01	3.170E+00	1.360E+00	5.000E-02	2.300E-01	5.040E+00
1.000E+02	2.500E+01	3.000E-02	6.000E-02	1.100E-01	1.130E+00	1.790E+00	1.000E-02	2.200E-01	3.360E+00
1.000E+02	5.000E+01	3.000E-02	7.000E-02	1.100E-01	1.130E+00	1.790E+00	1.000E-02	2.200E-01	3.350E+00
1.000E+02	7.500E+01	3.000E-02	5.000E-02	1.100E-01	1.000E-01	3.690E+00	0.000E+00	2.200E-01	4.200E+00
1.000E+02	1.000E+02	3.000E-02	5.000E-02	1.100E-01	9.000E-02	3.710E+00	0.000E+00	2.200E-01	4.210E+00

Uniform-Uniform Distribution (4)
uniform = 4225 (source points), uniform = 2025 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	1.000E-01	6.000E-02	8.900E-01	6.250E+00	4.540E+00	1.250E+00	1.400E-01	1.324E+01
5.000E+00	2.500E+01	1.000E-01	3.000E-02	8.700E-01	5.700E-01	1.540E+00	2.990E+00	1.200E-01	6.220E+00
5.000E+00	5.000E+01	1.000E-01	2.000E-02	8.700E-01	9.000E-02	1.340E+00	3.550E+00	1.100E-01	6.080E+00
5.000E+00	7.500E+01	1.000E-01	3.000E-02	8.600E-01	9.000E-02	1.340E+00	3.540E+00	1.100E-01	6.080E+00
5.000E+00	1.000E+02	1.000E-01	2.000E-02	8.700E-01	9.000E-02	1.340E+00	3.540E+00	1.100E-01	6.080E+00
2.500E+01	5.000E+00	7.000E-02	6.000E-02	2.900E-01	4.120E+00	1.450E+00	4.000E-02	1.200E-01	6.150E+00
2.500E+01	2.500E+01	7.000E-02	3.000E-02	2.900E-01	5.800E-01	1.350E+00	2.000E-01	1.100E-01	2.630E+00
2.500E+01	5.000E+01	7.000E-02	2.000E-02	2.900E-01	9.000E-02	1.790E+00	1.310E+00	1.100E-01	3.690E+00
2.500E+01	7.500E+01	7.000E-02	3.000E-02	3.000E-01	9.000E-02	1.790E+00	1.310E+00	1.100E-01	3.690E+00
2.500E+01	1.000E+02	7.000E-02	3.000E-02	2.900E-01	9.000E-02	1.780E+00	1.310E+00	1.100E-01	3.680E+00
5.000E+01	5.000E+00	7.000E-02	6.000E-02	2.900E-01	3.860E+00	1.450E+00	4.000E-02	1.100E-01	5.890E+00
5.000E+01	2.500E+01	7.000E-02	3.000E-02	2.900E-01	5.800E-01	1.350E+00	2.000E-01	1.100E-01	2.630E+00
5.000E+01	5.000E+01	7.000E-02	2.000E-02	2.900E-01	9.000E-02	1.790E+00	1.310E+00	1.100E-01	3.690E+00
5.000E+01	7.500E+01	7.000E-02	2.000E-02	3.000E-01	9.000E-02	1.790E+00	1.310E+00	1.100E-01	3.690E+00
5.000E+01	1.000E+02	7.000E-02	2.000E-02	2.900E-01	9.000E-02	1.790E+00	1.310E+00	1.100E-01	3.690E+00
7.500E+01	5.000E+00	5.000E-02	6.000E-02	1.900E-01	5.220E+00	1.260E+00	4.000E-02	1.200E-01	6.940E+00
7.500E+01	2.500E+01	5.000E-02	3.000E-02	1.900E-01	1.740E+00	2.060E+00	1.000E-02	1.100E-01	4.190E+00
7.500E+01	5.000E+01	5.000E-02	2.000E-02	1.900E-01	9.000E-02	3.680E+00	2.000E-02	1.100E-01	4.160E+00
7.500E+01	7.500E+01	5.000E-02	2.000E-02	1.900E-01	9.000E-02	3.690E+00	2.000E-02	1.100E-01	4.180E+00
7.500E+01	1.000E+02	5.000E-02	2.000E-02	1.900E-01	9.000E-02	3.680E+00	2.000E-02	1.100E-01	4.150E+00
1.000E+02	5.000E+00	5.000E-02	6.000E-02	1.900E-01	5.210E+00	1.270E+00	5.000E-02	1.300E-01	6.950E+00
1.000E+02	2.500E+01	5.000E-02	3.000E-02	1.800E-01	1.740E+00	2.060E+00	1.000E-02	1.100E-01	4.200E+00
1.000E+02	5.000E+01	5.000E-02	2.000E-02	1.900E-01	9.000E-02	3.700E+00	0.000E+00	1.100E-01	4.160E+00
1.000E+02	7.500E+01	5.000E-02	2.000E-02	1.900E-01	9.000E-02	3.700E+00	0.000E+00	1.100E-01	4.160E+00
1.000E+02	1.000E+02	5.000E-02	2.000E-02	1.900E-01	9.000E-02	3.700E+00	0.000E+00	1.100E-01	4.160E+00

Uniform-Uniform Distribution (5)
uniform = 2025 (source points), uniform = 8281(field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	6.000E–02	2.900E–01	7.900E–01	2.748E+01	1.443E+01	2.700E–01	5.500E–01	4.387E+01
5.000E+00	2.500E+01	6.000E–02	1.700E–01	7.800E–01	5.760E+00	3.990E+00	6.300E–01	4.700E–01	1.187E+01
5.000E+00	5.000E+01	6.000E–02	1.300E–01	7.600E–01	6.600E–01	1.940E+00	6.550E+00	4.600E–01	1.057E+01
5.000E+00	7.500E+01	6.000E–02	1.300E–01	7.600E–01	6.600E–01	1.940E+00	6.550E+00	4.500E–01	1.055E+01
5.000E+00	1.000E+02	6.000E–02	1.300E–01	7.700E–01	6.700E–01	1.930E+00	6.550E+00	4.600E–01	1.057E+01
2.500E+01	5.000E+00	4.000E–02	2.800E–01	1.900E–01	1.071E+01	4.160E+00	1.700E–01	4.800E–01	1.604E+01
2.500E+01	2.500E+01	4.000E–02	1.700E–01	1.900E–01	2.940E+00	1.990E+00	4.000E–02	4.500E–01	5.830E+00
2.500E+01	5.000E+01	3.000E–02	1.300E–01	1.900E–01	7.300E–01	2.460E+00	1.000E–02	4.500E–01	4.000E+00
2.500E+01	7.500E+01	4.000E–02	1.300E–01	2.000E–01	7.400E–01	2.460E+00	1.000E–02	4.400E–01	4.000E+00
2.500E+01	1.000E+02	4.000E–02	1.300E–01	2.000E–01	7.400E–01	2.460E+00	1.000E–02	5.000E–01	4.060E+00
5.000E+01	5.000E+00	3.000E–02	2.900E–01	1.100E–01	8.090E+00	2.600E+00	1.700E–01	4.700E–01	1.176E+01
5.000E+01	2.500E+01	3.000E–02	1.700E–01	1.100E–01	2.810E+00	2.450E+00	4.000E–02	4.500E–01	6.050E+00
5.000E+01	5.000E+01	3.000E–02	1.300E–01	1.100E–01	1.130E+00	3.450E+00	1.000E–02	4.400E–01	5.290E+00
5.000E+01	7.500E+01	3.000E–02	1.300E–01	1.100E–01	1.130E+00	3.450E+00	1.000E–02	4.400E–01	5.290E+00
5.000E+01	1.000E+02	3.000E–02	1.300E–01	1.100E–01	1.130E+00	3.450E+00	1.000E–02	4.900E–01	5.340E+00
7.500E+01	5.000E+00	3.000E–02	2.800E–01	1.100E–01	8.110E+00	2.600E+00	1.700E–01	4.700E–01	1.177E+01
7.500E+01	2.500E+01	3.000E–02	1.700E–01	1.100E–01	2.800E+00	2.450E+00	4.000E–02	4.500E–01	6.050E+00
7.500E+01	5.000E+01	3.000E–02	1.300E–01	1.100E–01	1.130E+00	3.450E+00	1.000E–02	4.500E–01	5.300E+00
7.500E+01	7.500E+01	3.000E–02	1.300E–01	1.100E–01	1.130E+00	3.450E+00	1.000E–02	4.400E–01	5.290E+00
7.500E+01	1.000E+02	3.000E–02	1.300E–01	1.100E–01	1.130E+00	3.450E+00	1.000E–02	4.900E–01	5.350E+00
1.000E+02	5.000E+00	3.000E–02	2.800E–01	1.100E–01	8.060E+00	2.590E+00	1.700E–01	4.700E–01	1.172E+01
1.000E+02	2.500E+01	3.000E–02	1.700E–01	1.100E–01	2.810E+00	2.450E+00	4.000E–02	4.500E–01	6.060E+00
1.000E+02	5.000E+01	3.000E–02	1.300E–01	1.100E–01	1.120E+00	3.450E+00	1.000E–02	4.400E–01	5.290E+00
1.000E+02	7.500E+01	3.000E–02	1.300E–01	1.100E–01	1.130E+00	3.450E+00	1.000E–02	4.400E–01	5.300E+00
1.000E+02	1.000E+02	3.000E–02	1.300E–01	1.100E–01	1.130E+00	3.460E+00	1.000E–02	4.500E–01	5.300E+00

Uniform-Uniform Distribution (6)
uniform = 8281 (source points), uniform = 2025 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	2.800E–01	6.000E–02	5.510E+00	6.170E+00	1.522E+01	1.969E+01	2.000E–01	4.713E+01
5.000E+00	2.500E+01	2.800E–01	3.000E–02	5.470E+00	5.700E–01	4.160E+00	9.230E+00	1.300E–01	1.988E+01
5.000E+00	5.000E+01	2.800E–01	2.000E–02	5.480E+00	9.000E–02	2.540E+00	7.480E+00	1.300E–01	1.603E+01
5.000E+00	7.500E+01	2.900E–01	2.000E–02	5.460E+00	9.000E–02	2.540E+00	7.480E+00	1.100E–01	1.599E+01
5.000E+00	1.000E+02	2.800E–01	2.000E–02	5.520E+00	9.000E–02	2.540E+00	7.480E+00	1.100E–01	1.605E+01
2.500E+01	5.000E+00	1.700E–01	6.000E–02	9.700E–01	6.190E+00	3.950E+00	1.200E–01	1.300E–01	1.160E+01
2.500E+01	2.500E+01	1.700E–01	3.000E–02	9.400E–01	5.700E–01	1.950E+00	2.460E+00	1.200E–01	6.250E+00
2.500E+01	5.000E+01	1.700E–01	2.000E–02	9.600E–01	9.000E–02	2.420E+00	3.160E+00	1.200E–01	6.950E+00
2.500E+01	7.500E+01	1.700E–01	2.000E–02	9.500E–01	9.000E–02	2.430E+00	3.150E+00	1.100E–01	6.920E+00
2.500E+01	1.000E+02	1.700E–01	2.000E–02	9.400E–01	9.000E–02	2.430E+00	3.160E+00	1.100E–01	6.920E+00
5.000E+01	5.000E+00	1.300E–01	6.000E–02	4.500E–01	6.050E+00	1.920E+00	5.000E–02	1.200E–01	8.780E+00
5.000E+01	2.500E+01	1.300E–01	3.000E–02	4.400E–01	5.800E–01	2.440E+00	2.000E–01	1.100E–01	3.930E+00
5.000E+01	5.000E+01	1.300E–01	2.000E–02	4.500E–01	9.000E–02	3.440E+00	1.310E+00	1.200E–01	5.570E+00
5.000E+01	7.500E+01	1.300E–01	2.000E–02	4.500E–01	9.000E–02	3.440E+00	1.310E+00	1.100E–01	5.560E+00
5.000E+01	1.000E+02	1.300E–01	2.000E–02	4.500E–01	9.000E–02	3.440E+00	1.310E+00	1.100E–01	5.560E+00
7.500E+01	5.000E+00	1.300E–01	6.000E–02	4.500E–01	6.060E+00	1.930E+00	5.000E–02	1.200E–01	8.790E+00
7.500E+01	2.500E+01	1.300E–01	3.000E–02	4.500E–01	5.800E–01	2.440E+00	2.000E–01	1.100E–01	3.930E+00
7.500E+01	5.000E+01	1.300E–01	2.000E–02	4.500E–01	9.000E–02	3.440E+00	1.310E+00	1.100E–01	5.560E+00
7.500E+01	7.500E+01	1.300E–01	2.000E–02	4.600E–01	9.000E–02	3.440E+00	1.310E+00	1.100E–01	5.570E+00
7.500E+01	1.000E+02	1.300E–01	2.000E–02	4.400E–01	9.000E–02	3.440E+00	1.310E+00	1.100E–01	5.550E+00
1.000E+02	5.000E+00	1.300E–01	6.000E–02	4.500E–01	6.060E+00	1.930E+00	5.000E–02	1.200E–01	8.780E+00
1.000E+02	2.500E+01	1.300E–01	3.000E–02	4.400E–01	5.800E–01	2.440E+00	2.000E–01	1.100E–01	3.930E+00
1.000E+02	5.000E+01	1.300E–01	2.000E–02	4.500E–01	9.000E–02	3.440E+00	1.310E+00	1.100E–01	5.560E+00
1.000E+02	7.500E+01	1.300E–01	2.000E–02	4.500E–01	9.000E–02	3.440E+00	1.310E+00	1.100E–01	5.560E+00
1.000E+02	1.000E+02	1.300E–01	2.000E–02	4.500E–01	9.000E–02	3.440E+00	1.310E+00	1.100E–01	5.560E+00

Uniform-Uniform Distribution (7)
uniform = 2025 (source points), uniform = 18225 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	6.000E–02	5.400E–01	7.900E–01	4.395E+01	2.422E+01	4.400E–01	1.160E+00	7.117E+01
5.000E+00	2.500E+01	6.000E–02	3.600E–01	8.000E–01	6.360E+00	4.930E+00	7.000E–02	1.030E+00	1.360E+01
5.000E+00	5.000E+01	6.000E–02	3.600E–01	8.000E–01	6.360E+00	4.930E+00	7.000E–02	1.010E+00	1.358E+01
5.000E+00	7.500E+01	7.000E–02	2.900E–01	8.100E–01	1.640E+00	3.490E+00	1.054E+01	1.010E+00	1.784E+01
5.000E+00	1.000E+02	6.000E–02	2.800E–01	8.100E–01	6.700E–01	3.140E+00	1.302E+01	9.900E–01	1.896E+01
2.500E+01	5.000E+00	4.000E–02	5.400E–01	2.000E–01	1.514E+01	7.300E+00	3.400E–01	1.170E+00	2.473E+01
2.500E+01	2.500E+01	4.000E–02	3.600E–01	2.000E–01	3.210E+00	3.230E+00	5.000E–02	9.800E–01	8.060E+00
2.500E+01	5.000E+01	4.000E–02	3.600E–01	2.000E–01	3.210E+00	3.220E+00	5.000E–02	9.900E–01	8.070E+00
2.500E+01	7.500E+01	4.000E–02	2.900E–01	2.000E–01	1.110E+00	4.740E+00	2.000E–02	9.700E–01	7.370E+00
2.500E+01	1.000E+02	4.000E–02	2.800E–01	2.000E–01	7.400E–01	5.100E+00	1.000E–02	9.700E–01	7.330E+00
5.000E+01	5.000E+00	3.000E–02	5.400E–01	1.100E–01	1.025E+01	5.010E+00	2.900E–01	1.040E+00	1.726E+01
5.000E+01	2.500E+01	3.000E–02	3.600E–01	1.100E–01	3.040E+00	4.630E+00	5.000E–02	1.000E+00	9.210E+00
5.000E+01	5.000E+01	3.000E–02	3.600E–01	1.100E–01	3.040E+00	4.630E+00	5.000E–02	9.800E–01	9.200E+00
5.000E+01	7.500E+01	3.000E–02	2.900E–01	1.100E–01	1.370E+00	7.000E+00	2.000E–02	1.110E+00	9.930E+00
5.000E+01	1.000E+02	3.000E–02	2.800E–01	1.100E–01	1.130E+00	7.490E+00	1.000E–02	9.800E–01	1.003E+01
7.500E+01	5.000E+00	3.000E–02	5.400E–01	1.100E–01	1.023E+01	4.990E+00	2.900E–01	1.040E+00	1.723E+01
7.500E+01	2.500E+01	3.000E–02	3.600E–01	1.100E–01	3.050E+00	4.630E+00	5.000E–02	9.800E–01	9.200E+00
7.500E+01	5.000E+01	3.000E–02	3.600E–01	1.100E–01	3.050E+00	4.630E+00	5.000E–02	9.800E–01	9.210E+00
7.500E+01	7.500E+01	3.000E–02	2.900E–01	1.100E–01	1.370E+00	7.000E+00	2.000E–02	9.700E–01	9.790E+00
7.500E+01	1.000E+02	3.000E–02	2.800E–01	1.100E–01	1.130E+00	7.490E+00	1.000E–02	9.700E–01	1.001E+01
1.000E+02	5.000E+00	3.000E–02	5.400E–01	1.100E–01	1.022E+01	4.990E+00	2.900E–01	1.040E+00	1.721E+01
1.000E+02	2.500E+01	3.000E–02	3.600E–01	1.100E–01	3.050E+00	4.630E+00	5.000E–02	1.000E+00	9.220E+00
1.000E+02	5.000E+01	3.000E–02	3.600E–01	1.100E–01	3.050E+00	4.630E+00	5.000E–02	1.000E+00	9.220E+00
1.000E+02	7.500E+01	3.000E–02	2.900E–01	1.100E–01	1.370E+00	6.990E+00	2.000E–02	1.090E+00	9.910E+00
1.000E+02	1.000E+02	3.000E–02	2.800E–01	1.100E–01	1.130E+00	7.490E+00	1.000E–02	9.700E–01	1.001E+01

Uniform-Uniform Distribution (8)
uniform = 18225 (source points), uniform = 2025 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	5.400E–01	6.000E–02	7.790E+00	6.180E+00	2.480E+01	3.611E+01	2.000E–01	7.568E+01
5.000E+00	2.500E+01	5.500E–01	3.000E–02	7.730E+00	5.800E–01	7.020E+00	1.345E+01	1.300E–01	2.949E+01
5.000E+00	5.000E+01	5.500E–01	2.000E–02	7.710E+00	9.000E–02	4.850E+00	9.360E+00	1.100E–01	2.270E+01
5.000E+00	7.500E+01	5.400E–01	2.000E–02	7.690E+00	9.000E–02	4.850E+00	9.360E+00	1.100E–01	2.267E+01
5.000E+00	1.000E+02	5.500E–01	2.000E–02	7.720E+00	9.000E–02	4.860E+00	9.360E+00	1.100E–01	2.271E+01
2.500E+01	5.000E+00	3.600E–01	6.000E–02	1.440E+00	6.240E+00	4.870E+00	1.400E–01	1.400E–01	1.325E+01
2.500E+01	2.500E+01	3.600E–01	3.000E–02	1.440E+00	5.700E–01	3.160E+00	2.720E+00	1.200E–01	8.400E+00
2.500E+01	5.000E+01	3.600E–01	2.000E–02	1.450E+00	9.000E–02	4.600E+00	3.410E+00	1.300E–01	1.006E+01
2.500E+01	7.500E+01	3.600E–01	2.000E–02	1.440E+00	9.000E–02	4.600E+00	3.410E+00	1.100E–01	1.004E+01
2.500E+01	1.000E+02	3.600E–01	2.000E–02	1.450E+00	9.000E–02	4.600E+00	3.410E+00	1.100E–01	1.004E+01
5.000E+01	5.000E+00	3.600E–01	6.000E–02	1.440E+00	6.260E+00	4.870E+00	1.400E–01	1.400E–01	1.327E+01
5.000E+01	2.500E+01	3.600E–01	4.000E–02	1.470E+00	5.800E–01	3.150E+00	2.720E+00	1.200E–01	8.440E+00
5.000E+01	5.000E+01	3.600E–01	2.000E–02	1.440E+00	9.000E–02	4.590E+00	3.410E+00	1.100E–01	1.004E+01
5.000E+01	7.500E+01	3.600E–01	2.000E–02	1.460E+00	9.000E–02	4.600E+00	3.410E+00	1.100E–01	1.006E+01
5.000E+01	1.000E+02	3.600E–01	2.000E–02	1.460E+00	9.000E–02	4.600E+00	3.410E+00	1.100E–01	1.006E+01
7.500E+01	5.000E+00	3.000E–01	6.000E–02	8.300E–01	9.850E+00	3.370E+00	6.000E–02	1.200E–01	1.459E+01
7.500E+01	2.500E+01	3.000E–01	3.000E–02	8.400E–01	5.900E–01	4.690E+00	6.400E–01	1.100E–01	7.210E+00
7.500E+01	5.000E+01	3.000E–01	2.000E–02	8.300E–01	9.000E–02	6.980E+00	1.640E+00	1.100E–01	9.970E+00
7.500E+01	7.500E+01	3.000E–01	2.000E–02	8.300E–01	9.000E–02	6.980E+00	1.640E+00	1.100E–01	9.970E+00
7.500E+01	1.000E+02	3.000E–01	2.000E–02	8.300E–01	9.000E–02	6.980E+00	1.640E+00	1.100E–01	9.960E+00
1.000E+02	5.000E+00	2.800E–01	6.000E–02	8.400E–01	1.072E+01	3.050E+00	5.000E–02	1.200E–01	1.512E+01
1.000E+02	2.500E+01	2.800E–01	4.000E–02	8.300E–01	5.800E–01	5.060E+00	2.000E–01	1.100E–01	7.110E+00
1.000E+02	5.000E+01	2.800E–01	2.000E–02	8.400E–01	9.000E–02	7.480E+00	1.320E+00	1.100E–01	1.015E+01
1.000E+02	7.500E+01	2.800E–01	2.000E–02	8.300E–01	9.000E–02	7.480E+00	1.320E+00	1.100E–01	1.014E+01
1.000E+02	1.000E+02	2.800E–01	2.000E–02	8.500E–01	1.000E–01	7.480E+00	1.320E+00	1.100E–01	1.016E+01

Spiral-Spiral Distribution (9)
spiral = 500 (source points), spiral = 500 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	2.000E-02	2.000E-02	7.000E-02	2.100E-01	1.200E-01	7.000E-02	3.000E-02	5.400E-01
5.000E+00	2.500E+01	2.000E-02	1.000E-02	6.000E-02	3.000E-02	8.000E-02	1.800E-01	3.000E-02	4.100E-01
5.000E+00	5.000E+01	2.000E-02	1.000E-02	7.000E-02	1.000E-02	1.200E-01	2.700E-01	3.000E-02	5.200E-01
5.000E+00	7.500E+01	2.000E-02	1.000E-02	6.000E-02	1.000E-02	1.300E-01	2.900E-01	3.000E-02	5.500E-01
5.000E+00	1.000E+02	2.000E-02	1.000E-02	7.000E-02	1.000E-02	1.500E-01	3.800E-01	3.000E-02	6.600E-01
2.500E+01	5.000E+00	1.000E-02	2.000E-02	3.000E-02	2.000E-01	9.000E-02	1.000E-02	3.000E-02	3.800E-01
2.500E+01	2.500E+01	1.000E-02	1.000E-02	3.000E-02	5.000E-02	1.300E-01	4.000E-02	3.000E-02	2.900E-01
2.500E+01	5.000E+01	1.000E-02	1.000E-02	3.000E-02	2.000E-02	1.900E-01	1.100E-01	3.000E-02	4.000E-01
2.500E+01	7.500E+01	1.000E-02	1.000E-02	3.000E-02	2.000E-02	2.100E-01	1.400E-01	3.000E-02	4.400E-01
2.500E+01	1.000E+02	1.000E-02	1.000E-02	3.000E-02	1.000E-02	2.500E-01	1.700E-01	3.000E-02	5.100E-01
5.000E+01	5.000E+00	1.000E-02	2.000E-02	2.000E-02	2.400E-01	1.200E-01	1.000E-02	3.000E-02	4.500E-01
5.000E+01	2.500E+01	1.000E-02	1.000E-02	2.000E-02	9.000E-02	1.900E-01	2.000E-02	3.000E-02	3.700E-01
5.000E+01	5.000E+01	1.000E-02	1.000E-02	2.000E-02	3.000E-02	3.200E-01	4.000E-02	3.000E-02	4.600E-01
5.000E+01	7.500E+01	1.000E-02	1.000E-02	2.000E-02	3.000E-02	3.500E-01	6.000E-02	3.000E-02	5.000E-01
5.000E+01	1.000E+02	1.000E-02	1.000E-02	2.000E-02	2.000E-02	4.200E-01	6.000E-02	3.000E-02	5.700E-01
7.500E+01	5.000E+00	1.000E-02	2.000E-02	2.000E-02	2.500E-01	1.400E-01	1.000E-02	3.000E-02	4.700E-01
7.500E+01	2.500E+01	1.000E-02	1.000E-02	2.000E-02	1.100E-01	2.100E-01	1.000E-02	3.000E-02	4.000E-01
7.500E+01	5.000E+01	1.000E-02	1.000E-02	2.000E-02	5.000E-02	3.500E-01	3.000E-02	3.000E-02	4.900E-01
7.500E+01	7.500E+01	1.000E-02	1.000E-02	2.000E-02	4.000E-02	3.700E-01	5.000E-02	3.000E-02	5.300E-01
7.500E+01	1.000E+02	1.000E-02	1.000E-02	2.000E-02	3.000E-02	4.400E-01	6.000E-02	3.000E-02	5.900E-01
1.000E+02	5.000E+00	1.000E-02	2.000E-02	2.000E-02	3.100E-01	1.600E-01	1.000E-02	3.000E-02	5.500E-01
1.000E+02	2.500E+01	1.000E-02	1.000E-02	2.000E-02	1.300E-01	2.500E-01	1.000E-02	3.000E-02	4.600E-01
1.000E+02	5.000E+01	1.000E-02	1.000E-02	2.000E-02	5.000E-02	4.200E-01	2.000E-02	3.000E-02	5.500E-01
1.000E+02	7.500E+01	1.000E-02	1.000E-02	2.000E-02	5.000E-02	4.400E-01	4.000E-02	3.000E-02	5.900E-01
1.000E+02	1.000E+02	1.000E-02	1.000E-02	2.000E-02	4.000E-02	5.000E-01	5.000E-02	3.000E-02	6.600E-01

Spiral-Spiral Distribution (10)
spiral = 1000 (source points), spiral = 1000 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	4.000E-02	3.000E-02	1.400E-01	6.300E-01	4.000E-01	2.300E-01	6.000E-02	1.530E+00
5.000E+00	2.500E+01	3.000E-02	2.000E-02	1.400E-01	9.000E-02	2.000E-01	3.700E-01	6.000E-02	9.100E-01
5.000E+00	5.000E+01	4.000E-02	2.000E-02	1.400E-01	4.000E-02	2.300E-01	4.900E-01	6.000E-02	1.010E+00
5.000E+00	7.500E+01	4.000E-02	2.000E-02	1.400E-01	2.000E-02	2.800E-01	5.300E-01	6.000E-02	1.090E+00
5.000E+00	1.000E+02	3.000E-02	1.000E-02	1.400E-01	1.000E-02	3.700E-01	7.100E-01	6.000E-02	1.340E+00
2.500E+01	5.000E+00	2.000E-02	3.000E-02	6.000E-02	4.400E-01	2.100E-01	2.000E-02	6.000E-02	8.400E-01
2.500E+01	2.500E+01	2.000E-02	2.000E-02	6.000E-02	9.000E-02	2.600E-01	6.000E-02	6.000E-02	5.600E-01
2.500E+01	5.000E+01	2.000E-02	2.000E-02	6.000E-02	4.000E-02	3.400E-01	1.700E-01	6.000E-02	7.000E-01
2.500E+01	7.500E+01	2.000E-02	2.000E-02	6.000E-02	3.000E-02	4.000E-01	2.300E-01	6.000E-02	8.100E-01
2.500E+01	1.000E+02	2.000E-02	1.000E-02	6.000E-02	2.000E-02	5.300E-01	3.600E-01	6.000E-02	1.060E+00
5.000E+01	5.000E+00	2.000E-02	3.000E-02	5.000E-02	4.600E-01	2.300E-01	1.000E-02	6.000E-02	8.700E-01
5.000E+01	2.500E+01	2.000E-02	2.000E-02	5.000E-02	1.500E-01	3.400E-01	2.000E-02	6.000E-02	6.600E-01
5.000E+01	5.000E+01	2.000E-02	2.000E-02	5.000E-02	7.000E-02	4.800E-01	7.000E-02	6.000E-02	7.600E-01
5.000E+01	7.500E+01	2.000E-02	2.000E-02	5.000E-02	5.000E-02	5.600E-01	1.300E-01	6.000E-02	8.800E-01
5.000E+01	1.000E+02	2.000E-02	1.000E-02	5.000E-02	3.000E-02	7.400E-01	2.200E-01	6.000E-02	1.130E+00
7.500E+01	5.000E+00	2.000E-02	3.000E-02	4.000E-02	4.800E-01	2.900E-01	1.000E-02	6.000E-02	9.400E-01
7.500E+01	2.500E+01	2.000E-02	2.000E-02	4.000E-02	1.900E-01	4.000E-01	1.000E-02	6.000E-02	7.500E-01
7.500E+01	5.000E+01	2.000E-02	2.000E-02	4.000E-02	1.100E-01	5.700E-01	5.000E-02	6.000E-02	8.600E-01
7.500E+01	7.500E+01	2.000E-02	2.000E-02	4.000E-02	7.000E-02	7.000E-01	8.000E-02	6.000E-02	9.900E-01
7.500E+01	1.000E+02	2.000E-02	1.000E-02	4.000E-02	4.000E-02	9.400E-01	1.400E-01	6.000E-02	1.250E+00
1.000E+02	5.000E+00	2.000E-02	3.000E-02	4.000E-02	5.900E-01	3.700E-01	1.000E-02	6.000E-02	1.130E+00
1.000E+02	2.500E+01	2.000E-02	2.000E-02	4.000E-02	2.800E-01	5.300E-01	1.000E-02	6.000E-02	9.500E-01
1.000E+02	5.000E+01	2.000E-02	2.000E-02	4.000E-02	1.700E-01	7.400E-01	3.000E-02	6.000E-02	1.080E+00
1.000E+02	7.500E+01	2.000E-02	2.000E-02	4.000E-02	1.100E-01	9.400E-01	4.000E-02	6.000E-02	1.220E+00
1.000E+02	1.000E+02	2.000E-02	1.000E-02	4.000E-02	6.000E-02	1.270E+00	7.000E-02	6.000E-02	1.530E+00

Spiral-Spiral Distribution (11)
spiral = 2000 (source points), spiral = 2000 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	8.000E-02	8.000E-02	3.100E-01	1.820E+00	1.250E+00	6.100E-01	1.200E-01	4.250E+00
5.000E+00	2.500E+01	8.000E-02	5.000E-02	3.100E-01	2.400E-01	5.400E-01	8.700E-01	1.300E-01	2.210E+00
5.000E+00	5.000E+01	8.000E-02	4.000E-02	3.100E-01	9.000E-02	5.300E-01	9.600E-01	1.100E-01	2.130E+00
5.000E+00	7.500E+01	8.000E-02	3.000E-02	3.100E-01	6.000E-02	6.200E-01	1.070E+00	1.100E-01	2.280E+00
5.000E+00	1.000E+02	8.000E-02	3.000E-02	3.100E-01	3.000E-02	7.200E-01	1.180E+00	1.100E-01	2.470E+00
2.500E+01	5.000E+00	5.000E-02	8.000E-02	1.100E-01	1.080E+00	5.500E-01	6.000E-02	1.200E-01	2.040E+00
2.500E+01	2.500E+01	5.000E-02	5.000E-02	1.200E-01	2.200E-01	5.600E-01	1.200E-01	1.100E-01	1.230E+00
2.500E+01	5.000E+01	5.000E-02	4.000E-02	1.200E-01	9.000E-02	7.000E-01	2.800E-01	1.100E-01	1.390E+00
2.500E+01	7.500E+01	5.000E-02	4.000E-02	1.200E-01	5.000E-02	8.300E-01	4.300E-01	1.100E-01	1.620E+00
2.500E+01	1.000E+02	5.000E-02	3.000E-02	1.200E-01	4.000E-02	9.500E-01	5.600E-01	1.100E-01	1.860E+00
5.000E+01	5.000E+00	4.000E-02	8.000E-02	9.000E-02	9.800E-01	5.500E-01	3.000E-02	1.200E-01	1.890E+00
5.000E+01	2.500E+01	4.000E-02	5.000E-02	9.000E-02	2.900E-01	7.000E-01	4.000E-02	1.100E-01	1.330E+00
5.000E+01	5.000E+01	4.000E-02	4.000E-02	9.000E-02	1.200E-01	9.500E-01	1.000E-01	1.100E-01	1.460E+00
5.000E+01	7.500E+01	4.000E-02	4.000E-02	1.000E-01	7.000E-02	1.130E+00	2.200E-01	1.100E-01	1.710E+00
5.000E+01	1.000E+02	4.000E-02	3.000E-02	9.000E-02	5.000E-02	1.320E+00	3.200E-01	1.100E-01	1.970E+00
7.500E+01	5.000E+00	4.000E-02	8.000E-02	9.000E-02	1.000E+00	6.300E-01	3.000E-02	1.200E-01	1.970E+00
7.500E+01	2.500E+01	4.000E-02	5.000E-02	9.000E-02	3.800E-01	8.300E-01	2.000E-02	1.300E-01	1.520E+00
7.500E+01	5.000E+01	4.000E-02	4.000E-02	9.000E-02	2.000E-01	1.130E+00	5.000E-02	1.100E-01	1.660E+00
7.500E+01	7.500E+01	4.000E-02	3.000E-02	9.000E-02	1.100E-01	1.420E+00	1.100E-01	1.100E-01	1.930E+00
7.500E+01	1.000E+02	4.000E-02	3.000E-02	9.000E-02	8.000E-02	1.640E+00	1.900E-01	1.100E-01	2.180E+00
1.000E+02	5.000E+00	4.000E-02	8.000E-02	8.000E-02	1.050E+00	7.300E-01	2.000E-02	1.200E-01	2.110E+00
1.000E+02	2.500E+01	4.000E-02	5.000E-02	8.000E-02	4.600E-01	9.600E-01	1.000E-02	1.100E-01	1.710E+00
1.000E+02	5.000E+01	4.000E-02	4.000E-02	8.000E-02	2.600E-01	1.320E+00	3.000E-02	1.100E-01	1.890E+00
1.000E+02	7.500E+01	4.000E-02	3.000E-02	8.000E-02	1.600E-01	1.640E+00	8.000E-02	1.100E-01	2.150E+00
1.000E+02	1.000E+02	4.000E-02	3.000E-02	8.000E-02	1.100E-01	1.890E+00	1.300E-01	1.100E-01	2.400E+00

Spiral-Spiral Distribution (12)
spiral = 3000 (source points), spiral = 3000 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	1.200E-01	1.200E-01	5.600E-01	3.670E+00	2.560E+00	1.130E+00	1.700E-01	8.330E+00
5.000E+00	2.500E+01	1.200E-01	8.000E-02	5.600E-01	4.900E-01	1.030E+00	1.530E+00	1.600E-01	3.970E+00
5.000E+00	5.000E+01	1.200E-01	6.000E-02	5.600E-01	1.700E-01	9.200E-01	1.540E+00	1.800E-01	3.550E+00
5.000E+00	7.500E+01	1.200E-01	6.000E-02	5.600E-01	9.000E-02	1.020E+00	1.630E+00	1.600E-01	3.650E+00
5.000E+00	1.000E+02	1.200E-01	5.000E-02	5.600E-01	7.000E-02	1.120E+00	1.770E+00	1.600E-01	3.860E+00
2.500E+01	5.000E+00	8.000E-02	1.200E-01	1.800E-01	2.010E+00	1.050E+00	1.100E-01	1.700E-01	3.710E+00
2.500E+01	2.500E+01	8.000E-02	8.000E-02	1.800E-01	4.100E-01	8.900E-01	2.200E-01	1.600E-01	2.020E+00
2.500E+01	5.000E+01	8.000E-02	6.000E-02	1.800E-01	1.600E-01	1.060E+00	4.500E-01	1.600E-01	2.150E+00
2.500E+01	7.500E+01	8.000E-02	6.000E-02	1.800E-01	9.000E-02	1.280E+00	6.300E-01	1.600E-01	2.470E+00
2.500E+01	1.000E+02	8.000E-02	5.000E-02	1.800E-01	7.000E-02	1.410E+00	8.000E-01	1.600E-01	2.750E+00
5.000E+01	5.000E+00	7.000E-02	1.200E-01	1.500E-01	1.650E+00	9.400E-01	4.000E-02	1.600E-01	3.130E+00
5.000E+01	2.500E+01	7.000E-02	8.000E-02	1.500E-01	4.800E-01	1.070E+00	6.000E-02	1.800E-01	2.080E+00
5.000E+01	5.000E+01	7.000E-02	6.000E-02	1.500E-01	1.800E-01	1.420E+00	1.200E-01	1.600E-01	2.160E+00
5.000E+01	7.500E+01	7.000E-02	6.000E-02	1.400E-01	1.000E-01	1.700E+00	2.900E-01	1.600E-01	2.510E+00
5.000E+01	1.000E+02	7.000E-02	5.000E-02	1.500E-01	7.000E-02	1.860E+00	4.500E-01	1.600E-01	2.800E+00
7.500E+01	5.000E+00	6.000E-02	1.200E-01	1.300E-01	1.590E+00	1.030E+00	4.000E-02	1.600E-01	3.130E+00
7.500E+01	2.500E+01	6.000E-02	7.000E-02	1.300E-01	5.700E-01	1.270E+00	3.000E-02	1.600E-01	2.310E+00
7.500E+01	5.000E+01	6.000E-02	7.000E-02	1.300E-01	2.800E-01	1.700E+00	5.000E-02	1.600E-01	2.450E+00
7.500E+01	7.500E+01	6.000E-02	6.000E-02	1.300E-01	1.500E-01	2.100E+00	1.400E-01	1.600E-01	2.800E+00
7.500E+01	1.000E+02	6.000E-02	5.000E-02	1.300E-01	1.100E-01	2.370E+00	2.600E-01	1.600E-01	3.150E+00
1.000E+02	5.000E+00	6.000E-02	1.200E-01	1.300E-01	1.650E+00	1.140E+00	4.000E-02	1.900E-01	3.320E+00
1.000E+02	2.500E+01	6.000E-02	8.000E-02	1.300E-01	6.700E-01	1.410E+00	3.000E-02	1.600E-01	2.530E+00
1.000E+02	5.000E+01	6.000E-02	6.000E-02	1.200E-01	3.900E-01	1.860E+00	4.000E-02	1.600E-01	2.680E+00
1.000E+02	7.500E+01	6.000E-02	6.000E-02	1.300E-01	2.300E-01	2.370E+00	1.000E-01	1.600E-01	3.100E+00
1.000E+02	1.000E+02	6.000E-02	5.000E-02	1.300E-01	1.800E-01	2.710E+00	2.000E-01	1.600E-01	3.480E+00

Spiral-Spiral Distribution (17)
spiral = 1000 (source points), spiral = 2000 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	3.000E-02	8.000E-02	1.400E-01	1.310E+00	7.000E-01	2.300E-01	1.200E-01	2.600E+00
5.000E+00	2.500E+01	4.000E-02	5.000E-02	1.400E-01	2.100E-01	3.200E-01	4.700E-01	1.100E-01	1.320E+00
5.000E+00	5.000E+01	3.000E-02	4.000E-02	1.400E-01	9.000E-02	3.100E-01	6.200E-01	1.100E-01	1.340E+00
5.000E+00	7.500E+01	4.000E-02	4.000E-02	1.400E-01	5.000E-02	3.500E-01	7.400E-01	1.100E-01	1.470E+00
5.000E+00	1.000E+02	4.000E-02	3.000E-02	1.400E-01	3.000E-02	4.000E-01	8.700E-01	1.100E-01	1.630E+00
2.500E+01	5.000E+00	2.000E-02	8.000E-02	5.000E-02	7.300E-01	3.600E-01	3.000E-02	1.100E-01	1.380E+00
2.500E+01	2.500E+01	2.000E-02	5.000E-02	6.000E-02	2.100E-01	3.700E-01	4.000E-02	1.100E-01	8.500E-01
2.500E+01	5.000E+01	2.000E-02	4.000E-02	6.000E-02	9.000E-02	4.900E-01	1.000E-01	1.100E-01	9.000E-01
2.500E+01	7.500E+01	2.000E-02	4.000E-02	5.000E-02	6.000E-02	5.800E-01	2.100E-01	1.100E-01	1.070E+00
2.500E+01	1.000E+02	2.000E-02	3.000E-02	6.000E-02	4.000E-02	6.700E-01	3.200E-01	1.100E-01	1.240E+00
5.000E+01	5.000E+00	2.000E-02	8.000E-02	5.000E-02	6.900E-01	4.100E-01	3.000E-02	1.100E-01	1.380E+00
5.000E+01	2.500E+01	2.000E-02	5.000E-02	5.000E-02	2.700E-01	4.900E-01	1.000E-02	1.100E-01	1.000E+00
5.000E+01	5.000E+01	2.000E-02	4.000E-02	5.000E-02	1.600E-01	6.700E-01	3.000E-02	1.100E-01	1.070E+00
5.000E+01	7.500E+01	2.000E-02	4.000E-02	5.000E-02	1.000E-01	8.300E-01	8.000E-02	1.100E-01	1.210E+00
5.000E+01	1.000E+02	2.000E-02	3.000E-02	5.000E-02	7.000E-02	9.500E-01	1.300E-01	1.100E-01	1.350E+00
7.500E+01	5.000E+00	2.000E-02	8.000E-02	4.000E-02	6.900E-01	5.200E-01	3.000E-02	1.100E-01	1.480E+00
7.500E+01	2.500E+01	2.000E-02	5.000E-02	4.000E-02	3.100E-01	6.100E-01	1.000E-02	1.100E-01	1.150E+00
7.500E+01	5.000E+01	2.000E-02	4.000E-02	4.000E-02	1.900E-01	7.900E-01	2.000E-02	1.100E-01	1.210E+00
7.500E+01	7.500E+01	2.000E-02	4.000E-02	4.000E-02	1.400E-01	9.800E-01	5.000E-02	1.100E-01	1.370E+00
7.500E+01	1.000E+02	2.000E-02	3.000E-02	4.000E-02	1.100E-01	1.120E+00	9.000E-02	1.100E-01	1.520E+00
1.000E+02	5.000E+00	2.000E-02	8.000E-02	4.000E-02	8.000E-01	6.800E-01	3.000E-02	1.100E-01	1.760E+00
1.000E+02	2.500E+01	2.000E-02	5.000E-02	4.000E-02	4.000E-01	8.400E-01	1.000E-02	1.100E-01	1.460E+00
1.000E+02	5.000E+01	2.000E-02	4.000E-02	4.000E-02	2.800E-01	1.040E+00	2.000E-02	1.100E-01	1.540E+00
1.000E+02	7.500E+01	2.000E-02	4.000E-02	4.000E-02	2.100E-01	1.300E+00	2.000E-02	1.100E-01	1.730E+00
1.000E+02	1.000E+02	2.000E-02	3.000E-02	4.000E-02	1.700E-01	1.480E+00	6.000E-02	1.100E-01	1.900E+00

Spiral-Spiral Distribution (18)
spiral = 2000 (source points), spiral = 1000 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	8.000E-02	3.000E-02	3.200E-01	8.000E-01	6.900E-01	6.400E-01	6.000E-02	2.630E+00
5.000E+00	2.500E+01	8.000E-02	2.000E-02	3.200E-01	9.000E-02	3.400E-01	6.100E-01	5.000E-02	1.510E+00
5.000E+00	5.000E+01	8.000E-02	2.000E-02	3.100E-01	3.000E-02	4.000E-01	6.700E-01	5.000E-02	1.570E+00
5.000E+00	7.500E+01	8.000E-02	2.000E-02	3.200E-01	2.000E-02	5.100E-01	7.000E-01	5.000E-02	1.700E+00
5.000E+00	1.000E+02	8.000E-02	1.000E-02	3.100E-01	1.000E-02	6.800E-01	9.000E-01	6.000E-02	2.050E+00
2.500E+01	5.000E+00	5.000E-02	3.000E-02	1.200E-01	6.000E-01	3.200E-01	5.000E-02	6.000E-02	1.220E+00
2.500E+01	2.500E+01	5.000E-02	2.000E-02	1.200E-01	9.000E-02	3.700E-01	1.600E-01	5.000E-02	8.600E-01
2.500E+01	5.000E+01	5.000E-02	2.000E-02	1.200E-01	4.000E-02	4.900E-01	3.000E-01	5.000E-02	1.060E+00
2.500E+01	7.500E+01	5.000E-02	2.000E-02	1.200E-01	2.000E-02	6.100E-01	3.600E-01	5.000E-02	1.230E+00
2.500E+01	1.000E+02	5.000E-02	1.000E-02	1.200E-01	2.000E-02	8.400E-01	5.000E-01	5.000E-02	1.590E+00
5.000E+01	5.000E+00	4.000E-02	3.000E-02	9.000E-02	6.100E-01	3.200E-01	2.000E-02	6.000E-02	1.170E+00
5.000E+01	2.500E+01	4.000E-02	2.000E-02	9.000E-02	1.200E-01	4.800E-01	6.000E-02	6.000E-02	8.900E-01
5.000E+01	5.000E+01	4.000E-02	2.000E-02	9.000E-02	5.000E-02	6.700E-01	1.700E-01	5.000E-02	1.090E+00
5.000E+01	7.500E+01	4.000E-02	2.000E-02	9.000E-02	3.000E-02	7.800E-01	2.300E-01	5.000E-02	1.250E+00
5.000E+01	1.000E+02	4.000E-02	1.000E-02	9.000E-02	2.000E-02	1.040E+00	3.600E-01	5.000E-02	1.620E+00
7.500E+01	5.000E+00	4.000E-02	3.000E-02	9.000E-02	6.700E-01	3.600E-01	2.000E-02	6.000E-02	1.260E+00
7.500E+01	2.500E+01	4.000E-02	2.000E-02	9.000E-02	1.900E-01	5.800E-01	3.000E-02	5.000E-02	1.000E+00
7.500E+01	5.000E+01	4.000E-02	2.000E-02	9.000E-02	8.000E-02	8.200E-01	1.000E-01	6.000E-02	1.210E+00
7.500E+01	7.500E+01	4.000E-02	2.000E-02	9.000E-02	5.000E-02	9.700E-01	1.600E-01	5.000E-02	1.380E+00
7.500E+01	1.000E+02	4.000E-02	1.000E-02	9.000E-02	3.000E-02	1.300E+00	2.700E-01	5.000E-02	1.790E+00
1.000E+02	5.000E+00	4.000E-02	3.000E-02	8.000E-02	7.400E-01	4.000E-01	1.000E-02	6.000E-02	1.360E+00
1.000E+02	2.500E+01	4.000E-02	2.000E-02	8.000E-02	2.600E-01	6.700E-01	2.000E-02	5.000E-02	1.140E+00
1.000E+02	5.000E+01	4.000E-02	2.000E-02	8.000E-02	1.100E-01	9.500E-01	7.000E-02	5.000E-02	1.320E+00
1.000E+02	7.500E+01	4.000E-02	2.000E-02	8.000E-02	8.000E-02	1.120E+00	1.300E-01	5.000E-02	1.510E+00
1.000E+02	1.000E+02	4.000E-02	1.000E-02	8.000E-02	5.000E-02	1.480E+00	2.200E-01	6.000E-02	1.940E+00

Spiral-Uniform Distribution (13)
spiral = 2000 (source points), uniform = 4225 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	8.000E–02	1.000E–01	3.100E–01	8.500E–01	2.380E+00	4.920E+00	2.500E–01	8.900E+00
5.000E+00	2.500E+01	8.000E–02	7.000E–02	3.100E–01	1.700E–01	6.500E–01	1.510E+00	2.300E–01	3.010E+00
5.000E+00	5.000E+01	8.000E–02	6.000E–02	3.200E–01	1.700E–01	6.600E–01	1.510E+00	2.600E–01	3.050E+00
5.000E+00	7.500E+01	8.000E–02	5.000E–02	3.200E–01	4.000E–02	7.200E–01	1.400E+00	2.200E–01	2.830E+00
5.000E+00	1.000E+02	8.000E–02	5.000E–02	3.100E–01	4.000E–02	7.200E–01	1.390E+00	2.300E–01	2.810E+00
2.500E+01	5.000E+00	5.000E–02	1.000E–01	1.200E–01	8.900E–01	7.600E–01	8.600E–01	2.300E–01	3.010E+00
2.500E+01	2.500E+01	5.000E–02	6.000E–02	1.100E–01	1.900E–01	4.300E–01	4.700E–01	2.300E–01	1.540E+00
2.500E+01	5.000E+01	5.000E–02	7.000E–02	1.100E–01	1.800E–01	4.400E–01	4.700E–01	2.200E–01	1.540E+00
2.500E+01	7.500E+01	5.000E–02	5.000E–02	1.200E–01	5.000E–02	9.300E–01	7.100E–01	2.200E–01	2.120E+00
2.500E+01	1.000E+02	5.000E–02	5.000E–02	1.200E–01	5.000E–02	9.300E–01	7.000E–01	2.200E–01	2.120E+00
5.000E+01	5.000E+00	4.000E–02	1.000E–01	9.000E–02	1.020E+00	6.000E–01	3.000E–01	2.300E–01	2.390E+00
5.000E+01	2.500E+01	4.000E–02	7.000E–02	9.000E–02	2.300E–01	5.600E–01	2.400E–01	2.200E–01	1.460E+00
5.000E+01	5.000E+01	4.000E–02	6.000E–02	1.000E–01	2.300E–01	5.600E–01	2.500E–01	2.500E–01	1.490E+00
5.000E+01	7.500E+01	4.000E–02	5.000E–02	9.000E–02	5.000E–02	1.260E+00	4.900E–01	2.200E–01	2.200E+00
5.000E+01	1.000E+02	4.000E–02	5.000E–02	9.000E–02	5.000E–02	1.260E+00	4.900E–01	2.200E–01	2.200E+00
7.500E+01	5.000E+00	4.000E–02	1.000E–01	9.000E–02	1.200E+00	6.000E–01	1.700E–01	2.300E–01	2.440E+00
7.500E+01	2.500E+01	4.000E–02	7.000E–02	9.000E–02	2.600E–01	7.300E–01	1.400E–01	2.300E–01	1.550E+00
7.500E+01	5.000E+01	4.000E–02	7.000E–02	9.000E–02	2.600E–01	7.300E–01	1.400E–01	2.200E–01	1.550E+00
7.500E+01	7.500E+01	4.000E–02	5.000E–02	9.000E–02	5.000E–02	1.580E+00	3.500E–01	2.200E–01	2.380E+00
7.500E+01	1.000E+02	4.000E–02	5.000E–02	9.000E–02	5.000E–02	1.570E+00	3.500E–01	2.200E–01	2.370E+00
1.000E+02	5.000E+00	4.000E–02	1.000E–01	8.000E–02	1.500E+00	7.100E–01	1.100E–01	2.300E–01	2.780E+00
1.000E+02	2.500E+01	4.000E–02	6.000E–02	8.000E–02	3.300E–01	9.800E–01	8.000E–02	2.300E–01	1.810E+00
1.000E+02	5.000E+01	4.000E–02	7.000E–02	8.000E–02	3.300E–01	9.800E–01	8.000E–02	2.500E–01	1.840E+00
1.000E+02	7.500E+01	4.000E–02	5.000E–02	8.000E–02	8.000E–02	1.910E+00	2.900E–01	2.200E–01	2.670E+00
1.000E+02	1.000E+02	4.000E–02	5.000E–02	8.000E–02	8.000E–02	1.910E+00	2.900E–01	2.200E–01	2.670E+00

Spiral-Uniform Distribution (14)
spiral = 2000 (source points), uniform = 8281 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	8.000E–02	2.900E–01	3.200E–01	4.330E+00	7.730E+00	1.458E+01	5.200E–01	2.783E+01
5.000E+00	2.500E+01	8.000E–02	1.700E–01	3.100E–01	6.800E–01	1.950E+00	4.170E+00	4.600E–01	7.830E+00
5.000E+00	5.000E+01	8.000E–02	1.300E–01	3.100E–01	1.700E–01	8.000E–01	1.930E+00	4.400E–01	3.850E+00
5.000E+00	7.500E+01	8.000E–02	1.300E–01	3.100E–01	1.700E–01	8.000E–01	1.930E+00	4.400E–01	3.850E+00
5.000E+00	1.000E+02	8.000E–02	1.300E–01	3.100E–01	1.700E–01	8.000E–01	1.930E+00	4.500E–01	3.860E+00
2.500E+01	5.000E+00	5.000E–02	2.800E–01	1.100E–01	4.290E+00	2.220E+00	1.640E+00	4.700E–01	9.070E+00
2.500E+01	2.500E+01	5.000E–02	1.700E–01	1.100E–01	7.200E–01	7.900E–01	8.500E–01	4.500E–01	3.150E+00
2.500E+01	5.000E+01	5.000E–02	1.300E–01	1.100E–01	1.800E–01	7.100E–01	7.000E–01	4.400E–01	2.330E+00
2.500E+01	7.500E+01	5.000E–02	1.300E–01	1.200E–01	1.800E–01	7.100E–01	7.000E–01	4.400E–01	2.320E+00
2.500E+01	1.000E+02	5.000E–02	1.200E–01	1.100E–01	1.900E–01	7.100E–01	7.000E–01	4.400E–01	2.330E+00
5.000E+01	5.000E+00	4.000E–02	2.800E–01	9.000E–02	4.440E+00	1.460E+00	5.800E–01	4.700E–01	7.360E+00
5.000E+01	2.500E+01	4.000E–02	1.700E–01	9.000E–02	8.600E–01	8.000E–01	3.100E–01	4.500E–01	2.720E+00
5.000E+01	5.000E+01	4.000E–02	1.300E–01	9.000E–02	2.300E–01	1.030E+00	3.800E–01	4.400E–01	2.340E+00
5.000E+01	7.500E+01	4.000E–02	1.200E–01	9.000E–02	2.200E–01	1.030E+00	3.800E–01	4.400E–01	2.330E+00
5.000E+01	1.000E+02	4.000E–02	1.200E–01	9.000E–02	2.200E–01	1.030E+00	3.800E–01	4.400E–01	2.330E+00
7.500E+01	5.000E+00	4.000E–02	2.900E–01	9.000E–02	4.640E+00	1.280E+00	4.500E–01	5.200E–01	7.310E+00
7.500E+01	2.500E+01	4.000E–02	1.700E–01	9.000E–02	1.030E+00	9.400E–01	1.600E–01	4.500E–01	2.870E+00
7.500E+01	5.000E+01	4.000E–02	1.300E–01	9.000E–02	2.500E–01	1.360E+00	2.200E–01	4.500E–01	2.530E+00
7.500E+01	7.500E+01	4.000E–02	1.300E–01	9.000E–02	2.500E–01	1.360E+00	2.200E–01	4.400E–01	2.520E+00
7.500E+01	1.000E+02	4.000E–02	1.200E–01	9.000E–02	2.500E–01	1.360E+00	2.200E–01	4.400E–01	2.520E+00
1.000E+02	5.000E+00	4.000E–02	2.800E–01	8.000E–02	4.990E+00	1.350E+00	2.800E–01	4.600E–01	7.480E+00
1.000E+02	2.500E+01	4.000E–02	1.700E–01	8.000E–02	1.320E+00	1.230E+00	1.100E–01	4.500E–01	3.390E+00
1.000E+02	5.000E+01	4.000E–02	1.300E–01	8.000E–02	3.300E–01	1.890E+00	1.200E–01	4.400E–01	3.020E+00
1.000E+02	7.500E+01	4.000E–02	1.200E–01	8.000E–02	3.300E–01	1.890E+00	1.200E–01	4.900E–01	3.070E+00
1.000E+02	1.000E+02	4.000E–02	1.200E–01	8.000E–02	3.300E–01	1.890E+00	1.200E–01	4.400E–01	3.020E+00

Spiral-Uniform Distribution (15)
spiral = 2025 (source points), uniform = 1024 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	8.000E–02	2.000E–02	3.200E–01	1.700E–01	5.300E–01	1.200E+00	6.000E–02	2.380E+00
5.000E+00	2.500E+01	8.000E–02	1.000E–02	3.200E–01	4.000E–02	2.800E–01	5.600E–01	6.000E–02	1.350E+00
5.000E+00	5.000E+01	8.000E–02	1.000E–02	3.200E–01	4.000E–02	2.800E–01	5.600E–01	6.000E–02	1.350E+00
5.000E+00	7.500E+01	8.000E–02	1.000E–02	3.200E–01	1.000E–02	5.700E–01	7.400E–01	5.000E–02	1.780E+00
5.000E+00	1.000E+02	8.000E–02	1.000E–02	3.200E–01	1.000E–02	5.700E–01	7.400E–01	5.000E–02	1.780E+00
2.500E+01	5.000E+00	5.000E–02	2.000E–02	1.200E–01	1.800E–01	2.100E–01	2.900E–01	6.000E–02	9.300E–01
2.500E+01	2.500E+01	5.000E–02	1.000E–02	1.200E–01	5.000E–02	2.600E–01	2.300E–01	5.000E–02	7.700E–01
2.500E+01	5.000E+01	5.000E–02	1.000E–02	1.200E–01	5.000E–02	2.600E–01	2.300E–01	5.000E–02	7.700E–01
2.500E+01	7.500E+01	5.000E–02	1.000E–02	1.200E–01	1.000E–02	7.200E–01	4.100E–01	5.000E–02	1.370E+00
2.500E+01	1.000E+02	5.000E–02	1.000E–02	1.200E–01	1.000E–02	7.200E–01	4.100E–01	5.000E–02	1.360E+00
5.000E+01	5.000E+00	4.000E–02	2.000E–02	9.000E–02	2.300E–01	2.000E–01	1.400E–01	6.000E–02	7.800E–01
5.000E+01	2.500E+01	4.000E–02	1.000E–02	9.000E–02	5.000E–02	3.300E–01	1.500E–01	5.000E–02	7.300E–01
5.000E+01	5.000E+01	4.000E–02	1.000E–02	9.000E–02	5.000E–02	3.300E–01	1.500E–01	5.000E–02	7.300E–01
5.000E+01	7.500E+01	4.000E–02	1.000E–02	9.000E–02	1.000E–02	9.000E–01	2.900E–01	5.000E–02	1.400E+00
5.000E+01	1.000E+02	4.000E–02	1.000E–02	9.000E–02	1.000E–02	9.000E–01	2.900E–01	5.000E–02	1.400E+00
7.500E+01	5.000E+00	4.000E–02	2.000E–02	9.000E–02	2.500E–01	2.200E–01	8.000E–02	6.000E–02	7.700E–01
7.500E+01	2.500E+01	4.000E–02	1.000E–02	9.000E–02	5.000E–02	4.100E–01	1.100E–01	5.000E–02	7.600E–01
7.500E+01	5.000E+01	4.000E–02	1.000E–02	9.000E–02	5.000E–02	4.100E–01	1.100E–01	6.000E–02	7.600E–01
7.500E+01	7.500E+01	4.000E–02	1.000E–02	9.000E–02	1.000E–02	1.050E+00	2.400E–01	5.000E–02	1.490E+00
7.500E+01	1.000E+02	4.000E–02	1.000E–02	9.000E–02	1.000E–02	1.050E+00	2.400E–01	5.000E–02	1.490E+00
1.000E+02	5.000E+00	4.000E–02	2.000E–02	8.000E–02	3.300E–01	2.800E–01	5.000E–02	6.000E–02	8.600E–01
1.000E+02	2.500E+01	4.000E–02	1.000E–02	8.000E–02	8.000E–02	4.900E–01	9.000E–02	5.000E–02	8.400E–01
1.000E+02	5.000E+01	4.000E–02	1.000E–02	8.000E–02	8.000E–02	4.900E–01	9.000E–02	5.000E–02	8.400E–01
1.000E+02	7.500E+01	4.000E–02	1.000E–02	8.000E–02	1.000E–02	1.180E+00	2.100E–01	6.000E–02	1.590E+00
1.000E+02	1.000E+02	4.000E–02	1.000E–02	8.000E–02	1.000E–02	1.180E+00	2.100E–01	5.000E–02	1.580E+00

Spiral-Uniform Distribution (16)
spiral = 1024 (source points), uniform = 2025 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	4.000E–02	6.000E–02	1.400E–01	7.300E–01	1.100E+00	2.150E+00	1.200E–01	4.340E+00
5.000E+00	2.500E+01	4.000E–02	3.000E–02	1.400E–01	1.500E–01	3.200E–01	7.300E–01	1.200E–01	1.540E+00
5.000E+00	5.000E+01	4.000E–02	2.000E–02	1.400E–01	4.000E–02	2.300E–01	6.000E–01	1.100E–01	1.190E+00
5.000E+00	7.500E+01	4.000E–02	2.000E–02	1.400E–01	4.000E–02	2.300E–01	6.000E–01	1.100E–01	1.180E+00
5.000E+00	1.000E+02	4.000E–02	2.000E–02	1.400E–01	4.000E–02	2.300E–01	6.000E–01	1.100E–01	1.190E+00
2.500E+01	5.000E+00	2.000E–02	6.000E–02	6.000E–02	8.000E–01	3.500E–01	2.600E–01	1.100E–01	1.660E+00
2.500E+01	2.500E+01	2.000E–02	3.000E–02	6.000E–02	1.700E–01	2.200E–01	1.800E–01	1.100E–01	7.900E–01
2.500E+01	5.000E+01	2.000E–02	2.000E–02	6.000E–02	4.000E–02	3.200E–01	2.500E–01	1.100E–01	8.300E–01
2.500E+01	7.500E+01	2.000E–02	2.000E–02	6.000E–02	4.000E–02	3.200E–01	2.500E–01	1.200E–01	8.400E–01
2.500E+01	1.000E+02	2.000E–02	2.000E–02	6.000E–02	4.000E–02	3.200E–01	2.500E–01	1.100E–01	8.300E–01
5.000E+01	5.000E+00	2.000E–02	6.000E–02	5.000E–02	1.010E+00	3.000E–01	1.000E–01	1.100E–01	1.650E+00
5.000E+01	2.500E+01	2.000E–02	3.000E–02	5.000E–02	2.100E–01	3.000E–01	7.000E–02	1.100E–01	8.000E–01
5.000E+01	5.000E+01	2.000E–02	2.000E–02	5.000E–02	6.000E–02	4.800E–01	1.500E–01	1.100E–01	8.800E–01
5.000E+01	7.500E+01	2.000E–02	2.000E–02	5.000E–02	6.000E–02	4.700E–01	1.500E–01	1.100E–01	8.800E–01
5.000E+01	1.000E+02	2.000E–02	2.000E–02	5.000E–02	6.000E–02	4.800E–01	1.500E–01	1.100E–01	8.800E–01
7.500E+01	5.000E+00	2.000E–02	6.000E–02	4.000E–02	1.150E+00	3.000E–01	8.000E–02	1.100E–01	1.770E+00
7.500E+01	2.500E+01	2.000E–02	3.000E–02	4.000E–02	2.800E–01	3.500E–01	5.000E–02	1.100E–01	8.800E–01
7.500E+01	5.000E+01	2.000E–02	2.000E–02	4.000E–02	6.000E–02	5.700E–01	1.000E–01	1.200E–01	9.400E–01
7.500E+01	7.500E+01	2.000E–02	2.000E–02	4.000E–02	6.000E–02	5.800E–01	1.000E–01	1.100E–01	9.300E–01
7.500E+01	1.000E+02	2.000E–02	2.000E–02	4.000E–02	6.000E–02	5.700E–01	1.000E–01	1.100E–01	9.300E–01
1.000E+02	5.000E+00	2.000E–02	6.000E–02	4.000E–02	1.340E+00	3.100E–01	5.000E–02	1.100E–01	1.930E+00
1.000E+02	2.500E+01	2.000E–02	3.000E–02	4.000E–02	3.600E–01	4.100E–01	3.000E–02	1.100E–01	1.010E+00
1.000E+02	5.000E+01	2.000E–02	2.000E–02	4.000E–02	6.000E–02	7.200E–01	5.000E–02	1.100E–01	1.020E+00
1.000E+02	7.500E+01	2.000E–02	2.000E–02	4.000E–02	6.000E–02	7.200E–01	5.000E–02	1.100E–01	1.020E+00
1.000E+02	1.000E+02	2.000E–02	2.000E–02	4.000E–02	6.000E–02	7.200E–01	5.000E–02	1.100E–01	1.020E+00

Spiral-Uniform Distribution (19)
spiral = 2025 (source points), uniform = 2025 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	8.000E–02	6.000E–02	3.200E–01	7.300E–01	1.940E+00	4.100E+00	1.300E–01	7.360E+00
5.000E+00	2.500E+01	8.000E–02	3.000E–02	3.200E–01	1.500E–01	5.600E–01	1.230E+00	1.100E–01	2.480E+00
5.000E+00	5.000E+01	8.000E–02	2.000E–02	3.200E–01	4.000E–02	4.100E–01	8.100E–01	1.100E–01	1.790E+00
5.000E+00	7.500E+01	8.000E–02	2.000E–02	3.200E–01	4.000E–02	4.100E–01	8.100E–01	1.100E–01	1.790E+00
5.000E+00	1.000E+02	8.000E–02	2.000E–02	3.200E–01	4.000E–02	4.100E–01	8.100E–01	1.100E–01	1.790E+00
2.500E+01	5.000E+00	5.000E–02	6.000E–02	1.200E–01	7.700E–01	5.900E–01	7.500E–01	1.300E–01	2.470E+00
2.500E+01	2.500E+01	5.000E–02	3.000E–02	1.200E–01	1.600E–01	3.100E–01	3.500E–01	1.100E–01	1.130E+00
2.500E+01	5.000E+01	5.000E–02	2.000E–02	1.200E–01	5.000E–02	4.600E–01	3.800E–01	1.100E–01	1.190E+00
2.500E+01	7.500E+01	5.000E–02	2.000E–02	1.200E–01	5.000E–02	4.600E–01	3.800E–01	1.100E–01	1.190E+00
2.500E+01	1.000E+02	5.000E–02	2.000E–02	1.200E–01	5.000E–02	4.600E–01	3.800E–01	1.100E–01	1.190E+00
5.000E+01	5.000E+00	4.000E–02	6.000E–02	9.000E–02	9.000E–01	4.100E–01	2.600E–01	1.100E–01	1.880E+00
5.000E+01	2.500E+01	4.000E–02	3.000E–02	9.000E–02	1.900E–01	3.600E–01	1.800E–01	1.100E–01	1.010E+00
5.000E+01	5.000E+01	4.000E–02	2.000E–02	9.000E–02	5.000E–02	6.100E–01	2.500E–01	1.100E–01	1.180E+00
5.000E+01	7.500E+01	4.000E–02	2.000E–02	9.000E–02	5.000E–02	6.100E–01	2.500E–01	1.200E–01	1.190E+00
5.000E+01	1.000E+02	4.000E–02	2.000E–02	9.000E–02	5.000E–02	6.100E–01	2.500E–01	1.100E–01	1.180E+00
7.500E+01	5.000E+00	4.000E–02	6.000E–02	9.000E–02	1.080E+00	3.800E–01	1.400E–01	1.100E–01	1.910E+00
7.500E+01	2.500E+01	4.000E–02	3.000E–02	9.000E–02	2.100E–01	4.400E–01	1.100E–01	1.100E–01	1.030E+00
7.500E+01	5.000E+01	4.000E–02	2.000E–02	9.000E–02	5.000E–02	7.600E–01	1.800E–01	1.100E–01	1.250E+00
7.500E+01	7.500E+01	4.000E–02	2.000E–02	9.000E–02	5.000E–02	7.600E–01	1.800E–01	1.100E–01	1.260E+00
7.500E+01	1.000E+02	4.000E–02	3.000E–02	9.000E–02	5.000E–02	7.600E–01	1.800E–01	1.100E–01	1.260E+00
1.000E+02	5.000E+00	4.000E–02	6.000E–02	8.000E–02	1.390E+00	4.400E–01	1.000E–01	1.300E–01	2.240E+00
1.000E+02	2.500E+01	4.000E–02	3.000E–02	8.000E–02	2.900E–01	5.600E–01	7.000E–02	1.100E–01	1.180E+00
1.000E+02	5.000E+01	4.000E–02	2.000E–02	8.000E–02	8.000E–02	9.300E–01	1.500E–01	1.100E–01	1.410E+00
1.000E+02	7.500E+01	4.000E–02	2.000E–02	8.000E–02	8.000E–02	9.300E–01	1.500E–01	1.100E–01	1.410E+00
1.000E+02	1.000E+02	4.000E–02	2.000E–02	8.000E–02	8.000E–02	9.300E–01	1.500E–01	1.100E–01	1.410E+00

Spiral-Uniform Distribution (20)
spiral = 3025 (source points), uniform = 3025 (field points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	1.200E–01	8.000E–02	5.600E–01	7.700E–01	2.890E+00	6.290E+00	1.900E–01	1.089E+01
5.000E+00	2.500E+01	1.200E–01	5.000E–02	5.500E–01	1.800E–01	8.800E–01	1.930E+00	1.900E–01	3.900E+00
5.000E+00	5.000E+01	1.200E–01	3.000E–02	5.500E–01	4.000E–02	7.700E–01	1.270E+00	1.600E–01	2.950E+00
5.000E+00	7.500E+01	1.200E–01	3.000E–02	5.500E–01	4.000E–02	7.700E–01	1.270E+00	1.600E–01	2.950E+00
5.000E+00	1.000E+02	1.200E–01	3.000E–02	5.500E–01	4.000E–02	7.700E–01	1.270E+00	1.600E–01	2.950E+00
2.500E+01	5.000E+00	8.000E–02	8.000E–02	1.800E–01	7.700E–01	8.900E–01	1.410E+00	1.700E–01	3.570E+00
2.500E+01	2.500E+01	8.000E–02	5.000E–02	1.800E–01	1.700E–01	4.400E–01	5.900E–01	1.600E–01	1.670E+00
2.500E+01	5.000E+01	8.000E–02	3.000E–02	1.800E–01	4.000E–02	8.200E–01	6.600E–01	1.600E–01	1.980E+00
2.500E+01	7.500E+01	8.000E–02	3.000E–02	1.800E–01	4.000E–02	8.200E–01	6.600E–01	1.600E–01	1.980E+00
2.500E+01	1.000E+02	8.000E–02	3.000E–02	1.800E–01	4.000E–02	8.200E–01	6.600E–01	1.800E–01	2.000E+00
5.000E+01	5.000E+00	7.000E–02	8.000E–02	1.400E–01	8.300E–01	5.900E–01	5.500E–01	1.700E–01	2.430E+00
5.000E+01	2.500E+01	7.000E–02	5.000E–02	1.500E–01	1.900E–01	4.600E–01	3.200E–01	1.600E–01	1.400E+00
5.000E+01	5.000E+01	7.000E–02	4.000E–02	1.500E–01	5.000E–02	1.030E+00	4.700E–01	1.600E–01	1.960E+00
5.000E+01	7.500E+01	7.000E–02	3.000E–02	1.500E–01	5.000E–02	1.030E+00	4.700E–01	1.600E–01	1.960E+00
5.000E+01	1.000E+02	7.000E–02	3.000E–02	1.500E–01	5.000E–02	1.030E+00	4.700E–01	1.600E–01	1.960E+00
7.500E+01	5.000E+00	6.000E–02	8.000E–02	1.300E–01	1.030E+00	5.600E–01	2.700E–01	1.700E–01	2.310E+00
7.500E+01	2.500E+01	6.000E–02	5.000E–02	1.300E–01	2.500E–01	5.800E–01	2.000E–01	1.600E–01	1.450E+00
7.500E+01	5.000E+01	6.000E–02	4.000E–02	1.300E–01	5.000E–02	1.320E+00	3.600E–01	1.800E–01	2.130E+00
7.500E+01	7.500E+01	6.000E–02	3.000E–02	1.300E–01	5.000E–02	1.310E+00	3.600E–01	1.600E–01	2.110E+00
7.500E+01	1.000E+02	6.000E–02	3.000E–02	1.300E–01	5.000E–02	1.320E+00	3.600E–01	1.600E–01	2.110E+00
1.000E+02	5.000E+00	6.000E–02	8.000E–02	1.300E–01	1.270E+00	5.800E–01	2.000E–01	1.700E–01	2.480E+00
1.000E+02	2.500E+01	6.000E–02	5.000E–02	1.200E–01	3.000E–01	7.100E–01	1.500E–01	1.600E–01	1.550E+00
1.000E+02	5.000E+01	6.000E–02	3.000E–02	1.200E–01	5.000E–02	1.590E+00	2.800E–01	1.600E–01	2.300E+00
1.000E+02	7.500E+01	6.000E–02	3.000E–02	1.200E–01	5.000E–02	1.580E+00	2.800E–01	1.600E–01	2.290E+00
1.000E+02	1.000E+02	6.000E–02	3.000E–02	1.200E–01	5.000E–02	1.580E+00	2.800E–01	1.800E–01	2.310E+00

Uniform-Spiral Distribution (23)
spiral = 2025 (field points), uniform = 4225 (source points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	1.000E–01	8.000E–02	9.100E–01	5.530E+00	2.430E+00	4.000E–02	1.200E–01	9.210E+00
5.000E+00	2.500E+01	1.000E–01	5.000E–02	8.700E–01	1.200E+00	7.300E–01	1.000E–01	1.100E–01	3.170E+00
5.000E+00	5.000E+01	1.000E–01	4.000E–02	8.700E–01	5.300E–01	5.600E–01	3.600E–01	1.100E–01	2.580E+00
5.000E+00	7.500E+01	1.000E–01	4.000E–02	8.800E–01	3.100E–01	5.600E–01	7.400E–01	1.100E–01	2.730E+00
5.000E+00	1.000E+02	1.000E–01	3.000E–02	8.700E–01	1.900E–01	6.800E–01	1.240E+00	1.100E–01	3.230E+00
2.500E+01	5.000E+00	7.000E–02	8.000E–02	3.000E–01	1.640E+00	7.000E–01	2.000E–02	1.100E–01	2.910E+00
2.500E+01	2.500E+01	7.000E–02	5.000E–02	2.900E–01	4.900E–01	4.400E–01	3.000E–02	1.100E–01	1.470E+00
2.500E+01	5.000E+01	7.000E–02	4.000E–02	3.000E–01	2.700E–01	5.600E–01	9.000E–02	1.100E–01	1.430E+00
2.500E+01	7.500E+01	7.000E–02	4.000E–02	2.900E–01	1.800E–01	7.300E–01	1.300E–01	1.100E–01	1.540E+00
2.500E+01	1.000E+02	7.000E–02	3.000E–02	3.000E–01	1.200E–01	9.800E–01	2.400E–01	1.100E–01	1.850E+00
5.000E+01	5.000E+00	7.000E–02	8.000E–02	3.000E–01	1.640E+00	7.000E–01	2.000E–02	1.100E–01	2.910E+00
5.000E+01	2.500E+01	7.000E–02	5.000E–02	3.000E–01	4.900E–01	4.400E–01	3.000E–02	1.100E–01	1.480E+00
5.000E+01	5.000E+01	7.000E–02	4.000E–02	2.900E–01	2.700E–01	5.600E–01	8.000E–02	1.100E–01	1.420E+00
5.000E+01	7.500E+01	7.000E–02	4.000E–02	2.900E–01	1.800E–01	7.300E–01	1.300E–01	1.100E–01	1.540E+00
5.000E+01	1.000E+02	7.000E–02	3.000E–02	3.000E–01	1.200E–01	9.800E–01	2.400E–01	1.100E–01	1.860E+00
7.500E+01	5.000E+00	5.000E–02	8.000E–02	1.900E–01	1.240E+00	7.300E–01	2.000E–02	1.100E–01	2.430E+00
7.500E+01	2.500E+01	5.000E–02	5.000E–02	1.900E–01	5.600E–01	9.400E–01	2.000E–02	1.100E–01	1.920E+00
7.500E+01	5.000E+01	5.000E–02	4.000E–02	1.900E–01	3.800E–01	1.270E+00	1.000E–02	1.100E–01	2.050E+00
7.500E+01	7.500E+01	5.000E–02	4.000E–02	1.900E–01	2.800E–01	1.590E+00	1.000E–02	1.100E–01	2.260E+00
7.500E+01	1.000E+02	5.000E–02	3.000E–02	1.900E–01	2.300E–01	1.930E+00	6.000E–02	1.100E–01	2.600E+00
1.000E+02	5.000E+00	5.000E–02	8.000E–02	1.900E–01	1.230E+00	7.300E–01	2.000E–02	1.200E–01	2.420E+00
1.000E+02	2.500E+01	5.000E–02	5.000E–02	1.900E–01	5.600E–01	9.400E–01	2.000E–02	1.100E–01	1.920E+00
1.000E+02	5.000E+01	5.000E–02	4.000E–02	1.900E–01	3.800E–01	1.270E+00	1.000E–02	1.100E–01	2.050E+00
1.000E+02	7.500E+01	5.000E–02	4.000E–02	1.900E–01	2.800E–01	1.590E+00	1.000E–02	1.100E–01	2.260E+00
1.000E+02	1.000E+02	5.000E–02	3.000E–02	1.900E–01	2.300E–01	1.930E+00	6.000E–02	1.100E–01	2.600E+00

Uniform-Spiral Distribution (24)
spiral = 2025 (field points), uniform = 8281 (source points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	2.800E–01	8.000E–02	5.540E+00	1.793E+01	8.440E+00	1.100E–01	1.400E–01	3.252E+01
5.000E+00	2.500E+01	2.800E–01	5.000E–02	5.450E+00	3.260E+00	2.250E+00	5.000E–01	1.200E–01	1.191E+01
5.000E+00	5.000E+01	2.800E–01	4.000E–02	5.460E+00	1.150E+00	1.370E+00	1.770E+00	1.200E–01	1.020E+01
5.000E+00	7.500E+01	2.900E–01	4.000E–02	5.450E+00	5.800E–01	1.150E+00	2.650E+00	1.100E–01	1.026E+01
5.000E+00	1.000E+02	2.800E–01	3.000E–02	5.490E+00	3.300E–01	1.260E+00	3.380E+00	1.200E–01	1.089E+01
2.500E+01	5.000E+00	1.700E–01	8.000E–02	9.500E–01	4.610E+00	2.010E+00	3.000E–02	1.300E–01	7.980E+00
2.500E+01	2.500E+01	1.700E–01	5.000E–02	9.500E–01	1.090E+00	7.700E–01	8.000E–02	1.200E–01	3.230E+00
2.500E+01	5.000E+01	1.700E–01	4.000E–02	9.600E–01	5.000E–01	7.800E–01	3.100E–01	1.100E–01	2.860E+00
2.500E+01	7.500E+01	1.700E–01	4.000E–02	9.600E–01	2.900E–01	9.200E–01	6.400E–01	1.100E–01	3.130E+00
2.500E+01	1.000E+02	1.700E–01	3.000E–02	9.500E–01	1.800E–01	1.220E+00	1.130E+00	1.100E–01	3.790E+00
5.000E+01	5.000E+00	1.300E–01	8.000E–02	4.400E–01	1.920E+00	8.300E–01	3.000E–02	1.200E–01	3.550E+00
5.000E+01	2.500E+01	1.300E–01	5.000E–02	4.500E–01	6.500E–01	7.200E–01	3.000E–02	1.100E–01	2.140E+00
5.000E+01	5.000E+01	1.300E–01	4.000E–02	4.500E–01	3.700E–01	1.040E+00	9.000E–02	1.100E–01	2.210E+00
5.000E+01	7.500E+01	1.300E–01	3.000E–02	4.400E–01	2.300E–01	1.380E+00	1.300E–01	1.100E–01	2.470E+00
5.000E+01	1.000E+02	1.300E–01	3.000E–02	4.500E–01	1.500E–01	1.920E+00	2.500E–01	1.100E–01	3.020E+00
7.500E+01	5.000E+00	1.300E–01	8.000E–02	4.400E–01	1.930E+00	8.300E–01	3.000E–02	1.200E–01	3.560E+00
7.500E+01	2.500E+01	1.300E–01	5.000E–02	4.500E–01	6.600E–01	7.200E–01	3.000E–02	1.100E–01	2.140E+00
7.500E+01	5.000E+01	1.300E–01	4.000E–02	4.400E–01	3.700E–01	1.040E+00	9.000E–02	1.100E–01	2.210E+00
7.500E+01	7.500E+01	1.300E–01	4.000E–02	4.500E–01	2.300E–01	1.380E+00	1.300E–01	1.100E–01	2.470E+00
7.500E+01	1.000E+02	1.300E–01	3.000E–02	4.500E–01	1.500E–01	1.920E+00	2.400E–01	1.100E–01	3.030E+00
1.000E+02	5.000E+00	1.300E–01	8.000E–02	4.500E–01	1.930E+00	8.300E–01	2.000E–02	1.100E–01	3.550E+00
1.000E+02	2.500E+01	1.300E–01	5.000E–02	4.500E–01	6.600E–01	7.200E–01	3.000E–02	1.100E–01	2.150E+00
1.000E+02	5.000E+01	1.300E–01	4.000E–02	4.500E–01	3.700E–01	1.040E+00	9.000E–02	1.100E–01	2.210E+00
1.000E+02	7.500E+01	1.300E–01	4.000E–02	4.400E–01	2.300E–01	1.380E+00	1.300E–01	1.100E–01	2.470E+00
1.000E+02	1.000E+02	1.300E–01	3.000E–02	4.500E–01	1.500E–01	1.920E+00	2.500E–01	1.100E–01	3.030E+00

Uniform-Spiral Distribution (22)
spiral = 2025 (field points), uniform = 2025 (source points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	6.000E–02	8.000E–02	7.900E–01	4.620E+00	2.000E+00	4.000E–02	1.200E–01	7.710E+00
5.000E+00	2.500E+01	6.000E–02	5.000E–02	7.600E–01	1.030E+00	5.600E–01	8.000E–02	1.100E–01	2.660E+00
5.000E+00	5.000E+01	6.000E–02	4.000E–02	7.700E–01	4.800E–01	3.900E–01	3.100E–01	1.100E–01	2.140E+00
5.000E+00	7.500E+01	6.000E–02	4.000E–02	7.600E–01	2.800E–01	3.600E–01	6.600E–01	1.100E–01	2.270E+00
5.000E+00	1.000E+02	6.000E–02	3.000E–02	7.700E–01	1.700E–01	4.100E–01	1.150E+00	1.100E–01	2.710E+00
2.500E+01	5.000E+00	3.000E–02	8.000E–02	1.900E–01	1.390E+00	6.000E–01	2.000E–02	1.100E–01	2.420E+00
2.500E+01	2.500E+01	4.000E–02	5.000E–02	2.000E–01	3.900E–01	3.200E–01	3.000E–02	1.100E–01	1.120E+00
2.500E+01	5.000E+01	4.000E–02	4.000E–02	2.000E–01	2.200E–01	3.600E–01	6.000E–02	1.100E–01	1.030E+00
2.500E+01	7.500E+01	4.000E–02	4.000E–02	2.000E–01	1.500E–01	4.300E–01	1.000E–01	1.100E–01	1.060E+00
2.500E+01	1.000E+02	4.000E–02	3.000E–02	2.000E–01	1.100E–01	5.500E–01	2.100E–01	1.100E–01	1.240E+00
5.000E+01	5.000E+00	3.000E–02	8.000E–02	1.100E–01	8.400E–01	4.300E–01	2.000E–02	1.100E–01	1.610E+00
5.000E+01	2.500E+01	3.000E–02	5.000E–02	1.100E–01	3.300E–01	4.700E–01	2.000E–02	1.100E–01	1.100E+00
5.000E+01	5.000E+01	3.000E–02	4.000E–02	1.100E–01	2.200E–01	6.100E–01	1.000E–02	1.100E–01	1.130E+00
5.000E+01	7.500E+01	3.000E–02	4.000E–02	1.100E–01	1.600E–01	7.700E–01	1.000E–02	1.100E–01	1.220E+00
5.000E+01	1.000E+02	3.000E–02	3.000E–02	1.100E–01	1.300E–01	9.300E–01	6.000E–02	1.100E–01	1.390E+00
7.500E+01	5.000E+00	3.000E–02	8.000E–02	1.100E–01	8.300E–01	4.300E–01	2.000E–02	1.100E–01	1.600E+00
7.500E+01	2.500E+01	3.000E–02	5.000E–02	1.100E–01	3.300E–01	4.700E–01	2.000E–02	1.100E–01	1.100E+00
7.500E+01	5.000E+01	3.000E–02	4.000E–02	1.100E–01	2.200E–01	6.100E–01	1.000E–02	1.100E–01	1.130E+00
7.500E+01	7.500E+01	3.000E–02	4.000E–02	1.100E–01	1.600E–01	7.700E–01	1.000E–02	1.100E–01	1.210E+00
7.500E+01	1.000E+02	3.000E–02	3.000E–02	1.100E–01	1.300E–01	9.300E–01	6.000E–02	1.100E–01	1.390E+00
1.000E+02	5.000E+00	3.000E–02	8.000E–02	1.100E–01	8.400E–01	4.300E–01	2.000E–02	1.100E–01	1.610E+00
1.000E+02	2.500E+01	3.000E–02	5.000E–02	1.100E–01	3.300E–01	4.700E–01	2.000E–02	1.100E–01	1.110E+00
1.000E+02	5.000E+01	3.000E–02	4.000E–02	1.100E–01	2.200E–01	6.100E–01	1.000E–02	1.100E–01	1.130E+00
1.000E+02	7.500E+01	3.000E–02	4.000E–02	1.100E–01	1.600E–01	7.700E–01	1.000E–02	1.100E–01	1.220E+00
1.000E+02	1.000E+02	3.000E–02	3.000E–02	1.100E–01	1.300E–01	9.300E–01	6.000E–02	1.100E–01	1.400E+00

Uniform-Spiral Distribution (21)
spiral = 3025 (field points), uniform = 3025 (source points)

Ns	Nf	t1	t2	t3	t4	t5	t6	t7	t8
5.000E+00	5.000E+00	8.000E–02	8.000E–02	8.600E–01	4.740E+00	2.060E+00	4.000E–02	1.200E–01	7.970E+00
5.000E+00	2.500E+01	8.000E–02	5.000E–02	8.500E–01	1.080E+00	6.100E–01	9.000E–02	1.100E–01	2.860E+00
5.000E+00	5.000E+01	8.000E–02	4.000E–02	8.500E–01	5.000E–01	4.500E–01	3.200E–01	1.100E–01	2.350E+00
5.000E+00	7.500E+01	8.000E–02	4.000E–02	8.500E–01	2.900E–01	4.300E–01	6.900E–01	1.100E–01	2.490E+00
5.000E+00	1.000E+02	8.000E–02	3.000E–02	8.400E–01	1.800E–01	5.200E–01	1.200E+00	1.100E–01	2.950E+00
2.500E+01	5.000E+00	5.000E–02	8.000E–02	2.500E–01	1.530E+00	6.500E–01	3.000E–02	1.200E–01	2.700E+00
2.500E+01	2.500E+01	5.000E–02	5.000E–02	2.500E–01	4.500E–01	3.500E–01	3.000E–02	1.100E–01	1.280E+00
2.500E+01	5.000E+01	5.000E–02	4.000E–02	2.500E–01	2.500E–01	4.100E–01	8.000E–02	1.100E–01	1.190E+00
2.500E+01	7.500E+01	5.000E–02	4.000E–02	2.500E–01	1.600E–01	5.200E–01	1.300E–01	1.100E–01	1.260E+00
2.500E+01	1.000E+02	5.000E–02	3.000E–02	2.600E–01	1.100E–01	7.000E–01	2.400E–01	1.100E–01	1.500E+00
5.000E+01	5.000E+00	4.000E–02	8.000E–02	1.400E–01	1.010E+00	5.500E–01	2.000E–02	1.200E–01	1.960E+00
5.000E+01	2.500E+01	4.000E–02	5.000E–02	1.500E–01	4.400E–01	6.500E–01	1.000E–02	1.100E–01	1.450E+00
5.000E+01	5.000E+01	4.000E–02	4.000E–02	1.500E–01	2.900E–01	8.800E–01	1.000E–02	1.100E–01	1.510E+00
5.000E+01	7.500E+01	4.000E–02	3.000E–02	1.400E–01	2.100E–01	1.100E+00	1.000E–02	1.100E–01	1.650E+00
5.000E+01	1.000E+02	4.000E–02	3.000E–02	1.500E–01	1.800E–01	1.350E+00	5.000E–02	1.100E–01	1.900E+00
7.500E+01	5.000E+00	4.000E–02	8.000E–02	1.400E–01	1.010E+00	5.500E–01	2.000E–02	1.200E–01	1.950E+00
7.500E+01	2.500E+01	4.000E–02	5.000E–02	1.400E–01	4.400E–01	6.500E–01	1.000E–02	1.100E–01	1.440E+00
7.500E+01	5.000E+01	4.000E–02	4.000E–02	1.500E–01	2.900E–01	8.800E–01	1.000E–02	1.100E–01	1.520E+00
7.500E+01	7.500E+01	4.000E–02	4.000E–02	1.400E–01	2.100E–01	1.100E+00	1.000E–02	1.100E–01	1.650E+00
7.500E+01	1.000E+02	4.000E–02	3.000E–02	1.400E–01	1.700E–01	1.350E+00	5.000E–02	1.100E–01	1.900E+00
1.000E+02	5.000E+00	4.000E–02	8.000E–02	1.400E–01	1.010E+00	5.500E–01	2.000E–02	1.200E–01	1.950E+00
1.000E+02	2.500E+01	4.000E–02	5.000E–02	1.400E–01	4.400E–01	6.600E–01	2.000E–02	1.100E–01	1.440E+00
1.000E+02	5.000E+01	4.000E–02	4.000E–02	1.400E–01	2.900E–01	8.800E–01	1.000E–02	1.100E–01	1.510E+00
1.000E+02	7.500E+01	4.000E–02	4.000E–02	1.400E–01	2.100E–01	1.100E+00	1.000E–02	1.100E–01	1.650E+00
1.000E+02	1.000E+02	4.000E–02	3.000E–02	1.500E–01	1.800E–01	1.350E+00	5.000E–02	1.100E–01	1.900E+00

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